Computer algebra independent integration tests

Summer 2022 edition

6-Hyperbolic-functions/6.1-Hyperbolic-sine/160-6.1.1-c+d-x-^m-a+b-sinh-^n

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [502]. This is test number [160].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (502)	0.00 (0)
Mathematica	94.62 (475)	5.38 (27)
Fricas	92.63 (465)	7.37 (37)
Maple	67.93 (341)	32.07 (161)
Maxima	57.57 (289)	42.43 (213)
Mupad	41.63 (209)	58.37 (293)
Giac	39.44 (198)	60.56 (304)
Sympy	24.50 (123)	75.50 (379)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

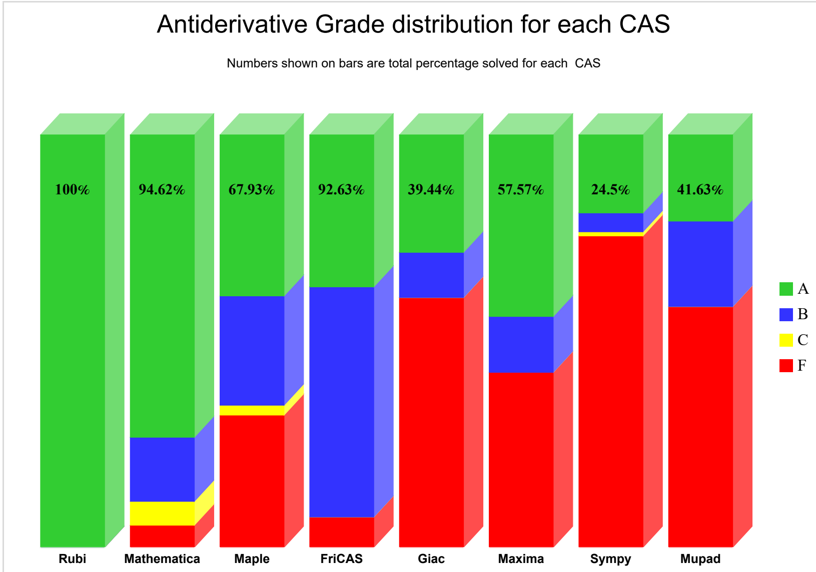
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

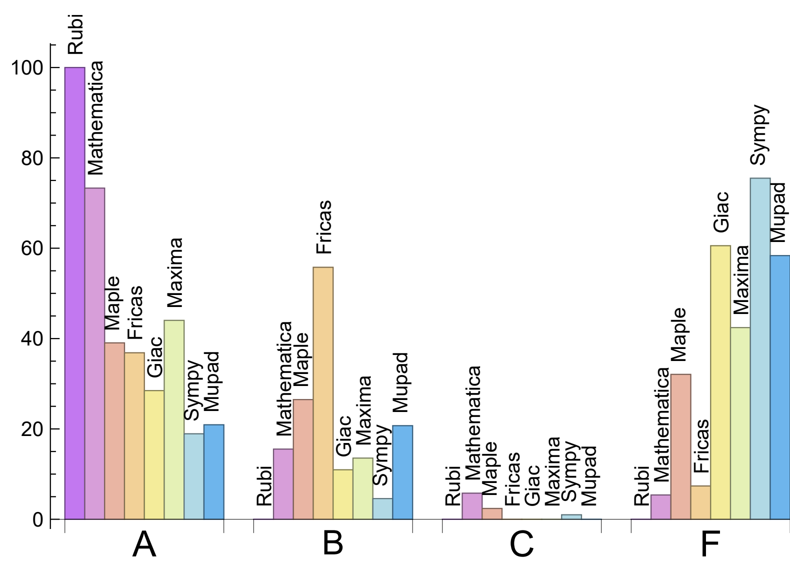
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	73.31	15.54	5.78	5.38
Maxima	44.02	13.55	0.00	42.43
Maple	39.04	26.49	2.39	32.07
Fricas	36.85	55.78	0.00	7.37
Giac	28.49	10.96	0.00	60.56
Mupad	N/A	20.72	0.00	58.37
Sympy	18.92	4.58	1.00	75.50

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	27	0.00 %	100.00 %	0.00 %
Maple	161	100.00 %	0.00 %	0.00 %
Fricas	37	24.32 %	2.70 %	72.97 %
Giac	304	62.50 %	36.84 %	0.66 %
Maxima	213	91.55 %	0.00 %	8.45 %
Sympy	379	56.99 %	33.51 %	9.50 %
Mupad	293	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

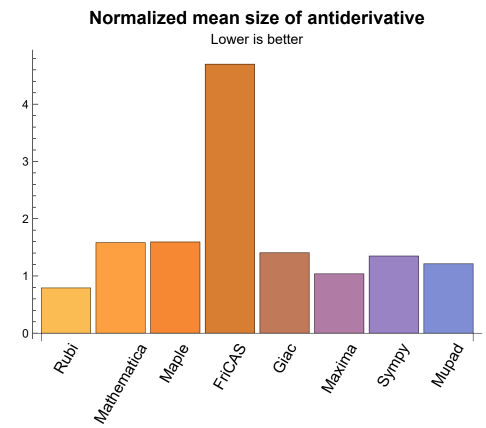
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

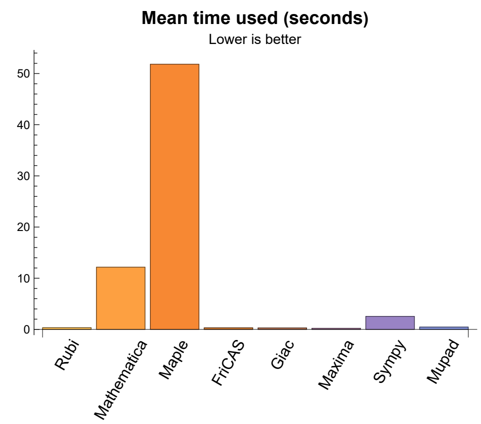
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.34	267.92	0.79	139.00	1.00
Mathematica	12.16	768.43	1.58	164.00	1.00
Maple	51.82	377.16	1.59	133.00	1.50
Maxima	0.22	136.54	1.04	95.00	0.93
Fricas	0.32	2562.02	4.70	385.00	2.58
Sympy	2.55	136.87	1.35	0.00	0.00
Giac	0.30	158.05	1.41	100.00	1.36
Mupad	0.46	115.33	1.21	-1.00	-0.02

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
10
20
30
40
50
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{26, 27, 31, 32, 36, 37, 65, 66, 67, 72, 76, 77, 111, 112, 116, 117, 139, 140, 144, 145, 149, 150, 151, 155, 156, 172, 173, 176, 177, 179, 180, 181, 185, 186, 191, 192, 197, 198, 203, 204, 209, 210, 215, 216, 221, 222, 227, 232, 237, 242, 247, 252, 257, 258, 275, 276, 281, 282, 287, 288, 293, 298, 303, 308, 313, 317, 318, 319, 320, 337, 342, 347, 352, 357, 361, 366, 371, 376, 381, 386, 390, 395, 400, 405, 410, 415, 419, 424, 429, 434, 439, 444, 448, 453, 458, 463, 468, 472, 475, 480, 485, 490, 495, 499, 502}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {211, 217, 218, 277, 283, 284, 310, 314, 338, 339, 340, 354, 358, 383, 387, 396, 397, 398, 401, 412, 416, 430, 435, 436, 441, 449, 454, 459, 464, 466, 469, 470, 476, 481, 486, 491, 492, 493, 496, 497, 500}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

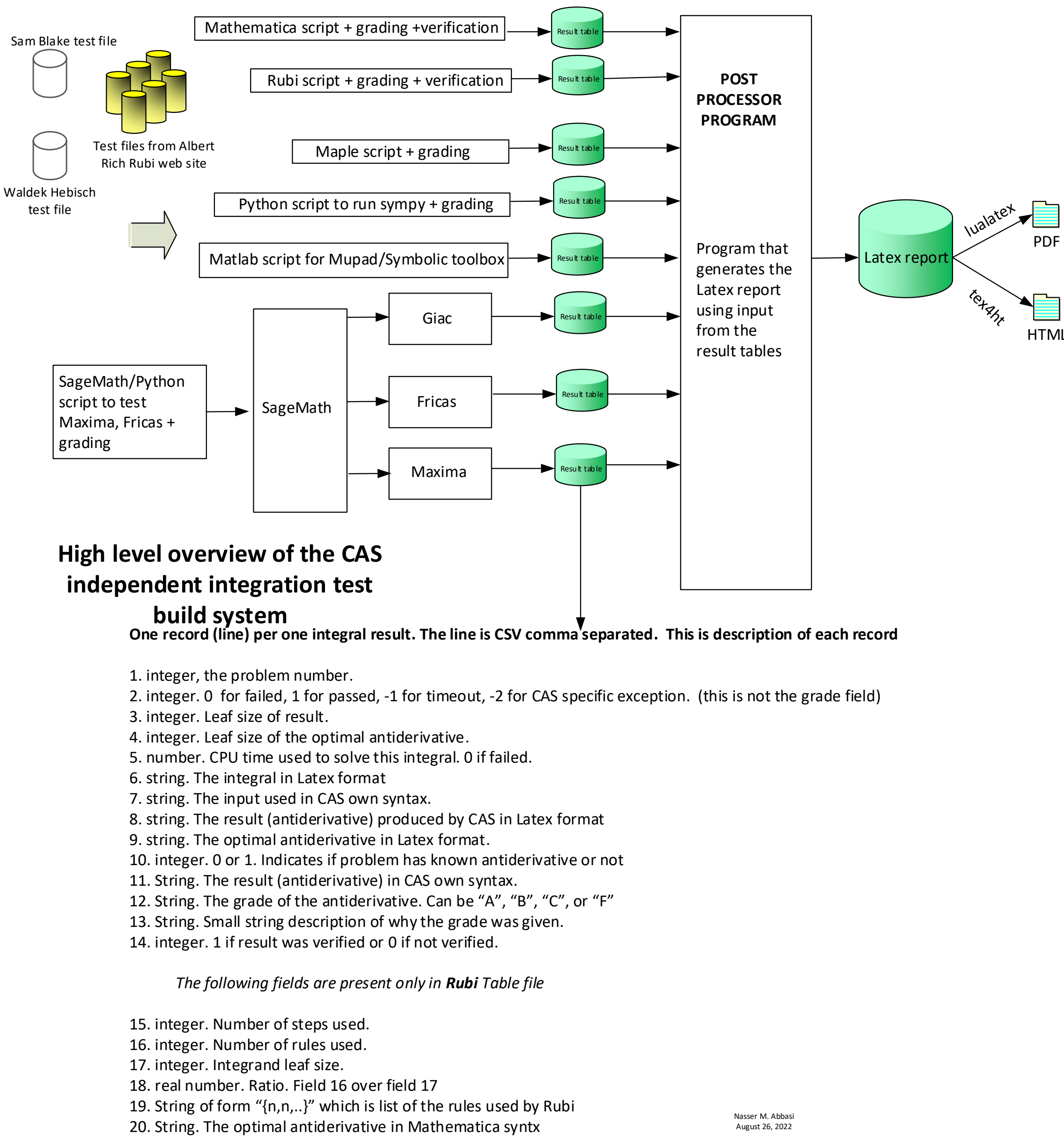
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 52, 53, 54, 55, 56, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 188, 191, 192, 194, 195, 196, 197, 198, 199, 201, 202, 204, 205, 206, 208, 209, 210, 214, 215, 220, 224, 225, 226, 227, 228, 229, 230, 231, 232, 235, 236, 237, 239, 240, 241, 242, 244, 245, 246, 247, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 263, 264, 265, 266, 267, 268, 269, 270, 271, 274, 275, 278, 279, 280, 281, 286, 287, 289, 290, 291, 292, 293, 295, 296, 298, 301, 302, 303, 306, 307, 308, 309, 311, 312, 313, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 330, 335, 336, 337, 341, 345, 346, 349, 350, 351, 352, 354, 355, 356, 357, 359, 360, 361, 364, 365, 368, 369, 370, 374, 375, 378, 379, 381, 382, 383, 384, 385, 388, 393, 394, 399, 402, 403, 404, 407, 408, 411, 412, 413, 414, 416, 417, 421, 422, 423, 424, 426, 427, 428, 429, 431, 432, 433, 439, 440, 442, 443, 444, 446, 447, 448, 450, 451, 452, 453, 455, 456, 457, 458, 460, 461, 462, 465, 466, 467, 468, 471, 472, 473, 478, 479, 484, 488, 489, 492, 493, 494, 495, 498, 499 }

B grade: { 34, 49, 51, 57, 59, 110, 130, 135, 182, 189, 190, 193, 200, 207, 211, 212, 213, 217, 218, 219, 223, 233, 234, 238, 243, 248, 249, 262, 272, 273, 277, 283, 284, 285, 294, 299, 300, 304, 305, 310, 314, 328, 329, 331, 332, 333, 334, 343, 344, 348, 353, 358, 362, 363, 367, 372, 373, 377, 387, 391, 392, 401, 406, 420, 425, 435, 436, 437, 441, 445, 464, 469, 477, 482, 487, 491, 496, 500 }

C grade: { 25, 29, 35, 71, 250, 297, 338, 339, 340, 380, 389, 396, 397, 398, 409, 418, 430, 438, 449, 454, 459, 470, 474, 476, 481, 483, 486, 497, 501 }

F grade: { 203, 216, 221, 222, 276, 282, 288, 342, 347, 366, 371, 376, 386, 390, 395, 400, 405, 410, 415, 419, 434, 463, 475, 480, 485, 490, 502 }

2.1.3 Maple

A grade: { 4, 5, 6, 12, 13, 19, 20, 21, 25, 26, 27, 30, 31, 32, 36, 37, 65, 66, 67, 72, 76, 77, 99, 100, 104, 105, 106, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 139, 140, 144, 145, 149, 150, 151, 155, 156, 160, 161, 165, 166, 167, 172, 173, 176, 177, 179, 180, 181, 185, 186, 189, 190, 191, 192, 195, 196, 197, 198, 201, 202, 203, 204, 207, 208, 209, 210, 214, 215, 216, 220, 221, 222, 226, 227, 231, 232, 235, 236, 237, 241, 242, 246, 247, 251, 252, 256, 257, 258, 261, 263, 264, 266, 267, 268, 269, 270, 273, 275, 276, 278, 279, 280, 281, 282, 286, 287, 288, 292, 293, 298, 303, 307, 308, 312, 313, 316, 317, 318, 319, 320, 336, 337, 342, 347, 351, 352, 356, 357, 360, 361, 365, 366, 370, 371, 376, 380, 381, 385, 386, 389, 390, 394, 395, 400, 405, 409, 410, 414, 415, 418, 419, 423, 424, 428, 429, 434, 438, 439, 443, 444, 447, 448, 452, 453, 457, 458, 463, 467, 468, 471, 472, 474, 475, 479, 480, 484, 485, 489, 490, 494, 495, 498, 499, 501, 502 }

B grade: { 1, 2, 3, 7, 8, 9, 10, 11, 14, 15, 16, 17, 18, 22, 23, 24, 28, 29, 33, 34, 35, 96, 97, 98, 101, 102, 103, 107, 108, 109, 113, 157, 158, 159, 162, 163, 164, 168, 171, 175, 178, 187, 188, 193, 194, 199, 200, 205, 206, 211, 212, 213, 217, 218, 219, 225, 230, 240, 245, 250, 253, 254, 255, 259, 260, 262, 265, 271, 272, 274, 277, 283, 284, 285, 291, 296, 297, 301, 302, 306, 311, 315, 321, 322, 324, 325, 327, 328, 330, 331, 335, 340, 341, 345, 346, 350, 355, 359, 364, 369, 374, 375, 379, 384, 388, 393, 398, 399, 403, 404, 408, 413, 417, 422, 427, 432, 433, 437, 442, 446, 451, 456, 461, 462, 466, 470, 473, 478, 483, 488, 493, 497, 500 }

C grade: { 60, 61, 62, 63, 64, 78, 79, 80, 81, 82, 83, 84 }

F grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 68, 69, 70, 71, 73, 74, 75, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 152, 153, 154, 169, 170, 174, 182, 183, 184, 223, 224, 228, 229, 233, 234, 238, 239, 243, 244, 248, 249, 289, 290, 294, 295, 299, 300, 304, 305, 309, 310, 314, 323, 326, 329, 332, 333, 334, 338, 339, 343, 344, 348, 349, 353, 354, 358, 362, 363, 367, 368, 372, 373, 377, 378, 382, 383, 387, 391, 392, 396, 397, 401, 402, 406, 407, 411, 412, 416, 420, 421, 425, 426, 430, 431, 435, 436, 440, 441, 445, 449, 450, 454, 455, 459, 460, 464, 465, 469, 476, 477, 481, 482, 486, 487, 491, 492, 496 }

2.1.4 Maxima

A grade: { 5, 6, 7, 10, 11, 12, 13, 14, 15, 20, 21, 22, 26, 27, 31, 32, 36, 37, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 63, 64, 65, 66, 67, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 98, 99, 100, 101, 104, 105, 106, 107, 110, 111, 112, 116, 117, 139, 140, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 159, 160, 161, 162, 164, 165, 166, 167, 168, 172, 173, 176, 177, 179, 180, 181, 182, 183, 184, 185, 186, 189, 190, 191, 192, 196, 197, 198, 202, 206, 208, 209, 210, 215, 216, 220, 221, 222, 226, 227, 231, 232, 236, 237, 241, 242, 246, 247, 251, 252, 254, 256, 257, 258, 263, 264, 268, 271, 272, 275, 276, 277, 279, 281, 282, 292, 293, 297, 298, 303, 307, 308, 312, 313, 316, 317, 318, 319, 320, 337, 341, 342, 347, 351, 352, 356, 357, 360, 361, 366, 370, 371, 376, 380, 381, 385, 386, 389, 390, 395, 399, 400, 405, 409, 410, 414, 415, 418, 419, 424, 428, 429, 434, 438, 439, 443, 444, 447, 448, 453, 457, 458, 463, 467, 468, 471, 472, 474, 475, 480, 485, 490, 494, 495, 498, 499, 502 }

B grade: { 1, 2, 3, 4, 8, 9, 16, 17, 18, 19, 23, 24, 28, 30, 33, 34, 38, 39, 40, 41, 60, 61, 62, 96, 97, 102, 103, 108, 113, 115, 157, 158, 163, 187, 193, 195, 205, 211, 212, 214, 217, 218, 253, 259, 260, 261, 262, 274, 280, 302, 321, 324, 327, 330, 336, 346, 365, 375, 394, 404, 423, 433, 452, 462, 479, 484, 489, 501 }

C grade: { }

F grade: { 25, 29, 35, 68, 69, 70, 71, 90, 91, 92, 93, 94, 95, 109, 114, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 169, 170, 171, 174, 175, 178, 188, 194, 199, 200, 201, 203, 204, 207, 213, 219, 223, 224, 225, 228, 229, 230, 233, 234, 235, 238, 239, 240, 243, 244, 245, 248, 249, 250, 255, 265, 266, 267, 269, 270, 273, 278, 283, 284, 285, 286, 287, 288, 289, 290, 291, 294, 295, 296, 299, 300, 301, 304, 305, 306, 309, 310, 311, 314, 315, 322, 323, 325, 326, 328, 329, 331, 332, 333, 334, 335, 338, 339, 340, 343, 344, 345, 348, 349, 350, 353, 354, 355, 358, 359, 362, 363, 364, 367, 368, 369, 372, 373, 374, 377, 378, 379, 382, 383, 384, 387, 388, 391, 392, 393, 396, 397, 398, 401, 402, 403, 406, 407, 408, 411, 412, 413, 416, 417, 420, 421, 422, 425, 426, 427, 430, 431, 432, 435, 436, 437, 440, 441, 442, 445, 446, 449, 450, 451, 454, 455, 456, 459, 460, 461, 464, 465, 466, 469, 470, 473, 476, 477, 478, 481, 482, 483, 486, 487, 488, 491, 492, 493, 496, 497, 500 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 9, 10, 11, 12, 18, 19, 20, 26, 27, 31, 32, 36, 37, 41, 48, 56, 62, 65, 66, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 98, 99, 100, 101, 104, 105, 106, 110, 111, 112, 115, 116, 117, 139, 140, 144, 145, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 172, 173, 176, 177, 179, 180, 181, 182, 185, 186, 189, 190, 191, 192, 195, 196, 197, 198, 201, 202, 203, 204, 208, 209, 210, 215, 216, 221, 222, 227, 232, 237, 242, 247, 252, 255, 256, 257, 258, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 275, 276, 279, 280, 281, 282, 287, 288, 293, 298, 303, 307, 308, 313, 317, 318, 319, 320, 337, 342, 347, 350, 351, 352, 357, 361, 366, 371, 376, 379, 380, 381, 386, 390, 395, 400, 405, 410, 415, 419, 423, 424, 429, 434, 438, 439, 444, 448, 453, 458, 463, 468, 472, 475, 480, 485, 490, 495, 499 }

B grade: { 6, 7, 8, 13, 14, 15, 16, 17, 21, 22, 23, 24, 25, 28, 29, 30, 33, 34, 35, 38, 39, 40, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 63, 64, 69, 96, 97, 102, 103, 107, 108, 109, 113, 114, 162, 167, 168, 169, 170, 171, 174, 175, 178, 183, 184, 187, 188, 193, 194, 199, 200, 205, 206, 207, 211, 212, 213, 214, 217, 218, 219, 220, 223, 224, 225, 226, 228, 229, 230, 231, 233, 234, 235, 236, 238, 239, 240, 241, 243, 244, 245, 246, 248, 249, 250, 251, 253, 254, 259, 260, 271, 272, 273, 274, 277, 278, 283, 284, 285, 286, 289, 290, 291, 292, 294, 295, 296, 297, 299, 300, 301, 302, 304, 305, 306, 309, 310, 311, 312, 314, 315, 316, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 343, 344, 345, 346, 348, 349, 353, 354, 355, 356, 358, 359, 360, 362, 363, 364, 365, 367, 368, 369, 370, 372, 373, 374, 375, 377, 378, 382, 383, 384, 385, 387, 388, 389, 391, 392, 393, 394, 396, 397, 398, 399, 401, 402, 403, 404, 406, 407, 408, 409, 411, 412, 413, 414, 416, 417, 418, 420, 421, 422, 425, 426, 427, 428, 430, 431, 432, 433, 435, 436, 437, 440, 441, 442, 443, 445, 446, 447, 449, 450, 451, 452, 454, 455, 456, 457, 459, 460, 461, 462, 464, 465, 466, 467, 469, 470, 471, 473, 474, 476, 477, 478, 479, 481, 482, 483, 484, 486, 487, 488, 489, 491, 492, 493, 494, 496, 497, 498, 500, 501 }

C grade: { }

F grade: { 67, 68, 70, 71, 92, 93, 94, 95, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 150, 502 }

2.1.6 Sympy

A grade: { 4, 19, 26, 27, 31, 32, 36, 37, 65, 66, 67, 72, 76, 77, 96, 97, 98, 102, 103, 104, 110, 111, 112, 115, 139, 140, 144, 145, 150, 155, 156, 159, 165, 172, 173, 185, 186, 189, 190, 191, 192, 195, 196, 197, 201, 202, 203, 209, 215, 221, 227, 242, 247, 252, 256, 257, 258, 259, 260, 261, 262, 275, 276, 281, 282, 287, 288, 293, 308, 313, 317, 318, 319, 320, 352, 357, 361, 365, 381, 386, 390, 394, 410, 415, 419, 424, 429, 434, 439, 453, 458, 463, 480, 485, 490 }

B grade: { 1, 2, 3, 8, 9, 10, 11, 16, 17, 18, 157, 158, 163, 164, 226, 231, 265, 266, 267, 268, 292, 297, 336 }

C grade: { 60, 61, 62, 63, 64 }

F grade: { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 28, 29, 30, 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 68, 69, 70, 71, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 105, 106, 107, 108, 109, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 149, 151, 152, 153, 154, 160, 161, 162, 166, 167, 168, 169, 170, 171, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 187, 188, 193, 194, 198, 199, 200, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 228, 229, 230, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 248, 249, 250, 251, 253, 254, 255, 263, 264, 269, 270, 271, 272, 273, 274, 277, 278, 279, 280, 283, 284, 285, 286, 289, 290, 291, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 309, 310, 311, 312, 314, 315, 316, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 355, 356, 358, 359, 360, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 382, 383, 384, 385, 387, 388, 389, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 411, 412, 413, 414, 416, 417, 418, 420, 421, 422, 423, 425, 426, 427, 428, 430, 431, 432, 433, 435, 436, 437, 438, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 454, 455, 456, 457, 459, 460, 461, 462, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 486, 487, 488, 489, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502 }

2.1.7 Giac

A grade: { 4, 5, 10, 11, 12, 19, 20, 26, 27, 31, 32, 36, 38, 39, 40, 41, 48, 60, 61, 62, 65, 66, 67, 72, 76, 77, 98, 99, 104, 105, 110, 111, 112, 115, 116, 117, 139, 140, 144, 145, 149, 150, 151, 155, 156, 159, 160, 165, 166, 172, 173, 176, 177, 179, 180, 181, 185, 186, 189, 190, 191, 192, 196, 197, 198, 202, 203, 204, 208, 209, 214, 220, 226, 227, 231, 232, 236, 237, 241, 246, 251, 256, 257, 258, 261, 263, 267, 268, 269, 275, 276, 279, 280, 281, 287, 292, 293, 297, 298, 302, 303, 307, 308, 312, 318, 319, 320, 336, 337, 341, 342, 346, 347, 351, 356, 365, 366, 370, 371, 375, 376, 380, 385, 394, 395, 399, 400, 404, 405, 409, 414, 423, 424, 428, 433, 438, 439, 443, 457, 467, 471, 484, 498 }

B grade: { 1, 2, 3, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 21, 22, 30, 96, 97, 100, 101, 102, 103, 106, 107, 157, 158, 161, 162, 163, 164, 167, 168, 195, 201, 259, 260, 262, 264, 265, 266, 270, 274, 286, 316, 360, 389, 418, 447, 452, 462, 474, 479, 489, 494, 501 }

C grade: { }

F grade: { 23, 24, 25, 28, 29, 33, 34, 35, 37, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 63, 64, 68, 69, 70, 71, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 108, 109, 113, 114, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 152, 153, 154, 169, 170, 171, 174, 175, 178, 182, 183, 184, 187, 188, 193, 194, 199, 200, 205, 206, 207, 210, 211, 212, 213, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 228, 229, 230, 233, 234, 235, 238, 239, 240, 242, 243, 244, 245, 247, 248, 249, 250, 252, 253, 254, 255, 271, 272, 273, 277, 278, 282, 283, 284, 285, 288, 289, 290, 291, 294, 295, 296, 299, 300, 301, 304, 305, 306, 309, 310, 311, 313, 314, 315, 317, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 338, 339, 340, 343, 344, 345, 348, 349, 350, 352, 353, 354, 355, 357, 358, 359, 361, 362, 363, 364, 367, 368, 369, 372, 373, 374, 377, 378, 379, 381, 382, 383, 384, 386, 387, 388, 390, 391, 392, 393, 396, 397, 398, 401, 402, 403, 406, 407, 408, 410, 411, 412, 413, 415, 416, 417, 419, 420, 421, 422, 425, 426, 427, 429, 430, 431, 432, 434, 435, 436, 437, 440, 441, 442, 444, 445, 446, 448, 449, 450, 451, 453, 454, 455, 456, 458, 459, 460, 461, 463, 464, 465, 466, 468, 469, 470, 472, 473, 475, 476, 477, 478, 480, 481, 482, 483, 485, 486, 487, 488, 490, 491, 492, 493, 495, 496, 497, 499, 500, 502 }

2.1.8 Mupad

A grade: { 26, 27, 31, 32, 36, 37, 65, 66, 67, 72, 76, 77, 111, 112, 116, 117, 139, 140, 144, 145, 149, 150, 151, 155, 156, 172, 173, 176, 177, 179, 180, 181, 185, 186, 191, 192, 197, 198, 203, 204, 209, 210, 215, 216, 221, 222, 227, 232, 237, 242, 247, 252, 257, 258, 275, 276, 281, 282, 287, 288, 293, 298, 303, 308, 313, 317, 318, 319, 320, 337, 342, 347, 352, 357, 361, 366, 371, 376, 381, 386, 390, 395, 400, 405, 410, 415, 419, 424, 429, 434, 439, 444, 448, 453, 458, 463, 468, 472, 475, 480, 485, 490, 495, 499, 502 }

B grade: { 1, 2, 3, 4, 8, 9, 10, 11, 16, 17, 18, 19, 30, 68, 69, 70, 96, 97, 98, 102, 103, 104, 110, 115, 118, 119, 120, 121, 157, 158, 159, 163, 164, 165, 189, 190, 195, 196, 201, 202, 208, 214, 220, 226, 231, 236, 241, 246, 251, 256, 259, 260, 261, 262, 265, 266, 267, 268, 274, 279, 280, 286, 292, 297, 302, 307, 312, 316, 321, 324, 336, 341, 346, 351, 356, 360, 365, 370, 375, 380, 385, 389, 394, 399, 404, 409, 414, 418, 423, 428, 433, 443, 452, 457, 462, 467, 471, 474, 479, 484, 489, 494, 498, 501 }

C grade: { }

F grade: { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 28, 29, 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 71, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 105, 106, 107, 108, 109, 113, 114, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 152, 153, 154, 160, 161, 162, 166, 167, 168, 169, 170, 171, 174, 175, 178, 182, 183, 184, 187, 188, 193, 194, 199, 200, 205, 206, 207, 211, 212, 213, 217, 218, 219, 223, 224, 225, 228, 229, 230, 233, 234, 235, 238, 239, 240, 243, 244, 245, 248, 249, 250, 253, 254, 255, 263, 264, 269, 270, 271, 272, 273, 277, 278, 283, 284, 285, 289, 290, 291, 294, 295, 296, 299, 300, 301, 304, 305, 306, 309, 310, 311, 314, 315, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 338, 339, 340, 343, 344, 345, 348, 349, 350, 353, 354, 355, 358, 359, 362, 363, 364, 367, 368, 369, 372, 373, 374, 377, 378, 379, 382, 383, 384, 387, 388, 391, 392, 393, 396, 397, 398, 401, 402, 403, 406, 407, 408, 411, 412, 413, 416, 417, 420, 421, 422, 425, 426, 427, 430, 431, 432, 435, 436, 437, 438, 440, 441, 442, 445, 446, 447, 449, 450, 451, 454, 455, 456, 459, 460, 461, 464, 465, 466, 469, 470, 473, 476, 477, 478, 481, 482, 483, 486, 487, 488, 491, 492, 493, 496, 497, 500 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	76	547	326	169	311	324	215
N.S.	1	1.00	0.84	6.01	3.58	1.86	3.42	3.56	2.36
time (sec)	N/A	0.085	0.180	0.300	0.278	0.351	0.364	0.431	0.452

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	61	308	222	109	202	204	143
N.S.	1	1.00	0.87	4.40	3.17	1.56	2.89	2.91	2.04
time (sec)	N/A	0.056	0.112	0.303	0.268	0.338	0.251	0.441	0.142

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	44	147	134	62	112	112	82
N.S.	1	1.00	0.90	3.00	2.73	1.27	2.29	2.29	1.67
time (sec)	N/A	0.034	0.088	0.303	0.275	0.405	0.181	0.414	0.120

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	53	68	29	46	46	35
N.S.	1	1.00	0.96	1.89	2.43	1.04	1.64	1.64	1.25
time (sec)	N/A	0.014	0.053	0.424	0.261	0.372	0.087	0.411	0.096

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	82	57	94	0	57	-1
N.S.	1	1.00	0.96	1.61	1.12	1.84	0.00	1.12	-0.02
time (sec)	N/A	0.077	0.057	0.365	0.295	0.333	0.000	0.409	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	65	133	80	148	0	615	-1
N.S.	1	1.00	0.92	1.87	1.13	2.08	0.00	8.66	-0.01
time (sec)	N/A	0.093	0.151	0.378	0.299	0.371	0.000	0.447	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	88	277	94	254	0	301	-1
N.S.	1	1.00	0.85	2.66	0.90	2.44	0.00	2.89	-0.01
time (sec)	N/A	0.125	0.376	0.388	0.317	0.369	0.000	0.415	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	132	910	382	312	660	374	334
N.S.	1	1.00	0.81	5.62	2.36	1.93	4.07	2.31	2.06
time (sec)	N/A	0.076	0.363	0.442	0.320	0.371	0.700	0.450	0.693

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	104	523	263	209	456	243	229
N.S.	1	1.00	0.78	3.90	1.96	1.56	3.40	1.81	1.71
time (sec)	N/A	0.050	0.250	0.372	0.283	0.351	0.384	0.430	0.376

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	75	262	165	123	264	136	127
N.S.	1	1.00	0.79	2.76	1.74	1.29	2.78	1.43	1.34
time (sec)	N/A	0.043	0.185	0.368	0.267	0.337	0.240	0.441	0.181

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	52	103	88	64	126	63	60
N.S.	1	1.00	0.95	1.87	1.60	1.16	2.29	1.15	1.09
time (sec)	N/A	0.018	0.127	0.418	0.263	0.350	0.147	0.438	0.093

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	66	97	72	104	0	68	-1
N.S.	1	1.00	0.85	1.24	0.92	1.33	0.00	0.87	-0.01
time (sec)	N/A	0.113	0.083	3.020	0.293	0.336	0.000	0.447	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	75	152	88	166	0	574	-1
N.S.	1	1.00	0.93	1.88	1.09	2.05	0.00	7.09	-0.01
time (sec)	N/A	0.108	0.291	3.308	0.318	0.391	0.000	0.483	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	102	299	99	280	0	330	-1
N.S.	1	1.00	0.91	2.67	0.88	2.50	0.00	2.95	-0.01
time (sec)	N/A	0.136	0.616	3.328	0.322	0.357	0.000	0.425	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	123	555	110	411	0	537	-1
N.S.	1	1.00	0.76	3.43	0.68	2.54	0.00	3.31	-0.01
time (sec)	N/A	0.130	0.611	3.246	0.308	0.358	0.000	0.422	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	150	1266	639	528	772	654	532
N.S.	1	1.00	0.67	5.63	2.84	2.35	3.43	2.91	2.36
time (sec)	N/A	0.250	0.621	0.493	0.301	0.370	0.810	0.449	0.547

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	127	710	435	345	495	414	364
N.S.	1	1.00	0.73	4.06	2.49	1.97	2.83	2.37	2.08
time (sec)	N/A	0.161	0.618	0.427	0.303	0.351	0.530	0.407	0.349

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	86	343	269	199	284	230	184
N.S.	1	1.00	0.70	2.79	2.19	1.62	2.31	1.87	1.50
time (sec)	N/A	0.093	0.244	0.421	0.282	0.351	0.383	0.413	0.408

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	59	125	141	97	126	98	79
N.S.	1	1.00	0.79	1.67	1.88	1.29	1.68	1.31	1.05
time (sec)	N/A	0.044	0.143	0.464	0.276	0.387	0.203	0.435	0.162

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	102	166	117	188	0	113	-1
N.S.	1	1.00	0.84	1.37	0.97	1.55	0.00	0.93	-0.01
time (sec)	N/A	0.192	0.162	1.604	0.322	0.376	0.000	0.417	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	160	271	145	301	0	1076	-1
N.S.	1	1.00	1.10	1.87	1.00	2.08	0.00	7.42	-0.01
time (sec)	N/A	0.193	0.817	1.620	0.337	0.340	0.000	0.475	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	220	562	145	529	0	601	-1
N.S.	1	1.00	1.20	3.05	0.79	2.88	0.00	3.27	-0.01
time (sec)	N/A	0.303	0.539	1.642	0.343	0.334	0.000	0.423	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	296	541	333	396	0	0	-1
N.S.	1	1.00	1.99	3.63	2.23	2.66	0.00	0.00	-0.01
time (sec)	N/A	0.099	2.052	0.880	0.345	0.356	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	170	306	195	242	0	0	-1
N.S.	1	1.00	1.72	3.09	1.97	2.44	0.00	0.00	-0.01
time (sec)	N/A	0.063	1.587	0.772	0.327	0.339	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	174	60	0	119	0	0	-1
N.S.	1	1.00	3.48	1.20	0.00	2.38	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.059	0.507	0.000	0.335	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.015	9.354	180.000	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.015	9.265	180.000	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	133	473	320	1159	0	0	-1
N.S.	1	1.00	1.29	4.59	3.11	11.25	0.00	0.00	-0.01
time (sec)	N/A	0.146	1.360	0.940	0.413	0.351	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	198	240	0	623	0	0	-1
N.S.	1	1.00	2.68	3.24	0.00	8.42	0.00	0.00	-0.01
time (sec)	N/A	0.102	3.680	0.826	0.000	0.412	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	52	56	91	166	0	80	49
N.S.	1	1.00	1.79	1.93	3.14	5.72	0.00	2.76	1.69
time (sec)	N/A	0.021	0.088	0.470	0.273	0.350	0.000	0.435	0.077

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.029	15.084	180.000	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.026	15.624	180.000	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	398	876	605	4008	0	0	-1
N.S.	1	1.00	1.55	3.42	2.36	15.66	0.00	0.00	-0.00
time (sec)	N/A	0.195	5.980	1.069	0.420	0.412	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	375	444	393	2218	0	0	-1
N.S.	1	1.00	2.44	2.88	2.55	14.40	0.00	0.00	-0.01
time (sec)	N/A	0.119	8.802	0.947	0.410	0.367	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	313	197	0	1026	0	0	-1
N.S.	1	1.00	3.40	2.14	0.00	11.15	0.00	0.00	-0.01
time (sec)	N/A	0.058	1.688	0.691	0.000	0.353	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.033	52.153	180.000	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.026	55.688	180.000	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	108	0	308	521	0	232	-1
N.S.	1	1.00	0.63	0.00	1.80	3.05	0.00	1.36	-0.01
time (sec)	N/A	0.261	0.040	180.000	0.278	0.352	0.000	0.471	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	106	0	268	385	0	202	-1
N.S.	1	1.00	0.73	0.00	1.84	2.64	0.00	1.38	-0.01
time (sec)	N/A	0.168	0.068	180.000	0.273	0.353	0.000	0.488	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	104	0	230	301	0	168	-1
N.S.	1	1.00	0.85	0.00	1.87	2.45	0.00	1.37	-0.01
time (sec)	N/A	0.126	0.063	180.000	0.283	0.359	0.000	0.451	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	0	181	122	0	90	-1
N.S.	1	1.00	1.00	0.00	1.74	1.17	0.00	0.87	-0.01
time (sec)	N/A	0.091	0.024	180.000	0.282	0.374	0.000	0.436	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	120	0	103	339	0	0	-1
N.S.	1	1.00	1.02	0.00	0.87	2.87	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.107	180.000	0.261	0.379	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	161	0	114	532	0	0	-1
N.S.	1	1.00	1.08	0.00	0.77	3.57	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.529	180.000	0.343	0.395	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	168	0	114	855	0	0	-1
N.S.	1	1.00	0.97	0.00	0.66	4.91	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.381	180.000	0.353	0.374	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	190	0	281	1001	0	0	-1
N.S.	1	1.00	0.79	0.00	1.18	4.19	0.00	0.00	-0.00
time (sec)	N/A	0.310	4.511	180.000	0.479	0.359	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	163	0	239	755	0	0	-1
N.S.	1	1.00	0.77	0.00	1.13	3.58	0.00	0.00	-0.00
time (sec)	N/A	0.224	1.558	180.000	0.492	0.624	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	129	0	189	590	0	0	-1
N.S.	1	1.00	0.78	0.00	1.14	3.55	0.00	0.00	-0.01
time (sec)	N/A	0.202	0.407	180.000	0.476	0.434	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	142	0	107	155	0	115	-1
N.S.	1	1.00	1.02	0.00	0.77	1.12	0.00	0.83	-0.01
time (sec)	N/A	0.156	0.081	180.000	0.478	0.462	0.000	0.534	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	570	0	116	571	0	0	-1
N.S.	1	1.00	4.01	0.00	0.82	4.02	0.00	0.00	-0.01
time (sec)	N/A	0.166	3.199	180.000	0.346	0.433	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	156	0	118	864	0	0	-1
N.S.	1	1.00	0.90	0.00	0.68	4.97	0.00	0.00	-0.01
time (sec)	N/A	0.226	0.795	180.000	0.344	0.541	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	825	0	118	1352	0	0	-1
N.S.	1	1.00	3.75	0.00	0.54	6.15	0.00	0.00	-0.00
time (sec)	N/A	0.222	6.082	180.000	0.330	0.463	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	222	0	118	1827	0	0	-1
N.S.	1	1.00	0.88	0.00	0.47	7.28	0.00	0.00	-0.00
time (sec)	N/A	0.283	0.369	180.000	0.326	0.407	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	243	0	513	2090	0	0	-1
N.S.	1	1.00	0.64	0.00	1.35	5.49	0.00	0.00	-0.00
time (sec)	N/A	0.798	6.676	180.000	0.506	0.462	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	243	0	430	1543	0	0	-1
N.S.	1	1.00	0.75	0.00	1.32	4.75	0.00	0.00	-0.00
time (sec)	N/A	0.612	2.702	180.000	0.483	0.440	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	209	0	333	1216	0	0	-1
N.S.	1	1.00	0.76	0.00	1.21	4.42	0.00	0.00	-0.00
time (sec)	N/A	0.382	0.207	180.000	0.482	0.381	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	191	0	178	252	0	0	-1
N.S.	1	1.00	0.84	0.00	0.78	1.11	0.00	0.00	-0.00
time (sec)	N/A	0.288	0.131	180.000	0.494	0.353	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	574	0	197	1346	0	0	-1
N.S.	1	1.00	2.33	0.00	0.80	5.47	0.00	0.00	-0.00
time (sec)	N/A	0.325	8.422	180.000	0.370	0.355	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	253	0	196	2059	0	0	-1
N.S.	1	1.00	0.91	0.00	0.71	7.43	0.00	0.00	-0.00
time (sec)	N/A	0.516	2.002	180.000	0.361	0.373	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	3211	0	197	3286	0	0	-1
N.S.	1	1.00	9.70	0.00	0.60	9.93	0.00	0.00	-0.00
time (sec)	N/A	0.575	14.151	180.000	0.357	0.469	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	50	132	175	189	133	146	-1
N.S.	1	1.00	0.45	1.19	1.58	1.70	1.20	1.32	-0.01
time (sec)	N/A	0.111	0.011	0.237	0.262	0.340	13.772	0.437	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	49	120	149	137	99	108	-1
N.S.	1	1.00	0.53	1.30	1.62	1.49	1.08	1.17	-0.01
time (sec)	N/A	0.078	0.012	0.196	0.290	0.581	1.019	0.420	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	47	71	116	58	70	61	-1
N.S.	1	1.00	0.61	0.92	1.51	0.75	0.91	0.79	-0.01
time (sec)	N/A	0.051	0.007	0.196	0.274	0.416	0.565	0.447	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	49	120	74	137	94	0	-1
N.S.	1	1.00	0.56	1.38	0.85	1.57	1.08	0.00	-0.01
time (sec)	N/A	0.079	0.015	0.198	0.269	0.442	1.989	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	84	132	57	178	129	0	-1
N.S.	1	1.00	0.74	1.16	0.50	1.56	1.13	0.00	-0.01
time (sec)	N/A	0.104	0.055	0.198	0.315	0.414	15.711	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.020	23.349	180.000	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.020	15.644	180.000	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.071	5.432	0.129	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	17	0	0	0	0	0	38
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	1.90
time (sec)	N/A	0.036	0.078	1.747	0.000	0.000	0.000	0.000	0.174

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	0	0	108	0	0	40
N.S.	1	1.00	0.92	0.00	0.00	4.50	0.00	0.00	1.67
time (sec)	N/A	0.038	0.043	1.652	0.000	0.347	0.000	0.000	0.153

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	33	0	0	0	0	0	111
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	2.36
time (sec)	N/A	0.052	0.077	1.813	0.000	0.000	0.000	0.000	0.291

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	68	0	0	0	0	0	-1
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.080	0.848	1.678	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.031	2.010	180.000	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	206	0	161	340	0	0	-1
N.S.	1	1.00	0.87	0.00	0.68	1.43	0.00	0.00	-0.00
time (sec)	N/A	0.225	0.144	180.000	0.082	0.089	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	131	0	102	241	0	0	-1
N.S.	1	1.00	0.91	0.00	0.71	1.67	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.123	180.000	0.070	0.085	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	101	0	79	168	0	0	-1
N.S.	1	1.00	0.92	0.00	0.72	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.044	180.000	0.060	0.086	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.015	5.970	180.000	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	2.439	180.000	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	73	55	86	0	0	-1
N.S.	1	1.00	0.92	1.24	0.93	1.46	0.00	0.00	-0.02
time (sec)	N/A	0.051	0.020	0.171	0.075	0.080	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	53	73	55	86	0	0	-1
N.S.	1	1.00	0.90	1.24	0.93	1.46	0.00	0.00	-0.02
time (sec)	N/A	0.051	0.016	0.173	0.079	0.088	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	73	55	86	0	0	-1
N.S.	1	1.00	0.92	1.24	0.93	1.46	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.017	0.161	0.087	0.080	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	53	73	55	78	0	0	-1
N.S.	1	1.00	0.90	1.24	0.93	1.32	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.014	0.155	0.077	0.106	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	67	43	78	0	0	-1
N.S.	1	1.00	1.00	1.37	0.88	1.59	0.00	0.00	-0.02
time (sec)	N/A	0.048	0.017	0.181	0.083	0.082	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	51	67	55	86	0	0	-1
N.S.	1	1.00	0.93	1.22	1.00	1.56	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.015	0.187	0.094	0.085	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	71	55	86	0	0	-1
N.S.	1	1.00	0.92	1.20	0.93	1.46	0.00	0.00	-0.02
time (sec)	N/A	0.050	0.017	0.168	0.093	0.083	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	79	0	71	136	0	0	-1
N.S.	1	1.00	0.92	0.00	0.83	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.101	0.285	0.082	0.089	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	78	0	71	136	0	0	-1
N.S.	1	1.00	0.92	0.00	0.84	1.60	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.100	0.225	0.075	0.138	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	79	0	71	136	0	0	-1
N.S.	1	1.00	0.92	0.00	0.83	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.093	0.228	0.074	0.136	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	76	0	71	122	0	0	-1
N.S.	1	1.00	0.89	0.00	0.84	1.44	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.080	0.215	0.081	0.088	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	63	0	55	117	0	0	-1
N.S.	1	1.00	0.88	0.00	0.76	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.065	0.284	0.094	0.087	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	72	0	0	136	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	1.64	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.088	0.287	0.000	0.084	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	136	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.082	0.286	0.000	0.102	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.075	0.059	1.598	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.078	0.084	1.587	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.086	0.094	1.566	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	63	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.103	1.587	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	128	494	248	287	517	262	196
N.S.	1	1.00	1.31	5.04	2.53	2.93	5.28	2.67	2.00
time (sec)	N/A	0.099	0.293	0.441	0.294	0.345	0.443	0.456	0.383

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	88	249	149	175	314	150	118
N.S.	1	1.00	1.19	3.36	2.01	2.36	4.24	2.03	1.59
time (sec)	N/A	0.071	0.254	0.368	0.269	0.373	0.281	0.440	0.246

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	96	70	84	162	69	56
N.S.	1	1.00	0.96	1.92	1.40	1.68	3.24	1.38	1.12
time (sec)	N/A	0.034	0.077	0.380	0.263	0.345	0.197	0.434	0.123

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	60	96	73	81	0	69	-1
N.S.	1	1.00	0.86	1.37	1.04	1.16	0.00	0.99	-0.01
time (sec)	N/A	0.116	0.174	0.543	0.320	0.357	0.000	0.417	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	83	153	90	139	0	630	-1
N.S.	1	1.00	0.87	1.61	0.95	1.46	0.00	6.63	-0.01
time (sec)	N/A	0.131	0.280	0.502	0.307	0.375	0.000	0.447	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	109	303	101	227	0	322	-1
N.S.	1	1.00	0.83	2.31	0.77	1.73	0.00	2.46	-0.01
time (sec)	N/A	0.169	0.393	0.503	0.320	0.357	0.000	0.436	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	220	1082	552	601	1134	580	393
N.S.	1	1.00	0.90	4.42	2.25	2.45	4.63	2.37	1.60
time (sec)	N/A	0.202	0.859	0.637	0.300	0.489	0.770	0.440	1.046

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	189	550	343	356	694	333	217
N.S.	1	1.00	1.09	3.16	1.97	2.05	3.99	1.91	1.25
time (sec)	N/A	0.140	0.401	0.631	0.280	0.401	0.569	0.469	0.676

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	86	215	176	169	357	155	104
N.S.	1	1.00	0.70	1.76	1.44	1.39	2.93	1.27	0.85
time (sec)	N/A	0.071	0.556	0.622	0.277	0.358	0.380	0.447	0.352

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	117	193	154	153	0	135	-1
N.S.	1	1.00	0.79	1.30	1.03	1.03	0.00	0.91	-0.01
time (sec)	N/A	0.256	0.209	3.452	0.319	0.366	0.000	0.422	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	214	313	187	285	0	1134	-1
N.S.	1	1.00	1.26	1.84	1.10	1.68	0.00	6.67	-0.01
time (sec)	N/A	0.245	0.383	3.392	0.315	0.404	0.000	0.513	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	198	625	209	472	0	682	-1
N.S.	1	1.00	0.84	2.65	0.89	2.00	0.00	2.89	-0.00
time (sec)	N/A	0.369	1.469	3.440	0.334	0.393	0.000	0.435	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	209	435	249	372	0	0	-1
N.S.	1	1.00	1.58	3.30	1.89	2.82	0.00	0.00	-0.01
time (sec)	N/A	0.208	1.960	1.244	0.372	0.330	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	139	227	0	212	0	0	-1
N.S.	1	1.00	1.38	2.25	0.00	2.10	0.00	0.00	-0.01
time (sec)	N/A	0.150	1.583	0.980	0.000	0.383	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	185	66	80	64	56	67	56
N.S.	1	1.00	2.94	1.05	1.27	1.02	0.89	1.06	0.89
time (sec)	N/A	0.052	0.309	0.805	0.270	0.381	0.129	0.411	0.340

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	17.568	180.000	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	18.055	180.000	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	508	723	659	938	0	0	-1
N.S.	1	1.00	1.67	2.37	2.16	3.08	0.00	0.00	-0.00
time (sec)	N/A	0.289	4.132	3.246	0.457	0.343	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	259	374	0	504	0	0	-1
N.S.	1	1.00	1.07	1.55	0.00	2.09	0.00	0.00	-0.00
time (sec)	N/A	0.210	2.544	2.588	0.000	0.341	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	241	113	271	170	167	195	160
N.S.	1	1.00	1.53	0.72	1.72	1.08	1.06	1.23	1.01
time (sec)	N/A	0.075	0.679	2.748	0.288	0.382	0.275	0.436	0.542

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	24.780	180.000	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	26.265	180.000	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	141	174	0	0	0	0	149
N.S.	1	1.00	0.78	0.96	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.150	0.171	0.302	0.000	0.000	0.000	0.000	0.722

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	125	151	0	0	0	0	126
N.S.	1	1.00	0.92	1.11	0.00	0.00	0.00	0.00	0.93
time (sec)	N/A	0.117	0.165	0.241	0.000	0.000	0.000	0.000	0.360

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	105	128	0	0	0	0	92
N.S.	1	1.00	0.95	1.15	0.00	0.00	0.00	0.00	0.83
time (sec)	N/A	0.096	0.130	0.234	0.000	0.000	0.000	0.000	0.289

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	87	105	0	0	0	0	80
N.S.	1	1.00	1.32	1.59	0.00	0.00	0.00	0.00	1.21
time (sec)	N/A	0.054	0.104	0.230	0.000	0.000	0.000	0.000	0.267

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	96	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.094	0.311	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	133	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.153	0.122	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	170	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.144	0.204	0.121	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	269	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.249	0.597	0.159	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	173	0	0	0	0	0	-1
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.182	0.518	0.118	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	138	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.363	0.126	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	146	0	0	0	0	0	-1
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.197	0.353	0.124	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	243	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.198	0.429	0.124	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	638	638	2918	0	0	0	0	0	-1
N.S.	1	1.00	4.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.444	6.318	0.380	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	506	506	300	0	0	0	0	0	-1
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.262	0.717	0.124	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	218	0	0	0	0	0	-1
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.135	0.573	0.122	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	242	0	0	0	0	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.287	0.615	0.123	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	347	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.304	1.130	0.123	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	536	536	4751	0	0	0	0	0	-1
N.S.	1	1.00	8.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.418	6.405	0.124	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	493	493	331	0	0	0	0	0	-1
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.182	0.823	0.187	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	276	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.143	0.663	0.154	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	221	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.075	0.341	0.155	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.055	2.813	0.183	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.049	2.871	0.186	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	807	807	546	0	0	0	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.340	1.784	0.189	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	506	506	384	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.234	1.197	0.185	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	332	0	0	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.124	0.607	0.184	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.060	15.756	0.188	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.056	17.520	0.185	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1016	1016	1200	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.499	2.699	0.187	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	689	689	482	0	0	0	0	0	-1
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.320	1.594	0.184	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	411	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.173	1.065	0.187	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	25.091	0.190	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.048	2.877	0.043	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	2.793	180.000	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	452	0	383	380	0	0	-1
N.S.	1	1.00	1.10	0.00	0.93	0.93	0.00	0.00	-0.00
time (sec)	N/A	0.418	4.366	180.000	0.123	0.100	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	383	0	214	263	0	0	-1
N.S.	1	1.00	1.43	0.00	0.80	0.98	0.00	0.00	-0.00
time (sec)	N/A	0.266	7.849	180.000	0.092	0.093	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	166	0	103	136	0	0	-1
N.S.	1	1.00	1.23	0.00	0.76	1.01	0.00	0.00	-0.01
time (sec)	N/A	0.104	4.665	180.000	0.063	0.098	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	3.034	180.000	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	12.226	180.000	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	123	482	247	174	264	258	187
N.S.	1	1.00	1.38	5.42	2.78	1.96	2.97	2.90	2.10
time (sec)	N/A	0.097	0.262	0.412	0.277	0.339	0.283	0.437	0.243

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	83	240	147	108	151	146	110
N.S.	1	1.00	1.24	3.58	2.19	1.61	2.25	2.18	1.64
time (sec)	N/A	0.065	0.188	0.355	0.274	0.325	0.155	0.430	0.107

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	91	69	57	68	64	49
N.S.	1	1.00	0.96	2.02	1.53	1.27	1.51	1.42	1.09
time (sec)	N/A	0.033	0.091	0.373	0.262	0.330	0.100	0.453	0.143

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	57	94	73	122	0	68	-1
N.S.	1	1.00	0.89	1.47	1.14	1.91	0.00	1.06	-0.02
time (sec)	N/A	0.102	0.087	0.424	0.297	0.376	0.000	0.426	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	71	149	90	178	0	630	-1
N.S.	1	1.00	0.82	1.71	1.03	2.05	0.00	7.24	-0.01
time (sec)	N/A	0.119	0.229	0.437	0.298	0.359	0.000	0.438	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	95	296	101	293	0	319	-1
N.S.	1	1.00	0.77	2.41	0.82	2.38	0.00	2.59	-0.01
time (sec)	N/A	0.156	0.395	0.447	0.310	0.357	0.000	0.433	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	235	1061	547	433	779	598	481
N.S.	1	1.00	0.94	4.24	2.19	1.73	3.12	2.39	1.92
time (sec)	N/A	0.195	0.719	0.613	0.289	0.345	0.460	0.445	1.858

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	249	535	339	262	456	344	281
N.S.	1	1.00	1.37	2.94	1.86	1.44	2.51	1.89	1.54
time (sec)	N/A	0.141	0.434	0.819	0.282	0.357	0.301	0.445	0.550

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	98	208	173	143	219	159	135
N.S.	1	1.00	0.84	1.79	1.49	1.23	1.89	1.37	1.16
time (sec)	N/A	0.071	0.421	0.934	0.279	0.342	0.167	0.427	0.147

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	134	201	152	255	0	144	-1
N.S.	1	1.00	0.86	1.29	0.97	1.63	0.00	0.92	-0.01
time (sec)	N/A	0.252	0.167	8.269	0.334	0.359	0.000	0.429	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	232	319	185	542	0	1135	-1
N.S.	1	1.00	1.27	1.74	1.01	2.96	0.00	6.20	-0.01
time (sec)	N/A	0.253	0.403	6.297	0.331	0.374	0.000	0.486	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	395	626	207	834	0	678	-1
N.S.	1	1.00	1.63	2.59	0.86	3.45	0.00	2.80	-0.00
time (sec)	N/A	0.324	0.570	7.543	0.357	0.373	0.000	0.439	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	318	0	0	1382	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	3.42	0.00	0.00	-0.00
time (sec)	N/A	0.557	0.184	180.000	0.000	0.364	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	233	0	0	910	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	3.07	0.00	0.00	-0.00
time (sec)	N/A	0.469	0.112	180.000	0.000	0.396	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	142	393	0	541	0	0	-1
N.S.	1	1.00	0.76	2.10	0.00	2.89	0.00	0.00	-0.01
time (sec)	N/A	0.253	0.028	1.033	0.000	0.349	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	0.676	180.000	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	0.673	180.000	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	549	549	428	0	0	5925	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	10.79	0.00	0.00	-0.00
time (sec)	N/A	0.706	1.119	1.838	0.000	0.442	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	194	519	0	2245	0	0	-1
N.S.	1	1.00	0.76	2.04	0.00	8.84	0.00	0.00	-0.00
time (sec)	N/A	0.318	0.752	2.875	0.000	0.408	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	33.846	180.000	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	35.115	180.000	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	774	1232	0	7272	0	0	-1
N.S.	1	1.00	1.42	2.26	0.00	13.37	0.00	0.00	-0.00
time (sec)	N/A	1.464	7.080	1.571	0.000	0.455	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	69.673	180.000	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	65.539	180.000	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	3.036	180.000	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	2639	0	385	899	0	0	-1
N.S.	1	1.00	4.86	0.00	0.71	1.66	0.00	0.00	-0.00
time (sec)	N/A	0.556	18.525	180.000	0.122	0.113	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	241	0	212	563	0	0	-1
N.S.	1	1.00	0.86	0.00	0.75	2.00	0.00	0.00	-0.00
time (sec)	N/A	0.257	6.477	180.000	0.083	0.150	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	201	0	103	271	0	0	-1
N.S.	1	1.00	1.53	0.00	0.79	2.07	0.00	0.00	-0.01
time (sec)	N/A	0.100	17.295	180.000	0.062	0.105	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	0.875	180.000	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	3.820	180.000	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	247	513	321	455	0	0	-1
N.S.	1	1.00	1.52	3.15	1.97	2.79	0.00	0.00	-0.01
time (sec)	N/A	0.241	2.201	1.546	0.361	0.420	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	183	281	0	263	0	0	-1
N.S.	1	1.00	1.41	2.16	0.00	2.02	0.00	0.00	-0.01
time (sec)	N/A	0.186	1.702	1.283	0.000	0.347	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	239	86	109	97	73	111	74
N.S.	1	1.00	2.66	0.96	1.21	1.08	0.81	1.23	0.82
time (sec)	N/A	0.076	0.409	1.273	0.286	0.396	0.165	0.429	0.548

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	84	57	36	33	24	33	27
N.S.	1	1.00	2.40	1.63	1.03	0.94	0.69	0.94	0.77
time (sec)	N/A	0.029	0.099	0.883	0.261	0.334	0.077	0.420	0.244

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.033	34.863	180.000	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.035	32.804	180.000	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	872	699	697	825	0	0	-1
N.S.	1	1.00	3.62	2.90	2.89	3.42	0.00	0.00	-0.00
time (sec)	N/A	0.375	4.516	3.165	0.438	0.408	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	249	385	0	475	0	0	-1
N.S.	1	1.00	1.35	2.09	0.00	2.58	0.00	0.00	-0.01
time (sec)	N/A	0.279	2.446	1.760	0.000	0.352	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	238	134	239	176	224	251	143
N.S.	1	1.00	2.00	1.13	2.01	1.48	1.88	2.11	1.20
time (sec)	N/A	0.130	0.667	1.617	0.332	0.443	0.320	0.450	0.582

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	59	86	74	69	99	63	59
N.S.	1	1.00	1.13	1.65	1.42	1.33	1.90	1.21	1.13
time (sec)	N/A	0.068	0.173	0.911	0.273	0.387	0.156	0.441	0.305

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.056	170.221	180.000	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.051	150.469	180.000	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	397	940	0	1045	0	0	-1
N.S.	1	1.00	1.01	2.39	0.00	2.66	0.00	0.00	-0.00
time (sec)	N/A	0.499	6.985	2.792	0.000	0.371	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	831	520	0	596	0	0	-1
N.S.	1	1.00	2.90	1.81	0.00	2.08	0.00	0.00	-0.00
time (sec)	N/A	0.379	4.235	2.596	0.000	0.407	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	325	197	0	237	396	341	215
N.S.	1	1.00	1.86	1.13	0.00	1.35	2.26	1.95	1.23
time (sec)	N/A	0.189	1.127	2.598	0.000	0.414	0.497	0.446	0.708

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	109	123	98	96	175	87	94
N.S.	1	1.00	1.31	1.48	1.18	1.16	2.11	1.05	1.13
time (sec)	N/A	0.058	0.128	1.061	0.270	0.385	0.238	0.465	0.353

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.055	180.000	180.000	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.058	161.130	180.000	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	501	1034	579	1013	0	0	-1
N.S.	1	1.00	1.60	3.30	1.85	3.24	0.00	0.00	-0.00
time (sec)	N/A	0.361	4.408	3.338	0.446	0.413	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	327	573	351	571	0	0	-1
N.S.	1	1.00	1.46	2.56	1.57	2.55	0.00	0.00	-0.00
time (sec)	N/A	0.253	3.759	3.059	0.434	0.487	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	345	211	0	217	0	0	-1
N.S.	1	1.00	2.74	1.67	0.00	1.72	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.794	3.030	0.000	0.399	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	52	36	62	57	0	48	56
N.S.	1	1.00	1.27	0.88	1.51	1.39	0.00	1.17	1.37
time (sec)	N/A	0.043	0.049	1.012	0.264	0.420	0.000	0.432	0.882

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.035	32.356	180.000	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.035	40.583	180.000	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	1005	1535	940	2568	0	0	-1
N.S.	1	1.00	2.40	3.66	2.24	6.13	0.00	0.00	-0.00
time (sec)	N/A	0.597	13.540	3.230	0.466	0.446	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	659	847	611	1381	0	0	-1
N.S.	1	1.00	2.23	2.86	2.06	4.67	0.00	0.00	-0.00
time (sec)	N/A	0.413	11.068	3.010	0.451	0.374	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	454	316	0	520	0	0	-1
N.S.	1	1.00	2.79	1.94	0.00	3.19	0.00	0.00	-0.01
time (sec)	N/A	0.177	3.465	3.069	0.000	0.471	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	61	63	109	146	0	90	122
N.S.	1	1.00	1.07	1.11	1.91	2.56	0.00	1.58	2.14
time (sec)	N/A	0.069	0.157	1.078	0.259	0.336	0.000	0.445	1.402

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.056	91.226	180.000	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.053	180.002	180.000	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	546	546	2394	2058	1353	4261	0	0	-1
N.S.	1	1.00	4.38	3.77	2.48	7.80	0.00	0.00	-0.00
time (sec)	N/A	0.869	52.330	3.435	0.598	0.428	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	1370	1107	897	2238	0	0	-1
N.S.	1	1.00	3.72	3.01	2.44	6.08	0.00	0.00	-0.00
time (sec)	N/A	0.603	21.342	3.252	0.525	0.379	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	541	423	0	827	0	0	-1
N.S.	1	1.00	2.53	1.98	0.00	3.86	0.00	0.00	-0.00
time (sec)	N/A	0.274	1.760	4.015	0.000	0.386	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	90	91	156	234	0	97	132
N.S.	1	1.00	1.03	1.05	1.79	2.69	0.00	1.11	1.52
time (sec)	N/A	0.097	0.351	1.174	0.271	0.394	0.000	0.418	0.610

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.054	180.017	180.000	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.053	180.026	180.000	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	453	453	1009	0	0	1614	0	0	-1
N.S.	1	1.00	2.23	0.00	0.00	3.56	0.00	0.00	-0.00
time (sec)	N/A	0.578	2.730	0.529	0.000	0.391	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	610	0	0	956	0	0	-1
N.S.	1	1.00	1.81	0.00	0.00	2.84	0.00	0.00	-0.00
time (sec)	N/A	0.494	2.041	0.467	0.000	0.357	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	163	440	0	532	0	0	-1
N.S.	1	1.00	0.74	2.00	0.00	2.42	0.00	0.00	-0.00
time (sec)	N/A	0.278	0.483	1.286	0.000	0.355	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	64	82	85	186	350	84	121
N.S.	1	1.00	1.19	1.52	1.57	3.44	6.48	1.56	2.24
time (sec)	N/A	0.052	0.083	0.542	0.489	0.394	36.208	0.429	0.823

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.032	6.150	180.000	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	551	551	1074	0	0	4491	0	0	-1
N.S.	1	1.00	1.95	0.00	0.00	8.15	0.00	0.00	-0.00
time (sec)	N/A	0.716	6.614	1.805	0.000	0.559	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	697	0	0	2365	0	0	-1
N.S.	1	1.00	1.71	0.00	0.00	5.81	0.00	0.00	-0.00
time (sec)	N/A	0.600	5.320	1.834	0.000	0.385	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	299	510	0	1078	0	0	-1
N.S.	1	1.00	1.13	1.93	0.00	4.08	0.00	0.00	-0.00
time (sec)	N/A	0.352	1.405	1.613	0.000	0.372	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	74	121	119	331	1748	111	166
N.S.	1	1.00	1.04	1.70	1.68	4.66	24.62	1.56	2.34
time (sec)	N/A	0.092	0.218	0.758	0.470	0.336	142.616	0.440	0.373

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.048	91.988	180.000	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	712	712	1948	0	0	9525	0	0	-1
N.S.	1	1.00	2.74	0.00	0.00	13.38	0.00	0.00	-0.00
time (sec)	N/A	0.841	7.789	0.987	0.000	0.447	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	1589	0	0	4857	0	0	-1
N.S.	1	1.00	3.04	0.00	0.00	9.30	0.00	0.00	-0.00
time (sec)	N/A	0.733	7.459	1.608	0.000	0.430	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	307	589	0	2068	0	0	-1
N.S.	1	1.00	0.92	1.76	0.00	6.17	0.00	0.00	-0.00
time (sec)	N/A	0.424	1.640	1.593	0.000	0.393	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	101	191	164	601	0	151	212
N.S.	1	1.00	0.94	1.79	1.53	5.62	0.00	1.41	1.98
time (sec)	N/A	0.159	0.252	0.780	0.486	0.382	0.000	0.438	0.438

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.050	178.145	180.000	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	605	605	1264	0	0	2509	0	0	-1
N.S.	1	1.00	2.09	0.00	0.00	4.15	0.00	0.00	-0.00
time (sec)	N/A	0.683	3.825	180.000	0.000	0.405	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	750	0	0	1379	0	0	-1
N.S.	1	1.00	1.73	0.00	0.00	3.18	0.00	0.00	-0.00
time (sec)	N/A	0.561	2.427	180.000	0.000	0.395	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	306	532	0	691	0	0	-1
N.S.	1	1.00	1.17	2.04	0.00	2.65	0.00	0.00	-0.00
time (sec)	N/A	0.326	1.155	1.585	0.000	0.421	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	69	63	112	223	0	102	347
N.S.	1	1.00	1.08	0.98	1.75	3.48	0.00	1.59	5.42
time (sec)	N/A	0.064	0.074	1.005	0.474	0.378	0.000	0.451	0.459

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.032	3.767	180.000	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	745	745	2152	0	0	11107	0	0	-1
N.S.	1	1.00	2.89	0.00	0.00	14.91	0.00	0.00	-0.00
time (sec)	N/A	0.964	14.154	0.875	0.000	0.516	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	535	535	1011	0	0	5257	0	0	-1
N.S.	1	1.00	1.89	0.00	0.00	9.83	0.00	0.00	-0.00
time (sec)	N/A	0.744	13.138	0.897	0.000	0.446	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	405	626	0	2027	0	0	-1
N.S.	1	1.00	1.32	2.05	0.00	6.62	0.00	0.00	-0.00
time (sec)	N/A	0.394	3.483	1.568	0.000	0.373	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	100	97	137	479	0	123	360
N.S.	1	1.00	1.25	1.21	1.71	5.99	0.00	1.54	4.50
time (sec)	N/A	0.105	0.536	0.981	0.478	0.370	0.000	0.474	0.393

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.052	77.437	180.000	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1053	1053	3202	0	0	32529	0	0	-1
N.S.	1	1.00	3.04	0.00	0.00	30.89	0.00	0.00	-0.00
time (sec)	N/A	1.276	29.933	1.724	0.000	0.761	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	725	725	1775	0	0	14973	0	0	-1
N.S.	1	1.00	2.45	0.00	0.00	20.65	0.00	0.00	-0.00
time (sec)	N/A	0.932	18.872	1.985	0.000	0.540	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	736	861	0	5442	0	0	-1
N.S.	1	1.00	1.75	2.05	0.00	12.96	0.00	0.00	-0.00
time (sec)	N/A	0.519	7.135	1.743	0.000	0.416	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	145	142	211	1203	0	176	776
N.S.	1	1.00	1.28	1.26	1.87	10.65	0.00	1.56	6.87
time (sec)	N/A	0.273	1.358	1.189	0.477	0.432	0.000	0.427	0.746

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.055	124.508	180.000	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	118	647	263	302	0	0	-1
N.S.	1	1.00	0.85	4.65	1.89	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.060	2.992	0.332	0.351	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	94	405	166	190	0	0	-1
N.S.	1	1.00	0.89	3.82	1.57	1.79	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.035	1.564	0.330	0.354	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	66	188	0	95	0	0	-1
N.S.	1	1.00	0.90	2.58	0.00	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.021	1.523	0.000	0.351	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	23	20	23	22	32	19
N.S.	1	1.00	1.00	1.00	0.87	1.00	0.96	1.39	0.83
time (sec)	N/A	0.020	0.014	0.725	0.259	0.340	0.101	0.499	0.239

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.034	19.991	180.000	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.036	26.228	180.000	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	106	448	372	260	518	355	269
N.S.	1	1.00	0.98	4.15	3.44	2.41	4.80	3.29	2.49
time (sec)	N/A	0.118	0.360	1.102	0.400	0.358	0.390	0.457	0.716

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	78	223	270	160	318	208	167
N.S.	1	1.00	0.95	2.72	3.29	1.95	3.88	2.54	2.04
time (sec)	N/A	0.092	0.272	1.096	0.367	0.352	0.297	0.455	0.523

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	84	189	79	167	96	87
N.S.	1	1.00	1.02	1.50	3.38	1.41	2.98	1.71	1.55
time (sec)	N/A	0.048	0.382	1.050	0.323	0.377	0.196	0.428	0.385

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	139	70	44	40	78	41	36
N.S.	1	1.00	6.32	3.18	2.00	1.82	3.55	1.86	1.64
time (sec)	N/A	0.030	0.117	1.113	0.273	0.325	0.118	0.446	0.213

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	62	103	81	84	0	76	-1
N.S.	1	1.00	0.82	1.36	1.07	1.11	0.00	1.00	-0.01
time (sec)	N/A	0.151	0.181	1.625	0.367	0.336	0.000	0.453	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	85	164	99	141	0	572	-1
N.S.	1	1.00	0.83	1.59	0.96	1.37	0.00	5.55	-0.01
time (sec)	N/A	0.156	0.300	1.521	0.405	0.357	0.000	0.486	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	134	726	0	401	1040	618	449
N.S.	1	1.00	0.58	3.14	0.00	1.74	4.50	2.68	1.94
time (sec)	N/A	0.182	0.712	1.210	0.000	0.337	0.652	0.462	1.284

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	99	241	0	227	631	338	271
N.S.	1	1.00	0.58	1.41	0.00	1.33	3.69	1.98	1.58
time (sec)	N/A	0.138	0.519	2.633	0.000	0.350	0.464	0.450	0.936

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	60	120	0	96	321	138	144
N.S.	1	1.00	0.61	1.22	0.00	0.98	3.28	1.41	1.47
time (sec)	N/A	0.070	0.730	1.202	0.000	0.335	0.328	0.448	0.537

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	30	60	49	133	55	29
N.S.	1	1.00	0.82	0.88	1.76	1.44	3.91	1.62	0.85
time (sec)	N/A	0.032	0.036	0.660	0.261	0.342	0.177	0.450	0.293

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	112	180	0	135	0	144	-1
N.S.	1	1.00	0.85	1.37	0.00	1.03	0.00	1.10	-0.01
time (sec)	N/A	0.233	0.240	2.430	0.000	0.336	0.000	0.477	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	212	299	0	257	0	1080	-1
N.S.	1	1.00	1.18	1.66	0.00	1.43	0.00	6.00	-0.01
time (sec)	N/A	0.268	0.381	2.471	0.000	0.375	0.000	0.548	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	866	1152	691	1462	0	0	-1
N.S.	1	1.00	1.87	2.49	1.49	3.16	0.00	0.00	-0.00
time (sec)	N/A	0.345	7.972	3.249	0.443	0.397	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	599	613	394	827	0	0	-1
N.S.	1	1.00	2.24	2.29	1.47	3.09	0.00	0.00	-0.00
time (sec)	N/A	0.190	7.254	3.211	0.440	0.403	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	710	268	0	359	0	0	-1
N.S.	1	1.00	4.41	1.66	0.00	2.23	0.00	0.00	-0.01
time (sec)	N/A	0.100	2.212	4.042	0.000	0.400	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	30	75	87	102	0	104	74
N.S.	1	1.00	0.71	1.79	2.07	2.43	0.00	2.48	1.76
time (sec)	N/A	0.040	0.039	1.225	0.259	0.369	0.000	0.440	1.098

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.034	45.475	180.000	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.033	180.006	180.000	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	450	450	1127	1001	749	1424	0	0	-1
N.S.	1	1.00	2.50	2.22	1.66	3.16	0.00	0.00	-0.00
time (sec)	N/A	0.422	10.078	5.179	0.508	0.374	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	576	509	0	724	0	0	-1
N.S.	1	1.00	1.77	1.57	0.00	2.23	0.00	0.00	-0.00
time (sec)	N/A	0.271	8.422	3.048	0.000	0.372	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	194	143	252	203	0	260	205
N.S.	1	1.00	1.23	0.91	1.59	1.28	0.00	1.65	1.30
time (sec)	N/A	0.122	0.751	3.476	0.290	0.372	0.000	0.426	2.483

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	75	104	54	0	59	43
N.S.	1	1.00	1.00	1.60	2.21	1.15	0.00	1.26	0.91
time (sec)	N/A	0.040	0.042	1.337	0.266	0.334	0.000	0.426	0.483

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.052	102.106	180.000	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.051	180.012	180.000	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	667	667	2072	2026	0	3847	0	0	-1
N.S.	1	1.00	3.11	3.04	0.00	5.77	0.00	0.00	-0.00
time (sec)	N/A	0.517	11.337	4.164	0.000	0.441	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	423	423	1396	1044	0	2117	0	0	-1
N.S.	1	1.00	3.30	2.47	0.00	5.00	0.00	0.00	-0.00
time (sec)	N/A	0.294	11.133	4.258	0.000	0.432	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	929	445	0	936	0	0	-1
N.S.	1	1.00	3.99	1.91	0.00	4.02	0.00	0.00	-0.00
time (sec)	N/A	0.143	5.020	4.917	0.000	0.371	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	101	141	0	287	0	177	137
N.S.	1	1.00	1.11	1.55	0.00	3.15	0.00	1.95	1.51
time (sec)	N/A	0.063	0.093	1.552	0.000	0.426	0.000	0.430	0.986

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.053	94.549	180.000	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	F(-2)	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.051	180.020	180.000	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	329	0	0	1278	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	3.59	0.00	0.00	-0.00
time (sec)	N/A	0.354	0.111	0.773	0.000	0.440	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	244	0	0	738	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	2.80	0.00	0.00	-0.00
time (sec)	N/A	0.311	0.108	0.773	0.000	0.361	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	157	412	0	401	0	0	-1
N.S.	1	1.00	0.92	2.42	0.00	2.36	0.00	0.00	-0.01
time (sec)	N/A	0.172	0.026	1.526	0.000	0.368	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	44	41	33	18
N.S.	1	1.00	1.00	1.06	1.00	2.44	2.28	1.83	1.00
time (sec)	N/A	0.020	0.008	0.427	0.275	0.355	0.393	0.423	0.080

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.032	8.759	180.000	0.000	0.000	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	527	527	1055	0	0	3291	0	0	-1
N.S.	1	1.00	2.00	0.00	0.00	6.24	0.00	0.00	-0.00
time (sec)	N/A	0.664	9.069	2.158	0.000	0.420	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	723	0	0	1741	0	0	-1
N.S.	1	1.00	1.86	0.00	0.00	4.48	0.00	0.00	-0.00
time (sec)	N/A	0.569	4.718	2.076	0.000	0.405	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	258	901	0	814	0	0	-1
N.S.	1	1.00	1.02	3.58	0.00	3.23	0.00	0.00	-0.00
time (sec)	N/A	0.328	1.221	3.320	0.000	0.426	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	458	129	116	259	503	110	121
N.S.	1	1.00	6.74	1.90	1.71	3.81	7.40	1.62	1.78
time (sec)	N/A	0.089	1.027	1.210	0.479	0.345	86.392	0.448	0.419

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.048	19.516	180.000	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	642	642	2833	0	0	8057	0	0	-1
N.S.	1	1.00	4.41	0.00	0.00	12.55	0.00	0.00	-0.00
time (sec)	N/A	0.562	16.604	1.450	0.000	0.507	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	477	477	1253	0	0	4069	0	0	-1
N.S.	1	1.00	2.63	0.00	0.00	8.53	0.00	0.00	-0.00
time (sec)	N/A	0.478	12.974	1.565	0.000	0.438	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	251	975	0	1682	0	0	-1
N.S.	1	1.00	0.84	3.27	0.00	5.64	0.00	0.00	-0.00
time (sec)	N/A	0.259	1.121	3.306	0.000	0.378	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	53	190	127	327	0	92	120
N.S.	1	1.00	0.90	3.22	2.15	5.54	0.00	1.56	2.03
time (sec)	N/A	0.048	0.097	1.103	0.268	0.402	0.000	0.456	0.333

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.050	63.280	180.000	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	786	786	3214	0	0	2794	0	0	-1
N.S.	1	1.00	4.09	0.00	0.00	3.55	0.00	0.00	-0.00
time (sec)	N/A	1.043	22.357	180.000	0.000	0.430	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	558	1639	0	0	1424	0	0	-1
N.S.	1	1.00	2.94	0.00	0.00	2.55	0.00	0.00	-0.00
time (sec)	N/A	0.719	17.097	180.000	0.000	0.429	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	439	954	0	632	0	0	-1
N.S.	1	1.00	1.31	2.86	0.00	1.89	0.00	0.00	-0.00
time (sec)	N/A	0.404	1.843	2.873	0.000	0.372	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	114	88	95	92	0	121	129
N.S.	1	1.00	1.65	1.28	1.38	1.33	0.00	1.75	1.87
time (sec)	N/A	0.045	0.080	1.125	0.479	0.405	0.000	0.424	1.231

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.032	14.789	180.000	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	780	780	1531	0	0	10547	0	0	-1
N.S.	1	1.00	1.96	0.00	0.00	13.52	0.00	0.00	-0.00
time (sec)	N/A	1.269	14.500	2.461	0.000	0.593	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	548	548	1180	0	0	4630	0	0	-1
N.S.	1	1.00	2.15	0.00	0.00	8.45	0.00	0.00	-0.00
time (sec)	N/A	0.933	9.443	2.132	0.000	0.454	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	284	1928	0	1411	0	0	-1
N.S.	1	1.00	0.96	6.54	0.00	4.78	0.00	0.00	-0.00
time (sec)	N/A	0.507	1.871	3.107	0.000	0.419	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	104	90	115	353	0	108	413
N.S.	1	1.00	1.35	1.17	1.49	4.58	0.00	1.40	5.36
time (sec)	N/A	0.074	0.174	1.248	0.475	0.338	0.000	0.599	1.381

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.048	50.058	180.000	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	928	928	3368	0	0	15652	0	0	-1
N.S.	1	1.00	3.63	0.00	0.00	16.87	0.00	0.00	-0.00
time (sec)	N/A	1.355	25.773	1.308	0.000	0.625	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	560	560	588	2051	0	5468	0	0	-1
N.S.	1	1.00	1.05	3.66	0.00	9.76	0.00	0.00	-0.00
time (sec)	N/A	0.729	4.960	3.009	0.000	0.443	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	104	205	216	893	0	282	381
N.S.	1	1.00	0.87	1.72	1.82	7.50	0.00	2.37	3.20
time (sec)	N/A	0.097	0.137	1.420	0.484	0.442	0.000	0.449	2.204

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.048	100.169	180.000	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	8.707	2.200	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	5.601	1.902	0.000	0.000	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.029	4.393	0.509	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	78	164	157	435	0	0	199
N.S.	1	1.00	1.05	2.22	2.12	5.88	0.00	0.00	2.69
time (sec)	N/A	0.056	0.424	7.704	0.536	0.389	0.000	0.000	0.560

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	175	491	0	1590	0	0	-1
N.S.	1	1.00	0.75	2.10	0.00	6.79	0.00	0.00	-0.00
time (sec)	N/A	0.314	1.143	6.450	0.000	0.376	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	634	0	0	3516	0	0	-1
N.S.	1	1.00	1.82	0.00	0.00	10.10	0.00	0.00	-0.00
time (sec)	N/A	0.516	4.728	5.196	0.000	0.385	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	78	164	157	435	0	0	199
N.S.	1	1.00	1.05	2.22	2.12	5.88	0.00	0.00	2.69
time (sec)	N/A	0.052	0.380	6.458	0.561	0.387	0.000	0.000	0.002

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	175	491	0	1590	0	0	-1
N.S.	1	1.00	0.75	2.10	0.00	6.79	0.00	0.00	-0.00
time (sec)	N/A	0.304	0.640	6.205	0.000	0.372	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	634	0	0	3516	0	0	-1
N.S.	1	1.00	1.82	0.00	0.00	10.10	0.00	0.00	-0.00
time (sec)	N/A	0.486	0.406	0.038	0.000	0.411	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	112	308	413	1293	0	0	-1
N.S.	1	1.00	1.00	2.75	3.69	11.54	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.910	6.909	0.587	0.382	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	623	805	0	6052	0	0	-1
N.S.	1	1.00	2.04	2.63	0.00	19.78	0.00	0.00	-0.00
time (sec)	N/A	0.371	13.017	6.938	0.000	0.472	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	631	631	5753	0	0	16887	0	0	-1
N.S.	1	1.00	9.12	0.00	0.00	26.76	0.00	0.00	-0.00
time (sec)	N/A	0.776	21.714	5.243	0.000	0.531	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	112	308	413	1293	0	0	-1
N.S.	1	1.00	1.00	2.75	3.69	11.54	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.788	6.151	0.588	0.374	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	623	805	0	6052	0	0	-1
N.S.	1	1.00	2.04	2.63	0.00	19.78	0.00	0.00	-0.00
time (sec)	N/A	0.363	7.268	6.324	0.000	0.421	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	631	631	5753	0	0	16887	0	0	-1
N.S.	1	1.00	9.12	0.00	0.00	26.76	0.00	0.00	-0.00
time (sec)	N/A	0.757	7.413	0.037	0.000	0.627	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	2945	0	0	3363	0	0	-1
N.S.	1	1.00	6.57	0.00	0.00	7.51	0.00	0.00	-0.00
time (sec)	N/A	0.464	15.282	0.842	0.000	0.494	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	1301	0	0	1745	0	0	-1
N.S.	1	1.00	3.94	0.00	0.00	5.29	0.00	0.00	-0.00
time (sec)	N/A	0.389	13.171	0.895	0.000	0.383	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	206	483	0	780	0	0	-1
N.S.	1	1.00	0.97	2.28	0.00	3.68	0.00	0.00	-0.00
time (sec)	N/A	0.226	0.668	1.290	0.000	0.372	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	33	33	83	132	65	60	31
N.S.	1	1.00	0.97	0.97	2.44	3.88	1.91	1.76	0.91
time (sec)	N/A	0.042	0.032	0.426	0.265	0.341	0.432	0.447	0.074

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.043	55.433	180.000	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	696	696	2963	0	0	6807	0	0	-1
N.S.	1	1.00	4.26	0.00	0.00	9.78	0.00	0.00	-0.00
time (sec)	N/A	0.796	12.580	1.937	0.000	0.451	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	510	510	2172	0	0	3459	0	0	-1
N.S.	1	1.00	4.26	0.00	0.00	6.78	0.00	0.00	-0.00
time (sec)	N/A	0.697	8.537	1.839	0.000	0.438	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	1551	1012	0	1484	0	0	-1
N.S.	1	1.00	4.74	3.09	0.00	4.54	0.00	0.00	-0.00
time (sec)	N/A	0.391	3.136	1.575	0.000	0.421	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	109	189	160	446	0	155	212
N.S.	1	1.00	1.15	1.99	1.68	4.69	0.00	1.63	2.23
time (sec)	N/A	0.137	0.423	0.928	0.493	0.344	0.000	0.464	0.488

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.052	180.002	180.000	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	864	864	5041	0	0	15138	0	0	-1
N.S.	1	1.00	5.83	0.00	0.00	17.52	0.00	0.00	-0.00
time (sec)	N/A	0.783	16.878	1.885	0.000	0.546	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	636	636	2455	0	0	7539	0	0	-1
N.S.	1	1.00	3.86	0.00	0.00	11.85	0.00	0.00	-0.00
time (sec)	N/A	0.603	8.929	1.878	0.000	0.427	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	551	1102	0	3011	0	0	-1
N.S.	1	1.00	1.38	2.76	0.00	7.53	0.00	0.00	-0.00
time (sec)	N/A	0.348	2.146	1.628	0.000	0.393	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	90	246	183	652	0	145	180
N.S.	1	1.00	1.06	2.89	2.15	7.67	0.00	1.71	2.12
time (sec)	N/A	0.089	0.231	0.889	0.271	0.392	0.000	0.448	0.428

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.057	180.002	180.000	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1021	1021	3214	0	0	2787	0	0	-1
N.S.	1	1.00	3.15	0.00	0.00	2.73	0.00	0.00	-0.00
time (sec)	N/A	1.081	19.420	180.000	0.000	0.441	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	716	716	872	0	0	1411	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	1.97	0.00	0.00	-0.00
time (sec)	N/A	0.793	6.781	180.000	0.000	0.412	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	438	1287	0	631	0	0	-1
N.S.	1	1.00	1.04	3.06	0.00	1.50	0.00	0.00	-0.00
time (sec)	N/A	0.440	1.977	3.087	0.000	0.417	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	51	97	95	92	0	121	130
N.S.	1	1.00	0.74	1.41	1.38	1.33	0.00	1.75	1.88
time (sec)	N/A	0.059	0.061	1.185	0.472	0.343	0.000	0.445	1.270

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.032	10.894	180.000	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	917	917	1928	0	0	10811	0	0	-1
N.S.	1	1.00	2.10	0.00	0.00	11.79	0.00	0.00	-0.00
time (sec)	N/A	1.381	14.522	2.276	0.000	0.570	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	648	648	1182	0	0	4750	0	0	-1
N.S.	1	1.00	1.82	0.00	0.00	7.33	0.00	0.00	-0.00
time (sec)	N/A	1.060	9.204	2.519	0.000	0.458	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	285	1858	0	1462	0	0	-1
N.S.	1	1.00	0.85	5.55	0.00	4.36	0.00	0.00	-0.00
time (sec)	N/A	0.542	2.039	3.199	0.000	0.411	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	104	101	117	350	0	106	170
N.S.	1	1.00	1.33	1.29	1.50	4.49	0.00	1.36	2.18
time (sec)	N/A	0.084	0.161	1.187	0.476	0.368	0.000	0.461	0.596

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.040	46.131	180.000	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1176	1176	3264	0	0	16498	0	0	-1
N.S.	1	1.00	2.78	0.00	0.00	14.03	0.00	0.00	-0.00
time (sec)	N/A	1.423	25.002	1.258	0.000	0.676	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	711	711	587	2074	0	5790	0	0	-1
N.S.	1	1.00	0.83	2.92	0.00	8.14	0.00	0.00	-0.00
time (sec)	N/A	0.823	6.893	3.826	0.000	0.463	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	105	212	218	926	0	286	337
N.S.	1	1.00	0.86	1.74	1.79	7.59	0.00	2.34	2.76
time (sec)	N/A	0.143	0.158	1.316	0.487	0.370	0.000	0.451	1.933

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.054	115.445	180.000	0.000	0.000	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	606	606	2518	0	0	7049	0	0	-1
N.S.	1	1.00	4.16	0.00	0.00	11.63	0.00	0.00	-0.00
time (sec)	N/A	0.611	11.054	2.781	0.000	0.470	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	1324	0	0	3541	0	0	-1
N.S.	1	1.00	2.95	0.00	0.00	7.89	0.00	0.00	-0.00
time (sec)	N/A	0.510	3.394	2.784	0.000	0.395	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	422	565	0	1466	0	0	-1
N.S.	1	1.00	1.52	2.03	0.00	5.27	0.00	0.00	-0.00
time (sec)	N/A	0.300	0.700	3.739	0.000	0.368	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	49	119	309	87	88	46
N.S.	1	1.00	0.89	0.89	2.16	5.62	1.58	1.60	0.84
time (sec)	N/A	0.056	0.108	0.569	0.277	0.365	0.499	0.431	0.111

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.054	180.001	180.000	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	897	897	2729	0	0	13024	0	0	-1
N.S.	1	1.00	3.04	0.00	0.00	14.52	0.00	0.00	-0.00
time (sec)	N/A	1.028	9.802	2.142	0.000	0.507	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	649	649	991	0	0	6483	0	0	-1
N.S.	1	1.00	1.53	0.00	0.00	9.99	0.00	0.00	-0.00
time (sec)	N/A	0.839	4.684	2.003	0.000	0.449	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	676	1128	0	2629	0	0	-1
N.S.	1	1.00	1.68	2.80	0.00	6.52	0.00	0.00	-0.00
time (sec)	N/A	0.498	1.794	3.097	0.000	0.407	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	123	247	209	745	0	211	278
N.S.	1	1.00	0.87	1.75	1.48	5.28	0.00	1.50	1.97
time (sec)	N/A	0.346	0.285	1.161	0.482	0.380	0.000	0.434	0.614

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.084	180.000	180.000	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1123	1123	6348	0	0	24245	0	0	-1
N.S.	1	1.00	5.65	0.00	0.00	21.59	0.00	0.00	-0.00
time (sec)	N/A	1.082	16.978	2.240	0.000	0.623	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	819	819	3947	0	0	11997	0	0	-1
N.S.	1	1.00	4.82	0.00	0.00	14.65	0.00	0.00	-0.00
time (sec)	N/A	0.803	9.640	2.224	0.000	0.509	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	499	499	848	1217	0	4707	0	0	-1
N.S.	1	1.00	1.70	2.44	0.00	9.43	0.00	0.00	-0.00
time (sec)	N/A	0.471	2.291	3.119	0.000	0.460	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	104	343	234	1069	0	202	238
N.S.	1	1.00	0.92	3.04	2.07	9.46	0.00	1.79	2.11
time (sec)	N/A	0.109	0.351	1.125	0.269	0.379	0.000	0.468	0.593

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.083	180.000	180.000	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1218	1218	2891	0	0	3310	0	0	-1
N.S.	1	1.00	2.37	0.00	0.00	2.72	0.00	0.00	-0.00
time (sec)	N/A	1.302	19.927	2.658	0.000	0.518	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	861	861	1202	0	0	1694	0	0	-1
N.S.	1	1.00	1.40	0.00	0.00	1.97	0.00	0.00	-0.00
time (sec)	N/A	0.990	15.240	2.524	0.000	0.415	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	516	516	441	3882	0	750	0	0	-1
N.S.	1	1.00	0.85	7.52	0.00	1.45	0.00	0.00	-0.00
time (sec)	N/A	0.564	1.943	5.742	0.000	0.398	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	78	132	110	111	0	121	174
N.S.	1	1.00	1.05	1.78	1.49	1.50	0.00	1.64	2.35
time (sec)	N/A	0.105	0.078	1.544	0.478	0.377	0.000	0.427	1.261

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.040	145.453	180.000	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1118	1118	1614	0	0	10804	0	0	-1
N.S.	1	1.00	1.44	0.00	0.00	9.66	0.00	0.00	-0.00
time (sec)	N/A	1.621	13.554	2.059	0.000	0.561	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	772	772	1183	0	0	4757	0	0	-1
N.S.	1	1.00	1.53	0.00	0.00	6.16	0.00	0.00	-0.00
time (sec)	N/A	1.234	9.405	2.357	0.000	0.452	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	284	1928	0	1460	0	0	-1
N.S.	1	1.00	0.74	5.01	0.00	3.79	0.00	0.00	-0.00
time (sec)	N/A	0.632	1.932	5.553	0.000	0.378	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	106	103	115	351	0	108	422
N.S.	1	1.00	1.18	1.14	1.28	3.90	0.00	1.20	4.69
time (sec)	N/A	0.086	0.156	1.156	0.471	0.378	0.000	0.476	0.804

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.049	180.002	180.000	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1256	1256	2735	0	0	16156	0	0	-1
N.S.	1	1.00	2.18	0.00	0.00	12.86	0.00	0.00	-0.00
time (sec)	N/A	1.645	20.907	1.351	0.000	0.636	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	760	760	588	2068	0	5682	0	0	-1
N.S.	1	1.00	0.77	2.72	0.00	7.48	0.00	0.00	-0.00
time (sec)	N/A	0.944	6.794	4.270	0.000	0.518	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	130	209	219	917	0	281	339
N.S.	1	1.00	1.07	1.73	1.81	7.58	0.00	2.32	2.80
time (sec)	N/A	0.167	0.275	1.477	0.489	0.351	0.000	0.490	2.019

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.054	180.002	180.000	0.000	0.000	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	792	792	3901	0	0	13122	0	0	-1
N.S.	1	1.00	4.93	0.00	0.00	16.57	0.00	0.00	-0.00
time (sec)	N/A	0.845	6.470	2.315	0.000	0.537	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	578	578	1945	0	0	6483	0	0	-1
N.S.	1	1.00	3.37	0.00	0.00	11.22	0.00	0.00	-0.00
time (sec)	N/A	0.656	3.299	2.297	0.000	0.422	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	460	671	0	2579	0	0	-1
N.S.	1	1.00	1.32	1.93	0.00	7.41	0.00	0.00	-0.00
time (sec)	N/A	0.376	1.724	3.076	0.000	0.396	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	71	65	171	602	105	117	63
N.S.	1	1.00	0.93	0.86	2.25	7.92	1.38	1.54	0.83
time (sec)	N/A	0.072	0.146	0.582	0.268	0.368	0.599	0.445	0.144

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.059	180.003	180.000	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1038	1038	6428	0	0	20130	0	0	-1
N.S.	1	1.00	6.19	0.00	0.00	19.39	0.00	0.00	-0.00
time (sec)	N/A	1.297	26.351	1.910	0.000	0.642	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	755	755	4023	0	0	9951	0	0	-1
N.S.	1	1.00	5.33	0.00	0.00	13.18	0.00	0.00	-0.00
time (sec)	N/A	1.048	14.829	1.878	0.000	0.533	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	2275	1213	0	3950	0	0	-1
N.S.	1	1.00	4.80	2.56	0.00	8.33	0.00	0.00	-0.00
time (sec)	N/A	0.617	10.132	1.618	0.000	0.454	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	153	355	257	1134	0	258	330
N.S.	1	1.00	0.83	1.93	1.40	6.16	0.00	1.40	1.79
time (sec)	N/A	0.554	1.464	1.176	0.497	0.411	0.000	0.463	0.727

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.084	180.004	180.000	0.000	0.000	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1443	1443	5157	0	0	36449	0	0	-1
N.S.	1	1.00	3.57	0.00	0.00	25.26	0.00	0.00	-0.00
time (sec)	N/A	1.520	17.912	2.496	0.000	0.838	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1049	1049	1170	0	0	17953	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	17.11	0.00	0.00	-0.00
time (sec)	N/A	1.113	7.541	2.596	0.000	0.629	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	641	641	958	1363	0	6966	0	0	-1
N.S.	1	1.00	1.49	2.13	0.00	10.87	0.00	0.00	-0.00
time (sec)	N/A	0.650	2.854	3.718	0.000	0.441	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	123	421	300	1660	0	258	307
N.S.	1	1.00	0.87	2.99	2.13	11.77	0.00	1.83	2.18
time (sec)	N/A	0.154	0.297	1.163	0.278	0.433	0.000	0.454	0.754

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.086	180.003	180.000	0.000	0.000	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1519	1519	3060	0	0	8301	0	0	-1
N.S.	1	1.00	2.01	0.00	0.00	5.46	0.00	0.00	-0.00
time (sec)	N/A	1.708	18.166	5.046	0.000	0.567	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1067	1067	1948	0	0	4087	0	0	-1
N.S.	1	1.00	1.83	0.00	0.00	3.83	0.00	0.00	-0.00
time (sec)	N/A	1.238	7.826	4.865	0.000	0.435	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	631	631	481	4066	0	1646	0	0	-1
N.S.	1	1.00	0.76	6.44	0.00	2.61	0.00	0.00	-0.00
time (sec)	N/A	0.703	3.260	5.851	0.000	0.441	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	91	169	147	288	0	145	249
N.S.	1	1.00	1.02	1.90	1.65	3.24	0.00	1.63	2.80
time (sec)	N/A	0.139	0.156	1.694	0.487	0.384	0.000	0.459	1.640

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.057	180.003	180.000	0.000	0.000	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1294	1294	1632	0	0	12510	0	0	-1
N.S.	1	1.00	1.26	0.00	0.00	9.67	0.00	0.00	-0.00
time (sec)	N/A	1.892	11.727	2.289	0.000	0.639	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	904	904	1211	0	0	5647	0	0	-1
N.S.	1	1.00	1.34	0.00	0.00	6.25	0.00	0.00	-0.00
time (sec)	N/A	1.449	9.485	2.109	0.000	0.524	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	454	317	1897	0	1794	0	0	-1
N.S.	1	1.00	0.70	4.18	0.00	3.95	0.00	0.00	-0.00
time (sec)	N/A	0.714	2.813	6.385	0.000	0.472	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	96	125	141	459	0	128	468
N.S.	1	1.00	0.79	1.03	1.17	3.79	0.00	1.06	3.87
time (sec)	N/A	0.176	0.378	1.443	0.459	0.440	0.000	0.481	2.002

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.067	180.003	180.000	0.000	0.000	0.000	0.000	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1479	1479	2713	0	0	15580	0	0	-1
N.S.	1	1.00	1.83	0.00	0.00	10.53	0.00	0.00	-0.00
time (sec)	N/A	2.028	20.911	2.730	0.000	0.709	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	894	894	588	2284	0	5468	0	0	-1
N.S.	1	1.00	0.66	2.55	0.00	6.12	0.00	0.00	-0.00
time (sec)	N/A	1.110	5.472	5.715	0.000	0.527	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	152	210	217	896	0	279	381
N.S.	1	1.00	1.27	1.75	1.81	7.47	0.00	2.32	3.18
time (sec)	N/A	0.144	0.312	1.470	0.487	0.502	0.000	0.476	2.368

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.055	180.003	180.000	0.000	0.000	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	451	1002	0	0	1816	0	0	-1
N.S.	1	1.00	2.22	0.00	0.00	4.03	0.00	0.00	-0.00
time (sec)	N/A	0.541	5.216	180.000	0.000	0.417	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	492	0	0	990	0	0	-1
N.S.	1	1.00	1.51	0.00	0.00	3.05	0.00	0.00	-0.00
time (sec)	N/A	0.507	4.249	180.000	0.000	0.404	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	236	451	0	500	0	0	-1
N.S.	1	1.00	1.15	2.20	0.00	2.44	0.00	0.00	-0.00
time (sec)	N/A	0.267	0.688	3.049	0.000	0.383	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	33	75	67	0	61	254
N.S.	1	1.00	0.82	0.97	2.21	1.97	0.00	1.79	7.47
time (sec)	N/A	0.035	0.028	0.745	0.275	0.402	0.000	0.481	0.409

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.030	21.318	180.000	0.000	0.000	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	638	638	1374	0	0	2193	0	0	-1
N.S.	1	1.00	2.15	0.00	0.00	3.44	0.00	0.00	-0.00
time (sec)	N/A	0.910	4.024	3.241	0.000	0.461	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	462	462	815	0	0	1227	0	0	-1
N.S.	1	1.00	1.76	0.00	0.00	2.66	0.00	0.00	-0.00
time (sec)	N/A	0.775	3.161	3.157	0.000	0.428	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	339	970	0	635	0	0	-1
N.S.	1	1.00	1.19	3.39	0.00	2.22	0.00	0.00	-0.00
time (sec)	N/A	0.426	1.311	13.155	0.000	0.398	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	80	109	126	209	0	113	384
N.S.	1	1.00	1.13	1.54	1.77	2.94	0.00	1.59	5.41
time (sec)	N/A	0.162	0.131	2.536	0.487	0.409	0.000	0.471	0.474

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.042	42.448	180.000	0.000	0.000	0.000	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	656	656	14209	0	0	5885	0	0	-1
N.S.	1	1.00	21.66	0.00	0.00	8.97	0.00	0.00	-0.00
time (sec)	N/A	0.908	27.736	5.767	0.000	0.522	0.000	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	486	486	700	0	0	2987	0	0	-1
N.S.	1	1.00	1.44	0.00	0.00	6.15	0.00	0.00	-0.00
time (sec)	N/A	0.720	5.178	5.743	0.000	0.479	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	296	932	0	1266	0	0	-1
N.S.	1	1.00	0.92	2.89	0.00	3.93	0.00	0.00	-0.00
time (sec)	N/A	0.416	1.198	10.707	0.000	0.441	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	48	138	130	203	0	94	360
N.S.	1	1.00	0.84	2.42	2.28	3.56	0.00	1.65	6.32
time (sec)	N/A	0.090	0.066	5.353	0.268	0.445	0.000	0.473	0.494

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.056	180.001	180.000	0.000	0.000	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1049	1049	9140	0	0	4128	0	0	-1
N.S.	1	1.00	8.71	0.00	0.00	3.94	0.00	0.00	-0.00
time (sec)	N/A	1.109	23.283	180.000	0.000	0.488	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	734	734	2950	0	0	2074	0	0	-1
N.S.	1	1.00	4.02	0.00	0.00	2.83	0.00	0.00	-0.00
time (sec)	N/A	0.842	22.466	180.000	0.000	0.472	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	439	1541	1065	0	888	0	0	-1
N.S.	1	1.00	3.51	2.43	0.00	2.02	0.00	0.00	-0.00
time (sec)	N/A	0.477	1.718	3.162	0.000	0.418	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	92	108	138	134	0	147	-1
N.S.	1	1.00	1.02	1.20	1.53	1.49	0.00	1.63	-0.01
time (sec)	N/A	0.125	0.133	1.337	0.467	0.418	0.000	0.470	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.043	19.710	180.000	0.000	0.000	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1164	1164	1991	0	0	16781	0	0	-1
N.S.	1	1.00	1.71	0.00	0.00	14.42	0.00	0.00	-0.00
time (sec)	N/A	1.761	16.315	2.719	0.000	0.721	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	795	795	1592	0	0	7596	0	0	-1
N.S.	1	1.00	2.00	0.00	0.00	9.55	0.00	0.00	-0.00
time (sec)	N/A	1.282	11.563	2.622	0.000	0.509	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	442	442	459	1815	0	2461	0	0	-1
N.S.	1	1.00	1.04	4.11	0.00	5.57	0.00	0.00	-0.00
time (sec)	N/A	0.639	4.373	5.826	0.000	0.442	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	171	106	168	581	0	146	668
N.S.	1	1.00	1.51	0.94	1.49	5.14	0.00	1.29	5.91
time (sec)	N/A	0.191	0.206	1.640	0.485	0.499	0.000	0.427	4.817

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.059	172.008	180.000	0.000	0.000	0.000	0.000	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1185	1185	3310	0	0	25021	0	0	-1
N.S.	1	1.00	2.79	0.00	0.00	21.11	0.00	0.00	-0.00
time (sec)	N/A	1.731	24.800	3.021	0.000	0.810	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	746	746	886	2580	0	8973	0	0	-1
N.S.	1	1.00	1.19	3.46	0.00	12.03	0.00	0.00	-0.00
time (sec)	N/A	0.877	9.061	8.084	0.000	0.624	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	196	230	265	1279	0	343	-1
N.S.	1	1.00	1.22	1.44	1.66	7.99	0.00	2.14	-0.01
time (sec)	N/A	0.186	0.595	1.756	0.490	0.562	0.000	0.460	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.056	123.538	180.000	0.000	0.000	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	601	601	7478	0	0	7283	0	0	-1
N.S.	1	1.00	12.44	0.00	0.00	12.12	0.00	0.00	-0.00
time (sec)	N/A	0.702	16.224	2.441	0.000	0.480	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	783	0	0	3408	0	0	-1
N.S.	1	1.00	1.87	0.00	0.00	8.13	0.00	0.00	-0.00
time (sec)	N/A	0.560	9.416	2.125	0.000	0.461	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	416	528	0	1339	0	0	-1
N.S.	1	1.00	1.71	2.17	0.00	5.51	0.00	0.00	-0.00
time (sec)	N/A	0.325	1.340	3.187	0.000	0.404	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	34	110	211	0	110	409
N.S.	1	1.00	1.00	0.68	2.20	4.22	0.00	2.20	8.18
time (sec)	N/A	0.052	0.042	0.756	0.270	0.418	0.000	0.431	0.873

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.043	140.656	180.000	0.000	0.000	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	721	721	4761	0	0	7711	0	0	-1
N.S.	1	1.00	6.60	0.00	0.00	10.69	0.00	0.00	-0.00
time (sec)	N/A	1.173	11.064	2.357	0.000	0.517	0.000	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	517	972	0	0	3633	0	0	-1
N.S.	1	1.00	1.88	0.00	0.00	7.03	0.00	0.00	-0.00
time (sec)	N/A	0.918	8.282	2.125	0.000	0.463	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	364	1017	0	1447	0	0	-1
N.S.	1	1.00	1.24	3.46	0.00	4.92	0.00	0.00	-0.00
time (sec)	N/A	0.486	2.610	5.404	0.000	0.407	0.000	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	98	105	134	360	0	120	380
N.S.	1	1.00	1.27	1.36	1.74	4.68	0.00	1.56	4.94
time (sec)	N/A	0.189	0.369	1.418	0.476	0.371	0.000	0.468	0.635

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.051	142.651	180.000	0.000	0.000	0.000	0.000	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	718	718	13888	0	0	10474	0	0	-1
N.S.	1	1.00	19.34	0.00	0.00	14.59	0.00	0.00	-0.00
time (sec)	N/A	1.164	10.195	3.014	0.000	0.560	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	518	518	978	0	0	5069	0	0	-1
N.S.	1	1.00	1.89	0.00	0.00	9.79	0.00	0.00	-0.00
time (sec)	N/A	0.888	10.128	1.579	0.000	0.453	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	313	938	0	2006	0	0	-1
N.S.	1	1.00	0.97	2.90	0.00	6.19	0.00	0.00	-0.00
time (sec)	N/A	0.519	1.775	13.298	0.000	0.445	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	52	135	131	299	0	121	356
N.S.	1	1.00	0.88	2.29	2.22	5.07	0.00	2.05	6.03
time (sec)	N/A	0.084	0.080	2.409	0.265	0.394	0.000	0.452	0.468

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.053	180.004	180.000	0.000	0.000	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1428	1428	8578	0	0	17737	0	0	-1
N.S.	1	1.00	6.01	0.00	0.00	12.42	0.00	0.00	-0.00
time (sec)	N/A	1.681	10.265	2.510	0.000	0.715	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	982	982	1467	0	0	8198	0	0	-1
N.S.	1	1.00	1.49	0.00	0.00	8.35	0.00	0.00	-0.00
time (sec)	N/A	1.264	9.856	2.245	0.000	0.574	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	591	591	591	1529	0	2955	0	0	-1
N.S.	1	1.00	1.00	2.59	0.00	5.00	0.00	0.00	-0.00
time (sec)	N/A	0.624	5.379	5.835	0.000	0.516	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	160	140	173	441	0	200	142
N.S.	1	1.00	1.54	1.35	1.66	4.24	0.00	1.92	1.37
time (sec)	N/A	0.120	0.494	1.541	0.485	0.433	0.000	0.463	2.869

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.058	56.516	180.000	0.000	0.000	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	914	914	3086	0	0	14354	0	0	-1
N.S.	1	1.00	3.38	0.00	0.00	15.70	0.00	0.00	-0.00
time (sec)	N/A	1.610	25.871	2.780	0.000	0.688	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	499	499	1862	1771	0	4647	0	0	-1
N.S.	1	1.00	3.73	3.55	0.00	9.31	0.00	0.00	-0.00
time (sec)	N/A	0.796	7.967	5.936	0.000	0.448	0.000	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	135	139	208	1040	0	185	768
N.S.	1	1.00	0.94	0.97	1.44	7.22	0.00	1.28	5.33
time (sec)	N/A	0.226	2.019	1.528	0.487	0.499	0.000	0.484	5.232

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.084	140.357	180.000	0.000	0.000	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	978	978	1337	3280	0	17940	0	0	-1
N.S.	1	1.00	1.37	3.35	0.00	18.34	0.00	0.00	-0.00
time (sec)	N/A	1.148	9.220	5.916	0.000	0.688	0.000	0.000	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	227	249	350	2568	0	458	398
N.S.	1	1.00	1.26	1.38	1.94	14.27	0.00	2.54	2.21
time (sec)	N/A	0.182	0.818	1.682	0.508	0.666	0.000	0.486	7.495

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.082	180.002	180.000	0.000	0.000	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	752	752	7955	0	0	20211	0	0	-1
N.S.	1	1.00	10.58	0.00	0.00	26.88	0.00	0.00	-0.00
time (sec)	N/A	0.921	48.618	2.743	0.000	0.557	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	502	502	1161	0	0	9084	0	0	-1
N.S.	1	1.00	2.31	0.00	0.00	18.10	0.00	0.00	-0.00
time (sec)	N/A	0.688	22.092	2.469	0.000	0.433	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	376	649	0	3272	0	0	-1
N.S.	1	1.00	1.26	2.18	0.00	10.98	0.00	0.00	-0.00
time (sec)	N/A	0.387	5.281	5.123	0.000	0.375	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	65	161	545	0	145	470
N.S.	1	1.00	0.83	0.90	2.24	7.57	0.00	2.01	6.53
time (sec)	N/A	0.071	0.088	0.932	0.276	0.336	0.000	0.451	0.996

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.055	180.001	180.000	0.000	0.000	0.000	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1038	1038	5829	0	0	24038	0	0	-1
N.S.	1	1.00	5.62	0.00	0.00	23.16	0.00	0.00	-0.00
time (sec)	N/A	1.520	39.987	3.341	0.000	0.622	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	714	714	1803	0	0	11068	0	0	-1
N.S.	1	1.00	2.53	0.00	0.00	15.50	0.00	0.00	-0.00
time (sec)	N/A	1.199	21.674	3.353	0.000	0.450	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	734	1284	0	4079	0	0	-1
N.S.	1	1.00	1.78	3.11	0.00	9.88	0.00	0.00	-0.00
time (sec)	N/A	0.649	7.358	11.734	0.000	0.401	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	145	140	217	892	0	182	628
N.S.	1	1.00	1.31	1.26	1.95	8.04	0.00	1.64	5.66
time (sec)	N/A	0.373	1.016	2.624	0.483	0.376	0.000	0.454	0.691

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.057	180.002	180.000	0.000	0.000	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	972	972	14876	0	0	24387	0	0	-1
N.S.	1	1.00	15.30	0.00	0.00	25.09	0.00	0.00	-0.00
time (sec)	N/A	1.572	55.318	3.122	0.000	0.658	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	689	689	1748	0	0	11172	0	0	-1
N.S.	1	1.00	2.54	0.00	0.00	16.21	0.00	0.00	-0.00
time (sec)	N/A	1.186	30.367	3.041	0.000	0.459	0.000	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	435	455	1098	0	4064	0	0	-1
N.S.	1	1.00	1.05	2.52	0.00	9.34	0.00	0.00	-0.00
time (sec)	N/A	0.663	3.399	8.258	0.000	0.386	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	64	143	173	617	0	184	1329
N.S.	1	1.00	0.80	1.79	2.16	7.71	0.00	2.30	16.61
time (sec)	N/A	0.077	0.120	2.151	0.282	0.352	0.000	0.476	1.013

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.050	180.002	180.000	0.000	0.000	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1795	1795	10559	0	0	43440	0	0	-1
N.S.	1	1.00	5.88	0.00	0.00	24.20	0.00	0.00	-0.00
time (sec)	N/A	2.421	62.923	2.481	0.000	1.110	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1219	1219	2337	0	0	19402	0	0	-1
N.S.	1	1.00	1.92	0.00	0.00	15.92	0.00	0.00	-0.00
time (sec)	N/A	1.730	30.439	2.986	0.000	0.652	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	762	762	913	1478	0	6634	0	0	-1
N.S.	1	1.00	1.20	1.94	0.00	8.71	0.00	0.00	-0.00
time (sec)	N/A	0.821	7.182	8.661	0.000	0.482	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	164	186	236	1035	0	263	196
N.S.	1	1.00	1.26	1.43	1.82	7.96	0.00	2.02	1.51
time (sec)	N/A	0.164	0.308	1.772	0.483	0.452	0.000	0.501	3.612

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.059	118.322	180.000	0.000	0.000	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1245	1245	3762	0	0	41729	0	0	-1
N.S.	1	1.00	3.02	0.00	0.00	33.52	0.00	0.00	-0.00
time (sec)	N/A	2.590	21.601	2.918	0.000	0.977	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	699	699	863	2767	0	12895	0	0	-1
N.S.	1	1.00	1.23	3.96	0.00	18.45	0.00	0.00	-0.00
time (sec)	N/A	1.023	9.431	11.232	0.000	0.491	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	185	183	334	2653	0	224	531
N.S.	1	1.00	0.90	0.89	1.62	12.88	0.00	1.09	2.58
time (sec)	N/A	0.288	1.757	2.787	0.494	0.567	0.000	0.434	3.470

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.082	138.796	180.000	0.000	0.000	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1122	1122	2870	3563	0	19958	0	0	-1
N.S.	1	1.00	2.56	3.18	0.00	17.79	0.00	0.00	-0.00
time (sec)	N/A	1.372	8.529	11.298	0.000	0.638	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	237	292	418	3148	0	464	554
N.S.	1	1.00	1.12	1.38	1.98	14.92	0.00	2.20	2.63
time (sec)	N/A	0.262	0.616	2.020	0.499	0.718	0.000	0.448	6.913

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	F(-1)	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.086	180.017	180.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [367] had the largest ratio of [36]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	2	1.00	14	0.143
2	A	4	2	1.00	14	0.143
3	A	3	2	1.00	14	0.143
4	A	2	2	1.00	12	0.167
5	A	3	3	1.00	14	0.214
6	A	4	4	1.00	14	0.286
7	A	5	4	1.00	14	0.286
8	A	6	4	1.00	16	0.250
9	A	4	3	1.00	16	0.188
10	A	4	4	1.00	16	0.250
11	A	2	1	1.00	14	0.071
12	A	5	4	1.00	16	0.250
13	A	5	5	1.00	16	0.312
14	A	7	6	1.00	16	0.375
15	A	7	7	1.00	16	0.438
16	A	12	4	1.00	16	0.250
17	A	8	4	1.00	16	0.250
18	A	6	4	1.00	16	0.250
19	A	3	3	1.00	14	0.214
20	A	8	4	1.00	16	0.250
21	A	8	4	1.00	16	0.250
22	A	12	5	1.00	16	0.312
23	A	9	5	1.00	14	0.357
24	A	7	4	1.00	14	0.286
25	A	5	3	1.00	12	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	0	0	0.00	0	0.000
27	A	0	0	0.00	0	0.000
28	A	6	6	1.00	16	0.375
29	A	5	5	1.00	16	0.312
30	A	2	2	1.00	14	0.143
31	A	0	0	0.00	0	0.000
32	A	0	0	0.00	0	0.000
33	A	15	8	1.00	16	0.500
34	A	9	6	1.00	16	0.375
35	A	6	4	1.00	14	0.286
36	A	0	0	0.00	0	0.000
37	A	0	0	0.00	0	0.000
38	A	8	5	1.00	16	0.312
39	A	7	5	1.00	16	0.312
40	A	6	5	1.00	16	0.312
41	A	5	4	1.00	16	0.250
42	A	6	5	1.00	16	0.312
43	A	7	5	1.00	16	0.312
44	A	8	5	1.00	16	0.312
45	A	10	8	1.00	18	0.444
46	A	9	7	1.00	18	0.389
47	A	8	6	1.00	18	0.333
48	A	7	5	1.00	18	0.278
49	A	7	6	1.00	18	0.333
50	A	9	7	1.00	18	0.389
51	A	9	8	1.00	18	0.444
52	A	11	7	1.00	18	0.389
53	A	23	7	1.00	18	0.389
54	A	20	7	1.00	18	0.389
55	A	14	6	1.00	18	0.333
56	A	12	5	1.00	18	0.278
57	A	12	5	1.00	18	0.278
58	A	18	6	1.00	18	0.333
59	A	19	7	1.00	18	0.389
60	A	7	5	1.00	12	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	6	5	1.00	12	0.417
62	A	5	4	1.00	12	0.333
63	A	6	5	1.00	12	0.417
64	A	7	5	1.00	12	0.417
65	A	0	0	0.00	0	0.000
66	A	0	0	0.00	0	0.000
67	A	0	0	0.00	0	0.000
68	A	2	1	1.00	18	0.056
69	A	2	1	1.00	20	0.050
70	A	3	1	1.00	20	0.050
71	A	4	3	1.00	22	0.136
72	A	0	0	0.00	0	0.000
73	A	8	3	1.00	16	0.188
74	A	5	3	1.00	16	0.188
75	A	3	2	1.00	14	0.143
76	A	0	0	0.00	0	0.000
77	A	0	0	0.00	0	0.000
78	A	3	2	1.00	12	0.167
79	A	3	2	1.00	12	0.167
80	A	3	2	1.00	12	0.167
81	A	3	2	1.00	10	0.200
82	A	3	2	1.00	12	0.167
83	A	3	2	1.00	12	0.167
84	A	3	2	1.00	12	0.167
85	A	5	3	1.00	14	0.214
86	A	5	3	1.00	14	0.214
87	A	5	3	1.00	14	0.214
88	A	5	3	1.00	12	0.250
89	A	5	3	1.00	14	0.214
90	A	5	3	1.00	14	0.214
91	A	5	3	1.00	14	0.214
92	A	4	2	1.00	20	0.100
93	A	4	2	1.00	20	0.100
94	A	5	2	1.00	20	0.100
95	A	7	5	1.00	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	6	3	1.00	21	0.143
97	A	5	3	1.00	21	0.143
98	A	4	3	1.00	19	0.158
99	A	5	4	1.00	21	0.190
100	A	6	5	1.00	21	0.238
101	A	7	5	1.00	21	0.238
102	A	10	6	1.00	23	0.261
103	A	9	7	1.00	23	0.304
104	A	6	4	1.00	21	0.190
105	A	9	5	1.00	23	0.217
106	A	9	5	1.00	23	0.217
107	A	15	6	1.00	23	0.261
108	A	7	7	1.00	23	0.304
109	A	6	6	1.00	23	0.261
110	A	3	3	1.00	21	0.143
111	A	0	0	0.00	0	0.000
112	A	0	0	0.00	0	0.000
113	A	10	9	1.00	23	0.391
114	A	9	9	1.00	23	0.391
115	A	4	4	1.00	21	0.190
116	A	0	0	0.00	0	0.000
117	A	0	0	0.00	0	0.000
118	A	6	3	1.00	21	0.143
119	A	5	3	1.00	21	0.143
120	A	4	3	1.00	21	0.143
121	A	3	3	1.00	19	0.158
122	A	4	4	1.00	21	0.190
123	A	5	5	1.00	21	0.238
124	A	6	5	1.00	21	0.238
125	A	9	5	1.00	21	0.238
126	A	7	5	1.00	21	0.238
127	A	4	4	1.00	19	0.210
128	A	9	5	1.00	21	0.238
129	A	9	5	1.00	21	0.238
130	A	14	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	10	5	1.00	21	0.238
132	A	5	4	1.00	19	0.210
133	A	12	5	1.00	21	0.238
134	A	12	5	1.00	21	0.238
135	A	21	6	1.00	21	0.286
136	A	10	6	1.00	21	0.286
137	A	8	5	1.00	21	0.238
138	A	6	4	1.00	19	0.210
139	A	0	0	0.00	0	0.000
140	A	0	0	0.00	0	0.000
141	A	16	9	1.00	21	0.429
142	A	10	7	1.00	21	0.333
143	A	7	5	1.00	19	0.263
144	A	0	0	0.00	0	0.000
145	A	0	0	0.00	0	0.000
146	A	23	10	1.00	21	0.476
147	A	13	8	1.00	21	0.381
148	A	8	5	1.00	19	0.263
149	A	0	0	0.00	0	0.000
150	A	0	0	0.00	0	0.000
151	A	0	0	0.00	0	0.000
152	A	12	5	1.00	23	0.217
153	A	9	5	1.00	23	0.217
154	A	5	3	1.00	21	0.143
155	A	0	0	0.00	0	0.000
156	A	0	0	0.00	0	0.000
157	A	6	3	1.00	18	0.167
158	A	5	3	1.00	18	0.167
159	A	4	3	1.00	16	0.188
160	A	5	4	1.00	18	0.222
161	A	6	5	1.00	18	0.278
162	A	7	5	1.00	18	0.278
163	A	10	6	1.00	20	0.300
164	A	9	7	1.00	20	0.350
165	A	6	4	1.00	18	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	10	5	1.00	20	0.250
167	A	11	7	1.00	20	0.350
168	A	14	8	1.00	20	0.400
169	A	12	7	1.00	20	0.350
170	A	10	6	1.00	20	0.300
171	A	8	5	1.00	18	0.278
172	A	0	0	0.00	0	0.000
173	A	0	0	0.00	0	0.000
174	A	18	10	1.00	20	0.500
175	A	11	8	1.00	18	0.444
176	A	0	0	0.00	0	0.000
177	A	0	0	0.00	0	0.000
178	A	35	11	1.00	18	0.611
179	A	0	0	0.00	0	0.000
180	A	0	0	0.00	0	0.000
181	A	0	0	0.00	0	0.000
182	A	18	5	1.00	20	0.250
183	A	10	5	1.00	20	0.250
184	A	5	3	1.00	18	0.167
185	A	0	0	0.00	0	0.000
186	A	0	0	0.00	0	0.000
187	A	9	9	1.00	29	0.310
188	A	8	8	1.00	29	0.276
189	A	5	4	1.00	27	0.148
190	A	2	2	1.00	22	0.091
191	A	0	0	0.00	0	0.000
192	A	0	0	0.00	0	0.000
193	A	14	11	1.00	31	0.355
194	A	12	10	1.00	31	0.323
195	A	8	6	1.00	29	0.207
196	A	4	4	1.00	24	0.167
197	A	0	0	0.00	0	0.000
198	A	0	0	0.00	0	0.000
199	A	19	13	1.00	31	0.419
200	A	17	13	1.00	31	0.419

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	11	7	1.00	29	0.241
202	A	2	2	1.00	24	0.083
203	A	0	0	0.00	0	0.000
204	A	0	0	0.00	0	0.000
205	A	17	10	1.00	29	0.345
206	A	14	11	1.00	29	0.379
207	A	9	7	1.00	27	0.259
208	A	3	3	1.00	22	0.136
209	A	0	0	0.00	0	0.000
210	A	0	0	0.00	0	0.000
211	A	24	10	1.00	31	0.323
212	A	20	11	1.00	31	0.355
213	A	12	7	1.00	29	0.241
214	A	5	5	1.00	24	0.208
215	A	0	0	0.00	0	0.000
216	A	0	0	0.00	0	0.000
217	A	40	13	1.00	31	0.419
218	A	30	13	1.00	31	0.419
219	A	19	8	1.00	29	0.276
220	A	6	6	1.00	24	0.250
221	A	0	0	0.00	0	0.000
222	A	0	0	0.00	0	0.000
223	A	14	9	1.00	26	0.346
224	A	12	8	1.00	26	0.308
225	A	10	6	1.00	24	0.250
226	A	4	4	1.00	19	0.210
227	A	0	0	0.00	0	0.000
228	A	19	11	1.00	28	0.393
229	A	16	10	1.00	28	0.357
230	A	13	8	1.00	26	0.308
231	A	6	6	1.00	21	0.286
232	A	0	0	0.00	0	0.000
233	A	24	13	1.00	28	0.464
234	A	21	13	1.00	28	0.464
235	A	16	9	1.00	26	0.346

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	6	6	1.00	21	0.286
237	A	0	0	0.00	0	0.000
238	A	22	9	1.00	26	0.346
239	A	18	8	1.00	26	0.308
240	A	14	7	1.00	24	0.292
241	A	5	5	1.00	19	0.263
242	A	0	0	0.00	0	0.000
243	A	29	11	1.00	28	0.393
244	A	24	12	1.00	28	0.429
245	A	17	9	1.00	26	0.346
246	A	7	7	1.00	21	0.333
247	A	0	0	0.00	0	0.000
248	A	45	14	1.00	28	0.500
249	A	34	14	1.00	28	0.500
250	A	24	10	1.00	26	0.385
251	A	7	7	1.00	21	0.333
252	A	0	0	0.00	0	0.000
253	A	6	6	1.00	29	0.207
254	A	5	5	1.00	29	0.172
255	A	4	4	1.00	27	0.148
256	A	2	2	1.00	22	0.091
257	A	0	0	0.00	0	0.000
258	A	0	0	0.00	0	0.000
259	A	6	4	1.00	31	0.129
260	A	5	4	1.00	31	0.129
261	A	4	3	1.00	29	0.103
262	A	2	2	1.00	24	0.083
263	A	5	5	1.00	31	0.161
264	A	6	6	1.00	31	0.194
265	A	10	8	1.00	31	0.258
266	A	7	5	1.00	31	0.161
267	A	6	6	1.00	29	0.207
268	A	2	1	1.00	24	0.042
269	A	9	6	1.00	31	0.194
270	A	11	7	1.00	31	0.226

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	22	13	1.00	29	0.448
272	A	13	10	1.00	29	0.345
273	A	10	8	1.00	27	0.296
274	A	4	3	1.00	22	0.136
275	A	0	0	0.00	0	0.000
276	A	0	0	0.00	0	0.000
277	A	20	12	1.00	31	0.387
278	A	16	12	1.00	31	0.387
279	A	7	7	1.00	29	0.241
280	A	3	3	1.00	24	0.125
281	A	0	0	0.00	0	0.000
282	A	0	0	0.00	0	0.000
283	A	32	16	1.00	31	0.516
284	A	17	12	1.00	31	0.387
285	A	11	7	1.00	29	0.241
286	A	4	3	1.00	24	0.125
287	A	0	0	0.00	0	0.000
288	A	0	0	0.00	0	0.000
289	A	11	6	1.00	26	0.231
290	A	9	5	1.00	26	0.192
291	A	7	4	1.00	24	0.167
292	A	2	2	1.00	19	0.105
293	A	0	0	0.00	0	0.000
294	A	18	11	1.00	28	0.393
295	A	15	10	1.00	28	0.357
296	A	12	8	1.00	26	0.308
297	A	5	5	1.00	21	0.238
298	A	0	0	0.00	0	0.000
299	A	21	14	1.00	28	0.500
300	A	16	10	1.00	28	0.357
301	A	13	10	1.00	26	0.385
302	A	3	2	1.00	21	0.095
303	A	0	0	0.00	0	0.000
304	A	29	10	1.00	26	0.385
305	A	24	9	1.00	26	0.346

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	19	8	1.00	24	0.333
307	A	6	6	1.00	19	0.316
308	A	0	0	0.00	0	0.000
309	A	29	13	1.00	28	0.464
310	A	24	14	1.00	28	0.500
311	A	15	11	1.00	26	0.423
312	A	5	5	1.00	21	0.238
313	A	0	0	0.00	0	0.000
314	A	39	14	1.00	28	0.500
315	A	31	12	1.00	26	0.462
316	A	7	6	1.00	21	0.286
317	A	0	0	0.00	0	0.000
318	A	0	0	0.00	0	0.000
319	A	0	0	0.00	0	0.000
320	A	0	0	0.00	0	0.000
321	A	4	4	1.00	24	0.167
322	A	9	6	1.00	26	0.231
323	A	11	7	1.00	26	0.269
324	A	4	4	1.00	24	0.167
325	A	9	6	1.00	26	0.231
326	A	11	7	1.00	26	0.269
327	A	6	6	1.00	24	0.250
328	A	12	9	1.00	26	0.346
329	A	19	11	1.00	26	0.423
330	A	6	6	1.00	24	0.250
331	A	12	9	1.00	26	0.346
332	A	19	11	1.00	26	0.423
333	A	16	9	1.00	32	0.281
334	A	13	8	1.00	32	0.250
335	A	10	7	1.00	30	0.233
336	A	4	3	1.00	25	0.120
337	A	0	0	0.00	0	0.000
338	A	23	14	1.00	34	0.412
339	A	20	14	1.00	34	0.412
340	A	15	10	1.00	32	0.312

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	5	5	1.00	27	0.185
342	A	0	0	0.00	0	0.000
343	A	30	16	1.00	34	0.471
344	A	23	13	1.00	34	0.382
345	A	17	12	1.00	32	0.375
346	A	4	3	1.00	27	0.111
347	A	0	0	0.00	0	0.000
348	A	39	11	1.00	26	0.423
349	A	32	10	1.00	26	0.385
350	A	25	9	1.00	24	0.375
351	A	6	5	1.00	19	0.263
352	A	0	0	0.00	0	0.000
353	A	36	14	1.00	32	0.438
354	A	30	15	1.00	32	0.469
355	A	18	12	1.00	30	0.400
356	A	5	5	1.00	25	0.200
357	A	0	0	0.00	0	0.000
358	A	49	15	1.00	34	0.441
359	A	38	13	1.00	32	0.406
360	A	8	7	1.00	27	0.259
361	A	0	0	0.00	0	0.000
362	A	22	14	1.00	34	0.412
363	A	17	10	1.00	34	0.294
364	A	14	10	1.00	32	0.312
365	A	4	3	1.00	27	0.111
366	A	0	0	0.00	0	0.000
367	A	31	16	1.00	36	0.444
368	A	25	16	1.00	36	0.444
369	A	19	12	1.00	34	0.353
370	A	8	8	1.00	29	0.276
371	A	0	0	0.00	0	0.000
372	A	40	17	1.00	36	0.472
373	A	28	14	1.00	36	0.389
374	A	22	13	1.00	34	0.382
375	A	4	3	1.00	29	0.103

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	0	0	0.00	0	0.000
377	A	46	12	1.00	32	0.375
378	A	38	11	1.00	32	0.344
379	A	30	10	1.00	30	0.333
380	A	7	6	1.00	25	0.240
381	A	0	0	0.00	0	0.000
382	A	45	15	1.00	28	0.536
383	A	37	16	1.00	28	0.571
384	A	21	13	1.00	26	0.500
385	A	8	7	1.00	21	0.333
386	A	0	0	0.00	0	0.000
387	A	53	15	1.00	34	0.441
388	A	42	13	1.00	32	0.406
389	A	8	7	1.00	27	0.259
390	A	0	0	0.00	0	0.000
391	A	30	15	1.00	34	0.441
392	A	22	10	1.00	34	0.294
393	A	18	11	1.00	32	0.344
394	A	4	3	1.00	27	0.111
395	A	0	0	0.00	0	0.000
396	A	38	18	1.00	36	0.500
397	A	31	18	1.00	36	0.500
398	A	24	14	1.00	34	0.412
399	A	9	8	1.00	29	0.276
400	A	0	0	0.00	0	0.000
401	A	55	18	1.00	36	0.500
402	A	40	15	1.00	36	0.417
403	A	31	14	1.00	34	0.412
404	A	4	3	1.00	29	0.103
405	A	0	0	0.00	0	0.000
406	A	61	15	1.00	34	0.441
407	A	50	14	1.00	34	0.412
408	A	39	13	1.00	32	0.406
409	A	7	6	1.00	27	0.222
410	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	53	18	1.00	34	0.529
412	A	44	19	1.00	34	0.559
413	A	25	15	1.00	32	0.469
414	A	9	8	1.00	27	0.296
415	A	0	0	0.00	0	0.000
416	A	71	17	1.00	28	0.607
417	A	55	15	1.00	26	0.577
418	A	7	6	1.00	21	0.286
419	A	0	0	0.00	0	0.000
420	A	18	8	1.00	26	0.308
421	A	15	7	1.00	26	0.269
422	A	12	6	1.00	24	0.250
423	A	4	4	1.00	19	0.210
424	A	0	0	0.00	0	0.000
425	A	33	14	1.00	32	0.438
426	A	27	13	1.00	32	0.406
427	A	21	11	1.00	30	0.367
428	A	6	6	1.00	25	0.240
429	A	0	0	0.00	0	0.000
430	A	34	17	1.00	34	0.500
431	A	26	13	1.00	34	0.382
432	A	22	13	1.00	32	0.406
433	A	4	3	1.00	27	0.111
434	A	0	0	0.00	0	0.000
435	A	40	13	1.00	32	0.406
436	A	33	12	1.00	32	0.375
437	A	26	11	1.00	30	0.367
438	A	7	6	1.00	25	0.240
439	A	0	0	0.00	0	0.000
440	A	53	22	1.00	34	0.647
441	A	44	23	1.00	34	0.676
442	A	26	19	1.00	32	0.594
443	A	10	9	1.00	27	0.333
444	A	0	0	0.00	0	0.000
445	A	57	23	1.00	34	0.676

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	43	20	1.00	32	0.625
447	A	9	7	1.00	27	0.259
448	A	0	0	0.00	0	0.000
449	A	27	11	1.00	32	0.344
450	A	22	12	1.00	32	0.375
451	A	15	9	1.00	30	0.300
452	A	4	3	1.00	25	0.120
453	A	0	0	0.00	0	0.000
454	A	41	17	1.00	28	0.607
455	A	34	18	1.00	28	0.643
456	A	25	14	1.00	26	0.538
457	A	7	7	1.00	21	0.333
458	A	0	0	0.00	0	0.000
459	A	48	19	1.00	34	0.559
460	A	37	17	1.00	34	0.500
461	A	28	15	1.00	32	0.469
462	A	4	3	1.00	27	0.111
463	A	0	0	0.00	0	0.000
464	A	64	20	1.00	34	0.588
465	A	53	21	1.00	34	0.618
466	A	37	18	1.00	32	0.562
467	A	7	6	1.00	27	0.222
468	A	0	0	0.00	0	0.000
469	A	51	25	1.00	36	0.694
470	A	30	20	1.00	34	0.588
471	A	13	11	1.00	29	0.379
472	A	0	0	0.00	0	0.000
473	A	57	27	1.00	34	0.794
474	A	9	7	1.00	29	0.241
475	A	0	0	0.00	0	0.000
476	A	34	14	1.00	34	0.412
477	A	26	14	1.00	34	0.412
478	A	19	11	1.00	32	0.344
479	A	4	3	1.00	27	0.111
480	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	67	22	1.00	34	0.647
482	A	52	22	1.00	34	0.647
483	A	38	17	1.00	32	0.531
484	A	8	8	1.00	27	0.296
485	A	0	0	0.00	0	0.000
486	A	62	23	1.00	28	0.821
487	A	47	20	1.00	28	0.714
488	A	36	18	1.00	26	0.692
489	A	3	2	1.00	21	0.095
490	A	0	0	0.00	0	0.000
491	A	87	28	1.00	34	0.824
492	A	71	26	1.00	34	0.765
493	A	49	23	1.00	32	0.719
494	A	7	6	1.00	27	0.222
495	A	0	0	0.00	0	0.000
496	A	88	33	1.00	36	0.917
497	A	44	22	1.00	34	0.647
498	A	17	12	1.00	29	0.414
499	A	0	0	0.00	0	0.000
500	A	65	28	1.00	34	0.824
501	A	9	7	1.00	29	0.241
502	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int (c+dx)^4 \sinh(a+bx)\, dx$

Optimal. Leaf size=91

$$\frac{24d^4\cosh(a+bx)}{b^5}+\frac{12d^2(c+dx)^2\cosh(a+bx)}{b^3}+\frac{(c+dx)^4\cosh(a+bx)}{b}-\frac{24d^3(c+dx)\sinh(a+bx)}{b^4}-\frac{4d(c$$

[Out] 24*d^4*cosh(b*x+a)/b^5+12*d^2*(d*x+c)^2*cosh(b*x+a)/b^3+(d*x+c)^4*cosh(b*x+a)/b-24*d^3*(d*x+c)*sinh(b*x+a)/b^4-4*d*(d*x+c)^3*sinh(b*x+a)/b^2

Rubi [A]
time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3377, 2718}

$$\frac{24d^4\cosh(a+bx)}{b^5}-\frac{24d^3(c+dx)\sinh(a+bx)}{b^4}+\frac{12d^2(c+dx)^2\cosh(a+bx)}{b^3}-\frac{4d(c+dx)^3\sinh(a+bx)}{b^2}+\frac{(c+dx)^4\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sinh[a + b*x],x]
```

```
[Out] (24*d^4*Cosh[a + b*x])/b^5 + (12*d^2*(c + d*x)^2*Cosh[a + b*x])/b^3 + ((c + d*x)^4*Cosh[a + b*x])/b - (24*d^3*(c + d*x)*Sinh[a + b*x])/b^4 - (4*d*(c + d*x)^3*Sinh[a + b*x])/b^2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^4\sinh(a+bx)\,dx &= \frac{(c+dx)^4\cosh(a+bx)}{b}-\frac{(4d)\int (c+dx)^3\cosh(a+bx)\,dx}{b}\\
&= \frac{(c+dx)^4\cosh(a+bx)}{b}-\frac{4d(c+dx)^3\sinh(a+bx)}{b^2}+\frac{(12d^2)\int (c+dx)^2\sinh}{b^2}\\
&= \frac{12d^2(c+dx)^2\cosh(a+bx)}{b^3}+\frac{(c+dx)^4\cosh(a+bx)}{b}-\frac{4d(c+dx)^3\sinh(a+}{b^2}\\
&= \frac{12d^2(c+dx)^2\cosh(a+bx)}{b^3}+\frac{(c+dx)^4\cosh(a+bx)}{b}-\frac{24d^3(c+dx)\sinh(a}{b^4}\\
&= \frac{24d^4\cosh(a+bx)}{b^5}+\frac{12d^2(c+dx)^2\cosh(a+bx)}{b^3}+\frac{(c+dx)^4\cosh(a+bx)}{b}-
\end{aligned}$$

Mathematica [A]
time = 0.18, size = 76, normalized size = 0.84

$$\frac{\left(24d^4+12b^2d^2(c+dx)^2+b^4(c+dx)^4\right)\cosh(a+bx)-4bd(c+dx)\left(6d^2+b^2(c+dx)^2\right)\sinh(a+bx)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Sinh[a + b*x],x]
```

```
[Out] ((24*d^4 + 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cosh[a + b*x] - 4*b*d*
(c + d*x)*(6*d^2 + b^2*(c + d*x)^2)*Sinh[a + b*x])/b^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 546 vs. 2(91) = 182.
time = 0.30, size = 547, normalized size = 6.01

method	result
risch	$\frac{\left(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2-4b^3d^4x^3+4b^4c^3dx-12b^3c\,d^3x^2+b^4c^4-12b^3c^2d^2x+12b^2d^4x^2-4b^3c^3d+24b^2c\,d^3x+12b^2c\right.}{2b^5}$
derivativedivides	$-\frac{4da\,c^3\cosh(bx+a)}{b}+\frac{6d^2a^2c^2\cosh(bx+a)}{b^2}-\frac{4d^3a^3c\cosh(bx+a)}{b^3}-\frac{4d^4a^3((bx+a)\cosh(bx+a)-\sinh(bx+a))}{b^4}+\frac{4d\,c^3((bx+a)\cosh(bx+a)}{b}$
default	$-\frac{4da\,c^3\cosh(bx+a)}{b}+\frac{6d^2a^2c^2\cosh(bx+a)}{b^2}-\frac{4d^3a^3c\cosh(bx+a)}{b^3}-\frac{4d^4a^3((bx+a)\cosh(bx+a)-\sinh(bx+a))}{b^4}+\frac{4d\,c^3((bx+a)\cosh(bx+a)}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-4*d/b*a*c^3*cosh(b*x+a)+6*d^2/b^2*a^2*c^2*cosh(b*x+a)-4*d^3/b^3*a^3*c
*cosh(b*x+a)-4*d^4/b^4*a^3*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))+4*d/b*c^3*((b*
x+a)*cosh(b*x+a)-sinh(b*x+a))+6*d^2/b^2*c^2*((b*x+a)^2*cosh(b*x+a)-2*(b*x+a
)*sinh(b*x+a)+2*cosh(b*x+a))-4*d^4/b^4*a*((b*x+a)^3*cosh(b*x+a)-3*(b*x+a)^2
*sinh(b*x+a)+6*(b*x+a)*cosh(b*x+a)-6*sinh(b*x+a))+4*d^3/b^3*c*((b*x+a)^3*co
sh(b*x+a)-3*(b*x+a)^2*sinh(b*x+a)+6*(b*x+a)*cosh(b*x+a)-6*sinh(b*x+a))+6*d^
4/b^4*a^2*((b*x+a)^2*cosh(b*x+a)-2*(b*x+a)*sinh(b*x+a)+2*cosh(b*x+a))+d^4/b
^4*((b*x+a)^4*cosh(b*x+a)-4*(b*x+a)^3*sinh(b*x+a)+12*(b*x+a)^2*cosh(b*x+a)-
24*(b*x+a)*sinh(b*x+a)+24*cosh(b*x+a))+d^4/b^4*a^4*cosh(b*x+a)-12*d^2/b^2*a
*c^2*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))-12*d^3/b^3*a*c*((b*x+a)^2*cosh(b*x+a
)-2*(b*x+a)*sinh(b*x+a)+2*cosh(b*x+a))+12*d^3/b^3*a^2*c*((b*x+a)*cosh(b*x+a
)-sinh(b*x+a))+c^4*cosh(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(91) = 182.
time = 0.28, size = 326, normalized size = 3.58

$$\frac{c^4e^{(bx+a)}}{2b}+\frac{2(bxe^a-e^a)c^3de^{(bx)}}{b^2}+\frac{c^4e^{(-bx-a)}}{2b}+\frac{2(bx+1)c^3de^{(-bx-a)}}{b^2}+\frac{3(b^2x^2e^a-2bxe^a+2e^a)c^2d^2e^{(bx)}}{b^3}+\frac{3(b^2x^2+2bx+2)c^2d^2e^{(-bx-a)}}{b^3}+\frac{2(b^3x^3e^a-3b^2x^2e^a+6bxe^a-6e^a)cd^3e^{(bx)}}{b^4}+\frac{2(b^3x^3+3b^2x^2+6bx+6)cd^3e^{(-bx-a)}}{b^4}+\frac{(b^4x^4e^a-4b^3x^3e^a+12b^2x^2e^a-24bxe^a+24e^a)d^4e^{(bx)}}{2b^5}+\frac{(b^4x^4+4b^3x^3+12b^2x^2+24bx+24)d^4e^{(-bx-a)}}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*c^4*e^(b*x + a)/b + 2*(b*x*e^a - e^a)*c^3*d*e^(b*x)/b^2 + 1/2*c^4*e^(-b
*x - a)/b + 2*(b*x + 1)*c^3*d*e^(-b*x - a)/b^2 + 3*(b^2*x^2*e^a - 2*b*x*e^a
 + 2*e^a)*c^2*d^2*e^(b*x)/b^3 + 3*(b^2*x^2 + 2*b*x + 2)*c^2*d^2*e^(-b*x - a
)/b^3 + 2*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*c*d^3*e^(b*x)/b
^4 + 2*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*c*d^3*e^(-b*x - a)/b^4 + 1/2*(b^4*
x^4*e^a - 4*b^3*x^3*e^a + 12*b^2*x^2*e^a - 24*b*x*e^a + 24*e^a)*d^4*e^(b*x)
/b^5 + 1/2*(b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*d^4*e^(-b*x - a
)/b^5
```

Fricas [A]
time = 0.35, size = 169, normalized size = 1.86

$$\frac{\left(b^4d^4x^4 + 4\,b^4cd^3x^3 + b^4c^4 + 12\,b^2c^2d^2 + 24\,d^4 + 6\,(b^4c^2d^2 + 2\,b^2d^4)x^2 + 4\,(b^4c^3d + 6\,b^2cd^3)x\right)\cosh\left(bx+a\right) - 4\,\left(b^3d^4x^3 + 3\,b^3cd^3x^2 + b^3c^3d + 6\,bcd^3 + 3\,(b^3c^2d^2 + 2\,bd^4)x\right)\sinh\left(bx+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 + 12*b^2*c^2*d^2 + 24*d^4 + 6*(b^
4*c^2*d^2 + 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d + 6*b^2*c*d^3)*x)*cosh(b*x + a) -
 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d + 6*b*c*d^3 + 3*(b^3*c^2*d^2
+ 2*b*d^4)*x)*sinh(b*x + a))/b^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(92) = 184.
time = 0.36, size = 311, normalized size = 3.42

$$\begin{cases} \frac{c^4\cosh\left(a+bx\right)}{b} + \frac{4c^3dx\cosh\left(a+bx\right)}{b} + \frac{6c^2d^2x^2\cosh\left(a+bx\right)}{b} + \frac{4cd^3x^3\cosh\left(a+bx\right)}{b} + \frac{d^4x^4\cosh\left(a+bx\right)}{b} - \frac{4c^3d\sinh\left(a+bx\right)}{b^2} - \frac{12c^2d^2x\sinh\left(a+bx\right)}{b^2} - \frac{12cd^3x^2\sinh\left(a+bx\right)}{b^2} - \frac{4d^4x^3\sinh\left(a+bx\right)}{b^2} + \frac{12c^2d^2\cosh\left(a+bx\right)}{b^3} + \frac{24cd^3x\cosh\left(a+bx\right)}{b^3} + \frac{12d^4x^2\cosh\left(a+bx\right)}{b^3} - \frac{24cd^3\sinh\left(a+bx\right)}{b^4} - \frac{24d^4x\sinh\left(a+bx\right)}{b^4} + \frac{24d^4\cosh\left(a+bx\right)}{b^5} & \text{for}\: b \neq 0 \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\sinh\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sinh(b*x+a),x)
```

```
[Out] Piecewise((c**4*cosh(a + b*x)/b + 4*c**3*d*x*cosh(a + b*x)/b + 6*c**2*d**2*
x**2*cosh(a + b*x)/b + 4*c*d**3*x**3*cosh(a + b*x)/b + d**4*x**4*cosh(a + b
*x)/b - 4*c**3*d*sinh(a + b*x)/b**2 - 12*c**2*d**2*x*sinh(a + b*x)/b**2 - 1
2*c*d**3*x**2*sinh(a + b*x)/b**2 - 4*d**4*x**3*sinh(a + b*x)/b**2 + 12*c**2
*d**2*cosh(a + b*x)/b**3 + 24*c*d**3*x*cosh(a + b*x)/b**3 + 12*d**4*x**2*co
sh(a + b*x)/b**3 - 24*c*d**3*sinh(a + b*x)/b**4 - 24*d**4*x*sinh(a + b*x)/b
**4 + 24*d**4*cosh(a + b*x)/b**5, Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c
**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sinh(a), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(91) = 182.
time = 0.43, size = 324, normalized size = 3.56

$$\frac{\left(b^4d^4x^4 + 4\,b^4cd^3x^3 + 6\,b^4c^2d^2x^2 - 4\,b^3d^4x^3 + 4\,b^4c^3dx - 12\,b^3cd^3x^2 + b^4c^4 - 12\,b^3c^2d^2x + 12\,b^2d^4x^2 - 4\,b^3c^3d + 24\,b^2cd^3x + 12\,b^2c^2d^2 - 24\,bd^4x - 24\,bcd^3 + 24\,d^4\right)e^{(bx+a)}}{2\,b^5} + \frac{\left(b^4d^4x^4 + 4\,b^4cd^3x^3 + 6\,b^4c^2d^2x^2 + 4\,b^3d^4x^3 + 4\,b^4c^3dx + 12\,b^3cd^3x^2 + b^4c^4 + 12\,b^3c^2d^2x + 12\,b^2d^4x^2 + 4\,b^3c^3d + 24\,b^2cd^3x + 12\,b^2c^2d^2 + 24\,bd^4x + 24\,bcd^3 + 24\,d^4\right)e^{(-bx-a)}}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 4*
b^4*c^3*d*x - 12*b^3*c*d^3*x^2 + b^4*c^4 - 12*b^3*c^2*d^2*x + 12*b^2*d^4*x^
2 - 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2*d^2 - 24*b*d^4*x - 24*b*c*d^3
 + 24*d^4)*e^(b*x + a)/b^5 + 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2
*d^2*x^2 + 4*b^3*d^4*x^3 + 4*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + b^4*c^4 + 12*
b^3*c^2*d^2*x + 12*b^2*d^4*x^2 + 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2*
d^2 + 24*b*d^4*x + 24*b*c*d^3 + 24*d^4)*e^(-b*x - a)/b^5
```

Mupad [B]
time = 0.45, size = 215, normalized size = 2.36

$$\frac{\cosh(a+b\,x)\,\left(b^4\,c^4+12\,b^2\,c^2\,d^2+24\,d^4\right)}{b^5}-\frac{4\,\sinh(a+b\,x)\,\left(b^2\,c^3\,d+6\,c\,d^3\right)}{b^4}+\frac{d^4\,x^4\,\cosh(a+b\,x)}{b}+\frac{4\,x\,\cosh(a+b\,x)\,\left(b^2\,c^3\,d+6\,c\,d^3\right)}{b^3}-\frac{4\,d^4\,x^3\,\sinh(a+b\,x)}{b^2}-\frac{12\,x\,\sinh(a+b\,x)\,\left(b^2\,c^2\,d^2+2\,d^4\right)}{b^4}+\frac{6\,x^2\,\cosh(a+b\,x)\,\left(b^2\,c^2\,d^2+2\,d^4\right)}{b^3}+\frac{4\,c\,d^3\,x^3\,\cosh(a+b\,x)}{b}-\frac{12\,c\,d^3\,x^2\,\sinh(a+b\,x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*(c + d*x)^4,x)
```

```
[Out] (cosh(a + b*x)*(24*d^4 + b^4*c^4 + 12*b^2*c^2*d^2))/b^5 - (4*sinh(a + b*x)*
(6*c*d^3 + b^2*c^3*d))/b^4 + (d^4*x^4*cosh(a + b*x))/b + (4*x*cosh(a + b*x)
*(6*c*d^3 + b^2*c^3*d))/b^3 - (4*d^4*x^3*sinh(a + b*x))/b^2 - (12*x*sinh(a
+ b*x)*(2*d^4 + b^2*c^2*d^2))/b^4 + (6*x^2*cosh(a + b*x)*(2*d^4 + b^2*c^2*d
^2))/b^3 + (4*c*d^3*x^3*cosh(a + b*x))/b - (12*c*d^3*x^2*sinh(a + b*x))/b^2
```

3.2 $\int (c+dx)^3 \sinh(a+bx)\,dx$

Optimal. Leaf size=70

$$\frac{6d^2(c+dx)\cosh(a+bx)}{b^3}+\frac{(c+dx)^3\cosh(a+bx)}{b}-\frac{6d^3\sinh(a+bx)}{b^4}-\frac{3d(c+dx)^2\sinh(a+bx)}{b^2}$$

```
[Out] 6*d^2*(d*x+c)*cosh(b*x+a)/b^3+(d*x+c)^3*cosh(b*x+a)/b-6*d^3*sinh(b*x+a)/b^4
-3*d*(d*x+c)^2*sinh(b*x+a)/b^2
```

Rubi [A]
time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3377, 2717}

$$-\frac{6d^3\sinh(a+bx)}{b^4}+\frac{6d^2(c+dx)\cosh(a+bx)}{b^3}-\frac{3d(c+dx)^2\sinh(a+bx)}{b^2}+\frac{(c+dx)^3\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sinh[a + b*x],x]
```

```
[Out] (6*d^2*(c + d*x)*Cosh[a + b*x])/b^3 + ((c + d*x)^3*Cosh[a + b*x])/b - (6*d^
3*Sinh[a + b*x])/b^4 - (3*d*(c + d*x)^2*Sinh[a + b*x])/b^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\sinh(a+bx)\,dx &= \frac{(c+dx)^3\cosh(a+bx)}{b}-\frac{(3d)\int (c+dx)^2\cosh(a+bx)\,dx}{b}\\ &= \frac{(c+dx)^3\cosh(a+bx)}{b}-\frac{3d(c+dx)^2\sinh(a+bx)}{b^2}+\frac{(6d^2)\int (c+dx)\sinh(a}{b^2}\\ &= \frac{6d^2(c+dx)\cosh(a+bx)}{b^3}+\frac{(c+dx)^3\cosh(a+bx)}{b}-\frac{3d(c+dx)^2\sinh(a+b}{b^2}\\ &= \frac{6d^2(c+dx)\cosh(a+bx)}{b^3}+\frac{(c+dx)^3\cosh(a+bx)}{b}-\frac{6d^3\sinh(a+bx)}{b^4}-3d\end{aligned}$$

Mathematica [A]
time = 0.11, size = 61, normalized size = 0.87

$$\frac{b(c+dx)\left(6d^2+b^2(c+dx)^2\right)\cosh(a+bx)-3d\left(2d^2+b^2(c+dx)^2\right)\sinh(a+bx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sinh[a + b*x],x]
```

```
[Out] (b*(c + d*x)*(6*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] - 3*d*(2*d^2 + b^2*(c
+ d*x)^2)*Sinh[a + b*x])/b^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(70) = 140$.
time = 0.30, size = 308, normalized size = 4.40

method	result
risch	$\frac{\left(d^3x^3b^3+3b^3c\,d^2x^2+3b^3c^2dx-3b^2d^3x^2+b^3c^3-6b^2c\,d^2x-3b^2c^2d+6b\,d^3x+6bc\,d^2-6d^3\right)e^{bx+a}}{2b^4}+\frac{\left(d^3x^3b^3+3b^3c\,d^2x^2+3b\right.}{}$
derivativedivides	$\frac{\frac{d^3\left((bx+a)^3\cosh(bx+a)-3(bx+a)^2\sinh(bx+a)+6(bx+a)\cosh(bx+a)-6\sinh(bx+a)\right)}{b^3}-\frac{3d^3a\left((bx+a)^2\cosh(bx+a)-2(bx+a)\sinh(bx+a)\right.}{b^3}}{}$
default	$\frac{\frac{d^3\left((bx+a)^3\cosh(bx+a)-3(bx+a)^2\sinh(bx+a)+6(bx+a)\cosh(bx+a)-6\sinh(bx+a)\right)}{b^3}-\frac{3d^3a\left((bx+a)^2\cosh(bx+a)-2(bx+a)\sinh(bx+a)\right.}{b^3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(d^3/b^3*((b*x+a)^3*cosh(b*x+a)-3*(b*x+a)^2*sinh(b*x+a)+6*(b*x+a)*cosh(
b*x+a)-6*sinh(b*x+a))-3*d^3/b^3*a*((b*x+a)^2*cosh(b*x+a)-2*(b*x+a)*sinh(b*x
+a)+2*cosh(b*x+a))+3*d^2/b^2*c*((b*x+a)^2*cosh(b*x+a)-2*(b*x+a)*sinh(b*x+a)
+2*cosh(b*x+a))+3*d^3/b^3*a^2*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))-6*d^2/b^2*a
*c*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))+3*d/b*c^2*((b*x+a)*cosh(b*x+a)-sinh(b*
x+a))-d^3/b^3*a^3*cosh(b*x+a)+3*d^2/b^2*a^2*c*cosh(b*x+a)-3*d/b*a*c^2*cosh(
b*x+a)+c^3*cosh(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(70) = 140$.
time = 0.27, size = 222, normalized size = 3.17

$$\frac{c^3e^{(bx+a)}}{2\,b}+\frac{3\,(bxe^a-e^a)c^2de^{(bx)}}{2\,b^2}+\frac{c^3e^{(-bx-a)}}{2\,b}+\frac{3\,(bx+1)c^2de^{(-bx-a)}}{2\,b^2}+\frac{3\,(b^2x^2e^a-2\,bxe^a+2\,e^a)cd^2e^{(bx)}}{2\,b^3}+\frac{3\,(b^2x^2+2\,bx+2)cd^2e^{(-bx-a)}}{2\,b^3}+\frac{(b^3x^3e^a-3\,b^2x^2e^a+6\,bxe^a-6\,e^a)d^3e^{(bx)}}{2\,b^4}+\frac{(b^3x^3+3\,b^2x^2+6\,bx+6)d^3e^{(-bx-a)}}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*e^(b*x + a)/b + 3/2*(b*x*e^a - e^a)*c^2*d*e^(b*x)/b^2 + 1/2*c^3*e^(
-b*x - a)/b + 3/2*(b*x + 1)*c^2*d*e^(-b*x - a)/b^2 + 3/2*(b^2*x^2*e^a - 2*b
```

```
*x*e^a + 2*e^a)*c*d^2*e^(b*x)/b^3 + 3/2*(b^2*x^2 + 2*b*x + 2)*c*d^2*e^(-b*x
 - a)/b^3 + 1/2*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*d^3*e^(b*
x)/b^4 + 1/2*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*d^3*e^(-b*x - a)/b^4
```

Fricas [**A**]
time = 0.34, size = 109, normalized size = 1.56

$$\frac{\left(b^{3} d^{3} x^{3} + 3 \, b^{3} c d^{2} x^{2} + b^{3} c^{3} + 6 \, b c d^{2} + 3 \, {\left(b^{3} c^{2} d + 2 \, b d^{3}\right)} x\right)} \cosh\left(b x + a\right) - 3 \, {\left(b^{2} d^{3} x^{2} + 2 \, b^{2} c d^{2} x + b^{2} c^{2} d + 2 \, d^{3}\right)} \sinh\left(b x + a\right)}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 6*b*c*d^2 + 3*(b^3*c^2*d + 2*b*
d^3)*x)*cosh(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d + 2*d^3)
*sinh(b*x + a))/b^4
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(70) = 140.
time = 0.25, size = 202, normalized size = 2.89

$$\begin{cases} \frac{c^{3} \cosh{\left(a + b x \right)}}{b} + \frac{3 c^{2} d x \cosh{\left(a + b x \right)}}{b} + \frac{3 c d^{2} x^{2} \cosh{\left(a + b x \right)}}{b} + \frac{d^{3} x^{3} \cosh{\left(a + b x \right)}}{b} - \frac{3 c^{2} d \sinh{\left(a + b x \right)}}{b^{2}} - \frac{6 c d^{2} x \sinh{\left(a + b x \right)}}{b^{2}} - \frac{3 d^{3} x^{2} \sinh{\left(a + b x \right)}}{b^{2}} + \frac{6 c d^{2} \cosh{\left(a + b x \right)}}{b^{3}} + \frac{6 d^{3} x \cosh{\left(a + b x \right)}}{b^{3}} - \frac{6 d^{3} \sinh{\left(a + b x \right)}}{b^{4}} & \text{for}\: b \neq 0 \\\left(c^{3} x + \frac{3 c^{2} d x^{2}}{2} + c d^{2} x^{3} + \frac{d^{3} x^{4}}{4}\right) \sinh{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((c**3*cosh(a + b*x)/b + 3*c**2*d*x*cosh(a + b*x)/b + 3*c*d**2*x**
2*cosh(a + b*x)/b + d**3*x**3*cosh(a + b*x)/b - 3*c**2*d*sinh(a + b*x)/b**2
 - 6*c*d**2*x*sinh(a + b*x)/b**2 - 3*d**3*x**2*sinh(a + b*x)/b**2 + 6*c*d**
2*cosh(a + b*x)/b**3 + 6*d**3*x*cosh(a + b*x)/b**3 - 6*d**3*sinh(a + b*x)/b
**4, Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin
h(a), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(70) = 140.
time = 0.44, size = 204, normalized size = 2.91

$$\frac{{\left(b^{3} d^{3} x^{3} + 3 \, b^{3} c d^{2} x^{2} + 3 \, b^{3} c^{2} d x - 3 \, b^{2} d^{3} x^{2} + b^{3} c^{3} - 6 \, b^{2} c d^{2} x - 3 \, b^{2} c^{2} d + 6 \, b d^{3} x + 6 \, b c d^{2} - 6 \, d^{3}\right)} e^{\left(b x + a\right)}}{2 \, b^{4}} + \frac{{\left(b^{3} d^{3} x^{3} + 3 \, b^{3} c d^{2} x^{2} + 3 \, b^{3} c^{2} d x + 3 \, b^{2} d^{3} x^{2} + b^{3} c^{3} + 6 \, b^{2} c d^{2} x + 3 \, b^{2} c^{2} d + 6 \, b d^{3} x + 6 \, b c d^{2} + 6 \, d^{3}\right)} e^{\left(-b x - a\right)}}{2 \, b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x - 3*b^2*d^3*x^2 + b^3*c^
3 - 6*b^2*c*d^2*x - 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 6*d^3)*e^(b*x + a
)/b^4 + 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*b^2*d^3*x^2
```

```
+ b^3*c^3 + 6*b^2*c*d^2*x + 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 + 6*d^3)*e^
(-b*x - a)/b^4
```

Mupad [B]
time = 0.14, size = 143, normalized size = 2.04

$$\frac{\cosh(a+b\,x)\,\left(b^2\,c^3+6\,c\,d^2\right)}{b^3}-\frac{3\sinh(a+b\,x)\,\left(b^2\,c^2\,d+2\,d^3\right)}{b^4}+\frac{d^3\,x^3\cosh(a+b\,x)}{b}-\frac{3\,d^3\,x^2\sinh(a+b\,x)}{b^2}+\frac{3\,x\cosh(a+b\,x)\,\left(b^2\,c^2\,d+2\,d^3\right)}{b^3}-\frac{6\,c\,d^2\,x\sinh(a+b\,x)}{b^2}+\frac{3\,c\,d^2\,x^2\cosh(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*(c + d*x)^3,x)
```

```
[Out] (cosh(a + b*x)*(6*c*d^2 + b^2*c^3))/b^3 - (3*sinh(a + b*x)*(2*d^3 + b^2*c^2
*d))/b^4 + (d^3*x^3*cosh(a + b*x))/b - (3*d^3*x^2*sinh(a + b*x))/b^2 + (3*x
*cosh(a + b*x)*(2*d^3 + b^2*c^2*d))/b^3 - (6*c*d^2*x*sinh(a + b*x))/b^2 + (
3*c*d^2*x^2*cosh(a + b*x))/b
```

3.3 $\int (c+dx)^2 \sinh(a+bx)\,dx$

Optimal. Leaf size=49

$$\frac{2d^2\cosh(a+bx)}{b^3}+\frac{(c+dx)^2\cosh(a+bx)}{b}-\frac{2d(c+dx)\sinh(a+bx)}{b^2}$$

```
[Out] 2*d^2*cosh(b*x+a)/b^3+(d*x+c)^2*cosh(b*x+a)/b-2*d*(d*x+c)*sinh(b*x+a)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3377, 2718}

$$\frac{2d^2\cosh(a+bx)}{b^3}-\frac{2d(c+dx)\sinh(a+bx)}{b^2}+\frac{(c+dx)^2\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sinh[a + b*x],x]
```

```
[Out] (2*d^2*Cosh[a + b*x])/b^3 + ((c + d*x)^2*Cosh[a + b*x])/b - (2*d*(c + d*x)*
Sinh[a + b*x])/b^2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\sinh(a+bx)\,dx &= \frac{(c+dx)^2\cosh(a+bx)}{b}-\frac{(2d)\int (c+dx)\cosh(a+bx)\,dx}{b}\\ &= \frac{(c+dx)^2\cosh(a+bx)}{b}-\frac{2d(c+dx)\sinh(a+bx)}{b^2}+\frac{(2d^2)\int \sinh(a+bx)\,dx}{b^2}\\ &= \frac{2d^2\cosh(a+bx)}{b^3}+\frac{(c+dx)^2\cosh(a+bx)}{b}-\frac{2d(c+dx)\sinh(a+bx)}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 44, normalized size = 0.90

$$\frac{(2d^2+b^2(c+dx)^2)\cosh(a+bx)-2bd(c+dx)\sinh(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sinh[a + b*x],x]
```

```
[Out] ((2*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] - 2*b*d*(c + d*x)*Sinh[a + b*x])/b
^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(49) = 98$.
time = 0.30, size = 147, normalized size = 3.00

method	result
risch	$\frac{(b^2d^2x^2+2b^2cdx+b^2c^2-2b\,d^2x-2bcd+2d^2)\,\mathrm{e}^{bx+a}}{2b^3}+\frac{(b^2d^2x^2+2b^2cdx+b^2c^2+2b\,d^2x+2bcd+2d^2)\,\mathrm{e}^{-bx-a}}{2b^3}$
derivativedivides	$\frac{d^2\left((bx+a)^2\cosh(bx+a)-2(bx+a)\sinh(bx+a)+2\cosh(bx+a)\right)}{b^2}-\frac{2d^2a((bx+a)\cosh(bx+a)-\sinh(bx+a))}{b^2}+\frac{2dc((bx+a)\cosh(bx+a)-\sinh}{b}$ … (over b)
default	$\frac{d^2\left((bx+a)^2\cosh(bx+a)-2(bx+a)\sinh(bx+a)+2\cosh(bx+a)\right)}{b^2}-\frac{2d^2a((bx+a)\cosh(bx+a)-\sinh(bx+a))}{b^2}+\frac{2dc((bx+a)\cosh(bx+a)-\sinh}{b}$ … (over b)

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(d^2/b^2*((b*x+a)^2*cosh(b*x+a)-2*(b*x+a)*sinh(b*x+a)+2*cosh(b*x+a))-2*
d^2/b^2*a*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))+2*d/b*c*((b*x+a)*cosh(b*x+a)-si
nh(b*x+a))+d^2/b^2*a^2*cosh(b*x+a)-2*d/b*a*c*cosh(b*x+a)+c^2*cosh(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(49) = 98$.
time = 0.27, size = 134, normalized size = 2.73

$$\frac{c^2e^{(bx+a)}}{2\,b}+\frac{(bxe^a-e^a)cde^{(bx)}}{b^2}+\frac{c^2e^{(-bx-a)}}{2\,b}+\frac{(bx+1)cde^{(-bx-a)}}{b^2}+\frac{(b^2x^2e^a-2\,bxe^a+2\,e^a)d^2e^{(bx)}}{2\,b^3}+\frac{(b^2x^2+2\,bx+2)d^2e^{(-bx-a)}}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*e^(b*x + a)/b + (b*x*e^a - e^a)*c*d*e^(b*x)/b^2 + 1/2*c^2*e^(-b*x -
 a)/b + (b*x + 1)*c*d*e^(-b*x - a)/b^2 + 1/2*(b^2*x^2*e^a - 2*b*x*e^a + 2*e
^a)*d^2*e^(b*x)/b^3 + 1/2*(b^2*x^2 + 2*b*x + 2)*d^2*e^(-b*x - a)/b^3
```

Fricas [A]
time = 0.41, size = 62, normalized size = 1.27

$$\frac{(b^2d^2x^2+2\,b^2cdx+b^2c^2+2\,d^2)\cosh(bx+a)-2\,(bd^2x+bcd)\sinh(bx+a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*d^2)*cosh(b*x + a) - 2*(b*d^2*x +
 b*c*d)*sinh(b*x + a))/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(48) = 96$.
time = 0.18, size = 112, normalized size = 2.29

$$\begin{cases} \frac{c^2\cosh{(a+bx)}}{b} + \frac{2cdx\cosh{(a+bx)}}{b} + \frac{d^2x^2\cosh{(a+bx)}}{b} - \frac{2cd\sinh{(a+bx)}}{b^2} - \frac{2d^2x\sinh{(a+bx)}}{b^2} + \frac{2d^2\cosh{(a+bx)}}{b^3} & \text{for}\ b \neq 0 \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sinh{(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((c**2*cosh(a + b*x)/b + 2*c*d*x*cosh(a + b*x)/b + d**2*x**2*cosh(
a + b*x)/b - 2*c*d*sinh(a + b*x)/b**2 - 2*d**2*x*sinh(a + b*x)/b**2 + 2*d**
2*cosh(a + b*x)/b**3, Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sinh(a)
, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(49) = 98$.
time = 0.41, size = 112, normalized size = 2.29

$$\frac{(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 - 2\,bd^2x - 2\,bcd + 2\,d^2)e^{(bx+a)}}{2\,b^3} + \frac{(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 + 2\,bd^2x + 2\,bcd + 2\,d^2)e^{(-bx-a)}}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*b*d^2*x - 2*b*c*d + 2*d^2)*e^(
b*x + a)/b^3 + 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*b*d^2*x + 2*b*c
*d + 2*d^2)*e^(-b*x - a)/b^3
```

Mupad [B]
time = 0.12, size = 82, normalized size = 1.67

$$\frac{\cosh(a+b\,x)\,\left(b^2\,c^2+2\,d^2\right)}{b^3} + \frac{d^2\,x^2\cosh(a+b\,x)}{b} - \frac{2\,c\,d\sinh(a+b\,x)}{b^2} - \frac{2\,d^2\,x\sinh(a+b\,x)}{b^2} + \frac{2\,c\,d\,x\cosh(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*(c + d*x)^2,x)
```

```
[Out] (cosh(a + b*x)*(2*d^2 + b^2*c^2))/b^3 + (d^2*x^2*cosh(a + b*x))/b - (2*c*d*
sinh(a + b*x))/b^2 - (2*d^2*x*sinh(a + b*x))/b^2 + (2*c*d*x*cosh(a + b*x))/
b
```

3.4 $\int (c+dx)\sinh(a+bx)\,dx$

Optimal. Leaf size=28

$$\frac{(c+dx)\cosh(a+bx)}{b}-\frac{d\sinh(a+bx)}{b^2}$$

```
[Out] (d*x+c)*cosh(b*x+a)/b-d*sinh(b*x+a)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3377, 2717}

$$\frac{(c+dx)\cosh(a+bx)}{b}-\frac{d\sinh(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sinh[a + b*x],x]
```

```
[Out] ((c + d*x)*Cosh[a + b*x])/b - (d*Sinh[a + b*x])/b^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\sinh(a+bx)\,dx &= \frac{(c+dx)\cosh(a+bx)}{b}-\frac{d\int \cosh(a+bx)\,dx}{b}\\ &= \frac{(c+dx)\cosh(a+bx)}{b}-\frac{d\sinh(a+bx)}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 27, normalized size = 0.96

$$\frac{b(c+dx)\cosh(a+bx)-d\sinh(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sinh[a + b*x],x]
```

```
[Out] (b*(c + d*x)*Cosh[a + b*x] - d*Sinh[a + b*x])/b^2
```

Maple [A]
time = 0.42, size = 53, normalized size = 1.89

method	result
risch	$\frac{(bdx+bc-d)\mathrm{e}^{bx+a}}{2b^2}+\frac{(bdx+bc+d)\mathrm{e}^{-bx-a}}{2b^2}$
derivativedivides	$\frac{\frac{d((bx+a)\cosh(bx+a)-\sinh(bx+a))}{b}-\frac{da\cosh(bx+a)}{b}+c\cosh(bx+a)}{b}$
default	$\frac{\frac{d((bx+a)\cosh(bx+a)-\sinh(bx+a))}{b}-\frac{da\cosh(bx+a)}{b}+c\cosh(bx+a)}{b}$
meijerg	$-\frac{2d\sinh(a)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh(bx)}{2\sqrt{\pi}}-\frac{bx\sinh(bx)}{2\sqrt{\pi}}\right)}{b^2}+\frac{d\cosh(a)(\cosh(bx)bx-\sinh(bx))}{b^2}+\frac{c\sinh(a)\sinh(bx)}{b}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(d/b*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))-d/b*a*cosh(b*x+a)+c*cosh(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(28) = 56$.
time = 0.26, size = 68, normalized size = 2.43

$$\frac{ce^{(bx+a)}}{2\,b}+\frac{(bxe^{a}-e^{a})de^{(bx)}}{2\,b^2}+\frac{ce^{(-bx-a)}}{2\,b}+\frac{(bx+1)de^{(-bx-a)}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*c*e^(b*x + a)/b + 1/2*(b*x*e^a - e^a)*d*e^(b*x)/b^2 + 1/2*c*e^(-b*x - a
)/b + 1/2*(b*x + 1)*d*e^(-b*x - a)/b^2
```

Fricas [A]
time = 0.37, size = 29, normalized size = 1.04

$$\frac{(bdx+bc)\cosh{(bx+a)}-d\sinh{(bx+a)}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*d*x + b*c)*cosh(b*x + a) - d*sinh(b*x + a))/b^2
```

Sympy [A]
time = 0.09, size = 46, normalized size = 1.64

$$\begin{cases} \frac{c\cosh(a+bx)}{b} + \frac{dx\cosh(a+bx)}{b} - \frac{d\sinh(a+bx)}{b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\sinh(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a),x)
```

```
[Out] Piecewise((c*cosh(a + b*x)/b + d*x*cosh(a + b*x)/b - d*sinh(a + b*x)/b**2,
Ne(b, 0)), ((c*x + d*x**2/2)*sinh(a), True))
```

Giac [A]
time = 0.41, size = 46, normalized size = 1.64

$$\frac{(bdx + bc - d)e^{(bx+a)}}{2\,b^2} + \frac{(bdx + bc + d)e^{(-bx-a)}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x + b*c - d)*e^(b*x + a)/b^2 + 1/2*(b*d*x + b*c + d)*e^(-b*x - a)/
b^2
```

Mupad [B]
time = 0.10, size = 35, normalized size = 1.25

$$\frac{c\cosh(a + b\,x) + d\,x\cosh(a + b\,x)}{b} - \frac{d\sinh(a + b\,x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*(c + d*x),x)
```

```
[Out] (c*cosh(a + b*x) + d*x*cosh(a + b*x))/b - (d*sinh(a + b*x))/b^2
```

3.5 $\int \frac{\sinh(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=51

$$\frac{\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{d}+\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] cosh(a-b*c/d)*Shi(b*c/d+b*x)/d+Chi(b*c/d+b*x)*sinh(a-b*c/d)/d
```

Rubi [A]
time = 0.08, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3384, 3379, 3382}

$$\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]/(c + d*x),x]
```

```
[Out] (CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*c)/d])/d + (Cosh[a - (b*c)/d]*Sinh
Integral[(b*c)/d + b*x])/d
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\sinh(a+bx)}{c+dx}\,dx = \cosh\left(a-\frac{bc}{d}\right)\int \frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx + \sinh\left(a-\frac{bc}{d}\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx$$
$$= \frac{\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{d} + \frac{\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}$$

Mathematica [A]
time = 0.06, size = 49, normalized size = 0.96

$$\frac{\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)+\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]/(c + d*x),x]
```

```
[Out] (CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*c)/d] + Cosh[a - (b*c)/d]*SinhInte
gral[(b*c)/d + b*x])/d
```

Maple [A]
time = 0.36, size = 82, normalized size = 1.61

method	result	size
risch	$\frac{e^{-\frac{ad-bc}{d}}\,\text{expIntegral}\left(1,bx+a-\frac{ad-bc}{d}\right)}{2d}-\frac{e^{\frac{ad-bc}{d}}\,\text{expIntegral}\left(1,-bx-a-\frac{-ad+bc}{d}\right)}{2d}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)-1/2/d*exp((a*d-b*c)/d)*Ei(1
,-b*x-a-(-a*d+b*c)/d)
```

Maxima [A]
time = 0.30, size = 57, normalized size = 1.12

$$\frac{e^{\left(-a+\frac{bc}{d}\right)}E_1\left(\frac{(dx+c)b}{d}\right)}{2\,d}-\frac{e^{\left(a-\frac{bc}{d}\right)}E_1\left(-\frac{(dx+c)b}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*e^(-a + b*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d - 1/2*e^(a - b*c/d)*e
xp_integral_e(1, -(d*x + c)*b/d)/d
```

Fricas [A]
time = 0.33, size = 94, normalized size = 1.84

$$\frac{\left(\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)-\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)+\left(\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)+\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\sinh\left(-\frac{bc-ad}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] 1/2*((Ei((b*d*x + b*c)/d) - Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) + (E
i((b*d*x + b*c)/d) + Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sinh\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c),x)
[Out] Integral(sinh(a + b*x)/(c + d*x), x)
```

Giac [A]
time = 0.41, size = 57, normalized size = 1.12

$$\frac{\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}-\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] 1/2*(Ei((b*d*x + b*c)/d)*e^(a - b*c/d) - Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d
))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\mathrm{sinh}(a+b\,x)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/(c + d*x),x)
[Out] int(sinh(a + b*x)/(c + d*x), x)
```

3.6 $\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=71

$$\frac{b\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{\sinh(a+bx)}{d(c+dx)}+\frac{b\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d^2}$$

```
[Out] b*Chi(b*c/d+b*x)*cosh(a-b*c/d)/d^2+b*Shi(b*c/d+b*x)*sinh(a-b*c/d)/d^2-sinh(
b*x+a)/d/(d*x+c)
```

Rubi [A]
time = 0.09, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3378, 3384, 3379, 3382}

$$\frac{b\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{b\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{\sinh(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]/(c + d*x)^2,x]
```

```
[Out] (b*Cosh[a - (b*c)/d]*CoshIntegral[(b*c)/d + b*x])/d^2 - Sinh[a + b*x]/(d*(c
 + d*x)) + (b*Sinh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx &= -\frac{\sinh(a+bx)}{d(c+dx)} + \frac{b\int \frac{\cosh(a+bx)}{c+dx}\,dx}{d}\\ &= -\frac{\sinh(a+bx)}{d(c+dx)} + \frac{\left(b\cosh\left(a-\frac{bc}{d}\right)\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d} + \frac{\left(b\sinh\left(a-\frac{bc}{d}\right)\right)\int \frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}\\ &= \frac{b\cosh\left(a-\frac{bc}{d}\right)\operatorname{Chi}\left(\frac{bc}{d}+bx\right)}{d^2} - \frac{\sinh(a+bx)}{d(c+dx)} + \frac{b\sinh\left(a-\frac{bc}{d}\right)\operatorname{Shi}\left(\frac{bc}{d}+bx\right)}{d^2}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 65, normalized size = 0.92

$$\frac{b\cosh\left(a-\frac{bc}{d}\right)\operatorname{Chi}\left(b\left(\frac{c}{d}+x\right)\right) - \frac{d\sinh(a+bx)}{c+dx} + b\sinh\left(a-\frac{bc}{d}\right)\operatorname{Shi}\left(b\left(\frac{c}{d}+x\right)\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]/(c + d*x)^2,x]
```

```
[Out] (b*Cosh[a - (b*c)/d]*CoshIntegral[b*(c/d + x)] - (d*Sinh[a + b*x])/(c + d*x
) + b*Sinh[a - (b*c)/d]*SinhIntegral[b*(c/d + x)])/d^2
```

Maple [A]
time = 0.38, size = 133, normalized size = 1.87

method	result	size
risch	$\frac{b\,e^{-bx-a}}{2d(bdx+bc)} - \frac{b\,e^{-\frac{ad-bc}{d}}\,\mathrm{expIntegral}\left(1,bx+a-\frac{ad-bc}{d}\right)}{2d^2} - \frac{b\,e^{bx+a}}{2d^2\left(\frac{bc}{d}+bx\right)} - \frac{b\,e^{\frac{ad-bc}{d}}\,\mathrm{expIntegral}\left(1,-bx-a-\frac{-ad+bc}{d}\right)}{2d^2}$	133

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*exp(-b*x-a)/d/(b*d*x+b*c)-1/2*b/d^2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d
-b*c)/d)-1/2*b/d^2*exp(b*x+a)/(b*c/d+b*x)-1/2*b/d^2*exp((a*d-b*c)/d)*Ei(1,-
b*x-a-(-a*d+b*c)/d)
```

Maxima [A]
time = 0.30, size = 80, normalized size = 1.13

$$-\frac{b\left(\frac{e^{\left(-a+\frac{bc}{d}\right)}E_1\left(\frac{(dx+c)b}{d}\right)}{d}+\frac{e^{\left(a-\frac{bc}{d}\right)}E_1\left(-\frac{(dx+c)b}{d}\right)}{d}\right)}{2\,d}-\frac{\sinh\left(bx+a\right)}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*(e^(-a + b*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d + e^(a - b*c/d)*e
xp_integral_e(1, -(d*x + c)*b/d)/d)/d - sinh(b*x + a)/((d*x + c)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(71) = 142.
time = 0.37, size = 148, normalized size = 2.08

$$\frac{((bdx+bc)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)+(bdx+bc)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right))\cosh\left(-\frac{bc-ad}{d}\right)-2\,d\sinh\left(bx+a\right)+((bdx+bc)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)-(bdx+bc)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right))\sinh\left(-\frac{bc-ad}{d}\right)}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(((b*d*x + b*c)*Ei((b*d*x + b*c)/d) + (b*d*x + b*c)*Ei(-(b*d*x + b*c)/d
))*cosh(-(b*c - a*d)/d) - 2*d*sinh(b*x + a) + ((b*d*x + b*c)*Ei((b*d*x + b*
c)/d) - (b*d*x + b*c)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d))/(d^3*x +
c*d^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 615 vs. 2(71) = 142.
time = 0.45, size = 615, normalized size = 8.66

$$\frac{\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\mathrm{Ei}\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}+b^3c\mathrm{Ei}\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}-ab^2d\mathrm{Ei}\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}+b^2de^{\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)}{d}\right)}\right)d^2}{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)d^4+bcd^4-ad^5\right)b}+\frac{\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\mathrm{Ei}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}+b^3c\mathrm{Ei}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}-ab^2d\mathrm{Ei}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}-b^2de^{\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)}{d}\right)}\right)d^2}{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)d^4+bcd^4-ad^5\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*Ei(-((d*x + c)*(b -
b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d) + b^3*c*Ei
(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c -
a*d)/d) - a*b^2*d*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c
- a*d)/d)*e^((b*c - a*d)/d) + b^2*d*e^(-(d*x + c)*(b - b*c/(d*x + c) + a*d/
(d*x + c))/d))*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*d^4 + b*
c*d^4 - a*d^5)*b) + 1/2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*
Ei(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-(b*c
- a*d)/d) + b^3*c*Ei(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c -
 a*d)/d)*e^(-(b*c - a*d)/d) - a*b^2*d*Ei(((d*x + c)*(b - b*c/(d*x + c) + a*
d/(d*x + c)) + b*c - a*d)/d)*e^(-(b*c - a*d)/d) - b^2*d*e^((d*x + c)*(b - b
*c/(d*x + c) + a*d/(d*x + c))/d))*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/
(d*x + c))*d^4 + b*c*d^4 - a*d^5)*b)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(a+b\,x)}{\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/(c + d*x)^2,x)
```

```
[Out] int(sinh(a + b*x)/(c + d*x)^2, x)
```

3.7 $\int \frac{\sinh(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=104

$$-\frac{b\cosh(a+bx)}{2d^2(c+dx)}+\frac{b^2\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{2d^3}-\frac{\sinh(a+bx)}{2d(c+dx)^2}+\frac{b^2\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{2d^3}$$

```
[Out] -1/2*b*cosh(b*x+a)/d^2/(d*x+c)+1/2*b^2*cosh(a-b*c/d)*Shi(b*c/d+b*x)/d^3+1/2
*b^2*Chi(b*c/d+b*x)*sinh(a-b*c/d)/d^3-1/2*sinh(b*x+a)/d/(d*x+c)^2
```

Rubi [A]
time = 0.13, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3378, 3384, 3379, 3382}

$$\frac{b^2\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{2d^3}+\frac{b^2\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{2d^3}-\frac{b\cosh(a+bx)}{2d^2(c+dx)}-\frac{\sinh(a+bx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]/(c + d*x)^3,x]
```

```
[Out] -1/2*(b*Cosh[a + b*x])/(d^2*(c + d*x)) + (b^2*CoshIntegral[(b*c)/d + b*x]*S
inh[a - (b*c)/d])/(2*d^3) - Sinh[a + b*x]/(2*d*(c + d*x)^2) + (b^2*Cosh[a -
 (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/(2*d^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(a+bx)}{(c+dx)^3}\,dx &= -\frac{\sinh(a+bx)}{2d(c+dx)^2}+\frac{b\int \frac{\cosh(a+bx)}{(c+dx)^2}\,dx}{2d}\\ &= -\frac{b\cosh(a+bx)}{2d^2(c+dx)}-\frac{\sinh(a+bx)}{2d(c+dx)^2}+\frac{b^2\int \frac{\sinh(a+bx)}{c+dx}\,dx}{2d^2}\\ &= -\frac{b\cosh(a+bx)}{2d^2(c+dx)}-\frac{\sinh(a+bx)}{2d(c+dx)^2}+\frac{\left(b^2\cosh\left(a-\frac{bc}{d}\right)\right)\int \frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{2d^2}+\frac{\left(b^2\sinh\left(a-\right.\right.}{}\\ &= -\frac{b\cosh(a+bx)}{2d^2(c+dx)}+\frac{b^2\mathrm{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{2d^3}-\frac{\sinh(a+bx)}{2d(c+dx)^2}+\frac{b^2\cosh\left(a-\frac{bc}{d}\right)\mathrm{Sh}}{2d^3}\end{aligned}$$

Mathematica [A]
time = 0.38, size = 88, normalized size = 0.85

$$\frac{b^2\mathrm{Chi}\left(b\left(\frac{c}{d}+x\right)\right)\sinh\left(a-\frac{bc}{d}\right)-\frac{d(b(c+dx)\cosh(a+bx)+d\sinh(a+bx))}{(c+dx)^2}+b^2\cosh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(b\left(\frac{c}{d}+x\right)\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]/(c + d*x)^3,x]
```

```
[Out] (b^2*CoshIntegral[b*(c/d + x)]*Sinh[a - (b*c)/d] - (d*(b*(c + d*x)*Cosh[a +
 b*x] + d*Sinh[a + b*x]))/(c + d*x)^2 + b^2*Cosh[a - (b*c)/d]*SinhIntegral[
b*(c/d + x)])/(2*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(96) = 192$.
time = 0.39, size = 277, normalized size = 2.66

method	result
risch	$-\frac{b^3 e^{-bx-a}x}{4d(b^2d^2x^2+2b^2cdx+b^2c^2)}-\frac{b^3 e^{-bx-a}c}{4d^2(b^2d^2x^2+2b^2cdx+b^2c^2)}+\frac{b^2 e^{-bx-a}}{4d(b^2d^2x^2+2b^2cdx+b^2c^2)}+\frac{b^2 e^{-\frac{ad-bc}{d}}\,\mathrm{expIntegral}\left(1,bx+a-\frac{ac}{}\right.}{4d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b^3*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*x-1/4*b^3*exp(-b*x
-a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c+1/4*b^2*exp(-b*x-a)/d/(b^2*d^2*
x^2+2*b^2*c*d*x+b^2*c^2)+1/4*b^2/d^3*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)
/d)-1/4*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)^2-1/4*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)
-1/4*b^2/d^3*exp((a*d-b*c)/d)*Ei(1,-b*x-a-(-a*d+b*c)/d)
```

Maxima [A]
time = 0.32, size = 94, normalized size = 0.90

$$-\frac{b\left(\frac{e^{\left(-a+\frac{bc}{d}\right)}E_2\left(\frac{(dx+c)b}{d}\right)}{(dx+c)d}+\frac{e^{\left(a-\frac{bc}{d}\right)}E_2\left(-\frac{(dx+c)b}{d}\right)}{(dx+c)d}\right)}{4\,d}-\frac{\sinh\left(bx+a\right)}{2\left(dx+c\right)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*(e^(-a + b*c/d)*exp_integral_e(2, (d*x + c)*b/d)/((d*x + c)*d) + e^(
a - b*c/d)*exp_integral_e(2, -(d*x + c)*b/d)/((d*x + c)*d))/d - 1/2*sinh(b*
x + a)/((d*x + c)^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(96) = 192.
time = 0.37, size = 254, normalized size = 2.44

$$-\frac{2\,d^2\sinh\left(bx+a\right)+2\left(bd^2x+bcd\right)\cosh\left(bx+a\right)-\left((b^2d^2x^2+2\,b^2cdx+b^2c^2)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)-(b^2d^2x^2+2\,b^2cdx+b^2c^2)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)-\left((b^2d^2x^2+2\,b^2cdx+b^2c^2)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)+(b^2d^2x^2+2\,b^2cdx+b^2c^2)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\sinh\left(-\frac{bc-ad}{d}\right)}{4\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d^2*sinh(b*x + a) + 2*(b*d^2*x + b*c*d)*cosh(b*x + a) - ((b^2*d^2*x
^2 + 2*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) - ((b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c
^2*d^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(96) = 192.
time = 0.41, size = 301, normalized size = 2.89

$$\frac{b^{2}d^{2}x^{2}\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}-b^{2}d^{2}x^{2}\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}+2\,b^{2}cdx\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}-2\,b^{2}cdx\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}+b^{2}c^{2}\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}-b^{2}c^{2}\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}-bd^{2}xe^{(bx+a)}-bd^{2}xe^{(-bx-a)}-bcde^{(bx+a)}-bcde^{(-bx-a)}-d^{2}e^{(bx+a)}+d^{2}e^{(-bx-a)}}{4\left(d^{5}x^{2}+2\,cd^{4}x+c^{2}d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^2*d^2*x^2*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) - b^2*d^2*x^2*Ei(-(b*d*x
 + b*c)/d)*e^(-a + b*c/d) + 2*b^2*c*d*x*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) -
 2*b^2*c*d*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + b^2*c^2*Ei((b*d*x + b*c)
/d)*e^(a - b*c/d) - b^2*c^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - b*d^2*x*e
^(b*x + a) - b*d^2*x*e^(-b*x - a) - b*c*d*e^(b*x + a) - b*c*d*e^(-b*x - a)
- d^2*e^(b*x + a) + d^2*e^(-b*x - a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(a+b\,x)}{{(c+d\,x)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/(c + d*x)^3,x)
```

```
[Out] int(sinh(a + b*x)/(c + d*x)^3, x)
```

3.8 $\int (c+dx)^4 \sinh^2(a+bx)\,dx$

Optimal. Leaf size=162

$$-\frac{3d^4x}{4b^4}-\frac{d(c+dx)^3}{2b^2}-\frac{(c+dx)^5}{10d}+\frac{3d^4\cosh(a+bx)\sinh(a+bx)}{4b^5}+\frac{3d^2(c+dx)^2\cosh(a+bx)\sinh(a+bx)}{2b^3}+\frac{(c$$

```
[Out] -3/4*d^4*x/b^4-1/2*d*(d*x+c)^3/b^2-1/10*(d*x+c)^5/d+3/4*d^4*cosh(b*x+a)*sin
h(b*x+a)/b^5+3/2*d^2*(d*x+c)^2*cosh(b*x+a)*sinh(b*x+a)/b^3+1/2*(d*x+c)^4*co
sh(b*x+a)*sinh(b*x+a)/b-3/2*d^3*(d*x+c)*sinh(b*x+a)^2/b^4-d*(d*x+c)^3*sinh(
b*x+a)^2/b^2
```

Rubi [A]
time = 0.08, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 32, 2715, 8}

$$\frac{3d^4\sinh(a+bx)\cosh(a+bx)}{4b^5}-\frac{3d^3(c+dx)\sinh^2(a+bx)}{2b^4}+\frac{3d^2(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b^3}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{3d^4x}{4b^4}-\frac{d(c+dx)^3}{2b^2}-\frac{(c+dx)^5}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sinh[a + b*x]^2,x]
```

```
[Out] (-3*d^4*x)/(4*b^4) - (d*(c + d*x)^3)/(2*b^2) - (c + d*x)^5/(10*d) + (3*d^4*
Cosh[a + b*x]*Sinh[a + b*x])/(4*b^5) + (3*d^2*(c + d*x)^2*Cosh[a + b*x]*Sin
h[a + b*x])/(2*b^3) + ((c + d*x)^4*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (3*
d^3*(c + d*x)*Sinh[a + b*x]^2)/(2*b^4) - (d*(c + d*x)^3*Sinh[a + b*x]^2)/b^
2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
```

```
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int (c+dx)^4 \sinh^2(a+bx)\,dx = \frac{(c+dx)^4\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{d(c+dx)^3\sinh^2(a+bx)}{b^2} - \frac{1}{2}\int (c+$$
$$= -\frac{(c+dx)^5}{10d} + \frac{3d^2(c+dx)^2\cosh(a+bx)\sinh(a+bx)}{2b^3} + \frac{(c+dx)^4\cosh(a+}{2b}$$
$$= -\frac{d(c+dx)^3}{2b^2} - \frac{(c+dx)^5}{10d} + \frac{3d^4\cosh(a+bx)\sinh(a+bx)}{4b^5} + \frac{3d^2(c+dx)^2 c}{}$$
$$= -\frac{3d^4x}{4b^4} - \frac{d(c+dx)^3}{2b^2} - \frac{(c+dx)^5}{10d} + \frac{3d^4\cosh(a+bx)\sinh(a+bx)}{4b^5} + \frac{3d^2(c}{}$$

Mathematica [A]
time = 0.36, size = 132, normalized size = 0.81

$$\frac{-8b^5x(5c^4+10c^3dx+10c^2d^2x^2+5cd^3x^3+d^4x^4)-20bd(c+dx)\left(3d^2+2b^2(c+dx)^2\right)\cosh(2(a+bx))+10\left(3d^4+6b^2d^2(c+dx)^2+2b^4(c+dx)^4\right)\sinh(2(a+bx))}{80b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Sinh[a + b*x]^2,x]
[Out] (-8*b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4) - 2
0*b*d*(c + d*x)*(3*d^2 + 2*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)] + 10*(3*d^4 +
 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Sinh[2*(a + b*x)])/(80*b^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 909 vs. $2(148) = 296$.
time = 0.44, size = 910, normalized size = 5.62

method	result
risch	$-\frac{d^4x^5}{10} - \frac{d^3cx^4}{2} - d^2c^2x^3 - dc^3x^2 - \frac{c^4x}{2} - \frac{c^5}{10d} + \frac{(2d^4x^4b^4+8b^4cd^3x^3+12b^4c^2d^2x^2-4b^3d^4x^3+8b^4c^3dx}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(d^4/b^4*(1/2*(b*x+a)^4*cosh(b*x+a)*sinh(b*x+a)-1/10*(b*x+a)^5-(b*x+a)^
3*cosh(b*x+a)^2+3/2*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)+1/2*(b*x+a)^3-3/2*(b*
x+a)*cosh(b*x+a)^2+3/4*cosh(b*x+a)*sinh(b*x+a)+3/4*b*x+3/4*a)+d^4/b^4*a^4*(
1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*x-1/2*a)-12*d^3/b^3*a*c*(1/2*(b*x+a)^2*co
sh(b*x+a)*sinh(b*x+a)-1/6*(b*x+a)^3-1/2*(b*x+a)*cosh(b*x+a)^2+1/4*cosh(b*x+
a)*sinh(b*x+a)+1/4*b*x+1/4*a)+12*d^3/b^3*a^2*c*(1/2*(b*x+a)*cosh(b*x+a)*sin
h(b*x+a)-1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)-12*d^2/b^2*a*c^2*(1/2*(b*x+a)*cos
h(b*x+a)*sinh(b*x+a)-1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)+c^4*(1/2*cosh(b*x+a)*
sinh(b*x+a)-1/2*b*x-1/2*a)-4*d^4/b^4*a*(1/2*(b*x+a)^3*cosh(b*x+a)*sinh(b*x+
a)-1/8*(b*x+a)^4-3/4*(b*x+a)^2*cosh(b*x+a)^2+3/4*(b*x+a)*cosh(b*x+a)*sinh(b
*x+a)+3/8*(b*x+a)^2-3/8*cosh(b*x+a)^2)+4*d^3/b^3*c*(1/2*(b*x+a)^3*cosh(b*x+
a)*sinh(b*x+a)-1/8*(b*x+a)^4-3/4*(b*x+a)^2*cosh(b*x+a)^2+3/4*(b*x+a)*cosh(b
*x+a)*sinh(b*x+a)+3/8*(b*x+a)^2-3/8*cosh(b*x+a)^2)+6*d^4/b^4*a^2*(1/2*(b*x+
a)^2*cosh(b*x+a)*sinh(b*x+a)-1/6*(b*x+a)^3-1/2*(b*x+a)*cosh(b*x+a)^2+1/4*co
sh(b*x+a)*sinh(b*x+a)+1/4*b*x+1/4*a)+6*d^2/b^2*c^2*(1/2*(b*x+a)^2*cosh(b*x+
a)*sinh(b*x+a)-1/6*(b*x+a)^3-1/2*(b*x+a)*cosh(b*x+a)^2+1/4*cosh(b*x+a)*sinh
(b*x+a)+1/4*b*x+1/4*a)-4*d^4/b^4*a^3*(1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-1
/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)+4*d/b*c^3*(1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x
+a)-1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)-4*d^3/b^3*a^3*c*(1/2*cosh(b*x+a)*sinh(
b*x+a)-1/2*b*x-1/2*a)+6*d^2/b^2*a^2*c^2*(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*
x-1/2*a)-4*d/b*a*c^3*(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*x-1/2*a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(148) = 296$.
time = 0.32, size = 382, normalized size = 2.36

$$-\frac{1}{4}\left(4x^{2}-\frac{\left(2bxe^{(2a)}-e^{(2a)}\right)e^{(2bx)}}{b^{2}}+\frac{(2bx+1)e^{(-2bx-2a)}}{b^{2}}\right)c^{3}d-\frac{1}{8}\left(8x^{3}-\frac{3\left(2b^{2}x^{2}e^{(2a)}-2bxe^{(2a)}+e^{(2a)}\right)e^{(2bx)}}{b^{3}}+\frac{3\left(2b^{2}x^{2}+2bx+1\right)e^{(-2bx-2a)}}{b^{3}}\right)c^{2}d^{2}-\frac{1}{8}\left(4x^{4}-\frac{\left(4b^{3}x^{3}e^{(2a)}-6b^{2}x^{2}e^{(2a)}+6bxe^{(2a)}-3e^{(2a)}\right)e^{(2bx)}}{b^{4}}+\frac{\left(4b^{3}x^{3}+6b^{2}x^{2}+6bx+3\right)e^{(-2bx-2a)}}{b^{4}}\right)cd^{3}-\frac{1}{80}\left(8x^{5}-\frac{5\left(2b^{4}x^{4}e^{(2a)}-4b^{3}x^{3}e^{(2a)}+6b^{2}x^{2}e^{(2a)}-6bxe^{(2a)}+3e^{(2a)}\right)e^{(2bx)}}{b^{5}}+\frac{5\left(2b^{4}x^{4}+4b^{3}x^{3}+6b^{2}x^{2}+6bx+3\right)e^{(-2bx-2a)}}{b^{5}}\right)d^{4}-\frac{1}{8}c^{4}\left(4x-\frac{e^{(2bx+2a)}}{b}+\frac{e^{(-2bx-2a)}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*x^2 - (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 + (2*b*x + 1)*e^(-2*b
*x - 2*a)/b^2)*c^3*d - 1/8*(8*x^3 - 3*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) +
e^(2*a))*e^(2*b*x)/b^3 + 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3)*c^
2*d^2 - 1/8*(4*x^4 - (4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(2*a)
 - 3*e^(2*a))*e^(2*b*x)/b^4 + (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x
 - 2*a)/b^4)*c*d^3 - 1/80*(8*x^5 - 5*(2*b^4*x^4*e^(2*a) - 4*b^3*x^3*e^(2*a)
 + 6*b^2*x^2*e^(2*a) - 6*b*x*e^(2*a) + 3*e^(2*a))*e^(2*b*x)/b^5 + 5*(2*b^4*
x^4 + 4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^5)*d^4 - 1/8*c^
4*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(148) = 296$.
time = 0.37, size = 312, normalized size = 1.93

$$-\frac{2b^{5}d^{4}x^{5}+10b^{5}cd^{3}x^{4}+20b^{5}c^{2}d^{2}x^{3}+20b^{5}c^{3}dx^{2}+10b^{5}c^{4}x+5\left(2b^{3}d^{4}x^{3}+6b^{3}cd^{3}x^{2}+2b^{3}c^{3}d+3bcd^{3}+3\left(2b^{3}c^{2}d^{2}+bd^{4}\right)x\right)\cosh\left(bx+a\right)^{2}-5\left(2b^{4}d^{4}x^{4}+8b^{4}cd^{3}x^{3}+2b^{4}c^{4}+6b^{2}c^{2}d^{2}+3d^{4}+6\left(2b^{4}c^{2}d^{2}+b^{2}d^{4}\right)x^{2}+4\left(2b^{4}c^{3}d+3b^{2}cd^{3}\right)x\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)+5\left(2b^{3}d^{4}x^{3}+6b^{3}cd^{3}x^{2}+2b^{3}c^{3}d+3bcd^{3}+3\left(2b^{3}c^{2}d^{2}+bd^{4}\right)x\right)\sinh\left(bx+a\right)^{2}}{20b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/20*(2*b^5*d^4*x^5 + 10*b^5*c*d^3*x^4 + 20*b^5*c^2*d^2*x^3 + 20*b^5*c^3*d
*x^2 + 10*b^5*c^4*x + 5*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d + 3*
b*c*d^3 + 3*(2*b^3*c^2*d^2 + b*d^4)*x)*cosh(b*x + a)^2 - 5*(2*b^4*d^4*x^4 +
 8*b^4*c*d^3*x^3 + 2*b^4*c^4 + 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 + b
^2*d^4)*x^2 + 4*(2*b^4*c^3*d + 3*b^2*c*d^3)*x)*cosh(b*x + a)*sinh(b*x + a)
+ 5*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d + 3*b*c*d^3 + 3*(2*b^3*c
^2*d^2 + b*d^4)*x)*sinh(b*x + a)^2)/b^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(156) = 312$.
time = 0.70, size = 660, normalized size = 4.07

$$\begin{cases} \frac{c^4 x \sinh^2(a+bx)}{2} - \frac{c^4 x \cosh^2(a+bx)}{2} + c^3dx^2\sinh^2(a+bx) - c^3dx^2\cosh^2(a+bx) + c^2d^2x^3\sinh^2(a+bx) - c^2d^2x^3\cosh^2(a+bx) + \frac{cd^3x^4\sinh^2(a+bx)}{2} - \frac{cd^3x^4\cosh^2(a+bx)}{2} + \frac{d^4x^5\sinh^2(a+bx)}{10} - \frac{d^4x^5\cosh^2(a+bx)}{10} + \frac{c^4\sinh(a+bx)\cosh(a+bx)}{2b} + \frac{2c^3dx\sinh(a+bx)\cosh(a+bx)}{b} + \frac{3c^2d^2x^2\sinh(a+bx)\cosh(a+bx)}{b} + \frac{2cd^3x^3\sinh(a+bx)\cosh(a+bx)}{b} + \frac{d^4x^4\sinh(a+bx)\cosh(a+bx)}{2b} - \frac{c^3d\cosh^2(a+bx)}{b^2} - \frac{3c^2d^2x\sinh^2(a+bx)}{2b^2} - \frac{3c^2d^2x\cosh^2(a+bx)}{2b^2} - \frac{3cd^3x^2\sinh^2(a+bx)}{2b^2} - \frac{3cd^3x^2\cosh^2(a+bx)}{2b^2} - \frac{d^4x^3\sinh^2(a+bx)}{2b^2} - \frac{d^4x^3\cosh^2(a+bx)}{2b^2} + \frac{3c^2d^2\sinh(a+bx)\cosh(a+bx)}{2b^3} + \frac{3cd^3x\sinh(a+bx)\cosh(a+bx)}{b^3} + \frac{3d^4x^2\sinh(a+bx)\cosh(a+bx)}{2b^3} - \frac{3cd^3\cosh^2(a+bx)}{2b^4} - \frac{3d^4x\sinh^2(a+bx)}{4b^4} - \frac{3d^4x\cosh^2(a+bx)}{4b^4} + \frac{3d^4\sinh(a+bx)\cosh(a+bx)}{4b^5} & \text{for } b \neq 0 \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\sinh^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((c**4*x*sinh(a + b*x)**2/2 - c**4*x*cosh(a + b*x)**2/2 + c**3*d*x
**2*sinh(a + b*x)**2 - c**3*d*x**2*cosh(a + b*x)**2 + c**2*d**2*x**3*sinh(a
 + b*x)**2 - c**2*d**2*x**3*cosh(a + b*x)**2 + c*d**3*x**4*sinh(a + b*x)**2
/2 - c*d**3*x**4*cosh(a + b*x)**2/2 + d**4*x**5*sinh(a + b*x)**2/10 - d**4*
x**5*cosh(a + b*x)**2/10 + c**4*sinh(a + b*x)*cosh(a + b*x)/(2*b) + 2*c**3*
d*x*sinh(a + b*x)*cosh(a + b*x)/b + 3*c**2*d**2*x**2*sinh(a + b*x)*cosh(a +
 b*x)/b + 2*c*d**3*x**3*sinh(a + b*x)*cosh(a + b*x)/b + d**4*x**4*sinh(a +
b*x)*cosh(a + b*x)/(2*b) - c**3*d*cosh(a + b*x)**2/b**2 - 3*c**2*d**2*x*sin
h(a + b*x)**2/(2*b**2) - 3*c**2*d**2*x*cosh(a + b*x)**2/(2*b**2) - 3*c*d**3
*x**2*sinh(a + b*x)**2/(2*b**2) - 3*c*d**3*x**2*cosh(a + b*x)**2/(2*b**2) -
 d**4*x**3*sinh(a + b*x)**2/(2*b**2) - d**4*x**3*cosh(a + b*x)**2/(2*b**2)
+ 3*c**2*d**2*sinh(a + b*x)*cosh(a + b*x)/(2*b**3) + 3*c*d**3*x*sinh(a + b*
x)*cosh(a + b*x)/b**3 + 3*d**4*x**2*sinh(a + b*x)*cosh(a + b*x)/(2*b**3) -
3*c*d**3*cosh(a + b*x)**2/(2*b**4) - 3*d**4*x*sinh(a + b*x)**2/(4*b**4) - 3
*d**4*x*cosh(a + b*x)**2/(4*b**4) + 3*d**4*sinh(a + b*x)*cosh(a + b*x)/(4*b
**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4
+ d**4*x**5/5)*sinh(a)**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(148) = 296$.
time = 0.45, size = 374, normalized size = 2.31

$$-\frac{1}{10}d^4x^5 - \frac{1}{2}cd^3x^4 - c^2d^2x^3 - c^3dx^2 - \frac{1}{2}c^4x + \frac{\left(2b^4d^4x^4 + 8b^4cd^3x^3 + 12b^4c^2d^2x^2 - 4b^3d^4x^3 + 8b^4c^3dx - 12b^3cd^3x^2 + 2b^4c^4 - 12b^3c^2d^2x + 6b^2d^4x^2 - 4b^3c^3d + 12b^2cd^3x + 6b^2c^2d^2 - 6bd^4x - 6bcd^3 + 3d^4\right)e^{(2bx+2a)}}{16b^5} - \frac{\left(2b^4d^4x^4 + 8b^4cd^3x^3 + 12b^4c^2d^2x^2 + 4b^3d^4x^3 + 8b^4c^3dx + 12b^3cd^3x^2 + 2b^4c^4 + 12b^3c^2d^2x + 6b^2d^4x^2 + 4b^3c^3d + 12b^2cd^3x + 6b^2c^2d^2 + 6bd^4x + 6bcd^3 + 3d^4\right)e^{(-2bx-2a)}}{16b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/10*d^4*x^5 - 1/2*c*d^3*x^4 - c^2*d^2*x^3 - c^3*d*x^2 - 1/2*c^4*x + 1/16*
(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 8*b
^4*c^3*d*x - 12*b^3*c*d^3*x^2 + 2*b^4*c^4 - 12*b^3*c^2*d^2*x + 6*b^2*d^4*x^
2 - 4*b^3*c^3*d + 12*b^2*c*d^3*x + 6*b^2*c^2*d^2 - 6*b*d^4*x - 6*b*c*d^3 +
3*d^4)*e^(2*b*x + 2*a)/b^5 - 1/16*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4
*c^2*d^2*x^2 + 4*b^3*d^4*x^3 + 8*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + 2*b^4*c^4
 + 12*b^3*c^2*d^2*x + 6*b^2*d^4*x^2 + 4*b^3*c^3*d + 12*b^2*c*d^3*x + 6*b^2*
c^2*d^2 + 6*b*d^4*x + 6*b*c*d^3 + 3*d^4)*e^(-2*b*x - 2*a)/b^5
```

Mupad [B]
time = 0.69, size = 334, normalized size = 2.06

$$\frac{c^4\sinh(2a+2bx)}{4b}-\frac{d^4x^5}{10}-c^3dx^2-\frac{cd^3x^4}{2}-\frac{c^4x}{2}+\frac{3d^4\sinh(2a+2bx)}{8b^5}-c^2d^2x^3-\frac{c^3d\cosh(2a+2bx)}{2b^2}-\frac{3cd^3\cosh(2a+2bx)}{4b^4}-\frac{3d^4x\cosh(2a+2bx)}{4b^4}+\frac{3c^2d^2\sinh(2a+2bx)}{4b^3}-\frac{d^4x^3\cosh(2a+2bx)}{2b^2}+\frac{d^4x^4\sinh(2a+2bx)}{4b}+\frac{3d^4x^2\sinh(2a+2bx)}{4b^3}+\frac{3c^2d^2x^2\sinh(2a+2bx)}{2b}+\frac{c^3dx\sinh(2a+2bx)}{b}+\frac{3cd^3x\sinh(2a+2bx)}{2b^3}-\frac{3c^2d^2x\cosh(2a+2bx)}{2b^2}-\frac{3cd^3x^2\cosh(2a+2bx)}{2b^2}+\frac{cd^3x^3\sinh(2a+2bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2*(c + d*x)^4,x)
```

```
[Out] (c^4*sinh(2*a + 2*b*x))/(4*b) - (d^4*x^5)/10 - c^3*d*x^2 - (c*d^3*x^4)/2 -
(c^4*x)/2 + (3*d^4*sinh(2*a + 2*b*x))/(8*b^5) - c^2*d^2*x^3 - (c^3*d*cosh(2
*a + 2*b*x))/(2*b^2) - (3*c*d^3*cosh(2*a + 2*b*x))/(4*b^4) - (3*d^4*x*cosh(
2*a + 2*b*x))/(4*b^4) + (3*c^2*d^2*sinh(2*a + 2*b*x))/(4*b^3) - (d^4*x^3*co
sh(2*a + 2*b*x))/(2*b^2) + (d^4*x^4*sinh(2*a + 2*b*x))/(4*b) + (3*d^4*x^2*s
inh(2*a + 2*b*x))/(4*b^3) + (3*c^2*d^2*x^2*sinh(2*a + 2*b*x))/(2*b) + (c^3*
d*x*sinh(2*a + 2*b*x))/b + (3*c*d^3*x*sinh(2*a + 2*b*x))/(2*b^3) - (3*c^2*d
^2*x*cosh(2*a + 2*b*x))/(2*b^2) - (3*c*d^3*x^2*cosh(2*a + 2*b*x))/(2*b^2) +
 (c*d^3*x^3*sinh(2*a + 2*b*x))/b
```

3.9 $\int (c+dx)^3 \sinh^2(a+bx)\,dx$

Optimal. Leaf size=134

$$-\frac{3cd^2x}{4b^2}-\frac{3d^3x^2}{8b^2}-\frac{(c+dx)^4}{8d}+\frac{3d^2(c+dx)\cosh(a+bx)\sinh(a+bx)}{4b^3}+\frac{(c+dx)^3\cosh(a+bx)\sinh(a+bx)}{2b}-$$

```
[Out] -3/4*c*d^2*x/b^2-3/8*d^3*x^2/b^2-1/8*(d*x+c)^4/d+3/4*d^2*(d*x+c)*cosh(b*x+a
)*sinh(b*x+a)/b^3+1/2*(d*x+c)^3*cosh(b*x+a)*sinh(b*x+a)/b-3/8*d^3*sinh(b*x+
a)^2/b^4-3/4*d*(d*x+c)^2*sinh(b*x+a)^2/b^2
```

Rubi [A]
time = 0.05, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3392, 32, 3391}

$$-\frac{3d^3\sinh^2(a+bx)}{8b^4}+\frac{3d^2(c+dx)\sinh(a+bx)\cosh(a+bx)}{4b^3}-\frac{3d(c+dx)^2\sinh^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{3cd^2x}{4b^2}-\frac{3d^3x^2}{8b^2}-\frac{(c+dx)^4}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sinh[a + b*x]^2,x]
```

```
[Out] (-3*c*d^2*x)/(4*b^2) - (3*d^3*x^2)/(8*b^2) - (c + d*x)^4/(8*d) + (3*d^2*(c
+ d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^3) + ((c + d*x)^3*Cosh[a + b*x]*Si
nh[a + b*x])/(2*b) - (3*d^3*Sinh[a + b*x]^2)/(8*b^4) - (3*d*(c + d*x)^2*Sin
h[a + b*x]^2)/(4*b^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
 - Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int (c+dx)^3 \sinh^2(a+bx)\,dx = \frac{(c+dx)^3\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{3d(c+dx)^2\sinh^2(a+bx)}{4b^2} - \frac{1}{2}\int (c+$$
$$= -\frac{(c+dx)^4}{8d} + \frac{3d^2(c+dx)\cosh(a+bx)\sinh(a+bx)}{4b^3} + \frac{(c+dx)^3\cosh(a+ba}{2b}$$
$$= -\frac{3cd^2x}{4b^2} - \frac{3d^3x^2}{8b^2} - \frac{(c+dx)^4}{8d} + \frac{3d^2(c+dx)\cosh(a+bx)\sinh(a+bx)}{4b^3} + \frac{(c-}{}$$

Mathematica [A]
time = 0.25, size = 104, normalized size = 0.78

$$\frac{-2b^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)-3d(d^2+2b^2(c+dx)^2)\cosh(2(a+bx))+2b(c+dx)\left(3d^2+2b^2(c+dx)^2\right)\sinh(2(a+bx))}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sinh[a + b*x]^2,x]
```

```
[Out] (-2*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 3*d*(d^2 + 2*b^2*(c
 + d*x)^2)*Cosh[2*(a + b*x)] + 2*b*(c + d*x)*(3*d^2 + 2*b^2*(c + d*x)^2)*Si
nh[2*(a + b*x)])/(16*b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 522 vs. 2(120) = 240.
time = 0.37, size = 523, normalized size = 3.90 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(d^3/b^3*(1/2*(b*x+a)^3*cosh(b*x+a)*sinh(b*x+a)-1/8*(b*x+a)^4-3/4*(b*x+
a)^2*cosh(b*x+a)^2+3/4*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+3/8*(b*x+a)^2-3/8*co
sh(b*x+a)^2)-3*d^3/b^3*a*(1/2*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)-1/6*(b*x+a)
^3-1/2*(b*x+a)*cosh(b*x+a)^2+1/4*cosh(b*x+a)*sinh(b*x+a)+1/4*b*x+1/4*a)+3*d
^2/b^2*c*(1/2*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)-1/6*(b*x+a)^3-1/2*(b*x+a)*c
osh(b*x+a)^2+1/4*cosh(b*x+a)*sinh(b*x+a)+1/4*b*x+1/4*a)+3*d^3/b^3*a^2*(1/2*
(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)-6*d^2/b^2*
a*c*(1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)+3
*d/b*c^2*(1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-1/4*(b*x+a)^2-1/4*cosh(b*x+a)
^2)-d^3/b^3*a^3*(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*x-1/2*a)+3*d^2/b^2*a^2*c
*(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*x-1/2*a)-3*d/b*a*c^2*(1/2*cosh(b*x+a)*s
inh(b*x+a)-1/2*b*x-1/2*a)+c^3*(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*x-1/2*a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(120) = 240$.
time = 0.28, size = 263, normalized size = 1.96

$$-\frac{3}{16}\left(4x^2 - \frac{\left(2bxe^{(2a)} - e^{(2a)}\right)e^{(2bx)}}{b^2} + \frac{(2bx+1)e^{(-2bx-2a)}}{b^2}\right)c^2d - \frac{1}{16}\left(8x^3 - \frac{3\left(2b^2x^2e^{(2a)} - 2bxe^{(2a)} + e^{(2a)}\right)e^{(2bx)}}{b^3} + \frac{3\left(2b^2x^2 + 2bx + 1\right)e^{(-2bx-2a)}}{b^3}\right)cd^2 - \frac{1}{32}\left(4x^4 - \frac{\left(4b^3x^3e^{(2a)} - 6b^2x^2e^{(2a)} + 6bxe^{(2a)} - 3e^{(2a)}\right)e^{(2bx)}}{b^4} + \frac{\left(4b^3x^3 + 6b^2x^2 + 6bx + 3\right)e^{(-2bx-2a)}}{b^4}\right)d^3 - \frac{1}{8}c^3\left(4x - \frac{e^{(2bx+2a)}}{b} + \frac{e^{(-2bx-2a)}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -3/16*(4*x^2 - (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 + (2*b*x + 1)*e^(-2*
b*x - 2*a)/b^2)*c^2*d - 1/16*(8*x^3 - 3*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a)
+ e^(2*a))*e^(2*b*x)/b^3 + 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3)*
c*d^2 - 1/32*(4*x^4 - (4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(2*a
) - 3*e^(2*a))*e^(2*b*x)/b^4 + (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*
x - 2*a)/b^4)*d^3 - 1/8*c^3*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)
```

Fricas [A]
time = 0.35, size = 209, normalized size = 1.56

$$-\frac{2b^4d^3x^4 + 8b^4cd^2x^3 + 12b^4c^2dx^2 + 8b^4c^3x + 3\left(2b^2d^3x^2 + 4b^2cd^2x + 2b^2c^2d + d^3\right)\cosh\left(bx+a\right)^2 - 4\left(2b^3d^3x^3 + 6b^3cd^2x^2 + 2b^3c^3 + 3bcd^2 + 3\left(2b^3c^2d + bd^3\right)x\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right) + 3\left(2b^2d^3x^2 + 4b^2cd^2x + 2b^2c^2d + d^3\right)\sinh\left(bx+a\right)^2}{16b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(2*b^4*d^3*x^4 + 8*b^4*c*d^2*x^3 + 12*b^4*c^2*d*x^2 + 8*b^4*c^3*x + 3
*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*cosh(b*x + a)^2 - 4*(2
*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 + 3*b*c*d^2 + 3*(2*b^3*c^2*d + b
*d^3)*x)*cosh(b*x + a)*sinh(b*x + a) + 3*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2
*b^2*c^2*d + d^3)*sinh(b*x + a)^2)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(131) = 262$.
time = 0.38, size = 456, normalized size = 3.40

$$\begin{cases} \frac{c^3x\sinh^2(a+bx)}{2} - \frac{c^3x\cosh^2(a+bx)}{2} + \frac{3c^2dx^2\sinh^2(a+bx)}{4} - \frac{3c^2dx^2\cosh^2(a+bx)}{4} + \frac{cd^2x^3\sinh^2(a+bx)}{2} - \frac{cd^2x^3\cosh^2(a+bx)}{2} + \frac{d^3x^4\sinh^2(a+bx)}{8} - \frac{d^3x^4\cosh^2(a+bx)}{8} + \frac{c^3\sinh(a+bx)\cosh(a+bx)}{2b} + \frac{3c^2dx\sinh(a+bx)\cosh(a+bx)}{2b} + \frac{3cd^2x^2\sinh(a+bx)\cosh(a+bx)}{2b} + \frac{d^3x^3\sinh(a+bx)\cosh(a+bx)}{2b} - \frac{3c^2d\cosh^2(a+bx)}{4b^2} - \frac{3cd^2x\sinh^2(a+bx)}{4b^2} - \frac{3cd^2x\cosh^2(a+bx)}{4b^2} - \frac{3d^3x^2\sinh^2(a+bx)}{8b^2} - \frac{3d^3x^2\cosh^2(a+bx)}{8b^2} + \frac{3cd^2\sinh(a+bx)\cosh(a+bx)}{4b^3} + \frac{3d^3x\sinh(a+bx)\cosh(a+bx)}{4b^3} - \frac{3d^3\cosh^2(a+bx)}{8b^4} & \text{for}\ b \neq 0 \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sinh^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((c**3*x*sinh(a + b*x)**2/2 - c**3*x*cosh(a + b*x)**2/2 + 3*c**2*d
*x**2*sinh(a + b*x)**2/4 - 3*c**2*d*x**2*cosh(a + b*x)**2/4 + c*d**2*x**3*s
inh(a + b*x)**2/2 - c*d**2*x**3*cosh(a + b*x)**2/2 + d**3*x**4*sinh(a + b*x
)**2/8 - d**3*x**4*cosh(a + b*x)**2/8 + c**3*sinh(a + b*x)*cosh(a + b*x)/(2
*b) + 3*c**2*d*x*sinh(a + b*x)*cosh(a + b*x)/(2*b) + 3*c*d**2*x**2*sinh(a +
 b*x)*cosh(a + b*x)/(2*b) + d**3*x**3*sinh(a + b*x)*cosh(a + b*x)/(2*b) - 3
```

```
*c**2*d*cosh(a + b*x)**2/(4*b**2) - 3*c*d**2*x*sinh(a + b*x)**2/(4*b**2) -
3*c*d**2*x*cosh(a + b*x)**2/(4*b**2) - 3*d**3*x**2*sinh(a + b*x)**2/(8*b**2
) - 3*d**3*x**2*cosh(a + b*x)**2/(8*b**2) + 3*c*d**2*sinh(a + b*x)*cosh(a +
 b*x)/(4*b**3) + 3*d**3*x*sinh(a + b*x)*cosh(a + b*x)/(4*b**3) - 3*d**3*cos
h(a + b*x)**2/(8*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3
 + d**3*x**4/4)*sinh(a)**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(120) = 240.
time = 0.43, size = 243, normalized size = 1.81

$$-\frac{1}{8}\,d^{3}x^{4}-\frac{1}{2}\,cd^{2}x^{3}-\frac{3}{4}\,c^{2}dx^{2}-\frac{1}{2}\,c^{3}x+\frac{\left(4\,b^{3}d^{3}x^{3}+12\,b^{3}cd^{2}x^{2}+12\,b^{3}c^{2}dx-6\,b^{2}d^{3}x^{2}+4\,b^{3}c^{3}-12\,b^{2}cd^{2}x-6\,b^{2}c^{2}d+6\,bd^{3}x+6\,bcd^{2}-3\,d^{3}\right)e^{\left(2\,bx+2\,a\right)}}{32\,b^{4}}-\frac{\left(4\,b^{3}d^{3}x^{3}+12\,b^{3}cd^{2}x^{2}+12\,b^{3}c^{2}dx+6\,b^{2}d^{3}x^{2}+4\,b^{3}c^{3}+12\,b^{2}cd^{2}x+6\,b^{2}c^{2}d+6\,bd^{3}x+6\,bcd^{2}+3\,d^{3}\right)e^{\left(-2\,bx-2\,a\right)}}{32\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*d^3*x^4 - 1/2*c*d^2*x^3 - 3/4*c^2*d*x^2 - 1/2*c^3*x + 1/32*(4*b^3*d^3*
x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x - 6*b^2*d^3*x^2 + 4*b^3*c^3 - 12*b^
2*c*d^2*x - 6*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 3*d^3)*e^(2*b*x + 2*a)/b^
4 - 1/32*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x + 6*b^2*d^3*x^2
 + 4*b^3*c^3 + 12*b^2*c*d^2*x + 6*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 + 3*d^3
)*e^(-2*b*x - 2*a)/b^4
```

Mupad [B]
time = 0.38, size = 229, normalized size = 1.71

$$-\frac{\frac{3\,d^3\,\mathrm{cosh}\left(2\,a+2\,b\,x\right)}{2}+4\,b^4\,c^3\,x-2\,b^3\,c^3\,\mathrm{sinh}\left(2\,a+2\,b\,x\right)+b^4\,d^3\,x^4+3\,b^2\,c^2\,d\,\mathrm{cosh}\left(2\,a+2\,b\,x\right)+6\,b^4\,c^2\,d\,x^2+4\,b^4\,c\,d^2\,x^3+3\,b^2\,d^3\,x^2\,\mathrm{cosh}\left(2\,a+2\,b\,x\right)-2\,b^3\,d^3\,x^3\,\mathrm{sinh}\left(2\,a+2\,b\,x\right)-3\,b\,c\,d^2\,\mathrm{sinh}\left(2\,a+2\,b\,x\right)-3\,b\,d^3\,x\,\mathrm{sinh}\left(2\,a+2\,b\,x\right)+6\,b^2\,c\,d^2\,x\,\mathrm{cosh}\left(2\,a+2\,b\,x\right)-6\,b^3\,c^2\,d\,x\,\mathrm{sinh}\left(2\,a+2\,b\,x\right)-6\,b^3\,c\,d^2\,x^2\,\mathrm{sinh}\left(2\,a+2\,b\,x\right)}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] -((3*d^3*cosh(2*a + 2*b*x))/2 + 4*b^4*c^3*x - 2*b^3*c^3*sinh(2*a + 2*b*x) +
 b^4*d^3*x^4 + 3*b^2*c^2*d*cosh(2*a + 2*b*x) + 6*b^4*c^2*d*x^2 + 4*b^4*c*d^
2*x^3 + 3*b^2*d^3*x^2*cosh(2*a + 2*b*x) - 2*b^3*d^3*x^3*sinh(2*a + 2*b*x) -
 3*b*c*d^2*sinh(2*a + 2*b*x) - 3*b*d^3*x*sinh(2*a + 2*b*x) + 6*b^2*c*d^2*x*
cosh(2*a + 2*b*x) - 6*b^3*c^2*d*x*sinh(2*a + 2*b*x) - 6*b^3*c*d^2*x^2*sinh(
2*a + 2*b*x))/(8*b^4)
```

3.10 $\int (c+dx)^2 \sinh^2(a+bx)\,dx$

Optimal. Leaf size=95

$$-\frac{d^2x}{4b^2}-\frac{(c+dx)^3}{6d}+\frac{d^2\cosh(a+bx)\sinh(a+bx)}{4b^3}+\frac{(c+dx)^2\cosh(a+bx)\sinh(a+bx)}{2b}-\frac{d(c+dx)\sinh^2(a+}{2b^2}$$

```
[Out] -1/4*d^2*x/b^2-1/6*(d*x+c)^3/d+1/4*d^2*cosh(b*x+a)*sinh(b*x+a)/b^3+1/2*(d*x
+c)^2*cosh(b*x+a)*sinh(b*x+a)/b-1/2*d*(d*x+c)*sinh(b*x+a)^2/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 32, 2715, 8}

$$\frac{d^2\sinh(a+bx)\cosh(a+bx)}{4b^3}-\frac{d(c+dx)\sinh^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{d^2x}{4b^2}-\frac{(c+dx)^3}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sinh[a + b*x]^2,x]
```

```
[Out] -1/4*(d^2*x)/b^2 - (c + d*x)^3/(6*d) + (d^2*Cosh[a + b*x]*Sinh[a + b*x])/(4
*b^3) + ((c + d*x)^2*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (d*(c + d*x)*Sinh
[a + b*x]^2)/(2*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
```

```
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int (c+dx)^2 \sinh^2(a+bx)\,dx = \frac{(c+dx)^2\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{d(c+dx)\sinh^2(a+bx)}{2b^2} - \frac{1}{2}\int (c+dx$$
$$= -\frac{(c+dx)^3}{6d} + \frac{d^2\cosh(a+bx)\sinh(a+bx)}{4b^3} + \frac{(c+dx)^2\cosh(a+bx)\sinh(a}{2b}$$
$$= -\frac{d^2x}{4b^2} - \frac{(c+dx)^3}{6d} + \frac{d^2\cosh(a+bx)\sinh(a+bx)}{4b^3} + \frac{(c+dx)^2\cosh(a+bx)\,s}{2b}$$

Mathematica [A]
time = 0.19, size = 75, normalized size = 0.79

$$\frac{-4b^3x(3c^2+3cdx+d^2x^2)-6bd(c+dx)\cosh(2(a+bx))+3(d^2+2b^2(c+dx)^2)\sinh(2(a+bx))}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sinh[a + b*x]^2,x]
[Out] (-4*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) - 6*b*d*(c + d*x)*Cosh[2*(a + b*x)] +
 3*(d^2 + 2*b^2*(c + d*x)^2)*Sinh[2*(a + b*x)])/(24*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(85) = 170$.
time = 0.37, size = 262, normalized size = 2.76

method	result
risch	$-\frac{d^2x^3}{6} - \frac{dcx^2}{2} - \frac{c^2x}{2} - \frac{c^3}{6d} + \frac{(2b^2d^2x^2+4b^2cdx+2b^2c^2-2bd^2x-2bcd+d^2)e^{2bx+2a}}{16b^3} - \frac{(2b^2d^2x^2+4b^2cdx+2b^2c^2}{1}$
derivativedivides	$\frac{d^2\left(\frac{(bx+a)^2\cosh(bx+a)\sinh(bx+a)}{2} - \frac{(bx+a)^3}{6} - \frac{(bx+a)\left(\cosh^2(bx+a)\right)}{2} + \frac{\cosh(bx+a)\sinh(bx+a)}{4} + \frac{bx}{4} + \frac{a}{4}\right)}{b^2} - \frac{2d^2a\left(\frac{(bx+a)\cosh(bx+}{2}\right.}{}$
default	$\frac{d^2\left(\frac{(bx+a)^2\cosh(bx+a)\sinh(bx+a)}{2} - \frac{(bx+a)^3}{6} - \frac{(bx+a)\left(\cosh^2(bx+a)\right)}{2} + \frac{\cosh(bx+a)\sinh(bx+a)}{4} + \frac{bx}{4} + \frac{a}{4}\right)}{b^2} - \frac{2d^2a\left(\frac{(bx+a)\cosh(bx+}{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(d^2/b^2*(1/2*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)-1/6*(b*x+a)^3-1/2*(b*x+
a)*cosh(b*x+a)^2+1/4*cosh(b*x+a)*sinh(b*x+a)+1/4*b*x+1/4*a)-2*d^2/b^2*a*(1/
```

```
2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)+2*d/b*c*
(1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)+d^2/b
^2*a^2*(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*x-1/2*a)-2*d/b*a*c*(1/2*cosh(b*x+
a)*sinh(b*x+a)-1/2*b*x-1/2*a)+c^2*(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*x-1/2*
a))
```

Maxima [A]
time = 0.27, size = 165, normalized size = 1.74

$$-\frac{1}{8}\left(4x^2-\frac{(2bxe^{(2a)}-e^{(2a)})e^{(2bx)}}{b^2}+\frac{(2bx+1)e^{(-2bx-2a)}}{b^2}\right)cd-\frac{1}{48}\left(8x^3-\frac{3(2b^2x^2e^{(2a)}-2bxe^{(2a)}+e^{(2a)})e^{(2bx)}}{b^3}+\frac{3(2b^2x^2+2bx+1)e^{(-2bx-2a)}}{b^3}\right)d^2-\frac{1}{8}c^2\left(4x-\frac{e^{(2bx+2a)}}{b}+\frac{e^{(-2bx-2a)}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(4*x^2 - (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 + (2*b*x + 1)*e^(-2*b
*x - 2*a)/b^2)*c*d - 1/48*(8*x^3 - 3*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e
^(2*a))*e^(2*b*x)/b^3 + 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3)*d^2
 - 1/8*c^2*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)
```

Fricas [A]
time = 0.34, size = 123, normalized size = 1.29

$$-\frac{2b^3d^2x^3+6b^3cdx^2+6b^3c^2x+3(bd^2x+bcd)\cosh(bx+a)^2-3(2b^2d^2x^2+4b^2cdx+2b^2c^2+d^2)\cosh(bx+a)\sinh(bx+a)+3(bd^2x+bcd)\sinh(bx+a)^2}{12b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/12*(2*b^3*d^2*x^3 + 6*b^3*c*d*x^2 + 6*b^3*c^2*x + 3*(b*d^2*x + b*c*d)*co
sh(b*x + a)^2 - 3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*cosh(b*x
+ a)*sinh(b*x + a) + 3*(b*d^2*x + b*c*d)*sinh(b*x + a)^2)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(85) = 170$.
time = 0.24, size = 264, normalized size = 2.78

$$\begin{cases}\frac{c^2x\sinh^2(a+bx)}{2}-\frac{c^2x\cosh^2(a+bx)}{2}+\frac{cdx^2\sinh^2(a+bx)}{2}-\frac{cdx^2\cosh^2(a+bx)}{2}+\frac{d^2x^3\sinh^2(a+bx)}{6}-\frac{d^2x^3\cosh^2(a+bx)}{6}+\frac{c^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{cdx\sinh(a+bx)\cosh(a+bx)}{b}+\frac{d^2x^2\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{cd\cosh^2(a+bx)}{2b^2}-\frac{d^2x\sinh^2(a+bx)}{4b^2}-\frac{d^2x\cosh^2(a+bx)}{4b^2}+\frac{d^2\sinh(a+bx)\cosh(a+bx)}{4b^3} & \text{for } b\neq 0\\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sinh^2(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((c**2*x*sinh(a + b*x)**2/2 - c**2*x*cosh(a + b*x)**2/2 + c*d*x**2
*sinh(a + b*x)**2/2 - c*d*x**2*cosh(a + b*x)**2/2 + d**2*x**3*sinh(a + b*x)
**2/6 - d**2*x**3*cosh(a + b*x)**2/6 + c**2*sinh(a + b*x)*cosh(a + b*x)/(2*
b) + c*d*x*sinh(a + b*x)*cosh(a + b*x)/b + d**2*x**2*sinh(a + b*x)*cosh(a +
 b*x)/(2*b) - c*d*cosh(a + b*x)**2/(2*b**2) - d**2*x*sinh(a + b*x)**2/(4*b*
```

```
*2) - d**2*x*cosh(a + b*x)**2/(4*b**2) + d**2*sinh(a + b*x)*cosh(a + b*x)/(
4*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sinh(a)**2, True))
```

Giac [A]
time = 0.44, size = 136, normalized size = 1.43

$$-\frac{1}{6}\,d^2x^3-\frac{1}{2}\,cdx^2-\frac{1}{2}\,c^2x+\frac{\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-2\,bd^2x-2\,bcd+d^2\right)e^{(2\,bx+2\,a)}}{16\,b^3}-\frac{\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2+2\,bd^2x+2\,bcd+d^2\right)e^{(-2\,bx-2\,a)}}{16\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/6*d^2*x^3 - 1/2*c*d*x^2 - 1/2*c^2*x + 1/16*(2*b^2*d^2*x^2 + 4*b^2*c*d*x
+ 2*b^2*c^2 - 2*b*d^2*x - 2*b*c*d + d^2)*e^(2*b*x + 2*a)/b^3 - 1/16*(2*b^2*
d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 2*b*d^2*x + 2*b*c*d + d^2)*e^(-2*b*x -
2*a)/b^3
```

Mupad [B]
time = 0.18, size = 127, normalized size = 1.34

$$\frac{c^2\sinh(2\,a+2\,b\,x)}{4\,b}-\frac{d^2\,x^3}{6}-\frac{c^2\,x}{2}+\frac{d^2\sinh(2\,a+2\,b\,x)}{8\,b^3}-\frac{c\,d\,x^2}{2}-\frac{d^2\,x\cosh(2\,a+2\,b\,x)}{4\,b^2}+\frac{d^2\,x^2\sinh(2\,a+2\,b\,x)}{4\,b}-\frac{c\,d\cosh(2\,a+2\,b\,x)}{4\,b^2}+\frac{c\,d\,x\sinh(2\,a+2\,b\,x)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] (c^2*sinh(2*a + 2*b*x))/(4*b) - (d^2*x^3)/6 - (c^2*x)/2 + (d^2*sinh(2*a + 2
*b*x))/(8*b^3) - (c*d*x^2)/2 - (d^2*x*cosh(2*a + 2*b*x))/(4*b^2) + (d^2*x^2
*sinh(2*a + 2*b*x))/(4*b) - (c*d*cosh(2*a + 2*b*x))/(4*b^2) + (c*d*x*sinh(2
*a + 2*b*x))/(2*b)
```

3.11 $\int (c+dx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=55

$$-\frac{cx}{2}-\frac{dx^2}{4}+\frac{(c+dx)\cosh(a+bx)\sinh(a+bx)}{2b}-\frac{d\sinh^2(a+bx)}{4b^2}$$

```
[Out] -1/2*c*x-1/4*d*x^2+1/2*(d*x+c)*cosh(b*x+a)*sinh(b*x+a)/b-1/4*d*sinh(b*x+a)^
2/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.071$, Rules used = {3391}

$$-\frac{d\sinh^2(a+bx)}{4b^2}+\frac{(c+dx)\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{cx}{2}-\frac{dx^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sinh[a + b*x]^2,x]
```

```
[Out] -1/2*(c*x) - (d*x^2)/4 + ((c + d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (d
*Sinh[a + b*x]^2)/(4*b^2)
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\sinh^2(a+bx)\,dx&=\frac{(c+dx)\cosh(a+bx)\sinh(a+bx)}{2b}-\frac{d\sinh^2(a+bx)}{4b^2}-\frac{1}{2}\int (c+dx)\,dx\\&=-\frac{cx}{2}-\frac{dx^2}{4}+\frac{(c+dx)\cosh(a+bx)\sinh(a+bx)}{2b}-\frac{d\sinh^2(a+bx)}{4b^2}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 52, normalized size = 0.95

$$\frac{-d\cosh(2(a+bx))+2b(-2ac-bx(2c+dx)+(c+dx)\sinh(2(a+bx)))}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sinh[a + b*x]^2,x]
```

```
[Out] (-(d*Cosh[2*(a + b*x)]) + 2*b*(-2*a*c - b*x*(2*c + d*x) + (c + d*x)*Sinh[2*
(a + b*x)]))/(8*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(47) = 94$.
time = 0.42, size = 103, normalized size = 1.87

method	result
risch	$-\frac{d\,x^{2}}{4}-\frac{cx}{2}+\frac{(2bdx+2bc-d)\mathrm{e}^{2bx+2a}}{16b^{2}}-\frac{(2bdx+2bc+d)\mathrm{e}^{-2bx-2a}}{16b^{2}}$
derivativedivides	$\frac{\frac{d\left(\frac{(bx+a)\cosh(bx+a)\sinh(bx+a)}{2}-\frac{(bx+a)^{2}}{4}-\frac{\left(\cosh^{2}(bx+a)\right)}{4}\right)}{b}-\frac{da\left(\frac{\cosh(bx+a)\sinh(bx+a)}{2}-\frac{bx}{2}-\frac{a}{2}\right)}{b}+c\left(\frac{\cosh(bx+a)\sinh(bx+a)}{2}\right.}{b}$
default	$\frac{\frac{d\left(\frac{(bx+a)\cosh(bx+a)\sinh(bx+a)}{2}-\frac{(bx+a)^{2}}{4}-\frac{\left(\cosh^{2}(bx+a)\right)}{4}\right)}{b}-\frac{da\left(\frac{\cosh(bx+a)\sinh(bx+a)}{2}-\frac{bx}{2}-\frac{a}{2}\right)}{b}+c\left(\frac{\cosh(bx+a)\sinh(bx+a)}{2}\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(d/b*(1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-1/4*(b*x+a)^2-1/4*cosh(b*x+a)
^2)-d/b*a*(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*x-1/2*a)+c*(1/2*cosh(b*x+a)*si
nh(b*x+a)-1/2*b*x-1/2*a))
```

Maxima [A]
time = 0.26, size = 88, normalized size = 1.60

$$-\frac{1}{16}\left(4\,x^{2}-\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{b^{2}}+\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{b^{2}}\right)d-\frac{1}{8}\,c\left(4\,x-\frac{e^{(2\,bx+2\,a)}}{b}+\frac{e^{(-2\,bx-2\,a)}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(4*x^2 - (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 + (2*b*x + 1)*e^(-2*
b*x - 2*a)/b^2)*d - 1/8*c*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)
```

Fricas [A]
time = 0.35, size = 64, normalized size = 1.16

$$-\frac{2\,b^{2}dx^{2}+4\,b^{2}cx+d\cosh\left(bx+a\right)^{2}-4\,(bdx+bc)\cosh\left(bx+a\right)\sinh\left(bx+a\right)+d\sinh\left(bx+a\right)^{2}}{8\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*b^2*d*x^2 + 4*b^2*c*x + d*cosh(b*x + a)^2 - 4*(b*d*x + b*c)*cosh(b*
x + a)*sinh(b*x + a) + d*sinh(b*x + a)^2)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(49) = 98$.
time = 0.15, size = 126, normalized size = 2.29

$$\begin{cases} \frac{cx\sinh^2{(a+bx)}}{2} - \frac{cx\cosh^2{(a+bx)}}{2} + \frac{dx^2\sinh^2{(a+bx)}}{4} - \frac{dx^2\cosh^2{(a+bx)}}{4} + \frac{c\sinh{(a+bx)}\cosh{(a+bx)}}{2b} + \frac{dx\sinh{(a+bx)}\cosh{(a+bx)}}{2b} - \frac{d\cosh^2{(a+bx)}}{4b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\sinh^2{(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((c*x*sinh(a + b*x)**2/2 - c*x*cosh(a + b*x)**2/2 + d*x**2*sinh(a
+ b*x)**2/4 - d*x**2*cosh(a + b*x)**2/4 + c*sinh(a + b*x)*cosh(a + b*x)/(2*
b) + d*x*sinh(a + b*x)*cosh(a + b*x)/(2*b) - d*cosh(a + b*x)**2/(4*b**2), N
e(b, 0)), ((c*x + d*x**2/2)*sinh(a)**2, True))
```

Giac [A]
time = 0.44, size = 63, normalized size = 1.15

$$-\frac{1}{4}\,dx^2 - \frac{1}{2}\,cx + \frac{(2\,bdx + 2\,bc - d)e^{(2\,bx+2\,a)}}{16\,b^2} - \frac{(2\,bdx + 2\,bc + d)e^{(-2\,bx-2\,a)}}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*d*x^2 - 1/2*c*x + 1/16*(2*b*d*x + 2*b*c - d)*e^(2*b*x + 2*a)/b^2 - 1/1
6*(2*b*d*x + 2*b*c + d)*e^(-2*b*x - 2*a)/b^2
```

Mupad [B]
time = 0.09, size = 60, normalized size = 1.09

$$-\frac{\frac{d\cosh(2\,a+2\,b\,x)}{2} + b^2\,d\,x^2 - b\,c\sinh(2\,a+2\,b\,x) + 2\,b^2\,c\,x - b\,d\,x\sinh(2\,a+2\,b\,x)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2*(c + d*x),x)
```

```
[Out] -((d*cosh(2*a + 2*b*x))/2 + b^2*d*x^2 - b*c*sinh(2*a + 2*b*x) + 2*b^2*c*x -
 b*d*x*sinh(2*a + 2*b*x))/(4*b^2)
```

3.12 $\int \frac{\sinh^2(a+bx)}{c+dx} \, dx$

Optimal. Leaf size=78

$$\frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{2d}-\frac{\log(c+dx)}{2d}+\frac{\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

```
[Out] 1/2*Chi(2*b*c/d+2*b*x)*cosh(2*a-2*b*c/d)/d-1/2*ln(d*x+c)/d+1/2*Shi(2*b*c/d+
2*b*x)*sinh(2*a-2*b*c/d)/d
```

Rubi [A]

time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3393, 3384, 3379, 3382}

$$\frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{2d}-\frac{\log(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2/(c + d*x),x]
```

```
[Out] (Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*c)/d + 2*b*x])/(2*d) - Log[c + d*x
]/(2*d) + (Sinh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(a+bx)}{c+dx}\,dx &= -\int \left(\frac{1}{2(c+dx)}-\frac{\cosh(2a+2bx)}{2(c+dx)}\right)dx\\ &= -\frac{\log(c+dx)}{2d}+\frac{1}{2}\int \frac{\cosh(2a+2bx)}{c+dx}\,dx\\ &= -\frac{\log(c+dx)}{2d}+\frac{1}{2}\cosh\left(2a-\frac{2bc}{d}\right)\int \frac{\cosh\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx+\frac{1}{2}\sinh\left(2a-\frac{2bc}{d}\right)\int\\ &= \frac{\cosh\left(2a-\frac{2bc}{d}\right)\mathrm{Chi}\left(\frac{2bc}{d}+2bx\right)}{2d}-\frac{\log(c+dx)}{2d}+\frac{\sinh\left(2a-\frac{2bc}{d}\right)\mathrm{Shi}\left(\frac{2bc}{d}+2bx\right)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 66, normalized size = 0.85

$$\frac{\cosh\left(2a-\frac{2bc}{d}\right)\mathrm{Chi}\left(\frac{2b(c+dx)}{d}\right)-\log(c+dx)+\sinh\left(2a-\frac{2bc}{d}\right)\mathrm{Shi}\left(\frac{2b(c+dx)}{d}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2/(c + d*x),x]
[Out] (Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*(c + d*x))/d] - Log[c + d*x] + Sin
h[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d])/(2*d)
```

Maple [A]
time = 3.02, size = 97, normalized size = 1.24

method	result	size
risch	$-\frac{\ln(dx+c)}{2d}-\frac{\mathrm{e}^{-\frac{2(ad-bc)}{d}}\,\mathrm{expIntegral}\left(1,2bx+2a-\frac{2(ad-bc)}{d}\right)}{4d}-\frac{\mathrm{e}^{\frac{2ad-2bc}{d}}\,\mathrm{expIntegral}\left(1,-2bx-2a-\frac{2(-ad+bc)}{d}\right)}{4d}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(d*x+c)/d-1/4/d*exp(-2*(a*d-b*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/
4/d*exp(2*(a*d-b*c)/d)*Ei(1,-2*b*x-2*a-2*(-a*d+b*c)/d)
```

Maxima [A]
time = 0.29, size = 72, normalized size = 0.92

$$-\frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_1\left(\frac{2\,(dx+c)b}{d}\right)}{4\,d}-\frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_1\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,d}-\frac{\log\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/4*e^(-2*a + 2*b*c/d)*exp_integral_e(1, 2*(d*x + c)*b/d)/d - 1/4*e^(2*a -
 2*b*c/d)*exp_integral_e(1, -2*(d*x + c)*b/d)/d - 1/2*log(d*x + c)/d
```

Fricas [A]
time = 0.34, size = 104, normalized size = 1.33

$$\frac{\left(\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)+\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{2\,(bc-ad)}{d}\right)+\left(\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)-\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sinh\left(-\frac{2\,(bc-ad)}{d}\right)-2\,\log\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*((Ei(2*(b*d*x + b*c)/d) + Ei(-2*(b*d*x + b*c)/d))*cosh(-2*(b*c - a*d)/d
) + (Ei(2*(b*d*x + b*c)/d) - Ei(-2*(b*d*x + b*c)/d))*sinh(-2*(b*c - a*d)/d)
 - 2*log(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sinh^2\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sinh(a + b*x)**2/(c + d*x), x)
```

Giac [A]
time = 0.45, size = 68, normalized size = 0.87

$$\frac{\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}+\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}-2\,\log\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) + Ei(-2*(b*d*x + b*c)/d)*e^(-2
*a + 2*b*c/d) - 2*log(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\mathrm{sinh}\left(a+b\,x\right)}^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2/(c + d*x),x)
```

```
[Out] int(sinh(a + b*x)^2/(c + d*x), x)
```

3.13 $\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=81

$$\frac{b\text{Chi}\left(\frac{2bc}{d}+2bx\right)\sinh\left(2a-\frac{2bc}{d}\right)}{d^2}-\frac{\sinh^2(a+bx)}{d(c+dx)}+\frac{b\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{d^2}$$

```
[Out] b*cosh(2*a-2*b*c/d)*Shi(2*b*c/d+2*b*x)/d^2+b*Chi(2*b*c/d+2*b*x)*sinh(2*a-2*
b*c/d)/d^2-sinh(b*x+a)^2/d/(d*x+c)
```

Rubi [A]
time = 0.11, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3394, 12, 3384, 3379, 3382}

$$\frac{b\sinh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{b\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{\sinh^2(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] (b*CoshIntegral[(2*b*c)/d + 2*b*x]*Sinh[2*a - (2*b*c)/d])/d^2 - Sinh[a + b*
x]^2/(d*(c + d*x)) + (b*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*
x])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx &= -\frac{\sinh^2(a+bx)}{d(c+dx)} - \frac{(2ib)\int \frac{i\sinh(2a+2bx)}{2(c+dx)}\,dx}{d} \\
&= -\frac{\sinh^2(a+bx)}{d(c+dx)} + \frac{b\int \frac{\sinh(2a+2bx)}{c+dx}\,dx}{d} \\
&= -\frac{\sinh^2(a+bx)}{d(c+dx)} + \frac{\left(b\cosh\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sinh\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d} + \frac{\left(b\sinh\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cosh\left(\frac{2b\ldots}{d}\right.}{c+\ldots}}{d} \\
&= \frac{b\text{Chi}\left(\frac{2bc}{d}+2bx\right)\sinh\left(2a-\frac{2bc}{d}\right)}{d^2} - \frac{\sinh^2(a+bx)}{d(c+dx)} + \frac{b\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{d^2}
\end{aligned}
$$

Mathematica [A]
time = 0.29, size = 75, normalized size = 0.93

$$
\frac{b\text{Chi}\left(\frac{2b(c+dx)}{d}\right)\sinh\left(2a-\frac{2bc}{d}\right) - \frac{d\sinh^2(a+bx)}{c+dx} + b\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2b(c+dx)}{d}\right)}{d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] (b*CoshIntegral[(2*b*(c + d*x))/d]*Sinh[2*a - (2*b*c)/d] - (d*Sinh[a + b*x]
^2)/(c + d*x) + b*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d])/d^
2
```

Maple [A]
time = 3.31, size = 152, normalized size = 1.88

method	result

risch	$\frac{1}{2(dx+c)d}-\frac{b\,\mathrm{e}^{-2bx-2a}}{4d(bdx+bc)}+\frac{b\,\mathrm{e}^{-\frac{2(ad-bc)}{d}}\operatorname{expIntegral}\left(1,2bx+2a-\frac{2(ad-bc)}{d}\right)}{2d^2}-\frac{b\,\mathrm{e}^{2bx+2a}}{4d^2\left(\frac{bc}{d}+bx\right)}-\frac{b\,\mathrm{e}^{\frac{2ad-2bc}{d}}\operatorname{expIntegral}\left(1,-2b\right.}{2d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(d*x+c)/d-1/4*b*exp(-2*b*x-2*a)/d/(b*d*x+b*c)+1/2*b/d^2*exp(-2*(a*d-b*c
)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/4*b/d^2*exp(2*b*x+2*a)/(b*c/d+b*x)-1/2
*b/d^2*exp(2*(a*d-b*c)/d)*Ei(1,-2*b*x-2*a-2*(-a*d+b*c)/d)
```

Maxima [A]
time = 0.32, size = 88, normalized size = 1.09

$$-\frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_2\left(\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)d}-\frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_2\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)d}+\frac{1}{2\,(d^2x+cd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*e^(-2*a + 2*b*c/d)*exp_integral_e(2, 2*(d*x + c)*b/d)/((d*x + c)*d) -
1/4*e^(2*a - 2*b*c/d)*exp_integral_e(2, -2*(d*x + c)*b/d)/((d*x + c)*d) + 1
/2/(d^2*x + c*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(81) = 162$.
time = 0.39, size = 166, normalized size = 2.05

$$-\frac{d\cosh\left(bx+a\right)^2+d\sinh\left(bx+a\right)^2-\left((bdx+bc)\operatorname{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)-(bdx+bc)\operatorname{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{2\,(bc-ad)}{d}\right)-\left((bdx+bc)\operatorname{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)+(bdx+bc)\operatorname{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sinh\left(-\frac{2\,(bc-ad)}{d}\right)-d}{2\,(d^3x+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(d*cosh(b*x + a)^2 + d*sinh(b*x + a)^2 - ((b*d*x + b*c)*Ei(2*(b*d*x +
b*c)/d) - (b*d*x + b*c)*Ei(-2*(b*d*x + b*c)/d))*cosh(-2*(b*c - a*d)/d) - ((
b*d*x + b*c)*Ei(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*Ei(-2*(b*d*x + b*c)/d))*
sinh(-2*(b*c - a*d)/d) - d)/(d^3*x + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sinh^2\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(sinh(a + b*x)**2/(c + d*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 574 vs. 2(81) = 162.
time = 0.48, size = 574, normalized size = 7.09

$$-\frac{\left(2\,(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\mathrm{Ei}\left(-\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)e^{\left(\frac{2\,(bc-ad)}{d}\right)}+2\,b^3c\mathrm{Ei}\left(-\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)e^{\left(\frac{2\,(bc-ad)}{d}\right)}-2\,ab^2d\mathrm{Ei}\left(-\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)e^{\left(\frac{2\,(bc-ad)}{d}\right)}-2\,(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\mathrm{Ei}\left(\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)e^{\left(-\frac{2\,(bc-ad)}{d}\right)}-2\,b^3c\mathrm{Ei}\left(\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)e^{\left(-\frac{2\,(bc-ad)}{d}\right)}+2\,ab^2d\mathrm{Ei}\left(\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)e^{\left(-\frac{2\,(bc-ad)}{d}\right)}+b^2de^{\left(\frac{2\,(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)}{d}\right)}+b^2de^{\left(-\frac{2\,(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)}{d}\right)}-2\,b^2d\right)d^2}{4\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)d^4+bcd^4-ad^5\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*Ei(-2*((d*x + c)*
(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(2*(b*c - a*d)/d) + 2
*b^3*c*Ei(-2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)
*e^(2*(b*c - a*d)/d) - 2*a*b^2*d*Ei(-2*((d*x + c)*(b - b*c/(d*x + c) + a*d/
(d*x + c)) + b*c - a*d)/d)*e^(2*(b*c - a*d)/d) - 2*(d*x + c)*(b - b*c/(d*x
+ c) + a*d/(d*x + c))*b^2*Ei(2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c
)) + b*c - a*d)/d)*e^(-2*(b*c - a*d)/d) - 2*b^3*c*Ei(2*((d*x + c)*(b - b*c/
(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-2*(b*c - a*d)/d) + 2*a*b^2*d
*Ei(2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-2*
(b*c - a*d)/d) + b^2*d*e^(2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d
) + b^2*d*e^(-2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d) - 2*b^2*d)
*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*d^4 + b*c*d^4 - a*d^5)
*b)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(a+b\,x)^2}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2/(c + d*x)^2,x)
```

```
[Out] int(sinh(a + b*x)^2/(c + d*x)^2, x)
```

3.14 $\int \frac{\sinh^2(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=112

$$\frac{b^2\cosh\left(2a-\frac{2bc}{d}\right)\mathrm{Chi}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)}-\frac{\sinh^2(a+bx)}{2d(c+dx)^2}+\frac{b^2\sinh\left(2a-\frac{2bc}{d}\right)\mathrm{Shi}\left(\frac{2bc}{d}-\right.}{d^3}$$

```
[Out] b^2*Chi(2*b*c/d+2*b*x)*cosh(2*a-2*b*c/d)/d^3+b^2*Shi(2*b*c/d+2*b*x)*sinh(2*
a-2*b*c/d)/d^3-b*cosh(b*x+a)*sinh(b*x+a)/d^2/(d*x+c)-1/2*sinh(b*x+a)^2/d/(d
*x+c)^2
```

Rubi [A]
time = 0.14, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3395, 31, 3393, 3384, 3379, 3382}

$$\frac{b^2\cosh\left(2a-\frac{2bc}{d}\right)\mathrm{Chi}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{b^2\sinh\left(2a-\frac{2bc}{d}\right)\mathrm{Shi}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)}-\frac{\sinh^2(a+bx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2/(c + d*x)^3,x]
```

```
[Out] (b^2*Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*c)/d + 2*b*x])/d^3 - (b*Cosh[a
 + b*x]*Sinh[a + b*x])/(d^2*(c + d*x)) - Sinh[a + b*x]^2/(2*d*(c + d*x)^2)
+ (b^2*Sinh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^3}\,dx = -\frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)} - \frac{\sinh^2(a+bx)}{2d(c+dx)^2} + \frac{b^2\int \frac{1}{c+dx}\,dx}{d^2} + \frac{(2b^2)\int \frac{\sinh^2(a+bx)}{c+dx}\,dx}{d^2}$$

$$= \frac{b^2\log(c+dx)}{d^3} - \frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)} - \frac{\sinh^2(a+bx)}{2d(c+dx)^2} - \frac{(2b^2)\int \left(\frac{1}{2(c+dx)} - \frac{\cos}{2}\right.}{d^2}$$

$$= -\frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)} - \frac{\sinh^2(a+bx)}{2d(c+dx)^2} + \frac{b^2\int \frac{\cosh(2a+2bx)}{c+dx}\,dx}{d^2}$$

$$= -\frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)} - \frac{\sinh^2(a+bx)}{2d(c+dx)^2} + \frac{\left(b^2\cosh\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cosh\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,d}{d^2}$$

$$= \frac{b^2\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{d^3} - \frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)} - \frac{\sinh^2(a+bx)}{2d(c+dx)^2} + \frac{b^2}{}$$

Mathematica [A]
time = 0.62, size = 102, normalized size = 0.91

$$\frac{2b^2\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2b(c+dx)}{d}\right) - \frac{d\left(d\sinh^2(a+bx)+b(c+dx)\sinh(2(a+bx))\right)}{(c+dx)^2} + 2b^2\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2b(c+dx)}{d}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2/(c + d*x)^3,x]
```

```
[Out] (2*b^2*Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*(c + d*x))/d] - (d*(d*Sinh[a
 + b*x]^2 + b*(c + d*x)*Sinh[2*(a + b*x)]))/(c + d*x)^2 + 2*b^2*Sinh[2*a -
(2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d])/(2*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(110) = 220$.
time = 3.33, size = 299, normalized size = 2.67

method	result
risch	$\frac{1}{4(dx+c)^2 d}+\frac{b^3 \mathrm{e}^{-2bx-2a} x}{4d(b^2d^2x^2+2b^2cdx+b^2c^2)}+\frac{b^3 \mathrm{e}^{-2bx-2a} c}{4d^2(b^2d^2x^2+2b^2cdx+b^2c^2)}-\frac{b^2 \mathrm{e}^{-2bx-2a}}{8d(b^2d^2x^2+2b^2cdx+b^2c^2)}-\frac{b^2 \mathrm{e}^{-\frac{2(ad-bc)}{d}} \mathrm{expInteg}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(d*x+c)^2/d+1/4*b^3*exp(-2*b*x-2*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)
*x+1/4*b^3*exp(-2*b*x-2*a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c-1/8*b^2*
exp(-2*b*x-2*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)-1/2*b^2/d^3*exp(-2*(a*d
-b*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/8*b^2/d^3*exp(2*b*x+2*a)/(b*c/d+b*
x)^2-1/4*b^2/d^3*exp(2*b*x+2*a)/(b*c/d+b*x)-1/2*b^2/d^3*exp(2*(a*d-b*c)/d)*
Ei(1,-2*b*x-2*a-2*(-a*d+b*c)/d)
```

Maxima [A]
time = 0.32, size = 99, normalized size = 0.88

$$\frac{1}{4\,(d^3x^2+2\,cd^2x+c^2d)}-\frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_3\!\left(\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^2d}-\frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_3\!\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4/(d^3*x^2 + 2*c*d^2*x + c^2*d) - 1/4*e^(-2*a + 2*b*c/d)*exp_integral_e(3
, 2*(d*x + c)*b/d)/((d*x + c)^2*d) - 1/4*e^(2*a - 2*b*c/d)*exp_integral_e(3
, -2*(d*x + c)*b/d)/((d*x + c)^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(110) = 220$.
time = 0.36, size = 280, normalized size = 2.50

$$-\frac{d^2\cosh\left(bx+a\right)^2+d^2\sinh\left(bx+a\right)^2+4\,(bd^2x+bcd)\cosh\left(bx+a\right)\sinh\left(bx+a\right)-d^2-2\left((b^2d^2x^2+2\,b^2cdx+b^2c^2)\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)+(b^2d^2x^2+2\,b^2cdx+b^2c^2)\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{2\,(bc-ad)}{d}\right)-2\left((b^2d^2x^2+2\,b^2cdx+b^2c^2)\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)-(b^2d^2x^2+2\,b^2cdx+b^2c^2)\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sinh\left(-\frac{2\,(bc-ad)}{d}\right)}{4\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(d^2*cosh(b*x + a)^2 + d^2*sinh(b*x + a)^2 + 4*(b*d^2*x + b*c*d)*cosh(
b*x + a)*sinh(b*x + a) - d^2 - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(
2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-2*(b*d*x + b
*c)/d))*cosh(-2*(b*c - a*d)/d) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*E
i(2*(b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-2*(b*d*x +
 b*c)/d))*sinh(-2*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] Integral(sinh(a + b*x)**2/(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(110) = 220.
time = 0.43, size = 330, normalized size = 2.95

$$\frac{4\,b^2d^2x^2\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}+4\,b^2d^2x^2\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}+8\,b^2cdx\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}+8\,b^2cdx\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}+4\,b^2c^2\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}+4\,b^2c^2\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}-2\,bd^2xe^{(2\,bx+2\,a)}+2\,bd^2xe^{(-2\,bx-2\,a)}-2\,bcde^{(2\,bx+2\,a)}+2\,bcde^{(-2\,bx-2\,a)}-d^2e^{(2\,bx+2\,a)}-d^2e^{(-2\,bx-2\,a)}+2\,d^2}{8\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*b^2*d^2*x^2*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) + 4*b^2*d^2*x^2*
Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) + 8*b^2*c*d*x*Ei(2*(b*d*x + b*c)/
d)*e^(2*a - 2*b*c/d) + 8*b^2*c*d*x*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d
) + 4*b^2*c^2*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) + 4*b^2*c^2*Ei(-2*(b*
d*x + b*c)/d)*e^(-2*a + 2*b*c/d) - 2*b*d^2*x*e^(2*b*x + 2*a) + 2*b*d^2*x*e^
(-2*b*x - 2*a) - 2*b*c*d*e^(2*b*x + 2*a) + 2*b*c*d*e^(-2*b*x - 2*a) - d^2*e
^(2*b*x + 2*a) - d^2*e^(-2*b*x - 2*a) + 2*d^2)/(d^5*x^2 + 2*c*d^4*x + c^2*d
^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{sinh}\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2/(c + d*x)^3,x)
```

```
[Out] int(sinh(a + b*x)^2/(c + d*x)^3, x)
```

3.15 $\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=162

$$-\frac{b^2}{3d^3(c+dx)}+\frac{2b^3\mathrm{Chi}\left(\frac{2bc}{d}+2bx\right)\sinh\left(2a-\frac{2bc}{d}\right)}{3d^4}-\frac{b\cosh(a+bx)\sinh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{2b^2\sinh^2(}{3d^3(c+}$$

```
[Out] -1/3*b^2/d^3/(d*x+c)+2/3*b^3*cosh(2*a-2*b*c/d)*Shi(2*b*c/d+2*b*x)/d^4+2/3*b
^3*Chi(2*b*c/d+2*b*x)*sinh(2*a-2*b*c/d)/d^4-1/3*b*cosh(b*x+a)*sinh(b*x+a)/d
^2/(d*x+c)^2-1/3*sinh(b*x+a)^2/d/(d*x+c)^3-2/3*b^2*sinh(b*x+a)^2/d^3/(d*x+c
)
```

Rubi [A]
time = 0.13, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3395, 32, 3394, 12, 3384, 3379, 3382}

$$\frac{2b^3\sinh\left(2a-\frac{2bc}{d}\right)\mathrm{Chi}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{2b^3\cosh\left(2a-\frac{2bc}{d}\right)\mathrm{Shi}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{2b^2\sinh^2(a+bx)}{3d^3(c+dx)}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{b^2}{3d^3(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2/(c + d*x)^4,x]
```

```
[Out] -1/3*b^2/(d^3*(c + d*x)) + (2*b^3*CoshIntegral[(2*b*c)/d + 2*b*x]*Sinh[2*a
- (2*b*c)/d])/(3*d^4) - (b*Cosh[a + b*x]*Sinh[a + b*x])/(3*d^2*(c + d*x)^2)
 - Sinh[a + b*x]^2/(3*d*(c + d*x)^3) - (2*b^2*Sinh[a + b*x]^2)/(3*d^3*(c +
d*x)) + (2*b^3*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/(3*d^
4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx &= -\frac{b\cosh(a+bx)\sinh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}+\frac{b^2\int\frac{1}{(c+dx)^2}\,dx}{3d^2}+\frac{(2b^2)\int\frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx}{3d^2}\\ &= -\frac{b^2}{3d^3(c+dx)}-\frac{b\cosh(a+bx)\sinh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{2b^2\sinh^2(a+bx)}{3d^3(c+dx)}-\\ &= -\frac{b^2}{3d^3(c+dx)}-\frac{b\cosh(a+bx)\sinh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{2b^2\sinh^2(a+bx)}{3d^3(c+dx)}+\\ &= -\frac{b^2}{3d^3(c+dx)}-\frac{b\cosh(a+bx)\sinh(a+bx)}{3d^2(c+dx)^2}-\frac{\sinh^2(a+bx)}{3d(c+dx)^3}-\frac{2b^2\sinh^2(a+bx)}{3d^3(c+dx)}+\\ &= -\frac{b^2}{3d^3(c+dx)}+\frac{2b^3\mathrm{Chi}\left(\frac{2bc}{d}+2bx\right)\sinh\left(2a-\frac{2bc}{d}\right)}{3d^4}-\frac{b\cosh(a+bx)\sinh(a+bx)}{3d^2(c+dx)^2}-\end{aligned}$$

Mathematica [A]
time = 0.61, size = 123, normalized size = 0.76

$$\frac{4b^3\mathrm{Chi}\left(\frac{2b(c+dx)}{d}\right)\sinh\left(2a-\frac{2bc}{d}\right)-\frac{d\left(\left(d^2+2b^2(c+dx)^2\right)\cosh(2(a+bx))+d(-d+b(c+dx)\sinh(2(a+bx)))\right)}{(c+dx)^3}+4b^3\cosh\left(2a-\frac{2bc}{d}\right)\mathrm{Shi}\left(\frac{2b(c+dx)}{d}\right)}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2/(c + d*x)^4,x]
```

```
[Out] (4*b^3*CoshIntegral[(2*b*(c + d*x))/d]*Sinh[2*a - (2*b*c)/d] - (d*((d^2 + 2
*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)] + d*(-d + b*(c + d*x)*Sinh[2*(a + b*x)]
)))/(c + d*x)^3 + 4*b^3*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/
d])/(6*d^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(150) = 300$.
time = 3.25, size = 555, normalized size = 3.43

method	result
risch	$\frac{1}{6(dx+c)^3 d}-\frac{b^5 \mathrm{e}^{-2bx-2a}x^2}{6d(d^3x^3b^3+3b^3c\,d^2x^2+3b^3c^2dx+b^3c^3)}-\frac{b^5\mathrm{e}^{-2bx-2a}cx}{3d^2(d^3x^3b^3+3b^3c\,d^2x^2+3b^3c^2dx+b^3c^3)}-\frac{b^5\mathrm{e}^{-2bx-2a}c^2}{6d^3(d^3x^3b^3+3b^3c\,d^2x^2+3b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/(d*x+c)^3/d-1/6*b^5*exp(-2*b*x-2*a)/d/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^
3*c^2*d*x+b^3*c^3)*x^2-1/3*b^5*exp(-2*b*x-2*a)/d^2/(b^3*d^3*x^3+3*b^3*c*d^2
*x^2+3*b^3*c^2*d*x+b^3*c^3)*c*x-1/6*b^5*exp(-2*b*x-2*a)/d^3/(b^3*d^3*x^3+3*
b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c^2+1/12*b^4*exp(-2*b*x-2*a)/d/(b^3*d^
3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*x+1/12*b^4*exp(-2*b*x-2*a)/d^2
/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c-1/12*b^3*exp(-2*b*x-
2*a)/d/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)+1/3*b^3/d^4*exp(
-2*(a*d-b*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/12*b^3/d^4*exp(2*b*x+2*a)/(
b*c/d+b*x)^3-1/12*b^3/d^4*exp(2*b*x+2*a)/(b*c/d+b*x)^2-1/6*b^3/d^4*exp(2*b*
x+2*a)/(b*c/d+b*x)-1/3*b^3/d^4*exp(2*(a*d-b*c)/d)*Ei(1,-2*b*x-2*a-2*(-a*d+b
*c)/d)
```

Maxima [A]
time = 0.31, size = 110, normalized size = 0.68

$$\frac{1}{6\left(d^4x^3+3\,cd^3x^2+3\,c^2d^2x+c^3d\right)}-\frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_4\left(\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^3d}-\frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_4\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/6/(d^4*x^3 + 3*c*d^3*x^2 + 3*c^2*d^2*x + c^3*d) - 1/4*e^(-2*a + 2*b*c/d)*
exp_integral_e(4, 2*(d*x + c)*b/d)/((d*x + c)^3*d) - 1/4*e^(2*a - 2*b*c/d)*
exp_integral_e(4, -2*(d*x + c)*b/d)/((d*x + c)^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(150) = 300$.
time = 0.36, size = 411, normalized size = 2.54

$$\frac{d^3 - (2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d + d^3)\cosh(bx+a)^2 - 2\,(bd^3x + bcd^2)\cosh(bx+a)\sinh(bx+a) - (2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d + d^3)\sinh(bx+a)^2 + 2\left((b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3)\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right) - (b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3)\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{2\,(bc-ad)}{d}\right) + 2\left((b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3)\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right) + (b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3)\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sinh\left(-\frac{2\,(bc-ad)}{d}\right)}{6\,(d^7x^3 + 3\,cd^6x^2 + 3\,c^2d^5x + c^3d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(d^3 - (2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*cosh(b*x + a
)^2 - 2*(b*d^3*x + b*c*d^2)*cosh(b*x + a)*sinh(b*x + a) - (2*b^2*d^3*x^2 +
4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*sinh(b*x + a)^2 + 2*((b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*Ei(2*(b*d*x + b*c)/d) - (b^3*d^3*x^3
 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*Ei(-2*(b*d*x + b*c)/d))*cosh(
-2*(b*c - a*d)/d) + 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3
*c^3)*Ei(2*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + b^3*c^3)*Ei(-2*(b*d*x + b*c)/d))*sinh(-2*(b*c - a*d)/d))/(d^7*x^3 + 3*c
*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2/(d*x+c)**4,x)
```

```
[Out] Integral(sinh(a + b*x)**2/(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(150) = 300$.
time = 0.42, size = 537, normalized size = 3.31

$$\frac{4\,b^3d^3x^3\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)} - 4\,b^3d^3x^3\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)} + 12\,b^3cd^2x^2\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)} - 12\,b^3cd^2x^2\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)} + 12\,b^3c^2dx\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)} - 12\,b^3c^2dx\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)} - 2\,b^2d^3x^2e^{(2\,bx+2\,a)} - 2\,b^2d^3x^2e^{(-2\,bx-2\,a)} + 4\,b^3c^3\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)} - 4\,b^3c^3\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)} - 4\,b^2cd^2xe^{(2\,bx+2\,a)} - 4\,b^2cd^2xe^{(-2\,bx-2\,a)} - 2\,b^2c^2de^{(2\,bx+2\,a)} - bd^3xe^{(2\,bx+2\,a)} - 2\,b^2c^2de^{(-2\,bx-2\,a)} + bd^3xe^{(-2\,bx-2\,a)} - bcd^2e^{(2\,bx+2\,a)} + bcd^2e^{(-2\,bx-2\,a)} - d^3e^{(2\,bx+2\,a)} - d^3e^{(-2\,bx-2\,a)} + 2\,d^3}{12\,(d^7x^3 + 3\,cd^6x^2 + 3\,c^2d^5x + c^3d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/12*(4*b^3*d^3*x^3*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) - 4*b^3*d^3*x^3
*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) + 12*b^3*c*d^2*x^2*Ei(2*(b*d*x +
```

```
 b*c)/d)*e^(2*a - 2*b*c/d) - 12*b^3*c*d^2*x^2*Ei(-2*(b*d*x + b*c)/d)*e^(-2*
a + 2*b*c/d) + 12*b^3*c^2*d*x*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) - 12*
b^3*c^2*d*x*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) - 2*b^2*d^3*x^2*e^(2*
b*x + 2*a) - 2*b^2*d^3*x^2*e^(-2*b*x - 2*a) + 4*b^3*c^3*Ei(2*(b*d*x + b*c)/
d)*e^(2*a - 2*b*c/d) - 4*b^3*c^3*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d)
- 4*b^2*c*d^2*x*e^(2*b*x + 2*a) - 4*b^2*c*d^2*x*e^(-2*b*x - 2*a) - 2*b^2*c^
2*d*e^(2*b*x + 2*a) - b*d^3*x*e^(2*b*x + 2*a) - 2*b^2*c^2*d*e^(-2*b*x - 2*a
) + b*d^3*x*e^(-2*b*x - 2*a) - b*c*d^2*e^(2*b*x + 2*a) + b*c*d^2*e^(-2*b*x
- 2*a) - d^3*e^(2*b*x + 2*a) - d^3*e^(-2*b*x - 2*a) + 2*d^3)/(d^7*x^3 + 3*c
*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(a+bx)^2}{(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2/(c + d*x)^4,x)
```

```
[Out] int(sinh(a + b*x)^2/(c + d*x)^4, x)
```

3.16 $\int (c+dx)^4 \sinh^3(a+bx)\,dx$

Optimal. Leaf size=225

$$-\frac{488d^4\cosh(a+bx)}{27b^5}-\frac{80d^2(c+dx)^2\cosh(a+bx)}{9b^3}-\frac{2(c+dx)^4\cosh(a+bx)}{3b}+\frac{8d^4\cosh^3(a+bx)}{81b^5}+\frac{160d^3(c+$$

```
[Out] -488/27*d^4*cosh(b*x+a)/b^5-80/9*d^2*(d*x+c)^2*cosh(b*x+a)/b^3-2/3*(d*x+c)^
4*cosh(b*x+a)/b+8/81*d^4*cosh(b*x+a)^3/b^5+160/9*d^3*(d*x+c)*sinh(b*x+a)/b^
4+8/3*d*(d*x+c)^3*sinh(b*x+a)/b^2+4/9*d^2*(d*x+c)^2*cosh(b*x+a)*sinh(b*x+a)
^2/b^3+1/3*(d*x+c)^4*cosh(b*x+a)*sinh(b*x+a)^2/b-8/27*d^3*(d*x+c)*sinh(b*x+
a)^3/b^4-4/9*d*(d*x+c)^3*sinh(b*x+a)^3/b^2
```

Rubi [A]
time = 0.25, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 3377, 2718, 2713}

$$\frac{8d^4\cosh^3(a+bx)}{81b^5}-\frac{488d^4\cosh(a+bx)}{27b^5}-\frac{8d^3(c+dx)\sinh^3(a+bx)}{27b^4}+\frac{160d^3(c+dx)\sinh(a+bx)}{9b^4}-\frac{80d^2(c+dx)^2\cosh(a+bx)}{9b^3}+\frac{4d^2(c+dx)^2\sinh^2(a+bx)\cosh(a+bx)}{9b^3}-\frac{4d(c+dx)^3\sinh^3(a+bx)}{9b^2}+\frac{8d(c+dx)^3\sinh(a+bx)}{3b^2}-\frac{2(c+dx)^4\cosh(a+bx)}{3b}+\frac{(c+dx)^4\sinh^2(a+bx)\cosh(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sinh[a + b*x]^3,x]
```

```
[Out] (-488*d^4*Cosh[a + b*x])/(27*b^5) - (80*d^2*(c + d*x)^2*Cosh[a + b*x])/(9*b
^3) - (2*(c + d*x)^4*Cosh[a + b*x])/(3*b) + (8*d^4*Cosh[a + b*x]^3)/(81*b^5
) + (160*d^3*(c + d*x)*Sinh[a + b*x])/(9*b^4) + (8*d*(c + d*x)^3*Sinh[a + b
*x])/(3*b^2) + (4*d^2*(c + d*x)^2*Cosh[a + b*x]*Sinh[a + b*x]^2)/(9*b^3) +
((c + d*x)^4*Cosh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (8*d^3*(c + d*x)*Sinh[a
 + b*x]^3)/(27*b^4) - (4*d*(c + d*x)^3*Sinh[a + b*x]^3)/(9*b^2)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int (c+dx)^4 \sinh^3(a+bx)\,dx = \frac{(c+dx)^4\cosh(a+bx)\sinh^2(a+bx)}{3b} - \frac{4d(c+dx)^3\sinh^3(a+bx)}{9b^2} - \frac{2}{3}\int (c$$

$$= -\frac{2(c+dx)^4\cosh(a+bx)}{3b} + \frac{4d^2(c+dx)^2\cosh(a+bx)\sinh^2(a+bx)}{9b^3} + \frac{(c+}{}$$

$$= -\frac{8d^2(c+dx)^2\cosh(a+bx)}{9b^3} - \frac{2(c+dx)^4\cosh(a+bx)}{3b} + \frac{8d(c+dx)^3\sinh(}{3b^2}$$

$$= -\frac{8d^4\cosh(a+bx)}{27b^5} - \frac{80d^2(c+dx)^2\cosh(a+bx)}{9b^3} - \frac{2(c+dx)^4\cosh(a+bx)}{3b}$$

$$= -\frac{56d^4\cosh(a+bx)}{27b^5} - \frac{80d^2(c+dx)^2\cosh(a+bx)}{9b^3} - \frac{2(c+dx)^4\cosh(a+b}{3b}$$

$$= -\frac{488d^4\cosh(a+bx)}{27b^5} - \frac{80d^2(c+dx)^2\cosh(a+bx)}{9b^3} - \frac{2(c+dx)^4\cosh(a+}{3b}$$

Mathematica [A]
time = 0.62, size = 150, normalized size = 0.67

$$\frac{-243(24d^4+12b^2d^2(c+dx)^2+b^4(c+dx)^4)\cosh(a+bx)+(8d^4+36b^2d^2(c+dx)^2+27b^4(c+dx)^4)\cosh(3(a+bx))-24bd(c+dx)\left(-242d^2-39b^2(c+dx)^2+(2d^2+3b^2(c+dx)^2)\cosh(2(a+bx))\right)\sinh(a+bx)}{324b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Sinh[a + b*x]^3,x]
```

```
[Out] (-243*(24*d^4 + 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cosh[a + b*x] + (
8*d^4 + 36*b^2*d^2*(c + d*x)^2 + 27*b^4*(c + d*x)^4)*Cosh[3*(a + b*x)] - 24
*b*d*(c + d*x)*(-242*d^2 - 39*b^2*(c + d*x)^2 + (2*d^2 + 3*b^2*(c + d*x)^2)
*Cosh[2*(a + b*x)])*Sinh[a + b*x])/(324*b^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1265 vs. $2(205) = 410$.
time = 0.49, size = 1266, normalized size = 5.63

method	result

risch	$\frac{(27d^4x^4b^4+108b^4c\,d^3x^3+162b^4c^2d^2x^2-36b^3d^4x^3+108b^4c^3dx-108b^3c\,d^3x^2+27b^4c^4-108b^3c^2d^2x+36b^2d^4x^2-36b^3c^3d+72b^2c\,d^3x+36}{648b^5}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4/b*(-d^4/b^4*((b*x+a)^4*cosh(b*x+a)-4*(b*x+a)^3*sinh(b*x+a)+12*(b*x+a)^2
*cosh(b*x+a)-24*(b*x+a)*sinh(b*x+a)+24*cosh(b*x+a))+4*d^4/b^4*a*((b*x+a)^3*
cosh(b*x+a)-3*(b*x+a)^2*sinh(b*x+a)+6*(b*x+a)*cosh(b*x+a)-6*sinh(b*x+a))-6*
d^4/b^4*a^2*((b*x+a)^2*cosh(b*x+a)-2*(b*x+a)*sinh(b*x+a)+2*cosh(b*x+a))+4*d
^4/b^4*a^3*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))-d^4/b^4*a^4*cosh(b*x+a)-4*d^3/
b^3*c*((b*x+a)^3*cosh(b*x+a)-3*(b*x+a)^2*sinh(b*x+a)+6*(b*x+a)*cosh(b*x+a)-
6*sinh(b*x+a))+12*d^3/b^3*a*c*((b*x+a)^2*cosh(b*x+a)-2*(b*x+a)*sinh(b*x+a)+
2*cosh(b*x+a))-12*d^3/b^3*a^2*c*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))+4*d^3/b^3
*a^3*c*cosh(b*x+a)-6*d^2/b^2*c^2*((b*x+a)^2*cosh(b*x+a)-2*(b*x+a)*sinh(b*x+
a)+2*cosh(b*x+a))+12*d^2/b^2*a*c^2*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))-6*d^2/
b^2*a^2*c^2*cosh(b*x+a)-4*d/b*c^3*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))+4*d/b*a
*c^3*cosh(b*x+a)-c^4*cosh(b*x+a))+1/972/b*(1/b^4*d^4*((3*b*x+3*a)^4*cosh(3*
b*x+3*a)-4*(3*b*x+3*a)^3*sinh(3*b*x+3*a)+12*(3*b*x+3*a)^2*cosh(3*b*x+3*a)-2
4*(3*b*x+3*a)*sinh(3*b*x+3*a)+24*cosh(3*b*x+3*a))-12/b^4*d^4*a*((3*b*x+3*a)
^3*cosh(3*b*x+3*a)-3*(3*b*x+3*a)^2*sinh(3*b*x+3*a)+6*(3*b*x+3*a)*cosh(3*b*x
+3*a)-6*sinh(3*b*x+3*a))+54/b^4*d^4*a^2*((3*b*x+3*a)^2*cosh(3*b*x+3*a)-2*(3
*b*x+3*a)*sinh(3*b*x+3*a)+2*cosh(3*b*x+3*a))-108/b^4*d^4*a^3*((3*b*x+3*a)*c
osh(3*b*x+3*a)-sinh(3*b*x+3*a))+81/b^4*d^4*a^4*cosh(3*b*x+3*a)+12/b^3*c*d^3
*((3*b*x+3*a)^3*cosh(3*b*x+3*a)-3*(3*b*x+3*a)^2*sinh(3*b*x+3*a)+6*(3*b*x+3*
a)*cosh(3*b*x+3*a)-6*sinh(3*b*x+3*a))-108/b^3*c*d^3*a*((3*b*x+3*a)^2*cosh(3
*b*x+3*a)-2*(3*b*x+3*a)*sinh(3*b*x+3*a)+2*cosh(3*b*x+3*a))+324/b^3*c*d^3*a^
2*((3*b*x+3*a)*cosh(3*b*x+3*a)-sinh(3*b*x+3*a))-324/b^3*c*d^3*a^3*cosh(3*b*
x+3*a)+54/b^2*c^2*d^2*((3*b*x+3*a)^2*cosh(3*b*x+3*a)-2*(3*b*x+3*a)*sinh(3*b
*x+3*a)+2*cosh(3*b*x+3*a))-324/b^2*c^2*d^2*a*((3*b*x+3*a)*cosh(3*b*x+3*a)-s
inh(3*b*x+3*a))+486/b^2*c^2*d^2*a^2*cosh(3*b*x+3*a)+108/b*c^3*d*((3*b*x+3*a
)*cosh(3*b*x+3*a)-sinh(3*b*x+3*a))-324/b*c^3*d*a*cosh(3*b*x+3*a)+81*c^4*cos
h(3*b*x+3*a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(205) = 410$.
time = 0.30, size = 639, normalized size = 2.84

$$\frac{1}{18}c^3d\left(\frac{(3bxe^{(3a)}-e^{(3a)})e^{(3bx)}}{b^2}-\frac{27(bxe^a-e^a)e^{(bx)}}{b^2}-\frac{27(bx+1)e^{(-bx-a)}}{b^2}+\frac{(3bx+1)e^{(-3bx-3a)}}{b^2}\right)+\frac{1}{24}c^4\left(\frac{e^{(3bx+3a)}}{b}-\frac{9e^{(bx+a)}}{b}-\frac{9e^{(-bx-a)}}{b}+\frac{e^{(-3bx-3a)}}{b}\right)+\frac{1}{36}c^2d^2\left(\frac{(9b^2x^2e^{(3a)}-6bxe^{(3a)}+2e^{(3a)})e^{(3bx)}}{b^3}-\frac{81(b^2x^2e^a-2bxe^a+2e^a)e^{(bx)}}{b^3}-\frac{81(b^2x^2+2bx+2)e^{(-bx-a)}}{b^3}+\frac{(9b^2x^2+6bx+2)e^{(-3bx-3a)}}{b^3}\right)+\frac{1}{54}cd^3\left(\frac{(9b^3x^3e^{(3a)}-9b^2x^2e^{(3a)}+6bxe^{(3a)}-2e^{(3a)})e^{(3bx)}}{b^4}-\cdots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*c^3*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 - 27*(b*x*e^a - e^a)*e^
(b*x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 + (3*b*x + 1)*e^(-3*b*x - 3*a)/b^
2) + 1/24*c^4*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b + e^(
-3*b*x - 3*a)/b) + 1/36*c^2*d^2*((9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(
3*a))*e^(3*b*x)/b^3 - 81*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 - 81
*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 + (9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x
 - 3*a)/b^3) + 1/54*c*d^3*((9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x*e
^(3*a) - 2*e^(3*a))*e^(3*b*x)/b^4 - 81*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x
*e^a - 6*e^a)*e^(b*x)/b^4 - 81*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x -
a)/b^4 + (9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4) + 1/648*
d^4*((27*b^4*x^4*e^(3*a) - 36*b^3*x^3*e^(3*a) + 36*b^2*x^2*e^(3*a) - 24*b*x
*e^(3*a) + 8*e^(3*a))*e^(3*b*x)/b^5 - 243*(b^4*x^4*e^a - 4*b^3*x^3*e^a + 12
*b^2*x^2*e^a - 24*b*x*e^a + 24*e^a)*e^(b*x)/b^5 - 243*(b^4*x^4 + 4*b^3*x^3
+ 12*b^2*x^2 + 24*b*x + 24)*e^(-b*x - a)/b^5 + (27*b^4*x^4 + 36*b^3*x^3 + 3
6*b^2*x^2 + 24*b*x + 8)*e^(-3*b*x - 3*a)/b^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(205) = 410$.
time = 0.37, size = 528, normalized size = 2.35

$$\frac{(27b^4d^4x^4 + 108b^4cd^3x^3 + 27b^4c^4 + 36b^2c^2d^2 + 8d^4 + 18(9b^4c^2d^2 + 2b^2d^4)x^2 + 36(3b^4c^3d + 2b^2cd^3)x)\cosh(bx+a)^3 + 3(27b^4d^4x^4 + 108b^4cd^3x^3 + 27b^4c^4 + 36b^2c^2d^2 + 8d^4 + 18(9b^4c^2d^2 + 2b^2d^4)x^2 + 36(3b^4c^3d + 2b^2cd^3)x)\cosh(bx+a)\sinh(bx+a)^2 - 12(3b^3d^4x^3 + 9b^3cd^3x^2 + 3b^3c^3d + 2bcd^3 + (9b^3c^2d^2 + 2bd^4)x)\sinh(bx+a)^3 - 243(b^4d^4x^4 + 4b^4cd^3x^3 + b^4c^4 + 12b^2c^2d^2 + 24d^4 + 6(b^4c^2d^2 + 2b^2d^4)x^2 + 4(b^4c^3d + 6b^2cd^3)x)\cosh(bx+a) + 36(27b^3d^4x^3 + 81b^3cd^3x^2 + 27b^3c^3d + 162bcd^3 - (3b^3d^4x^3 + 9b^3cd^3x^2 + 3b^3c^3d + 2bcd^3 + (9b^3c^2d^2 + 2bd^4)x)\cosh(bx+a)^2 + 81(b^3c^2d^2 + 2bd^4)x)\sinh(bx+a)}{324b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/324*((27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 + 36*b^2*c^2*d^2 +
8*d^4 + 18*(9*b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 36*(3*b^4*c^3*d + 2*b^2*c*d^3)
*x)*cosh(b*x + a)^3 + 3*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 +
36*b^2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 36*(3*b^4*c^3
*d + 2*b^2*c*d^3)*x)*cosh(b*x + a)*sinh(b*x + a)^2 - 12*(3*b^3*d^4*x^3 + 9*
b^3*c*d^3*x^2 + 3*b^3*c^3*d + 2*b*c*d^3 + (9*b^3*c^2*d^2 + 2*b*d^4)*x)*sinh
(b*x + a)^3 - 243*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 + 12*b^2*c^2*d^2
 + 24*d^4 + 6*(b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d + 6*b^2*c*d^3)*x
)*cosh(b*x + a) + 36*(27*b^3*d^4*x^3 + 81*b^3*c*d^3*x^2 + 27*b^3*c^3*d + 16
2*b*c*d^3 - (3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d + 2*b*c*d^3 + (9
*b^3*c^2*d^2 + 2*b*d^4)*x)*cosh(b*x + a)^2 + 81*(b^3*c^2*d^2 + 2*b*d^4)*x)*
sinh(b*x + a))/b^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(226) = 452$.
time = 0.81, size = 772, normalized size = 3.43

$$\begin{cases} \frac{c^4\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{2c^4\cosh^3(a+bx)}{3b} + \frac{4c^3dx\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{8c^3dx\cosh^3(a+bx)}{3b} + \frac{6c^2d^2x^2\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{4c^2d^2x^2\cosh^3(a+bx)}{b} + \frac{4cd^3x^3\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{8cd^3x^3\cosh^3(a+bx)}{3b} + \frac{d^4x^4\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{2d^4x^4\cosh^3(a+bx)}{3b} - \frac{28c^3d\sinh^3(a+bx)}{9b^2} + \frac{8c^3d\sinh(a+bx)\cosh^2(a+bx)}{3b^2} - \frac{28c^2d^2x\sinh^3(a+bx)}{3b^2} + \frac{8c^2d^2x\sinh(a+bx)\cosh^2(a+bx)}{b^2} - \frac{28cd^3x^2\sinh^3(a+bx)}{3b^2} + \frac{8cd^3x^2\sinh(a+bx)\cosh^2(a+bx)}{b^2} - \frac{28d^4x^3\sinh^3(a+bx)}{9b^2} + \frac{8d^4x^3\sinh(a+bx)\cosh^2(a+bx)}{3b^2} + \frac{28c^2d^2\sinh^2(a+bx)\cosh(a+bx)}{3b^3} - \frac{80c^2d^2\cosh^3(a+bx)}{9b^3} + \frac{56cd^3x\sinh^2(a+bx)\cosh(a+bx)}{3b^3} - \frac{160cd^3x\cosh^3(a+bx)}{9b^3} + \frac{28d^4x^2\sinh^2(a+bx)\cosh(a+bx)}{3b^3} - \frac{80d^4x^2\cosh^3(a+bx)}{9b^3} - \frac{488cd^3\sinh^3(a+bx)}{27b^4} + \frac{160cd^3\sinh(a+bx)\cosh^2(a+bx)}{9b^4} - \frac{488d^4x\sinh^3(a+bx)}{27b^4} + \frac{160d^4x\sinh(a+bx)\cosh^2(a+bx)}{9b^4} + \frac{488d^4\sinh^2(a+bx)\cosh(a+bx)}{27b^5} - \frac{1456d^4\cosh^3(a+bx)}{81b^5} & \text{for}\ b \neq 0 \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\sinh^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((c**4*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c**4*cosh(a + b*x)**3/
(3*b) + 4*c**3*d*x*sinh(a + b*x)**2*cosh(a + b*x)/b - 8*c**3*d*x*cosh(a + b
*x)**3/(3*b) + 6*c**2*d**2*x**2*sinh(a + b*x)**2*cosh(a + b*x)/b - 4*c**2*d
**2*x**2*cosh(a + b*x)**3/b + 4*c*d**3*x**3*sinh(a + b*x)**2*cosh(a + b*x)/
b - 8*c*d**3*x**3*cosh(a + b*x)**3/(3*b) + d**4*x**4*sinh(a + b*x)**2*cosh(
a + b*x)/b - 2*d**4*x**4*cosh(a + b*x)**3/(3*b) - 28*c**3*d*sinh(a + b*x)**
3/(9*b**2) + 8*c**3*d*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**2) - 28*c**2*d**
2*x*sinh(a + b*x)**3/(3*b**2) + 8*c**2*d**2*x*sinh(a + b*x)*cosh(a + b*x)**
2/b**2 - 28*c*d**3*x**2*sinh(a + b*x)**3/(3*b**2) + 8*c*d**3*x**2*sinh(a +
b*x)*cosh(a + b*x)**2/b**2 - 28*d**4*x**3*sinh(a + b*x)**3/(9*b**2) + 8*d**
4*x**3*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**2) + 28*c**2*d**2*sinh(a + b*x)
**2*cosh(a + b*x)/(3*b**3) - 80*c**2*d**2*cosh(a + b*x)**3/(9*b**3) + 56*c*
d**3*x*sinh(a + b*x)**2*cosh(a + b*x)/(3*b**3) - 160*c*d**3*x*cosh(a + b*x)
**3/(9*b**3) + 28*d**4*x**2*sinh(a + b*x)**2*cosh(a + b*x)/(3*b**3) - 80*d*
*4*x**2*cosh(a + b*x)**3/(9*b**3) - 488*c*d**3*sinh(a + b*x)**3/(27*b**4) +
 160*c*d**3*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**4) - 488*d**4*x*sinh(a + b
*x)**3/(27*b**4) + 160*d**4*x*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**4) + 488
*d**4*sinh(a + b*x)**2*cosh(a + b*x)/(27*b**5) - 1456*d**4*cosh(a + b*x)**3
/(81*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3
*x**4 + d**4*x**5/5)*sinh(a)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 654 vs. 2(205) = 410.
time = 0.45, size = 654, normalized size = 2.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/648*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 - 36*b^3*d^
4*x^3 + 108*b^4*c^3*d*x - 108*b^3*c*d^3*x^2 + 27*b^4*c^4 - 108*b^3*c^2*d^2*
x + 36*b^2*d^4*x^2 - 36*b^3*c^3*d + 72*b^2*c*d^3*x + 36*b^2*c^2*d^2 - 24*b*
d^4*x - 24*b*c*d^3 + 8*d^4)*e^(3*b*x + 3*a)/b^5 - 3/8*(b^4*d^4*x^4 + 4*b^4*
c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 4*b^4*c^3*d*x - 12*b^3*c*d^
3*x^2 + b^4*c^4 - 12*b^3*c^2*d^2*x + 12*b^2*d^4*x^2 - 4*b^3*c^3*d + 24*b^2*
c*d^3*x + 12*b^2*c^2*d^2 - 24*b*d^4*x - 24*b*c*d^3 + 24*d^4)*e^(b*x + a)/b^
5 - 3/8*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^3*d^4*x^3
+ 4*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + b^4*c^4 + 12*b^3*c^2*d^2*x + 12*b^2*d^
4*x^2 + 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2*d^2 + 24*b*d^4*x + 24*b*c
*d^3 + 24*d^4)*e^(-b*x - a)/b^5 + 1/648*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3
 + 162*b^4*c^2*d^2*x^2 + 36*b^3*d^4*x^3 + 108*b^4*c^3*d*x + 108*b^3*c*d^3*x
^2 + 27*b^4*c^4 + 108*b^3*c^2*d^2*x + 36*b^2*d^4*x^2 + 36*b^3*c^3*d + 72*b^
2*c*d^3*x + 36*b^2*c^2*d^2 + 24*b*d^4*x + 24*b*c*d^3 + 8*d^4)*e^(-3*b*x - 3
*a)/b^5
```

Mupad [B]
time = 0.55, size = 532, normalized size = 2.36

$$\frac{\cosh(a+b\,x)\,\sinh(a+b\,x)^2\,\left(27\,b^4\,c^4+252\,b^2\,c^2\,d^2+488\,d^4\right)}{27\,b^5}-\frac{2\,\cosh(a+b\,x)^3\,\left(27\,b^4\,c^4+360\,b^2\,c^2\,d^2+728\,d^4\right)}{81\,b^5}-\frac{4\,\sinh(a+b\,x)^3\,\left(21\,b^2\,c^3\,d+122\,c\,d^3\right)}{27\,b^4}+\frac{8\,\cosh(a+b\,x)^2\,\sinh(a+b\,x)\,\left(3\,b^2\,c^3\,d+20\,c\,d^3\right)}{9\,b^4}-\frac{2\,d^4\,x^4\,\cosh(a+b\,x)^3}{3\,b}-\frac{8\,x\,\cosh(a+b\,x)^3\,\left(3\,b^2\,c^3\,d+20\,c\,d^3\right)}{9\,b^3}-\frac{28\,d^4\,x^3\,\sinh(a+b\,x)^3}{9\,b^2}-\frac{4\,x\,\sinh(a+b\,x)^3\,\left(63\,b^2\,c^2\,d^2+122\,d^4\right)}{27\,b^4}-\frac{4\,x^2\,\cosh(a+b\,x)^3\,\left(9\,b^2\,c^2\,d^2+20\,d^4\right)}{9\,b^3}+\frac{2\,x^2\,\cosh(a+b\,x)\,\sinh(a+b\,x)^2\,\left(9\,b^2\,c^2\,d^2+14\,d^4\right)}{3\,b^3}-\frac{8\,c\,d^3\,x^3\,\cosh(a+b\,x)^3}{3\,b}+\frac{d^4\,x^4\,\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{b}+\frac{8\,d^4\,x^3\,\cosh(a+b\,x)^2\,\sinh(a+b\,x)}{3\,b^2}-\frac{28\,c\,d^3\,x^2\,\sinh(a+b\,x)^3}{3\,b^2}+\frac{8\,x\,\cosh(a+b\,x)^2\,\sinh(a+b\,x)\,\left(9\,b^2\,c^2\,d^2+20\,d^4\right)}{9\,b^4}+\frac{4\,x\,\cosh(a+b\,x)\,\sinh(a+b\,x)^2\,\left(3\,b^2\,c^3\,d+14\,c\,d^3\right)}{3\,b^3}+\frac{4\,c\,d^3\,x^3\,\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{b}+\frac{8\,c\,d^3\,x^2\,\cosh(a+b\,x)^2\,\sinh(a+b\,x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3*(c + d*x)^4,x)
```

```
[Out] (cosh(a + b*x)*sinh(a + b*x)^2*(488*d^4 + 27*b^4*c^4 + 252*b^2*c^2*d^2))/(2
7*b^5) - (2*cosh(a + b*x)^3*(728*d^4 + 27*b^4*c^4 + 360*b^2*c^2*d^2))/(81*b
^5) - (4*sinh(a + b*x)^3*(122*c*d^3 + 21*b^2*c^3*d))/(27*b^4) + (8*cosh(a +
 b*x)^2*sinh(a + b*x)*(20*c*d^3 + 3*b^2*c^3*d))/(9*b^4) - (2*d^4*x^4*cosh(a
 + b*x)^3)/(3*b) - (8*x*cosh(a + b*x)^3*(20*c*d^3 + 3*b^2*c^3*d))/(9*b^3) -
 (28*d^4*x^3*sinh(a + b*x)^3)/(9*b^2) - (4*x*sinh(a + b*x)^3*(122*d^4 + 63*
b^2*c^2*d^2))/(27*b^4) - (4*x^2*cosh(a + b*x)^3*(20*d^4 + 9*b^2*c^2*d^2))/(
9*b^3) + (2*x^2*cosh(a + b*x)*sinh(a + b*x)^2*(14*d^4 + 9*b^2*c^2*d^2))/(3*
b^3) - (8*c*d^3*x^3*cosh(a + b*x)^3)/(3*b) + (d^4*x^4*cosh(a + b*x)*sinh(a
+ b*x)^2)/b + (8*d^4*x^3*cosh(a + b*x)^2*sinh(a + b*x))/(3*b^2) - (28*c*d^3
*x^2*sinh(a + b*x)^3)/(3*b^2) + (8*x*cosh(a + b*x)^2*sinh(a + b*x)*(20*d^4
+ 9*b^2*c^2*d^2))/(9*b^4) + (4*x*cosh(a + b*x)*sinh(a + b*x)^2*(14*c*d^3 +
3*b^2*c^3*d))/(3*b^3) + (4*c*d^3*x^3*cosh(a + b*x)*sinh(a + b*x)^2)/b + (8*
c*d^3*x^2*cosh(a + b*x)^2*sinh(a + b*x))/b^2
```

3.17 $\int (c+dx)^3 \sinh^3(a+bx)\,dx$

Optimal. Leaf size=175

$$-\frac{40d^2(c+dx)\cosh(a+bx)}{9b^3}-\frac{2(c+dx)^3\cosh(a+bx)}{3b}+\frac{40d^3\sinh(a+bx)}{9b^4}+\frac{2d(c+dx)^2\sinh(a+bx)}{b^2}+\frac{2d^2(c$$

```
[Out] -40/9*d^2*(d*x+c)*cosh(b*x+a)/b^3-2/3*(d*x+c)^3*cosh(b*x+a)/b+40/9*d^3*sinh
(b*x+a)/b^4+2*d*(d*x+c)^2*sinh(b*x+a)/b^2+2/9*d^2*(d*x+c)*cosh(b*x+a)*sinh(
b*x+a)^2/b^3+1/3*(d*x+c)^3*cosh(b*x+a)*sinh(b*x+a)^2/b-2/27*d^3*sinh(b*x+a)
^3/b^4-1/3*d*(d*x+c)^2*sinh(b*x+a)^3/b^2
```

Rubi [A]
time = 0.16, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 3377, 2717, 3391}

$$-\frac{2d^3\sinh^3(a+bx)}{27b^4}+\frac{40d^3\sinh(a+bx)}{9b^4}-\frac{40d^2(c+dx)\cosh(a+bx)}{9b^3}+\frac{2d^2(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{9b^3}-\frac{d(c+dx)^2\sinh^3(a+bx)}{3b^2}+\frac{2d(c+dx)^2\sinh(a+bx)}{b^2}-\frac{2(c+dx)^3\cosh(a+bx)}{3b}+\frac{(c+dx)^3\sinh^2(a+bx)\cosh(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sinh[a + b*x]^3,x]
```

```
[Out] (-40*d^2*(c + d*x)*Cosh[a + b*x])/(9*b^3) - (2*(c + d*x)^3*Cosh[a + b*x])/(
3*b) + (40*d^3*Sinh[a + b*x])/(9*b^4) + (2*d*(c + d*x)^2*Sinh[a + b*x])/b^2
 + (2*d^2*(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^2)/(9*b^3) + ((c + d*x)^3*C
osh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (2*d^3*Sinh[a + b*x]^3)/(27*b^4) - (d
*(c + d*x)^2*Sinh[a + b*x]^3)/(3*b^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3 \sinh^3(a+bx)\,dx &= \frac{(c+dx)^3\cosh(a+bx)\sinh^2(a+bx)}{3b} - \frac{d(c+dx)^2\sinh^3(a+bx)}{3b^2} - \frac{2}{3}\int (c+ \\
&= -\frac{2(c+dx)^3\cosh(a+bx)}{3b} + \frac{2d^2(c+dx)\cosh(a+bx)\sinh^2(a+bx)}{9b^3} + \frac{(c+ \\
&= -\frac{4d^2(c+dx)\cosh(a+bx)}{9b^3} - \frac{2(c+dx)^3\cosh(a+bx)}{3b} + \frac{2d(c+dx)^2\sinh(a}{b^2} \\
&= -\frac{40d^2(c+dx)\cosh(a+bx)}{9b^3} - \frac{2(c+dx)^3\cosh(a+bx)}{3b} + \frac{4d^3\sinh(a+bx)}{9b^4} \\
&= -\frac{40d^2(c+dx)\cosh(a+bx)}{9b^3} - \frac{2(c+dx)^3\cosh(a+bx)}{3b} + \frac{40d^3\sinh(a+bx}{9b^4}
\end{aligned}$$

Mathematica [A]
time = 0.62, size = 127, normalized size = 0.73

$$\frac{-162b(c+dx)\left(6d^2+b^2(c+dx)^2\right)\cosh(a+bx)+6b(c+dx)\left(2d^2+3b^2(c+dx)^2\right)\cosh(3(a+bx))-4d\left(-242d^2-117b^2(c+dx)^2+\left(2d^2+9b^2(c+dx)^2\right)\cosh(2(a+bx))\right)\sinh(a+bx)}{216b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sinh[a + b*x]^3,x]
```

```
[Out] (-162*b*(c + d*x)*(6*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] + 6*b*(c + d*x)*(
2*d^2 + 3*b^2*(c + d*x)^2)*Cosh[3*(a + b*x)] - 4*d*(-242*d^2 - 117*b^2*(c +
 d*x)^2 + (2*d^2 + 9*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])/(21
6*b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(161) = 322$.
time = 0.43, size = 710, normalized size = 4.06

method	result
risch	$\frac{\left(9d^3x^3b^3+27b^3c\,d^2x^2+27b^3c^2dx-9b^2d^3x^2+9b^3c^3-18b^2c\,d^2x-9b^2c^2d+6b\,d^3x+6bc\,d^2-2d^3\right)e^{3bx+3a}}{216b^4} - \frac{3\left(d^3x^3b^3+3b^3c\,d^2x^2+3b^3c\right.}{}$

default	$-\frac{3d^3\left((bx+a)^3\cosh(bx+a)-3(bx+a)^2\sinh(bx+a)+6(bx+a)\cosh(bx+a)-6\sinh(bx+a)\right)}{4b^3}+\frac{9d^3a\left((bx+a)^2\cosh(bx+a)-2(bx+a)\sinh(bx+a)+2\cosh(bx \ldots}{4b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4/b*(-1/b^3*d^3*((b*x+a)^3*cosh(b*x+a)-3*(b*x+a)^2*sinh(b*x+a)+6*(b*x+a)*
cosh(b*x+a)-6*sinh(b*x+a))+3/b^3*d^3*a*((b*x+a)^2*cosh(b*x+a)-2*(b*x+a)*sin
h(b*x+a)+2*cosh(b*x+a))-3/b^3*d^3*a^2*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))+1/b
^3*d^3*a^3*cosh(b*x+a)-3/b^2*c*d^2*((b*x+a)^2*cosh(b*x+a)-2*(b*x+a)*sinh(b*
x+a)+2*cosh(b*x+a))+6/b^2*c*d^2*a*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))-3/b^2*c
*d^2*a^2*cosh(b*x+a)-3/b*c^2*d*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))+3/b*c^2*d*
a*cosh(b*x+a)-c^3*cosh(b*x+a))+1/324/b*(1/b^3*d^3*((3*b*x+3*a)^3*cosh(3*b*x
+3*a)-3*(3*b*x+3*a)^2*sinh(3*b*x+3*a)+6*(3*b*x+3*a)*cosh(3*b*x+3*a)-6*sinh(
3*b*x+3*a))-9/b^3*d^3*a*((3*b*x+3*a)^2*cosh(3*b*x+3*a)-2*(3*b*x+3*a)*sinh(3
*b*x+3*a)+2*cosh(3*b*x+3*a))+27/b^3*d^3*a^2*((3*b*x+3*a)*cosh(3*b*x+3*a)-si
nh(3*b*x+3*a))-27/b^3*d^3*a^3*cosh(3*b*x+3*a)+9/b^2*c*d^2*((3*b*x+3*a)^2*co
sh(3*b*x+3*a)-2*(3*b*x+3*a)*sinh(3*b*x+3*a)+2*cosh(3*b*x+3*a))-54/b^2*c*d^2
*a*((3*b*x+3*a)*cosh(3*b*x+3*a)-sinh(3*b*x+3*a))+81/b^2*c*d^2*a^2*cosh(3*b*
x+3*a)+27/b*c^2*d*((3*b*x+3*a)*cosh(3*b*x+3*a)-sinh(3*b*x+3*a))-81/b*c^2*d*
a*cosh(3*b*x+3*a)+27*c^3*cosh(3*b*x+3*a))
```

<u>Maxima</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(161) = 322$.
time = 0.30, size = 435, normalized size = 2.49

$$\frac{1}{24}c^2d\left(\frac{\left(3bxe^{(3a)}-e^{(3a)}\right)e^{(3bx)}}{b^2}-\frac{27\left(bxe^a-e^a\right)e^{(bx)}}{b^2}-\frac{27(bx+1)e^{(-bx-a)}}{b^2}+\frac{(3bx+1)e^{(-3bx-3a)}}{b^2}\right)+\frac{1}{24}c^3\left(\frac{e^{(3bx+3a)}}{b}-\frac{9e^{(bx+a)}}{b}-\frac{9e^{(-bx-a)}}{b}+\frac{e^{(-3bx-3a)}}{b}\right)+\frac{1}{72}cd^2\left(\frac{\left(9b^2x^2e^{(3a)}-6bxe^{(3a)}+2e^{(3a)}\right)e^{(3bx)}}{b^3}-\frac{81\left(b^2x^2e^a-2bxe^a+2e^a\right)e^{(bx)}}{b^3}-\frac{81\left(b^2x^2+2bx+2\right)e^{(-bx-a)}}{b^3}+\frac{\left(9b^2x^2+6bx+2\right)e^{(-3bx-3a)}}{b^3}\right)+\frac{1}{216}d^3\left(\frac{\left(9b^3x^3e^{(3a)}-9b^2x^2e^{(3a)}+6bxe^{(3a)}-2e^{(3a)}\right)e^{(3bx)}}{b^4}-\frac{81\left(b^3x^3e^a-3b^2x^2e^a+6bxe^a-6e^a\right)e^{(bx)}}{b^4}-\frac{81\left(b^3x^3+3b^2x^2+6bx+6\right)e^{(-bx-a)}}{b^4}+\frac{\left(9b^3x^3+9b^2x^2+6bx+2\right)e^{(-3bx-3a)}}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*c^2*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 - 27*(b*x*e^a - e^a)*e^
(b*x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 + (3*b*x + 1)*e^(-3*b*x - 3*a)/b^
2) + 1/24*c^3*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b + e^(
-3*b*x - 3*a)/b) + 1/72*c*d^2*((9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*
a))*e^(3*b*x)/b^3 - 81*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 - 81*(
b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 + (9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x -
 3*a)/b^3) + 1/216*d^3*((9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x*e^(3
*a) - 2*e^(3*a))*e^(3*b*x)/b^4 - 81*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^
a - 6*e^a)*e^(b*x)/b^4 - 81*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x - a)/
b^4 + (9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4)
```

<u>Fricas</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(161) = 322$.
time = 0.35, size = 345, normalized size = 1.97

$$\frac{3\left(3b^3d^3x^3+9b^3cd^2x^2+3b^3c^3+2bcd^2+\left(9b^3c^2d+2bd^3\right)x\right)\cosh(bx+a)^3+9\left(3b^3d^3x^3+9b^3cd^2x^2+3b^3c^3+2bcd^2+\left(9b^3c^2d+2bd^3\right)x\right)\cosh(bx+a)\sinh(bx+a)^2-\left(9b^2d^3x^2+18b^2cd^2x+9b^2c^2d+2d^3\right)\sinh(bx+a)^3-81\left(b^3d^3x^3+3b^3cd^2x^2+b^3c^3+6bcd^2+3\left(b^3c^2d+2bd^3\right)x\right)\cosh(bx+a)+3\left(81b^2d^3x^2+162b^2cd^2x+81b^2c^2d+162d^3-\left(9b^2d^3x^2+18b^2cd^2x+9b^2c^2d+2d^3\right)\cosh(bx+a)^2\right)\sinh(bx+a)}{108b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 + 2*b*c*d^2 + (9*b^3*
c^2*d + 2*b*d^3)*x)*cosh(b*x + a)^3 + 9*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 +
3*b^3*c^3 + 2*b*c*d^2 + (9*b^3*c^2*d + 2*b*d^3)*x)*cosh(b*x + a)*sinh(b*x +
 a)^2 - (9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d + 2*d^3)*sinh(b*x + a
)^3 - 81*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 6*b*c*d^2 + 3*(b^3*c^2*
d + 2*b*d^3)*x)*cosh(b*x + a) + 3*(81*b^2*d^3*x^2 + 162*b^2*c*d^2*x + 81*b^
2*c^2*d + 162*d^3 - (9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d + 2*d^3)*
cosh(b*x + a)^2)*sinh(b*x + a))/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(173) = 346$.
time = 0.53, size = 495, normalized size = 2.83

$$\begin{cases} \frac{c^3 \sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{2c^3\cosh^3(a+bx)}{3b} + \frac{3c^2dx\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{2c^2dx\cosh^3(a+bx)}{b} + \frac{3cd^2x^2\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{2cd^2x^2\cosh^3(a+bx)}{b} + \frac{d^3x^3\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{2d^3x^3\cosh^3(a+bx)}{3b} - \frac{7c^2d\sinh^3(a+bx)}{3b^2} + \frac{2c^2d\sinh(a+bx)\cosh^2(a+bx)}{b^2} - \frac{14cd^2x\sinh^3(a+bx)}{3b^2} + \frac{4cd^2x\sinh(a+bx)\cosh^2(a+bx)}{b^2} - \frac{7d^3x^2\sinh^3(a+bx)}{3b^2} + \frac{2d^3x^2\sinh(a+bx)\cosh^2(a+bx)}{b^2} + \frac{14cd^2\sinh^2(a+bx)\cosh(a+bx)}{3b^3} - \frac{40cd^2\cosh^3(a+bx)}{9b^3} + \frac{14d^3x\sinh^2(a+bx)\cosh(a+bx)}{3b^3} - \frac{40d^3x\cosh^3(a+bx)}{9b^3} - \frac{122d^3\sinh^3(a+bx)}{27b^4} + \frac{40d^3\sinh(a+bx)\cosh^2(a+bx)}{9b^4} & \text{for}\ b \neq 0 \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sinh^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((c**3*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c**3*cosh(a + b*x)**3/
(3*b) + 3*c**2*d*x*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c**2*d*x*cosh(a + b
*x)**3/b + 3*c*d**2*x**2*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c*d**2*x**2*c
osh(a + b*x)**3/b + d**3*x**3*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*d**3*x**
3*cosh(a + b*x)**3/(3*b) - 7*c**2*d*sinh(a + b*x)**3/(3*b**2) + 2*c**2*d*si
nh(a + b*x)*cosh(a + b*x)**2/b**2 - 14*c*d**2*x*sinh(a + b*x)**3/(3*b**2) +
 4*c*d**2*x*sinh(a + b*x)*cosh(a + b*x)**2/b**2 - 7*d**3*x**2*sinh(a + b*x)
**3/(3*b**2) + 2*d**3*x**2*sinh(a + b*x)*cosh(a + b*x)**2/b**2 + 14*c*d**2*
sinh(a + b*x)**2*cosh(a + b*x)/(3*b**3) - 40*c*d**2*cosh(a + b*x)**3/(9*b**
3) + 14*d**3*x*sinh(a + b*x)**2*cosh(a + b*x)/(3*b**3) - 40*d**3*x*cosh(a +
 b*x)**3/(9*b**3) - 122*d**3*sinh(a + b*x)**3/(27*b**4) + 40*d**3*sinh(a +
b*x)*cosh(a + b*x)**2/(9*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d
**2*x**3 + d**3*x**4/4)*sinh(a)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(161) = 322$.
time = 0.41, size = 414, normalized size = 2.37

$$\frac{(9\,b^3d^3x^3 + 27\,b^3cd^2x^2 + 27\,b^3c^2dx - 9\,b^2d^3x^2 + 9\,b^3c^3 - 18\,b^2cd^2x - 9\,b^2c^2d + 6\,bd^3x + 6\,bcd^2 - 2\,d^3)e^{(3\,bx+3\,a)}}{216\,b^4} - \frac{3\,(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx - 3\,b^2d^3x^2 + b^3c^3 - 6\,b^2cd^2x - 3\,b^2c^2d + 6\,bd^3x + 6\,bcd^2 - 6\,d^3)e^{(bx+a)}}{8\,b^4} - \frac{3\,(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + 3\,b^2d^3x^2 + b^3c^3 + 6\,b^2cd^2x + 3\,b^2c^2d + 6\,bd^3x + 6\,bcd^2 + 6\,d^3)e^{(-bx-a)}}{8\,b^4} + \frac{(9\,b^3d^3x^3 + 27\,b^3cd^2x^2 + 27\,b^3c^2dx + 9\,b^2d^3x^2 + 9\,b^3c^3 + 18\,b^2cd^2x + 9\,b^2c^2d + 6\,bd^3x + 6\,bcd^2 + 2\,d^3)e^{(-3\,bx-3\,a)}}{216\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/216*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x^2 + 27*b^3*c^2*d*x - 9*b^2*d^3*x^2 +
9*b^3*c^3 - 18*b^2*c*d^2*x - 9*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 2*d^3)*e
```

```
^(3*b*x + 3*a)/b^4 - 3/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x - 3
*b^2*d^3*x^2 + b^3*c^3 - 6*b^2*c*d^2*x - 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^
2 - 6*d^3)*e^(b*x + a)/b^4 - 3/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2
*d*x + 3*b^2*d^3*x^2 + b^3*c^3 + 6*b^2*c*d^2*x + 3*b^2*c^2*d + 6*b*d^3*x +
6*b*c*d^2 + 6*d^3)*e^(-b*x - a)/b^4 + 1/216*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x
^2 + 27*b^3*c^2*d*x + 9*b^2*d^3*x^2 + 9*b^3*c^3 + 18*b^2*c*d^2*x + 9*b^2*c^
2*d + 6*b*d^3*x + 6*b*c*d^2 + 2*d^3)*e^(-3*b*x - 3*a)/b^4
```

Mupad [B]
time = 0.35, size = 364, normalized size = 2.08

$$\frac{\cosh(a+b\,x)\,\sinh(a+b\,x)^2\,\left(3\,b^2\,c^3+14\,c\,d^2\right)}{3\,b^3}-\frac{\sinh(a+b\,x)^3\,\left(63\,b^2\,c^2\,d+122\,d^3\right)}{27\,b^4}-\frac{2\cosh(a+b\,x)^3\,\left(3\,b^2\,c^3+20\,c\,d^2\right)}{9\,b^3}+\frac{2\cosh(a+b\,x)^2\sinh(a+b\,x)\,\left(9\,b^2\,c^2\,d+20\,d^3\right)}{9\,b^4}-\frac{2\,x\cosh(a+b\,x)^3\,\left(9\,b^2\,c^2\,d+20\,d^3\right)}{9\,b^3}-\frac{2\,d^3\,x^3\cosh(a+b\,x)^3}{3\,b}-\frac{7\,d^3\,x^2\sinh(a+b\,x)^3}{3\,b^2}-\frac{14\,c\,d^2\,x\sinh(a+b\,x)^3}{3\,b^2}+\frac{x\cosh(a+b\,x)\,\sinh(a+b\,x)^2\,\left(9\,b^2\,c^2\,d+14\,d^3\right)}{3\,b^3}-\frac{2\,c\,d^2\,x^2\cosh(a+b\,x)^3}{b}+\frac{d^3\,x^3\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{b}+\frac{2\,d^3\,x^2\cosh(a+b\,x)^2\sinh(a+b\,x)}{b^2}+\frac{3\,c\,d^2\,x^2\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{b}+\frac{4\,c\,d^2\,x\cosh(a+b\,x)^2\sinh(a+b\,x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3*(c + d*x)^3,x)
```

```
[Out] (cosh(a + b*x)*sinh(a + b*x)^2*(14*c*d^2 + 3*b^2*c^3))/(3*b^3) - (sinh(a +
b*x)^3*(122*d^3 + 63*b^2*c^2*d))/(27*b^4) - (2*cosh(a + b*x)^3*(20*c*d^2 +
3*b^2*c^3))/(9*b^3) + (2*cosh(a + b*x)^2*sinh(a + b*x)*(20*d^3 + 9*b^2*c^2*
d))/(9*b^4) - (2*x*cosh(a + b*x)^3*(20*d^3 + 9*b^2*c^2*d))/(9*b^3) - (2*d^3
*x^3*cosh(a + b*x)^3)/(3*b) - (7*d^3*x^2*sinh(a + b*x)^3)/(3*b^2) - (14*c*d
^2*x*sinh(a + b*x)^3)/(3*b^2) + (x*cosh(a + b*x)*sinh(a + b*x)^2*(14*d^3 +
9*b^2*c^2*d))/(3*b^3) - (2*c*d^2*x^2*cosh(a + b*x)^3)/b + (d^3*x^3*cosh(a +
 b*x)*sinh(a + b*x)^2)/b + (2*d^3*x^2*cosh(a + b*x)^2*sinh(a + b*x))/b^2 +
(3*c*d^2*x^2*cosh(a + b*x)*sinh(a + b*x)^2)/b + (4*c*d^2*x*cosh(a + b*x)^2*
sinh(a + b*x))/b^2
```

3.18 $\int (c+dx)^2 \sinh^3(a+bx)\,dx$

Optimal. Leaf size=123

$$-\frac{14d^2\cosh(a+bx)}{9b^3}-\frac{2(c+dx)^2\cosh(a+bx)}{3b}+\frac{2d^2\cosh^3(a+bx)}{27b^3}+\frac{4d(c+dx)\sinh(a+bx)}{3b^2}+\frac{(c+dx)^2\cosh$$

```
[Out] -14/9*d^2*cosh(b*x+a)/b^3-2/3*(d*x+c)^2*cosh(b*x+a)/b+2/27*d^2*cosh(b*x+a)^
3/b^3+4/3*d*(d*x+c)*sinh(b*x+a)/b^2+1/3*(d*x+c)^2*cosh(b*x+a)*sinh(b*x+a)^2
/b-2/9*d*(d*x+c)*sinh(b*x+a)^3/b^2
```

Rubi [A]
time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 3377, 2718, 2713}

$$\frac{2d^2\cosh^3(a+bx)}{27b^3}-\frac{14d^2\cosh(a+bx)}{9b^3}-\frac{2d(c+dx)\sinh^3(a+bx)}{9b^2}+\frac{4d(c+dx)\sinh(a+bx)}{3b^2}-\frac{2(c+dx)^2\cosh(a+bx)}{3b}+\frac{(c+dx)^2\sinh^2(a+bx)\cosh(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sinh[a + b*x]^3,x]
```

```
[Out] (-14*d^2*Cosh[a + b*x])/(9*b^3) - (2*(c + d*x)^2*Cosh[a + b*x])/(3*b) + (2*
d^2*Cosh[a + b*x]^3)/(27*b^3) + (4*d*(c + d*x)*Sinh[a + b*x])/(3*b^2) + ((c
 + d*x)^2*Cosh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (2*d*(c + d*x)*Sinh[a + b*
x]^3)/(9*b^2)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
```

```
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int (c+dx)^2 \sinh^3(a+bx)\,dx = \frac{(c+dx)^2\cosh(a+bx)\sinh^2(a+bx)}{3b} - \frac{2d(c+dx)\sinh^3(a+bx)}{9b^2} - \frac{2}{3}\int (c+$$
$$= -\frac{2(c+dx)^2\cosh(a+bx)}{3b} + \frac{(c+dx)^2\cosh(a+bx)\sinh^2(a+bx)}{3b} - \frac{2d(c+d}{}$$
$$= -\frac{2d^2\cosh(a+bx)}{9b^3} - \frac{2(c+dx)^2\cosh(a+bx)}{3b} + \frac{2d^2\cosh^3(a+bx)}{27b^3} + \frac{4d(c+}{}$$
$$= -\frac{14d^2\cosh(a+bx)}{9b^3} - \frac{2(c+dx)^2\cosh(a+bx)}{3b} + \frac{2d^2\cosh^3(a+bx)}{27b^3} + \frac{4d(c+}{}$$

Mathematica [A]
time = 0.24, size = 86, normalized size = 0.70

$$\frac{-81(2d^2+b^2(c+dx)^2)\cosh(a+bx)+(2d^2+9b^2(c+dx)^2)\cosh(3(a+bx))-6bd(c+dx)(-27\sinh(a+bx)+\sinh(3(a+bx)))}{108b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sinh[a + b*x]^3,x]
```

```
[Out] (-81*(2*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] + (2*d^2 + 9*b^2*(c + d*x)^2)*
Cosh[3*(a + b*x)] - 6*b*d*(c + d*x)*(-27*Sinh[a + b*x] + Sinh[3*(a + b*x)])
)/(108*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(111) = 222$.
time = 0.42, size = 343, normalized size = 2.79

method	result
risch	$\frac{(9b^2d^2x^2+18b^2cdx+9b^2c^2-6b\,d^2x-6bcd+2d^2)\mathrm{e}^{3bx+3a}}{216b^3} - \frac{3(b^2d^2x^2+2b^2cdx+b^2c^2-2b\,d^2x-2bcd+2d^2)\mathrm{e}^{bx+a}}{8b^3} - \frac{3(b^2d^2x^2+2b^2cdx}{}$
default	$\frac{-\frac{3d^2((bx+a)^2\cosh(bx+a)-2(bx+a)\sinh(bx+a)+2\cosh(bx+a))}{4b^2}+\frac{3d^2a((bx+a)\cosh(bx+a)-\sinh(bx+a))}{2b^2}-\frac{3d^2a^2\cosh(bx+a)}{4b^2}-\frac{3cd((bx+a)\cosh(bx}{2b}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4/b*(-d^2/b^2*((b*x+a)^2*cosh(b*x+a)-2*(b*x+a)*sinh(b*x+a)+2*cosh(b*x+a))
+2/b^2*d^2*a*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))-1/b^2*d^2*a^2*cosh(b*x+a)-2/
b*c*d*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))+2/b*c*d*a*cosh(b*x+a)-c^2*cosh(b*x+
a))+1/108/b*(1/b^2*d^2*((3*b*x+3*a)^2*cosh(3*b*x+3*a)-2*(3*b*x+3*a)*sinh(3*
b*x+3*a)+2*cosh(3*b*x+3*a))-6/b^2*d^2*a*((3*b*x+3*a)*cosh(3*b*x+3*a)-sinh(3
*b*x+3*a))+9/b^2*d^2*a^2*cosh(3*b*x+3*a)+6/b*c*d*((3*b*x+3*a)*cosh(3*b*x+3*
a)-sinh(3*b*x+3*a))-18/b*c*d*a*cosh(3*b*x+3*a)+9*c^2*cosh(3*b*x+3*a))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(111) = 222$.
time = 0.28, size = 269, normalized size = 2.19

$$\frac{1}{36}cd\left(\frac{(3\,bxe^{(3\,a)}-e^{(3\,a)})e^{(3\,bx)}}{b^2}-\frac{27\,(bxe^a-e^a)e^{(bx)}}{b^2}-\frac{27\,(bx+1)e^{(-bx-a)}}{b^2}+\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{b^2}\right)+\frac{1}{24}c^2\left(\frac{e^{(3\,bx+3\,a)}}{b}-\frac{9\,e^{(bx+a)}}{b}-\frac{9\,e^{(-bx-a)}}{b}+\frac{e^{(-3\,bx-3\,a)}}{b}\right)+\frac{1}{216}d^2\left(\frac{(9\,b^2x^2e^{(3\,a)}-6\,bxe^{(3\,a)}+2\,e^{(3\,a)})e^{(3\,bx)}}{b^3}-\frac{81\,(b^2x^2e^a-2\,bxe^a+2\,e^a)e^{(bx)}}{b^3}-\frac{81\,(b^2x^2+2\,bx+2)e^{(-bx-a)}}{b^3}+\frac{(9\,b^2x^2+6\,bx+2)e^{(-3\,bx-3\,a)}}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36*c*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 - 27*(b*x*e^a - e^a)*e^(b
*x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 + (3*b*x + 1)*e^(-3*b*x - 3*a)/b^2)
 + 1/24*c^2*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b + e^(-3
*b*x - 3*a)/b) + 1/216*d^2*((9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*a))
*e^(3*b*x)/b^3 - 81*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 - 81*(b^2
*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 + (9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*
a)/b^3)
```

Fricas [**A**]
time = 0.35, size = 199, normalized size = 1.62

$$\frac{(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2+2\,d^2)\cosh(bx+a)^3+3\,(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2+2\,d^2)\cosh(bx+a)\sinh(bx+a)^2-6\,(bd^2x+bcd)\sinh(bx+a)^3-81\,(b^2d^2x^2+2\,b^2cdx+b^2c^2+2\,d^2)\cosh(bx+a)+18\left(9\,bd^2x+9\,bcd-(bd^2x+bcd)\cosh(bx+a)^2\right)\sinh(bx+a)}{108\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*((9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 2*d^2)*cosh(b*x + a)^3 +
 3*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 2*d^2)*cosh(b*x + a)*sinh(b*
x + a)^2 - 6*(b*d^2*x + b*c*d)*sinh(b*x + a)^3 - 81*(b^2*d^2*x^2 + 2*b^2*c*
d*x + b^2*c^2 + 2*d^2)*cosh(b*x + a) + 18*(9*b*d^2*x + 9*b*c*d - (b*d^2*x +
 b*c*d)*cosh(b*x + a)^2)*sinh(b*x + a))/b^3
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(121) = 242$.
time = 0.38, size = 284, normalized size = 2.31

$$\begin{cases} \frac{c^2\sinh^2(a+bx)\cosh(a+bx)}{b}-\frac{2c^2\cosh^3(a+bx)}{3b}+\frac{2cdx\sinh^2(a+bx)\cosh(a+bx)}{b}-\frac{4cdx\cosh^3(a+bx)}{3b}+\frac{d^2x^2\sinh^2(a+bx)\cosh(a+bx)}{b}-\frac{2d^2x^2\cosh^3(a+bx)}{3b}-\frac{14cd\sinh^3(a+bx)}{9b^2}+\frac{4cd\sinh(a+bx)\cosh^2(a+bx)}{3b^2}-\frac{14d^2x\sinh^3(a+bx)}{9b^2}+\frac{4d^2x\sinh(a+bx)\cosh^2(a+bx)}{3b^2}+\frac{14d^2\sinh^2(a+bx)\cosh(a+bx)}{9b^3}-\frac{40d^2\cosh^3(a+bx)}{27b^3} & \text{for}\ b\neq 0 \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sinh^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((c**2*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c**2*cosh(a + b*x)**3/
(3*b) + 2*c*d*x*sinh(a + b*x)**2*cosh(a + b*x)/b - 4*c*d*x*cosh(a + b*x)**3
/(3*b) + d**2*x**2*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*d**2*x**2*cosh(a +
b*x)**3/(3*b) - 14*c*d*sinh(a + b*x)**3/(9*b**2) + 4*c*d*sinh(a + b*x)*cosh
(a + b*x)**2/(3*b**2) - 14*d**2*x*sinh(a + b*x)**3/(9*b**2) + 4*d**2*x*sinh
(a + b*x)*cosh(a + b*x)**2/(3*b**2) + 14*d**2*sinh(a + b*x)**2*cosh(a + b*x
)/(9*b**3) - 40*d**2*cosh(a + b*x)**3/(27*b**3), Ne(b, 0)), ((c**2*x + c*d*
x**2 + d**2*x**3/3)*sinh(a)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(111) = 222$.
time = 0.41, size = 230, normalized size = 1.87

$$\frac{\left(9\,b^{2}d^{2}x^{2}+18\,b^{2}cdx+9\,b^{2}c^{2}-6\,bd^{2}x-6\,bcd+2\,d^{2}\right)e^{\left(3\,bx+3\,a\right)}}{216\,b^{3}}-\frac{3\,\left(b^{2}d^{2}x^{2}+2\,b^{2}cdx+b^{2}c^{2}-2\,bd^{2}x-2\,bcd+2\,d^{2}\right)e^{\left(bx+a\right)}}{8\,b^{3}}-\frac{3\,\left(b^{2}d^{2}x^{2}+2\,b^{2}cdx+b^{2}c^{2}+2\,bd^{2}x+2\,bcd+2\,d^{2}\right)e^{\left(-bx-a\right)}}{8\,b^{3}}+\frac{\left(9\,b^{2}d^{2}x^{2}+18\,b^{2}cdx+9\,b^{2}c^{2}+6\,bd^{2}x+6\,bcd+2\,d^{2}\right)e^{\left(-3\,bx-3\,a\right)}}{216\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/216*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 6*b*d^2*x - 6*b*c*d + 2*d
^2)*e^(3*b*x + 3*a)/b^3 - 3/8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*b*d^
2*x - 2*b*c*d + 2*d^2)*e^(b*x + a)/b^3 - 3/8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2 + 2*b*d^2*x + 2*b*c*d + 2*d^2)*e^(-b*x - a)/b^3 + 1/216*(9*b^2*d^2*x
^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 6*b*d^2*x + 6*b*c*d + 2*d^2)*e^(-3*b*x - 3*
a)/b^3
```

Mupad [B]
time = 0.41, size = 184, normalized size = 1.50

$$-\frac{\frac{3\,d^2\,\mathrm{cosh}\left(a+b\,x\right)}{2}-\frac{d^2\,\mathrm{cosh}\left(3\,a+3\,b\,x\right)}{54}+\frac{3\,b^2\,c^2\,\mathrm{cosh}\left(a+b\,x\right)}{4}-\frac{b^2\,c^2\,\mathrm{cosh}\left(3\,a+3\,b\,x\right)}{12}+\frac{3\,b^2\,d^2\,x^2\,\mathrm{cosh}\left(a+b\,x\right)}{4}+\frac{b\,c\,d\,\mathrm{sinh}\left(3\,a+3\,b\,x\right)}{18}-\frac{3\,b\,d^2\,x\,\mathrm{sinh}\left(a+b\,x\right)}{2}-\frac{b^2\,d^2\,x^2\,\mathrm{cosh}\left(3\,a+3\,b\,x\right)}{12}+\frac{b\,d^2\,x\,\mathrm{sinh}\left(3\,a+3\,b\,x\right)}{18}-\frac{3\,b\,c\,d\,\mathrm{sinh}\left(a+b\,x\right)}{2}-\frac{b^2\,c\,d\,x\,\mathrm{cosh}\left(3\,a+3\,b\,x\right)}{6}+\frac{3\,b^2\,c\,d\,x\,\mathrm{cosh}\left(a+b\,x\right)}{2}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] -((3*d^2*cosh(a + b*x))/2 - (d^2*cosh(3*a + 3*b*x))/54 + (3*b^2*c^2*cosh(a
 + b*x))/4 - (b^2*c^2*cosh(3*a + 3*b*x))/12 + (3*b^2*d^2*x^2*cosh(a + b*x))/
4 + (b*c*d*sinh(3*a + 3*b*x))/18 - (3*b*d^2*x*sinh(a + b*x))/2 - (b^2*d^2*x
^2*cosh(3*a + 3*b*x))/12 + (b*d^2*x*sinh(3*a + 3*b*x))/18 - (3*b*c*d*sinh(a
 + b*x))/2 - (b^2*c*d*x*cosh(3*a + 3*b*x))/6 + (3*b^2*c*d*x*cosh(a + b*x))/
2)/b^3
```

3.19 $\int (c+dx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=75

$$-\frac{2(c+dx)\cosh(a+bx)}{3b}+\frac{2d\sinh(a+bx)}{3b^2}+\frac{(c+dx)\cosh(a+bx)\sinh^2(a+bx)}{3b}-\frac{d\sinh^3(a+bx)}{9b^2}$$

```
[Out] -2/3*(d*x+c)*cosh(b*x+a)/b+2/3*d*sinh(b*x+a)/b^2+1/3*(d*x+c)*cosh(b*x+a)*si
nh(b*x+a)^2/b-1/9*d*sinh(b*x+a)^3/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3391, 3377, 2717}

$$-\frac{d\sinh^3(a+bx)}{9b^2}+\frac{2d\sinh(a+bx)}{3b^2}-\frac{2(c+dx)\cosh(a+bx)}{3b}+\frac{(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sinh[a + b*x]^3,x]
```

```
[Out] (-2*(c + d*x)*Cosh[a + b*x])/(3*b) + (2*d*Sinh[a + b*x])/(3*b^2) + ((c + d*
x)*Cosh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (d*Sinh[a + b*x]^3)/(9*b^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)\sinh^3(a+bx)\,dx = \frac{(c+dx)\cosh(a+bx)\sinh^2(a+bx)}{3b} - \frac{d\sinh^3(a+bx)}{9b^2} - \frac{2}{3}\int (c+dx)\sinh(a$$

$$= -\frac{2(c+dx)\cosh(a+bx)}{3b} + \frac{(c+dx)\cosh(a+bx)\sinh^2(a+bx)}{3b} - \frac{d\sinh^3(a+}{9b^2}$$

$$= -\frac{2(c+dx)\cosh(a+bx)}{3b} + \frac{2d\sinh(a+bx)}{3b^2} + \frac{(c+dx)\cosh(a+bx)\sinh^2(a+}{3b}$$

Mathematica [A]
time = 0.14, size = 59, normalized size = 0.79

$$\frac{-27b(c+dx)\cosh(a+bx)+3b(c+dx)\cosh(3(a+bx))+d(27\sinh(a+bx)-\sinh(3(a+bx)))}{36b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sinh[a + b*x]^3,x]
```

```
[Out] (-27*b*(c + d*x)*Cosh[a + b*x] + 3*b*(c + d*x)*Cosh[3*(a + b*x)] + d*(27*Si
nh[a + b*x] - Sinh[3*(a + b*x)]))/(36*b^2)
```

Maple [A]
time = 0.46, size = 125, normalized size = 1.67

method	result
risch	$\frac{(3bdx+3bc-d)e^{3bx+3a}}{72b^2} - \frac{3(bdx+bc-d)e^{bx+a}}{8b^2} - \frac{3(bdx+bc+d)e^{-bx-a}}{8b^2} + \frac{(3bdx+3bc+d)e^{-3bx-3a}}{72b^2}$
default	$\frac{-\frac{3d((bx+a)\cosh(bx+a)-\sinh(bx+a))}{4b}+\frac{3ad\cosh(bx+a)}{4b}-\frac{3c\cosh(bx+a)}{4}}{b} + \frac{\frac{d((3bx+3a)\cosh(3bx+3a)-\sinh(3bx+3a))}{b}-\frac{3ad\cosh(3bx+3a)}{b}+3c\cos}{36b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4/b*(-d/b*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))+a*d/b*cosh(b*x+a)-c*cosh(b*x+
a))+1/36/b*(d/b*((3*b*x+3*a)*cosh(3*b*x+3*a)-sinh(3*b*x+3*a))-3*a*d/b*cosh(
3*b*x+3*a)+3*c*cosh(3*b*x+3*a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(67) = 134.
time = 0.28, size = 141, normalized size = 1.88

$$\frac{1}{72}d\left(\frac{(3bxe^{(3a)}-e^{(3a)})e^{(3bx)}}{b^2} - \frac{27(bxe^{a}-e^{a})e^{(bx)}}{b^2} - \frac{27(bx+1)e^{(-bx-a)}}{b^2} + \frac{(3bx+1)e^{(-3bx-3a)}}{b^2}\right) + \frac{1}{24}c\left(\frac{e^{(3bx+3a)}}{b} - \frac{9e^{(bx+a)}}{b} - \frac{9e^{(-bx-a)}}{b} + \frac{e^{(-3bx-3a)}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/72*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 - 27*(b*x*e^a - e^a)*e^(b*x
)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 + (3*b*x + 1)*e^(-3*b*x - 3*a)/b^2) +
 1/24*c*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b + e^(-3*b*x
 - 3*a)/b)
```

Fricas [A]
time = 0.39, size = 97, normalized size = 1.29

$$\frac{3\,(bdx+bc)\cosh{(bx+a)}^{3}+9\,(bdx+bc)\cosh{(bx+a)}\sinh{(bx+a)}^{2}-d\sinh{(bx+a)}^{3}-27\,(bdx+bc)\cosh{(bx+a)}-3\left(d\cosh{(bx+a)}^{2}-9\,d\right)\sinh{(bx+a)}}{36\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/36*(3*(b*d*x + b*c)*cosh(b*x + a)^3 + 9*(b*d*x + b*c)*cosh(b*x + a)*sinh(
b*x + a)^2 - d*sinh(b*x + a)^3 - 27*(b*d*x + b*c)*cosh(b*x + a) - 3*(d*cosh
(b*x + a)^2 - 9*d)*sinh(b*x + a))/b^2
```

Sympy [A]
time = 0.20, size = 126, normalized size = 1.68

$$\begin{cases} \frac{c\sinh^{2}{(a+bx)}\cosh{(a+bx)}}{b}-\frac{2c\cosh^{3}{(a+bx)}}{3b}+\frac{dx\sinh^{2}{(a+bx)}\cosh{(a+bx)}}{b}-\frac{2dx\cosh^{3}{(a+bx)}}{3b}-\frac{7d\sinh^{3}{(a+bx)}}{9b^{2}}+\frac{2d\sinh{(a+bx)}\cosh^{2}{(a+bx)}}{3b^{2}} & \text{for}\ b\neq 0 \\ \left(cx+\frac{dx^{2}}{2}\right)\sinh^{3}{(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((c*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c*cosh(a + b*x)**3/(3*b)
+ d*x*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*d*x*cosh(a + b*x)**3/(3*b) - 7*d
*sinh(a + b*x)**3/(9*b**2) + 2*d*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**2), N
e(b, 0)), ((c*x + d*x**2/2)*sinh(a)**3, True))
```

Giac [A]
time = 0.44, size = 98, normalized size = 1.31

$$\frac{(3\,bdx+3\,bc-d)e^{(3\,bx+3\,a)}}{72\,b^{2}}-\frac{3\,(bdx+bc-d)e^{(bx+a)}}{8\,b^{2}}-\frac{3\,(bdx+bc+d)e^{(-bx-a)}}{8\,b^{2}}+\frac{(3\,bdx+3\,bc+d)e^{(-3\,bx-3\,a)}}{72\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/72*(3*b*d*x + 3*b*c - d)*e^(3*b*x + 3*a)/b^2 - 3/8*(b*d*x + b*c - d)*e^(b
*x + a)/b^2 - 3/8*(b*d*x + b*c + d)*e^(-b*x - a)/b^2 + 1/72*(3*b*d*x + 3*b*
c + d)*e^(-3*b*x - 3*a)/b^2
```

Mupad [B]
time = 0.16, size = 79, normalized size = 1.05

$$\frac{7\,d\sinh(a+b\,x)}{9\,b^2}-\frac{c\cosh(a+b\,x)-\frac{c\cosh(a+b\,x)^3}{3}+d\,x\cosh(a+b\,x)-\frac{d\,x\cosh(a+b\,x)^3}{3}}{b}-\frac{d\cosh(a+b\,x)^2\sinh(a+b\,x)}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3*(c + d*x),x)
```

```
[Out] (7*d*sinh(a + b*x))/(9*b^2) - (c*cosh(a + b*x) - (c*cosh(a + b*x)^3)/3 + d*
x*cosh(a + b*x) - (d*x*cosh(a + b*x)^3)/3)/b - (d*cosh(a + b*x)^2*sinh(a +
b*x))/(9*b^2)
```

3.20 $\int \frac{\sinh^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=121

$$\frac{\text{Chi}\left(\frac{3bc}{d}+3bx\right)\sinh\left(3a-\frac{3bc}{d}\right)}{4d}-\frac{3\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{4d}-\frac{3\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cosh\left(3a-\frac{3bc}{d}\right.}{}$$

```
[Out] -3/4*cosh(a-b*c/d)*Shi(b*c/d+b*x)/d+1/4*cosh(3*a-3*b*c/d)*Shi(3*b*c/d+3*b*x
)/d+1/4*Chi(3*b*c/d+3*b*x)*sinh(3*a-3*b*c/d)/d-3/4*Chi(b*c/d+b*x)*sinh(a-b*
c/d)/d
```

Rubi [A]
time = 0.19, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3393, 3384, 3379, 3382}

$$\frac{\sinh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d}-\frac{3\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{3\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cosh\left(3a-\frac{3bc}{d}\right)\text{Shi}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3/(c + d*x),x]
```

```
[Out] (CoshIntegral[(3*b*c)/d + 3*b*x]*Sinh[3*a - (3*b*c)/d])/(4*d) - (3*CoshInte
gral[(b*c)/d + b*x]*Sinh[a - (b*c)/d])/(4*d) - (3*Cosh[a - (b*c)/d]*SinhInt
egral[(b*c)/d + b*x])/(4*d) + (Cosh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d
 + 3*b*x])/(4*d)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(a+bx)}{c+dx}\,dx &= i\int \left(\frac{3i\sinh(a+bx)}{4(c+dx)}-\frac{i\sinh(3a+3bx)}{4(c+dx)}\right)dx\\ &= \frac{1}{4}\int \frac{\sinh(3a+3bx)}{c+dx}\,dx-\frac{3}{4}\int \frac{\sinh(a+bx)}{c+dx}\,dx\\ &= \frac{1}{4}\cosh\left(3a-\frac{3bc}{d}\right)\int \frac{\sinh\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx-\frac{1}{4}\left(3\cosh\left(a-\frac{bc}{d}\right)\right)\int \frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}\,d\\ &= \frac{\text{Chi}\left(\frac{3bc}{d}+3bx\right)\sinh\left(3a-\frac{3bc}{d}\right)}{4d}-\frac{3\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{4d}-\frac{3\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{b}{d}\right.}{4d}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 102, normalized size = 0.84

$$\frac{\text{Chi}\left(\frac{3b(c+dx)}{d}\right)\sinh\left(3a-\frac{3bc}{d}\right)-3\text{Chi}\left(b\left(\frac{c}{d}+x\right)\right)\sinh\left(a-\frac{bc}{d}\right)-3\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(b\left(\frac{c}{d}+x\right)\right)+\cosh\left(3a-\frac{3bc}{d}\right)\text{Shi}\left(\frac{3b(c+dx)}{d}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3/(c + d*x),x]
```

```
[Out] (CoshIntegral[(3*b*(c + d*x))/d]*Sinh[3*a - (3*b*c)/d] - 3*CoshIntegral[b*(
c/d + x)]*Sinh[a - (b*c)/d] - 3*Cosh[a - (b*c)/d]*SinhIntegral[b*(c/d + x)]
 + Cosh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*(c + d*x))/d])/(4*d)
```

Maple [A]
time = 1.60, size = 166, normalized size = 1.37

method	result
risch	$\frac{e^{-\frac{3(ad-bc)}{d}}\text{expIntegral}\left(1,3bx+3a-\frac{3(ad-bc)}{d}\right)}{8d}-\frac{3\,e^{-\frac{ad-bc}{d}}\text{expIntegral}\left(1,bx+a-\frac{ad-bc}{d}\right)}{8d}+\frac{3\,e^{\frac{ad-bc}{d}}\text{expIntegral}\left(1,-bx-a-\frac{-a}{}\right.}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*exp(-3*(a*d-b*c)/d)*Ei(1,3*b*x+3*a-3*(a*d-b*c)/d)-3/8/d*exp(-(a*d-b*c
)/d)*Ei(1,b*x+a-(a*d-b*c)/d)+3/8/d*exp((a*d-b*c)/d)*Ei(1,-b*x-a-(-a*d+b*c)/
d)-1/8/d*exp(3*(a*d-b*c)/d)*Ei(1,-3*b*x-3*a-3*(-a*d+b*c)/d)
```

Maxima [A]
time = 0.32, size = 117, normalized size = 0.97

$$\frac{e^{\left(-3\,a+\frac{3\,bc}{d}\right)}E_1\left(\frac{3\,(dx+c)b}{d}\right)}{8\,d}-\frac{3\,e^{\left(-a+\frac{bc}{d}\right)}E_1\left(\frac{(dx+c)b}{d}\right)}{8\,d}+\frac{3\,e^{\left(a-\frac{bc}{d}\right)}E_1\left(-\frac{(dx+c)b}{d}\right)}{8\,d}-\frac{e^{\left(3\,a-\frac{3\,bc}{d}\right)}E_1\left(-\frac{3\,(dx+c)b}{d}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/8*e^(-3*a + 3*b*c/d)*exp_integral_e(1, 3*(d*x + c)*b/d)/d - 3/8*e^(-a + b
*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d + 3/8*e^(a - b*c/d)*exp_integral_e
(1, -(d*x + c)*b/d)/d - 1/8*e^(3*a - 3*b*c/d)*exp_integral_e(1, -3*(d*x + c
)*b/d)/d
```

Fricas [A]
time = 0.38, size = 188, normalized size = 1.55

$$-\frac{3\,\left(\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)-\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)-\left(\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)-\mathrm{Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{3\,(bc-ad)}{d}\right)+3\,\left(\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)+\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\sinh\left(-\frac{bc-ad}{d}\right)-\left(\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)+\mathrm{Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\sinh\left(-\frac{3\,(bc-ad)}{d}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/8*(3*(Ei((b*d*x + b*c)/d) - Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) -
 (Ei(3*(b*d*x + b*c)/d) - Ei(-3*(b*d*x + b*c)/d))*cosh(-3*(b*c - a*d)/d) +
3*(Ei((b*d*x + b*c)/d) + Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d) - (Ei(3
*(b*d*x + b*c)/d) + Ei(-3*(b*d*x + b*c)/d))*sinh(-3*(b*c - a*d)/d))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(sinh(a + b*x)**3/(c + d*x), x)
```

Giac [A]
time = 0.42, size = 113, normalized size = 0.93

$$\frac{\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)}-3\,\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}+3\,\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}-\mathrm{Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)e^{\left(-3\,a+\frac{3\,bc}{d}\right)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/8*(Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) - 3*Ei((b*d*x + b*c)/d)*e^(a -
 b*c/d) + 3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - Ei(-3*(b*d*x + b*c)/d)*e^
(-3*a + 3*b*c/d))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(a+b\,x)^3}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3/(c + d*x),x)
```

```
[Out] int(sinh(a + b*x)^3/(c + d*x), x)
```

3.21 $\int \frac{\sinh^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=145

$$-\frac{3b\cosh\left(a-\frac{bc}{d}\right)\operatorname{Chi}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cosh\left(3a-\frac{3bc}{d}\right)\operatorname{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{\sinh^3(a+bx)}{d(c+dx)}-\frac{3b\sinh\left(a-\frac{bc}{d}\right)\operatorname{Shi}\left(\frac{bc}{d}\right.}{4d^2}$$

```
[Out] 3/4*b*Chi(3*b*c/d+3*b*x)*cosh(3*a-3*b*c/d)/d^2-3/4*b*Chi(b*c/d+b*x)*cosh(a-
b*c/d)/d^2+3/4*b*Shi(3*b*c/d+3*b*x)*sinh(3*a-3*b*c/d)/d^2-3/4*b*Shi(b*c/d+b
*x)*sinh(a-b*c/d)/d^2-sinh(b*x+a)^3/d/(d*x+c)
```

Rubi [A]
time = 0.19, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3394, 3384, 3379, 3382}

$$-\frac{3b\cosh\left(a-\frac{bc}{d}\right)\operatorname{Chi}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cosh\left(3a-\frac{3bc}{d}\right)\operatorname{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{3b\sinh\left(a-\frac{bc}{d}\right)\operatorname{Shi}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\sinh\left(3a-\frac{3bc}{d}\right)\operatorname{Shi}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{\sinh^3(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] (-3*b*Cosh[a - (b*c)/d]*CoshIntegral[(b*c)/d + b*x])/(4*d^2) + (3*b*Cosh[3*
a - (3*b*c)/d]*CoshIntegral[(3*b*c)/d + 3*b*x])/(4*d^2) - Sinh[a + b*x]^3/(
d*(c + d*x)) - (3*b*Sinh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/(4*d^2)
+ (3*b*Sinh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d + 3*b*x])/(4*d^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(a+bx)}{(c+dx)^2}\,dx &= -\frac{\sinh^3(a+bx)}{d(c+dx)} - \frac{(3b)\int\left(\frac{\cosh(a+bx)}{4(c+dx)} - \frac{\cosh(3a+3bx)}{4(c+dx)}\right)dx}{d} \\
&= -\frac{\sinh^3(a+bx)}{d(c+dx)} - \frac{(3b)\int\frac{\cosh(a+bx)}{c+dx}\,dx}{4d} + \frac{(3b)\int\frac{\cosh(3a+3bx)}{c+dx}\,dx}{4d} \\
&= -\frac{\sinh^3(a+bx)}{d(c+dx)} + \frac{\left(3b\cosh\left(3a-\frac{3bc}{d}\right)\right)\int\frac{\cosh\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx}{4d} - \frac{\left(3b\cosh\left(a-\frac{bc}{d}\right)\right)\int\frac{\cosh\left(\frac{b}{c}\right.}{c+}}{4d} \\
&= -\frac{3b\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{4d^2} + \frac{3b\cosh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d^2} - \frac{\sinh^3(a+bx)}{d(c+dx)}
\end{aligned}$$

Mathematica [A]

time = 0.82, size = 160, normalized size = 1.10

$$\frac{6d\cosh(bx)\sinh(a)-2d\cosh(3bx)\sinh(3a)+6d\cosh(a)\sinh(bx)-2d\cosh(3a)\sinh(3bx)+6b(c+dx)\left(-\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(b\left(\frac{c}{d}+x\right)\right)+\cosh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3b(c+dx)}{d}\right)-\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(b\left(\frac{c}{d}+x\right)\right)+\sinh\left(3a-\frac{3bc}{d}\right)\text{Shi}\left(\frac{3b(c+dx)}{d}\right)\right)}{8d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] (6*d*Cosh[b*x]*Sinh[a] - 2*d*Cosh[3*b*x]*Sinh[3*a] + 6*d*Cosh[a]*Sinh[b*x]
- 2*d*Cosh[3*a]*Sinh[3*b*x] + 6*b*(c + d*x)*(-(Cosh[a - (b*c)/d]*CoshIntegr
al[b*(c/d + x)]) + Cosh[3*a - (3*b*c)/d]*CoshIntegral[(3*b*(c + d*x))/d] -
Sinh[a - (b*c)/d]*SinhIntegral[b*(c/d + x)] + Sinh[3*a - (3*b*c)/d]*SinhInt
egral[(3*b*(c + d*x))/d]))/(8*d^2*(c + d*x))
```

Maple [A]

time = 1.62, size = 271, normalized size = 1.87

method	result
risch	$\frac{b\,\mathrm{e}^{-3bx-3a}}{8d(bdx+bc)} - \frac{3b\,\mathrm{e}^{-\frac{3(ad-bc)}{d}}\,\text{expIntegral}\left(1,3bx+3a-\frac{3(ad-bc)}{d}\right)}{8d^2} - \frac{3b\,\mathrm{e}^{-bx-a}}{8d(bdx+bc)} + \frac{3b\,\mathrm{e}^{-\frac{ad-bc}{d}}\,\text{expIntegral}\left(1,bx+a-\frac{ad-bc}{d}\right)}{8d^2} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b*exp(-3*b*x-3*a)/d/(b*d*x+b*c)-3/8*b/d^2*exp(-3*(a*d-b*c)/d)*Ei(1,3*b*
x+3*a-3*(a*d-b*c)/d)-3/8*b*exp(-b*x-a)/d/(b*d*x+b*c)+3/8*b/d^2*exp(-(a*d-b*
c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)+3/8*b/d^2*exp(b*x+a)/(b*c/d+b*x)+3/8*b/d^2*ex
p((a*d-b*c)/d)*Ei(1,-b*x-a-(-a*d+b*c)/d)-1/8*b/d^2*exp(3*b*x+3*a)/(b*c/d+b*
x)-3/8*b/d^2*exp(3*(a*d-b*c)/d)*Ei(1,-3*b*x-3*a-3*(-a*d+b*c)/d)
```

Maxima [A]
time = 0.34, size = 145, normalized size = 1.00

$$\frac{e^{\left(-3\,a+\frac{3\,bc}{d}\right)}E_2\left(\frac{3\,(dx+c)b}{d}\right)}{8\,(dx+c)d}-\frac{3\,e^{\left(-a+\frac{bc}{d}\right)}E_2\left(\frac{(dx+c)b}{d}\right)}{8\,(dx+c)d}+\frac{3\,e^{\left(a-\frac{bc}{d}\right)}E_2\left(-\frac{(dx+c)b}{d}\right)}{8\,(dx+c)d}-\frac{e^{\left(3\,a-\frac{3\,bc}{d}\right)}E_2\left(-\frac{3\,(dx+c)b}{d}\right)}{8\,(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/8*e^(-3*a + 3*b*c/d)*exp_integral_e(2, 3*(d*x + c)*b/d)/((d*x + c)*d) - 3
/8*e^(-a + b*c/d)*exp_integral_e(2, (d*x + c)*b/d)/((d*x + c)*d) + 3/8*e^(a
 - b*c/d)*exp_integral_e(2, -(d*x + c)*b/d)/((d*x + c)*d) - 1/8*e^(3*a - 3*
b*c/d)*exp_integral_e(2, -3*(d*x + c)*b/d)/((d*x + c)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(137) = 274$.
time = 0.34, size = 301, normalized size = 2.08

$$-\frac{2\,d\sinh\left(bx+a\right)^3+3\left((bdx+bc)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)+(bdx+bc)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)-3\left((bdx+bc)\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)+(bdx+bc)\mathrm{Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{3\,(bc-ad)}{d}\right)+6\left(d\cosh\left(bx+a\right)^2-d\right)\sinh\left(bx+a\right)+3\left((bdx+bc)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)-(bdx+bc)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\sinh\left(-\frac{bc-ad}{d}\right)-3\left((bdx+bc)\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)-(bdx+bc)\mathrm{Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\sinh\left(-\frac{3\,(bc-ad)}{d}\right)}{8\,(d^3x+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*d*sinh(b*x + a)^3 + 3*((b*d*x + b*c)*Ei((b*d*x + b*c)/d) + (b*d*x +
 b*c)*Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) - 3*((b*d*x + b*c)*Ei(3*(b
*d*x + b*c)/d) + (b*d*x + b*c)*Ei(-3*(b*d*x + b*c)/d))*cosh(-3*(b*c - a*d)/
d) + 6*(d*cosh(b*x + a)^2 - d)*sinh(b*x + a) + 3*((b*d*x + b*c)*Ei((b*d*x +
 b*c)/d) - (b*d*x + b*c)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d) - 3*((b
*d*x + b*c)*Ei(3*(b*d*x + b*c)/d) - (b*d*x + b*c)*Ei(-3*(b*d*x + b*c)/d))*s
inh(-3*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(sinh(a + b*x)**3/(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. $2(137) = 274$.
time = 0.48, size = 1076, normalized size = 7.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*Ei(-3*((d*x + c)*(
b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(3*(b*c - a*d)/d) + 3*
b^3*c*Ei(-3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*
e^(3*(b*c - a*d)/d) - 3*a*b^2*d*Ei(-3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(
d*x + c)) + b*c - a*d)/d)*e^(3*(b*c - a*d)/d) - 3*(d*x + c)*(b - b*c/(d*x +
 c) + a*d/(d*x + c))*b^2*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))
 + b*c - a*d)/d)*e^((b*c - a*d)/d) - 3*b^3*c*Ei(-((d*x + c)*(b - b*c/(d*x +
 c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d) + 3*a*b^2*d*Ei(-((d*
x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d
) - 3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*Ei(((d*x + c)*(b -
b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-(b*c - a*d)/d) - 3*b^3*c
*Ei(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-(b*c
 - a*d)/d) + 3*a*b^2*d*Ei(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) +
b*c - a*d)/d)*e^(-(b*c - a*d)/d) + 3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*
x + c))*b^2*Ei(3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d
)/d)*e^(-3*(b*c - a*d)/d) + 3*b^3*c*Ei(3*((d*x + c)*(b - b*c/(d*x + c) + a*
d/(d*x + c)) + b*c - a*d)/d)*e^(-3*(b*c - a*d)/d) - 3*a*b^2*d*Ei(3*((d*x +
c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-3*(b*c - a*d)/d)
 - b^2*d*e^(3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d) + 3*b^2*d*e^
((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d) - 3*b^2*d*e^(-(d*x + c)*(
b - b*c/(d*x + c) + a*d/(d*x + c))/d) + b^2*d*e^(-3*(d*x + c)*(b - b*c/(d*x
 + c) + a*d/(d*x + c))/d))*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x +
c))*d^4 + b*c*d^4 - a*d^5)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(a+b\,x)^3}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3/(c + d*x)^2,x)
```

```
[Out] int(sinh(a + b*x)^3/(c + d*x)^2, x)
```

3.22 $\int \frac{\sinh^3(a+bx)}{(c+dx)^3} \, dx$

Optimal. Leaf size=184

$$\frac{9b^2\text{Chi}\left(\frac{3bc}{d}+3bx\right)\sinh\left(3a-\frac{3bc}{d}\right)}{8d^3}-\frac{3b^2\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{8d^3}-\frac{3b\cosh(a+bx)\sinh^2(a+bx)}{2d^2(c+dx)}-\frac{\sinh^3(a}{2d(c+}$$

```
[Out] -3/8*b^2*cosh(a-b*c/d)*Shi(b*c/d+b*x)/d^3+9/8*b^2*cosh(3*a-3*b*c/d)*Shi(3*b
*c/d+3*b*x)/d^3+9/8*b^2*Chi(3*b*c/d+3*b*x)*sinh(3*a-3*b*c/d)/d^3-3/8*b^2*Ch
i(b*c/d+b*x)*sinh(a-b*c/d)/d^3-3/2*b*cosh(b*x+a)*sinh(b*x+a)^2/d^2/(d*x+c)-
1/2*sinh(b*x+a)^3/d/(d*x+c)^2
```

Rubi [A]
time = 0.30, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3395, 3384, 3379, 3382, 3393}

$$\frac{9b^2\sinh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{8d^3}-\frac{3b^2\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{3b^2\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2\cosh\left(3a-\frac{3bc}{d}\right)\text{Shi}\left(\frac{3bc}{d}+3bx\right)}{8d^3}-\frac{3b\sinh^2(a+bx)\cosh(a+bx)}{2d^2(c+dx)}-\frac{\sinh^3(a+bx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3/(c + d*x)^3,x]
```

```
[Out] (9*b^2*CoshIntegral[(3*b*c)/d + 3*b*x]*Sinh[3*a - (3*b*c)/d])/(8*d^3) - (3*
b^2*CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*c)/d])/(8*d^3) - (3*b*Cosh[a +
b*x]*Sinh[a + b*x]^2)/(2*d^2*(c + d*x)) - Sinh[a + b*x]^3/(2*d*(c + d*x)^2)
 - (3*b^2*Cosh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/(8*d^3) + (9*b^2*C
osh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx &= -\frac{3b\cosh(a+bx)\sinh^2(a+bx)}{2d^2(c+dx)} - \frac{\sinh^3(a+bx)}{2d(c+dx)^2} + \frac{(3b^2)\int \frac{\sinh(a+bx)}{c+dx}\,dx}{d^2} + \frac{(9b^2)\int \frac{\sinh^3}{c+}}{2d^2} \\
&= -\frac{3b\cosh(a+bx)\sinh^2(a+bx)}{2d^2(c+dx)} - \frac{\sinh^3(a+bx)}{2d(c+dx)^2} + \frac{(9ib^2)\int \left(\frac{3i\sinh(a+bx)}{4(c+dx)} - \frac{i\sinh(3a+3bx)}{4(c+dx)}\right.}{2d^2} \\
&= \frac{3b^2\operatorname{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{d^3} - \frac{3b\cosh(a+bx)\sinh^2(a+bx)}{2d^2(c+dx)} - \frac{\sinh^3(a+bx)}{2d(c+dx)^2} + \frac{3b^2}{} \\
&= \frac{3b^2\operatorname{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{d^3} - \frac{3b\cosh(a+bx)\sinh^2(a+bx)}{2d^2(c+dx)} - \frac{\sinh^3(a+bx)}{2d(c+dx)^2} + \frac{3b^2}{} \\
&= \frac{9b^2\operatorname{Chi}\left(\frac{3bc}{d}+3bx\right)\sinh\left(3a-\frac{3bc}{d}\right)}{8d^3} - \frac{3b^2\operatorname{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{8d^3} - \frac{3b\cosh(a+bx)}{2d^2(c+}
\end{aligned}$$

Mathematica [A]
time = 0.54, size = 220, normalized size = 1.20

$$\frac{6d\cosh(bx)(b(c+dx)\cosh(a)+d\sinh(a)) - 2d\cosh(3bx)(3b(c+dx)\cosh(3a)+d\sinh(3a)) + 6d(d\cosh(a)+b(c+dx)\sinh(a))\sinh(bx) - 2d(d\cosh(3a)+3b(c+dx)\sinh(3a))\sinh(3bx) + 6b^2(c+dx)^2\left(3\operatorname{Chi}\left(\frac{3b(c+dx)}{d}\right)\sinh\left(3a-\frac{3bc}{d}\right) - \operatorname{Chi}\left(b\left(\frac{c}{d}+x\right)\right)\sinh\left(a-\frac{bc}{d}\right) - \cosh\left(a-\frac{bc}{d}\right)\operatorname{Shi}\left(b\left(\frac{c}{d}+x\right)\right) + 3\cosh\left(3a-\frac{3bc}{d}\right)\operatorname{Shi}\left(\frac{3b(c+dx)}{d}\right)\right)}{16d^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3/(c + d*x)^3,x]
```

```
[Out] (6*d*Cosh[b*x]*(b*(c + d*x)*Cosh[a] + d*Sinh[a]) - 2*d*Cosh[3*b*x]*(3*b*(c
+ d*x)*Cosh[3*a] + d*Sinh[3*a]) + 6*d*(d*Cosh[a] + b*(c + d*x)*Sinh[a])*Sin
h[b*x] - 2*d*(d*Cosh[3*a] + 3*b*(c + d*x)*Sinh[3*a])*Sinh[3*b*x] + 6*b^2*(c
 + d*x)^2*(3*CoshIntegral[(3*b*(c + d*x))/d]*Sinh[3*a - (3*b*c)/d] - CoshIn
tegral[b*(c/d + x)]*Sinh[a - (b*c)/d] - Cosh[a - (b*c)/d]*SinhIntegral[b*(c
/d + x)] + 3*Cosh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*(c + d*x))/d]))/(16*d^
3*(c + d*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(172) = 344$.
time = 1.64, size = 562, normalized size = 3.05

method	result
risch	$-\frac{3b^3 \mathrm{e}^{-3bx-3a} x}{16d(b^2d^2x^2+2b^2cdx+b^2c^2)} - \frac{3b^3 \mathrm{e}^{-3bx-3a} c}{16d^2(b^2d^2x^2+2b^2cdx+b^2c^2)} + \frac{b^2 \mathrm{e}^{-3bx-3a}}{16d(b^2d^2x^2+2b^2cdx+b^2c^2)} + \frac{9b^2 \mathrm{e}^{-\frac{3(ad-bc)}{d}} \mathrm{expIntegral}\left(1,3b\right.}{16d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/16*b^3*exp(-3*b*x-3*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*x-3/16*b^3*ex
p(-3*b*x-3*a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c+1/16*b^2*exp(-3*b*x-3
*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)+9/16*b^2/d^3*exp(-3*(a*d-b*c)/d)*Ei
(1,3*b*x+3*a-3*(a*d-b*c)/d)+3/16*b^3*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*d*x
+b^2*c^2)*x+3/16*b^3*exp(-b*x-a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c-3/
16*b^2*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)-3/16*b^2/d^3*exp(-(a
*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)+3/16*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)^2+3/
16*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)+3/16*b^2/d^3*exp((a*d-b*c)/d)*Ei(1,-b*x-a
-(-a*d+b*c)/d)-1/16*b^2/d^3*exp(3*b*x+3*a)/(b*c/d+b*x)^2-3/16*b^2/d^3*exp(3
*b*x+3*a)/(b*c/d+b*x)-9/16*b^2/d^3*exp(3*(a*d-b*c)/d)*Ei(1,-3*b*x-3*a-3*(-a
*d+b*c)/d)
```

Maxima [A]
time = 0.34, size = 145, normalized size = 0.79

$$\frac{e^{\left(-3a+\frac{3bc}{d}\right)}E_3\left(\frac{3(dx+c)b}{d}\right)}{8(dx+c)^2d} - \frac{3e^{\left(-a+\frac{bc}{d}\right)}E_3\left(\frac{(dx+c)b}{d}\right)}{8(dx+c)^2d} + \frac{3e^{\left(a-\frac{bc}{d}\right)}E_3\left(-\frac{(dx+c)b}{d}\right)}{8(dx+c)^2d} - \frac{e^{\left(3a-\frac{3bc}{d}\right)}E_3\left(-\frac{3(dx+c)b}{d}\right)}{8(dx+c)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*e^(-3*a + 3*b*c/d)*exp_integral_e(3, 3*(d*x + c)*b/d)/((d*x + c)^2*d) -
 3/8*e^(-a + b*c/d)*exp_integral_e(3, (d*x + c)*b/d)/((d*x + c)^2*d) + 3/8*
e^(a - b*c/d)*exp_integral_e(3, -(d*x + c)*b/d)/((d*x + c)^2*d) - 1/8*e^(3*
a - 3*b*c/d)*exp_integral_e(3, -3*(d*x + c)*b/d)/((d*x + c)^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(172) = 344$.
time = 0.33, size = 529, normalized size = 2.88

$$-\frac{2\,d^{2}\sinh\left(bx+a\right)^{3}+6\,\left(bd^{2}x+bcd\right)\cosh\left(bx+a\right)^{3}+18\,\left(bd^{2}x+bcd\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}-6\,\left(bd^{2}x+bcd\right)\cosh\left(bx+a\right)+3\,\left(\left(b^{2}d^{2}x^{2}+2\,b^{2}cdx+b^{2}c^{2}\right)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)-\left(b^{2}d^{2}x^{2}+2\,b^{2}cdx+b^{2}c^{2}\right)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)-9\,\left(\left(b^{2}d^{2}x^{2}+2\,b^{2}cdx+b^{2}c^{2}\right)\mathrm{Ei}\left(\frac{3\,\left(bdx+bc\right)}{d}\right)-\left(b^{2}d^{2}x^{2}+2\,b^{2}cdx+b^{2}c^{2}\right)\mathrm{Ei}\left(-\frac{3\,\left(bdx+bc\right)}{d}\right)\right)\cosh\left(-\frac{3\,\left(bc-ad\right)}{d}\right)+6\,\left(d^{2}\cosh\left(bx+a\right)^{2}-d^{2}\right)\sinh\left(bx+a\right)+3\,\left(\left(b^{2}d^{2}x^{2}+2\,b^{2}cdx+b^{2}c^{2}\right)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)+\left(b^{2}d^{2}x^{2}+2\,b^{2}cdx+b^{2}c^{2}\right)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\sinh\left(-\frac{bc-ad}{d}\right)-9\,\left(\left(b^{2}d^{2}x^{2}+2\,b^{2}cdx+b^{2}c^{2}\right)\mathrm{Ei}\left(\frac{3\,\left(bdx+bc\right)}{d}\right)+\left(b^{2}d^{2}x^{2}+2\,b^{2}cdx+b^{2}c^{2}\right)\mathrm{Ei}\left(-\frac{3\,\left(bdx+bc\right)}{d}\right)\right)\sinh\left(-\frac{3\,\left(bc-ad\right)}{d}\right)}{16\,\left(d^{5}x^{2}+2\,cd^{4}x+c^{2}d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*d^2*sinh(b*x + a)^3 + 6*(b*d^2*x + b*c*d)*cosh(b*x + a)^3 + 18*(b*
d^2*x + b*c*d)*cosh(b*x + a)*sinh(b*x + a)^2 - 6*(b*d^2*x + b*c*d)*cosh(b*x
 + a) + 3*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) - (b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d
) - 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(3*(b*d*x + b*c)/d) - (b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-3*(b*d*x + b*c)/d))*cosh(-3*(b*c - a*d)
/d) + 6*(d^2*cosh(b*x + a)^2 - d^2)*sinh(b*x + a) + 3*((b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d) - 9*((b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2)*Ei(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*Ei(-3*(b*d*x + b*c)/d))*sinh(-3*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2
*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{3}\left(a+bx\right)}{\left(c+dx\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Integral(sinh(a + b*x)**3/(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(172) = 344$.
time = 0.42, size = 601, normalized size = 3.27

$$\frac{9\,b^{2}d^{2}x^{2}\mathrm{Ei}\left(\frac{3\,\left(bdx+bc\right)}{d}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)}-3\,b^{2}d^{2}x^{2}\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}+3\,b^{2}d^{2}x^{2}\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}-9\,b^{2}d^{2}x^{2}\mathrm{Ei}\left(-\frac{3\,\left(bdx+bc\right)}{d}\right)e^{\left(-3\,a+\frac{3\,bc}{d}\right)}+18\,b^{2}cdx\mathrm{Ei}\left(\frac{3\,\left(bdx+bc\right)}{d}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)}-6\,b^{2}cdx\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}+6\,b^{2}cdx\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}-18\,b^{2}cdx\mathrm{Ei}\left(-\frac{3\,\left(bdx+bc\right)}{d}\right)e^{\left(-3\,a+\frac{3\,bc}{d}\right)}+9\,b^{2}c^{2}\mathrm{Ei}\left(\frac{3\,\left(bdx+bc\right)}{d}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)}-3\,b^{2}c^{2}\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}+3\,b^{2}c^{2}\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}-9\,b^{2}c^{2}\mathrm{Ei}\left(-\frac{3\,\left(bdx+bc\right)}{d}\right)e^{\left(-3\,a+\frac{3\,bc}{d}\right)}-3\,bd^{2}xe^{\left(3\,bx+3\,a\right)}+3\,bd^{2}xe^{\left(bx+a\right)}+3\,bd^{2}xe^{\left(-bx-a\right)}-3\,bd^{2}xe^{\left(-3\,bx-3\,a\right)}-3\,bcde^{\left(3\,bx+3\,a\right)}+3\,bcde^{\left(bx+a\right)}+3\,bcde^{\left(-bx-a\right)}-3\,bcde^{\left(-3\,bx-3\,a\right)}-d^{2}e^{\left(3\,bx+3\,a\right)}+3\,d^{2}e^{\left(bx+a\right)}-3\,d^{2}e^{\left(-bx-a\right)}+d^{2}e^{\left(-3\,bx-3\,a\right)}}{16\,\left(d^{5}x^{2}+2\,cd^{4}x+c^{2}d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*(9*b^2*d^2*x^2*Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) - 3*b^2*d^2*x^2
*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) + 3*b^2*d^2*x^2*Ei(-(b*d*x + b*c)/d)*e^(
-a + b*c/d) - 9*b^2*d^2*x^2*Ei(-3*(b*d*x + b*c)/d)*e^(-3*a + 3*b*c/d) + 18*
```

```
b^2*c*d*x*Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) - 6*b^2*c*d*x*Ei((b*d*x +
 b*c)/d)*e^(a - b*c/d) + 6*b^2*c*d*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) -
18*b^2*c*d*x*Ei(-3*(b*d*x + b*c)/d)*e^(-3*a + 3*b*c/d) + 9*b^2*c^2*Ei(3*(b*
d*x + b*c)/d)*e^(3*a - 3*b*c/d) - 3*b^2*c^2*Ei((b*d*x + b*c)/d)*e^(a - b*c/
d) + 3*b^2*c^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 9*b^2*c^2*Ei(-3*(b*d*x
 + b*c)/d)*e^(-3*a + 3*b*c/d) - 3*b*d^2*x*e^(3*b*x + 3*a) + 3*b*d^2*x*e^(b*
x + a) + 3*b*d^2*x*e^(-b*x - a) - 3*b*d^2*x*e^(-3*b*x - 3*a) - 3*b*c*d*e^(3
*b*x + 3*a) + 3*b*c*d*e^(b*x + a) + 3*b*c*d*e^(-b*x - a) - 3*b*c*d*e^(-3*b*
x - 3*a) - d^2*e^(3*b*x + 3*a) + 3*d^2*e^(b*x + a) - 3*d^2*e^(-b*x - a) + d
^2*e^(-3*b*x - 3*a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(a+b\,x)^3}{(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3/(c + d*x)^3,x)
```

```
[Out] int(sinh(a + b*x)^3/(c + d*x)^3, x)
```

3.23 $\int (c+dx)^3 \mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=149

$$-\frac{2(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{3d(c+dx)^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{3d(c+dx)^2\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{6d^2(c+dx)\text{P}}{}$$

```
[Out] -2*(d*x+c)^3*arctanh(exp(b*x+a))/b-3*d*(d*x+c)^2*polylog(2,-exp(b*x+a))/b^2
+3*d*(d*x+c)^2*polylog(2,exp(b*x+a))/b^2+6*d^2*(d*x+c)*polylog(3,-exp(b*x+a
))/b^3-6*d^2*(d*x+c)*polylog(3,exp(b*x+a))/b^3-6*d^3*polylog(4,-exp(b*x+a))
/b^4+6*d^3*polylog(4,exp(b*x+a))/b^4
```

Rubi [A]
time = 0.10, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4267, 2611, 6744, 2320, 6724}

$$-\frac{6d^3\text{Li}_4\left(-e^{a+bx}\right)}{b^4}+\frac{6d^3\text{Li}_4\left(e^{a+bx}\right)}{b^4}+\frac{6d^2(c+dx)\text{Li}_3\left(-e^{a+bx}\right)}{b^3}-\frac{6d^2(c+dx)\text{Li}_3\left(e^{a+bx}\right)}{b^3}-\frac{3d(c+dx)^2\text{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{3d(c+dx)^2\text{Li}_2\left(e^{a+bx}\right)}{b^2}-\frac{2(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csch[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^3*ArcTanh[E^(a + b*x)])/b - (3*d*(c + d*x)^2*PolyLog[2, -E^(a
 + b*x)])/b^2 + (3*d*(c + d*x)^2*PolyLog[2, E^(a + b*x)])/b^2 + (6*d^2*(c +
 d*x)*PolyLog[3, -E^(a + b*x)])/b^3 - (6*d^2*(c + d*x)*PolyLog[3, E^(a + b*
x)])/b^3 - (6*d^3*PolyLog[4, -E^(a + b*x)])/b^4 + (6*d^3*PolyLog[4, E^(a +
b*x)])/b^4
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3\text{csch}(a+bx)\,dx &= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{(3d)\int (c+dx)^2\log\left(1-e^{a+bx}\right)\,dx}{b}+\frac{(3d)\int (c
\\
&= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{3d(c+dx)^2\text{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{3d(c+dx)^2\text{Li}_2\left(e^{a-}\right.}{b^2}
\\
&= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{3d(c+dx)^2\text{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{3d(c+dx)^2\text{Li}_2\left(e^{a-}\right.}{b^2}
\\
&= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{3d(c+dx)^2\text{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{3d(c+dx)^2\text{Li}_2\left(e^{a-}\right.}{b^2}
\\
&= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{3d(c+dx)^2\text{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{3d(c+dx)^2\text{Li}_2\left(e^{a-}\right.}{b^2}
\end{aligned}$$

Mathematica [A]
time = 2.05, size = 296, normalized size = 1.99

$$\frac{-2b^3c^3\tanh^{-1}\left(e^{a+bx}\right)+3b^3c^2dx\log\left(1-e^{a+bx}\right)+3b^3cd^2x^2\log\left(1-e^{a+bx}\right)+b^3d^3x^3\log\left(1-e^{a+bx}\right)-3b^3c^2dx\log\left(1+e^{a+bx}\right)-3b^3cd^2x^2\log\left(1+e^{a+bx}\right)-b^3d^3x^3\log\left(1+e^{a+bx}\right)-3b^2d(c+dx)^2\text{PolyLog}\left(2,-e^{a+bx}\right)+3b^2d(c+dx)^2\text{PolyLog}\left(2,e^{a+bx}\right)+6bcd^2\text{PolyLog}\left(3,-e^{a+bx}\right)+6bd^3x\text{PolyLog}\left(3,-e^{a+bx}\right)-6bcd^2\text{PolyLog}\left(3,e^{a+bx}\right)-6bd^3x\text{PolyLog}\left(3,e^{a+bx}\right)-6d^3\text{PolyLog}\left(4,-e^{a+bx}\right)+6d^3\text{PolyLog}\left(4,e^{a+bx}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csch[a + b*x],x]
```

```
[Out] (-2*b^3*c^3*ArcTanh[E^(a + b*x)] + 3*b^3*c^2*d*x*Log[1 - E^(a + b*x)] + 3*b
^3*c*d^2*x^2*Log[1 - E^(a + b*x)] + b^3*d^3*x^3*Log[1 - E^(a + b*x)] - 3*b^
3*c^2*d*x*Log[1 + E^(a + b*x)] - 3*b^3*c*d^2*x^2*Log[1 + E^(a + b*x)] - b^3
*d^3*x^3*Log[1 + E^(a + b*x)] - 3*b^2*d*(c + d*x)^2*PolyLog[2, -E^(a + b*x)
] + 3*b^2*d*(c + d*x)^2*PolyLog[2, E^(a + b*x)] + 6*b*c*d^2*PolyLog[3, -E^(
a + b*x)] + 6*b*d^3*x*PolyLog[3, -E^(a + b*x)] - 6*b*c*d^2*PolyLog[3, E^(a
+ b*x)] - 6*b*d^3*x*PolyLog[3, E^(a + b*x)] - 6*d^3*PolyLog[4, -E^(a + b*x)
] + 6*d^3*PolyLog[4, E^(a + b*x)])/b^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(142) = 284$.
time = 0.88, size = 541, normalized size = 3.63

method	result
risch	$-\frac{6d^2c\,\mathrm{polylog}(2,-\mathrm{e}^{bx+a})x}{b^2}+\frac{d^3\ln(1-\mathrm{e}^{bx+a})x^3}{b}+\frac{d^3\ln(1-\mathrm{e}^{bx+a})a^3}{b^4}+\frac{6d^3\,\mathrm{polylog}(3,-\mathrm{e}^{bx+a})x}{b^3}+\frac{3d^3\,\mathrm{polylog}(2,\mathrm{e}^{bx+a})x^2}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csch(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*d^3*ln(1-exp(b*x+a))*x^3+6/b^3*d^3*polylog(3,-exp(b*x+a))*x+1/b^4*d^3*l
n(1-exp(b*x+a))*a^3+3/b^2*d^3*polylog(2,exp(b*x+a))*x^2-6/b^3*d^3*polylog(3
,exp(b*x+a))*x+3/b^2*d*c^2*polylog(2,exp(b*x+a))-3/b^2*d*c^2*polylog(2,-exp
(b*x+a))-6*d^3*polylog(4,-exp(b*x+a))/b^4+6*d^3*polylog(4,exp(b*x+a))/b^4+2
/b^4*d^3*a^3*arctanh(exp(b*x+a))-6/b^3*d^2*c*polylog(3,exp(b*x+a))+6/b^3*d^
2*c*polylog(3,-exp(b*x+a))-1/b*d^3*ln(exp(b*x+a)+1)*x^3-1/b^4*d^3*ln(exp(b*
x+a)+1)*a^3-3/b^2*d^3*polylog(2,-exp(b*x+a))*x^2-6/b^2*d^2*c*polylog(2,-exp
(b*x+a))*x-3/b*d*c^2*ln(exp(b*x+a)+1)*x-3/b^2*d*c^2*ln(exp(b*x+a)+1)*a+3/b^
3*d^2*a^2*c*ln(exp(b*x+a)+1)+3/b*d*c^2*ln(1-exp(b*x+a))*x+3/b^2*d*c^2*ln(1-
exp(b*x+a))*a-3/b^3*d^2*a^2*c*ln(1-exp(b*x+a))+6/b^2*d*a*c^2*arctanh(exp(b*
x+a))-6/b^3*d^2*a^2*c*arctanh(exp(b*x+a))+3/b*d^2*c*ln(1-exp(b*x+a))*x^2+6/
b^2*d^2*c*polylog(2,exp(b*x+a))*x-3/b*d^2*c*ln(exp(b*x+a)+1)*x^2-2/b*c^3*ar
ctanh(exp(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(140) = 280$.
time = 0.35, size = 333, normalized size = 2.23

$$-c^3\left(\frac{\log\left(e^{(-bx-a)}+1\right)}{b}-\frac{\log\left(e^{(-bx-a)}-1\right)}{b}\right)-\frac{3\left(bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2(-e^{(bx+a)})\right)c^2d}{b^2}+\frac{3\left(bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2(e^{(bx+a)})\right)c^2d}{b^2}-\frac{3\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2(-e^{(bx+a)})-2\,\mathrm{Li}_3(-e^{(bx+a)})\right)cd^2}{b^3}+\frac{3\left(b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2(e^{(bx+a)})-2\,\mathrm{Li}_3(e^{(bx+a)})\right)cd^2}{b^3}-\frac{\left(b^3x^3\log\left(e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2(-e^{(bx+a)})-6\,bx\mathrm{Li}_3(-e^{(bx+a)})+6\,\mathrm{Li}_4(-e^{(bx+a)})\right)d^3}{b^4}+\frac{\left(b^3x^3\log\left(-e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2(e^{(bx+a)})-6\,bx\mathrm{Li}_3(e^{(bx+a)})+6\,\mathrm{Li}_4(e^{(bx+a)})\right)d^3}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -c^3*(log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b) - 3*(b*x*log(e^(b*
x + a) + 1) + dilog(-e^(b*x + a)))*c^2*d/b^2 + 3*(b*x*log(-e^(b*x + a) + 1)
 + dilog(e^(b*x + a)))*c^2*d/b^2 - 3*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*
```

```
dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))*c*d^2/b^3 + 3*(b^2*x^2*lo
g(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))
*c*d^2/b^3 - (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a))
- 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))*d^3/b^4 + (b
^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog
(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))*d^3/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(140) = 280$.
time = 0.36, size = 396, normalized size = 2.66

$$\frac{6\,d^3\mathrm{polylog}(4,\cosh(bx+a)+\sinh(bx+a)) - 6\,d^3\mathrm{polylog}(4,-\cosh(bx+a)-\sinh(bx+a)) + 3\,(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d)\mathrm{Li}_2(\cosh(bx+a)+\sinh(bx+a)) - 3\,(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d)\mathrm{Li}_2(-\cosh(bx+a)-\sinh(bx+a)) - (b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3)\log(\cosh(bx+a)+\sinh(bx+a)+1) + (b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log(\cosh(bx+a)+\sinh(bx+a)-1) + (b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+3\,ab^2c^2d-3\,a^2bcd^2+a^3d^3)\log(-\cosh(bx+a)-\sinh(bx+a)+1) - 6\,(bd^3x+bcd^2)\mathrm{polylog}(3,\cosh(bx+a)+\sinh(bx+a)) + 6\,(bd^3x+bcd^2)\mathrm{polylog}(3,-\cosh(bx+a)-\sinh(bx+a))}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] (6*d^3*polylog(4, cosh(b*x + a) + sinh(b*x + a)) - 6*d^3*polylog(4, -cosh(b
*x + a) - sinh(b*x + a)) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*dilo
g(cosh(b*x + a) + sinh(b*x + a)) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2
*d)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
+ 3*b^3*c^2*d*x + b^3*c^3)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + (b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cosh(b*x + a) + sinh(b*x +
 a) - 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2 + a^3*d^3)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - 6*(b*d^
3*x + b*c*d^2)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 6*(b*d^3*x + b*c
*d^2)*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3\,\mathrm{csch}\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csch(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*csch(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csch(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^3}{\sinh\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/sinh(a + b*x),x)
```

```
[Out] int((c + d*x)^3/sinh(a + b*x), x)
```

3.24 $\int (c+dx)^2 \mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=99

$$-\frac{2(c+dx)^2\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{2d(c+dx)\mathrm{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{2d(c+dx)\mathrm{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{2d^2\mathrm{PolyLog}(3}{b^3}$$

```
[Out] -2*(d*x+c)^2*arctanh(exp(b*x+a))/b-2*d*(d*x+c)*polylog(2,-exp(b*x+a))/b^2+2
*d*(d*x+c)*polylog(2,exp(b*x+a))/b^2+2*d^2*polylog(3,-exp(b*x+a))/b^3-2*d^2
*polylog(3,exp(b*x+a))/b^3
```

Rubi [A]
time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4267, 2611, 2320, 6724}

$$\frac{2d^2\mathrm{Li}_3\left(-e^{a+bx}\right)}{b^3}-\frac{2d^2\mathrm{Li}_3\left(e^{a+bx}\right)}{b^3}-\frac{2d(c+dx)\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{2d(c+dx)\mathrm{Li}_2\left(e^{a+bx}\right)}{b^2}-\frac{2(c+dx)^2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csch[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^2*ArcTanh[E^(a + b*x)])/b - (2*d*(c + d*x)*PolyLog[2, -E^(a +
 b*x)])/b^2 + (2*d*(c + d*x)*PolyLog[2, E^(a + b*x)])/b^2 + (2*d^2*PolyLog[
3, -E^(a + b*x)])/b^3 - (2*d^2*PolyLog[3, E^(a + b*x)])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
```

```
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\mathrm{csch}(a+bx)\,dx = -\frac{2(c+dx)^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{(2d)\int (c+dx)\log\left(1-e^{a+bx}\right)\,dx}{b} + \frac{(2d)\int (c+$$
$$= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{2d(c+dx)\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{2d(c+dx)\mathrm{Li}_2\left(e^{a+bx}\right)}{b^2}$$
$$= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{2d(c+dx)\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{2d(c+dx)\mathrm{Li}_2\left(e^{a+bx}\right)}{b^2}$$
$$= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{2d(c+dx)\mathrm{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{2d(c+dx)\mathrm{Li}_2\left(e^{a+bx}\right)}{b^2}$$

Mathematica [A]
time = 1.59, size = 170, normalized size = 1.72

$$\frac{-2b^2c^2\tanh^{-1}\left(e^{a+bx}\right)+2b^2cdx\log\left(1-e^{a+bx}\right)+b^2d^2x^2\log\left(1-e^{a+bx}\right)-2b^2cdx\log\left(1+e^{a+bx}\right)-b^2d^2x^2\log\left(1+e^{a+bx}\right)-2bd(c+dx)\mathrm{PolyLog}\left(2,-e^{a+bx}\right)+2bd(c+dx)\mathrm{PolyLog}\left(2,e^{a+bx}\right)+2d^2\mathrm{PolyLog}\left(3,-e^{a+bx}\right)-2d^2\mathrm{PolyLog}\left(3,e^{a+bx}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csch[a + b*x],x]
```

```
[Out] (-2*b^2*c^2*ArcTanh[E^(a + b*x)] + 2*b^2*c*d*x*Log[1 - E^(a + b*x)] + b^2*d
^2*x^2*Log[1 - E^(a + b*x)] - 2*b^2*c*d*x*Log[1 + E^(a + b*x)] - b^2*d^2*x^
2*Log[1 + E^(a + b*x)] - 2*b*d*(c + d*x)*PolyLog[2, -E^(a + b*x)] + 2*b*d*(
c + d*x)*PolyLog[2, E^(a + b*x)] + 2*d^2*PolyLog[3, -E^(a + b*x)] - 2*d^2*P
olyLog[3, E^(a + b*x)])/b^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(94) = 188$.
time = 0.77, size = 306, normalized size = 3.09

method	result
risch	$-\frac{2d^2a^2\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)}{b^3}+\frac{2dc\,\mathrm{polylog}\left(2,\mathrm{e}^{bx+a}\right)}{b^2}-\frac{2dc\,\mathrm{polylog}\left(2,-\mathrm{e}^{bx+a}\right)}{b^2}+\frac{d^2\ln\left(1-\mathrm{e}^{bx+a}\right)x^2}{b}-\frac{d^2\ln\left(1-\mathrm{e}^{bx+a}\right)a^2}{b^3}+\frac{2d^2\,\mathrm{p}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csch(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^3*d^2*a^2*arctanh(exp(b*x+a))+2/b^2*d*c*polylog(2,exp(b*x+a))-2/b^2*d*
c*polylog(2,-exp(b*x+a))+1/b*d^2*ln(1-exp(b*x+a))*x^2-1/b^3*d^2*ln(1-exp(b*
x+a))*a^2+2/b^2*d^2*polylog(2,exp(b*x+a))*x-1/b*d^2*ln(exp(b*x+a)+1)*x^2+1/
b^3*d^2*ln(exp(b*x+a)+1)*a^2-2/b^2*d^2*polylog(2,-exp(b*x+a))*x-2*d^2*polyl
og(3,exp(b*x+a))/b^3+2*d^2*polylog(3,-exp(b*x+a))/b^3+2/b*d*c*ln(1-exp(b*x+
a))*x+2/b^2*d*c*ln(1-exp(b*x+a))*a-2/b*d*c*ln(exp(b*x+a)+1)*x-2/b^2*d*c*ln(
exp(b*x+a)+1)*a+4/b^2*d*a*c*arctanh(exp(b*x+a))-2/b*c^2*arctanh(exp(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(92) = 184$.
time = 0.33, size = 195, normalized size = 1.97

$$-c^2\left(\frac{\log\left(e^{(-bx-a)}+1\right)}{b}-\frac{\log\left(e^{(-bx-a)}-1\right)}{b}\right)-\frac{2\left(bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)cd}{b^2}+\frac{2\left(bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)\right)cd}{b^2}-\frac{\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3\left(-e^{(bx+a)}\right)\right)d^2}{b^3}+\frac{\left(b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right)-2\,\mathrm{Li}_3\left(e^{(bx+a)}\right)\right)d^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -c^2*(log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b) - 2*(b*x*log(e^(b*
x + a) + 1) + dilog(-e^(b*x + a)))*c*d/b^2 + 2*(b*x*log(-e^(b*x + a) + 1) +
 dilog(e^(b*x + a)))*c*d/b^2 - (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(
-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))*d^2/b^3 + (b^2*x^2*log(-e^(b*x
+ a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))*d^2/b^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(92) = 184$.
time = 0.34, size = 242, normalized size = 2.44

$$-\frac{2\,d^2\mathrm{polylog}(3,\cosh\left(bx+a\right)+\sinh\left(bx+a\right))-2\,d^2\mathrm{polylog}(3,-\cosh\left(bx+a\right)-\sinh\left(bx+a\right))-2\left(bd^2x+bcd\right)\mathrm{Li}_2(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))+2\left(bd^2x+bcd\right)\mathrm{Li}_2(-\cosh\left(bx+a\right)-\sinh\left(bx+a\right))+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)-\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-1\right)-\left(b^2d^2x^2+2\,b^2cdx+2\,abcd-a^2d^2\right)\log\left(-\cosh\left(bx+a\right)-\sinh\left(bx+a\right)+1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*d^2*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 2*d^2*polylog(3, -cosh(
b*x + a) - sinh(b*x + a)) - 2*(b*d^2*x + b*c*d)*dilog(cosh(b*x + a) + sinh(
b*x + a)) + 2*(b*d^2*x + b*c*d)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b^
2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cosh(b*x + a) + sinh(b*x + a) + 1) -
 (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - (
b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cosh(b*x + a) - sinh(
b*x + a) + 1))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \operatorname{csch}(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csch(b*x+a),x)
[Out] Integral((c + d*x)**2*csch(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csch(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^2*csch(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + d\,x)^2}{\sinh(a + b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/sinh(a + b*x),x)
[Out] int((c + d*x)^2/sinh(a + b*x), x)
```

3.25 $\int (c+dx)\text{csch}(a+bx)\,dx$

Optimal. Leaf size=50

$$-\frac{2(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{d\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{d\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}$$

```
[Out] -2*(d*x+c)*arctanh(exp(b*x+a))/b-d*polylog(2,-exp(b*x+a))/b^2+d*polylog(2,e
xp(b*x+a))/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4267, 2317, 2438}

$$-\frac{d\text{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{d\text{Li}_2\left(e^{a+bx}\right)}{b^2}-\frac{2(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csch[a + b*x],x]
```

```
[Out] (-2*(c + d*x)*ArcTanh[E^(a + b*x)])/b - (d*PolyLog[2, -E^(a + b*x)])/b^2 +
(d*PolyLog[2, E^(a + b*x)])/b^2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int (c+dx)\text{csch}(a+bx)\,dx = -\frac{2(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{d\int \log\left(1-e^{a+bx}\right)\,dx}{b} + \frac{d\int \log\left(1+e^{a+bx}\right)\,dx}{b}$$
$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{d\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{a+bx}\right)}{b^2} + \frac{d\text{Subst}\left(\int \frac{\log(1+}{x}\right.}{b}$$
$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{d\text{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{d\text{Li}_2\left(e^{a+bx}\right)}{b^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.06, size = 174, normalized size = 3.48

$$-\frac{c\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} + \frac{c\log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} + \frac{d\left(-a\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right) - i\left((ia+ibx)\left(\log\left(1-e^{i(ia+ibx)}\right) - \log\left(1+e^{i(ia+ibx)}\right)\right) + i\left(\text{PolyLog}\left(2,-e^{i(ia+ibx)}\right) - \text{PolyLog}\left(2,e^{i(ia+ibx)}\right)\right)\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csch[a + b*x],x]
```

```
[Out] -((c*Log[Cosh[a/2 + (b*x)/2]])/b) + (c*Log[Sinh[a/2 + (b*x)/2]])/b + (d*(-(
a*Log[Tanh[(a + b*x)/2]]) - I*((I*a + I*b*x)*(Log[1 - E^(I*(I*a + I*b*x))]
- Log[1 + E^(I*(I*a + I*b*x))]) + I*(PolyLog[2, -E^(I*(I*a + I*b*x))] - Pol
yLog[2, E^(I*(I*a + I*b*x))]))))/b^2
```

Maple [A]
time = 0.51, size = 60, normalized size = 1.20

method	result
derivativedivides	$\frac{\frac{d\left(2\,\text{dilog}\left(e^{-bx-a}\right)-\frac{\text{dilog}\left(e^{-2bx-2a}\right)}{2}\right)}{b}+\frac{2da\,\text{arctanh}\left(e^{bx+a}\right)}{b}-2c\,\text{arctanh}\left(e^{bx+a}\right)}{b}$
default	$\frac{\frac{d\left(2\,\text{dilog}\left(e^{-bx-a}\right)-\frac{\text{dilog}\left(e^{-2bx-2a}\right)}{2}\right)}{b}+\frac{2da\,\text{arctanh}\left(e^{bx+a}\right)}{b}-2c\,\text{arctanh}\left(e^{bx+a}\right)}{b}$
risch	$-\frac{2c\,\text{arctanh}\left(e^{bx+a}\right)}{b}+\frac{d\ln\left(1-e^{bx+a}\right)x}{b}+\frac{d\ln\left(1-e^{bx+a}\right)a}{b^2}+\frac{d\,\text{polylog}\left(2,e^{bx+a}\right)}{b^2}-\frac{d\ln\left(e^{bx+a}+1\right)x}{b}-\frac{d\ln\left(e^{bx+}\right.}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csch(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(d/b*(2*dilog(exp(-b*x-a))-1/2*dilog(exp(-2*b*x-2*a)))+2*d/b*a*arctanh(
exp(b*x+a))-2*c*arctanh(exp(b*x+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -c*(log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b) + 2*d*(integrate(1/2
*x/(e^(b*x + a) + 1), x) + integrate(1/2*x/(e^(b*x + a) - 1), x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(45) = 90$.
time = 0.34, size = 119, normalized size = 2.38

$$\frac{d\text{Li}_2(\cosh(bx+a)+\sinh(bx+a)) - d\text{Li}_2(-\cosh(bx+a)-\sinh(bx+a)) - (bdx+bc)\log(\cosh(bx+a)+\sinh(bx+a)+1) + (bc-ad)\log(\cosh(bx+a)+\sinh(bx+a)-1) + (bdx+ad)\log(-\cosh(bx+a)-\sinh(bx+a)+1)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] (d*dilog(cosh(b*x + a) + sinh(b*x + a)) - d*dilog(-cosh(b*x + a) - sinh(b*x
 + a)) - (b*d*x + b*c)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + (b*c - a*d)
*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (b*d*x + a*d)*log(-cosh(b*x + a)
- sinh(b*x + a) + 1))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\operatorname{csch}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a),x)
```

```
[Out] Integral((c + d*x)*csch(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csch(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{c+d\,x}{\sinh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/sinh(a + b*x),x)
```

```
[Out] int((c + d*x)/sinh(a + b*x), x)
```

3.26 $\int \frac{\mathbf{csch}(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=17

$$\mathrm{Int}\left(\frac{\mathrm{csch}(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(csch(b*x+a)/(d*x+c),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\mathrm{csch}(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csch[a + b*x]/(c + d*x),x]
```

```
[Out] Defer[Int][Csch[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\mathrm{csch}(a+bx)}{c+dx}\,dx = \int \frac{\mathrm{csch}(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 9.35, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{csch}(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csch[a + b*x]/(c + d*x),x]
```

```
[Out] Integrate[Csch[a + b*x]/(c + d*x), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{csch}(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)/(d*x+c),x)
[Out] int(csch(b*x+a)/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate(csch(b*x + a)/(d*x + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(csch(b*x + a)/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c),x)
[Out] Integral(csch(a + b*x)/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(csch(b*x + a)/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sinh\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(a + b*x)*(c + d*x)),x)
[Out] int(1/(sinh(a + b*x)*(c + d*x)), x)
```

3.27 $\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=17

$$\operatorname{Int}\left(\frac{\operatorname{csch}(a+bx)}{(c+dx)^2},x\right)$$

[Out] Unintegrable(csch(b*x+a)/(d*x+c)^2,x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2} dx$$

Verification is not applicable to the result.

[In] Int[Csch[a + b*x]/(c + d*x)^2,x]

[Out] Defer[Int][Csch[a + b*x]/(c + d*x)^2, x]

Rubi steps

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2} dx = \int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2} dx$$

Mathematica [A]
time = 9.26, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2} dx$$

Verification is not applicable to the result.

[In] Integrate[Csch[a + b*x]/(c + d*x)^2,x]

[Out] Integrate[Csch[a + b*x]/(c + d*x)^2, x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(csch(b*x+a)/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(csch(b*x + a)/(d*x + c)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(a+bx\right)}{\left(c+dx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(csch(a + b*x)/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sinh\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(a + b*x)*(c + d*x)^2),x)
[Out] int(1/(sinh(a + b*x)*(c + d*x)^2), x)
```

3.28 $\int (c+dx)^3 \operatorname{csch}^2(a+bx)\,dx$

Optimal. Leaf size=103

$$-\frac{(c+dx)^3}{b}-\frac{(c+dx)^3\coth(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{3d^2(c+dx)\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}-\frac{3d^3\text{P}}{}$$

```
[Out] -(d*x+c)^3/b-(d*x+c)^3*coth(b*x+a)/b+3*d*(d*x+c)^2*ln(1-exp(2*b*x+2*a))/b^2
+3*d^2*(d*x+c)*polylog(2,exp(2*b*x+2*a))/b^3-3/2*d^3*polylog(3,exp(2*b*x+2*
a))/b^4
```

Rubi [A]
time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4269, 3797, 2221, 2611, 2320, 6724}

$$-\frac{3d^3\text{Li}_3\left(e^{2(a+bx)}\right)}{2b^4}+\frac{3d^2(c+dx)\text{Li}_2\left(e^{2(a+bx)}\right)}{b^3}+\frac{3d(c+dx)^2\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\coth(a+bx)}{b}-\frac{(c+dx)^3}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csch[a + b*x]^2,x]
```

```
[Out] -((c + d*x)^3/b) - ((c + d*x)^3*Coth[a + b*x])/b + (3*d*(c + d*x)^2*Log[1 -
 E^(2*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[2, E^(2*(a + b*x))])/b^3
- (3*d^3*PolyLog[3, E^(2*(a + b*x))])/(2*b^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^3\text{csch}^2(a+bx)\,dx &= -\frac{(c+dx)^3\coth(a+bx)}{b}+\frac{(3d)\int (c+dx)^2\coth(a+bx)\,dx}{b}\\
&= -\frac{(c+dx)^3}{b}-\frac{(c+dx)^3\coth(a+bx)}{b}-\frac{(6d)\int \frac{e^{2(a+bx)}(c+dx)^2}{1-e^{2(a+bx)}}\,dx}{b}\\
&= -\frac{(c+dx)^3}{b}-\frac{(c+dx)^3\coth(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{(6d\\
&= -\frac{(c+dx)^3}{b}-\frac{(c+dx)^3\coth(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{3d^2\\
&= -\frac{(c+dx)^3}{b}-\frac{(c+dx)^3\coth(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{3d^2\\
&= -\frac{(c+dx)^3}{b}-\frac{(c+dx)^3\coth(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{3d^2
\end{aligned}
$$

Mathematica [A]
time = 1.36, size = 133, normalized size = 1.29

$$
\frac{d\left(-\frac{4b^3e^{2a}x(3c^2+3cdx+d^2x^2)}{-1+e^{2a}}+6b^2(c+dx)^2\log\left(1-e^{2(a+bx)}\right)+6bd(c+dx)\text{PolyLog}\left(2,e^{2(a+bx)}\right)-3d^2\text{PolyLog}\left(3,e^{2(a+bx)}\right)\right)}{2b^4}+\frac{(c+dx)^3\text{csch}(a)\text{csch}(a+bx)\sinh(bx)}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csch[a + b*x]^2,x]
```

```
[Out] (d*((-4*b^3*E^(2*a)*x*(3*c^2 + 3*c*d*x + d^2*x^2))/(-1 + E^(2*a)) + 6*b^2*(
c + d*x)^2*Log[1 - E^(2*(a + b*x))] + 6*b*d*(c + d*x)*PolyLog[2, E^(2*(a +
b*x))] - 3*d^2*PolyLog[3, E^(2*(a + b*x))]))/(2*b^4) + ((c + d*x)^3*Csch[a]
*Csch[a + b*x]*Sinh[b*x])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(101) = 202$.
time = 0.94, size = 473, normalized size = 4.59

method	result
risch	$-\frac{12d^2acx}{b^2}+\frac{6d^2c\ln(1-e^{bx+a})x}{b^2}+\frac{6d^2c\ln(1-e^{bx+a})a}{b^3}+\frac{6d^2c\ln(e^{bx+a}+1)x}{b^2}-\frac{6d^2ac\ln(e^{bx+a}-1)}{b^3}+\frac{12d^2ac\ln(e^{bx+a})}{b^3}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -12/b^2*d^2*a*c*x+6/b^2*d^2*c*ln(1-exp(b*x+a))*x+6/b^3*d^2*c*ln(1-exp(b*x+a
))*a+6/b^2*d^2*c*ln(exp(b*x+a)+1)*x-6/b^3*d^2*a*c*ln(exp(b*x+a)-1)+12/b^3*d
^2*a*c*ln(exp(b*x+a))-2/b*d^3*x^3+4/b^4*d^3*a^3-6/b^4*d^3*polylog(3,exp(b*x
+a))-6/b^4*d^3*polylog(3,-exp(b*x+a))-2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)
/(exp(2*b*x+2*a)-1)/b+3/b^2*d^3*ln(exp(b*x+a)+1)*x^2+6/b^3*d^3*polylog(2,-e
xp(b*x+a))*x+3/b^2*d*c^2*ln(exp(b*x+a)+1)-6/b*d^2*c*x^2-6/b^3*d^2*c*a^2+6/b
^3*d^3*a^2*x+3/b^2*d*c^2*ln(exp(b*x+a)-1)+3/b^4*d^3*a^2*ln(exp(b*x+a)-1)-6/
b^4*d^3*a^2*ln(exp(b*x+a))+6/b^3*d^2*c*polylog(2,exp(b*x+a))+6/b^3*d^2*c*po
lylog(2,-exp(b*x+a))+3/b^2*d^3*ln(1-exp(b*x+a))*x^2-3/b^4*d^3*ln(1-exp(b*x+
a))*a^2+6/b^3*d^3*polylog(2,exp(b*x+a))*x-6/b^2*d*c^2*ln(exp(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(100) = 200$.
time = 0.41, size = 320, normalized size = 3.11

$$-3c^2d\left(\frac{2xe^{(2bx+2a)}}{be^{(2bx+2a)}-b}-\frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2}-\frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}\right)+\frac{6\left(bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)cd^2}{b^3}+\frac{6\left(bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(e^{(bx+a)}\right)\right)cd^2}{b^3}+\frac{2c^3}{b\left(e^{(-2bx-2a)}-1\right)}-\frac{2\left(d^3x^3+3cd^2x^2\right)}{be^{(2bx+2a)}-b}+\frac{3\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\mathrm{Li}_3\left(-e^{(bx+a)}\right)\right)d^3}{b^4}+\frac{3\left(b^2x^2\log\left(-e^{(bx+a)}+1\right)+2bx\mathrm{Li}_2\left(e^{(bx+a)}\right)-2\mathrm{Li}_3\left(e^{(bx+a)}\right)\right)d^3}{b^4}-\frac{2\left(b^3d^3x^3+3b^3cd^2x^2\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -3*c^2*d*(2*x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) - b) - log((e^(b*x + a) +
1)*e^(-a))/b^2 - log((e^(b*x + a) - 1)*e^(-a))/b^2) + 6*(b*x*log(e^(b*x + a
) + 1) + dilog(-e^(b*x + a)))*c*d^2/b^3 + 6*(b*x*log(-e^(b*x + a) + 1) + di
log(e^(b*x + a)))*c*d^2/b^3 + 2*c^3/(b*(e^(-2*b*x - 2*a) - 1)) - 2*(d^3*x^3
 + 3*c*d^2*x^2)/(b*e^(2*b*x + 2*a) - b) + 3*(b^2*x^2*log(e^(b*x + a) + 1) +
 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))*d^3/b^4 + 3*(b^2*x
```

```
^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x +
 a)))*d^3/b^4 - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2)/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1159 vs. $2(100) = 200$.
time = 0.35, size = 1159, normalized size = 11.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 2*a^3*d^3 + 2*(b^3*d^3*x^3 +
3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*
cosh(b*x + a)^2 + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^
2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cosh(b*x + a)*sinh(b*x + a) + 2*(b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^
3*d^3)*sinh(b*x + a)^2 + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cosh(b*
x + a)^2 - 2*(b*d^3*x + b*c*d^2)*cosh(b*x + a)*sinh(b*x + a) - (b*d^3*x + b
*c*d^2)*sinh(b*x + a)^2)*dilog(cosh(b*x + a) + sinh(b*x + a)) + 6*(b*d^3*x
+ b*c*d^2 - (b*d^3*x + b*c*d^2)*cosh(b*x + a)^2 - 2*(b*d^3*x + b*c*d^2)*cos
h(b*x + a)*sinh(b*x + a) - (b*d^3*x + b*c*d^2)*sinh(b*x + a)^2)*dilog(-cosh
(b*x + a) - sinh(b*x + a)) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - (
b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cosh(b*x + a)^2 - 2*(b^2*d^3*x^2 +
 2*b^2*c*d^2*x + b^2*c^2*d)*cosh(b*x + a)*sinh(b*x + a) - (b^2*d^3*x^2 + 2*
b^2*c*d^2*x + b^2*c^2*d)*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a)
 + 1) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3 - (b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*cosh(b*x + a)^2 - 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cosh(b*x +
a)*sinh(b*x + a) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sinh(b*x + a)^2)*log
(cosh(b*x + a) + sinh(b*x + a) - 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*
b*c*d^2 - a^2*d^3 - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*c
osh(b*x + a)^2 - 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*co
sh(b*x + a)*sinh(b*x + a) - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^
2*d^3)*sinh(b*x + a)^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*(d^3*co
sh(b*x + a)^2 + 2*d^3*cosh(b*x + a)*sinh(b*x + a) + d^3*sinh(b*x + a)^2 - d
^3)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 6*(d^3*cosh(b*x + a)^2 + 2*
d^3*cosh(b*x + a)*sinh(b*x + a) + d^3*sinh(b*x + a)^2 - d^3)*polylog(3, -co
sh(b*x + a) - sinh(b*x + a)))/(b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x + a)*si
nh(b*x + a) + b^4*sinh(b*x + a)^2 - b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \operatorname{csch}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csch(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*csch(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csch(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^3}{\sinh\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/sinh(a + b*x)^2,x)
```

```
[Out] int((c + d*x)^3/sinh(a + b*x)^2, x)
```

3.29 $\int (c+dx)^2 \mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=74

$$-\frac{(c+dx)^2}{b}-\frac{(c+dx)^2\coth(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}$$

```
[Out] -(d*x+c)^2/b-(d*x+c)^2*coth(b*x+a)/b+2*d*(d*x+c)*ln(1-exp(2*b*x+2*a))/b^2+d
^2*polylog(2,exp(2*b*x+2*a))/b^3
```

Rubi [A]
time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {4269, 3797, 2221, 2317, 2438}

$$\frac{d^2\text{Li}_2\left(e^{2(a+bx)}\right)}{b^3}+\frac{2d(c+dx)\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\coth(a+bx)}{b}-\frac{(c+dx)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csch[a + b*x]^2,x]
```

```
[Out] -((c + d*x)^2/b) - ((c + d*x)^2*Coth[a + b*x])/b + (2*d*(c + d*x)*Log[1 - E
^(2*(a + b*x))])/b^2 + (d^2*PolyLog[2, E^(2*(a + b*x))])/b^3
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
```

```
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\operatorname{csch}^2(a+bx)\,dx &= -\frac{(c+dx)^2\coth(a+bx)}{b}+\frac{(2d)\int (c+dx)\coth(a+bx)\,dx}{b}\\ &= -\frac{(c+dx)^2}{b}-\frac{(c+dx)^2\coth(a+bx)}{b}-\frac{(4d)\int \frac{e^{2(a+bx)}(c+dx)}{1-e^{2(a+bx)}}\,dx}{b}\\ &= -\frac{(c+dx)^2}{b}-\frac{(c+dx)^2\coth(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{(2d^2)}{}\\ &= -\frac{(c+dx)^2}{b}-\frac{(c+dx)^2\coth(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{d^2\mathrm{Sul}}{}\\ &= -\frac{(c+dx)^2}{b}-\frac{(c+dx)^2\coth(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{d^2\mathrm{Li}_2}{}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.68, size = 198, normalized size = 2.68

$$\frac{\operatorname{csch}(a)\left(-2bcd(bx\cosh(a)-\log(\sinh(a+bx))\sinh(a))+d^2\left(-b^2e^{-\tanh^{-1}(\tanh(a))}x^2\cosh(a)\sqrt{\operatorname{sech}^2(a)}+ib\pi x\sinh(a)-i\pi\log\left(1+e^{2bx}\right)\sinh(a)+2bx\log\left(1-e^{-2\left(bx+\tanh^{-1}(\tanh(a))\right)}\right)\sinh(a)+i\pi\log(\cosh(bx))\sinh(a)+2\tanh^{-1}(\tanh(a))\left(bx+\log\left(1-e^{-2\left(bx+\tanh^{-1}(\tanh(a))\right)}\right)-\log\left(i\sinh\left(bx+\tanh^{-1}(\tanh(a))\right)\right)\right)\sinh(a)-\operatorname{PolyLog}\left(2,e^{-2\left(bx+\tanh^{-1}(\tanh(a))\right)}\right)\sinh(a)\right)+b^2(c+dx)^2\operatorname{csch}(a+bx)\sinh(bx)\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csch[a + b*x]^2,x]
```

```
[Out] (Csch[a]*(-2*b*c*d*(b*x*Cosh[a] - Log[Sinh[a + b*x]]*Sinh[a]) + d^2*(-((b^2
*x^2*Cosh[a]*Sqrt[Sech[a]^2])/E^ArcTanh[Tanh[a]]) + I*b*Pi*x*Sinh[a] - I*Pi
*Log[1 + E^(2*b*x)]*Sinh[a] + 2*b*x*Log[1 - E^(-2*(b*x + ArcTanh[Tanh[a]]))
]*Sinh[a] + I*Pi*Log[Cosh[b*x]]*Sinh[a] + 2*ArcTanh[Tanh[a]]*(b*x + Log[1 -
 E^(-2*(b*x + ArcTanh[Tanh[a]]))] - Log[I*Sinh[b*x + ArcTanh[Tanh[a]]]])*Si
nh[a] - PolyLog[2, E^(-2*(b*x + ArcTanh[Tanh[a]]))]*Sinh[a]) + b^2*(c + d*x
)^2*Csch[a + b*x]*Sinh[b*x]))/b^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(74) = 148.
time = 0.83, size = 240, normalized size = 3.24

method	result
risch	$-\frac{2(d^2x^2+2cdx+c^2)}{b(e^{2bx+2a}-1)}+\frac{2dc\ln(e^{bx+a}-1)}{b^2}+\frac{2dc\ln(e^{bx+a}+1)}{b^2}-\frac{4dc\ln(e^{bx+a})}{b^2}-\frac{2d^2x^2}{b}-\frac{4d^2ax}{b^2}-\frac{2d^2a^2}{b^3}+\frac{2d^2\ln(1-e^{b}}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b*(d^2*x^2+2*c*d*x+c^2)/(exp(2*b*x+2*a)-1)+2/b^2*d*c*ln(exp(b*x+a)-1)+2/
b^2*d*c*ln(exp(b*x+a)+1)-4/b^2*d*c*ln(exp(b*x+a))-2/b*d^2*x^2-4/b^2*d^2*a*x
-2/b^3*d^2*a^2+2/b^2*d^2*ln(1-exp(b*x+a))*x+2/b^3*d^2*ln(1-exp(b*x+a))*a+2/
b^3*d^2*polylog(2,exp(b*x+a))+2/b^2*d^2*ln(exp(b*x+a)+1)*x+2/b^3*d^2*polylo
g(2,-exp(b*x+a))-2/b^3*d^2*a*ln(exp(b*x+a)-1)+4/b^3*d^2*a*ln(exp(b*x+a))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*d^2*(x^2/(b*e^(2*b*x + 2*a) - b) + 2*integrate(1/2*x/(b*e^(b*x + a) + b)
, x) - 2*integrate(1/2*x/(b*e^(b*x + a) - b), x)) - 2*c*d*(2*x*e^(2*b*x + 2
*a)/(b*e^(2*b*x + 2*a) - b) - log((e^(b*x + a) + 1)*e^(-a))/b^2 - log((e^(b
*x + a) - 1)*e^(-a))/b^2) + 2*c^2/(b*(e^(-2*b*x - 2*a) - 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(73) = 146$.
time = 0.41, size = 623, normalized size = 8.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d
- a^2*d^2)*cosh(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2
*d^2)*cosh(b*x + a)*sinh(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d
- a^2*d^2)*sinh(b*x + a)^2 - (d^2*cosh(b*x + a)^2 + 2*d^2*cosh(b*x + a)*sin
h(b*x + a) + d^2*sinh(b*x + a)^2 - d^2)*dilog(cosh(b*x + a) + sinh(b*x + a)
) - (d^2*cosh(b*x + a)^2 + 2*d^2*cosh(b*x + a)*sinh(b*x + a) + d^2*sinh(b*x
 + a)^2 - d^2)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*d^2*x + b*c*d - (
b*d^2*x + b*c*d)*cosh(b*x + a)^2 - 2*(b*d^2*x + b*c*d)*cosh(b*x + a)*sinh(b
*x + a) - (b*d^2*x + b*c*d)*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x +
```

```
 a) + 1) + (b*c*d - a*d^2 - (b*c*d - a*d^2)*cosh(b*x + a)^2 - 2*(b*c*d - a*
d^2)*cosh(b*x + a)*sinh(b*x + a) - (b*c*d - a*d^2)*sinh(b*x + a)^2)*log(cos
h(b*x + a) + sinh(b*x + a) - 1) + (b*d^2*x + a*d^2 - (b*d^2*x + a*d^2)*cosh
(b*x + a)^2 - 2*(b*d^2*x + a*d^2)*cosh(b*x + a)*sinh(b*x + a) - (b*d^2*x +
a*d^2)*sinh(b*x + a)^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1))/(b^3*cosh(
b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2 - b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \operatorname{csch}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csch(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*csch(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csch(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^2}{\sinh(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/sinh(a + b*x)^2,x)
```

```
[Out] int((c + d*x)^2/sinh(a + b*x)^2, x)
```

3.30 $\int (c+dx)\text{csch}^2(a+bx)\,dx$

Optimal. Leaf size=29

$$-\frac{(c+dx)\coth(a+bx)}{b}+\frac{d\log(\sinh(a+bx))}{b^2}$$

```
[Out] -(d*x+c)*coth(b*x+a)/b+d*ln(sinh(b*x+a))/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4269, 3556}

$$\frac{d\log(\sinh(a+bx))}{b^2}-\frac{(c+dx)\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csch[a + b*x]^2,x]
```

```
[Out] -(((c + d*x)*Coth[a + b*x])/b) + (d*Log[Sinh[a + b*x]])/b^2
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\text{csch}^2(a+bx)\,dx &= -\frac{(c+dx)\coth(a+bx)}{b}+\frac{d\int \coth(a+bx)\,dx}{b}\\ &= -\frac{(c+dx)\coth(a+bx)}{b}+\frac{d\log(\sinh(a+bx))}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 52, normalized size = 1.79

$$-\frac{dx\coth(a)}{b}-\frac{c\coth(a+bx)}{b}+\frac{d\log(\sinh(a+bx))}{b^2}+\frac{dx\text{csch}(a)\text{csch}(a+bx)\sinh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csch[a + b*x]^2,x]
```

```
[Out] -((d*x*Coth[a])/b) - (c*Coth[a + b*x])/b + (d*Log[Sinh[a + b*x]])/b^2 + (d*
x*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b
```

Maple [A]
time = 0.47, size = 56, normalized size = 1.93

method	result	size
risch	$-\frac{2dx}{b}-\frac{2da}{b^{2}}-\frac{2(dx+c)}{(\mathrm{e}^{2bx+2a}-1)b}+\frac{d\ln(\mathrm{e}^{2bx+2a}-1)}{b^{2}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*d/b*x-2*d/b^2*a-2*(d*x+c)/(exp(2*b*x+2*a)-1)/b+d/b^2*ln(exp(2*b*x+2*a)-1
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(29) = 58.
time = 0.27, size = 91, normalized size = 3.14

$$-d\left(\frac{2\,xe^{(2\,bx+2\,a)}}{be^{(2\,bx+2\,a)}-b}-\frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^{2}}-\frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^{2}}\right)+\frac{2\,c}{b\left(e^{(-2\,bx-2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -d*(2*x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) - b) - log((e^(b*x + a) + 1)*e^(
-a))/b^2 - log((e^(b*x + a) - 1)*e^(-a))/b^2) + 2*c/(b*(e^(-2*b*x - 2*a) -
1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(29) = 58.
time = 0.35, size = 166, normalized size = 5.72

$$-\frac{2\,bdx\cosh\left(bx+a\right)^{2}+4\,bdx\cosh\left(bx+a\right)\sinh\left(bx+a\right)+2\,bdx\sinh\left(bx+a\right)^{2}+2\,bc-\left(d\cosh\left(bx+a\right)^{2}+2\,d\cosh\left(bx+a\right)\sinh\left(bx+a\right)+d\sinh\left(bx+a\right)^{2}-d\right)\log\left(\frac{2\,\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b^{2}\cosh\left(bx+a\right)^{2}+2\,b^{2}\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b^{2}\sinh\left(bx+a\right)^{2}-b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b*d*x*cosh(b*x + a)^2 + 4*b*d*x*cosh(b*x + a)*sinh(b*x + a) + 2*b*d*x*s
inh(b*x + a)^2 + 2*b*c - (d*cosh(b*x + a)^2 + 2*d*cosh(b*x + a)*sinh(b*x +
a) + d*sinh(b*x + a)^2 - d)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x +
```

```
a))))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(b*x + a) + b^2*sinh(
b*x + a)^2 - b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\operatorname{csch}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*csch(a + b*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(29) = 58.
time = 0.44, size = 80, normalized size = 2.76

$$-\frac{2\,bdxe^{(2\,bx+2\,a)}-de^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)}-1\right)+2\,bc+d\log\left(e^{(2\,bx+2\,a)}-1\right)}{b^2e^{(2\,bx+2\,a)}-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(2*b*d*x*e^(2*b*x + 2*a) - d*e^(2*b*x + 2*a)*log(e^(2*b*x + 2*a) - 1) + 2*
b*c + d*log(e^(2*b*x + 2*a) - 1))/(b^2*e^(2*b*x + 2*a) - b^2)
```

Mupad [B]
time = 0.08, size = 49, normalized size = 1.69

$$\frac{d\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b^2}-\frac{2\,(c+d\,x)}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}-\frac{2\,d\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/sinh(a + b*x)^2,x)
```

```
[Out] (d*log(exp(2*a)*exp(2*b*x) - 1))/b^2 - (2*(c + d*x))/(b*(exp(2*a + 2*b*x) -
 1)) - (2*d*x)/b
```

3.31 $\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{\operatorname{csch}^2(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(csch(b*x+a)^2/(d*x+c),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csch[a + b*x]^2/(c + d*x),x]
[Out] Defer[Int][Csch[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 15.08, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csch[a + b*x]^2/(c + d*x),x]
[Out] Integrate[Csch[a + b*x]^2/(c + d*x), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(csch(b*x+a)^2/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 4*d*integrate(1/4/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2*e^a + 2*b*c*d
*x*e^a + b*c^2*e^a)*e^(b*x)), x) - 4*d*integrate(-1/4/(b*d^2*x^2 + 2*b*c*d*
x + b*c^2 - (b*d^2*x^2*e^a + 2*b*c*d*x*e^a + b*c^2*e^a)*e^(b*x)), x) + 2/(b
*d*x + b*c - (b*d*x*e^(2*a) + b*c*e^(2*a))*e^(2*b*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^2/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(csch(a + b*x)**2/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^2/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sinh (a+b\,x)^2\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(a + b*x)^2*(c + d*x)),x)
[Out] int(1/(sinh(a + b*x)^2*(c + d*x)), x)
```

3.32 $\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(csch(b*x+a)^2/(d*x+c)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[Csch[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Csch[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2} dx = \int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2} dx$$

Mathematica [A]
time = 15.62, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csch[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Integrate[Csch[a + b*x]^2/(c + d*x)^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(csch(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 4*d*integrate(1/2/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3
*x^3*e^a + 3*b*c*d^2*x^2*e^a + 3*b*c^2*d*x*e^a + b*c^3*e^a)*e^(b*x)), x) -
4*d*integrate(-1/2/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 - (b*d^
3*x^3*e^a + 3*b*c*d^2*x^2*e^a + 3*b*c^2*d*x*e^a + b*c^3*e^a)*e^(b*x)), x) +
 2/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 - (b*d^2*x^2*e^(2*a) + 2*b*c*d*x*e^(2*a)
+ b*c^2*e^(2*a))*e^(2*b*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{2}\left(a + bx\right)}{\left(c + dx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(csch(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sinh\left(a+b\,x\right)^2\,{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(a + b*x)^2*(c + d*x)^2),x)
```

```
[Out] int(1/(sinh(a + b*x)^2*(c + d*x)^2), x)
```

3.33 $\int (c+dx)^3 \textbf{csch}^3(a+bx)\,dx$

Optimal. Leaf size=256

$$-\frac{6d^2(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b^3}+\frac{(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{3d(c+dx)^2\text{csch}(a+bx)}{2b^2}-\frac{(c+dx)^3\coth(a+bx)\text{cs}}{2b}$$

[Out] -6*d^2*(d*x+c)*arctanh(exp(b*x+a))/b^3+(d*x+c)^3*arctanh(exp(b*x+a))/b-3/2*d*(d*x+c)^2*csch(b*x+a)/b^2-1/2*(d*x+c)^3*coth(b*x+a)*csch(b*x+a)/b-3*d^3*polylog(2,-exp(b*x+a))/b^4+3/2*d*(d*x+c)^2*polylog(2,-exp(b*x+a))/b^2+3*d^3*polylog(2,exp(b*x+a))/b^4-3/2*d*(d*x+c)^2*polylog(2,exp(b*x+a))/b^2-3*d^2*(d*x+c)*polylog(3,-exp(b*x+a))/b^3+3*d^2*(d*x+c)*polylog(3,exp(b*x+a))/b^3+3*d^3*polylog(4,-exp(b*x+a))/b^4-3*d^3*polylog(4,exp(b*x+a))/b^4

Rubi [A]
time = 0.20, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4271, 4267, 2317, 2438, 2611, 6744, 2320, 6724}

$$-\frac{3d^3\text{Li}_2\left(-e^{a+bx}\right)}{b^4}+\frac{3d^3\text{Li}_2\left(e^{a+bx}\right)}{b^4}+\frac{3d^3\text{Li}_4\left(-e^{a+bx}\right)}{b^4}-\frac{3d^3\text{Li}_4\left(e^{a+bx}\right)}{b^4}-\frac{3d^2(c+dx)\text{Li}_3\left(-e^{a+bx}\right)}{b^3}+\frac{3d^2(c+dx)\text{Li}_3\left(e^{a+bx}\right)}{b^3}-\frac{6d^2(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b^3}+\frac{3d(c+dx)^2\text{Li}_2\left(-e^{a+bx}\right)}{2b^2}-\frac{3d(c+dx)^2\text{Li}_2\left(e^{a+bx}\right)}{2b^2}-\frac{3d(c+dx)^2\text{csch}(a+bx)}{2b^2}+\frac{(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{(c+dx)^3\coth(a+bx)\text{csch}(a+bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^3*Csch[a + b*x]^3,x]

[Out] (-6*d^2*(c + d*x)*ArcTanh[E^(a + b*x)])/b^3 + ((c + d*x)^3*ArcTanh[E^(a + b*x)])/b - (3*d*(c + d*x)^2*Csch[a + b*x])/(2*b^2) - ((c + d*x)^3*Coth[a + b*x]*Csch[a + b*x])/(2*b) - (3*d^3*PolyLog[2, -E^(a + b*x)])/b^4 + (3*d*(c + d*x)^2*PolyLog[2, -E^(a + b*x)])/(2*b^2) + (3*d^3*PolyLog[2, E^(a + b*x)])/b^4 - (3*d*(c + d*x)^2*PolyLog[2, E^(a + b*x)])/(2*b^2) - (3*d^2*(c + d*x)*PolyLog[3, -E^(a + b*x)])/b^3 + (3*d^2*(c + d*x)*PolyLog[3, E^(a + b*x)])/b^3 + (3*d^3*PolyLog[4, -E^(a + b*x)])/b^4 - (3*d^3*PolyLog[4, E^(a + b*x)])/b^4

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3\operatorname{csch}^3(a+bx)\,dx = -\frac{3d(c+dx)^2\operatorname{csch}(a+bx)}{2b^2} - \frac{(c+dx)^3\coth(a+bx)\operatorname{csch}(a+bx)}{2b} - \frac{1}{2}\int (c+$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3d(c+dx)^2\operatorname{csch}(}{2b^2}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3d(c+dx)^2\operatorname{csch}(}{2b^2}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3d(c+dx)^2\operatorname{csch}(}{2b^2}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3d(c+dx)^2\operatorname{csch}(}{2b^2}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3d(c+dx)^2\operatorname{csch}(}{2b^2}$$

Mathematica [**A**]
time = 5.98, size = 398, normalized size = 1.55

$$-\frac{-2b^3c^3\tanh^{-1}\left(e^{a+bx}\right) + 12bcd^2\tanh^{-1}\left(e^{a+bx}\right) + b^2(c+dx)^2(3d+b(c+dx)\coth(a+bx))\operatorname{csch}(a+bx) + 3b^3c^2dx\log\left(1-e^{a+bx}\right) - 6bd^3x\log\left(1-e^{a+bx}\right) + 3b^3cd^2x^2\log\left(1-e^{a+bx}\right) + b^3d^3x^3\log\left(1-e^{a+bx}\right) - 3b^3c^2dx\log\left(1+e^{a+bx}\right) + 6bd^3x\log\left(1+e^{a+bx}\right) - 3b^3cd^2x^2\log\left(1+e^{a+bx}\right) - b^3d^3x^3\log\left(1+e^{a+bx}\right) - 3d(-2d^2+b^2(c+dx)^2)\operatorname{PolyLog}\left(2,-e^{a+bx}\right) + 3d(-2d^2+b^2(c+dx)^2)\operatorname{PolyLog}\left(2,e^{a+bx}\right) + 6bcd^2\operatorname{PolyLog}\left(3,-e^{a+bx}\right) + 6bd^3x\operatorname{PolyLog}\left(3,-e^{a+bx}\right) - 6bcd^2\operatorname{PolyLog}\left(3,e^{a+bx}\right) - 6bd^3x\operatorname{PolyLog}\left(3,e^{a+bx}\right) - 6d^3\operatorname{PolyLog}\left(4,-e^{a+bx}\right) + 6d^3\operatorname{PolyLog}\left(4,e^{a+bx}\right)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csch[a + b*x]^3,x]
```

```
[Out] -1/2*(-2*b^3*c^3*ArcTanh[E^(a + b*x)] + 12*b*c*d^2*ArcTanh[E^(a + b*x)] + b
^2*(c + d*x)^2*(3*d + b*(c + d*x)*Coth[a + b*x])*Csch[a + b*x] + 3*b^3*c^2*
d*x*Log[1 - E^(a + b*x)] - 6*b*d^3*x*Log[1 - E^(a + b*x)] + 3*b^3*c*d^2*x^2
*Log[1 - E^(a + b*x)] + b^3*d^3*x^3*Log[1 - E^(a + b*x)] - 3*b^3*c^2*d*x*Lo
g[1 + E^(a + b*x)] + 6*b*d^3*x*Log[1 + E^(a + b*x)] - 3*b^3*c*d^2*x^2*Log[1
 + E^(a + b*x)] - b^3*d^3*x^3*Log[1 + E^(a + b*x)] - 3*d*(-2*d^2 + b^2*(c +
 d*x)^2)*PolyLog[2, -E^(a + b*x)] + 3*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[
2, E^(a + b*x)] + 6*b*c*d^2*PolyLog[3, -E^(a + b*x)] + 6*b*d^3*x*PolyLog[3,
 -E^(a + b*x)] - 6*b*c*d^2*PolyLog[3, E^(a + b*x)] - 6*b*d^3*x*PolyLog[3, E
^(a + b*x)] - 6*d^3*PolyLog[4, -E^(a + b*x)] + 6*d^3*PolyLog[4, E^(a + b*x)
])/b^4
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(238) = 476$.
time = 1.07, size = 876, normalized size = 3.42

method	result
risch	$\frac{3d^2c\,\operatorname{polylog}(2,-e^{bx+a})x}{b^2} - \frac{d^3\ln(1-e^{bx+a})x^3}{2b} - \frac{d^3\ln(1-e^{bx+a})a^3}{2b^4} - \frac{3d^2c\,\operatorname{polylog}(2,e^{bx+a})x}{b^2} - \frac{3da\,c^2\operatorname{arctanh}(e^{bx+a})}{b^2} - \frac{e^{b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*d^3*ln(1-exp(b*x+a))*x^3-3/b^3*d^3*polylog(3,-exp(b*x+a))*x-1/2/b^4*
d^3*ln(1-exp(b*x+a))*a^3-3/2/b^2*d^3*polylog(2,exp(b*x+a))*x^2+3/b^3*d^3*po
lylog(3,exp(b*x+a))*x-3/2/b^2*d*c^2*polylog(2,exp(b*x+a))+3/2/b^2*d*c^2*pol
ylog(2,-exp(b*x+a))-exp(b*x+a)*(b*d^3*x^3*exp(2*b*x+2*a)+3*b*c*d^2*x^2*exp(
2*b*x+2*a)+3*b*c^2*d*x*exp(2*b*x+2*a)+b*d^3*x^3+3*d^3*x^2*exp(2*b*x+2*a)+b*
c^3*exp(2*b*x+2*a)+3*b*c*d^2*x^2+6*c*d^2*x*exp(2*b*x+2*a)+3*b*c^2*d*x+3*c^2
*d*exp(2*b*x+2*a)-3*d^3*x^2+b*c^3-6*c*d^2*x-3*c^2*d)/b^2/(exp(2*b*x+2*a)-1)
^2+3*d^3*polylog(4,-exp(b*x+a))/b^4-3*d^3*polylog(4,exp(b*x+a))/b^4-3*d^3*p
olylog(2,-exp(b*x+a))/b^4-1/b^4*d^3*a^3*arctanh(exp(b*x+a))+3/b^3*d^2*c*pol
ylog(3,exp(b*x+a))-3/b^3*d^2*c*polylog(3,-exp(b*x+a))+1/2/b*d^3*ln(exp(b*x+
a)+1)*x^3+1/2/b^4*d^3*ln(exp(b*x+a)+1)*a^3+3/2/b^2*d^3*polylog(2,-exp(b*x+a
))*x^2+3/b^2*d^2*c*polylog(2,-exp(b*x+a))*x+3/2/b*d*c^2*ln(exp(b*x+a)+1)*x+
3/2/b^2*d*c^2*ln(exp(b*x+a)+1)*a-3/2/b^3*d^2*a^2*c*ln(exp(b*x+a)+1)-3/2/b*d
*c^2*ln(1-exp(b*x+a))*x-3/2/b^2*d*c^2*ln(1-exp(b*x+a))*a+3/2/b^3*d^2*a^2*c*
ln(1-exp(b*x+a))-3/b^2*d*a*c^2*arctanh(exp(b*x+a))+3/b^3*d^2*a^2*c*arctanh(
exp(b*x+a))-3/2/b*d^2*c*ln(1-exp(b*x+a))*x^2-3/b^2*d^2*c*polylog(2,exp(b*x+
a))*x+3/2/b*d^2*c*ln(exp(b*x+a)+1)*x^2+1/b*c^3*arctanh(exp(b*x+a))-6/b^3*c*
d^2*arctanh(exp(b*x+a))+6/b^4*d^3*a*arctanh(exp(b*x+a))+3/b^3*d^3*ln(1-exp(
b*x+a))*x-3/b^3*d^3*ln(exp(b*x+a)+1)*x+3/b^4*d^3*a*ln(1-exp(b*x+a))-3/b^4*d
^3*a*ln(exp(b*x+a)+1)+3*d^3*polylog(2,exp(b*x+a))/b^4
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(234) = 468$.
time = 0.42, size = 605, normalized size = 2.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*c^3*(log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b + 2*(e^(-b*x - a
) + e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))) + 3/
2*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3,
-e^(b*x + a)))*c*d^2/b^3 - 3/2*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog
(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))*c*d^2/b^3 - 3*c*d^2*log(e^(b*x +
 a) + 1)/b^3 + 3*c*d^2*log(e^(b*x + a) - 1)/b^3 - ((b*d^3*x^3*e^(3*a) + 3*c
^2*d*e^(3*a) + 3*(b*c*d^2 + d^3)*x^2*e^(3*a) + 3*(b*c^2*d + 2*c*d^2)*x*e^(3
*a))*e^(3*b*x) + (b*d^3*x^3*e^a - 3*c^2*d*e^a + 3*(b*c*d^2 - d^3)*x^2*e^a +
 3*(b*c^2*d - 2*c*d^2)*x*e^a)*e^(b*x))/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*
x + 2*a) + b^2) + 1/2*(b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b
```

```
*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))*d^3
/b^4 - 1/2*(b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) -
6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))*d^3/b^4 + 3/2*(b
^2*c^2*d - 2*d^3)*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^4 - 3/
2*(b^2*c^2*d - 2*d^3)*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4008 vs. $2(234) = 468$.
time = 0.41, size = 4008, normalized size = 15.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b^3*d^3*x^3 + b^3*c^3 + 3*b^2*c^2*d + 3*(b^3*c*d^2 + b^2*d^3)*x^2
+ 3*(b^3*c^2*d + 2*b^2*c*d^2)*x)*cosh(b*x + a)^3 + 6*(b^3*d^3*x^3 + b^3*c^3
 + 3*b^2*c^2*d + 3*(b^3*c*d^2 + b^2*d^3)*x^2 + 3*(b^3*c^2*d + 2*b^2*c*d^2)*
x)*cosh(b*x + a)*sinh(b*x + a)^2 + 2*(b^3*d^3*x^3 + b^3*c^3 + 3*b^2*c^2*d +
 3*(b^3*c*d^2 + b^2*d^3)*x^2 + 3*(b^3*c^2*d + 2*b^2*c*d^2)*x)*sinh(b*x + a)
^3 + 2*(b^3*d^3*x^3 + b^3*c^3 - 3*b^2*c^2*d + 3*(b^3*c*d^2 - b^2*d^3)*x^2 +
 3*(b^3*c^2*d - 2*b^2*c*d^2)*x)*cosh(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^
2*x + b^2*c^2*d + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh(b*
x + a)^4 + 4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh(b*x + a
)*sinh(b*x + a)^3 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sinh(
b*x + a)^4 - 2*d^3 - 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*co
sh(b*x + a)^2 - 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3 - 3*(b^2
*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh(b*x + a)^2)*sinh(b*x + a
)^2 + 4*((b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh(b*x + a)^3
- (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh(b*x + a))*sinh(b*x
 + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*
x + b^2*c^2*d + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh(b*x
+ a)^4 + 4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh(b*x + a)*
sinh(b*x + a)^3 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sinh(b*
x + a)^4 - 2*d^3 - 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh
(b*x + a)^2 - 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3 - 3*(b^2*d
^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh(b*x + a)^2)*sinh(b*x + a)^
2 + 4*((b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh(b*x + a)^3 -
(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh(b*x + a))*sinh(b*x +
 a))*dilog(-cosh(b*x + a) - sinh(b*x + a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
 + b^3*c^3 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*
c^2*d - 2*b*d^3)*x)*cosh(b*x + a)^4 + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^
3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)*sinh(b*x + a)^
3 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2
*b*d^3)*x)*sinh(b*x + a)^4 - 6*b*c*d^2 - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
```

```
 b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)^2 - 2*(b^3*
d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 - 3*(b^3*d^3*x^3 + 3*b^3*c*
d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)^2
+ 3*(b^3*c^2*d - 2*b*d^3)*x)*sinh(b*x + a)^2 + 3*(b^3*c^2*d - 2*b*d^3)*x +
4*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*
b*d^3)*x)*cosh(b*x + a)^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*
c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b
*x + a) + sinh(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d
^2 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cosh
(b*x + a)^4 + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a
)*d^3)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 -
2)*b*c*d^2 - (a^3 - 6*a)*d^3)*sinh(b*x + a)^4 - (a^3 - 6*a)*d^3 - 2*(b^3*c^
3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cosh(b*x + a)^2
- 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3 - 3*(b
^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cosh(b*x +
a)^2)*sinh(b*x + a)^2 + 4*((b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 -
 (a^3 - 6*a)*d^3)*cosh(b*x + a)^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*
b*c*d^2 - (a^3 - 6*a)*d^3)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a)
+ sinh(b*x + a) - 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)^4 + 4*(b^3*d^
3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 +
 3*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*d^3*x^3 +
3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*
c^2*d - 2*b*d^3)*x)*sinh(b*x + a)^4 + (a^3 - 6*a)*d^3 - 2*(b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^
2*d - 2*b*d^3)*x)*cosh(b*x + a)^2 - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*
x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*
d^3)*x)*cosh(b*x + a)^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*sinh(b*x + a)^2 + 3*(b
^3*c^2*d - 2*b*d^3)*x + 4*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)
^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3
- 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a))*sinh(b*x + a))*log(-
cosh(b*x + a) - sinh(b*x + a) + 1) + 6*(d^3*cosh(b*x + a)^4 + 4*d^3*cosh(b*
x + a)*sinh(b*x + a)^3 + d^3*sinh(b*x + a)^4 - 2*d^3*cosh(b*x + a)^2 + d^3
+ 2*(3*d^3*cosh(b*x + a)^2 - d^3)*sinh(b*x + a)...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \operatorname{csch}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csch(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**3*csch(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csch(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^3}{\sinh\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/sinh(a + b*x)^3,x)
```

```
[Out] int((c + d*x)^3/sinh(a + b*x)^3, x)
```

3.34 $\int (c+dx)^2 \operatorname{csch}^3(a+bx)\,dx$

Optimal. Leaf size=154

$$\frac{(c+dx)^2\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{d^2\tanh^{-1}(\cosh(a+bx))}{b^3}-\frac{d(c+dx)\operatorname{csch}(a+bx)}{b^2}-\frac{(c+dx)^2\coth(a+bx)\operatorname{csch}(a}{2b}$$

```
[Out] (d*x+c)^2*arctanh(exp(b*x+a))/b-d^2*arctanh(cosh(b*x+a))/b^3-d*(d*x+c)*csch
(b*x+a)/b^2-1/2*(d*x+c)^2*coth(b*x+a)*csch(b*x+a)/b+d*(d*x+c)*polylog(2,-ex
p(b*x+a))/b^2-d*(d*x+c)*polylog(2,exp(b*x+a))/b^2-d^2*polylog(3,-exp(b*x+a)
)/b^3+d^2*polylog(3,exp(b*x+a))/b^3
```

Rubi [A]
time = 0.12, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4271, 3855, 4267, 2611, 2320, 6724}

$$-\frac{d^2\text{Li}_3\left(-e^{a+bx}\right)}{b^3}+\frac{d^2\text{Li}_3\left(e^{a+bx}\right)}{b^3}-\frac{d^2\tanh^{-1}(\cosh(a+bx))}{b^3}+\frac{d(c+dx)\text{Li}_2\left(-e^{a+bx}\right)}{b^2}-\frac{d(c+dx)\text{Li}_2\left(e^{a+bx}\right)}{b^2}-\frac{d(c+dx)\operatorname{csch}(a+bx)}{b^2}+\frac{(c+dx)^2\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{(c+dx)^2\coth(a+bx)\operatorname{csch}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csch[a + b*x]^3,x]
```

```
[Out] ((c + d*x)^2*ArcTanh[E^(a + b*x)])/b - (d^2*ArcTanh[Cosh[a + b*x]])/b^3 - (
d*(c + d*x)*Csch[a + b*x])/b^2 - ((c + d*x)^2*Coth[a + b*x]*Csch[a + b*x])/
(2*b) + (d*(c + d*x)*PolyLog[2, -E^(a + b*x)])/b^2 - (d*(c + d*x)*PolyLog[2
, E^(a + b*x)])/b^2 - (d^2*PolyLog[3, -E^(a + b*x)])/b^3 + (d^2*PolyLog[3,
E^(a + b*x)])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\text{csch}^3(a+bx)\,dx &= -\frac{d(c+dx)\text{csch}(a+bx)}{b^2}-\frac{(c+dx)^2\coth(a+bx)\text{csch}(a+bx)}{2b}-\frac{1}{2}\int (c+dx\\ &= \frac{(c+dx)^2\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{d^2\tanh^{-1}(\cosh(a+bx))}{b^3}-\frac{d(c+dx)\text{csch}(a+bx)}{b^2}\\ &= \frac{(c+dx)^2\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{d^2\tanh^{-1}(\cosh(a+bx))}{b^3}-\frac{d(c+dx)\text{csch}(a+bx)}{b^2}\\ &= \frac{(c+dx)^2\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{d^2\tanh^{-1}(\cosh(a+bx))}{b^3}-\frac{d(c+dx)\text{csch}(a+bx)}{b^2}\\ &= \frac{(c+dx)^2\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{d^2\tanh^{-1}(\cosh(a+bx))}{b^3}-\frac{d(c+dx)\text{csch}(a+bx)}{b^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 375 vs. 2(154) = 308.

time = 8.80, size = 375, normalized size = 2.44

$$-\frac{d(c+dx)\operatorname{csch}(a)}{b^2}+\frac{(-c^2-2cdx-d^2x^2)\operatorname{csch}^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}+\frac{bc^2\tanh^{-1}\left(e^{a+bx}\right)-\frac{2d^2\tanh^{-1}\left(e^{a+bx}\right)}{b}-bcdx\log\left(1-e^{a+bx}\right)-\frac{1}{2}bd^2x^2\log\left(1-e^{a+bx}\right)+bcdx\log\left(1+e^{a+bx}\right)+\frac{1}{2}bd^2x^2\log\left(1+e^{a+bx}\right)+d(c+dx)\text{PolyLog}\left(2,-e^{a+bx}\right)-d(c+dx)\text{PolyLog}\left(2,e^{a+bx}\right)-\frac{d^2\text{PolyLog}\left(3,-e^{a+bx}\right)}{b}+\frac{d^2\text{PolyLog}\left(3,e^{a+bx}\right)}{b}}{b^2}+\frac{(-c^2-2cdx-d^2x^2)\operatorname{sech}^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}+\frac{\operatorname{csch}\left(\frac{a}{2}\right)\operatorname{csch}\left(\frac{a}{2}+\frac{bx}{2}\right)\left(cd\sinh\left(\frac{bx}{2}\right)+d^2x\sinh\left(\frac{bx}{2}\right)\right)}{2b^2}+\frac{\operatorname{sech}\left(\frac{a}{2}\right)\operatorname{sech}\left(\frac{a}{2}+\frac{bx}{2}\right)\left(cd\sinh\left(\frac{bx}{2}\right)+d^2x\sinh\left(\frac{bx}{2}\right)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csch[a + b*x]^3,x]
```

```
[Out] -((d*(c + d*x)*Csch[a])/b^2) + ((-c^2 - 2*c*d*x - d^2*x^2)*Csch[a/2 + (b*x)
/2]^2)/(8*b) + (b*c^2*ArcTanh[E^(a + b*x)] - (2*d^2*ArcTanh[E^(a + b*x)])/b
 - b*c*d*x*Log[1 - E^(a + b*x)] - (b*d^2*x^2*Log[1 - E^(a + b*x)])/2 + b*c*
d*x*Log[1 + E^(a + b*x)] + (b*d^2*x^2*Log[1 + E^(a + b*x)])/2 + d*(c + d*x)
*PolyLog[2, -E^(a + b*x)] - d*(c + d*x)*PolyLog[2, E^(a + b*x)] - (d^2*Poly
Log[3, -E^(a + b*x)])/b + (d^2*PolyLog[3, E^(a + b*x)])/b)/b^2 + ((-c^2 - 2
*c*d*x - d^2*x^2)*Sech[a/2 + (b*x)/2]^2)/(8*b) + (Csch[a/2]*Csch[a/2 + (b*x
)/2]*(c*d*Sinh[(b*x)/2] + d^2*x*Sinh[(b*x)/2]))/(2*b^2) + (Sech[a/2]*Sech[a
/2 + (b*x)/2]*(c*d*Sinh[(b*x)/2] + d^2*x*Sinh[(b*x)/2]))/(2*b^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(147) = 294$.
time = 0.95, size = 444, normalized size = 2.88

method	result
risch	$-\frac{e^{bx+a}\left(b\,d^2x^2e^{2bx+2a}+2bcdx\,e^{2bx+2a}+b\,c^2e^{2bx+2a}+b\,d^2x^2+2d^2x\,e^{2bx+2a}+2bcdx+2cd\,e^{2bx+2a}+b\,c^2-2d^2x-2cd\right)}{b^2\left(e^{2bx+2a}-1\right)^2}+\frac{d^2\,\text{polylog}(}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -exp(b*x+a)*(b*d^2*x^2*exp(2*b*x+2*a)+2*b*c*d*x*exp(2*b*x+2*a)+b*c^2*exp(2*
b*x+2*a)+b*d^2*x^2+2*d^2*x*exp(2*b*x+2*a)+2*b*c*d*x+2*c*d*exp(2*b*x+2*a)+b*
c^2-2*d^2*x-2*c*d)/b^2/(exp(2*b*x+2*a)-1)^2+d^2*polylog(3,exp(b*x+a))/b^3-1
/2/b*ln(1-exp(b*x+a))*d^2*x^2-1/b^2*polylog(2,exp(b*x+a))*d^2*x+1/2/b*ln(ex
p(b*x+a)+1)*d^2*x^2+1/b^2*polylog(2,-exp(b*x+a))*d^2*x+1/2/b^3*ln(1-exp(b*x
+a))*a^2*d^2-1/2/b^3*d^2*ln(exp(b*x+a)+1)*a^2+1/b^3*a^2*d^2*arctanh(exp(b*x
+a))+1/b*c^2*arctanh(exp(b*x+a))-1/b*ln(1-exp(b*x+a))*c*d*x-1/b^2*ln(1-exp(
b*x+a))*a*c*d+1/b*d*c*ln(exp(b*x+a)+1)*x+1/b^2*ln(exp(b*x+a)+1)*a*c*d-2/b^2
*c*d*a*arctanh(exp(b*x+a))+1/b^2*c*d*polylog(2,-exp(b*x+a))-1/b^2*c*d*polyl
og(2,exp(b*x+a))-d^2*polylog(3,-exp(b*x+a))/b^3-2/b^3*d^2*arctanh(exp(b*x+a
))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(145) = 290$.
time = 0.41, size = 393, normalized size = 2.55

$$\frac{1}{2}c^2\left(\frac{\log\left(e^{(-bx-a)}+1\right)}{b}-\frac{\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{2\left(e^{(-bx-a)}+e^{(-3bx-3a)}\right)}{b(2e^{(-2bx-2a)}-e^{(-4bx-4a)}-1)}\right)+\frac{\left(bx\log\left(e^{(bx+a)}+1\right)+\text{Li}_2(-e^{(bx+a)})\right)cd}{b^2}-\frac{\left(bx\log\left(-e^{(bx+a)}+1\right)+\text{Li}_2(e^{(bx+a)})\right)cd}{b^2}-\frac{\left(bd^2x^2e^{(3a)}+2cde^{(3a)}+2(bcd+d^2)xe^{(3a)}\right)e^{(3bx)}+\left(bd^2x^2e^a-2cde^a+2(bcd-d^2)xe^a\right)e^{(bx)}}{b^2e^{(4bx+4a)}-2b^2e^{(2bx+2a)}+b^2}+\frac{\left(b^2x^2\log\left(e^{(bx+a)}+1\right)+2bx\text{Li}_2(-e^{(bx+a)})-2\text{Li}_3(-e^{(bx+a)})\right)d^2}{2b^3}-\frac{\left(b^2x^2\log\left(-e^{(bx+a)}+1\right)+2bx\text{Li}_2(e^{(bx+a)})-2\text{Li}_3(e^{(bx+a)})\right)d^2}{2b^3}-\frac{d^2\log\left(e^{(bx+a)}+1\right)}{b^3}+\frac{d^2\log\left(e^{(bx+a)}-1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*(log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b + 2*(e^(-b*x - a
) + e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))) + (b
*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))*c*d/b^2 - (b*x*log(-e^(b*x +
 a) + 1) + dilog(e^(b*x + a)))*c*d/b^2 - ((b*d^2*x^2*e^(3*a) + 2*c*d*e^(3*a
) + 2*(b*c*d + d^2)*x*e^(3*a))*e^(3*b*x) + (b*d^2*x^2*e^a - 2*c*d*e^a + 2*(
b*c*d - d^2)*x*e^a)*e^(b*x))/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) +
 b^2) + 1/2*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*p
olylog(3, -e^(b*x + a)))*d^2/b^3 - 1/2*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b
*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))*d^2/b^3 - d^2*log(e^(b*x
 + a) + 1)/b^3 + d^2*log(e^(b*x + a) - 1)/b^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2218 vs. $2(145) = 290$.
time = 0.37, size = 2218, normalized size = 14.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d + b*d^2)*x)*cosh(b*x
+ a)^3 + 6*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d + b*d^2)*x)*cosh(b
*x + a)*sinh(b*x + a)^2 + 2*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d +
 b*d^2)*x)*sinh(b*x + a)^3 + 2*(b^2*d^2*x^2 + b^2*c^2 - 2*b*c*d + 2*(b^2*c*
d - b*d^2)*x)*cosh(b*x + a) + 2*((b*d^2*x + b*c*d)*cosh(b*x + a)^4 + 4*(b*d
^2*x + b*c*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*d^2*x + b*c*d)*sinh(b*x +
a)^4 + b*d^2*x + b*c*d - 2*(b*d^2*x + b*c*d)*cosh(b*x + a)^2 - 2*(b*d^2*x +
 b*c*d - 3*(b*d^2*x + b*c*d)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4*((b*d^2*x
 + b*c*d)*cosh(b*x + a)^3 - (b*d^2*x + b*c*d)*cosh(b*x + a))*sinh(b*x + a))
*dilog(cosh(b*x + a) + sinh(b*x + a)) - 2*((b*d^2*x + b*c*d)*cosh(b*x + a)^
4 + 4*(b*d^2*x + b*c*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*d^2*x + b*c*d)*s
inh(b*x + a)^4 + b*d^2*x + b*c*d - 2*(b*d^2*x + b*c*d)*cosh(b*x + a)^2 - 2*
(b*d^2*x + b*c*d - 3*(b*d^2*x + b*c*d)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4
*((b*d^2*x + b*c*d)*cosh(b*x + a)^3 - (b*d^2*x + b*c*d)*cosh(b*x + a))*sinh
(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x + a)) - (b^2*d^2*x^2 + 2*b^2*c*d
*x + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cosh(b*x + a)^4 + 4*(b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sinh(b*x + a)^4 + b^2*c^2 - 2*
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cosh(b*x + a)^2 - 2*(b^2*d^2*
x^2 + 2*b^2*c*d*x + b^2*c^2 - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^
2)*cosh(b*x + a)^2 - 2*d^2)*sinh(b*x + a)^2 - 2*d^2 + 4*((b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2 - 2*d^2)*cosh(b*x + a)^3 - (b^2*d^2*x^2 + 2*b^2*c*d*x +
```

```
b^2*c^2 - 2*d^2)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x
 + a) + 1) + ((b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cosh(b*x + a)^4 + 4*(b^
2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*c^2
 - 2*a*b*c*d + (a^2 - 2)*d^2)*sinh(b*x + a)^4 + b^2*c^2 - 2*a*b*c*d + (a^2
- 2)*d^2 - 2*(b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cosh(b*x + a)^2 - 2*(b^2
*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2 - 3*(b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*
cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4*((b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)
*cosh(b*x + a)^3 - (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cosh(b*x + a))*sin
h(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (b^2*d^2*x^2 + 2*b^2*c
*d*x + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cosh(b*x + a)^4 +
4*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cosh(b*x + a)*sinh(b*x
+ a)^3 + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*sinh(b*x + a)^4
+ 2*a*b*c*d - a^2*d^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)
*cosh(b*x + a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - 3*(
b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cosh(b*x + a)^2)*sinh(b*x
+ a)^2 + 4*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cosh(b*x + a)
^3 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cosh(b*x + a))*sinh(
b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - 2*(d^2*cosh(b*x + a)^4
+ 4*d^2*cosh(b*x + a)*sinh(b*x + a)^3 + d^2*sinh(b*x + a)^4 - 2*d^2*cosh(b*
x + a)^2 + 2*(3*d^2*cosh(b*x + a)^2 - d^2)*sinh(b*x + a)^2 + d^2 + 4*(d^2*c
osh(b*x + a)^3 - d^2*cosh(b*x + a))*sinh(b*x + a))*polylog(3, cosh(b*x + a)
 + sinh(b*x + a)) + 2*(d^2*cosh(b*x + a)^4 + 4*d^2*cosh(b*x + a)*sinh(b*x +
 a)^3 + d^2*sinh(b*x + a)^4 - 2*d^2*cosh(b*x + a)^2 + 2*(3*d^2*cosh(b*x + a
)^2 - d^2)*sinh(b*x + a)^2 + d^2 + 4*(d^2*cosh(b*x + a)^3 - d^2*cosh(b*x +
a))*sinh(b*x + a))*polylog(3, -cosh(b*x + a) - sinh(b*x + a)) + 2*(b^2*d^2*
x^2 + b^2*c^2 - 2*b*c*d + 3*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d +
 b*d^2)*x)*cosh(b*x + a)^2 + 2*(b^2*c*d - b*d^2)*x)*sinh(b*x + a))/(b^3*cos
h(b*x + a)^4 + 4*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 -
2*b^3*cosh(b*x + a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 - b^3)*sinh(b*x + a)
^2 + 4*(b^3*cosh(b*x + a)^3 - b^3*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \operatorname{csch}^3(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csch(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**2*csch(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csch(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^2}{\sinh\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/sinh(a + b*x)^3,x)
```

```
[Out] int((c + d*x)^2/sinh(a + b*x)^3, x)
```

3.35 $\int (c+dx)\text{csch}^3(a+bx)\,dx$

Optimal. Leaf size=92

$$\frac{(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{d\text{csch}(a+bx)}{2b^2}-\frac{(c+dx)\coth(a+bx)\text{csch}(a+bx)}{2b}+\frac{d\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}-\frac{d\text{Poly}}{}$$

```
[Out] (d*x+c)*arctanh(exp(b*x+a))/b-1/2*d*csch(b*x+a)/b^2-1/2*(d*x+c)*coth(b*x+a)
*csch(b*x+a)/b+1/2*d*polylog(2,-exp(b*x+a))/b^2-1/2*d*polylog(2,exp(b*x+a))
/b^2
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4270, 4267, 2317, 2438}

$$\frac{d\text{Li}_2\left(-e^{a+bx}\right)}{2b^2}-\frac{d\text{Li}_2\left(e^{a+bx}\right)}{2b^2}-\frac{d\text{csch}(a+bx)}{2b^2}+\frac{(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{(c+dx)\coth(a+bx)\text{csch}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csch[a + b*x]^3,x]
```

```
[Out] ((c + d*x)*ArcTanh[E^(a + b*x)])/b - (d*Csch[a + b*x])/(2*b^2) - ((c + d*x)
*Coth[a + b*x]*Csch[a + b*x])/(2*b) + (d*PolyLog[2, -E^(a + b*x)])/(2*b^2)
- (d*PolyLog[2, E^(a + b*x)])/(2*b^2)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\int (c+dx)\text{csch}^3(a+bx)\,dx = -\frac{d\text{csch}(a+bx)}{2b^2} - \frac{(c+dx)\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{1}{2}\int (c+dx)\text{csch}(a+$$
$$= \frac{(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{d\text{csch}(a+bx)}{2b^2} - \frac{(c+dx)\coth(a+bx)\text{csch}(a+bx)}{2b}$$
$$= \frac{(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{d\text{csch}(a+bx)}{2b^2} - \frac{(c+dx)\coth(a+bx)\text{csch}(a+bx)}{2b}$$
$$= \frac{(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{d\text{csch}(a+bx)}{2b^2} - \frac{(c+dx)\coth(a+bx)\text{csch}(a+bx)}{2b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.69, size = 313, normalized size = 3.40

$$-\frac{dx\text{csch}^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} - \frac{c\text{csch}^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{c\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{d\left(-a\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right) - i\left((ia+ibx)\left(\log\left(1-e^{i(ia+ibx)}\right) - \log\left(1+e^{i(ia+ibx)}\right)\right) + i\left(\text{PolyLog}\left(2,-e^{i(ia+ibx)}\right) - \text{PolyLog}\left(2,e^{i(ia+ibx)}\right)\right)\right)\right)}{2b^2} - \frac{dx\text{sech}^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} - \frac{c\text{sech}^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{d\text{csch}\left(\frac{a}{2}\right)\text{csch}\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{bx}{2}\right)}{4b^2} + \frac{d\text{sech}\left(\frac{a}{2}\right)\text{sech}\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{bx}{2}\right)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csch[a + b*x]^3,x]
```

```
[Out] -1/8*(d*x*Csch[a/2 + (b*x)/2]^2)/b - (c*Csch[(a + b*x)/2]^2)/(8*b) - (c*Log
[Tanh[(a + b*x)/2]])/(2*b) - (d*(-(a*Log[Tanh[(a + b*x)/2]]) - I*((I*a + I*
b*x)*(Log[1 - E^(I*(I*a + I*b*x))] - Log[1 + E^(I*(I*a + I*b*x))]) + I*(Pol
yLog[2, -E^(I*(I*a + I*b*x))] - PolyLog[2, E^(I*(I*a + I*b*x))]))))/(2*b^2)
 - (d*x*Sech[a/2 + (b*x)/2]^2)/(8*b) - (c*Sech[(a + b*x)/2]^2)/(8*b) + (d*C
sch[a/2]*Csch[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(4*b^2) + (d*Sech[a/2]*Sech[a/2
 + (b*x)/2]*Sinh[(b*x)/2])/(4*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(81) = 162$.
time = 0.69, size = 197, normalized size = 2.14

method	result
risch	$-\frac{e^{bx+a}\left(bdx\,e^{2bx+2a}+bc\,e^{2bx+2a}+bdx+e^{2bx+2a}d+bc-d\right)}{b^2\left(e^{2bx+2a}-1\right)^2} + \frac{c\,\text{arctanh}\left(e^{bx+a}\right)}{b} - \frac{d\ln\left(1-e^{bx+a}\right)x}{2b} - \frac{d\ln\left(1-e^{bx+a}\right)a}{2b^2} - \frac{d\,\text{polylo}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -exp(b*x+a)*(b*d*x*exp(2*b*x+2*a)+b*c*exp(2*b*x+2*a)+b*d*x+exp(2*b*x+2*a)*d
+b*c-d)/b^2/(exp(2*b*x+2*a)-1)^2+1/b*c*arctanh(exp(b*x+a))-1/2/b*d*ln(1-exp
(b*x+a))*x-1/2/b^2*d*ln(1-exp(b*x+a))*a-1/2*d*polylog(2,exp(b*x+a))/b^2+1/2
/b*d*ln(exp(b*x+a)+1)*x+1/2/b^2*d*ln(exp(b*x+a)+1)*a+1/2*d*polylog(2,-exp(b
*x+a))/b^2-1/b^2*d*a*arctanh(exp(b*x+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -d*(((b*x*e^(3*a) + e^(3*a))*e^(3*b*x) + (b*x*e^a - e^a)*e^(b*x))/(b^2*e^(4
*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) + 8*integrate(1/16*x/(e^(b*x + a
) + 1), x) + 8*integrate(1/16*x/(e^(b*x + a) - 1), x)) + 1/2*c*(log(e^(-b*x
 - a) + 1)/b - log(e^(-b*x - a) - 1)/b + 2*(e^(-b*x - a) + e^(-3*b*x - 3*a)
)/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1026 vs. $2(79) = 158$.
time = 0.35, size = 1026, normalized size = 11.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*d*x + b*c + d)*cosh(b*x + a)^3 + 6*(b*d*x + b*c + d)*cosh(b*x +
a)*sinh(b*x + a)^2 + 2*(b*d*x + b*c + d)*sinh(b*x + a)^3 + 2*(b*d*x + b*c -
 d)*cosh(b*x + a) + (d*cosh(b*x + a)^4 + 4*d*cosh(b*x + a)*sinh(b*x + a)^3
+ d*sinh(b*x + a)^4 - 2*d*cosh(b*x + a)^2 + 2*(3*d*cosh(b*x + a)^2 - d)*sin
h(b*x + a)^2 + 4*(d*cosh(b*x + a)^3 - d*cosh(b*x + a))*sinh(b*x + a) + d)*d
ilog(cosh(b*x + a) + sinh(b*x + a)) - (d*cosh(b*x + a)^4 + 4*d*cosh(b*x + a
)*sinh(b*x + a)^3 + d*sinh(b*x + a)^4 - 2*d*cosh(b*x + a)^2 + 2*(3*d*cosh(b
*x + a)^2 - d)*sinh(b*x + a)^2 + 4*(d*cosh(b*x + a)^3 - d*cosh(b*x + a))*si
nh(b*x + a) + d)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - ((b*d*x + b*c)*cos
h(b*x + a)^4 + 4*(b*d*x + b*c)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*d*x + b*c
)*sinh(b*x + a)^4 + b*d*x - 2*(b*d*x + b*c)*cosh(b*x + a)^2 - 2*(b*d*x - 3*
```

```
(b*d*x + b*c)*cosh(b*x + a)^2 + b*c)*sinh(b*x + a)^2 + b*c + 4*((b*d*x + b*
c)*cosh(b*x + a)^3 - (b*d*x + b*c)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b
*x + a) + sinh(b*x + a) + 1) + ((b*c - a*d)*cosh(b*x + a)^4 + 4*(b*c - a*d)
*cosh(b*x + a)*sinh(b*x + a)^3 + (b*c - a*d)*sinh(b*x + a)^4 - 2*(b*c - a*d
)*cosh(b*x + a)^2 + 2*(3*(b*c - a*d)*cosh(b*x + a)^2 - b*c + a*d)*sinh(b*x
+ a)^2 + b*c - a*d + 4*((b*c - a*d)*cosh(b*x + a)^3 - (b*c - a*d)*cosh(b*x
+ a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1) + ((b*d*x + a*d
)*cosh(b*x + a)^4 + 4*(b*d*x + a*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*d*x
+ a*d)*sinh(b*x + a)^4 + b*d*x - 2*(b*d*x + a*d)*cosh(b*x + a)^2 - 2*(b*d*x
 - 3*(b*d*x + a*d)*cosh(b*x + a)^2 + a*d)*sinh(b*x + a)^2 + a*d + 4*((b*d*x
 + a*d)*cosh(b*x + a)^3 - (b*d*x + a*d)*cosh(b*x + a))*sinh(b*x + a))*log(-
cosh(b*x + a) - sinh(b*x + a) + 1) + 2*(b*d*x + 3*(b*d*x + b*c + d)*cosh(b*
x + a)^2 + b*c - d)*sinh(b*x + a))/(b^2*cosh(b*x + a)^4 + 4*b^2*cosh(b*x +
a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 - 2*b^2*cosh(b*x + a)^2 + 2*(3*b^2
*cosh(b*x + a)^2 - b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^2*cosh(b*x + a)^3 - b^
2*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)\,\mathrm{csch}^3\,(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*csch(a + b*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csch(b*x + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c + d\,x}{{\sinh\left(a + b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/sinh(a + b*x)^3,x)
```

```
[Out] int((c + d*x)/sinh(a + b*x)^3, x)
```

3.36 $\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{\operatorname{csch}^3(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(csch(b*x+a)^3/(d*x+c),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csch[a + b*x]^3/(c + d*x),x]
```

```
[Out] Defer[Int][Csch[a + b*x]^3/(c + d*x), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 52.15, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csch[a + b*x]^3/(c + d*x),x]
```

```
[Out] Integrate[Csch[a + b*x]^3/(c + d*x), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(csch(b*x+a)^3/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] -((b*d*x*e^(3*a) + (b*c - d)*e^(3*a))*e^(3*b*x) + (b*d*x*e^a + (b*c + d)*e^
a)*e^(b*x))/(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2*e^(4*a) + 2
*b^2*c*d*x*e^(4*a) + b^2*c^2*e^(4*a))*e^(4*b*x) - 2*(b^2*d^2*x^2*e^(2*a) +
2*b^2*c*d*x*e^(2*a) + b^2*c^2*e^(2*a))*e^(2*b*x)) - 8*integrate(1/16*(b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3*e^a + 3*b^2*c*d^2*x^2*e^a + 3*b^2*c^2*
d*x*e^a + b^2*c^3*e^a)*e^(b*x)), x) - 8*integrate(-1/16*(b^2*d^2*x^2 + 2*b^
2*c*d*x + b^2*c^2 - 2*d^2)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
 b^2*c^3 - (b^2*d^3*x^3*e^a + 3*b^2*c*d^2*x^2*e^a + 3*b^2*c^2*d*x*e^a + b^2
*c^3*e^a)*e^(b*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^3/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{3}(a + bx)}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(csch(a + b*x)**3/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sinh\left(a+b\,x\right)^3\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(a + b*x)^3*(c + d*x)),x)
```

```
[Out] int(1/(sinh(a + b*x)^3*(c + d*x)), x)
```

3.37 $\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(csch(b*x+a)^3/(d*x+c)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csch[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Csch[a + b*x]^3/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A]
time = 55.69, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csch[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] Integrate[Csch[a + b*x]^3/(c + d*x)^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)^3}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(csch(b*x+a)^3/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((b*d*x*e^(3*a) + (b*c - 2*d)*e^(3*a))*e^(3*b*x) + (b*d*x*e^a + (b*c + 2*d
)*e^a)*e^(b*x))/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 +
(b^2*d^3*x^3*e^(4*a) + 3*b^2*c*d^2*x^2*e^(4*a) + 3*b^2*c^2*d*x*e^(4*a) + b^
2*c^3*e^(4*a))*e^(4*b*x) - 2*(b^2*d^3*x^3*e^(2*a) + 3*b^2*c*d^2*x^2*e^(2*a)
 + 3*b^2*c^2*d*x*e^(2*a) + b^2*c^3*e^(2*a))*e^(2*b*x)) - 8*integrate(1/16*(
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 6*d^2)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3
 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4*e^a + 4*b^2*c
*d^3*x^3*e^a + 6*b^2*c^2*d^2*x^2*e^a + 4*b^2*c^3*d*x*e^a + b^2*c^4*e^a)*e^(
b*x)), x) - 8*integrate(-1/16*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 6*d^2)
/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c
^4 - (b^2*d^4*x^4*e^a + 4*b^2*c*d^3*x^3*e^a + 6*b^2*c^2*d^2*x^2*e^a + 4*b^2
*c^3*d*x*e^a + b^2*c^4*e^a)*e^(b*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(csch(a + b*x)**3/(c + d*x)**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sinh\left(a+b\,x\right)^3\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(a + b*x)^3*(c + d*x)^2),x)
```

```
[Out] int(1/(sinh(a + b*x)^3*(c + d*x)^2), x)
```

3.38 $\int (c+dx)^{5/2}\sinh(a+bx)\,dx$

Optimal. Leaf size=171

$$\frac{15d^2\sqrt{c+dx}\,\cosh(a+bx)}{4b^3}+\frac{(c+dx)^{5/2}\cosh(a+bx)}{b}-\frac{15d^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{15d^{5/2}e^{a-\frac{bc}{d}}$$

[Out] (d*x+c)^(5/2)*cosh(b*x+a)/b-5/2*d*(d*x+c)^(3/2)*sinh(b*x+a)/b^2-15/16*d^(5/2)*exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(7/2)-15/16*d^(5/2)*exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(7/2)+15/4*d^2*cosh(b*x+a)*(d*x+c)^(1/2)/b^3

Rubi [A]
time = 0.26, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3377, 3388, 2211, 2235, 2236}

$$-\frac{15\sqrt{\pi}\,d^{5/2}e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{15\sqrt{\pi}\,d^{5/2}e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\cosh(a+bx)}{4b^3}-\frac{5d(c+dx)^{3/2}\sinh(a+bx)}{2b^2}+\frac{(c+dx)^{5/2}\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(5/2)*Sinh[a + b*x],x]

[Out] (15*d^2*Sqrt[c + d*x]*Cosh[a + b*x])/(4*b^3) + ((c + d*x)^(5/2)*Cosh[a + b*x])/b - (15*d^(5/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (15*d^(5/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d*(c + d*x)^(3/2)*Sinh[a + b*x])/(2*b^2)

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\int (c+dx)^{5/2}\sinh(a+bx)\,dx = \frac{(c+dx)^{5/2}\cosh(a+bx)}{b} - \frac{(5d)\int (c+dx)^{3/2}\cosh(a+bx)\,dx}{2b}$$

$$= \frac{(c+dx)^{5/2}\cosh(a+bx)}{b} - \frac{5d(c+dx)^{3/2}\sinh(a+bx)}{2b^2} + \frac{(15d^2)\int \sqrt{c+dx}}{4b^2}$$

$$= \frac{15d^2\sqrt{c+dx}\cosh(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\cosh(a+bx)}{b} - \frac{5d(c+dx)^{3/2}\sinh}{2b^2}$$

$$= \frac{15d^2\sqrt{c+dx}\cosh(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\cosh(a+bx)}{b} - \frac{5d(c+dx)^{3/2}\sinh}{2b^2}$$

$$= \frac{15d^2\sqrt{c+dx}\cosh(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\cosh(a+bx)}{b} - \frac{5d(c+dx)^{3/2}\sinh}{2b^2}$$

$$= \frac{15d^2\sqrt{c+dx}\cosh(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\cosh(a+bx)}{b} - \frac{15d^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,e}{16}$$

Mathematica [A]
time = 0.04, size = 108, normalized size = 0.63

$$\frac{d^3e^{-a-\frac{bc}{d}}\left(-e^{2a}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{7}{2},-\frac{b(c+dx)}{d}\right)+e^{\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{7}{2},\frac{b(c+dx)}{d}\right)\right)}{2b^4\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Sinh[a + b*x],x]
```

[Out] (d^3*E^(-a - (b*c)/d)*(-(E^(2*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[7/2, -((b*(
c + d*x))/d)]) + E^((2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[7/2, (b*(c + d*x
))/d]))/(2*b^4*Sqrt[c + d*x])

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^{\frac{5}{2}} \sinh (bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(5/2)*sinh(b*x+a),x)

[Out] int((d*x+c)^(5/2)*sinh(b*x+a),x)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(131) = 262.
time = 0.28, size = 308, normalized size = 1.80

$$\frac{32\,(dx+c)^{\frac{7}{2}}\sinh(bx+a) - \frac{\left(\frac{105\sqrt{\pi}\,d^4\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^4\sqrt{-\frac{b}{d}}} + \frac{105\sqrt{\pi}\,d^4\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^4\sqrt{\frac{b}{d}}} - \frac{2\left(8\,(dx+c)^{\frac{7}{2}}b^3de^{\left(\frac{bc}{d}\right)} + 28\,(dx+c)^{\frac{5}{2}}b^2d^2e^{\left(\frac{bc}{d}\right)} + 70\,(dx+c)^{\frac{3}{2}}bd^3e^{\left(\frac{bc}{d}\right)} + 105\sqrt{dx+c}\,d^4e^{\left(\frac{bc}{d}\right)}\right)e^{\left(-a-\frac{(dx+c)b}{d}\right)}}{b^4} + \frac{2\left(8\,(dx+c)^{\frac{7}{2}}b^3de^a - 28\,(dx+c)^{\frac{5}{2}}b^2d^2e^a + 70\,(dx+c)^{\frac{3}{2}}bd^3e^a - 105\sqrt{dx+c}\,d^4e^a\right)e^{\left(\frac{(dx+c)b}{d}-\frac{bc}{d}\right)}}{b^4}\right)b}{d}}{112\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*sinh(b*x+a),x, algorithm="maxima")

[Out] 1/112*(32*(d*x + c)^(7/2)*sinh(b*x + a) - (105*sqrt(pi)*d^4*erf(sqrt(d*x +
c)*sqrt(-b/d))*e^(a - b*c/d)/(b^4*sqrt(-b/d)) + 105*sqrt(pi)*d^4*erf(sqrt(d
*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^4*sqrt(b/d)) - 2*(8*(d*x + c)^(7/2)*b^
3*d*e^(b*c/d) + 28*(d*x + c)^(5/2)*b^2*d^2*e^(b*c/d) + 70*(d*x + c)^(3/2)*b
*d^3*e^(b*c/d) + 105*sqrt(d*x + c)*d^4*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^
4 + 2*(8*(d*x + c)^(7/2)*b^3*d*e^a - 28*(d*x + c)^(5/2)*b^2*d^2*e^a + 70*(d
*x + c)^(3/2)*b*d^3*e^a - 105*sqrt(d*x + c)*d^4*e^a)*e^((d*x + c)*b/d - b*c
/d)/b^4)*b/d)/d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(131) = 262.
time = 0.35, size = 521, normalized size = 3.05

$$-\frac{15\sqrt{\pi}\left(d^3\cosh(bx+a)\cosh\left(-\frac{bc-ad}{d}\right) - d^3\cosh(bx+a)\sinh\left(-\frac{bc-ad}{d}\right) + \left(d^3\cosh\left(-\frac{bc-ad}{d}\right) - d^3\sinh\left(-\frac{bc-ad}{d}\right)\right)\sinh(bx+a)\right)\sqrt{\frac{b}{d}}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right) - 15\sqrt{\pi}\left(d^3\cosh(bx+a)\cosh\left(-\frac{bc-ad}{d}\right) + d^3\cosh(bx+a)\sinh\left(-\frac{bc-ad}{d}\right) + \left(d^3\cosh\left(-\frac{bc-ad}{d}\right) + d^3\sinh\left(-\frac{bc-ad}{d}\right)\right)\sinh(bx+a)\right)\sqrt{-\frac{b}{d}}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right) - 2\left(4b^3d^2x^2 + 4b^3c^2 + 10b^2cd + 15bd^2 + \left(4b^3d^2x^2 + 4b^3c^2 - 10b^2cd + 15bd^2 + 2\left(4b^3cd - 5b^2d^2\right)x\right)\cosh(bx+a)^2 + 2\left(4b^3d^2x^2 + 4b^3c^2 - 10b^2cd + 15bd^2 + 2\left(4b^3cd - 5b^2d^2\right)x\right)\cosh(bx+a)\sinh(bx+a) + \left(4b^3d^2x^2 + 4b^3c^2 - 10b^2cd + 15bd^2 + 2\left(4b^3cd - 5b^2d^2\right)x\right)\sinh(bx+a)^2 + 2\left(4b^3cd + 5b^2d^2\right)x\right)\sqrt{dx+c}}{16\left(b^4\cosh(bx+a) + b^4\sinh(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)*sinh(b*x+a),x, algorithm="fricas")

[Out] -1/16*(15*sqrt(pi)*(d^3*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d^3*cosh(b*x +
 a)*sinh(-(b*c - a*d)/d) + (d^3*cosh(-(b*c - a*d)/d) - d^3*sinh(-(b*c - a*d
)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 15*sqrt(pi)*(

```
d^3*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d^3*cosh(b*x + a)*sinh(-(b*c - a*d
)/d) + (d^3*cosh(-(b*c - a*d)/d) + d^3*sinh(-(b*c - a*d)/d))*sinh(b*x + a))
*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) - 2*(4*b^3*d^2*x^2 + 4*b^3*c^2 +
10*b^2*c*d + 15*b*d^2 + (4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2
+ 2*(4*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^2 + 2*(4*b^3*d^2*x^2 + 4*b^3*c
^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)*sin
h(b*x + a) + (4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*
c*d - 5*b^2*d^2)*x)*sinh(b*x + a)^2 + 2*(4*b^3*c*d + 5*b^2*d^2)*x)*sqrt(d*x
 + c))/(b^4*cosh(b*x + a) + b^4*sinh(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{5}{2}} \sinh(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*sinh(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(5/2)*sinh(a + b*x), x)
```

Giac [A]
time = 0.47, size = 232, normalized size = 1.36

$$\frac{\frac{15\sqrt{\pi}\,d^4\operatorname{erf}\left(-\frac{\sqrt{bd}\,\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}\,b^3}+\frac{15\sqrt{\pi}\,d^4\operatorname{erf}\left(-\frac{\sqrt{-bd}\,\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}\,b^3}+\frac{2\left(4\,(dx+c)^{\frac{5}{2}}b^2d-10\,(dx+c)^{\frac{3}{2}}bd^2+15\,\sqrt{dx+c}\,d^3\right)e^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b^3}+\frac{2\left(4\,(dx+c)^{\frac{5}{2}}b^2d+10\,(dx+c)^{\frac{3}{2}}bd^2+15\,\sqrt{dx+c}\,d^3\right)e^{\left(-\frac{(dx+c)b-bc+ad}{d}\right)}}{b^3}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/16*(15*sqrt(pi)*d^4*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/(sq
rt(b*d)*b^3) + 15*sqrt(pi)*d^4*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c -
a*d)/d)/(sqrt(-b*d)*b^3) + 2*(4*(d*x + c)^(5/2)*b^2*d - 10*(d*x + c)^(3/2)*
b*d^2 + 15*sqrt(d*x + c)*d^3)*e^(((d*x + c)*b - b*c + a*d)/d)/b^3 + 2*(4*(d
*x + c)^(5/2)*b^2*d + 10*(d*x + c)^(3/2)*b*d^2 + 15*sqrt(d*x + c)*d^3)*e^(-
((d*x + c)*b - b*c + a*d)/d)/b^3)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh(a+b\,x)\,\left(c+d\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*(c + d*x)^(5/2),x)
```

```
[Out] int(sinh(a + b*x)*(c + d*x)^(5/2), x)
```

3.39 $\int (c+dx)^{3/2}\sinh(a+bx)\,dx$

Optimal. Leaf size=146

$$\frac{(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3d^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{3d^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-3d\sqrt{}$$

```
[Out] (d*x+c)^(3/2)*cosh(b*x+a)/b-3/8*d^(3/2)*exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(
1/2)/d^(1/2))*Pi^(1/2)/b^(5/2)+3/8*d^(3/2)*exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*Pi^(1/2)/b^(5/2)-3/2*d*sinh(b*x+a)*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.17, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3377, 3389, 2211, 2235, 2236}

$$-\frac{3\sqrt{\pi}\,d^{3/2}e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{3\sqrt{\pi}\,d^{3/2}e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3d\sqrt{c+dx}\,\sinh(a+bx)}{2b^2}+\frac{(c+dx)^{3/2}\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Sinh[a + b*x],x]
```

```
[Out] ((c + d*x)^(3/2)*Cosh[a + b*x])/b - (3*d^(3/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Er
f[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (3*d^(3/2)*E^(a - (b*c)/d
)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (3*d*Sqrt[c
 + d*x]*Sinh[a + b*x])/(2*b^2)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\sinh(a+bx)\,dx = \frac{(c+dx)^{3/2}\cosh(a+bx)}{b} - \frac{(3d)\int \sqrt{c+dx}\,\cosh(a+bx)\,dx}{2b}$$
$$= \frac{(c+dx)^{3/2}\cosh(a+bx)}{b} - \frac{3d\sqrt{c+dx}\,\sinh(a+bx)}{2b^2} + \frac{(3d^2)\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx}{4b^2}$$
$$= \frac{(c+dx)^{3/2}\cosh(a+bx)}{b} - \frac{3d\sqrt{c+dx}\,\sinh(a+bx)}{2b^2} + \frac{(3d^2)\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{8b^2}$$
$$= \frac{(c+dx)^{3/2}\cosh(a+bx)}{b} - \frac{3d\sqrt{c+dx}\,\sinh(a+bx)}{2b^2} - \frac{(3d)\text{Subst}\left(\int e^{i\left(ia-\right.}\right.}{}$$
$$= \frac{(c+dx)^{3/2}\cosh(a+bx)}{b} - \frac{3d^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} + \frac{3d^{3/2}e^{a-}}{}$$

Mathematica [A]
time = 0.07, size = 106, normalized size = 0.73

$$\frac{de^{-a-\frac{bc}{d}}\sqrt{c+dx}\left(-\frac{e^{2a}\Gamma\left(\frac{5}{2},-\frac{b(c+dx)}{d}\right)}{\sqrt{-\frac{b(c+dx)}{d}}}+\frac{e^{\frac{2bc}{d}}\Gamma\left(\frac{5}{2},\frac{b(c+dx)}{d}\right)}{\sqrt{\frac{b(c+dx)}{d}}}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Sinh[a + b*x],x]
```

```
[Out] (d*E^(-a - (b*c)/d)*Sqrt[c + d*x]*(-((E^(2*a)*Gamma[5/2, -((b*(c + d*x))/d)
])/Sqrt[-((b*(c + d*x))/d)]) + (E^((2*b*c)/d)*Gamma[5/2, (b*(c + d*x))/d])/
Sqrt[(b*(c + d*x))/d]))/(2*b^2)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^{\frac{3}{2}} \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*sinh(b*x+a),x)
```

```
[Out] int((d*x+c)^(3/2)*sinh(b*x+a),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(110) = 220.
time = 0.27, size = 268, normalized size = 1.84

$$\frac{16\,(dx+c)^{\frac{5}{2}}\sinh(bx+a)+\frac{\left(\frac{15\sqrt{\pi}\,d^3\,\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}}-\frac{15\sqrt{\pi}\,d^3\,\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}}+\frac{2\left(4\,(dx+c)^{\frac{5}{2}}b^2de^{\left(\frac{bc}{d}\right)}+10\,(dx+c)^{\frac{3}{2}}bd^2e^{\left(\frac{bc}{d}\right)}+15\sqrt{dx+c}\,d^3e^{\left(\frac{bc}{d}\right)}\right)e^{\left(-a-\frac{(dx+c)b}{d}\right)}}{b^3}-\frac{2\left(4\,(dx+c)^{\frac{5}{2}}b^2de^a-10\,(dx+c)^{\frac{3}{2}}bd^2e^a+15\sqrt{dx+c}\,d^3e^a\right)e^{\left(\frac{(dx+c)b}{d}-\frac{bc}{d}\right)}}{b^3}\right)b}{d}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/40*(16*(d*x + c)^(5/2)*sinh(b*x + a) + (15*sqrt(pi)*d^3*erf(sqrt(d*x + c)
*sqrt(-b/d))*e^(a - b*c/d)/(b^3*sqrt(-b/d)) - 15*sqrt(pi)*d^3*erf(sqrt(d*x
+ c)*sqrt(b/d))*e^(-a + b*c/d)/(b^3*sqrt(b/d)) + 2*(4*(d*x + c)^(5/2)*b^2*d
*e^(b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(b*c/d) + 15*sqrt(d*x + c)*d^3*e^(b
*c/d))*e^(-a - (d*x + c)*b/d)/b^3 - 2*(4*(d*x + c)^(5/2)*b^2*d*e^a - 10*(d*
x + c)^(3/2)*b*d^2*e^a + 15*sqrt(d*x + c)*d^3*e^a)*e^((d*x + c)*b/d - b*c/d
)/b^3)*b/d)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(110) = 220.
time = 0.35, size = 385, normalized size = 2.64

$$-\frac{3\sqrt{\pi}\left(d^2\cosh(bx+a)\cosh\left(-\frac{bc-ad}{d}\right)-d^2\cosh(bx+a)\sinh\left(-\frac{bc-ad}{d}\right)+\left(d^2\cosh\left(-\frac{bc-ad}{d}\right)-d^2\sinh\left(-\frac{bc-ad}{d}\right)\right)\sinh(bx+a)\right)\sqrt{\frac{b}{d}}\,\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{\frac{b}{d}}\right)+3\sqrt{\pi}\left(d^2\cosh(bx+a)\cosh\left(-\frac{bc-ad}{d}\right)+d^2\cosh(bx+a)\sinh\left(-\frac{bc-ad}{d}\right)+\left(d^2\cosh\left(-\frac{bc-ad}{d}\right)+d^2\sinh\left(-\frac{bc-ad}{d}\right)\right)\sinh(bx+a)\right)\sqrt{-\frac{b}{d}}\,\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}\right)-2\left(2b^2dx+2b^2c+\left(2b^2dx+2b^2c-3bd\right)\cosh(bx+a)^2+2\left(2b^2dx+2b^2c-3bd\right)\cosh(bx+a)\sinh(bx+a)+\left(2b^2dx+2b^2c-3bd\right)\sinh(bx+a)^2+3bd\right)\sqrt{dx+c}}{8\left(b^3\cosh(bx+a)+b^3\sinh(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(3*sqrt(pi)*(d^2*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d^2*cosh(b*x + a
)*sinh(-(b*c - a*d)/d) + (d^2*cosh(-(b*c - a*d)/d) - d^2*sinh(-(b*c - a*d)/
d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 3*sqrt(pi)*(d^2
*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x + a)*sinh(-(b*c - a*d)/d
) + (d^2*cosh(-(b*c - a*d)/d) + d^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sq
rt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) - 2*(2*b^2*d*x + 2*b^2*c + (2*b^2*d*
x + 2*b^2*c - 3*b*d)*cosh(b*x + a)^2 + 2*(2*b^2*d*x + 2*b^2*c - 3*b*d)*cosh
```

```
(b*x + a)*sinh(b*x + a) + (2*b^2*d*x + 2*b^2*c - 3*b*d)*sinh(b*x + a)^2 + 3
*b*d)*sqrt(d*x + c))/(b^3*cosh(b*x + a) + b^3*sinh(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}} \sinh{(a+bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*sinh(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(3/2)*sinh(a + b*x), x)
```

Giac [A]
time = 0.49, size = 202, normalized size = 1.38

$$\frac{\frac{3\sqrt{\pi}\, d^3 \operatorname{erf}\left(-\frac{\sqrt{bd}\,\sqrt{dx+c}}{d}\right) e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}\, b^2} - \frac{3\sqrt{\pi}\, d^3 \operatorname{erf}\left(-\frac{\sqrt{-bd}\,\sqrt{dx+c}}{d}\right) e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}\, b^2} + \frac{2\left(2\,(dx+c)^{\frac{3}{2}} bd - 3\sqrt{dx+c}\, d^2\right) e^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b^2} + \frac{2\left(2\,(dx+c)^{\frac{3}{2}} bd + 3\sqrt{dx+c}\, d^2\right) e^{\left(-\frac{(dx+c)b-bc+ad}{d}\right)}}{b^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*(3*sqrt(pi)*d^3*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/(sqrt
(b*d)*b^2) - 3*sqrt(pi)*d^3*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c - a*d
)/d)/(sqrt(-b*d)*b^2) + 2*(2*(d*x + c)^(3/2)*b*d - 3*sqrt(d*x + c)*d^2)*e^(
((d*x + c)*b - b*c + a*d)/d)/b^2 + 2*(2*(d*x + c)^(3/2)*b*d + 3*sqrt(d*x +
c)*d^2)*e^(-((d*x + c)*b - b*c + a*d)/d)/b^2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh(a+b\,x)\,{\left(c+d\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*(c + d*x)^(3/2),x)
```

```
[Out] int(sinh(a + b*x)*(c + d*x)^(3/2), x)
```

3.40 $\int \sqrt{c+dx}\,\sinh(a+bx)\,dx$

Optimal. Leaf size=123

$$\frac{\sqrt{c+dx}\,\cosh(a+bx)}{b}-\frac{\sqrt{d}\,e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{d}\,e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}$$

```
[Out] -1/4*exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*Pi^(1/2)/b^(3
/2)-1/4*exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*Pi^(1/2)/b
^(3/2)+cosh(b*x+a)*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.13, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3377, 3388, 2211, 2235, 2236}

$$-\frac{\sqrt{\pi}\,\sqrt{d}\,e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\pi}\,\sqrt{d}\,e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{c+dx}\,\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Sinh[a + b*x],x]
```

```
[Out] (Sqrt[c + d*x]*Cosh[a + b*x])/b - (Sqrt[d]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(S
qrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*b^(3/2)) - (Sqrt[d]*E^(a - (b*c)/d)*Sqrt
[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*b^(3/2))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\int \sqrt{c+dx}\,\sinh(a+bx)\,dx = \frac{\sqrt{c+dx}\,\cosh(a+bx)}{b} - \frac{d\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx}{2b}$$

$$= \frac{\sqrt{c+dx}\,\cosh(a+bx)}{b} - \frac{d\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{4b} - \frac{d\int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{4b}$$

$$= \frac{\sqrt{c+dx}\,\cosh(a+bx)}{b} - \frac{\operatorname{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{2b} - \frac{\operatorname{Subst}\left(\cdots\right.}{\cdots}$$

$$= \frac{\sqrt{c+dx}\,\cosh(a+bx)}{b} - \frac{\sqrt{d}\,e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{\sqrt{d}\,e^{a-\frac{bc}{d}}\sqrt{\pi}\cdots}{\cdots}$$

Mathematica [A]
time = 0.06, size = 104, normalized size = 0.85

$$\frac{e^{-a-\frac{bc}{d}}\sqrt{c+dx}\left(\frac{e^{2a}\Gamma\left(\frac{3}{2},-\frac{b(c+dx)}{d}\right)}{\sqrt{-\frac{b(c+dx)}{d}}}+\frac{e^{\frac{2bc}{d}}\Gamma\left(\frac{3}{2},\frac{b(c+dx)}{d}\right)}{\sqrt{\frac{b(c+dx)}{d}}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Sinh[a + b*x],x]
```

```
[Out] (E^(-a - (b*c)/d)*Sqrt[c + d*x]*((E^(2*a)*Gamma[3/2, -((b*(c + d*x))/d)])/S
qrt[-((b*(c + d*x))/d)] + (E^((2*b*c)/d)*Gamma[3/2, (b*(c + d*x))/d])/Sqrt[
(b*(c + d*x))/d]))/(2*b)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \sinh(bx+a)\sqrt{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*(d*x+c)^(1/2),x)
```

```
[Out] int(sinh(b*x+a)*(d*x+c)^(1/2),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(91) = 182$.
time = 0.28, size = 230, normalized size = 1.87

$$\frac{8(dx+c)^{\frac{3}{2}}\sinh(bx+a) - \frac{\left(\frac{3\sqrt{\pi}d^2\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}} + \frac{3\sqrt{\pi}d^2\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}} - \frac{2\left(2(dx+c)^{\frac{3}{2}}bde^{\left(\frac{bc}{d}\right)}+3\sqrt{dx+c}\,d^2e^{\left(\frac{bc}{d}\right)}\right)e^{\left(-a-\frac{(dx+c)b}{d}\right)}}{b^2} + \frac{2\left(2(dx+c)^{\frac{3}{2}}bde^a-3\sqrt{dx+c}\,d^2e^a\right)e^{\left(\frac{(dx+c)b}{d}-\frac{bc}{d}\right)}}{b^2}\right)b}{d}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(8*(d*x + c)^(3/2)*sinh(b*x + a) - (3*sqrt(pi)*d^2*erf(sqrt(d*x + c)*s
qrt(-b/d))*e^(a - b*c/d)/(b^2*sqrt(-b/d)) + 3*sqrt(pi)*d^2*erf(sqrt(d*x + c
)*sqrt(b/d))*e^(-a + b*c/d)/(b^2*sqrt(b/d)) - 2*(2*(d*x + c)^(3/2)*b*d*e^(b
*c/d) + 3*sqrt(d*x + c)*d^2*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^2 + 2*(2*(d
*x + c)^(3/2)*b*d*e^a - 3*sqrt(d*x + c)*d^2*e^a)*e^((d*x + c)*b/d - b*c/d)/
b^2)*b/d)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(91) = 182$.
time = 0.36, size = 301, normalized size = 2.45

$$-\frac{\sqrt{\pi}\left(d\cosh(bx+a)\cosh\left(-\frac{bc-ad}{d}\right) - d\cosh(bx+a)\sinh\left(-\frac{bc-ad}{d}\right) + \left(d\cosh\left(-\frac{bc-ad}{d}\right) - d\sinh\left(-\frac{bc-ad}{d}\right)\right)\sinh(bx+a)\right)\sqrt{\frac{b}{d}}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right) - \sqrt{\pi}\left(d\cosh(bx+a)\cosh\left(-\frac{bc-ad}{d}\right) + d\cosh(bx+a)\sinh\left(-\frac{bc-ad}{d}\right) + \left(d\cosh\left(-\frac{bc-ad}{d}\right) + d\sinh\left(-\frac{bc-ad}{d}\right)\right)\sinh(bx+a)\right)\sqrt{-\frac{b}{d}}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right) - 2\left(b\cosh(bx+a)^2 + 2b\cosh(bx+a)\sinh(bx+a) + b\sinh(bx+a)^2 + b\right)\sqrt{dx+c}}{4\left(b^2\cosh(bx+a) + b^2\sinh(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*(d*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)*sinh
(-(b*c - a*d)/d) + (d*cosh(-(b*c - a*d)/d) - d*sinh(-(b*c - a*d)/d))*sinh(b
*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - sqrt(pi)*(d*cosh(b*x + a)
*cosh(-(b*c - a*d)/d) + d*cosh(b*x + a)*sinh(-(b*c - a*d)/d) + (d*cosh(-(b*
c - a*d)/d) + d*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*
x + c)*sqrt(-b/d)) - 2*(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a)
 + b*sinh(b*x + a)^2 + b)*sqrt(d*x + c))/(b^2*cosh(b*x + a) + b^2*sinh(b*x
+ a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sinh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*sinh(a + b*x), x)
```

Giac [A]
time = 0.45, size = 168, normalized size = 1.37

$$\frac{\frac{\sqrt{\pi}\,d^2\operatorname{erf}\left(-\frac{\sqrt{bd}\,\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}\,b}+\frac{\sqrt{\pi}\,d^2\operatorname{erf}\left(-\frac{\sqrt{-bd}\,\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}\,b}+\frac{2\sqrt{dx+c}\,de^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b}+\frac{2\sqrt{dx+c}\,de^{\left(-\frac{(dx+c)b-bc+ad}{d}\right)}}{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/(sqrt(b
*d)*b) + sqrt(pi)*d^2*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c - a*d)/d)/(
sqrt(-b*d)*b) + 2*sqrt(d*x + c)*d*e^(((d*x + c)*b - b*c + a*d)/d)/b + 2*sqr
t(d*x + c)*d*e^(-((d*x + c)*b - b*c + a*d)/d)/b)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh(a+b\,x)\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*(c + d*x)^(1/2),x)
```

```
[Out] int(sinh(a + b*x)*(c + d*x)^(1/2), x)
```

3.41 $\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=104

$$-\frac{e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\,\sqrt{d}}+\frac{e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\,\sqrt{d}}$$

```
[Out] -1/2*exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(1/2)/d^(1
/2)+1/2*exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(1/2)/d
^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3389, 2211, 2235, 2236}

$$\frac{\sqrt{\pi}\,e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\,\sqrt{d}}-\frac{\sqrt{\pi}\,e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] -1/2*(E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt
[b]*Sqrt[d]) + (E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[
d]])/(2*Sqrt[b]*Sqrt[d])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx &= \frac{1}{2}\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx - \frac{1}{2}\int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx\\ &= -\frac{\text{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d} + \frac{\text{Subst}\left(\int e^{-i\left(ia-\frac{ibc}{d}\right)+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d}\\ &= -\frac{e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}} + \frac{e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 104, normalized size = 1.00

$$\frac{e^{-a-\frac{bc}{d}}\left(e^{2a}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)+e^{\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{b(c+dx)}{d}\right)\right)}{2b\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] (E^(-a - (b*c)/d)*(E^(2*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*
x))/d)] + E^((2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (b*(c + d*x))/d]))
/(2*b*Sqrt[c + d*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(bx+a\right)}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)/(d*x+c)^(1/2),x)
```

```
[Out] int(sinh(b*x+a)/(d*x+c)^(1/2),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(74) = 148.
time = 0.28, size = 181, normalized size = 1.74

$$4\sqrt{dx+c}\sinh(bx+a)+\frac{\left(\frac{\sqrt{\pi}\,d\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}}-\frac{\sqrt{\pi}\,d\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b\sqrt{\frac{b}{d}}}-\frac{2\sqrt{dx+c}\,de^{\left(a+\frac{(dx+c)b}{d}-\frac{bc}{d}\right)}}{b}+\frac{2\sqrt{dx+c}\,de^{\left(-a-\frac{(dx+c)b}{d}+\frac{bc}{d}\right)}}{b}\right)b}{d} \over 2d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*sqrt(d*x + c)*sinh(b*x + a) + (sqrt(pi)*d*erf(sqrt(d*x + c)*sqrt(-b/
d))*e^(a - b*c/d)/(b*sqrt(-b/d)) - sqrt(pi)*d*erf(sqrt(d*x + c)*sqrt(b/d))*
e^(-a + b*c/d)/(b*sqrt(b/d)) - 2*sqrt(d*x + c)*d*e^(a + (d*x + c)*b/d - b*c
/d)/b + 2*sqrt(d*x + c)*d*e^(-a - (d*x + c)*b/d + b*c/d)/b)*b/d)/d
```

Fricas [A]
time = 0.37, size = 122, normalized size = 1.17

$$-\frac{\sqrt{\pi}\sqrt{\frac{b}{d}}\left(\cosh\left(-\frac{bc-ad}{d}\right)-\sinh\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)+\sqrt{\pi}\sqrt{-\frac{b}{d}}\left(\cosh\left(-\frac{bc-ad}{d}\right)+\sinh\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(pi)*sqrt(b/d)*(cosh(-(b*c - a*d)/d) - sinh(-(b*c - a*d)/d))*erf(
sqrt(d*x + c)*sqrt(b/d)) + sqrt(pi)*sqrt(-b/d)*(cosh(-(b*c - a*d)/d) + sinh
(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-b/d)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(sinh(a + b*x)/sqrt(c + d*x), x)
```

Giac [A]
time = 0.44, size = 90, normalized size = 0.87

$$\frac{\left(\frac{\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc}{d}\right)}}{\sqrt{bd}}-\frac{\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-2ad}{d}\right)}}{\sqrt{-bd}}\right)e^{(-a)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^(b*c/d)/sqrt(b*d) - sqrt(
pi)*d*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c - 2*a*d)/d)/sqrt(-b*d))*e^(
-a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(a+b\,x)}{\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/(c + d*x)^(1/2),x)
```

```
[Out] int(sinh(a + b*x)/(c + d*x)^(1/2), x)
```

3.42 $\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{\sqrt{b}\,e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{b}\,e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sinh(a+bx)}{d\sqrt{c+dx}}$$

```
[Out] exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^(1/2)*Pi^(1/2)/d^(3/2)+e
xp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^(1/2)*Pi^(1/2)/d^(3/2)-2*
sinh(b*x+a)/d/(d*x+c)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.312$, Rules used = {3378, 3388, 2211, 2235, 2236}

$$\frac{\sqrt{\pi}\,\sqrt{b}\,e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{\pi}\,\sqrt{b}\,e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sinh(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (Sqrt[b]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/d^
(3/2) + (Sqrt[b]*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]])/d^(3/2) - (2*Sinh[a + b*x])/(d*Sqrt[c + d*x])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx &= -\frac{2\sinh(a+bx)}{d\sqrt{c+dx}}+\frac{(2b)\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx}{d}\\ &= -\frac{2\sinh(a+bx)}{d\sqrt{c+dx}}+\frac{b\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{d}+\frac{b\int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{d}\\ &= -\frac{2\sinh(a+bx)}{d\sqrt{c+dx}}+\frac{(2b)\text{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{d^2}+\frac{(2b)\text{Subst}\left(\int e^{-i\left(ia-\right.}\right.}{}\\ &= \frac{\sqrt{b}\,e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{b}\,e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sinh(a+}{d\sqrt{c+dx}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 120, normalized size = 1.02

$$\frac{e^{-a-\frac{bc}{d}}\left(e^{2a}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)-e^{\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{b(c+dx)}{d}\right)-2e^{a+\frac{bc}{d}}\sinh(a+bx)\right)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]/(c + d*x)^(3/2),x]
[Out] (E^(-a - (b*c)/d)*(E^(2*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*
x))/d)] - E^((2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (b*(c + d*x))/d] -
 2*E^(a + (b*c)/d)*Sinh[a + b*x]))/(d*Sqrt[c + d*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh (bx+a)}{(dx+c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)/(d*x+c)^(3/2),x)
```

```
[Out] int(sinh(b*x+a)/(d*x+c)^(3/2),x)
```

Maxima [A]
time = 0.26, size = 103, normalized size = 0.87

$$\frac{\frac{\left(\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{dx+c}\,\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{\sqrt{\frac{b}{d}}}\right)b}{d}-\frac{2\,\sinh(bx+a)}{\sqrt{dx+c}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] ((sqrt(pi)*erf(sqrt(d*x + c)*sqrt(-b/d))*e^(a - b*c/d)/sqrt(-b/d) + sqrt(pi
)*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/sqrt(b/d))*b/d - 2*sinh(b*x +
 a)/sqrt(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(90) = 180.
time = 0.38, size = 339, normalized size = 2.87

$$\frac{\sqrt{\pi}\left((dx+c)\cosh(bx+a)\cosh\left(-\frac{bc-ad}{d}\right)-(dx+c)\cosh(bx+a)\sinh\left(-\frac{bc-ad}{d}\right)+\left((dx+c)\cosh\left(-\frac{bc-ad}{d}\right)-(dx+c)\sinh\left(-\frac{bc-ad}{d}\right)\right)\sinh(bx+a)\right)\sqrt{\frac{b}{d}}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)-\sqrt{\pi}\left((dx+c)\cosh(bx+a)\cosh\left(-\frac{bc-ad}{d}\right)+(dx+c)\cosh(bx+a)\sinh\left(-\frac{bc-ad}{d}\right)+\left((dx+c)\cosh\left(-\frac{bc-ad}{d}\right)+(dx+c)\sinh\left(-\frac{bc-ad}{d}\right)\right)\sinh(bx+a)\right)\sqrt{-\frac{b}{d}}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)-\sqrt{dx+c}\left(\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1\right)}{(d^2x+cd)\cosh(bx+a)+(d^2x+cd)\sinh(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(pi)*((d*x + c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (d*x + c)*cosh(b*
x + a)*sinh(-(b*c - a*d)/d) + ((d*x + c)*cosh(-(b*c - a*d)/d) - (d*x + c)*s
inh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d))
- sqrt(pi)*((d*x + c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + (d*x + c)*cosh(b
*x + a)*sinh(-(b*c - a*d)/d) + ((d*x + c)*cosh(-(b*c - a*d)/d) + (d*x + c)*
sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d
)) - sqrt(d*x + c)*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(
```

```
b*x + a)^2 - 1))/((d^2*x + c*d)*cosh(b*x + a) + (d^2*x + c*d)*sinh(b*x + a)
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh{(a+bx)}}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(sinh(a + b*x)/(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)/(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(a+b\,x)}{{(c+d\,x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/(c + d*x)^(3/2),x)
```

```
[Out] int(sinh(a + b*x)/(c + d*x)^(3/2), x)
```

3.43 $\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{4b\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{2\sinh(a+}{3d(c+dx)}$$

```
[Out] -2/3*sinh(b*x+a)/d/(d*x+c)^(3/2)-2/3*b^(3/2)*exp(-a+b*c/d)*erf(b^(1/2)*(d*x
+c)^(1/2)/d^(1/2))*Pi^(1/2)/d^(5/2)+2/3*b^(3/2)*exp(a-b*c/d)*erfi(b^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/d^(5/2)-4/3*b*cosh(b*x+a)/d^2/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3378, 3389, 2211, 2235, 2236}

$$-\frac{2\sqrt{\pi}\,b^{3/2}e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2\sqrt{\pi}\,b^{3/2}e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4b\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (-4*b*Cosh[a + b*x])/(3*d^2*Sqrt[c + d*x]) - (2*b^(3/2)*E^(-a + (b*c)/d)*Sq
rt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) + (2*b^(3/2)*E^(a
- (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) - (2
*Sinh[a + b*x])/(3*d*(c + d*x)^(3/2))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx &= -\frac{2\sinh(a+bx)}{3d(c+dx)^{3/2}}+\frac{(2b)\int\frac{\cosh(a+bx)}{(c+dx)^{3/2}}\,dx}{3d}\\
&= -\frac{4b\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh(a+bx)}{3d(c+dx)^{3/2}}+\frac{(4b^2)\int\frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx}{3d^2}\\
&= -\frac{4b\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh(a+bx)}{3d(c+dx)^{3/2}}+\frac{(2b^2)\int\frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{3d^2}-\frac{(2b^2)\int\frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{3d^2}\\
&= -\frac{4b\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh(a+bx)}{3d(c+dx)^{3/2}}-\frac{(4b^2)\operatorname{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{3d^3}+\\
&= -\frac{4b\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{}}\right.}{3d^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.53, size = 161, normalized size = 1.08

$$\frac{2b\left(\frac{e^a\left(-e^{bx}+e^{-\frac{bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)\right)}{d\sqrt{c+dx}}+\frac{e^{-a-bx}\left(-1+e^{b\left(\frac{c}{d}+x\right)}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{b(c+dx)}{d}\right)\right)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\sinh(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (2*b*((E^a*(-E^(b*x) + (Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*x))
/d)])/E^((b*c)/d)))/(d*Sqrt[c + d*x]) + (E^(-a - b*x)*(-1 + E^(b*(c/d + x))
*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (b*(c + d*x))/d]))/(d*Sqrt[c + d*x])))/(3
*d) - (2*Sinh[a + b*x])/(3*d*(c + d*x)^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(bx+a\right)}{\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)/(d*x+c)^(5/2),x)
```

```
[Out] int(sinh(b*x+a)/(d*x+c)^(5/2),x)
```

Maxima [A]
time = 0.34, size = 114, normalized size = 0.77

$$-\frac{\frac{\left(\frac{\sqrt{\frac{(dx+c)b}{d}}\,e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}}+\frac{\sqrt{-\frac{(dx+c)b}{d}}\,e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}}\right)b}{d}+\frac{2\sinh(bx+a)}{(dx+c)^{\frac{3}{2}}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*((sqrt((d*x + c)*b/d)*e^(-a + b*c/d)*gamma(-1/2, (d*x + c)*b/d)/sqrt(d
*x + c) + sqrt(-(d*x + c)*b/d)*e^(a - b*c/d)*gamma(-1/2, -(d*x + c)*b/d)/sq
rt(d*x + c))*b/d + 2*sinh(b*x + a)/(d*x + c)^(3/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 532 vs. 2(111) = 222.
time = 0.39, size = 532, normalized size = 3.57

$$-\frac{2\sqrt{\pi}\left(\left(bd^2x^2+2bcdx+bc^2\right)\cosh\left(bx+a\right)\cosh\left(-\frac{bc-ad}{d}\right)-\left(bd^2x^2+2bcdx+bc^2\right)\cosh\left(bx+a\right)\sinh\left(-\frac{bc-ad}{d}\right)+\left(\left(bd^2x^2+2bcdx+bc^2\right)\cosh\left(-\frac{bc-ad}{d}\right)-\left(bd^2x^2+2bcdx+bc^2\right)\sinh\left(-\frac{bc-ad}{d}\right)\right)\sinh\left(bx+a\right)\right)\sqrt{\frac{b}{d}}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)+2\sqrt{\pi}\left(\left(bd^2x^2+2bcdx+bc^2\right)\cosh\left(bx+a\right)\cosh\left(-\frac{bc-ad}{d}\right)+\left(bd^2x^2+2bcdx+bc^2\right)\cosh\left(bx+a\right)\sinh\left(-\frac{bc-ad}{d}\right)+\left(\left(bd^2x^2+2bcdx+bc^2\right)\cosh\left(-\frac{bc-ad}{d}\right)+\left(bd^2x^2+2bcdx+bc^2\right)\sinh\left(-\frac{bc-ad}{d}\right)\right)\sinh\left(bx+a\right)\right)\sqrt{-\frac{b}{d}}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)+\left(2bdx+\left(2bdx+2bc+d\right)\cosh\left(bx+a\right)^2+2\left(2bdx+2bc+d\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\left(2bdx+2bc+d\right)\sinh\left(bx+a\right)^2+2bc-d\right)\sqrt{dx+c}}{3\left(\left(d^4x^2+2cd^3x+c^2d^2\right)\cosh\left(bx+a\right)+\left(d^4x^2+2cd^3x+c^2d^2\right)\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-(b*c
- a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-(b*c - a*d)
/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-(b*c - a*d)/d) - (b*d^2*x^2 +
2*b*c*d*x + b*c^2)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(
```

```
d*x + c)*sqrt(b/d)) + 2*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x
+ a)*cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*s
inh(-(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-(b*c - a*d)/d)
 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqr
t(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) + (2*b*d*x + (2*b*d*x + 2*b*c + d)*co
sh(b*x + a)^2 + 2*(2*b*d*x + 2*b*c + d)*cosh(b*x + a)*sinh(b*x + a) + (2*b*
d*x + 2*b*c + d)*sinh(b*x + a)^2 + 2*b*c - d)*sqrt(d*x + c))/((d^4*x^2 + 2*
c*d^3*x + c^2*d^2)*cosh(b*x + a) + (d^4*x^2 + 2*c*d^3*x + c^2*d^2)*sinh(b*x
 + a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh{\left(a+bx \right)}}{\left(c+dx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(sinh(a + b*x)/(c + d*x)**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)/(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{sinh}\left(a+b\,x\right)}{{\left(c+d\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/(c + d*x)^(5/2),x)
```

```
[Out] int(sinh(a + b*x)/(c + d*x)^(5/2), x)
```

3.44 $\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{4b\cosh(a+bx)}{15d^2(c+dx)^{3/2}}+\frac{4b^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{4b^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}-\frac{2\sinh(a+}{5d(c+dx)}$$

```
[Out] -4/15*b*cosh(b*x+a)/d^2/(d*x+c)^(3/2)-2/5*sinh(b*x+a)/d/(d*x+c)^(5/2)+4/15*
b^(5/2)*exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/d^(7/2)+4
/15*b^(5/2)*exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/d^(7/
2)-8/15*b^2*sinh(b*x+a)/d^3/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3378, 3388, 2211, 2235, 2236}

$$\frac{4\sqrt{\pi}\,b^{5/2}e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{4\sqrt{\pi}\,b^{5/2}e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}-\frac{8b^2\sinh(a+bx)}{15d^3\sqrt{c+dx}}-\frac{4b\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh(a+bx)}{5d(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]/(c + d*x)^(7/2),x]
```

```
[Out] (-4*b*Cosh[a + b*x])/(15*d^2*(c + d*x)^(3/2)) + (4*b^(5/2)*E^(-a + (b*c)/d)
*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(15*d^(7/2)) + (4*b^(5/2)*E
^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(15*d^(7/2))
 - (2*Sinh[a + b*x])/(5*d*(c + d*x)^(5/2)) - (8*b^2*Sinh[a + b*x])/(15*d^3*
Sqrt[c + d*x])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx &= -\frac{2\sinh(a+bx)}{5d(c+dx)^{5/2}} + \frac{(2b)\int \frac{\cosh(a+bx)}{(c+dx)^{5/2}}\,dx}{5d} \\
&= -\frac{4b\cosh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh(a+bx)}{5d(c+dx)^{5/2}} + \frac{(4b^2)\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx}{15d^2} \\
&= -\frac{4b\cosh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2\sinh(a+bx)}{15d^3\sqrt{c+dx}} + \frac{(8b^3)\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx}{15d^3} \\
&= -\frac{4b\cosh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2\sinh(a+bx)}{15d^3\sqrt{c+dx}} + \frac{(4b^3)\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{15d^3} + \frac{(4b^3)}{} \\
&= -\frac{4b\cosh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2\sinh(a+bx)}{15d^3\sqrt{c+dx}} + \frac{(8b^3)\,\mathrm{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\right.}{15d^4} \\
&= -\frac{4b\cosh(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{4b^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{4b^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{}}\right.}{15d^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.38, size = 168, normalized size = 0.97

$$\frac{2\left(-b(c+dx)\left(e^{a-\frac{bc}{d}}\left(e^{b\left(\frac{c}{d}+x\right)}(d+2b(c+dx))+2d\left(-\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)\right)+e^{-a-bx}\left(d-2b(c+dx)+2de^{b\left(\frac{c}{d}+x\right)}\left(\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},\frac{b(c+dx)}{d}\right)\right)\right)-3d^2\sinh(a+bx)\right)}{15d^3(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]/(c + d*x)^(7/2),x]
```

```
[Out] (2*(-(b*(c + d*x)*(E^(a - (b*c)/d)*(E^(b*(c/d + x))*(d + 2*b*(c + d*x)) + 2
*d*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, -((b*(c + d*x))/d)]) + E^(-a - b*x
)*(d - 2*b*(c + d*x) + 2*d*E^(b*(c/d + x))*((b*(c + d*x))/d)^(3/2)*Gamma[1/
2, (b*(c + d*x))/d]))) - 3*d^2*Sinh[a + b*x]))/(15*d^3*(c + d*x)^(5/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(bx+a\right)}{\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)/(d*x+c)^(7/2),x)
```

```
[Out] int(sinh(b*x+a)/(d*x+c)^(7/2),x)
```

Maxima [A]
time = 0.35, size = 114, normalized size = 0.66

$$-\frac{\frac{\left(\frac{\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}+\frac{\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}\right)b}{d}+\frac{2\,\sinh(bx+a)}{(dx+c)^{\frac{5}{2}}}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/5*((((d*x + c)*b/d)^(3/2)*e^(-a + b*c/d)*gamma(-3/2, (d*x + c)*b/d)/(d*x
 + c)^(3/2) + (-(d*x + c)*b/d)^(3/2)*e^(a - b*c/d)*gamma(-3/2, -(d*x + c)*b
/d)/(d*x + c)^(3/2))*b/d + 2*sinh(b*x + a)/(d*x + c)^(5/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 855 vs. 2(132) = 264.
time = 0.37, size = 855, normalized size = 4.91

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(4*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)
*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3
*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-(b*c - a*d)/d) - (b^2*d^3*x
```

```
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*sinh(
b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 4*sqrt(pi)*((b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-(b*c - a*
d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x
+ a)*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
 + b^2*c^3)*cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(
d*x + c)*sqrt(-b/d)) + (4*b^2*d^2*x^2 + 4*b^2*c^2 - 2*b*c*d - (4*b^2*d^2*x^
2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^2
- 2*(4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)
*cosh(b*x + a)*sinh(b*x + a) - (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2
 + 2*(4*b^2*c*d + b*d^2)*x)*sinh(b*x + a)^2 + 3*d^2 + 2*(4*b^2*c*d - b*d^2)
*x)*sqrt(d*x + c))/((d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*cosh(b*
x + a) + (d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*sinh(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a+bx)}{(c+dx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)**(7/2),x)
[Out] Integral(sinh(a + b*x)/(c + d*x)**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(7/2),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)/(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(a+b\,x)}{(c+d\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/(c + d*x)^(7/2),x)
[Out] int(sinh(a + b*x)/(c + d*x)^(7/2), x)
```

3.45 $\int (c+dx)^{5/2}\sinh^2(a+bx)\,dx$

Optimal. Leaf size=239

$$-\frac{5d(c+dx)^{3/2}}{16b^2}-\frac{(c+dx)^{7/2}}{7d}+\frac{15d^{5/2}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}}-\frac{15d^{5/2}e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{}\right.}{256b^{7/2}}$$

```
[Out] -5/16*d*(d*x+c)^(3/2)/b^2-1/7*(d*x+c)^(7/2)/d+1/2*(d*x+c)^(5/2)*cosh(b*x+a)
*sinh(b*x+a)/b-5/8*d*(d*x+c)^(3/2)*sinh(b*x+a)^2/b^2+15/512*d^(5/2)*exp(-2*
a+2*b*c/d)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7
/2)-15/512*d^(5/2)*exp(2*a-2*b*c/d)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/64*d^2*sinh(2*b*x+2*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A]
time = 0.31, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3392, 32, 3393, 3377, 3389, 2211, 2235, 2236}

$$\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}e^{\frac{2bc}{d}-2a}\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}e^{2a-\frac{2bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\sinh(2a+2bx)}{64b^3}-\frac{5d(c+dx)^{3/2}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{5d(c+dx)^{3/2}}{16b^2}-\frac{(c+dx)^{7/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Sinh[a + b*x]^2,x]
```

```
[Out] (-5*d*(c + d*x)^(3/2))/(16*b^2) - (c + d*x)^(7/2)/(7*d) + (15*d^(5/2)*E^(-2
*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2
56*b^(7/2)) - (15*d^(5/2)*E^(2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt
[b]*Sqrt[c + d*x])/Sqrt[d]])/(256*b^(7/2)) + ((c + d*x)^(5/2)*Cosh[a + b*x]
*Sinh[a + b*x])/(2*b) - (5*d*(c + d*x)^(3/2)*Sinh[a + b*x]^2)/(8*b^2) + (15
*d^2*Sqrt[c + d*x]*Sinh[2*a + 2*b*x])/(64*b^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int (c+dx)^{5/2}\sinh^2(a+bx)\,dx &= \frac{(c+dx)^{5/2}\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{5d(c+dx)^{3/2}\sinh^2(a+bx)}{8b^2} - \frac{1}{2} \\
&= -\frac{(c+dx)^{7/2}}{7d} + \frac{(c+dx)^{5/2}\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{5d(c+dx)^{3/2}\sin}{8b^2} \\
&= -\frac{5d(c+dx)^{3/2}}{16b^2} - \frac{(c+dx)^{7/2}}{7d} + \frac{(c+dx)^{5/2}\cosh(a+bx)\sinh(a+bx)}{2b} - \\
&= -\frac{5d(c+dx)^{3/2}}{16b^2} - \frac{(c+dx)^{7/2}}{7d} + \frac{(c+dx)^{5/2}\cosh(a+bx)\sinh(a+bx)}{2b} - \\
&= -\frac{5d(c+dx)^{3/2}}{16b^2} - \frac{(c+dx)^{7/2}}{7d} + \frac{(c+dx)^{5/2}\cosh(a+bx)\sinh(a+bx)}{2b} - \\
&= -\frac{5d(c+dx)^{3/2}}{16b^2} - \frac{(c+dx)^{7/2}}{7d} + \frac{(c+dx)^{5/2}\cosh(a+bx)\sinh(a+bx)}{2b} - \\
&= -\frac{5d(c+dx)^{3/2}}{16b^2} - \frac{(c+dx)^{7/2}}{7d} + \frac{15d^{5/2}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right.}{256b^{7/2}}\end{aligned}$$

Mathematica [A]
time = 4.51, size = 190, normalized size = 0.79

$$\frac{\sqrt{c+dx}\left(-b(c+dx)\left(64b^3(c+dx)^3\sqrt{\frac{b(c+dx)}{d}}+7\sqrt{2}\,d^3\Gamma\left(\frac{7}{2},\frac{2b(c+dx)}{d}\right)\left(\cosh\left(2a-\frac{2bc}{d}\right)-\sinh\left(2a-\frac{2bc}{d}\right)\right)\right)-7\sqrt{2}\,d^4\sqrt{-\frac{b^2(c+dx)^2}{d^2}}\,\Gamma\left(\frac{7}{2},-\frac{2b(c+dx)}{d}\right)\left(\cosh\left(2a-\frac{2bc}{d}\right)+\sinh\left(2a-\frac{2bc}{d}\right)\right)\right)}{448b^3d^2\left(\frac{b(c+dx)}{d}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Sinh[a + b*x]^2,x]
```

```
[Out] (Sqrt[c + d*x]*(-(b*(c + d*x)*(64*b^3*(c + d*x)^3*Sqrt[(b*(c + d*x))/d] + 7
*Sqrt[2]*d^3*Gamma[7/2, (2*b*(c + d*x))/d]*(Cosh[2*a - (2*b*c)/d] - Sinh[2*
a - (2*b*c)/d]))) - 7*Sqrt[2]*d^4*Sqrt[-((b^2*(c + d*x)^2)/d^2)]*Gamma[7/2,
 (-2*b*(c + d*x))/d]*(Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d])))/(448
*b^3*d^2*((b*(c + d*x))/d)^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^{\frac{5}{2}}\left(\sinh^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*sinh(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^(5/2)*sinh(b*x+a)^2,x)
```

Maxima [A]
time = 0.48, size = 281, normalized size = 1.18

$$-\frac{512\,(dx+c)^{\frac{7}{2}}+\frac{105\,\sqrt{2}\,\sqrt{\pi}\,d^3\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}}-\frac{105\,\sqrt{2}\,\sqrt{\pi}\,d^3\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}}+\frac{28\left(16\,(dx+c)^{\frac{5}{2}}b^2de^{\left(\frac{2\,bc}{d}\right)}+20\,(dx+c)^{\frac{3}{2}}bd^2e^{\left(\frac{2\,bc}{d}\right)}+15\,\sqrt{dx+c}\,d^3e^{\left(\frac{2\,bc}{d}\right)}\right)e^{\left(-2\,a-\frac{2\,(dx+c)b}{d}\right)}}{b^3}-\frac{28\left(16\,(dx+c)^{\frac{5}{2}}b^2de^{(2\,a)}-20\,(dx+c)^{\frac{3}{2}}bd^2e^{(2\,a)}+15\,\sqrt{dx+c}\,d^3e^{(2\,a)}\right)e^{\left(\frac{2\,(dx+c)b}{d}-\frac{2\,bc}{d}\right)}}{b^3}}{3584\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3584*(512*(d*x + c)^(7/2) + 105*sqrt(2)*sqrt(pi)*d^3*erf(sqrt(2)*sqrt(d*
x + c)*sqrt(-b/d))*e^(2*a - 2*b*c/d)/(b^3*sqrt(-b/d)) - 105*sqrt(2)*sqrt(pi
)*d^3*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(-2*a + 2*b*c/d)/(b^3*sqrt(b/d
)) + 28*(16*(d*x + c)^(5/2)*b^2*d*e^(2*b*c/d) + 20*(d*x + c)^(3/2)*b*d^2*e^
(2*b*c/d) + 15*sqrt(d*x + c)*d^3*e^(2*b*c/d))*e^(-2*a - 2*(d*x + c)*b/d)/b^
3 - 28*(16*(d*x + c)^(5/2)*b^2*d*e^(2*a) - 20*(d*x + c)^(3/2)*b*d^2*e^(2*a)
 + 15*sqrt(d*x + c)*d^3*e^(2*a))*e^(2*(d*x + c)*b/d - 2*b*c/d)/b^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(183) = 366$.
time = 0.36, size = 1001, normalized size = 4.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3584*(105*sqrt(2)*sqrt(pi)*(d^4*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) -
d^4*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^4*cosh(-2*(b*c - a*d)/d) -
d^4*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^4*cosh(b*x + a)*cosh(-2*
(b*c - a*d)/d) - d^4*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*s
qrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) + 105*sqrt(2)*sqrt(pi)*(d^4*c
osh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + d^4*cosh(b*x + a)^2*sinh(-2*(b*c -
a*d)/d) + (d^4*cosh(-2*(b*c - a*d)/d) + d^4*sinh(-2*(b*c - a*d)/d))*sinh(b*
x + a)^2 + 2*(d^4*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + d^4*cosh(b*x + a)*
sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)
*sqrt(-b/d)) - 4*(112*b^3*d^3*x^2 + 112*b^3*c^2*d + 140*b^2*c*d^2 - 7*(16*b
^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^
2*d^3)*x)*cosh(b*x + a)^4 - 28*(16*b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^
2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^2*d^3)*x)*cosh(b*x + a)*sinh(b*x + a)^3
 - 7*(16*b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*
d^2 - 5*b^2*d^3)*x)*sinh(b*x + a)^4 + 105*b*d^3 + 128*(b^4*d^3*x^3 + 3*b^4*
c*d^2*x^2 + 3*b^4*c^2*d*x + b^4*c^3)*cosh(b*x + a)^2 + 2*(64*b^4*d^3*x^3 +
192*b^4*c*d^2*x^2 + 192*b^4*c^2*d*x + 64*b^4*c^3 - 21*(16*b^3*d^3*x^2 + 16*
b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^2*d^3)*x)*cosh(b
```

```
*x + a)^2)*sinh(b*x + a)^2 + 28*(8*b^3*c*d^2 + 5*b^2*d^3)*x - 4*(7*(16*b^3*
d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^2*d
^3)*x)*cosh(b*x + a)^3 - 64*(b^4*d^3*x^3 + 3*b^4*c*d^2*x^2 + 3*b^4*c^2*d*x
+ b^4*c^3)*cosh(b*x + a))*sinh(b*x + a))*sqrt(d*x + c))/(b^4*d*cosh(b*x + a
)^2 + 2*b^4*d*cosh(b*x + a)*sinh(b*x + a) + b^4*d*sinh(b*x + a)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{5}{2}}\sinh^2{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*sinh(b*x+a)**2,x)
[Out] Integral((c + d*x)**(5/2)*sinh(a + b*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sinh(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^(5/2)*sinh(b*x + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{sinh}(a+b\,x)^2\,{(c+d\,x)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2*(c + d*x)^(5/2),x)
[Out] int(sinh(a + b*x)^2*(c + d*x)^(5/2), x)
```

3.46 $\int (c+dx)^{3/2} \sinh^2(a+bx)\, dx$

Optimal. Leaf size=211

$$-\frac{3d\sqrt{c+dx}}{16b^2}-\frac{(c+dx)^{5/2}}{5d}+\frac{3d^{3/2}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}}+\frac{3d^{3/2}e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c}}{\sqrt{d}}\right.}{64b^{5/2}}$$

```
[Out] -1/5*(d*x+c)^(5/2)/d+1/2*(d*x+c)^(3/2)*cosh(b*x+a)*sinh(b*x+a)/b+3/128*d^(3
/2)*exp(-2*a+2*b*c/d)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi
^(1/2)/b^(5/2)+3/128*d^(3/2)*exp(2*a-2*b*c/d)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)-3/16*d*(d*x+c)^(1/2)/b^2-3/8*d*sinh
(b*x+a)^2*(d*x+c)^(1/2)/b^2
```

Rubi [A]

time = 0.22, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3392, 32, 3393, 3388, 2211, 2235, 2236}

$$\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}e^{\frac{2bc}{d}-2a}\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}e^{2a-\frac{2bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}}-\frac{3d\sqrt{c+dx}\,\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{3d\sqrt{c+dx}}{16b^2}-\frac{(c+dx)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Sinh[a + b*x]^2,x]
```

```
[Out] (-3*d*Sqrt[c + d*x])/(16*b^2) - (c + d*x)^(5/2)/(5*d) + (3*d^(3/2)*E^(-2*a
+ (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(64*b
^(5/2)) + (3*d^(3/2)*E^(2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*S
qrt[c + d*x])/Sqrt[d]])/(64*b^(5/2)) + ((c + d*x)^(3/2)*Cosh[a + b*x]*Sinh[
a + b*x])/(2*b) - (3*d*Sqrt[c + d*x]*Sinh[a + b*x]^2)/(8*b^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int (c+dx)^{3/2}\sinh^2(a+bx)\,dx = \frac{(c+dx)^{3/2}\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{3d\sqrt{c+dx}\,\sinh^2(a+bx)}{8b^2} - \frac{1}{2}\int (c$$

$$= -\frac{(c+dx)^{5/2}}{5d} + \frac{(c+dx)^{3/2}\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{3d\sqrt{c+dx}\,\sinh^2(}{8b^2}$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} - \frac{(c+dx)^{5/2}}{5d} + \frac{(c+dx)^{3/2}\cosh(a+bx)\sinh(a+bx)}{2b} - 3d$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} - \frac{(c+dx)^{5/2}}{5d} + \frac{(c+dx)^{3/2}\cosh(a+bx)\sinh(a+bx)}{2b} - 3d$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} - \frac{(c+dx)^{5/2}}{5d} + \frac{(c+dx)^{3/2}\cosh(a+bx)\sinh(a+bx)}{2b} - 3d$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} - \frac{(c+dx)^{5/2}}{5d} + \frac{3d^{3/2}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}}$$

Mathematica [A]
time = 1.56, size = 163, normalized size = 0.77

$$\frac{-32b^3(c+dx)^3 + 5\sqrt{2}\,d^3\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{5}{2},\frac{2b(c+dx)}{d}\right)\left(-\cosh\left(2a-\frac{2bc}{d}\right)+\sinh\left(2a-\frac{2bc}{d}\right)\right) + 5\sqrt{2}\,d^3\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{5}{2},-\frac{2b(c+dx)}{d}\right)\left(\cosh\left(2a-\frac{2bc}{d}\right)+\sinh\left(2a-\frac{2bc}{d}\right)\right)}{160b^3d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Sinh[a + b*x]^2,x]
```

```
[Out] (-32*b^3*(c + d*x)^3 + 5*Sqrt[2]*d^3*Sqrt[(b*(c + d*x))/d]*Gamma[5/2, (2*b*
(c + d*x))/d]*(-Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d]) + 5*Sqrt[2]*
d^3*Sqrt[-((b*(c + d*x))/d)]*Gamma[5/2, (-2*b*(c + d*x))/d]*(Cosh[2*a - (2*
b*c)/d] + Sinh[2*a - (2*b*c)/d]))/(160*b^3*d*Sqrt[c + d*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^{\frac{3}{2}}\left(\sinh^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*sinh(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^(3/2)*sinh(b*x+a)^2,x)
```

Maxima [A]
time = 0.49, size = 239, normalized size = 1.13

$$-\frac{128\,(dx+c)^{\frac{5}{2}} - \frac{15\,\sqrt{2}\,\sqrt{\pi}\,d^2\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}} - \frac{15\,\sqrt{2}\,\sqrt{\pi}\,d^2\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}} + \frac{20\left(4\,(dx+c)^{\frac{3}{2}}\,bde^{\left(\frac{2\,bc}{d}\right)}+3\,\sqrt{dx+c}\,d^2e^{\left(\frac{2\,bc}{d}\right)}\right)e^{\left(-2\,a-\frac{2\,(dx+c)b}{d}\right)}}{b^2} - \frac{20\left(4\,(dx+c)^{\frac{3}{2}}\,bde^{(2\,a)}-3\,\sqrt{dx+c}\,d^2e^{(2\,a)}\right)e^{\left(\frac{2\,(dx+c)b}{d}-\frac{2\,bc}{d}\right)}}{b^2}}{640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*sinh(b*x+a)^2,x, algorithm="maxima")

[Out] -1/640*(128*(d*x + c)^(5/2) - 15*sqrt(2)*sqrt(pi)*d^2*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))*e^(2*a - 2*b*c/d)/(b^2*sqrt(-b/d)) - 15*sqrt(2)*sqrt(pi)*d^2*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(-2*a + 2*b*c/d)/(b^2*sqrt(b/d)) + 20*(4*(d*x + c)^(3/2)*b*d*e^(2*b*c/d) + 3*sqrt(d*x + c)*d^2*e^(2*b*c/d))*e^(-2*a - 2*(d*x + c)*b/d)/b^2 - 20*(4*(d*x + c)^(3/2)*b*d*e^(2*a) - 3*sqrt(d*x + c)*d^2*e^(2*a))*e^(2*(d*x + c)*b/d - 2*b*c/d)/b^2)/d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(159) = 318$.
time = 0.62, size = 755, normalized size = 3.58

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)*sinh(b*x+a)^2,x, algorithm="fricas")

[Out] 1/640*(15*sqrt(2)*sqrt(pi)*(d^3*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - d^3*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^3*cosh(-2*(b*c - a*d)/d) - d^3*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^3*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) - d^3*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 15*sqrt(2)*sqrt(pi)*(d^3*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + d^3*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^3*cosh(-2*(b*c - a*d)/d) + d^3*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^3*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + d^3*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d)) - 4*(20*b^2*d^2*x - 5*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*cosh(b*x + a)^4 - 20*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*cosh(b*x + a)*sinh(b*x + a)^3 - 5*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*sinh(b*x + a)^4 + 20*b^2*c*d + 15*b*d^2 + 32*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*cosh(b*x + a)^2 + 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c^2 - 15*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 4*(5*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*cosh(b*x + a)^3 - 16*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*cosh(b*x + a))*sinh(b*x + a))*sqrt(d*x + c))/(b^3*d*cosh(b*x + a)^2 + 2*b^3*d*cosh(b*x + a)*sinh(b*x + a) + b^3*d*sinh(b*x + a)^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}} \sinh^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*sinh(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**(3/2)*sinh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(3/2)*sinh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sinh(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2*(c + d*x)^(3/2),x)
```

```
[Out] int(sinh(a + b*x)^2*(c + d*x)^(3/2), x)
```

3.47 $\int \sqrt{c+dx} \sinh^2(a+bx)\, dx$

Optimal. Leaf size=166

$$-\frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{d}\, e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\, \text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{d}\, e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\, \text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{c}}{}$$

```
[Out] -1/3*(d*x+c)^(3/2)/d+1/32*exp(-2*a+2*b*c/d)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/
2)/d^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/32*exp(2*a-2*b*c/d)*erfi(2^(
1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/4*si
nh(2*b*x+2*a)*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.20, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3393, 3377, 3389, 2211, 2235, 2236}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\, e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\, e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{c+dx}\, \sinh(2a+2bx)}{4b}-\frac{(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Sinh[a + b*x]^2,x]
```

```
[Out] -1/3*(c + d*x)^(3/2)/d + (Sqrt[d]*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt
[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)) - (Sqrt[d]*E^(2*a - (2*b*
c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)
) + (Sqrt[c + d*x]*Sinh[2*a + 2*b*x])/(4*b)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\int \sqrt{c+dx}\,\sinh^2(a+bx)\,dx &= -\int \left(\frac{1}{2}\sqrt{c+dx}-\frac{1}{2}\sqrt{c+dx}\,\cosh(2a+2bx)\right)dx \\
&= -\frac{(c+dx)^{3/2}}{3d}+\frac{1}{2}\int \sqrt{c+dx}\,\cosh(2a+2bx)\,dx \\
&= -\frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{c+dx}\,\sinh(2a+2bx)}{4b}-\frac{d\int \frac{\sinh(2a+2bx)}{\sqrt{c+dx}}\,dx}{8b} \\
&= -\frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{c+dx}\,\sinh(2a+2bx)}{4b}-\frac{d\int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx}{16b}+\frac{d\int \frac{e^{i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx}{16b} \\
&= -\frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{c+dx}\,\sinh(2a+2bx)}{4b}+\frac{\text{Subst}\left(\int e^{i\left(2ia-\frac{2ibc}{d}\right)-\frac{2bx^2}{d}}\,dx,x,\right.}{8b} \\
&= -\frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{d}\,e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{d}\,e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}}{}
\end{aligned}$$

Mathematica [A]
time = 0.41, size = 129, normalized size = 0.78

$$\frac{1}{48}\sqrt{c+dx}\left(-\frac{16(c+dx)}{d}+\frac{3\sqrt{2}\,e^{2a-\frac{2bc}{d}}\Gamma\left(\frac{3}{2},-\frac{2b(c+dx)}{d}\right)}{b\sqrt{-\frac{b(c+dx)}{d}}}-\frac{3\sqrt{2}\,e^{-2a+\frac{2bc}{d}}\Gamma\left(\frac{3}{2},\frac{2b(c+dx)}{d}\right)}{b\sqrt{\frac{b(c+dx)}{d}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (Sqrt[c + d*x]*((-16*(c + d*x))/d + (3*Sqrt[2]*E^(2*a - (2*b*c)/d)*Gamma[3/
2, (-2*b*(c + d*x))/d])/(b*Sqrt[-((b*(c + d*x))/d)]) - (3*Sqrt[2]*E^(-2*a +
 (2*b*c)/d)*Gamma[3/2, (2*b*(c + d*x))/d])/(b*Sqrt[(b*(c + d*x))/d])))/48
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(\sinh^2 (bx+a)\right) \sqrt{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2*(d*x+c)^(1/2),x)
```

```
[Out] int(sinh(b*x+a)^2*(d*x+c)^(1/2),x)
```

Maxima [A]
time = 0.48, size = 189, normalized size = 1.14

$$-\frac{\frac{3\sqrt{2}\sqrt{\pi}\, d \operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2a-\frac{2bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}} - \frac{3\sqrt{2}\sqrt{\pi}\, d \operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2a+\frac{2bc}{d}\right)}}{b\sqrt{\frac{b}{d}}} + 32\,(dx+c)^{\frac{3}{2}} - \frac{12\sqrt{dx+c}\, de^{\left(2a+\frac{2(dx+c)b}{d}-\frac{2bc}{d}\right)}}{b} + \frac{12\sqrt{dx+c}\, de^{\left(-2a-\frac{2(dx+c)b}{d}+\frac{2bc}{d}\right)}}{b}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/96*(3*sqrt(2)*sqrt(pi)*d*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))*e^(2*a -
2*b*c/d)/(b*sqrt(-b/d)) - 3*sqrt(2)*sqrt(pi)*d*erf(sqrt(2)*sqrt(d*x + c)*sq
rt(b/d))*e^(-2*a + 2*b*c/d)/(b*sqrt(b/d)) + 32*(d*x + c)^(3/2) - 12*sqrt(d*
x + c)*d*e^(2*a + 2*(d*x + c)*b/d - 2*b*c/d)/b + 12*sqrt(d*x + c)*d*e^(-2*a
 - 2*(d*x + c)*b/d + 2*b*c/d)/b)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 590 vs. 2(122) = 244.
time = 0.43, size = 590, normalized size = 3.55

$$\frac{3\sqrt{2}\sqrt{\pi}\left(d^2\cosh(bx+a)^2\cosh\left(-\frac{2(bc-ad)}{d}\right) - d^2\cosh(bx+a)^2\sinh\left(-\frac{2(bc-ad)}{d}\right) + \left(d^2\cosh\left(-\frac{2(bc-ad)}{d}\right) - d^2\sinh\left(-\frac{2(bc-ad)}{d}\right)\right)\sinh(bx+a)^2 + 2\left(d^2\cosh(bx+a)\cosh\left(-\frac{2(bc-ad)}{d}\right) - d^2\cosh(bx+a)\sinh\left(-\frac{2(bc-ad)}{d}\right)\right)\sinh(bx+a)\right)\sqrt{\frac{b}{d}}\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right) + 3\sqrt{2}\sqrt{\pi}\left(d^2\cosh(bx+a)^2\cosh\left(-\frac{2(bc-ad)}{d}\right) + d^2\cosh(bx+a)^2\sinh\left(-\frac{2(bc-ad)}{d}\right) + \left(d^2\cosh\left(-\frac{2(bc-ad)}{d}\right) + d^2\sinh\left(-\frac{2(bc-ad)}{d}\right)\right)\sinh(bx+a)^2 + 2\left(d^2\cosh(bx+a)\cosh\left(-\frac{2(bc-ad)}{d}\right) + d^2\cosh(bx+a)\sinh\left(-\frac{2(bc-ad)}{d}\right)\right)\sinh(bx+a)\right)\sqrt{-\frac{b}{d}}\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right) + 4\left(3bd\cosh(bx+a)^4 + 12bd\cosh(bx+a)\sinh(bx+a)^3 + 3bd\sinh(bx+a)^4 - 8(b^2dx+b^2c)\cosh(bx+a)^2 - 2\left(4b^2dx - 9bd\cosh(bx+a)^2 + 4b^2c\right)\sinh(bx+a)^2 - 3bd + 4\left(3bd\cosh(bx+a)^3 - 4(b^2dx+b^2c)\cosh(bx+a)\right)\sinh(bx+a)\right)\sqrt{dx+c}}{96\left(b^2d\cosh(bx+a)^2 + 2b^2d\cosh(bx+a)\sinh(bx+a) + b^2d\sinh(bx+a)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(3*sqrt(2)*sqrt(pi)*(d^2*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - d^2*
cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^2*cosh(-2*(b*c - a*d)/d) - d^2*
sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^2*cosh(b*x + a)*cosh(-2*(b*c
```

```
 - a*d)/d) - d^2*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(
b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) + 3*sqrt(2)*sqrt(pi)*(d^2*cosh(b*
x + a)^2*cosh(-2*(b*c - a*d)/d) + d^2*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d
) + (d^2*cosh(-2*(b*c - a*d)/d) + d^2*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)
^2 + 2*(d^2*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + d^2*cosh(b*x + a)*sinh(-
2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(
-b/d)) + 4*(3*b*d*cosh(b*x + a)^4 + 12*b*d*cosh(b*x + a)*sinh(b*x + a)^3 +
3*b*d*sinh(b*x + a)^4 - 8*(b^2*d*x + b^2*c)*cosh(b*x + a)^2 - 2*(4*b^2*d*x
- 9*b*d*cosh(b*x + a)^2 + 4*b^2*c)*sinh(b*x + a)^2 - 3*b*d + 4*(3*b*d*cosh(
b*x + a)^3 - 4*(b^2*d*x + b^2*c)*cosh(b*x + a))*sinh(b*x + a))*sqrt(d*x + c
))/(b^2*d*cosh(b*x + a)^2 + 2*b^2*d*cosh(b*x + a)*sinh(b*x + a) + b^2*d*sin
h(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sinh^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2*(d*x+c)**(1/2),x)
[Out] Integral(sqrt(c + d*x)*sinh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x + c)*sinh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{sinh}(a+b\,x)^2\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2*(c + d*x)^(1/2),x)
[Out] int(sinh(a + b*x)^2*(c + d*x)^(1/2), x)
```

3.48 $\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=139

$$-\frac{\sqrt{c+dx}}{d}+\frac{e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\,\sqrt{d}}+\frac{e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\,\sqrt{d}}$$

```
[Out] 1/8*exp(-2*a+2*b*c/d)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi
^(1/2)/b^(1/2)/d^(1/2)+1/8*exp(2*a-2*b*c/d)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)-(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3393, 3388, 2211, 2235, 2236}

$$\frac{\sqrt{\frac{\pi}{2}}\,e^{\frac{2bc}{d}-2a}\mathrm{Erf}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\,\sqrt{d}}+\frac{\sqrt{\frac{\pi}{2}}\,e^{2a-\frac{2bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\,\sqrt{d}}-\frac{\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2/Sqrt[c + d*x],x]
```

```
[Out] -(Sqrt[c + d*x]/d) + (E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*
Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d]) + (E^(2*a - (2*b*c)/d)*Sqrt[Pi
/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx &= -\int \left(\frac{1}{2\sqrt{c+dx}}-\frac{\cosh(2a+2bx)}{2\sqrt{c+dx}}\right)dx\\
&= -\frac{\sqrt{c+dx}}{d}+\frac{1}{2}\int \frac{\cosh(2a+2bx)}{\sqrt{c+dx}}\,dx\\
&= -\frac{\sqrt{c+dx}}{d}+\frac{1}{4}\int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx+\frac{1}{4}\int \frac{e^{i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx\\
&= -\frac{\sqrt{c+dx}}{d}+\frac{\text{Subst}\left(\int e^{i\left(2ia-\frac{2ibc}{d}\right)-\frac{2bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{2d}+\frac{\text{Subst}\left(\int e^{-i\left(2ia-\frac{2ibc}{d}\right)+\frac{2bx^2}{d}} \cdots\right)}{2d}\\
&= -\frac{\sqrt{c+dx}}{d}+\frac{e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+ \cdots}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 142, normalized size = 1.02

$$
-\frac{\sqrt{c+dx}}{d}+\frac{e^{2a-\frac{2bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)}{4\sqrt{2}\,b\sqrt{c+dx}}-\frac{e^{-2a+\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{2b(c+dx)}{d}\right)}{4\sqrt{2}\,b\sqrt{c+dx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2/Sqrt[c + d*x],x]
```

```
[Out] -(Sqrt[c + d*x]/d) + (E^(2*a - (2*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/
2, (-2*b*(c + d*x))/d])/(4*Sqrt[2]*b*Sqrt[c + d*x]) - (E^(-2*a + (2*b*c)/d)
*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (2*b*(c + d*x))/d])/(4*Sqrt[2]*b*Sqrt[c +
 d*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(bx+a)}{\sqrt{dx+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2/(d*x+c)^(1/2),x)
```

```
[Out] int(sinh(b*x+a)^2/(d*x+c)^(1/2),x)
```

Maxima [A]
time = 0.48, size = 107, normalized size = 0.77

$$\frac{\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}+\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}}{\sqrt{\frac{b}{d}}}-8\,\sqrt{dx+c}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))*e^(2*a - 2*b*c/
d)/sqrt(-b/d) + sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(-2
*a + 2*b*c/d)/sqrt(b/d) - 8*sqrt(d*x + c))/d
```

Fricas [A]
time = 0.46, size = 155, normalized size = 1.12

$$\frac{\sqrt{2}\,\sqrt{\pi}\left(d\cosh\left(-\frac{2\,(bc-ad)}{d}\right)-d\sinh\left(-\frac{2\,(bc-ad)}{d}\right)\right)\sqrt{\frac{b}{d}}\operatorname{erf}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}\right)-\sqrt{2}\,\sqrt{\pi}\left(d\cosh\left(-\frac{2\,(bc-ad)}{d}\right)+d\sinh\left(-\frac{2\,(bc-ad)}{d}\right)\right)\sqrt{-\frac{b}{d}}\operatorname{erf}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}\right)-8\,\sqrt{dx+c}\,b}{8\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*sqrt(pi)*(d*cosh(-2*(b*c - a*d)/d) - d*sinh(-2*(b*c - a*d)/d))
*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - sqrt(2)*sqrt(pi)*(d*cosh(
-2*(b*c - a*d)/d) + d*sinh(-2*(b*c - a*d)/d))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d
*x + c)*sqrt(-b/d)) - 8*sqrt(d*x + c)*b)/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2/(d*x+c)**(1/2),x)
```

```
[Out] Integral(sinh(a + b*x)**2/sqrt(c + d*x), x)
```

Giac [A]
time = 0.53, size = 115, normalized size = 0.83

$$-\frac{\left(\frac{\sqrt{2}\sqrt{\pi}\,d\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{2bc}{d}\right)}}{\sqrt{bd}}+\frac{\sqrt{2}\sqrt{\pi}\,d\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{2(bc-2ad)}{d}\right)}}{\sqrt{-bd}}+8\sqrt{dx+c}\,e^{(2a)}\right)e^{(-2a)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)/d)*e^(2*b*c/d
)/sqrt(b*d) + sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(-b*d)*sqrt(d*x + c)/d)*e
^(-2*(b*c - 2*a*d)/d)/sqrt(-b*d) + 8*sqrt(d*x + c)*e^(2*a))*e^(-2*a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(a+b\,x)^2}{\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2/(c + d*x)^(1/2),x)
```

```
[Out] int(sinh(a + b*x)^2/(c + d*x)^(1/2), x)
```

3.49 $\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=142

$$-\frac{\sqrt{b}\,e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{b}\,e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\text{Erfi}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}$$

```
[Out] -1/2*exp(-2*a+2*b*c/d)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^(1/2)*2
^(1/2)*Pi^(1/2)/d^(3/2)+1/2*exp(2*a-2*b*c/d)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(
1/2)/d^(1/2))*b^(1/2)*2^(1/2)*Pi^(1/2)/d^(3/2)-2*sinh(b*x+a)^2/d/(d*x+c)^(1
/2)
```

Rubi [A]
time = 0.17, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3394, 12, 3389, 2211, 2235, 2236}

$$-\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{b}\,e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{b}\,e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2/(c + d*x)^(3/2),x]
```

```
[Out] -((Sqrt[b]*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*
x])/Sqrt[d]])/d^(3/2)) + (Sqrt[b]*E^(2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt
[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2) - (2*Sinh[a + b*x]^2)/(d*Sqrt[
c + d*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,dx &= -\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}} - \frac{(4ib)\int \frac{i\sinh(2a+2bx)}{2\sqrt{c+dx}}\,dx}{d} \\ &= -\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}} + \frac{(2b)\int \frac{\sinh(2a+2bx)}{\sqrt{c+dx}}\,dx}{d} \\ &= -\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}} + \frac{b\int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx}{d} - \frac{b\int \frac{e^{i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx}{d} \\ &= -\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}} - \frac{(2b)\mathrm{Subst}\left(\int e^{i\left(2ia-\frac{2ibc}{d}\right)-\frac{2bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d^2} + \frac{(2b)\mathrm{Subst}\left(\int e^{\cdots}\right.}{\cdots} \\ &= -\frac{\sqrt{b}\,e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{b}\,e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right.}{d^{3/2}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 570 vs. 2(142) = 284.

time = 3.20, size = 570, normalized size = 4.01

$$\frac{e^{-\frac{2b(c+dx)}{d}}\left(2\sqrt{d}\,e^{\frac{2b(c+dx)}{d}}-\sqrt{d}\,\cosh(2a)\cosh\left(\frac{2bc}{d}\right)-\sqrt{d}\,e^{\frac{4b(c+dx)}{d}}\cosh(2a)\cosh\left(\frac{2bc}{d}\right)+\sqrt{d}\,\cosh\left(\frac{2bc}{d}\right)\sinh(2a)-\sqrt{d}\,e^{\frac{4b(c+dx)}{d}}\cosh\left(\frac{2bc}{d}\right)\sinh(2a)+\sqrt{2}\,\sqrt{d}\,e^{\frac{2b(c+dx)}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)\left(\cosh\left(2a-\frac{2bc}{d}\right)+\cosh\left(\frac{2bc}{d}\right)\sinh(2a)\right)-\sqrt{d}\,\cosh(2a)\sinh\left(\frac{2bc}{d}\right)+\sqrt{d}\,e^{\frac{4b(c+dx)}{d}}\cosh(2a)\sinh\left(\frac{2bc}{d}\right)-\sqrt{b}\,e^{\frac{2b(c+dx)}{d}}\sqrt{2\pi}\,\sqrt{c+dx}\,\cosh(2a)\mathrm{Erf}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)\sinh\left(\frac{2bc}{d}\right)-\sqrt{b}\,e^{\frac{2b(c+dx)}{d}}\sqrt{2\pi}\,\sqrt{c+dx}\,\cosh(2a)\mathrm{Erfi}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)\sinh\left(\frac{2bc}{d}\right)+\sqrt{d}\,\sinh(2a)\sinh\left(\frac{2bc}{d}\right)+\sqrt{d}\,e^{\frac{4b(c+dx)}{d}}\sinh(2a)\sinh\left(\frac{2bc}{d}\right)+\sqrt{2}\,\sqrt{d}\,e^{\frac{2b(c+dx)}{d}}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{2b(c+dx)}{d}\right)\left(\cosh(2a)\cosh\left(\frac{2bc}{d}\right)-\sinh(2a)\left(\cosh\left(\frac{2bc}{d}\right)+\sinh\left(\frac{2bc}{d}\right)\right)\right)\right)}{2d^{3/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2/(c + d*x)^(3/2),x]
```

```
[Out] (2*Sqrt[d]*E^((2*b*(c + d*x))/d) - Sqrt[d]*Cosh[2*a]*Cosh[(2*b*c)/d] - Sqrt
[d]*E^((4*b*(c + d*x))/d)*Cosh[2*a]*Cosh[(2*b*c)/d] + Sqrt[d]*Cosh[(2*b*c)/
d]*Sinh[2*a] - Sqrt[d]*E^((4*b*(c + d*x))/d)*Cosh[(2*b*c)/d]*Sinh[2*a] + Sq
rt[2]*Sqrt[d]*E^((2*b*(c + d*x))/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-2
*b*(c + d*x))/d]*(Cosh[2*a - (2*b*c)/d] + Cosh[(2*b*c)/d]*Sinh[2*a]) - Sqrt
[d]*Cosh[2*a]*Sinh[(2*b*c)/d] + Sqrt[d]*E^((4*b*(c + d*x))/d)*Cosh[2*a]*Sin
h[(2*b*c)/d] - Sqrt[b]*E^((2*b*(c + d*x))/d)*Sqrt[2*Pi]*Sqrt[c + d*x]*Cosh[
2*a]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]*Sinh[(2*b*c)/d] - Sqrt[b]
*E^((2*b*(c + d*x))/d)*Sqrt[2*Pi]*Sqrt[c + d*x]*Cosh[2*a]*Erfi[(Sqrt[2]*Sqr
t[b]*Sqrt[c + d*x])/Sqrt[d]]*Sinh[(2*b*c)/d] + Sqrt[d]*Sinh[2*a]*Sinh[(2*b*
c)/d] + Sqrt[d]*E^((4*b*(c + d*x))/d)*Sinh[2*a]*Sinh[(2*b*c)/d] + Sqrt[2]*S
qrt[d]*E^((2*b*(c + d*x))/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (2*b*(c + d*x
))/d]*(Cosh[2*a]*Cosh[(2*b*c)/d] - Sinh[2*a]*(Cosh[(2*b*c)/d] + Sinh[(2*b*c
)/d])))/(2*d^(3/2)*E^((2*b*(c + d*x))/d)*Sqrt[c + d*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(bx+a)}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2/(d*x+c)^(3/2),x)
```

```
[Out] int(sinh(b*x+a)^2/(d*x+c)^(3/2),x)
```

Maxima [A]
time = 0.35, size = 116, normalized size = 0.82

$$-\frac{\frac{\sqrt{2}\sqrt{\frac{(dx+c)b}{d}}\,e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{2(dx+c)b}{d}\right)}{\sqrt{dx+c}}+\frac{\sqrt{2}\sqrt{-\frac{(dx+c)b}{d}}\,e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{2(dx+c)b}{d}\right)}{\sqrt{dx+c}}-\frac{4}{\sqrt{dx+c}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(sqrt(2)*sqrt((d*x + c)*b/d)*e^(2*(b*c - a*d)/d)*gamma(-1/2, 2*(d*x +
c)*b/d)/sqrt(d*x + c) + sqrt(2)*sqrt(-(d*x + c)*b/d)*e^(-2*(b*c - a*d)/d)*g
amma(-1/2, -2*(d*x + c)*b/d)/sqrt(d*x + c) - 4/sqrt(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(109) = 218$.
time = 0.43, size = 571, normalized size = 4.02

$$-\frac{\sqrt{2}\sqrt{\pi}\left((dx+c)\cosh\left(bx+a\right)^{2}\cosh\left(-\frac{2(bc-ad)}{d}\right)-(dx+c)\cosh\left(bx+a\right)^{2}\sinh\left(-\frac{2(bc-ad)}{d}\right)+\left((dx+c)\cosh\left(-\frac{2(bc-ad)}{d}\right)-(dx+c)\sinh\left(-\frac{2(bc-ad)}{d}\right)\right)\sinh\left(bx+a\right)^{2}+2\left((dx+c)\cosh\left(bx+a\right)\cosh\left(-\frac{2(bc-ad)}{d}\right)-(dx+c)\cosh\left(bx+a\right)\sinh\left(-\frac{2(bc-ad)}{d}\right)\right)\sinh\left(bx+a\right)\right)\sqrt{\frac{b}{d}}\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)+\sqrt{2}\sqrt{\pi}\left((dx+c)\cosh\left(bx+a\right)^{2}\cosh\left(-\frac{2(bc-ad)}{d}\right)+(dx+c)\cosh\left(bx+a\right)^{2}\sinh\left(-\frac{2(bc-ad)}{d}\right)+\left((dx+c)\cosh\left(-\frac{2(bc-ad)}{d}\right)+(dx+c)\sinh\left(-\frac{2(bc-ad)}{d}\right)\right)\sinh\left(bx+a\right)^{2}+2\left((dx+c)\cosh\left(bx+a\right)\cosh\left(-\frac{2(bc-ad)}{d}\right)+(dx+c)\cosh\left(bx+a\right)\sinh\left(-\frac{2(bc-ad)}{d}\right)\right)\sinh\left(bx+a\right)\right)\sqrt{-\frac{b}{d}}\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)+\left(\cosh\left(bx+a\right)^{4}+4\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+\sinh\left(bx+a\right)^{4}+2\left(3\cosh\left(bx+a\right)^{2}-1\right)\sinh\left(bx+a\right)^{2}-2\cosh\left(bx+a\right)^{2}+4\left(\cosh\left(bx+a\right)^{3}-\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)+1\right)\sqrt{dx+c}}{2\left((d^{2}x+cd)\cosh\left(bx+a\right)^{2}+2(d^{2}x+cd)\cosh\left(bx+a\right)\sinh\left(bx+a\right)+(d^{2}x+cd)\sinh\left(bx+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*sqrt(pi)*((d*x + c)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) -
(d*x + c)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((d*x + c)*cosh(-2*(b*c
- a*d)/d) - (d*x + c)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((d*x + c
)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)*sinh(-2*(b
*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))
 + sqrt(2)*sqrt(pi)*((d*x + c)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + (d*
x + c)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((d*x + c)*cosh(-2*(b*c - a
*d)/d) + (d*x + c)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((d*x + c)*c
osh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)*sinh(-2*(b*c
- a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))
+ (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*
(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x +
 a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*sqrt(d*x + c))/((d^2*x + c*d)*cos
h(b*x + a)^2 + 2*(d^2*x + c*d)*cosh(b*x + a)*sinh(b*x + a) + (d^2*x + c*d)*
sinh(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{2}\left(a+bx\right)}{\left(c+dx\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2/(d*x+c)**(3/2),x)
```

```
[Out] Integral(sinh(a + b*x)**2/(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^2/(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(a+b\,x)^2}{(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2/(c + d*x)^(3/2),x)
[Out] int(sinh(a + b*x)^2/(c + d*x)^(3/2), x)
```

3.50 $\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=174

$$\frac{2b^{3/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2b^{3/2}e^{2a-\frac{2bc}{d}}\sqrt{2\pi}\,\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{8b\cosh(a+bx)\sin}{3d^2\sqrt{c+d}}$$

```
[Out] -2/3*sinh(b*x+a)^2/d/(d*x+c)^(3/2)+2/3*b^(3/2)*exp(-2*a+2*b*c/d)*erf(2^(1/2
)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(5/2)+2/3*b^(3/2)*exp(2
*a-2*b*c/d)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^
(5/2)-8/3*b*cosh(b*x+a)*sinh(b*x+a)/d^2/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3395, 32, 3393, 3388, 2211, 2235, 2236}

$$\frac{2\sqrt{2\pi}\,b^{3/2}e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2\sqrt{2\pi}\,b^{3/2}e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2/(c + d*x)^(5/2),x]
```

```
[Out] (2*b^(3/2)*E^(-2*a + (2*b*c)/d)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*
x])/Sqrt[d]])/(3*d^(5/2)) + (2*b^(3/2)*E^(2*a - (2*b*c)/d)*Sqrt[2*Pi]*Erfi[
(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) - (8*b*Cosh[a + b*x]*
Sinh[a + b*x])/(3*d^2*Sqrt[c + d*x]) - (2*Sinh[a + b*x]^2)/(3*d*(c + d*x)^(
3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{(8b^2)\int \frac{1}{\sqrt{c+dx}}\,dx}{3d^2} + \frac{(16b^2)\int \cdots}{3\cdots}$$

$$= \frac{16b^2\sqrt{c+dx}}{3d^3} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{(16b^2)\int\left(\frac{1}{2\sqrt{c+\cdots}}\cdots\right.}{\cdots}$$

$$= -\frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{(8b^2)\int \frac{\cosh(2a+2bx)}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{(4b^2)\int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx}{3d^2} + \frac{(4b^2)\int \frac{e^{i\cdots}}{\sqrt{\cdots}}}{3\cdots}$$

$$= -\frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{(8b^2)\,\mathrm{Subst}\left(\int e^{i\left(2ia-\frac{2ibc}{d}\right)-\frac{2bx^2}{d}}\,dx\cdots\right.}{3d^3}$$

$$= \frac{2b^{3/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2b^{3/2}e^{2a-\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\cdots\right.}{3d^{5/2}}$$

Mathematica [A]
time = 0.79, size = 156, normalized size = 0.90

$$-\frac{2e^{-2\left(a+\frac{bc}{d}\right)}\left(\sqrt{2}\,de^{4a}\left(-\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)+\sqrt{2}\,de^{\frac{4bc}{d}}\left(\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},\frac{2b(c+dx)}{d}\right)+e^{2\left(a+\frac{bc}{d}\right)}\left(d\sinh^2(a+bx)+2b(c+dx)\sinh(2(a+bx))\right)\right)}{3d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(Sqrt[2]*d*E^(4*a)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, (-2*b*(c + d*x
))/d] + Sqrt[2]*d*E^((4*b*c)/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (2*b*(c
+ d*x))/d] + E^(2*(a + (b*c)/d))*(d*Sinh[a + b*x]^2 + 2*b*(c + d*x)*Sinh[2*
(a + b*x)])))/(3*d^2*E^(2*(a + (b*c)/d))*(c + d*x)^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(bx+a)}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2/(d*x+c)^(5/2),x)
```

```
[Out] int(sinh(b*x+a)^2/(d*x+c)^(5/2),x)
```

Maxima [A]
time = 0.34, size = 118, normalized size = 0.68

$$-\frac{\frac{3\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}+\frac{3\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}-\frac{2}{(dx+c)^{\frac{3}{2}}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*(3*sqrt(2)*((d*x + c)*b/d)^(3/2)*e^(2*(b*c - a*d)/d)*gamma(-3/2, 2*(d*
x + c)*b/d)/(d*x + c)^(3/2) + 3*sqrt(2)*(-(d*x + c)*b/d)^(3/2)*e^(-2*(b*c -
 a*d)/d)*gamma(-3/2, -2*(d*x + c)*b/d)/(d*x + c)^(3/2) - 2/(d*x + c)^(3/2))
/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 864 vs. 2(134) = 268.
time = 0.54, size = 864, normalized size = 4.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(2)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*co
sh(-2*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh
(-2*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-2*(b*c - a*d)/d
) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2
 + 2*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d)
- (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sin
h(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 4*sqrt(2)*sqrt
(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d
) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d)
+ ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-2*(b*c - a*d)/d) + (b*d^2*x^2 + 2*
b*c*d*x + b*c^2)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b*d^2*x^2 +
2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*
c*d*x + b*c^2)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b
/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d)) - ((4*b*d*x + 4*b*c + d)*cosh(b*x
 + a)^4 + 4*(4*b*d*x + 4*b*c + d)*cosh(b*x + a)*sinh(b*x + a)^3 + (4*b*d*x
+ 4*b*c + d)*sinh(b*x + a)^4 - 4*b*d*x - 2*d*cosh(b*x + a)^2 + 2*(3*(4*b*d*
x + 4*b*c + d)*cosh(b*x + a)^2 - d)*sinh(b*x + a)^2 - 4*b*c + 4*((4*b*d*x +
 4*b*c + d)*cosh(b*x + a)^3 - d*cosh(b*x + a))*sinh(b*x + a) + d)*sqrt(d*x
+ c))/((d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a)^2 + 2*(d^4*x^2 + 2*c*d
```

```
^3*x + c^2*d^2)*cosh(b*x + a)*sinh(b*x + a) + (d^4*x^2 + 2*c*d^3*x + c^2*d^
2)*sinh(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2/(d*x+c)**(5/2),x)
[Out] Integral(sinh(a + b*x)**2/(c + d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^2/(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(a+b\,x)^2}{(c+d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2/(c + d*x)^(5/2),x)
[Out] int(sinh(a + b*x)^2/(c + d*x)^(5/2), x)
```

3.51 $\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal. Leaf size=220

$$-\frac{16b^2}{15d^3\sqrt{c+dx}}-\frac{8b^{5/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{Erf}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{8b^{5/2}e^{2a-\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}$$

```
[Out] -8/15*b*cosh(b*x+a)*sinh(b*x+a)/d^2/(d*x+c)^(3/2)-2/5*sinh(b*x+a)^2/d/(d*x+
c)^(5/2)-8/15*b^(5/2)*exp(-2*a+2*b*c/d)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d
^(1/2))*2^(1/2)*Pi^(1/2)/d^(7/2)+8/15*b^(5/2)*exp(2*a-2*b*c/d)*erfi(2^(1/2)
*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(7/2)-16/15*b^2/d^3/(d*x
+c)^(1/2)-32/15*b^2*sinh(b*x+a)^2/d^3/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3395, 32, 3394, 12, 3389, 2211, 2235, 2236}

$$-\frac{8\sqrt{2\pi}\,b^{5/2}e^{\frac{2bc}{d}-2a}\mathrm{Erf}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{8\sqrt{2\pi}\,b^{5/2}e^{2a-\frac{2bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}-\frac{32b^2\sinh^2(a+bx)}{15d^3\sqrt{c+dx}}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2/(c + d*x)^(7/2),x]
```

```
[Out] (-16*b^2)/(15*d^3*Sqrt[c + d*x]) - (8*b^(5/2)*E^(-2*a + (2*b*c)/d)*Sqrt[2*P
i]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(15*d^(7/2)) + (8*b^(5/2)*
E^(2*a - (2*b*c)/d)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]
])/(15*d^(7/2)) - (8*b*Cosh[a + b*x]*Sinh[a + b*x])/(15*d^2*(c + d*x)^(3/2)
) - (2*Sinh[a + b*x]^2)/(5*d*(c + d*x)^(5/2)) - (32*b^2*Sinh[a + b*x]^2)/(1
5*d^3*Sqrt[c + d*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{8b\cosh(a+bx)\sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{(8b^2)\int \frac{1}{(c+dx)^{3/2}}\,dx}{15d^2} + \frac{(16b^2)\int \frac{\sin}{(}}{15}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2\sinh^2(a+}{15d^3\sqrt{c+dx}}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2\sinh^2(a+}{15d^3\sqrt{c+dx}}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2\sinh^2(a+}{15d^3\sqrt{c+dx}}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2\sinh^2(a+}{15d^3\sqrt{c+dx}}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{8b^{5/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{8b^{5/2}e^{2a-\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{er}}{1}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 825 vs. 2(220) = 440.
time = 6.08, size = 825, normalized size = 3.75

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2/(c + d*x)^(7/2),x]
```

```
[Out] (6*d^2*E^((2*b*(c + d*x))/d) - 16*b^2*c^2*Cosh[2*a - (2*b*c)/d] + 4*b*c*d*C
osh[2*a - (2*b*c)/d] - 3*d^2*Cosh[2*a - (2*b*c)/d] - 16*b^2*c^2*E^((4*b*(c
+ d*x))/d)*Cosh[2*a - (2*b*c)/d] - 4*b*c*d*E^((4*b*(c + d*x))/d)*Cosh[2*a -
 (2*b*c)/d] - 3*d^2*E^((4*b*(c + d*x))/d)*Cosh[2*a - (2*b*c)/d] - 32*b^2*c*
d*x*Cosh[2*a - (2*b*c)/d] + 4*b*d^2*x*Cosh[2*a - (2*b*c)/d] - 32*b^2*c*d*E^
((4*b*(c + d*x))/d)*x*Cosh[2*a - (2*b*c)/d] - 4*b*d^2*E^((4*b*(c + d*x))/d)
*x*Cosh[2*a - (2*b*c)/d] - 16*b^2*d^2*x^2*Cosh[2*a - (2*b*c)/d] - 16*b^2*d^
2*E^((4*b*(c + d*x))/d)*x^2*Cosh[2*a - (2*b*c)/d] + 16*Sqrt[2]*d^2*E^((2*b*
(c + d*x))/d)*((b*(c + d*x))/d)^(5/2)*Gamma[1/2, (2*b*(c + d*x))/d]*(Cosh[2
*a - (2*b*c)/d] - Sinh[2*a - (2*b*c)/d]) + 16*b^2*c^2*Sinh[2*a - (2*b*c)/d]
 - 4*b*c*d*Sinh[2*a - (2*b*c)/d] + 3*d^2*Sinh[2*a - (2*b*c)/d] - 16*b^2*c^2
*E^((4*b*(c + d*x))/d)*Sinh[2*a - (2*b*c)/d] - 4*b*c*d*E^((4*b*(c + d*x))/d
)*Sinh[2*a - (2*b*c)/d] - 3*d^2*E^((4*b*(c + d*x))/d)*Sinh[2*a - (2*b*c)/d]
 + 32*b^2*c*d*x*Sinh[2*a - (2*b*c)/d] - 4*b*d^2*x*Sinh[2*a - (2*b*c)/d] - 3
```

```
2*b^2*c*d*E^((4*b*(c + d*x))/d)*x*Sinh[2*a - (2*b*c)/d] - 4*b*d^2*E^((4*b*(
c + d*x))/d)*x*Sinh[2*a - (2*b*c)/d] + 16*b^2*d^2*x^2*Sinh[2*a - (2*b*c)/d]
 - 16*b^2*d^2*E^((4*b*(c + d*x))/d)*x^2*Sinh[2*a - (2*b*c)/d] + 16*Sqrt[2]*
d^2*E^((2*b*(c + d*x))/d)*(-((b*(c + d*x))/d))^(5/2)*Gamma[1/2, (-2*b*(c +
d*x))/d]*(Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d]))/(30*d^3*E^((2*b*(
c + d*x))/d)*(c + d*x)^(5/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(bx+a)}{(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2/(d*x+c)^(7/2),x)
```

```
[Out] int(sinh(b*x+a)^2/(d*x+c)^(7/2),x)
```

Maxima [A]
time = 0.33, size = 118, normalized size = 0.54

$$-\frac{\frac{5\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}+\frac{5\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},-\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}-\frac{1}{(dx+c)^{\frac{5}{2}}}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/5*(5*sqrt(2)*((d*x + c)*b/d)^(5/2)*e^(2*(b*c - a*d)/d)*gamma(-5/2, 2*(d*
x + c)*b/d)/(d*x + c)^(5/2) + 5*sqrt(2)*(-(d*x + c)*b/d)^(5/2)*e^(-2*(b*c -
 a*d)/d)*gamma(-5/2, -2*(d*x + c)*b/d)/(d*x + c)^(5/2) - 1/(d*x + c)^(5/2))
/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1352 vs. 2(172) = 344.
time = 0.46, size = 1352, normalized size = 6.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/30*(16*sqrt(2)*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*
d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) +
 ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-2*(b*c -
```

```
a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-2
*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) - (b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-2*(b*c - a*d
)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) + 16*sq
rt(2)*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*c
osh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((b^2*d^3*x
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-2*(b*c - a*d)/d) + (b
^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-2*(b*c - a*d)
/d))*sinh(b*x + a)^2 + 2*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
b^2*c^3)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b
*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d)) + (16*b^2*d^2*x^2
 + (16*b^2*d^2*x^2 + 16*b^2*c^2 + 4*b*c*d + 3*d^2 + 4*(8*b^2*c*d + b*d^2)*x
)*cosh(b*x + a)^4 + 4*(16*b^2*d^2*x^2 + 16*b^2*c^2 + 4*b*c*d + 3*d^2 + 4*(8
*b^2*c*d + b*d^2)*x)*cosh(b*x + a)*sinh(b*x + a)^3 + (16*b^2*d^2*x^2 + 16*b
^2*c^2 + 4*b*c*d + 3*d^2 + 4*(8*b^2*c*d + b*d^2)*x)*sinh(b*x + a)^4 + 16*b^
2*c^2 - 6*d^2*cosh(b*x + a)^2 - 4*b*c*d + 6*((16*b^2*d^2*x^2 + 16*b^2*c^2 +
 4*b*c*d + 3*d^2 + 4*(8*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^2 - d^2)*sinh(b*x
 + a)^2 + 3*d^2 + 4*(8*b^2*c*d - b*d^2)*x + 4*((16*b^2*d^2*x^2 + 16*b^2*c^2
 + 4*b*c*d + 3*d^2 + 4*(8*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^3 - 3*d^2*cosh(
b*x + a))*sinh(b*x + a))*sqrt(d*x + c))/((d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4
*x + c^3*d^3)*cosh(b*x + a)^2 + 2*(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^
3*d^3)*cosh(b*x + a)*sinh(b*x + a) + (d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x +
 c^3*d^3)*sinh(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2/(d*x+c)**(7/2),x)
[Out] Integral(sinh(a + b*x)**2/(c + d*x)**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^2/(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(a+b\,x)^2}{(c+d\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2/(c + d*x)^(7/2),x)
[Out] int(sinh(a + b*x)^2/(c + d*x)^(7/2), x)
```

3.52 $\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}} \, dx$

Optimal. Leaf size=251

$$-\frac{16b^2}{105d^3(c+dx)^{3/2}}+\frac{32b^{7/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}}+\frac{32b^{7/2}e^{2a-\frac{2bc}{d}}\sqrt{2\pi}\,\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+d}}{\sqrt{d}}\right.}{105d^{9/2}}$$

```
[Out] -16/105*b^2/d^3/(d*x+c)^(3/2)-8/35*b*cosh(b*x+a)*sinh(b*x+a)/d^2/(d*x+c)^(5
/2)-2/7*sinh(b*x+a)^2/d/(d*x+c)^(7/2)-32/105*b^2*sinh(b*x+a)^2/d^3/(d*x+c)^
(3/2)+32/105*b^(7/2)*exp(-2*a+2*b*c/d)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*2^(1/2)*Pi^(1/2)/d^(9/2)+32/105*b^(7/2)*exp(2*a-2*b*c/d)*erfi(2^(1/2
)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(9/2)-128/105*b^3*cosh(
b*x+a)*sinh(b*x+a)/d^4/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3395, 32, 3393, 3388, 2211, 2235, 2236}

$$\frac{32\sqrt{2\pi}\,b^{7/2}e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}}+\frac{32\sqrt{2\pi}\,b^{7/2}e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}}-\frac{128b^3\sinh(a+bx)\cosh(a+bx)}{105d^4\sqrt{c+dx}}-\frac{32b^2\sinh^2(a+bx)}{105d^3(c+dx)^{3/2}}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2/(c + d*x)^(9/2),x]
```

```
[Out] (-16*b^2)/(105*d^3*(c + d*x)^(3/2)) + (32*b^(7/2)*E^(-2*a + (2*b*c)/d)*Sqrt
[2*Pi]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(105*d^(9/2)) + (32*b^
(7/2)*E^(2*a - (2*b*c)/d)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/S
qrt[d]])/(105*d^(9/2)) - (8*b*Cosh[a + b*x]*Sinh[a + b*x])/(35*d^2*(c + d*x
)^(5/2)) - (128*b^3*Cosh[a + b*x]*Sinh[a + b*x])/(105*d^4*Sqrt[c + d*x]) -
(2*Sinh[a + b*x]^2)/(7*d*(c + d*x)^(7/2)) - (32*b^2*Sinh[a + b*x]^2)/(105*d
^3*(c + d*x)^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}\,dx = -\frac{8b\cosh(a+bx)\sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{(8b^2)\int \frac{1}{(c+dx)^{5/2}}\,dx}{35d^2} + \frac{(16b^2)\int \frac{\sin}{(}}{35}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3\cosh(a+bx)\sinh(a+bx}{105d^4\sqrt{c+dx}}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} + \frac{256b^4\sqrt{c+dx}}{105d^5} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3\cosh(a}{105}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3\cosh(a+bx)\sinh(a+bx}{105d^4\sqrt{c+dx}}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3\cosh(a+bx)\sinh(a+bx}{105d^4\sqrt{c+dx}}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3\cosh(a+bx)\sinh(a+bx}{105d^4\sqrt{c+dx}}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} + \frac{32b^{7/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}} + \frac{32b^{7/2}e^{2a-\frac{2bc}{d}}\sqrt{}}{}$$

Mathematica [A]
time = 0.37, size = 222, normalized size = 0.88

$$\frac{2\left(-8b^2d(c+dx)^2+16\sqrt{2}\,b^3e^{2a-\frac{2bc}{d}}(c+dx)^3\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)-16\sqrt{2}\,b^3e^{-2a+\frac{2bc}{d}}(c+dx)^3\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{2b(c+dx)}{d}\right)-15d^3\sinh^2(a+bx)-16b^2d(c+dx)^2\sinh^2(a+bx)-6bd^2(c+dx)\sinh(2(a+bx))-32b^3(c+dx)^3\sinh(2(a+bx))\right)}{105d^4(c+dx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2/(c + d*x)^(9/2),x]
```

```
[Out] (2*(-8*b^2*d*(c + d*x)^2 + 16*Sqrt[2]*b^3*E^(2*a - (2*b*c)/d)*(c + d*x)^3*S
qrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-2*b*(c + d*x))/d] - 16*Sqrt[2]*b^3*E^(
-2*a + (2*b*c)/d)*(c + d*x)^3*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (2*b*(c + d*
x))/d] - 15*d^3*Sinh[a + b*x]^2 - 16*b^2*d*(c + d*x)^2*Sinh[a + b*x]^2 - 6*
b*d^2*(c + d*x)*Sinh[2*(a + b*x)] - 32*b^3*(c + d*x)^3*Sinh[2*(a + b*x)]))/
(105*d^4*(c + d*x)^(7/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(bx+a)}{(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2/(d*x+c)^(9/2),x)
```

```
[Out] int(sinh(b*x+a)^2/(d*x+c)^(9/2),x)
```

Maxima [A]
time = 0.33, size = 118, normalized size = 0.47

$$-\frac{\frac{14\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{7}{2}}e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{7}{2},\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{7}{2}}}+\frac{14\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{7}{2}}e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{7}{2},-\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{7}{2}}}-\frac{1}{(dx+c)^{\frac{7}{2}}}}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/7*(14*sqrt(2)*((d*x + c)*b/d)^(7/2)*e^(2*(b*c - a*d)/d)*gamma(-7/2, 2*(d
*x + c)*b/d)/(d*x + c)^(7/2) + 14*sqrt(2)*(-(d*x + c)*b/d)^(7/2)*e^(-2*(b*c
 - a*d)/d)*gamma(-7/2, -2*(d*x + c)*b/d)/(d*x + c)^(7/2) - 1/(d*x + c)^(7/2
))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1827 vs. 2(199) = 398.
time = 0.41, size = 1827, normalized size = 7.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/210*(64*sqrt(2)*sqrt(pi)*((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*
x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - (b^
3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*
cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 +
6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(-2*(b*c - a*d)/d) - (b^3*
d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*si
nh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 +
 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)*cosh(-2*(b*c -
a*d)/d) - (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*
x + b^3*c^4)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)
*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 64*sqrt(2)*sqrt(pi)*((b^3*d^4*x^4 +
 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x +
a)^2*cosh(-2*(b*c - a*d)/d) + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^
2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (
(b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^
4)*cosh(-2*(b*c - a*d)/d) + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*
```

```
x^2 + 4*b^3*c^3*d*x + b^3*c^4)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*
((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c
^4)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 +
 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)*sinh(-2*(b*c -
a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d)) +
(64*b^3*d^3*x^3 + 64*b^3*c^3 - 16*b^2*c^2*d + 30*d^3*cosh(b*x + a)^2 - (64*
b^3*d^3*x^3 + 64*b^3*c^3 + 16*b^2*c^2*d + 12*b*c*d^2 + 15*d^3 + 16*(12*b^3*
c*d^2 + b^2*d^3)*x^2 + 4*(48*b^3*c^2*d + 8*b^2*c*d^2 + 3*b*d^3)*x)*cosh(b*x
 + a)^4 - 4*(64*b^3*d^3*x^3 + 64*b^3*c^3 + 16*b^2*c^2*d + 12*b*c*d^2 + 15*d
^3 + 16*(12*b^3*c*d^2 + b^2*d^3)*x^2 + 4*(48*b^3*c^2*d + 8*b^2*c*d^2 + 3*b*
d^3)*x)*cosh(b*x + a)*sinh(b*x + a)^3 - (64*b^3*d^3*x^3 + 64*b^3*c^3 + 16*b
^2*c^2*d + 12*b*c*d^2 + 15*d^3 + 16*(12*b^3*c*d^2 + b^2*d^3)*x^2 + 4*(48*b^
3*c^2*d + 8*b^2*c*d^2 + 3*b*d^3)*x)*sinh(b*x + a)^4 + 12*b*c*d^2 - 15*d^3 +
 16*(12*b^3*c*d^2 - b^2*d^3)*x^2 + 6*(5*d^3 - (64*b^3*d^3*x^3 + 64*b^3*c^3
+ 16*b^2*c^2*d + 12*b*c*d^2 + 15*d^3 + 16*(12*b^3*c*d^2 + b^2*d^3)*x^2 + 4*
(48*b^3*c^2*d + 8*b^2*c*d^2 + 3*b*d^3)*x)*cosh(b*x + a)^2)*sinh(b*x + a)^2
+ 4*(48*b^3*c^2*d - 8*b^2*c*d^2 + 3*b*d^3)*x + 4*(15*d^3*cosh(b*x + a) - (6
4*b^3*d^3*x^3 + 64*b^3*c^3 + 16*b^2*c^2*d + 12*b*c*d^2 + 15*d^3 + 16*(12*b^
3*c*d^2 + b^2*d^3)*x^2 + 4*(48*b^3*c^2*d + 8*b^2*c*d^2 + 3*b*d^3)*x)*cosh(b
*x + a)^3)*sinh(b*x + a))*sqrt(d*x + c))/((d^8*x^4 + 4*c*d^7*x^3 + 6*c^2*d^
6*x^2 + 4*c^3*d^5*x + c^4*d^4)*cosh(b*x + a)^2 + 2*(d^8*x^4 + 4*c*d^7*x^3 +
 6*c^2*d^6*x^2 + 4*c^3*d^5*x + c^4*d^4)*cosh(b*x + a)*sinh(b*x + a) + (d^8*
x^4 + 4*c*d^7*x^3 + 6*c^2*d^6*x^2 + 4*c^3*d^5*x + c^4*d^4)*sinh(b*x + a)^2)
```

Sympy [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2/(d*x+c)**(9/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^2/(d*x + c)^(9/2), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(a+b\,x)^2}{(c+d\,x)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2/(c + d*x)^(9/2),x)
```

```
[Out] int(sinh(a + b*x)^2/(c + d*x)^(9/2), x)
```

3.53 $\int (c+dx)^{5/2} \sinh^3(a+bx)\,dx$

Optimal. Leaf size=381

$$-\frac{45d^2\sqrt{c+dx}\,\cosh(a+bx)}{16b^3}-\frac{2(c+dx)^{5/2}\cosh(a+bx)}{3b}+\frac{5d^2\sqrt{c+dx}\,\cosh(3a+3bx)}{144b^3}+\frac{45d^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}}{6}$$

```
[Out] -2/3*(d*x+c)^(5/2)*cosh(b*x+a)/b+5/3*d*(d*x+c)^(3/2)*sinh(b*x+a)/b^2+1/3*(d
*x+c)^(5/2)*cosh(b*x+a)*sinh(b*x+a)^2/b-5/18*d*(d*x+c)^(3/2)*sinh(b*x+a)^3/
b^2-5/1728*d^(5/2)*exp(-3*a+3*b*c/d)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*3^(1/2)*Pi^(1/2)/b^(7/2)-5/1728*d^(5/2)*exp(3*a-3*b*c/d)*erfi(3^(1/2)*
b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)+45/64*d^(5/2)*exp(-
a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(7/2)+45/64*d^(5/2)*
exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(7/2)-45/16*d^2
*cosh(b*x+a)*(d*x+c)^(1/2)/b^3+5/144*d^2*cosh(3*b*x+3*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A]
time = 0.80, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3392, 3377, 3388, 2211, 2235, 2236, 3393}

$$\frac{45\sqrt{\pi}\,d^{5/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}e^{\frac{3bc}{d}-3a}\text{Erf}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{45\sqrt{\pi}\,d^{5/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}e^{3a-\frac{3bc}{d}}\text{Erfi}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}-\frac{45d^2\sqrt{c+dx}\,\cosh(a+bx)}{16b^3}+\frac{5d^2\sqrt{c+dx}\,\cosh(3a+3bx)}{144b^3}-\frac{5d(c+dx)^{3/2}\sinh^3(a+bx)}{18b^2}+\frac{5d(c+dx)^{3/2}\sinh(a+bx)}{3b^2}-\frac{2(c+dx)^{5/2}\cosh(a+bx)}{3b}+\frac{(c+dx)^{5/2}\sinh^2(a+bx)\cosh(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Sinh[a + b*x]^3,x]
```

```
[Out] (-45*d^2*Sqrt[c + d*x]*Cosh[a + b*x])/(16*b^3) - (2*(c + d*x)^(5/2)*Cosh[a
+ b*x])/(3*b) + (5*d^2*Sqrt[c + d*x]*Cosh[3*a + 3*b*x])/(144*b^3) + (45*d^(
5/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^
(7/2)) - (5*d^(5/2)*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b]*Sq
rt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) + (45*d^(5/2)*E^(a - (b*c)/d)*Sqrt[Pi]
*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(7/2)) - (5*d^(5/2)*E^(3*a -
(3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(576*b
^(7/2)) + (5*d*(c + d*x)^(3/2)*Sinh[a + b*x])/(3*b^2) + ((c + d*x)^(5/2)*Co
sh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (5*d*(c + d*x)^(3/2)*Sinh[a + b*x]^3)/
(18*b^2)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx = \frac{(c+dx)^{5/2}\cosh(a+bx)\sinh^2(a+bx)}{3b} - \frac{5d(c+dx)^{3/2}\sinh^3(a+bx)}{18b^2} - \frac{2}{3}$$

$$= -\frac{2(c+dx)^{5/2}\cosh(a+bx)}{3b} + \frac{(c+dx)^{5/2}\cosh(a+bx)\sinh^2(a+bx)}{3b} - 5$$

$$= -\frac{2(c+dx)^{5/2}\cosh(a+bx)}{3b} + \frac{5d(c+dx)^{3/2}\sinh(a+bx)}{3b^2} + \frac{(c+dx)^{5/2}\mathrm{co}}{}$$

$$= -\frac{45d^2\sqrt{c+dx}\,\cosh(a+bx)}{16b^3} - \frac{2(c+dx)^{5/2}\cosh(a+bx)}{3b} + \frac{5d^2\sqrt{c+dx}}{1}$$

$$= -\frac{45d^2\sqrt{c+dx}\,\cosh(a+bx)}{16b^3} - \frac{2(c+dx)^{5/2}\cosh(a+bx)}{3b} + \frac{5d^2\sqrt{c+dx}}{1}$$

$$= -\frac{45d^2\sqrt{c+dx}\,\cosh(a+bx)}{16b^3} - \frac{2(c+dx)^{5/2}\cosh(a+bx)}{3b} + \frac{5d^2\sqrt{c+dx}}{1}$$

$$= -\frac{45d^2\sqrt{c+dx}\,\cosh(a+bx)}{16b^3} - \frac{2(c+dx)^{5/2}\cosh(a+bx)}{3b} + \frac{5d^2\sqrt{c+dx}}{1}$$

Mathematica [A]
time = 6.68, size = 243, normalized size = 0.64

$$-\frac{d^3\left(\sqrt{3}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{7}{2},-\frac{3b(c+dx)}{d}\right)\left(\cosh\left(3a-\frac{3bc}{d}\right)+\sinh\left(3a-\frac{3bc}{d}\right)\right)+\left(\sqrt{\frac{b(c+dx)}{d}}\left(-243\Gamma\left(\frac{7}{2},\frac{b(c+dx)}{d}\right)+\sqrt{3}\,\Gamma\left(\frac{7}{2},\frac{3b(c+dx)}{d}\right)\left(\cosh\left(2a-\frac{2bc}{d}\right)-\sinh\left(2a-\frac{2bc}{d}\right)\right)\right)+243\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{7}{2},-\frac{b(c+dx)}{d}\right)\left(\cosh\left(2a-\frac{2bc}{d}\right)+\sinh\left(2a-\frac{2bc}{d}\right)\right)\right)\left(-\cosh\left(a-\frac{bc}{d}\right)+\sinh\left(a-\frac{bc}{d}\right)\right)\right)}{648b^4\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Sinh[a + b*x]^3,x]
```

```
[Out] -1/648*(d^3*(Sqrt[3]*Sqrt[-((b*(c + d*x))/d)]*Gamma[7/2, (-3*b*(c + d*x))/d
]*(Cosh[3*a - (3*b*c)/d] + Sinh[3*a - (3*b*c)/d]) + (Sqrt[(b*(c + d*x))/d]*
(-243*Gamma[7/2, (b*(c + d*x))/d] + Sqrt[3]*Gamma[7/2, (3*b*(c + d*x))/d]*(
Cosh[2*a - (2*b*c)/d] - Sinh[2*a - (2*b*c)/d])) + 243*Sqrt[-((b*(c + d*x))/
d)]*Gamma[7/2, -((b*(c + d*x))/d)]*(Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b
*c)/d]))*(-Cosh[a - (b*c)/d] + Sinh[a - (b*c)/d])))/(b^4*Sqrt[c + d*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^{\frac{5}{2}}\left(\sinh^3(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*sinh(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^(5/2)*sinh(b*x+a)^3,x)
```

Maxima [A]
time = 0.51, size = 513, normalized size = 1.35

$$-\frac{\frac{5\sqrt{3}\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}}+\frac{5\sqrt{3}\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}}-\frac{1215\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}}-\frac{1215\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}}+\frac{162\left(4(dx+c)^{\frac{5}{2}}b^2de^{\left(\frac{bc}{d}\right)}+10(dx+c)^{\frac{3}{2}}bd^2e^{\left(\frac{bc}{d}\right)}+15\sqrt{dx+c}\,d^3e^{\left(\frac{bc}{d}\right)}\right)e^{\left(-a-\frac{(dx+c)b}{d}\right)}}{b^3}-\frac{6\left(12(dx+c)^{\frac{5}{2}}b^2de^{\left(\frac{3bc}{d}\right)}+10(dx+c)^{\frac{3}{2}}bd^2e^{\left(\frac{3bc}{d}\right)}+5\sqrt{dx+c}\,d^3e^{\left(\frac{3bc}{d}\right)}\right)e^{\left(-3a-\frac{3(dx+c)b}{d}\right)}}{b^3}-\frac{6\left(12(dx+c)^{\frac{5}{2}}b^2de^{(3a)}-10(dx+c)^{\frac{3}{2}}bd^2e^{(3a)}+5\sqrt{dx+c}\,d^3e^{(3a)}\right)e^{\left(\frac{3(dx+c)b}{d}-\frac{3bc}{d}\right)}}{b^3}+\frac{162\left(4(dx+c)^{\frac{5}{2}}b^2de^a-10(dx+c)^{\frac{3}{2}}bd^2e^a+15\sqrt{dx+c}\,d^3e^a\right)e^{\left(\frac{(dx+c)b}{d}-\frac{bc}{d}\right)}}{b^3}}{1728\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/1728*(5*sqrt(3)*sqrt(pi)*d^3*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*
a - 3*b*c/d)/(b^3*sqrt(-b/d)) + 5*sqrt(3)*sqrt(pi)*d^3*erf(sqrt(3)*sqrt(d*x
 + c)*sqrt(b/d))*e^(-3*a + 3*b*c/d)/(b^3*sqrt(b/d)) - 1215*sqrt(pi)*d^3*erf
(sqrt(d*x + c)*sqrt(-b/d))*e^(a - b*c/d)/(b^3*sqrt(-b/d)) - 1215*sqrt(pi)*d
^3*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^3*sqrt(b/d)) + 162*(4*(d*
x + c)^(5/2)*b^2*d*e^(b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(b*c/d) + 15*sqrt
(d*x + c)*d^3*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^3 - 6*(12*(d*x + c)^(5/2)
*b^2*d*e^(3*b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(3*b*c/d) + 5*sqrt(d*x + c)
*d^3*e^(3*b*c/d))*e^(-3*a - 3*(d*x + c)*b/d)/b^3 - 6*(12*(d*x + c)^(5/2)*b^
2*d*e^(3*a) - 10*(d*x + c)^(3/2)*b*d^2*e^(3*a) + 5*sqrt(d*x + c)*d^3*e^(3*a
))*e^(3*(d*x + c)*b/d - 3*b*c/d)/b^3 + 162*(4*(d*x + c)^(5/2)*b^2*d*e^a - 1
0*(d*x + c)^(3/2)*b*d^2*e^a + 15*sqrt(d*x + c)*d^3*e^a)*e^((d*x + c)*b/d -
b*c/d)/b^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2090 vs. 2(291) = 582.
time = 0.46, size = 2090, normalized size = 5.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/1728*(5*sqrt(3)*sqrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - d
^3*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^3*cosh(-3*(b*c - a*d)/d) - d
^3*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^3*cosh(b*x + a)*cosh(-3*(
b*c - a*d)/d) - d^3*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 +
 3*(d^3*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) - d^3*cosh(b*x + a)^2*sinh(-
3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b
/d)) - 5*sqrt(3)*sqrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + d^3
*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^3*cosh(-3*(b*c - a*d)/d) + d^3
*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^3*cosh(b*x + a)*cosh(-3*(b*
c - a*d)/d) + d^3*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3
*(d^3*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + d^3*cosh(b*x + a)^2*sinh(-3*
```

```
(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b
/d)) - 1215*sqrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) - d^3*cosh(b
*x + a)^3*sinh(-(b*c - a*d)/d) + (d^3*cosh(-(b*c - a*d)/d) - d^3*sinh(-(b*c
 - a*d)/d))*sinh(b*x + a)^3 + 3*(d^3*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d
^3*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d^3*cosh(b*x +
a)^2*cosh(-(b*c - a*d)/d) - d^3*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(
b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 1215*sqrt(pi)*(d^3*cosh(
b*x + a)^3*cosh(-(b*c - a*d)/d) + d^3*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d)
+ (d^3*cosh(-(b*c - a*d)/d) + d^3*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3
*(d^3*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d^3*cosh(b*x + a)*sinh(-(b*c - a
*d)/d))*sinh(b*x + a)^2 + 3*(d^3*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) + d^3
*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d
*x + c)*sqrt(-b/d)) - 6*(12*b^3*d^2*x^2 + (12*b^3*d^2*x^2 + 12*b^3*c^2 - 10
*b^2*c*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^6 + 6*(12*
b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2
)*x)*cosh(b*x + a)*sinh(b*x + a)^5 + (12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*
c*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x)*sinh(b*x + a)^6 + 12*b^3*c^2
- 27*(4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*
b^2*d^2)*x)*cosh(b*x + a)^4 - 3*(36*b^3*d^2*x^2 + 36*b^3*c^2 - 90*b^2*c*d +
 135*b*d^2 - 5*(12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c*d + 5*b*d^2 + 2*(12*
b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^2 + 18*(4*b^3*c*d - 5*b^2*d^2)*x)*sin
h(b*x + a)^4 + 10*b^2*c*d + 4*(5*(12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c*d
+ 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^3 - 27*(4*b^3*d^2*x
^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x)*cosh(
b*x + a))*sinh(b*x + a)^3 + 5*b*d^2 - 27*(4*b^3*d^2*x^2 + 4*b^3*c^2 + 10*b^
2*c*d + 15*b*d^2 + 2*(4*b^3*c*d + 5*b^2*d^2)*x)*cosh(b*x + a)^2 - 3*(36*b^3
*d^2*x^2 + 36*b^3*c^2 - 5*(12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c*d + 5*b*d
^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^4 + 90*b^2*c*d + 135*b*d^2
 + 54*(4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5
*b^2*d^2)*x)*cosh(b*x + a)^2 + 18*(4*b^3*c*d + 5*b^2*d^2)*x)*sinh(b*x + a)^
2 + 2*(12*b^3*c*d + 5*b^2*d^2)*x + 6*((12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2
*c*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^5 - 18*(4*b^3*
d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x)*
cosh(b*x + a)^3 - 9*(4*b^3*d^2*x^2 + 4*b^3*c^2 + 10*b^2*c*d + 15*b*d^2 + 2*
(4*b^3*c*d + 5*b^2*d^2)*x)*cosh(b*x + a))*sinh(b*x + a))*sqrt(d*x + c))/(b^
4*cosh(b*x + a)^3 + 3*b^4*cosh(b*x + a)^2*sinh(b*x + a) + 3*b^4*cosh(b*x +
a)*sinh(b*x + a)^2 + b^4*sinh(b*x + a)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/2)*sinh(b*x + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sinh(a+b\,x)^3\,(c+d\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] int(sinh(a + b*x)^3*(c + d*x)^(5/2), x)
```

3.54 $\int (c+dx)^{3/2}\sinh^3(a+bx)\,dx$

Optimal. Leaf size=325

$$-\frac{2(c+dx)^{3/2}\cosh(a+bx)}{3b}+\frac{9d^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}}-\frac{d^{3/2}e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}$$

```
[Out] -2/3*(d*x+c)^(3/2)*cosh(b*x+a)/b+1/3*(d*x+c)^(3/2)*cosh(b*x+a)*sinh(b*x+a)^
2/b-1/288*d^(3/2)*exp(-3*a+3*b*c/d)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/
2))*3^(1/2)*Pi^(1/2)/b^(5/2)+1/288*d^(3/2)*exp(3*a-3*b*c/d)*erfi(3^(1/2)*b^
(1/2)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(5/2)+9/32*d^(3/2)*exp(-a+b
*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(5/2)-9/32*d^(3/2)*exp(
a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(5/2)+d*sinh(b*x+a)
*(d*x+c)^(1/2)/b^2-1/6*d*sinh(b*x+a)^3*(d*x+c)^(1/2)/b^2
```

Rubi [A]

time = 0.61, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3392, 3377, 3389, 2211, 2235, 2236, 3393}

$$\frac{9\sqrt{\pi}\,d^{3/2}e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}}-\frac{\sqrt{\frac{\pi}{3}}\,d^{3/2}e^{\frac{3bc}{d}-3a}\mathrm{Erf}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-\frac{9\sqrt{\pi}\,d^{3/2}e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}}+\frac{\sqrt{\frac{\pi}{3}}\,d^{3/2}e^{3a-\frac{3bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-\frac{d\sqrt{c+dx}\,\sinh^3(a+bx)}{6b^2}+\frac{d\sqrt{c+dx}\,\sinh(a+bx)}{b^2}-\frac{2(c+dx)^{3/2}\cosh(a+bx)}{3b}+\frac{(c+dx)^{3/2}\sinh^2(a+bx)\cosh(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Sinh[a + b*x]^3,x]
```

```
[Out] (-2*(c + d*x)^(3/2)*Cosh[a + b*x])/(3*b) + (9*d^(3/2)*E^(-a + (b*c)/d)*Sqrt
[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(32*b^(5/2)) - (d^(3/2)*E^(-3*a
 + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(96*b
^(5/2)) - (9*d^(3/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/
Sqrt[d]])/(32*b^(5/2)) + (d^(3/2)*E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt
[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (d*Sqrt[c + d*x]*Sinh[a
 + b*x])/b^2 + ((c + d*x)^(3/2)*Cosh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (d*S
qrt[c + d*x]*Sinh[a + b*x]^3)/(6*b^2)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int (c+dx)^{3/2}\sinh^3(a+bx)\,dx = \frac{(c+dx)^{3/2}\cosh(a+bx)\sinh^2(a+bx)}{3b} - \frac{d\sqrt{c+dx}\,\sinh^3(a+bx)}{6b^2} - \frac{2}{3}\int$$

$$= -\frac{2(c+dx)^{3/2}\cosh(a+bx)}{3b} + \frac{(c+dx)^{3/2}\cosh(a+bx)\sinh^2(a+bx)}{3b} - d$$

$$= -\frac{2(c+dx)^{3/2}\cosh(a+bx)}{3b} + \frac{d\sqrt{c+dx}\,\sinh(a+bx)}{b^2} + \frac{(c+dx)^{3/2}\cosh}{}$$

$$= -\frac{2(c+dx)^{3/2}\cosh(a+bx)}{3b} + \frac{d\sqrt{c+dx}\,\sinh(a+bx)}{b^2} + \frac{(c+dx)^{3/2}\cosh}{}$$

$$= -\frac{2(c+dx)^{3/2}\cosh(a+bx)}{3b} + \frac{d\sqrt{c+dx}\,\sinh(a+bx)}{b^2} + \frac{(c+dx)^{3/2}\cosh}{}$$

$$= -\frac{2(c+dx)^{3/2}\cosh(a+bx)}{3b} + \frac{9d^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}} - \frac{d^{3/2}}{}$$

Mathematica [A]
time = 2.70, size = 243, normalized size = 0.75

$$\frac{d^2\left(\sqrt{3}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{5}{2},-\frac{3b(c+dx)}{d}\right)\left(\cosh\left(3a-\frac{3bc}{d}\right)+\sinh\left(3a-\frac{3bc}{d}\right)\right)+\left(81\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{5}{2},-\frac{b(c+dx)}{d}\right)\left(\cosh\left(2a-\frac{2bc}{d}\right)+\sinh\left(2a-\frac{2bc}{d}\right)\right)+\sqrt{\frac{b(c+dx)}{d}}\left(81\Gamma\left(\frac{5}{2},\frac{b(c+dx)}{d}\right)+\sqrt{3}\,\Gamma\left(\frac{5}{2},\frac{3b(c+dx)}{d}\right)\left(-\cosh\left(2a-\frac{2bc}{d}\right)+\sinh\left(2a-\frac{2bc}{d}\right)\right)\right)\right)\left(-\cosh\left(a-\frac{bc}{d}\right)+\sinh\left(a-\frac{bc}{d}\right)\right)\right)}{216b^3\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Sinh[a + b*x]^3,x]
```

```
[Out] (d^2*(Sqrt[3]*Sqrt[-((b*(c + d*x))/d)]*Gamma[5/2, (-3*b*(c + d*x))/d]*(Cosh
[3*a - (3*b*c)/d] + Sinh[3*a - (3*b*c)/d]) + (81*Sqrt[-((b*(c + d*x))/d)]*G
amma[5/2, -((b*(c + d*x))/d)]*(Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d
]) + Sqrt[(b*(c + d*x))/d]*(81*Gamma[5/2, (b*(c + d*x))/d] + Sqrt[3]*Gamma[
5/2, (3*b*(c + d*x))/d]*(-Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d]))*
(-Cosh[a - (b*c)/d] + Sinh[a - (b*c)/d])))/(216*b^3*Sqrt[c + d*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^{\frac{3}{2}}\left(\sinh^3(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*sinh(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^(3/2)*sinh(b*x+a)^3,x)
```

Maxima [A]
time = 0.48, size = 430, normalized size = 1.32

$$\frac{\frac{\sqrt{3}\sqrt{\pi}d^2\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}}-\frac{\sqrt{3}\sqrt{\pi}d^2\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}}-\frac{81\sqrt{\pi}d^2\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}}+\frac{81\sqrt{\pi}d^2\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}}-\frac{54\left(2(dx+c)^{\frac{3}{2}}bde^{\left(\frac{bc}{d}\right)}+3\sqrt{dx+c}\,d^2e^{\left(\frac{bc}{d}\right)}\right)e^{\left(-a-\frac{(dx+c)b}{d}\right)}}{b^2}+\frac{6\left(2(dx+c)^{\frac{3}{2}}bde^{\left(\frac{3bc}{d}\right)}+\sqrt{dx+c}\,d^2e^{\left(\frac{3bc}{d}\right)}\right)e^{\left(-3a-\frac{3(dx+c)b}{d}\right)}}{b^2}+\frac{6\left(2(dx+c)^{\frac{3}{2}}bde^{(3a)}-\sqrt{dx+c}\,d^2e^{(3a)}\right)e^{\left(\frac{3(dx+c)b}{d}-\frac{3bc}{d}\right)}}{b^2}-\frac{54\left(2(dx+c)^{\frac{3}{2}}bde^{a}-3\sqrt{dx+c}\,d^2e^{a}\right)e^{\left(\frac{(dx+c)b}{d}-\frac{bc}{d}\right)}}{b^2}}{288\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/288*(sqrt(3)*sqrt(pi)*d^2*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a -
3*b*c/d)/(b^2*sqrt(-b/d)) - sqrt(3)*sqrt(pi)*d^2*erf(sqrt(3)*sqrt(d*x + c)*
sqrt(b/d))*e^(-3*a + 3*b*c/d)/(b^2*sqrt(b/d)) - 81*sqrt(pi)*d^2*erf(sqrt(d*
x + c)*sqrt(-b/d))*e^(a - b*c/d)/(b^2*sqrt(-b/d)) + 81*sqrt(pi)*d^2*erf(sqr
t(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^2*sqrt(b/d)) - 54*(2*(d*x + c)^(3/2
)*b*d*e^(b*c/d) + 3*sqrt(d*x + c)*d^2*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^2
 + 6*(2*(d*x + c)^(3/2)*b*d*e^(3*b*c/d) + sqrt(d*x + c)*d^2*e^(3*b*c/d))*e^
(-3*a - 3*(d*x + c)*b/d)/b^2 + 6*(2*(d*x + c)^(3/2)*b*d*e^(3*a) - sqrt(d*x
+ c)*d^2*e^(3*a))*e^(3*(d*x + c)*b/d - 3*b*c/d)/b^2 - 54*(2*(d*x + c)^(3/2)
*b*d*e^a - 3*sqrt(d*x + c)*d^2*e^a)*e^((d*x + c)*b/d - b*c/d)/b^2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1543 vs. $2(245) = 490$.
time = 0.44, size = 1543, normalized size = 4.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/288*(sqrt(3)*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - d^2*
cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^2*cosh(-3*(b*c - a*d)/d) - d^2*
sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-3*(b*c
 - a*d)/d) - d^2*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*
(d^2*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) - d^2*cosh(b*x + a)^2*sinh(-3*(
b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)
) + sqrt(3)*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + d^2*cosh
(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^2*cosh(-3*(b*c - a*d)/d) + d^2*sinh
(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-3*(b*c - a
*d)/d) + d^2*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d^2
*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + d^2*cosh(b*x + a)^2*sinh(-3*(b*c
- a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))
- 81*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) - d^2*cosh(b*x + a)
^3*sinh(-(b*c - a*d)/d) + (d^2*cosh(-(b*c - a*d)/d) - d^2*sinh(-(b*c - a*d)
/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d^2*cosh
```

```
(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d^2*cosh(b*x + a)^2*co
sh(-(b*c - a*d)/d) - d^2*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a
))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 81*sqrt(pi)*(d^2*cosh(b*x + a)^
3*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + (d^2*co
sh(-(b*c - a*d)/d) + d^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cos
h(b*x + a)*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*s
inh(b*x + a)^2 + 3*(d^2*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x
 + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*x + c)*s
qrt(-b/d)) - 6*((2*b^2*d*x + 2*b^2*c - b*d)*cosh(b*x + a)^6 + 6*(2*b^2*d*x
+ 2*b^2*c - b*d)*cosh(b*x + a)*sinh(b*x + a)^5 + (2*b^2*d*x + 2*b^2*c - b*d
)*sinh(b*x + a)^6 - 9*(2*b^2*d*x + 2*b^2*c - 3*b*d)*cosh(b*x + a)^4 - 3*(6*
b^2*d*x + 6*b^2*c - 5*(2*b^2*d*x + 2*b^2*c - b*d)*cosh(b*x + a)^2 - 9*b*d)*
sinh(b*x + a)^4 + 2*b^2*d*x + 4*(5*(2*b^2*d*x + 2*b^2*c - b*d)*cosh(b*x + a
)^3 - 9*(2*b^2*d*x + 2*b^2*c - 3*b*d)*cosh(b*x + a))*sinh(b*x + a)^3 + 2*b^
2*c - 9*(2*b^2*d*x + 2*b^2*c + 3*b*d)*cosh(b*x + a)^2 + 3*(5*(2*b^2*d*x + 2
*b^2*c - b*d)*cosh(b*x + a)^4 - 6*b^2*d*x - 6*b^2*c - 18*(2*b^2*d*x + 2*b^2
*c - 3*b*d)*cosh(b*x + a)^2 - 9*b*d)*sinh(b*x + a)^2 + b*d + 6*((2*b^2*d*x
+ 2*b^2*c - b*d)*cosh(b*x + a)^5 - 6*(2*b^2*d*x + 2*b^2*c - 3*b*d)*cosh(b*x
 + a)^3 - 3*(2*b^2*d*x + 2*b^2*c + 3*b*d)*cosh(b*x + a))*sinh(b*x + a))*sqr
t(d*x + c))/(b^3*cosh(b*x + a)^3 + 3*b^3*cosh(b*x + a)^2*sinh(b*x + a) + 3*
b^3*cosh(b*x + a)*sinh(b*x + a)^2 + b^3*sinh(b*x + a)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\sinh^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*sinh(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**(3/2)*sinh(a + b*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(3/2)*sinh(b*x + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sinh(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3*(c + d*x)^(3/2),x)
[Out] int(sinh(a + b*x)^3*(c + d*x)^(3/2), x)
```

3.55 $\int \sqrt{c+dx}\, \sinh^3(a+bx)\, dx$

Optimal. Leaf size=275

$$-\frac{3\sqrt{c+dx}\,\cosh(a+bx)}{4b}+\frac{\sqrt{c+dx}\,\cosh(3a+3bx)}{12b}+\frac{3\sqrt{d}\,e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{d}\,e^{-3a+\frac{3bc}{d}}}{}$$

```
[Out] -1/144*exp(-3*a+3*b*c/d)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)
*3^(1/2)*Pi^(1/2)/b^(3/2)-1/144*exp(3*a-3*b*c/d)*erfi(3^(1/2)*b^(1/2)*(d*x+
c)^(1/2)/d^(1/2))*d^(1/2)*3^(1/2)*Pi^(1/2)/b^(3/2)+3/16*exp(-a+b*c/d)*erf(b
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*Pi^(1/2)/b^(3/2)+3/16*exp(a-b*c/d)*er
fi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*Pi^(1/2)/b^(3/2)-3/4*cosh(b*x+a)*
(d*x+c)^(1/2)/b+1/12*cosh(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.38, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3393, 3377, 3388, 2211, 2235, 2236}

$$\frac{3\sqrt{\pi}\,\sqrt{d}\,e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}\,\sqrt{d}\,e^{\frac{3bc}{d}-3a}\mathrm{Erf}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}+\frac{3\sqrt{\pi}\,\sqrt{d}\,e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}\,\sqrt{d}\,e^{3a-\frac{3bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{3\sqrt{c+dx}\,\cosh(a+bx)}{4b}+\frac{\sqrt{c+dx}\,\cosh(3a+3bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (-3*Sqrt[c + d*x]*Cosh[a + b*x])/(4*b) + (Sqrt[c + d*x]*Cosh[3*a + 3*b*x])/
(12*b) + (3*Sqrt[d]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/S
qrt[d]])/(16*b^(3/2)) - (Sqrt[d]*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[
3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) + (3*Sqrt[d]*E^(a - (b*c)/
d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)) - (Sqrt[d]*
E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]
])/(48*b^(3/2))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\int \sqrt{c+dx}\,\sinh^3(a+bx)\,dx &= i\int \left(\frac{3}{4}i\sqrt{c+dx}\,\sinh(a+bx)-\frac{1}{4}i\sqrt{c+dx}\,\sinh(3a+3bx)\right)dx\\
&= \frac{1}{4}\int \sqrt{c+dx}\,\sinh(3a+3bx)\,dx-\frac{3}{4}\int \sqrt{c+dx}\,\sinh(a+bx)\,dx\\
&= -\frac{3\sqrt{c+dx}\,\cosh(a+bx)}{4b}+\frac{\sqrt{c+dx}\,\cosh(3a+3bx)}{12b}-\frac{d\int \frac{\cosh(3a+3bx)}{\sqrt{c+dx}}\,dx}{24b}\\
&= -\frac{3\sqrt{c+dx}\,\cosh(a+bx)}{4b}+\frac{\sqrt{c+dx}\,\cosh(3a+3bx)}{12b}-\frac{d\int \frac{e^{-i(3ia+3ibx)}}{\sqrt{c+dx}}\,dx}{48b}\\
&= -\frac{3\sqrt{c+dx}\,\cosh(a+bx)}{4b}+\frac{\sqrt{c+dx}\,\cosh(3a+3bx)}{12b}-\frac{\text{Subst}\left(\int e^{i\left(3ia-\frac{3ibc}{d}\right.}\right.}{}\\
&= -\frac{3\sqrt{c+dx}\,\cosh(a+bx)}{4b}+\frac{\sqrt{c+dx}\,\cosh(3a+3bx)}{12b}+\frac{3\sqrt{d}\,e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}}{16b}
\end{aligned}$$

Mathematica [A]

time = 0.21, size = 209, normalized size = 0.76

$$\frac{e^{-3\left(a+\frac{bc}{d}\right)}\sqrt{c+dx}\left(\sqrt{3}\,e^{6a}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{3}{2},-\frac{3b(c+dx)}{d}\right)-27e^{4a+\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{3}{2},-\frac{b(c+dx)}{d}\right)+e^{\frac{4bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\left(-27e^{2a}\Gamma\left(\frac{3}{2},\frac{b(c+dx)}{d}\right)+\sqrt{3}\,e^{\frac{2bc}{d}}\Gamma\left(\frac{3}{2},\frac{3b(c+dx)}{d}\right)\right)\right)}{72b\sqrt{-\frac{b^2(c+dx)^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (Sqrt[c + d*x]*(Sqrt[3]*E^(6*a)*Sqrt[(b*(c + d*x))/d]*Gamma[3/2, (-3*b*(c +
 d*x))/d] - 27*E^(4*a + (2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[3/2, -((b*(c
 + d*x))/d)] + E^((4*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*(-27*E^(2*a)*Gamma[3/
2, (b*(c + d*x))/d] + Sqrt[3]*E^((2*b*c)/d)*Gamma[3/2, (3*b*(c + d*x))/d]))
)/(72*b*E^(3*(a + (b*c)/d))*Sqrt[-((b^2*(c + d*x)^2)/d^2)])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(\sinh^3(bx+a)\right)\sqrt{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3*(d*x+c)^(1/2),x)
```

```
[Out] int(sinh(b*x+a)^3*(d*x+c)^(1/2),x)
```

Maxima [A]
time = 0.48, size = 333, normalized size = 1.21

$$-\frac{\frac{\sqrt{3}\sqrt{\pi}\,d\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}}+\frac{\sqrt{3}\sqrt{\pi}\,d\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{b\sqrt{\frac{b}{d}}}-\frac{27\sqrt{\pi}\,d\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}}-\frac{27\sqrt{\pi}\,d\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b\sqrt{\frac{b}{d}}}-\frac{6\sqrt{dx+c}\,de^{\left(3a+\frac{3(dx+c)b}{d}-\frac{3bc}{d}\right)}}{b}+\frac{54\sqrt{dx+c}\,de^{\left(a+\frac{(dx+c)b}{d}-\frac{bc}{d}\right)}}{b}+\frac{54\sqrt{dx+c}\,de^{\left(-a-\frac{(dx+c)b}{d}+\frac{bc}{d}\right)}}{b}-\frac{6\sqrt{dx+c}\,de^{\left(-3a-\frac{3(dx+c)b}{d}+\frac{3bc}{d}\right)}}{b}}{144\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/144*(sqrt(3)*sqrt(pi)*d*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a - 3
*b*c/d)/(b*sqrt(-b/d)) + sqrt(3)*sqrt(pi)*d*erf(sqrt(3)*sqrt(d*x + c)*sqrt(
b/d))*e^(-3*a + 3*b*c/d)/(b*sqrt(b/d)) - 27*sqrt(pi)*d*erf(sqrt(d*x + c)*sq
rt(-b/d))*e^(a - b*c/d)/(b*sqrt(-b/d)) - 27*sqrt(pi)*d*erf(sqrt(d*x + c)*sq
rt(b/d))*e^(-a + b*c/d)/(b*sqrt(b/d)) - 6*sqrt(d*x + c)*d*e^(3*a + 3*(d*x +
 c)*b/d - 3*b*c/d)/b + 54*sqrt(d*x + c)*d*e^(a + (d*x + c)*b/d - b*c/d)/b +
 54*sqrt(d*x + c)*d*e^(-a - (d*x + c)*b/d + b*c/d)/b - 6*sqrt(d*x + c)*d*e^
(-3*a - 3*(d*x + c)*b/d + 3*b*c/d)/b)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1216 vs. $2(201) = 402$.
time = 0.38, size = 1216, normalized size = 4.42

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b*x+a)^3*(d*x+c)^(1/2),x, algorithm="fricas")

[Out] -1/144*(sqrt(3)*sqrt(pi)*(d*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - d*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d*cosh(-3*(b*c - a*d)/d) - d*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) - d*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) - d*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) - sqrt(3)*sqrt(pi)*(d*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + d*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d*cosh(-3*(b*c - a*d)/d) + d*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) + d*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + d*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d)) - 27*sqrt(pi)*(d*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + (d*cosh(-(b*c - a*d)/d) - d*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 27*sqrt(pi)*(d*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) + d*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + (d*cosh(-(b*c - a*d)/d) + d*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) + d*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) - 6*(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 - 9*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 - 3*b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 - 9*b*cosh(b*x + a))*sinh(b*x + a)^3 - 9*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 - 18*b*cosh(b*x + a)^2 - 3*b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^5 - 6*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a) + b)*sqrt(d*x + c))/(b^2*cosh(b*x + a)^3 + 3*b^2*cosh(b*x + a)^2*sinh(b*x + a) + 3*b^2*cosh(b*x + a)*sinh(b*x + a)^2 + b^2*sinh(b*x + a)^3)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx} \sinh^3(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b*x+a)**3*(d*x+c)**(1/2),x)

[Out] Integral(sqrt(c + d*x)*sinh(a + b*x)**3, x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x + c)*sinh(b*x + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sinh(a+b\,x)^3\,\sqrt{c+d\,x}\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3*(c + d*x)^(1/2),x)
```

```
[Out] int(sinh(a + b*x)^3*(c + d*x)^(1/2), x)
```

3.56 $\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=228

$$\frac{3e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\,\sqrt{d}}-\frac{e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\,\sqrt{d}}-\frac{3e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\,\sqrt{d}}-$$

```
[Out] -1/24*exp(-3*a+3*b*c/d)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*
Pi^(1/2)/b^(1/2)/d^(1/2)+1/24*exp(3*a-3*b*c/d)*erfi(3^(1/2)*b^(1/2)*(d*x+c)
^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)+3/8*exp(-a+b*c/d)*erf(b^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(1/2)/d^(1/2)-3/8*exp(a-b*c/d)*erfi(b
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3393, 3389, 2211, 2235, 2236}

$$\frac{3\sqrt{\pi}\,e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\,\sqrt{d}}-\frac{\sqrt{\frac{\pi}{3}}\,e^{\frac{3bc}{d}-3a}\mathrm{Erf}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\,\sqrt{d}}-\frac{3\sqrt{\pi}\,e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\,\sqrt{d}}+\frac{\sqrt{\frac{\pi}{3}}\,e^{3a-\frac{3bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3/Sqrt[c + d*x],x]
```

```
[Out] (3*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[
b]*Sqrt[d]) - (E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c
+ d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) - (3*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(S
qrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) + (E^(3*a - (3*b*c)/d)*
Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d
])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx &= i\int \left(\frac{3i\sinh(a+bx)}{4\sqrt{c+dx}} - \frac{i\sinh(3a+3bx)}{4\sqrt{c+dx}}\right)dx \\
&= \frac{1}{4}\int \frac{\sinh(3a+3bx)}{\sqrt{c+dx}}\,dx - \frac{3}{4}\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx \\
&= \frac{1}{8}\int \frac{e^{-i(3ia+3ibx)}}{\sqrt{c+dx}}\,dx - \frac{1}{8}\int \frac{e^{i(3ia+3ibx)}}{\sqrt{c+dx}}\,dx - \frac{3}{8}\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx + \frac{3}{8}\int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx \\
&= -\frac{\operatorname{Subst}\left(\int e^{i\left(3ia-\frac{3ibc}{d}\right)-\frac{3bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{4d} + \frac{\operatorname{Subst}\left(\int e^{-i\left(3ia-\frac{3ibc}{d}\right)+\frac{3bx^2}{d}}\,dx, x, \sqrt{c\right.}}{4d} \\
&= \frac{3e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} - \frac{e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} - \frac{3e^{a-\frac{bc}{d}}\sqrt{\right.}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 191, normalized size = 0.84

$$
\frac{e^{-3\left(a+\frac{bc}{d}\right)}\left(\sqrt{3}\,e^{6a}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{3b(c+dx)}{d}\right) - 9e^{4a+\frac{2bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right) + e^{\frac{4bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\left(-9e^{2a}\Gamma\left(\frac{1}{2},\frac{b(c+dx)}{d}\right) + \sqrt{3}\,e^{\frac{2bc}{d}}\Gamma\left(\frac{1}{2},\frac{3b(c+dx)}{d}\right)\right)\right)}{24b\sqrt{c+dx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[3]*E^(6*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-3*b*(c + d*x))/d] -
9*E^(4*a + (2*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*x))/d
```

```
)] + E^((4*b*c)/d)*Sqrt[(b*(c + d*x))/d]*(-9*E^(2*a)*Gamma[1/2, (b*(c + d*x
))/d] + Sqrt[3]*E^((2*b*c)/d)*Gamma[1/2, (3*b*(c + d*x))/d]))/(24*b*E^(3*(a
 + (b*c)/d))*Sqrt[c + d*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(bx+a)}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3/(d*x+c)^(1/2),x)
```

```
[Out] int(sinh(b*x+a)^3/(d*x+c)^(1/2),x)
```

Maxima [A]
time = 0.49, size = 178, normalized size = 0.78

$$\frac{\frac{\sqrt{3}\,\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{3}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}-\frac{\sqrt{3}\,\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{3}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}\right)e^{\left(-3\,a+\frac{3\,bc}{d}\right)}}{\sqrt{\frac{b}{d}}}-\frac{9\,\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}+\frac{9\,\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{\sqrt{\frac{b}{d}}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/24*(sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a - 3*b*c
/d)/sqrt(-b/d) - sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d))*e^(-
3*a + 3*b*c/d)/sqrt(b/d) - 9*sqrt(pi)*erf(sqrt(d*x + c)*sqrt(-b/d))*e^(a -
b*c/d)/sqrt(-b/d) + 9*sqrt(pi)*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/
sqrt(b/d))/d
```

Fricas [A]
time = 0.35, size = 252, normalized size = 1.11

$$-\frac{\sqrt{3}\,\sqrt{\pi}\,\sqrt{\frac{b}{d}}\left(\cosh\left(-\frac{3\,(bc-ad)}{d}\right)-\sinh\left(-\frac{3\,(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{3}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}\right)+\sqrt{3}\,\sqrt{\pi}\,\sqrt{-\frac{b}{d}}\left(\cosh\left(-\frac{3\,(bc-ad)}{d}\right)+\sinh\left(-\frac{3\,(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{3}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}\right)-9\,\sqrt{\pi}\,\sqrt{\frac{b}{d}}\left(\cosh\left(-\frac{bc-ad}{d}\right)-\sinh\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{\frac{b}{d}}\right)-9\,\sqrt{\pi}\,\sqrt{-\frac{b}{d}}\left(\cosh\left(-\frac{bc-ad}{d}\right)+\sinh\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(sqrt(3)*sqrt(pi)*sqrt(b/d)*(cosh(-3*(b*c - a*d)/d) - sinh(-3*(b*c -
a*d)/d))*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) + sqrt(3)*sqrt(pi)*sqrt(-b/d)
*(cosh(-3*(b*c - a*d)/d) + sinh(-3*(b*c - a*d)/d))*erf(sqrt(3)*sqrt(d*x + c
)*sqrt(-b/d)) - 9*sqrt(pi)*sqrt(b/d)*(cosh(-(b*c - a*d)/d) - sinh(-(b*c - a
*d)/d))*erf(sqrt(d*x + c)*sqrt(b/d)) - 9*sqrt(pi)*sqrt(-b/d)*(cosh(-(b*c -
a*d)/d) + sinh(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-b/d)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3/(d*x+c)**(1/2),x)
[Out] Integral(sinh(a + b*x)**3/sqrt(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^3/sqrt(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\sinh(a+b\,x)}^3}{\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3/(c + d*x)^(1/2),x)
[Out] int(sinh(a + b*x)^3/(c + d*x)^(1/2), x)
```

3.57 $\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=246

$$-\frac{3\sqrt{b}\,e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}+\frac{\sqrt{b}\,e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\,\mathrm{Erf}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}-\frac{3\sqrt{b}\,e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\right.}{4d^{3/2}}$$

```
[Out] -3/4*exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^(1/2)*Pi^(1/2)/d^(3
/2)-3/4*exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^(1/2)*Pi^(1/2)/d
^(3/2)+1/4*exp(-3*a+3*b*c/d)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^(
1/2)*3^(1/2)*Pi^(1/2)/d^(3/2)+1/4*exp(3*a-3*b*c/d)*erfi(3^(1/2)*b^(1/2)*(d*
x+c)^(1/2)/d^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/d^(3/2)-2*sinh(b*x+a)^3/d/(d*x
+c)^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3394, 3388, 2211, 2235, 2236}

$$-\frac{3\sqrt{\pi}\,\sqrt{b}\,e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}+\frac{\sqrt{3\pi}\,\sqrt{b}\,e^{\frac{3bc}{d}-3a}\mathrm{Erf}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}-\frac{3\sqrt{\pi}\,\sqrt{b}\,e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}+\frac{\sqrt{3\pi}\,\sqrt{b}\,e^{3a-\frac{3bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}-\frac{2\sinh^3(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3/(c + d*x)^(3/2),x]
```

```
[Out] (-3*Sqrt[b]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])
/(4*d^(3/2)) + (Sqrt[b]*E^(-3*a + (3*b*c)/d)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[b
]*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/2)) - (3*Sqrt[b]*E^(a - (b*c)/d)*Sqrt[Pi
]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/2)) + (Sqrt[b]*E^(3*a - (3
*b*c)/d)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/
2)) - (2*Sinh[a + b*x]^3)/(d*Sqrt[c + d*x])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}\,dx &= -\frac{2\sinh^3(a+bx)}{d\sqrt{c+dx}} - \frac{(6b)\int \left(\frac{\cosh(a+bx)}{4\sqrt{c+dx}} - \frac{\cosh(3a+3bx)}{4\sqrt{c+dx}}\right) dx}{d} \\
&= -\frac{2\sinh^3(a+bx)}{d\sqrt{c+dx}} - \frac{(3b)\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx}{2d} + \frac{(3b)\int \frac{\cosh(3a+3bx)}{\sqrt{c+dx}}\,dx}{2d} \\
&= -\frac{2\sinh^3(a+bx)}{d\sqrt{c+dx}} - \frac{(3b)\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{4d} - \frac{(3b)\int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{4d} + \frac{(3b)\int \frac{e^{-i(3ia+3ibx)}}{\sqrt{c+dx}}\,d}{4d} \\
&= -\frac{2\sinh^3(a+bx)}{d\sqrt{c+dx}} + \frac{(3b)\text{Subst}\left(\int e^{i\left(3ia-\frac{3ibc}{d}\right)-\frac{3bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{2d^2} - \frac{(3b)\text{Subst}\left(\int \right.}{} \\
&= -\frac{3\sqrt{b}\,e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} + \frac{\sqrt{b}\,e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 574 vs. 2(246) = 492.

time = 8.42, size = 574, normalized size = 2.33

$$\frac{\sqrt{b}\sqrt{3\pi}\cosh\left(3a-\frac{3bc}{d}\right)\text{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)+\sqrt{b}\sqrt{3\pi}\cosh\left(3a-\frac{3bc}{d}\right)\text{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)+\frac{\sqrt{d}\,e^{-\frac{3b(c+dx)}{d}}\left(\cosh\left(3a-\frac{3bc}{d}\right)-e^{\frac{6b(c+dx)}{d}}\cosh\left(3a-\frac{3bc}{d}\right)-3e^{\frac{2b(c+dx)}{d}}\cosh\left(a-\frac{bc}{d}\right)+3e^{\frac{4b(c+dx)}{d}}\cosh\left(a-\frac{bc}{d}\right)-\sinh\left(3a-\frac{3bc}{d}\right)-e^{\frac{6b(c+dx)}{d}}\sinh\left(3a-\frac{3bc}{d}\right)+\sqrt{3}\,e^{\frac{3b(c+dx)}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{3b(c+dx)}{d}\right)\sinh\left(3a-\frac{3bc}{d}\right)+\sqrt{3}\,e^{\frac{3b(c+dx)}{d}}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{3b(c+dx)}{d}\right)\sinh\left(3a-\frac{3bc}{d}\right)+3e^{\frac{3b(c+dx)}{d}}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{b(c+dx)}{d}\right)\left(\cosh\left(a-\frac{bc}{d}\right)-\sinh\left(a-\frac{bc}{d}\right)\right)+3e^{\frac{2b(c+dx)}{d}}\sinh\left(a-\frac{bc}{d}\right)+3e^{\frac{4b(c+dx)}{d}}\sinh\left(a-\frac{bc}{d}\right)-3e^{\frac{3b(c+dx)}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)\left(\cosh\left(a-\frac{bc}{d}\right)+\sinh\left(a-\frac{bc}{d}\right)\right)\right)}{\sqrt{c+dx}}}{4d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3/(c + d*x)^(3/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[3*Pi]*Cosh[3*a - (3*b*c)/d]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x
])/Sqrt[d]] + Sqrt[b]*Sqrt[3*Pi]*Cosh[3*a - (3*b*c)/d]*Erfi[(Sqrt[3]*Sqrt[b
]*Sqrt[c + d*x])/Sqrt[d]] + (Sqrt[d]*(Cosh[3*a - (3*b*c)/d] - E^((6*b*(c +
d*x))/d)*Cosh[3*a - (3*b*c)/d] - 3*E^((2*b*(c + d*x))/d)*Cosh[a - (b*c)/d]
+ 3*E^((4*b*(c + d*x))/d)*Cosh[a - (b*c)/d] - Sinh[3*a - (3*b*c)/d] - E^((6
*b*(c + d*x))/d)*Sinh[3*a - (3*b*c)/d] + Sqrt[3]*E^((3*b*(c + d*x))/d)*Sqrt
[-((b*(c + d*x))/d)]*Gamma[1/2, (-3*b*(c + d*x))/d]*Sinh[3*a - (3*b*c)/d] +
 Sqrt[3]*E^((3*b*(c + d*x))/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (3*b*(c + d
*x))/d]*Sinh[3*a - (3*b*c)/d] + 3*E^((3*b*(c + d*x))/d)*Sqrt[(b*(c + d*x))/
d]*Gamma[1/2, (b*(c + d*x))/d]*(Cosh[a - (b*c)/d] - Sinh[a - (b*c)/d]) + 3*
E^((2*b*(c + d*x))/d)*Sinh[a - (b*c)/d] + 3*E^((4*b*(c + d*x))/d)*Sinh[a -
(b*c)/d] - 3*E^((3*b*(c + d*x))/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b
*(c + d*x))/d)]*(Cosh[a - (b*c)/d] + Sinh[a - (b*c)/d])))/(E^((3*b*(c + d*x
))/d)*Sqrt[c + d*x]))/(4*d^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{3}\left(bx+a\right)}{\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3/(d*x+c)^(3/2),x)
```

```
[Out] int(sinh(b*x+a)^3/(d*x+c)^(3/2),x)
```

Maxima [A]
time = 0.37, size = 197, normalized size = 0.80

$$\frac{\frac{\sqrt{3}\sqrt{\frac{(dx+c)b}{d}}\,e^{\left(\frac{3\,(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{3\,(dx+c)b}{d}\right)}{\sqrt{dx+c}}-\frac{\sqrt{3}\sqrt{-\frac{(dx+c)b}{d}}\,e^{\left(-\frac{3\,(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{3\,(dx+c)b}{d}\right)}{\sqrt{dx+c}}-\frac{3\sqrt{\frac{(dx+c)b}{d}}\,e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}}+\frac{3\sqrt{-\frac{(dx+c)b}{d}}\,e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(3)*sqrt((d*x + c)*b/d)*e^(3*(b*c - a*d)/d)*gamma(-1/2, 3*(d*x + c
)*b/d)/sqrt(d*x + c) - sqrt(3)*sqrt(-(d*x + c)*b/d)*e^(-3*(b*c - a*d)/d)*ga
mma(-1/2, -3*(d*x + c)*b/d)/sqrt(d*x + c) - 3*sqrt((d*x + c)*b/d)*e^(-a + b
*c/d)*gamma(-1/2, (d*x + c)*b/d)/sqrt(d*x + c) + 3*sqrt(-(d*x + c)*b/d)*e^(
a - b*c/d)*gamma(-1/2, -(d*x + c)*b/d)/sqrt(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1346 vs. $2(182) = 364$.

time = 0.36, size = 1346, normalized size = 5.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
[Out] 1/4*(sqrt(3)*sqrt(pi)*((d*x + c)*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - (
d*x + c)*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + ((d*x + c)*cosh(-3*(b*c -
 a*d)/d) - (d*x + c)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((d*x + c)
*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)*sinh(-3*(b*
c - a*d)/d))*sinh(b*x + a)^2 + 3*((d*x + c)*cosh(b*x + a)^2*cosh(-3*(b*c -
a*d)/d) - (d*x + c)*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*
sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) - sqrt(3)*sqrt(pi)*((d*x + c
)*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)^3*sinh(-
3*(b*c - a*d)/d) + ((d*x + c)*cosh(-3*(b*c - a*d)/d) + (d*x + c)*sinh(-3*(b
*c - a*d)/d))*sinh(b*x + a)^3 + 3*((d*x + c)*cosh(b*x + a)*cosh(-3*(b*c - a
*d)/d) + (d*x + c)*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 +
3*((d*x + c)*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + (d*x + c)*cosh(b*x +
a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x
 + c)*sqrt(-b/d)) - 3*sqrt(pi)*((d*x + c)*cosh(b*x + a)^3*cosh(-(b*c - a*d)
/d) - (d*x + c)*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + ((d*x + c)*cosh(-(b*
c - a*d)/d) - (d*x + c)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((d*x + c
)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)*sinh(-(b*c -
 a*d)/d))*sinh(b*x + a)^2 + 3*((d*x + c)*cosh(b*x + a)^2*cosh(-(b*c - a*d)/
d) - (d*x + c)*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/
d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 3*sqrt(pi)*((d*x + c)*cosh(b*x + a)^3*cos
h(-(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + ((d*x
+ c)*cosh(-(b*c - a*d)/d) + (d*x + c)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3
 + 3*((d*x + c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a
)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((d*x + c)*cosh(b*x + a)^2*cosh
(-(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x
 + a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) - (cosh(b*x + a)^6 + 6*cosh
(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 - 1)*sin
h(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - 3*cosh(b*x + a))*
sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*sinh(b*x +
a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 + cosh(b*
x + a))*sinh(b*x + a) - 1)*sqrt(d*x + c))/((d^2*x + c*d)*cosh(b*x + a)^3 +
3*(d^2*x + c*d)*cosh(b*x + a)^2*sinh(b*x + a) + 3*(d^2*x + c*d)*cosh(b*x +
a)*sinh(b*x + a)^2 + (d^2*x + c*d)*sinh(b*x + a)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3/(d*x+c)**(3/2),x)
```

```
[Out] Integral(sinh(a + b*x)**3/(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^3/(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(a+b\,x)^3}{(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3/(c + d*x)^(3/2),x)
```

```
[Out] int(sinh(a + b*x)^3/(c + d*x)^(3/2), x)
```

3.58 $\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=277

$$\frac{b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}-\frac{b^{3/2}e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\,\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}-\frac{b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{b}\sqrt{}}{\sqrt{}}\right.}{2d^{5/2}}$$

```
[Out] -2/3*sinh(b*x+a)^3/d/(d*x+c)^(3/2)+1/2*b^(3/2)*exp(-a+b*c/d)*erf(b^(1/2)*(d
*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/d^(5/2)-1/2*b^(3/2)*exp(a-b*c/d)*erfi(b^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/d^(5/2)-1/2*b^(3/2)*exp(-3*a+3*b*c/d)*erf(
3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/d^(5/2)+1/2*b^(3/2)
*exp(3*a-3*b*c/d)*erfi(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1
/2)/d^(5/2)-4*b*cosh(b*x+a)*sinh(b*x+a)^2/d^2/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.52, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3395, 3389, 2211, 2235, 2236, 3393}

$$\frac{\sqrt{\pi}\,b^{3/2}e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}-\frac{\sqrt{3\pi}\,b^{3/2}e^{\frac{3bc}{d}-3a}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}-\frac{\sqrt{\pi}\,b^{3/2}e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}+\frac{\sqrt{3\pi}\,b^{3/2}e^{3a-\frac{3bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}-\frac{4b\sinh^2(a+bx)\cosh(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\sinh^3(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3/(c + d*x)^(5/2),x]
```

```
[Out] (b^(3/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2
*d^(5/2)) - (b^(3/2)*E^(-3*a + (3*b*c)/d)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[b]*S
qrt[c + d*x])/Sqrt[d]])/(2*d^(5/2)) - (b^(3/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erf
i[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*d^(5/2)) + (b^(3/2)*E^(3*a - (3*b*c)
/d)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*d^(5/2)) -
 (4*b*Cosh[a + b*x]*Sinh[a + b*x]^2)/(d^2*Sqrt[c + d*x]) - (2*Sinh[a + b*x]
^3)/(3*d*(c + d*x)^(3/2))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{4b\cosh(a+bx)\sinh^2(a+bx)}{d^2\sqrt{c+dx}} - \frac{2\sinh^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{(8b^2)\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx}{d^2} + \frac{(12b^2)\int}{}$$

$$= -\frac{4b\cosh(a+bx)\sinh^2(a+bx)}{d^2\sqrt{c+dx}} - \frac{2\sinh^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{(12ib^2)\int\left(\frac{3i\sinh(a+bx)}{4\sqrt{c+dx}} - \frac{i\sinh(3a}{4\sqrt{c+}\right.}{d^2}$$

$$= -\frac{4b\cosh(a+bx)\sinh^2(a+bx)}{d^2\sqrt{c+dx}} - \frac{2\sinh^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{(8b^2)\,\mathrm{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx\right.}{d^3}$$

$$= -\frac{4b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} + \frac{4b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} - \frac{4b\,\mathrm{co}}{}$$

$$= -\frac{4b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} + \frac{4b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} - \frac{4b\,\mathrm{co}}{}$$

$$= \frac{b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} - \frac{b^{3/2}e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} - \frac{b^{3}}{}$$

Mathematica [A]
time = 2.00, size = 253, normalized size = 0.91

$$\frac{e^{-3\left(a+\frac{bc}{d}\right)}\left(-3\sqrt{3}\,de^{6a}\left(-\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{3b(c+dx)}{d}\right)+3de^{4a+\frac{2bc}{d}}\left(-\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)-3de^{2a+\frac{4bc}{d}}\left(\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},\frac{b(c+dx)}{d}\right)+3\sqrt{3}\,de^{\frac{6bc}{d}}\left(\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},\frac{3b(c+dx)}{d}\right)-4e^{3\left(a+\frac{bc}{d}\right)}\sinh^2(a+bx)(6b(c+dx)\cosh(a+bx)+d\sinh(a+bx))\right)}{6d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3/(c + d*x)^(5/2),x]
```

```
[Out] (-3*Sqrt[3]*d*E^(6*a)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, (-3*b*(c + d*x)
)/d] + 3*d*E^(4*a + (2*b*c)/d)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, -((b*(
c + d*x))/d)] - 3*d*E^(2*a + (4*b*c)/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2,
(b*(c + d*x))/d] + 3*Sqrt[3]*d*E^((6*b*c)/d)*((b*(c + d*x))/d)^(3/2)*Gamma[
1/2, (3*b*(c + d*x))/d] - 4*E^(3*(a + (b*c)/d))*Sinh[a + b*x]^2*(6*b*(c + d
*x)*Cosh[a + b*x] + d*Sinh[a + b*x]))/(6*d^2*E^(3*(a + (b*c)/d))*(c + d*x)^
(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(bx+a)}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3/(d*x+c)^(5/2),x)
```

```
[Out] int(sinh(b*x+a)^3/(d*x+c)^(5/2),x)
```

Maxima [A]
time = 0.36, size = 196, normalized size = 0.71

$$\frac{3\left(\frac{\sqrt{3}\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}-\frac{\sqrt{3}\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}-\frac{\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}+\frac{\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 3/8*(sqrt(3)*((d*x + c)*b/d)^(3/2)*e^(3*(b*c - a*d)/d)*gamma(-3/2, 3*(d*x +
 c)*b/d)/(d*x + c)^(3/2) - sqrt(3)*(-(d*x + c)*b/d)^(3/2)*e^(-3*(b*c - a*d)
/d)*gamma(-3/2, -3*(d*x + c)*b/d)/(d*x + c)^(3/2) - ((d*x + c)*b/d)^(3/2)*e
^(-a + b*c/d)*gamma(-3/2, (d*x + c)*b/d)/(d*x + c)^(3/2) + (-(d*x + c)*b/d)
^(3/2)*e^(a - b*c/d)*gamma(-3/2, -(d*x + c)*b/d)/(d*x + c)^(3/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2059 vs. 2(209) = 418.
time = 0.37, size = 2059, normalized size = 7.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(6*sqrt(3)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*
cosh(-3*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*si
nh(-3*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-3*(b*c - a*d)
/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)
^3 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d
) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*s
inh(b*x + a)^2 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*cosh(-3
*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh(-3*(
b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)
) + 6*sqrt(3)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*cos
h(-3*(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*sinh(
-3*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-3*(b*c - a*d)/d)
 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3
 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) +
 (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh
(b*x + a)^2 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*cosh(-3*(b
```

```
*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh(-3*(b*c
 - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))
 - 6*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*cosh(-(b*c -
 a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*sinh(-(b*c - a*d
)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-(b*c - a*d)/d) - (b*d^2*x^2 +
 2*b*c*d*x + b*c^2)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((b*d^2*x^2 +
 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c
*d*x + b*c^2)*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((b*d
^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) - (b*d^2*x
^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a)
)*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 6*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d
*x + b*c^2)*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x +
 b*c^2)*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*
c^2)*cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-(b*c - a*
d)/d))*sinh(b*x + a)^3 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*c
osh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-(
b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*
x + a)^2*cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x +
a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt
(-b/d)) + ((6*b*d*x + 6*b*c + d)*cosh(b*x + a)^6 + 6*(6*b*d*x + 6*b*c + d)*
cosh(b*x + a)*sinh(b*x + a)^5 + (6*b*d*x + 6*b*c + d)*sinh(b*x + a)^6 - 3*(
2*b*d*x + 2*b*c + d)*cosh(b*x + a)^4 - 3*(2*b*d*x - 5*(6*b*d*x + 6*b*c + d)
*cosh(b*x + a)^2 + 2*b*c + d)*sinh(b*x + a)^4 + 4*(5*(6*b*d*x + 6*b*c + d)*
cosh(b*x + a)^3 - 3*(2*b*d*x + 2*b*c + d)*cosh(b*x + a))*sinh(b*x + a)^3 +
6*b*d*x - 3*(2*b*d*x + 2*b*c - d)*cosh(b*x + a)^2 + 3*(5*(6*b*d*x + 6*b*c +
 d)*cosh(b*x + a)^4 - 2*b*d*x - 6*(2*b*d*x + 2*b*c + d)*cosh(b*x + a)^2 - 2
*b*c + d)*sinh(b*x + a)^2 + 6*b*c + 6*((6*b*d*x + 6*b*c + d)*cosh(b*x + a)^
5 - 2*(2*b*d*x + 2*b*c + d)*cosh(b*x + a)^3 - (2*b*d*x + 2*b*c - d)*cosh(b*
x + a))*sinh(b*x + a) - d)*sqrt(d*x + c))/((d^4*x^2 + 2*c*d^3*x + c^2*d^2)*
cosh(b*x + a)^3 + 3*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a)^2*sinh(b*
x + a) + 3*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a)*sinh(b*x + a)^2 +
(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*sinh(b*x + a)^3)
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3/(d*x+c)**(5/2),x)
```

```
[Out] Integral(sinh(a + b*x)**3/(c + d*x)**(5/2), x)
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^3/(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(a+b\,x)^3}{(c+d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3/(c + d*x)^(5/2),x)
```

```
[Out] int(sinh(a + b*x)^3/(c + d*x)^(5/2), x)
```

3.59 $\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal. Leaf size=331

$$-\frac{b^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{3b^{5/2}e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\,\mathrm{Erf}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{b^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{b}\right.}{\ldots}}{5d^{7/2}}$$

```
[Out] -4/5*b*cosh(b*x+a)*sinh(b*x+a)^2/d^2/(d*x+c)^(3/2)-2/5*sinh(b*x+a)^3/d/(d*x
+c)^(5/2)-1/5*b^(5/2)*exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(
1/2)/d^(7/2)-1/5*b^(5/2)*exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*P
i^(1/2)/d^(7/2)+3/5*b^(5/2)*exp(-3*a+3*b*c/d)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(
1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/d^(7/2)+3/5*b^(5/2)*exp(3*a-3*b*c/d)*erfi(3^
(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/d^(7/2)-16/5*b^2*sinh
(b*x+a)/d^3/(d*x+c)^(1/2)-24/5*b^2*sinh(b*x+a)^3/d^3/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3395, 3378, 3388, 2211, 2235, 2236, 3394}

$$-\frac{\sqrt{\pi}\,b^{5/2}e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{3\sqrt{3\pi}\,b^{5/2}e^{\frac{3bc}{d}-3a}\mathrm{Erf}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{\sqrt{\pi}\,b^{5/2}e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{3\sqrt{3\pi}\,b^{5/2}e^{3a-\frac{3bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{24b^2\sinh^3(a+bx)}{5d^3\sqrt{c+dx}}-\frac{16b^2\sinh(a+bx)}{5d^3\sqrt{c+dx}}-\frac{4b\sinh^2(a+bx)\cosh(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\sinh^3(a+bx)}{5d(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3/(c + d*x)^(7/2),x]
```

```
[Out] -1/5*(b^(5/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]
])/d^(7/2) + (3*b^(5/2)*E^(-3*a + (3*b*c)/d)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[b
]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/2)) - (b^(5/2)*E^(a - (b*c)/d)*Sqrt[Pi]*
Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/2)) + (3*b^(5/2)*E^(3*a - (3
*b*c)/d)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/
2)) - (16*b^2*Sinh[a + b*x])/(5*d^3*Sqrt[c + d*x]) - (4*b*Cosh[a + b*x]*Sin
h[a + b*x]^2)/(5*d^2*(c + d*x)^(3/2)) - (2*Sinh[a + b*x]^3)/(5*d*(c + d*x)^
(5/2)) - (24*b^2*Sinh[a + b*x]^3)/(5*d^3*Sqrt[c + d*x])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{4b\cosh(a+bx)\sinh^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2\sinh^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{(8b^2)\int\frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx}{5d^2} + \frac{(12b^2)\int}{}$$

$$= -\frac{16b^2\sinh(a+bx)}{5d^3\sqrt{c+dx}} - \frac{4b\cosh(a+bx)\sinh^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2\sinh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2\sinh^3(}{5d^3\sqrt{c-}}$$

$$= -\frac{16b^2\sinh(a+bx)}{5d^3\sqrt{c+dx}} - \frac{4b\cosh(a+bx)\sinh^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2\sinh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2\sinh^3(}{5d^3\sqrt{c-}}$$

$$= -\frac{16b^2\sinh(a+bx)}{5d^3\sqrt{c+dx}} - \frac{4b\cosh(a+bx)\sinh^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2\sinh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2\sinh^3(}{5d^3\sqrt{c-}}$$

$$= \frac{8b^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{8b^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{16b^2\,\text{si}}{5d^3\,}$$

$$= -\frac{b^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{3b^{5/2}e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} -$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 3211 vs. $2(331) = 662$.
time = 14.15, size = 3211, normalized size = 9.70

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3/(c + d*x)^(7/2),x]
```

```
[Out] (-3*(Cosh[a]*(-1/30*((-2*E^((b*(c + d*x))/d)*(3*d^2 + 2*b*d*(c + d*x) + 4*b
^2*(c + d*x)^2) + 8*d^2*(-((b*(c + d*x))/d))^(5/2)*Gamma[1/2, -((b*(c + d*x
))/d)] + (-6*d^2 + 4*b*d*(c + d*x) - 8*b^2*(c + d*x)^2 + 8*b*d*E^((b*(c + d
*x))/d)*(c + d*x)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (b*(c + d*x))/d])/E^((
b*(c + d*x))/d))*Sinh[(b*c)/d])/(d^3*(c + d*x)^(5/2)) + (2*Cosh[(b*c)/d]*(-
1/2*(b*(c + d*x)*(2*E^((b*(c + d*x))/d)*(d + 2*b*(c + d*x)) + 4*d*(-((b*(c
+ d*x))/d))^(3/2)*Gamma[1/2, -((b*(c + d*x))/d)] + (2*(d - 2*b*(c + d*x) +
2*d*E^((b*(c + d*x))/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (b*(c + d*x))/d]
))/E^((b*(c + d*x))/d))) - 3*d^2*Sinh[(b*(c + d*x))/d]))/(15*d^3*(c + d*x)^
(5/2))) + Sinh[a]*((Cosh[(b*c)/d]*(-2*E^((b*(c + d*x))/d)*(3*d^2 + 2*b*d*(c
 + d*x) + 4*b^2*(c + d*x)^2) + 8*d^2*(-((b*(c + d*x))/d))^(5/2)*Gamma[1/2,
-((b*(c + d*x))/d)] + (-6*d^2 + 4*b*d*(c + d*x) - 8*b^2*(c + d*x)^2 + 8*b*d
*E^((b*(c + d*x))/d)*(c + d*x)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (b*(c + d
```

x))/d])/E^((b(c + d*x))/d)))/(30*d^3*(c + d*x)^(5/2)) - (2*Sinh[(b*c)/d]*
(-1/2*(b*(c + d*x)*(2*E^((b*(c + d*x))/d)*(d + 2*b*(c + d*x)) + 4*d*(-((b*(
c + d*x))/d))^(3/2)*Gamma[1/2, -((b*(c + d*x))/d)] + (2*(d - 2*b*(c + d*x)
+ 2*d*E^((b*(c + d*x))/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (b*(c + d*x))/
d]))/E^((b*(c + d*x))/d))) - 3*d^2*Sinh[(b*(c + d*x))/d]))/(15*d^3*(c + d*x
)^(5/2)))))/4 + (-(Sinh[3*a]*(-1/10*((1 + 2*Cosh[(2*b*c)/d])*(-2*E^((3*b*(c
 + d*x))/d)*(d^2 + 2*b*d*(c + d*x) + 12*b^2*(c + d*x)^2) + 24*Sqrt[3]*d^2*(
-((b*(c + d*x))/d))^(5/2)*Gamma[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*
(c + d*x) - 24*b^2*(c + d*x)^2 + 24*Sqrt[3]*d^2*E^((3*b*(c + d*x))/d)*((b*(
c + d*x))/d)^(5/2)*Gamma[1/2, (3*b*(c + d*x))/d])/E^((3*b*(c + d*x))/d))*Si
nh[(b*c)/d])/(d^3*(c + d*x)^(5/2)) - (2*Cosh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/
d])*(-6*b^(5/2)*Sqrt[3*Pi]*(c + d*x)^(5/2)*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*
x])/Sqrt[d]] - 6*b^(5/2)*Sqrt[3*Pi]*(c + d*x)^(5/2)*Erfi[(Sqrt[3]*Sqrt[b]*S
qrt[c + d*x])/Sqrt[d]] + Sqrt[d]*(2*b*d*(c + d*x)*Cosh[(3*b*(c + d*x))/d] +
 (d^2 + 12*b^2*(c + d*x)^2)*Sinh[(3*b*(c + d*x))/d])))/(5*d^(7/2)*(c + d*x)
^(5/2)))) - Cosh[3*a]*((Cosh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*(-2*E^((3*b*
(c + d*x))/d)*(d^2 + 2*b*d*(c + d*x) + 12*b^2*(c + d*x)^2) + 24*Sqrt[3]*d^2
(-((b(c + d*x))/d))^(5/2)*Gamma[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*
d*(c + d*x) - 24*b^2*(c + d*x)^2 + 24*Sqrt[3]*d^2*E^((3*b*(c + d*x))/d)*((b
*(c + d*x))/d)^(5/2)*Gamma[1/2, (3*b*(c + d*x))/d])/E^((3*b*(c + d*x))/d)))
/(10*d^3*(c + d*x)^(5/2)) + (2*(1 + 2*Cosh[(2*b*c)/d])*Sinh[(b*c)/d]*(-6*b^
(5/2)*Sqrt[3*Pi]*(c + d*x)^(5/2)*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d
]] - 6*b^(5/2)*Sqrt[3*Pi]*(c + d*x)^(5/2)*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*
x])/Sqrt[d]] + Sqrt[d]*(2*b*d*(c + d*x)*Cosh[(3*b*(c + d*x))/d] + (d^2 + 12
b^2(c + d*x)^2)*Sinh[(3*b*(c + d*x))/d])))/(5*d^(7/2)*(c + d*x)^(5/2))))/
8 + (Sinh[3*a]*(-1/10*((1 + 2*Cosh[(2*b*c)/d])*(-2*E^((3*b*(c + d*x))/d)*(d
^2 + 2*b*d*(c + d*x) + 12*b^2*(c + d*x)^2) + 24*Sqrt[3]*d^2*(-((b*(c + d*x)
)/d))^(5/2)*Gamma[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24
b^2(c + d*x)^2 + 24*Sqrt[3]*d^2*E^((3*b*(c + d*x))/d)*((b*(c + d*x))/d)^(
5/2)*Gamma[1/2, (3*b*(c + d*x))/d])/E^((3*b*(c + d*x))/d))*Sinh[(b*c)/d])/(
d^3*(c + d*x)^(5/2)) - (2*Cosh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*(-6*b^(5/2
)*Sqrt[3*Pi]*(c + d*x)^(5/2)*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] -
 6*b^(5/2)*Sqrt[3*Pi]*(c + d*x)^(5/2)*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/
Sqrt[d]] + Sqrt[d]*(2*b*d*(c + d*x)*Cosh[(3*b*(c + d*x))/d] + (d^2 + 12*b^2
*(c + d*x)^2)*Sinh[(3*b*(c + d*x))/d])))/(5*d^(7/2)*(c + d*x)^(5/2))) + Cos
h[3*a]*((Cosh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*(-2*E^((3*b*(c + d*x))/d)*(
d^2 + 2*b*d*(c + d*x) + 12*b^2*(c + d*x)^2) + 24*Sqrt[3]*d^2*(-((b*(c + d*x
))/d))^(5/2)*Gamma[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 2
4*b^2*(c + d*x)^2 + 24*Sqrt[3]*d^2*E^((3*b*(c + d*x))/d)*((b*(c + d*x))/d)^
(5/2)*Gamma[1/2, (3*b*(c + d*x))/d])/E^((3*b*(c + d*x))/d)))/(10*d^3*(c + d
x)^(5/2)) + (2(1 + 2*Cosh[(2*b*c)/d])*Sinh[(b*c)/d]*(-6*b^(5/2)*Sqrt[3*Pi
]*(c + d*x)^(5/2)*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - 6*b^(5/2)*
Sqrt[3*Pi]*(c + d*x)^(5/2)*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] +
Sqrt[d]*(2*b*d*(c + d*x)*Cosh[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*(c + d*x)^
2)*Sinh[(3*b*(c + d*x))/d])))/(5*d^(7/2)*(c + d*x)^(5/2))))/8 + (Cosh[3*a]*

```
(-1/10*((1 + 2*Cosh[(2*b*c)/d])*(-2*E^((3*b*(c + d*x))/d)*(d^2 + 2*b*d*(c +
 d*x) + 12*b^2*(c + d*x)^2) + 24*Sqrt[3]*d^2*(-((b*(c + d*x))/d))^(5/2)*Gam
ma[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24*b^2*(c + d*x)^
2 + 24*Sqrt[3]*d^2*E^((3*b*(c + d*x))/d)*((b*(c + d*x))/d)^(5/2)*Gamma[1/2,
 (3*b*(c + d*x))/d])/E^((3*b*(c + d*x))/d))*Sinh[(b*c)/d])/(d^3*(c + d*x)^(
5/2)) - (2*Cosh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*(-6*b^(5/2)*Sqrt[3*Pi]*(c
 + d*x)^(5/2)*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - 6*b^(5/2)*Sqrt
[3*Pi]*(c + d*x)^(5/2)*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + Sqrt
[d]*(2*b*d*(c + d*x)*Cosh[(3*b*(c + d*x))/d] + ...
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(bx+a)}{(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3/(d*x+c)^(7/2),x)
```

```
[Out] int(sinh(b*x+a)^3/(d*x+c)^(7/2),x)
```

Maxima [A]
time = 0.36, size = 197, normalized size = 0.60

$$\frac{3\left(\frac{3\sqrt{3}\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}-\frac{3\sqrt{3}\left(-\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(-\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},-\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}-\frac{\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{5}{2},\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}+\frac{\left(-\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{5}{2},-\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 3/8*(3*sqrt(3)*((d*x + c)*b/d)^(5/2)*e^(3*(b*c - a*d)/d)*gamma(-5/2, 3*(d*x
 + c)*b/d)/(d*x + c)^(5/2) - 3*sqrt(3)*(-(d*x + c)*b/d)^(5/2)*e^(-3*(b*c -
a*d)/d)*gamma(-5/2, -3*(d*x + c)*b/d)/(d*x + c)^(5/2) - ((d*x + c)*b/d)^(5/
2)*e^(-a + b*c/d)*gamma(-5/2, (d*x + c)*b/d)/(d*x + c)^(5/2) + (-(d*x + c)*
b/d)^(5/2)*e^(a - b*c/d)*gamma(-5/2, -(d*x + c)*b/d)/(d*x + c)^(5/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3286 vs. $2(253) = 506$.
time = 0.47, size = 3286, normalized size = 9.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/20*(12*sqrt(3)*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
 b^2*c^3)*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d
^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) +
((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-3*(b*c - a
*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-3*
(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2
*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) - (b^2*d^3*x^3 + 3
*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-3*(b*c - a*d)
/d))*sinh(b*x + a)^2 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
b^2*c^3)*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^
2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*si
nh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) - 12*sqrt(3)*sq
rt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x
+ a)^3*cosh(-3*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*
d*x + b^2*c^3)*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b
^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-3*(b*c - a*d)/d) + (b^2*d^3*x
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-3*(b*c - a*d)/d))*sin
h(b*x + a)^3 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)
*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^
2 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x +
 a)^2*cosh(-3*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-
b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d)) - 4*sqrt(pi)*((b^2*d^3*x^3 + 3*b
^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d
) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)
^3*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
 b^2*c^3)*cosh(-(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2
*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((b^2*d^3*x^3 + 3
*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-(b*c - a*d)/d
) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)
*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
+ 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) - (b^2*d^3*
x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-(b*c
 - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 4*sqrt(
pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a
)^3*cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
 b^2*c^3)*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b^2*c*d^
2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^
2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^
3 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x +
 a)*cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
 b^2*c^3)*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((b^2*d^3
*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*cosh(-(b*
c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos
```

```
h(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*x +
 c)*sqrt(-b/d)) - ((12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^2
*c*d + b*d^2)*x)*cosh(b*x + a)^6 + 6*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d
 + d^2 + 2*(12*b^2*c*d + b*d^2)*x)*cosh(b*x + a)*sinh(b*x + a)^5 + (12*b^2*
d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^2*c*d + b*d^2)*x)*sinh(b*x +
 a)^6 - 12*b^2*d^2*x^2 - (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(
4*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^4 - (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*
d - 15*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^2*c*d + b*d^2
)*x)*cosh(b*x + a)^2 + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*sinh(b*x + a)^4 - 1
2*b^2*c^2 + 4*(5*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^2*c
*d + b*d^2)*x)*cosh(b*x + a)^3 - (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d
^2 + 2*(4*b^2*c*d + b*d^2)*x)*cosh(b*x + a))*sinh(b*x + a)^3 + 2*b*c*d + (4
*b^2*d^2*x^2 + 4*b^2*c^2 - 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d - b*d^2)*x)*cosh(
b*x + a)^2 + (4*b^2*d^2*x^2 + 15*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d
^2 + 2*(12*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^4 + 4*b^2*c^2 - 2*b*c*d - 6*(4
*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*cosh(
b*x + a)^2 + 3*d^2 + 2*(4*b^2*c*d - b*d^2)*x)*sinh(b*x + a)^2 - d^2 - 2*(12
*b^2*c*d - b*d^2)*x + 2*(3*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2
*(12*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^5 - 2*(4*b^2*d^2*x^2 + 4*b^2*c^2 + 2
*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^3 + (4*b^2*d^2*x^2
+ 4*b^2*c^2 - 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d - b*d^2)*x)*cosh(b*x + a))*sin
h(b*x + a))*sqrt(d*x + c))/((d^6*x^3 + 3*c*d^5*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3/(d*x+c)**(7/2),x)
[Out] Integral(sinh(a + b*x)**3/(c + d*x)**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^3/(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(a+b\,x)^3}{(c+d\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3/(c + d*x)^(7/2),x)
[Out] int(sinh(a + b*x)^3/(c + d*x)^(7/2), x)
```

3.60 $\int (dx)^{3/2} \sinh(fx)\,dx$

Optimal. Leaf size=111

$$\frac{(dx)^{3/2}\cosh(fx)}{f}-\frac{3d^{3/2}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}}+\frac{3d^{3/2}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}}-\frac{3d\sqrt{dx}\,\sinh(fx)}{2f^2}$$

```
[Out] (d*x)^(3/2)*cosh(f*x)/f-3/8*d^(3/2)*erf(f^(1/2)*(d*x)^(1/2)/d^(1/2))*Pi^(1/
2)/f^(5/2)+3/8*d^(3/2)*erfi(f^(1/2)*(d*x)^(1/2)/d^(1/2))*Pi^(1/2)/f^(5/2)-3
/2*d*sinh(f*x)*(d*x)^(1/2)/f^2
```

Rubi [A]
time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3377, 3389, 2211, 2235, 2236}

$$-\frac{3\sqrt{\pi}\,d^{3/2}\mathrm{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}}+\frac{3\sqrt{\pi}\,d^{3/2}\mathrm{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}}-\frac{3d\sqrt{dx}\,\sinh(fx)}{2f^2}+\frac{(dx)^{3/2}\cosh(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*Sinh[f*x],x]
```

```
[Out] ((d*x)^(3/2)*Cosh[f*x])/f - (3*d^(3/2)*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqr
t[d]])/(8*f^(5/2)) + (3*d^(3/2)*Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])
/(8*f^(5/2)) - (3*d*Sqrt[d*x]*Sinh[f*x])/(2*f^2)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\int (dx)^{3/2}\sinh(fx)\,dx &= \frac{(dx)^{3/2}\cosh(fx)}{f} - \frac{(3d)\int \sqrt{dx}\,\cosh(fx)\,dx}{2f} \\
&= \frac{(dx)^{3/2}\cosh(fx)}{f} - \frac{3d\sqrt{dx}\,\sinh(fx)}{2f^2} + \frac{(3d^2)\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx}{4f^2} \\
&= \frac{(dx)^{3/2}\cosh(fx)}{f} - \frac{3d\sqrt{dx}\,\sinh(fx)}{2f^2} - \frac{(3d^2)\int \frac{e^{-fx}}{\sqrt{dx}}\,dx}{8f^2} + \frac{(3d^2)\int \frac{e^{fx}}{\sqrt{dx}}\,dx}{8f^2} \\
&= \frac{(dx)^{3/2}\cosh(fx)}{f} - \frac{3d\sqrt{dx}\,\sinh(fx)}{2f^2} - \frac{(3d)\text{Subst}\left(\int e^{-\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{4f^2} + \frac{(3d)\text{Sul}}{} \\
&= \frac{(dx)^{3/2}\cosh(fx)}{f} - \frac{3d^{3/2}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} + \frac{3d^{3/2}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} - \frac{3d\sqrt{}}{}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 0.45

$$\frac{d^2\left(\sqrt{-fx}\,\Gamma\left(\frac{5}{2},-fx\right)+\sqrt{fx}\,\Gamma\left(\frac{5}{2},fx\right)\right)}{2f^3\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*Sinh[f*x],x]
[Out] (d^2*(Sqrt[-(f*x)]*Gamma[5/2, -(f*x)] + Sqrt[f*x]*Gamma[5/2, f*x]))/(2*f^3*
Sqrt[d*x])
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 132, normalized size = 1.19

method	result
meijerg	$-\frac{2(dx)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\left(-\frac{\sqrt{x}\,\sqrt{2}\,(if)^{\frac{7}{2}}(-14fx+21)e^{fx}}{112\sqrt{\pi}\,f^3}+\frac{\sqrt{x}\,\sqrt{2}\,(if)^{\frac{7}{2}}(14fx+21)e^{-fx}}{112\sqrt{\pi}\,f^3}-\frac{3(if)^{\frac{7}{2}}\sqrt{2}\,\mathrm{erf}\left(\sqrt{x}\,\sqrt{f}\right)}{32f^{\frac{7}{2}}}+\frac{3(if}{}\right.}{x^{\frac{3}{2}}(if)^{\frac{3}{2}}f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*sinh(f*x),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(d*x)^(3/2)/x^(3/2)*2^(1/2)/(I*f)^(3/2)*Pi^(1/2)/f*(-1/112/Pi^(1/2)*x^(1
/2)*2^(1/2)*(I*f)^(7/2)*(-14*f*x+21)/f^3*exp(f*x)+1/112/Pi^(1/2)*x^(1/2)*2^
(1/2)*(I*f)^(7/2)*(14*f*x+21)/f^3*exp(-f*x)-3/32*(I*f)^(7/2)*2^(1/2)/f^(7/2
)*erf(x^(1/2)*f^(1/2))+3/32*(I*f)^(7/2)*2^(1/2)/f^(7/2)*erfi(x^(1/2)*f^(1/2
)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(77) = 154$.
time = 0.26, size = 175, normalized size = 1.58

$$\frac{16\,(dx)^{\frac{5}{2}}\sinh\left(fx\right)-\frac{f\left(\frac{15\sqrt{\pi}\,d^3\,\mathrm{erf}\left(\sqrt{dx}\,\sqrt{\frac{f}{d}}\right)}{f^3\sqrt{\frac{f}{d}}}-\frac{15\sqrt{\pi}\,d^3\,\mathrm{erf}\left(\sqrt{dx}\,\sqrt{-\frac{f}{d}}\right)}{f^3\sqrt{-\frac{f}{d}}}+\frac{2\left(4\,(dx)^{\frac{5}{2}}df^2-10\,(dx)^{\frac{3}{2}}d^2f+15\sqrt{dx}\,d^3\right)e^{(fx)}}{f^3}-\frac{2\left(4\,(dx)^{\frac{5}{2}}df^2+10\,(dx)^{\frac{3}{2}}d^2f+15\sqrt{dx}\,d^3\right)e^{(-fx)}}{f^3}\right)}{d}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*sinh(f*x),x, algorithm="maxima")
```

```
[Out] 1/40*(16*(d*x)^(5/2)*sinh(f*x) - f*(15*sqrt(pi)*d^3*erf(sqrt(d*x)*sqrt(f/d)
)/(f^3*sqrt(f/d)) - 15*sqrt(pi)*d^3*erf(sqrt(d*x)*sqrt(-f/d))/(f^3*sqrt(-f/
d)) + 2*(4*(d*x)^(5/2)*d*f^2 - 10*(d*x)^(3/2)*d^2*f + 15*sqrt(d*x)*d^3)*e^(
f*x)/f^3 - 2*(4*(d*x)^(5/2)*d*f^2 + 10*(d*x)^(3/2)*d^2*f + 15*sqrt(d*x)*d^3
)*e^(-f*x)/f^3)/d)/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(77) = 154$.
time = 0.34, size = 189, normalized size = 1.70

$$-\frac{3\sqrt{\pi}\left(d^2\cosh\left(fx\right)+d^2\sinh\left(fx\right)\right)\sqrt{\frac{f}{d}}\,\mathrm{erf}\left(\sqrt{dx}\,\sqrt{\frac{f}{d}}\right)+3\sqrt{\pi}\left(d^2\cosh\left(fx\right)+d^2\sinh\left(fx\right)\right)\sqrt{-\frac{f}{d}}\,\mathrm{erf}\left(\sqrt{dx}\,\sqrt{-\frac{f}{d}}\right)-2\left(2\,df^2x+\left(2\,df^2x-3\,df\right)\cosh\left(fx\right)^2+2\left(2\,df^2x-3\,df\right)\cosh\left(fx\right)\sinh\left(fx\right)+\left(2\,df^2x-3\,df\right)\sinh\left(fx\right)^2+3\,df\right)\sqrt{dx}}{8\left(f^3\cosh\left(fx\right)+f^3\sinh\left(fx\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*sinh(f*x),x, algorithm="fricas")
```

```
[Out] -1/8*(3*sqrt(pi)*(d^2*cosh(f*x) + d^2*sinh(f*x))*sqrt(f/d)*erf(sqrt(d*x)*sq
rt(f/d)) + 3*sqrt(pi)*(d^2*cosh(f*x) + d^2*sinh(f*x))*sqrt(-f/d)*erf(sqrt(d
```

```
*x)*sqrt(-f/d)) - 2*(2*d*f^2*x + (2*d*f^2*x - 3*d*f)*cosh(f*x)^2 + 2*(2*d*f
^2*x - 3*d*f)*cosh(f*x)*sinh(f*x) + (2*d*f^2*x - 3*d*f)*sinh(f*x)^2 + 3*d*f
)*sqrt(d*x))/(f^3*cosh(f*x) + f^3*sinh(f*x))
```

Sympy [C] Result contains complex when optimal does not.
time = 13.77, size = 133, normalized size = 1.20

$$\frac{7d^{\frac{3}{2}}x^{\frac{3}{2}}\cosh{(fx)}\Gamma\left(\frac{7}{4}\right)}{4f\Gamma\left(\frac{11}{4}\right)}-\frac{21d^{\frac{3}{2}}\sqrt{x}\sinh{(fx)}\Gamma\left(\frac{7}{4}\right)}{8f^2\Gamma\left(\frac{11}{4}\right)}+\frac{21\sqrt{2}\sqrt{\pi}d^{\frac{3}{2}}e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{7}{4}\right)}{16f^{\frac{5}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*sinh(f*x),x)
```

```
[Out] 7*d**(3/2)*x**(3/2)*cosh(f*x)*gamma(7/4)/(4*f*gamma(11/4)) - 21*d**(3/2)*sq
rt(x)*sinh(f*x)*gamma(7/4)/(8*f**2*gamma(11/4)) + 21*sqrt(2)*sqrt(pi)*d**(3
/2)*exp(-3*I*pi/4)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)*exp(I*pi/4)/sqrt(pi))*g
amma(7/4)/(16*f**(5/2)*gamma(11/4))
```

Giac [A]
time = 0.44, size = 146, normalized size = 1.32

$$\frac{1}{8}d\left(\frac{\frac{3\sqrt{\pi}d^3\operatorname{erf}\left(-\frac{\sqrt{df}\sqrt{dx}}{d}\right)}{\sqrt{df}f^2}+\frac{2\left(2\sqrt{dx}d^2fx+3\sqrt{dx}d^2\right)e^{(-fx)}}{f^2}}{d^2}-\frac{\frac{3\sqrt{\pi}d^3\operatorname{erf}\left(-\frac{\sqrt{-df}\sqrt{dx}}{d}\right)}{\sqrt{-df}f^2}-\frac{2\left(2\sqrt{dx}d^2fx-3\sqrt{dx}d^2\right)e^{(fx)}}{f^2}}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*sinh(f*x),x, algorithm="giac")
```

```
[Out] 1/8*d*((3*sqrt(pi)*d^3*erf(-sqrt(d*f)*sqrt(d*x)/d)/(sqrt(d*f)*f^2) + 2*(2*s
qrt(d*x)*d^2*f*x + 3*sqrt(d*x)*d^2)*e^(-f*x)/f^2)/d^2 - (3*sqrt(pi)*d^3*erf
(-sqrt(-d*f)*sqrt(d*x)/d)/(sqrt(-d*f)*f^2) - 2*(2*sqrt(d*x)*d^2*f*x - 3*sqr
t(d*x)*d^2)*e^(f*x)/f^2)/d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh(f\,x)\,{(d\,x)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x)*(d*x)^(3/2),x)
```

```
[Out] int(sinh(f*x)*(d*x)^(3/2), x)
```

3.61 $\int \sqrt{dx} \sinh(fx) \, dx$

Optimal. Leaf size=92

$$\frac{\sqrt{dx}\cosh(fx)}{f} - \frac{\sqrt{d}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}} - \frac{\sqrt{d}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}}$$

```
[Out] -1/4*erf(f^(1/2)*(d*x)^(1/2)/d^(1/2))*d^(1/2)*Pi^(1/2)/f^(3/2)-1/4*erfi(f^(
1/2)*(d*x)^(1/2)/d^(1/2))*d^(1/2)*Pi^(1/2)/f^(3/2)+cosh(f*x)*(d*x)^(1/2)/f
```

Rubi [A]
time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3377, 3388, 2211, 2235, 2236}

$$-\frac{\sqrt{\pi}\sqrt{d}\,\mathrm{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}} - \frac{\sqrt{\pi}\sqrt{d}\,\mathrm{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}} + \frac{\sqrt{dx}\cosh(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*Sinh[f*x],x]
```

```
[Out] (Sqrt[d*x]*Cosh[f*x])/f - (Sqrt[d]*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]
])/(4*f^(3/2)) - (Sqrt[d]*Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(4*f^
(3/2))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{dx}\,\sinh(fx)\,dx &= \frac{\sqrt{dx}\,\cosh(fx)}{f} - \frac{d\int \frac{\cosh(fx)}{\sqrt{dx}}\,dx}{2f}\\
&= \frac{\sqrt{dx}\,\cosh(fx)}{f} - \frac{d\int \frac{e^{-fx}}{\sqrt{dx}}\,dx}{4f} - \frac{d\int \frac{e^{fx}}{\sqrt{dx}}\,dx}{4f}\\
&= \frac{\sqrt{dx}\,\cosh(fx)}{f} - \frac{\text{Subst}\left(\int e^{-\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{2f} - \frac{\text{Subst}\left(\int e^{\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{2f}\\
&= \frac{\sqrt{dx}\,\cosh(fx)}{f} - \frac{\sqrt{d}\,\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{f}\,\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}} - \frac{\sqrt{d}\,\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{f}\,\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 0.53

$$\frac{d\left(-\sqrt{-fx}\,\Gamma\left(\frac{3}{2},-fx\right)+\sqrt{fx}\,\Gamma\left(\frac{3}{2},fx\right)\right)}{2f^2\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*Sinh[f*x],x]
```

```
[Out] (d*(-(Sqrt[-(f*x)]*Gamma[3/2, -(f*x)]) + Sqrt[f*x]*Gamma[3/2, f*x]))/(2*f^2
*Sqrt[d*x])
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 120, normalized size = 1.30

method	result

meijerg	$-\frac{\sqrt{\pi}\,\sqrt{dx}\,\sqrt{2}\left(\frac{\sqrt{x}\,\sqrt{2}\,(if)^{\frac{5}{2}}\mathrm{e}^{-fx}}{4\sqrt{\pi}\,f^{2}}+\frac{\sqrt{x}\,\sqrt{2}\,(if)^{\frac{5}{2}}\mathrm{e}^{fx}}{4\sqrt{\pi}\,f^{2}}-\frac{(if)^{\frac{5}{2}}\sqrt{2}\,\mathrm{erf}\left(\sqrt{x}\,\sqrt{f}\right)}{8f^{\frac{5}{2}}}-\frac{(if)^{\frac{5}{2}}\sqrt{2}\,\mathrm{erfi}\left(\sqrt{x}\,\sqrt{f}\right.}{8f^{\frac{5}{2}}}\right.}{\sqrt{x}\,\sqrt{if}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x)*(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -Pi^(1/2)*(d*x)^(1/2)/x^(1/2)*2^(1/2)/(I*f)^(1/2)/f*(1/4/Pi^(1/2)*x^(1/2)*2
^(1/2)*(I*f)^(5/2)/f^2*exp(-f*x)+1/4/Pi^(1/2)*x^(1/2)*2^(1/2)*(I*f)^(5/2)/f
^2*exp(f*x)-1/8*(I*f)^(5/2)*2^(1/2)/f^(5/2)*erf(x^(1/2)*f^(1/2))-1/8*(I*f)^
(5/2)*2^(1/2)/f^(5/2)*erfi(x^(1/2)*f^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(62) = 124$.
time = 0.29, size = 149, normalized size = 1.62

$$\frac{8\,(dx)^{\frac{3}{2}}\sinh(fx)-\frac{f\left(\frac{3\sqrt{\pi}\,d^{2}\,\mathrm{erf}\left(\sqrt{dx}\,\sqrt{\frac{f}{d}}\right)}{f^{2}\sqrt{\frac{f}{d}}}+\frac{3\sqrt{\pi}\,d^{2}\,\mathrm{erf}\left(\sqrt{dx}\,\sqrt{-\frac{f}{d}}\right)}{f^{2}\sqrt{-\frac{f}{d}}}+\frac{2\left(2\,(dx)^{\frac{3}{2}}df-3\sqrt{dx}\,d^{2}\right)e^{(fx)}}{f^{2}}-\frac{2\left(2\,(dx)^{\frac{3}{2}}df+3\sqrt{dx}\,d^{2}\right)e^{(-fx)}}{f^{2}}\right)}{d}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(8*(d*x)^(3/2)*sinh(f*x) - f*(3*sqrt(pi)*d^2*erf(sqrt(d*x)*sqrt(f/d))/
(f^2*sqrt(f/d)) + 3*sqrt(pi)*d^2*erf(sqrt(d*x)*sqrt(-f/d))/(f^2*sqrt(-f/d))
 + 2*(2*(d*x)^(3/2)*d*f - 3*sqrt(d*x)*d^2)*e^(f*x)/f^2 - 2*(2*(d*x)^(3/2)*d
*f + 3*sqrt(d*x)*d^2)*e^(-f*x)/f^2)/d)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(62) = 124$.
time = 0.58, size = 137, normalized size = 1.49

$$-\frac{\sqrt{\pi}\,(d\cosh(fx)+d\sinh(fx))\sqrt{\frac{f}{d}}\,\mathrm{erf}\left(\sqrt{dx}\,\sqrt{\frac{f}{d}}\right)-\sqrt{\pi}\,(d\cosh(fx)+d\sinh(fx))\sqrt{-\frac{f}{d}}\,\mathrm{erf}\left(\sqrt{dx}\,\sqrt{-\frac{f}{d}}\right)-2\left(f\cosh(fx)^{2}+2\,f\cosh(fx)\sinh(fx)+f\sinh(fx)^{2}+f\right)\sqrt{dx}}{4\left(f^{2}\cosh(fx)+f^{2}\sinh(fx)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*(d*cosh(f*x) + d*sinh(f*x))*sqrt(f/d)*erf(sqrt(d*x)*sqrt(f/d
)) - sqrt(pi)*(d*cosh(f*x) + d*sinh(f*x))*sqrt(-f/d)*erf(sqrt(d*x)*sqrt(-f/
d)) - 2*(f*cosh(f*x)^2 + 2*f*cosh(f*x)*sinh(f*x) + f*sinh(f*x)^2 + f)*sqrt(
d*x))/(f^2*cosh(f*x) + f^2*sinh(f*x))
```

Sympy [C] Result contains complex when optimal does not.
time = 1.02, size = 99, normalized size = 1.08

$$\frac{5\sqrt{d}\,\sqrt{x}\,\cosh{(fx)}\Gamma\left(\frac{5}{4}\right)}{4f\Gamma\left(\frac{9}{4}\right)}-\frac{5\sqrt{2}\,\sqrt{\pi}\,\sqrt{d}\,e^{-\frac{i\pi}{4}}C\left(\frac{\sqrt{2}\,\sqrt{f}\,\sqrt{x}\,e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{8f^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)*(d*x)**(1/2),x)
```

```
[Out] 5*sqrt(d)*sqrt(x)*cosh(f*x)*gamma(5/4)/(4*f*gamma(9/4)) - 5*sqrt(2)*sqrt(pi
)*sqrt(d)*exp(-I*pi/4)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)*exp(I*pi/4)/sqrt(pi
))*gamma(5/4)/(8*f**(3/2)*gamma(9/4))
```

Giac [A]
time = 0.42, size = 108, normalized size = 1.17

$$\frac{\frac{\sqrt{\pi}\,d^2\operatorname{erf}\left(-\frac{\sqrt{df}\,\sqrt{dx}}{d}\right)}{\sqrt{df}\,f}+\frac{2\sqrt{dx}\,de^{(-fx)}}{f}}{4\,d}+\frac{\frac{\sqrt{\pi}\,d^2\operatorname{erf}\left(-\frac{\sqrt{-df}\,\sqrt{dx}}{d}\right)}{\sqrt{-df}\,f}+\frac{2\sqrt{dx}\,de^{(fx)}}{f}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(pi)*d^2*erf(-sqrt(d*f)*sqrt(d*x)/d)/(sqrt(d*f)*f) + 2*sqrt(d*x)*d
*e^(-f*x)/f)/d + 1/4*(sqrt(pi)*d^2*erf(-sqrt(-d*f)*sqrt(d*x)/d)/(sqrt(-d*f)
*f) + 2*sqrt(d*x)*d*e^(f*x)/f)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh(f\,x)\,\sqrt{d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x)*(d*x)^(1/2),x)
```

```
[Out] int(sinh(f*x)*(d*x)^(1/2), x)
```

3.62 $\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx$

Optimal. Leaf size=77

$$-\frac{\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{f}\,\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\,\sqrt{f}}+\frac{\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{f}\,\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\,\sqrt{f}}$$

```
[Out] -1/2*erf(f^(1/2)*(d*x)^(1/2)/d^(1/2))*Pi^(1/2)/d^(1/2)/f^(1/2)+1/2*erfi(f^(
1/2)*(d*x)^(1/2)/d^(1/2))*Pi^(1/2)/d^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3389, 2211, 2235, 2236}

$$\frac{\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{f}\,\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\,\sqrt{f}}-\frac{\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{f}\,\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[f*x]/Sqrt[d*x],x]
```

```
[Out] -1/2*(Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[d]*Sqrt[f]) + (Sqrt[
Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[d]*Sqrt[f])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx &= -\left(\frac{1}{2}\int \frac{e^{-fx}}{\sqrt{dx}}\,dx\right)+\frac{1}{2}\int \frac{e^{fx}}{\sqrt{dx}}\,dx\\ &= -\frac{\text{Subst}\left(\int e^{-\frac{fx^2}{d}}\,dx,x,\sqrt{dx}\right)}{d}+\frac{\text{Subst}\left(\int e^{\frac{fx^2}{d}}\,dx,x,\sqrt{dx}\right)}{d}\\ &= -\frac{\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\,\sqrt{f}}+\frac{\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\,\sqrt{f}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 0.61

$$\frac{\sqrt{-fx}\,\Gamma\left(\frac{1}{2},-fx\right)+\sqrt{fx}\,\Gamma\left(\frac{1}{2},fx\right)}{2f\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[f*x]/Sqrt[d*x],x]
```

```
[Out] (Sqrt[-(f*x)]*Gamma[1/2, -(f*x)] + Sqrt[f*x]*Gamma[1/2, f*x])/(2*f*Sqrt[d*x
])
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 71, normalized size = 0.92

method	result	size
meijerg	$-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{if}\left(-\frac{(if)^{\frac{3}{2}}\sqrt{2}\,\text{erf}\left(\sqrt{x}\,\sqrt{f}\right)}{2f^{\frac{3}{2}}}+\frac{(if)^{\frac{3}{2}}\sqrt{2}\,\text{erfi}\left(\sqrt{x}\,\sqrt{f}\right)}{2f^{\frac{3}{2}}}\right)}{2\sqrt{dx}\,f}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*Pi^(1/2)/(d*x)^(1/2)*x^(1/2)*2^(1/2)*(I*f)^(1/2)/f*(-1/2*(I*f)^(3/2)*2
^(1/2)/f^(3/2)*erf(x^(1/2)*f^(1/2))+1/2*(I*f)^(3/2)*2^(1/2)/f^(3/2)*erfi(x^
(1/2)*f^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(49) = 98$.
time = 0.27, size = 116, normalized size = 1.51

$$\frac{4\sqrt{dx}\,\sinh\left(fx\right)-\frac{\left(\frac{2\sqrt{dx}\,de^{(fx)}}{f}-\frac{2\sqrt{dx}\,de^{(-fx)}}{f}+\frac{\sqrt{\pi}\,d\,\mathrm{erf}\left(\sqrt{dx}\,\sqrt{\frac{f}{d}}\right)}{f\sqrt{\frac{f}{d}}}-\frac{\sqrt{\pi}\,d\,\mathrm{erf}\left(\sqrt{dx}\,\sqrt{-\frac{f}{d}}\right)}{f\sqrt{-\frac{f}{d}}}\right)f}{d}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*sqrt(d*x)*sinh(f*x) - (2*sqrt(d*x)*d*e^(f*x)/f - 2*sqrt(d*x)*d*e^(-f
*x)/f + sqrt(pi)*d*erf(sqrt(d*x)*sqrt(f/d))/(f*sqrt(f/d)) - sqrt(pi)*d*erf(
sqrt(d*x)*sqrt(-f/d))/(f*sqrt(-f/d)))*f/d)/d
```

Fricas [A]
time = 0.42, size = 58, normalized size = 0.75

$$-\frac{\sqrt{\pi}\,\sqrt{\frac{f}{d}}\,\mathrm{erf}\left(\sqrt{dx}\,\sqrt{\frac{f}{d}}\right)+\sqrt{\pi}\,\sqrt{-\frac{f}{d}}\,\mathrm{erf}\left(\sqrt{dx}\,\sqrt{-\frac{f}{d}}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(pi)*sqrt(f/d)*erf(sqrt(d*x)*sqrt(f/d)) + sqrt(pi)*sqrt(-f/d)*erf
(sqrt(d*x)*sqrt(-f/d)))/f
```

Sympy [C] Result contains complex when optimal does not.
time = 0.56, size = 70, normalized size = 0.91

$$\frac{3\sqrt{2}\,\sqrt{\pi}\,e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\,\sqrt{f}\,\sqrt{x}\,e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{3}{4}\right)}{4\sqrt{d}\,\sqrt{f}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)**(1/2),x)
```

```
[Out] 3*sqrt(2)*sqrt(pi)*exp(-3*I*pi/4)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)*exp(I*pi
/4)/sqrt(pi))*gamma(3/4)/(4*sqrt(d)*sqrt(f)*gamma(7/4))
```

Giac [A]
time = 0.45, size = 61, normalized size = 0.79

$$\frac{\frac{\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{df}\,\sqrt{dx}}{d}\right)}{\sqrt{df}}-\frac{\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{-df}\,\sqrt{dx}}{d}\right)}{\sqrt{-df}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(pi)*d*erf(-sqrt(d*f)*sqrt(d*x)/d)/sqrt(d*f) - sqrt(pi)*d*erf(-sqr
t(-d*f)*sqrt(d*x)/d)/sqrt(-d*f))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(f\,x)}{\sqrt{d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x)/(d*x)^(1/2),x)
```

```
[Out] int(sinh(f*x)/(d*x)^(1/2), x)
```

3.63 $\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{f}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{f}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sinh(fx)}{d\sqrt{dx}}$$

```
[Out] erf(f^(1/2)*(d*x)^(1/2)/d^(1/2))*f^(1/2)*Pi^(1/2)/d^(3/2)+erfi(f^(1/2)*(d*x
)^(1/2)/d^(1/2))*f^(1/2)*Pi^(1/2)/d^(3/2)-2*sinh(f*x)/d/(d*x)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3378, 3388, 2211, 2235, 2236}

$$\frac{\sqrt{\pi}\sqrt{f}\,\mathrm{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{\pi}\sqrt{f}\,\mathrm{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sinh(fx)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[f*x]/(d*x)^(3/2),x]
```

```
[Out] (Sqrt[f]*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (Sqrt[f]*Sqrt
[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/d^(3/2) - (2*Sinh[f*x])/(d*Sqrt[d*x
])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx &= -\frac{2\sinh(fx)}{d\sqrt{dx}} + \frac{(2f)\int \frac{\cosh(fx)}{\sqrt{dx}}\,dx}{d} \\
&= -\frac{2\sinh(fx)}{d\sqrt{dx}} + \frac{f\int \frac{e^{-fx}}{\sqrt{dx}}\,dx}{d} + \frac{f\int \frac{e^{fx}}{\sqrt{dx}}\,dx}{d} \\
&= -\frac{2\sinh(fx)}{d\sqrt{dx}} + \frac{(2f)\mathrm{Subst}\left(\int e^{-\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{d^2} + \frac{(2f)\mathrm{Subst}\left(\int e^{\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{d^2} \\
&= \frac{\sqrt{f}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{f}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sinh(fx)}{d\sqrt{dx}}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 0.56

$$\frac{x\left(\sqrt{-fx}\,\Gamma\left(\frac{1}{2},-fx\right)-\sqrt{fx}\,\Gamma\left(\frac{1}{2},fx\right)-2\sinh(fx)\right)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[f*x]/(d*x)^(3/2),x]
```

```
[Out] (x*(Sqrt[-(f*x)]*Gamma[1/2, -(f*x)] - Sqrt[f*x]*Gamma[1/2, f*x] - 2*Sinh[f*
x]))/(d*x)^(3/2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 120, normalized size = 1.38

method	result
meijerg	$-\frac{\sqrt{\pi}\, x^{\frac{3}{2}} \sqrt{2}\, (if)^{\frac{3}{2}} \left(\frac{2\sqrt{2}\,\sqrt{if}\, e^{-fx}}{\sqrt{\pi}\,\sqrt{x}\, f} - \frac{2\sqrt{2}\,\sqrt{if}\, e^{fx}}{\sqrt{\pi}\,\sqrt{x}\, f} + \frac{2\sqrt{if}\,\sqrt{2}\, \operatorname{erf}\left(\sqrt{x}\,\sqrt{f}\right)}{\sqrt{f}} + \frac{2\sqrt{if}\,\sqrt{2}\, \operatorname{erfi}\left(\sqrt{x}\,\sqrt{\right.}\right.}{\sqrt{f}}}{4(dx)^{\frac{3}{2}} f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)/(d*x)^(3/2)*x^(3/2)*2^(1/2)*(I*f)^(3/2)/f*(2/Pi^(1/2)/x^(1/2)
*2^(1/2)*(I*f)^(1/2)/f*exp(-f*x)-2/Pi^(1/2)/x^(1/2)*2^(1/2)*(I*f)^(1/2)/f*e
xp(f*x)+2*(I*f)^(1/2)*2^(1/2)/f^(1/2)*erf(x^(1/2)*f^(1/2))+2*(I*f)^(1/2)*2^
(1/2)/f^(1/2)*erfi(x^(1/2)*f^(1/2)))
```

Maxima [A]
time = 0.27, size = 74, normalized size = 0.85

$$\frac{\frac{f\left(\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{dx}\,\sqrt{\frac{f}{d}}\right)}{\sqrt{\frac{f}{d}}} + \frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{dx}\,\sqrt{-\frac{f}{d}}\right)}{\sqrt{-\frac{f}{d}}}\right)}{d} - \frac{2\sinh(fx)}{\sqrt{dx}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] (f*(sqrt(pi)*erf(sqrt(d*x)*sqrt(f/d))/sqrt(f/d) + sqrt(pi)*erf(sqrt(d*x)*sq
rt(-f/d))/sqrt(-f/d))/d - 2*sinh(f*x)/sqrt(d*x))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(61) = 122$.
time = 0.44, size = 137, normalized size = 1.57

$$\frac{\sqrt{\pi}\,(dx\cosh(fx) + dx\sinh(fx))\sqrt{\frac{f}{d}}\,\operatorname{erf}\left(\sqrt{dx}\,\sqrt{\frac{f}{d}}\right) - \sqrt{\pi}\,(dx\cosh(fx) + dx\sinh(fx))\sqrt{-\frac{f}{d}}\,\operatorname{erf}\left(\sqrt{dx}\,\sqrt{-\frac{f}{d}}\right) - \sqrt{dx}\,\left(\cosh(fx)^2 + 2\cosh(fx)\sinh(fx) + \sinh(fx)^2 - 1\right)}{d^2x\cosh(fx) + d^2x\sinh(fx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(pi)*(d*x*cosh(f*x) + d*x*sinh(f*x))*sqrt(f/d)*erf(sqrt(d*x)*sqrt(f/d)
) - sqrt(pi)*(d*x*cosh(f*x) + d*x*sinh(f*x))*sqrt(-f/d)*erf(sqrt(d*x)*sqrt(
-f/d)) - sqrt(d*x)*(cosh(f*x)^2 + 2*cosh(f*x)*sinh(f*x) + sinh(f*x)^2 - 1))
/(d^2*x*cosh(f*x) + d^2*x*sinh(f*x))
```

Sympy [C] Result contains complex when optimal does not.
time = 1.99, size = 94, normalized size = 1.08

$$\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{f}\,e^{-\frac{i\pi}{4}}C\left(\frac{\sqrt{2}\,\sqrt{f}\,\sqrt{x}\,e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{2d^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}-\frac{\sinh\left(fx\right)\Gamma\left(\frac{1}{4}\right)}{2d^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)**(3/2),x)
```

```
[Out] sqrt(2)*sqrt(pi)*sqrt(f)*exp(-I*pi/4)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)*exp(
I*pi/4)/sqrt(pi))*gamma(1/4)/(2*d**(3/2)*gamma(5/4)) - sinh(f*x)*gamma(1/4)
/(2*d**(3/2)*sqrt(x)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x)/(d*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(f\,x)}{{(d\,x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x)/(d*x)^(3/2),x)
```

```
[Out] int(sinh(f*x)/(d*x)^(3/2), x)
```

3.64 $\int \frac{\sinh(fx)}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{4f\cosh(fx)}{3d^2\sqrt{dx}}-\frac{2f^{3/2}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2f^{3/2}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{2\sinh(fx)}{3d(dx)^{3/2}}$$

```
[Out] -2/3*sinh(f*x)/d/(d*x)^(3/2)-2/3*f^(3/2)*erf(f^(1/2)*(d*x)^(1/2)/d^(1/2))*P
i^(1/2)/d^(5/2)+2/3*f^(3/2)*erfi(f^(1/2)*(d*x)^(1/2)/d^(1/2))*Pi^(1/2)/d^(5
/2)-4/3*f*cosh(f*x)/d^2/(d*x)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3378, 3389, 2211, 2235, 2236}

$$-\frac{2\sqrt{\pi}f^{3/2}\mathrm{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2\sqrt{\pi}f^{3/2}\mathrm{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4f\cosh(fx)}{3d^2\sqrt{dx}}-\frac{2\sinh(fx)}{3d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[f*x]/(d*x)^(5/2),x]
```

```
[Out] (-4*f*Cosh[f*x])/(3*d^2*Sqrt[d*x]) - (2*f^(3/2)*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[
d*x])/Sqrt[d]])/(3*d^(5/2)) + (2*f^(3/2)*Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/
Sqrt[d]])/(3*d^(5/2)) - (2*Sinh[f*x])/(3*d*(d*x)^(3/2))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(fx)}{(dx)^{5/2}}\,dx &= -\frac{2\sinh(fx)}{3d(dx)^{3/2}}+\frac{(2f)\int\frac{\cosh(fx)}{(dx)^{3/2}}\,dx}{3d}\\
&= -\frac{4f\cosh(fx)}{3d^2\sqrt{dx}}-\frac{2\sinh(fx)}{3d(dx)^{3/2}}+\frac{(4f^2)\int\frac{\sinh(fx)}{\sqrt{dx}}\,dx}{3d^2}\\
&= -\frac{4f\cosh(fx)}{3d^2\sqrt{dx}}-\frac{2\sinh(fx)}{3d(dx)^{3/2}}-\frac{(2f^2)\int\frac{e^{-fx}}{\sqrt{dx}}\,dx}{3d^2}+\frac{(2f^2)\int\frac{e^{fx}}{\sqrt{dx}}\,dx}{3d^2}\\
&= -\frac{4f\cosh(fx)}{3d^2\sqrt{dx}}-\frac{2\sinh(fx)}{3d(dx)^{3/2}}-\frac{(4f^2)\,\mathrm{Subst}\left(\int e^{-\frac{fx^2}{d}}\,dx,x,\sqrt{dx}\right)}{3d^3}+\frac{(4f^2)\,\mathrm{Subst}\left(\int e^{\frac{fx^2}{d}}\,dx\right.}{3d^3}\\
&= -\frac{4f\cosh(fx)}{3d^2\sqrt{dx}}-\frac{2f^{3/2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2f^{3/2}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{2\sinh(fx)}{3d(dx)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 84, normalized size = 0.74

$$-\frac{e^{-fx}x\left(-1+e^{2fx}+2fx+2e^{2fx}fx+2e^{fx}(-fx)^{3/2}\Gamma\left(\frac{1}{2},-fx\right)-2e^{fx}(fx)^{3/2}\Gamma\left(\frac{1}{2},fx\right)\right)}{3(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[f*x]/(d*x)^(5/2),x]
```

```
[Out] -1/3*(x*(-1 + E^(2*f*x) + 2*f*x + 2*E^(2*f*x)*f*x + 2*E^(f*x)*(-(f*x))^(3/2
)*Gamma[1/2, -(f*x)] - 2*E^(f*x)*(f*x)^(3/2)*Gamma[1/2, f*x]))/(E^(f*x)*(d*
x)^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 132, normalized size = 1.16

method	result
meijerg	$-\frac{\sqrt{\pi}\, x^{\frac{5}{2}} \sqrt{2}\, (if)^{\frac{5}{2}} \left(-\frac{4\sqrt{2}\,(2fx+1)\mathrm{e}^{fx}}{3\sqrt{\pi}\, x^{\frac{3}{2}} \sqrt{if}\, f}+\frac{4\sqrt{2}\,(-2fx+1)\mathrm{e}^{-fx}}{3\sqrt{\pi}\, x^{\frac{3}{2}} \sqrt{if}\, f}-\frac{8\sqrt{2}\,\sqrt{f}\,\mathrm{erf}\left(\sqrt{x}\,\sqrt{f}\right)}{3\sqrt{if}}+\frac{8\sqrt{2}\,\sqrt{f}\,\mathrm{erfi}\left(\sqrt{x}\,\sqrt{f}\right)}{3\sqrt{if}}\right)}{8(dx)^{\frac{5}{2}} f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*Pi^(1/2)/(d*x)^(5/2)*x^(5/2)*2^(1/2)*(I*f)^(5/2)/f*(-4/3/Pi^(1/2)/x^(3
/2)*2^(1/2)/(I*f)^(1/2)*(2*f*x+1)/f*exp(f*x)+4/3/Pi^(1/2)/x^(3/2)*2^(1/2)/(
I*f)^(1/2)*(-2*f*x+1)/f*exp(-f*x)-8/3/(I*f)^(1/2)*2^(1/2)*f^(1/2)*erf(x^(1/
2)*f^(1/2))+8/3/(I*f)^(1/2)*2^(1/2)*f^(1/2)*erfi(x^(1/2)*f^(1/2)))
```

Maxima [A]
time = 0.31, size = 57, normalized size = 0.50

$$-\frac{\frac{f\left(\frac{\sqrt{fx}\,\Gamma\left(-\frac{1}{2},fx\right)}{\sqrt{dx}}+\frac{\sqrt{-fx}\,\Gamma\left(-\frac{1}{2},-fx\right)}{\sqrt{dx}}\right)}{d}+\frac{2\,\sinh(fx)}{(dx)^{\frac{3}{2}}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(f*(sqrt(f*x)*gamma(-1/2, f*x)/sqrt(d*x) + sqrt(-f*x)*gamma(-1/2, -f*x
)/sqrt(d*x))/d + 2*sinh(f*x)/(d*x)^(3/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(78) = 156.
time = 0.41, size = 178, normalized size = 1.56

$$-\frac{2\sqrt{\pi}\left(dfx^2\cosh(fx)+dfx^2\sinh(fx)\right)\sqrt{\frac{f}{d}}\,\mathrm{erf}\left(\sqrt{dx}\,\sqrt{\frac{f}{d}}\right)+2\sqrt{\pi}\left(dfx^2\cosh(fx)+dfx^2\sinh(fx)\right)\sqrt{-\frac{f}{d}}\,\mathrm{erf}\left(\sqrt{dx}\,\sqrt{-\frac{f}{d}}\right)+\left((2fx+1)\cosh(fx)^2+2(2fx+1)\cosh(fx)\sinh(fx)+(2fx+1)\sinh(fx)^2+2fx-1\right)\sqrt{dx}}{3\left(d^3x^2\cosh(fx)+d^3x^2\sinh(fx)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(pi)*(d*f*x^2*cosh(f*x) + d*f*x^2*sinh(f*x))*sqrt(f/d)*erf(sqrt
(d*x)*sqrt(f/d)) + 2*sqrt(pi)*(d*f*x^2*cosh(f*x) + d*f*x^2*sinh(f*x))*sqrt(
-f/d)*erf(sqrt(d*x)*sqrt(-f/d)) + ((2*f*x + 1)*cosh(f*x)^2 + 2*(2*f*x + 1)*
cosh(f*x)*sinh(f*x) + (2*f*x + 1)*sinh(f*x)^2 + 2*f*x - 1)*sqrt(d*x))/(d^3*
x^2*cosh(f*x) + d^3*x^2*sinh(f*x))
```

Sympy [C] Result contains complex when optimal does not.
time = 15.71, size = 129, normalized size = 1.13

$$-\frac{\sqrt{2}\,\sqrt{\pi}\,f^{\frac{3}{2}}e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\,\sqrt{f}\,\sqrt{x}\,e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(-\frac{1}{4}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{3}{4}\right)}+\frac{f\cosh\left(fx\right)\Gamma\left(-\frac{1}{4}\right)}{3d^{\frac{5}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}+\frac{\sinh\left(fx\right)\Gamma\left(-\frac{1}{4}\right)}{6d^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)**(5/2),x)
```

```
[Out] -sqrt(2)*sqrt(pi)*f**(3/2)*exp(-3*I*pi/4)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)*
exp(I*pi/4)/sqrt(pi))*gamma(-1/4)/(3*d**(5/2)*gamma(3/4)) + f*cosh(f*x)*gam
ma(-1/4)/(3*d**(5/2)*sqrt(x)*gamma(3/4)) + sinh(f*x)*gamma(-1/4)/(6*d**(5/2
)*x**(3/2)*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x)/(d*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(f\,x)}{{(d\,x)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x)/(d*x)^(5/2),x)
```

```
[Out] int(sinh(f*x)/(d*x)^(5/2), x)
```

3.65 $\int \sqrt{c+dx}\,\mathrm{csch}(a+bx)\,dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left(\sqrt{c+dx}\,\mathrm{csch}(a+bx),x\right)$$

```
[Out] Unintegrable(csch(b*x+a)*(d*x+c)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{c+dx}\,\mathrm{csch}(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + d*x]*Csch[a + b*x],x]
[Out] Defer[Int][Sqrt[c + d*x]*Csch[a + b*x], x]
```

Rubi steps

$$\int \sqrt{c+dx}\,\mathrm{csch}(a+bx)\,dx = \int \sqrt{c+dx}\,\mathrm{csch}(a+bx)\,dx$$

Mathematica [A]
time = 23.35, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\mathrm{csch}(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + d*x]*Csch[a + b*x],x]
[Out] Integrate[Sqrt[c + d*x]*Csch[a + b*x], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \mathrm{csch}(bx+a)\sqrt{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*(d*x+c)^(1/2),x)
```

```
[Out] int(csch(b*x+a)*(d*x+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*x + c)*csch(b*x + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*x + c)*csch(b*x + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\operatorname{csch}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*(d*x+c)**(1/2),x)
[Out] Integral(sqrt(c + d*x)*csch(a + b*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x + c)*csch(b*x + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{c+d\,x}}{\mathrm{sinh}\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/sinh(a + b*x),x)
[Out] int((c + d*x)^(1/2)/sinh(a + b*x), x)
```

3.66 $\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}},x\right)$$

```
[Out] Unintegrable(csch(b*x+a)/(d*x+c)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csch[a + b*x]/Sqrt[c + d*x],x]
[Out] Defer[Int][Csch[a + b*x]/Sqrt[c + d*x], x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx$$

Mathematica [A]
time = 15.64, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csch[a + b*x]/Sqrt[c + d*x],x]
[Out] Integrate[Csch[a + b*x]/Sqrt[c + d*x], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(bx+a)}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)/(d*x+c)^(1/2),x)
[Out] int(csch(b*x+a)/(d*x+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(csch(b*x + a)/sqrt(d*x + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral(csch(b*x + a)/sqrt(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c)**(1/2),x)
[Out] Integral(csch(a + b*x)/sqrt(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(csch(b*x + a)/sqrt(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sinh\left(a+b\,x\right)\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(a + b*x)*(c + d*x)^(1/2)),x)
[Out] int(1/(sinh(a + b*x)*(c + d*x)^(1/2)), x)
```

3.67 $\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3} dx$

Optimal. Leaf size=63

$$-\frac{3\cosh(x)\sqrt{\sinh(x)}}{4x} - \frac{\sinh^{\frac{3}{2}}(x)}{2x^2} + \frac{3}{8}\text{Int}\left(\frac{1}{x\sqrt{\sinh(x)}}, x\right) + \frac{9}{8}\text{Int}\left(\frac{\sinh^{\frac{3}{2}}(x)}{x}, x\right)$$

```
[Out] -1/2*sinh(x)^(3/2)/x^2-3/4*cosh(x)*sinh(x)^(1/2)/x+9/8*Unintegrable(sinh(x)
^(3/2)/x,x)+3/8*Unintegrable(1/x/sinh(x)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3} dx$$

Verification is not applicable to the result.

```
[In] Int[Sinh[x]^(3/2)/x^3,x]
```

```
[Out] (-3*Cosh[x]*Sqrt[Sinh[x]])/(4*x) - Sinh[x]^(3/2)/(2*x^2) + (3*Defer[Int][1/
(x*Sqrt[Sinh[x]]), x])/8 + (9*Defer[Int][Sinh[x]^(3/2)/x, x])/8
```

Rubi steps

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3} dx = -\frac{3\cosh(x)\sqrt{\sinh(x)}}{4x} - \frac{\sinh^{\frac{3}{2}}(x)}{2x^2} + \frac{3}{8}\int \frac{1}{x\sqrt{\sinh(x)}} dx + \frac{9}{8}\int \frac{\sinh^{\frac{3}{2}}(x)}{x} dx$$

Mathematica [A]
time = 5.43, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sinh[x]^(3/2)/x^3,x]
```

```
[Out] Integrate[Sinh[x]^(3/2)/x^3, x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^(3/2)/x^3,x)
```

```
[Out] int(sinh(x)^(3/2)/x^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sinh(x)^(3/2)/x^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**(3/2)/x**3,x)
```

```
[Out] Integral(sinh(x)**(3/2)/x**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sinh(x)^(3/2)/x^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sinh(x)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^(3/2)/x^3,x)
```

```
[Out] int(sinh(x)^(3/2)/x^3, x)
```

3.68 $\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)} - x\sqrt{\sinh(x)}\right) dx$

Optimal. Leaf size=20

$$-\frac{2x\cosh(x)}{\sqrt{\sinh(x)}} + 4\sqrt{\sinh(x)}$$

```
[Out] -2*x*cosh(x)/sinh(x)^(1/2)+4*sinh(x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {3396}

$$4\sqrt{\sinh(x)} - \frac{2x\cosh(x)}{\sqrt{\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sinh[x]^(3/2) - x*Sqrt[Sinh[x]],x]
```

```
[Out] (-2*x*Cosh[x])/Sqrt[Sinh[x]] + 4*Sqrt[Sinh[x]]
```

Rule 3396

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1))), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[d*((b*Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)} - x\sqrt{\sinh(x)}\right) dx &= \int \frac{x}{\sinh^{\frac{3}{2}}(x)}\, dx - \int x\sqrt{\sinh(x)}\, dx\\ &= -\frac{2x\cosh(x)}{\sqrt{\sinh(x)}} + 4\sqrt{\sinh(x)}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 17, normalized size = 0.85

$$\frac{-2x\cosh(x) + 4\sinh(x)}{\sqrt{\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sinh[x]^(3/2) - x*Sqrt[Sinh[x]],x]
```

```
[Out] (-2*x*Cosh[x] + 4*Sinh[x])/Sqrt[Sinh[x]]
```

Maple [F]
time = 1.75, size = 0, normalized size = 0.00

$$\int \frac{x}{\sinh(x)^{\frac{3}{2}}} - x\left(\sqrt{\sinh}(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x)
```

```
[Out] int(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-x*sqrt(sinh(x)) + x/sinh(x)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{x}{\sinh^{\frac{3}{2}}(x)}\right) dx - \int x\sqrt{\sinh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)**(3/2)-x*sinh(x)**(1/2),x)
```

```
[Out] -Integral(-x/sinh(x)**(3/2), x) - Integral(x*sqrt(sinh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x, algorithm="giac")
[Out] integrate(-x*sqrt(sinh(x)) + x/sinh(x)^(3/2), x)
```

Mupad [B]
time = 0.17, size = 38, normalized size = 1.90

$$-\frac{2\sqrt{\frac{e^x}{2}-\frac{e^{-x}}{2}}\,\left(x-2\,e^{2\,x}+x\,e^{2\,x}+2\right)}{e^{2\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sinh(x)^(3/2) - x*sinh(x)^(1/2),x)
[Out] -(2*(exp(x)/2 - exp(-x)/2)^(1/2)*(x - 2*exp(2*x) + x*exp(2*x) + 2))/(exp(2*
x) - 1)
```

3.69 $\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right) dx$

Optimal. Leaf size=24

$$-\frac{2x\cosh(x)}{3\sinh^{\frac{3}{2}}(x)}-\frac{4}{3\sqrt{\sinh(x)}}$$

```
[Out] -2/3*x*cosh(x)/sinh(x)^(3/2)-4/3/sinh(x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3396}

$$-\frac{4}{3\sqrt{\sinh(x)}}-\frac{2x\cosh(x)}{3\sinh^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sinh[x]^(5/2) + x/(3*Sqrt[Sinh[x]]),x]
```

```
[Out] (-2*x*Cosh[x])/(3*Sinh[x]^(3/2)) - 4/(3*Sqrt[Sinh[x]])
```

Rule 3396

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1))), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[d*((b*Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right) dx &= \frac{1}{3}\int \frac{x}{\sqrt{\sinh(x)}}\,dx+\int \frac{x}{\sinh^{\frac{5}{2}}(x)}\,dx\\ &= -\frac{2x\cosh(x)}{3\sinh^{\frac{3}{2}}(x)}-\frac{4}{3\sqrt{\sinh(x)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 22, normalized size = 0.92

$$\frac{1}{6}(-8\text{csch}(x)-4x\coth(x)\text{csch}(x))\sqrt{\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sinh[x]^(5/2) + x/(3*Sqrt[Sinh[x]]),x]
```

```
[Out] ((-8*Csch[x] - 4*x*Coth[x]*Csch[x])*Sqrt[Sinh[x]])/6
```

Maple [**F**]
time = 1.65, size = 0, normalized size = 0.00

$$\int \frac{x}{\sinh(x)^{\frac{5}{2}}} + \frac{x}{3\sqrt{\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x)
```

```
[Out] int(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/3*x/sqrt(sinh(x)) + x/sinh(x)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(16) = 32$.
time = 0.35, size = 108, normalized size = 4.50

$$-\frac{4\left((x+2)\cosh(x)^{3}+3(x+2)\cosh(x)\sinh(x)^{2}+(x+2)\sinh(x)^{3}+(x-2)\cosh(x)+\left(3(x+2)\cosh(x)^{2}+x-2\right)\sinh(x)\right)\sqrt{\sinh(x)}}{3\left(\cosh(x)^{4}+4\cosh(x)\sinh(x)^{3}+\sinh(x)^{4}+2\left(3\cosh(x)^{2}-1\right)\sinh(x)^{2}-2\cosh(x)^{2}+4\left(\cosh(x)^{3}-\cosh(x)\right)\sinh(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] -4/3*((x + 2)*cosh(x)^3 + 3*(x + 2)*cosh(x)*sinh(x)^2 + (x + 2)*sinh(x)^3 +
 (x - 2)*cosh(x) + (3*(x + 2)*cosh(x)^2 + x - 2)*sinh(x))*sqrt(sinh(x))/(co
sh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 -
 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{3x}{\sinh^{\frac{5}{2}}(x)}\,dx + \int \frac{x}{\sqrt{\sinh(x)}}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)**(5/2)+1/3*x/sinh(x)**(1/2),x)
```

```
[Out] (Integral(3*x/sinh(x)**(5/2), x) + Integral(x/sqrt(sinh(x)), x))/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/3*x/sqrt(sinh(x)) + x/sinh(x)^(5/2), x)
```

Mupad [B]
time = 0.15, size = 40, normalized size = 1.67

$$-\frac{4\,{\mathrm{e}}^x\,\sqrt{\frac{{\mathrm{e}}^x}{2}-\frac{{\mathrm{e}}^{-x}}{2}}\,\left(x+2\,{\mathrm{e}}^{2\,x}+x\,{\mathrm{e}}^{2\,x}-2\right)}{3\,{\left({\mathrm{e}}^{2\,x}-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3*sinh(x)^(1/2)) + x/sinh(x)^(5/2),x)
```

```
[Out] -(4*exp(x)*(exp(x)/2 - exp(-x)/2)^(1/2)*(x + 2*exp(2*x) + x*exp(2*x) - 2))/
(3*(exp(2*x) - 1)^2)
```

3.70 $\int \left(\frac{x}{\sinh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\sinh(x)}\right)\,dx$

Optimal. Leaf size=47

$$-\frac{2x\cosh(x)}{5\sinh^{\frac{5}{2}}(x)}-\frac{4}{15\sinh^{\frac{3}{2}}(x)}+\frac{6x\cosh(x)}{5\sqrt{\sinh(x)}}-\frac{12\sqrt{\sinh(x)}}{5}$$

```
[Out] -2/5*x*cosh(x)/sinh(x)^(5/2)-4/15/sinh(x)^(3/2)+6/5*x*cosh(x)/sinh(x)^(1/2)
-12/5*sinh(x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3396}

$$-\frac{4}{15\sinh^{\frac{3}{2}}(x)}-\frac{12\sqrt{\sinh(x)}}{5}-\frac{2x\cosh(x)}{5\sinh^{\frac{5}{2}}(x)}+\frac{6x\cosh(x)}{5\sqrt{\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sinh[x]^(7/2) + (3*x*Sqrt[Sinh[x]])/5,x]
```

```
[Out] (-2*x*Cosh[x])/(5*Sinh[x]^(5/2)) - 4/(15*Sinh[x]^(3/2)) + (6*x*Cosh[x])/(5*
Sqrt[Sinh[x]]) - (12*Sqrt[Sinh[x]])/5
```

Rule 3396

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1))), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[d*((b*Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int\left(\frac{x}{\sinh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\sinh(x)}\right)dx &= \frac{3}{5}\int x\sqrt{\sinh(x)}\,dx+\int\frac{x}{\sinh^{\frac{7}{2}}(x)}\,dx\\ &= -\frac{2x\cosh(x)}{5\sinh^{\frac{5}{2}}(x)}-\frac{4}{15\sinh^{\frac{3}{2}}(x)}-\frac{3}{5}\int\frac{x}{\sinh^{\frac{3}{2}}(x)}\,dx+\frac{3}{5}\int x\sqrt{\sinh(x)}\,d\\ &= -\frac{2x\cosh(x)}{5\sinh^{\frac{5}{2}}(x)}-\frac{4}{15\sinh^{\frac{3}{2}}(x)}+\frac{6x\cosh(x)}{5\sqrt{\sinh(x)}}-\frac{12\sqrt{\sinh(x)}}{5}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 33, normalized size = 0.70

$$\frac{-21x\cosh(x)+9x\cosh(3x)+46\sinh(x)-18\sinh(3x)}{30\sinh^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sinh[x]^(7/2) + (3*x*Sqrt[Sinh[x]])/5,x]
```

```
[Out] (-21*x*Cosh[x] + 9*x*Cosh[3*x] + 46*Sinh[x] - 18*Sinh[3*x])/(30*Sinh[x]^(5/
2))
```

Maple [F]
time = 1.81, size = 0, normalized size = 0.00

$$\int \frac{x}{\sinh\left(x\right)^{\frac{7}{2}}}+\frac{3x\left(\sqrt{\sinh}\left(x\right)\right)}{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x)
```

```
[Out] int(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(3/5*x*sqrt(sinh(x)) + x/sinh(x)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{5x}{\sinh^{\frac{7}{2}}(x)}\,dx + \int 3x\sqrt{\sinh(x)}\,dx}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)**(7/2)+3/5*x*sinh(x)**(1/2),x)
```

```
[Out] (Integral(5*x/sinh(x)**(7/2), x) + Integral(3*x*sqrt(sinh(x)), x))/5
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(3/5*x*sqrt(sinh(x)) + x/sinh(x)^(7/2), x)
```

Mupad [**B**]
time = 0.29, size = 111, normalized size = 2.36

$$\frac{12\,x\,e^{2\,x}\,\sqrt{\frac{e^{x}}{2}-\frac{e^{-x}}{2}}}{5\,\left(e^{2\,x}-1\right)}-\frac{e^{2\,x}\,\left(\frac{8\,x}{5}+\frac{16}{15}\right)\,\sqrt{\frac{e^{x}}{2}-\frac{e^{-x}}{2}}}{{\left(e^{2\,x}-1\right)}^2}-\left(\frac{6\,x}{5}+\frac{12}{5}\right)\,\sqrt{\frac{e^{x}}{2}-\frac{e^{-x}}{2}}-\frac{16\,x\,e^{2\,x}\,\sqrt{\frac{e^{x}}{2}-\frac{e^{-x}}{2}}}{5\,{\left(e^{2\,x}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x*sinh(x)^(1/2))/5 + x/sinh(x)^(7/2),x)
```

```
[Out] (12*x*exp(2*x)*(exp(x)/2 - exp(-x)/2)^(1/2))/(5*(exp(2*x) - 1)) - (exp(2*x)
*((8*x)/5 + 16/15)*(exp(x)/2 - exp(-x)/2)^(1/2))/(exp(2*x) - 1)^2 - ((6*x)/
5 + 12/5)*(exp(x)/2 - exp(-x)/2)^(1/2) - (16*x*exp(2*x)*(exp(x)/2 - exp(-x)
/2)^(1/2))/(5*(exp(2*x) - 1)^3)
```

3.71 $\int \left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)} - x^2\sqrt{\sinh(x)}\right) dx$

Optimal. Leaf size=58

$$-\frac{2x^2\cosh(x)}{\sqrt{\sinh(x)}} + 8x\sqrt{\sinh(x)} - \frac{16iE\left(\frac{\pi}{4} - \frac{ix}{2}\middle|2\right)\sqrt{\sinh(x)}}{\sqrt{i\sinh(x)}}$$

```
[Out] -2*x^2*cosh(x)/sinh(x)^(1/2)+8*x*sinh(x)^(1/2)-16*I*(sin(1/4*Pi+1/2*I*x)^2)
^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2))*sinh(x)^(
1/2)/(I*sinh(x))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3397, 2721, 2719}

$$-\frac{2x^2\cosh(x)}{\sqrt{\sinh(x)}} + 8x\sqrt{\sinh(x)} - \frac{16i\sqrt{\sinh(x)}\,E\left(\frac{\pi}{4} - \frac{ix}{2}\middle|2\right)}{\sqrt{i\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sinh[x]^(3/2) - x^2*Sqrt[Sinh[x]],x]
```

```
[Out] (-2*x^2*Cosh[x])/Sqrt[Sinh[x]] + 8*x*Sqrt[Sinh[x]] - ((16*I)*EllipticE[Pi/4
 - (I/2)*x, 2]*Sqrt[Sinh[x]])/Sqrt[I*Sinh[x]]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3397

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1)
)), x] + (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n +
 2), x], x] + Dist[d^2*m*((m - 1)/(b^2*f^2*(n + 1)*(n + 2))), Int[(c + d*x)
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] - Simp[d*m*(c + d*x)^(m - 1)*((b*
Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; FreeQ[{b, c, d, e,
```

```
f}, x] && LtQ[n, -1] && NeQ[n, -2] && GtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\int \left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)}-x^2\sqrt{\sinh(x)}\right)dx &= \int \frac{x^2}{\sinh^{\frac{3}{2}}(x)}\,dx-\int x^2\sqrt{\sinh(x)}\,dx\\
&= -\frac{2x^2\cosh(x)}{\sqrt{\sinh(x)}}+8x\sqrt{\sinh(x)}-8\int \sqrt{\sinh(x)}\,dx\\
&= -\frac{2x^2\cosh(x)}{\sqrt{\sinh(x)}}+8x\sqrt{\sinh(x)}-\frac{\left(8\sqrt{\sinh(x)}\right)\int \sqrt{i\sinh(x)}\,dx}{\sqrt{i\sinh(x)}}\\
&= -\frac{2x^2\cosh(x)}{\sqrt{\sinh(x)}}+8x\sqrt{\sinh(x)}-\frac{16iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{\sinh(x)}}{\sqrt{i\sinh(x)}}
\end{aligned}
$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.85, size = 68, normalized size = 1.17

$$
-\frac{2\left(x^2\cosh(x)-4(-2+x)\sinh(x)-8\sqrt{2}\ {}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};\cosh(2x)+\sinh(2x)\right)(-\cosh(x)+\sinh(x))\sqrt{-\sinh(x)(\cosh(x)+\sinh(x))}\right)}{\sqrt{\sinh(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sinh[x]^(3/2) - x^2*Sqrt[Sinh[x]],x]
```

```
[Out] (-2*(x^2*Cosh[x] - 4*(-2 + x)*Sinh[x] - 8*Sqrt[2]*Hypergeometric2F1[-1/4, 1
/2, 3/4, Cosh[2*x] + Sinh[2*x]]*(-Cosh[x] + Sinh[x])*Sqrt[-(Sinh[x]*(Cosh[x
] + Sinh[x]))]))/Sqrt[Sinh[x]]
```

Maple [**F**]
time = 1.68, size = 0, normalized size = 0.00

$$
\int \frac{x^2}{\sinh(x)^{\frac{3}{2}}}-x^2\left(\sqrt{\sinh}\,(x)\right)dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x)
```

```
[Out] int(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-x^2*sqrt(sinh(x)) + x^2/sinh(x)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{x^2}{\sinh^{\frac{3}{2}}(x)}\right)dx-\int x^2\sqrt{\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/sinh(x)**(3/2)-x**2*sinh(x)**(1/2),x)
```

```
[Out] -Integral(-x**2/sinh(x)**(3/2), x) - Integral(x**2*sqrt(sinh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x^2*sqrt(sinh(x)) + x^2/sinh(x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int x^2\,\sqrt{\sinh(x)}-\frac{x^2}{{\sinh(x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/sinh(x)^(3/2) - x^2*sinh(x)^(1/2),x)
```

```
[Out] -int(x^2*sinh(x)^(1/2) - x^2/sinh(x)^(3/2), x)
```

3.72 $\int (c+dx)^m(b\sinh(e+fx))^n\,dx$

Optimal. Leaf size=21

$$\text{Int}((c+dx)^m(b\sinh(e+fx))^n,x)$$

```
[Out] Unintegrable((d*x+c)^m*(b*sinh(f*x+e))^n,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m(b\sinh(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*(b*Sinh[e + f*x])^n,x]
[Out] Defer[Int][(c + d*x)^m*(b*Sinh[e + f*x])^n, x]
```

Rubi steps

$$\int (c+dx)^m(b\sinh(e+fx))^n\,dx = \int (c+dx)^m(b\sinh(e+fx))^n\,dx$$

Mathematica [A]
time = 2.01, size = 0, normalized size = 0.00

$$\int (c+dx)^m(b\sinh(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(b*Sinh[e + f*x])^n,x]
[Out] Integrate[(c + d*x)^m*(b*Sinh[e + f*x])^n, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m\,(b\sinh(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(b*sinh(f*x+e))^n,x)
```

```
[Out] int((d*x+c)^m*(b*sinh(f*x+e))^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*sinh(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*(b*sinh(f*x + e))^n, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*sinh(f*x+e))^n,x, algorithm="fricas")
[Out] integral((d*x + c)^m*(b*sinh(f*x + e))^n, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sinh(e+fx))^n (c+dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(b*sinh(f*x+e))**n,x)
[Out] Integral((b*sinh(e + f*x))**n*(c + d*x)**m, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*sinh(f*x+e))^n,x, algorithm="giac")
[Out] integrate((d*x + c)^m*(b*sinh(f*x + e))^n, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (b\sinh(e+fx))^n (c+dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(e + f*x))^n*(c + d*x)^m,x)
[Out] int((b*sinh(e + f*x))^n*(c + d*x)^m, x)
```

3.73 $\int (c+dx)^m \sinh^3(a+bx)\,dx$

Optimal. Leaf size=237

$$\frac{3^{-1-m}e^{3a-\frac{3bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3b(c+dx)}{d}\right)}{8b}-\frac{3e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{b(c\cdots\right.}{}}{8b}$$

```
[Out] 1/8*3^(-1-m)*exp(3*a-3*b*c/d)*(d*x+c)^m*GAMMA(1+m,-3*b*(d*x+c)/d)/b/((-b*(d
*x+c)/d)^m)-3/8*exp(a-b*c/d)*(d*x+c)^m*GAMMA(1+m,-b*(d*x+c)/d)/b/((-b*(d*x+
c)/d)^m)-3/8*exp(-a+b*c/d)*(d*x+c)^m*GAMMA(1+m,b*(d*x+c)/d)/b/((b*(d*x+c)/d
)^m)+1/8*3^(-1-m)*exp(-3*a+3*b*c/d)*(d*x+c)^m*GAMMA(1+m,3*b*(d*x+c)/d)/b/((
b*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.22, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3393, 3389, 2212}

$$\frac{3^{-m-1}e^{3a-\frac{3bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{3b(c+dx)}{d}\right)}{8b}-\frac{3e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{b(c+dx)}{d}\right)}{8b}-\frac{3e^{\frac{bc}{d}-a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{b(c+dx)}{d}\right)}{8b}+\frac{3^{-m-1}e^{\frac{3bc}{d}-3a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{3b(c+dx)}{d}\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Sinh[a + b*x]^3,x]
```

```
[Out] (3^(-1 - m)*E^(3*a - (3*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (-3*b*(c + d*x))/d
])/(8*b*(-((b*(c + d*x))/d))^m) - (3*E^(a - (b*c)/d)*(c + d*x)^m*Gamma[1 +
m, -((b*(c + d*x))/d)])/(8*b*(-((b*(c + d*x))/d))^m) - (3*E^(-a + (b*c)/d)*
(c + d*x)^m*Gamma[1 + m, (b*(c + d*x))/d])/(8*b*((b*(c + d*x))/d)^m) + (3^(
-1 - m)*E^(-3*a + (3*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (3*b*(c + d*x))/d])/(
8*b*((b*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \sinh^3(a+bx)\,dx &= i\int \left(\frac{3}{4}i(c+dx)^m \sinh(a+bx) - \frac{1}{4}i(c+dx)^m \sinh(3a+3bx)\right)\,dx\\ &= \frac{1}{4}\int (c+dx)^m \sinh(3a+3bx)\,dx - \frac{3}{4}\int (c+dx)^m \sinh(a+bx)\,dx\\ &= \frac{1}{8}\int e^{-i(3ia+3ibx)}(c+dx)^m\,dx - \frac{1}{8}\int e^{i(3ia+3ibx)}(c+dx)^m\,dx - \frac{3}{8}\int e^{-i(ia+ibx)}\\ &= \frac{3^{-1-m}e^{3a-\frac{3bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3b(c+dx)}{d}\right)}{8b} - \frac{3e^{a-\frac{bc}{d}}(c+dx)}{}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 206, normalized size = 0.87

$$\frac{3^{-1-m}e^{-3\left(a+\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(e^{6a}\left(b\left(\frac{c}{d}+x\right)\right)^m\Gamma\left(1+m,-\frac{3b(c+dx)}{d}\right)-3^{2+m}e^{4a+\frac{2bc}{d}}\left(b\left(\frac{c}{d}+x\right)\right)^m\Gamma\left(1+m,-\frac{b(c+dx)}{d}\right)+e^{\frac{4bc}{d}}\left(-\frac{b(c+dx)}{d}\right)^m\left(-3^{2+m}e^{2a}\Gamma\left(1+m,\frac{b(c+dx)}{d}\right)+e^{\frac{2bc}{d}}\Gamma\left(1+m,\frac{3b(c+dx)}{d}\right)\right)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Sinh[a + b*x]^3,x]
```

```
[Out] (3^(-1 - m)*(c + d*x)^m*(E^(6*a)*(b*(c/d + x))^m*Gamma[1 + m, (-3*b*(c + d*
x))/d] - 3^(2 + m)*E^(4*a + (2*b*c)/d)*(b*(c/d + x))^m*Gamma[1 + m, -((b*(c
 + d*x))/d)] + E^((4*b*c)/d)*(-((b*(c + d*x))/d))^m*(-(3^(2 + m)*E^(2*a)*Ga
mma[1 + m, (b*(c + d*x))/d]) + E^((2*b*c)/d)*Gamma[1 + m, (3*b*(c + d*x))/d
])))/(8*b*E^(3*(a + (b*c)/d))*(-((b^2*(c + d*x)^2)/d^2))^m)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\sinh^3(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sinh(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*sinh(b*x+a)^3,x)
```

Maxima [A]
time = 0.08, size = 161, normalized size = 0.68

$$\frac{(dx+c)^{m+1}e^{\left(-3\,a+\frac{3\,bc}{d}\right)}E_{-m}\left(\frac{3\,(dx+c)b}{d}\right)}{8\,d} - \frac{3\,(dx+c)^{m+1}e^{\left(-a+\frac{bc}{d}\right)}E_{-m}\left(\frac{(dx+c)b}{d}\right)}{8\,d} + \frac{3\,(dx+c)^{m+1}e^{\left(a-\frac{bc}{d}\right)}E_{-m}\left(-\frac{(dx+c)b}{d}\right)}{8\,d} - \frac{(dx+c)^{m+1}e^{\left(3\,a-\frac{3\,bc}{d}\right)}E_{-m}\left(-\frac{3\,(dx+c)b}{d}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x + c)^(m + 1)*e^(-3*a + 3*b*c/d)*exp_integral_e(-m, 3*(d*x + c)*b/d
)/d - 3/8*(d*x + c)^(m + 1)*e^(-a + b*c/d)*exp_integral_e(-m, (d*x + c)*b/d
)/d + 3/8*(d*x + c)^(m + 1)*e^(a - b*c/d)*exp_integral_e(-m, -(d*x + c)*b/d
)/d - 1/8*(d*x + c)^(m + 1)*e^(3*a - 3*b*c/d)*exp_integral_e(-m, -3*(d*x +
c)*b/d)/d
```

Fricas [A]
time = 0.09, size = 340, normalized size = 1.43

$$\frac{\cosh\left(\frac{dm\log\left(\frac{3b}{d}\right)-3bc+3ad}{d}\right)\Gamma\left(m+1,\frac{3(bdx+bc)}{d}\right)-9\cosh\left(\frac{dm\log\left(\frac{b}{d}\right)-bc+ad}{d}\right)\Gamma\left(m+1,\frac{bdx+bc}{d}\right)-9\cosh\left(\frac{dm\log\left(-\frac{b}{d}\right)+bc-ad}{d}\right)\Gamma\left(m+1,-\frac{bdx+bc}{d}\right)+\cosh\left(\frac{dm\log\left(-\frac{3b}{d}\right)+3bc-3ad}{d}\right)\Gamma\left(m+1,-\frac{3(bdx+bc)}{d}\right)-\Gamma\left(m+1,\frac{3(bdx+bc)}{d}\right)\sinh\left(\frac{dm\log\left(\frac{3b}{d}\right)-3bc+3ad}{d}\right)+9\Gamma\left(m+1,\frac{bdx+bc}{d}\right)\sinh\left(\frac{dm\log\left(\frac{b}{d}\right)-bc+ad}{d}\right)+9\Gamma\left(m+1,-\frac{bdx+bc}{d}\right)\sinh\left(\frac{dm\log\left(-\frac{b}{d}\right)+bc-ad}{d}\right)-\Gamma\left(m+1,-\frac{3(bdx+bc)}{d}\right)\sinh\left(\frac{dm\log\left(-\frac{3b}{d}\right)+3bc-3ad}{d}\right)}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(cosh((d*m*log(3*b/d) - 3*b*c + 3*a*d)/d)*gamma(m + 1, 3*(b*d*x + b*c)
/d) - 9*cosh((d*m*log(b/d) - b*c + a*d)/d)*gamma(m + 1, (b*d*x + b*c)/d) -
9*cosh((d*m*log(-b/d) + b*c - a*d)/d)*gamma(m + 1, -(b*d*x + b*c)/d) + cosh
((d*m*log(-3*b/d) + 3*b*c - 3*a*d)/d)*gamma(m + 1, -3*(b*d*x + b*c)/d) - ga
mma(m + 1, 3*(b*d*x + b*c)/d)*sinh((d*m*log(3*b/d) - 3*b*c + 3*a*d)/d) + 9*
gamma(m + 1, (b*d*x + b*c)/d)*sinh((d*m*log(b/d) - b*c + a*d)/d) + 9*gamma(
m + 1, -(b*d*x + b*c)/d)*sinh((d*m*log(-b/d) + b*c - a*d)/d) - gamma(m + 1,
 -3*(b*d*x + b*c)/d)*sinh((d*m*log(-3*b/d) + 3*b*c - 3*a*d)/d))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sinh^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sinh(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**m*sinh(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sinh(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sinh(a+b\,x)^3\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3*(c + d*x)^m,x)
```

```
[Out] int(sinh(a + b*x)^3*(c + d*x)^m, x)
```

3.74 $\int (c+dx)^m \sinh^2(a+bx)\,dx$

Optimal. Leaf size=144

$$-\frac{(c+dx)^{1+m}}{2d(1+m)}+\frac{2^{-3-m}e^{2a-\frac{2bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2b(c+dx)}{d}\right)}{b}-\frac{2^{-3-m}e^{-2a+\frac{2bc}{d}}(c+dx)^m\left(\frac{b(c+\ldots}{d}\right.\ldots}{b}$$

```
[Out] -1/2*(d*x+c)^(1+m)/d/(1+m)+2^(-3-m)*exp(2*a-2*b*c/d)*(d*x+c)^m*GAMMA(1+m,-2
*b*(d*x+c)/d)/b/((-b*(d*x+c)/d)^m)-2^(-3-m)*exp(-2*a+2*b*c/d)*(d*x+c)^m*GAM
MA(1+m,2*b*(d*x+c)/d)/b/((b*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.14, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3393, 3388, 2212}

$$\frac{2^{-m-3}e^{2a-\frac{2bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2b(c+dx)}{d}\right)}{b}-\frac{2^{-m-3}e^{\frac{2bc}{d}-2a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2b(c+dx)}{d}\right)}{b}-\frac{(c+dx)^{m+1}}{2d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Sinh[a + b*x]^2,x]
```

```
[Out] -1/2*(c + d*x)^(1 + m)/(d*(1 + m)) + (2^(-3 - m)*E^(2*a - (2*b*c)/d)*(c + d
*x)^m*Gamma[1 + m, (-2*b*(c + d*x))/d])/(b*(-((b*(c + d*x))/d))^m) - (2^(-3
 - m)*E^(-2*a + (2*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (2*b*(c + d*x))/d])/(b*
((b*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int (c+dx)^m \sinh^2(a+bx)\,dx = -\int \left(\frac{1}{2}(c+dx)^m - \frac{1}{2}(c+dx)^m \cosh(2a+2bx)\right) dx$$
$$= -\frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{1}{2}\int (c+dx)^m \cosh(2a+2bx)\,dx$$
$$= -\frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{1}{4}\int e^{-i(2ia+2ibx)}(c+dx)^m\,dx + \frac{1}{4}\int e^{i(2ia+2ibx)}(c+dx)^m\,dx$$
$$= -\frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{2^{-3-m}e^{2a-\frac{2bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2b(c+dx)}{d}\right)}{b}$$

Mathematica [A]
time = 0.12, size = 131, normalized size = 0.91

$$\frac{1}{8}(c+dx)^m\left(-\frac{4(c+dx)}{d(1+m)}+\frac{2^{-m}e^{2a-\frac{2bc}{d}}\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2b(c+dx)}{d}\right)}{b}-\frac{2^{-m}e^{-2a+\frac{2bc}{d}}\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2b(c+dx)}{d}\right)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Sinh[a + b*x]^2,x]
[Out] ((c + d*x)^m*((-4*(c + d*x))/(d*(1 + m)) + (E^(2*a - (2*b*c)/d)*Gamma[1 + m
, (-2*b*(c + d*x))/d])/(2^m*b*(-((b*(c + d*x))/d))^m) - (E^(-2*a + (2*b*c)/
d)*Gamma[1 + m, (2*b*(c + d*x))/d])/(2^m*b*((b*(c + d*x))/d)^m)))/8
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\sinh^2(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sinh(b*x+a)^2,x)
[Out] int((d*x+c)^m*sinh(b*x+a)^2,x)
```

Maxima [A]
time = 0.07, size = 102, normalized size = 0.71

$$-\frac{(dx+c)^{m+1}e^{\left(-2a+\frac{2bc}{d}\right)}E_{-m}\left(\frac{2(dx+c)b}{d}\right)}{4d}-\frac{(dx+c)^{m+1}e^{\left(2a-\frac{2bc}{d}\right)}E_{-m}\left(-\frac{2(dx+c)b}{d}\right)}{4d}-\frac{(dx+c)^{m+1}}{2d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(d*x + c)^(m + 1)*e^(-2*a + 2*b*c/d)*exp_integral_e(-m, 2*(d*x + c)*b/
d)/d - 1/4*(d*x + c)^(m + 1)*e^(2*a - 2*b*c/d)*exp_integral_e(-m, -2*(d*x +
 c)*b/d)/d - 1/2*(d*x + c)^(m + 1)/(d*(m + 1))
```

Fricas [A]
time = 0.09, size = 241, normalized size = 1.67

$$-\frac{(dm+d)\cosh\left(\frac{dm\log\left(\frac{2b}{d}\right)-2bc+2ad}{d}\right)\Gamma\left(m+1,\frac{2(bdx+bc)}{d}\right)-(dm+d)\cosh\left(\frac{dm\log\left(-\frac{2b}{d}\right)+2bc-2ad}{d}\right)\Gamma\left(m+1,-\frac{2(bdx+bc)}{d}\right)-(dm+d)\Gamma\left(m+1,\frac{2(bdx+bc)}{d}\right)\sinh\left(\frac{dm\log\left(\frac{2b}{d}\right)-2bc+2ad}{d}\right)+(dm+d)\Gamma\left(m+1,-\frac{2(bdx+bc)}{d}\right)\sinh\left(\frac{dm\log\left(-\frac{2b}{d}\right)+2bc-2ad}{d}\right)+4(bdx+bc)\cosh(m\log(dx+c))+4(bdx+bc)\sinh(m\log(dx+c))}{8(bdm+bd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((d*m + d)*cosh((d*m*log(2*b/d) - 2*b*c + 2*a*d)/d)*gamma(m + 1, 2*(b*
d*x + b*c)/d) - (d*m + d)*cosh((d*m*log(-2*b/d) + 2*b*c - 2*a*d)/d)*gamma(m
 + 1, -2*(b*d*x + b*c)/d) - (d*m + d)*gamma(m + 1, 2*(b*d*x + b*c)/d)*sinh(
(d*m*log(2*b/d) - 2*b*c + 2*a*d)/d) + (d*m + d)*gamma(m + 1, -2*(b*d*x + b*
c)/d)*sinh((d*m*log(-2*b/d) + 2*b*c - 2*a*d)/d) + 4*(b*d*x + b*c)*cosh(m*lo
g(d*x + c)) + 4*(b*d*x + b*c)*sinh(m*log(d*x + c)))/(b*d*m + b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sinh^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sinh(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*sinh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sinh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh(a+b\,x)^2\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2*(c + d*x)^m,x)
[Out] int(sinh(a + b*x)^2*(c + d*x)^m, x)
```

3.75 $\int (c+dx)^m \sinh(a+bx)\,dx$

Optimal. Leaf size=110

$$\frac{e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{b(c+dx)}{d}\right)}{2b}+\frac{e^{-a+\frac{bc}{d}}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{b(c+dx)}{d}\right)}{2b}$$

```
[Out] 1/2*exp(a-b*c/d)*(d*x+c)^m*GAMMA(1+m,-b*(d*x+c)/d)/b/((-b*(d*x+c)/d)^m)+1/2
*exp(-a+b*c/d)*(d*x+c)^m*GAMMA(1+m,b*(d*x+c)/d)/b/((b*(d*x+c)/d)^m)
```

Rubi [A]

time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3389, 2212}

$$\frac{e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{b(c+dx)}{d}\right)}{2b}+\frac{e^{\frac{bc}{d}-a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{b(c+dx)}{d}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Sinh[a + b*x],x]
```

```
[Out] (E^(a - (b*c)/d)*(c + d*x)^m*Gamma[1 + m, -((b*(c + d*x))/d)])/(2*b*(-((b*(
c + d*x))/d))^m) + (E^(-a + (b*c)/d)*(c + d*x)^m*Gamma[1 + m, (b*(c + d*x))
/d])/(2*b*((b*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m\sinh(a+bx)\,dx &= \frac{1}{2}\int e^{-i(ia+ibx)}(c+dx)^m\,dx-\frac{1}{2}\int e^{i(ia+ibx)}(c+dx)^m\,dx\\ &= \frac{e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{b(c+dx)}{d}\right)}{2b}+\frac{e^{-a+\frac{bc}{d}}(c+dx)^m\left(\frac{b(c+\end{aligned}$$

Mathematica [A]
time = 0.04, size = 101, normalized size = 0.92

$$\frac{e^{-a-\frac{bc}{d}}(c+dx)^m\left(e^{2a}\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{b(c+dx)}{d}\right)+e^{\frac{2bc}{d}}\left(b\left(\frac{c}{d}+x\right)\right)^{-m}\Gamma\left(1+m,\frac{b(c+dx)}{d}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Sinh[a + b*x],x]
```

```
[Out] (E^(-a - (b*c)/d)*(c + d*x)^m*((E^(2*a)*Gamma[1 + m, -((b*(c + d*x))/d)])/(
-((b*(c + d*x))/d))^m + (E^((2*b*c)/d)*Gamma[1 + m, (b*(c + d*x))/d])/(b*(c
/d + x))^m))/(2*b)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sinh (bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sinh(b*x+a),x)
```

```
[Out] int((d*x+c)^m*sinh(b*x+a),x)
```

Maxima [A]
time = 0.06, size = 79, normalized size = 0.72

$$\frac{(dx+c)^{m+1}e^{\left(-a+\frac{bc}{d}\right)}E_{-m}\left(\frac{(dx+c)b}{d}\right)}{2\,d}-\frac{(dx+c)^{m+1}e^{\left(a-\frac{bc}{d}\right)}E_{-m}\left(-\frac{(dx+c)b}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(d*x + c)^(m + 1)*e^(-a + b*c/d)*exp_integral_e(-m, (d*x + c)*b/d)/d -
1/2*(d*x + c)^(m + 1)*e^(a - b*c/d)*exp_integral_e(-m, -(d*x + c)*b/d)/d
```

Fricas [A]
time = 0.09, size = 168, normalized size = 1.53

$$\frac{\cosh\left(\frac{dm\log\left(\frac{b}{d}\right)-bc+ad}{d}\right)\Gamma\left(m+1,\frac{bdx+bc}{d}\right)+\cosh\left(\frac{dm\log\left(-\frac{b}{d}\right)+bc-ad}{d}\right)\Gamma\left(m+1,-\frac{bdx+bc}{d}\right)-\Gamma\left(m+1,\frac{bdx+bc}{d}\right)\sinh\left(\frac{dm\log\left(\frac{b}{d}\right)-bc+ad}{d}\right)-\Gamma\left(m+1,-\frac{bdx+bc}{d}\right)\sinh\left(\frac{dm\log\left(-\frac{b}{d}\right)+bc-ad}{d}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh((d*m*log(b/d) - b*c + a*d)/d)*gamma(m + 1, (b*d*x + b*c)/d) + cos
h((d*m*log(-b/d) + b*c - a*d)/d)*gamma(m + 1, -(b*d*x + b*c)/d) - gamma(m +
 1, (b*d*x + b*c)/d)*sinh((d*m*log(b/d) - b*c + a*d)/d) - gamma(m + 1, -(b*
d*x + b*c)/d)*sinh((d*m*log(-b/d) + b*c - a*d)/d))/b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sinh(b*x+a),x)
```

```
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sinh(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh(a + b\,x)\,\left(c + d\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*(c + d*x)^m,x)
```

```
[Out] int(sinh(a + b*x)*(c + d*x)^m, x)
```

3.76 $\int (c+dx)^m \operatorname{csch}(a+bx)\,dx$

Optimal. Leaf size=17

$$\operatorname{Int}((c+dx)^m \operatorname{csch}(a+bx),x)$$

```
[Out] Unintegrable((d*x+c)^m*csch(b*x+a),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \operatorname{csch}(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Csch[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Csch[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \operatorname{csch}(a+bx)\,dx = \int (c+dx)^m \operatorname{csch}(a+bx)\,dx$$

Mathematica [A]
time = 5.97, size = 0, normalized size = 0.00

$$\int (c+dx)^m \operatorname{csch}(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csch[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Csch[a + b*x], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \operatorname{csch}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csch(b*x+a),x)
```

```
[Out] int((d*x+c)^m*csch(b*x+a),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csch(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*csch(b*x + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csch(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*csch(b*x + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^m \operatorname{csch}(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csch(b*x+a),x)
[Out] Integral((c + d*x)**m*csch(a + b*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csch(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*csch(b*x + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{(c + d\,x)^m}{\sinh(a + b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/sinh(a + b*x),x)
[Out] int((c + d*x)^m/sinh(a + b*x), x)
```

3.77 $\int (c+dx)^m \mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=19
$$\text{Int}\left((c+dx)^m\text{csch}^2(a+bx),x\right)$$

```
[Out] Unintegrable((d*x+c)^m*csch(b*x+a)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m\text{csch}^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Csch[a + b*x]^2,x]
[Out] Defer[Int][(c + d*x)^m*Csch[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m\text{csch}^2(a+bx)\,dx = \int (c+dx)^m\text{csch}^2(a+bx)\,dx$$

Mathematica [A]
time = 2.44, size = 0, normalized size = 0.00

$$\int (c+dx)^m\text{csch}^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csch[a + b*x]^2,x]
[Out] Integrate[(c + d*x)^m*Csch[a + b*x]^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m\,\text{csch}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csch(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*csch(b*x+a)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csch(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*csch(b*x + a)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csch(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m*csch(b*x + a)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \operatorname{csch}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csch(b*x+a)**2,x)
[Out] Integral((c + d*x)**m*csch(a + b*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csch(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*csch(b*x + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(c+d\,x)^m}{\sinh(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/sinh(a + b*x)^2,x)
[Out] int((c + d*x)^m/sinh(a + b*x)^2, x)
```

3.78 $\int x^{3+m} \sinh(a+bx)\,dx$

Optimal. Leaf size=59

$$-\frac{e^a x^m(-bx)^{-m}\Gamma(4+m,-bx)}{2b^4}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(4+m,bx)}{2b^4}$$

```
[Out] -1/2*exp(a)*x^m*GAMMA(4+m,-b*x)/b^4/((-b*x)^m)+1/2*x^m*GAMMA(4+m,b*x)/b^4/e
xp(a)/((b*x)^m)
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3389, 2212}

$$\frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m+4,bx)}{2b^4}-\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+4,-bx)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 + m)*Sinh[a + b*x],x]
```

```
[Out] -1/2*(E^a*x^m*Gamma[4 + m, -(b*x)])/(b^4*(-(b*x))^m) + (x^m*Gamma[4 + m, b*
x])/(2*b^4*E^a*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int x^{3+m}\sinh(a+bx)\,dx &= \frac{1}{2}\int e^{-i(ia+ibx)}x^{3+m}\,dx-\frac{1}{2}\int e^{i(ia+ibx)}x^{3+m}\,dx\\ &= -\frac{e^a x^m(-bx)^{-m}\Gamma(4+m,-bx)}{2b^4}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(4+m,bx)}{2b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 0.92

$$\frac{e^{-a}x^m(-e^{2a}(-bx)^{-m}\Gamma(4+m,-bx)+(bx)^{-m}\Gamma(4+m,bx))}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 + m)*Sinh[a + b*x],x]
```

```
[Out] (x^m*(-((E^(2*a)*Gamma[4 + m, -(b*x)])/(-(b*x))^m) + Gamma[4 + m, b*x]/(b*x
)^m))/(2*b^4*E^a)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.17, size = 73, normalized size = 1.24

method	result	size
meijerg	$\frac{x^{4+m}\,\mathrm{hypergeom}\left(\left[2+\frac{m}{2}\right],\left[\frac{1}{2},\frac{m}{2}+3\right],\frac{b^2x^2}{4}\right)\sinh(a)}{4+m}+\frac{b\,x^{m+5}\,\mathrm{hypergeom}\left(\left[\frac{m}{2}+\frac{5}{2}\right],\left[\frac{3}{2},\frac{7}{2}+\frac{m}{2}\right],\frac{b^2x^2}{4}\right)\cosh(a)}{m+5}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m+3)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(4+m)*x^(4+m)*hypergeom([2+1/2*m],[1/2,1/2*m+3],1/4*b^2*x^2)*sinh(a)+b/(m
+5)*x^(m+5)*hypergeom([1/2*m+5/2],[3/2,7/2+1/2*m],1/4*b^2*x^2)*cosh(a)
```

Maxima [A]
time = 0.07, size = 55, normalized size = 0.93

$$\frac{1}{2}\,(bx)^{-m-4}\,x^{m+4}e^{(-a)}\Gamma(m+4,bx)-\frac{1}{2}\,(-bx)^{-m-4}\,x^{m+4}e^{a}\Gamma(m+4,-bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x)^(-m - 4)*x^(m + 4)*e^(-a)*gamma(m + 4, b*x) - 1/2*(-b*x)^(-m - 4)
*x^(m + 4)*e^a*gamma(m + 4, -b*x)
```

Fricas [A]
time = 0.08, size = 86, normalized size = 1.46

$$\frac{\cosh\left((m+3)\log\left(b\right)+a\right)\Gamma(m+4,bx)+\cosh\left((m+3)\log\left(-b\right)-a\right)\Gamma(m+4,-bx)-\Gamma(m+4,-bx)\sinh\left((m+3)\log\left(-b\right)-a\right)-\Gamma(m+4,bx)\sinh\left((m+3)\log\left(b\right)+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh((m + 3)*log(b) + a)*gamma(m + 4, b*x) + cosh((m + 3)*log(-b) - a)
*gamma(m + 4, -b*x) - gamma(m + 4, -b*x)*sinh((m + 3)*log(-b) - a) - gamma(
m + 4, b*x)*sinh((m + 3)*log(b) + a))/b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3+m)*sinh(b*x+a),x)
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x^(m + 3)*sinh(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{m+3}\sinh(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 3)*sinh(a + b*x),x)
[Out] int(x^(m + 3)*sinh(a + b*x), x)
```

3.79 $\int x^{2+m}\sinh(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{e^a x^m(-bx)^{-m}\Gamma(3+m,-bx)}{2b^3}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(3+m,bx)}{2b^3}$$

```
[Out] 1/2*exp(a)*x^m*GAMMA(3+m,-b*x)/b^3/((-b*x)^m)+1/2*x^m*GAMMA(3+m,b*x)/b^3/ex
p(a)/((b*x)^m)
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3389, 2212}

$$\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+3,-bx)}{2b^3}+\frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m+3,bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + m)*Sinh[a + b*x],x]
```

```
[Out] (E^a*x^m*Gamma[3 + m, -(b*x)])/(2*b^3*(-(b*x))^m) + (x^m*Gamma[3 + m, b*x])
/(2*b^3*E^a*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int x^{2+m}\sinh(a+bx)\,dx &= \frac{1}{2}\int e^{-i(ia+ibx)}x^{2+m}\,dx-\frac{1}{2}\int e^{i(ia+ibx)}x^{2+m}\,dx\\ &= \frac{e^a x^m(-bx)^{-m}\Gamma(3+m,-bx)}{2b^3}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(3+m,bx)}{2b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 0.90

$$\frac{e^{-a}x^m(e^{2a}(-bx)^{-m}\Gamma(3+m,-bx)+(bx)^{-m}\Gamma(3+m,bx))}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + m)*Sinh[a + b*x],x]
```

```
[Out] (x^m*((E^(2*a)*Gamma[3 + m, -(b*x)])/(-(b*x))^m + Gamma[3 + m, b*x]/(b*x)^m
))/(2*b^3*E^a)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.17, size = 73, normalized size = 1.24

method	result	size
meijerg	$\frac{x^{m+3}\,\mathrm{hypergeom}\left(\left[\frac{3}{2}+\frac{m}{2}\right],\left[\frac{1}{2},\frac{m}{2}+\frac{5}{2}\right],\frac{b^2x^2}{4}\right)\sinh(a)}{m+3}+\frac{b\,x^{4+m}\,\mathrm{hypergeom}\left(\left[2+\frac{m}{2}\right],\left[\frac{3}{2},\frac{m}{2}+3\right],\frac{b^2x^2}{4}\right)\cosh(a)}{4+m}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+m)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(m+3)*x^(m+3)*hypergeom([3/2+1/2*m],[1/2,1/2*m+5/2],1/4*b^2*x^2)*sinh(a)+
b/(4+m)*x^(4+m)*hypergeom([2+1/2*m],[3/2,1/2*m+3],1/4*b^2*x^2)*cosh(a)
```

Maxima [A]
time = 0.08, size = 55, normalized size = 0.93

$$\frac{1}{2}\,(bx)^{-m-3}\,x^{m+3}e^{(-a)}\Gamma(m+3,bx)-\frac{1}{2}\,(-bx)^{-m-3}\,x^{m+3}e^{a}\Gamma(m+3,-bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x)^(-m - 3)*x^(m + 3)*e^(-a)*gamma(m + 3, b*x) - 1/2*(-b*x)^(-m - 3)
*x^(m + 3)*e^a*gamma(m + 3, -b*x)
```

Fricas [A]
time = 0.09, size = 86, normalized size = 1.46

$$\frac{\cosh\left((m+2)\log\left(b\right)+a\right)\Gamma(m+3,bx)+\cosh\left((m+2)\log\left(-b\right)-a\right)\Gamma(m+3,-bx)-\Gamma(m+3,-bx)\sinh\left((m+2)\log\left(-b\right)-a\right)-\Gamma(m+3,bx)\sinh\left((m+2)\log\left(b\right)+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh((m + 2)*log(b) + a)*gamma(m + 3, b*x) + cosh((m + 2)*log(-b) - a)
*gamma(m + 3, -b*x) - gamma(m + 3, -b*x)*sinh((m + 2)*log(-b) - a) - gamma(
m + 3, b*x)*sinh((m + 2)*log(b) + a))/b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+m)*sinh(b*x+a),x)
```

```
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2)*sinh(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{m+2}\,\sinh(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 2)*sinh(a + b*x),x)
```

```
[Out] int(x^(m + 2)*sinh(a + b*x), x)
```

3.80 $\int x^{1+m} \sinh(a + bx)\, dx$

Optimal. Leaf size=59

$$-\frac{e^a x^m (-bx)^{-m} \Gamma(2+m, -bx)}{2b^2} + \frac{e^{-a} x^m (bx)^{-m} \Gamma(2+m, bx)}{2b^2}$$

```
[Out] -1/2*exp(a)*x^m*GAMMA(2+m,-b*x)/b^2/((-b*x)^m)+1/2*x^m*GAMMA(2+m,b*x)/b^2/e
xp(a)/((b*x)^m)
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3389, 2212}

$$\frac{e^{-a} x^m (bx)^{-m} \text{Gamma}(m+2, bx)}{2b^2} - \frac{e^a x^m (-bx)^{-m} \text{Gamma}(m+2, -bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + m)*Sinh[a + b*x],x]
```

```
[Out] -1/2*(E^a*x^m*Gamma[2 + m, -(b*x)])/(b^2*(-(b*x))^m) + (x^m*Gamma[2 + m, b*
x])/(2*b^2*E^a*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\int x^{1+m} \sinh(a+bx)\, dx &= \frac{1}{2} \int e^{-i(ia+ibx)} x^{1+m}\, dx - \frac{1}{2} \int e^{i(ia+ibx)} x^{1+m}\, dx \\
&= -\frac{e^a x^m (-bx)^{-m} \Gamma(2+m, -bx)}{2b^2} + \frac{e^{-a} x^m (bx)^{-m} \Gamma(2+m, bx)}{2b^2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 0.92

$$\frac{e^{-a}x^m(-e^{2a}(-bx)^{-m}\Gamma(2+m,-bx)+(bx)^{-m}\Gamma(2+m,bx))}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + m)*Sinh[a + b*x],x]
```

```
[Out] (x^m*(-((E^(2*a)*Gamma[2 + m, -(b*x)])/(-(b*x))^m) + Gamma[2 + m, b*x]/(b*x
)^m))/(2*b^2*E^a)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.16, size = 73, normalized size = 1.24

method	result	size
meijerg	$\frac{x^{2+m}\,\mathrm{hypergeom}\left(\left[1+\frac{m}{2}\right],\left[\frac{1}{2},2+\frac{m}{2}\right],\frac{b^2x^2}{4}\right)\sinh(a)}{2+m}+\frac{b\,x^{m+3}\,\mathrm{hypergeom}\left(\left[\frac{3}{2}+\frac{m}{2}\right],\left[\frac{3}{2},\frac{m}{2}+\frac{5}{2}\right],\frac{b^2x^2}{4}\right)\cosh(a)}{m+3}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(2+m)*x^(2+m)*hypergeom([1+1/2*m],[1/2,2+1/2*m],1/4*b^2*x^2)*sinh(a)+b/(m
+3)*x^(m+3)*hypergeom([3/2+1/2*m],[3/2,1/2*m+5/2],1/4*b^2*x^2)*cosh(a)
```

Maxima [A]
time = 0.09, size = 55, normalized size = 0.93

$$\frac{1}{2}\,(bx)^{-m-2}\,x^{m+2}e^{(-a)}\Gamma(m+2,bx)-\frac{1}{2}\,(-bx)^{-m-2}\,x^{m+2}e^{a}\Gamma(m+2,-bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x)^(-m - 2)*x^(m + 2)*e^(-a)*gamma(m + 2, b*x) - 1/2*(-b*x)^(-m - 2)
*x^(m + 2)*e^a*gamma(m + 2, -b*x)
```

Fricas [A]
time = 0.08, size = 86, normalized size = 1.46

$$\frac{\cosh\left((m+1)\log\left(b\right)+a\right)\Gamma(m+2,bx)+\cosh\left((m+1)\log\left(-b\right)-a\right)\Gamma(m+2,-bx)-\Gamma(m+2,-bx)\sinh\left((m+1)\log\left(-b\right)-a\right)-\Gamma(m+2,bx)\sinh\left((m+1)\log\left(b\right)+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh((m + 1)*log(b) + a)*gamma(m + 2, b*x) + cosh((m + 1)*log(-b) - a)
*gamma(m + 2, -b*x) - gamma(m + 2, -b*x)*sinh((m + 1)*log(-b) - a) - gamma(
m + 2, b*x)*sinh((m + 1)*log(b) + a))/b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)*sinh(b*x+a),x)
```

```
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 1)*sinh(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{m+1}\sinh(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 1)*sinh(a + b*x),x)
```

```
[Out] int(x^(m + 1)*sinh(a + b*x), x)
```

3.81 $\int x^m \sinh(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b}$$

```
[Out] 1/2*exp(a)*x^m*GAMMA(1+m,-b*x)/b/((-b*x)^m)+1/2*x^m*GAMMA(1+m,b*x)/b/exp(a)
/((b*x)^m)
```

Rubi [A]

time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3389, 2212}

$$\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+1,-bx)}{2b}+\frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m+1,bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sinh[a + b*x],x]
```

```
[Out] (E^a*x^m*Gamma[1 + m, -(b*x)])/(2*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(
2*b*E^a*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int x^m\sinh(a+bx)\,dx&=\frac{1}{2}\int e^{-i(ia+ibx)}x^m\,dx-\frac{1}{2}\int e^{i(ia+ibx)}x^m\,dx\\&=\frac{e^a x^m(-bx)^{-m}\Gamma(1+m,-bx)}{2b}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(1+m,bx)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 0.90

$$\frac{e^{-a}x^m(e^{2a}(-bx)^{-m}\Gamma(1+m,-bx)+(bx)^{-m}\Gamma(1+m,bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sinh[a + b*x],x]
```

```
[Out] (x^m*((E^(2*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m + Gamma[1 + m, b*x]/(b*x)^m
))/(2*b*E^a)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.16, size = 73, normalized size = 1.24

method	result	size
meijerg	$\frac{x^{1+m}\,\mathrm{hypergeom}\left(\left[\frac{1}{2}+\frac{m}{2}\right],\left[\frac{1}{2},\frac{3}{2}+\frac{m}{2}\right],\frac{b^2x^2}{4}\right)\sinh(a)}{1+m}+\frac{b\,x^{2+m}\,\mathrm{hypergeom}\left(\left[1+\frac{m}{2}\right],\left[\frac{3}{2},2+\frac{m}{2}\right],\frac{b^2x^2}{4}\right)\cosh(a)}{2+m}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1+m)*x^(1+m)*hypergeom([1/2+1/2*m],[1/2,3/2+1/2*m],1/4*b^2*x^2)*sinh(a)+
b/(2+m)*x^(2+m)*hypergeom([1+1/2*m],[3/2,2+1/2*m],1/4*b^2*x^2)*cosh(a)
```

Maxima [A]
time = 0.08, size = 55, normalized size = 0.93

$$\frac{1}{2}\,(bx)^{-m-1}\,x^{m+1}e^{(-a)}\Gamma(m+1,bx)-\frac{1}{2}\,(-bx)^{-m-1}\,x^{m+1}e^{a}\Gamma(m+1,-bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x)^(-m - 1)*x^(m + 1)*e^(-a)*gamma(m + 1, b*x) - 1/2*(-b*x)^(-m - 1)
*x^(m + 1)*e^a*gamma(m + 1, -b*x)
```

Fricas [A]
time = 0.11, size = 78, normalized size = 1.32

$$\frac{\cosh\left(m\log\left(b\right)+a\right)\Gamma(m+1,bx)+\cosh\left(m\log\left(-b\right)-a\right)\Gamma(m+1,-bx)-\Gamma(m+1,-bx)\sinh\left(m\log\left(-b\right)-a\right)-\Gamma(m+1,bx)\sinh\left(m\log\left(b\right)+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(m*log(b) + a)*gamma(m + 1, b*x) + cosh(m*log(-b) - a)*gamma(m + 1
, -b*x) - gamma(m + 1, -b*x)*sinh(m*log(-b) - a) - gamma(m + 1, b*x)*sinh(m
*log(b) + a))/b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(b*x+a),x)
```

```
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*sinh(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \sinh(a + b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(a + b*x),x)
```

```
[Out] int(x^m*sinh(a + b*x), x)
```

3.82 $\int x^{-1+m}\sinh(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{1}{2}e^a x^m(-bx)^{-m}\Gamma(m,-bx)+\frac{1}{2}e^{-a}x^m(bx)^{-m}\Gamma(m,bx)$$

```
[Out] -1/2*exp(a)*x^m*GAMMA(m,-b*x)/((-b*x)^m)+1/2*x^m*GAMMA(m,b*x)/exp(a)/((b*x)
^m)
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {3389, 2212}

$$\frac{1}{2}e^{-a}x^m(bx)^{-m}\text{Gamma}(m,bx)-\frac{1}{2}e^a x^m(-bx)^{-m}\text{Gamma}(m,-bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)*Sinh[a + b*x],x]
```

```
[Out] -1/2*(E^a*x^m*Gamma[m, -(b*x)])/(-(b*x))^m + (x^m*Gamma[m, b*x])/(2*E^a*(b*
x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int x^{-1+m}\sinh(a+bx)\,dx&=\frac{1}{2}\int e^{-i(ia+ibx)}x^{-1+m}\,dx-\frac{1}{2}\int e^{i(ia+ibx)}x^{-1+m}\,dx\\&=-\frac{1}{2}e^a x^m(-bx)^{-m}\Gamma(m,-bx)+\frac{1}{2}e^{-a}x^m(bx)^{-m}\Gamma(m,bx)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 1.00

$$-\frac{1}{2}e^a x^m(-bx)^{-m}\Gamma(m,-bx)+\frac{1}{2}e^{-a}x^m(bx)^{-m}\Gamma(m,bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)*Sinh[a + b*x],x]
```

```
[Out] -1/2*(E^a*x^m*Gamma[m, -(b*x)])/(-(b*x))^m + (x^m*Gamma[m, b*x])/(2*E^a*(b*
x)^m)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.18, size = 67, normalized size = 1.37

method	result	size
meijerg	$\frac{x^m \operatorname{hypergeom}\left(\left[\frac{m}{2}\right],\left[\frac{1}{2},1+\frac{m}{2}\right],\frac{b^2x^2}{4}\right)\sinh(a)}{m}+\frac{b\,x^{1+m}\operatorname{hypergeom}\left(\left[\frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2},\frac{3}{2}+\frac{m}{2}\right],\frac{b^2x^2}{4}\right)\cosh(a)}{1+m}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/m*x^m*hypergeom([1/2*m],[1/2,1+1/2*m],1/4*b^2*x^2)*sinh(a)+b/(1+m)*x^(1+m
)*hypergeom([1/2+1/2*m],[3/2,3/2+1/2*m],1/4*b^2*x^2)*cosh(a)
```

Maxima [A]
time = 0.08, size = 43, normalized size = 0.88

$$\frac{x^m e^{(-a)}\Gamma(m,bx)}{2\,(bx)^m}-\frac{x^m e^a\Gamma(m,-bx)}{2\,(-bx)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^m*e^(-a)*gamma(m, b*x)/(b*x)^m - 1/2*x^m*e^a*gamma(m, -b*x)/(-b*x)^m
```

Fricas [A]
time = 0.08, size = 78, normalized size = 1.59

$$\frac{\cosh\left((m-1)\log\left(b\right)+a\right)\Gamma(m,bx)+\cosh\left((m-1)\log\left(-b\right)-a\right)\Gamma(m,-bx)-\Gamma(m,-bx)\sinh\left((m-1)\log\left(-b\right)-a\right)-\Gamma(m,bx)\sinh\left((m-1)\log\left(b\right)+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh((m - 1)*log(b) + a)*gamma(m, b*x) + cosh((m - 1)*log(-b) - a)*gam
ma(m, -b*x) - gamma(m, -b*x)*sinh((m - 1)*log(-b) - a) - gamma(m, b*x)*sinh
((m - 1)*log(b) + a))/b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*sinh(b*x+a),x)
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x^(m - 1)*sinh(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{m-1}\sinh(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 1)*sinh(a + b*x),x)
[Out] int(x^(m - 1)*sinh(a + b*x), x)
```

3.83 $\int x^{-2+m} \sinh(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}be^a x^m(-bx)^{-m}\Gamma(-1+m,-bx)+\frac{1}{2}be^{-a}x^m(bx)^{-m}\Gamma(-1+m,bx)$$

```
[Out] 1/2*b*exp(a)*x^m*GAMMA(-1+m,-b*x)/((-b*x)^m)+1/2*b*x^m*GAMMA(-1+m,b*x)/exp(
a)/((b*x)^m)
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3389, 2212}

$$\frac{1}{2}e^a bx^m(-bx)^{-m}\text{Gamma}(m-1,-bx)+\frac{1}{2}e^{-a}bx^m(bx)^{-m}\text{Gamma}(m-1,bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + m)*Sinh[a + b*x],x]
```

```
[Out] (b*E^a*x^m*Gamma[-1 + m, -(b*x)])/(2*(-(b*x))^m) + (b*x^m*Gamma[-1 + m, b*x
])/(2*E^a*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int x^{-2+m}\sinh(a+bx)\,dx &= \frac{1}{2}\int e^{-i(ia+ibx)}x^{-2+m}\,dx-\frac{1}{2}\int e^{i(ia+ibx)}x^{-2+m}\,dx\\ &= \frac{1}{2}be^a x^m(-bx)^{-m}\Gamma(-1+m,-bx)+\frac{1}{2}be^{-a}x^m(bx)^{-m}\Gamma(-1+m,bx)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 51, normalized size = 0.93

$$\frac{1}{2}be^{-a}x^m\left(e^{2a}(-bx)^{-m}\Gamma(-1+m,-bx)+(bx)^{-m}\Gamma(-1+m,bx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + m)*Sinh[a + b*x],x]
```

```
[Out] (b*x^m*((E^(2*a)*Gamma[-1 + m, -(b*x)])/(-(b*x))^m + Gamma[-1 + m, b*x]/(b*
x)^m))/(2*E^a)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.19, size = 67, normalized size = 1.22

method	result	size
meijerg	$\frac{x^{-1+m}\,\mathrm{hypergeom}\left([-\frac{1}{2}+\frac{m}{2}],[\frac{1}{2},\frac{1}{2}+\frac{m}{2}],\frac{b^2x^2}{4}\right)\sinh(a)}{-1+m}+\frac{b\,x^m\,\mathrm{hypergeom}\left([\frac{m}{2}],[\frac{3}{2},1+\frac{m}{2}],\frac{b^2x^2}{4}\right)\cosh(a)}{m}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2+m)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-1+m)*x^(-1+m)*hypergeom([-1/2+1/2*m],[1/2,1/2+1/2*m],1/4*b^2*x^2)*sinh(
a)+b/m*x^m*hypergeom([1/2*m],[3/2,1+1/2*m],1/4*b^2*x^2)*cosh(a)
```

Maxima [A]
time = 0.09, size = 55, normalized size = 1.00

$$\frac{1}{2}(bx)^{-m+1}x^{m-1}e^{(-a)}\Gamma(m-1,bx)-\frac{1}{2}(-bx)^{-m+1}x^{m-1}e^{a}\Gamma(m-1,-bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x)^(-m + 1)*x^(m - 1)*e^(-a)*gamma(m - 1, b*x) - 1/2*(-b*x)^(-m + 1)
*x^(m - 1)*e^a*gamma(m - 1, -b*x)
```

Fricas [A]
time = 0.08, size = 86, normalized size = 1.56

$$\frac{\cosh\left((m-2)\log(b)+a\right)\Gamma(m-1,bx)+\cosh\left((m-2)\log(-b)-a\right)\Gamma(m-1,-bx)-\Gamma(m-1,-bx)\sinh\left((m-2)\log(-b)-a\right)-\Gamma(m-1,bx)\sinh\left((m-2)\log(b)+a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh((m - 2)*log(b) + a)*gamma(m - 1, b*x) + cosh((m - 2)*log(-b) - a)
*gamma(m - 1, -b*x) - gamma(m - 1, -b*x)*sinh((m - 2)*log(-b) - a) - gamma(
m - 1, b*x)*sinh((m - 2)*log(b) + a))/b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+m)*sinh(b*x+a),x)
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x^(m - 2)*sinh(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{m-2}\sinh(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 2)*sinh(a + b*x),x)
[Out] int(x^(m - 2)*sinh(a + b*x), x)
```

3.84 $\int x^{-3+m}\sinh(a+bx)\,dx$

Optimal. Leaf size=59

$$-\frac{1}{2}b^2e^ax^m(-bx)^{-m}\Gamma(-2+m,-bx)+\frac{1}{2}b^2e^{-a}x^m(bx)^{-m}\Gamma(-2+m,bx)$$

```
[Out] -1/2*b^2*exp(a)*x^m*GAMMA(-2+m,-b*x)/((-b*x)^m)+1/2*b^2*x^m*GAMMA(-2+m,b*x)
/exp(a)/((b*x)^m)
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3389, 2212}

$$\frac{1}{2}e^{-a}b^2x^m(bx)^{-m}\text{Gamma}(m-2,bx)-\frac{1}{2}e^ab^2x^m(-bx)^{-m}\text{Gamma}(m-2,-bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + m)*Sinh[a + b*x],x]
```

```
[Out] -1/2*(b^2*E^a*x^m*Gamma[-2 + m, -(b*x)])/(-(b*x))^m + (b^2*x^m*Gamma[-2 + m
, b*x])/(2*E^a*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int x^{-3+m}\sinh(a+bx)\,dx &= \frac{1}{2}\int e^{-i(ia+ibx)}x^{-3+m}\,dx-\frac{1}{2}\int e^{i(ia+ibx)}x^{-3+m}\,dx\\ &= -\frac{1}{2}b^2e^ax^m(-bx)^{-m}\Gamma(-2+m,-bx)+\frac{1}{2}b^2e^{-a}x^m(bx)^{-m}\Gamma(-2+m,bx)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 0.92

$$\frac{1}{2}b^2e^{-a}x^m\left(-e^{2a}(-bx)^{-m}\Gamma(-2+m,-bx)+(bx)^{-m}\Gamma(-2+m,bx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + m)*Sinh[a + b*x],x]
```

```
[Out] (b^2*x^m*(-((E^(2*a)*Gamma[-2 + m, -(b*x)])/(-(b*x))^m) + Gamma[-2 + m, b*x
]/(b*x)^m))/(2*E^a)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.17, size = 71, normalized size = 1.20

method	result	size
meijerg	$\frac{x^{-2+m}\,\mathrm{hypergeom}\left(\left[-1+\frac{m}{2}\right],\left[\frac{1}{2},\frac{m}{2}\right],\frac{b^2x^2}{4}\right)\sinh(a)}{-2+m}+\frac{b\,x^{-1+m}\,\mathrm{hypergeom}\left(\left[-\frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2},\frac{1}{2}+\frac{m}{2}\right],\frac{b^2x^2}{4}\right)\cosh(a)}{-1+m}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3+m)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-2+m)*x^(-2+m)*hypergeom([-1+1/2*m],[1/2,1/2*m],1/4*b^2*x^2)*sinh(a)+b/(
-1+m)*x^(-1+m)*hypergeom([-1/2+1/2*m],[3/2,1/2+1/2*m],1/4*b^2*x^2)*cosh(a)
```

Maxima [A]
time = 0.09, size = 55, normalized size = 0.93

$$\frac{1}{2}\left(bx\right)^{-m+2}x^{m-2}e^{(-a)}\Gamma(m-2,bx)-\frac{1}{2}\left(-bx\right)^{-m+2}x^{m-2}e^{a}\Gamma(m-2,-bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x)^(-m + 2)*x^(m - 2)*e^(-a)*gamma(m - 2, b*x) - 1/2*(-b*x)^(-m + 2)
*x^(m - 2)*e^a*gamma(m - 2, -b*x)
```

Fricas [A]
time = 0.08, size = 86, normalized size = 1.46

$$\frac{\cosh\left((m-3)\log\left(b\right)+a\right)\Gamma(m-2,bx)+\cosh\left((m-3)\log\left(-b\right)-a\right)\Gamma(m-2,-bx)-\Gamma(m-2,-bx)\sinh\left((m-3)\log\left(-b\right)-a\right)-\Gamma(m-2,bx)\sinh\left((m-3)\log\left(b\right)+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh((m - 3)*log(b) + a)*gamma(m - 2, b*x) + cosh((m - 3)*log(-b) - a)
*gamma(m - 2, -b*x) - gamma(m - 2, -b*x)*sinh((m - 3)*log(-b) - a) - gamma(
m - 2, b*x)*sinh((m - 3)*log(b) + a))/b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+m)*sinh(b*x+a),x)
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sinh(b*x+a),x, algorithm="giac")
[Out] integrate(x^(m - 3)*sinh(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{m-3}\sinh(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 3)*sinh(a + b*x),x)
[Out] int(x^(m - 3)*sinh(a + b*x), x)
```

3.85 $\int x^{3+m}\sinh^2(a+bx)\,dx$

Optimal. Leaf size=86

$$-\frac{x^{4+m}}{2(4+m)}-\frac{2^{-6-m}e^{2a}x^m(-bx)^{-m}\Gamma(4+m,-2bx)}{b^4}-\frac{2^{-6-m}e^{-2a}x^m(bx)^{-m}\Gamma(4+m,2bx)}{b^4}$$

```
[Out] -1/2*x^(4+m)/(4+m)-2^(-6-m)*exp(2*a)*x^m*GAMMA(4+m,-2*b*x)/b^4/((-b*x)^m)-2
^(-6-m)*x^m*GAMMA(4+m,2*b*x)/b^4/exp(2*a)/((b*x)^m)
```

Rubi [A]
time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3393, 3388, 2212}

$$-\frac{e^{2a}2^{-m-6}x^m(-bx)^{-m}\text{Gamma}(m+4,-2bx)}{b^4}-\frac{e^{-2a}2^{-m-6}x^m(bx)^{-m}\text{Gamma}(m+4,2bx)}{b^4}-\frac{x^{m+4}}{2(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 + m)*Sinh[a + b*x]^2,x]
```

```
[Out] -1/2*x^(4 + m)/(4 + m) - (2^(-6 - m)*E^(2*a)*x^m*Gamma[4 + m, -2*b*x])/(b^4
*(-(b*x))^m) - (2^(-6 - m)*x^m*Gamma[4 + m, 2*b*x])/(b^4*E^(2*a)*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int x^{3+m}\sinh^2(a+bx)\,dx = -\int\left(\frac{x^{3+m}}{2}-\frac{1}{2}x^{3+m}\cosh(2a+2bx)\right)dx$$
$$= -\frac{x^{4+m}}{2(4+m)}+\frac{1}{2}\int x^{3+m}\cosh(2a+2bx)\,dx$$
$$= -\frac{x^{4+m}}{2(4+m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{3+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{3+m}\,dx$$
$$= -\frac{x^{4+m}}{2(4+m)}-\frac{2^{-6-m}e^{2a}x^m(-bx)^{-m}\Gamma(4+m,-2bx)}{b^4}-\frac{2^{-6-m}e^{-2a}x^m(bx)^{-m}\Gamma(4+$$

Mathematica [A]
time = 0.10, size = 79, normalized size = 0.92

$$\frac{1}{64}x^m\left(-\frac{32x^4}{4+m}-\frac{2^{-m}e^{2a}(-bx)^{-m}\Gamma(4+m,-2bx)}{b^4}-\frac{2^{-m}e^{-2a}(bx)^{-m}\Gamma(4+m,2bx)}{b^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 + m)*Sinh[a + b*x]^2,x]
[Out] (x^m*((-32*x^4)/(4 + m) - (E^(2*a)*Gamma[4 + m, -2*b*x])/(2^m*b^4*(-(b*x))^
m) - Gamma[4 + m, 2*b*x]/(2^m*b^4*E^(2*a)*(b*x)^m)))/64
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int x^{m+3}\left(\sinh^2(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m+3)*sinh(b*x+a)^2,x)
[Out] int(x^(m+3)*sinh(b*x+a)^2,x)
```

Maxima [A]
time = 0.08, size = 71, normalized size = 0.83

$$-\frac{1}{4}(2\,bx)^{-m-4}\,x^{m+4}e^{(-2\,a)}\Gamma(m+4,2\,bx)-\frac{1}{4}(-2\,bx)^{-m-4}\,x^{m+4}e^{(2\,a)}\Gamma(m+4,-2\,bx)-\frac{x^{m+4}}{2\,(m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] -1/4*(2*b*x)^(-m - 4)*x^(m + 4)*e^(-2*a)*gamma(m + 4, 2*b*x) - 1/4*(-2*b*x)
^(-m - 4)*x^(m + 4)*e^(2*a)*gamma(m + 4, -2*b*x) - 1/2*x^(m + 4)/(m + 4)
```

Fricas [A]
time = 0.09, size = 136, normalized size = 1.58

$$-\frac{4\,bx\cosh\left((m+3)\log\left(x\right)\right)+(m+4)\cosh\left((m+3)\log\left(2\,b\right)+2\,a\right)\Gamma(m+4,2\,bx)-(m+4)\cosh\left((m+3)\log\left(-2\,b\right)-2\,a\right)\Gamma(m+4,-2\,bx)-(m+4)\Gamma(m+4,2\,bx)\sinh\left((m+3)\log\left(2\,b\right)+2\,a\right)+(m+4)\Gamma(m+4,-2\,bx)\sinh\left((m+3)\log\left(-2\,b\right)-2\,a\right)+4\,bx\sinh\left((m+3)\log\left(x\right)\right)}{8\,(bm+4\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*x*cosh((m + 3)*log(x)) + (m + 4)*cosh((m + 3)*log(2*b) + 2*a)*gam
ma(m + 4, 2*b*x) - (m + 4)*cosh((m + 3)*log(-2*b) - 2*a)*gamma(m + 4, -2*b*
x) - (m + 4)*gamma(m + 4, 2*b*x)*sinh((m + 3)*log(2*b) + 2*a) + (m + 4)*gam
ma(m + 4, -2*b*x)*sinh((m + 3)*log(-2*b) - 2*a) + 4*b*x*sinh((m + 3)*log(x)
))/(b*m + 4*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+3}\sinh^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3+m)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**(m + 3)*sinh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 3)*sinh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m+3}\,\mathrm{sinh}(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 3)*sinh(a + b*x)^2,x)
```

```
[Out] int(x^(m + 3)*sinh(a + b*x)^2, x)
```

3.86 $\int x^{2+m} \sinh^2(a+bx)\,dx$

Optimal. Leaf size=85

$$-\frac{x^{3+m}}{2(3+m)}+\frac{2^{-5-m}e^{2a}x^m(-bx)^{-m}\Gamma(3+m,-2bx)}{b^3}-\frac{2^{-5-m}e^{-2a}x^m(bx)^{-m}\Gamma(3+m,2bx)}{b^3}$$

```
[Out] -1/2*x^(3+m)/(3+m)+2^(-5-m)*exp(2*a)*x^m*GAMMA(3+m,-2*b*x)/b^3/((-b*x)^m)-2
^(-5-m)*x^m*GAMMA(3+m,2*b*x)/b^3/exp(2*a)/((b*x)^m)
```

Rubi [A]

time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3393, 3388, 2212}

$$\frac{e^{2a}2^{-m-5}x^m(-bx)^{-m}\text{Gamma}(m+3,-2bx)}{b^3}-\frac{e^{-2a}2^{-m-5}x^m(bx)^{-m}\text{Gamma}(m+3,2bx)}{b^3}-\frac{x^{m+3}}{2(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + m)*Sinh[a + b*x]^2,x]
```

```
[Out] -1/2*x^(3 + m)/(3 + m) + (2^(-5 - m)*E^(2*a)*x^m*Gamma[3 + m, -2*b*x])/(b^3
*(-(b*x))^m) - (2^(-5 - m)*x^m*Gamma[3 + m, 2*b*x])/(b^3*E^(2*a)*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int x^{2+m}\sinh^2(a+bx)\,dx &= -\int\left(\frac{x^{2+m}}{2}-\frac{1}{2}x^{2+m}\cosh(2a+2bx)\right)\,dx\\ &= -\frac{x^{3+m}}{2(3+m)}+\frac{1}{2}\int x^{2+m}\cosh(2a+2bx)\,dx\\ &= -\frac{x^{3+m}}{2(3+m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{2+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{2+m}\,dx\\ &= -\frac{x^{3+m}}{2(3+m)}+\frac{2^{-5-m}e^{2a}x^m(-bx)^{-m}\Gamma(3+m,-2bx)}{b^3}-\frac{2^{-5-m}e^{-2a}x^m(bx)^{-m}\Gamma(3\end{aligned}$$

Mathematica [A]
time = 0.10, size = 78, normalized size = 0.92

$$\frac{1}{32}x^m\left(-\frac{16x^3}{3+m}+\frac{2^{-m}e^{2a}(-bx)^{-m}\Gamma(3+m,-2bx)}{b^3}-\frac{2^{-m}e^{-2a}(bx)^{-m}\Gamma(3+m,2bx)}{b^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + m)*Sinh[a + b*x]^2,x]
```

```
[Out] (x^m*((-16*x^3)/(3 + m) + (E^(2*a)*Gamma[3 + m, -2*b*x])/(2^m*b^3*(-(b*x))^
m) - Gamma[3 + m, 2*b*x]/(2^m*b^3*E^(2*a)*(b*x)^m)))/32
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int x^{2+m}\left(\sinh^2\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+m)*sinh(b*x+a)^2,x)
```

```
[Out] int(x^(2+m)*sinh(b*x+a)^2,x)
```

Maxima [A]
time = 0.07, size = 71, normalized size = 0.84

$$-\frac{1}{4}\left(2\,bx\right)^{-m-3}x^{m+3}e^{(-2\,a)}\Gamma(m+3,2\,bx)-\frac{1}{4}\left(-2\,bx\right)^{-m-3}x^{m+3}e^{(2\,a)}\Gamma(m+3,-2\,bx)-\frac{x^{m+3}}{2\,(m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*x)^(-m - 3)*x^(m + 3)*e^(-2*a)*gamma(m + 3, 2*b*x) - 1/4*(-2*b*x)
^(-m - 3)*x^(m + 3)*e^(2*a)*gamma(m + 3, -2*b*x) - 1/2*x^(m + 3)/(m + 3)
```

Fricas [A]
time = 0.14, size = 136, normalized size = 1.60

$$-\frac{4\,bx\cosh\left((m+2)\log\left(x\right)\right)+(m+3)\cosh\left((m+2)\log\left(2\,b\right)+2\,a\right)\Gamma(m+3,2\,bx)-(m+3)\cosh\left((m+2)\log\left(-2\,b\right)-2\,a\right)\Gamma(m+3,-2\,bx)-(m+3)\Gamma(m+3,2\,bx)\sinh\left((m+2)\log\left(2\,b\right)+2\,a\right)+(m+3)\Gamma(m+3,-2\,bx)\sinh\left((m+2)\log\left(-2\,b\right)-2\,a\right)+4\,bx\sinh\left((m+2)\log\left(x\right)\right)}{8\,(bm+3\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*x*cosh((m + 2)*log(x)) + (m + 3)*cosh((m + 2)*log(2*b) + 2*a)*gam
ma(m + 3, 2*b*x) - (m + 3)*cosh((m + 2)*log(-2*b) - 2*a)*gamma(m + 3, -2*b*
x) - (m + 3)*gamma(m + 3, 2*b*x)*sinh((m + 2)*log(2*b) + 2*a) + (m + 3)*gam
ma(m + 3, -2*b*x)*sinh((m + 2)*log(-2*b) - 2*a) + 4*b*x*sinh((m + 2)*log(x)
))/(b*m + 3*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+2}\sinh^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+m)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**(m + 2)*sinh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2)*sinh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m+2}\,{\mathrm{sinh}\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 2)*sinh(a + b*x)^2,x)
```

```
[Out] int(x^(m + 2)*sinh(a + b*x)^2, x)
```

3.87 $\int x^{1+m} \sinh^2(a+bx)\,dx$

Optimal. Leaf size=86

$$-\frac{x^{2+m}}{2(2+m)}-\frac{2^{-4-m}e^{2a}x^m(-bx)^{-m}\Gamma(2+m,-2bx)}{b^2}-\frac{2^{-4-m}e^{-2a}x^m(bx)^{-m}\Gamma(2+m,2bx)}{b^2}$$

```
[Out] -1/2*x^(2+m)/(2+m)-2^(-4-m)*exp(2*a)*x^m*GAMMA(2+m,-2*b*x)/b^2/((-b*x)^m)-2
^(-4-m)*x^m*GAMMA(2+m,2*b*x)/b^2/exp(2*a)/((b*x)^m)
```

Rubi [A]
time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3393, 3388, 2212}

$$-\frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\text{Gamma}(m+2,-2bx)}{b^2}-\frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\text{Gamma}(m+2,2bx)}{b^2}-\frac{x^{m+2}}{2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + m)*Sinh[a + b*x]^2,x]
```

```
[Out] -1/2*x^(2 + m)/(2 + m) - (2^(-4 - m)*E^(2*a)*x^m*Gamma[2 + m, -2*b*x])/(b^2
*(-(b*x))^m) - (2^(-4 - m)*x^m*Gamma[2 + m, 2*b*x])/(b^2*E^(2*a)*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int x^{1+m}\sinh^2(a+bx)\,dx = -\int\left(\frac{x^{1+m}}{2}-\frac{1}{2}x^{1+m}\cosh(2a+2bx)\right)dx$$
$$= -\frac{x^{2+m}}{2(2+m)}+\frac{1}{2}\int x^{1+m}\cosh(2a+2bx)\,dx$$
$$= -\frac{x^{2+m}}{2(2+m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{1+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{1+m}\,dx$$
$$= -\frac{x^{2+m}}{2(2+m)}-\frac{2^{-4-m}e^{2a}x^m(-bx)^{-m}\Gamma(2+m,-2bx)}{b^2}-\frac{2^{-4-m}e^{-2a}x^m(bx)^{-m}\Gamma(2+}{b^2}$$

Mathematica [A]
time = 0.09, size = 79, normalized size = 0.92

$$\frac{1}{16}x^m\left(-\frac{8x^2}{2+m}-\frac{2^{-m}e^{2a}(-bx)^{-m}\Gamma(2+m,-2bx)}{b^2}-\frac{2^{-m}e^{-2a}(bx)^{-m}\Gamma(2+m,2bx)}{b^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + m)*Sinh[a + b*x]^2,x]
```

```
[Out] (x^m*((-8*x^2)/(2 + m) - (E^(2*a)*Gamma[2 + m, -2*b*x])/(2^m*b^2*(-(b*x))^m
) - Gamma[2 + m, 2*b*x]/(2^m*b^2*E^(2*a)*(b*x)^m)))/16
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int x^{1+m}\left(\sinh^2(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)*sinh(b*x+a)^2,x)
```

```
[Out] int(x^(1+m)*sinh(b*x+a)^2,x)
```

Maxima [A]
time = 0.07, size = 71, normalized size = 0.83

$$-\frac{1}{4}(2\,bx)^{-m-2}\,x^{m+2}e^{(-2\,a)}\Gamma(m+2,2\,bx)-\frac{1}{4}(-2\,bx)^{-m-2}\,x^{m+2}e^{(2\,a)}\Gamma(m+2,-2\,bx)-\frac{x^{m+2}}{2\,(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*x)^(-m - 2)*x^(m + 2)*e^(-2*a)*gamma(m + 2, 2*b*x) - 1/4*(-2*b*x)
^(-m - 2)*x^(m + 2)*e^(2*a)*gamma(m + 2, -2*b*x) - 1/2*x^(m + 2)/(m + 2)
```

Fricas [A]
time = 0.14, size = 136, normalized size = 1.58

$$-\frac{4\,bx\cosh\left((m+1)\log\left(x\right)\right)+(m+2)\cosh\left((m+1)\log\left(2\,b\right)+2\,a\right)\Gamma(m+2,2\,bx)-(m+2)\cosh\left((m+1)\log\left(-2\,b\right)-2\,a\right)\Gamma(m+2,-2\,bx)-(m+2)\Gamma(m+2,2\,bx)\sinh\left((m+1)\log\left(2\,b\right)+2\,a\right)+(m+2)\Gamma(m+2,-2\,bx)\sinh\left((m+1)\log\left(-2\,b\right)-2\,a\right)+4\,bx\sinh\left((m+1)\log\left(x\right)\right)}{8\,(bm+2\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*x*cosh((m + 1)*log(x)) + (m + 2)*cosh((m + 1)*log(2*b) + 2*a)*gam
ma(m + 2, 2*b*x) - (m + 2)*cosh((m + 1)*log(-2*b) - 2*a)*gamma(m + 2, -2*b*
x) - (m + 2)*gamma(m + 2, 2*b*x)*sinh((m + 1)*log(2*b) + 2*a) + (m + 2)*gam
ma(m + 2, -2*b*x)*sinh((m + 1)*log(-2*b) - 2*a) + 4*b*x*sinh((m + 1)*log(x)
))/(b*m + 2*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+1}\sinh^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**(m + 1)*sinh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 1)*sinh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m+1}\,{\mathrm{sinh}\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 1)*sinh(a + b*x)^2,x)
```

```
[Out] int(x^(m + 1)*sinh(a + b*x)^2, x)
```

3.88 $\int x^m \sinh^2(a+bx)\,dx$

Optimal. Leaf size=85

$$-\frac{x^{1+m}}{2(1+m)}+\frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b}-\frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b}$$

```
[Out] -1/2*x^(1+m)/(1+m)+2^(-3-m)*exp(2*a)*x^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)-2^(
-3-m)*x^m*GAMMA(1+m,2*b*x)/b/exp(2*a)/((b*x)^m)
```

Rubi [A]
time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3393, 3388, 2212}

$$\frac{e^{2a}2^{-m-3}x^m(-bx)^{-m}\text{Gamma}(m+1,-2bx)}{b}-\frac{e^{-2a}2^{-m-3}x^m(bx)^{-m}\text{Gamma}(m+1,2bx)}{b}-\frac{x^{m+1}}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sinh[a + b*x]^2,x]
```

```
[Out] -1/2*x^(1 + m)/(1 + m) + (2^(-3 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(
-(b*x))^m) - (2^(-3 - m)*x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int x^m \sinh^2(a+bx)\,dx &= -\int \left(\frac{x^m}{2}-\frac{1}{2}x^m\cosh(2a+2bx)\right)\,dx\\ &= -\frac{x^{1+m}}{2(1+m)}+\frac{1}{2}\int x^m\cosh(2a+2bx)\,dx\\ &= -\frac{x^{1+m}}{2(1+m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^m\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^m\,dx\\ &= -\frac{x^{1+m}}{2(1+m)}+\frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b}-\frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+\end{aligned}$$

Mathematica [A]
time = 0.08, size = 76, normalized size = 0.89

$$\frac{1}{8}x^m\left(-\frac{4x}{1+m}+\frac{2^{-m}e^{2a}(-bx)^{-m}\Gamma(1+m,-2bx)}{b}-\frac{2^{-m}e^{-2a}(bx)^{-m}\Gamma(1+m,2bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sinh[a + b*x]^2,x]
```

```
[Out] (x^m*((-4*x)/(1 + m) + (E^(2*a)*Gamma[1 + m, -2*b*x])/(2^m*b*(-(b*x))^m) -
Gamma[1 + m, 2*b*x]/(2^m*b*E^(2*a)*(b*x)^m)))/8
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int x^m\left(\sinh^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(b*x+a)^2,x)
```

```
[Out] int(x^m*sinh(b*x+a)^2,x)
```

Maxima [A]
time = 0.08, size = 71, normalized size = 0.84

$$-\frac{1}{4}\left(2\,bx\right)^{-m-1}x^{m+1}e^{(-2\,a)}\Gamma(m+1,2\,bx)-\frac{1}{4}\left(-2\,bx\right)^{-m-1}x^{m+1}e^{(2\,a)}\Gamma(m+1,-2\,bx)-\frac{x^{m+1}}{2\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*x)^(-m - 1)*x^(m + 1)*e^(-2*a)*gamma(m + 1, 2*b*x) - 1/4*(-2*b*x)
^(-m - 1)*x^(m + 1)*e^(2*a)*gamma(m + 1, -2*b*x) - 1/2*x^(m + 1)/(m + 1)
```

Fricas [A]
time = 0.09, size = 122, normalized size = 1.44

$$-\frac{4\,bx\cosh\left(m\log\left(x\right)\right)+\left(m+1\right)\cosh\left(m\log\left(2\,b\right)+2\,a\right)\Gamma(m+1,2\,bx)-\left(m+1\right)\cosh\left(m\log\left(-2\,b\right)-2\,a\right)\Gamma(m+1,-2\,bx)-\left(m+1\right)\Gamma(m+1,2\,bx)\sinh\left(m\log\left(2\,b\right)+2\,a\right)+\left(m+1\right)\Gamma(m+1,-2\,bx)\sinh\left(m\log\left(-2\,b\right)-2\,a\right)+4\,bx\sinh\left(m\log\left(x\right)\right)}{8\,\left(bm+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*x*cosh(m*log(x)) + (m + 1)*cosh(m*log(2*b) + 2*a)*gamma(m + 1, 2*
b*x) - (m + 1)*cosh(m*log(-2*b) - 2*a)*gamma(m + 1, -2*b*x) - (m + 1)*gamma
(m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) + (m + 1)*gamma(m + 1, -2*b*x)*sinh(m
*log(-2*b) - 2*a) + 4*b*x*sinh(m*log(x)))/(b*m + b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m}\sinh^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**m*sinh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*sinh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,{\mathrm{sinh}\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(a + b*x)^2,x)
```

```
[Out] int(x^m*sinh(a + b*x)^2, x)
```

3.89 $\int x^{-1+m} \sinh^2(a+bx)\,dx$

Optimal. Leaf size=72

$$-\frac{x^m}{2m} - 2^{-2-m}e^{2a}x^m(-bx)^{-m}\Gamma(m,-2bx) - 2^{-2-m}e^{-2a}x^m(bx)^{-m}\Gamma(m,2bx)$$

```
[Out] -1/2*x^m/m-2^(-2-m)*exp(2*a)*x^m*GAMMA(m,-2*b*x)/((-b*x)^m)-2^(-2-m)*x^m*GA
MMA(m,2*b*x)/exp(2*a)/((b*x)^m)
```

Rubi [A]

time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3393, 3388, 2212}

$$e^{2a}\left(-2^{-m-2}\right)x^m(-bx)^{-m}\text{Gamma}(m,-2bx) - e^{-2a}2^{-m-2}x^m(bx)^{-m}\text{Gamma}(m,2bx) - \frac{x^m}{2m}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)*Sinh[a + b*x]^2,x]
```

```
[Out] -1/2*x^m/m - (2^(-2 - m)*E^(2*a)*x^m*Gamma[m, -2*b*x])/(-(b*x))^m - (2^(-2
- m)*x^m*Gamma[m, 2*b*x])/(E^(2*a)*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int x^{-1+m}\sinh^2(a+bx)\,dx &= -\int \left(\frac{x^{-1+m}}{2}-\frac{1}{2}x^{-1+m}\cosh(2a+2bx)\right)\,dx\\ &= -\frac{x^m}{2m}+\frac{1}{2}\int x^{-1+m}\cosh(2a+2bx)\,dx\\ &= -\frac{x^m}{2m}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{-1+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{-1+m}\,dx\\ &= -\frac{x^m}{2m}-2^{-2-m}e^{2a}x^m(-bx)^{-m}\Gamma(m,-2bx)-2^{-2-m}e^{-2a}x^m(bx)^{-m}\Gamma(m,2bx)\end{aligned}$$

Mathematica [A]
time = 0.07, size = 63, normalized size = 0.88

$$-\frac{x^m(2+2^{-m}e^{2a}m(-bx)^{-m}\Gamma(m,-2bx)+2^{-m}e^{-2a}m(bx)^{-m}\Gamma(m,2bx))}{4m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)*Sinh[a + b*x]^2,x]
```

```
[Out] -1/4*(x^m*(2 + (E^(2*a)*m*Gamma[m, -2*b*x])/(2^m*(-(b*x))^m) + (m*Gamma[m,
2*b*x])/(2^m*E^(2*a)*(b*x)^m)))/m
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int x^{-1+m}\left(\sinh^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*sinh(b*x+a)^2,x)
```

```
[Out] int(x^(-1+m)*sinh(b*x+a)^2,x)
```

Maxima [A]
time = 0.09, size = 55, normalized size = 0.76

$$-\frac{x^m e^{(-2a)}\Gamma(m,2bx)}{4(2bx)^m}-\frac{x^m e^{(2a)}\Gamma(m,-2bx)}{4(-2bx)^m}-\frac{x^m}{2m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*x^m*e^(-2*a)*gamma(m, 2*b*x)/(2*b*x)^m - 1/4*x^m*e^(2*a)*gamma(m, -2*b
*x)/(-2*b*x)^m - 1/2*x^m/m
```

Fricas [A]
time = 0.09, size = 117, normalized size = 1.62

$$-\frac{4\,bx\cosh\left((m-1)\log\left(x\right)\right)+m\cosh\left((m-1)\log\left(2\,b\right)+2\,a\right)\Gamma\left(m,2\,bx\right)-m\cosh\left((m-1)\log\left(-2\,b\right)-2\,a\right)\Gamma\left(m,-2\,bx\right)-m\Gamma\left(m,2\,bx\right)\sinh\left((m-1)\log\left(2\,b\right)+2\,a\right)+m\Gamma\left(m,-2\,bx\right)\sinh\left((m-1)\log\left(-2\,b\right)-2\,a\right)+4\,bx\sinh\left((m-1)\log\left(x\right)\right)}{8\,bm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*x*cosh((m - 1)*log(x)) + m*cosh((m - 1)*log(2*b) + 2*a)*gamma(m,
2*b*x) - m*cosh((m - 1)*log(-2*b) - 2*a)*gamma(m, -2*b*x) - m*gamma(m, 2*b*
x)*sinh((m - 1)*log(2*b) + 2*a) + m*gamma(m, -2*b*x)*sinh((m - 1)*log(-2*b)
 - 2*a) + 4*b*x*sinh((m - 1)*log(x)))/(b*m)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-1}\sinh^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**(m - 1)*sinh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 1)*sinh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m-1}\,{\mathrm{sinh}\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 1)*sinh(a + b*x)^2,x)
```

```
[Out] int(x^(m - 1)*sinh(a + b*x)^2, x)
```

3.90 $\int x^{-2+m}\sinh^2(a+bx)\,dx$

Optimal. Leaf size=83

$$\frac{x^{-1+m}}{2(1-m)}+2^{-1-m}be^{2a}x^m(-bx)^{-m}\Gamma(-1+m,-2bx)-2^{-1-m}be^{-2a}x^m(bx)^{-m}\Gamma(-1+m,2bx)$$

```
[Out] 1/2*x^(-1+m)/(1-m)+2^(-1-m)*b*exp(2*a)*x^m*GAMMA(-1+m,-2*b*x)/((-b*x)^m)-2^
(-1-m)*b*x^m*GAMMA(-1+m,2*b*x)/exp(2*a)/((b*x)^m)
```

Rubi [A]
time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3393, 3388, 2212}

$$e^{2a}b2^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m-1,-2bx)-e^{-2a}b2^{-m-1}x^m(bx)^{-m}\text{Gamma}(m-1,2bx)+\frac{x^{m-1}}{2(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + m)*Sinh[a + b*x]^2,x]
```

```
[Out] x^(-1 + m)/(2*(1 - m)) + (2^(-1 - m)*b*E^(2*a)*x^m*Gamma[-1 + m, -2*b*x])/(
-(b*x))^m - (2^(-1 - m)*b*x^m*Gamma[-1 + m, 2*b*x])/(E^(2*a)*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int x^{-2+m}\sinh^2(a+bx)\,dx = -\int \left(\frac{x^{-2+m}}{2}-\frac{1}{2}x^{-2+m}\cosh(2a+2bx)\right)dx$$
$$= \frac{x^{-1+m}}{2(1-m)}+\frac{1}{2}\int x^{-2+m}\cosh(2a+2bx)\,dx$$
$$= \frac{x^{-1+m}}{2(1-m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{-2+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{-2+m}\,dx$$
$$= \frac{x^{-1+m}}{2(1-m)}+2^{-1-m}be^{2a}x^m(-bx)^{-m}\Gamma(-1+m,-2bx)-2^{-1-m}be^{-2a}x^m(bx)^{-m}\Gamma$$

Mathematica [A]
time = 0.09, size = 72, normalized size = 0.87

$$\frac{1}{2}x^m\left(\frac{1}{x-mx}+2^{-m}be^{2a}(-bx)^{-m}\Gamma(-1+m,-2bx)-2^{-m}be^{-2a}(bx)^{-m}\Gamma(-1+m,2bx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + m)*Sinh[a + b*x]^2,x]
[Out] (x^m*((x - m*x)^(-1) + (b*E^(2*a)*Gamma[-1 + m, -2*b*x])/(2^m*(-(b*x))^m) -
 (b*Gamma[-1 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m)))/2
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int x^{-2+m}\left(\sinh^2(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2+m)*sinh(b*x+a)^2,x)
[Out] int(x^(-2+m)*sinh(b*x+a)^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(m-2>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 0.08, size = 136, normalized size = 1.64

$$-\frac{4\,bx\cosh\left((m-2)\log\left(x\right)\right)+(m-1)\cosh\left((m-2)\log\left(2\,b\right)+2\,a\right)\Gamma(m-1,2\,bx)-(m-1)\cosh\left((m-2)\log\left(-2\,b\right)-2\,a\right)\Gamma(m-1,-2\,bx)-(m-1)\Gamma(m-1,2\,bx)\sinh\left((m-2)\log\left(2\,b\right)+2\,a\right)+(m-1)\Gamma(m-1,-2\,bx)\sinh\left((m-2)\log\left(-2\,b\right)-2\,a\right)+4\,bx\sinh\left((m-2)\log\left(x\right)\right)}{8\,(bm-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*x*cosh((m - 2)*log(x)) + (m - 1)*cosh((m - 2)*log(2*b) + 2*a)*gam
ma(m - 1, 2*b*x) - (m - 1)*cosh((m - 2)*log(-2*b) - 2*a)*gamma(m - 1, -2*b*
x) - (m - 1)*gamma(m - 1, 2*b*x)*sinh((m - 2)*log(2*b) + 2*a) + (m - 1)*gam
ma(m - 1, -2*b*x)*sinh((m - 2)*log(-2*b) - 2*a) + 4*b*x*sinh((m - 2)*log(x)
))/(b*m - b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-2}\sinh^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+m)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**(m - 2)*sinh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 2)*sinh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m-2}\,{\sinh\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 2)*sinh(a + b*x)^2,x)
```

```
[Out] int(x^(m - 2)*sinh(a + b*x)^2, x)
```

3.91 $\int x^{-3+m} \sinh^2(a+bx)\,dx$

Optimal. Leaf size=84

$$\frac{x^{-2+m}}{2(2-m)} - 2^{-m}b^2e^{2a}x^m(-bx)^{-m}\Gamma(-2+m,-2bx) - 2^{-m}b^2e^{-2a}x^m(bx)^{-m}\Gamma(-2+m,2bx)$$

```
[Out] 1/2*x^(-2+m)/(2-m)-b^2*exp(2*a)*x^m*GAMMA(-2+m,-2*b*x)/(2^m)/((-b*x)^m)-b^2
*x^m*GAMMA(-2+m,2*b*x)/(2^m)/exp(2*a)/((b*x)^m)
```

Rubi [A]
time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3393, 3388, 2212}

$$-e^{2a}b^2 2^{-m}x^m(-bx)^{-m}\text{Gamma}(m-2,-2bx) - e^{-2a}b^2 2^{-m}x^m(bx)^{-m}\text{Gamma}(m-2,2bx) + \frac{x^{m-2}}{2(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + m)*Sinh[a + b*x]^2,x]
```

```
[Out] x^(-2 + m)/(2*(2 - m)) - (b^2*E^(2*a)*x^m*Gamma[-2 + m, -2*b*x])/(2^m*(-(b*
x))^m) - (b^2*x^m*Gamma[-2 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int x^{-3+m}\sinh^2(a+bx)\,dx &= -\int\left(\frac{x^{-3+m}}{2}-\frac{1}{2}x^{-3+m}\cosh(2a+2bx)\right)\,dx\\ &= \frac{x^{-2+m}}{2(2-m)}+\frac{1}{2}\int x^{-3+m}\cosh(2a+2bx)\,dx\\ &= \frac{x^{-2+m}}{2(2-m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{-3+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{-3+m}\,dx\\ &= \frac{x^{-2+m}}{2(2-m)}-2^{-m}b^2e^{2a}x^m(-bx)^{-m}\Gamma(-2+m,-2bx)-2^{-m}b^2e^{-2a}x^m(bx)^{-m}\Gamma(-2\end{aligned}$$

Mathematica [A]
time = 0.08, size = 77, normalized size = 0.92

$$x^m\left(\frac{1}{(4-2m)x^2}-2^{-m}b^2e^{2a}(-bx)^{-m}\Gamma(-2+m,-2bx)-2^{-m}b^2e^{-2a}(bx)^{-m}\Gamma(-2+m,2bx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + m)*Sinh[a + b*x]^2,x]
```

```
[Out] x^m*(1/((4 - 2*m)*x^2) - (b^2*E^(2*a)*Gamma[-2 + m, -2*b*x])/(2^m*(-(b*x))^
m) - (b^2*Gamma[-2 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m))
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int x^{-3+m}\left(\sinh^2\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3+m)*sinh(b*x+a)^2,x)
```

```
[Out] int(x^(-3+m)*sinh(b*x+a)^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(m-3>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 0.10, size = 136, normalized size = 1.62

$$-\frac{4\,bx\cosh\left((m-3)\log\left(x\right)\right)+(m-2)\cosh\left((m-3)\log\left(2\,b\right)+2\,a\right)\Gamma(m-2,2\,bx)-(m-2)\cosh\left((m-3)\log\left(-2\,b\right)-2\,a\right)\Gamma(m-2,-2\,bx)-(m-2)\Gamma(m-2,2\,bx)\sinh\left((m-3)\log\left(2\,b\right)+2\,a\right)+(m-2)\Gamma(m-2,-2\,bx)\sinh\left((m-3)\log\left(-2\,b\right)-2\,a\right)+4\,bx\sinh\left((m-3)\log\left(x\right)\right)}{8\,(bm-2\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*x*cosh((m - 3)*log(x)) + (m - 2)*cosh((m - 3)*log(2*b) + 2*a)*gam
ma(m - 2, 2*b*x) - (m - 2)*cosh((m - 3)*log(-2*b) - 2*a)*gamma(m - 2, -2*b*
x) - (m - 2)*gamma(m - 2, 2*b*x)*sinh((m - 3)*log(2*b) + 2*a) + (m - 2)*gam
ma(m - 2, -2*b*x)*sinh((m - 3)*log(-2*b) - 2*a) + 4*b*x*sinh((m - 3)*log(x)
))/(b*m - 2*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-3}\sinh^{2}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+m)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**(m - 3)*sinh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 3)*sinh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m-3}\,{\mathrm{sinh}\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 3)*sinh(a + b*x)^2,x)
```

```
[Out] int(x^(m - 3)*sinh(a + b*x)^2, x)
```

3.92 $\int \left(\frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x\sqrt{\operatorname{csch}(x)}\right)\,dx$

Optimal. Leaf size=24

$$-\frac{4}{9\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{2x\cosh(x)}{3\sqrt{\operatorname{csch}(x)}}$$

```
[Out] -4/9/csch(x)^(3/2)+2/3*x*cosh(x)/csch(x)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4272, 4274}

$$\frac{2x\cosh(x)}{3\sqrt{\operatorname{csch}(x)}}-\frac{4}{9\operatorname{csch}^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Csch[x]^(3/2) + (x*Sqrt[Csch[x]])/3,x]
[Out] -4/(9*Csch[x]^(3/2)) + (2*x*Cosh[x])/(3*Sqrt[Csch[x]])
```

Rule 4272

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[d*((b*Csc[e + f*x])^n/(f^2*n^2)), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[(c + d*x)*Cos[e + f*x]*((b*C
sc[e + f*x])^(n + 1)/(b*f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x\sqrt{\operatorname{csch}(x)}\right)dx &= \frac{1}{3}\int x\sqrt{\operatorname{csch}(x)}\,dx+\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}\,dx\\ &= -\frac{4}{9\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{2x\cosh(x)}{3\sqrt{\operatorname{csch}(x)}}-\frac{1}{3}\int x\sqrt{\operatorname{csch}(x)}\,dx+\frac{1}{3}\Big(\sqrt{\operatorname{csch}(x)}\ \sqrt{}\\ &= -\frac{4}{9\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{2x\cosh(x)}{3\sqrt{\operatorname{csch}(x)}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 17, normalized size = 0.71

$$\frac{2(-2+3x\coth(x))}{9\mathrm{csch}^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Csch[x]^(3/2) + (x*Sqrt[Csch[x]])/3,x]
```

```
[Out] (2*(-2 + 3*x*Coth[x]))/(9*Csch[x]^(3/2))
```

Maple [F]
time = 1.60, size = 0, normalized size = 0.00

$$\int \frac{x}{\mathrm{csch}\left(x\right)^{\frac{3}{2}}}+\frac{x\sqrt{\mathrm{csch}\left(x\right)}}{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/csch(x)^(3/2)+1/3*x*csch(x)^(1/2),x)
```

```
[Out] int(x/csch(x)^(3/2)+1/3*x*csch(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)^(3/2)+1/3*x*csch(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/3*x*sqrt(csch(x)) + x/csch(x)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)^(3/2)+1/3*x*csch(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{3x}{\mathrm{csch}^{\frac{3}{2}}\left(x\right)}\,dx+\int x\sqrt{\mathrm{csch}\left(x\right)}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)**(3/2)+1/3*x*csch(x)**(1/2),x)
```

```
[Out] (Integral(3*x/csch(x)**(3/2), x) + Integral(x*sqrt(csch(x)), x))/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)^(3/2)+1/3*x*csch(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/3*x*sqrt(csch(x)) + x/csch(x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,\sqrt{\frac{1}{\sinh\left(x\right)}}}{3}+\frac{x}{{\left(\frac{1}{\sinh\left(x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1/sinh(x))^(1/2))/3 + x/(1/sinh(x))^(3/2),x)
```

```
[Out] int((x*(1/sinh(x))^(1/2))/3 + x/(1/sinh(x))^(3/2), x)
```

3.93 $\int \left(\frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\operatorname{csch}(x)}}\right) dx$

Optimal. Leaf size=24

$$-\frac{4}{25\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{2x\cosh(x)}{5\operatorname{csch}^{\frac{3}{2}}(x)}$$

```
[Out] -4/25/csch(x)^(5/2)+2/5*x*cosh(x)/csch(x)^(3/2)
```

Rubi [A]

time = 0.08, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4272, 4274}

$$\frac{2x\cosh(x)}{5\operatorname{csch}^{\frac{3}{2}}(x)}-\frac{4}{25\operatorname{csch}^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Csch[x]^(5/2) + (3*x)/(5*Sqrt[Csch[x]]),x]
```

```
[Out] -4/(25*Csch[x]^(5/2)) + (2*x*Cosh[x])/(5*Csch[x]^(3/2))
```

Rule 4272

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[d*((b*Csc[e + f*x])^n/(f^2*n^2)), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[(c + d*x)*Cos[e + f*x]*((b*C
sc[e + f*x])^(n + 1)/(b*f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\operatorname{csch}(x)}}\right) dx &= \frac{3}{5}\int \frac{x}{\sqrt{\operatorname{csch}(x)}}\,dx+\int \frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}\,dx\\ &= -\frac{4}{25\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{2x\cosh(x)}{5\operatorname{csch}^{\frac{3}{2}}(x)}-\frac{3}{5}\int \frac{x}{\sqrt{\operatorname{csch}(x)}}\,dx+\frac{3\int x\sqrt{-\sinh(}{5\sqrt{\operatorname{csch}(x)}\sqrt{-}}\\ &= -\frac{4}{25\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{2x\cosh(x)}{5\operatorname{csch}^{\frac{3}{2}}(x)}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 17, normalized size = 0.71

$$\frac{2(-2+5x\coth(x))}{25\mathrm{csch}^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Csch[x]^(5/2) + (3*x)/(5*Sqrt[Csch[x]]),x]
[Out] (2*(-2 + 5*x*Coth[x]))/(25*Csch[x]^(5/2))
```

Maple [F]
time = 1.59, size = 0, normalized size = 0.00

$$\int \frac{x}{\mathrm{csch}\left(x\right)^{\frac{5}{2}}}+\frac{3x}{5\sqrt{\mathrm{csch}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x)
[Out] int(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x, algorithm="maxima")
[Out] integrate(3/5*x/sqrt(csch(x)) + x/csch(x)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{5x}{\mathrm{csch}^{\frac{5}{2}}\left(x\right)}\,dx+\int \frac{3x}{\sqrt{\mathrm{csch}\left(x\right)}}\,dx}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)**(5/2)+3/5*x/csch(x)**(1/2),x)
```

```
[Out] (Integral(5*x/csch(x)**(5/2), x) + Integral(3*x/sqrt(csch(x)), x))/5
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(3/5*x/sqrt(csch(x)) + x/csch(x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{3\,x}{5\,\sqrt{\frac{1}{\sinh\left(x\right)}}}+\frac{x}{{\left(\frac{1}{\sinh\left(x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x)/(5*(1/sinh(x))^(1/2)) + x/(1/sinh(x))^(5/2),x)
```

```
[Out] int((3*x)/(5*(1/sinh(x))^(1/2)) + x/(1/sinh(x))^(5/2), x)
```

3.94 $\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)}-\frac{5}{21} x \sqrt{\operatorname{csch}(x)}\right) dx$

Optimal. Leaf size=47

$$-\frac{4}{49\operatorname{csch}^{\frac{7}{2}}(x)}+\frac{2x\cosh(x)}{7\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{20}{63\operatorname{csch}^{\frac{3}{2}}(x)}-\frac{10x\cosh(x)}{21\sqrt{\operatorname{csch}(x)}}$$

```
[Out] -4/49/csch(x)^(7/2)+2/7*x*cosh(x)/csch(x)^(5/2)+20/63/csch(x)^(3/2)-10/21*x
*cosh(x)/csch(x)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4272, 4274}

$$\frac{20}{63\operatorname{csch}^{\frac{3}{2}}(x)}-\frac{4}{49\operatorname{csch}^{\frac{7}{2}}(x)}+\frac{2x\cosh(x)}{7\operatorname{csch}^{\frac{5}{2}}(x)}-\frac{10x\cosh(x)}{21\sqrt{\operatorname{csch}(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Csch[x]^(7/2) - (5*x*Sqrt[Csch[x]])/21,x]
```

```
[Out] -4/(49*Csch[x]^(7/2)) + (2*x*Cosh[x])/(7*Csch[x]^(5/2)) + 20/(63*Csch[x]^(3
/2)) - (10*x*Cosh[x])/(21*Sqrt[Csch[x]])
```

Rule 4272

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[d*((b*Csc[e + f*x])^n/(f^2*n^2)), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[(c + d*x)*Cos[e + f*x]*((b*C
sc[e + f*x])^(n + 1)/(b*f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)} - \frac{5}{21}x\sqrt{\operatorname{csch}(x)}\right) dx = -\left(\frac{5}{21}\int x\sqrt{\operatorname{csch}(x)}\, dx\right) + \int \frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)}\, dx$$
$$= -\frac{4}{49\operatorname{csch}^{\frac{7}{2}}(x)} + \frac{2x\cosh(x)}{7\operatorname{csch}^{\frac{5}{2}}(x)} - \frac{5}{7}\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}\, dx - \frac{1}{21}\Big(5\sqrt{\operatorname{csch}(x)}$$
$$= -\frac{4}{49\operatorname{csch}^{\frac{7}{2}}(x)} + \frac{2x\cosh(x)}{7\operatorname{csch}^{\frac{5}{2}}(x)} + \frac{20}{63\operatorname{csch}^{\frac{3}{2}}(x)} - \frac{10x\cosh(x)}{21\sqrt{\operatorname{csch}(x)}} + \frac{5}{21}\int$$
$$= -\frac{4}{49\operatorname{csch}^{\frac{7}{2}}(x)} + \frac{2x\cosh(x)}{7\operatorname{csch}^{\frac{5}{2}}(x)} + \frac{20}{63\operatorname{csch}^{\frac{3}{2}}(x)} - \frac{10x\cosh(x)}{21\sqrt{\operatorname{csch}(x)}}$$

Mathematica [A]
time = 0.09, size = 45, normalized size = 0.96

$$\sqrt{\operatorname{csch}(x)}\left(-\frac{167}{882} + \frac{88}{441}\cosh(2x) - \frac{1}{98}\cosh(4x) - \frac{13}{42}x\sinh(2x) + \frac{1}{28}x\sinh(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Csch[x]^(7/2) - (5*x*Sqrt[Csch[x]])/21,x]
```

```
[Out] Sqrt[Csch[x]]*(-167/882 + (88*Cosh[2*x])/441 - Cosh[4*x]/98 - (13*x*Sinh[2*
x])/42 + (x*Sinh[4*x])/28)
```

Maple [F]
time = 1.57, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{csch}(x)^{\frac{7}{2}}} - \frac{5x\sqrt{\operatorname{csch}(x)}}{21}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x)
```

```
[Out] int(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-5/21*x*sqrt(csch(x)) + x/csch(x)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{21x}{\operatorname{csch}^{\frac{7}{2}}(x)}\right)dx+\int 5x\sqrt{\operatorname{csch}(x)}\,dx}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)**(7/2)-5/21*x*csch(x)**(1/2),x)
```

```
[Out] -(Integral(-21*x/csch(x)**(7/2), x) + Integral(5*x*sqrt(csch(x)), x))/21
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-5/21*x*sqrt(csch(x)) + x/csch(x)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int\frac{5\,x\,\sqrt{\frac{1}{\sinh(x)}}}{21}-\frac{x}{\left(\frac{1}{\sinh(x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1/sinh(x))^(7/2) - (5*x*(1/sinh(x))^(1/2))/21,x)
```

```
[Out] -int((5*x*(1/sinh(x))^(1/2))/21 - x/(1/sinh(x))^(7/2), x)
```

3.95 $\int \left(\frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x^2\sqrt{\operatorname{csch}(x)}\right)dx$

Optimal. Leaf size=76

$$-\frac{8x}{9\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{16\cosh(x)}{27\sqrt{\operatorname{csch}(x)}}+\frac{2x^2\cosh(x)}{3\sqrt{\operatorname{csch}(x)}}-\frac{16}{27}i\sqrt{\operatorname{csch}(x)}\,F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{i\sinh(x)}$$

```
[Out] -8/9*x/csch(x)^(3/2)+16/27*cosh(x)/csch(x)^(1/2)+2/3*x^2*cosh(x)/csch(x)^(1
/2)-16/27*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos
(1/4*Pi+1/2*I*x),2^(1/2))*csch(x)^(1/2)*(I*sinh(x))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4273, 4274, 3854, 3856, 2720}

$$\frac{2x^2\cosh(x)}{3\sqrt{\operatorname{csch}(x)}}-\frac{8x}{9\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{16\cosh(x)}{27\sqrt{\operatorname{csch}(x)}}-\frac{16}{27}i\sqrt{i\sinh(x)}\sqrt{\operatorname{csch}(x)}\,F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Csch[x]^(3/2) + (x^2*Sqrt[Csch[x]])/3,x]
```

```
[Out] (-8*x)/(9*Csch[x]^(3/2)) + (16*Cosh[x])/(27*Sqrt[Csch[x]]) + (2*x^2*Cosh[x]
)/(3*Sqrt[Csch[x]]) - ((16*I)/27)*Sqrt[Csch[x]]*EllipticF[Pi/4 - (I/2)*x, 2
]*Sqrt[I*Sinh[x]]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4273

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^n/(f^2*n^2)), x] + (Dist
[(n + 1)/(b^2*n), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n + 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^n, x], x]
+ Simp[(c + d*x)^m*Cos[e + f*x]*((b*Csc[e + f*x])^(n + 1)/(b*f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
```

Rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \left(\frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x^2\sqrt{\operatorname{csch}(x)}\right)dx=\frac{1}{3}\int x^2\sqrt{\operatorname{csch}(x)}\,dx+\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}\,dx$$
$$=-\frac{8x}{9\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{2x^2\cosh(x)}{3\sqrt{\operatorname{csch}(x)}}-\frac{1}{3}\int x^2\sqrt{\operatorname{csch}(x)}\,dx+\frac{8}{9}\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(x)}$$
$$=-\frac{8x}{9\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{16\cosh(x)}{27\sqrt{\operatorname{csch}(x)}}+\frac{2x^2\cosh(x)}{3\sqrt{\operatorname{csch}(x)}}-\frac{8}{27}\int \sqrt{\operatorname{csch}(x)}\,dx$$
$$=-\frac{8x}{9\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{16\cosh(x)}{27\sqrt{\operatorname{csch}(x)}}+\frac{2x^2\cosh(x)}{3\sqrt{\operatorname{csch}(x)}}-\frac{1}{27}\Big(8\sqrt{\operatorname{csch}(x)}\,\sqrt{i\,\mathrm{si}}$$
$$=-\frac{8x}{9\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{16\cosh(x)}{27\sqrt{\operatorname{csch}(x)}}+\frac{2x^2\cosh(x)}{3\sqrt{\operatorname{csch}(x)}}-\frac{16}{27}i\sqrt{\operatorname{csch}(x)}\,F\bigg(\frac{\pi}{4}$$

Mathematica [A]
time = 0.10, size = 63, normalized size = 0.83

$$\frac{1}{27}\sqrt{\operatorname{csch}(x)}\left(12x-12x\cosh(2x)-16iF\left(\frac{1}{4}(\pi-2ix)\middle|2\right)\sqrt{i\sinh(x)}+8\sinh(2x)+9x^2\sinh(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Csch[x]^(3/2) + (x^2*Sqrt[Csch[x]])/3,x]
```

```
[Out] (Sqrt[Csch[x]]*(12*x - 12*x*Cosh[2*x] - (16*I)*EllipticF[(Pi - (2*I)*x)/4,
2]*Sqrt[I*Sinh[x]] + 8*Sinh[2*x] + 9*x^2*Sinh[2*x]))/27
```

Maple [F]
time = 1.59, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{csch}(x)^{\frac{3}{2}}} + \frac{x^2\sqrt{\operatorname{csch}(x)}}{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/csch(x)^(3/2)+1/3*x^2*csch(x)^(1/2),x)
[Out] int(x^2/csch(x)^(3/2)+1/3*x^2*csch(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(x)^(3/2)+1/3*x^2*csch(x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/3*x^2*sqrt(csch(x)) + x^2/csch(x)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(x)^(3/2)+1/3*x^2*csch(x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{3x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}\,dx + \int x^2\sqrt{\operatorname{csch}(x)}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/csch(x)**(3/2)+1/3*x**2*csch(x)**(1/2),x)
[Out] (Integral(3*x**2/csch(x)**(3/2), x) + Integral(x**2*sqrt(csch(x)), x))/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(x)^(3/2)+1/3*x^2*csch(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/3*x^2*sqrt(csch(x)) + x^2/csch(x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\sqrt{\frac{1}{\sinh\left(x\right)}}}{3}+\frac{x^2}{{\left(\frac{1}{\sinh\left(x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1/sinh(x))^(1/2))/3 + x^2/(1/sinh(x))^(3/2),x)
```

```
[Out] int((x^2*(1/sinh(x))^(1/2))/3 + x^2/(1/sinh(x))^(3/2), x)
```

3.96 $\int (c+dx)^3(a+ia\sinh(e+fx))\,dx$

Optimal. Leaf size=98

$$\frac{a(c+dx)^4}{4d}+\frac{6iad^2(c+dx)\cosh(e+fx)}{f^3}+\frac{ia(c+dx)^3\cosh(e+fx)}{f}-\frac{6iad^3\sinh(e+fx)}{f^4}-\frac{3iad(c+dx)^2\sinh(e+fx)}{f^2}$$

```
[Out] 1/4*a*(d*x+c)^4/d+6*I*a*d^2*(d*x+c)*cosh(f*x+e)/f^3+I*a*(d*x+c)^3*cosh(f*x+
e)/f-6*I*a*d^3*sinh(f*x+e)/f^4-3*I*a*d*(d*x+c)^2*sinh(f*x+e)/f^2
```

Rubi [A]
time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3398, 3377, 2717}

$$\frac{6iad^2(c+dx)\cosh(e+fx)}{f^3}-\frac{3iad(c+dx)^2\sinh(e+fx)}{f^2}+\frac{ia(c+dx)^3\cosh(e+fx)}{f}+\frac{a(c+dx)^4}{4d}-\frac{6iad^3\sinh(e+fx)}{f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) + ((6*I)*a*d^2*(c + d*x)*Cosh[e + f*x])/f^3 + (I*a*(c
 + d*x)^3*Cosh[e + f*x])/f - ((6*I)*a*d^3*Sinh[e + f*x])/f^4 - ((3*I)*a*d*(
c + d*x)^2*Sinh[e + f*x])/f^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3(a+ia\sinh(e+fx))\,dx &= \int \left(a(c+dx)^3+ia(c+dx)^3\sinh(e+fx)\right)\,dx\\ &= \frac{a(c+dx)^4}{4d}+(ia)\int (c+dx)^3\sinh(e+fx)\,dx\\ &= \frac{a(c+dx)^4}{4d}+\frac{ia(c+dx)^3\cosh(e+fx)}{f}-\frac{(3iad)\int (c+dx)^2\cosh(e+}{f}\\ &= \frac{a(c+dx)^4}{4d}+\frac{ia(c+dx)^3\cosh(e+fx)}{f}-\frac{3iad(c+dx)^2\sinh(e+fx)}{f^2}\\ &= \frac{a(c+dx)^4}{4d}+\frac{6iad^2(c+dx)\cosh(e+fx)}{f^3}+\frac{ia(c+dx)^3\cosh(e+fx)}{f}\\ &= \frac{a(c+dx)^4}{4d}+\frac{6iad^2(c+dx)\cosh(e+fx)}{f^3}+\frac{ia(c+dx)^3\cosh(e+fx)}{f}\end{aligned}$$

Mathematica [A]
time = 0.29, size = 128, normalized size = 1.31

$$\frac{a(f^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)+4if(c+dx)\left(c^2f^2+2cdf^2x+d^2(6+f^2x^2)\right)\cosh(e+fx)-12id(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\sinh(e+fx))}{4f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (a*(f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + (4*I)*f*(c + d*x)*(
c^2*f^2 + 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Cosh[e + f*x] - (12*I)*d*(c^2*f^
2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x]))/(4*f^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(92) = 184$.
time = 0.44, size = 494, normalized size = 5.04

method	result
risch	$\frac{a\,d^3x^4}{4}+ac\,d^2x^3+\frac{3a\,c^2d\,x^2}{2}+c^3ax+\frac{a\,c^4}{4d}+\frac{ia(d^3x^3f^3+3c\,d^2f^3x^2+3c^2d\,f^3x-3d^3f^2x^2+c^3f^3-6c\,d^2f^2x-3}{2f^4}$
derivativedivides	$\frac{\frac{d^3a(fx+e)^4}{4f^3}-\frac{id^3e^3a\cosh(fx+e)}{f^3}-\frac{d^3ea(fx+e)^3}{f^3}-\frac{3ide\,c^2a\cosh(fx+e)}{f}+\frac{d^2ca(fx+e)^3}{f^2}+\frac{3id^3e^2a((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f^3}}{}$
default	$\frac{\frac{d^3a(fx+e)^4}{4f^3}-\frac{id^3e^3a\cosh(fx+e)}{f^3}-\frac{d^3ea(fx+e)^3}{f^3}-\frac{3ide\,c^2a\cosh(fx+e)}{f}+\frac{d^2ca(fx+e)^3}{f^2}+\frac{3id^3e^2a((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f^3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+I*a*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4*d^3/f^3*a*(f*x+e)^4-I*d^3/f^3*e^3*a*cosh(f*x+e)-d^3/f^3*e*a*(f*x+e
)^3-3*I*d/f*e*c^2*a*cosh(f*x+e)+d^2/f^2*c*a*(f*x+e)^3+3*I*d^3/f^3*e^2*a*((f
*x+e)*cosh(f*x+e)-sinh(f*x+e))+3/2*d^3/f^3*e^2*a*(f*x+e)^2-6*I*d^2/f^2*e*c*
a*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))-3*d^2/f^2*e*c*a*(f*x+e)^2+I*d^3/f^3*a*(
(f*x+e)^3*cosh(f*x+e)-3*(f*x+e)^2*sinh(f*x+e)+6*(f*x+e)*cosh(f*x+e)-6*sinh(
f*x+e))+3/2*d/f*c^2*a*(f*x+e)^2+3*I*d^2/f^2*e^2*c*a*cosh(f*x+e)-d^3/f^3*e^3
*a*(f*x+e)-3*I*d^3/f^3*e*a*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*sinh(f*x+e)+2*c
osh(f*x+e))+3*d^2/f^2*e^2*c*a*(f*x+e)+3*I*d^2/f^2*c*a*((f*x+e)^2*cosh(f*x+e
)-2*(f*x+e)*sinh(f*x+e)+2*cosh(f*x+e))-3*d/f*e*c^2*a*(f*x+e)+3*I*d/f*c^2*a*
((f*x+e)*cosh(f*x+e)-sinh(f*x+e))+c^3*a*(f*x+e)+I*c^3*a*cosh(f*x+e))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(92) = 184$.
time = 0.29, size = 248, normalized size = 2.53

$$\frac{1}{4}ad^3x^4 + acd^2x^3 + \frac{3}{2}ac^2dx^2 + ac^3x + \frac{3}{2}i\,ac^2d\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{3}{2}i\,acd^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2 + 2\,fx + 2)e^{(-fx-e)}}{f^3}\right) + \frac{1}{2}i\,ad^3\left(\frac{(f^3x^3e^e - 3\,f^2x^2e^e + 6\,fxe^e - 6\,e^e)e^{(fx)}}{f^4} + \frac{(f^3x^3 + 3\,f^2x^2 + 6\,fx + 6)e^{(-fx-e)}}{f^4}\right) + \frac{i\,ac^3\cosh(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 3/2*I*a*c^2*d*((f
*x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^(-f*x - e)/f^2) + 3/2*I*a*c*d^2*((f
^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x
 - e)/f^3) + 1/2*I*a*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e^e + 6*f*x*e^e - 6*e^e)
*e^(f*x)/f^4 + (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(-f*x - e)/f^4) + I*a*c^
3*cosh(f*x + e)/f
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(92) = 184$.
time = 0.34, size = 287, normalized size = 2.93

$$\frac{\left(2i\,ad^3f^3x^3 + 2i\,ac^3f^3 + 6i\,ac^2df^2 + 12i\,acd^2f + 12i\,ad^3 - 6\left(-i\,acd^2f^3 - i\,ad^3f^2\right)x^2 - 6\left(-i\,ac^2df^3 - 2i\,acd^2f^2 - 2i\,ad^3f\right)x - 2\left(-i\,ad^3f^3x^3 - i\,ac^3f^3 + 3i\,ac^2df^2 - 6i\,acd^2f + 6i\,ad^3 + 3\left(-i\,acd^2f^3 + i\,ad^3f^2\right)x^2 + 3\left(-i\,ac^2df^3 + 2i\,acd^2f^2 - 2i\,ad^3f\right)x\right)e^{(2\,fx+2\,e)} + \left(ad^3f^4x^4 + 4\,acd^2f^4x^3 + 6\,ac^2df^4x^2 + 4\,ac^3f^4x\right)e^{(fx+e)}\right)e^{(-fx-e)}}{4\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*I*a*d^3*f^3*x^3 + 2*I*a*c^3*f^3 + 6*I*a*c^2*d*f^2 + 12*I*a*c*d^2*f +
 12*I*a*d^3 - 6*(-I*a*c*d^2*f^3 - I*a*d^3*f^2)*x^2 - 6*(-I*a*c^2*d*f^3 - 2*
I*a*c*d^2*f^2 - 2*I*a*d^3*f)*x - 2*(-I*a*d^3*f^3*x^3 - I*a*c^3*f^3 + 3*I*a*
c^2*d*f^2 - 6*I*a*c*d^2*f + 6*I*a*d^3 + 3*(-I*a*c*d^2*f^3 + I*a*d^3*f^2)*x^
2 + 3*(-I*a*c^2*d*f^3 + 2*I*a*c*d^2*f^2 - 2*I*a*d^3*f)*x)*e^(2*f*x + 2*e) +
 (a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4*x)*e^
(f*x + e))*e^(-f*x - e)/f^4
```

Sympy [**A**]
time = 0.44, size = 517, normalized size = 5.28

$$ac^3x + \frac{3ac^2dx^2}{2} + acd^2x^3 + \frac{ad^3x^4}{4} + \begin{cases} \frac{\left(\left(2iac^3f^7 + 6iac^2df^7x + 6iac^2df^6 + 6iacd^2f^7x^2 + 12iacd^2f^6x + 12iacd^2f^5 + 2iad^3f^7x^3 + 6iad^3f^6x^2 + 12iad^3f^5x + 12iad^3f^4\right)e^{-fx} + \left(2iac^3f^7e^{2e} + 6iac^2df^7xe^{2e} - 6iac^2df^6e^{2e} + 6iacd^2f^7x^2e^{2e} - 12iacd^2f^6xe^{2e} + 12iacd^2f^5e^{2e} + 2iad^3f^7x^3e^{2e} - 6iad^3f^6x^2e^{2e} + 12iad^3f^5xe^{2e} - 12iad^3f^4e^{2e}\right)e^{fx}\right)e^{-e}}{4f^8} & \text{for}\: f^8e^e \neq 0 \\ \frac{x^4\left(iad^3e^{2e} - iad^3\right)e^{-e}}{8} + \frac{x^3\left(iacd^2e^{2e} - iacd^2\right)e^{-e}}{2} + \frac{x^2\cdot\left(3iac^2de^{2e} - 3iac^2d\right)e^{-e}}{4} + \frac{x\left(iac^3e^{2e} - iac^3\right)e^{-e}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+I*a*sinh(f*x+e)),x)
```

```
[Out] a*c**3*x + 3*a*c**2*d*x**2/2 + a*c*d**2*x**3 + a*d**3*x**4/4 + Piecewise(((
(2*I*a*c**3*f**7 + 6*I*a*c**2*d*f**7*x + 6*I*a*c**2*d*f**6 + 6*I*a*c*d**2*f
**7*x**2 + 12*I*a*c*d**2*f**6*x + 12*I*a*c*d**2*f**5 + 2*I*a*d**3*f**7*x**3
 + 6*I*a*d**3*f**6*x**2 + 12*I*a*d**3*f**5*x + 12*I*a*d**3*f**4)*exp(-f*x)
+ (2*I*a*c**3*f**7*exp(2*e) + 6*I*a*c**2*d*f**7*x*exp(2*e) - 6*I*a*c**2*d*f
**6*exp(2*e) + 6*I*a*c*d**2*f**7*x**2*exp(2*e) - 12*I*a*c*d**2*f**6*x*exp(2
*e) + 12*I*a*c*d**2*f**5*exp(2*e) + 2*I*a*d**3*f**7*x**3*exp(2*e) - 6*I*a*d
**3*f**6*x**2*exp(2*e) + 12*I*a*d**3*f**5*x*exp(2*e) - 12*I*a*d**3*f**4*exp
(2*e))*exp(f*x))*exp(-e)/(4*f**8), Ne(f**8*exp(e), 0)), (x**4*(I*a*d**3*exp
(2*e) - I*a*d**3)*exp(-e)/8 + x**3*(I*a*c*d**2*exp(2*e) - I*a*c*d**2)*exp(-
e)/2 + x**2*(3*I*a*c**2*d*exp(2*e) - 3*I*a*c**2*d)*exp(-e)/4 + x*(I*a*c**3*
exp(2*e) - I*a*c**3)*exp(-e)/2, True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(88) = 176$.
time = 0.46, size = 262, normalized size = 2.67

$$\frac{1}{4}ad^3x^4 + acd^2x^3 + \frac{3}{2}ac^2dx^2 + ac^3x - \frac{(-i\,ad^3f^3x^3 - 3i\,acd^2f^3x^2 - 3i\,ac^2df^3x + 3i\,ad^3f^2x^2 - i\,ac^3f^3 + 6i\,acd^2f^2x + 3i\,ac^2df^2 - 6i\,ad^3fx - 6i\,acd^2f + 6i\,ad^3)e^{(fx+e)}}{2\,f^4} - \frac{(-i\,ad^3f^3x^3 - 3i\,acd^2f^3x^2 - 3i\,ac^2df^3x - 3i\,ad^3f^2x^2 - i\,ac^3f^3 - 6i\,acd^2f^2x - 3i\,ac^2df^2 - 6i\,ad^3fx - 6i\,acd^2f - 6i\,ad^3)e^{(-fx-e)}}{2\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x - 1/2*(-I*a*d^3*f^3
*x^3 - 3*I*a*c*d^2*f^3*x^2 - 3*I*a*c^2*d*f^3*x + 3*I*a*d^3*f^2*x^2 - I*a*c^
3*f^3 + 6*I*a*c*d^2*f^2*x + 3*I*a*c^2*d*f^2 - 6*I*a*d^3*f*x - 6*I*a*c*d^2*f
 + 6*I*a*d^3)*e^(f*x + e)/f^4 - 1/2*(-I*a*d^3*f^3*x^3 - 3*I*a*c*d^2*f^3*x^2
 - 3*I*a*c^2*d*f^3*x - 3*I*a*d^3*f^2*x^2 - I*a*c^3*f^3 - 6*I*a*c*d^2*f^2*x
- 3*I*a*c^2*d*f^2 - 6*I*a*d^3*f*x - 6*I*a*c*d^2*f - 6*I*a*d^3)*e^(-f*x - e)
/f^4
```

Mupad [B]
time = 0.38, size = 196, normalized size = 2.00

$$\frac{\cosh(e+f\,x)\,\left(a\,c^3\,f^2+6\,a\,c\,d^2\right)\,1\mathrm{i}}{f^3} - \frac{\sinh(e+f\,x)\,\left(a\,c^2\,d\,f^2+2\,a\,d^3\right)\,3\mathrm{i}}{f^4} + \frac{a\,d^3\,x^4}{4} + a\,c^3\,x + \frac{x\cosh(e+f\,x)\,\left(a\,c^2\,d\,f^2+2\,a\,d^3\right)\,3\mathrm{i}}{f^3} + \frac{3\,a\,c^2\,d\,x^2}{2} + a\,c\,d^2\,x^3 + \frac{a\,d^3\,x^3\cosh(e+f\,x)\,1\mathrm{i}}{f} - \frac{a\,d^3\,x^2\sinh(e+f\,x)\,3\mathrm{i}}{f^2} - \frac{a\,c\,d^2\,x\sinh(e+f\,x)\,6\mathrm{i}}{f^2} + \frac{a\,c\,d^2\,x^2\cosh(e+f\,x)\,3\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(e + f*x)*1i)*(c + d*x)^3,x)
```

```
[Out] (cosh(e + f*x)*(a*c^3*f^2 + 6*a*c*d^2)*1i)/f^3 - (sinh(e + f*x)*(2*a*d^3 +
a*c^2*d*f^2)*3i)/f^4 + (a*d^3*x^4)/4 + a*c^3*x + (x*cosh(e + f*x)*(2*a*d^3
+ a*c^2*d*f^2)*3i)/f^3 + (3*a*c^2*d*x^2)/2 + a*c*d^2*x^3 + (a*d^3*x^3*cosh(
e + f*x)*1i)/f - (a*d^3*x^2*sinh(e + f*x)*3i)/f^2 - (a*c*d^2*x*sinh(e + f*x
)*6i)/f^2 + (a*c*d^2*x^2*cosh(e + f*x)*3i)/f
```

3.97 $\int (c+dx)^2(a+ia\sinh(e+fx))\,dx$

Optimal. Leaf size=74

$$\frac{a(c+dx)^3}{3d}+\frac{2iad^2\cosh(e+fx)}{f^3}+\frac{ia(c+dx)^2\cosh(e+fx)}{f}-\frac{2iad(c+dx)\sinh(e+fx)}{f^2}$$

```
[Out] 1/3*a*(d*x+c)^3/d+2*I*a*d^2*cosh(f*x+e)/f^3+I*a*(d*x+c)^2*cosh(f*x+e)/f-2*I
*a*d*(d*x+c)*sinh(f*x+e)/f^2
```

Rubi [A]
time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3398, 3377, 2718}

$$-\frac{2iad(c+dx)\sinh(e+fx)}{f^2}+\frac{ia(c+dx)^2\cosh(e+fx)}{f}+\frac{a(c+dx)^3}{3d}+\frac{2iad^2\cosh(e+fx)}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^3)/(3*d) + ((2*I)*a*d^2*Cosh[e + f*x])/f^3 + (I*a*(c + d*x)^2*
Cosh[e + f*x])/f - ((2*I)*a*d*(c + d*x)*Sinh[e + f*x])/f^2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^2(a+ia\sinh(e+fx))\,dx &= \int \left(a(c+dx)^2+ia(c+dx)^2\sinh(e+fx)\right)\,dx \\
&= \frac{a(c+dx)^3}{3d}+(ia)\int (c+dx)^2\sinh(e+fx)\,dx \\
&= \frac{a(c+dx)^3}{3d}+\frac{ia(c+dx)^2\cosh(e+fx)}{f}-\frac{(2iad)\int (c+dx)\cosh(e+f
}{f} \\
&= \frac{a(c+dx)^3}{3d}+\frac{ia(c+dx)^2\cosh(e+fx)}{f}-\frac{2iad(c+dx)\sinh(e+fx)}{f^2} \\
&= \frac{a(c+dx)^3}{3d}+\frac{2iad^2\cosh(e+fx)}{f^3}+\frac{ia(c+dx)^2\cosh(e+fx)}{f}-\frac{2iad(
}{}
\end{aligned}
$$

Mathematica [A]
time = 0.25, size = 88, normalized size = 1.19

$$
\frac{a(f^3x(3c^2+3cdx+d^2x^2)+3i(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\cosh(e+fx)-6idf(c+dx)\sinh(e+fx))}{3f^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (a*(f^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) + (3*I)*(c^2*f^2 + 2*c*d*f^2*x + d^2*
(2 + f^2*x^2))*Cosh[e + f*x] - (6*I)*d*f*(c + d*x)*Sinh[e + f*x]))/(3*f^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(69) = 138.
time = 0.37, size = 249, normalized size = 3.36

method	result
risch	$\frac{a\,d^2x^3}{3}+adc\,x^2+a\,c^2x+\frac{a\,c^3}{3d}+\frac{ia\left(d^2x^2f^2+2cd\,f^2x+c^2f^2-2d^2fx-2cdf+2d^2\right)\mathrm{e}^{fx+e}}{2f^3}+\frac{ia\left(d^2x^2f^2+2cd\,f^2x\right.}{}$
derivativedivides	$\frac{\frac{d^2a(fx+e)^3}{3f^2}+\frac{id^2a\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2}-\frac{d^2ea(fx+e)^2}{f^2}-\frac{2id^2ea((fx+e)\cosh(fx+e)-\sinh(fx}{f^2}}{}$
default	$\frac{\frac{d^2a(fx+e)^3}{3f^2}+\frac{id^2a\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2}-\frac{d^2ea(fx+e)^2}{f^2}-\frac{2id^2ea((fx+e)\cosh(fx+e)-\sinh(fx}{f^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+I*a*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*d^2/f^2*a*(f*x+e)^3+I*d^2/f^2*a*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*s
inh(f*x+e)+2*cosh(f*x+e))-d^2/f^2*e*a*(f*x+e)^2-2*I*d^2/f^2*e*a*((f*x+e)*co
sh(f*x+e)-sinh(f*x+e))+d/f*c*a*(f*x+e)^2+2*I*d/f*c*a*((f*x+e)*cosh(f*x+e)-s
```

inh(f*x+e))+d^2/f^2*e^2*a*(f*x+e)+I*d^2/f^2*e^2*a*cosh(f*x+e)-2*d/f*e*c*a*(f*x+e)-2*I*d/f*e*c*a*cosh(f*x+e)+a*c^2*(f*x+e)+I*c^2*a*cosh(f*x+e))

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(69) = 138$.
time = 0.27, size = 149, normalized size = 2.01

$$\frac{1}{3}ad^2x^3 + acdx^2 + ac^2x + iacd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{1}{2}iad^2\left(\frac{(f^2x^2e^e - 2fxe^e + 2e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2 + 2fx + 2)e^{(-fx-e)}}{f^3}\right) + \frac{iac^2\cosh(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*(a+I*a*sinh(f*x+e)),x, algorithm="maxima")

[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + I*a*c*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^(-f*x - e)/f^2) + 1/2*I*a*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + I*a*c^2*cosh(f*x + e)/f

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(69) = 138$.
time = 0.37, size = 175, normalized size = 2.36

$$\frac{\left(3iad^2f^2x^2 + 3iac^2f^2 + 6iacdf + 6iad^2 - 6(-iacdf^2 - iad^2f)x - 3(-iad^2f^2x^2 - iac^2f^2 + 2iacdf - 2iad^2 + 2(-iacdf^2 + iad^2f)x)e^{(2fx+2e)} + 2(ad^2f^3x^3 + 3acdf^3x^2 + 3ac^2f^3x)e^{(fx+e)}\right)e^{(-fx-e)}}{6f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*(a+I*a*sinh(f*x+e)),x, algorithm="fricas")

[Out] 1/6*(3*I*a*d^2*f^2*x^2 + 3*I*a*c^2*f^2 + 6*I*a*c*d*f + 6*I*a*d^2 - 6*(-I*a*c*d*f^2 - I*a*d^2*f)*x - 3*(-I*a*d^2*f^2*x^2 - I*a*c^2*f^2 + 2*I*a*c*d*f - 2*I*a*d^2 + 2*(-I*a*c*d*f^2 + I*a*d^2*f)*x)*e^(2*f*x + 2*e) + 2*(a*d^2*f^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x)*e^(f*x + e))*e^(-f*x - e)/f^3

Sympy [A]
time = 0.28, size = 314, normalized size = 4.24

$$ac^2x + acdx^2 + \frac{ad^2x^3}{3} + \begin{cases} \frac{\left((2iac^2f^5 + 4iacdf^5x + 4iacdf^4 + 2iad^2f^5x^2 + 4iad^2f^4x + 4iad^2f^3)e^{-fx} + (2iac^2f^5e^{2e} + 4iacdf^5xe^{2e} - 4iacdf^4e^{2e} + 2iad^2f^5x^2e^{2e} - 4iad^2f^4xe^{2e} + 4iad^2f^3e^{2e})e^{fx}\right)e^{-e}}{4f^6} & \text{for } f^6e^e \neq 0 \\ \frac{x^3(iad^2e^{2e} - iad^2)e^{-e}}{6} + \frac{x^2(iacde^{2e} - iacd)e^{-e}}{2} + \frac{x(iac^2e^{2e} - iac^2)e^{-e}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*(a+I*a*sinh(f*x+e)),x)

[Out] a*c**2*x + a*c*d*x**2 + a*d**2*x**3/3 + Piecewise((((2*I*a*c**2*f**5 + 4*I*a*c*d*f**5*x + 4*I*a*c*d*f**4 + 2*I*a*d**2*f**5*x**2 + 4*I*a*d**2*f**4*x + 4*I*a*d**2*f**3)*exp(-f*x) + (2*I*a*c**2*f**5*exp(2*e) + 4*I*a*c*d*f**5*x*exp(2*e) - 4*I*a*c*d*f**4*exp(2*e) + 2*I*a*d**2*f**5*x**2*exp(2*e) - 4*I*a*d**2*f**4*x*exp(2*e) + 4*I*a*d**2*f**3*exp(2*e))*exp(f*x))*exp(-e)/(4*f**6), Ne(f**6*exp(e), 0)), (x**3*(I*a*d**2*exp(2*e) - I*a*d**2)*exp(-e)/6 + x**2

```
*(I*a*c*d*exp(2*e) - I*a*c*d)*exp(-e)/2 + x*(I*a*c**2*exp(2*e) - I*a*c**2)*
exp(-e)/2, True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(66) = 132$.
time = 0.44, size = 150, normalized size = 2.03

$$\frac{1}{3}\,ad^2x^3 + acdx^2 + ac^2x - \frac{(-i\,ad^2f^2x^2 - 2i\,acdf^2x - i\,ac^2f^2 + 2i\,ad^2fx + 2i\,acdf - 2i\,ad^2)e^{(fx+e)}}{2\,f^3} + \frac{(i\,ad^2f^2x^2 + 2i\,acdf^2x + i\,ac^2f^2 + 2i\,ad^2fx + 2i\,acdf + 2i\,ad^2)e^{(-fx-e)}}{2\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x - 1/2*(-I*a*d^2*f^2*x^2 - 2*I*a*c*d*f^2
*x - I*a*c^2*f^2 + 2*I*a*d^2*f*x + 2*I*a*c*d*f - 2*I*a*d^2)*e^(f*x + e)/f^3
 + 1/2*(I*a*d^2*f^2*x^2 + 2*I*a*c*d*f^2*x + I*a*c^2*f^2 + 2*I*a*d^2*f*x + 2
*I*a*c*d*f + 2*I*a*d^2)*e^(-f*x - e)/f^3
```

Mupad [B]
time = 0.25, size = 118, normalized size = 1.59

$$\frac{-\frac{a\,f\,\left(6\mathrm{i}\,x\,\sinh(e+f\,x)\,d^2+6\mathrm{i}\,c\,\sinh(e+f\,x)\,d\right)}{3}+\frac{a\,f^2\,\left(c^2\,\cosh(e+f\,x)\,3\mathrm{i}+d^2\,x^2\,\cosh(e+f\,x)\,3\mathrm{i}+c\,d\,x\,\cosh(e+f\,x)\,6\mathrm{i}\right)}{3}+a\,d^2\,\cosh\left(e+f\,x\right)\,2\mathrm{i}}{f^3}+\frac{a\,\left(3\,c^2\,x+3\,c\,d\,x^2+d^2\,x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(e + f*x)*1i)*(c + d*x)^2,x)
```

```
[Out] ((a*f^2*(c^2*cosh(e + f*x)*3i + d^2*x^2*cosh(e + f*x)*3i + c*d*x*cosh(e + f
*x)*6i))/3 - (a*f*(d^2*x*sinh(e + f*x)*6i + c*d*sinh(e + f*x)*6i))/3 + a*d^
2*cosh(e + f*x)*2i)/f^3 + (a*(3*c^2*x + d^2*x^3 + 3*c*d*x^2))/3
```

3.98 $\int (c+dx)(a+ia\sinh(e+fx))\,dx$

Optimal. Leaf size=50

$$\frac{a(c+dx)^2}{2d}+\frac{ia(c+dx)\cosh(e+fx)}{f}-\frac{iad\sinh(e+fx)}{f^2}$$

```
[Out] 1/2*a*(d*x+c)^2/d+I*a*(d*x+c)*cosh(f*x+e)/f-I*a*d*sinh(f*x+e)/f^2
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3398, 3377, 2717}

$$\frac{ia(c+dx)\cosh(e+fx)}{f}+\frac{a(c+dx)^2}{2d}-\frac{iad\sinh(e+fx)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^2)/(2*d) + (I*a*(c + d*x)*Cosh[e + f*x])/f - (I*a*d*Sinh[e + f
*x])/f^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int (c+dx)(a+ia\sinh(e+fx))\,dx &= \int (a(c+dx)+ia(c+dx)\sinh(e+fx))\,dx \\ &= \frac{a(c+dx)^2}{2d}+(ia)\int (c+dx)\sinh(e+fx)\,dx \\ &= \frac{a(c+dx)^2}{2d}+\frac{ia(c+dx)\cosh(e+fx)}{f}-\frac{(iad)\int \cosh(e+fx)\,dx}{f} \\ &= \frac{a(c+dx)^2}{2d}+\frac{ia(c+dx)\cosh(e+fx)}{f}-\frac{iad\sinh(e+fx)}{f^2}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 48, normalized size = 0.96

$$\frac{a(f^2x(2c+dx)+2if(c+dx)\cosh(e+fx)-2id\sinh(e+fx))}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (a*(f^2*x*(2*c + d*x) + (2*I)*f*(c + d*x)*Cosh[e + f*x] - (2*I)*d*Sinh[e +
f*x]))/(2*f^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(46) = 92.
time = 0.38, size = 96, normalized size = 1.92

method	result	size
risch	$\frac{ad\,x^2}{2}+acx+\frac{ia(dxf+cf-d)\mathrm{e}^{fx+e}}{2f^2}+\frac{ia(dxf+cf+d)\mathrm{e}^{-fx-e}}{2f^2}$	62
derivativedivides	$\frac{\frac{da(fx+e)^2}{2f}+\frac{ida((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{dea(fx+e)}{f}-\frac{idea\cosh(fx+e)}{f}+ac(fx+e)+iac\cosh(fx+e)}{f}$	96
default	$\frac{\frac{da(fx+e)^2}{2f}+\frac{ida((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{dea(fx+e)}{f}-\frac{idea\cosh(fx+e)}{f}+ac(fx+e)+iac\cosh(fx+e)}{f}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+I*a*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*d/f*a*(f*x+e)^2+I*d/f*a*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))-d/f*e*a*
(f*x+e)-I*d/f*e*a*cosh(f*x+e)+a*c*(f*x+e)+I*a*c*cosh(f*x+e))
```

Maxima [A]
time = 0.26, size = 70, normalized size = 1.40

$$\frac{1}{2}\,adx^2+acx+\frac{1}{2}i\,ad\left(\frac{(fxe^e-e^e)e^{(fx)}}{f^2}+\frac{(fx+1)e^{(-fx-e)}}{f^2}\right)+\frac{i\,ac\cosh{(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + 1/2*I*a*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^
(-f*x - e)/f^2) + I*a*c*cosh(f*x + e)/f
```

Fricas [A]
time = 0.34, size = 84, normalized size = 1.68

$$\frac{\left(i\,adfx+i\,acf+i\,ad+(i\,adfx+i\,acf-i\,ad)e^{(2\,fx+2\,e)}+(adf^2x^2+2\,acf^2x)e^{(fx+e)}\right)e^{(-fx-e)}}{2\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*a*d*f*x + I*a*c*f + I*a*d + (I*a*d*f*x + I*a*c*f - I*a*d)*e^(2*f*x +
 2*e) + (a*d*f^2*x^2 + 2*a*c*f^2*x)*e^(f*x + e))*e^(-f*x - e)/f^2
```

Sympy [A]
time = 0.20, size = 162, normalized size = 3.24

$$acx+\frac{adx^2}{2}+\begin{cases}\frac{((2iacf^3+2iadf^3x+2iadf^2)e^{-fx}+(2iacf^3e^{2e}+2iadf^3xe^{2e}-2iadf^2e^{2e})e^{fx})e^{-e}}{4f^4} & \text{for } f^4e^e\neq 0\\ \frac{x^2(iade^{2e}-iad)e^{-e}}{4}+\frac{x(iace^{2e}-iac)e^{-e}}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+I*a*sinh(f*x+e)),x)
```

```
[Out] a*c*x + a*d*x**2/2 + Piecewise((((2*I*a*c*f**3 + 2*I*a*d*f**3*x + 2*I*a*d*f
**2)*exp(-f*x) + (2*I*a*c*f**3*exp(2*e) + 2*I*a*d*f**3*x*exp(2*e) - 2*I*a*d
*f**2*exp(2*e))*exp(f*x))*exp(-e)/(4*f**4), Ne(f**4*exp(e), 0)), (x**2*(I*a
*d*exp(2*e) - I*a*d)*exp(-e)/4 + x*(I*a*c*exp(2*e) - I*a*c)*exp(-e)/2, True
))
```

Giac [A]
time = 0.43, size = 69, normalized size = 1.38

$$\frac{1}{2}\,adx^2+acx-\frac{(-i\,adfx-i\,acf+i\,ad)e^{(fx+e)}}{2\,f^2}-\frac{(-i\,adfx-i\,acf-i\,ad)e^{(-fx-e)}}{2\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a*d*x^2 + a*c*x - 1/2*(-I*a*d*f*x - I*a*c*f + I*a*d)*e^(f*x + e)/f^2 -
1/2*(-I*a*d*f*x - I*a*c*f - I*a*d)*e^(-f*x - e)/f^2
```

Mupad [B]
time = 0.12, size = 56, normalized size = 1.12

$$\frac{\frac{a\,f\,\left(c\,\mathrm{cosh}\left(e+f\,x\right)\,2\mathrm{i}+d\,x\,\mathrm{cosh}\left(e+f\,x\right)\,2\mathrm{i}\right)}{2}-a\,d\,\mathrm{sinh}\left(e+f\,x\right)\,1\mathrm{i}}{f^2}+\frac{a\,\left(d\,x^2+2\,c\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(e + f*x)*1i)*(c + d*x),x)
```

```
[Out] ((a*f*(c*cosh(e + f*x)*2i + d*x*cosh(e + f*x)*2i))/2 - a*d*sinh(e + f*x)*1i
)/f^2 + (a*(2*c*x + d*x^2))/2
```

3.99 $\int \frac{a+ia\sinh(e+fx)}{c+dx}\,dx$

Optimal. Leaf size=70

$$\frac{a\log(c+dx)}{d}+\frac{ia\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{ia\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d}$$

```
[Out] a*ln(d*x+c)/d+I*a*cosh(-e+c*f/d)*Shi(c*f/d+f*x)/d-I*a*Chi(c*f/d+f*x)*sinh(-
e+c*f/d)/d
```

Rubi [A]
time = 0.12, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3398, 3384, 3379, 3382}

$$\frac{ia\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{ia\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}+\frac{a\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[e + f*x])/(c + d*x),x]
```

```
[Out] (a*Log[c + d*x])/d + (I*a*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d
+ (I*a*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
```

```
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+ia\sinh(e+fx)}{c+dx}\,dx &= \int \left(\frac{a}{c+dx}+\frac{ia\sinh(e+fx)}{c+dx}\right)dx\\ &= \frac{a\log(c+dx)}{d}+(ia)\int \frac{\sinh(e+fx)}{c+dx}\,dx\\ &= \frac{a\log(c+dx)}{d}+\left(ia\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\sinh\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx+\left(ia\sinh\left(e-\frac{cf}{d}\right.\right.\\ &= \frac{a\log(c+dx)}{d}+\frac{ia\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{ia\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 60, normalized size = 0.86

$$\frac{a\left(\log(c+dx)+i\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+i\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[e + f*x])/(c + d*x),x]
[Out] (a*(Log[c + d*x] + I*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] + I*Cosh[e
 - (c*f)/d]*SinhIntegral[f*(c/d + x)]))/d
```

Maple [A]
time = 0.54, size = 96, normalized size = 1.37

method	result	size
risch	$\frac{a\ln(dx+c)}{d}+\frac{ia\,\mathrm{e}^{\frac{cf-de}{d}}\,\text{expIntegral}\left(1,fx+e+\frac{cf-de}{d}\right)}{2d}-\frac{ia\,\mathrm{e}^{-\frac{cf-de}{d}}\,\text{expIntegral}\left(1,-fx-e-\frac{cf-de}{d}\right)}{2d}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))/(d*x+c),x,method=_RETURNVERBOSE)
[Out] a*ln(d*x+c)/d+1/2*I*a/d*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*I*a/d*
exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Maxima [A]
time = 0.32, size = 73, normalized size = 1.04

$$\frac{1}{2}i\,a\left(\frac{e^{\left(\frac{cf}{d}-e\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d}-\frac{e^{\left(-\frac{cf}{d}+e\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right)+\frac{a\log\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*I*a*(e^(c*f/d - e)*exp_integral_e(1, (d*x + c)*f/d)/d - e^(-c*f/d + e)*
exp_integral_e(1, -(d*x + c)*f/d)/d) + a*log(d*x + c)/d
```

Fricas [A]
time = 0.36, size = 81, normalized size = 1.16

$$\frac{-i\,a\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf-de}{d}\right)}+i\,a\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf-de}{d}\right)}+2\,a\log\left(\frac{dx+c}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*a*Ei(-(d*f*x + c*f)/d)*e^((c*f - d*e)/d) + I*a*Ei((d*f*x + c*f)/d)*
e^(-(c*f - d*e)/d) + 2*a*log((d*x + c)/d))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ia\left(\int\left(-\frac{i}{c+dx}\right)dx+\int\frac{\sinh\left(e+fx\right)}{c+dx}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c),x)
```

```
[Out] I*a*(Integral(-I/(c + d*x), x) + Integral(sinh(e + f*x)/(c + d*x), x))
```

Giac [A]
time = 0.42, size = 69, normalized size = 0.99

$$-\frac{-i\,a\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}+i\,a\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}-2\,a\log\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(-I*a*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + I*a*Ei(-(d*f*x + c*f)/d)*e^(
-e + c*f/d) - 2*a*log(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+a\,\mathrm{sinh}(e+f\,x)\,1{}\mathrm{i}}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(e + f*x)*1i)/(c + d*x),x)
```

```
[Out] int((a + a*sinh(e + f*x)*1i)/(c + d*x), x)
```

3.100 $\int \frac{a+ia\sinh(e+fx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=95

$$-\frac{a}{d(c+dx)}+\frac{iaf\cosh\left(e-\frac{cf}{d}\right)\operatorname{Chi}\left(\frac{cf}{d}+fx\right)}{d^2}-\frac{ia\sinh(e+fx)}{d(c+dx)}+\frac{iaf\sinh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(\frac{cf}{d}+fx\right)}{d^2}$$

```
[Out] -a/d/(d*x+c)+I*a*f*Chi(c*f/d+f*x)*cosh(-e+c*f/d)/d^2-I*a*f*Shi(c*f/d+f*x)*s
inh(-e+c*f/d)/d^2-I*a*sinh(f*x+e)/d/(d*x+c)
```

Rubi [A]
time = 0.13, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3398, 3378, 3384, 3379, 3382}

$$\frac{iaf\operatorname{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{iaf\sinh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{ia\sinh(e+fx)}{d(c+dx)}-\frac{a}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[e + f*x])/(c + d*x)^2,x]
```

```
[Out] -(a/(d*(c + d*x))) + (I*a*f*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/
d^2 - (I*a*Sinh[e + f*x])/(d*(c + d*x)) + (I*a*f*Sinh[e - (c*f)/d]*SinhInte
gral[(c*f)/d + f*x])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+ia\sinh(e+fx)}{(c+dx)^2}\,dx &= \int \left(\frac{a}{(c+dx)^2}+\frac{ia\sinh(e+fx)}{(c+dx)^2}\right)dx\\ &= -\frac{a}{d(c+dx)}+(ia)\int \frac{\sinh(e+fx)}{(c+dx)^2}\,dx\\ &= -\frac{a}{d(c+dx)}-\frac{ia\sinh(e+fx)}{d(c+dx)}+\frac{(iaf)\int \frac{\cosh(e+fx)}{c+dx}\,dx}{d}\\ &= -\frac{a}{d(c+dx)}-\frac{ia\sinh(e+fx)}{d(c+dx)}+\frac{\left(iaf\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\cosh\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx}{d}+\frac{(iaf\,\text{s}}{}\\ &= -\frac{a}{d(c+dx)}+\frac{iaf\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d^2}-\frac{ia\sinh(e+fx)}{d(c+dx)}+\frac{iaf\sinh\left(e\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 83, normalized size = 0.87

$$\frac{ia\left(f(c+dx)\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)-d(-i+\sinh(e+fx))+f(c+dx)\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)\right)}{d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[e + f*x])/(c + d*x)^2,x]
[Out] (I*a*(f*(c + d*x)*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] - d*(-I + Sin
h[e + f*x]) + f*(c + d*x)*Sinh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)]))/(d^
2*(c + d*x))
```

Maple [A]
time = 0.50, size = 153, normalized size = 1.61

method	result
risch	$-\frac{a}{d(dx+c)}+\frac{iaf\,\mathrm{e}^{-fx-e}}{2d(dxf+cf)}-\frac{iaf\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{expIntegral}\left(1,fx+e+\frac{cf-de}{d}\right)}{2d^2}-\frac{iaf\,\mathrm{e}^{fx+e}}{2d^2\left(\frac{cf}{d}+fx\right)}-\frac{iaf\,\mathrm{e}^{-\frac{cf-de}{d}}\,\mathrm{expIntegral}\left(1,-fx-e\right.}{2d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/d/(d*x+c)+1/2*I*a*f*exp(-f*x-e)/d/(d*f*x+c*f)-1/2*I*a*f/d^2*exp((c*f-d*e
)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*I*a*f/d^2*exp(f*x+e)/(c*f/d+f*x)-1/2*I*a*f
/d^2*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Maxima [A]
time = 0.31, size = 90, normalized size = 0.95

$$\frac{1}{2}i\,a\left(\frac{e^{\left(\frac{cf}{d}-e\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d}-\frac{e^{\left(-\frac{cf}{d}+e\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right)-\frac{a}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*I*a*(e^(c*f/d - e)*exp_integral_e(2, (d*x + c)*f/d)/((d*x + c)*d) - e^(
-c*f/d + e)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d)) - a/(d^2*x + c
*d)
```

Fricas [A]
time = 0.37, size = 139, normalized size = 1.46

$$\frac{\left(-i\,ade^{(2\,fx+2\,e)}+i\,ad+\left((i\,adfx+i\,acf)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf-de}{d}\right)}+(i\,adfx+i\,acf)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf-de}{d}\right)}-2\,ad\right)e^{(fx+e)}\right)e^{(-fx-e)}}{2\,(d^3x+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(-I*a*d*e^(2*f*x + 2*e) + I*a*d + ((I*a*d*f*x + I*a*c*f)*Ei(-(d*f*x + c
*f)/d)*e^((c*f - d*e)/d) + (I*a*d*f*x + I*a*c*f)*Ei((d*f*x + c*f)/d)*e^(-(c
*f - d*e)/d) - 2*a*d)*e^(f*x + e))*e^(-f*x - e)/(d^3*x + c*d^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(91) = 182$.
time = 0.45, size = 630, normalized size = 6.63

$$\frac{1}{2}ia\left(\frac{\left((dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)f^2\mathrm{Ei}\left(\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)}-def^2\mathrm{Ei}\left(\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)}+cf^3\mathrm{Ei}\left(\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)}-df^2e^{\left(\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)}{d}\right)}\right)d^2}{\left((dx+c)d^4\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-d^5e+cd^4f\right)f}+\frac{\left((dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)f^2\mathrm{Ei}\left(-\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(-\frac{de-cf}{d}\right)}-def^2\mathrm{Ei}\left(-\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(-\frac{de-cf}{d}\right)}+cf^3\mathrm{Ei}\left(-\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(-\frac{de-cf}{d}\right)}+df^2e^{\left(-\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)}{d}\right)}\right)d^2}{\left((dx+c)d^4\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-d^5e+cd^4f\right)f}\right)-\frac{a}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*I*a*(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(((d*x + c)*(
d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) - d*e*
f^2*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d
*e - c*f)/d) + c*f^3*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*
e + c*f)/d)*e^((d*e - c*f)/d) - d*f^2*e^((d*x + c)*(d*e/(d*x + c) - c*f/(d*
x + c) + f)/d))*d^2/(((d*x + c)*d^4*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d
^5*e + c*d^4*f)*f) + ((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(
-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e -
c*f)/d) - d*e*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e
+ c*f)/d)*e^(-(d*e - c*f)/d) + c*f^3*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d
*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) + d*f^2*e^(-(d*x + c)*(d*e/
(d*x + c) - c*f/(d*x + c) + f)/d))*d^2/(((d*x + c)*d^4*(d*e/(d*x + c) - c*f
/(d*x + c) + f) - d^5*e + c*d^4*f)*f)) - a/((d*x + c)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\sinh(e+f\,x)\ 1\mathrm{i}}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(e + f*x)*1i)/(c + d*x)^2,x)
```

```
[Out] int((a + a*sinh(e + f*x)*1i)/(c + d*x)^2, x)
```

3.101 $\int \frac{a+ia\sinh(e+fx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=131

$$-\frac{a}{2d(c+dx)^2}-\frac{iaf\cosh(e+fx)}{2d^2(c+dx)}+\frac{iaf^2\mathrm{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{ia\sinh(e+fx)}{2d(c+dx)^2}+\frac{iaf^2\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}$$

```
[Out] -1/2*a/d/(d*x+c)^2-1/2*I*a*f*cosh(f*x+e)/d^2/(d*x+c)+1/2*I*a*f^2*cosh(-e+c*
f/d)*Shi(c*f/d+f*x)/d^3-1/2*I*a*f^2*Chi(c*f/d+f*x)*sinh(-e+c*f/d)/d^3-1/2*I
*a*sinh(f*x+e)/d/(d*x+c)^2
```

Rubi [A]
time = 0.17, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3398, 3378, 3384, 3379, 3382}

$$\frac{iaf^2\mathrm{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{2d^3}+\frac{iaf^2\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{iaf\cosh(e+fx)}{2d^2(c+dx)}-\frac{ia\sinh(e+fx)}{2d(c+dx)^2}-\frac{a}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[e + f*x])/(c + d*x)^3,x]
```

```
[Out] -1/2*a/(d*(c + d*x)^2) - ((I/2)*a*f*Cosh[e + f*x])/(d^2*(c + d*x)) + ((I/2)
*a*f^2*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d^3 - ((I/2)*a*Sinh[e
 + f*x])/(d*(c + d*x)^2) + ((I/2)*a*f^2*Cosh[e - (c*f)/d]*SinhIntegral[(c*f
)/d + f*x])/d^3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+ia\sinh(e+fx)}{(c+dx)^3}\,dx &= \int \left(\frac{a}{(c+dx)^3}+\frac{ia\sinh(e+fx)}{(c+dx)^3}\right)dx \\
&= -\frac{a}{2d(c+dx)^2}+(ia)\int \frac{\sinh(e+fx)}{(c+dx)^3}\,dx \\
&= -\frac{a}{2d(c+dx)^2}-\frac{ia\sinh(e+fx)}{2d(c+dx)^2}+\frac{(iaf)\int \frac{\cosh(e+fx)}{(c+dx)^2}\,dx}{2d} \\
&= -\frac{a}{2d(c+dx)^2}-\frac{iaf\cosh(e+fx)}{2d^2(c+dx)}-\frac{ia\sinh(e+fx)}{2d(c+dx)^2}+\frac{(iaf^2)\int \frac{\sinh(e+fx)}{c+dx}\,dx}{2d^2} \\
&= -\frac{a}{2d(c+dx)^2}-\frac{iaf\cosh(e+fx)}{2d^2(c+dx)}-\frac{ia\sinh(e+fx)}{2d(c+dx)^2}+\frac{\left(iaf^2\cosh\left(e-\frac{cf}{d}\right)\right)\int}{2d^2} \\
&= -\frac{a}{2d(c+dx)^2}-\frac{iaf\cosh(e+fx)}{2d^2(c+dx)}+\frac{iaf^2\mathrm{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{ia\sinh}{2d(c}
\end{aligned}$$

Mathematica [A]
time = 0.39, size = 109, normalized size = 0.83

$$\frac{ia\left(f^2(c+dx)^2\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)-d(f(c+dx)\cosh(e+fx)+d(-i+\sinh(e+fx)))+f^2(c+dx)^2\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}+x\right)\right)\right)}{2d^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[e + f*x])/(c + d*x)^3,x]
```

```
[Out] ((I/2)*a*(f^2*(c + d*x)^2*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] - d*(
f*(c + d*x)*Cosh[e + f*x] + d*(-I + Sinh[e + f*x])) + f^2*(c + d*x)^2*Cosh[
e - (c*f)/d]*SinhIntegral[f*(c/d + x)]))/(d^3*(c + d*x)^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(119) = 238.
time = 0.50, size = 303, normalized size = 2.31

method	result
risch	$-\frac{a}{2d(dx+c)^2}-\frac{ia\,f^3\mathrm{e}^{-fx-e}x}{4d(d^2x^2f^2+2cd\,f^2x+c^2f^2)}-\frac{ia\,f^3\mathrm{e}^{-fx-e}c}{4d^2(d^2x^2f^2+2cd\,f^2x+c^2f^2)}+\frac{ia\,f^2\mathrm{e}^{-fx-e}}{4d(d^2x^2f^2+2cd\,f^2x+c^2f^2)}+\frac{ia\,f^2\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{expI}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d/(d*x+c)^2-1/4*I*a*f^3*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f
^2)*x-1/4*I*a*f^3*exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c+1/4*I
*a*f^2*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)+1/4*I*a*f^2/d^3*exp(
(c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/4*I*a*f^2/d^3*exp(f*x+e)/(c*f/d+f*x)
^2-1/4*I*a*f^2/d^3*exp(f*x+e)/(c*f/d+f*x)-1/4*I*a*f^2/d^3*exp(-(c*f-d*e)/d)
*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Maxima [A]
time = 0.32, size = 101, normalized size = 0.77

$$\frac{1}{2}i\,a\left(\frac{e^{\left(\frac{cf}{d}-e\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}-\frac{e^{\left(-\frac{cf}{d}+e\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}\right)-\frac{a}{2\,(d^3x^2+2\,cd^2x+c^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*I*a*(e^(c*f/d - e)*exp_integral_e(3, (d*x + c)*f/d)/((d*x + c)^2*d) - e
^(-c*f/d + e)*exp_integral_e(3, -(d*x + c)*f/d)/((d*x + c)^2*d)) - 1/2*a/(d
^3*x^2 + 2*c*d^2*x + c^2*d)
```

Fricas [A]
time = 0.36, size = 227, normalized size = 1.73

$$\frac{\left(-i\,ad^2fx-i\,acdf+i\,ad^2+(-i\,ad^2fx-i\,acdf-i\,ad^2)e^{(2\,fx+2\,e)}-\left(2\,ad^2-(-i\,ad^2f^2x^2-2i\,acdf^2x-i\,ac^2f^2)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf-de}{d}\right)}-(i\,ad^2f^2x^2+2i\,acdf^2x+i\,ac^2f^2)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf-de}{d}\right)}\right)e^{(fx+e)}\right)e^{(-fx-e)}}{4\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(-I*a*d^2*f*x - I*a*c*d*f + I*a*d^2 + (-I*a*d^2*f*x - I*a*c*d*f - I*a*d
^2)*e^(2*f*x + 2*e) - (2*a*d^2 - (-I*a*d^2*f^2*x^2 - 2*I*a*c*d*f^2*x - I*a*
c^2*f^2)*Ei(-(d*f*x + c*f)/d)*e^((c*f - d*e)/d) - (I*a*d^2*f^2*x^2 + 2*I*a*
```

```
c*d*f^2*x + I*a*c^2*f^2)*Ei((d*f*x + c*f)/d)*e^(-(c*f - d*e)/d))*e^(f*x + e
))*e^(-f*x - e)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(115) = 230.
time = 0.44, size = 322, normalized size = 2.46

$$\frac{i\,ad^2f^2x^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}-i\,ad^2f^2x^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+2i\,acdf^2x\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}-2i\,acdf^2x\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+i\,ac^2f^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}-i\,ac^2f^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}-i\,ad^2fxe^{(fx+e)}-i\,ad^2fxe^{(-fx-e)}-i\,acdfe^{(fx+e)}-i\,acdfe^{(-fx-e)}-i\,ad^2e^{(fx+e)}+i\,ad^2e^{(-fx-e)}-2\,ad^2}{4\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(I*a*d^2*f^2*x^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) - I*a*d^2*f^2*x^2*Ei
(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + 2*I*a*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(
e - c*f/d) - 2*I*a*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + I*a*c^2*
f^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) - I*a*c^2*f^2*Ei(-(d*f*x + c*f)/d)*e^
(-e + c*f/d) - I*a*d^2*f*x*e^(f*x + e) - I*a*d^2*f*x*e^(-f*x - e) - I*a*c*d
*f*e^(f*x + e) - I*a*c*d*f*e^(-f*x - e) - I*a*d^2*e^(f*x + e) + I*a*d^2*e^(
-f*x - e) - 2*a*d^2)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\sinh(e+f\,x)\,1\mathrm{i}}{(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(e + f*x)*1i)/(c + d*x)^3,x)
```

```
[Out] int((a + a*sinh(e + f*x)*1i)/(c + d*x)^3, x)
```

3.102 $\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx$

Optimal. Leaf size=245

$$\frac{3a^2cd^2x}{4f^2}+\frac{3a^2d^3x^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d}+\frac{12ia^2d^2(c+dx)\cosh(e+fx)}{f^3}+\frac{2ia^2(c+dx)^3\cosh(e+fx)}{f}-\frac{12ia^2d^3\sinh}{f^4}$$

```
[Out] 3/4*a^2*c*d^2*x/f^2+3/8*a^2*d^3*x^2/f^2+3/8*a^2*(d*x+c)^4/d+12*I*a^2*d^2*(d
*x+c)*cosh(f*x+e)/f^3+2*I*a^2*(d*x+c)^3*cosh(f*x+e)/f-12*I*a^2*d^3*sinh(f*x
+e)/f^4-6*I*a^2*d*(d*x+c)^2*sinh(f*x+e)/f^2-3/4*a^2*d^2*(d*x+c)*cosh(f*x+e)
*sinh(f*x+e)/f^3-1/2*a^2*(d*x+c)^3*cosh(f*x+e)*sinh(f*x+e)/f+3/8*a^2*d^3*si
nh(f*x+e)^2/f^4+3/4*a^2*d*(d*x+c)^2*sinh(f*x+e)^2/f^2
```

Rubi [A]

time = 0.20, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3398, 3377, 2717, 3392, 32, 3391}

$$\frac{12ia^2d^2(c+dx)\cosh(e+fx)}{f^3}-\frac{3a^2d^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{4f^3}+\frac{3a^2cd^2x}{4f^2}+\frac{3a^2d(c+dx)^2\sinh^2(e+fx)}{4f^2}-\frac{6ia^2d(c+dx)^2\sinh(e+fx)}{f^2}+\frac{2ia^2(c+dx)^3\cosh(e+fx)}{f}-\frac{a^2(c+dx)^3\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{3a^2(c+dx)^4}{8d}+\frac{3a^2d^3\sinh^2(e+fx)}{8f^4}-\frac{12ia^2d^3\sinh(e+fx)}{f^4}+\frac{3a^2d^3x^2}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] (3*a^2*c*d^2*x)/(4*f^2) + (3*a^2*d^3*x^2)/(8*f^2) + (3*a^2*(c + d*x)^4)/(8*
d) + ((12*I)*a^2*d^2*(c + d*x)*Cosh[e + f*x])/f^3 + ((2*I)*a^2*(c + d*x)^3*
Cosh[e + f*x])/f - ((12*I)*a^2*d^3*Sinh[e + f*x])/f^4 - ((6*I)*a^2*d*(c + d
*x)^2*Sinh[e + f*x])/f^2 - (3*a^2*d^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x]
)/(4*f^3) - (a^2*(c + d*x)^3*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) + (3*a^2*d^
3*Sinh[e + f*x]^2)/(8*f^4) + (3*a^2*d*(c + d*x)^2*Sinh[e + f*x]^2)/(4*f^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx = \int \left(a^2(c+dx)^3+2ia^2(c+dx)^3\sinh(e+fx)-a^2(c+dx)^3\sinh^2(e\right.$$

$$= \frac{a^2(c+dx)^4}{4d}+\left(2ia^2\right)\int (c+dx)^3\sinh(e+fx)\,dx-a^2\int (c+dx)^3$$

$$= \frac{a^2(c+dx)^4}{4d}+\frac{2ia^2(c+dx)^3\cosh(e+fx)}{f}-\frac{a^2(c+dx)^3\cosh(e+}{2f}$$

$$= \frac{3a^2(c+dx)^4}{8d}+\frac{2ia^2(c+dx)^3\cosh(e+fx)}{f}-\frac{6ia^2d(c+dx)^2\sinh(e}{f^2}$$

$$= \frac{3a^2cd^2x}{4f^2}+\frac{3a^2d^3x^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d}+\frac{12ia^2d^2(c+dx)\cosh(e+fx}{f^3}$$

$$= \frac{3a^2cd^2x}{4f^2}+\frac{3a^2d^3x^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d}+\frac{12ia^2d^2(c+dx)\cosh(e+fx}{f^3}$$

Mathematica [A]
time = 0.86, size = 220, normalized size = 0.90

$$\frac{a^2\left(6f^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)+32if(c+dx)\left(c^2f^2+2cdf^2x+d^2(6+f^2x^2)\right)\cosh(e+fx)+3d\left(2c^2f^2+4cdf^2x+d^2(1+2f^2x^2)\right)\cosh(2(e+fx))-96id\left(c^2f^2+2cdf^2x+d^2(2+f^2x^2)\right)\sinh(e+fx)-2f(c+dx)\left(2c^2f^2+4cdf^2x+d^2(3+2f^2x^2)\right)\sinh(2(e+fx))\right)}{16f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] (a^2*(6*f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + (32*I)*f*(c + d
*x)*(c^2*f^2 + 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Cosh[e + f*x] + 3*d*(2*c^2*
f^2 + 4*c*d*f^2*x + d^2*(1 + 2*f^2*x^2))*Cosh[2*(e + f*x)] - (96*I)*d*(c^2*
f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x] - 2*f*(c + d*x)*(2*c^2
*f^2 + 4*c*d*f^2*x + d^2*(3 + 2*f^2*x^2))*Sinh[2*(e + f*x)]))/(16*f^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1081 vs. $2(227) = 454$.
time = 0.64, size = 1082, normalized size = 4.42

method	result
risch	$\frac{3a^2d^3x^4}{8}+\frac{3a^2c\,d^2x^3}{2}+\frac{9a^2c^2d\,x^2}{4}+\frac{3c^3a^2x}{2}+\frac{3a^2c^4}{8d}-\frac{a^2\left(4d^3x^3f^3+12c\,d^2f^3x^2+12c^2d\,f^3x-6d^3f^2x^2+4c^3f^3-\right.}{32f^4}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+I*a*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*I*d^3/f^3*a^2*((f*x+e)^3*cosh(f*x+e)-3*(f*x+e)^2*sinh(f*x+e)+6*(f*x+
e)*cosh(f*x+e)-6*sinh(f*x+e))+d^2/f^2*c*a^2*(f*x+e)^3-3*d/f*e*c^2*a^2*(f*x+
e)-3*d^2/f^2*e^2*c*a^2*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)+3*d/f*e*
c^2*a^2*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)+6*d^2/f^2*e*c*a^2*(1/2*
(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)-6*I*d^3/f^
3*e*a^2*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*sinh(f*x+e)+2*cosh(f*x+e))+6*I*d^2
/f^2*c*a^2*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*sinh(f*x+e)+2*cosh(f*x+e))+6*I*
d^3/f^3*e^2*a^2*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))+6*I*d/f*c^2*a^2*((f*x+e)*
cosh(f*x+e)-sinh(f*x+e))-3*d^2/f^2*e*c*a^2*(f*x+e)^2-2*I*d^3/f^3*e^3*a^2*co
sh(f*x+e)+3*d^2/f^2*e^2*c*a^2*(f*x+e)-c^3*a^2*(1/2*cosh(f*x+e)*sinh(f*x+e)-
1/2*f*x-1/2*e)+c^3*a^2*(f*x+e)+1/4*d^3/f^3*a^2*(f*x+e)^4-d^3/f^3*a^2*(1/2*(
f*x+e)^3*cosh(f*x+e)*sinh(f*x+e)-1/8*(f*x+e)^4-3/4*(f*x+e)^2*cosh(f*x+e)^2+
3/4*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+3/8*(f*x+e)^2-3/8*cosh(f*x+e)^2)+2*I*c^
3*a^2*cosh(f*x+e)-3*d^2/f^2*c*a^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)-1/
6*(f*x+e)^3-1/2*(f*x+e)*cosh(f*x+e)^2+1/4*cosh(f*x+e)*sinh(f*x+e)+1/4*f*x+1
/4*e)+3*d^3/f^3*e*a^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)-1/6*(f*x+e)^3-
1/2*(f*x+e)*cosh(f*x+e)^2+1/4*cosh(f*x+e)*sinh(f*x+e)+1/4*f*x+1/4*e)-d^3/f^
3*e*a^2*(f*x+e)^3-d^3/f^3*e^3*a^2*(f*x+e)+3/2*d^3/f^3*e^2*a^2*(f*x+e)^2+3/2
*d/f*c^2*a^2*(f*x+e)^2+d^3/f^3*e^3*a^2*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x
-1/2*e)-3*d^3/f^3*e^2*a^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^
2-1/4*cosh(f*x+e)^2)-3*d/f*c^2*a^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4
*(f*x+e)^2-1/4*cosh(f*x+e)^2)-6*I*d/f*e*c^2*a^2*cosh(f*x+e)+6*I*d^2/f^2*e^2
*c*a^2*cosh(f*x+e)-12*I*d^2/f^2*e*c*a^2*((f*x+e)*cosh(f*x+e)-sinh(f*x+e)))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(233) = 466.
time = 0.30, size = 552, normalized size = 2.25

$$\frac{1}{4}a^2d^3x^4 + a^2cd^2x^3 + \frac{3}{2}a^2c^2dx^2 + \frac{3}{16}\left(4x^2 - \frac{(2fxe^{(2e)} - e^{(2e)})e^{(2fx)}}{f^2} + \frac{(2fx+1)e^{(-2fx-2e)}}{f^2}\right)a^2c^2d + \frac{1}{16}\left(8x^3 - \frac{3(2f^2x^2e^{(2e)} - 2fxe^{(2e)} + e^{(2e)})e^{(2fx)}}{f^3} + \frac{3(2f^2x^2+2fx+1)e^{(-2fx-2e)}}{f^3}\right)a^2cd^2 + \frac{1}{32}\left(4x^4 - \frac{(4f^3x^3e^{(2e)} - 6f^2x^2e^{(2e)} + 6fxe^{(2e)} - 3e^{(2e)})e^{(2fx)}}{f^4} + \frac{(4f^3x^3+6f^2x^2+6fx+3)e^{(-2fx-2e)}}{f^4}\right)a^2d^3 + \frac{1}{8}a^2c^3\left(4x - \frac{e^{(2fx+2e)}}{f} + \frac{e^{(-2fx-2e)}}{f}\right) + a^2c^3x + 3ia^2c^2d\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + 3ia^2cd^2\left(\frac{(f^2x^2e^e - 2fxe^e + 2e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2+2fx+2)e^{(-fx-e)}}{f^3}\right) + ia^2d^3\left(\frac{(f^3x^3e^e - 3f^2x^2e^e + 6fxe^e - 6e^e)e^{(fx)}}{f^4} + \frac{(f^3x^3+3f^2x^2+6fx+6)e^{(-fx-e)}}{f^4}\right) + \frac{2ia^2c^3\cosh(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 + 3/16*(4*x^2 - (2*f*x*
e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 + (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*a^2*c^
2*d + 1/16*(8*x^3 - 3*(2*f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*
x)/f^3 + 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*a^2*c*d^2 + 1/32*(
4*x^4 - (4*f^3*x^3*e^(2*e) - 6*f^2*x^2*e^(2*e) + 6*f*x*e^(2*e) - 3*e^(2*e))
*e^(2*f*x)/f^4 + (4*f^3*x^3 + 6*f^2*x^2 + 6*f*x + 3)*e^(-2*f*x - 2*e)/f^4)*
a^2*d^3 + 1/8*a^2*c^3*(4*x - e^(2*f*x + 2*e)/f + e^(-2*f*x - 2*e)/f) + a^2*
c^3*x + 3*I*a^2*c^2*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^(-f*x - e)
/f^2) + 3*I*a^2*c*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 + (f^2
*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + I*a^2*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e
^e + 6*f*x*e^e - 6*e^e)*e^(f*x)/f^4 + (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(
-f*x - e)/f^4) + 2*I*a^2*c^3*cosh(f*x + e)/f
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 601 vs. 2(233) = 466.
time = 0.49, size = 601, normalized size = 2.45

$$\frac{\left(4a^2d^3f^3x^3 + 4a^2c^3f^3 + 6a^2c^2df^2 + 6a^2cd^2f + 3a^2d^3 + 6(2a^2cd^2f^3 + a^2d^3f^2)x^2 + 6(2a^2c^2df^3 + 2a^2cd^2f^2 + a^2d^3f)x - (4a^2d^3f^3x^3 + 4a^2c^3f^3 - 6a^2c^2df^2 + 6a^2cd^2f - 3a^2d^3 + 6(2a^2cd^2f^3 - a^2d^3f^2)x^2 + 6(2a^2c^2df^3 - 2a^2cd^2f^2 + a^2d^3f)x)e^{(4fx+4e)} - 32(-ia^2d^3f^3x^3 - ia^2c^3f^3 + 3ia^2c^2df^2 - 6ia^2cd^2f + 6ia^2d^3 + 3(-ia^2cd^2f^3 + ia^2d^3f^2)x^2 + 3(-ia^2c^2df^3 + 2ia^2cd^2f^2 - 2ia^2d^3f)x)e^{(3fx+3e)} + 12(a^2d^3f^4x^4 + 4a^2cd^2f^4x^3 + 6a^2c^2df^4x^2 + 4a^2c^3f^4x)e^{(2fx+2e)} - 32(-ia^2d^3f^3x^3 - ia^2c^3f^3 - 3ia^2c^2df^2 - 6ia^2cd^2f - 6ia^2d^3 + 3(-ia^2cd^2f^3 - ia^2d^3f^2)x^2 + 3(-ia^2c^2df^3 - 2ia^2cd^2f^2 - 2ia^2d^3f)x)e^{(fx+e)}\right)e^{(-2fx-2e)}}{32f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/32*(4*a^2*d^3*f^3*x^3 + 4*a^2*c^3*f^3 + 6*a^2*c^2*d*f^2 + 6*a^2*c*d^2*f +
 3*a^2*d^3 + 6*(2*a^2*c*d^2*f^3 + a^2*d^3*f^2)*x^2 + 6*(2*a^2*c^2*d*f^3 + 2
*a^2*c*d^2*f^2 + a^2*d^3*f)*x - (4*a^2*d^3*f^3*x^3 + 4*a^2*c^3*f^3 - 6*a^2*
c^2*d*f^2 + 6*a^2*c*d^2*f - 3*a^2*d^3 + 6*(2*a^2*c*d^2*f^3 - a^2*d^3*f^2)*x
^2 + 6*(2*a^2*c^2*d*f^3 - 2*a^2*c*d^2*f^2 + a^2*d^3*f)*x)*e^(4*f*x + 4*e) -
 32*(-I*a^2*d^3*f^3*x^3 - I*a^2*c^3*f^3 + 3*I*a^2*c^2*d*f^2 - 6*I*a^2*c*d^2
*f + 6*I*a^2*d^3 + 3*(-I*a^2*c*d^2*f^3 + I*a^2*d^3*f^2)*x^2 + 3*(-I*a^2*c^2
*d*f^3 + 2*I*a^2*c*d^2*f^2 - 2*I*a^2*d^3*f)*x)*e^(3*f*x + 3*e) + 12*(a^2*d^
3*f^4*x^4 + 4*a^2*c*d^2*f^4*x^3 + 6*a^2*c^2*d*f^4*x^2 + 4*a^2*c^3*f^4*x)*e^
(2*f*x + 2*e) - 32*(-I*a^2*d^3*f^3*x^3 - I*a^2*c^3*f^3 - 3*I*a^2*c^2*d*f^2
- 6*I*a^2*c*d^2*f - 6*I*a^2*d^3 + 3*(-I*a^2*c*d^2*f^3 - I*a^2*d^3*f^2)*x^2
+ 3*(-I*a^2*c^2*d*f^3 - 2*I*a^2*c*d^2*f^2 - 2*I*a^2*d^3*f)*x)*e^(f*x + e))*
e^(-2*f*x - 2*e)/f^4
```

Sympy [A]
time = 0.77, size = 1134, normalized size = 4.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+I*a*sinh(f*x+e))**2,x)
```

```
[Out] 3*a**2*c**3*x/2 + 9*a**2*c**2*d*x**2/4 + 3*a**2*c*d**2*x**3/2 + 3*a**2*d**3
*x**4/8 + Piecewise((((128*a**2*c**3*f**15*exp(e) + 384*a**2*c**2*d*f**15*x
*exp(e) + 192*a**2*c**2*d*f**14*exp(e) + 384*a**2*c*d**2*f**15*x**2*exp(e)
+ 384*a**2*c*d**2*f**14*x*exp(e) + 192*a**2*c*d**2*f**13*exp(e) + 128*a**2*
d**3*f**15*x**3*exp(e) + 192*a**2*d**3*f**14*x**2*exp(e) + 192*a**2*d**3*f*
*13*x*exp(e) + 96*a**2*d**3*f**12*exp(e))*exp(-2*f*x) + (-128*a**2*c**3*f**
15*exp(5*e) - 384*a**2*c**2*d*f**15*x*exp(5*e) + 192*a**2*c**2*d*f**14*exp(
5*e) - 384*a**2*c*d**2*f**15*x**2*exp(5*e) + 384*a**2*c*d**2*f**14*x*exp(5*
e) - 192*a**2*c*d**2*f**13*exp(5*e) - 128*a**2*d**3*f**15*x**3*exp(5*e) + 1
92*a**2*d**3*f**14*x**2*exp(5*e) - 192*a**2*d**3*f**13*x*exp(5*e) + 96*a**2
*d**3*f**12*exp(5*e))*exp(2*f*x) + (1024*I*a**2*c**3*f**15*exp(2*e) + 3072*
I*a**2*c**2*d*f**15*x*exp(2*e) + 3072*I*a**2*c**2*d*f**14*exp(2*e) + 3072*I
*a**2*c*d**2*f**15*x**2*exp(2*e) + 6144*I*a**2*c*d**2*f**14*x*exp(2*e) + 61
44*I*a**2*c*d**2*f**13*exp(2*e) + 1024*I*a**2*d**3*f**15*x**3*exp(2*e) + 30
72*I*a**2*d**3*f**14*x**2*exp(2*e) + 6144*I*a**2*d**3*f**13*x*exp(2*e) + 61
44*I*a**2*d**3*f**12*exp(2*e))*exp(-f*x) + (1024*I*a**2*c**3*f**15*exp(4*e)
 + 3072*I*a**2*c**2*d*f**15*x*exp(4*e) - 3072*I*a**2*c**2*d*f**14*exp(4*e)
+ 3072*I*a**2*c*d**2*f**15*x**2*exp(4*e) - 6144*I*a**2*c*d**2*f**14*x*exp(4
*e) + 6144*I*a**2*c*d**2*f**13*exp(4*e) + 1024*I*a**2*d**3*f**15*x**3*exp(4
*e) - 3072*I*a**2*d**3*f**14*x**2*exp(4*e) + 6144*I*a**2*d**3*f**13*x*exp(4
*e) - 6144*I*a**2*d**3*f**12*exp(4*e))*exp(f*x))*exp(-3*e)/(1024*f**16), Ne
(f**16*exp(3*e), 0)), (x**4*(-a**2*d**3*exp(4*e) + 4*I*a**2*d**3*exp(3*e) -
 4*I*a**2*d**3*exp(e) - a**2*d**3)*exp(-2*e)/16 + x**3*(-a**2*c*d**2*exp(4*
e) + 4*I*a**2*c*d**2*exp(3*e) - 4*I*a**2*c*d**2*exp(e) - a**2*c*d**2)*exp(-
2*e)/4 + x**2*(-3*a**2*c**2*d*exp(4*e) + 12*I*a**2*c**2*d*exp(3*e) - 12*I*a
**2*c**2*d*exp(e) - 3*a**2*c**2*d)*exp(-2*e)/8 + x*(-a**2*c**3*exp(4*e) + 4
*I*a**2*c**3*exp(3*e) - 4*I*a**2*c**3*exp(e) - a**2*c**3)*exp(-2*e)/4, True
))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(223) = 446.
time = 0.44, size = 580, normalized size = 2.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/8*a^2*d^3*x^4 + 3/2*a^2*c*d^2*x^3 + 9/4*a^2*c^2*d*x^2 + 3/2*a^2*c^3*x - 1
/32*(4*a^2*d^3*f^3*x^3 + 12*a^2*c*d^2*f^3*x^2 + 12*a^2*c^2*d*f^3*x - 6*a^2*
d^3*f^2*x^2 + 4*a^2*c^3*f^3 - 12*a^2*c*d^2*f^2*x - 6*a^2*c^2*d*f^2 + 6*a^2*
d^3*f*x + 6*a^2*c*d^2*f - 3*a^2*d^3)*e^(2*f*x + 2*e)/f^4 + (I*a^2*d^3*f^3*x
^3 + 3*I*a^2*c*d^2*f^3*x^2 + 3*I*a^2*c^2*d*f^3*x - 3*I*a^2*d^3*f^2*x^2 + I*
a^2*c^3*f^3 - 6*I*a^2*c*d^2*f^2*x - 3*I*a^2*c^2*d*f^2 + 6*I*a^2*d^3*f*x + 6
*I*a^2*c*d^2*f - 6*I*a^2*d^3)*e^(f*x + e)/f^4 + (I*a^2*d^3*f^3*x^3 + 3*I*a^
2*c*d^2*f^3*x^2 + 3*I*a^2*c^2*d*f^3*x + 3*I*a^2*d^3*f^2*x^2 + I*a^2*c^3*f^3
 + 6*I*a^2*c*d^2*f^2*x + 3*I*a^2*c^2*d*f^2 + 6*I*a^2*d^3*f*x + 6*I*a^2*c*d^
2*f + 6*I*a^2*d^3)*e^(-f*x - e)/f^4 + 1/32*(4*a^2*d^3*f^3*x^3 + 12*a^2*c*d^
2*f^3*x^2 + 12*a^2*c^2*d*f^3*x + 6*a^2*d^3*f^2*x^2 + 4*a^2*c^3*f^3 + 12*a^2
*c*d^2*f^2*x + 6*a^2*c^2*d*f^2 + 6*a^2*d^3*f*x + 6*a^2*c*d^2*f + 3*a^2*d^3)
*e^(-2*f*x - 2*e)/f^4
```

Mupad [B]
time = 1.05, size = 393, normalized size = 1.60

$$\frac{a^2\,\left(3\,d^3\,\mathrm{cosh}\left(2\,e+2\,f\,x\right)+24\,c^3\,f^4\,x-4\,c^3\,f^3\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)+6\,d^3\,f^4\,x^4+6\,c^2\,d\,f^2\,\mathrm{cosh}\left(2\,e+2\,f\,x\right)+36\,c^2\,d\,f^4\,x^2+24\,c\,d^2\,f^4\,x^3+6\,d^3\,f^2\,x^2\,\mathrm{cosh}\left(2\,e+2\,f\,x\right)-4\,d^3\,f^3\,x^3\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)-6\,c\,d^2\,f\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)-6\,d^3\,f\,x\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)+12\,c\,d^2\,f^2\,x\,\mathrm{cosh}\left(2\,e+2\,f\,x\right)-12\,c^2\,d\,f^3\,x\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)-12\,c\,d^2\,f^3\,x^2\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)-d^3\,\mathrm{sinh}\left(e+f\,x\right)\,192{}\mathrm{i}+c^3\,f^3\,\mathrm{cosh}\left(e+f\,x\right)\,32{}\mathrm{i}+d^3\,f^3\,x^3\,\mathrm{cosh}\left(e+f\,x\right)\,32{}\mathrm{i}-d^3\,f^2\,x^2\,\mathrm{sinh}\left(e+f\,x\right)\,96{}\mathrm{i}+c\,d^2\,f\,\mathrm{cosh}\left(e+f\,x\right)\,192{}\mathrm{i}+d^3\,f\,x\,\mathrm{cosh}\left(e+f\,x\right)\,192{}\mathrm{i}-c^2\,d\,f^2\,\mathrm{sinh}\left(e+f\,x\right)\,96{}\mathrm{i}+c^2\,d\,f^3\,x\,\mathrm{cosh}\left(e+f\,x\right)\,96{}\mathrm{i}-c\,d^2\,f^2\,x\,\mathrm{sinh}\left(e+f\,x\right)\,192{}\mathrm{i}+c\,d^2\,f^3\,x^2\,\mathrm{cosh}\left(e+f\,x\right)\,96{}\mathrm{i}\right)}{16\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(e + f*x)*1i)^2*(c + d*x)^3,x)
```

```
[Out] (a^2*(3*d^3*cosh(2*e + 2*f*x) - d^3*sinh(e + f*x)*192i + c^3*f^3*cosh(e + f
*x)*32i + 24*c^3*f^4*x - 4*c^3*f^3*sinh(2*e + 2*f*x) + 6*d^3*f^4*x^4 + 6*c^
2*d*f^2*cosh(2*e + 2*f*x) + 36*c^2*d*f^4*x^2 + 24*c*d^2*f^4*x^3 + d^3*f^3*x
^3*cosh(e + f*x)*32i - d^3*f^2*x^2*sinh(e + f*x)*96i + c*d^2*f*cosh(e + f*x
)*192i + d^3*f*x*cosh(e + f*x)*192i + 6*d^3*f^2*x^2*cosh(2*e + 2*f*x) - 4*d
^3*f^3*x^3*sinh(2*e + 2*f*x) - 6*c*d^2*f*sinh(2*e + 2*f*x) - c^2*d*f^2*sinh
(e + f*x)*96i - 6*d^3*f*x*sinh(2*e + 2*f*x) + c^2*d*f^3*x*cosh(e + f*x)*96i
 - c*d^2*f^2*x*sinh(e + f*x)*192i + 12*c*d^2*f^2*x*cosh(2*e + 2*f*x) + c*d^
2*f^3*x^2*cosh(e + f*x)*96i - 12*c^2*d*f^3*x*sinh(2*e + 2*f*x) - 12*c*d^2*f
^3*x^2*sinh(2*e + 2*f*x)))/(16*f^4)
```

3.103 $\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx$

Optimal. Leaf size=174

$$\frac{a^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{2d}+\frac{4ia^2d^2\cosh(e+fx)}{f^3}+\frac{2ia^2(c+dx)^2\cosh(e+fx)}{f}-\frac{4ia^2d(c+dx)\sinh(e+fx)}{f^2}-\frac{a^2d^2\,\mathrm{c}}{}$$

```
[Out] 1/4*a^2*d^2*x/f^2+1/2*a^2*(d*x+c)^3/d+4*I*a^2*d^2*cosh(f*x+e)/f^3+2*I*a^2*(
d*x+c)^2*cosh(f*x+e)/f-4*I*a^2*d*(d*x+c)*sinh(f*x+e)/f^2-1/4*a^2*d^2*cosh(f
*x+e)*sinh(f*x+e)/f^3-1/2*a^2*(d*x+c)^2*cosh(f*x+e)*sinh(f*x+e)/f+1/2*a^2*d
*(d*x+c)*sinh(f*x+e)^2/f^2
```

Rubi [A]
time = 0.14, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3398, 3377, 2718, 3392, 32, 2715, 8}

$$\frac{a^2d(c+dx)\sinh^2(e+fx)}{2f^2}-\frac{4ia^2d(c+dx)\sinh(e+fx)}{f^2}+\frac{2ia^2(c+dx)^2\cosh(e+fx)}{f}-\frac{a^2(c+dx)^2\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{a^2(c+dx)^3}{2d}+\frac{4ia^2d^2\cosh(e+fx)}{f^3}-\frac{a^2d^2\sinh(e+fx)\cosh(e+fx)}{4f^3}+\frac{a^2d^2x}{4f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] (a^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(2*d) + ((4*I)*a^2*d^2*Cosh[e + f*x
])/f^3 + ((2*I)*a^2*(c + d*x)^2*Cosh[e + f*x])/f - ((4*I)*a^2*d*(c + d*x)*S
inh[e + f*x])/f^2 - (a^2*d^2*Cosh[e + f*x]*Sinh[e + f*x])/(4*f^3) - (a^2*(c
 + d*x)^2*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) + (a^2*d*(c + d*x)*Sinh[e + f*
x]^2)/(2*f^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx = \int \left(a^2(c+dx)^2+2ia^2(c+dx)^2\sinh(e+fx)-a^2(c+dx)^2\sinh^2(e\right.$$

$$= \frac{a^2(c+dx)^3}{3d}+\left(2ia^2\right)\int (c+dx)^2\sinh(e+fx)\,dx-a^2\int (c+dx)^2$$

$$= \frac{a^2(c+dx)^3}{3d}+\frac{2ia^2(c+dx)^2\cosh(e+fx)}{f}-\frac{a^2(c+dx)^2\cosh(e+}{2f}$$

$$= \frac{a^2(c+dx)^3}{2d}+\frac{2ia^2(c+dx)^2\cosh(e+fx)}{f}-\frac{4ia^2d(c+dx)\sinh(e+}{f^2}$$

$$= \frac{a^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{2d}+\frac{4ia^2d^2\cosh(e+fx)}{f^3}+\frac{2ia^2(c+dx)^2\cosh(e}{f}$$

Mathematica [A]
time = 0.40, size = 189, normalized size = 1.09

$$\frac{a^2(12c^2f^3x+12cdf^3x^2+4d^2f^3x^3+16i(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\cosh(e+fx)+2df(c+dx)\cosh(2(e+fx))-32icdf\sinh(e+fx)-32id^2fx\sinh(e+fx)-d^2\sinh(2(e+fx))-2c^2f^2\sinh(2(e+fx))-4cdf^2x\sinh(2(e+fx))-2d^2f^2x^2\sinh(2(e+fx)))}{8f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] (a^2*(12*c^2*f^3*x + 12*c*d*f^3*x^2 + 4*d^2*f^3*x^3 + (16*I)*(c^2*f^2 + 2*c
*d*f^2*x + d^2*(2 + f^2*x^2))*Cosh[e + f*x] + 2*d*f*(c + d*x)*Cosh[2*(e + f
*x)] - (32*I)*c*d*f*Sinh[e + f*x] - (32*I)*d^2*f*x*Sinh[e + f*x] - d^2*Sinh
[2*(e + f*x)] - 2*c^2*f^2*Sinh[2*(e + f*x)] - 4*c*d*f^2*x*Sinh[2*(e + f*x)]
 - 2*d^2*f^2*x^2*Sinh[2*(e + f*x)]))/(8*f^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(161) = 322$.
time = 0.63, size = 550, normalized size = 3.16

method	result
risch	$\frac{a^2 d^2 x^3}{2} + \frac{3a^2 cd\,x^2}{2} + \frac{3a^2 c^2 x}{2} + \frac{a^2 c^3}{2d} - \frac{a^2\left(2d^2x^2f^2+4cd\,f^2x+2c^2f^2-2d^2fx-2cdf+d^2\right)\mathrm{e}^{2fx+2e}}{16f^3} + \frac{ia^2\left(d^2x^2f^2+\right.}{}$
derivativedivides	$\frac{\frac{d^2a^2(fx+e)^3}{3f^2} + 2ic^2a^2\cosh(fx+e) - \frac{d^2a^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2} - \frac{(fx+e)^3}{6} - \frac{(fx+e)\left(\cosh^2(fx+e)\right)}{2} + \frac{\cosh(fx+e)\sinh(}{4}\right.}{f^2}}{}$
default	$\frac{\frac{d^2a^2(fx+e)^3}{3f^2} + 2ic^2a^2\cosh(fx+e) - \frac{d^2a^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2} - \frac{(fx+e)^3}{6} - \frac{(fx+e)\left(\cosh^2(fx+e)\right)}{2} + \frac{\cosh(fx+e)\sinh(}{4}\right.}{f^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+I*a*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*d^2/f^2*a^2*(f*x+e)^3+2*I*c^2*a^2*cosh(f*x+e)-d^2/f^2*a^2*(1/2*(f*
x+e)^2*cosh(f*x+e)*sinh(f*x+e)-1/6*(f*x+e)^3-1/2*(f*x+e)*cosh(f*x+e)^2+1/4*
cosh(f*x+e)*sinh(f*x+e)+1/4*f*x+1/4*e)-d^2/f^2*e*a^2*(f*x+e)^2+2*I*d^2/f^2*
a^2*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*sinh(f*x+e)+2*cosh(f*x+e))+2*d^2/f^2*e
*a^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)+
d/f*c*a^2*(f*x+e)^2-4*I*d/f*e*c*a^2*cosh(f*x+e)-2*d/f*c*a^2*(1/2*(f*x+e)*co
sh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)+d^2/f^2*e^2*a^2*(f*x
+e)+4*I*d/f*c*a^2*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))-d^2/f^2*e^2*a^2*(1/2*co
sh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)-2*d/f*e*c*a^2*(f*x+e)-4*I*d^2/f^2*e*a^
2*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))+2*d/f*e*c*a^2*(1/2*cosh(f*x+e)*sinh(f*x
+e)-1/2*f*x-1/2*e)+a^2*c^2*(f*x+e)+2*I*d^2/f^2*e^2*a^2*cosh(f*x+e)-a^2*c^2*
(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(166) = 332$.
time = 0.28, size = 343, normalized size = 1.97

$$\frac{1}{3}a^2d^2x^3 + a^2cdx^2 + \frac{1}{8}\left(4x^2 - \frac{\left(2fxe^{(2e)} - e^{(2e)}\right)e^{(2fx)}}{f^2} + \frac{(2fx+1)e^{(-2fx-2e)}}{f^2}\right)a^2cd + \frac{1}{48}\left(8x^3 - \frac{3\left(2f^2x^2e^{(2e)} - 2fxe^{(2e)} + e^{(2e)}\right)e^{(2fx)}}{f^3} + \frac{3\left(2f^2x^2 + 2fx + 1\right)e^{(-2fx-2e)}}{f^3}\right)a^2d^2 + \frac{1}{8}a^2c^2\left(4x - \frac{e^{(2fx+2e)}}{f} + \frac{e^{(-2fx-2e)}}{f}\right) + a^2c^2x + 2i\,a^2cd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + i\,a^2d^2\left(\frac{(f^2x^2e^e - 2fxe^e + 2e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2 + 2fx + 2)e^{(-fx-e)}}{f^3}\right) + \frac{2i\,a^2c^2\cosh(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*d^2*x^3 + a^2*c*d*x^2 + 1/8*(4*x^2 - (2*f*x*e^(2*e) - e^(2*e))*e^(2
*f*x)/f^2 + (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*a^2*c*d + 1/48*(8*x^3 - 3*(2*
f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*x)/f^3 + 3*(2*f^2*x^2 + 2
*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*a^2*d^2 + 1/8*a^2*c^2*(4*x - e^(2*f*x + 2*e
)/f + e^(-2*f*x - 2*e)/f) + a^2*c^2*x + 2*I*a^2*c*d*((f*x*e^e - e^e)*e^(f*x
)/f^2 + (f*x + 1)*e^(-f*x - e)/f^2) + I*a^2*d^2*((f^2*x^2*e^e - 2*f*x*e^e +
 2*e^e)*e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + 2*I*a^2*c^2
*cosh(f*x + e)/f
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(166) = 332$.
time = 0.40, size = 356, normalized size = 2.05

$$\frac{(2\,a^2d^2f^2x^2 + 2\,a^2c^2f^2 + 2\,a^2cdf + a^2d^2 + 2\,(2\,a^2cdf^2 + a^2d^2f)x - (2\,a^2d^2f^2x^2 + 2\,a^2c^2f^2 - 2\,a^2cdf + a^2d^2 + 2\,(2\,a^2cdf^2 - a^2d^2f)x)e^{(4\,fx+4\,e)} - 16\,(-i\,a^2d^2f^2x^2 - i\,a^2c^2f^2 + 2i\,a^2cdf - 2i\,a^2d^2 + 2\,(-i\,a^2cdf^2 + i\,a^2d^2f)x)e^{(3\,fx+3\,e)} + 8\,(a^2d^2f^3x^3 + 3\,a^2cdf^3x^2 + 3\,a^2c^2f^3x)e^{(2\,fx+2\,e)} - 16\,(-i\,a^2d^2f^2x^2 - i\,a^2c^2f^2 - 2i\,a^2cdf - 2i\,a^2d^2 + 2\,(-i\,a^2cdf^2 - i\,a^2d^2f)x)e^{(fx+e)})e^{(-2\,fx-2\,e)}}{16\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(2*a^2*d^2*f^2*x^2 + 2*a^2*c^2*f^2 + 2*a^2*c*d*f + a^2*d^2 + 2*(2*a^2*
c*d*f^2 + a^2*d^2*f)*x - (2*a^2*d^2*f^2*x^2 + 2*a^2*c^2*f^2 - 2*a^2*c*d*f +
 a^2*d^2 + 2*(2*a^2*c*d*f^2 - a^2*d^2*f)*x)*e^(4*f*x + 4*e) - 16*(-I*a^2*d^
2*f^2*x^2 - I*a^2*c^2*f^2 + 2*I*a^2*c*d*f - 2*I*a^2*d^2 + 2*(-I*a^2*c*d*f^2
 + I*a^2*d^2*f)*x)*e^(3*f*x + 3*e) + 8*(a^2*d^2*f^3*x^3 + 3*a^2*c*d*f^3*x^2
 + 3*a^2*c^2*f^3*x)*e^(2*f*x + 2*e) - 16*(-I*a^2*d^2*f^2*x^2 - I*a^2*c^2*f^
2 - 2*I*a^2*c*d*f - 2*I*a^2*d^2 + 2*(-I*a^2*c*d*f^2 - I*a^2*d^2*f)*x)*e^(f*
x + e))*e^(-2*f*x - 2*e)/f^3
```

Sympy [**A**]
time = 0.57, size = 694, normalized size = 3.99

$$\frac{3a^2c^2x}{2} + \frac{3a^2cdx^2}{2} + \frac{a^2d^2x^3}{2} + \begin{cases} \frac{\left(\left(32a^2c^2f^{11}e^{e}+64a^2cdf^{11}xe^{e}+32a^2cdf^{10}e^{e}+32a^2d^2f^{11}x^2e^{e}+32a^2d^2f^{10}xe^{e}+16a^2d^2f^9e^{e}\right)e^{-2fx}+\left(-32a^2c^2f^{11}e^{5e}-64a^2cdf^{11}xe^{5e}+32a^2cdf^{10}e^{5e}-32a^2d^2f^{11}x^2e^{5e}+32a^2d^2f^{10}xe^{5e}-16a^2d^2f^9e^{5e}\right)e^{2fx}+\left(256ia^2c^2f^{11}e^{2e}+512ia^2cdf^{11}xe^{2e}+512ia^2cdf^{10}e^{2e}+256ia^2d^2f^{11}x^2e^{2e}+512ia^2d^2f^{10}xe^{2e}+512ia^2d^2f^9e^{2e}\right)e^{-fx}+\left(256ia^2c^2f^{11}e^{4e}+512ia^2cdf^{11}xe^{4e}-512ia^2cdf^{10}e^{4e}+256ia^2d^2f^{11}x^2e^{4e}-512ia^2d^2f^{10}xe^{4e}+512ia^2d^2f^9e^{4e}\right)e^{fx}\right)e^{-3e}}{256f^{12}} & \text{for}\: f^{12}e^{3e} \neq 0 \\ \frac{x^3\left(-a^2d^2e^{4e}+4ia^2d^2e^{3e}-4ia^2d^2e^{e}-a^2d^2\right)e^{-2e}}{12} + \frac{x^2\left(-a^2cde^{4e}+4ia^2cde^{3e}-4ia^2cde^{e}-a^2cd\right)e^{-2e}}{4} + \frac{x\left(-a^2c^2e^{4e}+4ia^2c^2e^{3e}-4ia^2c^2e^{e}-a^2c^2\right)e^{-2e}}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+I*a*sinh(f*x+e))**2,x)
```

```
[Out] 3*a**2*c**2*x/2 + 3*a**2*c*d*x**2/2 + a**2*d**2*x**3/2 + Piecewise((((32*a*
*2*c**2*f**11*exp(e) + 64*a**2*c*d*f**11*x*exp(e) + 32*a**2*c*d*f**10*exp(e
) + 32*a**2*d**2*f**11*x**2*exp(e) + 32*a**2*d**2*f**10*x*exp(e) + 16*a**2*
d**2*f**9*exp(e))*exp(-2*f*x) + (-32*a**2*c**2*f**11*exp(5*e) - 64*a**2*c*d
*f**11*x*exp(5*e) + 32*a**2*c*d*f**10*exp(5*e) - 32*a**2*d**2*f**11*x**2*ex
p(5*e) + 32*a**2*d**2*f**10*x*exp(5*e) - 16*a**2*d**2*f**9*exp(5*e))*exp(2*
f*x) + (256*I*a**2*c**2*f**11*exp(2*e) + 512*I*a**2*c*d*f**11*x*exp(2*e) +
512*I*a**2*c*d*f**10*exp(2*e) + 256*I*a**2*d**2*f**11*x**2*exp(2*e) + 512*I
*a**2*d**2*f**10*x*exp(2*e) + 512*I*a**2*d**2*f**9*exp(2*e))*exp(-f*x) + (2
```

```
56*I*a**2*c**2*f**11*exp(4*e) + 512*I*a**2*c*d*f**11*x*exp(4*e) - 512*I*a**
2*c*d*f**10*exp(4*e) + 256*I*a**2*d**2*f**11*x**2*exp(4*e) - 512*I*a**2*d**
2*f**10*x*exp(4*e) + 512*I*a**2*d**2*f**9*exp(4*e))*exp(f*x))*exp(-3*e)/(25
6*f**12), Ne(f**12*exp(3*e), 0)), (x**3*(-a**2*d**2*exp(4*e) + 4*I*a**2*d**
2*exp(3*e) - 4*I*a**2*d**2*exp(e) - a**2*d**2)*exp(-2*e)/12 + x**2*(-a**2*c
*d*exp(4*e) + 4*I*a**2*c*d*exp(3*e) - 4*I*a**2*c*d*exp(e) - a**2*c*d)*exp(-
2*e)/4 + x*(-a**2*c**2*exp(4*e) + 4*I*a**2*c**2*exp(3*e) - 4*I*a**2*c**2*ex
p(e) - a**2*c**2)*exp(-2*e)/4, True))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(158) = 316$.
time = 0.47, size = 333, normalized size = 1.91

$$\frac{1}{2}a^2d^2x^3+\frac{3}{2}a^2cdx^2+\frac{3}{2}a^2c^2x-\frac{\left(2a^2d^2f^2x^2+4a^2cdf^2x+2a^2c^2f^2-2a^2d^2fx-2a^2cdf+a^2d^2\right)e^{(2fx+2e)}}{16f^3}+\frac{\left(ia^2d^2f^2x^2+2ia^2cdf^2x+ia^2c^2f^2-2ia^2d^2fx-2ia^2cdf+2ia^2d^2\right)e^{(fx+e)}}{f^3}-\frac{\left(-ia^2d^2f^2x^2-2ia^2cdf^2x-ia^2c^2f^2-2ia^2d^2fx-2ia^2cdf-2ia^2d^2\right)e^{(-fx-e)}}{f^3}+\frac{\left(2a^2d^2f^2x^2+4a^2cdf^2x+2a^2c^2f^2+2a^2d^2fx+2a^2cdf+a^2d^2\right)e^{(-2fx-2e)}}{16f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*d^2*x^3 + 3/2*a^2*c*d*x^2 + 3/2*a^2*c^2*x - 1/16*(2*a^2*d^2*f^2*x^2
 + 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^2 - 2*a^2*d^2*f*x - 2*a^2*c*d*f + a^2*d^2)
*e^(2*f*x + 2*e)/f^3 + (I*a^2*d^2*f^2*x^2 + 2*I*a^2*c*d*f^2*x + I*a^2*c^2*f
^2 - 2*I*a^2*d^2*f*x - 2*I*a^2*c*d*f + 2*I*a^2*d^2)*e^(f*x + e)/f^3 - (-I*a
^2*d^2*f^2*x^2 - 2*I*a^2*c*d*f^2*x - I*a^2*c^2*f^2 - 2*I*a^2*d^2*f*x - 2*I*
a^2*c*d*f - 2*I*a^2*d^2)*e^(-f*x - e)/f^3 + 1/16*(2*a^2*d^2*f^2*x^2 + 4*a^2
*c*d*f^2*x + 2*a^2*c^2*f^2 + 2*a^2*d^2*f*x + 2*a^2*c*d*f + a^2*d^2)*e^(-2*f
*x - 2*e)/f^3
```

Mupad [**B**]
time = 0.68, size = 217, normalized size = 1.25

$$\frac{a^2\left(12c^2x+12cdx^2+4d^2x^3\right)}{8}+\frac{\frac{a^2\left(-d^2\sinh(2e+2fx)+d^2\cosh(e+fx)\,32\mathrm{i}\right)}{8}+\frac{a^2f^2\left(-2c^2\sinh(2e+2fx)-2d^2x^2\sinh(2e+2fx)-4cdx\sinh(2e+2fx)+c^2\cosh(e+fx)\,16\mathrm{i}+d^2x^2\cosh(e+fx)\,16\mathrm{i}+cdx\cosh(e+fx)\,32\mathrm{i}\right)}{8}-\frac{a^2f\left(-2d^2x\cosh(2e+2fx)-2cd\cosh(2e+2fx)+d^2x\sinh(e+fx)\,32\mathrm{i}+cd\sinh(e+fx)\,32\mathrm{i}\right)}{8}}{f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(e + f*x)*1i)^2*(c + d*x)^2,x)
```

```
[Out] (a^2*(12*c^2*x + 4*d^2*x^3 + 12*c*d*x^2))/8 + ((a^2*(d^2*cosh(e + f*x)*32i
- d^2*sinh(2*e + 2*f*x)))/8 + (a^2*f^2*(c^2*cosh(e + f*x)*16i - 2*c^2*sinh(
2*e + 2*f*x) + d^2*x^2*cosh(e + f*x)*16i - 2*d^2*x^2*sinh(2*e + 2*f*x) + c*
d*x*cosh(e + f*x)*32i - 4*c*d*x*sinh(2*e + 2*f*x)))/8 - (a^2*f*(d^2*x*sinh(
e + f*x)*32i - 2*d^2*x*cosh(2*e + 2*f*x) + c*d*sinh(e + f*x)*32i - 2*c*d*co
sh(2*e + 2*f*x)))/8)/f^3
```

3.104 $\int (c+dx)(a+ia\sinh(e+fx))^2\,dx$

Optimal. Leaf size=122

$$\frac{1}{2}a^2cx+\frac{1}{4}a^2dx^2+\frac{a^2(c+dx)^2}{2d}+\frac{2ia^2(c+dx)\cosh(e+fx)}{f}-\frac{2ia^2d\sinh(e+fx)}{f^2}-\frac{a^2(c+dx)\cosh(e+fx)\sin}{2f}$$

```
[Out] 1/2*a^2*c*x+1/4*a^2*d*x^2+1/2*a^2*(d*x+c)^2/d+2*I*a^2*(d*x+c)*cosh(f*x+e)/f
-2*I*a^2*d*sinh(f*x+e)/f^2-1/2*a^2*(d*x+c)*cosh(f*x+e)*sinh(f*x+e)/f+1/4*a^
2*d*sinh(f*x+e)^2/f^2
```

Rubi [A]
time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3398, 3377, 2717, 3391}

$$\frac{2ia^2(c+dx)\cosh(e+fx)}{f}-\frac{a^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{a^2(c+dx)^2}{2d}+\frac{1}{2}a^2cx+\frac{a^2d\sinh^2(e+fx)}{4f^2}-\frac{2ia^2d\sinh(e+fx)}{f^2}+\frac{1}{4}a^2dx^2$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] (a^2*c*x)/2 + (a^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) + ((2*I)*a^2*(c + d*x
)*Cosh[e + f*x])/f - ((2*I)*a^2*d*Sinh[e + f*x])/f^2 - (a^2*(c + d*x)*Cosh[
e + f*x]*Sinh[e + f*x])/(2*f) + (a^2*d*Sinh[e + f*x]^2)/(4*f^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (c+dx)(a+ia\sinh(e+fx))^2\,dx = \int \left(a^2(c+dx)+2ia^2(c+dx)\sinh(e+fx)-a^2(c+dx)\sinh^2(e+fx\right.$$
$$= \frac{a^2(c+dx)^2}{2d}+\left(2ia^2\right)\int (c+dx)\sinh(e+fx)\,dx-a^2\int (c+dx)\sinh^2$$
$$= \frac{a^2(c+dx)^2}{2d}+\frac{2ia^2(c+dx)\cosh(e+fx)}{f}-\frac{a^2(c+dx)\cosh(e+fx)\,\mathrm{s}}{2f}$$
$$= \frac{1}{2}a^2cx+\frac{1}{4}a^2dx^2+\frac{a^2(c+dx)^2}{2d}+\frac{2ia^2(c+dx)\cosh(e+fx)}{f}-\frac{2ia^2d}{}$$

Mathematica [A]
time = 0.56, size = 86, normalized size = 0.70

$$\frac{a^2(16if(c+dx)\cosh(e+fx)+d\cosh(2(e+fx))-2(3(e+fx)(de-2cf-dfx)+8id\sinh(e+fx)+f(c+dx)\sinh(2(e+fx))))}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] (a^2*((16*I)*f*(c + d*x)*Cosh[e + f*x] + d*Cosh[2*(e + f*x)] - 2*(3*(e + f*
x)*(d*e - 2*c*f - d*f*x) + (8*I)*d*Sinh[e + f*x] + f*(c + d*x)*Sinh[2*(e +
f*x)])))/(8*f^2)
```

Maple [A]
time = 0.62, size = 215, normalized size = 1.76

method	result
risch	$\frac{3d\,a^2x^2}{4}+\frac{3a^2cx}{2}-\frac{a^2(2dxf+2cf-d)\mathrm{e}^{2fx+2e}}{16f^2}+\frac{ia^2(dxf+cf-d)\mathrm{e}^{fx+e}}{f^2}+\frac{ia^2(dxf+cf+d)\mathrm{e}^{-fx-e}}{f^2}+\frac{a^2(2dxf+2c}{}$
derivativedivides	$\frac{\frac{d\,a^2(fx+e)^2}{2f}+\frac{2id\,a^2((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{d\,a^2\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^2}{4}-\frac{\left(\cosh^2(fx+e)\right)}{4}\right)}{f}-\frac{de\,a^2}{}}{}$
default	$\frac{\frac{d\,a^2(fx+e)^2}{2f}+\frac{2id\,a^2((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{d\,a^2\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^2}{4}-\frac{\left(\cosh^2(fx+e)\right)}{4}\right)}{f}-\frac{de\,a^2}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+I*a*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*d/f*a^2*(f*x+e)^2+2*I*d/f*a^2*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))-d/
f*a^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)
-d/f*e*a^2*(f*x+e)-2*I*d/f*e*a^2*cosh(f*x+e)+d/f*e*a^2*(1/2*cosh(f*x+e)*sin
h(f*x+e)-1/2*f*x-1/2*e)+a^2*c*(f*x+e)+2*I*c*a^2*cosh(f*x+e)-a^2*c*(1/2*cosh
(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e))
```

Maxima [A]
time = 0.28, size = 176, normalized size = 1.44

$$\frac{1}{2}\,a^2dx^2+\frac{1}{16}\left(4\,x^2-\frac{(2\,fxe^{(2\,e)}-e^{(2\,e)})e^{(2\,fx)}}{f^2}+\frac{(2\,fx+1)e^{(-2\,fx-2\,e)}}{f^2}\right)a^2d+\frac{1}{8}\,a^2c\left(4\,x-\frac{e^{(2\,fx+2\,e)}}{f}+\frac{e^{(-2\,fx-2\,e)}}{f}\right)+a^2cx+i\,a^2d\left(\frac{(fxe^e-e^e)e^{(fx)}}{f^2}+\frac{(fx+1)e^{(-fx-e)}}{f^2}\right)+\frac{2i\,a^2c\cosh(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*d*x^2 + 1/16*(4*x^2 - (2*f*x*e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 + (2*
f*x + 1)*e^(-2*f*x - 2*e)/f^2)*a^2*d + 1/8*a^2*c*(4*x - e^(2*f*x + 2*e)/f +
 e^(-2*f*x - 2*e)/f) + a^2*c*x + I*a^2*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f*
x + 1)*e^(-f*x - e)/f^2) + 2*I*a^2*c*cosh(f*x + e)/f
```

Fricas [A]
time = 0.36, size = 169, normalized size = 1.39

$$\frac{(2\,a^2dfx+2\,a^2cf+a^2d-(2\,a^2dfx+2\,a^2cf-a^2d)e^{(4\,fx+4\,e)}-16\,(-i\,a^2dfx-i\,a^2cf+i\,a^2d)e^{(3\,fx+3\,e)}+12\,(a^2df^2x^2+2\,a^2cf^2x)e^{(2\,fx+2\,e)}-16\,(-i\,a^2dfx-i\,a^2cf-i\,a^2d)e^{(fx+e)})e^{(-2\,fx-2\,e)}}{16\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(2*a^2*d*f*x + 2*a^2*c*f + a^2*d - (2*a^2*d*f*x + 2*a^2*c*f - a^2*d)*e
^(4*f*x + 4*e) - 16*(-I*a^2*d*f*x - I*a^2*c*f + I*a^2*d)*e^(3*f*x + 3*e) +
12*(a^2*d*f^2*x^2 + 2*a^2*c*f^2*x)*e^(2*f*x + 2*e) - 16*(-I*a^2*d*f*x - I*a
^2*c*f - I*a^2*d)*e^(f*x + e))*e^(-2*f*x - 2*e)/f^2
```

Sympy [A]
time = 0.38, size = 357, normalized size = 2.93

$$\frac{3a^2cx}{2}+\frac{3a^2dx^2}{4}+\begin{cases}\frac{((32a^2cf^7e^e+32a^2df^7xe^e+16a^2df^6e^e)e^{-2fx}+(-32a^2cf^7e^{5e}-32a^2df^7xe^{5e}+16a^2df^6e^{5e})e^{2fx}+(256ia^2cf^7e^{2e}+256ia^2df^7xe^{2e}+256ia^2df^6e^{2e})e^{-fx}+(256ia^2cf^7e^{4e}+256ia^2df^7xe^{4e}-256ia^2df^6e^{4e})e^{fx})e^{-3e}}{256f^8} & \text{for } f^8e^{3e}\neq 0\\ \frac{x^2(-a^2de^{4e}+4ia^2de^{3e}-4ia^2de^e-a^2d)e^{-2e}}{8}+\frac{x(-a^2ce^{4e}+4ia^2ce^{3e}-4ia^2ce^e-a^2c)e^{-2e}}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+I*a*sinh(f*x+e))**2,x)
```

```
[Out] 3*a**2*c*x/2 + 3*a**2*d*x**2/4 + Piecewise((((32*a**2*c*f**7*exp(e) + 32*a*
*2*d*f**7*x*exp(e) + 16*a**2*d*f**6*exp(e))*exp(-2*f*x) + (-32*a**2*c*f**7*
exp(5*e) - 32*a**2*d*f**7*x*exp(5*e) + 16*a**2*d*f**6*exp(5*e))*exp(2*f*x)
+ (256*I*a**2*c*f**7*exp(2*e) + 256*I*a**2*d*f**7*x*exp(2*e) + 256*I*a**2*d
*f**6*exp(2*e))*exp(-f*x) + (256*I*a**2*c*f**7*exp(4*e) + 256*I*a**2*d*f**7
*x*exp(4*e) - 256*I*a**2*d*f**6*exp(4*e))*exp(f*x))*exp(-3*e)/(256*f**8), N
e(f**8*exp(3*e), 0)), (x**2*(-a**2*d*exp(4*e) + 4*I*a**2*d*exp(3*e) - 4*I*a
**2*d*exp(e) - a**2*d)*exp(-2*e)/8 + x*(-a**2*c*exp(4*e) + 4*I*a**2*c*exp(3
*e) - 4*I*a**2*c*exp(e) - a**2*c)*exp(-2*e)/4, True))
```

Giac [A]
time = 0.45, size = 155, normalized size = 1.27

$$\frac{3}{4}a^2dx^2 + \frac{3}{2}a^2cx - \frac{(2a^2dfx + 2a^2cf - a^2d)e^{(2fx+2e)}}{16f^2} + \frac{(ia^2dfx + ia^2cf - ia^2d)e^{(fx+e)}}{f^2} + \frac{(ia^2dfx + ia^2cf + ia^2d)e^{(-fx-e)}}{f^2} + \frac{(2a^2dfx + 2a^2cf + a^2d)e^{(-2fx-2e)}}{16f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/4*a^2*d*x^2 + 3/2*a^2*c*x - 1/16*(2*a^2*d*f*x + 2*a^2*c*f - a^2*d)*e^(2*f
*x + 2*e)/f^2 + (I*a^2*d*f*x + I*a^2*c*f - I*a^2*d)*e^(f*x + e)/f^2 + (I*a^
2*d*f*x + I*a^2*c*f + I*a^2*d)*e^(-f*x - e)/f^2 + 1/16*(2*a^2*d*f*x + 2*a^2
*c*f + a^2*d)*e^(-2*f*x - 2*e)/f^2
```

Mupad [B]
time = 0.35, size = 104, normalized size = 0.85

$$\frac{a^2\,\left(6\,d\,x^2+12\,c\,x\right)}{8}-\frac{\frac{a^2\,\left(-d\,\mathrm{cosh}\left(2\,e+2\,f\,x\right)+d\,\mathrm{sinh}\left(e+f\,x\right)\,16{}\mathrm{i}\right)}{8}-\frac{a^2\,f\,\left(c\,\mathrm{cosh}\left(e+f\,x\right)\,16{}\mathrm{i}-2\,c\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)-2\,d\,x\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)+d\,x\,\mathrm{cosh}\left(e+f\,x\right)\,16{}\mathrm{i}\right)}{8}}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(e + f*x)*1i)^2*(c + d*x),x)
```

```
[Out] (a^2*(12*c*x + 6*d*x^2))/8 - ((a^2*(d*sinh(e + f*x)*16i - d*cosh(2*e + 2*f*
x)))/8 - (a^2*f*(c*cosh(e + f*x)*16i - 2*c*sinh(2*e + 2*f*x) - 2*d*x*sinh(2
*e + 2*f*x) + d*x*cosh(e + f*x)*16i))/8)/f^2
```

3.105 $\int \frac{(a+ia\sinh(e+fx))^2}{c+dx}\,dx$

Optimal. Leaf size=149

$$-\frac{a^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{3a^2\log(c+dx)}{2d}+\frac{2ia^2\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{2ia^2\cosh\left(e-\frac{cf}{d}\right)}{d}$$

```
[Out] -1/2*a^2*Chi(2*c*f/d+2*f*x)*cosh(-2*e+2*c*f/d)/d+3/2*a^2*ln(d*x+c)/d+2*I*a^
2*cosh(-e+c*f/d)*Shi(c*f/d+f*x)/d+1/2*a^2*Shi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*
f/d)/d-2*I*a^2*Chi(c*f/d+f*x)*sinh(-e+c*f/d)/d
```

Rubi [A]
time = 0.26, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3399, 3393, 3384, 3379, 3382}

$$\frac{2ia^2\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d}-\frac{a^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{2d}-\frac{a^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{2d}+\frac{2ia^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}+\frac{3a^2\log(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[e + f*x])^2/(c + d*x),x]
```

```
[Out] -1/2*(a^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/d + (3*a^2
*Log[c + d*x])/(2*d) + ((2*I)*a^2*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f
)/d])/d + ((2*I)*a^2*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d - (a^
2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2*d)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\sinh(e+fx))^2}{c+dx}\,dx &= \left(4a^2\right)\int \frac{\sin^4\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right)}{c+dx}\,dx\\ &= \left(4a^2\right)\int \left(\frac{3}{8(c+dx)}-\frac{\cosh(2e+2fx)}{8(c+dx)}+\frac{i\sinh(e+fx)}{2(c+dx)}\right)dx\\ &= \frac{3a^2\log(c+dx)}{2d}+\left(2ia^2\right)\int \frac{\sinh(e+fx)}{c+dx}\,dx-\frac{1}{2}a^2\int \frac{\cosh(2e+2fx)}{c+dx}\,dx\\ &= \frac{3a^2\log(c+dx)}{2d}-\frac{1}{2}\left(a^2\cosh\left(2e-\frac{2cf}{d}\right)\right)\int \frac{\cosh\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx+\Bigg(2ia^2 \\ &= -\frac{a^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{3a^2\log(c+dx)}{2d}+\frac{2ia^2\text{Chi}\left(\frac{cf}{d}+fx\right)\text{s}}{d}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 117, normalized size = 0.79

$$-\frac{a^2\left(\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2f(c+dx)}{d}\right)-3\log(c+dx)-4i\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)-4i\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2f(c+dx)}{d}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[e + f*x])^2/(c + d*x),x]
```

```
[Out] -1/2*(a^2*(Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*f*(c + d*x))/d] - 3*Log[c
+ d*x] - (4*I)*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] - (4*I)*Cosh[e -
 (c*f)/d]*SinhIntegral[f*(c/d + x)] + Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2
*f*(c + d*x))/d]))/d
```

Maple [A]
time = 3.45, size = 193, normalized size = 1.30

method	result
risch	$-\frac{i a^{2} \mathrm{e}^{-\frac{cf-de}{d}} \operatorname{expIntegral}\left(1,-fx-e-\frac{cf-de}{d}\right)}{d}+\frac{3 a^{2} \ln (dx+c)}{2 d}+\frac{a^{2} \mathrm{e}^{-\frac{2(cf-de)}{d}} \operatorname{expIntegral}\left(1,-2fx-2e-\frac{2(cf-de)}{d}\right)}{4 d}+\frac{a^{2} \mathrm{e}^{\frac{2c}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))^2/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -I*a^2/d*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)+3/2*a^2*ln(d*x+c)/d+1/4
*a^2/d*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)+1/4*a^2/d*exp(2*(
c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)+I*a^2/d*exp((c*f-d*e)/d)*Ei(1,f*x
+e+(c*f-d*e)/d)
```

Maxima [A]
time = 0.32, size = 154, normalized size = 1.03

$$\frac{1}{4}\,a^{2}\left(\frac{e^{\left(\frac{2\,cf}{d}-2\,e\right)}E_{1}\left(\frac{2\,(dx+c)f}{d}\right)}{d}+\frac{e^{\left(-\frac{2\,cf}{d}+2\,e\right)}E_{1}\left(-\frac{2\,(dx+c)f}{d}\right)}{d}+\frac{2\,\log\left(dx+c\right)}{d}\right)+i\,a^{2}\left(\frac{e^{\left(\frac{cf}{d}-e\right)}E_{1}\left(\frac{(dx+c)f}{d}\right)}{d}-\frac{e^{\left(-\frac{cf}{d}+e\right)}E_{1}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)+\frac{a^{2}\log\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*a^2*(e^(2*c*f/d - 2*e)*exp_integral_e(1, 2*(d*x + c)*f/d)/d + e^(-2*c*f
/d + 2*e)*exp_integral_e(1, -2*(d*x + c)*f/d)/d + 2*log(d*x + c)/d) + I*a^2
*(e^(c*f/d - e)*exp_integral_e(1, (d*x + c)*f/d)/d - e^(-c*f/d + e)*exp_int
egral_e(1, -(d*x + c)*f/d)/d) + a^2*log(d*x + c)/d
```

Fricas [A]
time = 0.37, size = 153, normalized size = 1.03

$$-\frac{a^{2}\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(\frac{2\,(cf-de)}{d}\right)}+4i\,a^{2}\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf-de}{d}\right)}-4i\,a^{2}\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf-de}{d}\right)}+a^{2}\mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)e^{\left(-\frac{2\,(cf-de)}{d}\right)}-6\,a^{2}\log\left(\frac{dx+c}{d}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/4*(a^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*(c*f - d*e)/d) + 4*I*a^2*Ei(-(d*f*x +
 c*f)/d)*e^((c*f - d*e)/d) - 4*I*a^2*Ei((d*f*x + c*f)/d)*e^(-(c*f - d*e)/d)
 + a^2*Ei(2*(d*f*x + c*f)/d)*e^(-2*(c*f - d*e)/d) - 6*a^2*log((d*x + c)/d))
/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^{2}\left(\int\frac{\sinh^{2}\left(e+fx\right)}{c+dx}\,dx+\int\left(-\frac{2i\sinh\left(e+fx\right)}{c+dx}\right)dx+\int\left(-\frac{1}{c+dx}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))**2/(d*x+c),x)
```

```
[Out] -a**2*(Integral(sinh(e + f*x)**2/(c + d*x), x) + Integral(-2*I*sinh(e + f*x
)/(c + d*x), x) + Integral(-1/(c + d*x), x))
```

Giac [A]
time = 0.42, size = 135, normalized size = 0.91

$$-\frac{a^2\mathrm{Ei}\left(\frac{2(dfx+cf)}{d}\right)e^{\left(2e-\frac{2cf}{d}\right)}-4i\,a^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}+4i\,a^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+a^2\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)e^{\left(-2e+\frac{2cf}{d}\right)}-6a^2\log(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

```
[Out] -1/4*(a^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) - 4*I*a^2*Ei((d*f*x + c*f
)/d)*e^(e - c*f/d) + 4*I*a^2*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + a^2*Ei(-
2*(d*f*x + c*f)/d)*e^(-2*e + 2*c*f/d) - 6*a^2*log(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2}{c+d\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(e + f*x)*1i)^2/(c + d*x),x)
```

```
[Out] int((a + a*sinh(e + f*x)*1i)^2/(c + d*x), x)
```

3.106 $\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^2}\,dx$

Optimal. Leaf size=170

$$-\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)}+\frac{2ia^2f\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(\frac{cf}{d}+fx\right)}{d^2}-\frac{a^2f\,\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2}+\frac{2ia^2f\,\mathrm{s}}{}$$

```
[Out] 2*I*a^2*f*Chi(c*f/d+f*x)*cosh(-e+c*f/d)/d^2-4*a^2*cosh(1/2*e+1/4*I*Pi+1/2*f
*x)^4/d/(d*x+c)-a^2*f*cosh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/d^2+a^2*f*Chi(2
*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/d^2-2*I*a^2*f*Shi(c*f/d+f*x)*sinh(-e+c*f/d
)/d^2
```

Rubi [A]
time = 0.24, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3399, 3394, 3384, 3379, 3382}

$$-\frac{a^2f\,\mathrm{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2}+\frac{2ia^2f\,\mathrm{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{2ia^2f\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{a^2f\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(2xf+\frac{2cf}{d}\right)}{d^2}-\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] (-4*a^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^4)/(d*(c + d*x)) + ((2*I)*a^2*f*Cosh
[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d^2 - (a^2*f*CoshIntegral[(2*c*f
)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/d^2 + ((2*I)*a^2*f*Sinh[e - (c*f)/d]*Si
nhIntegral[(c*f)/d + f*x])/d^2 - (a^2*f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[
(2*c*f)/d + 2*f*x])/d^2
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^2}\,dx &= \left(4a^2\right)\int \frac{\sin^4\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right)}{(c+dx)^2}\,dx \\
&= -\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)} + \frac{\left(8ia^2f\right)\int\left(\frac{\cosh(e+fx)}{4(c+dx)}+\frac{i\sinh(2e+2fx)}{8(c+dx)}\right)dx}{d} \\
&= -\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)} + \frac{\left(2ia^2f\right)\int\frac{\cosh(e+fx)}{c+dx}\,dx}{d} - \frac{\left(a^2f\right)\int\frac{\sinh(2e+2fx)}{c+dx}\,dx}{d} \\
&= -\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)} - \frac{\left(a^2f\cosh\left(2e-\frac{2cf}{d}\right)\right)\int\frac{\sinh\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx}{d} + \frac{\left(2ia^2\right.}{} \\
&= -\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)} + \frac{2ia^2f\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(\frac{cf}{d}+fx\right)}{d^2} - \frac{a^2f\mathrm{Chi}\left(\frac{2cf}{d}\right.}{}
\end{aligned}$$

Mathematica [A]
time = 0.38, size = 214, normalized size = 1.26

$$\frac{a^2\left(-3d+d\cosh(2(e+fx))+4if(c+dx)\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)-2f(c+dx)\mathrm{Chi}\left(\frac{2f(c+dx)}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)-4id\sinh(e+fx)+4icf\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+4idfx\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}+x\right)\right)-2cf\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(\frac{2f(c+dx)}{d}\right)-2dfx\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(\frac{2f(c+dx)}{d}\right)\right)}{2d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] (a^2*(-3*d + d*Cosh[2*(e + f*x)] + (4*I)*f*(c + d*x)*Cosh[e - (c*f)/d]*Cosh
Integral[f*(c/d + x)] - 2*f*(c + d*x)*CoshIntegral[(2*f*(c + d*x))/d]*Sinh[
```

```
2*e - (2*c*f)/d] - (4*I)*d*Sinh[e + f*x] + (4*I)*c*f*Sinh[e - (c*f)/d]*Sinh
Integral[f*(c/d + x)] + (4*I)*d*f*x*Sinh[e - (c*f)/d]*SinhIntegral[f*(c/d +
 x)] - 2*c*f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d] - 2*d*f*
x*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d]))/(2*d^2*(c + d*x))
```

Maple [A]
time = 3.39, size = 313, normalized size = 1.84

method	result
risch	$-\frac{i a^{2} f \,{\mathrm e}^{f x +e}}{d^{2} \left(\frac{c f}{d}+f x \right)}-\frac{i a^{2} f \,{\mathrm e}^{-\frac{c f -d e}{d}} \operatorname{expIntegral}\left(1,-f x -e -\frac{c f -d e}{d}\right)}{d^{2}}-\frac{3 a^{2}}{2 d \left(d x +c \right)}+\frac{f \,a^{2} {\mathrm e}^{-2 f x -2 e}}{4 d \left(d x f +c f \right)}-\frac{f \,a^{2} {\mathrm e}^{\frac{2 c f -2 d e}{d}} \operatorname{expIntegral}\left(1\right.}{2 d^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -I*a^2*f/d^2*exp(f*x+e)/(c*f/d+f*x)-I*a^2*f/d^2*exp(-(c*f-d*e)/d)*Ei(1,-f*x
-e-(c*f-d*e)/d)-3/2*a^2/d/(d*x+c)+1/4*f*a^2*exp(-2*f*x-2*e)/d/(d*f*x+c*f)-1
/2*f*a^2/d^2*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)+1/4/d^2*f*a^2
*exp(2*f*x+2*e)/(c*f/d+f*x)+1/2/d^2*f*a^2*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2
*e-2*(c*f-d*e)/d)+I*a^2*f*exp(-f*x-e)/d/(d*f*x+c*f)-I*a^2*f/d^2*exp((c*f-d*
e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)
```

Maxima [A]
time = 0.32, size = 187, normalized size = 1.10

$$\frac{1}{4}\,a^2\left(\frac{e^{\left(\frac{2\,cf}{d}-2\,e\right)}E_2\left(\frac{2\,(dx+c)f}{d}\right)}{(dx+c)d}+\frac{e^{\left(-\frac{2\,cf}{d}+2\,e\right)}E_2\left(-\frac{2\,(dx+c)f}{d}\right)}{(dx+c)d}-\frac{2}{d^2x+cd}\right)+i\,a^2\left(\frac{e^{\left(\frac{cf}{d}-e\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d}-\frac{e^{\left(-\frac{cf}{d}+e\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right)-\frac{a^2}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*(e^(2*c*f/d - 2*e)*exp_integral_e(2, 2*(d*x + c)*f/d)/((d*x + c)*d)
 + e^(-2*c*f/d + 2*e)*exp_integral_e(2, -2*(d*x + c)*f/d)/((d*x + c)*d) - 2
/(d^2*x + c*d)) + I*a^2*(e^(c*f/d - e)*exp_integral_e(2, (d*x + c)*f/d)/((d
*x + c)*d) - e^(-c*f/d + e)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d)
) - a^2/(d^2*x + c*d)
```

Fricas [A]
time = 0.40, size = 285, normalized size = 1.68

$$\frac{\left(a^2de^{(4\,fx+4\,e)}-4i\,a^2de^{(3\,fx+3\,e)}+4i\,a^2de^{(fx+e)}+a^2d-2\,(a^2dfx+a^2cf)\mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)e^{\left(2\,fx-\frac{2\,(cf-de)}{d}+2\,e\right)}-2\left(3\,a^2d-(a^2dfx+a^2cf)\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(\frac{2\,(cf-de)}{d}\right)}+2\,(-i\,a^2dfx-i\,a^2cf)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf-de}{d}\right)}+2\,(-i\,a^2dfx-i\,a^2cf)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf-de}{d}\right)}\right)e^{(2\,fx+2\,e)}\right)e^{(-2\,fx-2\,e)}}{4\,(d^3x+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(a^2*d*e^(4*f*x + 4*e) - 4*I*a^2*d*e^(3*f*x + 3*e) + 4*I*a^2*d*e^(f*x +
 e) + a^2*d - 2*(a^2*d*f*x + a^2*c*f)*Ei(2*(d*f*x + c*f)/d)*e^(2*f*x - 2*(c
*f - d*e)/d + 2*e) - 2*(3*a^2*d - (a^2*d*f*x + a^2*c*f)*Ei(-2*(d*f*x + c*f)
/d)*e^(2*(c*f - d*e)/d) + 2*(-I*a^2*d*f*x - I*a^2*c*f)*Ei(-(d*f*x + c*f)/d)
*e^((c*f - d*e)/d) + 2*(-I*a^2*d*f*x - I*a^2*c*f)*Ei((d*f*x + c*f)/d)*e^(-(
c*f - d*e)/d))*e^(2*f*x + 2*e))*e^(-2*f*x - 2*e)/(d^3*x + c*d^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int \frac{\sinh^2(e+fx)}{c^2+2cdx+d^2x^2}\,dx+\int\left(-\frac{2i\sinh(e+fx)}{c^2+2cdx+d^2x^2}\right)dx+\int\left(-\frac{1}{c^2+2cdx+d^2x^2}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))**2/(d*x+c)**2,x)
```

```
[Out] -a**2*(Integral(sinh(e + f*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integra
l(-2*I*sinh(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(-1/(c**2 +
 2*c*d*x + d**2*x**2), x))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(159) = 318$.
time = 0.51, size = 1134, normalized size = 6.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*(d*x + c)*a^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(2*((d*x +
c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(2*(d*e - c*f)/d)
- 2*a^2*d*e*f^2*Ei(2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e +
 c*f)/d)*e^(2*(d*e - c*f)/d) + 2*a^2*c*f^3*Ei(2*((d*x + c)*(d*e/(d*x + c) -
 c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(2*(d*e - c*f)/d) - 4*I*(d*x + c)*a^2
*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(((d*x + c)*(d*e/(d*x + c) - c*f
/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) + 4*I*a^2*d*e*f^2*Ei(((d*
x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d
) - 4*I*a^2*c*f^3*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e +
 c*f)/d)*e^((d*e - c*f)/d) - 4*I*(d*x + c)*a^2*(d*e/(d*x + c) - c*f/(d*x +
c) + f)*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)
/d)*e^(-(d*e - c*f)/d) + 4*I*a^2*d*e*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*
f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) - 4*I*a^2*c*f^3*Ei(-((d
*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f)
/d) - 2*(d*x + c)*a^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-2*((d*x +
 c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d
) + 2*a^2*d*e*f^2*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*
```

```
e + c*f)/d)*e^(-2*(d*e - c*f)/d) - 2*a^2*c*f^3*Ei(-2*((d*x + c)*(d*e/(d*x +
 c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) - a^2*d*f^2*e
^(2*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d) + 4*I*a^2*d*f^2*e^((d*
x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d) - 4*I*a^2*d*f^2*e^(-(d*x + c)
*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d) - a^2*d*f^2*e^(-2*(d*x + c)*(d*e/(d
*x + c) - c*f/(d*x + c) + f)/d) + 6*a^2*d*f^2)*d^2/(((d*x + c)*d^4*(d*e/(d*
x + c) - c*f/(d*x + c) + f) - d^5*e + c*d^4*f)*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2}{{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(e + f*x)*1i)^2/(c + d*x)^2,x)
```

```
[Out] int((a + a*sinh(e + f*x)*1i)^2/(c + d*x)^2, x)
```

3.107 $\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^3}\,dx$

Optimal. Leaf size=236

$$-\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)^2}-\frac{a^2f^2\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)}{d^3}+\frac{ia^2f^2\mathrm{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d^3}-\frac{4a^2f\,\mathrm{co}}{}$$

```
[Out] -a^2*f^2*Chi(2*c*f/d+2*f*x)*cosh(-2*e+2*c*f/d)/d^3-2*a^2*cosh(1/2*e+1/4*I*P
i+1/2*f*x)^4/d/(d*x+c)^2+I*a^2*f^2*cosh(-e+c*f/d)*Shi(c*f/d+f*x)/d^3+a^2*f^
2*Shi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/d^3-I*a^2*f^2*Chi(c*f/d+f*x)*sinh(-
e+c*f/d)/d^3-4*a^2*f*cosh(1/2*e+1/4*I*Pi+1/2*f*x)^3*sinh(1/2*e+1/4*I*Pi+1/2
*f*x)/d^2/(d*x+c)
```

Rubi [A]

time = 0.37, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3399, 3395, 3393, 3384, 3379, 3382}

$$\frac{ia^2f^2\mathrm{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d^3}-\frac{a^2f^2\mathrm{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{d^3}-\frac{a^2f^2\sinh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(2xf+\frac{2cf}{d}\right)}{d^3}+\frac{ia^2f^2\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(xf+\frac{cf}{d}\right)}{d^3}-\frac{4a^2f\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\cosh^3\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{d^2(c+dx)}-\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[e + f*x])^2/(c + d*x)^3,x]
```

```
[Out] (-2*a^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^4)/(d*(c + d*x)^2) - (a^2*f^2*Cosh[2
*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/d^3 + (I*a^2*f^2*CoshInteg
ral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d^3 - (4*a^2*f*Cosh[e/2 + (I/4)*Pi +
(f*x)/2]^3*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/(d^2*(c + d*x)) + (I*a^2*f^2*Cos
h[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d^3 - (a^2*f^2*Sinh[2*e - (2*c*
f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d^3
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^3}\,dx = \left(4a^2\right)\int \frac{\sin^4\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right)}{(c+dx)^3}\,dx$$

$$= -\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)^2} - \frac{4a^2f\cosh^3\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d^2(c+dx)} +$$

$$= -\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)^2} - \frac{4a^2f\cosh^3\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d^2(c+dx)} -$$

$$= -\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)^2} - \frac{4a^2f\cosh^3\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d^2(c+dx)} -$$

$$= -\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)^2} - \frac{4a^2f\cosh^3\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d^2(c+dx)} -$$

$$= -\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)^2} - \frac{a^2f^2\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)}{d^3} + \frac{ia^2f^2\mathrm{C}}{}$$

Mathematica [A]
time = 1.47, size = 198, normalized size = 0.84

$$\frac{a^2\left(-4f^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2f(c+dx)}{d}\right)+4if^2\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+\frac{d(-3d-4if(c+dx)\cosh(e+fx)+d\cosh(2(e+fx))-4id\sinh(e+fx)+2cf\sinh(2(e+fx))+2dfx\sinh(2(e+fx)))}{(c+dx)^2}+4if^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)-4f^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2f(c+dx)}{d}\right)\right)}{4d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[e + f*x])^2/(c + d*x)^3,x]
```

```
[Out] (a^2*(-4*f^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*f*(c + d*x))/d] + (4*I)*
f^2*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] + (d*(-3*d - (4*I)*f*(c + d
*x)*Cosh[e + f*x] + d*Cosh[2*(e + f*x)] - (4*I)*d*Sinh[e + f*x] + 2*c*f*Sin
h[2*(e + f*x)] + 2*d*f*x*Sinh[2*(e + f*x)]))/(c + d*x)^2 + (4*I)*f^2*Cosh[e
 - (c*f)/d]*SinhIntegral[f*(c/d + x)] - 4*f^2*Sinh[2*e - (2*c*f)/d]*SinhInt
egral[(2*f*(c + d*x))/d]))/(4*d^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(214) = 428$.
time = 3.44, size = 625, normalized size = 2.65

method	result
risch	$-\frac{ia^2f^2\mathrm{e}^{fx+e}}{2d^3\left(\frac{cf}{d}+fx\right)^2}-\frac{ia^2f^2\mathrm{e}^{fx+e}}{2d^3\left(\frac{cf}{d}+fx\right)}-\frac{ia^2f^2\mathrm{e}^{-\frac{cf-de}{d}}\,\text{expIntegral}\left(1,-fx-e-\frac{cf-de}{d}\right)}{2d^3}-\frac{3a^2}{4d(dx+c)^2}-\frac{f^3a^2\mathrm{e}^{-2fx-2e}x}{4d(d^2x^2f^2+2cd\,f^2x+c^2f^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*a^2*f^2/d^3*exp(f*x+e)/(c*f/d+f*x)^2-1/2*I*a^2*f^2/d^3*exp(f*x+e)/(c
*f/d+f*x)-1/2*I*a^2*f^2/d^3*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)-3/4*
a^2/d/(d*x+c)^2-1/4*f^3*a^2*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*
f^2)*x-1/4*f^3*a^2*exp(-2*f*x-2*e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c+
1/8*f^2*a^2*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)+1/2*f^2*a^2
/d^3*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)+1/8*f^2*a^2/d^3*exp(2
*f*x+2*e)/(c*f/d+f*x)^2+1/4*f^2*a^2/d^3*exp(2*f*x+2*e)/(c*f/d+f*x)+1/2*f^2*
a^2/d^3*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)-1/2*I*a^2*f^3*ex
p(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x-1/2*I*a^2*f^3*exp(-f*x-e)/d
^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c+1/2*I*a^2*f^2*exp(-f*x-e)/d/(d^2*f^2
*x^2+2*c*d*f^2*x+c^2*f^2)+1/2*I*a^2*f^2/d^3*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*
f-d*e)/d)
```

Maxima [A]
time = 0.33, size = 209, normalized size = 0.89

$$-\frac{1}{4}a^2\left(\frac{1}{d^3x^2+2\,cd^2x+c^2d}-\frac{e^{\left(\frac{2\,cf}{d}-2\,e\right)}E_3\left(\frac{2\,(dx+c)f}{d}\right)}{(dx+c)^2d}-\frac{e^{\left(-\frac{2\,cf}{d}+2\,e\right)}E_3\left(-\frac{2\,(dx+c)f}{d}\right)}{(dx+c)^2d}\right)+i\,a^2\left(\frac{e^{\left(\frac{cf}{d}-e\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}-\frac{e^{\left(-\frac{cf}{d}+e\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}\right)-\frac{a^2}{2\,(d^3x^2+2\,cd^2x+c^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a^2*(1/(d^3*x^2 + 2*c*d^2*x + c^2*d) - e^(2*c*f/d - 2*e)*exp_integral_
e(3, 2*(d*x + c)*f/d)/((d*x + c)^2*d) - e^(-2*c*f/d + 2*e)*exp_integral_e(3
, -2*(d*x + c)*f/d)/((d*x + c)^2*d)) + I*a^2*(e^(c*f/d - e)*exp_integral_e(
3, (d*x + c)*f/d)/((d*x + c)^2*d) - e^(-c*f/d + e)*exp_integral_e(3, -(d*x
+ c)*f/d)/((d*x + c)^2*d)) - 1/2*a^2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(216) = 432$.
time = 0.39, size = 472, normalized size = 2.00

$$-\frac{\left(2\,a^2d^2fx+2\,a^2cdf-a^2d^2+4\,(a^2d^2f^2x^2+2\,a^2cdf^2x+a^2c^2f^2){\rm Ei}\left(\frac{2\,(dfx+cf)}{d}\right)e^{\left(2\,fx-\frac{2\,(cf-de)}{d}+2\,e\right)}-(2\,a^2d^2fx+2\,a^2cdf+a^2d^2)e^{(4\,fx+4\,e)}+4\,(i\,a^2d^2fx+i\,a^2cdf+i\,a^2d^2)e^{(3\,fx+3\,e)}+2\left(3\,a^2d^2+2\,(a^2d^2f^2x^2+2\,a^2cdf^2x+a^2c^2f^2){\rm Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(\frac{2\,(cf-de)}{d}\right)}+2\,(i\,a^2d^2f^2x^2+2i\,a^2cdf^2x+i\,a^2c^2f^2){\rm Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf-de}{d}\right)}+2\,(-i\,a^2d^2f^2x^2-2i\,a^2cdf^2x-i\,a^2c^2f^2){\rm Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf-de}{d}\right)}\right)e^{(2\,fx+2\,e)}+4\,(i\,a^2d^2fx+i\,a^2cdf-i\,a^2d^2)e^{(fx+e)}\right)e^{(-2\,fx-2\,e)}}{8\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a^2*d^2*f*x + 2*a^2*c*d*f - a^2*d^2 + 4*(a^2*d^2*f^2*x^2 + 2*a^2*c*
d*f^2*x + a^2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d)*e^(2*f*x - 2*(c*f - d*e)/d + 2
*e) - (2*a^2*d^2*f*x + 2*a^2*c*d*f + a^2*d^2)*e^(4*f*x + 4*e) + 4*(I*a^2*d^
2*f*x + I*a^2*c*d*f + I*a^2*d^2)*e^(3*f*x + 3*e) + 2*(3*a^2*d^2 + 2*(a^2*d^
2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d)*e^(2*(c*f
 - d*e)/d) + 2*(I*a^2*d^2*f^2*x^2 + 2*I*a^2*c*d*f^2*x + I*a^2*c^2*f^2)*Ei(-
(d*f*x + c*f)/d)*e^((c*f - d*e)/d) + 2*(-I*a^2*d^2*f^2*x^2 - 2*I*a^2*c*d*f^
2*x - I*a^2*c^2*f^2)*Ei((d*f*x + c*f)/d)*e^(-(c*f - d*e)/d))*e^(2*f*x + 2*e
) + 4*(I*a^2*d^2*f*x + I*a^2*c*d*f - I*a^2*d^2)*e^(f*x + e))*e^(-2*f*x - 2*
e)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\int\frac{\sinh^2(e+fx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int\left(-\frac{2i\sinh(e+fx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\right)dx+\int\left(-\frac{1}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))**2/(d*x+c)**3,x)
```

```
[Out] -a**2*(Integral(sinh(e + f*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*
x**3), x) + Integral(-2*I*sinh(e + f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
+ d**3*x**3), x) + Integral(-1/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x*
*3), x))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(209) = 418$.
time = 0.43, size = 682, normalized size = 2.89

$$-\frac{4\,a^2d^2f^2x^2{\rm Ei}\left(\frac{2\,(dfx+cf)}{d}\right)e^{\left(2\,e-\frac{2\,cf}{d}\right)}-4i\,a^2d^2f^2x^2{\rm Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}+4i\,a^2d^2f^2x^2{\rm Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+4\,a^2d^2f^2x^2{\rm Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(-2\,e+\frac{2\,cf}{d}\right)}+8\,a^2cdf^2x{\rm Ei}\left(\frac{2\,(dfx+cf)}{d}\right)e^{\left(2\,e-\frac{2\,cf}{d}\right)}-8i\,a^2cdf^2x{\rm Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}+8i\,a^2cdf^2x{\rm Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+8\,a^2cdf^2x{\rm Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(-2\,e+\frac{2\,cf}{d}\right)}+4\,a^2c^2f^2{\rm Ei}\left(\frac{2\,(dfx+cf)}{d}\right)e^{\left(2\,e-\frac{2\,cf}{d}\right)}-4i\,a^2c^2f^2{\rm Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}+4i\,a^2c^2f^2{\rm Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+4\,a^2c^2f^2{\rm Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(-2\,e+\frac{2\,cf}{d}\right)}-2\,a^2d^2fxe^{(2\,fx+2\,e)}+4i\,a^2d^2fxe^{(fx+e)}+4i\,a^2d^2fxe^{(-fx-e)}+2\,a^2d^2fxe^{(-2\,fx-2\,e)}-2\,a^2cdfe^{(2\,fx+2\,e)}+4i\,a^2cdfe^{(fx+e)}+4i\,a^2cdfe^{(-fx-e)}+2\,a^2cdfe^{(-2\,fx-2\,e)}-a^2d^2e^{(2\,fx+2\,e)}+4i\,a^2d^2e^{(fx+e)}-4i\,a^2d^2e^{(-fx-e)}-a^2d^2e^{(-2\,fx-2\,e)}+6\,a^2d^2}{8\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*a^2*d^2*f^2*x^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) - 4*I*a^2*d
^2*f^2*x^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + 4*I*a^2*d^2*f^2*x^2*Ei(-(d*f
*x + c*f)/d)*e^(-e + c*f/d) + 4*a^2*d^2*f^2*x^2*Ei(-2*(d*f*x + c*f)/d)*e^(-
2*e + 2*c*f/d) + 8*a^2*c*d*f^2*x*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) -
8*I*a^2*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + 8*I*a^2*c*d*f^2*x*Ei(
-(d*f*x + c*f)/d)*e^(-e + c*f/d) + 8*a^2*c*d*f^2*x*Ei(-2*(d*f*x + c*f)/d)*e
^(-2*e + 2*c*f/d) + 4*a^2*c^2*f^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) -
 4*I*a^2*c^2*f^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + 4*I*a^2*c^2*f^2*Ei(-(d
*f*x + c*f)/d)*e^(-e + c*f/d) + 4*a^2*c^2*f^2*Ei(-2*(d*f*x + c*f)/d)*e^(-2*
e + 2*c*f/d) - 2*a^2*d^2*f*x*e^(2*f*x + 2*e) + 4*I*a^2*d^2*f*x*e^(f*x + e)
+ 4*I*a^2*d^2*f*x*e^(-f*x - e) + 2*a^2*d^2*f*x*e^(-2*f*x - 2*e) - 2*a^2*c*d
*f*e^(2*f*x + 2*e) + 4*I*a^2*c*d*f*e^(f*x + e) + 4*I*a^2*c*d*f*e^(-f*x - e)
 + 2*a^2*c*d*f*e^(-2*f*x - 2*e) - a^2*d^2*e^(2*f*x + 2*e) + 4*I*a^2*d^2*e^(
f*x + e) - 4*I*a^2*d^2*e^(-f*x - e) - a^2*d^2*e^(-2*f*x - 2*e) + 6*a^2*d^2)
/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + a\sinh(e + f\,x)\ 1\mathrm{i})^2}{(c + d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(e + f*x)*1i)^2/(c + d*x)^3,x)
```

```
[Out] int((a + a*sinh(e + f*x)*1i)^2/(c + d*x)^3, x)
```

3.108 $\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx$

Optimal. Leaf size=132

$$\frac{(c+dx)^3}{af}-\frac{6d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{af^2}-\frac{12d^2(c+dx)\mathrm{PolyLog}\left(2,-ie^{e+fx}\right)}{af^3}+\frac{12d^3\mathrm{PolyLog}\left(3,-ie^{e+fx}\right)}{af^4}+\frac{(c$$

```
[Out] (d*x+c)^3/a/f-6*d*(d*x+c)^2*ln(1+I*exp(f*x+e))/a/f^2-12*d^2*(d*x+c)*polylog
(2,-I*exp(f*x+e))/a/f^3+12*d^3*polylog(3,-I*exp(f*x+e))/a/f^4+(d*x+c)^3*tan
h(1/2*e+1/4*I*Pi+1/2*f*x)/a/f
```

Rubi [A]
time = 0.21, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3399, 4269, 3797, 2221, 2611, 2320, 6724}

$$-\frac{12d^2(c+dx)\mathrm{Li}_2\left(-ie^{e+fx}\right)}{af^3}-\frac{6d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{af^2}+\frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{af}+\frac{(c+dx)^3}{af}+\frac{12d^3\mathrm{Li}_3\left(-ie^{e+fx}\right)}{af^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (c + d*x)^3/(a*f) - (6*d*(c + d*x)^2*Log[1 + I*E^(e + f*x)])/(a*f^2) - (12*
d^2*(c + d*x)*PolyLog[2, (-I)*E^(e + f*x)])/(a*f^3) + (12*d^3*PolyLog[3, (-
I)*E^(e + f*x)])/(a*f^4) + ((c + d*x)^3*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(a*
f)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx &= \frac{\int (c+dx)^3\csc^2\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right)\,dx}{2a}\\
&= \frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}-\frac{(3d)\int (c+dx)^2\coth\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\,dx}{af}\\
&= \frac{(c+dx)^3}{af}+\frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}-\frac{(6id)\int \frac{e^{2\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)^2}{1+ie^{2\left(\frac{e}{2}+\frac{fx}{2}\right)}}\,dx}{af}\\
&= \frac{(c+dx)^3}{af}-\frac{6d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{af^2}+\frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{(1\ldots}{\ldots}\\
&= \frac{(c+dx)^3}{af}-\frac{6d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{af^2}-\frac{12d^2(c+dx)\mathrm{Li}_2\left(-ie^{e+fx}\right)}{af^3}+\frac{(c+\ldots}{\ldots}\\
&= \frac{(c+dx)^3}{af}-\frac{6d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{af^2}-\frac{12d^2(c+dx)\mathrm{Li}_2\left(-ie^{e+fx}\right)}{af^3}+\frac{(c+\ldots}{\ldots}\\
&= \frac{(c+dx)^3}{af}-\frac{6d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{af^2}-\frac{12d^2(c+dx)\mathrm{Li}_2\left(-ie^{e+fx}\right)}{af^3}+\frac{12d^3\ldots}{\ldots}\end{aligned}$$

Mathematica [**A**]
time = 1.96, size = 209, normalized size = 1.58

$$\frac{2\left(\frac{d\left(f^2\left(-ie^efx(3c^2+3cdx+d^2x^2)+3(1+ie^e)(c+dx)^2\log\left(1+ie^{e+fx}\right)\right)+6d(1+ie^e)f(c+dx)\mathrm{PolyLog}\left(2,-ie^{e+fx}\right)-6id^2(-i+e^e)\mathrm{PolyLog}\left(3,-ie^{e+fx}\right)\right)}{-1-ie^e}+\frac{f^3(c+dx)^3\sinh\left(\frac{fx}{2}\right)}{\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}\right)}{af^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (2*((d*(f^2*((-I)*E^e*f*x*(3*c^2 + 3*c*d*x + d^2*x^2) + 3*(1 + I*E^e)*(c +
d*x)^2*Log[1 + I*E^(e + f*x)]) + 6*d*(1 + I*E^e)*f*(c + d*x)*PolyLog[2, (-I
)*E^(e + f*x)] - (6*I)*d^2*(-I + E^e)*PolyLog[3, (-I)*E^(e + f*x)]))/(-1 -
I*E^e) + (f^3*(c + d*x)^3*Sinh[(f*x)/2])/((Cosh[e/2] + I*Sinh[e/2])*(Cosh[(
e + f*x)/2] + I*Sinh[(e + f*x)/2]))))/(a*f^4)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(119) = 238.
time = 1.24, size = 435, normalized size = 3.30

method	result

risch	$\frac{2i(d^3x^3+3c\,d^2x^2+3c^2dx+c^3)}{fa(e^{fx+e}-i)}+\frac{6d^2c\,e^2}{a\,f^3}-\frac{4d^3e^3}{a\,f^4}-\frac{6d^3\ln(1+ie^{fx+e})x^2}{a\,f^2}+\frac{6d^3\ln(1+ie^{fx+e})e^2}{a\,f^4}+\frac{6d^3e^2\ln(e^{fx+e})}{a\,f^4}+\frac{12d^2e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+I*a*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/f/a/(exp(f*x+e)-I)+6/a/f^3*d^2*c*e^
2-4/a/f^4*d^3*e^3-6/a/f^2*d^3*ln(1+I*exp(f*x+e))*x^2+6/a/f^4*d^3*ln(1+I*exp
(f*x+e))*e^2+6/a/f^4*d^3*e^2*ln(exp(f*x+e))+12/a/f^3*d^2*e*c*ln(exp(f*x+e)-
I)+12/a/f^2*d^2*c*e*x+6/a/f*d^2*c*x^2-12/a/f^2*d^2*c*ln(1+I*exp(f*x+e))*x-1
2/a/f^3*d^2*c*ln(1+I*exp(f*x+e))*e-12/a/f^3*d^2*c*polylog(2,-I*exp(f*x+e))-
12/a/f^3*d^2*e*c*ln(exp(f*x+e))+12*d^3*polylog(3,-I*exp(f*x+e))/a/f^4+6/a/f
^2*d*ln(exp(f*x+e))*c^2-6/a/f^4*d^3*e^2*ln(exp(f*x+e)-I)-6/a/f^3*d^3*e^2*x+
2/a/f*d^3*x^3-6/a/f^2*d*ln(exp(f*x+e)-I)*c^2-12/a/f^3*d^3*polylog(2,-I*exp(
f*x+e))*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(118) = 236.
time = 0.37, size = 249, normalized size = 1.89

$$6\,c^2d\left(\frac{xe^{(fx+e)}}{afe^{(fx+e)}-i\,af}-\frac{\log\left(\left(e^{(fx+e)}-i\right)e^{(-e)}\right)}{af^2}\right)-\frac{2\,c^3}{\left(i\,ae^{(-fx-e)}-a\right)f}-\frac{2\left(-i\,d^3x^3-3i\,cd^2x^2\right)}{afe^{(fx+e)}-i\,af}-\frac{12\left(fx\log\left(i\,e^{(fx+e)}+1\right)+\mathrm{Li}_2(-i\,e^{(fx+e)})\right)cd^2}{af^3}-\frac{6\left(f^2x^2\log\left(i\,e^{(fx+e)}+1\right)+2\,fx\mathrm{Li}_2(-i\,e^{(fx+e)})-2\,\mathrm{Li}_3(-i\,e^{(fx+e)})\right)d^3}{af^4}+\frac{2\left(d^3f^3x^3+3\,cd^2f^3x^2\right)}{af^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 6*c^2*d*(x*e^(f*x + e)/(a*f*e^(f*x + e) - I*a*f) - log((e^(f*x + e) - I)*e^
(-e))/(a*f^2)) - 2*c^3/((I*a*e^(-f*x - e) - a)*f) - 2*(-I*d^3*x^3 - 3*I*c*d
^2*x^2)/(a*f*e^(f*x + e) - I*a*f) - 12*(f*x*log(I*e^(f*x + e) + 1) + dilog(
-I*e^(f*x + e)))*c*d^2/(a*f^3) - 6*(f^2*x^2*log(I*e^(f*x + e) + 1) + 2*f*x*
dilog(-I*e^(f*x + e)) - 2*polylog(3, -I*e^(f*x + e)))*d^3/(a*f^4) + 2*(d^3*
f^3*x^3 + 3*c*d^2*f^3*x^2)/(a*f^4)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(118) = 236.
time = 0.33, size = 372, normalized size = 2.82

$$-\frac{2\left(-i\,c^3f^3+3i\,c^2df^2e-3i\,cd^2fe^2+i\,d^3e^3+6\left(-i\,d^3fx-i\,cd^2f+\left(d^3fx+cd^2f\right)e^{(fx+e)}\right)\mathrm{Li}_2(-i\,e^{(fx+e)})-\left(d^3f^3x^3+3\,cd^2f^3x^2+3\,c^2df^3x+3\,c^2df^2e-3\,cd^2fe^2+d^3e^3\right)e^{(fx+e)}+3\left(-i\,c^2df^2+2i\,cd^2fe-i\,d^3e^2+\left(c^2df^2-2\,cd^2fe+d^3e^2\right)e^{(fx+e)}\right)\log\left(e^{(fx+e)}-i\right)+3\left(-i\,d^3f^2x^2-2i\,cd^2f^2x-2i\,cd^2fe+i\,d^3e^2+\left(d^3f^2x^2+2\,cd^2f^2x+2\,cd^2fe-d^3e^2\right)e^{(fx+e)}\right)\log\left(i\,e^{(fx+e)}+1\right)-6\left(d^3e^{(fx+e)}-i\,d^3\right)\mathrm{polylog}\left(3,-i\,e^{(fx+e)}\right)\right)}{af^4e^{(fx+e)}-i\,af^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2*(-I*c^3*f^3 + 3*I*c^2*d*f^2*e - 3*I*c*d^2*f*e^2 + I*d^3*e^3 + 6*(-I*d^3*
f*x - I*c*d^2*f + (d^3*f*x + c*d^2*f)*e^(f*x + e))*dilog(-I*e^(f*x + e)) -
(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + 3*c^2*d*f^2*e - 3*c*d^2*f*
```

```
e^2 + d^3*e^3)*e^(f*x + e) + 3*(-I*c^2*d*f^2 + 2*I*c*d^2*f*e - I*d^3*e^2 +
(c^2*d*f^2 - 2*c*d^2*f*e + d^3*e^2)*e^(f*x + e))*log(e^(f*x + e) - I) + 3*(
-I*d^3*f^2*x^2 - 2*I*c*d^2*f^2*x - 2*I*c*d^2*f*e + I*d^3*e^2 + (d^3*f^2*x^2
 + 2*c*d^2*f^2*x + 2*c*d^2*f*e - d^3*e^2)*e^(f*x + e))*log(I*e^(f*x + e) +
1) - 6*(d^3*e^(f*x + e) - I*d^3)*polylog(3, -I*e^(f*x + e)))/(a*f^4*e^(f*x
+ e) - I*a*f^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{2ic^3+6ic^2dx+6icd^2x^2+2id^3x^3}{afe^ee^{fx}-iaf}-\frac{6id\left(\int\frac{c^2}{e^ee^{fx}-i}\,dx+\int\frac{d^2x^2}{e^ee^{fx}-i}\,dx+\int\frac{2cdx}{e^ee^{fx}-i}\,dx\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+I*a*sinh(f*x+e)),x)
```

```
[Out] (2*I*c**3 + 6*I*c**2*d*x + 6*I*c*d**2*x**2 + 2*I*d**3*x**3)/(a*f*exp(e)*exp
(f*x) - I*a*f) - 6*I*d*(Integral(c**2/(exp(e)*exp(f*x) - I), x) + Integral(
d**2*x**2/(exp(e)*exp(f*x) - I), x) + Integral(2*c*d*x/(exp(e)*exp(f*x) - I
), x))/(a*f)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(I*a*sinh(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^3}{a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + a*sinh(e + f*x)*1i),x)
```

```
[Out] int((c + d*x)^3/(a + a*sinh(e + f*x)*1i), x)
```

3.109 $\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}\,dx$

Optimal. Leaf size=101

$$\frac{(c+dx)^2}{af}-\frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{af^2}-\frac{4d^2\mathrm{PolyLog}\left(2,-ie^{e+fx}\right)}{af^3}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}$$

```
[Out] (d*x+c)^2/a/f-4*d*(d*x+c)*ln(1+I*exp(f*x+e))/a/f^2-4*d^2*polylog(2,-I*exp(f
*x+e))/a/f^3+(d*x+c)^2*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a/f
```

Rubi [A]
time = 0.15, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3399, 4269, 3797, 2221, 2317, 2438}

$$-\frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{af^2}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{af}+\frac{(c+dx)^2}{af}-\frac{4d^2\mathrm{Li}_2\left(-ie^{e+fx}\right)}{af^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (c + d*x)^2/(a*f) - (4*d*(c + d*x)*Log[1 + I*E^(e + f*x)])/(a*f^2) - (4*d^2
*PolyLog[2, (-I)*E^(e + f*x)])/(a*f^3) + ((c + d*x)^2*Tanh[e/2 + (I/4)*Pi +
 (f*x)/2])/(a*f)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}\,dx &= \frac{\int (c+dx)^2\csc^2\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right)\,dx}{2a}\\
&= \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}-\frac{(2d)\int (c+dx)\coth\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\,dx}{af}\\
&= \frac{(c+dx)^2}{af}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}-\frac{(4id)\int \frac{e^{2\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)}{1+ie^{2\left(\frac{e}{2}+\frac{fx}{2}\right)}}\,dx}{af}\\
&= \frac{(c+dx)^2}{af}-\frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{af^2}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{(4d\cdots}{\cdots}\\
&= \frac{(c+dx)^2}{af}-\frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{af^2}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{(4d\cdots}{\cdots}\\
&= \frac{(c+dx)^2}{af}-\frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{af^2}-\frac{4d^2\text{Li}_2\left(-ie^{e+fx}\right)}{af^3}+\frac{(c+dx)^2\tanh\left(\cdots\right)}{af}
\end{aligned}$$

Mathematica [A]
time = 1.58, size = 139, normalized size = 1.38

$$\frac{2\left(\frac{de^ef^2x(2c+dx)}{-i+e^e}-2df(c+dx)\log\left(1+ie^{e+fx}\right)-2d^2\text{PolyLog}\left(2,-ie^{e+fx}\right)+\frac{f^2(c+dx)^2\sinh\left(\frac{fx}{2}\right)}{\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}\right)}{af^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (2*((d*E^e*f^2*x*(2*c + d*x))/(-I + E^e) - 2*d*f*(c + d*x)*Log[1 + I*E^(e +
 f*x)] - 2*d^2*PolyLog[2, (-I)*E^(e + f*x)] + (f^2*(c + d*x)^2*Sinh[(f*x)/2
])/((Cosh[e/2] + I*Sinh[e/2])*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))))/
(a*f^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(90) = 180$.
time = 0.98, size = 227, normalized size = 2.25

method	result
risch	$\frac{2i(d^2x^2+2cdx+c^2)}{fa(e^{fx+e}-i)} - \frac{4d\ln(e^{fx+e}-i)c}{a\,f^2} + \frac{4d\ln(e^{fx+e})c}{a\,f^2} + \frac{2d^2x^2}{af} + \frac{4d^2ex}{a\,f^2} + \frac{2d^2e^2}{a\,f^3} - \frac{4d^2\ln(1+ie^{fx+e})x}{a\,f^2} - \frac{4d^2\ln(1+ie^{fx}}{a\,f^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+I*a*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I*(d^2*x^2+2*c*d*x+c^2)/f/a/(exp(f*x+e)-I)-4/a/f^2*d*ln(exp(f*x+e)-I)*c+4
/a/f^2*d*ln(exp(f*x+e))*c+2/a/f*d^2*x^2+4/a/f^2*d^2*e*x+2/a/f^3*d^2*e^2-4/a
/f^2*d^2*ln(1+I*exp(f*x+e))*x-4/a/f^3*d^2*ln(1+I*exp(f*x+e))*e-4*d^2*polylo
g(2,-I*exp(f*x+e))/a/f^3+4/a/f^3*d^2*e*ln(exp(f*x+e)-I)-4/a/f^3*d^2*e*ln(ex
p(f*x+e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*d^2*(-I*x^2/(a*f*e^(f*x + e) - I*a*f) + 2*I*integrate(x/(a*f*e^(f*x + e)
 - I*a*f), x)) + 4*c*d*(x*e^(f*x + e)/(a*f*e^(f*x + e) - I*a*f) - log((e^(f
*x + e) - I)*e^(-e))/(a*f^2)) - 2*c^2/((I*a*e^(-f*x - e) - a)*f)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(89) = 178$.
time = 0.38, size = 212, normalized size = 2.10

$$-\frac{2\left(-i\,c^2f^2+2i\,cdfe-i\,d^2e^2+2\left(d^2e^{(fx+e)}-i\,d^2\right)\mathrm{Li}_2\left(-i\,e^{(fx+e)}\right)-(d^2f^2x^2+2\,cdf^2x+2\,cdfe-d^2e^2)e^{(fx+e)}+2\left(-i\,cdf+i\,d^2e+(cdf-d^2e)e^{(fx+e)}\right)\log\left(e^{(fx+e)}-i\right)+2\left(-i\,d^2fx-i\,d^2e+(d^2fx+d^2e)e^{(fx+e)}\right)\log\left(i\,e^{(fx+e)}+1\right)\right)}{af^3e^{(fx+e)}-i\,af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2*(-I*c^2*f^2 + 2*I*c*d*f*e - I*d^2*e^2 + 2*(d^2*e^(f*x + e) - I*d^2)*dilo
g(-I*e^(f*x + e)) - (d^2*f^2*x^2 + 2*c*d*f^2*x + 2*c*d*f*e - d^2*e^2)*e^(f*
x + e) + 2*(-I*c*d*f + I*d^2*e + (c*d*f - d^2*e)*e^(f*x + e))*log(e^(f*x +
e) - I) + 2*(-I*d^2*f*x - I*d^2*e + (d^2*f*x + d^2*e)*e^(f*x + e))*log(I*e^
(f*x + e) + 1))/(a*f^3*e^(f*x + e) - I*a*f^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{2ic^2 + 4icdx + 2id^2x^2}{afe^e e^{fx} - iaf} - \frac{4id\left(\int \frac{c}{e^e e^{fx} - i}\, dx + \int \frac{dx}{e^e e^{fx} - i}\, dx\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+I*a*sinh(f*x+e)),x)
```

```
[Out] (2*I*c**2 + 4*I*c*d*x + 2*I*d**2*x**2)/(a*f*exp(e)*exp(f*x) - I*a*f) - 4*I*
d*(Integral(c/(exp(e)*exp(f*x) - I), x) + Integral(d*x/(exp(e)*exp(f*x) - I
), x))/(a*f)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(I*a*sinh(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^2}{a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + a*sinh(e + f*x)*1i),x)
```

```
[Out] int((c + d*x)^2/(a + a*sinh(e + f*x)*1i), x)
```

3.110 $\int \frac{c+dx}{a+ia\sinh(e+fx)}\,dx$

Optimal. Leaf size=63

$$-\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{af^2}+\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}$$

```
[Out] -2*d*ln(cosh(1/2*e+1/4*I*Pi+1/2*f*x))/a/f^2+(d*x+c)*tanh(1/2*e+1/4*I*Pi+1/2
*f*x)/a/f
```

Rubi [A]
time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3399, 4269, 3556}

$$\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{af}-\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{af^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (-2*d*Log[Cosh[e/2 + (I/4)*Pi + (f*x)/2]])/(a*f^2) + ((c + d*x)*Tanh[e/2 +
(I/4)*Pi + (f*x)/2])/(a*f)
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx}{a+ia\sinh(e+fx)}\,dx = \frac{\int (c+dx)\csc^2\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right)\,dx}{2a}$$
$$= \frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af} - \frac{d\int \coth\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\,dx}{af}$$
$$= -\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{af^2} + \frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 185 vs. $2(63) = 126$.
time = 0.31, size = 185, normalized size = 2.94

$$\frac{idfx\cosh\left(e+\frac{fx}{2}\right)+\cosh\left(\frac{fx}{2}\right)\left(-2id\mathrm{ArcTan}\left(\mathrm{sech}\left(e+\frac{fx}{2}\right)\sinh\left(\frac{fx}{2}\right)\right)-d\log(\cosh(e+fx))\right)+2cf\sinh\left(\frac{fx}{2}\right)+dfx\sinh\left(\frac{fx}{2}\right)+2d\mathrm{ArcTan}\left(\mathrm{sech}\left(e+\frac{fx}{2}\right)\sinh\left(\frac{fx}{2}\right)\right)\sinh\left(e+\frac{fx}{2}\right)-id\log(\cosh(e+fx))\sinh\left(e+\frac{fx}{2}\right)}{af^2\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (I*d*f*x*Cosh[e + (f*x)/2] + Cosh[(f*x)/2]*((-2*I)*d*ArcTan[Sech[e + (f*x)/
2]*Sinh[(f*x)/2]] - d*Log[Cosh[e + f*x]]) + 2*c*f*Sinh[(f*x)/2] + d*f*x*Sin
h[(f*x)/2] + 2*d*ArcTan[Sech[e + (f*x)/2]*Sinh[(f*x)/2]]*Sinh[e + (f*x)/2]
- I*d*Log[Cosh[e + f*x]]*Sinh[e + (f*x)/2])/(a*f^2*(Cosh[e/2] + I*Sinh[e/2]
)*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))
```

Maple [**A**]
time = 0.80, size = 66, normalized size = 1.05

method	result	size
risch	$\frac{2dx}{af}+\frac{2de}{a\,f^2}+\frac{2i(dx+c)}{fa\left(\mathrm{e}^{fx+e}-i\right)}-\frac{2d\ln\left(\mathrm{e}^{fx+e}-i\right)}{a\,f^2}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+I*a*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d/a/f*x+2*d/a/f^2*e+2*I*(d*x+c)/f/a/(exp(f*x+e)-I)-2*d/a/f^2*ln(exp(f*x+e
)-I)
```

Maxima [**A**]
time = 0.27, size = 80, normalized size = 1.27

$$2\,d\left(\frac{xe^{(fx+e)}}{afe^{(fx+e)}-i\,af}-\frac{\log\left(\left(e^{(fx+e)}-i\right)e^{(-e)}\right)}{af^2}\right)-\frac{2\,c}{\left(i\,ae^{(-fx-e)}-a\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*d*(x*e^(f*x + e)/(a*f*e^(f*x + e) - I*a*f) - log((e^(f*x + e) - I)*e^(-e)
)/(a*f^2)) - 2*c/((I*a*e^(-f*x - e) - a)*f)
```

Fricas [A]
time = 0.38, size = 64, normalized size = 1.02

$$\frac{2\left(dfxe^{(fx+e)}+i\,cf-\left(de^{(fx+e)}-i\,d\right)\log\left(e^{(fx+e)}-i\right)\right)}{af^2e^{(fx+e)}-i\,af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 2*(d*f*x*e^(f*x + e) + I*c*f - (d*e^(f*x + e) - I*d)*log(e^(f*x + e) - I))/
(a*f^2*e^(f*x + e) - I*a*f^2)
```

Sympy [A]
time = 0.13, size = 56, normalized size = 0.89

$$\frac{2ic+2idx}{afe^ee^{fx}-iaf}+\frac{2dx}{af}-\frac{2d\log\left(e^{fx}-ie^{-e}\right)}{af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

```
[Out] (2*I*c + 2*I*d*x)/(a*f*exp(e)*exp(f*x) - I*a*f) + 2*d*x/(a*f) - 2*d*log(exp
(f*x) - I*exp(-e))/(a*f**2)
```

Giac [A]
time = 0.41, size = 67, normalized size = 1.06

$$\frac{2\left(dfxe^{(fx+e)}-de^{(fx+e)}\log\left(e^{(fx+e)}-i\right)+i\,cf+i\,d\log\left(e^{(fx+e)}-i\right)\right)}{af^2e^{(fx+e)}-i\,af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] 2*(d*f*x*e^(f*x + e) - d*e^(f*x + e)*log(e^(f*x + e) - I) + I*c*f + I*d*log
(e^(f*x + e) - I))/(a*f^2*e^(f*x + e) - I*a*f^2)
```

Mupad [B]
time = 0.34, size = 56, normalized size = 0.89

$$\frac{(c+d\,x)\,2\mathrm{i}}{a\,f\,\left(\mathrm{e}^{e+f\,x}-\mathrm{i}\right)}+\frac{2\,d\,x}{a\,f}-\frac{2\,d\,\ln\left(\mathrm{e}^{f\,x}\,\mathrm{e}^{e}-\mathrm{i}\right)}{a\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + a*sinh(e + f*x)*1i),x)
```

```
[Out] ((c + d*x)*2i)/(a*f*(exp(e + f*x) - 1i)) + (2*d*x)/(a*f) - (2*d*log(exp(f*x
)*exp(e) - 1i))/(a*f^2)
```

3.111 $\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{1}{(c+dx)(a+ia\sinh(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + I*a*Sinh[e + f*x])),x]
[Out] Defer[Int][1/((c + d*x)*(a + I*a*Sinh[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx$$

Mathematica [A]
time = 17.57, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + I*a*Sinh[e + f*x])),x]
[Out] Integrate[1/((c + d*x)*(a + I*a*Sinh[e + f*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\,(a+ia\sinh(fx+e))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*I*d*integrate(1/(-I*a*d^2*f*x^2 - 2*I*a*c*d*f*x - I*a*c^2*f + (a*d^2*f*x^
2*e^e + 2*a*c*d*f*x*e^e + a*c^2*f*e^e)*e^(f*x)), x) + 2*I/(-I*a*d*f*x - I*a
*c*f + (a*d*f*x*e^e + a*c*f*e^e)*e^(f*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] ((-I*a*d*f*x - I*a*c*f + (a*d*f*x + a*c*f)*e^(f*x + e))*integral(2*I*d/(-I*
a*d^2*f*x^2 - 2*I*a*c*d*f*x - I*a*c^2*f + (a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^
2*f)*e^(f*x + e)), x) + 2*I)/(-I*a*d*f*x - I*a*c*f + (a*d*f*x + a*c*f)*e^(f
*x + e))
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{2i}{-iacf - iadfx + (acfe^e + adfxe^e)\,e^{fx}} + \frac{2id\int \frac{1}{c^2e^ee^{fx} - ic^2 + 2cdxe^ee^{fx} - 2icdx + d^2x^2e^ee^{fx} - id^2x^2}\,dx}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

```
[Out] 2*I/(-I*a*c*f - I*a*d*f*x + (a*c*f*exp(e) + a*d*f*x*exp(e))*exp(f*x)) + 2*I
*d*Integral(1/(c**2*exp(e)*exp(f*x) - I*c**2 + 2*c*d*x*exp(e)*exp(f*x) - 2*
I*c*d*x + d**2*x**2*exp(e)*exp(f*x) - I*d**2*x**2), x)/(a*f)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
[Out] integrate(1/((d*x + c)*(I*a*sinh(f*x + e) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a + a\sinh(e + f\,x)\ 1\mathrm{i})\ (c + d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sinh(e + f*x)*1i)*(c + d*x)),x)
[Out] int(1/((a + a*sinh(e + f*x)*1i)*(c + d*x)), x)
```

3.112 $\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+ia\sinh(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])),x]
[Out] Defer[Int][1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx$$

Mathematica [A]
time = 18.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])),x]
[Out] Integrate[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\,(a+ia\sinh(fx+e))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 4*I*d*integrate(1/(-I*a*d^3*f*x^3 - 3*I*a*c*d^2*f*x^2 - 3*I*a*c^2*d*f*x - I
*a*c^3*f + (a*d^3*f*x^3*e^e + 3*a*c*d^2*f*x^2*e^e + 3*a*c^2*d*f*x*e^e + a*c
^3*f*e^e)*e^(f*x)), x) + 2*I/(-I*a*d^2*f*x^2 - 2*I*a*c*d*f*x - I*a*c^2*f +
(a*d^2*f*x^2*e^e + 2*a*c*d*f*x*e^e + a*c^2*f*e^e)*e^(f*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] ((-I*a*d^2*f*x^2 - 2*I*a*c*d*f*x - I*a*c^2*f + (a*d^2*f*x^2 + 2*a*c*d*f*x +
 a*c^2*f)*e^(f*x + e))*integral(4*I*d/(-I*a*d^3*f*x^3 - 3*I*a*c*d^2*f*x^2 -
 3*I*a*c^2*d*f*x - I*a*c^3*f + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f
*x + a*c^3*f)*e^(f*x + e)), x) + 2*I)/(-I*a*d^2*f*x^2 - 2*I*a*c*d*f*x - I*a
*c^2*f + (a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*e^(f*x + e))
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{2i}{-iac^2f - 2iacdfx - iad^2fx^2 + (ac^2fe^e + 2acdfxe^e + ad^2fx^2e^e)\,e^{fx}} + \frac{4id\int \frac{1}{c^3e^ee^{fx} - ic^3 + 3c^2dxe^ee^{fx} - 3ic^2dx + 3cd^2x^2e^ee^{fx} - 3icd^2x^2 + d^3x^3e^ee^{fx} - id^3x^3}\,dx}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+I*a*sinh(f*x+e)),x)
```

```
[Out] 2*I/(-I*a*c**2*f - 2*I*a*c*d*f*x - I*a*d**2*f*x**2 + (a*c**2*f*exp(e) + 2*a
*c*d*f*x*exp(e) + a*d**2*f*x**2*exp(e))*exp(f*x)) + 4*I*d*Integral(1/(c**3*
exp(e)*exp(f*x) - I*c**3 + 3*c**2*d*x*exp(e)*exp(f*x) - 3*I*c**2*d*x + 3*c*
d**2*x**2*exp(e)*exp(f*x) - 3*I*c*d**2*x**2 + d**3*x**3*exp(e)*exp(f*x) - I
*d**3*x**3), x)/(a*f)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(I*a*sinh(f*x + e) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1\mathrm{i}\right)\,{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sinh(e + f*x)*1i)*(c + d*x)^2),x)
```

```
[Out] int(1/((a + a*sinh(e + f*x)*1i)*(c + d*x)^2), x)
```

3.113 $\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx$

Optimal. Leaf size=305

$$\frac{(c+dx)^3}{3a^2f}-\frac{2d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{a^2f^2}+\frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{a^2f^4}-\frac{4d^2(c+dx)\text{PolyLog}\left(2,-ie^{e+fx}\right)}{a^2f^3}+$$

```
[Out] 1/3*(d*x+c)^3/a^2/f-2*d*(d*x+c)^2*ln(1+I*exp(f*x+e))/a^2/f^2+4*d^3*ln(cosh(
1/2*e+1/4*I*Pi+1/2*f*x))/a^2/f^4-4*d^2*(d*x+c)*polylog(2,-I*exp(f*x+e))/a^2
/f^3+4*d^3*polylog(3,-I*exp(f*x+e))/a^2/f^4+1/2*d*(d*x+c)^2*sech(1/2*e+1/4*
I*Pi+1/2*f*x)^2/a^2/f^2-2*d^2*(d*x+c)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a^2/f^3+
1/3*(d*x+c)^3*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a^2/f+1/6*(d*x+c)^3*sech(1/2*e+1
/4*I*Pi+1/2*f*x)^2*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a^2/f
```

Rubi [A]
time = 0.29, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3399, 4271, 4269, 3556, 3797, 2221, 2611, 2320, 6724}

$$-\frac{4d^2(c+dx)\text{Li}_2\left(-ie^{e+fx}\right)}{a^2f^3}-\frac{2d^2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{a^2f^3}-\frac{2d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{a^2f^2}+\frac{d(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{2a^2f^2}+\frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3a^2f}+\frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{6a^2f}+\frac{(c+dx)^3}{3a^2f}+\frac{4d^3\text{Li}_3\left(-ie^{e+fx}\right)}{a^2f^4}+\frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{a^2f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] (c + d*x)^3/(3*a^2*f) - (2*d*(c + d*x)^2*Log[1 + I*E^(e + f*x)])/(a^2*f^2)
+ (4*d^3*Log[Cosh[e/2 + (I/4)*Pi + (f*x)/2]])/(a^2*f^4) - (4*d^2*(c + d*x)*
PolyLog[2, (-I)*E^(e + f*x)])/(a^2*f^3) + (4*d^3*PolyLog[3, (-I)*E^(e + f*x
)])/(a^2*f^4) + (d*(c + d*x)^2*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2)/(2*a^2*f^2
) - (2*d^2*(c + d*x)*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(a^2*f^3) + ((c + d*x)
^3*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*a^2*f) + ((c + d*x)^3*Sech[e/2 + (I/4
)*Pi + (f*x)/2]^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(6*a^2*f)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx = \frac{\int (c+dx)^3\csc^4\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right)\,dx}{4a^2}$$

$$= \frac{d(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2a^2f^2} + \frac{(c+dx)^3\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}\right.}{6a^2f}$$

$$= \frac{d(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2a^2f^2} - \frac{2d^2(c+dx)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{a^2f^3} + \frac{(c+dx)}{}$$

$$= \frac{(c+dx)^3}{3a^2f} + \frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{a^2f^4} + \frac{d(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2a^2f^2} -$$

$$= \frac{(c+dx)^3}{3a^2f} - \frac{2d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{a^2f^2} + \frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{a^2f^4} +$$

$$= \frac{(c+dx)^3}{3a^2f} - \frac{2d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{a^2f^2} + \frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{a^2f^4} -$$

$$= \frac{(c+dx)^3}{3a^2f} - \frac{2d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{a^2f^2} + \frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{a^2f^4} -$$

$$= \frac{(c+dx)^3}{3a^2f} - \frac{2d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{a^2f^2} + \frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{a^2f^4} -$$

Mathematica [A]

time = 4.13, size = 508, normalized size = 1.67

$$\frac{\frac{2d\left(-6d^2e^efx+3c^2e^ef^3x+3cde^ef^3x^2+d^2e^ef^3x^3-6id^2\log\left(i-e^{e+fx}\right)+6d^2e^e\log\left(i-e^{e+fx}\right)+3ic^2f^2\log\left(i-e^{e+fx}\right)-3c^2e^ef^2\log\left(i-e^{e+fx}\right)+6icdf^2x\log\left(1+ie^{e+fx}\right)-6cde^ef^2x\log\left(1+ie^{e+fx}\right)+3id^2f^2x^2\log\left(1+ie^{e+fx}\right)-3d^2e^ef^2x^2\log\left(1+ie^{e+fx}\right)-6d(-i+e^e)f(c+dx)\text{PolyLog}\left(2,-ie^{e+fx}\right)+6d^2(-i+e^e)\text{PolyLog}\left(3,-ie^{e+fx}\right)\right)}{-i+e^e}+\frac{f(c+dx)\left(3df(c+dx)\cosh\left(\frac{fx}{2}\right)+6id^2\cosh\left(e+\frac{fx}{2}\right)+i\left(c^2f^2+2cdf^2x+d^2\left(-6+f^2x^2\right)\right)\cosh\left(e+\frac{3fx}{2}\right)+3\left(c^2f^2+2cdf^2x+d^2\left(-4+f^2x^2\right)\right)\sinh\left(\frac{fx}{2}\right)+3idf(c+dx)\sinh\left(e+\frac{fx}{2}\right)\right)}{\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)^3}}{3a^2f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] ((2*d*(-6*d^2*E^e*f*x + 3*c^2*E^e*f^3*x + 3*c*d*E^e*f^3*x^2 + d^2*E^e*f^3*x
^3 - (6*I)*d^2*Log[I - E^(e + f*x)] + 6*d^2*E^e*Log[I - E^(e + f*x)] + (3*I
)*c^2*f^2*Log[I - E^(e + f*x)] - 3*c^2*E^e*f^2*Log[I - E^(e + f*x)] + (6*I)
```

```
*c*d*f^2*x*Log[1 + I*E^(e + f*x)] - 6*c*d*E^e*f^2*x*Log[1 + I*E^(e + f*x)]
+ (3*I)*d^2*f^2*x^2*Log[1 + I*E^(e + f*x)] - 3*d^2*E^e*f^2*x^2*Log[1 + I*E^
(e + f*x)] - 6*d*(-I + E^e)*f*(c + d*x)*PolyLog[2, (-I)*E^(e + f*x)] + 6*d^
2*(-I + E^e)*PolyLog[3, (-I)*E^(e + f*x)]))/(-I + E^e) + (f*(c + d*x)*(3*d*
f*(c + d*x)*Cosh[(f*x)/2] + (6*I)*d^2*Cosh[e + (f*x)/2] + I*(c^2*f^2 + 2*c*
d*f^2*x + d^2*(-6 + f^2*x^2))*Cosh[e + (3*f*x)/2] + 3*(c^2*f^2 + 2*c*d*f^2*
x + d^2*(-4 + f^2*x^2))*Sinh[(f*x)/2] + (3*I)*d*f*(c + d*x)*Sinh[e + (f*x)/
2]))/((Cosh[e/2] + I*Sinh[e/2])*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^3
))/(3*a^2*f^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(249) = 498$.
time = 3.25, size = 723, normalized size = 2.37

method	result
risch	$\frac{2f^2d^3x^3e^{fx+e}-2f\,d^3x^2e^{fx+e}-2f\,c^2d\,e^{fx+e}-4id^3x\,e^{2fx+2e}+4id^3x+4ic\,d^2+6f^2c\,d^2x^2e^{fx+e}+6f^2c^2dx\,e^{fx+e}-4fc\,d^2x\,e^{fx+e}-4ifc}{(e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+I*a*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(3*f^2*d^3*x^3*exp(f*x+e)-3*f*d^3*x^2*exp(f*x+e)-3*f*c^2*d*exp(f*x+e)-6
*I*d^3*x*exp(2*f*x+2*e)+6*I*d^3*x+6*I*c*d^2+9*f^2*c*d^2*x^2*exp(f*x+e)+9*f^
2*c^2*d*x*exp(f*x+e)-6*f*c*d^2*x*exp(f*x+e)-6*I*f*c*d^2*x*exp(2*f*x+2*e)-I*
f^2*c^3-3*I*f^2*c*d^2*x^2-3*I*f^2*c^2*d*x-3*I*f*c^2*d*exp(2*f*x+2*e)-6*I*c*
d^2*exp(2*f*x+2*e)-12*d^3*x*exp(f*x+e)-12*c*d^2*exp(f*x+e)-3*I*f*d^3*x^2*ex
p(2*f*x+2*e)+3*f^2*c^3*exp(f*x+e)-I*f^2*d^3*x^3)/(exp(f*x+e)-I)^3/f^3/a^2+2
/a^2/f*d^2*c*x^2+2/a^2/f^3*d^2*c*e^2+2/3/a^2/f*d^3*x^3-2/a^2/f^2*d*ln(exp(f
*x+e)-I)*c^2+4/a^2/f^2*d^2*c*e*x-4/a^2/f^3*d^2*ln(exp(f*x+e))*c*e+4/a^2/f^3
*d^2*ln(exp(f*x+e)-I)*c*e+2/a^2/f^4*d^3*ln(1+I*exp(f*x+e))*e^2-2/a^2/f^3*d^
3*e^2*x-4/a^2/f^2*d^2*ln(1+I*exp(f*x+e))*c*x-4/a^2/f^3*d^2*ln(1+I*exp(f*x+e
))*c*e+2/a^2/f^2*d*ln(exp(f*x+e))*c^2-2/a^2/f^4*d^3*ln(exp(f*x+e)-I)*e^2+2/
a^2/f^4*d^3*ln(exp(f*x+e))*e^2-4/a^2/f^3*d^2*c*polylog(2,-I*exp(f*x+e))-4/a
^2/f^3*d^3*polylog(2,-I*exp(f*x+e))*x-2/a^2/f^2*d^3*ln(1+I*exp(f*x+e))*x^2-
4/3/a^2/f^4*d^3*e^3-4/a^2/f^4*d^3*ln(exp(f*x+e))+4/a^2/f^4*d^3*ln(exp(f*x+e
)-I)+4*d^3*polylog(3,-I*exp(f*x+e))/a^2/f^4
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 659 vs. $2(248) = 496$.
time = 0.46, size = 659, normalized size = 2.16

$$2c^2d\left(\frac{fxe^{(3fx+3e)}-(3ifxe^{(2e)}+ie^{(2e)})e^{(2fx)}-e^{(fx+e)}}{a^2f^2e^{(3fx+3e)}-3ia^2f^2e^{(2fx+2e)}-3a^2f^2e^{(fx+e)}+ia^2f^2}-\frac{\log\left(-i\left(ie^{(fx+e)}+1\right)e^{(-e)}\right)}{a^2f^2}\right)+\frac{2}{3}c^3\left(\frac{3e^{(-fx-e)}}{(3a^2e^{(-fx-e)}-3ia^2e^{(-2fx-2e)}-a^2e^{(-3fx-3e)}+ia^2)f}+\frac{i}{(3a^2e^{(-fx-e)}-3ia^2e^{(-2fx-2e)}-a^2e^{(-3fx-3e)}+ia^2)f}\right)-\frac{2\left(id^3f^2x^3+3icd^2f^2x^2-6id^3x-6icd^2-3\left(-id^3fx^2e^{(2e)}-2icd^2e^{(2e)}+2(-icd^2f-id^3)xe^{(2e)}\right)e^{(2fx)}-3\left(d^3f^2x^3e^e-4cd^2e^e+(3cd^2f^2-d^3f)x^2e^e-2(cd^2f+2d^3)xe^e\right)e^{(fx)}\right)}{3\left(a^2f^3e^{(3fx+3e)}-3ia^2f^3e^{(2fx+2e)}-3a^2f^3e^{(fx+e)}+ia^2f^3\right)}-\frac{4\left(fx\log\left(ie^{(fx+e)}+1\right)+\mathrm{Li}_2(-ie^{(fx+e)})\right)cd^2}{a^2f^3}-\frac{4d^3x}{a^2f^3}-\frac{2\left(f^2x^2\log\left(ie^{(fx+e)}+1\right)+2fx\mathrm{Li}_2(-ie^{(fx+e)})-2\mathrm{Li}_3(-ie^{(fx+e)})\right)d^3}{a^2f^4}+\frac{4d^3\log\left(e^{(fx+e)}-i\right)}{a^2f^4}+\frac{2\left(d^3f^3x^3+3cd^2f^3x^2\right)}{3a^2f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2*c^2*d*((f*x*e^(3*f*x + 3*e) - (3*I*f*x*e^(2*e) + I*e^(2*e))*e^(2*f*x) - e
^(f*x + e))/(a^2*f^2*e^(3*f*x + 3*e) - 3*I*a^2*f^2*e^(2*f*x + 2*e) - 3*a^2*
f^2*e^(f*x + e) + I*a^2*f^2) - log(-I*(I*e^(f*x + e) + 1)*e^(-e))/(a^2*f^2)
) + 2/3*c^3*(3*e^(-f*x - e)/((3*a^2*e^(-f*x - e) - 3*I*a^2*e^(-2*f*x - 2*e)
 - a^2*e^(-3*f*x - 3*e) + I*a^2)*f) + I/((3*a^2*e^(-f*x - e) - 3*I*a^2*e^(-
2*f*x - 2*e) - a^2*e^(-3*f*x - 3*e) + I*a^2)*f)) - 2/3*(I*d^3*f^2*x^3 + 3*I
*c*d^2*f^2*x^2 - 6*I*d^3*x - 6*I*c*d^2 - 3*(-I*d^3*f*x^2*e^(2*e) - 2*I*c*d^
2*e^(2*e) + 2*(-I*c*d^2*f - I*d^3)*x*e^(2*e))*e^(2*f*x) - 3*(d^3*f^2*x^3*e^
e - 4*c*d^2*e^e + (3*c*d^2*f^2 - d^3*f)*x^2*e^e - 2*(c*d^2*f + 2*d^3)*x*e^e
)*e^(f*x))/(a^2*f^3*e^(3*f*x + 3*e) - 3*I*a^2*f^3*e^(2*f*x + 2*e) - 3*a^2*f
^3*e^(f*x + e) + I*a^2*f^3) - 4*(f*x*log(I*e^(f*x + e) + 1) + dilog(-I*e^(f
*x + e)))*c*d^2/(a^2*f^3) - 4*d^3*x/(a^2*f^3) - 2*(f^2*x^2*log(I*e^(f*x + e
) + 1) + 2*f*x*dilog(-I*e^(f*x + e)) - 2*polylog(3, -I*e^(f*x + e)))*d^3/(a
^2*f^4) + 4*d^3*log(e^(f*x + e) - I)/(a^2*f^4) + 2/3*(d^3*f^3*x^3 + 3*c*d^2
*f^3*x^2)/(a^2*f^4)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 938 vs. $2(248) = 496$.
time = 0.34, size = 938, normalized size = 3.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(I*c^3*f^3 + 3*I*c*d^2*f*e^2 - 6*I*c*d^2*f - I*d^3*e^3 + 6*(I*d^3*f*x
+ I*c*d^2*f + (d^3*f*x + c*d^2*f)*e^(3*f*x + 3*e) + 3*(-I*d^3*f*x - I*c*d^2
*f)*e^(2*f*x + 2*e) - 3*(d^3*f*x + c*d^2*f)*e^(f*x + e))*dilog(-I*e^(f*x +
e)) + 3*(-I*c^2*d*f^2 + 2*I*d^3)*e - (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 - 3*c*d
^2*f*e^2 + d^3*e^3 + 3*(c^2*d*f^3 - 2*d^3*f)*x + 3*(c^2*d*f^2 - 2*d^3)*e)*e
^(3*f*x + 3*e) + 3*(I*d^3*f^3*x^3 + I*c^2*d*f^2 - 3*I*c*d^2*f*e^2 + 2*I*c*d
^2*f + I*d^3*e^3 + (3*I*c*d^2*f^3 + I*d^3*f^2)*x^2 + (3*I*c^2*d*f^3 + 2*I*c
*d^2*f^2 - 4*I*d^3*f)*x + 3*(I*c^2*d*f^2 - 2*I*d^3)*e)*e^(2*f*x + 2*e) + 3*
(d^3*f^2*x^2 - c^3*f^3 + c^2*d*f^2 - 3*c*d^2*f*e^2 + 4*c*d^2*f + d^3*e^3 +
2*(c*d^2*f^2 - d^3*f)*x + 3*(c^2*d*f^2 - 2*d^3)*e)*e^(f*x + e) + 3*(I*c^2*d
*f^2 - 2*I*c*d^2*f*e + I*d^3*e^2 - 2*I*d^3 + (c^2*d*f^2 - 2*c*d^2*f*e + d^3
*e^2 - 2*d^3)*e^(3*f*x + 3*e) + 3*(-I*c^2*d*f^2 + 2*I*c*d^2*f*e - I*d^3*e^2
 + 2*I*d^3)*e^(2*f*x + 2*e) - 3*(c^2*d*f^2 - 2*c*d^2*f*e + d^3*e^2 - 2*d^3)
*e^(f*x + e))*log(e^(f*x + e) - I) + 3*(I*d^3*f^2*x^2 + 2*I*c*d^2*f^2*x + 2
*I*c*d^2*f*e - I*d^3*e^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + 2*c*d^2*f*e - d^3
*e^2)*e^(3*f*x + 3*e) + 3*(-I*d^3*f^2*x^2 - 2*I*c*d^2*f^2*x - 2*I*c*d^2*f*e
 + I*d^3*e^2)*e^(2*f*x + 2*e) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + 2*c*d^2*f*
e - d^3*e^2)*e^(f*x + e))*log(I*e^(f*x + e) + 1) - 6*(d^3*e^(3*f*x + 3*e) -
 3*I*d^3*e^(2*f*x + 2*e) - 3*d^3*e^(f*x + e) + I*d^3)*polylog(3, -I*e^(f*x
+ e)))/(a^2*f^4*e^(3*f*x + 3*e) - 3*I*a^2*f^4*e^(2*f*x + 2*e) - 3*a^2*f^4*e
^(f*x + e) + I*a^2*f^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{-2ic^3f^2 - 6ic^2df^2x - 6icd^2f^2x^2 + 12icd^2 - 2id^3f^2x^3 + 12id^3x + (-6ic^2dfe^{2e} - 12icd^2fxe^{2e} - 12icd^2e^{2e} - 6id^3fx^2e^{2e} - 12id^3xe^{2e})\,e^{2fx} + (6c^3f^2e^e + 18c^2df^2xe^e - 6c^2dfe^e + 18cd^2f^2x^2e^e - 12cd^2fxe^e - 24cd^2e^e + 6d^3f^2x^3e^e - 6d^3fx^2e^e - 24d^3xe^e)\,e^{fx}}{3a^2f^3e^{3e}e^{3fx} - 9ia^2f^3e^{2e}e^{2fx} - 9a^2f^3e^ee^{fx} + 3ia^2f^3} - \frac{2id\left(\int\left(-\frac{2d^2}{e^ee^{fx}-i}\right)dx + \int\frac{c^2f^2}{e^ee^{fx}-i}\,dx + \int\frac{d^2f^2x^2}{e^ee^{fx}-i}\,dx + \int\frac{2cdf^2x}{e^ee^{fx}-i}\,dx\right)}{a^2f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+I*a*sinh(f*x+e))**2,x)
```

```
[Out] (-2*I*c**3*f**2 - 6*I*c**2*d*f**2*x - 6*I*c*d**2*f**2*x**2 + 12*I*c*d**2 -
2*I*d**3*f**2*x**3 + 12*I*d**3*x + (-6*I*c**2*d*f*exp(2*e) - 12*I*c*d**2*f*
x*exp(2*e) - 12*I*c*d**2*exp(2*e) - 6*I*d**3*f*x**2*exp(2*e) - 12*I*d**3*x*
exp(2*e))*exp(2*f*x) + (6*c**3*f**2*exp(e) + 18*c**2*d*f**2*x*exp(e) - 6*c*
*2*d*f*exp(e) + 18*c*d**2*f**2*x**2*exp(e) - 12*c*d**2*f*x*exp(e) - 24*c*d*
*2*exp(e) + 6*d**3*f**2*x**3*exp(e) - 6*d**3*f*x**2*exp(e) - 24*d**3*x*exp(
e))*exp(f*x))/(3*a**2*f**3*exp(3*e)*exp(3*f*x) - 9*I*a**2*f**3*exp(2*e)*exp
(2*f*x) - 9*a**2*f**3*exp(e)*exp(f*x) + 3*I*a**2*f**3) - 2*I*d*(Integral(-2
*d**2/(exp(e)*exp(f*x) - I), x) + Integral(c**2*f**2/(exp(e)*exp(f*x) - I),
 x) + Integral(d**2*f**2*x**2/(exp(e)*exp(f*x) - I), x) + Integral(2*c*d*f*
*2*x/(exp(e)*exp(f*x) - I), x))/(a**2*f**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(I*a*sinh(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,x\right)}^3}{{\left(a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + a*sinh(e + f*x)*1i)^2,x)
```

```
[Out] int((c + d*x)^3/(a + a*sinh(e + f*x)*1i)^2, x)
```

3.114 $\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx$

Optimal. Leaf size=241

$$\frac{(c+dx)^2}{3a^2f}-\frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{3a^2f^2}-\frac{4d^2\text{PolyLog}\left(2,-ie^{e+fx}\right)}{3a^2f^3}+\frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f^2}-\frac{2d^2\tanh}{}$$

[Out] 1/3*(d*x+c)^2/a^2/f-4/3*d*(d*x+c)*ln(1+I*exp(f*x+e))/a^2/f^2-4/3*d^2*polylog(2,-I*exp(f*x+e))/a^2/f^3+1/3*d*(d*x+c)*sech(1/2*e+1/4*I*Pi+1/2*f*x)^2/a^2/f^2-2/3*d^2*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a^2/f^3+1/3*(d*x+c)^2*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a^2/f+1/6*(d*x+c)^2*sech(1/2*e+1/4*I*Pi+1/2*f*x)^2*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a^2/f

Rubi [A]
time = 0.21, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3399, 4271, 3852, 8, 4269, 3797, 2221, 2317, 2438}

$$-\frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{3a^2f^2}+\frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3a^2f^2}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3a^2f}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{6a^2f}+\frac{(c+dx)^2}{3a^2f}-\frac{4d^2\text{Li}_2\left(-ie^{e+fx}\right)}{3a^2f^3}-\frac{2d^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3a^2f^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2/(a + I*a*Sinh[e + f*x])^2,x]

[Out] (c + d*x)^2/(3*a^2*f) - (4*d*(c + d*x)*Log[1 + I*E^(e + f*x)])/(3*a^2*f^2) - (4*d^2*PolyLog[2, (-I)*E^(e + f*x)])/(3*a^2*f^3) + (d*(c + d*x)*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2)/(3*a^2*f^2) - (2*d^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*a^2*f^3) + ((c + d*x)^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*a^2*f) + ((c + d*x)^2*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(6*a^2*f)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx = \frac{\int (c+dx)^2\csc^4\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right)\,dx}{4a^2}$$

$$= \frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\right.}{6a^2f}$$

$$= \frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)^2\text{se}}{}$$

$$= \frac{(c+dx)^2}{3a^2f} + \frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f^2} - \frac{2d^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f^3} + \frac{(c+}{}$$

$$= \frac{(c+dx)^2}{3a^2f} - \frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{3a^2f^2} + \frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f^2} - $$

$$= \frac{(c+dx)^2}{3a^2f} - \frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{3a^2f^2} + \frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f^2} - $$

$$= \frac{(c+dx)^2}{3a^2f} - \frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{3a^2f^2} - \frac{4d^2\text{Li}_2\left(-ie^{e+fx}\right)}{3a^2f^3} + \frac{d(c+dx)\text{sec}}{3}$$

Mathematica [A]
time = 2.54, size = 259, normalized size = 1.07

$$\frac{\frac{2de^ef^2x(2c+dx)}{-i+e^e} - 4df(c+dx)\log\left(1+ie^{e+fx}\right) - 4d^2\text{PolyLog}(2,-ie^{e+fx}) + \frac{2df(c+dx)\cosh\left(\frac{fx}{2}\right)+2id^2\cosh\left(e+\frac{fx}{2}\right)+i(c^2f^2+2cdf^2x+d^2(-2+f^2x^2))\cosh\left(e+\frac{3fx}{2}\right)+(3c^2f^2+6cdf^2x+d^2(-4+3f^2x^2))\sinh\left(\frac{fx}{2}\right)+2idf(c+dx)\sinh\left(e+\frac{fx}{2}\right)}{\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)^3}}{3a^2f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] ((2*d*E^e*f^2*x*(2*c + d*x))/(-I + E^e) - 4*d*f*(c + d*x)*Log[1 + I*E^(e +
f*x)] - 4*d^2*PolyLog[2, (-I)*E^(e + f*x)] + (2*d*f*(c + d*x)*Cosh[(f*x)/2]
 + (2*I)*d^2*Cosh[e + (f*x)/2] + I*(c^2*f^2 + 2*c*d*f^2*x + d^2*(-2 + f^2*x
^2))*Cosh[e + (3*f*x)/2] + (3*c^2*f^2 + 6*c*d*f^2*x + d^2*(-4 + 3*f^2*x^2))
*Sinh[(f*x)/2] + (2*I)*d*f*(c + d*x)*Sinh[e + (f*x)/2])/((Cosh[e/2] + I*Sin
h[e/2])*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^3))/(3*a^2*f^3)
```

Maple [A]
time = 2.59, size = 374, normalized size = 1.55

method	result
risch	$\frac{-\frac{4if^2cdx}{3}-\frac{4if\,d^2x\,e^{2fx+2e}}{3}-\frac{4ifcd\,e^{2fx+2e}}{3}-\frac{4id^2e^{2fx+2e}}{3}-\frac{4f\,d^2x\,e^{fx+e}}{3}-\frac{4fcd\,e^{fx+e}}{3}-\frac{2if^2d^2x^2}{3}-\frac{2if^2c^2}{3}-\frac{8d^2e^{fx+e}}{3}+\frac{4id^2}{3}+2f^2d^2x}{\left(e^{fx+e}-i\right)^3f^3a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(-2*I*f^2*c*d*x-2*I*f*d^2*x*exp(2*f*x+2*e)-2*I*f*c*d*exp(2*f*x+2*e)-2*I
*d^2*exp(2*f*x+2*e)-2*f*d^2*x*exp(f*x+e)-2*f*c*d*exp(f*x+e)-I*f^2*d^2*x^2-I
*f^2*c^2-4*d^2*exp(f*x+e)+2*I*d^2+3*f^2*d^2*x^2*exp(f*x+e)+6*f^2*c*d*x*exp(
f*x+e)+3*f^2*c^2*exp(f*x+e))/(exp(f*x+e)-I)^3/f^3/a^2-4/3/a^2/f^2*d*ln(exp(
f*x+e)-I)*c+4/3/a^2/f^2*d*ln(exp(f*x+e))*c+2/3/a^2/f*d^2*x^2+4/3/a^2/f^2*d^
2*e*x+2/3/a^2/f^3*d^2*e^2-4/3/a^2/f^2*d^2*ln(1+I*exp(f*x+e))*x-4/3/a^2/f^3*
d^2*ln(1+I*exp(f*x+e))*e-4/3*d^2*polylog(2,-I*exp(f*x+e))/a^2/f^3+4/3/a^2/f
^3*d^2*e*ln(exp(f*x+e)-I)-4/3/a^2/f^3*d^2*e*ln(exp(f*x+e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*d^2*((I*f^2*x^2 - 2*(-I*f*x*e^(2*e) - I*e^(2*e))*e^(2*f*x) - (3*f^2*x^
2*e^e - 2*f*x*e^e - 4*e^e)*e^(f*x) - 2*I)/(a^2*f^3*e^(3*f*x + 3*e) - 3*I*a^
2*f^3*e^(2*f*x + 2*e) - 3*a^2*f^3*e^(f*x + e) + I*a^2*f^3) + 6*I*integrate(
1/3*x/(a^2*f*e^(f*x + e) - I*a^2*f), x)) + 4/3*c*d*((f*x*e^(3*f*x + 3*e) -
(3*I*f*x*e^(2*e) + I*e^(2*e))*e^(2*f*x) - e^(f*x + e))/(a^2*f^2*e^(3*f*x +
3*e) - 3*I*a^2*f^2*e^(2*f*x + 2*e) - 3*a^2*f^2*e^(f*x + e) + I*a^2*f^2) - l
og(-I*(I*e^(f*x + e) + 1)*e^(-e))/(a^2*f^2)) + 2/3*c^2*(3*e^(-f*x - e)/((3*
a^2*e^(-f*x - e) - 3*I*a^2*e^(-2*f*x - 2*e) - a^2*e^(-3*f*x - 3*e) + I*a^2)
*f) + I/((3*a^2*e^(-f*x - e) - 3*I*a^2*e^(-2*f*x - 2*e) - a^2*e^(-3*f*x - 3
*e) + I*a^2)*f))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(187) = 374$.
time = 0.34, size = 504, normalized size = 2.09

$$-\frac{2\left(i\,c^2f^2-2i\,cdfe+i\,d^2e^2-2i\,d^2+2\left(d^2e^{(3fx+3e)}-3i\,d^2e^{(2fx+2e)}-3\,d^2e^{(fx+e)}+i\,d^2\right)\mathrm{Li}_2\left(-i\,e^{(fx+e)}\right)-\left(d^2f^2x^2+2\,cdf^2x+2\,cdfe-d^2e^2\right)e^{(3fx+3e)}+\left(3i\,d^2f^2x^2+6i\,cdfe+2i\,cdf-3i\,d^2e^2+2i\,d^2+2\left(3i\,cdf^2+i\,d^2f\right)x\right)e^{(2fx+2e)}-\left(3\,c^2f^2-2\,d^2fx-6\,cdfe-2\,cdf+3\,d^2e^2-4\,d^2\right)e^{(fx+e)}+2\left(i\,cdf-i\,d^2e+(cdf-d^2e)e^{(3fx+3e)}+3\left(-i\,cdf+i\,d^2e\right)e^{(2fx+2e)}-3\left(cdf-d^2e\right)e^{(fx+e)}\right)\log\left(e^{(fx+e)}-i\right)+2\left(i\,d^2fx+i\,d^2e+(d^2fx+d^2e)e^{(3fx+3e)}+3\left(-i\,d^2fx-i\,d^2e\right)e^{(2fx+2e)}-3\left(d^2fx+d^2e\right)e^{(fx+e)}\right)\log\left(i\,e^{(fx+e)}+1\right)\right)}{3\left(a^2f^3e^{(3fx+3e)}-3i\,a^2f^3e^{(2fx+2e)}-3\,a^2f^3e^{(fx+e)}+i\,a^2f^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(I*c^2*f^2 - 2*I*c*d*f*e + I*d^2*e^2 - 2*I*d^2 + 2*(d^2*e^(3*f*x + 3*e
) - 3*I*d^2*e^(2*f*x + 2*e) - 3*d^2*e^(f*x + e) + I*d^2)*dilog(-I*e^(f*x +
e)) - (d^2*f^2*x^2 + 2*c*d*f^2*x + 2*c*d*f*e - d^2*e^2)*e^(3*f*x + 3*e) + (
```

```
3*I*d^2*f^2*x^2 + 6*I*c*d*f*e + 2*I*c*d*f - 3*I*d^2*e^2 + 2*I*d^2 + 2*(3*I*
c*d*f^2 + I*d^2*f)*x)*e^(2*f*x + 2*e) - (3*c^2*f^2 - 2*d^2*f*x - 6*c*d*f*e
- 2*c*d*f + 3*d^2*e^2 - 4*d^2)*e^(f*x + e) + 2*(I*c*d*f - I*d^2*e + (c*d*f
- d^2*e)*e^(3*f*x + 3*e) + 3*(-I*c*d*f + I*d^2*e)*e^(2*f*x + 2*e) - 3*(c*d*
f - d^2*e)*e^(f*x + e))*log(e^(f*x + e) - I) + 2*(I*d^2*f*x + I*d^2*e + (d^
2*f*x + d^2*e)*e^(3*f*x + 3*e) + 3*(-I*d^2*f*x - I*d^2*e)*e^(2*f*x + 2*e) -
 3*(d^2*f*x + d^2*e)*e^(f*x + e))*log(I*e^(f*x + e) + 1))/(a^2*f^3*e^(3*f*x
 + 3*e) - 3*I*a^2*f^3*e^(2*f*x + 2*e) - 3*a^2*f^3*e^(f*x + e) + I*a^2*f^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{-2ic^2f^2 - 4icdf^2x - 2id^2f^2x^2 + 4id^2 + \left(-4icdfe^{2e} - 4id^2fxe^{2e} - 4id^2e^{2e}\right)e^{2fx} + \left(6c^2f^2e^e + 12cdf^2xe^e - 4cdfe^e + 6d^2f^2x^2e^e - 4d^2fxe^e - 8d^2e^e\right)e^{fx}}{3a^2f^3e^{3e}e^{3fx} - 9ia^2f^3e^{2e}e^{2fx} - 9a^2f^3e^ee^{fx} + 3ia^2f^3} - \frac{4id\left(\int \frac{c}{e^ee^{fx}-i}\,dx + \int \frac{dx}{e^ee^{fx}-i}\,dx\right)}{3a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+I*a*sinh(f*x+e))**2,x)
```

```
[Out] (-2*I*c**2*f**2 - 4*I*c*d*f**2*x - 2*I*d**2*f**2*x**2 + 4*I*d**2 + (-4*I*c*
d*f*exp(2*e) - 4*I*d**2*f*x*exp(2*e) - 4*I*d**2*exp(2*e))*exp(2*f*x) + (6*c
**2*f**2*exp(e) + 12*c*d*f**2*x*exp(e) - 4*c*d*f*exp(e) + 6*d**2*f**2*x**2*
exp(e) - 4*d**2*f*x*exp(e) - 8*d**2*exp(e))*exp(f*x))/(3*a**2*f**3*exp(3*e)
*exp(3*f*x) - 9*I*a**2*f**3*exp(2*e)*exp(2*f*x) - 9*a**2*f**3*exp(e)*exp(f*
x) + 3*I*a**2*f**3) - 4*I*d*(Integral(c/(exp(e)*exp(f*x) - I), x) + Integra
l(d*x/(exp(e)*exp(f*x) - I), x))/(3*a**2*f)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(I*a*sinh(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,x\right)}^2}{{\left(a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + a*sinh(e + f*x)*1i)^2,x)
```

```
[Out] int((c + d*x)^2/(a + a*sinh(e + f*x)*1i)^2, x)
```

3.115 $\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}\,dx$

Optimal. Leaf size=158

$$-\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{3a^2f^2}+\frac{d\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{6a^2f^2}+\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\cdot\right.}{6a^2}$$

```
[Out] -2/3*d*ln(cosh(1/2*e+1/4*I*Pi+1/2*f*x))/a^2/f^2+1/6*d*sech(1/2*e+1/4*I*Pi+1
/2*f*x)^2/a^2/f^2+1/3*(d*x+c)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a^2/f+1/6*(d*x+c
)*sech(1/2*e+1/4*I*Pi+1/2*f*x)^2*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a^2/f
```

Rubi [A]
time = 0.08, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3399, 4270, 4269, 3556}

$$\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3a^2f}+\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{6a^2f}+\frac{d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{6a^2f^2}-\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{3a^2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] (-2*d*Log[Cosh[e/2 + (I/4)*Pi + (f*x)/2]])/(3*a^2*f^2) + (d*Sech[e/2 + (I/4
)*Pi + (f*x)/2]^2)/(6*a^2*f^2) + ((c + d*x)*Tanh[e/2 + (I/4)*Pi + (f*x)/2])
/(3*a^2*f) + ((c + d*x)*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2*Tanh[e/2 + (I/4)*P
i + (f*x)/2])/(6*a^2*f)
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}\,dx = \frac{\int (c+dx)\csc^4\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right)\,dx}{4a^2}$$
$$= \frac{d\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{6a^2f^2} + \frac{(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{6a^2f} - \int$$
$$= \frac{d\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{6a^2f^2} + \frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}\right.}{}$$
$$= -\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{3a^2f^2} + \frac{d\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{6a^2f^2} + \frac{(c+dx)\tanh\left(\frac{e}{2}+\right.}{3a^2f}$$

Mathematica [A]
time = 0.68, size = 241, normalized size = 1.53

$$\frac{\left(-i\cosh\left(\frac{1}{2}(e+fx)\right)+\sinh\left(\frac{1}{2}(e+fx)\right)\right)\left(d\cosh\left(\frac{1}{2}(e+fx)\right)\left(-2i+3e+3fx-6\text{ArcTan}\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right)+3i\log(\cosh(e+fx))\right)+\cosh\left(\frac{3}{2}(e+fx)\right)\left(-de+2cf+dfx+2d\text{ArcTan}\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right)-id\log(\cosh(e+fx))\right)+2i\left(-id+2de-3cf-dfx-4d\text{ArcTan}\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right)+d\cosh(e+fx)\left(e+fx-2\text{ArcTan}\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right)+i\log(\cosh(e+fx))\right)+2id\log(\cosh(e+fx))\right)\sinh\left(\frac{1}{2}(e+fx)\right)\right)}{6a^2f^2(-i+\sinh(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] (((-I)*Cosh[(e + f*x)/2] + Sinh[(e + f*x)/2])*(d*Cosh[(e + f*x)/2]*(-2*I +
3*e + 3*f*x - 6*ArcTan[Tanh[(e + f*x)/2]] + (3*I)*Log[Cosh[e + f*x]]) + Cos
h[(3*(e + f*x))/2]*(-(d*e) + 2*c*f + d*f*x + 2*d*ArcTan[Tanh[(e + f*x)/2]]
- I*d*Log[Cosh[e + f*x]]) + (2*I)*((-I)*d + 2*d*e - 3*c*f - d*f*x - 4*d*Arc
Tan[Tanh[(e + f*x)/2]] + d*Cosh[e + f*x]*(e + f*x - 2*ArcTan[Tanh[(e + f*x)
/2]] + I*Log[Cosh[e + f*x]]) + (2*I)*d*Log[Cosh[e + f*x]])*Sinh[(e + f*x)/2
]))/(6*a^2*f^2*(-I + Sinh[e + f*x])^2)
```

Maple [A]
time = 2.75, size = 113, normalized size = 0.72

method	result	size
risch	$\frac{2dx}{3f\,a^2}+\frac{2de}{3f^2a^2}-\frac{2i\left(3ifdx\,\mathrm{e}^{fx+e}+3ifc\,\mathrm{e}^{fx+e}-id\,\mathrm{e}^{fx+e}+dxf+d\,\mathrm{e}^{2fx+2e}+cf\right)}{3\left(\mathrm{e}^{fx+e}-i\right)^3f^2a^2}-\frac{2d\ln\left(\mathrm{e}^{fx+e}-i\right)}{3f^2a^2}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+I*a*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*d/f/a^2*x+2/3*d/f^2/a^2*e-2/3*I*(3*I*f*d*x*exp(f*x+e)+3*I*f*c*exp(f*x+e
)-I*d*exp(f*x+e)+d*x*f+d*exp(2*f*x+2*e)+c*f)/(exp(f*x+e)-I)^3/f^2/a^2-2/3*d
/f^2/a^2*ln(exp(f*x+e)-I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(115) = 230$.
time = 0.29, size = 271, normalized size = 1.72

$$\frac{2}{3}\,d\left(\frac{fxe^{(3\,fx+3\,e)}-\left(3i\,fxe^{(2\,e)}+i\,e^{(2\,e)}\right)e^{(2\,fx)}-e^{(fx+e)}}{a^2f^2e^{(3\,fx+3\,e)}-3i\,a^2f^2e^{(2\,fx+2\,e)}-3\,a^2f^2e^{(fx+e)}+i\,a^2f^2}-\frac{\log\left(-i\left(i\,e^{(fx+e)}+1\right)e^{(-e)}\right)}{a^2f^2}\right)+\frac{2}{3}\,c\left(\frac{3\,e^{(-fx-e)}}{\left(3\,a^2e^{(-fx-e)}-3i\,a^2e^{(-2\,fx-2\,e)}-a^2e^{(-3\,fx-3\,e)}+i\,a^2\right)f}+\frac{i}{\left(3\,a^2e^{(-fx-e)}-3i\,a^2e^{(-2\,fx-2\,e)}-a^2e^{(-3\,fx-3\,e)}+i\,a^2\right)f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*d*((f*x*e^(3*f*x + 3*e) - (3*I*f*x*e^(2*e) + I*e^(2*e))*e^(2*f*x) - e^(
f*x + e))/(a^2*f^2*e^(3*f*x + 3*e) - 3*I*a^2*f^2*e^(2*f*x + 2*e) - 3*a^2*f^
2*e^(f*x + e) + I*a^2*f^2) - log(-I*(I*e^(f*x + e) + 1)*e^(-e))/(a^2*f^2))
+ 2/3*c*(3*e^(-f*x - e)/((3*a^2*e^(-f*x - e) - 3*I*a^2*e^(-2*f*x - 2*e) - a
^2*e^(-3*f*x - 3*e) + I*a^2)*f) + I/((3*a^2*e^(-f*x - e) - 3*I*a^2*e^(-2*f*
x - 2*e) - a^2*e^(-3*f*x - 3*e) + I*a^2)*f))
```

Fricas [A]
time = 0.38, size = 170, normalized size = 1.08

$$\frac{2\left(dfxe^{(3\,fx+3\,e)}-i\,cf-(3i\,dfx+i\,d)e^{(2\,fx+2\,e)}+(3\,cf-d)e^{(fx+e)}-\left(de^{(3\,fx+3\,e)}-3i\,de^{(2\,fx+2\,e)}-3\,de^{(fx+e)}+i\,d\right)\log\left(e^{(fx+e)}-i\right)\right)}{3\left(a^2f^2e^{(3\,fx+3\,e)}-3i\,a^2f^2e^{(2\,fx+2\,e)}-3\,a^2f^2e^{(fx+e)}+i\,a^2f^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 2/3*(d*f*x*e^(3*f*x + 3*e) - I*c*f - (3*I*d*f*x + I*d)*e^(2*f*x + 2*e) + (3
*c*f - d)*e^(f*x + e) - (d*e^(3*f*x + 3*e) - 3*I*d*e^(2*f*x + 2*e) - 3*d*e^
(f*x + e) + I*d)*log(e^(f*x + e) - I))/(a^2*f^2*e^(3*f*x + 3*e) - 3*I*a^2*f
^2*e^(2*f*x + 2*e) - 3*a^2*f^2*e^(f*x + e) + I*a^2*f^2)
```

Sympy [A]
time = 0.28, size = 167, normalized size = 1.06

$$\frac{-2icf-2idfx-2ide^{2e}e^{2fx}+\left(6cfe^{e}+6dfxe^{e}-2de^{e}\right)e^{fx}}{3a^2f^2e^{3e}e^{3fx}-9ia^2f^2e^{2e}e^{2fx}-9a^2f^2e^{e}e^{fx}+3ia^2f^2}+\frac{2dx}{3a^2f}-\frac{2d\log\left(e^{fx}-ie^{-e}\right)}{3a^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*sinh(f*x+e))**2,x)
```

```
[Out] (-2*I*c*f - 2*I*d*f*x - 2*I*d*exp(2*e)*exp(2*f*x) + (6*c*f*exp(e) + 6*d*f*x
*exp(e) - 2*d*exp(e))*exp(f*x))/(3*a**2*f**2*exp(3*e)*exp(3*f*x) - 9*I*a**2
```

```
*f**2*exp(2*e)*exp(2*f*x) - 9*a**2*f**2*exp(e)*exp(f*x) + 3*I*a**2*f**2) +
2*d*x/(3*a**2*f) - 2*d*log(exp(f*x) - I*exp(-e))/(3*a**2*f**2)
```

Giac [A]
time = 0.44, size = 195, normalized size = 1.23

$$\frac{2\left(dfxe^{(3\,fx+3\,e)}-3i\,dfxe^{(2\,fx+2\,e)}+3\,cfe^{(fx+e)}-de^{(3\,fx+3\,e)}\log\left(e^{(fx+e)}-i\right)+3i\,de^{(2\,fx+2\,e)}\log\left(e^{(fx+e)}-i\right)+3\,de^{(fx+e)}\log\left(e^{(fx+e)}-i\right)-i\,cf-i\,de^{(2\,fx+2\,e)}-de^{(fx+e)}-i\,d\log\left(e^{(fx+e)}-i\right)\right)}{3\left(a^2f^2e^{(3\,fx+3\,e)}-3i\,a^2f^2e^{(2\,fx+2\,e)}-3\,a^2f^2e^{(fx+e)}+i\,a^2f^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2/3*(d*f*x*e^(3*f*x + 3*e) - 3*I*d*f*x*e^(2*f*x + 2*e) + 3*c*f*e^(f*x + e)
- d*e^(3*f*x + 3*e)*log(e^(f*x + e) - I) + 3*I*d*e^(2*f*x + 2*e)*log(e^(f*x
 + e) - I) + 3*d*e^(f*x + e)*log(e^(f*x + e) - I) - I*c*f - I*d*e^(2*f*x +
2*e) - d*e^(f*x + e) - I*d*log(e^(f*x + e) - I))/(a^2*f^2*e^(3*f*x + 3*e) -
 3*I*a^2*f^2*e^(2*f*x + 2*e) - 3*a^2*f^2*e^(f*x + e) + I*a^2*f^2)
```

Mupad [B]
time = 0.54, size = 160, normalized size = 1.01

$$-\frac{\frac{2\,d\,\ln\left(\mathrm{e}^{f\,x}\,\mathrm{e}^{e}-\mathrm{i}\right)}{3}+\mathrm{e}^{e+f\,x}\,\left(-\frac{d\,2\mathrm{i}}{3}+c\,f\,2\mathrm{i}+d\,\ln\left(\mathrm{e}^{f\,x}\,\mathrm{e}^{e}-\mathrm{i}\right)\,2\mathrm{i}\right)+\frac{2\,d\,\mathrm{e}^{2\,e+2\,f\,x}}{3}+f\,\left(\frac{2\,c}{3}+2\,d\,x\,\mathrm{e}^{2\,e+2\,f\,x}+\frac{d\,x\,\mathrm{e}^{3\,e+3\,f\,x}\,2\mathrm{i}}{3}\right)-2\,d\,\mathrm{e}^{2\,e+2\,f\,x}\,\ln\left(\mathrm{e}^{f\,x}\,\mathrm{e}^{e}-\mathrm{i}\right)-\frac{d\,\mathrm{e}^{3\,e+3\,f\,x}\,\ln\left(\mathrm{e}^{f\,x}\,\mathrm{e}^{e}-\mathrm{i}\right)\,2\mathrm{i}}{3}}{a^2\,f^2\,{\left(1+\mathrm{e}^{e+f\,x}\,1\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + a*sinh(e + f*x)*1i)^2,x)
```

```
[Out] -((2*d*log(exp(f*x)*exp(e) - 1i))/3 + exp(e + f*x)*(c*f*2i - (d*2i)/3 + d*l
og(exp(f*x)*exp(e) - 1i)*2i) + (2*d*exp(2*e + 2*f*x))/3 + f*((2*c)/3 + 2*d*
x*exp(2*e + 2*f*x) + (d*x*exp(3*e + 3*f*x)*2i)/3) - 2*d*exp(2*e + 2*f*x)*lo
g(exp(f*x)*exp(e) - 1i) - (d*exp(3*e + 3*f*x)*log(exp(f*x)*exp(e) - 1i)*2i)
/3)/(a^2*f^2*(exp(e + f*x)*1i + 1)^3)
```

3.116 $\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{1}{(c+dx)(a+ia\sinh(e+fx))^2},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + I*a*Sinh[e + f*x])^2),x]
[Out] Defer[Int][1/((c + d*x)*(a + I*a*Sinh[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx$$

Mathematica [A]
time = 24.78, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + I*a*Sinh[e + f*x])^2),x]
[Out] Integrate[1/((c + d*x)*(a + I*a*Sinh[e + f*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(a+ia\sinh\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x)

[Out] int(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")

[Out] -2*(I*d^2*f^2*x^2 + 2*I*c*d*f^2*x + I*c^2*f^2 - 2*I*d^2 + (-I*d^2*f*x*e^(2*e) + (-I*c*d*f + 2*I*d^2)*e^(2*e))*e^(2*f*x) - (3*d^2*f^2*x^2*e^e + (6*c*d*f^2 + d^2*f)*x*e^e + (3*c^2*f^2 + c*d*f - 4*d^2)*e^e)*e^(f*x))/(3*I*a^2*d^3*f^3*x^3 + 9*I*a^2*c*d^2*f^3*x^2 + 9*I*a^2*c^2*d*f^3*x + 3*I*a^2*c^3*f^3 + 3*(a^2*d^3*f^3*x^3*e^(3*e) + 3*a^2*c*d^2*f^3*x^2*e^(3*e) + 3*a^2*c^2*d*f^3*x*e^(3*e) + a^2*c^3*f^3*e^(3*e))*e^(3*f*x) - 9*(I*a^2*d^3*f^3*x^3*e^(2*e) + 3*I*a^2*c*d^2*f^3*x^2*e^(2*e) + 3*I*a^2*c^2*d*f^3*x*e^(2*e) + I*a^2*c^3*f^3*e^(2*e))*e^(2*f*x) - 9*(a^2*d^3*f^3*x^3*e^e + 3*a^2*c*d^2*f^3*x^2*e^e + 3*a^2*c^2*d*f^3*x*e^e + a^2*c^3*f^3*e^e)*e^(f*x)) - integrate(2/3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 6*d^3)/(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 - (-I*a^2*d^4*f^3*x^4*e^e - 4*I*a^2*c*d^3*f^3*x^3*e^e - 6*I*a^2*c^2*d^2*f^3*x^2*e^e - 4*I*a^2*c^3*d*f^3*x*e^e - I*a^2*c^4*f^3*e^e)*e^(f*x)), x)

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")

[Out] (-2*I*d^2*f^2*x^2 - 4*I*c*d*f^2*x - 2*I*c^2*f^2 + 4*I*d^2 - 2*(-I*d^2*f*x - I*c*d*f + 2*I*d^2)*e^(2*f*x + 2*e) + 2*(3*d^2*f^2*x^2 + 3*c^2*f^2 + c*d*f - 4*d^2 + (6*c*d*f^2 + d^2*f)*x)*e^(f*x + e) - 3*(-I*a^2*d^3*f^3*x^3 - 3*I*a^2*c*d^2*f^3*x^2 - 3*I*a^2*c^2*d*f^3*x - I*a^2*c^3*f^3 - (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e^(3*f*x + 3*e) + 3*(I*a^2*d^3*f^3*x^3 + 3*I*a^2*c*d^2*f^3*x^2 + 3*I*a^2*c^2*d*f^3*x + I*a^2*c^3*f^3)*e^(2*f*x + 2*e) + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e^(f*x + e))*integral(-2*(-I*d^3*f^2*x^2 - 2*I*c*d^2*f^2*x - I*c^2*d*f^2 + 6*I*d^3)/(-3*I*a^2*d^4*f^3*x^4 - 12*I*a^2*c*d^3*f^3*x^3 - 18*I*a^2*c^2*d^2*f^3*x^2 - 12*I*a^2*c^3*d*f^3*x - 3*I*a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*e^(f*x + e)), x))/(3*I*a^2*d^3*f^3*x^3 + 9*I*a

```
^2*c*d^2*f^3*x^2 + 9*I*a^2*c^2*d*f^3*x + 3*I*a^2*c^3*f^3 + 3*(a^2*d^3*f^3*x
^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e^(3*f*x + 3*e)
 - 9*(I*a^2*d^3*f^3*x^3 + 3*I*a^2*c*d^2*f^3*x^2 + 3*I*a^2*c^2*d*f^3*x + I*a
^2*c^3*f^3)*e^(2*f*x + 2*e) - 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*
a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e^(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(I*a*sinh(f*x + e) + a)^2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sinh(e + f*x)*1i)^2*(c + d*x)),x)
```

```
[Out] int(1/((a + a*sinh(e + f*x)*1i)^2*(c + d*x)), x)
```

3.117 $\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])^2),x]
[Out] Defer[Int][1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx$$

Mathematica [A]
time = 26.27, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])^2),x]
[Out] Integrate[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\left(a+ia\sinh(fx+e)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2*(I*d^2*f^2*x^2 + 2*I*c*d*f^2*x + I*c^2*f^2 - 6*I*d^2 + 2*(-I*d^2*f*x*e^(
2*e) + (-I*c*d*f + 3*I*d^2)*e^(2*e))*e^(2*f*x) - (3*d^2*f^2*x^2*e^e + 2*(3*
c*d*f^2 + d^2*f)*x*e^e + (3*c^2*f^2 + 2*c*d*f - 12*d^2)*e^e)*e^(f*x))/(3*I*
a^2*d^4*f^3*x^4 + 12*I*a^2*c*d^3*f^3*x^3 + 18*I*a^2*c^2*d^2*f^3*x^2 + 12*I*
a^2*c^3*d*f^3*x + 3*I*a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4*e^(3*e) + 4*a^2*c*d^
3*f^3*x^3*e^(3*e) + 6*a^2*c^2*d^2*f^3*x^2*e^(3*e) + 4*a^2*c^3*d*f^3*x*e^(3*
e) + a^2*c^4*f^3*e^(3*e))*e^(3*f*x) - 9*(I*a^2*d^4*f^3*x^4*e^(2*e) + 4*I*a^
2*c*d^3*f^3*x^3*e^(2*e) + 6*I*a^2*c^2*d^2*f^3*x^2*e^(2*e) + 4*I*a^2*c^3*d*f
^3*x*e^(2*e) + I*a^2*c^4*f^3*e^(2*e))*e^(2*f*x) - 9*(a^2*d^4*f^3*x^4*e^e +
4*a^2*c*d^3*f^3*x^3*e^e + 6*a^2*c^2*d^2*f^3*x^2*e^e + 4*a^2*c^3*d*f^3*x*e^e
 + a^2*c^4*f^3*e^e)*e^(f*x)) - integrate(4/3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x +
 c^2*d*f^2 - 12*d^3)/(a^2*d^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*c^2*d^
3*f^3*x^3 + 10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3 - (-I*
a^2*d^5*f^3*x^5*e^e - 5*I*a^2*c*d^4*f^3*x^4*e^e - 10*I*a^2*c^2*d^3*f^3*x^3*
e^e - 10*I*a^2*c^3*d^2*f^3*x^2*e^e - 5*I*a^2*c^4*d*f^3*x*e^e - I*a^2*c^5*f^
3*e^e)*e^(f*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] (-2*I*d^2*f^2*x^2 - 4*I*c*d*f^2*x - 2*I*c^2*f^2 + 12*I*d^2 - 4*(-I*d^2*f*x
- I*c*d*f + 3*I*d^2)*e^(2*f*x + 2*e) + 2*(3*d^2*f^2*x^2 + 3*c^2*f^2 + 2*c*d
*f - 12*d^2 + 2*(3*c*d*f^2 + d^2*f)*x)*e^(f*x + e) - 3*(-I*a^2*d^4*f^3*x^4
- 4*I*a^2*c*d^3*f^3*x^3 - 6*I*a^2*c^2*d^2*f^3*x^2 - 4*I*a^2*c^3*d*f^3*x - I
*a^2*c^4*f^3 - (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x
^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*e^(3*f*x + 3*e) + 3*(I*a^2*d^4*f^3*x^
4 + 4*I*a^2*c*d^3*f^3*x^3 + 6*I*a^2*c^2*d^2*f^3*x^2 + 4*I*a^2*c^3*d*f^3*x +
 I*a^2*c^4*f^3)*e^(2*f*x + 2*e) + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3
+ 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*e^(f*x + e))*int
```

```
egral(-4*(-I*d^3*f^2*x^2 - 2*I*c*d^2*f^2*x - I*c^2*d*f^2 + 12*I*d^3)/(-3*I*
a^2*d^5*f^3*x^5 - 15*I*a^2*c*d^4*f^3*x^4 - 30*I*a^2*c^2*d^3*f^3*x^3 - 30*I*
a^2*c^3*d^2*f^3*x^2 - 15*I*a^2*c^4*d*f^3*x - 3*I*a^2*c^5*f^3 + 3*(a^2*d^5*f
^3*x^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*c^2*d^3*f^3*x^3 + 10*a^2*c^3*d^2*f^3*
x^2 + 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3)*e^(f*x + e)), x))/(3*I*a^2*d^4*f^3*x
^4 + 12*I*a^2*c*d^3*f^3*x^3 + 18*I*a^2*c^2*d^2*f^3*x^2 + 12*I*a^2*c^3*d*f^3
*x + 3*I*a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2
*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*e^(3*f*x + 3*e) - 9*(I*a^2*
d^4*f^3*x^4 + 4*I*a^2*c*d^3*f^3*x^3 + 6*I*a^2*c^2*d^2*f^3*x^2 + 4*I*a^2*c^3
*d*f^3*x + I*a^2*c^4*f^3)*e^(2*f*x + 2*e) - 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^
3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*e^(f*x
 + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+I*a*sinh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(I*a*sinh(f*x + e) + a)^2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2\,{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*sinh(e + f*x)*1i)^2*(c + d*x)^2),x)
```

```
[Out] int(1/((a + a*sinh(e + f*x)*1i)^2*(c + d*x)^2), x)
```

3.118 $\int x^4 \sqrt{a + ia \sinh(e + fx)}\, dx$

Optimal. Leaf size=181

$$-\frac{384x\sqrt{a+ia\sinh(e+fx)}}{f^4}-\frac{16x^3\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{768\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f^5}+$$

```
[Out] -384*x*(a+I*a*sinh(f*x+e))^(1/2)/f^4-16*x^3*(a+I*a*sinh(f*x+e))^(1/2)/f^2+7
68*(a+I*a*sinh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f^5+96*x^2*(a+I*a
*sinh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f^3+2*x^4*(a+I*a*sinh(f*x+
e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f
```

Rubi [A]
time = 0.15, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3400, 3377, 2718}

$$\frac{768\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{f^5}-\frac{384x\sqrt{a+ia\sinh(e+fx)}}{f^4}+\frac{96x^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{f^3}-\frac{16x^3\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{2x^4\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] (-384*x*Sqrt[a + I*a*Sinh[e + f*x]])/f^4 - (16*x^3*Sqrt[a + I*a*Sinh[e + f*
x]])/f^2 + (768*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])
/f^5 + (96*x^2*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/
f^3 + (2*x^4*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/f
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx = \left(\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int x^4\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right.$$

$$=\frac{2x^4\sqrt{a+ia\sinh(e+fx)}\,\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f}-\frac{\left(8\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{\right.}}{}$$

$$=-\frac{16x^3\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{2x^4\sqrt{a+ia\sinh(e+fx)}\,\tanh\left(\frac{e}{2}+\frac{i\pi}{4}\right.}{f}$$

$$=-\frac{16x^3\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{96x^2\sqrt{a+ia\sinh(e+fx)}\,\tanh\left(\frac{e}{2}+\frac{i\pi}{4}\right.}{f^3}$$

$$=-\frac{384x\sqrt{a+ia\sinh(e+fx)}}{f^4}-\frac{16x^3\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{96x^2\sqrt{a}}{}$$

$$=-\frac{384x\sqrt{a+ia\sinh(e+fx)}}{f^4}-\frac{16x^3\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{768\sqrt{a+}}{}$$

Mathematica [A]
time = 0.17, size = 141, normalized size = 0.78

$$\frac{2\left(i(384+192ifx+48f^2x^2+8if^3x^3+f^4x^4)\cosh\left(\frac{1}{2}(e+fx)\right)+(384-192ifx+48f^2x^2-8if^3x^3+f^4x^4)\sinh\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a+ia\sinh(e+fx)}}{f^5\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] (2*(I*(384 + (192*I)*f*x + 48*f^2*x^2 + (8*I)*f^3*x^3 + f^4*x^4)*Cosh[(e +
f*x)/2] + (384 - (192*I)*f*x + 48*f^2*x^2 - (8*I)*f^3*x^3 + f^4*x^4)*Sinh[(
e + f*x)/2])*Sqrt[a + I*a*Sinh[e + f*x]])/(f^5*(Cosh[(e + f*x)/2] + I*Sinh[
(e + f*x)/2]))
```

Maple [A]
time = 0.30, size = 174, normalized size = 0.96

method	result
risch	$\frac{i\sqrt{2}\,\sqrt{a\left(ie^{2fx+2e}+2\,e^{fx+e}-i\right)e^{-fx-e}}\,(ix^4f^4+f^4x^4e^{fx+e}+8ix^3f^3-8f^3x^3e^{fx+e}+48ix^2f^2+48f^2x^2e^{fx+e}+192ixf}{\left(ie^{2fx+2e}+2\,e^{fx+e}-i\right)f^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+I*a*sinh(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*2^(1/2)*(a*(I*exp(2*f*x+2*e)+2*exp(f*x+e)-I)*exp(-f*x-e))^(1/2)/(I*exp(2*
f*x+2*e)+2*exp(f*x+e)-I)*(I*x^4*f^4+f^4*x^4*exp(f*x+e)+8*I*x^3*f^3-8*f^3*x^
3*exp(f*x+e)+48*I*x^2*f^2+48*f^2*x^2*exp(f*x+e)+192*I*x*f-192*f*x*exp(f*x+e
)+384*I+384*exp(f*x+e))*(exp(f*x+e)-I)/f^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)*x^4, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+I*a*sinh(f*x+e))**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(I*a*(sinh(e + f*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)*x^4, x)
```

Mupad [B]
time = 0.72, size = 149, normalized size = 0.82

$$\frac{\sqrt{2}\,\left(\mathrm{e}^{e+f\,x}+1{}\mathrm{i}\right)\,\sqrt{a\,\mathrm{e}^{-e-f\,x}\,{\left(\mathrm{e}^{e+f\,x}-\mathrm{i}\right)}^2\,1{}\mathrm{i}}\,\left(384\,\mathrm{e}^{e+f\,x}+f\,x\,192{}\mathrm{i}+f^2\,x^2\,48{}\mathrm{i}+f^3\,x^3\,8{}\mathrm{i}+f^4\,x^4\,1{}\mathrm{i}+48\,f^2\,x^2\,\mathrm{e}^{e+f\,x}-8\,f^3\,x^3\,\mathrm{e}^{e+f\,x}+f^4\,x^4\,\mathrm{e}^{e+f\,x}-192\,f\,x\,\mathrm{e}^{e+f\,x}+384{}\mathrm{i}\right)}{f^5\,\left(\mathrm{e}^{2\,e+2\,f\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + a*sinh(e + f*x)*1i)^(1/2),x)
```

```
[Out] (2^(1/2)*(exp(e + f*x) + 1i)*(a*exp(- e - f*x)*(exp(e + f*x) - 1i)^2*1i)^(1
/2)*(384*exp(e + f*x) + f*x*192i + f^2*x^2*48i + f^3*x^3*8i + f^4*x^4*1i +
48*f^2*x^2*exp(e + f*x) - 8*f^3*x^3*exp(e + f*x) + f^4*x^4*exp(e + f*x) - 1
92*f*x*exp(e + f*x) + 384i))/(f^5*(exp(2*e + 2*f*x) + 1))
```

3.119 $\int x^3 \sqrt{a+ia\sinh(e+fx)}\,dx$

Optimal. Leaf size=136

$$-\frac{96\sqrt{a+ia\sinh(e+fx)}}{f^4}-\frac{12x^2\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{48x\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f^3}+2$$

```
[Out] -96*(a+I*a*sinh(f*x+e))^(1/2)/f^4-12*x^2*(a+I*a*sinh(f*x+e))^(1/2)/f^2+48*x
*(a+I*a*sinh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f^3+2*x^3*(a+I*a*si
nh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f
```

Rubi [A]
time = 0.12, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3400, 3377, 2718}

$$-\frac{96\sqrt{a+ia\sinh(e+fx)}}{f^4}+\frac{48x\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{f^3}-\frac{12x^2\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{2x^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] (-96*Sqrt[a + I*a*Sinh[e + f*x]])/f^4 - (12*x^2*Sqrt[a + I*a*Sinh[e + f*x]]
)/f^2 + (48*x*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/f
^3 + (2*x^3*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/f
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x^3\sqrt{a+ia\sinh(e+fx)}\,dx = \left(\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int x^3\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right.$$

$$=\frac{2x^3\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f}-\frac{\left(6\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{\right.}{}$$

$$=-\frac{12x^2\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{2x^3\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}\right.}{f}$$

$$=-\frac{12x^2\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{48x\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}\right.}{f^3}$$

$$=-\frac{96\sqrt{a+ia\sinh(e+fx)}}{f^4}-\frac{12x^2\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{48x\sqrt{a+ia}}{}$$

Mathematica [A]
time = 0.17, size = 125, normalized size = 0.92

$$\frac{2\left(i(48i+24fx+6if^2x^2+f^3x^3)\cosh\left(\frac{1}{2}(e+fx)\right)+(-48i+24fx-6if^2x^2+f^3x^3)\sinh\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a+ia\sinh(e+fx)}}{f^4\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] (2*(I*(48*I + 24*f*x + (6*I)*f^2*x^2 + f^3*x^3)*Cosh[(e + f*x)/2] + (-48*I
+ 24*f*x - (6*I)*f^2*x^2 + f^3*x^3)*Sinh[(e + f*x)/2])*Sqrt[a + I*a*Sinh[e
+ f*x]])/(f^4*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))
```

Maple [A]
time = 0.24, size = 151, normalized size = 1.11

method	result
risch	$\frac{i\sqrt{2}\,\sqrt{a\left(ie^{2fx+2e}+2\,e^{fx+e}-i\right)e^{-fx-e}}\,(ix^3f^3+f^3x^3e^{fx+e}+6ix^2f^2-6f^2x^2e^{fx+e}+24ixf+24fx\,e^{fx+e}+48i-48\,e^{fx}}{\left(ie^{2fx+2e}+2\,e^{fx+e}-i\right)f^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+I*a*sinh(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*2^(1/2)*(a*(I*exp(2*f*x+2*e)+2*exp(f*x+e)-I)*exp(-f*x-e))^(1/2)/(I*exp(2*
f*x+2*e)+2*exp(f*x+e)-I)*(I*x^3*f^3+f^3*x^3*exp(f*x+e)+6*I*x^2*f^2-6*f^2*x^
2*exp(f*x+e)+24*I*x*f+24*f*x*exp(f*x+e)+48*I-48*exp(f*x+e))*(exp(f*x+e)-I)/
f^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)*x^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+I*a*sinh(f*x+e))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(I*a*(sinh(e + f*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)*x^3, x)
```

Mupad [B]
time = 0.36, size = 126, normalized size = 0.93

$$\frac{\sqrt{2}\,\left(\mathrm{e}^{e+f\,x}+1\mathrm{i}\right)\,\sqrt{a\,\mathrm{e}^{-e-f\,x}\,{\left(\mathrm{e}^{e+f\,x}-\mathrm{i}\right)}^2\,1\mathrm{i}}\,\left(f^3\,x^3\,\mathrm{e}^{e+f\,x}+f\,x\,24\mathrm{i}+f^2\,x^2\,6\mathrm{i}+f^3\,x^3\,1\mathrm{i}-6\,f^2\,x^2\,\mathrm{e}^{e+f\,x}-48\,\mathrm{e}^{e+f\,x}+24\,f\,x\,\mathrm{e}^{e+f\,x}+48\mathrm{i}\right)}{f^4\,\left(\mathrm{e}^{2\,e+2\,f\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + a*sinh(e + f*x)*1i)^(1/2),x)
```

```
[Out] (2^(1/2)*(exp(e + f*x) + 1i)*(a*exp(- e - f*x)*(exp(e + f*x) - 1i)^2*1i)^(1
/2)*(f*x*24i - 48*exp(e + f*x) + f^2*x^2*6i + f^3*x^3*1i - 6*f^2*x^2*exp(e
+ f*x) + f^3*x^3*exp(e + f*x) + 24*f*x*exp(e + f*x) + 48i))/(f^4*(exp(2*e +
 2*f*x) + 1))
```

3.120 $\int x^2 \sqrt{a+ia\sinh(e+fx)}\, dx$

Optimal. Leaf size=111

$$-\frac{8x\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{16\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f^3}+\frac{2x^2\sqrt{a+ia\sinh(e+fx)}\tanh}{f}$$

```
[Out] -8*x*(a+I*a*sinh(f*x+e))^(1/2)/f^2+16*(a+I*a*sinh(f*x+e))^(1/2)*tanh(1/2*e+
1/4*I*Pi+1/2*f*x)/f^3+2*x^2*(a+I*a*sinh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1
/2*f*x)/f
```

Rubi [A]

time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3400, 3377, 2718}

$$\frac{16\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{f^3}-\frac{8x\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{2x^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] (-8*x*Sqrt[a + I*a*Sinh[e + f*x]])/f^2 + (16*Sqrt[a + I*a*Sinh[e + f*x]]*Ta
nh[e/2 + (I/4)*Pi + (f*x)/2])/f^3 + (2*x^2*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh
[e/2 + (I/4)*Pi + (f*x)/2])/f
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x^2 \sqrt{a+ia\sinh(e+fx)}\,dx = \left(\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int x^2\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right.$$

$$= \frac{2x^2\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f} - \frac{\left(4\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{\right.}{}$$

$$= -\frac{8x\sqrt{a+ia\sinh(e+fx)}}{f^2} + \frac{2x^2\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\right.}{f}$$

$$= -\frac{8x\sqrt{a+ia\sinh(e+fx)}}{f^2} + \frac{16\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{f}{2}\right.}{f^3}$$

Mathematica [A]
time = 0.13, size = 105, normalized size = 0.95

$$\frac{2\left(i(8+4ifx+f^2x^2)\cosh\left(\frac{1}{2}(e+fx)\right)+(8-4ifx+f^2x^2)\sinh\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a+ia\sinh(e+fx)}}{f^3\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] (2*(I*(8 + (4*I)*f*x + f^2*x^2)*Cosh[(e + f*x)/2] + (8 - (4*I)*f*x + f^2*x^
2)*Sinh[(e + f*x)/2])*Sqrt[a + I*a*Sinh[e + f*x]])/(f^3*(Cosh[(e + f*x)/2]
+ I*Sinh[(e + f*x)/2]))
```

Maple [A]
time = 0.23, size = 128, normalized size = 1.15

method	result	size
risch	$\frac{i\sqrt{2}\,\sqrt{a\left(i\mathrm{e}^{2fx+2e}+2\,\mathrm{e}^{fx+e}-i\right)\mathrm{e}^{-fx-e}}\,\left(ix^2f^2+f^2x^2\mathrm{e}^{fx+e}+4ixf-4fx\,\mathrm{e}^{fx+e}+8i+8\,\mathrm{e}^{fx+e}\right)\left(\mathrm{e}^{fx+e}-i\right)}{\left(i\mathrm{e}^{2fx+2e}+2\,\mathrm{e}^{fx+e}-i\right)f^3}$	128

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+I*a*sinh(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*2^(1/2)*(a*(I*exp(2*f*x+2*e)+2*exp(f*x+e)-I)*exp(-f*x-e))^(1/2)/(I*exp(2*
f*x+2*e)+2*exp(f*x+e)-I)*(I*x^2*f^2+f^2*x^2*exp(f*x+e)+4*I*x*f-4*f*x*exp(f*
x+e)+8*I+8*exp(f*x+e))*(exp(f*x+e)-I)/f^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)*x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+I*a*sinh(f*x+e))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(I*a*(sinh(e + f*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)*x^2, x)
```

Mupad [B]
time = 0.29, size = 92, normalized size = 0.83

$$\frac{\sqrt{2}\,\sqrt{a\,\mathrm{e}^{-e-f\,x}\,{\left(\mathrm{e}^{e+f\,x}-\mathrm{i}\right)}^2\,1\mathrm{i}}\,\left(8\,\mathrm{e}^{e+f\,x}+f\,x\,4\mathrm{i}+f^2\,x^2\,1\mathrm{i}+f^2\,x^2\,\mathrm{e}^{e+f\,x}-4\,f\,x\,\mathrm{e}^{e+f\,x}+8\mathrm{i}\right)}{f^3\,\left(\mathrm{e}^{e+f\,x}-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + a*sinh(e + f*x)*1i)^(1/2),x)
```

```
[Out] (2^(1/2)*(a*exp(- e - f*x)*(exp(e + f*x) - 1i)^2*1i)^(1/2)*(8*exp(e + f*x)
+ f*x*4i + f^2*x^2*1i + f^2*x^2*exp(e + f*x) - 4*f*x*exp(e + f*x) + 8i))/(f
^3*(exp(e + f*x) - 1i))
```

3.121 $\int x\sqrt{a+ia\sinh(e+fx)}\,dx$

Optimal. Leaf size=66

$$-\frac{4\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{2x\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f}$$

```
[Out] -4*(a+I*a*sinh(f*x+e))^(1/2)/f^2+2*x*(a+I*a*sinh(f*x+e))^(1/2)*tanh(1/2*e+1
/4*I*Pi+1/2*f*x)/f
```

Rubi [A]
time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {3400, 3377, 2718}

$$\frac{2x\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{f}-\frac{4\sqrt{a+ia\sinh(e+fx)}}{f^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] (-4*Sqrt[a + I*a*Sinh[e + f*x]])/f^2 + (2*x*Sqrt[a + I*a*Sinh[e + f*x]]*Tan
h[e/2 + (I/4)*Pi + (f*x)/2])/f
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x\sqrt{a+ia\sinh(e+fx)}\,dx = \left(\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int x\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)$$

$$= \frac{2x\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f} - \frac{\left(2\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+\right.}{}$$

$$= -\frac{4\sqrt{a+ia\sinh(e+fx)}}{f^2} + \frac{2x\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f}$$

Mathematica [**A**]
time = 0.10, size = 87, normalized size = 1.32

$$\frac{2\left((-2+ifx)\cosh\left(\frac{1}{2}(e+fx)\right)+(-2i+fx)\sinh\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a+ia\sinh(e+fx)}}{f^2\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] (2*((-2 + I*f*x)*Cosh[(e + f*x)/2] + (-2*I + f*x)*Sinh[(e + f*x)/2])*Sqrt[a
 + I*a*Sinh[e + f*x]])/(f^2*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))
```

Maple [**A**]
time = 0.23, size = 105, normalized size = 1.59

method	result	size
risch	$\frac{i\sqrt{2}\,\sqrt{a\,(i\mathrm{e}^{2fx+2e}+2\,\mathrm{e}^{fx+e}-i)\,\mathrm{e}^{-fx-e}}\,(ixf+fx\,\mathrm{e}^{fx+e}+2i-2\,\mathrm{e}^{fx+e})\,(\mathrm{e}^{fx+e}-i)}{(i\mathrm{e}^{2fx+2e}+2\,\mathrm{e}^{fx+e}-i)\,f^2}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+I*a*sinh(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*2^(1/2)*(a*(I*exp(2*f*x+2*e)+2*exp(f*x+e)-I)*exp(-f*x-e))^(1/2)/(I*exp(2*
f*x+2*e)+2*exp(f*x+e)-I)*(I*x*f+f*x*exp(f*x+e)+2*I-2*exp(f*x+e))*(exp(f*x+e
)-I)/f^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)*x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(f*x+e))**(1/2),x)
```

```
[Out] Integral(x*sqrt(I*a*(sinh(e + f*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)*x, x)
```

Mupad [B]
time = 0.27, size = 80, normalized size = 1.21

$$\frac{\sqrt{2}\,\left(\mathrm{e}^{e+f\,x}+1\mathrm{i}\right)\,\left(f\,x\,\mathrm{e}^{e+f\,x}+f\,x\,1\mathrm{i}-2\,\mathrm{e}^{e+f\,x}+2\mathrm{i}\right)\,\sqrt{a\,\mathrm{e}^{-e-f\,x}\,{\left(\mathrm{e}^{e+f\,x}-\mathrm{i}\right)}^2\,1\mathrm{i}}}{f^2\,\left(\mathrm{e}^{2\,e+2\,f\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + a*sinh(e + f*x)*1i)^(1/2),x)
```

```
[Out] (2^(1/2)*(exp(e + f*x) + 1i)*(f*x*1i - 2*exp(e + f*x) + f*x*exp(e + f*x) +
2i)*(a*exp(- e - f*x)*(exp(e + f*x) - 1i)^2*1i)^(1/2))/(f^2*(exp(2*e + 2*f*
x) + 1))
```

3.122 $\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x}\,dx$

Optimal. Leaf size=125

$$i\text{Chi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(2e-i\pi)\right)\sqrt{a+ia\sinh(e+fx)}+i\cosh\left(\frac{1}{4}(2e-i\pi)\right)\text{sech}\left(\frac{e}{2}+\right.$$

```
[Out] sinh(1/2*e+1/4*I*Pi)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*Shi(1/2*f*x)*(a+I*a*sinh(
f*x+e))^(1/2)+Chi(1/2*f*x)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*cosh(1/2*e+1/4*I*Pi
)*(a+I*a*sinh(f*x+e))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3400, 3384, 3379, 3382}

$$i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\text{Chi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}+i\cosh\left(\frac{1}{4}(2e-i\pi)\right)\text{Shi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Sinh[e + f*x]]/x,x]
```

```
[Out] I*CoshIntegral[(f*x)/2]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[(2*e - I*Pi)/4]
*Sqrt[a + I*a*Sinh[e + f*x]] + I*Cosh[(2*e - I*Pi)/4]*Sech[e/2 + (I/4)*Pi +
 (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*SinhIntegral[(f*x)/2]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x}\,dx &= \left(\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int \frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)}{x}\,dx\\ &= \left(\cosh\left(\frac{1}{4}(2e-i\pi)\right)\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int \frac{\sinh}{}\\ &= i\operatorname{Chi}\left(\frac{fx}{2}\right)\operatorname{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(2e-i\pi)\right)\sqrt{a+ia\sinh(e+fx)}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 96, normalized size = 0.77

$$\frac{\sqrt{a+ia\sinh(e+fx)}\left(\operatorname{Chi}\left(\frac{fx}{2}\right)\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)+\left(i\cosh\left(\frac{e}{2}\right)+\sinh\left(\frac{e}{2}\right)\right)\operatorname{Shi}\left(\frac{fx}{2}\right)\right)}{\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Sinh[e + f*x]]/x,x]
```

```
[Out] (Sqrt[a + I*a*Sinh[e + f*x]]*(CoshIntegral[(f*x)/2]*(Cosh[e/2] + I*Sinh[e/2
]) + (I*Cosh[e/2] + Sinh[e/2])*SinhIntegral[(f*x)/2]))/(Cosh[(e + f*x)/2] +
 I*Sinh[(e + f*x)/2])
```

Maple [F]
time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+ia\sinh(fx+e)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))^(1/2)/x,x)
```

```
[Out] int((a+I*a*sinh(f*x+e))^(1/2)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)/x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(I*a*(sinh(e + f*x) - I))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\sinh\left(e+f\,x\right)\,1{}\mathrm{i}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(e + f*x)*1i)^(1/2)/x,x)
```

```
[Out] int((a + a*sinh(e + f*x)*1i)^(1/2)/x, x)
```

3.123 $\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^2}\,dx$

Optimal. Leaf size=149

$$-\frac{\sqrt{a+ia\sinh(e+fx)}}{x}+\frac{1}{2}f\text{Chi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(2e+i\pi)\right)\sqrt{a+ia\sinh(e+fx)}+$$

```
[Out] -(a+I*a*sinh(f*x+e))^(1/2)/x+1/2*f*cosh(1/2*e+1/4*I*Pi)*sech(1/2*e+1/4*I*Pi
+1/2*f*x)*Shi(1/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)+1/2*f*Chi(1/2*f*x)*sech(1/
2*e+1/4*I*Pi+1/2*f*x)*sinh(1/2*e+1/4*I*Pi)*(a+I*a*sinh(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3400, 3378, 3384, 3379, 3382}

$$\frac{1}{2}f\sinh\left(\frac{1}{4}(2e+i\pi)\right)\text{Chi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}+\frac{1}{2}f\cosh\left(\frac{1}{4}(2e+i\pi)\right)\text{Shi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}-\frac{\sqrt{a+ia\sinh(e+fx)}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Sinh[e + f*x]]/x^2,x]
```

```
[Out] -(Sqrt[a + I*a*Sinh[e + f*x]]/x) + (f*CoshIntegral[(f*x)/2]*Sech[e/2 + (I/4
)*Pi + (f*x)/2]*Sinh[(2*e + I*Pi)/4]*Sqrt[a + I*a*Sinh[e + f*x]])/2 + (f*Co
sh[(2*e + I*Pi)/4]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x
]]*SinhIntegral[(f*x)/2])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^2}\,dx &= \left(\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int \frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)}{x^2}\,dx\\ &= -\frac{\sqrt{a+ia\sinh(e+fx)}}{x}+\frac{1}{2}\left(f\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right.\\ &= -\frac{\sqrt{a+ia\sinh(e+fx)}}{x}-\frac{1}{2}\left(if\cosh\left(\frac{1}{4}(2e+i\pi)\right)\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\right.\\ &= -\frac{\sqrt{a+ia\sinh(e+fx)}}{x}+\frac{1}{2}f\operatorname{Chi}\left(\frac{fx}{2}\right)\operatorname{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(2e+\right.\end{aligned}$$

Mathematica [A]
time = 0.15, size = 133, normalized size = 0.89

$$\frac{\sqrt{a+ia\sinh(e+fx)}\left(fx\operatorname{Chi}\left(\frac{fx}{2}\right)\left(i\cosh\left(\frac{e}{2}\right)+\sinh\left(\frac{e}{2}\right)\right)-2\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)+fx\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)\operatorname{Shi}\left(\frac{fx}{2}\right)\right)}{2x\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Sinh[e + f*x]]/x^2,x]
```

```
[Out] (Sqrt[a + I*a*Sinh[e + f*x]]*(f*x*CoshIntegral[(f*x)/2]*(I*Cosh[e/2] + Sinh
[e/2]) - 2*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]) + f*x*(Cosh[e/2] + I*S
inh[e/2])*SinhIntegral[(f*x)/2]))/(2*x*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x
)/2]))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+ia\sinh\left(fx+e\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))^(1/2)/x^2,x)
```

```
[Out] int((a+I*a*sinh(f*x+e))^(1/2)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)/x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(I*a*(sinh(e + f*x) - I))/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(e + f*x)*1i)^(1/2)/x^2,x)
[Out] int((a + a*sinh(e + f*x)*1i)^(1/2)/x^2, x)
```

3.124 $\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^3}\,dx$

Optimal. Leaf size=204

$$-\frac{\sqrt{a+ia\sinh(e+fx)}}{2x^2}+\frac{1}{8}if^2\text{Chi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(2e-i\pi)\right)\sqrt{a+ia\sinh(e+fx)}$$

```
[Out] -1/2*(a+I*a*sinh(f*x+e))^(1/2)/x^2+1/8*f^2*sinh(1/2*e+1/4*I*Pi)*sech(1/2*e+
1/4*I*Pi+1/2*f*x)*Shi(1/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)+1/8*f^2*Chi(1/2*f*
x)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*cosh(1/2*e+1/4*I*Pi)*(a+I*a*sinh(f*x+e))^(1
/2)-1/4*f*(a+I*a*sinh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/x
```

Rubi [A]
time = 0.14, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3400, 3378, 3384, 3379, 3382}

$$\frac{1}{8}if^2\sinh\left(\frac{1}{4}(2e-i\pi)\right)\text{Chi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}+\frac{1}{8}if^2\cosh\left(\frac{1}{4}(2e-i\pi)\right)\text{Shi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}-\frac{\sqrt{a+ia\sinh(e+fx)}}{2x^2}-\frac{f\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Sinh[e + f*x]]/x^3,x]
```

```
[Out] -1/2*Sqrt[a + I*a*Sinh[e + f*x]]/x^2 + (I/8)*f^2*CoshIntegral[(f*x)/2]*Sech
[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[(2*e - I*Pi)/4]*Sqrt[a + I*a*Sinh[e + f*x]]
 + (I/8)*f^2*Cosh[(2*e - I*Pi)/4]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I
*a*Sinh[e + f*x]]*SinhIntegral[(f*x)/2] - (f*Sqrt[a + I*a*Sinh[e + f*x]]*Ta
nh[e/2 + (I/4)*Pi + (f*x)/2])/(4*x)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^3}\,dx &= \left(\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int \frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)}{x^3}\,dx \\
&= -\frac{\sqrt{a+ia\sinh(e+fx)}}{2x^2}+\frac{1}{4}\left(f\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right. \\
&= -\frac{\sqrt{a+ia\sinh(e+fx)}}{2x^2}-\frac{f\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{4x}-\frac{1}{8} \\
&= -\frac{\sqrt{a+ia\sinh(e+fx)}}{2x^2}-\frac{f\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{4x}+\frac{1}{8} \\
&= -\frac{\sqrt{a+ia\sinh(e+fx)}}{2x^2}+\frac{1}{8}if^2\operatorname{Chi}\left(\frac{fx}{2}\right)\operatorname{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(2e\right.
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 170, normalized size = 0.83

$$\frac{\sqrt{a+ia\sinh(e+fx)}\left(-4\cosh\left(\frac{1}{2}(e+fx)\right)-2ifx\cosh\left(\frac{1}{2}(e+fx)\right)+f^2x^2\operatorname{Chi}\left(\frac{fx}{2}\right)\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)-4i\sinh\left(\frac{1}{2}(e+fx)\right)-2fx\sinh\left(\frac{1}{2}(e+fx)\right)+f^2x^2\left(i\cosh\left(\frac{e}{2}\right)+\sinh\left(\frac{e}{2}\right)\right)\operatorname{Shi}\left(\frac{fx}{2}\right)\right)}{8x^2\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Sinh[e + f*x]]/x^3,x]
```

```
[Out] (Sqrt[a + I*a*Sinh[e + f*x]]*(-4*Cosh[(e + f*x)/2] - (2*I)*f*x*Cosh[(e + f*
x)/2] + f^2*x^2*CoshIntegral[(f*x)/2]*(Cosh[e/2] + I*Sinh[e/2]) - (4*I)*Sin
```

```
h[(e + f*x)/2] - 2*f*x*Sinh[(e + f*x)/2] + f^2*x^2*(I*Cosh[e/2] + Sinh[e/2]
)*SinhIntegral[(f*x)/2]))/(8*x^2*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))
```

Maple [**F**]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+ia\sinh\left(fx+e\right)}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))^(1/2)/x^3,x)
```

```
[Out] int((a+I*a*sinh(f*x+e))^(1/2)/x^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)/x^3, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(I*a*(sinh(e + f*x) - I))/x**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/2)/x^3,x, algorithm="giac")
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)/x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+a\sinh\left(e+f\,x\right)\,1{}\mathrm{i}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(e + f*x)*1i)^(1/2)/x^3,x)
[Out] int((a + a*sinh(e + f*x)*1i)^(1/2)/x^3, x)
```

3.125 $\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx$

Optimal. Leaf size=377

$$-\frac{1280a\sqrt{a+ia\sinh(e+fx)}}{9f^4}-\frac{16ax^2\sqrt{a+ia\sinh(e+fx)}}{f^2}-\frac{64a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{27f^4}$$

```
[Out] -1280/9*a*(a+I*a*sinh(f*x+e))^(1/2)/f^4-16*a*x^2*(a+I*a*sinh(f*x+e))^(1/2)/
f^2-64/27*a*cosh(1/2*e+1/4*I*Pi+1/2*f*x)^2*(a+I*a*sinh(f*x+e))^(1/2)/f^4-8/
3*a*x^2*cosh(1/2*e+1/4*I*Pi+1/2*f*x)^2*(a+I*a*sinh(f*x+e))^(1/2)/f^2+32/9*a
*x*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*sinh(1/2*e+1/4*I*Pi+1/2*f*x)*(a+I*a*sinh(f*
x+e))^(1/2)/f^3+4/3*a*x^3*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*sinh(1/2*e+1/4*I*Pi+
1/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)/f+640/9*a*x*(a+I*a*sinh(f*x+e))^(1/2)*ta
nh(1/2*e+1/4*I*Pi+1/2*f*x)/f^3+8/3*a*x^3*(a+I*a*sinh(f*x+e))^(1/2)*tanh(1/2
*e+1/4*I*Pi+1/2*f*x)/f
```

Rubi [A]
time = 0.25, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.238$, Rules used = {3400, 3392, 3377, 2718, 3391}

$$-\frac{1280a\sqrt{a+ia\sinh(e+fx)}}{9f^4}-\frac{64a\cosh^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{27f^4}+\frac{640ax\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^3}+\frac{32ax\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^3}-\frac{16ax^2\sqrt{a+ia\sinh(e+fx)}}{f^2}-\frac{8ax^2\cosh^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{3f^2}+\frac{8ax^3\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{3f}+\frac{4ax^3\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] (-1280*a*Sqrt[a + I*a*Sinh[e + f*x]])/(9*f^4) - (16*a*x^2*Sqrt[a + I*a*Sinh
[e + f*x]])/f^2 - (64*a*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*a*Sinh[
e + f*x]])/(27*f^4) - (8*a*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*
a*Sinh[e + f*x]])/(3*f^2) + (32*a*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[e/2
 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]])/(9*f^3) + (4*a*x^3*Cosh
[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh
[e + f*x]])/(3*f) + (640*a*x*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*P
i + (f*x)/2])/(9*f^3) + (8*a*x^3*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/
4)*Pi + (f*x)/2])/(3*f)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx = -\left(\left(2a\text{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int x^3\sinh^3\left(\frac{e}{2}-\frac{i\pi}{4}\right.\right.$$

$$= -\frac{8ax^2\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{3f^2}+\frac{4ax^3\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\right.}{}$$

$$= -\frac{64a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{27f^4}-\frac{8ax^2\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\right.}{}$$

$$= -\frac{16ax^2\sqrt{a+ia\sinh(e+fx)}}{f^2}-\frac{64a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e}{27f^4}$$

$$= -\frac{128a\sqrt{a+ia\sinh(e+fx)}}{9f^4}-\frac{16ax^2\sqrt{a+ia\sinh(e+fx)}}{f^2}-\frac{64a\cosh}{}$$

$$= -\frac{1280a\sqrt{a+ia\sinh(e+fx)}}{9f^4}-\frac{16ax^2\sqrt{a+ia\sinh(e+fx)}}{f^2}-\frac{64a\cos}{}$$

Mathematica [**A**]
time = 0.60, size = 269, normalized size = 0.71

$$-\frac{a(-i+\sinh(e+fx))\sqrt{a+ia\sinh(e+fx)}\left(81(48i+24fx+6if^2x^2+f^3x^3)\cosh\left(\frac{1}{2}(e+fx)\right)+(-16i+24fx-18if^2x^2+9f^3x^3)\cosh\left(\frac{3}{2}(e+fx)\right)-3888\sinh\left(\frac{1}{2}(e+fx)\right)-1944ifx\sinh\left(\frac{1}{2}(e+fx)\right)-486f^2x^2\sinh\left(\frac{1}{2}(e+fx)\right)-81if^3x^3\sinh\left(\frac{1}{2}(e+fx)\right)-16\sinh\left(\frac{3}{2}(e+fx)\right)+24ifx\sinh\left(\frac{3}{2}(e+fx)\right)-18f^2x^2\sinh\left(\frac{3}{2}(e+fx)\right)+9if^3x^3\sinh\left(\frac{3}{2}(e+fx)\right)\right)}{27f^4\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] -1/27*(a*(-I + Sinh[e + f*x])*Sqrt[a + I*a*Sinh[e + f*x]]*(81*(48*I + 24*f*
x + (6*I)*f^2*x^2 + f^3*x^3)*Cosh[(e + f*x)/2] + (-16*I + 24*f*x - (18*I)*f
^2*x^2 + 9*f^3*x^3)*Cosh[(3*(e + f*x))/2] - 3888*Sinh[(e + f*x)/2] - (1944*
I)*f*x*Sinh[(e + f*x)/2] - 486*f^2*x^2*Sinh[(e + f*x)/2] - (81*I)*f^3*x^3*S
inh[(e + f*x)/2] - 16*Sinh[(3*(e + f*x))/2] + (24*I)*f*x*Sinh[(3*(e + f*x))
/2] - 18*f^2*x^2*Sinh[(3*(e + f*x))/2] + (9*I)*f^3*x^3*Sinh[(3*(e + f*x))/2
]))/(f^4*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^3)
```

Maple [**F**]
time = 0.16, size = 0, normalized size = 0.00

$$\int x^3(a+ia\sinh(fx+e))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+I*a*sinh(f*x+e))^(3/2),x)
```

```
[Out] int(x^3*(a+I*a*sinh(f*x+e))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)*x^3, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}(ia(\sinh{(e+fx)}-i))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+I*a*sinh(f*x+e))**(3/2),x)
```

```
[Out] Integral(x**3*(I*a*(sinh(e + f*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + a*sinh(e + f*x)*1i)^(3/2),x)
```

```
[Out] int(x^3*(a + a*sinh(e + f*x)*1i)^(3/2), x)
```

3.126 $\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx$

Optimal. Leaf size=303

$$-\frac{32ax\sqrt{a+ia\sinh(e+fx)}}{3f^2}-\frac{16ax\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^2}+\frac{4ax^2\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{3f}$$

```
[Out] -32/3*a*x*(a+I*a*sinh(f*x+e))^(1/2)/f^2-16/9*a*x*cosh(1/2*e+1/4*I*Pi+1/2*f*
x)^2*(a+I*a*sinh(f*x+e))^(1/2)/f^2+4/3*a*x^2*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*s
inh(1/2*e+1/4*I*Pi+1/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)/f+224/9*a*(a+I*a*sinh
(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f^3+8/3*a*x^2*(a+I*a*sinh(f*x+e
))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f+32/27*a*sinh(1/2*e+1/4*I*Pi+1/2*f*x
)^2*(a+I*a*sinh(f*x+e))^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f^3
```

Rubi [A]

time = 0.18, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3400, 3392, 3377, 2718, 2713}

$$\frac{32a\sinh^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{27f^3}+\frac{224a\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^3}-\frac{32ax\sqrt{a+ia\sinh(e+fx)}}{3f^2}-\frac{16ax\cosh^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^2}+\frac{8ax^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{3f}+\frac{4ax^2\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] (-32*a*x*Sqrt[a + I*a*Sinh[e + f*x]])/(3*f^2) - (16*a*x*Cosh[e/2 + (I/4)*Pi
 + (f*x)/2]^2*Sqrt[a + I*a*Sinh[e + f*x]])/(9*f^2) + (4*a*x^2*Cosh[e/2 + (I
/4)*Pi + (f*x)/2]*Sinh[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]
])/(3*f) + (224*a*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2
])/(9*f^3) + (8*a*x^2*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*
x)/2])/(3*f) + (32*a*Sinh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*a*Sinh[e +
 f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(27*f^3)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx = -\left(\left(2a\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int x^2\sinh^3\left(\frac{e}{2}-\frac{i\pi}{4}\right.\right.$$

$$= -\frac{16ax\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^2}+\frac{4ax^2\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\right.}{}$$

$$= -\frac{16ax\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^2}+\frac{4ax^2\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\right.}{}$$

$$= -\frac{32ax\sqrt{a+ia\sinh(e+fx)}}{3f^2}-\frac{16ax\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e}}{9f^2}$$

$$= -\frac{32ax\sqrt{a+ia\sinh(e+fx)}}{3f^2}-\frac{16ax\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e}}{9f^2}$$

Mathematica [A]
time = 0.52, size = 173, normalized size = 0.57

$$-\frac{a\left(81(8+4ifx+f^2x^2)\cosh\left(\frac{1}{2}(e+fx)\right)+(8-12ifx+9f^2x^2)\cosh\left(\frac{3}{2}(e+fx)\right)+2i(-4(80-42ifx+9f^2x^2)+(8+12ifx+9f^2x^2)\cosh(e+fx))\sinh\left(\frac{1}{2}(e+fx)\right)\right)(-i+\sinh(e+fx))\sqrt{a+ia\sinh(e+fx)}}{27f^3\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] -1/27*(a*(81*(8 + (4*I)*f*x + f^2*x^2)*Cosh[(e + f*x)/2] + (8 - (12*I)*f*x
+ 9*f^2*x^2)*Cosh[(3*(e + f*x))/2] + (2*I)*(-4*(80 - (42*I)*f*x + 9*f^2*x^2
) + (8 + (12*I)*f*x + 9*f^2*x^2)*Cosh[e + f*x])*Sinh[(e + f*x)/2])*(-I + Si
nh[e + f*x])*Sqrt[a + I*a*Sinh[e + f*x]])/(f^3*(Cosh[(e + f*x)/2] + I*Sinh[
(e + f*x)/2])^3)
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int x^{2}(a+ia\sinh(fx+e))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+I*a*sinh(f*x+e))^(3/2),x)
```

```
[Out] int(x^2*(a+I*a*sinh(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)*x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}(ia(\sinh(e+fx)-i))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+I*a*sinh(f*x+e))**(3/2),x)
```

```
[Out] Integral(x**2*(I*a*(sinh(e + f*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1\mathrm{i}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + a*sinh(e + f*x)*1i)^(3/2),x)
```

```
[Out] int(x^2*(a + a*sinh(e + f*x)*1i)^(3/2), x)
```

3.127 $\int x(a+ia\sinh(e+fx))^{3/2}\,dx$

Optimal. Leaf size=185

$$-\frac{16a\sqrt{a+ia\sinh(e+fx)}}{3f^2}-\frac{8a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^2}+\frac{4ax\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\right.}{}$$

```
[Out] -16/3*a*(a+I*a*sinh(f*x+e))^(1/2)/f^2-8/9*a*cosh(1/2*e+1/4*I*Pi+1/2*f*x)^2*
(a+I*a*sinh(f*x+e))^(1/2)/f^2+4/3*a*x*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*sinh(1/2
*e+1/4*I*Pi+1/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)/f+8/3*a*x*(a+I*a*sinh(f*x+e)
)^(1/2)*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/f
```

Rubi [A]
time = 0.10, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3400, 3391, 3377, 2718}

$$-\frac{16a\sqrt{a+ia\sinh(e+fx)}}{3f^2}-\frac{8a\cosh^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^2}+\frac{8ax\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{3f}+\frac{4ax\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] (-16*a*Sqrt[a + I*a*Sinh[e + f*x]])/(3*f^2) - (8*a*Cosh[e/2 + (I/4)*Pi + (f
*x)/2]^2*Sqrt[a + I*a*Sinh[e + f*x]])/(9*f^2) + (4*a*x*Cosh[e/2 + (I/4)*Pi
+ (f*x)/2]*Sinh[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]])/(3*f
) + (8*a*x*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*f
)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x(a+ia\sinh(e+fx))^{3/2}\,dx = -\left(\left(2a\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int x\sinh^3\left(\frac{e}{2}-\frac{i\pi}{4}\right.\right.$$
$$= -\frac{8a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^2}+\frac{4ax\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{}$$
$$= -\frac{8a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^2}+\frac{4ax\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{}$$
$$= -\frac{16a\sqrt{a+ia\sinh(e+fx)}}{3f^2}-\frac{8a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+f}}{9f^2}$$

Mathematica [A]
time = 0.36, size = 138, normalized size = 0.75

$$-\frac{a\left(27(2i+fx)\cosh\left(\frac{1}{2}(e+fx)\right)+(-2i+3fx)\cosh\left(\frac{3}{2}(e+fx)\right)+2i(28i-12fx+(2i+3fx)\cosh(e+fx))\sinh\left(\frac{1}{2}(e+fx)\right)\right)(-i+\sinh(e+fx))\sqrt{a+ia\sinh(e+fx)}}{9f^2\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] -1/9*(a*(27*(2*I + f*x)*Cosh[(e + f*x)/2] + (-2*I + 3*f*x)*Cosh[(3*(e + f*x
))/2] + (2*I)*(28*I - 12*f*x + (2*I + 3*f*x)*Cosh[e + f*x])*Sinh[(e + f*x)/
2])*(-I + Sinh[e + f*x])*Sqrt[a + I*a*Sinh[e + f*x]])/(f^2*(Cosh[(e + f*x)/
2] + I*Sinh[(e + f*x)/2])^3)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int x(a+ia\sinh(fx+e))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+I*a*sinh(f*x+e))^(3/2),x)
```

```
[Out] int(x*(a+I*a*sinh(f*x+e))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)*x, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(ia(\sinh{(e+fx)}-i))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(f*x+e))**(3/2),x)
[Out] Integral(x*(I*a*(sinh(e + f*x) - I))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)*x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + a*sinh(e + f*x)*1i)^(3/2),x)
[Out] int(x*(a + a*sinh(e + f*x)*1i)^(3/2), x)
```

3.128 $\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x}\,dx$

Optimal. Leaf size=261

$$\frac{3}{2}ia\text{Chi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(2e-i\pi)\right)\sqrt{a+ia\sinh(e+fx)}+\frac{1}{2}ia\text{Chi}\left(\frac{3fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{i\pi}{4}\right.$$

```
[Out] 3/2*a*sinh(1/2*e+1/4*I*Pi)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*Shi(1/2*f*x)*(a+I*a
*sinh(f*x+e))^(1/2)+1/2*I*a*cosh(3/2*e+1/4*I*Pi)*sech(1/2*e+1/4*I*Pi+1/2*f*
x)*Shi(3/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)+3/2*a*Chi(1/2*f*x)*sech(1/2*e+1/4
*I*Pi+1/2*f*x)*cosh(1/2*e+1/4*I*Pi)*(a+I*a*sinh(f*x+e))^(1/2)+1/2*I*a*Chi(3
/2*f*x)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*sinh(3/2*e+1/4*I*Pi)*(a+I*a*sinh(f*x+e
))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3400, 3393, 3384, 3379, 3382}

$$\frac{3}{2}ia\sinh\left(\frac{1}{4}(2e-i\pi)\right)\text{Chi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}+\frac{1}{2}ia\sinh\left(\frac{1}{4}(6e+i\pi)\right)\text{Chi}\left(\frac{3fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}+\frac{3}{2}ia\cosh\left(\frac{1}{4}(2e-i\pi)\right)\text{Shi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}+\frac{1}{2}ia\cosh\left(\frac{1}{4}(6e+i\pi)\right)\text{Shi}\left(\frac{3fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[e + f*x])^(3/2)/x,x]
```

```
[Out] ((3*I)/2)*a*CoshIntegral[(f*x)/2]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[(2*e
- I*Pi)/4]*Sqrt[a + I*a*Sinh[e + f*x]] + (I/2)*a*CoshIntegral[(3*f*x)/2]*Se
ch[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[(6*e + I*Pi)/4]*Sqrt[a + I*a*Sinh[e + f*x
]] + ((3*I)/2)*a*Cosh[(2*e - I*Pi)/4]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a
 + I*a*Sinh[e + f*x]]*SinhIntegral[(f*x)/2] + (I/2)*a*Cosh[(6*e + I*Pi)/4]*
Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*SinhIntegral[(3*
f*x)/2]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x}\,dx &= -\left(\left(2a\text{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int \frac{\sinh^3\left(\frac{e}{2}-\frac{i\pi}{4}+\right.}{x}\right.\\
&= -\left(\left(2ia\text{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int \left(\frac{3i\sinh\left(\frac{1}{4}(2e\right.}{4x}\right.\right.\\
&= \frac{1}{2}\left(a\text{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int \frac{\sinh\left(\frac{1}{4}(6e+i\pi)+\right.}{x}\\
&= \frac{1}{2}\left(3a\cosh\left(\frac{1}{4}(2e-i\pi)\right)\text{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int\\
&= \frac{3}{2}ia\text{Chi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(2e-i\pi)\right)\sqrt{a+ia\sinh(e+}
\end{aligned}$$

Mathematica [A]
time = 0.35, size = 146, normalized size = 0.56

$$\frac{a\sqrt{a+ia\sinh(e+fx)}\left(3\text{Chi}\left(\frac{fx}{2}\right)\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)-\text{Chi}\left(\frac{3fx}{2}\right)\left(\cosh\left(\frac{3e}{2}\right)-i\sinh\left(\frac{3e}{2}\right)\right)+\left(i\cosh\left(\frac{e}{2}\right)+\sinh\left(\frac{e}{2}\right)\right)\left(3\text{Shi}\left(\frac{fx}{2}\right)+(1+2i\sinh(e))\text{Shi}\left(\frac{3fx}{2}\right)\right)\right)}{2\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[e + f*x])^(3/2)/x,x]
```

```
[Out] (a*Sqrt[a + I*a*Sinh[e + f*x]]*(3*CoshIntegral[(f*x)/2]*(Cosh[e/2] + I*Sinh
[e/2]) - CoshIntegral[(3*f*x)/2]*(Cosh[(3*e)/2] - I*Sinh[(3*e)/2]) + (I*Cos
h[e/2] + Sinh[e/2])*(3*SinhIntegral[(f*x)/2] + (1 + (2*I)*Sinh[e])*SinhInte
gral[(3*f*x)/2])))/(2*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(a + ia \sinh(fx + e))^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))^(3/2)/x,x)
```

```
[Out] int((a+I*a*sinh(f*x+e))^(3/2)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)/x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\sinh(e + fx) - i))^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))**(3/2)/x,x)
```

```
[Out] Integral((I*a*(sinh(e + f*x) - I))**(3/2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(3/2)/x,x, algorithm="giac")
[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+a\sinh(e+f\,x)\ 1\mathrm{i})^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(e + f*x)*1i)^(3/2)/x,x)
[Out] int((a + a*sinh(e + f*x)*1i)^(3/2)/x, x)
```

3.129 $\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x^2}\,dx$

Optimal. Leaf size=302

$$-\frac{2a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{x}-\frac{3}{4}af\,\text{Chi}\left(\frac{3fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(6e-i\pi)\right)\sqrt{}$$

```
[Out] -2*a*cosh(1/2*e+1/4*I*Pi+1/2*f*x)^2*(a+I*a*sinh(f*x+e))^(1/2)/x+3/4*a*f*cos
h(1/2*e+1/4*I*Pi)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*Shi(1/2*f*x)*(a+I*a*sinh(f*x
+e))^(1/2)+3/4*I*a*f*sinh(3/2*e+1/4*I*Pi)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*Shi(
3/2*f*x)*(a+I*a*sinh(f*x+e))^(1/2)+3/4*I*a*f*Chi(3/2*f*x)*sech(1/2*e+1/4*I*
Pi+1/2*f*x)*cosh(3/2*e+1/4*I*Pi)*(a+I*a*sinh(f*x+e))^(1/2)+3/4*a*f*Chi(1/2*
f*x)*sech(1/2*e+1/4*I*Pi+1/2*f*x)*sinh(1/2*e+1/4*I*Pi)*(a+I*a*sinh(f*x+e))^
(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3400, 3394, 3384, 3379, 3382}

$$-\frac{3}{4}af\sinh\left(\frac{1}{4}(6e-i\pi)\right)\text{Chi}\left(\frac{3fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}+\frac{3}{4}af\sinh\left(\frac{1}{4}(2e+i\pi)\right)\text{Chi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}+\frac{3}{4}af\cosh\left(\frac{1}{4}(2e+i\pi)\right)\text{Shi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}-\frac{3}{4}af\cosh\left(\frac{1}{4}(6e-i\pi)\right)\text{Shi}\left(\frac{3fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}-\frac{2a\cosh^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[e + f*x])^(3/2)/x^2,x]
```

```
[Out] (-2*a*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*a*Sinh[e + f*x]])/x - (3*
a*f*CoshIntegral[(3*f*x)/2]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[(6*e - I*Pi
)/4]*Sqrt[a + I*a*Sinh[e + f*x]])/4 + (3*a*f*CoshIntegral[(f*x)/2]*Sech[e/2
 + (I/4)*Pi + (f*x)/2]*Sinh[(2*e + I*Pi)/4]*Sqrt[a + I*a*Sinh[e + f*x]])/4
+ (3*a*f*Cosh[(2*e + I*Pi)/4]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*S
inh[e + f*x]]*SinhIntegral[(f*x)/2])/4 - (3*a*f*Cosh[(6*e - I*Pi)/4]*Sech[e
/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*SinhIntegral[(3*f*x)/2
])/4
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x^2}\,dx &= -\left(\left(2a\text{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int \frac{\sinh^3\left(\frac{e}{2}-\frac{i\pi}{4}+\cdots\right.}{x^2}\right.\\ &= -\frac{2a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{x}+\left(3af\text{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\cdots\right.\right.\\ &= -\frac{2a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{x}+\frac{1}{4}\left(3af\text{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\cdots\right.\right.\\ &= -\frac{2a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{x}+\frac{1}{4}\left(3iaf\cosh\left(\frac{1}{4}(6e-\cdots\right.\right.\\ &= -\frac{2a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{x}-\frac{3}{4}af\text{Chi}\left(\frac{3fx}{2}\right)\text{sech}\left(\cdots\right.\end{aligned}$$

Mathematica [A]
time = 0.43, size = 243, normalized size = 0.80

$$\frac{a(-i+\sinh(e+fx))\sqrt{a+ia\sinh(e+fx)}\left(-6i\cosh\left(\frac{1}{2}(e+fx)\right)+2i\cosh\left(\frac{3}{2}(e+fx)\right)-3fx\text{Chi}\left(\frac{fx}{2}\right)\left(\cosh\left(\frac{e}{2}\right)-i\sinh\left(\frac{e}{2}\right)\right)-3fx\text{Chi}\left(\frac{3fx}{2}\right)\left(\cosh\left(\frac{3e}{2}\right)+i\sinh\left(\frac{3e}{2}\right)\right)+6\sinh\left(\frac{1}{2}(e+fx)\right)+2\sinh\left(\frac{3}{2}(e+fx)\right)+3ifx\cosh\left(\frac{e}{2}\right)\text{Shi}\left(\frac{fx}{2}\right)-3fx\sinh\left(\frac{e}{2}\right)\text{Shi}\left(\frac{fx}{2}\right)-3ifx\cosh\left(\frac{3e}{2}\right)\text{Shi}\left(\frac{3fx}{2}\right)-3fx\sinh\left(\frac{3e}{2}\right)\text{Shi}\left(\frac{3fx}{2}\right)\right)}{4x\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[e + f*x])^(3/2)/x^2,x]
```

```
[Out] (a*(-I + Sinh[e + f*x])*Sqrt[a + I*a*Sinh[e + f*x]]*((-6*I)*Cosh[(e + f*x)/
2] + (2*I)*Cosh[(3*(e + f*x))/2] - 3*f*x*CoshIntegral[(f*x)/2]*(Cosh[e/2] -
 I*Sinh[e/2]) - 3*f*x*CoshIntegral[(3*f*x)/2]*(Cosh[(3*e)/2] + I*Sinh[(3*e)
/2]) + 6*Sinh[(e + f*x)/2] + 2*Sinh[(3*(e + f*x))/2] + (3*I)*f*x*Cosh[e/2]*
SinhIntegral[(f*x)/2] - 3*f*x*Sinh[e/2]*SinhIntegral[(f*x)/2] - (3*I)*f*x*C
osh[(3*e)/2]*SinhIntegral[(3*f*x)/2] - 3*f*x*Sinh[(3*e)/2]*SinhIntegral[(3*
f*x)/2]))/(4*x*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^3)
```

Maple [**F**]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(a + ia\sinh(fx + e))^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))^(3/2)/x^2,x)
```

```
[Out] int((a+I*a*sinh(f*x+e))^(3/2)/x^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)/x^2, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\sinh(e + fx) - i))^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))**(3/2)/x**2,x)
```

```
[Out] Integral((I*a*(sinh(e + f*x) - I))**(3/2)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + a\sinh(e + f\,x)\ 1\mathrm{i})^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(e + f*x)*1i)^(3/2)/x^2,x)
```

```
[Out] int((a + a*sinh(e + f*x)*1i)^(3/2)/x^2, x)
```

3.130 $\int x^3(a + ia\sinh(c + dx))^{5/2}\,dx$

Optimal. Leaf size=638

$$-\frac{265216a^2\sqrt{a+ia\sinh(c+dx)}}{1125d^4}-\frac{128a^2x^2\sqrt{a+ia\sinh(c+dx)}}{5d^2}-\frac{17408a^2\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sin}}{3375d^4}$$

```
[Out] -265216/1125*a^2*(a+I*a*sinh(d*x+c))^(1/2)/d^4-128/5*a^2*x^2*(a+I*a*sinh(d*
x+c))^(1/2)/d^2-17408/3375*a^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^2*(a+I*a*sinh(d
*x+c))^(1/2)/d^4-64/15*a^2*x^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^2*(a+I*a*sinh(d
*x+c))^(1/2)/d^2-384/625*a^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^4*(a+I*a*sinh(d*x
+c))^(1/2)/d^4-48/25*a^2*x^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^4*(a+I*a*sinh(d*x
+c))^(1/2)/d^2+8704/1125*a^2*x*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*sinh(1/2*c+1/4*
I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)/d^3+32/15*a^2*x^3*cosh(1/2*c+1/4*I*
Pi+1/2*d*x)*sinh(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)/d+192/12
5*a^2*x*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^3*sinh(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*
sinh(d*x+c))^(1/2)/d^3+8/5*a^2*x^3*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^3*sinh(1/2*
c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)/d+132608/1125*a^2*x*(a+I*a*si
nh(d*x+c))^(1/2)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/d^3+64/15*a^2*x^3*(a+I*a*sinh
(d*x+c))^(1/2)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/d
```

Rubi [A]
time = 0.44, antiderivative size = 638, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3400, 3392, 3377, 2718, 3391}

$$-\frac{265216a^2\sqrt{a+ia\sinh(c+dx)}}{1125d^4}-\frac{384a^2\cosh^4\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{625d^4}-\frac{17408a^2\cosh^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{3375d^4}+\frac{132608a^2x\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{1125d^3}+\frac{192a^2x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{125d^3}+\frac{8704a^2x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{1125d^3}-\frac{128a^2x^2\sqrt{a+ia\sinh(c+dx)}}{5d^2}-\frac{48a^2x^2\cosh^4\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{25d^2}-\frac{64a^2x^2\cosh^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{15d^2}+\frac{64a^2x^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{15d}+\frac{8a^2x^3\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{5d}+\frac{32a^2x^3\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (-265216*a^2*Sqrt[a + I*a*Sinh[c + d*x]])/(1125*d^4) - (128*a^2*x^2*Sqrt[a
+ I*a*Sinh[c + d*x]])/(5*d^2) - (17408*a^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^2
*Sqrt[a + I*a*Sinh[c + d*x]])/(3375*d^4) - (64*a^2*x^2*Cosh[c/2 + (I/4)*Pi
+ (d*x)/2]^2*Sqrt[a + I*a*Sinh[c + d*x]])/(15*d^2) - (384*a^2*Cosh[c/2 + (I
/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]])/(625*d^4) - (48*a^2*x^2*Co
sh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]])/(25*d^2) + (870
4*a^2*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[
a + I*a*Sinh[c + d*x]])/(1125*d^3) + (32*a^2*x^3*Cosh[c/2 + (I/4)*Pi + (d*x
)/2]*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]])/(15*d) + (
192*a^2*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^3*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*S
qrt[a + I*a*Sinh[c + d*x]])/(125*d^3) + (8*a^2*x^3*Cosh[c/2 + (I/4)*Pi + (d
*x)/2]^3*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]])/(5*d)
+ (132608*a^2*x*Sqrt[a + I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])
/(1125*d^3) + (64*a^2*x^3*Sqrt[a + I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi +
 (d*x)/2])/(15*d)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx = \left(4a^2\text{csch}\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\right)\int x^3\sinh^5\left(\frac{c}{2}-\frac{i\pi}{4}+\right.$$

$$= -\frac{48a^2x^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{25d^2}+\frac{8a^2x^3\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}\right.}{}$$

$$= -\frac{64a^2x^2\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{15d^2}-\frac{384a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}\right.}{}$$

$$= -\frac{17408a^2\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{3375d^4}-\frac{64a^2x^2\cosh^2\left(\frac{c}{2}+\right.}{}$$

$$= -\frac{128a^2x^2\sqrt{a+ia\sinh(c+dx)}}{5d^2}-\frac{17408a^2\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\,\text{s}}}{3375d^4}$$

$$= -\frac{34816a^2\sqrt{a+ia\sinh(c+dx)}}{1125d^4}-\frac{128a^2x^2\sqrt{a+ia\sinh(c+dx)}}{5d^2}-\frac{174}{}$$

$$= -\frac{265216a^2\sqrt{a+ia\sinh(c+dx)}}{1125d^4}-\frac{128a^2x^2\sqrt{a+ia\sinh(c+dx)}}{5d^2}-\frac{17}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2918 vs. $2(638) = 1276$.
time = 6.32, size = 2918, normalized size = 4.57

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (2*(((-1/135000 - I/135000)*Cosh[5*(c/2 + (d*x)/2)])/d^3 + ((1/135000 + I/1
35000)*Sinh[5*(c/2 + (d*x)/2)])/d^3)*(1296*I - (3240*I)*c + (4050*I)*c^2 -
(3375*I)*c^3 + (6480*I)*(c/2 + (d*x)/2) - (16200*I)*c*(c/2 + (d*x)/2) + (20
250*I)*c^2*(c/2 + (d*x)/2) + (16200*I)*(c/2 + (d*x)/2)^2 - (40500*I)*c*(c/2
 + (d*x)/2)^2 + (27000*I)*(c/2 + (d*x)/2)^3 - 50000*Cosh[2*(c/2 + (d*x)/2)]
 + 75000*c*Cosh[2*(c/2 + (d*x)/2)] - 56250*c^2*Cosh[2*(c/2 + (d*x)/2)] + 28
125*c^3*Cosh[2*(c/2 + (d*x)/2)] - 150000*(c/2 + (d*x)/2)*Cosh[2*(c/2 + (d*x
)/2)] + 225000*c*(c/2 + (d*x)/2)*Cosh[2*(c/2 + (d*x)/2)] - 168750*c^2*(c/2
+ (d*x)/2)*Cosh[2*(c/2 + (d*x)/2)] - 225000*(c/2 + (d*x)/2)^2*Cosh[2*(c/2 +
 (d*x)/2)] + 337500*c*(c/2 + (d*x)/2)^2*Cosh[2*(c/2 + (d*x)/2)] - 225000*(c
/2 + (d*x)/2)^3*Cosh[2*(c/2 + (d*x)/2)] - (8100000*I)*Cosh[4*(c/2 + (d*x)/2
)] + (4050000*I)*c*Cosh[4*(c/2 + (d*x)/2)] - (1012500*I)*c^2*Cosh[4*(c/2 +
(d*x)/2)] + (168750*I)*c^3*Cosh[4*(c/2 + (d*x)/2)] - (8100000*I)*(c/2 + (d*
x)/2)*Cosh[4*(c/2 + (d*x)/2)] + (4050000*I)*c*(c/2 + (d*x)/2)*Cosh[4*(c/2 +
```

```
 (d*x)/2)] - (1012500*I)*c^2*(c/2 + (d*x)/2)*Cosh[4*(c/2 + (d*x)/2)] - (405
0000*I)*(c/2 + (d*x)/2)^2*Cosh[4*(c/2 + (d*x)/2)] + (2025000*I)*c*(c/2 + (d
*x)/2)^2*Cosh[4*(c/2 + (d*x)/2)] - (1350000*I)*(c/2 + (d*x)/2)^3*Cosh[4*(c/
2 + (d*x)/2)] + 8100000*Cosh[6*(c/2 + (d*x)/2)] + 4050000*c*Cosh[6*(c/2 + (
d*x)/2)] + 1012500*c^2*Cosh[6*(c/2 + (d*x)/2)] + 168750*c^3*Cosh[6*(c/2 + (
d*x)/2)] - 8100000*(c/2 + (d*x)/2)*Cosh[6*(c/2 + (d*x)/2)] - 4050000*c*(c/2
 + (d*x)/2)*Cosh[6*(c/2 + (d*x)/2)] - 1012500*c^2*(c/2 + (d*x)/2)*Cosh[6*(c
/2 + (d*x)/2)] + 4050000*(c/2 + (d*x)/2)^2*Cosh[6*(c/2 + (d*x)/2)] + 202500
0*c*(c/2 + (d*x)/2)^2*Cosh[6*(c/2 + (d*x)/2)] - 1350000*(c/2 + (d*x)/2)^3*C
osh[6*(c/2 + (d*x)/2)] + (50000*I)*Cosh[8*(c/2 + (d*x)/2)] + (75000*I)*c*Co
sh[8*(c/2 + (d*x)/2)] + (56250*I)*c^2*Cosh[8*(c/2 + (d*x)/2)] + (28125*I)*c
^3*Cosh[8*(c/2 + (d*x)/2)] - (150000*I)*(c/2 + (d*x)/2)*Cosh[8*(c/2 + (d*x)
/2)] - (225000*I)*c*(c/2 + (d*x)/2)*Cosh[8*(c/2 + (d*x)/2)] - (168750*I)*c^
2*(c/2 + (d*x)/2)*Cosh[8*(c/2 + (d*x)/2)] + (225000*I)*(c/2 + (d*x)/2)^2*Co
sh[8*(c/2 + (d*x)/2)] + (337500*I)*c*(c/2 + (d*x)/2)^2*Cosh[8*(c/2 + (d*x)/
2)] - (225000*I)*(c/2 + (d*x)/2)^3*Cosh[8*(c/2 + (d*x)/2)] - 1296*Cosh[10*(
c/2 + (d*x)/2)] - 3240*c*Cosh[10*(c/2 + (d*x)/2)] - 4050*c^2*Cosh[10*(c/2 +
 (d*x)/2)] - 3375*c^3*Cosh[10*(c/2 + (d*x)/2)] + 6480*(c/2 + (d*x)/2)*Cosh[
10*(c/2 + (d*x)/2)] + 16200*c*(c/2 + (d*x)/2)*Cosh[10*(c/2 + (d*x)/2)] + 20
250*c^2*(c/2 + (d*x)/2)*Cosh[10*(c/2 + (d*x)/2)] - 16200*(c/2 + (d*x)/2)^2*
Cosh[10*(c/2 + (d*x)/2)] - 40500*c*(c/2 + (d*x)/2)^2*Cosh[10*(c/2 + (d*x)/2
)] + 27000*(c/2 + (d*x)/2)^3*Cosh[10*(c/2 + (d*x)/2)] - 50000*Sinh[2*(c/2 +
 (d*x)/2)] + 75000*c*Sinh[2*(c/2 + (d*x)/2)] - 56250*c^2*Sinh[2*(c/2 + (d*x
)/2)] + 28125*c^3*Sinh[2*(c/2 + (d*x)/2)] - 150000*(c/2 + (d*x)/2)*Sinh[2*(
c/2 + (d*x)/2)] + 225000*c*(c/2 + (d*x)/2)*Sinh[2*(c/2 + (d*x)/2)] - 168750
*c^2*(c/2 + (d*x)/2)*Sinh[2*(c/2 + (d*x)/2)] - 225000*(c/2 + (d*x)/2)^2*Sin
h[2*(c/2 + (d*x)/2)] + 337500*c*(c/2 + (d*x)/2)^2*Sinh[2*(c/2 + (d*x)/2)] -
 225000*(c/2 + (d*x)/2)^3*Sinh[2*(c/2 + (d*x)/2)] - (8100000*I)*Sinh[4*(c/2
 + (d*x)/2)] + (4050000*I)*c*Sinh[4*(c/2 + (d*x)/2)] - (1012500*I)*c^2*Sinh
[4*(c/2 + (d*x)/2)] + (168750*I)*c^3*Sinh[4*(c/2 + (d*x)/2)] - (8100000*I)*
(c/2 + (d*x)/2)*Sinh[4*(c/2 + (d*x)/2)] + (4050000*I)*c*(c/2 + (d*x)/2)*Sin
h[4*(c/2 + (d*x)/2)] - (1012500*I)*c^2*(c/2 + (d*x)/2)*Sinh[4*(c/2 + (d*x)/
2)] - (4050000*I)*(c/2 + (d*x)/2)^2*Sinh[4*(c/2 + (d*x)/2)] + (2025000*I)*c
*(c/2 + (d*x)/2)^2*Sinh[4*(c/2 + (d*x)/2)] - (1350000*I)*(c/2 + (d*x)/2)^3*
Sinh[4*(c/2 + (d*x)/2)] + 8100000*Sinh[6*(c/2 + (d*x)/2)] + 4050000*c*Sinh[
6*(c/2 + (d*x)/2)] + 1012500*c^2*Sinh[6*(c/2 + (d*x)/2)] + 168750*c^3*Sinh[
6*(c/2 + (d*x)/2)] - 8100000*(c/2 + (d*x)/2)*Sinh[6*(c/2 + (d*x)/2)] - 4050
000*c*(c/2 + (d*x)/2)*Sinh[6*(c/2 + (d*x)/2)] - 1012500*c^2*(c/2 + (d*x)/2)
*Sinh[6*(c/2 + (d*x)/2)] + 4050000*(c/2 + (d*x)/2)^2*Sinh[6*(c/2 + (d*x)/2)
] + 2025000*c*(c/2 + (d*x)/2)^2*Sinh[6*(c/2 + (d*x)/2)] - 1350000*(c/2 + (d
*x)/2)^3*Sinh[6*(c/2 + (d*x)/2)] + (50000*I)*Sinh[8*(c/2 + (d*x)/2)] + (750
00*I)*c*Sinh[8*(c/2 + (d*x)/2)] + (56250*I)*c^2*Sinh[8*(c/2 + (d*x)/2)] + (
28125*I)*c^3*Sinh[8*(c/2 + (d*x)/2)] - (150000*I)*(c/2 + (d*x)/2)*Sinh[8*(c
/2 + (d*x)/2)] - (225000*I)*c*(c/2 + (d*x)/2)*Sinh[8*(c/2 + (d*x)/2)] - (16
8750*I)*c^2*(c/2 + (d*x)/2)*Sinh[8*(c/2 + (d*x)/2)] + (225000*I)*(c/2 + (d*
```

```
x)/2)^2*Sinh[8*(c/2 + (d*x)/2)] + (337500*I)*c*(c/2 + (d*x)/2)^2*Sinh[8*(c/
2 + (d*x)/2)] - (225000*I)*(c/2 + (d*x)/2)^3*Sinh[8*(c/2 + (d*x)/2)] - 1296
*Sinh[10*(c/2 + (d*x)/2)] - 3240*c*Sinh[10*(c/2 + (d*x)/2)] - 4050*c^2*Sinh
[10*(c/2 + (d*x)/2)] - 3375*c^3*Sinh[10*(c/2 + (d*x)/2)] + 6480*(c/2 + (d*x
)/2)*Sinh[10*(c/2 + (d*x)/2)] + 16200*c*(c/2 + (d*x)/2)*Sinh[10*(c/2 + (d*x
)/2)] + 20250*c^2*(c/2 + (d*x)/2)*Sinh[10*(c/2 + (d*x)/2)] - 16200*(c/2 + (
d*x)/2)^2*Sinh[10*(c/2 + (d*x)/2)] - 40500*c*(c/2 + (d*x)/2)^2*Sinh[10*(c/2
 + (d*x)/2)] + 27000*(c/2 + (d*x)/2)^3*Sinh[10*...
```

Maple [F]
time = 0.38, size = 0, normalized size = 0.00

$$\int x^{3}(a+ia\sinh(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+I*a*sinh(d*x+c))^(5/2),x)
```

```
[Out] int(x^3*(a+I*a*sinh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)*x^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+I*a*sinh(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + a*sinh(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(x^3*(a + a*sinh(c + d*x)*1i)^(5/2), x)
```

3.131 $\int x^2(a+ia\sinh(c+dx))^{5/2}\,dx$

Optimal. Leaf size=506

$$-\frac{256a^2x\sqrt{a+ia\sinh(c+dx)}}{15d^2}-\frac{128a^2x\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{45d^2}-\frac{32a^2x\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\right.}{}$$

[Out] -256/15*a^2*x*(a+I*a*sinh(d*x+c))^(1/2)/d^2-128/45*a^2*x*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^2*(a+I*a*sinh(d*x+c))^(1/2)/d^2-32/25*a^2*x*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^4*(a+I*a*sinh(d*x+c))^(1/2)/d^2+32/15*a^2*x^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*sinh(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)/d+8/5*a^2*x^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^3*sinh(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)/d+9536/225*a^2*(a+I*a*sinh(d*x+c))^(1/2)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/d^3+64/15*a^2*x^2*(a+I*a*sinh(d*x+c))^(1/2)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/d+2432/675*a^2*sinh(1/2*c+1/4*I*Pi+1/2*d*x)^2*(a+I*a*sinh(d*x+c))^(1/2)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/d^3+64/125*a^2*sinh(1/2*c+1/4*I*Pi+1/2*d*x)^4*(a+I*a*sinh(d*x+c))^(1/2)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/d^3

Rubi [A]
time = 0.26, antiderivative size = 506, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3400, 3392, 3377, 2718, 2713}

$$\frac{64a^2\sinh^4\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{125d^3}+\frac{2432a^2\sinh^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{675d^3}+\frac{9536a^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{225d^3}-\frac{256a^2x\sqrt{a+ia\sinh(c+dx)}}{15d^2}-\frac{32a^2x\cosh^4\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{25d^2}-\frac{128a^2x\cosh^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{45d^2}+\frac{64a^2x^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{15d}+\frac{8a^2x^2\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{5d}+\frac{32a^2x^2\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{15d}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + I*a*Sinh[c + d*x])^(5/2),x]

[Out] (-256*a^2*x*Sqrt[a + I*a*Sinh[c + d*x]])/(15*d^2) - (128*a^2*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^2*Sqrt[a + I*a*Sinh[c + d*x]])/(45*d^2) - (32*a^2*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]])/(25*d^2) + (32*a^2*x^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]])/(15*d) + (8*a^2*x^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^3*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]])/(5*d) + (9536*a^2*Sqrt[a + I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(225*d^3) + (64*a^2*x^2*Sqrt[a + I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(15*d) + (2432*a^2*Sinh[c/2 + (I/4)*Pi + (d*x)/2]^2*Sqrt[a + I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(675*d^3) + (64*a^2*Sinh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(125*d^3)

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x^2(a+ia\sinh(c+dx))^{5/2}\,dx = \left(4a^2\operatorname{csch}\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\right)\int x^2\sinh^5\left(\frac{c}{2}-\frac{i\pi}{4}+\right.$$
$$= -\frac{32a^2x\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{25d^2}+\frac{8a^2x^2\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}\right.}{}$$
$$= -\frac{128a^2x\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{45d^2}-\frac{32a^2x\cosh^4\left(\frac{c}{2}+\right.}{}$$
$$= -\frac{128a^2x\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{45d^2}-\frac{32a^2x\cosh^4\left(\frac{c}{2}+\right.}{}$$
$$= -\frac{256a^2x\sqrt{a+ia\sinh(c+dx)}}{15d^2}-\frac{128a^2x\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sin}}{45d^2}$$
$$= -\frac{256a^2x\sqrt{a+ia\sinh(c+dx)}}{15d^2}-\frac{128a^2x\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sin}}{45d^2}$$

Mathematica [A]
time = 0.72, size = 300, normalized size = 0.59

$$\frac{a^2\sqrt{a+ia\sinh(c+dx)}\left(33750i(8+4idx+d^2x^2)\cosh\left(\frac{1}{2}(c+dx)\right)+625(8i+12dx+9id^2x^2)\cosh\left(\frac{3}{2}(c+dx)\right)-216i\cosh\left(\frac{5}{2}(c+dx)\right)+540dx\cosh\left(\frac{5}{2}(c+dx)\right)-675id^2x^2\cosh\left(\frac{5}{2}(c+dx)\right)+270000\sinh\left(\frac{1}{2}(c+dx)\right)-135000idx\sinh\left(\frac{1}{2}(c+dx)\right)+33750d^2x^2\sinh\left(\frac{1}{2}(c+dx)\right)-5000\sinh\left(\frac{3}{2}(c+dx)\right)-7500idx\sinh\left(\frac{3}{2}(c+dx)\right)-5625d^2x^2\sinh\left(\frac{3}{2}(c+dx)\right)-216\sinh\left(\frac{5}{2}(c+dx)\right)+540idx\sinh\left(\frac{5}{2}(c+dx)\right)-675d^2x^2\sinh\left(\frac{5}{2}(c+dx)\right)\right)}{6750d^3\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[a + I*a*Sinh[c + d*x]]*((33750*I)*(8 + (4*I)*d*x + d^2*x^2)*Cosh[
(c + d*x)/2] + 625*(8*I + 12*d*x + (9*I)*d^2*x^2)*Cosh[(3*(c + d*x))/2] - (
216*I)*Cosh[(5*(c + d*x))/2] + 540*d*x*Cosh[(5*(c + d*x))/2] - (675*I)*d^2*
x^2*Cosh[(5*(c + d*x))/2] + 270000*Sinh[(c + d*x)/2] - (135000*I)*d*x*Sinh[
(c + d*x)/2] + 33750*d^2*x^2*Sinh[(c + d*x)/2] - 5000*Sinh[(3*(c + d*x))/2]
 - (7500*I)*d*x*Sinh[(3*(c + d*x))/2] - 5625*d^2*x^2*Sinh[(3*(c + d*x))/2]
- 216*Sinh[(5*(c + d*x))/2] + (540*I)*d*x*Sinh[(5*(c + d*x))/2] - 675*d^2*x
^2*Sinh[(5*(c + d*x))/2]))/(6750*d^3*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/
2]))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int x^2(a+ia\sinh(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+I*a*sinh(d*x+c))^(5/2),x)
```

```
[Out] int(x^2*(a+I*a*sinh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)*x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+I*a*sinh(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + a*sinh(c + d*x)*1i)^(5/2),x)
[Out] int(x^2*(a + a*sinh(c + d*x)*1i)^(5/2), x)
```

3.132 $\int x(a + ia\sinh(c + dx))^{5/2}\,dx$

Optimal. Leaf size=312

$$-\frac{128a^2\sqrt{a+ia\sinh(c+dx)}}{15d^2}-\frac{64a^2\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{45d^2}-\frac{16a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{}}{25d^2}$$

```
[Out] -128/15*a^2*(a+I*a*sinh(d*x+c))^(1/2)/d^2-64/45*a^2*cosh(1/2*c+1/4*I*Pi+1/2
*d*x)^2*(a+I*a*sinh(d*x+c))^(1/2)/d^2-16/25*a^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x
)^4*(a+I*a*sinh(d*x+c))^(1/2)/d^2+32/15*a^2*x*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*
sinh(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)/d+8/5*a^2*x*cosh(1/2
*c+1/4*I*Pi+1/2*d*x)^3*sinh(1/2*c+1/4*I*Pi+1/2*d*x)*(a+I*a*sinh(d*x+c))^(1/
2)/d+64/15*a^2*x*(a+I*a*sinh(d*x+c))^(1/2)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/d
```

Rubi [A]
time = 0.13, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3400, 3391, 3377, 2718}

$$-\frac{128a^2\sqrt{a+ia\sinh(c+dx)}}{15d^2}-\frac{16a^2\cosh^4\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{25d^2}-\frac{64a^2\cosh^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{45d^2}+\frac{64a^2x\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{15d}+\frac{8a^2x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\cosh^3\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{5d}+\frac{32a^2x\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (-128*a^2*Sqrt[a + I*a*Sinh[c + d*x]])/(15*d^2) - (64*a^2*Cosh[c/2 + (I/4)*
Pi + (d*x)/2]^2*Sqrt[a + I*a*Sinh[c + d*x]])/(45*d^2) - (16*a^2*Cosh[c/2 +
(I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]])/(25*d^2) + (32*a^2*x*Cos
h[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sin
h[c + d*x]])/(15*d) + (8*a^2*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^3*Sinh[c/2 +
(I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]])/(5*d) + (64*a^2*x*Sqrt[a +
 I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(15*d)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
```

```
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x(a+ia\sinh(c+dx))^{5/2}\,dx = \left(4a^2\text{csch}\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\right)\int x\sinh^5\left(\frac{c}{2}-\frac{i\pi}{4}+\right.$$

$$= -\frac{16a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{25d^2}+\frac{8a^2x\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}+\right.}{}$$

$$= -\frac{64a^2\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{45d^2}-\frac{16a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\right.}{}$$

$$= -\frac{64a^2\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{45d^2}-\frac{16a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\right.}{}$$

$$= -\frac{128a^2\sqrt{a+ia\sinh(c+dx)}}{15d^2}-\frac{64a^2\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c}{45d^2}$$

Mathematica [A]
time = 0.57, size = 218, normalized size = 0.70

$$\frac{a^2(-i+\sinh(c+dx))^2\sqrt{a+ia\sinh(c+dx)}\left(2250(2-idx)\cosh\left(\frac{1}{2}(c+dx)\right)+(-250-375idx)\cosh\left(\frac{3}{2}(c+dx)\right)-18\cosh\left(\frac{5}{2}(c+dx)\right)+45idx\cosh\left(\frac{5}{2}(c+dx)\right)+4500i\sinh\left(\frac{1}{2}(c+dx)\right)-2250dx\sinh\left(\frac{1}{2}(c+dx)\right)+250i\sinh\left(\frac{3}{2}(c+dx)\right)+375dx\sinh\left(\frac{3}{2}(c+dx)\right)-18i\sinh\left(\frac{5}{2}(c+dx)\right)+45dx\sinh\left(\frac{5}{2}(c+dx)\right)\right)}{450d^2\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (a^2*(-I + Sinh[c + d*x])^2*Sqrt[a + I*a*Sinh[c + d*x]]*(2250*(2 - I*d*x)*C
osh[(c + d*x)/2] + (-250 - (375*I)*d*x)*Cosh[(3*(c + d*x))/2] - 18*Cosh[(5*
(c + d*x))/2] + (45*I)*d*x*Cosh[(5*(c + d*x))/2] + (4500*I)*Sinh[(c + d*x)/
2] - 2250*d*x*Sinh[(c + d*x)/2] + (250*I)*Sinh[(3*(c + d*x))/2] + 375*d*x*S
inh[(3*(c + d*x))/2] - (18*I)*Sinh[(5*(c + d*x))/2] + 45*d*x*Sinh[(5*(c + d
*x))/2]))/(450*d^2*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^5)
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int x(a+ia\sinh(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+I*a*sinh(d*x+c))^(5/2),x)
```

```
[Out] int(x*(a+I*a*sinh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)*x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+a\sinh(c+d\,x)\;1\mathrm{i})^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + a*sinh(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(x*(a + a*sinh(c + d*x)*1i)^(5/2), x)
```

3.133 $\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x}\,dx$

Optimal. Leaf size=403

$$-\frac{1}{4}ia^2\text{Chi}\left(\frac{5dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{5c}{2}-\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}+\frac{5}{2}ia^2\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{i\pi}{4}\right.$$

```
[Out] 5/2*a^2*sinh(1/2*c+1/4*I*Pi)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*Shi(1/2*d*x)*(a+I
*a*sinh(d*x+c))^(1/2)+5/4*I*a^2*cosh(3/2*c+1/4*I*Pi)*sech(1/2*c+1/4*I*Pi+1/
2*d*x)*Shi(3/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)-1/4*a^2*sinh(5/2*c+1/4*I*Pi)*
sech(1/2*c+1/4*I*Pi+1/2*d*x)*Shi(5/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)-1/4*a^2
*Chi(5/2*d*x)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*cosh(5/2*c+1/4*I*Pi)*(a+I*a*sinh
(d*x+c))^(1/2)+5/2*a^2*Chi(1/2*d*x)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*cosh(1/2*c
+1/4*I*Pi)*(a+I*a*sinh(d*x+c))^(1/2)+5/4*I*a^2*Chi(3/2*d*x)*sech(1/2*c+1/4*
I*Pi+1/2*d*x)*sinh(3/2*c+1/4*I*Pi)*(a+I*a*sinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3400, 3393, 3384, 3379, 3382}

$$-\frac{1}{4}ia^2\sinh\left(\frac{5c}{2}-\frac{i\pi}{4}\right)\text{Chi}\left(\frac{5dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}+\frac{5}{2}ia^2\sinh\left(\frac{1}{4}(2c-i\pi)\right)\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}+\frac{5}{4}ia^2\sinh\left(\frac{1}{4}(6c+i\pi)\right)\text{Chi}\left(\frac{3dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}+\frac{5}{2}ia^2\cosh\left(\frac{1}{4}(2c-i\pi)\right)\text{Shi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}+\frac{5}{4}ia^2\cosh\left(\frac{1}{4}(6c+i\pi)\right)\text{Shi}\left(\frac{3dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}-\frac{1}{4}ia^2\cosh\left(\frac{5c}{2}-\frac{i\pi}{4}\right)\text{Shi}\left(\frac{5dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[c + d*x])^(5/2)/x,x]
```

```
[Out] (-1/4*I)*a^2*CoshIntegral[(5*d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(5
*c)/2 - (I/4)*Pi]*Sqrt[a + I*a*Sinh[c + d*x]] + ((5*I)/2)*a^2*CoshIntegral[
(d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(2*c - I*Pi)/4]*Sqrt[a + I*a*S
inh[c + d*x]] + ((5*I)/4)*a^2*CoshIntegral[(3*d*x)/2]*Sech[c/2 + (I/4)*Pi +
 (d*x)/2]*Sinh[(6*c + I*Pi)/4]*Sqrt[a + I*a*Sinh[c + d*x]] + ((5*I)/2)*a^2*
Cosh[(2*c - I*Pi)/4]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d
*x]]*SinhIntegral[(d*x)/2] + ((5*I)/4)*a^2*Cosh[(6*c + I*Pi)/4]*Sech[c/2 +
(I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhIntegral[(3*d*x)/2] - (
I/4)*a^2*Cosh[(5*c)/2 - (I/4)*Pi]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I
*a*Sinh[c + d*x]]*SinhIntegral[(5*d*x)/2]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x}\,dx = \left(4a^2\operatorname{csch}\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\right)\int \frac{\sinh^5\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)}{x}$$

$$= -\left(\left(4ia^2\operatorname{csch}\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\right)\int\left(\frac{5i\sinh\left(\frac{1}{4}(2c\right.}{8x}\right.\right.$$

$$= -\left(\frac{1}{4}\left(a^2\operatorname{csch}\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\right)\int\frac{\sinh\left(\frac{1}{4}(10c-i\right.}{x}\right.$$

$$= -\left(\frac{1}{4}\left(a^2\cosh\left(\frac{5c}{2}-\frac{i\pi}{4}\right)\operatorname{csch}\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\right)\int\right.$$

$$= -\frac{1}{4}ia^2\operatorname{Chi}\left(\frac{5dx}{2}\right)\operatorname{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{5c}{2}-\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+}$$

Mathematica [A]
time = 0.62, size = 242, normalized size = 0.60

$$\frac{a^2(-i+\sinh(c+dx))^2\sqrt{a+ia\sinh(c+dx)}\left(\cosh\left(\frac{5c}{2}\right)\operatorname{Chi}\left(\frac{5dx}{2}\right)-10\operatorname{Chi}\left(\frac{dx}{2}\right)\left(\cosh\left(\frac{c}{2}\right)+i\sinh\left(\frac{c}{2}\right)\right)+5\operatorname{Chi}\left(\frac{3dx}{2}\right)\left(\cosh\left(\frac{3c}{2}\right)-i\sinh\left(\frac{3c}{2}\right)\right)+i\operatorname{Chi}\left(\frac{5dx}{2}\right)\sinh\left(\frac{5c}{2}\right)-10i\cosh\left(\frac{c}{2}\right)\operatorname{Shi}\left(\frac{dx}{2}\right)-10\sinh\left(\frac{c}{2}\right)\operatorname{Shi}\left(\frac{dx}{2}\right)-5i\cosh\left(\frac{3c}{2}\right)\operatorname{Shi}\left(\frac{3dx}{2}\right)+5\sinh\left(\frac{3c}{2}\right)\operatorname{Shi}\left(\frac{3dx}{2}\right)+i\cosh\left(\frac{5c}{2}\right)\operatorname{Shi}\left(\frac{5dx}{2}\right)+\sinh\left(\frac{5c}{2}\right)\operatorname{Shi}\left(\frac{5dx}{2}\right)\right)}{4\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(5/2)/x,x]
```

```
[Out] (a^2*(-I + Sinh[c + d*x])^2*Sqrt[a + I*a*Sinh[c + d*x]]*(Cosh[(5*c)/2]*Cosh
Integral[(5*d*x)/2] - 10*CoshIntegral[(d*x)/2]*(Cosh[c/2] + I*Sinh[c/2]) +
5*CoshIntegral[(3*d*x)/2]*(Cosh[(3*c)/2] - I*Sinh[(3*c)/2]) + I*CoshIntegra
l[(5*d*x)/2]*Sinh[(5*c)/2] - (10*I)*Cosh[c/2]*SinhIntegral[(d*x)/2] - 10*Si
nh[c/2]*SinhIntegral[(d*x)/2] - (5*I)*Cosh[(3*c)/2]*SinhIntegral[(3*d*x)/2]
 + 5*Sinh[(3*c)/2]*SinhIntegral[(3*d*x)/2] + I*Cosh[(5*c)/2]*SinhIntegral[(
5*d*x)/2] + Sinh[(5*c)/2]*SinhIntegral[(5*d*x)/2]))/(4*(Cosh[(c + d*x)/2] +
 I*Sinh[(c + d*x)/2])^5)
```

Maple [**F**]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(a + ia\sinh(dx + c))^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(d*x+c))^(5/2)/x,x)
```

```
[Out] int((a+I*a*sinh(d*x+c))^(5/2)/x,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)/x, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ia(\sinh(c + dx) - i))^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))**(5/2)/x,x)
```

```
[Out] Integral((I*a*(sinh(c + d*x) - I))**(5/2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(c + d*x)*1i)^(5/2)/x,x)
```

```
[Out] int((a + a*sinh(c + d*x)*1i)^(5/2)/x, x)
```

3.134 $\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^2}\,dx$

Optimal. Leaf size=444

$$-\frac{4a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x}-\frac{5}{8}a^2d\,\text{Chi}\left(\frac{5dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{5c}{2}+\frac{i\pi}{4}\right)\sqrt{a}$$

```
[Out] -4*a^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^4*(a+I*a*sinh(d*x+c))^(1/2)/x+5/4*a^2*d
*cosh(1/2*c+1/4*I*Pi)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*Shi(1/2*d*x)*(a+I*a*sinh
(d*x+c))^(1/2)+15/8*I*a^2*d*sinh(3/2*c+1/4*I*Pi)*sech(1/2*c+1/4*I*Pi+1/2*d*
x)*Shi(3/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)-5/8*a^2*d*cosh(5/2*c+1/4*I*Pi)*se
ch(1/2*c+1/4*I*Pi+1/2*d*x)*Shi(5/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)-5/8*a^2*d
*Chi(5/2*d*x)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*sinh(5/2*c+1/4*I*Pi)*(a+I*a*sinh
(d*x+c))^(1/2)+15/8*I*a^2*d*Chi(3/2*d*x)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*cosh(
3/2*c+1/4*I*Pi)*(a+I*a*sinh(d*x+c))^(1/2)+5/4*a^2*d*Chi(1/2*d*x)*sech(1/2*c
+1/4*I*Pi+1/2*d*x)*sinh(1/2*c+1/4*I*Pi)*(a+I*a*sinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3400, 3394, 3384, 3379, 3382}

$$-\frac{5}{8}a^2d\sinh\left(\frac{5c}{2}+\frac{i\pi}{4}\right)\text{Chi}\left(\frac{5dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}-\frac{15}{8}a^2d\sinh\left(\frac{1}{4}(6c-i\pi)\right)\text{Chi}\left(\frac{3dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}+\frac{5}{4}a^2d\sinh\left(\frac{1}{4}(2c+i\pi)\right)\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}+\frac{5}{4}a^2d\cosh\left(\frac{1}{4}(2c+i\pi)\right)\text{Shi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}-\frac{15}{8}a^2d\cosh\left(\frac{1}{4}(6c-i\pi)\right)\text{Shi}\left(\frac{3dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}-\frac{5}{8}a^2d\cosh\left(\frac{5c}{2}+\frac{i\pi}{4}\right)\text{Shi}\left(\frac{5dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}-\frac{4a^2\cosh^4\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[c + d*x])^(5/2)/x^2,x]
```

```
[Out] (-4*a^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]])/x - (
5*a^2*d*CoshIntegral[(5*d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(5*c)/2
 + (I/4)*Pi]*Sqrt[a + I*a*Sinh[c + d*x]])/8 - (15*a^2*d*CoshIntegral[(3*d*x
)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(6*c - I*Pi)/4]*Sqrt[a + I*a*Sinh[
c + d*x]])/8 + (5*a^2*d*CoshIntegral[(d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2
]*Sinh[(2*c + I*Pi)/4]*Sqrt[a + I*a*Sinh[c + d*x]])/4 + (5*a^2*d*Cosh[(2*c
+ I*Pi)/4]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhI
ntegral[(d*x)/2])/4 - (15*a^2*d*Cosh[(6*c - I*Pi)/4]*Sech[c/2 + (I/4)*Pi +
(d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhIntegral[(3*d*x)/2])/8 - (5*a^2*d*
Cosh[(5*c)/2 + (I/4)*Pi]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c
 + d*x]]*SinhIntegral[(5*d*x)/2])/8
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^2}\,dx = \left(4a^2\text{csch}\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\right)\int \frac{\sinh^5\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)}{x^2}$$

$$= -\frac{4a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x} + \left(10a^2d\text{csch}\left(\frac{c}{2}-\frac{i\pi}{4}+\right.\right.$$

$$= -\frac{4a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x} - \frac{1}{8}\left(5a^2d\text{csch}\left(\frac{c}{2}-\frac{i\pi}{4}\right.\right.$$

$$= -\frac{4a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x} + \frac{1}{8}\left(5ia^2d\cosh\left(\frac{5c}{2}+\right.\right.$$

$$= -\frac{4a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x} - \frac{5}{8}a^2d\text{Chi}\left(\frac{5dx}{2}\right)\text{sech}\left(\right.$$

Mathematica [A]
time = 1.13, size = 347, normalized size = 0.78

$$\frac{a^2(-i+\sinh(c+dx))^2\sqrt{a+ia\sinh(c+dx)}\left(20\cosh\left(\frac{1}{2}(c+dx)\right)-10\cosh\left(\frac{3}{2}(c+dx)\right)-2\cosh\left(\frac{5}{2}(c+dx)\right)+5idx\cosh\left(\frac{5c}{2}\right)\text{Chi}\left(\frac{5dx}{2}\right)-10idx\text{Chi}\left(\frac{dx}{2}\right)\left(\cosh\left(\frac{c}{2}\right)-i\sinh\left(\frac{c}{2}\right)\right)+15dx\text{Chi}\left(\frac{3dx}{2}\right)\left(-i\cosh\left(\frac{3c}{2}\right)+\sinh\left(\frac{3c}{2}\right)\right)+5dx\text{Chi}\left(\frac{5dx}{2}\right)\sinh\left(\frac{5c}{2}\right)+20i\sinh\left(\frac{1}{2}(c+dx)\right)+10i\sinh\left(\frac{3}{2}(c+dx)\right)-2i\sinh\left(\frac{5}{2}(c+dx)\right)-10dx\cosh\left(\frac{c}{2}\right)\text{Shi}\left(\frac{dx}{2}\right)-10idx\sinh\left(\frac{c}{2}\right)\text{Shi}\left(\frac{dx}{2}\right)+15dx\cosh\left(\frac{3c}{2}\right)\text{Shi}\left(\frac{3dx}{2}\right)-15idx\sinh\left(\frac{3c}{2}\right)\text{Shi}\left(\frac{3dx}{2}\right)+5dx\cosh\left(\frac{5c}{2}\right)\text{Shi}\left(\frac{5dx}{2}\right)+5idx\sinh\left(\frac{5c}{2}\right)\text{Shi}\left(\frac{5dx}{2}\right)\right)}{8x\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(5/2)/x^2,x]
```

```
[Out] (a^2*(-I + Sinh[c + d*x])^2*Sqrt[a + I*a*Sinh[c + d*x]]*(20*Cosh[(c + d*x)/
2] - 10*Cosh[(3*(c + d*x))/2] - 2*Cosh[(5*(c + d*x))/2] + (5*I)*d*x*Cosh[(5
*c)/2]*CoshIntegral[(5*d*x)/2] - (10*I)*d*x*CoshIntegral[(d*x)/2]*(Cosh[c/2
] - I*Sinh[c/2]) + 15*d*x*CoshIntegral[(3*d*x)/2]*((-I)*Cosh[(3*c)/2] + Sin
h[(3*c)/2]) + 5*d*x*CoshIntegral[(5*d*x)/2]*Sinh[(5*c)/2] + (20*I)*Sinh[(c
+ d*x)/2] + (10*I)*Sinh[(3*(c + d*x))/2] - (2*I)*Sinh[(5*(c + d*x))/2] - 10
*d*x*Cosh[c/2]*SinhIntegral[(d*x)/2] - (10*I)*d*x*Sinh[c/2]*SinhIntegral[(d
*x)/2] + 15*d*x*Cosh[(3*c)/2]*SinhIntegral[(3*d*x)/2] - (15*I)*d*x*Sinh[(3*
c)/2]*SinhIntegral[(3*d*x)/2] + 5*d*x*Cosh[(5*c)/2]*SinhIntegral[(5*d*x)/2]
 + (5*I)*d*x*Sinh[(5*c)/2]*SinhIntegral[(5*d*x)/2]))/(8*x*(Cosh[(c + d*x)/2
] + I*Sinh[(c + d*x)/2])^5)
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(a + ia\sinh(dx + c))^{\frac{5}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(d*x+c))^(5/2)/x^2,x)
```

```
[Out] int((a+I*a*sinh(d*x+c))^(5/2)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)/x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))**(5/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + a\sinh(c + d\,x)\ 1\mathrm{i})^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(c + d*x)*1i)^(5/2)/x^2,x)
```

```
[Out] int((a + a*sinh(c + d*x)*1i)^(5/2)/x^2, x)
```

3.135 $\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^3}\,dx$

Optimal. Leaf size=536

$$-\frac{2a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x^2}-\frac{25}{32}ia^2d^2\text{Chi}\left(\frac{5dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{5c}{2}-\frac{i\pi}{4}\right)$$

```
[Out] -2*a^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^4*(a+I*a*sinh(d*x+c))^(1/2)/x^2+5/16*a^
2*d^2*sinh(1/2*c+1/4*I*Pi)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*Shi(1/2*d*x)*(a+I*a
*sinh(d*x+c))^(1/2)+45/32*I*a^2*d^2*cosh(3/2*c+1/4*I*Pi)*sech(1/2*c+1/4*I*P
i+1/2*d*x)*Shi(3/2*d*x)*(a+I*a*sinh(d*x+c))^(1/2)-25/32*a^2*d^2*sinh(5/2*c+
1/4*I*Pi)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*Shi(5/2*d*x)*(a+I*a*sinh(d*x+c))^(1/
2)-25/32*a^2*d^2*Chi(5/2*d*x)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*cosh(5/2*c+1/4*I
*Pi)*(a+I*a*sinh(d*x+c))^(1/2)+5/16*a^2*d^2*Chi(1/2*d*x)*sech(1/2*c+1/4*I*P
i+1/2*d*x)*cosh(1/2*c+1/4*I*Pi)*(a+I*a*sinh(d*x+c))^(1/2)+45/32*I*a^2*d^2*C
hi(3/2*d*x)*sech(1/2*c+1/4*I*Pi+1/2*d*x)*sinh(3/2*c+1/4*I*Pi)*(a+I*a*sinh(d
*x+c))^(1/2)-5*a^2*d*cosh(1/2*c+1/4*I*Pi+1/2*d*x)^3*sinh(1/2*c+1/4*I*Pi+1/2
*d*x)*(a+I*a*sinh(d*x+c))^(1/2)/x
```

Rubi [A]
time = 0.42, antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3400, 3395, 3393, 3384, 3379, 3382}

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[c + d*x])^(5/2)/x^3,x]
```

```
[Out] (-2*a^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]])/x^2 -
 ((25*I)/32)*a^2*d^2*CoshIntegral[(5*d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]
*Sinh[(5*c)/2 - (I/4)*Pi]*Sqrt[a + I*a*Sinh[c + d*x]] + ((5*I)/16)*a^2*d^2*
CoshIntegral[(d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(2*c - I*Pi)/4]*S
qrt[a + I*a*Sinh[c + d*x]] + ((45*I)/32)*a^2*d^2*CoshIntegral[(3*d*x)/2]*Se
ch[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(6*c + I*Pi)/4]*Sqrt[a + I*a*Sinh[c + d*x
]] - (5*a^2*d*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^3*Sinh[c/2 + (I/4)*Pi + (d*x)/
2]*Sqrt[a + I*a*Sinh[c + d*x]])/x + ((5*I)/16)*a^2*d^2*Cosh[(2*c - I*Pi)/4]
*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhIntegral[(d
*x)/2] + ((45*I)/32)*a^2*d^2*Cosh[(6*c + I*Pi)/4]*Sech[c/2 + (I/4)*Pi + (d*
x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhIntegral[(3*d*x)/2] - ((25*I)/32)*a^2
*d^2*Cosh[(5*c)/2 - (I/4)*Pi]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*S
inh[c + d*x]]*SinhIntegral[(5*d*x)/2]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^3}\,dx = \left(4a^2\text{csch}\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\right)\int \frac{\sinh^5\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)}{x^3}\,d$$

$$= -\frac{2a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x^2} - \frac{5a^2d\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right.}{}$$

$$= -\frac{2a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x^2} - \frac{5a^2d\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right.}{}$$

$$= -\frac{2a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x^2} - \frac{5a^2d\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right.}{}$$

$$= -\frac{2a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x^2} - \frac{5a^2d\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right.}{}$$

$$= -\frac{2a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x^2} - \frac{25}{32}ia^2d^2\text{Chi}\left(\frac{5dx}{2}\right)\text{sech}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4751 vs. $2(536) = 1072$.
time = 6.40, size = 4751, normalized size = 8.86

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(5/2)/x^3,x]
```

```
[Out] (2*((1/128 + I/128)*Cosh[5*(c/2 + (d*x)/2)] - (1/128 + I/128)*Sinh[5*(c/2 +
 (d*x)/2)])*(a + I*a*Sinh[c + d*x])^(5/2)*((-4*I)*d^3 - (10*I)*c*d^3 + (20*
I)*d^3*(c/2 + (d*x)/2) + 20*d^3*Cosh[2*(c/2 + (d*x)/2)] + 30*c*d^3*Cosh[2*(
c/2 + (d*x)/2)] - 60*d^3*(c/2 + (d*x)/2)*Cosh[2*(c/2 + (d*x)/2)] + (40*I)*d
^3*Cosh[4*(c/2 + (d*x)/2)] + (20*I)*c*d^3*Cosh[4*(c/2 + (d*x)/2)] - (40*I)*
d^3*(c/2 + (d*x)/2)*Cosh[4*(c/2 + (d*x)/2)] - 40*d^3*Cosh[6*(c/2 + (d*x)/2)
] + 20*c*d^3*Cosh[6*(c/2 + (d*x)/2)] - 40*d^3*(c/2 + (d*x)/2)*Cosh[6*(c/2 +
 (d*x)/2)] - (20*I)*d^3*Cosh[8*(c/2 + (d*x)/2)] + (30*I)*c*d^3*Cosh[8*(c/2
+ (d*x)/2)] - (60*I)*d^3*(c/2 + (d*x)/2)*Cosh[8*(c/2 + (d*x)/2)] + 4*d^3*Co
sh[10*(c/2 + (d*x)/2)] - 10*c*d^3*Cosh[10*(c/2 + (d*x)/2)] + 20*d^3*(c/2 +
(d*x)/2)*Cosh[10*(c/2 + (d*x)/2)] - (10*I)*c^2*d^3*Cosh[c/2 - 5*(c/2 + (d*x
)/2)]*CoshIntegral[(d*x)/2] + (40*I)*c*d^3*(c/2 + (d*x)/2)*Cosh[c/2 - 5*(c/
2 + (d*x)/2)]*CoshIntegral[(d*x)/2] - (40*I)*d^3*(c/2 + (d*x)/2)^2*Cosh[c/2
 - 5*(c/2 + (d*x)/2)]*CoshIntegral[(d*x)/2] + 10*c^2*d^3*Cosh[c/2 + 5*(c/2
+ (d*x)/2)]*CoshIntegral[(d*x)/2] - 40*c*d^3*(c/2 + (d*x)/2)*Cosh[c/2 + 5*(
c/2 + (d*x)/2)]*CoshIntegral[(d*x)/2] + 40*d^3*(c/2 + (d*x)/2)^2*Cosh[c/2 +
 5*(c/2 + (d*x)/2)]*CoshIntegral[(d*x)/2] - 45*c^2*d^3*Cosh[(3*c)/2 - 5*(c/
```

2 + (d*x)/2)]*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)] + 180*c*d^3*(c/2 + (d*x)/2)*Cosh[(3*c)/2 - 5*(c/2 + (d*x)/2)]*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)] - 180*d^3*(c/2 + (d*x)/2)^2*Cosh[(3*c)/2 - 5*(c/2 + (d*x)/2)]*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)] + (45*I)*c^2*d^3*Cosh[(3*c)/2 + 5*(c/2 + (d*x)/2)]*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)] - (180*I)*c*d^3*(c/2 + (d*x)/2)*Cosh[(3*c)/2 + 5*(c/2 + (d*x)/2)]*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)] + (180*I)*d^3*(c/2 + (d*x)/2)^2*Cosh[(3*c)/2 + 5*(c/2 + (d*x)/2)]*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)] + (25*I)*c^2*d^3*Cosh[(5*c)/2 - 5*(c/2 + (d*x)/2)]*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)] - (100*I)*c*d^3*(c/2 + (d*x)/2)*Cosh[(5*c)/2 - 5*(c/2 + (d*x)/2)]*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)] + (100*I)*d^3*(c/2 + (d*x)/2)^2*Cosh[(5*c)/2 - 5*(c/2 + (d*x)/2)]*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)] - 25*c^2*d^3*Cosh[(5*c)/2 + 5*(c/2 + (d*x)/2)]*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)] + 100*c*d^3*(c/2 + (d*x)/2)*Cosh[(5*c)/2 + 5*(c/2 + (d*x)/2)]*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)] - 100*d^3*(c/2 + (d*x)/2)^2*Cosh[(5*c)/2 + 5*(c/2 + (d*x)/2)]*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)] + 20*d^3*Sinh[2*(c/2 + (d*x)/2)] + 30*c*d^3*Sinh[2*(c/2 + (d*x)/2)] - 60*d^3*(c/2 + (d*x)/2)*Sinh[2*(c/2 + (d*x)/2)] + (40*I)*d^3*Sinh[4*(c/2 + (d*x)/2)] + (20*I)*c*d^3*Sinh[4*(c/2 + (d*x)/2)] - (40*I)*d^3*(c/2 + (d*x)/2)*Sinh[4*(c/2 + (d*x)/2)] - 40*d^3*Sinh[6*(c/2 + (d*x)/2)] + 20*c*d^3*Sinh[6*(c/2 + (d*x)/2)] - 40*d^3*(c/2 + (d*x)/2)*Sinh[6*(c/2 + (d*x)/2)] - (20*I)*d^3*Sinh[8*(c/2 + (d*x)/2)] + (30*I)*c*d^3*Sinh[8*(c/2 + (d*x)/2)] - (60*I)*d^3*(c/2 + (d*x)/2)*Sinh[8*(c/2 + (d*x)/2)] + 4*d^3*Sinh[10*(c/2 + (d*x)/2)] - 10*c*d^3*Sinh[10*(c/2 + (d*x)/2)] + 20*d^3*(c/2 + (d*x)/2)*Sinh[10*(c/2 + (d*x)/2)] + (10*I)*c^2*d^3*CoshIntegral[(d*x)/2]*Sinh[c/2 - 5*(c/2 + (d*x)/2)] - (40*I)*c*d^3*(c/2 + (d*x)/2)*CoshIntegral[(d*x)/2]*Sinh[c/2 - 5*(c/2 + (d*x)/2)] + (40*I)*d^3*(c/2 + (d*x)/2)^2*CoshIntegral[(d*x)/2]*Sinh[c/2 - 5*(c/2 + (d*x)/2)] + 45*c^2*d^3*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)]*Sinh[(3*c)/2 - 5*(c/2 + (d*x)/2)] - 180*c*d^3*(c/2 + (d*x)/2)*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)]*Sinh[(3*c)/2 - 5*(c/2 + (d*x)/2)] + 180*d^3*(c/2 + (d*x)/2)^2*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)]*Sinh[(3*c)/2 - 5*(c/2 + (d*x)/2)] - (25*I)*c^2*d^3*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)]*Sinh[(5*c)/2 - 5*(c/2 + (d*x)/2)] + (100*I)*c*d^3*(c/2 + (d*x)/2)*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)]*Sinh[(5*c)/2 - 5*(c/2 + (d*x)/2)] - (100*I)*d^3*(c/2 + (d*x)/2)^2*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)]*Sinh[(5*c)/2 - 5*(c/2 + (d*x)/2)] + 10*c^2*d^3*CoshIntegral[(d*x)/2]*Sinh[c/2 + 5*(c/2 + (d*x)/2)] - 40*c*d^3*(c/2 + (d*x)/2)*CoshIntegral[(d*x)/2]*Sinh[c/2 + 5*(c/2 + (d*x)/2)] + 40*d^3*(c/2 + (d*x)/2)^2*CoshIntegral[(d*x)/2]*Sinh[c/2 + 5*(c/2 + (d*x)/2)] + (45*I)*c^2*d^3*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)]*Sinh[(3*c)/2 + 5*(c/2 + (d*x)/2)] - (180*I)*c*d^3*(c/2 + (d*x)/2)*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)]*Sinh[(3*c)/2 + 5*(c/2 + (d*x)/2)] + (180*I)*d^3*(c/2 + (d*x)/2)^2*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)]*Sinh[(3*c)/2 + 5*(c/2 + (d*x)/2)] - 25*c^2*d^3*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)]*Sinh[(5*c)/2 + 5*(c/2 + (d*x)/2)] + 100*c*d^3*(c/2 + (d*x)/2)*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)]*Sinh[(5*c)/2 + 5*(c/2 + (d*x)/

```
2)] - 100*d^3*(c/2 + (d*x)/2)^2*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)]*
Sinh[(5*c)/2 + 5*(c/2 + (d*x)/2)] + (10*I)*c^2*d^3*Cosh[c/2 - 5*(c/2 + (d*x
)/2)]*SinhIntegral[(d*x)/2] - (40*I)*c*d^3*(c/2 + (d*x)/2)*Cosh[c/2 - 5*(c/
2 + (d*x)/2)]*SinhIntegral[(d*x)/2] + (40*I)*d^3*(c/2 + (d*x)/2)^2*Cosh[c/2
 - 5*(c/2 + (d*x)/2)]*SinhIntegral[(d*x)/2] + 1...
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(a + ia\sinh(dx + c))^{\frac{5}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(d*x+c))^(5/2)/x^3,x)
```

```
[Out] int((a+I*a*sinh(d*x+c))^(5/2)/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)/x^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))**(5/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)/x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + a\sinh(c + d\,x)\ 1\mathrm{i})^{5/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(c + d*x)*1i)^(5/2)/x^3,x)
```

```
[Out] int((a + a*sinh(c + d*x)*1i)^(5/2)/x^3, x)
```

3.136 $\int \frac{x^3}{\sqrt{a+ia\sinh(e+fx)}}\,dx$

Optimal. Leaf size=493

$$\frac{4ix^3\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}}+\frac{12ix^2\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}-\frac{12ix^2}{}$$

```
[Out] -4*I*x^3*arctanh(exp(1/2*e+3/4*I*Pi+1/2*f*x))*cosh(1/2*e+1/4*I*Pi+1/2*f*x)/
f/(a+I*a*sinh(f*x+e))^(1/2)+12*I*x^2*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(2
,exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^2/(a+I*a*sinh(f*x+e))^(1/2)-12*I*x^2*cosh(1
/2*e+1/4*I*Pi+1/2*f*x)*polylog(2,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^2/(a+I*a*s
inh(f*x+e))^(1/2)-48*I*x*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(3,exp(1/2*e+3
/4*I*Pi+1/2*f*x))/f^3/(a+I*a*sinh(f*x+e))^(1/2)+48*I*x*cosh(1/2*e+1/4*I*Pi+
1/2*f*x)*polylog(3,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^3/(a+I*a*sinh(f*x+e))^(1
/2)+96*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(4,exp(1/2*e+3/4*I*Pi+1/2*f*x)
)/f^4/(a+I*a*sinh(f*x+e))^(1/2)-96*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(4
,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^4/(a+I*a*sinh(f*x+e))^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 493, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3400, 4267, 2611, 6744, 2320, 6724}

$$\frac{96i\text{Li}_4\left(-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^4\sqrt{a+ia\sinh(e+fx)}}-\frac{96i\text{Li}_4\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^4\sqrt{a+ia\sinh(e+fx)}}-\frac{48ix\text{Li}_3\left(-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^3\sqrt{a+ia\sinh(e+fx)}}+\frac{48ix\text{Li}_3\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^3\sqrt{a+ia\sinh(e+fx)}}+\frac{12ix^2\text{Li}_2\left(-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}-\frac{12ix^2\text{Li}_2\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}+\frac{4ix^3\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\tanh^{-1}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f\sqrt{a+ia\sinh(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] ((4*I)*x^3*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f*x
)/2])/(f*Sqrt[a + I*a*Sinh[e + f*x]]) + ((12*I)*x^2*Cosh[e/2 + (I/4)*Pi + (
f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e
 + f*x]]) - ((12*I)*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e -
 I*Pi)/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((48*I)*x*Cosh[e/
2 + (I/4)*Pi + (f*x)/2]*PolyLog[3, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^3*Sqr
t[a + I*a*Sinh[e + f*x]]) + ((48*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLo
g[3, E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^3*Sqrt[a + I*a*Sinh[e + f*x]]) + ((9
6*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[4, -E^((2*e - I*Pi)/4 + (f*x)/2
)])/(f^4*Sqrt[a + I*a*Sinh[e + f*x]]) - ((96*I)*Cosh[e/2 + (I/4)*Pi + (f*x)
/2]*PolyLog[4, E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^4*Sqrt[a + I*a*Sinh[e + f*
x]])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\int x^3\text{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\,dx}{\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{4ix^3\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} - \frac{\left(6\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\right)\int x^2}{f\sqrt{a+ia\,\text{si}}}$$

$$= \frac{4ix^3\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} + \frac{12ix^2\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{Li}}{f^2\sqrt{a+ia\sinh(e}}$$

$$= \frac{4ix^3\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} + \frac{12ix^2\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{Li}}{f^2\sqrt{a+ia\sinh(e}}$$

$$= \frac{4ix^3\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} + \frac{12ix^2\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{Li}}{f^2\sqrt{a+ia\sinh(e}}$$

$$= \frac{4ix^3\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} + \frac{12ix^2\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{Li}}{f^2\sqrt{a+ia\sinh(e}}$$

Mathematica [A]
time = 0.82, size = 331, normalized size = 0.67

$$\frac{(1-i)(-1)^{3/4}\left(2ie^3\text{ArcTan}\left(\sqrt[4]{-1}e^{\frac{1}{2}(e+fx)}\right)+e^3\log\left(1-(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)+f^3x^3\log\left(1-(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)-e^3\log\left(1+(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)-f^3x^3\log\left(1+(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)-6f^2x^2\text{PolyLog}\left(2,-(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)+6f^2x^2\text{PolyLog}\left(2,(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)+24fx\text{PolyLog}\left(3,-(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)-24fx\text{PolyLog}\left(3,(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)-48\text{PolyLog}\left(4,-(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)+48\text{PolyLog}\left(4,(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)\right)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}{f^4\sqrt{a+ia\sinh(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] ((1 - I)*(-1)^(3/4)*((2*I)*e^3*ArcTan[(-1)^(1/4)*E^((e + f*x)/2)] + e^3*Log
[1 - (-1)^(3/4)*E^((e + f*x)/2)] + f^3*x^3*Log[1 - (-1)^(3/4)*E^((e + f*x)/
2)] - e^3*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] - f^3*x^3*Log[1 + (-1)^(3/4)*
E^((e + f*x)/2)] - 6*f^2*x^2*PolyLog[2, -((-1)^(3/4)*E^((e + f*x)/2))] + 6*
f^2*x^2*PolyLog[2, (-1)^(3/4)*E^((e + f*x)/2)] + 24*f*x*PolyLog[3, -((-1)^(
3/4)*E^((e + f*x)/2))] - 24*f*x*PolyLog[3, (-1)^(3/4)*E^((e + f*x)/2)] - 48
*PolyLog[4, -((-1)^(3/4)*E^((e + f*x)/2))] + 48*PolyLog[4, (-1)^(3/4)*E^((e
 + f*x)/2)])*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))/(f^4*Sqrt[a + I*a*S
inh[e + f*x]])
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+ia\sinh\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+I*a*sinh(f*x+e))^(1/2),x)
```

```
[Out] int(x^3/(a+I*a*sinh(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(I*a*sinh(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-2*I*sqrt(1/2*I*a*e^(-f*x - e))*x^3*e^(f*x + e)/(a*e^(f*x + e) - I
*a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+I*a*sinh(f*x+e))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(I*a*(sinh(e + f*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(I*a*sinh(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{a + a\sinh\left(e + f\,x\right)\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + a*sinh(e + f*x)*1i)^(1/2),x)
[Out] int(x^3/(a + a*sinh(e + f*x)*1i)^(1/2), x)
```

3.137 $\int \frac{x^2}{\sqrt{a+ia\sinh(e+fx)}}\,dx$

Optimal. Leaf size=349

$$\frac{4ix^2\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}}+\frac{8ix\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}-\frac{8ix\cos}{}$$

```
[Out] -4*I*x^2*arctanh(exp(1/2*e+3/4*I*Pi+1/2*f*x))*cosh(1/2*e+1/4*I*Pi+1/2*f*x)/
f/(a+I*a*sinh(f*x+e))^(1/2)+8*I*x*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(2,ex
p(1/2*e+3/4*I*Pi+1/2*f*x))/f^2/(a+I*a*sinh(f*x+e))^(1/2)-8*I*x*cosh(1/2*e+1
/4*I*Pi+1/2*f*x)*polylog(2,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^2/(a+I*a*sinh(f*
x+e))^(1/2)-16*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(3,exp(1/2*e+3/4*I*Pi+
1/2*f*x))/f^3/(a+I*a*sinh(f*x+e))^(1/2)+16*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*p
olylog(3,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^3/(a+I*a*sinh(f*x+e))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3400, 4267, 2611, 2320, 6724}

$$-\frac{16i\text{Li}_3\left(-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^3\sqrt{a+ia\sinh(e+fx)}}+\frac{16i\text{Li}_3\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^3\sqrt{a+ia\sinh(e+fx)}}+\frac{8ix\text{Li}_2\left(-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}-\frac{8ix\text{Li}_2\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}+\frac{4ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\tanh^{-1}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f\sqrt{a+ia\sinh(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] ((4*I)*x^2*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f*x
)/2])/(f*Sqrt[a + I*a*Sinh[e + f*x]]) + ((8*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x
)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e +
f*x]]) - ((8*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e - I*Pi)
/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((16*I)*Cosh[e/2 + (I/4
)*Pi + (f*x)/2]*PolyLog[3, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^3*Sqrt[a + I*
a*Sinh[e + f*x]]) + ((16*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[3, E^((2
*e - I*Pi)/4 + (f*x)/2)])/(f^3*Sqrt[a + I*a*Sinh[e + f*x]])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\int x^2\text{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\,dx}{\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{4ix^2\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} - \frac{\left(4\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\right)\int x\,\mathrm{l}}{f\sqrt{a+ia\sin}}$$

$$= \frac{4ix^2\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} + \frac{8ix\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{Li}_2(}{f^2\sqrt{a+ia\sinh(e}}$$

$$= \frac{4ix^2\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} + \frac{8ix\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{Li}_2(}{f^2\sqrt{a+ia\sinh(e}}$$

$$= \frac{4ix^2\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} + \frac{8ix\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{Li}_2(}{f^2\sqrt{a+ia\sinh(e}}$$

Mathematica [A]
time = 0.66, size = 276, normalized size = 0.79

$$\frac{(1+i)(-1)^{3/4}\left(-2ie^2\text{ArcTan}\left(\sqrt[4]{-1}\,e^{\frac{1}{2}(e+fx)}\right)-e^2\log\left(1-(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)+f^2x^2\log\left(1-(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)+e^2\log\left(1+(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)-f^2x^2\log\left(1+(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)-4fx\text{PolyLog}\left(2,-(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)+4fx\text{PolyLog}\left(2,(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)+8\text{PolyLog}\left(3,-(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)-8\text{PolyLog}\left(3,(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)\right)\left(-i\cosh\left(\frac{1}{2}(e+fx)\right)+\sinh\left(\frac{1}{2}(e+fx)\right)\right)}{f^3\sqrt{a+ia\sinh(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + I*a*Sinh[e + f*x]],x]
[Out] ((1 + I)*(-1)^(3/4)*((-2*I)*e^2*ArcTan[(-1)^(1/4)*E^((e + f*x)/2)] - e^2*Lo
g[1 - (-1)^(3/4)*E^((e + f*x)/2)] + f^2*x^2*Log[1 - (-1)^(3/4)*E^((e + f*x)
/2)] + e^2*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] - f^2*x^2*Log[1 + (-1)^(3/4)
*E^((e + f*x)/2)] - 4*f*x*PolyLog[2, -((-1)^(3/4)*E^((e + f*x)/2))] + 4*f*x
*PolyLog[2, (-1)^(3/4)*E^((e + f*x)/2)] + 8*PolyLog[3, -((-1)^(3/4)*E^((e +
 f*x)/2))] - 8*PolyLog[3, (-1)^(3/4)*E^((e + f*x)/2)])*((-I)*Cosh[(e + f*x)
/2] + Sinh[(e + f*x)/2]))/(f^3*Sqrt[a + I*a*Sinh[e + f*x]])
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+ia\sinh(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+I*a*sinh(f*x+e))^(1/2),x)
[Out] int(x^2/(a+I*a*sinh(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(I*a*sinh(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-2*I*sqrt(1/2*I*a*e^(-f*x - e))*x^2*e^(f*x + e)/(a*e^(f*x + e) - I
*a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+I*a*sinh(f*x+e))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(I*a*(sinh(e + f*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(I*a*sinh(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{a+a\sinh\left(e+f\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + a*sinh(e + f*x)*1i)^(1/2),x)
```

```
[Out] int(x^2/(a + a*sinh(e + f*x)*1i)^(1/2), x)
```

3.138 $\int \frac{x}{\sqrt{a+ia\sinh(e+fx)}}\,dx$

Optimal. Leaf size=207

$$\frac{4ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}}+\frac{4i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}-\frac{4i\cosh\left(\right.}{}$$

```
[Out] -4*I*x*arctanh(exp(1/2*e+3/4*I*Pi+1/2*f*x))*cosh(1/2*e+1/4*I*Pi+1/2*f*x)/f/
(a+I*a*sinh(f*x+e))^(1/2)+4*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(2,exp(1/
2*e+3/4*I*Pi+1/2*f*x))/f^2/(a+I*a*sinh(f*x+e))^(1/2)-4*I*cosh(1/2*e+1/4*I*P
i+1/2*f*x)*polylog(2,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/f^2/(a+I*a*sinh(f*x+e))^
(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3400, 4267, 2317, 2438}

$$\frac{4i\text{Li}_2\left(-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}-\frac{4i\text{Li}_2\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}+\frac{4ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\tanh^{-1}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f\sqrt{a+ia\sinh(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] ((4*I)*x*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f*x)/
2])/(f*Sqrt[a + I*a*Sinh[e + f*x]]) + ((4*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]
*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e + f*x]
]) - ((4*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e - I*Pi)/4 + (
f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e + f*x]])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
```

```
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\int x\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\,dx}{\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{4ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} - \frac{\left(2\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\right)\int \log}{f\sqrt{a+ia\sinh}}$$

$$= \frac{4ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} - \frac{\left(4\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\right)\text{Subst}}{f^2\sqrt{a+ia}}$$

$$= \frac{4ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} + \frac{4i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{Li}_2\Big(-}{f^2\sqrt{a+ia\sinh(e+}}$$

Mathematica [A]
time = 0.34, size = 221, normalized size = 1.07

$$\frac{\sqrt{2}\left(-2e\text{ArcTan}\left(\frac{i+\tanh\left(\frac{1}{4}(e+fx)\right)}{\sqrt{2}}\right)+i\pi\text{ArcTan}\left(\frac{i+\tanh\left(\frac{1}{4}(e+fx)\right)}{\sqrt{2}}\right)-\frac{1}{2}(2ie+\pi+2ifx)\left(\log\left(1-\sqrt[4]{-1}\,e^{-\frac{e}{2}-\frac{fx}{2}}\right)-\log\left(1+\sqrt[4]{-1}\,e^{-\frac{e}{2}-\frac{fx}{2}}\right)\right)-2i\left(\text{PolyLog}\left(2,-\sqrt[4]{-1}\,e^{-\frac{e}{2}-\frac{fx}{2}}\right)-\text{PolyLog}\left(2,\sqrt[4]{-1}\,e^{-\frac{e}{2}-\frac{fx}{2}}\right)\right)\right)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] (Sqrt[2]*(-2*e*ArcTan[(I + Tanh[(e + f*x)/4])/Sqrt[2]] + I*Pi*ArcTan[(I + T
anh[(e + f*x)/4])/Sqrt[2]] - (((2*I)*e + Pi + (2*I)*f*x)*(Log[1 - (-1)^(1/4
)*E^(-1/2*e - (f*x)/2)] - Log[1 + (-1)^(1/4)*E^(-1/2*e - (f*x)/2)]))/2 - (2
*I)*(PolyLog[2, -((-1)^(1/4)*E^(-1/2*e - (f*x)/2))] - PolyLog[2, (-1)^(1/4)
*E^(-1/2*e - (f*x)/2)]))*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))/(f^2*Sq
rt[a + I*a*Sinh[e + f*x]])
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+ia\sinh\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+I*a*sinh(f*x+e))^(1/2),x)
[Out] int(x/(a+I*a*sinh(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(I*a*sinh(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(-2*I*sqrt(1/2*I*a*e^(-f*x - e))*x*e^(f*x + e)/(a*e^(f*x + e) - I*a
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(f*x+e))**(1/2),x)
[Out] Integral(x/sqrt(I*a*(sinh(e + f*x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(I*a*sinh(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{a + a \sinh(e + f\,x)\ 1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + a*sinh(e + f*x)*1i)^(1/2),x)
```

```
[Out] int(x/(a + a*sinh(e + f*x)*1i)^(1/2), x)
```

3.139 $\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{1}{x\sqrt{a+ia\sinh(e+fx)}},x\right)$$

```
[Out] Unintegrable(1/x/(a+I*a*sinh(f*x+e))^(1/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Sqrt[a + I*a*Sinh[e + f*x]]),x]
[Out] Defer[Int][1/(x*Sqrt[a + I*a*Sinh[e + f*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx = \int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx$$

Mathematica [A]
time = 2.81, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[a + I*a*Sinh[e + f*x]]),x]
[Out] Integrate[1/(x*Sqrt[a + I*a*Sinh[e + f*x]]), x]
```

Maple [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+ia\sinh(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+I*a*sinh(f*x+e))^(1/2),x)
```

```
[Out] int(1/x/(a+I*a*sinh(f*x+e))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(I*a*sinh(f*x + e) + a)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-2*I*sqrt(1/2*I*a*e^(-f*x - e))*e^(f*x + e)/(a*x*e^(f*x + e) - I*a
*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(I*a*(sinh(e + f*x) - I))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(I*a*sinh(f*x + e) + a)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\,\sqrt{a+a\sinh \left(e+f\,x\right)\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + a*sinh(e + f*x)*1i)^(1/2)),x)
[Out] int(1/(x*(a + a*sinh(e + f*x)*1i)^(1/2)), x)
```

3.140 $\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[a + I*a*Sinh[e + f*x]]),x]
[Out] Defer[Int][1/(x^2*Sqrt[a + I*a*Sinh[e + f*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx=\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx$$

Mathematica [A]
time = 2.87, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[a + I*a*Sinh[e + f*x]]),x]
[Out] Integrate[1/(x^2*Sqrt[a + I*a*Sinh[e + f*x]]), x]
```

Maple [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+ia\sinh\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x)
```

```
[Out] int(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(I*a*sinh(f*x + e) + a)*x^2), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-2*I*sqrt(1/2*I*a*e^(-f*x - e))*e^(f*x + e)/(a*x^2*e^(f*x + e) - I
*a*x^2), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{ia\left(\sinh\left(e+fx\right)-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+I*a*sinh(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(I*a*(sinh(e + f*x) - I))), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(I*a*sinh(f*x + e) + a)*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2 \sqrt{a + a \sinh(e + f x) \, 1\mathrm{i}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + a*sinh(e + f*x)*1i)^(1/2)),x)
[Out] int(1/(x^2*(a + a*sinh(e + f*x)*1i)^(1/2)), x)
```

3.141 $\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=807

$$\frac{3x^2}{af^2\sqrt{a+ia\sinh(e+fx)}}-\frac{24ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af^3\sqrt{a+ia\sinh(e+fx)}}+\frac{ix^3\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh}{af\sqrt{a+ia\sinh(e+}$$

```
[Out] 3*x^2/a/f^2/(a+I*a*sinh(f*x+e))^(1/2)+24*I*x*arctanh(exp(1/2*e+3/4*I*Pi+1/2
*f*x))*cosh(1/2*e+1/4*I*Pi+1/2*f*x)/a/f^3/(a+I*a*sinh(f*x+e))^(1/2)-I*x^3*a
rctanh(exp(1/2*e+3/4*I*Pi+1/2*f*x))*cosh(1/2*e+1/4*I*Pi+1/2*f*x)/a/f/(a+I*a
*sinh(f*x+e))^(1/2)-24*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(2,exp(1/2*e+3
/4*I*Pi+1/2*f*x))/a/f^4/(a+I*a*sinh(f*x+e))^(1/2)+3*I*x^2*cosh(1/2*e+1/4*I*
Pi+1/2*f*x)*polylog(2,exp(1/2*e+3/4*I*Pi+1/2*f*x))/a/f^2/(a+I*a*sinh(f*x+e)
)^(1/2)+24*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(2,-exp(1/2*e+3/4*I*Pi+1/2
*f*x))/a/f^4/(a+I*a*sinh(f*x+e))^(1/2)-3*I*x^2*cosh(1/2*e+1/4*I*Pi+1/2*f*x)
*polylog(2,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/a/f^2/(a+I*a*sinh(f*x+e))^(1/2)-12
*I*x*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(3,exp(1/2*e+3/4*I*Pi+1/2*f*x))/a/
f^3/(a+I*a*sinh(f*x+e))^(1/2)+12*I*x*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(3
,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/a/f^3/(a+I*a*sinh(f*x+e))^(1/2)+24*I*cosh(1/
2*e+1/4*I*Pi+1/2*f*x)*polylog(4,exp(1/2*e+3/4*I*Pi+1/2*f*x))/a/f^4/(a+I*a*s
inh(f*x+e))^(1/2)-24*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(4,-exp(1/2*e+3/
4*I*Pi+1/2*f*x))/a/f^4/(a+I*a*sinh(f*x+e))^(1/2)+1/2*x^3*tanh(1/2*e+1/4*I*P
i+1/2*f*x)/a/f/(a+I*a*sinh(f*x+e))^(1/2)
```

Rubi [A]

time = 0.34, antiderivative size = 807, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3400, 4271, 4267, 2317, 2438, 2611, 6744, 2320, 6724}

$$\frac{\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)x^3}{2af\sqrt{i\sinh(e+fx)a+a}}+\frac{i\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)x^3}{af\sqrt{i\sinh(e+fx)a+a}}+\frac{3i\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{Li}_2\left(-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)x^2}{af^2\sqrt{i\sinh(e+fx)a+a}}-\frac{3i\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{Li}_2\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)x^2}{af^2\sqrt{i\sinh(e+fx)a+a}}+\frac{3x^2}{af^2\sqrt{i\sinh(e+fx)a+a}}-\frac{24i\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)x}{af^3\sqrt{i\sinh(e+fx)a+a}}-\frac{12i\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{Li}_3\left(-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)x}{af^3\sqrt{i\sinh(e+fx)a+a}}+\frac{12i\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{Li}_3\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)x}{af^3\sqrt{i\sinh(e+fx)a+a}}-\frac{24i\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{Li}_2\left(-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^4\sqrt{i\sinh(e+fx)a+a}}+\frac{24i\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{Li}_2\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^4\sqrt{i\sinh(e+fx)a+a}}+\frac{24i\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{Li}_4\left(-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^4\sqrt{i\sinh(e+fx)a+a}}-\frac{24i\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{Li}_4\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^4\sqrt{i\sinh(e+fx)a+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] (3*x^2)/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((24*I)*x*ArcTanh[E^((2*e - I
*Pi)/4 + (f*x)/2]*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/(a*f^3*Sqrt[a + I*a*Sinh
[e + f*x]]) + (I*x^3*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)
*Pi + (f*x)/2])/(a*f*Sqrt[a + I*a*Sinh[e + f*x]]) - ((24*I)*Cosh[e/2 + (I/4
)*Pi + (f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^4*Sqrt[a +
I*a*Sinh[e + f*x]]) + ((3*I)*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2,
-E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) + ((24*
I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])
/(a*f^4*Sqrt[a + I*a*Sinh[e + f*x]]) - ((3*I)*x^2*Cosh[e/2 + (I/4)*Pi + (f*
x)/2]*PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sinh[e
```

```
+ f*x]]) - ((12*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[3, -E^((2*e - I
*Pi)/4 + (f*x)/2)])/(a*f^3*Sqrt[a + I*a*Sinh[e + f*x]]) + ((12*I)*x*Cosh[e/
2 + (I/4)*Pi + (f*x)/2]*PolyLog[3, E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^3*Sq
rt[a + I*a*Sinh[e + f*x]]) + ((24*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog
[4, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^4*Sqrt[a + I*a*Sinh[e + f*x]]) - (
(24*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[4, E^((2*e - I*Pi)/4 + (f*x)/
2)])/(a*f^4*Sqrt[a + I*a*Sinh[e + f*x]]) + (x^3*Tanh[e/2 + (I/4)*Pi + (f*x)
/2])/(2*a*f*Sqrt[a + I*a*Sinh[e + f*x]])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx = -\frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\int x^3\text{csch}^3\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\,dx}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{3x^2}{af^2\sqrt{a+ia\sinh(e+fx)}} + \frac{x^3\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+ia\sinh(e+fx)}} + \frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\right.}{4a\sqrt{a}}$$

$$= \frac{3x^2}{af^2\sqrt{a+ia\sinh(e+fx)}} - \frac{24ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right.}{af^3\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{3x^2}{af^2\sqrt{a+ia\sinh(e+fx)}} - \frac{24ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right.}{af^3\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{3x^2}{af^2\sqrt{a+ia\sinh(e+fx)}} - \frac{24ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right.}{af^3\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{3x^2}{af^2\sqrt{a+ia\sinh(e+fx)}} - \frac{24ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right.}{af^3\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{3x^2}{af^2\sqrt{a+ia\sinh(e+fx)}} - \frac{24ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right.}{af^3\sqrt{a+ia\sinh(e+fx)}}$$

Mathematica [A]
time = 1.78, size = 546, normalized size = 0.68

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] ((Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])*(f^2*x^2*(6 + I*f*x)*(Cosh[(e +
f*x)/2] + I*Sinh[(e + f*x)/2]) + (1/2 - I/2)*(-1)^(3/4)*(-48*e*ArcTanh[(-1)
^(3/4)*E^((e + f*x)/2)] + 2*e^3*ArcTanh[(-1)^(3/4)*E^((e + f*x)/2)] - 24*e*
Log[1 - (-1)^(3/4)*E^((e + f*x)/2)] + e^3*Log[1 - (-1)^(3/4)*E^((e + f*x)/2
)] - 24*f*x*Log[1 - (-1)^(3/4)*E^((e + f*x)/2)] + f^3*x^3*Log[1 - (-1)^(3/4
)*E^((e + f*x)/2)] + 24*e*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] - e^3*Log[1 +
 (-1)^(3/4)*E^((e + f*x)/2)] + 24*f*x*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] -
 f^3*x^3*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] - 6*(-8 + f^2*x^2)*PolyLog[2,
-((-1)^(3/4)*E^((e + f*x)/2))] + 6*(-8 + f^2*x^2)*PolyLog[2, (-1)^(3/4)*E^(
(e + f*x)/2)] + 24*f*x*PolyLog[3, -((-1)^(3/4)*E^((e + f*x)/2))] - 24*f*x*P
olyLog[3, (-1)^(3/4)*E^((e + f*x)/2)] - 48*PolyLog[4, -((-1)^(3/4)*E^((e +
f*x)/2))] + 48*PolyLog[4, (-1)^(3/4)*E^((e + f*x)/2)])*(Cosh[(e + f*x)/2] +
```

```
 I*Sinh[(e + f*x)/2])^2 + 2*f^3*x^3*Sinh[(e + f*x)/2]))/(2*f^4*(a + I*a*Sin
h[e + f*x])^(3/2))
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a + ia\sinh\left(fx + e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+I*a*sinh(f*x+e))^(3/2),x)
```

```
[Out] int(x^3/(a+I*a*sinh(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] ((a^2*f^2*e^(2*f*x + 2*e) - 2*I*a^2*f^2*e^(f*x + e) - a^2*f^2)*integral(1/2
*(-I*f^2*x^3 + 24*I*x)*sqrt(1/2*I*a*e^(-f*x - e))*e^(f*x + e)/(a^2*f^2*e^(f
*x + e) - I*a^2*f^2), x) + ((-I*f*x^3 - 6*I*x^2)*e^(2*f*x + 2*e) + (f*x^3 -
 6*x^2)*e^(f*x + e))*sqrt(1/2*I*a*e^(-f*x - e)))/(a^2*f^2*e^(2*f*x + 2*e) -
 2*I*a^2*f^2*e^(f*x + e) - a^2*f^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(ia\left(\sinh\left(e + fx\right) - i\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+I*a*sinh(f*x+e))**(3/2),x)
```

```
[Out] Integral(x**3/(I*a*(sinh(e + f*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(x^3/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{{\left(a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + a*sinh(e + f*x)*1i)^(3/2),x)
[Out] int(x^3/(a + a*sinh(e + f*x)*1i)^(3/2), x)
```

3.142 $\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=506

$$\frac{2x}{af^2\sqrt{a+ia\sinh(e+fx)}}-\frac{4\text{ArcTan}\left(\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af^3\sqrt{a+ia\sinh(e+fx)}}+\frac{ix^2\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\text{c}}{af\sqrt{a+ia\sinh(e}}$$

```
[Out] 2*x/a/f^2/(a+I*a*sinh(f*x+e))^(1/2)-4*arctan(sinh(1/2*e+1/4*I*Pi+1/2*f*x))*
cosh(1/2*e+1/4*I*Pi+1/2*f*x)/a/f^3/(a+I*a*sinh(f*x+e))^(1/2)-I*x^2*arctanh(
exp(1/2*e+3/4*I*Pi+1/2*f*x))*cosh(1/2*e+1/4*I*Pi+1/2*f*x)/a/f/(a+I*a*sinh(f
*x+e))^(1/2)+2*I*x*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(2,exp(1/2*e+3/4*I*P
i+1/2*f*x))/a/f^2/(a+I*a*sinh(f*x+e))^(1/2)-2*I*x*cosh(1/2*e+1/4*I*Pi+1/2*f
*x)*polylog(2,-exp(1/2*e+3/4*I*Pi+1/2*f*x))/a/f^2/(a+I*a*sinh(f*x+e))^(1/2)
-4*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(3,exp(1/2*e+3/4*I*Pi+1/2*f*x))/a/
f^3/(a+I*a*sinh(f*x+e))^(1/2)+4*I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(3,-e
xp(1/2*e+3/4*I*Pi+1/2*f*x))/a/f^3/(a+I*a*sinh(f*x+e))^(1/2)+1/2*x^2*tanh(1/
2*e+1/4*I*Pi+1/2*f*x)/a/f/(a+I*a*sinh(f*x+e))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 506, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3400, 4271, 3855, 4267, 2611, 2320, 6724}

$$-\frac{4\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{ArcTan}\left(\sinh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{af^3\sqrt{a+ia\sinh(e+fx)}}-\frac{4i\text{Li}_3\left(-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{af^3\sqrt{a+ia\sinh(e+fx)}}+\frac{4i\text{Li}_3\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{af^3\sqrt{a+ia\sinh(e+fx)}}+\frac{2ix\text{Li}_2\left(-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{af^2\sqrt{a+ia\sinh(e+fx)}}-\frac{2ix\text{Li}_2\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{af^2\sqrt{a+ia\sinh(e+fx)}}+\frac{2x}{af^2\sqrt{a+ia\sinh(e+fx)}}+\frac{x^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{2af\sqrt{a+ia\sinh(e+fx)}}+\frac{ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\tanh^{-1}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{af\sqrt{a+ia\sinh(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] (2*x)/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - (4*ArcTan[Sinh[e/2 + (I/4)*Pi +
 (f*x)/2]]*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/(a*f^3*Sqrt[a + I*a*Sinh[e + f*x
]]) + (I*x^2*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f
*x)/2])/(a*f*Sqrt[a + I*a*Sinh[e + f*x]]) + ((2*I)*x*Cosh[e/2 + (I/4)*Pi +
(f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sin
h[e + f*x]]) - ((2*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e -
 I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((4*I)*Cosh[e/2
 + (I/4)*Pi + (f*x)/2]*PolyLog[3, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^3*Sq
rt[a + I*a*Sinh[e + f*x]]) + ((4*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[
3, E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^3*Sqrt[a + I*a*Sinh[e + f*x]]) + (x^
2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(2*a*f*Sqrt[a + I*a*Sinh[e + f*x]])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx = -\frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\int x^2\text{csch}^3\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\,dx}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{2x}{af^2\sqrt{a+ia\sinh(e+fx)}} + \frac{x^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+ia\sinh(e+fx)}} + \frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\right.}{4a\sqrt{}}$$

$$= \frac{2x}{af^2\sqrt{a+ia\sinh(e+fx)}} - \frac{4\tan^{-1}\left(\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\right.}{af^3\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{2x}{af^2\sqrt{a+ia\sinh(e+fx)}} - \frac{4\tan^{-1}\left(\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\right.}{af^3\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{2x}{af^2\sqrt{a+ia\sinh(e+fx)}} - \frac{4\tan^{-1}\left(\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\right.}{af^3\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{2x}{af^2\sqrt{a+ia\sinh(e+fx)}} - \frac{4\tan^{-1}\left(\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\right.}{af^3\sqrt{a+ia\sinh(e+fx)}}$$

Mathematica [A]
time = 1.20, size = 384, normalized size = 0.76

$$\frac{\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)\left(fx(4+ifx)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)-\left(\frac{1}{2}-\frac{i}{2}\right)(-1)^{3/4}\left(-16\tanh^{-1}\left((-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)+2e^2\tanh^{-1}\left((-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)+e^2\log\left(1-(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)-f^2x^2\log\left(1-(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)-e^2\log\left(1+(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)+f^2x^2\log\left(1+(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)+4fx\text{PolyLog}\left(2,-(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)-4fx\text{PolyLog}\left(2,(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)-8\text{PolyLog}\left(3,-(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)+8\text{PolyLog}\left(3,(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)\right)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)^2+2f^2x^2\sinh\left(\frac{1}{2}(e+fx)\right)\right)}{2f^3(a+ia\sinh(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] ((Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])*(f*x*(4 + I*f*x)*(Cosh[(e + f*x)
/2] + I*Sinh[(e + f*x)/2]) - (1/2 - I/2)*(-1)^(3/4)*(-16*ArcTanh[(-1)^(3/4)
*E^((e + f*x)/2)] + 2*e^2*ArcTanh[(-1)^(3/4)*E^((e + f*x)/2)] + e^2*Log[1 -
 (-1)^(3/4)*E^((e + f*x)/2)] - f^2*x^2*Log[1 - (-1)^(3/4)*E^((e + f*x)/2)]
- e^2*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] + f^2*x^2*Log[1 + (-1)^(3/4)*E^((
e + f*x)/2)] + 4*f*x*PolyLog[2, -((-1)^(3/4)*E^((e + f*x)/2))] - 4*f*x*Poly
Log[2, (-1)^(3/4)*E^((e + f*x)/2)] - 8*PolyLog[3, -((-1)^(3/4)*E^((e + f*x)
/2))] + 8*PolyLog[3, (-1)^(3/4)*E^((e + f*x)/2)])*(Cosh[(e + f*x)/2] + I*Si
nh[(e + f*x)/2])^2 + 2*f^2*x^2*Sinh[(e + f*x)/2]))/(2*f^3*(a + I*a*Sinh[e +
 f*x])^(3/2))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + ia \sinh (fx + e))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+I*a*sinh(f*x+e))^(3/2),x)
```

```
[Out] int(x^2/(a+I*a*sinh(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] ((a^2*f^2*e^(2*f*x + 2*e) - 2*I*a^2*f^2*e^(f*x + e) - a^2*f^2)*integral(1/2
*(-I*f^2*x^2 + 8*I)*sqrt(1/2*I*a*e^(-f*x - e))*e^(f*x + e)/(a^2*f^2*e^(f*x
+ e) - I*a^2*f^2), x) + ((-I*f*x^2 - 4*I*x)*e^(2*f*x + 2*e) + (f*x^2 - 4*x)
*e^(f*x + e))*sqrt(1/2*I*a*e^(-f*x - e)))/(a^2*f^2*e^(2*f*x + 2*e) - 2*I*a^
2*f^2*e^(f*x + e) - a^2*f^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(ia (\sinh (e + fx) - i))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+I*a*sinh(f*x+e))**(3/2),x)
```

```
[Out] Integral(x**2/(I*a*(sinh(e + f*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(x^2/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(a + a\sinh(e + f\,x)\ 1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + a*sinh(e + f*x)*1i)^(3/2),x)
[Out] int(x^2/(a + a*sinh(e + f*x)*1i)^(3/2), x)
```

3.143 $\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=288

$$\frac{1}{af^2\sqrt{a+ia\sinh(e+fx)}}+\frac{ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af\sqrt{a+ia\sinh(e+fx)}}+\frac{i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{PolyLog}\Big(2,-}{af^2\sqrt{a+ia\sinh(e+f}}$$

```
[Out] 1/a/f^2/(a+I*a*sinh(f*x+e))^(1/2)-I*x*arctanh(exp(1/2*e+3/4*I*Pi+1/2*f*x))*
cosh(1/2*e+1/4*I*Pi+1/2*f*x)/a/f/(a+I*a*sinh(f*x+e))^(1/2)+I*cosh(1/2*e+1/4
*I*Pi+1/2*f*x)*polylog(2,exp(1/2*e+3/4*I*Pi+1/2*f*x))/a/f^2/(a+I*a*sinh(f*x
+e))^(1/2)-I*cosh(1/2*e+1/4*I*Pi+1/2*f*x)*polylog(2,-exp(1/2*e+3/4*I*Pi+1/2
*f*x))/a/f^2/(a+I*a*sinh(f*x+e))^(1/2)+1/2*x*tanh(1/2*e+1/4*I*Pi+1/2*f*x)/a
/f/(a+I*a*sinh(f*x+e))^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3400, 4270, 4267, 2317, 2438}

$$\frac{i\text{Li}_2\left(-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{af^2\sqrt{a+ia\sinh(e+fx)}}-\frac{i\text{Li}_2\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{af^2\sqrt{a+ia\sinh(e+fx)}}+\frac{1}{af^2\sqrt{a+ia\sinh(e+fx)}}+\frac{x\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{2af\sqrt{a+ia\sinh(e+fx)}}+\frac{ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\tanh^{-1}\left(e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{af\sqrt{a+ia\sinh(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] 1/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) + (I*x*ArcTanh[E^((2*e - I*Pi)/4 + (f
*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/(a*f*Sqrt[a + I*a*Sinh[e + f*x]]) +
 (I*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)
])/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - (I*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*
PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]
]) + (x*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(2*a*f*Sqrt[a + I*a*Sinh[e + f*x]])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
```

```
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx &= -\frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\int x\operatorname{csch}^3\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\,dx}{2a\sqrt{a+ia\sinh(e+fx)}}\\
&= \frac{1}{af^2\sqrt{a+ia\sinh(e+fx)}}+\frac{x\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+ia\sinh(e+fx)}}+\frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\right.}{4a\sqrt{}}\\
&= \frac{1}{af^2\sqrt{a+ia\sinh(e+fx)}}+\frac{ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af\sqrt{a+ia\sinh(e+fx)}}\\
&= \frac{1}{af^2\sqrt{a+ia\sinh(e+fx)}}+\frac{ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af\sqrt{a+ia\sinh(e+fx)}}\\
&= \frac{1}{af^2\sqrt{a+ia\sinh(e+fx)}}+\frac{ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af\sqrt{a+ia\sinh(e+fx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.61, size = 332, normalized size = 1.15

$$
\frac{\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)\left((2+ifx)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)-\sqrt{2}\,e\operatorname{ArcTan}\left(\frac{i+\tanh\left(\frac{1}{4}(e+fx)\right)}{\sqrt{2}}\right)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)^2+\frac{i\left(\pi\operatorname{ArcTan}\left(\frac{i+\tanh\left(\frac{1}{4}(e+fx)\right)}{\sqrt{2}}\right)+\frac{1}{2}i(2ie+\pi+2ifx)\left(\log\left(1-\sqrt[4]{-1}\,e^{-\frac{e}{2}-\frac{fx}{2}}\right)-\log\left(1+\sqrt[4]{-1}\,e^{-\frac{e}{2}-\frac{fx}{2}}\right)\right)-2\operatorname{PolyLog}\left(2,-\sqrt[4]{-1}\,e^{-\frac{e}{2}-\frac{fx}{2}}\right)+2\operatorname{PolyLog}\left(2,\sqrt[4]{-1}\,e^{-\frac{e}{2}-\frac{fx}{2}}\right)\right)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)^2}{\sqrt{2}}+2fx\sinh\left(\frac{1}{2}(e+fx)\right)\right)}{2f^2(a+ia\sinh(e+fx))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] ((Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])*((2 + I*f*x)*(Cosh[(e + f*x)/2]
+ I*Sinh[(e + f*x)/2]) - Sqrt[2]*e*ArcTan[(I + Tanh[(e + f*x)/4])/Sqrt[2]]*
(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^2 + (I*(Pi*ArcTan[(I + Tanh[(e +
f*x)/4])/Sqrt[2]] + (I/2)*((2*I)*e + Pi + (2*I)*f*x)*(Log[1 - (-1)^(1/4)*E^
(-1/2*e - (f*x)/2)] - Log[1 + (-1)^(1/4)*E^(-1/2*e - (f*x)/2)]) - 2*PolyLog
[2, -((-1)^(1/4)*E^(-1/2*e - (f*x)/2))] + 2*PolyLog[2, (-1)^(1/4)*E^(-1/2*e
 - (f*x)/2)])*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^2)/Sqrt[2] + 2*f*x*
Sinh[(e + f*x)/2]))/(2*f^2*(a + I*a*Sinh[e + f*x])^(3/2))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+ia\sinh\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+I*a*sinh(f*x+e))^(3/2),x)
```

```
[Out] int(x/(a+I*a*sinh(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] ((a^2*f^2*e^(2*f*x + 2*e) - 2*I*a^2*f^2*e^(f*x + e) - a^2*f^2)*integral(-1/
2*I*sqrt(1/2*I*a*e^(-f*x - e))*x*e^(f*x + e)/(a^2*e^(f*x + e) - I*a^2), x)
+ ((-I*f*x - 2*I)*e^(2*f*x + 2*e) + (f*x - 2)*e^(f*x + e))*sqrt(1/2*I*a*e^(
-f*x - e)))/(a^2*f^2*e^(2*f*x + 2*e) - 2*I*a^2*f^2*e^(f*x + e) - a^2*f^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(ia\left(\sinh\left(e+fx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(f*x+e))**(3/2),x)
```

```
[Out] Integral(x/(I*a*(sinh(e + f*x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + a*sinh(e + f*x)*1i)^(3/2),x)
```

```
[Out] int(x/(a + a*sinh(e + f*x)*1i)^(3/2), x)
```

3.144 $\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{1}{x(a+ia\sinh(e+fx))^{3/2}},x\right)$$

```
[Out] Unintegrable(1/x/(a+I*a*sinh(f*x+e))^(3/2),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(a + I*a*Sinh[e + f*x])^(3/2)),x]
[Out] Defer[Int][1/(x*(a + I*a*Sinh[e + f*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx = \int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx$$

Mathematica [A]
time = 15.76, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(a + I*a*Sinh[e + f*x])^(3/2)),x]
[Out] Integrate[1/(x*(a + I*a*Sinh[e + f*x])^(3/2)), x]
```

Maple [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+ia\sinh\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+I*a*sinh(f*x+e))^(3/2),x)
```

```
[Out] int(1/x/(a+I*a*sinh(f*x+e))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*sinh(f*x + e) + a)^(3/2)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] ((a^2*f^2*x^2*e^(2*f*x + 2*e) - 2*I*a^2*f^2*x^2*e^(f*x + e) - a^2*f^2*x^2)*
integral(1/2*(-I*f^2*x^2 + 8*I)*sqrt(1/2*I*a*e^(-f*x - e))*e^(f*x + e)/(a^2
*f^2*x^3*e^(f*x + e) - I*a^2*f^2*x^3), x) + ((-I*f*x + 2*I)*e^(2*f*x + 2*e)
 + (f*x + 2)*e^(f*x + e))*sqrt(1/2*I*a*e^(-f*x - e)))/(a^2*f^2*x^2*e^(2*f*x
 + 2*e) - 2*I*a^2*f^2*x^2*e^(f*x + e) - a^2*f^2*x^2)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(ia\left(\sinh\left(e+fx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(x*(I*a*(sinh(e + f*x) - I))**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*sinh(f*x + e) + a)^(3/2)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\left(a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + a*sinh(e + f*x)*1i)^(3/2)),x)
[Out] int(1/(x*(a + a*sinh(e + f*x)*1i)^(3/2)), x)
```

3.145 $\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(a + I*a*Sinh[e + f*x])^(3/2)),x]
```

```
[Out] Defer[Int][1/(x^2*(a + I*a*Sinh[e + f*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx=\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx$$

Mathematica [A]
time = 17.52, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(a + I*a*Sinh[e + f*x])^(3/2)),x]
```

```
[Out] Integrate[1/(x^2*(a + I*a*Sinh[e + f*x])^(3/2)), x]
```

Maple [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+ia\sinh\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x)
```

```
[Out] int(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*sinh(f*x + e) + a)^(3/2)*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] ((a^2*f^2*x^3*e^(2*f*x + 2*e) - 2*I*a^2*f^2*x^3*e^(f*x + e) - a^2*f^2*x^3)*
integral(1/2*(-I*f^2*x^2 + 24*I)*sqrt(1/2*I*a*e^(-f*x - e))*e^(f*x + e)/(a^
2*f^2*x^4*e^(f*x + e) - I*a^2*f^2*x^4), x) + ((-I*f*x + 4*I)*e^(2*f*x + 2*e
) + (f*x + 4)*e^(f*x + e))*sqrt(1/2*I*a*e^(-f*x - e)))/(a^2*f^2*x^3*e^(2*f*
x + 2*e) - 2*I*a^2*f^2*x^3*e^(f*x + e) - a^2*f^2*x^3)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(ia\left(\sinh\left(e+fx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+I*a*sinh(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(x**2*(I*a*(sinh(e + f*x) - I))**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*sinh(f*x + e) + a)^(3/2)*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2\,{\left(a+a\,\mathrm{sinh}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + a*sinh(e + f*x)*1i)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a + a*sinh(e + f*x)*1i)^(3/2)), x)
```

3.146 $\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=1016

$$-\frac{1}{a^2d^4\sqrt{a+ia\sinh(c+dx)}}+\frac{9x^2}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}}-\frac{10ix\tanh^{-1}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{a^2d^3\sqrt{a+ia\sinh(c+dx)}}-$$

```
[Out] -1/a^2/d^4/(a+I*a*sinh(d*x+c))^(1/2)+9/8*x^2/a^2/d^2/(a+I*a*sinh(d*x+c))^(1
/2)+10*I*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(2,-exp(1/2*c+3/4*I*Pi+1/2*d*x
))/a^2/d^4/(a+I*a*sinh(d*x+c))^(1/2)-9/8*I*x^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)
*polylog(2,-exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)-
10*I*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(2,exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^
2/d^4/(a+I*a*sinh(d*x+c))^(1/2)-3/8*I*x^3*arctanh(exp(1/2*c+3/4*I*Pi+1/2*d*
x))*cosh(1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d/(a+I*a*sinh(d*x+c))^(1/2)+9/2*I*x*co
sh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(3,-exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^3/
(a+I*a*sinh(d*x+c))^(1/2)+10*I*x*arctanh(exp(1/2*c+3/4*I*Pi+1/2*d*x))*cosh(
1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d^3/(a+I*a*sinh(d*x+c))^(1/2)-9/2*I*x*cosh(1/2*
c+1/4*I*Pi+1/2*d*x)*polylog(3,exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^3/(a+I*a*s
inh(d*x+c))^(1/2)-9*I*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(4,-exp(1/2*c+3/4
*I*Pi+1/2*d*x))/a^2/d^4/(a+I*a*sinh(d*x+c))^(1/2)+9*I*cosh(1/2*c+1/4*I*Pi+1
/2*d*x)*polylog(4,exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^4/(a+I*a*sinh(d*x+c))^
(1/2)+9/8*I*x^2*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(2,exp(1/2*c+3/4*I*Pi+1
/2*d*x))/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)+1/4*x^2*sech(1/2*c+1/4*I*Pi+1/2*
d*x)^2/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)-1/2*x*tanh(1/2*c+1/4*I*Pi+1/2*d*x)
/a^2/d^3/(a+I*a*sinh(d*x+c))^(1/2)+3/16*x^3*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a^
2/d/(a+I*a*sinh(d*x+c))^(1/2)+1/8*x^3*sech(1/2*c+1/4*I*Pi+1/2*d*x)^2*tanh(1
/2*c+1/4*I*Pi+1/2*d*x)/a^2/d/(a+I*a*sinh(d*x+c))^(1/2)
```

Rubi [A]

time = 0.50, antiderivative size = 1016, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3400, 4271, 4270, 4267, 2317, 2438, 2611, 6744, 2320, 6724}

$$\frac{\operatorname{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)x^3}{8a^2d\sqrt{i\sinh(c+dx)a+a}}+\frac{3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)x^3}{16a^2d\sqrt{i\sinh(c+dx)a+a}}+\frac{3i\tanh^{-1}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)x^3}{8a^2d\sqrt{i\sinh(c+dx)a+a}}+\frac{\operatorname{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)x^2}{4a^2d^2\sqrt{i\sinh(c+dx)a+a}}+\frac{9i\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\operatorname{Li}_2\left(-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)x^2}{8a^2d^2\sqrt{i\sinh(c+dx)a+a}}-\frac{9i\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\operatorname{Li}_2\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)x^2}{8a^2d^2\sqrt{i\sinh(c+dx)a+a}}+\frac{9x^2}{8a^2d^2\sqrt{i\sinh(c+dx)a+a}}-\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)x}{2a^2d^3\sqrt{i\sinh(c+dx)a+a}}-\frac{10i\tanh^{-1}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)x}{a^2d^3\sqrt{i\sinh(c+dx)a+a}}-\frac{9i\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\operatorname{Li}_3\left(-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)x}{2a^2d^3\sqrt{i\sinh(c+dx)a+a}}+\frac{9i\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\operatorname{Li}_3\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)x}{2a^2d^3\sqrt{i\sinh(c+dx)a+a}}-\frac{10i\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\operatorname{Li}_2\left(-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{a^2d^4\sqrt{i\sinh(c+dx)a+a}}+\frac{10i\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\operatorname{Li}_2\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{a^2d^4\sqrt{i\sinh(c+dx)a+a}}+\frac{9i\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\operatorname{Li}_4\left(-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{a^2d^4\sqrt{i\sinh(c+dx)a+a}}-\frac{9i\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\operatorname{Li}_4\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)}{a^2d^4\sqrt{i\sinh(c+dx)a+a}}-\frac{1}{a^2d^4\sqrt{i\sinh(c+dx)a+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] -(1/(a^2*d^4*Sqrt[a + I*a*Sinh[c + d*x]])) + (9*x^2)/(8*a^2*d^2*Sqrt[a + I*
a*Sinh[c + d*x]]) - ((10*I)*x*ArcTanh[E^((2*c - I*Pi)/4 + (d*x)/2)]*Cosh[c/
2 + (I/4)*Pi + (d*x)/2])/(a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (((3*I)/8)
*x^3*ArcTanh[E^((2*c - I*Pi)/4 + (d*x)/2)]*Cosh[c/2 + (I/4)*Pi + (d*x)/2])/
(a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) - ((10*I)*Cosh[c/2 + (I/4)*Pi + (d*x)/2
]*PolyLog[2, -E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*d^4*Sqrt[a + I*a*Sinh[c +
```

```
 d*x]]) + (((9*I)/8)*x^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[2, -E^((2*c
 - I*Pi)/4 + (d*x)/2)])/(a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) + ((10*I)*Cos
h[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[2, E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*
d^4*Sqrt[a + I*a*Sinh[c + d*x]]) - (((9*I)/8)*x^2*Cosh[c/2 + (I/4)*Pi + (d*
x)/2]*PolyLog[2, E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*d^2*Sqrt[a + I*a*Sinh[
c + d*x]]) - (((9*I)/2)*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[3, -E^((2*
c - I*Pi)/4 + (d*x)/2)])/(a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (((9*I)/2)
*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[3, E^((2*c - I*Pi)/4 + (d*x)/2)])
/(a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + ((9*I)*Cosh[c/2 + (I/4)*Pi + (d*x)
/2]*PolyLog[4, -E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*d^4*Sqrt[a + I*a*Sinh[c
 + d*x]]) - ((9*I)*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[4, E^((2*c - I*Pi
)/4 + (d*x)/2)])/(a^2*d^4*Sqrt[a + I*a*Sinh[c + d*x]]) + (x^2*Sech[c/2 + (I
/4)*Pi + (d*x)/2]^2)/(4*a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - (x*Tanh[c/2
+ (I/4)*Pi + (d*x)/2])/(2*a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (3*x^3*Tan
h[c/2 + (I/4)*Pi + (d*x)/2])/(16*a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) + (x^3*
Sech[c/2 + (I/4)*Pi + (d*x)/2]^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(8*a^2*d*S
qrt[a + I*a*Sinh[c + d*x]])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \frac{\sinh\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\int x^3\operatorname{csch}^5\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\,dx}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$= \frac{x^2\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}} + \frac{x^3\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}} -$$

$$= -\frac{1}{a^2d^4\sqrt{a+ia\sinh(c+dx)}} + \frac{9x^2}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} + \frac{x^2\operatorname{sech}^2}{4a^2d^2\sqrt{a}}$$

$$= -\frac{1}{a^2d^4\sqrt{a+ia\sinh(c+dx)}} + \frac{9x^2}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} - \frac{10ix\tanh^{-}}{a}$$

$$= -\frac{1}{a^2d^4\sqrt{a+ia\sinh(c+dx)}} + \frac{9x^2}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} - \frac{10ix\tanh^{-}}{a}$$

$$= -\frac{1}{a^2d^4\sqrt{a+ia\sinh(c+dx)}} + \frac{9x^2}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} - \frac{10ix\tanh^{-}}{a}$$

$$= -\frac{1}{a^2d^4\sqrt{a+ia\sinh(c+dx)}} + \frac{9x^2}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} - \frac{10ix\tanh^{-}}{a}$$

$$= -\frac{1}{a^2d^4\sqrt{a+ia\sinh(c+dx)}} + \frac{9x^2}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} - \frac{10ix\tanh^{-}}{a}$$

Mathematica [**A**]
time = 2.70, size = 1200, normalized size = 1.18

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] ((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(-48*Cosh[(c + d*x)/2] + (8*I)*c
*Cosh[(c + d*x)/2] + 70*c^2*Cosh[(c + d*x)/2] - (11*I)*c^3*Cosh[(c + d*x)/2
] - (8*I)*(c + d*x)*Cosh[(c + d*x)/2] - 140*c*(c + d*x)*Cosh[(c + d*x)/2] +
 (33*I)*c^2*(c + d*x)*Cosh[(c + d*x)/2] + 70*(c + d*x)^2*Cosh[(c + d*x)/2]
- (33*I)*c*(c + d*x)^2*Cosh[(c + d*x)/2] + (11*I)*(c + d*x)^3*Cosh[(c + d*x
)/2] + 16*Cosh[(3*(c + d*x))/2] + (8*I)*c*Cosh[(3*(c + d*x))/2] - 18*c^2*Co
sh[(3*(c + d*x))/2] - (3*I)*c^3*Cosh[(3*(c + d*x))/2] - (8*I)*(c + d*x)*Cos
h[(3*(c + d*x))/2] + 36*c*(c + d*x)*Cosh[(3*(c + d*x))/2] + (9*I)*c^2*(c +
d*x)*Cosh[(3*(c + d*x))/2] - 18*(c + d*x)^2*Cosh[(3*(c + d*x))/2] - (9*I)*c
```

```
*(c + d*x)^2*Cosh[(3*(c + d*x))/2] + (3*I)*(c + d*x)^3*Cosh[(3*(c + d*x))/2
] + (1 - I)*(-1)^(3/4)*(-160*c*ArcTanh[(-1)^(3/4)*E^((c + d*x)/2)] + 6*c^3*
ArcTanh[(-1)^(3/4)*E^((c + d*x)/2)] - 80*c*Log[1 - (-1)^(3/4)*E^((c + d*x)/
2)] + 3*c^3*Log[1 - (-1)^(3/4)*E^((c + d*x)/2)] - 80*d*x*Log[1 - (-1)^(3/4)
*E^((c + d*x)/2)] + 3*d^3*x^3*Log[1 - (-1)^(3/4)*E^((c + d*x)/2)] + 80*c*Lo
g[1 + (-1)^(3/4)*E^((c + d*x)/2)] - 3*c^3*Log[1 + (-1)^(3/4)*E^((c + d*x)/2
)] + 80*d*x*Log[1 + (-1)^(3/4)*E^((c + d*x)/2)] - 3*d^3*x^3*Log[1 + (-1)^(3
/4)*E^((c + d*x)/2)] - 2*(-80 + 9*d^2*x^2)*PolyLog[2, -((-1)^(3/4)*E^((c +
d*x)/2))] + 2*(-80 + 9*d^2*x^2)*PolyLog[2, (-1)^(3/4)*E^((c + d*x)/2)] + 72
*d*x*PolyLog[3, -((-1)^(3/4)*E^((c + d*x)/2))] - 72*d*x*PolyLog[3, (-1)^(3/
4)*E^((c + d*x)/2)] - 144*PolyLog[4, -((-1)^(3/4)*E^((c + d*x)/2))] + 144*P
olyLog[4, (-1)^(3/4)*E^((c + d*x)/2)])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x
)/2])^4 - (48*I)*Sinh[(c + d*x)/2] + 8*c*Sinh[(c + d*x)/2] + (70*I)*c^2*Sin
h[(c + d*x)/2] - 11*c^3*Sinh[(c + d*x)/2] - 8*(c + d*x)*Sinh[(c + d*x)/2] -
 (140*I)*c*(c + d*x)*Sinh[(c + d*x)/2] + 33*c^2*(c + d*x)*Sinh[(c + d*x)/2]
 + (70*I)*(c + d*x)^2*Sinh[(c + d*x)/2] - 33*c*(c + d*x)^2*Sinh[(c + d*x)/2
] + 11*(c + d*x)^3*Sinh[(c + d*x)/2] - (16*I)*Sinh[(3*(c + d*x))/2] - 8*c*S
inh[(3*(c + d*x))/2] + (18*I)*c^2*Sinh[(3*(c + d*x))/2] + 3*c^3*Sinh[(3*(c
+ d*x))/2] + 8*(c + d*x)*Sinh[(3*(c + d*x))/2] - (36*I)*c*(c + d*x)*Sinh[(3
*(c + d*x))/2] - 9*c^2*(c + d*x)*Sinh[(3*(c + d*x))/2] + (18*I)*(c + d*x)^2
*Sinh[(3*(c + d*x))/2] + 9*c*(c + d*x)^2*Sinh[(3*(c + d*x))/2] - 3*(c + d*x
)^3*Sinh[(3*(c + d*x))/2]))/(32*d^4*(a + I*a*Sinh[c + d*x])^(5/2))
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(a +i a \sinh \left(d x +c \right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+I*a*sinh(d*x+c))^(5/2),x)
[Out] int(x^3/(a+I*a*sinh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(x^3/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/8*(8*(a^3*d^4*e^(4*d*x + 4*c) - 4*I*a^3*d^4*e^(3*d*x + 3*c) - 6*a^3*d^4*e
^(2*d*x + 2*c) + 4*I*a^3*d^4*e^(d*x + c) + a^3*d^4)*integral(1/16*(-3*I*d^2
*x^3 + 80*I*x)*sqrt(1/2*I*a*e^(-d*x - c))*e^(d*x + c)/(a^3*d^2*e^(d*x + c)
- I*a^3*d^2), x) + ((-3*I*d^3*x^3 - 18*I*d^2*x^2 + 8*I*d*x + 16*I)*e^(4*d*x
 + 4*c) - (11*d^3*x^3 + 70*d^2*x^2 - 8*d*x - 48)*e^(3*d*x + 3*c) + (-11*I*d
^3*x^3 + 70*I*d^2*x^2 + 8*I*d*x - 48*I)*e^(2*d*x + 2*c) - (3*d^3*x^3 - 18*d
^2*x^2 - 8*d*x + 16)*e^(d*x + c))*sqrt(1/2*I*a*e^(-d*x - c)))/(a^3*d^4*e^(4
*d*x + 4*c) - 4*I*a^3*d^4*e^(3*d*x + 3*c) - 6*a^3*d^4*e^(2*d*x + 2*c) + 4*I
*a^3*d^4*e^(d*x + c) + a^3*d^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+I*a*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + a*sinh(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(x^3/(a + a*sinh(c + d*x)*1i)^(5/2), x)
```

3.147 $\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=689

$$\frac{3x}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}}-\frac{5\text{ArcTan}\left(\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{3a^2d^3\sqrt{a+ia\sinh(c+dx)}}+\frac{3ix^2\tanh^{-1}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cdots}{8a^2d\sqrt{a+ia\sinh\cdots}}$$

[Out] 3/4*x/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)-5/3*arctan(sinh(1/2*c+1/4*I*Pi+1/2*d*x))*cosh(1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d^3/(a+I*a*sinh(d*x+c))^(1/2)-3/8*I*x^2*arctanh(exp(1/2*c+3/4*I*Pi+1/2*d*x))*cosh(1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d/(a+I*a*sinh(d*x+c))^(1/2)+3/4*I*x*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(2,exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)-3/4*I*x*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(2,-exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)-3/2*I*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(3,exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^3/(a+I*a*sinh(d*x+c))^(1/2)+3/2*I*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(3,-exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^3/(a+I*a*sinh(d*x+c))^(1/2)+1/6*x*sech(1/2*c+1/4*I*Pi+1/2*d*x)^2/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)-1/6*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d^3/(a+I*a*sinh(d*x+c))^(1/2)+3/16*x^2*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d/(a+I*a*sinh(d*x+c))^(1/2)+1/8*x^2*sech(1/2*c+1/4*I*Pi+1/2*d*x)^2*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d/(a+I*a*sinh(d*x+c))^(1/2)

Rubi [A]
time = 0.32, antiderivative size = 689, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3400, 4271, 3853, 3855, 4267, 2611, 2320, 6724}

$$-\frac{5\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{ArcTan}\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{3a^2d^3\sqrt{a+ia\sinh(c+dx)}}-\frac{3i\text{Li}_3\left(-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{2a^2d^3\sqrt{a+ia\sinh(c+dx)}}+\frac{3i\text{Li}_3\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{2a^2d^3\sqrt{a+ia\sinh(c+dx)}}-\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{6a^2d^3\sqrt{a+ia\sinh(c+dx)}}+\frac{3ix\text{Li}_2\left(-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}}-\frac{3ix\text{Li}_2\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}}+\frac{3x}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}}+\frac{x\,\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{6a^2d^2\sqrt{a+ia\sinh(c+dx)}}+\frac{3x^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{16a^2d\sqrt{a+ia\sinh(c+dx)}}+\frac{3ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\tanh^{-1}\left(e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}}+\frac{x^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + I*a*Sinh[c + d*x])^(5/2),x]

[Out] (3*x)/(4*a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - (5*ArcTan[Sinh[c/2 + (I/4)*Pi + (d*x)/2]]*Cosh[c/2 + (I/4)*Pi + (d*x)/2])/(3*a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (((3*I)/8)*x^2*ArcTanh[E^((2*c - I*Pi)/4 + (d*x)/2)]*Cosh[c/2 + (I/4)*Pi + (d*x)/2])/(a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) + (((3*I)/4)*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[2, -E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - (((3*I)/4)*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[2, E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - (((3*I)/2)*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[3, -E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (((3*I)/2)*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[3, E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (x*Sech[c/2 + (I/4)*Pi + (d*x)/2]^2)/(6*a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - Tanh[c/2 + (I/4)*Pi + (d*x)/2]/(

```
6*a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (3*x^2*Tanh[c/2 + (I/4)*Pi + (d*x)
/2])/(16*a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) + (x^2*Sech[c/2 + (I/4)*Pi + (d
*x)/2]^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(8*a^2*d*Sqrt[a + I*a*Sinh[c + d*x
]])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx &= \frac{\sinh\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\int x^2\operatorname{csch}^5\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\,dx}{4a^2\sqrt{a+ia\sinh(c+dx)}}\\
&= \frac{x\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{6a^2d^2\sqrt{a+ia\sinh(c+dx)}}+\frac{x^2\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}}-\\
&= \frac{3x}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}}+\frac{x\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{6a^2d^2\sqrt{a+ia\sinh(c+dx)}}-\frac{\tanh\left(\frac{c}{2}\right.}{6a^2d^3\sqrt{a+}}\\
&= \frac{3x}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}}-\frac{5\tan^{-1}\left(\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\right.}{3a^2d^3\sqrt{a+ia\sinh(c+dx)}}\\
&= \frac{3x}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}}-\frac{5\tan^{-1}\left(\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\right.}{3a^2d^3\sqrt{a+ia\sinh(c+dx)}}\\
&= \frac{3x}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}}-\frac{5\tan^{-1}\left(\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\right.}{3a^2d^3\sqrt{a+ia\sinh(c+dx)}}\\
&= \frac{3x}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}}-\frac{5\tan^{-1}\left(\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\right.}{3a^2d^3\sqrt{a+ia\sinh(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 1.59, size = 482, normalized size = 0.70

$$
\frac{\left(\cosh\left(\tfrac{1}{2}(c+dx)\right)+i\sinh\left(\tfrac{1}{2}(c+dx)\right)\right)\left(4dx(4+3idx)\left(\cosh\left(\tfrac{1}{2}(c+dx)\right)+i\sinh\left(\tfrac{1}{2}(c+dx)\right)\right)+\left(-8i+36dx+9id^2x^2\right)\left(\cosh\left(\tfrac{1}{2}(c+dx)\right)+i\sinh\left(\tfrac{1}{2}(c+dx)\right)\right)^3-\left(\tfrac{1}{2}-\tfrac{i}{2}\right)(-1)^{3/4}\left(-160\tanh^{-1}\left((-1)^{3/4}e^{\frac{1}{2}(c+dx)}\right)+18c^2\tanh^{-1}\left((-1)^{3/4}e^{\frac{1}{2}(c+dx)}\right)+9c^2\log\left(1-(-1)^{3/4}e^{\frac{1}{2}(c+dx)}\right)-9d^2x^2\log\left(1-(-1)^{3/4}e^{\frac{1}{2}(c+dx)}\right)-9c^2\log\left(1+(-1)^{3/4}e^{\frac{1}{2}(c+dx)}\right)+9d^2x^2\log\left(1+(-1)^{3/4}e^{\frac{1}{2}(c+dx)}\right)+36dx\,\text{PolyLog}\left(2,-(-1)^{3/4}e^{\frac{1}{2}(c+dx)}\right)-36dx\,\text{PolyLog}\left(2,(-1)^{3/4}e^{\frac{1}{2}(c+dx)}\right)-72\,\text{PolyLog}\left(3,-(-1)^{3/4}e^{\frac{1}{2}(c+dx)}\right)+72\,\text{PolyLog}\left(3,(-1)^{3/4}e^{\frac{1}{2}(c+dx)}\right)\right)\left(\cosh\left(\tfrac{1}{2}(c+dx)\right)+i\sinh\left(\tfrac{1}{2}(c+dx)\right)\right)^4+24d^2x^2\sinh\left(\tfrac{1}{2}(c+dx)\right)+2\left(-8+9d^2x^2\right)\left(\cosh\left(\tfrac{1}{2}(c+dx)\right)+i\sinh\left(\tfrac{1}{2}(c+dx)\right)\right)^2\sinh\left(\tfrac{1}{2}(c+dx)\right)\right)}{48d^3(a+ia\sinh(c+dx))^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] ((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(4*d*x*(4 + (3*I)*d*x)*(Cosh[(c
+ d*x)/2] + I*Sinh[(c + d*x)/2]) + (-8*I + 36*d*x + (9*I)*d^2*x^2)*(Cosh[(c
 + d*x)/2] + I*Sinh[(c + d*x)/2])^3 - (1/2 - I/2)*(-1)^(3/4)*(-160*ArcTanh[
(-1)^(3/4)*E^((c + d*x)/2)] + 18*c^2*ArcTanh[(-1)^(3/4)*E^((c + d*x)/2)] +
9*c^2*Log[1 - (-1)^(3/4)*E^((c + d*x)/2)] - 9*d^2*x^2*Log[1 - (-1)^(3/4)*E^
((c + d*x)/2)] - 9*c^2*Log[1 + (-1)^(3/4)*E^((c + d*x)/2)] + 9*d^2*x^2*Log[
1 + (-1)^(3/4)*E^((c + d*x)/2)] + 36*d*x*PolyLog[2, -((-1)^(3/4)*E^((c + d*
x)/2))] - 36*d*x*PolyLog[2, (-1)^(3/4)*E^((c + d*x)/2)] - 72*PolyLog[3, -((
-1)^(3/4)*E^((c + d*x)/2))] + 72*PolyLog[3, (-1)^(3/4)*E^((c + d*x)/2)])*(C
osh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^4 + 24*d^2*x^2*Sinh[(c + d*x)/2] +
2*(-8 + 9*d^2*x^2)*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2*Sinh[(c + d*
x)/2]))/(48*d^3*(a + I*a*Sinh[c + d*x])^(5/2))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(a +i a \sinh \left(d x +c \right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+I*a*sinh(d*x+c))^(5/2),x)
```

```
[Out] int(x^2/(a+I*a*sinh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(24*(a^3*d^3*e^(4*d*x + 4*c) - 4*I*a^3*d^3*e^(3*d*x + 3*c) - 6*a^3*d^3
*e^(2*d*x + 2*c) + 4*I*a^3*d^3*e^(d*x + c) + a^3*d^3)*integral(1/48*(-9*I*d
^2*x^2 + 80*I)*sqrt(1/2*I*a*e^(-d*x - c))*e^(d*x + c)/(a^3*d^2*e^(d*x + c)
- I*a^3*d^2), x) + ((-9*I*d^2*x^2 - 36*I*d*x + 8*I)*e^(4*d*x + 4*c) - (33*d
^2*x^2 + 140*d*x - 8)*e^(3*d*x + 3*c) + (-33*I*d^2*x^2 + 140*I*d*x + 8*I)*e
^(2*d*x + 2*c) - (9*d^2*x^2 - 36*d*x - 8)*e^(d*x + c))*sqrt(1/2*I*a*e^(-d*x
 - c)))/(a^3*d^3*e^(4*d*x + 4*c) - 4*I*a^3*d^3*e^(3*d*x + 3*c) - 6*a^3*d^3*
e^(2*d*x + 2*c) + 4*I*a^3*d^3*e^(d*x + c) + a^3*d^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+I*a*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + a*sinh(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(x^2/(a + a*sinh(c + d*x)*1i)^(5/2), x)
```

3.148 $\int \frac{x}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=416

$$\frac{3}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}}+\frac{3ix\tanh^{-1}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}}+\frac{3i\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\text{PolyLog}}{8a^2d^2\sqrt{a+ia\sinh(}}$$

```
[Out] 3/8/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)-3/8*I*x*arctanh(exp(1/2*c+3/4*I*Pi+1/
2*d*x))*cosh(1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d/(a+I*a*sinh(d*x+c))^(1/2)+3/8*I*
cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(2,exp(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^2
/(a+I*a*sinh(d*x+c))^(1/2)-3/8*I*cosh(1/2*c+1/4*I*Pi+1/2*d*x)*polylog(2,-ex
p(1/2*c+3/4*I*Pi+1/2*d*x))/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)+1/12*sech(1/2*
c+1/4*I*Pi+1/2*d*x)^2/a^2/d^2/(a+I*a*sinh(d*x+c))^(1/2)+3/16*x*tanh(1/2*c+1
/4*I*Pi+1/2*d*x)/a^2/d/(a+I*a*sinh(d*x+c))^(1/2)+1/8*x*sech(1/2*c+1/4*I*Pi+
1/2*d*x)^2*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a^2/d/(a+I*a*sinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.263$, Rules used = {3400, 4270, 4267, 2317, 2438}

$$\frac{3i\text{Li}_2\left(-e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}}-\frac{3i\text{Li}_2\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}}+\frac{3}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}}+\frac{\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{12a^2d^2\sqrt{a+ia\sinh(c+dx)}}+\frac{3x\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{16a^2d\sqrt{a+ia\sinh(c+dx)}}+\frac{3ix\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\tanh^{-1}\left(e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}}+\frac{x\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] 3/(8*a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) + (((3*I)/8)*x*ArcTanh[E^((2*c -
I*Pi)/4 + (d*x)/2)]*Cosh[c/2 + (I/4)*Pi + (d*x)/2])/(a^2*d*Sqrt[a + I*a*Sin
h[c + d*x]]) + (((3*I)/8)*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[2, -E^((2*
c - I*Pi)/4 + (d*x)/2)])/(a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - (((3*I)/8)
*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[2, E^((2*c - I*Pi)/4 + (d*x)/2)])/(
a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) + Sech[c/2 + (I/4)*Pi + (d*x)/2]^2/(12
*a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) + (3*x*Tanh[c/2 + (I/4)*Pi + (d*x)/2]
)/(16*a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) + (x*Sech[c/2 + (I/4)*Pi + (d*x)/2
]^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(8*a^2*d*Sqrt[a + I*a*Sinh[c + d*x]])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\int \frac{x}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \frac{\sinh\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\int x\operatorname{csch}^5\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\,dx}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$= \frac{\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{12a^2d^2\sqrt{a+ia\sinh(c+dx)}} + \frac{x\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}} -$$

$$= \frac{3}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} + \frac{\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{12a^2d^2\sqrt{a+ia\sinh(c+dx)}} + \frac{3x\tanh}{16a^2d\sqrt{a+}}$$

$$= \frac{3}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} + \frac{3ix\tanh^{-1}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right.}{8a^2d\sqrt{a+ia\sinh(c+dx)}}$$

$$= \frac{3}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} + \frac{3ix\tanh^{-1}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right.}{8a^2d\sqrt{a+ia\sinh(c+dx)}}$$

$$= \frac{3}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} + \frac{3ix\tanh^{-1}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right.}{8a^2d\sqrt{a+ia\sinh(c+dx)}}$$

Mathematica [A]
time = 1.06, size = 411, normalized size = 0.99

$$\frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(4(2+3idx)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)+9(2+idx)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^3-9\sqrt{2}\,c\text{ArcTan}\left(\frac{i+\tanh\left(\frac{1}{4}(c+dx)\right)}{\sqrt{2}}\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^4+\frac{9i\left(\pi\text{ArcTan}\left(\frac{i+\tanh\left(\frac{1}{4}(c+dx)\right)}{\sqrt{2}}\right)+\frac{1}{2}i(2ic+\pi+2idx)\left(\log\left(1-\sqrt[4]{-1}\,e^{-\frac{c}{2}-\frac{dx}{2}}\right)-\log\left(1+\sqrt[4]{-1}\,e^{-\frac{c}{2}-\frac{dx}{2}}\right)\right)-2\text{PolyLog}\left(2,-\sqrt[4]{-1}\,e^{-\frac{c}{2}-\frac{dx}{2}}\right)+2\text{PolyLog}\left(2,\sqrt[4]{-1}\,e^{-\frac{c}{2}-\frac{dx}{2}}\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^4}{\sqrt{2}}+24dx\sinh\left(\frac{1}{2}(c+dx)\right)+18dx\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{48d^2(a+ia\sinh(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] ((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(4*(2 + (3*I)*d*x)*(Cosh[(c + d*
x)/2] + I*Sinh[(c + d*x)/2]) + 9*(2 + I*d*x)*(Cosh[(c + d*x)/2] + I*Sinh[(c
 + d*x)/2])^3 - 9*Sqrt[2]*c*ArcTan[(I + Tanh[(c + d*x)/4])/Sqrt[2]]*(Cosh[(
c + d*x)/2] + I*Sinh[(c + d*x)/2])^4 + ((9*I)*(Pi*ArcTan[(I + Tanh[(c + d*x
)/4])/Sqrt[2]] + (I/2)*((2*I)*c + Pi + (2*I)*d*x)*(Log[1 - (-1)^(1/4)*E^(-1
/2*c - (d*x)/2)] - Log[1 + (-1)^(1/4)*E^(-1/2*c - (d*x)/2)]) - 2*PolyLog[2,
 -((-1)^(1/4)*E^(-1/2*c - (d*x)/2))] + 2*PolyLog[2, (-1)^(1/4)*E^(-1/2*c -
(d*x)/2)])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^4)/Sqrt[2] + 24*d*x*Si
nh[(c + d*x)/2] + 18*d*x*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2*Sinh[(
c + d*x)/2]))/(48*d^2*(a + I*a*Sinh[c + d*x])^(5/2))
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+ia\sinh\left(dx+c\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+I*a*sinh(d*x+c))^(5/2),x)
```

```
[Out] int(x/(a+I*a*sinh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(24*(a^3*d^2*e^(4*d*x + 4*c) - 4*I*a^3*d^2*e^(3*d*x + 3*c) - 6*a^3*d^2
*e^(2*d*x + 2*c) + 4*I*a^3*d^2*e^(d*x + c) + a^3*d^2)*integral(-3/16*I*sqrt
(1/2*I*a*e^(-d*x - c))*x*e^(d*x + c)/(a^3*e^(d*x + c) - I*a^3), x) - (9*(I*
d*x + 2*I)*e^(4*d*x + 4*c) + (33*d*x + 70)*e^(3*d*x + 3*c) - (-33*I*d*x + 7
0*I)*e^(2*d*x + 2*c) + 9*(d*x - 2)*e^(d*x + c))*sqrt(1/2*I*a*e^(-d*x - c)))
/(a^3*d^2*e^(4*d*x + 4*c) - 4*I*a^3*d^2*e^(3*d*x + 3*c) - 6*a^3*d^2*e^(2*d*
x + 2*c) + 4*I*a^3*d^2*e^(d*x + c) + a^3*d^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{{\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + a*sinh(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(x/(a + a*sinh(c + d*x)*1i)^(5/2), x)
```

3.149 $\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=24

$$\operatorname{Int}\left(\frac{1}{x(a+ia\sinh(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable(1/x/(a+I*a*sinh(d*x+c))^(5/2),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(a + I*a*Sinh[c + d*x])^(5/2)),x]
[Out] Defer[Int][1/(x*(a + I*a*Sinh[c + d*x])^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx$$

Mathematica [A]
time = 25.09, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(a + I*a*Sinh[c + d*x])^(5/2)),x]
[Out] Integrate[1/(x*(a + I*a*Sinh[c + d*x])^(5/2)), x]
```

Maple [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+ia\sinh\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+I*a*sinh(d*x+c))^(5/2),x)
```

```
[Out] int(1/x/(a+I*a*sinh(d*x+c))^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*sinh(d*x + c) + a)^(5/2)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(24*(a^3*d^4*x^4*e^(4*d*x + 4*c) - 4*I*a^3*d^4*x^4*e^(3*d*x + 3*c) - 6
*a^3*d^4*x^4*e^(2*d*x + 2*c) + 4*I*a^3*d^4*x^4*e^(d*x + c) + a^3*d^4*x^4)*i
ntegral(1/48*(-9*I*d^4*x^4 + 80*I*d^2*x^2 - 384*I)*sqrt(1/2*I*a*e^(-d*x - c
))*e^(d*x + c)/(a^3*d^4*x^5*e^(d*x + c) - I*a^3*d^4*x^5), x) + ((-9*I*d^3*x
^3 + 18*I*d^2*x^2 + 8*I*d*x - 48*I)*e^(4*d*x + 4*c) - (33*d^3*x^3 - 70*d^2*
x^2 - 8*d*x + 144)*e^(3*d*x + 3*c) + (-33*I*d^3*x^3 - 70*I*d^2*x^2 + 8*I*d*
x + 144*I)*e^(2*d*x + 2*c) - (9*d^3*x^3 + 18*d^2*x^2 - 8*d*x - 48)*e^(d*x +
 c))*sqrt(1/2*I*a*e^(-d*x - c)))/(a^3*d^4*x^4*e^(4*d*x + 4*c) - 4*I*a^3*d^4
*x^4*e^(3*d*x + 3*c) - 6*a^3*d^4*x^4*e^(2*d*x + 2*c) + 4*I*a^3*d^4*x^4*e^(d
*x + c) + a^3*d^4*x^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*sinh(d*x + c) + a)^(5/2)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\,{\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + a*sinh(c + d*x)*1i)^(5/2)),x)
```

```
[Out] int(1/(x*(a + a*sinh(c + d*x)*1i)^(5/2)), x)
```

3.150 $\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx$

Optimal. Leaf size=24

$$\operatorname{Int}\left(\frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x},x\right)$$

```
[Out] Unintegrable((a+I*a*sinh(f*x+e))^(1/3)/x,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + I*a*Sinh[e + f*x])^(1/3)/x,x]
[Out] Defer[Int][(a + I*a*Sinh[e + f*x])^(1/3)/x, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx = \int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx$$

Mathematica [A]
time = 2.88, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + I*a*Sinh[e + f*x])^(1/3)/x,x]
[Out] Integrate[(a + I*a*Sinh[e + f*x])^(1/3)/x, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(a+ia\sinh(fx+e))^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))^(1/3)/x,x)
```

```
[Out] int((a+I*a*sinh(f*x+e))^(1/3)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/3)/x,x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^(1/3)/x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/3)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{ia\left(\sinh\left(e+fx\right)-i\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))**(1/3)/x,x)
```

```
[Out] Integral((I*a*(sinh(e + f*x) - I))**(1/3)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/3)/x,x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^(1/3)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a + a\sinh(e + f\,x)\ 1\mathrm{i})^{1/3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(e + f*x)*1i)^(1/3)/x,x)
[Out] int((a + a*sinh(e + f*x)*1i)^(1/3)/x, x)
```

3.151 $\int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx$

Optimal. Leaf size=26

$$\text{Int}((c+dx)^m(a+ia\sinh(e+fx))^n,x)$$

```
[Out] Unintegrable((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^n,x]
[Out] Defer[Int][(c + d*x)^m*(a + I*a*Sinh[e + f*x])^n, x]
```

Rubi steps

$$\int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx = \int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx$$

Mathematica [A]
time = 2.79, size = 0, normalized size = 0.00

$$\int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^n,x]
[Out] Integrate[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^n, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m\,(a+ia\sinh(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x)
[Out] int((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*(I*a*sinh(f*x + e) + a)^n, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x, algorithm="fricas")
[Out] integral((d*x + c)^m*(1/2*(I*a*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - I*a)*e^(
-f*x - e))^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+I*a*sinh(f*x+e))**n,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x, algorithm="giac")
[Out] integrate((d*x + c)^m*(I*a*sinh(f*x + e) + a)^n, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (a+a\sinh(e+f\,x)\ 1\mathrm{i})^n\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(e + f*x)*1i)^n*(c + d*x)^m,x)
[Out] int((a + a*sinh(e + f*x)*1i)^n*(c + d*x)^m, x)
```

3.152 $\int (c+dx)^m (a+ia\sinh(e+fx))^3\,dx$

Optimal. Leaf size=410

$$\frac{5a^3(c+dx)^{1+m}}{2d(1+m)}-\frac{i3^{-1-m}a^3e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3f(c+dx)}{d}\right)}{8f}-\frac{3\;2^{-3-m}a^3e^{2e-\frac{2cf}{d}}(c+dx)^m}{}$$

```
[Out] 5/2*a^3*(d*x+c)^(1+m)/d/(1+m)-1/8*I*3^(-1-m)*a^3*exp(3*e-3*c*f/d)*(d*x+c)^m
*GAMMA(1+m,-3*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)-3*2^(-3-m)*a^3*exp(2*e-2*c*
f/d)*(d*x+c)^m*GAMMA(1+m,-2*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+15/8*I*a^3*ex
p(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+15/8*I*a^
3*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)+3*2^(-
3-m)*a^3*exp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA(1+m,2*f*(d*x+c)/d)/f/((f*(d*x+c)
/d)^m)-1/8*I*3^(-1-m)*a^3*exp(-3*e+3*c*f/d)*(d*x+c)^m*GAMMA(1+m,3*f*(d*x+c)
/d)/f/((f*(d*x+c)/d)^m)
```

Rubi [A]

time = 0.42, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3399, 3393, 3388, 2212, 3389}

$$-\frac{ia^3 3^{-m-1}e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{3f(c+dx)}{d}\right)}{8f}-\frac{3a^3 2^{-m-3}e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2f(c+dx)}{d}\right)}{f}+\frac{15ia^3e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{8f}+\frac{15ia^3e^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{8f}+\frac{3a^3 2^{-m-3}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2f(c+dx)}{d}\right)}{f}-\frac{ia^3 3^{-m-1}e^{\frac{3cf}{d}-3e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{3f(c+dx)}{d}\right)}{8f}+\frac{5a^3(c+dx)^{m+1}}{2d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^3,x]
```

```
[Out] (5*a^3*(c + d*x)^(1 + m))/(2*d*(1 + m)) - ((I/8)*3^(-1 - m)*a^3*E^(3*e - (3
*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-3*f*(c + d*x))/d])/(f*(-((f*(c + d*x))/
d))^m) - (3*2^(-3 - m)*a^3*E^(2*e - (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2
*f*(c + d*x))/d])/(f*(-((f*(c + d*x))/d))^m) + (((15*I)/8)*a^3*E^(e - (c*f)
/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m
) + (((15*I)/8)*a^3*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))
/d])/(f*((f*(c + d*x))/d)^m) + (3*2^(-3 - m)*a^3*E^(-2*e + (2*c*f)/d)*(c +
d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) - ((I/8)*3^
(-1 - m)*a^3*E^(-3*e + (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (3*f*(c + d*x))/
d])/(f*((f*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^m(a+ia\sinh(e+fx))^3\,dx &= \left(8a^3\right)\int (c+dx)^m\sin^6\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right)dx \\
&= \left(8a^3\right)\int \left(\frac{5}{16}(c+dx)^m-\frac{3}{16}(c+dx)^m\cosh(2e+2fx)+\frac{15}{32}i(c+ \right. \\
&= \frac{5a^3(c+dx)^{1+m}}{2d(1+m)}-\frac{1}{4}\left(ia^3\right)\int (c+dx)^m\sinh(3e+3fx)\,dx+\frac{1}{4}(15ia \\
&= \frac{5a^3(c+dx)^{1+m}}{2d(1+m)}-\frac{1}{8}\left(ia^3\right)\int e^{-i(3ie+3ifx)}(c+dx)^m\,dx+\frac{1}{8}\left(ia^3\right)\int e \\
&= \frac{5a^3(c+dx)^{1+m}}{2d(1+m)}-\frac{i3^{-1-m}a^3e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\Big(1+}{8f}
\end{aligned}
$$

Mathematica [A]
time = 4.37, size = 452, normalized size = 1.10

$$
\frac{2^{-3-m}3^{-1-m}a^3e^{-3\left(e+\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}\left(5\ 2^m3^{2+m}de^{2e+\frac{4cf}{d}}(1+m)\left(-\frac{f(c+dx)}{d}\right)^m\Gamma\left(1+m,f\left(\frac{c}{d}+x\right)\right)-2^mde^{6e}(1+m)\left(\frac{f(c+dx)}{d}\right)^m\Gamma\left(1+m,-\frac{3f(c+dx)}{d}\right)+i3^{2+m}de^{5e+\frac{cf}{d}}(1+m)\left(f\left(\frac{c}{d}+x\right)\right)^m\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)+5\ 2^m3^{2+m}de^{4e+\frac{2cf}{d}}(1+m)\left(\frac{f(c+dx)}{d}\right)^m\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)-i3^{2+m}de^{e+\frac{5cf}{d}}(1+m)\left(-\frac{f(c+dx)}{d}\right)^m\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)-2^me^{\frac{3cf}{d}}\left(20i3^{1+m}e^{3e}f(c+dx)\left(-\frac{f^2(c+dx)^2}{d^2}\right)^m+de^{\frac{3cf}{d}}(1+m)\left(-\frac{f(c+dx)}{d}\right)^m\Gamma\left(1+m,\frac{3f(c+dx)}{d}\right)\right)\right)(-i+\sinh(e+fx))^3}{df(1+m)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^3,x]
```

```
[Out] (2^(-3 - m)*3^(-1 - m)*a^3*(c + d*x)^m*(5*2^m*3^(2 + m)*d*E^(2*e + (4*c*f)/
d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, f*(c/d + x)] - 2^m*d*E^(6*e)
*(1 + m)*((f*(c + d*x))/d)^m*Gamma[1 + m, (-3*f*(c + d*x))/d] + I*3^(2 + m)
*d*E^(5*e + (c*f)/d)*(1 + m)*(f*(c/d + x))^m*Gamma[1 + m, (-2*f*(c + d*x))/
d] + 5*2^m*3^(2 + m)*d*E^(4*e + (2*c*f)/d)*(1 + m)*((f*(c + d*x))/d)^m*Gamm
a[1 + m, -((f*(c + d*x))/d)] - I*3^(2 + m)*d*E^(e + (5*c*f)/d)*(1 + m)*(-((
f*(c + d*x))/d))^m*Gamma[1 + m, (2*f*(c + d*x))/d] - 2^m*E^((3*c*f)/d)*((20
*I)*3^(1 + m)*E^(3*e)*f*(c + d*x)*(-((f^2*(c + d*x)^2)/d^2))^m + d*E^((3*c*
f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (3*f*(c + d*x))/d]))*(-I
+ Sinh[e + f*x])^3)/(d*E^(3*(e + (c*f)/d))*f*(1 + m)*(-((f^2*(c + d*x)^2)/d
^2))^m*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^6)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m (a + ia \sinh(fx + e))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x)
```

```
[Out] int((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x)
```

Maxima [A]
time = 0.12, size = 383, normalized size = 0.93

$$-\frac{1}{8}i\left(\frac{(dx+c)^{m+1}e^{\left(\frac{3cf}{d}-3e\right)}E_{-m}\left(\frac{3(dx+c)f}{d}\right)}{d}-\frac{3(dx+c)^{m+1}e^{\left(\frac{cf}{d}-e\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{3(dx+c)^{m+1}e^{\left(-\frac{cf}{d}+e\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}-\frac{(dx+c)^{m+1}e^{\left(-\frac{3cf}{d}+3e\right)}E_{-m}\left(-\frac{3(dx+c)f}{d}\right)}{d}\right)a^3+\frac{3}{4}\left(\frac{(dx+c)^{m+1}e^{\left(\frac{2cf}{d}-2e\right)}E_{-m}\left(\frac{2(dx+c)f}{d}\right)}{d}+\frac{(dx+c)^{m+1}e^{\left(-\frac{2cf}{d}+2e\right)}E_{-m}\left(-\frac{2(dx+c)f}{d}\right)}{d}+\frac{2(dx+c)^{m+1}}{d(m+1)}\right)a^3+\frac{3}{2}i\left(\frac{(dx+c)^{m+1}e^{\left(\frac{cf}{d}-e\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}-\frac{(dx+c)^{m+1}e^{\left(-\frac{cf}{d}+e\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)a^3+\frac{(dx+c)^{m+1}a^3}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/8*I*((d*x + c)^(m + 1)*e^(3*c*f/d - 3*e)*exp_integral_e(-m, 3*(d*x + c)*
f/d)/d - 3*(d*x + c)^(m + 1)*e^(c*f/d - e)*exp_integral_e(-m, (d*x + c)*f/d
)/d + 3*(d*x + c)^(m + 1)*e^(-c*f/d + e)*exp_integral_e(-m, -(d*x + c)*f/d)
/d - (d*x + c)^(m + 1)*e^(-3*c*f/d + 3*e)*exp_integral_e(-m, -3*(d*x + c)*f
/d)/d)*a^3 + 3/4*((d*x + c)^(m + 1)*e^(2*c*f/d - 2*e)*exp_integral_e(-m, 2*
(d*x + c)*f/d)/d + (d*x + c)^(m + 1)*e^(-2*c*f/d + 2*e)*exp_integral_e(-m,
-2*(d*x + c)*f/d)/d + 2*(d*x + c)^(m + 1)/(d*(m + 1)))*a^3 + 3/2*I*((d*x +
c)^(m + 1)*e^(c*f/d - e)*exp_integral_e(-m, (d*x + c)*f/d)/d - (d*x + c)^(m
 + 1)*e^(-c*f/d + e)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a^3 + (d*x + c)^
(m + 1)*a^3/(d*(m + 1))
```

Fricas [A]
time = 0.10, size = 380, normalized size = 0.93

$$\frac{(-i\,a^3dm-i\,a^3d)e^{\left(-\frac{dm\log\left(\frac{3\,f}{d}\right)-3\,cf+3\,de}{d}\right)}\Gamma\left(m+1,\frac{3\,(dfx+cf)}{d}\right)+9\,(a^3dm+a^3d)e^{\left(-\frac{dm\log\left(\frac{2\,f}{d}\right)-2\,cf+2\,de}{d}\right)}\Gamma\left(m+1,\frac{2\,(dfx+cf)}{d}\right)-45\,(-i\,a^3dm-i\,a^3d)e^{\left(-\frac{dm\log\left(\frac{f}{d}\right)-cf+de}{d}\right)}\Gamma\left(m+1,\frac{dfx+cf}{d}\right)-45\,(-i\,a^3dm-i\,a^3d)e^{\left(-\frac{dm\log\left(-\frac{f}{d}\right)+cf-de}{d}\right)}\Gamma\left(m+1,-\frac{dfx+cf}{d}\right)-9\,(a^3dm+a^3d)e^{\left(-\frac{dm\log\left(-\frac{2\,f}{d}\right)+2\,cf-2\,de}{d}\right)}\Gamma\left(m+1,-\frac{2\,(dfx+cf)}{d}\right)+(-i\,a^3dm-i\,a^3d)e^{\left(-\frac{dm\log\left(-\frac{3\,f}{d}\right)+3\,cf-3\,de}{d}\right)}\Gamma\left(m+1,-\frac{3\,(dfx+cf)}{d}\right)+60\,(a^3dfx+a^3cf)(dx+c)^m}{24\,(dfm+df)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/24*((-I*a^3*d*m - I*a^3*d)*e^(-(d*m*log(3*f/d) - 3*c*f + 3*d*e)/d)*gamma(
m + 1, 3*(d*f*x + c*f)/d) + 9*(a^3*d*m + a^3*d)*e^(-(d*m*log(2*f/d) - 2*c*f
 + 2*d*e)/d)*gamma(m + 1, 2*(d*f*x + c*f)/d) - 45*(-I*a^3*d*m - I*a^3*d)*e^
(-(d*m*log(f/d) - c*f + d*e)/d)*gamma(m + 1, (d*f*x + c*f)/d) - 45*(-I*a^3*
d*m - I*a^3*d)*e^(-(d*m*log(-f/d) + c*f - d*e)/d)*gamma(m + 1, -(d*f*x + c*
f)/d) - 9*(a^3*d*m + a^3*d)*e^(-(d*m*log(-2*f/d) + 2*c*f - 2*d*e)/d)*gamma(
m + 1, -2*(d*f*x + c*f)/d) + (-I*a^3*d*m - I*a^3*d)*e^(-(d*m*log(-3*f/d) +
3*c*f - 3*d*e)/d)*gamma(m + 1, -3*(d*f*x + c*f)/d) + 60*(a^3*d*f*x + a^3*c*
f)*(d*x + c)^m)/(d*f*m + d*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+I*a*sinh(f*x+e))**3,x)
```

```
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^3*(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+a\sinh(e+f\,x)\ 1\mathrm{i})^3\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(e + f*x)*1i)^3*(c + d*x)^m,x)
```

```
[Out] int((a + a*sinh(e + f*x)*1i)^3*(c + d*x)^m, x)
```

3.153 $\int (c+dx)^m (a+ia\sinh(e+fx))^2 \, dx$

Optimal. Leaf size=268

$$\frac{3a^2(c+dx)^{1+m}}{2d(1+m)}-\frac{2^{-3-m}a^2e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)}{f}+\frac{ia^2e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+d}{d}\right.}{f}$$

```
[Out] 3/2*a^2*(d*x+c)^(1+m)/d/(1+m)-2^(-3-m)*a^2*exp(2*e-2*c*f/d)*(d*x+c)^m*GAMMA
(1+m,-2*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+I*a^2*exp(e-c*f/d)*(d*x+c)^m*GAMM
A(1+m,-f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+I*a^2*exp(-e+c*f/d)*(d*x+c)^m*GAMM
A(1+m,f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)+2^(-3-m)*a^2*exp(-2*e+2*c*f/d)*(d*x+
c)^m*GAMMA(1+m,2*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.27, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3399, 3393, 3388, 2212, 3389}

$$-\frac{a^2 2^{-m-3}e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2f(c+dx)}{d}\right)}{f}+\frac{ia^2e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{f}+\frac{ia^2e^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{f}+\frac{a^2 2^{-m-3}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2f(c+dx)}{d}\right)}{f}+\frac{3a^2(c+dx)^{m+1}}{2d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] (3*a^2*(c + d*x)^(1 + m))/(2*d*(1 + m)) - (2^(-3 - m)*a^2*E^(2*e - (2*c*f)/
d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c + d*x))/d])/(f*(-((f*(c + d*x))/d))^m)
 + (I*a^2*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c + d*x))/d)])/(f*
(-((f*(c + d*x))/d))^m) + (I*a^2*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m,
(f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) + (2^(-3 - m)*a^2*E^(-2*e + (2*c*
f)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int (c+dx)^m(a+ia\sinh(e+fx))^2\,dx = \left(4a^2\right)\int (c+dx)^m\sin^4\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right)dx$$

$$= \left(4a^2\right)\int \left(\frac{3}{8}(c+dx)^m-\frac{1}{8}(c+dx)^m\cosh(2e+2fx)+\frac{1}{2}i(c+dx)^{\cdots}\right.$$

$$= \frac{3a^2(c+dx)^{1+m}}{2d(1+m)}+\left(2ia^2\right)\int (c+dx)^m\sinh(e+fx)\,dx-\frac{1}{2}a^2\int (c\cdots$$

$$= \frac{3a^2(c+dx)^{1+m}}{2d(1+m)}+\left(ia^2\right)\int e^{-i(ie+ifx)}(c+dx)^m\,dx-\left(ia^2\right)\int e^{i(ie+i\cdots}$$

$$= \frac{3a^2(c+dx)^{1+m}}{2d(1+m)}-\frac{2^{-3-m}a^2e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+\cdots\right.}{f}$$

Mathematica [A]

time = 7.85, size = 383, normalized size = 1.43

$$\frac{(c+dx)^m\left(8ie^{-e-fx}+e^{-2(e+fx)}+8ie^{e+fx}-e^{2(e+fx)}+12(e+fx)+\frac{2^{-m}e^{-2\left(e+\frac{cf}{d}\right)}\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}\left(-32^{2+m}e^{2\left(e+\frac{cf}{d}\right)}\left(-\frac{f^2(c+dx)^2}{d^2}\right)^m(-cf+d(e+em+fmx))+i2^{3+m}de^{e+\frac{3cf}{d}}m(1+m)\left(-\frac{f(c+dx)}{d}\right)^m\Gamma\left(m,f\left(\frac{c}{d}+x\right)\right)-de^{4e}m(1+m)\left(f\left(\frac{c}{d}+x\right)\right)^m\Gamma\left(m,-\frac{2f(c+dx)}{d}\right)+i2^{3+m}de^{3e+\frac{cf}{d}}m(1+m)\left(f\left(\frac{c}{d}+x\right)\right)^m\Gamma\left(m,-\frac{f(c+dx)}{d}\right)+de^{\frac{4cf}{d}}m(1+m)\left(-\frac{f(c+dx)}{d}\right)^m\Gamma\left(m,\frac{2f(c+dx)}{d}\right)\right)}{d(1+m)}\right)(a+ia\sinh(e+fx))^2}{8f\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] ((c + d*x)^m*((8*I)*E^(-e - f*x) + E^(-2*(e + f*x)) + (8*I)*E^(e + f*x) - E
^(2*(e + f*x)) + 12*(e + f*x) + (-3*2^(2 + m)*E^(2*(e + (c*f)/d))*(-((f^2*(
```

```
c + d*x)^2)/d^2))^m*(-(c*f) + d*(e + e*m + f*m*x)) + I*2^(3 + m)*d*E^(e + (
3*c*f)/d)*m*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[m, f*(c/d + x)] - d*E^(4*e
)*m*(1 + m)*(f*(c/d + x))^m*Gamma[m, (-2*f*(c + d*x))/d] + I*2^(3 + m)*d*E^
(3*e + (c*f)/d)*m*(1 + m)*(f*(c/d + x))^m*Gamma[m, -((f*(c + d*x))/d)] + d*
E^((4*c*f)/d)*m*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[m, (2*f*(c + d*x))/d])
/(2^m*d*E^(2*(e + (c*f)/d))*(1 + m)*(-((f^2*(c + d*x)^2)/d^2))^m))*(a + I*a
*Sinh[e + f*x])^2)/(8*f*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^4)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(dx+c\right)^{m} \left(a+ia\sinh \left(fx+e\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+I*a*sinh(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m*(a+I*a*sinh(f*x+e))^2,x)
```

Maxima [A]
time = 0.09, size = 214, normalized size = 0.80

$$\frac{1}{4}\left(\frac{(dx+c)^{m+1}e^{\left(\frac{2\,cf}{d}-2\,e\right)}E_{-m}\left(\frac{2\,(dx+c)f}{d}\right)}{d}+\frac{(dx+c)^{m+1}e^{\left(-\frac{2\,cf}{d}+2\,e\right)}E_{-m}\left(-\frac{2\,(dx+c)f}{d}\right)}{d}+\frac{2\,(dx+c)^{m+1}}{d(m+1)}\right)a^{2}+i\left(\frac{(dx+c)^{m+1}e^{\left(\frac{cf}{d}-e\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}-\frac{(dx+c)^{m+1}e^{\left(-\frac{cf}{d}+e\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)a^{2}+\frac{(dx+c)^{m+1}a^{2}}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((d*x + c)^(m + 1)*e^(2*c*f/d - 2*e)*exp_integral_e(-m, 2*(d*x + c)*f/d
)/d + (d*x + c)^(m + 1)*e^(-2*c*f/d + 2*e)*exp_integral_e(-m, -2*(d*x + c)*
f/d)/d + 2*(d*x + c)^(m + 1)/(d*(m + 1)))*a^2 + I*((d*x + c)^(m + 1)*e^(c*f
/d - e)*exp_integral_e(-m, (d*x + c)*f/d)/d - (d*x + c)^(m + 1)*e^(-c*f/d +
 e)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a^2 + (d*x + c)^(m + 1)*a^2/(d*(m
 + 1))
```

Fricas [A]
time = 0.09, size = 263, normalized size = 0.98

$$\frac{(a^{2}dm+a^{2}d)e^{\left(-\frac{dm\log\left(\frac{2\,f}{d}\right)-2\,cf+2\,de}{d}\right)}\Gamma\left(m+1,\frac{2\,(dfx+cf)}{d}\right)-8\,(-i\,a^{2}dm-i\,a^{2}d)e^{\left(-\frac{dm\log\left(\frac{f}{d}\right)-cf+de}{d}\right)}\Gamma\left(m+1,\frac{dfx+cf}{d}\right)-8\,(-i\,a^{2}dm-i\,a^{2}d)e^{\left(-\frac{dm\log\left(-\frac{f}{d}\right)+cf-de}{d}\right)}\Gamma\left(m+1,-\frac{dfx+cf}{d}\right)-(a^{2}dm+a^{2}d)e^{\left(-\frac{dm\log\left(-\frac{2\,f}{d}\right)+2\,cf-2\,de}{d}\right)}\Gamma\left(m+1,-\frac{2\,(dfx+cf)}{d}\right)+12\,(a^{2}dfx+a^{2}cf)(dx+c)^{m}}{8\,(dfm+df)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((a^2*d*m + a^2*d)*e^(-(d*m*log(2*f/d) - 2*c*f + 2*d*e)/d)*gamma(m + 1,
 2*(d*f*x + c*f)/d) - 8*(-I*a^2*d*m - I*a^2*d)*e^(-(d*m*log(f/d) - c*f + d*
e)/d)*gamma(m + 1, (d*f*x + c*f)/d) - 8*(-I*a^2*d*m - I*a^2*d)*e^(-(d*m*log
(-f/d) + c*f - d*e)/d)*gamma(m + 1, -(d*f*x + c*f)/d) - (a^2*d*m + a^2*d)*e
```

```
^(-(d*m*log(-2*f/d) + 2*c*f - 2*d*e)/d)*gamma(m + 1, -2*(d*f*x + c*f)/d) +
12*(a^2*d*f*x + a^2*c*f)*(d*x + c)^m)/(d*f*m + d*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+I*a*sinh(f*x+e))**2,x)
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
[Out] integrate((I*a*sinh(f*x + e) + a)^2*(d*x + c)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + a \sinh(e + f\,x)\ 1\mathrm{i})^2\,(c + d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(e + f*x)*1i)^2*(c + d*x)^m,x)
[Out] int((a + a*sinh(e + f*x)*1i)^2*(c + d*x)^m, x)
```

3.154 $\int (c+dx)^m(a+ia\sinh(e+fx))\,dx$

Optimal. Leaf size=135

$$\frac{a(c+dx)^{1+m}}{d(1+m)}+\frac{iae^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{2f}+\frac{iae^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+\right.}{2f}$$

```
[Out] a*(d*x+c)^(1+m)/d/(1+m)+1/2*I*a*exp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)
/d)/f/((-f*(d*x+c)/d)^m)+1/2*I*a*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x+c
)/d)/f/((f*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.10, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3398, 3389, 2212}

$$\frac{iae^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}+\frac{iae^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+\frac{a(c+dx)^{m+1}}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^(1 + m))/(d*(1 + m)) + ((I/2)*a*E^(e - (c*f)/d)*(c + d*x)^m*Ga
mma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) + ((I/2)*a*E^(-e
 + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(f*((f*(c + d*x))/d)
^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
```

```
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (c+dx)^m(a+ia\sinh(e+fx))\,dx = \int \left(a(c+dx)^m + ia(c+dx)^m\sinh(e+fx)\right)\,dx$$
$$= \frac{a(c+dx)^{1+m}}{d(1+m)} + (ia)\int (c+dx)^m\sinh(e+fx)\,dx$$
$$= \frac{a(c+dx)^{1+m}}{d(1+m)} + \frac{1}{2}(ia)\int e^{-i(ie+ifx)}(c+dx)^m\,dx - \frac{1}{2}(ia)\int e^{i(ie+ifx)}$$
$$= \frac{a(c+dx)^{1+m}}{d(1+m)} + \frac{iae^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right.}{2f}$$

Mathematica [A]
time = 4.67, size = 166, normalized size = 1.23

$$\frac{a(c+dx)^m\left(2f(c+dx)+ide^{-e+\frac{cf}{d}}(1+m)\left(f\left(\frac{c}{d}+x\right)\right)^{-m}\Gamma\left(1+m,f\left(\frac{c}{d}+x\right)\right)+ide^{e-\frac{cf}{d}}(1+m)\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)\right)(1+i\sinh(e+fx))}{2df(1+m)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + I*a*Sinh[e + f*x]),x]
[Out] (a*(c + d*x)^m*(2*f*(c + d*x) + (I*d*E^(-e + (c*f)/d)*(1 + m)*Gamma[1 + m,
f*(c/d + x)])/(f*(c/d + x))^m + (I*d*E^(e - (c*f)/d)*(1 + m)*Gamma[1 + m, -
((f*(c + d*x))/d)])/(-((f*(c + d*x))/d))^m)*(1 + I*Sinh[e + f*x]))/(2*d*f*(
1 + m)*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^2)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m\,(a+ia\sinh(fx+e))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+I*a*sinh(f*x+e)),x)
[Out] int((d*x+c)^m*(a+I*a*sinh(f*x+e)),x)
```

Maxima [A]
time = 0.06, size = 103, normalized size = 0.76

$$\frac{1}{2}i\left(\frac{(dx+c)^{m+1}e^{\left(\frac{cf}{d}-e\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d} - \frac{(dx+c)^{m+1}e^{\left(-\frac{cf}{d}+e\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)a + \frac{(dx+c)^{m+1}a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*I*((d*x + c)^(m + 1)*e^(c*f/d - e)*exp_integral_e(-m, (d*x + c)*f/d)/d
- (d*x + c)^(m + 1)*e^(-c*f/d + e)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a
+ (d*x + c)^(m + 1)*a/(d*(m + 1))
```

Fricas [A]
time = 0.10, size = 136, normalized size = 1.01

$$\frac{(i\,adm+i\,ad)e^{\left(-\frac{dm\log\left(\frac{f}{d}\right)-cf+de}{d}\right)}\Gamma\left(m+1,\frac{dfx+cf}{d}\right)+(i\,adm+i\,ad)e^{\left(-\frac{dm\log\left(-\frac{f}{d}\right)+cf-de}{d}\right)}\Gamma\left(m+1,-\frac{dfx+cf}{d}\right)+2\,(adfx+acf)(dx+c)^m}{2\,(dfm+df)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*((I*a*d*m + I*a*d)*e^(-(d*m*log(f/d) - c*f + d*e)/d)*gamma(m + 1, (d*f*
x + c*f)/d) + (I*a*d*m + I*a*d)*e^(-(d*m*log(-f/d) + c*f - d*e)/d)*gamma(m
+ 1, -(d*f*x + c*f)/d) + 2*(a*d*f*x + a*c*f)*(d*x + c)^m)/(d*f*m + d*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+I*a*sinh(f*x+e)),x)
```

```
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)*(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\sinh(e+f\,x)\;1\mathrm{i})\;(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(e + f*x)*1i)*(c + d*x)^m,x)
```

```
[Out] int((a + a*sinh(e + f*x)*1i)*(c + d*x)^m, x)
```

3.155 $\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{(c+dx)^m}{a+ia\sinh(e+fx)},x\right)$$

```
[Out] Unintegrable((d*x+c)^m/(a+I*a*sinh(f*x+e)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + I*a*Sinh[e + f*x]),x]
[Out] Defer[Int][(c + d*x)^m/(a + I*a*Sinh[e + f*x]), x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx = \int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx$$

Mathematica [A]
time = 3.03, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + I*a*Sinh[e + f*x]),x]
[Out] Integrate[(c + d*x)^m/(a + I*a*Sinh[e + f*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{a+ia\sinh(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+I*a*sinh(f*x+e)),x)
```

```
[Out] int((d*x+c)^m/(a+I*a*sinh(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m/(I*a*sinh(f*x + e) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] ((a*f*e^(f*x + e) - I*a*f)*integral(-2*I*(d*x + c)^m*d*m/(-I*a*d*f*x - I*a*
c*f + (a*d*f*x + a*c*f)*e^(f*x + e)), x) + 2*I*(d*x + c)^m)/(a*f*e^(f*x + e
) - I*a*f)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{(c+dx)^m}{\sinh{(e+fx)}-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+I*a*sinh(f*x+e)),x)
```

```
[Out] -I*Integral((c + d*x)**m/(sinh(e + f*x) - I), x)/a
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(I*a*sinh(f*x + e) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{a+a\sinh\left(e+f\,x\right)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + a*sinh(e + f*x)*1i),x)
[Out] int((c + d*x)^m/(a + a*sinh(e + f*x)*1i), x)
```

3.156 $\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2},x\right)$$

```
[Out] Unintegrable((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m/(a + I*a*Sinh[e + f*x])^2, x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx = \int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx$$

Mathematica [A]
time = 12.23, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] Integrate[(c + d*x)^m/(a + I*a*Sinh[e + f*x])^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{(a+ia\sinh(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m/(I*a*sinh(f*x + e) + a)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -(2*(I*d^2*f^2*x^2 + 2*I*c*d*f^2*x + I*c^2*f^2 - I*d^2*m^2 + I*d^2*m + (I*d
^2*f*m*x + I*d^2*m^2 + (I*c*d*f - I*d^2)*m)*e^(2*f*x + 2*e) - (3*d^2*f^2*x^
2 + 3*c^2*f^2 - 2*d^2*m^2 - (c*d*f - 2*d^2)*m + (6*c*d*f^2 - d^2*f*m)*x)*e^
(f*x + e))*(d*x + c)^m + 3*(-I*a^2*d^2*f^3*x^2 - 2*I*a^2*c*d*f^3*x - I*a^2*
c^2*f^3 - (a^2*d^2*f^3*x^2 + 2*a^2*c*d*f^3*x + a^2*c^2*f^3)*e^(3*f*x + 3*e)
 + 3*(I*a^2*d^2*f^3*x^2 + 2*I*a^2*c*d*f^3*x + I*a^2*c^2*f^3)*e^(2*f*x + 2*e
) + 3*(a^2*d^2*f^3*x^2 + 2*a^2*c*d*f^3*x + a^2*c^2*f^3)*e^(f*x + e))*integr
al(-2*(I*d^3*f^2*m*x^2 + 2*I*c*d^2*f^2*m*x - I*d^3*m^3 + 3*I*d^3*m^2 + (I*c
^2*d*f^2 - 2*I*d^3)*m)*(d*x + c)^m/(-3*I*a^2*d^3*f^3*x^3 - 9*I*a^2*c*d^2*f^
3*x^2 - 9*I*a^2*c^2*d*f^3*x - 3*I*a^2*c^3*f^3 + 3*(a^2*d^3*f^3*x^3 + 3*a^2*
c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e^(f*x + e)), x))/(3*I*a^2
*d^2*f^3*x^2 + 6*I*a^2*c*d*f^3*x + 3*I*a^2*c^2*f^3 + 3*(a^2*d^2*f^3*x^2 + 2
*a^2*c*d*f^3*x + a^2*c^2*f^3)*e^(3*f*x + 3*e) - 9*(I*a^2*d^2*f^3*x^2 + 2*I*
a^2*c*d*f^3*x + I*a^2*c^2*f^3)*e^(2*f*x + 2*e) - 9*(a^2*d^2*f^3*x^2 + 2*a^2
*c*d*f^3*x + a^2*c^2*f^3)*e^(f*x + e))
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(c+dx)^m}{\sinh^2(e+fx)-2i\sinh(e+fx)-1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+I*a*sinh(f*x+e))**2,x)
```

```
[Out] -Integral((c + d*x)**m/(sinh(e + f*x)**2 - 2*I*sinh(e + f*x) - 1), x)/a**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(I*a*sinh(f*x + e) + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{(a+a\sinh\left(e+f\,x\right)\,1\mathrm{i})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + a*sinh(e + f*x)*1i)^2,x)
```

```
[Out] int((c + d*x)^m/(a + a*sinh(e + f*x)*1i)^2, x)
```

3.157 $\int (c+dx)^3(a+b\sinh(e+fx))\,dx$

Optimal. Leaf size=89

$$\frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\cosh(e+fx)}{f^3}+\frac{b(c+dx)^3\cosh(e+fx)}{f}-\frac{6bd^3\sinh(e+fx)}{f^4}-\frac{3bd(c+dx)^2\sinh(e}{f^2}$$

```
[Out] 1/4*a*(d*x+c)^4/d+6*b*d^2*(d*x+c)*cosh(f*x+e)/f^3+b*(d*x+c)^3*cosh(f*x+e)/f
-6*b*d^3*sinh(f*x+e)/f^4-3*b*d*(d*x+c)^2*sinh(f*x+e)/f^2
```

Rubi [A]
time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3398, 3377, 2717}

$$\frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\cosh(e+fx)}{f^3}-\frac{3bd(c+dx)^2\sinh(e+fx)}{f^2}+\frac{b(c+dx)^3\cosh(e+fx)}{f}-\frac{6bd^3\sinh(e+fx)}{f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Sinh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) + (6*b*d^2*(c + d*x)*Cosh[e + f*x])/f^3 + (b*(c + d*x
)^3*Cosh[e + f*x])/f - (6*b*d^3*Sinh[e + f*x])/f^4 - (3*b*d*(c + d*x)^2*Sin
h[e + f*x])/f^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (c+dx)^3(a+b\sinh(e+fx))\,dx = \int \left(a(c+dx)^3+b(c+dx)^3\sinh(e+fx)\right)\,dx$$
$$= \frac{a(c+dx)^4}{4d}+b\int (c+dx)^3\sinh(e+fx)\,dx$$
$$= \frac{a(c+dx)^4}{4d}+\frac{b(c+dx)^3\cosh(e+fx)}{f}-\frac{(3bd)\int (c+dx)^2\cosh(e+fx)}{f}$$
$$= \frac{a(c+dx)^4}{4d}+\frac{b(c+dx)^3\cosh(e+fx)}{f}-\frac{3bd(c+dx)^2\sinh(e+fx)}{f^2}+$$
$$= \frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\cosh(e+fx)}{f^3}+\frac{b(c+dx)^3\cosh(e+fx)}{f}-$$
$$= \frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\cosh(e+fx)}{f^3}+\frac{b(c+dx)^3\cosh(e+fx)}{f}-$$

Mathematica [A]
time = 0.26, size = 123, normalized size = 1.38

$$\frac{1}{4}ax\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)+\frac{b(c+dx)\left(c^2f^2+2cdf^2x+d^2(6+f^2x^2)\right)\cosh(e+fx)}{f^3}-\frac{3bd(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\sinh(e+fx)}{f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*Sinh[e + f*x]),x]
```

```
[Out] (a*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 + (b*(c + d*x)*(c^2*f^2
 + 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Cosh[e + f*x])/f^3 - (3*b*d*(c^2*f^2 +
2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x])/f^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(87) = 174$.
time = 0.41, size = 482, normalized size = 5.42

method	result
risch	$\frac{a\,d^3x^4}{4}+ac\,d^2x^3+\frac{3a\,c^2d\,x^2}{2}+c^3ax+\frac{a\,c^4}{4d}+\frac{b\left(d^3x^3f^3+3c\,d^2f^3x^2+3c^2d\,f^3x-3d^3f^2x^2+c^3f^3-6c\,d^2f^2x-3c\right.}{2f^4}$
derivativedivides	$\frac{\frac{d^3a(fx+e)^4}{4f^3}+\frac{d^3b\left((fx+e)^3\cosh(fx+e)-3(fx+e)^2\sinh(fx+e)+6(fx+e)\cosh(fx+e)-6\sinh(fx+e)\right)}{f^3}-\frac{d^3ea(fx+e)^3}{f^3}-\frac{3d^3eb\left((fx+e)\right.}{}}{}$
default	$\frac{\frac{d^3a(fx+e)^4}{4f^3}+\frac{d^3b\left((fx+e)^3\cosh(fx+e)-3(fx+e)^2\sinh(fx+e)+6(fx+e)\cosh(fx+e)-6\sinh(fx+e)\right)}{f^3}-\frac{d^3ea(fx+e)^3}{f^3}-\frac{3d^3eb\left((fx+e)\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4*d^3/f^3*a*(f*x+e)^4+d^3/f^3*b*((f*x+e)^3*cosh(f*x+e)-3*(f*x+e)^2*s
inh(f*x+e)+6*(f*x+e)*cosh(f*x+e)-6*sinh(f*x+e))-d^3/f^3*e*a*(f*x+e)^3-3*d^3
/f^3*e*b*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*sinh(f*x+e)+2*cosh(f*x+e))+d^2/f^
2*c*a*(f*x+e)^3+3*d^2/f^2*c*b*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*sinh(f*x+e)+
2*cosh(f*x+e))+3/2*d^3/f^3*e^2*a*(f*x+e)^2+3*d^3/f^3*e^2*b*((f*x+e)*cosh(f*
x+e)-sinh(f*x+e))-3*d^2/f^2*e*c*a*(f*x+e)^2-6*d^2/f^2*e*c*b*((f*x+e)*cosh(f
*x+e)-sinh(f*x+e))+3/2*d/f*c^2*a*(f*x+e)^2+3*d/f*c^2*b*((f*x+e)*cosh(f*x+e)
-sinh(f*x+e))-d^3/f^3*e^3*a*(f*x+e)-d^3/f^3*e^3*b*cosh(f*x+e)+3*d^2/f^2*e^2
*c*a*(f*x+e)+3*d^2/f^2*e^2*c*b*cosh(f*x+e)-3*d/f*e*c^2*a*(f*x+e)-3*d/f*e*c^
2*b*cosh(f*x+e)+c^3*a*(f*x+e)+b*c^3*cosh(f*x+e))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(91) = 182$.
time = 0.28, size = 247, normalized size = 2.78

$$\frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x + \frac{3}{2}\,bc^2d\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{3}{2}\,bcd^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2 + 2\,fx + 2)e^{(-fx-e)}}{f^3}\right) + \frac{1}{2}\,bd^3\left(\frac{(f^3x^3e^e - 3\,f^2x^2e^e + 6\,fxe^e - 6\,e^e)e^{(fx)}}{f^4} + \frac{(f^3x^3 + 3\,f^2x^2 + 6\,fx + 6)e^{(-fx-e)}}{f^4}\right) + \frac{bc^3\cosh(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 3/2*b*c^2*d*((f*x
*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^(-f*x - e)/f^2) + 3/2*b*c*d^2*((f^2*x
^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e
)/f^3) + 1/2*b*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e^e + 6*f*x*e^e - 6*e^e)*e^(f*
x)/f^4 + (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(-f*x - e)/f^4) + b*c^3*cosh(f
*x + e)/f
```

Fricas [**A**]
time = 0.34, size = 174, normalized size = 1.96

$$\frac{ad^3f^4x^4 + 4\,acd^2f^4x^3 + 6\,ac^2df^4x^2 + 4\,ac^3f^4x + 4\,(bd^3f^3x^3 + 3\,bcd^2f^3x^2 + bc^3f^3 + 6\,bcd^2f + 3\,(bc^2df^3 + 2\,bd^3f)x)\cosh(fx + \cosh(1) + \sinh(1)) - 12\,(bd^3f^2x^2 + 2\,bcd^2f^2x + bc^2df^2 + 2\,bd^3)\sinh(fx + \cosh(1) + \sinh(1))}{4\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4*x
+ 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + b*c^3*f^3 + 6*b*c*d^2*f + 3*(b*c^2
*d*f^3 + 2*b*d^3*f)*x)*cosh(f*x + cosh(1) + sinh(1)) - 12*(b*d^3*f^2*x^2 +
2*b*c*d^2*f^2*x + b*c^2*d*f^2 + 2*b*d^3)*sinh(f*x + cosh(1) + sinh(1)))/f^4
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(88) = 176$.
time = 0.28, size = 264, normalized size = 2.97

$$\begin{cases} ac^3x + \frac{3ac^2dx^2}{2} + acd^2x^3 + \frac{ad^3x^4}{4} + \frac{bc^3\cosh(e+fx)}{f} + \frac{3bc^2dx\cosh(e+fx)}{f} - \frac{3bc^2d\sinh(e+fx)}{f^2} + \frac{3bcd^2x^2\cosh(e+fx)}{f} - \frac{6bcd^2x\sinh(e+fx)}{f^2} + \frac{6bcd^2\cosh(e+fx)}{f^3} + \frac{bd^3x^3\cosh(e+fx)}{f} - \frac{3bd^3x^2\sinh(e+fx)}{f^2} + \frac{6bd^3x\cosh(e+fx)}{f^3} - \frac{6bd^3\sinh(e+fx)}{f^4} & \text{for}\ f \neq 0 \\ (a + b\sinh(e))\left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*sinh(f*x+e)),x)
```

```
[Out] Piecewise((a*c**3*x + 3*a*c**2*d*x**2/2 + a*c*d**2*x**3 + a*d**3*x**4/4 + b
*c**3*cosh(e + f*x)/f + 3*b*c**2*d*x*cosh(e + f*x)/f - 3*b*c**2*d*sinh(e +
f*x)/f**2 + 3*b*c*d**2*x**2*cosh(e + f*x)/f - 6*b*c*d**2*x*sinh(e + f*x)/f*
*2 + 6*b*c*d**2*cosh(e + f*x)/f**3 + b*d**3*x**3*cosh(e + f*x)/f - 3*b*d**3
*x**2*sinh(e + f*x)/f**2 + 6*b*d**3*x*cosh(e + f*x)/f**3 - 6*b*d**3*sinh(e
+ f*x)/f**4, Ne(f, 0)), ((a + b*sinh(e))*(c**3*x + 3*c**2*d*x**2/2 + c*d**2
*x**3 + d**3*x**4/4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(87) = 174$.
time = 0.44, size = 258, normalized size = 2.90

$$\frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x + \frac{(bd^3f^3x^3 + 3\,bcd^2f^3x^2 + 3\,bc^2df^3x - 3\,bd^3f^2x^2 + bc^3f^3 - 6\,bcd^2f^2x - 3\,bc^2df^2 + 6\,bd^3fx + 6\,bcd^2f - 6\,bd^3)e^{(fx+e)}}{2\,f^4} + \frac{(bd^3f^3x^3 + 3\,bcd^2f^3x^2 + 3\,bc^2df^3x + 3\,bd^3f^2x^2 + bc^3f^3 + 6\,bcd^2f^2x + 3\,bc^2df^2 + 6\,bd^3fx + 6\,bcd^2f + 6\,bd^3)e^{(-fx-e)}}{2\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 1/2*(b*d^3*f^3*x^
3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x - 3*b*d^3*f^2*x^2 + b*c^3*f^3 - 6*b
*c*d^2*f^2*x - 3*b*c^2*d*f^2 + 6*b*d^3*f*x + 6*b*c*d^2*f - 6*b*d^3)*e^(f*x
+ e)/f^4 + 1/2*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + 3*b*d
^3*f^2*x^2 + b*c^3*f^3 + 6*b*c*d^2*f^2*x + 3*b*c^2*d*f^2 + 6*b*d^3*f*x + 6*
b*c*d^2*f + 6*b*d^3)*e^(-f*x - e)/f^4
```

Mupad [B]
time = 0.24, size = 187, normalized size = 2.10

$$\frac{\cosh(e+f\,x)\,\left(b\,c^3\,f^2+6\,b\,c\,d^2\right)}{f^3}-\frac{3\sinh(e+f\,x)\,\left(b\,c^2\,d\,f^2+2\,b\,d^3\right)}{f^4}+\frac{a\,d^3\,x^4}{4}+a\,c^3\,x+\frac{3\,x\cosh(e+f\,x)\,\left(b\,c^2\,d\,f^2+2\,b\,d^3\right)}{f^3}+\frac{3\,a\,c^2\,d\,x^2}{2}+a\,c\,d^2\,x^3+\frac{b\,d^3\,x^3\cosh(e+f\,x)}{f}-\frac{3\,b\,d^3\,x^2\sinh(e+f\,x)}{f^2}-\frac{6\,b\,c\,d^2\,x\sinh(e+f\,x)}{f^2}+\frac{3\,b\,c\,d^2\,x^2\cosh(e+f\,x)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x))*(c + d*x)^3,x)
```

```
[Out] (cosh(e + f*x)*(b*c^3*f^2 + 6*b*c*d^2))/f^3 - (3*sinh(e + f*x)*(2*b*d^3 + b
*c^2*d*f^2))/f^4 + (a*d^3*x^4)/4 + a*c^3*x + (3*x*cosh(e + f*x)*(2*b*d^3 +
b*c^2*d*f^2))/f^3 + (3*a*c^2*d*x^2)/2 + a*c*d^2*x^3 + (b*d^3*x^3*cosh(e + f
*x))/f - (3*b*d^3*x^2*sinh(e + f*x))/f^2 - (6*b*c*d^2*x*sinh(e + f*x))/f^2
+ (3*b*c*d^2*x^2*cosh(e + f*x))/f
```

3.158 $\int (c+dx)^2(a+b\sinh(e+fx))\,dx$

Optimal. Leaf size=67

$$\frac{a(c+dx)^3}{3d}+\frac{2bd^2\cosh(e+fx)}{f^3}+\frac{b(c+dx)^2\cosh(e+fx)}{f}-\frac{2bd(c+dx)\sinh(e+fx)}{f^2}$$

```
[Out] 1/3*a*(d*x+c)^3/d+2*b*d^2*cosh(f*x+e)/f^3+b*(d*x+c)^2*cosh(f*x+e)/f-2*b*d*(
d*x+c)*sinh(f*x+e)/f^2
```

Rubi [A]
time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3398, 3377, 2718}

$$\frac{a(c+dx)^3}{3d}-\frac{2bd(c+dx)\sinh(e+fx)}{f^2}+\frac{b(c+dx)^2\cosh(e+fx)}{f}+\frac{2bd^2\cosh(e+fx)}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Sinh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^3)/(3*d) + (2*b*d^2*Cosh[e + f*x])/f^3 + (b*(c + d*x)^2*Cosh[e
 + f*x])/f - (2*b*d*(c + d*x)*Sinh[e + f*x])/f^2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2(a+b\sinh(e+fx))\,dx &= \int \left(a(c+dx)^2+b(c+dx)^2\sinh(e+fx)\right)\,dx \\
&= \frac{a(c+dx)^3}{3d}+b\int (c+dx)^2\sinh(e+fx)\,dx \\
&= \frac{a(c+dx)^3}{3d}+\frac{b(c+dx)^2\cosh(e+fx)}{f}-\frac{(2bd)\int (c+dx)\cosh(e+fx)}{f} \\
&= \frac{a(c+dx)^3}{3d}+\frac{b(c+dx)^2\cosh(e+fx)}{f}-\frac{2bd(c+dx)\sinh(e+fx)}{f^2}+\frac{(2
\\
&= \frac{a(c+dx)^3}{3d}+\frac{2bd^2\cosh(e+fx)}{f^3}+\frac{b(c+dx)^2\cosh(e+fx)}{f}-\frac{2bd(c+
\end{aligned}$$

Mathematica [A]
time = 0.19, size = 83, normalized size = 1.24

$$\frac{1}{3}ax\left(3c^2+3cdx+d^2x^2\right)+\frac{b(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\cosh(e+fx)}{f^3}-\frac{2bd(c+dx)\sinh(e+fx)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + b*Sinh[e + f*x]),x]
```

```
[Out] (a*x*(3*c^2 + 3*c*d*x + d^2*x^2))/3 + (b*(c^2*f^2 + 2*c*d*f^2*x + d^2*(2 +
f^2*x^2))*Cosh[e + f*x])/f^3 - (2*b*d*(c + d*x)*Sinh[e + f*x])/f^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(65) = 130$.
time = 0.36, size = 240, normalized size = 3.58

method	result
risch	$\frac{a\,d^2x^3}{3}+adc\,x^2+a\,c^2x+\frac{a\,c^3}{3d}+\frac{b\left(d^2x^2f^2+2cd\,f^2x+c^2f^2-2d^2fx-2cdf+2d^2\right)\mathrm{e}^{fx+e}}{2f^3}+\frac{b\left(d^2x^2f^2+2cd\,f^2x+\right.}{}$
derivativedivides	$\frac{\frac{d^2a(fx+e)^3}{3f^2}+\frac{d^2b\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2}-\frac{d^2ea(fx+e)^2}{f^2}-\frac{2d^2eb((fx+e)\cosh(fx+e)-\sinh(fx+e}{f^2}}{}$
default	$\frac{\frac{d^2a(fx+e)^3}{3f^2}+\frac{d^2b\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2}-\frac{d^2ea(fx+e)^2}{f^2}-\frac{2d^2eb((fx+e)\cosh(fx+e)-\sinh(fx+e}{f^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*d^2/f^2*a*(f*x+e)^3+d^2/f^2*b*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*sin
h(f*x+e)+2*cosh(f*x+e))-d^2/f^2*e*a*(f*x+e)^2-2*d^2/f^2*e*b*((f*x+e)*cosh(f
*x+e)-sinh(f*x+e))+d/f*c*a*(f*x+e)^2+2*d/f*c*b*((f*x+e)*cosh(f*x+e)-sinh(f*
```

```
x+e))+d^2/f^2*e^2*a*(f*x+e)+d^2/f^2*e^2*b*cosh(f*x+e)-2*d/f*e*c*a*(f*x+e)-2
*d/f*e*c*b*cosh(f*x+e)+a*c^2*(f*x+e)+b*c^2*cosh(f*x+e))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(68) = 136$.
time = 0.27, size = 147, normalized size = 2.19

$$\frac{1}{3}\,ad^2x^3 + acdx^2 + ac^2x + bcd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{1}{2}\,bd^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2 + 2\,fx + 2)e^{(-fx-e)}}{f^3}\right) + \frac{bc^2\cosh(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + b*c*d*((f*x*e^e - e^e)*e^(f*x)/f^2 +
(f*x + 1)*e^(-f*x - e)/f^2) + 1/2*b*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*
e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + b*c^2*cosh(f*x + e)
/f
```

Fricas [**A**]
time = 0.33, size = 108, normalized size = 1.61

$$\frac{ad^2f^3x^3 + 3\,acdf^3x^2 + 3\,ac^2f^3x + 3\,(bd^2f^2x^2 + 2\,bcdf^2x + bc^2f^2 + 2\,bd^2)\cosh(fx + \cosh(1) + \sinh(1)) - 6\,(bd^2fx + bcdf)\sinh(fx + \cosh(1) + \sinh(1))}{3\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/3*(a*d^2*f^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x + 3*(b*d^2*f^2*x^2 + 2
*b*c*d*f^2*x + b*c^2*f^2 + 2*b*d^2)*cosh(f*x + cosh(1) + sinh(1)) - 6*(b*d^
2*f*x + b*c*d*f)*sinh(f*x + cosh(1) + sinh(1)))/f^3
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(65) = 130$.
time = 0.15, size = 151, normalized size = 2.25

$$\begin{cases} ac^2x + acdx^2 + \frac{ad^2x^3}{3} + \frac{bc^2\cosh(e+fx)}{f} + \frac{2bcdx\cosh(e+fx)}{f} - \frac{2bcd\sinh(e+fx)}{f^2} + \frac{bd^2x^2\cosh(e+fx)}{f} - \frac{2bd^2x\sinh(e+fx)}{f^2} + \frac{2bd^2\cosh(e+fx)}{f^3} & \text{for } f \neq 0 \\ (a + b\sinh(e))\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*sinh(f*x+e)),x)
```

```
[Out] Piecewise((a*c**2*x + a*c*d*x**2 + a*d**2*x**3/3 + b*c**2*cosh(e + f*x)/f +
 2*b*c*d*x*cosh(e + f*x)/f - 2*b*c*d*sinh(e + f*x)/f**2 + b*d**2*x**2*cosh(
e + f*x)/f - 2*b*d**2*x*sinh(e + f*x)/f**2 + 2*b*d**2*cosh(e + f*x)/f**3, N
e(f, 0)), ((a + b*sinh(e))*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(65) = 130$.
time = 0.43, size = 146, normalized size = 2.18

$$\frac{1}{3}\,ad^2x^3 + acdx^2 + ac^2x + \frac{(bd^2f^2x^2 + 2\,bcdf^2x + bc^2f^2 - 2\,bd^2fx - 2\,bcdf + 2\,bd^2)e^{(fx+e)}}{2\,f^3} + \frac{(bd^2f^2x^2 + 2\,bcdf^2x + bc^2f^2 + 2\,bd^2fx + 2\,bcdf + 2\,bd^2)e^{(-fx-e)}}{2\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + 1/2*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x +
b*c^2*f^2 - 2*b*d^2*f*x - 2*b*c*d*f + 2*b*d^2)*e^(f*x + e)/f^3 + 1/2*(b*d^2
*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2 + 2*b*d^2*f*x + 2*b*c*d*f + 2*b*d^2)*e
^(-f*x - e)/f^3
```

Mupad [B]
time = 0.11, size = 110, normalized size = 1.64

$$\frac{a\,d^2\,x^3}{3}+\frac{\cosh(e+f\,x)\,\left(b\,c^2\,f^2+2\,b\,d^2\right)}{f^3}+a\,c^2\,x+a\,c\,d\,x^2-\frac{2\,b\,d^2\,x\sinh(e+f\,x)}{f^2}+\frac{b\,d^2\,x^2\cosh(e+f\,x)}{f}-\frac{2\,b\,c\,d\sinh(e+f\,x)}{f^2}+\frac{2\,b\,c\,d\,x\cosh(e+f\,x)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x))*(c + d*x)^2,x)
```

```
[Out] (a*d^2*x^3)/3 + (cosh(e + f*x)*(2*b*d^2 + b*c^2*f^2))/f^3 + a*c^2*x + a*c*d
*x^2 - (2*b*d^2*x*sinh(e + f*x))/f^2 + (b*d^2*x^2*cosh(e + f*x))/f - (2*b*c
*d*sinh(e + f*x))/f^2 + (2*b*c*d*x*cosh(e + f*x))/f
```

3.159 $\int (c+dx)(a+b\sinh(e+fx))\,dx$

Optimal. Leaf size=45

$$\frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\cosh(e+fx)}{f}-\frac{bd\sinh(e+fx)}{f^2}$$

```
[Out] 1/2*a*(d*x+c)^2/d+b*(d*x+c)*cosh(f*x+e)/f-b*d*sinh(f*x+e)/f^2
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3398, 3377, 2717}

$$\frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\cosh(e+fx)}{f}-\frac{bd\sinh(e+fx)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Sinh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^2)/(2*d) + (b*(c + d*x)*Cosh[e + f*x])/f - (b*d*Sinh[e + f*x])
/f^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)(a+b\sinh(e+fx))\,dx &= \int (a(c+dx)+b(c+dx)\sinh(e+fx))\,dx \\
&= \frac{a(c+dx)^2}{2d} + b\int (c+dx)\sinh(e+fx)\,dx \\
&= \frac{a(c+dx)^2}{2d} + \frac{b(c+dx)\cosh(e+fx)}{f} - \frac{(bd)\int \cosh(e+fx)\,dx}{f} \\
&= \frac{a(c+dx)^2}{2d} + \frac{b(c+dx)\cosh(e+fx)}{f} - \frac{bd\sinh(e+fx)}{f^2}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 43, normalized size = 0.96

$$
\frac{1}{2}ax(2c+dx) + \frac{b(c+dx)\cosh(e+fx)}{f} - \frac{bd\sinh(e+fx)}{f^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Sinh[e + f*x]),x]
```

```
[Out] (a*x*(2*c + d*x))/2 + (b*(c + d*x)*Cosh[e + f*x])/f - (b*d*Sinh[e + f*x])/f
^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(43) = 86.
time = 0.37, size = 91, normalized size = 2.02

method	result	size
risch	$\frac{ad\,x^2}{2}+acx+\frac{b(dxf+cf-d)\mathrm{e}^{fx+e}}{2f^2}+\frac{b(dxf+cf+d)\mathrm{e}^{-fx-e}}{2f^2}$	60
derivativedivides	$\frac{\frac{da(fx+e)^2}{2f}+\frac{db((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{dea(fx+e)}{f}-\frac{deb\cosh(fx+e)}{f}+ac(fx+e)+bc\cosh(fx+e)}{f}$	91
default	$\frac{\frac{da(fx+e)^2}{2f}+\frac{db((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}-\frac{dea(fx+e)}{f}-\frac{deb\cosh(fx+e)}{f}+ac(fx+e)+bc\cosh(fx+e)}{f}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*d/f*a*(f*x+e)^2+d/f*b*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))-d/f*e*a*(f
*x+e)-d/f*e*b*cosh(f*x+e)+a*c*(f*x+e)+b*c*cosh(f*x+e))
```

Maxima [A]
time = 0.26, size = 69, normalized size = 1.53

$$
\frac{1}{2}\,adx^2 + acx + \frac{1}{2}\,bd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{bc\cosh\left(fx+e\right)}{f}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + 1/2*b*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^(-
f*x - e)/f^2) + b*c*cosh(f*x + e)/f
```

Fricas [A]
time = 0.33, size = 57, normalized size = 1.27

$$\frac{adf^2x^2 + 2\,acf^2x - 2\,bd\sinh\left(fx + \cosh\left(1\right) + \sinh\left(1\right)\right) + 2\,(bdfx + bcf)\cosh\left(fx + \cosh\left(1\right) + \sinh\left(1\right)\right)}{2\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*f^2*x^2 + 2*a*c*f^2*x - 2*b*d*sinh(f*x + cosh(1) + sinh(1)) + 2*(b
*d*f*x + b*c*f)*cosh(f*x + cosh(1) + sinh(1)))/f^2
```

Sympy [A]
time = 0.10, size = 68, normalized size = 1.51

$$\begin{cases} acx + \frac{adx^2}{2} + \frac{bc\cosh\left(e+fx\right)}{f} + \frac{bdx\cosh\left(e+fx\right)}{f} - \frac{bd\sinh\left(e+fx\right)}{f^2} & \text{for}\ f \neq 0 \\ \left(a + b\sinh\left(e\right)\right)\left(cx + \frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sinh(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x + a*d*x**2/2 + b*c*cosh(e + f*x)/f + b*d*x*cosh(e + f*x)/f
 - b*d*sinh(e + f*x)/f**2, Ne(f, 0)), ((a + b*sinh(e))*(c*x + d*x**2/2), Tr
ue))
```

Giac [A]
time = 0.45, size = 64, normalized size = 1.42

$$\frac{1}{2}\,adx^2 + acx + \frac{(bdfx + bcf - bd)e^{(fx+e)}}{2\,f^2} + \frac{(bdfx + bcf + bd)e^{(-fx-e)}}{2\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + 1/2*(b*d*f*x + b*c*f - b*d)*e^(f*x + e)/f^2 + 1/2*(b*
d*f*x + b*c*f + b*d)*e^(-f*x - e)/f^2
```

Mupad [B]
time = 0.14, size = 49, normalized size = 1.09

$$\frac{f\,\left(b\,c\cosh(e+f\,x)+b\,d\,x\cosh(e+f\,x)\right)-b\,d\sinh(e+f\,x)}{f^2}+a\,c\,x+\frac{a\,d\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x))*(c + d*x),x)
```

```
[Out] (f*(b*c*cosh(e + f*x) + b*d*x*cosh(e + f*x)) - b*d*sinh(e + f*x))/f^2 + a*c
*x + (a*d*x^2)/2
```

3.160 $\int \frac{a+b\sinh(e+fx)}{c+dx}\,dx$

Optimal. Leaf size=64

$$\frac{a\log(c+dx)}{d}+\frac{b\mathrm{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{b\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(\frac{cf}{d}+fx\right)}{d}$$

```
[Out] a*ln(d*x+c)/d+b*cosh(-e+c*f/d)*Shi(c*f/d+f*x)/d-b*Chi(c*f/d+f*x)*sinh(-e+c*
f/d)/d
```

Rubi [A]
time = 0.10, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {3398, 3384, 3379, 3382}

$$\frac{a\log(c+dx)}{d}+\frac{b\mathrm{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{b\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(xf+\frac{cf}{d}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x])/(c + d*x),x]
```

```
[Out] (a*Log[c + d*x])/d + (b*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d +
(b*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sinh(e+fx)}{c+dx}\,dx &= \int \left(\frac{a}{c+dx}+\frac{b\sinh(e+fx)}{c+dx}\right)dx\\
&= \frac{a\log(c+dx)}{d}+b\int \frac{\sinh(e+fx)}{c+dx}\,dx\\
&= \frac{a\log(c+dx)}{d}+\left(b\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\sinh\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx+\left(b\sinh\left(e-\frac{cf}{d}\right)\right)\\
&= \frac{a\log(c+dx)}{d}+\frac{b\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{b\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 57, normalized size = 0.89

$$\frac{a\log(c+dx)+b\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+b\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x])/(c + d*x),x]
[Out] (a*Log[c + d*x] + b*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] + b*Cosh[e
- (c*f)/d]*SinhIntegral[f*(c/d + x)])/d
```

Maple [A]
time = 0.42, size = 94, normalized size = 1.47

method	result	size
risch	$\frac{a\ln(dx+c)}{d}+\frac{b\,\mathrm{e}^{\frac{cf-de}{d}}\,\text{expIntegral}\left(1,fx+e+\frac{cf-de}{d}\right)}{2d}-\frac{b\,\mathrm{e}^{-\frac{cf-de}{d}}\,\text{expIntegral}\left(1,-fx-e-\frac{cf-de}{d}\right)}{2d}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e))/(d*x+c),x,method=_RETURNVERBOSE)
[Out] a*ln(d*x+c)/d+1/2*b/d*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*b/d*exp(
-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Maxima [A]
time = 0.30, size = 73, normalized size = 1.14

$$\frac{1}{2}\,b\left(\frac{e^{\left(\frac{cf}{d}-e\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d}-\frac{e^{\left(-\frac{cf}{d}+e\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right)+\frac{a\log\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*b*(e^(c*f/d - e)*exp_integral_e(1, (d*x + c)*f/d)/d - e^(-c*f/d + e)*ex
p_integral_e(1, -(d*x + c)*f/d)/d) + a*log(d*x + c)/d
```

Fricas [A]
time = 0.38, size = 122, normalized size = 1.91

$$\frac{\left(b\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-b\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)+2\,a\log\left(dx+c\right)+\left(b\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+b\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\sinh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*((b*Ei((d*f*x + c*f)/d) - b*Ei(-(d*f*x + c*f)/d))*cosh(-(c*f - d*cosh(1
) - d*sinh(1))/d) + 2*a*log(d*x + c) + (b*Ei((d*f*x + c*f)/d) + b*Ei(-(d*f*
x + c*f)/d))*sinh(-(c*f - d*cosh(1) - d*sinh(1))/d))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\sinh\left(e+fx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c),x)
```

```
[Out] Integral((a + b*sinh(e + f*x))/(c + d*x), x)
```

Giac [A]
time = 0.43, size = 68, normalized size = 1.06

$$\frac{b\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}-b\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+2\,a\log\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) - b*Ei(-(d*f*x + c*f)/d)*e^(-e + c
*f/d) + 2*a*log(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{a+b\sinh(e+f\,x)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x))/(c + d*x),x)
```

```
[Out] int((a + b*sinh(e + f*x))/(c + d*x), x)
```

3.161 $\int \frac{a+b\sinh(e+fx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=87

$$-\frac{a}{d(c+dx)}+\frac{bf\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d^2}-\frac{b\sinh(e+fx)}{d(c+dx)}+\frac{bf\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d^2}$$

```
[Out] -a/d/(d*x+c)+b*f*Chi(c*f/d+f*x)*cosh(-e+c*f/d)/d^2-b*f*Shi(c*f/d+f*x)*sinh(
-e+c*f/d)/d^2-b*sinh(f*x+e)/d/(d*x+c)
```

Rubi [A]
time = 0.12, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3398, 3378, 3384, 3379, 3382}

$$-\frac{a}{d(c+dx)}+\frac{bf\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{bf\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{b\sinh(e+fx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x])/(c + d*x)^2,x]
```

```
[Out] -(a/(d*(c + d*x))) + (b*f*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d^
2 - (b*Sinh[e + f*x])/(d*(c + d*x)) + (b*f*Sinh[e - (c*f)/d]*SinhIntegral[(
c*f)/d + f*x])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sinh(e+fx)}{(c+dx)^2}\,dx &= \int \left(\frac{a}{(c+dx)^2}+\frac{b\sinh(e+fx)}{(c+dx)^2}\right)dx\\ &= -\frac{a}{d(c+dx)}+b\int \frac{\sinh(e+fx)}{(c+dx)^2}\,dx\\ &= -\frac{a}{d(c+dx)}-\frac{b\sinh(e+fx)}{d(c+dx)}+\frac{(bf)\int \frac{\cosh(e+fx)}{c+dx}\,dx}{d}\\ &= -\frac{a}{d(c+dx)}-\frac{b\sinh(e+fx)}{d(c+dx)}+\frac{\left(bf\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\cosh\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx}{d}+\frac{(bf\sinh\\ &= -\frac{a}{d(c+dx)}+\frac{bf\cosh\left(e-\frac{cf}{d}\right)\operatorname{Chi}\left(\frac{cf}{d}+fx\right)}{d^2}-\frac{b\sinh(e+fx)}{d(c+dx)}+\frac{bf\sinh\left(e-\frac{c}{}\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 71, normalized size = 0.82

$$\frac{bf\cosh\left(e-\frac{cf}{d}\right)\operatorname{Chi}\left(f\left(\frac{c}{d}+x\right)\right)-\frac{d(a+b\sinh(e+fx))}{c+dx}+bf\sinh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(f\left(\frac{c}{d}+x\right)\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x])/(c + d*x)^2,x]
```

```
[Out] (b*f*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] - (d*(a + b*Sinh[e + f*x])
)/(c + d*x) + b*f*Sinh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)])/d^2
```

Maple [A]
time = 0.44, size = 149, normalized size = 1.71

method	result
risch	$-\frac{a}{d(dx+c)}+\frac{fb\,\mathrm{e}^{-fx-e}}{2d(dxf+cf)}-\frac{fb\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{expIntegral}\left(1,fx+e+\frac{cf-de}{d}\right)}{2d^2}-\frac{bf\,\mathrm{e}^{fx+e}}{2d^2\left(\frac{cf}{d}+fx\right)}-\frac{bf\,\mathrm{e}^{-\frac{cf-de}{d}}\,\mathrm{expIntegral}\left(1,-fx-e-\right.}{2d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e))/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/d/(d*x+c)+1/2*f*b*exp(-f*x-e)/d/(d*f*x+c*f)-1/2*f*b/d^2*exp((c*f-d*e)/d)
*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*b*f/d^2*exp(f*x+e)/(c*f/d+f*x)-1/2*b*f/d^2*exp
(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Maxima [A]
time = 0.30, size = 90, normalized size = 1.03

$$\frac{1}{2}\,b\left(\frac{e^{\left(\frac{cf}{d}-e\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d}-\frac{e^{\left(-\frac{cf}{d}+e\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right)-\frac{a}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*(e^(c*f/d - e)*exp_integral_e(2, (d*x + c)*f/d)/((d*x + c)*d) - e^(-c
*f/d + e)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d)) - a/(d^2*x + c*d
)
```

Fricas [A]
time = 0.36, size = 178, normalized size = 2.05

$$-\frac{2\,bd\sinh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)+2\,ad-\left((bdfx+bcf)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+(bdfx+bcf)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)-\left((bdfx+bcf)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-(bdfx+bcf)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\sinh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*sinh(f*x + cosh(1) + sinh(1)) + 2*a*d - ((b*d*f*x + b*c*f)*Ei((
d*f*x + c*f)/d) + (b*d*f*x + b*c*f)*Ei(-(d*f*x + c*f)/d))*cosh(-(c*f - d*co
sh(1) - d*sinh(1))/d) - ((b*d*f*x + b*c*f)*Ei((d*f*x + c*f)/d) - (b*d*f*x +
 b*c*f)*Ei(-(d*f*x + c*f)/d))*sinh(-(c*f - d*cosh(1) - d*sinh(1))/d))/(d^3*
x + c*d^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 630 vs. 2(90) = 180.
time = 0.44, size = 630, normalized size = 7.24

$$\frac{1}{2}\,b\left(\frac{\left(\left(dx+c\right)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)f^{2}\mathrm{Ei}\left(\frac{\left(dx+c\right)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)}-def^{2}\mathrm{Ei}\left(\frac{\left(dx+c\right)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)}+cf^{3}\mathrm{Ei}\left(\frac{\left(dx+c\right)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)}-df^{2}e^{\left(\frac{\left(dx+c\right)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)}{d}\right)}\right)d^{2}}{\left(\left(dx+c\right)d^{4}\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-d^{5}e+cd^{4}f\right)f}+\frac{\left(\left(dx+c\right)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)f^{2}\mathrm{Ei}\left(-\frac{\left(dx+c\right)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(-\frac{de-cf}{d}\right)}-def^{2}\mathrm{Ei}\left(-\frac{\left(dx+c\right)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(-\frac{de-cf}{d}\right)}+cf^{3}\mathrm{Ei}\left(-\frac{\left(dx+c\right)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(-\frac{de-cf}{d}\right)}+df^{2}e^{\left(-\frac{\left(dx+c\right)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)}{d}\right)}\right)d^{2}}{\left(\left(dx+c\right)d^{4}\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-d^{5}e+cd^{4}f\right)f}\right)-\frac{a}{\left(dx+c\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*b*(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(((d*x + c)*(d*
e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) - d*e*f^
2*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e
 - c*f)/d) + c*f^3*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e
+ c*f)/d)*e^((d*e - c*f)/d) - d*f^2*e^((d*x + c)*(d*e/(d*x + c) - c*f/(d*x
+ c) + f)/d))*d^2/(((d*x + c)*d^4*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d^5
*e + c*d^4*f)*f) + ((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-(
(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*
f)/d) - d*e*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e +
c*f)/d)*e^(-(d*e - c*f)/d) + c*f^3*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x
 + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) + d*f^2*e^(-(d*x + c)*(d*e/(d
*x + c) - c*f/(d*x + c) + f)/d))*d^2/(((d*x + c)*d^4*(d*e/(d*x + c) - c*f/(
d*x + c) + f) - d^5*e + c*d^4*f)*f)) - a/((d*x + c)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\sinh(e+f\,x)}{{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x))/(c + d*x)^2,x)
```

```
[Out] int((a + b*sinh(e + f*x))/(c + d*x)^2, x)
```

3.162 $\int \frac{a+b\sinh(e+fx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=123

$$-\frac{a}{2d(c+dx)^2}-\frac{bf\cosh(e+fx)}{2d^2(c+dx)}+\frac{bf^2\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{b\sinh(e+fx)}{2d(c+dx)^2}+\frac{bf^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}\right.}{2d^3}$$

```
[Out] -1/2*a/d/(d*x+c)^2-1/2*b*f*cosh(f*x+e)/d^2/(d*x+c)+1/2*b*f^2*cosh(-e+c*f/d)
*Shi(c*f/d+f*x)/d^3-1/2*b*f^2*Chi(c*f/d+f*x)*sinh(-e+c*f/d)/d^3-1/2*b*sinh(
f*x+e)/d/(d*x+c)^2
```

Rubi [A]
time = 0.16, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3398, 3378, 3384, 3379, 3382}

$$-\frac{a}{2d(c+dx)^2}+\frac{bf^2\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{2d^3}+\frac{bf^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{bf\cosh(e+fx)}{2d^2(c+dx)}-\frac{b\sinh(e+fx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x])/(c + d*x)^3,x]
```

```
[Out] -1/2*a/(d*(c + d*x)^2) - (b*f*Cosh[e + f*x])/(2*d^2*(c + d*x)) + (b*f^2*Cos
hIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/(2*d^3) - (b*Sinh[e + f*x])/(2*
d*(c + d*x)^2) + (b*f^2*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/(2*d
^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sinh(e+fx)}{(c+dx)^3}\,dx &= \int \left(\frac{a}{(c+dx)^3}+\frac{b\sinh(e+fx)}{(c+dx)^3}\right)dx\\
&= -\frac{a}{2d(c+dx)^2}+b\int \frac{\sinh(e+fx)}{(c+dx)^3}\,dx\\
&= -\frac{a}{2d(c+dx)^2}-\frac{b\sinh(e+fx)}{2d(c+dx)^2}+\frac{(bf)\int \frac{\cosh(e+fx)}{(c+dx)^2}\,dx}{2d}\\
&= -\frac{a}{2d(c+dx)^2}-\frac{bf\cosh(e+fx)}{2d^2(c+dx)}-\frac{b\sinh(e+fx)}{2d(c+dx)^2}+\frac{(bf^2)\int \frac{\sinh(e+fx)}{c+dx}\,dx}{2d^2}\\
&= -\frac{a}{2d(c+dx)^2}-\frac{bf\cosh(e+fx)}{2d^2(c+dx)}-\frac{b\sinh(e+fx)}{2d(c+dx)^2}+\frac{\left(bf^2\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\sinh(}{}}{2d^2}\\
&= -\frac{a}{2d(c+dx)^2}-\frac{bf\cosh(e+fx)}{2d^2(c+dx)}+\frac{bf^2\mathrm{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{b\sinh(e-}{2d(c+}
\end{aligned}$$

Mathematica [A]
time = 0.39, size = 95, normalized size = 0.77

$$\frac{bf^2\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)-\frac{d(bf(c+dx)\cosh(e+fx)+d(a+b\sinh(e+fx)))}{(c+dx)^2}+bf^2\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}+x\right)\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x])/(c + d*x)^3,x]
```

```
[Out] (b*f^2*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] - (d*(b*f*(c + d*x)*Cosh
[e + f*x] + d*(a + b*Sinh[e + f*x])))/(c + d*x)^2 + b*f^2*Cosh[e - (c*f)/d]
*SinhIntegral[f*(c/d + x)])/(2*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(115) = 230$.
time = 0.45, size = 296, normalized size = 2.41

method	result
risch	$-\frac{a}{2d(dx+c)^2} - \frac{f^3 b\,\mathrm{e}^{-fx-e} x}{4d(d^2x^2f^2+2cd\,f^2x+c^2f^2)} - \frac{f^3 b\,\mathrm{e}^{-fx-e} c}{4d^2(d^2x^2f^2+2cd\,f^2x+c^2f^2)} + \frac{f^2 b\,\mathrm{e}^{-fx-e}}{4d(d^2x^2f^2+2cd\,f^2x+c^2f^2)} + \frac{f^2 b\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{expIn}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e))/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d/(d*x+c)^2-1/4*f^3*b*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2
)*x-1/4*f^3*b*exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c+1/4*f^2*b
*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)+1/4*f^2*b/d^3*exp((c*f-d*e
)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/4*b*f^2/d^3*exp(f*x+e)/(c*f/d+f*x)^2-1/4*b*f
^2/d^3*exp(f*x+e)/(c*f/d+f*x)-1/4*b*f^2/d^3*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(
c*f-d*e)/d)
```

Maxima [A]
time = 0.31, size = 101, normalized size = 0.82

$$\frac{1}{2}\,b\left(\frac{e^{\left(\frac{cf}{d}-e\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d} - \frac{e^{\left(-\frac{cf}{d}+e\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}\right) - \frac{a}{2\left(d^3x^2+2\,cd^2x+c^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*(e^(c*f/d - e)*exp_integral_e(3, (d*x + c)*f/d)/((d*x + c)^2*d) - e^(
-c*f/d + e)*exp_integral_e(3, -(d*x + c)*f/d)/((d*x + c)^2*d)) - 1/2*a/(d^3
*x^2 + 2*c*d^2*x + c^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(119) = 238$.
time = 0.36, size = 293, normalized size = 2.38

$$-\frac{2\,bd^2\sinh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)+2\,ad^2+2\left(bd^2fx+bcdf\right)\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)-\left(\left(bd^2f^2x^2+2\,bcdf^2x+bc^2f^2\right)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-\left(bd^2f^2x^2+2\,bcdf^2x+bc^2f^2\right)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)-\left(\left(bd^2f^2x^2+2\,bcdf^2x+bc^2f^2\right)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+\left(bd^2f^2x^2+2\,bcdf^2x+bc^2f^2\right)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\sinh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)}{4\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*d^2*sinh(f*x + cosh(1) + sinh(1)) + 2*a*d^2 + 2*(b*d^2*f*x + b*c*
d*f)*cosh(f*x + cosh(1) + sinh(1)) - ((b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^
2*f^2)*Ei((d*f*x + c*f)/d) - (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*Ei
```

```
(-(d*f*x + c*f)/d))*cosh(-(c*f - d*cosh(1) - d*sinh(1))/d) - ((b*d^2*f^2*x^
2 + 2*b*c*d*f^2*x + b*c^2*f^2)*Ei((d*f*x + c*f)/d) + (b*d^2*f^2*x^2 + 2*b*c
*d*f^2*x + b*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*sinh(-(c*f - d*cosh(1) - d*sinh
(1))/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(115) = 230.
time = 0.43, size = 319, normalized size = 2.59

$$\frac{bd^2f^2x^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} - bd^2f^2x^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)} + 2\,bcdf^2x\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} - 2\,bcdf^2x\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)} + bc^2f^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} - bc^2f^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)} - bd^2fxe^{(fx+e)} - bd^2fxe^{(-fx-e)} - bcdfe^{(fx+e)} - bcdfe^{(-fx-e)} - bd^2e^{(fx+e)} + bd^2e^{(-fx-e)} - 2\,ad^2}{4\,(d^5x^2 + 2\,cd^4x + c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(b*d^2*f^2*x^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) - b*d^2*f^2*x^2*Ei(-(d
*f*x + c*f)/d)*e^(-e + c*f/d) + 2*b*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(e - c*
f/d) - 2*b*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + b*c^2*f^2*Ei((d*
f*x + c*f)/d)*e^(e - c*f/d) - b*c^2*f^2*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d)
 - b*d^2*f*x*e^(f*x + e) - b*d^2*f*x*e^(-f*x - e) - b*c*d*f*e^(f*x + e) - b
*c*d*f*e^(-f*x - e) - b*d^2*e^(f*x + e) + b*d^2*e^(-f*x - e) - 2*a*d^2)/(d^
5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\sinh(e+f\,x)}{{(c+d\,x)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x))/(c + d*x)^3,x)
```

```
[Out] int((a + b*sinh(e + f*x))/(c + d*x)^3, x)
```

3.163 $\int (c+dx)^3 (a+b\sinh(e+fx))^2 \, dx$

Optimal. Leaf size=250

$$-\frac{3b^2cd^2x}{4f^2}-\frac{3b^2d^3x^2}{8f^2}+\frac{a^2(c+dx)^4}{4d}-\frac{b^2(c+dx)^4}{8d}+\frac{12abd^2(c+dx)\cosh(e+fx)}{f^3}+\frac{2ab(c+dx)^3\cosh(e+fx)}{f}-$$

```
[Out] -3/4*b^2*c*d^2*x/f^2-3/8*b^2*d^3*x^2/f^2+1/4*a^2*(d*x+c)^4/d-1/8*b^2*(d*x+c
)^4/d+12*a*b*d^2*(d*x+c)*cosh(f*x+e)/f^3+2*a*b*(d*x+c)^3*cosh(f*x+e)/f-12*a
*b*d^3*sinh(f*x+e)/f^4-6*a*b*d*(d*x+c)^2*sinh(f*x+e)/f^2+3/4*b^2*d^2*(d*x+c
)*cosh(f*x+e)*sinh(f*x+e)/f^3+1/2*b^2*(d*x+c)^3*cosh(f*x+e)*sinh(f*x+e)/f-3
/8*b^2*d^3*sinh(f*x+e)^2/f^4-3/4*b^2*d*(d*x+c)^2*sinh(f*x+e)^2/f^2
```

Rubi [A]
time = 0.20, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3398, 3377, 2717, 3392, 32, 3391}

$$\frac{a^2(c+dx)^4}{4d}+\frac{12abd^2(c+dx)\cosh(e+fx)}{f^3}-\frac{6abd(c+dx)^2\sinh(e+fx)}{f^2}+\frac{2ab(c+dx)^3\cosh(e+fx)}{f}-\frac{12abd^3\sinh(e+fx)}{f^4}+\frac{3b^2d^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{4f^3}-\frac{3b^2cd^2x}{4f^2}-\frac{3b^2d(c+dx)^2\sinh^2(e+fx)}{4f^2}+\frac{b^2(c+dx)^3\sinh(e+fx)\cosh(e+fx)}{2f}-\frac{b^2(c+dx)^4}{8d}-\frac{3b^2d^3\sinh^2(e+fx)}{8f^4}-\frac{3b^2d^3x^2}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] (-3*b^2*c*d^2*x)/(4*f^2) - (3*b^2*d^3*x^2)/(8*f^2) + (a^2*(c + d*x)^4)/(4*d
) - (b^2*(c + d*x)^4)/(8*d) + (12*a*b*d^2*(c + d*x)*Cosh[e + f*x])/f^3 + (2
*a*b*(c + d*x)^3*Cosh[e + f*x])/f - (12*a*b*d^3*Sinh[e + f*x])/f^4 - (6*a*b
*d*(c + d*x)^2*Sinh[e + f*x])/f^2 + (3*b^2*d^2*(c + d*x)*Cosh[e + f*x]*Sinh
[e + f*x])/(4*f^3) + (b^2*(c + d*x)^3*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) -
(3*b^2*d^3*Sinh[e + f*x]^2)/(8*f^4) - (3*b^2*d*(c + d*x)^2*Sinh[e + f*x]^2)
/(4*f^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx &= \int \left(a^2(c+dx)^3+2ab(c+dx)^3\sinh(e+fx)+b^2(c+dx)^3\sinh^2(e+\right.\\ &= \frac{a^2(c+dx)^4}{4d}+(2ab)\int (c+dx)^3\sinh(e+fx)\,dx+b^2\int (c+dx)^3\sin\\ &= \frac{a^2(c+dx)^4}{4d}+\frac{2ab(c+dx)^3\cosh(e+fx)}{f}+\frac{b^2(c+dx)^3\cosh(e+fx}{2f}\\ &= \frac{a^2(c+dx)^4}{4d}-\frac{b^2(c+dx)^4}{8d}+\frac{2ab(c+dx)^3\cosh(e+fx)}{f}-\frac{6abd(c+}{}\\ &= -\frac{3b^2cd^2x}{4f^2}-\frac{3b^2d^3x^2}{8f^2}+\frac{a^2(c+dx)^4}{4d}-\frac{b^2(c+dx)^4}{8d}+\frac{12abd^2(c+dx}{f}\\ &= -\frac{3b^2cd^2x}{4f^2}-\frac{3b^2d^3x^2}{8f^2}+\frac{a^2(c+dx)^4}{4d}-\frac{b^2(c+dx)^4}{8d}+\frac{12abd^2(c+dx}{f}\end{aligned}$$

Mathematica [A]
time = 0.72, size = 235, normalized size = 0.94

$$\frac{32abf(c+dx)\left(c^2f^2+2cdf^2x+d^2(6+f^2x^2)\right)\cosh(e+fx)-3b^2d(2c^2f^2+4cdf^2x+d^2(1+2f^2x^2))\cosh(2(e+fx))+2\left((2a^2-b^2)f^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)-48abd(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\sinh(e+fx)+b^2f(c+dx)\left(2c^2f^2+4cdf^2x+d^2(3+2f^2x^2)\right)\sinh(2(e+fx))\right)}{16f^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3*(a + b*Sinh[e + f*x])^2,x]

[Out] (32*a*b*f*(c + d*x)*(c^2*f^2 + 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Cosh[e + f*x] - 3*b^2*d*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(1 + 2*f^2*x^2))*Cosh[2*(e + f*x)] + 2*((2*a^2 - b^2)*f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 48*a*b*d*(c^2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x] + b^2*f*(c + d*x)*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(3 + 2*f^2*x^2))*Sinh[2*(e + f*x)]))/(16*f^4)

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1060 vs. $2(234) = 468$.
time = 0.61, size = 1061, normalized size = 4.24

method	result
risch	$\frac{a^2 d^3 x^4}{4} - \frac{d^3 b^2 x^4}{8} + a^2 c\, d^2 x^3 - \frac{d^2 b^2 c\, x^3}{2} + \frac{3 a^2 c^2 d\, x^2}{2} - \frac{3 d\, b^2 c^2 x^2}{4} + c^3 a^2 x - \frac{b^2 c^3 x}{2} + \frac{a^2 c^4}{4d} - \frac{b^2 c^4}{8d} +$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*(a+b*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] 1/f*(6*d^3/f^3*e^2*a*b*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))+d^2/f^2*c*a^2*(f*x+e)^3+3*d^2/f^2*e^2*c*b^2*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)-3*d/f*e*c^2*b^2*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)-2*d^3/f^3*e^3*a*b*cosh(f*x+e)+6*d/f*c^2*a*b*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))-6*d^2/f^2*e*c*b^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)-6*d^3/f^3*e*a*b*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*sinh(f*x+e)+2*cosh(f*x+e))+6*d^2/f^2*c*a*b*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*sinh(f*x+e)+2*cosh(f*x+e))-3*d/f*e*c^2*a^2*(f*x+e)-3*d^2/f^2*e*c*a^2*(f*x+e)^2+3*d^2/f^2*e^2*c*a^2*(f*x+e)+c^3*a^2*(f*x+e)+1/4*d^3/f^3*a^2*(f*x+e)^4+d^3/f^3*b^2*(1/2*(f*x+e)^3*cosh(f*x+e)*sinh(f*x+e)-1/8*(f*x+e)^4-3/4*(f*x+e)^2*cosh(f*x+e)^2+3/4*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+3/8*(f*x+e)^2-3/8*cosh(f*x+e)^2)+2*c^3*a*b*cosh(f*x+e)+c^3*b^2*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)-d^3/f^3*e*a^2*(f*x+e)^3-d^3/f^3*e^3*a^2*(f*x+e)+3/2*d^3/f^3*e^2*a^2*(f*x+e)^2+3/2*d/f*c^2*a^2*(f*x+e)^2-12*d^2/f^2*e*c*a*b*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))-6*d/f*e*c^2*a*b*cosh(f*x+e)+6*d^2/f^2*e^2*c*a*b*cosh(f*x+e)-d^3/f^3*e^3*b^2*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)+3*d^2/f^2*c*b^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)-1/6*(f*x+e)^3-1/2*(f*x+e)*cosh(f*x+e)^2+1/4*cosh(f*x+e)*sinh(f*x+e)+1/4*f*x+1/4*e)+3*d^3/f^3*e^2*b^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)+3*d/f*c^2*b^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)+2*d^3/f^3*a*b*((f*x+e)^3*cosh(f*x+e)-3*(f*x+e)^2*sinh(f*x+e)+6*(f*x+e)*cosh(f*x+e)-6*sinh(f*x+e))-3*d^3/f^3

```
*e*b^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)-1/6*(f*x+e)^3-1/2*(f*x+e)*cos
h(f*x+e)^2+1/4*cosh(f*x+e)*sinh(f*x+e)+1/4*f*x+1/4*e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(244) = 488$.
time = 0.29, size = 547, normalized size = 2.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 - 3/16*(4*x^2 - (2*f*x*
e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 + (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*b^2*c^
2*d - 1/16*(8*x^3 - 3*(2*f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*
x)/f^3 + 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*b^2*c*d^2 - 1/32*(
4*x^4 - (4*f^3*x^3*e^(2*e) - 6*f^2*x^2*e^(2*e) + 6*f*x*e^(2*e) - 3*e^(2*e))
*e^(2*f*x)/f^4 + (4*f^3*x^3 + 6*f^2*x^2 + 6*f*x + 3)*e^(-2*f*x - 2*e)/f^4)*
b^2*d^3 - 1/8*b^2*c^3*(4*x - e^(2*f*x + 2*e)/f + e^(-2*f*x - 2*e)/f) + a^2*
c^3*x + 3*a*b*c^2*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^(-f*x - e)/f
^2) + 3*a*b*c*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 + (f^2*x^2
 + 2*f*x + 2)*e^(-f*x - e)/f^3) + a*b*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e^e + 6
*f*x*e^e - 6*e^e)*e^(f*x)/f^4 + (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(-f*x -
 e)/f^4) + 2*a*b*c^3*cosh(f*x + e)/f
```

Fricas [A]
time = 0.34, size = 433, normalized size = 1.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(2*a^2 - b^2)*d^3*f^4*x^4 + 8*(2*a^2 - b^2)*c*d^2*f^4*x^3 + 12*(2*a
^2 - b^2)*c^2*d*f^4*x^2 + 8*(2*a^2 - b^2)*c^3*f^4*x - 3*(2*b^2*d^3*f^2*x^2
+ 4*b^2*c*d^2*f^2*x + 2*b^2*c^2*d*f^2 + b^2*d^3)*cosh(f*x + cosh(1) + sinh(
1))^2 - 3*(2*b^2*d^3*f^2*x^2 + 4*b^2*c*d^2*f^2*x + 2*b^2*c^2*d*f^2 + b^2*d^
3)*sinh(f*x + cosh(1) + sinh(1))^2 + 32*(a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*
x^2 + a*b*c^3*f^3 + 6*a*b*c*d^2*f + 3*(a*b*c^2*d*f^3 + 2*a*b*d^3*f)*x)*cosh
(f*x + cosh(1) + sinh(1)) - 4*(24*a*b*d^3*f^2*x^2 + 48*a*b*c*d^2*f^2*x + 24
*a*b*c^2*d*f^2 + 48*a*b*d^3 - (2*b^2*d^3*f^3*x^3 + 6*b^2*c*d^2*f^3*x^2 + 2*
b^2*c^3*f^3 + 3*b^2*c*d^2*f + 3*(2*b^2*c^2*d*f^3 + b^2*d^3*f)*x)*cosh(f*x +
 cosh(1) + sinh(1)))*sinh(f*x + cosh(1) + sinh(1)))/f^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(255) = 510$.

time = 0.46, size = 779, normalized size = 3.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*sinh(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**3*x + 3*a**2*c**2*d*x**2/2 + a**2*c*d**2*x**3 + a**2*d**
3*x**4/4 + 2*a*b*c**3*cosh(e + f*x)/f + 6*a*b*c**2*d*x*cosh(e + f*x)/f - 6*
a*b*c**2*d*sinh(e + f*x)/f**2 + 6*a*b*c*d**2*x**2*cosh(e + f*x)/f - 12*a*b*
c*d**2*x*sinh(e + f*x)/f**2 + 12*a*b*c*d**2*cosh(e + f*x)/f**3 + 2*a*b*d**3
*x**3*cosh(e + f*x)/f - 6*a*b*d**3*x**2*sinh(e + f*x)/f**2 + 12*a*b*d**3*x*
cosh(e + f*x)/f**3 - 12*a*b*d**3*sinh(e + f*x)/f**4 + b**2*c**3*x*sinh(e +
f*x)**2/2 - b**2*c**3*x*cosh(e + f*x)**2/2 + b**2*c**3*sinh(e + f*x)*cosh(e
 + f*x)/(2*f) + 3*b**2*c**2*d*x**2*sinh(e + f*x)**2/4 - 3*b**2*c**2*d*x**2*
cosh(e + f*x)**2/4 + 3*b**2*c**2*d*x*sinh(e + f*x)*cosh(e + f*x)/(2*f) - 3*
b**2*c**2*d*cosh(e + f*x)**2/(4*f**2) + b**2*c*d**2*x**3*sinh(e + f*x)**2/2
 - b**2*c*d**2*x**3*cosh(e + f*x)**2/2 + 3*b**2*c*d**2*x**2*sinh(e + f*x)*c
osh(e + f*x)/(2*f) - 3*b**2*c*d**2*x*sinh(e + f*x)**2/(4*f**2) - 3*b**2*c*d
**2*x*cosh(e + f*x)**2/(4*f**2) + 3*b**2*c*d**2*sinh(e + f*x)*cosh(e + f*x)
/(4*f**3) + b**2*d**3*x**4*sinh(e + f*x)**2/8 - b**2*d**3*x**4*cosh(e + f*x
)**2/8 + b**2*d**3*x**3*sinh(e + f*x)*cosh(e + f*x)/(2*f) - 3*b**2*d**3*x**
2*sinh(e + f*x)**2/(8*f**2) - 3*b**2*d**3*x**2*cosh(e + f*x)**2/(8*f**2) +
3*b**2*d**3*x*sinh(e + f*x)*cosh(e + f*x)/(4*f**3) - 3*b**2*d**3*cosh(e + f
*x)**2/(8*f**4), Ne(f, 0)), ((a + b*sinh(e))**2*(c**3*x + 3*c**2*d*x**2/2 +
 c*d**2*x**3 + d**3*x**4/4), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 598 vs. 2(234) = 468.
time = 0.45, size = 598, normalized size = 2.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/4*a^2*d^3*x^4 - 1/8*b^2*d^3*x^4 + a^2*c*d^2*x^3 - 1/2*b^2*c*d^2*x^3 + 3/2
*a^2*c^2*d*x^2 - 3/4*b^2*c^2*d*x^2 + a^2*c^3*x - 1/2*b^2*c^3*x + 1/32*(4*b^
2*d^3*f^3*x^3 + 12*b^2*c*d^2*f^3*x^2 + 12*b^2*c^2*d*f^3*x - 6*b^2*d^3*f^2*x
^2 + 4*b^2*c^3*f^3 - 12*b^2*c*d^2*f^2*x - 6*b^2*c^2*d*f^2 + 6*b^2*d^3*f*x +
 6*b^2*c*d^2*f - 3*b^2*d^3)*e^(2*f*x + 2*e)/f^4 + (a*b*d^3*f^3*x^3 + 3*a*b*
c*d^2*f^3*x^2 + 3*a*b*c^2*d*f^3*x - 3*a*b*d^3*f^2*x^2 + a*b*c^3*f^3 - 6*a*b
*c*d^2*f^2*x - 3*a*b*c^2*d*f^2 + 6*a*b*d^3*f*x + 6*a*b*c*d^2*f - 6*a*b*d^3)
*e^(f*x + e)/f^4 + (a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*x^2 + 3*a*b*c^2*d*f^3
*x + 3*a*b*d^3*f^2*x^2 + a*b*c^3*f^3 + 6*a*b*c*d^2*f^2*x + 3*a*b*c^2*d*f^2
```

```
+ 6*a*b*d^3*f*x + 6*a*b*c*d^2*f + 6*a*b*d^3)*e^(-f*x - e)/f^4 - 1/32*(4*b^2
*d^3*f^3*x^3 + 12*b^2*c*d^2*f^3*x^2 + 12*b^2*c^2*d*f^3*x + 6*b^2*d^3*f^2*x^
2 + 4*b^2*c^3*f^3 + 12*b^2*c*d^2*f^2*x + 6*b^2*c^2*d*f^2 + 6*b^2*d^3*f*x +
6*b^2*c*d^2*f + 3*b^2*d^3)*e^(-2*f*x - 2*e)/f^4
```

Mupad [B]
time = 1.86, size = 481, normalized size = 1.92

$$a^2\,c^3\,x-\frac{b^2\,c^3\,x}{2}+\frac{a^2\,d^3\,x^4}{4}-\frac{b^2\,d^3\,x^4}{8}+\frac{3\,a^2\,c^2\,d\,x^2}{2}+a^2\,c\,d^2\,x^3-\frac{3\,b^2\,c^2\,d\,x^2}{4}-\frac{b^2\,c\,d^2\,x^3}{2}-\frac{3\,b^2\,d^3\,\mathrm{cosh}\left(2\,e+2\,f\,x\right)}{16\,f^4}+\frac{b^2\,c^3\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)}{4\,f}+\frac{2\,a\,b\,c^3\,\mathrm{cosh}\left(e+f\,x\right)}{f}-\frac{12\,a\,b\,d^3\,\mathrm{sinh}\left(e+f\,x\right)}{f^4}-\frac{3\,b^2\,d^3\,x^2\,\mathrm{cosh}\left(2\,e+2\,f\,x\right)}{8\,f^2}+\frac{b^2\,d^3\,x^3\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)}{4\,f}-\frac{3\,b^2\,c^2\,d\,\mathrm{cosh}\left(2\,e+2\,f\,x\right)}{8\,f^2}+\frac{3\,b^2\,c\,d^2\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)}{8\,f^3}+\frac{3\,b^2\,d^3\,x\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)}{8\,f^3}-\frac{3\,b^2\,c\,d^2\,x\,\mathrm{cosh}\left(2\,e+2\,f\,x\right)}{4\,f^2}+\frac{3\,b^2\,c^2\,d\,x\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)}{4\,f}+\frac{12\,a\,b\,c\,d^2\,\mathrm{cosh}\left(e+f\,x\right)}{f^3}-\frac{6\,a\,b\,c^2\,d\,\mathrm{sinh}\left(e+f\,x\right)}{f^2}+\frac{12\,a\,b\,d^3\,x\,\mathrm{cosh}\left(e+f\,x\right)}{f^3}+\frac{3\,b^2\,c\,d^2\,x^2\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)}{4\,f}+\frac{2\,a\,b\,d^3\,x^3\,\mathrm{cosh}\left(e+f\,x\right)}{f}-\frac{6\,a\,b\,d^3\,x^2\,\mathrm{sinh}\left(e+f\,x\right)}{f^2}+\frac{6\,a\,b\,c\,d^2\,x^2\,\mathrm{cosh}\left(e+f\,x\right)}{f}+\frac{6\,a\,b\,c^2\,d\,x\,\mathrm{cosh}\left(e+f\,x\right)}{f}-\frac{12\,a\,b\,c\,d^2\,x\,\mathrm{sinh}\left(e+f\,x\right)}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x))^2*(c + d*x)^3,x)
```

```
[Out] a^2*c^3*x - (b^2*c^3*x)/2 + (a^2*d^3*x^4)/4 - (b^2*d^3*x^4)/8 + (3*a^2*c^2*
d*x^2)/2 + a^2*c*d^2*x^3 - (3*b^2*c^2*d*x^2)/4 - (b^2*c*d^2*x^3)/2 - (3*b^2
*d^3*cosh(2*e + 2*f*x))/(16*f^4) + (b^2*c^3*sinh(2*e + 2*f*x))/(4*f) + (2*a
*b*c^3*cosh(e + f*x))/f - (12*a*b*d^3*sinh(e + f*x))/f^4 - (3*b^2*d^3*x^2*c
osh(2*e + 2*f*x))/(8*f^2) + (b^2*d^3*x^3*sinh(2*e + 2*f*x))/(4*f) - (3*b^2*
c^2*d*cosh(2*e + 2*f*x))/(8*f^2) + (3*b^2*c*d^2*sinh(2*e + 2*f*x))/(8*f^3)
+ (3*b^2*d^3*x*sinh(2*e + 2*f*x))/(8*f^3) - (3*b^2*c*d^2*x*cosh(2*e + 2*f*x
))/(4*f^2) + (3*b^2*c^2*d*x*sinh(2*e + 2*f*x))/(4*f) + (12*a*b*c*d^2*cosh(e
 + f*x))/f^3 - (6*a*b*c^2*d*sinh(e + f*x))/f^2 + (12*a*b*d^3*x*cosh(e + f*x
))/f^3 + (3*b^2*c*d^2*x^2*sinh(2*e + 2*f*x))/(4*f) + (2*a*b*d^3*x^3*cosh(e
+ f*x))/f - (6*a*b*d^3*x^2*sinh(e + f*x))/f^2 + (6*a*b*c*d^2*x^2*cosh(e + f
*x))/f + (6*a*b*c^2*d*x*cosh(e + f*x))/f - (12*a*b*c*d^2*x*sinh(e + f*x))/f
^2
```

3.164 $\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx$

Optimal. Leaf size=182

$$-\frac{b^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{3d}-\frac{b^2(c+dx)^3}{6d}+\frac{4abd^2\cosh(e+fx)}{f^3}+\frac{2ab(c+dx)^2\cosh(e+fx)}{f}-\frac{4abd(c+dx)\sinh(e+\ldots}{f^2}$$

```
[Out] -1/4*b^2*d^2*x/f^2+1/3*a^2*(d*x+c)^3/d-1/6*b^2*(d*x+c)^3/d+4*a*b*d^2*cosh(f
*x+e)/f^3+2*a*b*(d*x+c)^2*cosh(f*x+e)/f-4*a*b*d*(d*x+c)*sinh(f*x+e)/f^2+1/4
*b^2*d^2*cosh(f*x+e)*sinh(f*x+e)/f^3+1/2*b^2*(d*x+c)^2*cosh(f*x+e)*sinh(f*x
+e)/f-1/2*b^2*d*(d*x+c)*sinh(f*x+e)^2/f^2
```

Rubi [A]
time = 0.14, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3398, 3377, 2718, 3392, 32, 2715, 8}

$$\frac{a^2(c+dx)^3}{3d}-\frac{4abd(c+dx)\sinh(e+fx)}{f^2}+\frac{2ab(c+dx)^2\cosh(e+fx)}{f}+\frac{4abd^2\cosh(e+fx)}{f^3}-\frac{b^2d(c+dx)\sinh^2(e+fx)}{2f^2}+\frac{b^2(c+dx)^2\sinh(e+fx)\cosh(e+fx)}{2f}-\frac{b^2(c+dx)^3}{6d}+\frac{b^2d^2\sinh(e+fx)\cosh(e+fx)}{4f^3}-\frac{b^2d^2x}{4f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] -1/4*(b^2*d^2*x)/f^2 + (a^2*(c + d*x)^3)/(3*d) - (b^2*(c + d*x)^3)/(6*d) +
(4*a*b*d^2*Cosh[e + f*x])/f^3 + (2*a*b*(c + d*x)^2*Cosh[e + f*x])/f - (4*a*
b*d*(c + d*x)*Sinh[e + f*x])/f^2 + (b^2*d^2*Cosh[e + f*x]*Sinh[e + f*x])/(4
*f^3) + (b^2*(c + d*x)^2*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) - (b^2*d*(c + d
*x)*Sinh[e + f*x]^2)/(2*f^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx = \int \left(a^2(c+dx)^2+2ab(c+dx)^2\sinh(e+fx)+b^2(c+dx)^2\sinh^2(e+\right.$$
$$= \frac{a^2(c+dx)^3}{3d}+(2ab)\int (c+dx)^2\sinh(e+fx)\,dx+b^2\int (c+dx)^2\sin$$
$$= \frac{a^2(c+dx)^3}{3d}+\frac{2ab(c+dx)^2\cosh(e+fx)}{f}+\frac{b^2(c+dx)^2\cosh(e+fx}{2f}$$
$$= \frac{a^2(c+dx)^3}{3d}-\frac{b^2(c+dx)^3}{6d}+\frac{2ab(c+dx)^2\cosh(e+fx)}{f}-\frac{4abd(c+}{}$$
$$= -\frac{b^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{3d}-\frac{b^2(c+dx)^3}{6d}+\frac{4abd^2\cosh(e+fx)}{f^3}+\frac{2ab(c}{}$$

Mathematica [A]
time = 0.43, size = 249, normalized size = 1.37

$$\frac{24a^2c^2f^3x-12b^2c^2f^3x+24a^2cdf^3x^2-12b^2cdf^3x^2+8a^2d^2f^3x^3-4b^2d^2f^3x^3+48ab(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\cosh(e+fx)-6b^2df(c+dx)\cosh(2(e+fx))-96abcdf\sinh(e+fx)-96abd^2fx\sinh(e+fx)+3b^2d^2\sinh(2(e+fx))+6b^2c^2f^2\sinh(2(e+fx))+12b^2cdf^2x\sinh(2(e+fx))+6b^2d^2f^2x^2\sinh(2(e+fx))}{24f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] (24*a^2*c^2*f^3*x - 12*b^2*c^2*f^3*x + 24*a^2*c*d*f^3*x^2 - 12*b^2*c*d*f^3*
x^2 + 8*a^2*d^2*f^3*x^3 - 4*b^2*d^2*f^3*x^3 + 48*a*b*(c^2*f^2 + 2*c*d*f^2*x
 + d^2*(2 + f^2*x^2))*Cosh[e + f*x] - 6*b^2*d*f*(c + d*x)*Cosh[2*(e + f*x)]
 - 96*a*b*c*d*f*Sinh[e + f*x] - 96*a*b*d^2*f*x*Sinh[e + f*x] + 3*b^2*d^2*Si
nh[2*(e + f*x)] + 6*b^2*c^2*f^2*Sinh[2*(e + f*x)] + 12*b^2*c*d*f^2*x*Sinh[2
*(e + f*x)] + 6*b^2*d^2*f^2*x^2*Sinh[2*(e + f*x)])/(24*f^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(170) = 340$.
time = 0.82, size = 535, normalized size = 2.94

method	result
risch	$\frac{d^2a^2x^3}{3} - \frac{d^2b^2x^3}{6} + a^2cd\,x^2 - \frac{d\,b^2c\,x^2}{2} + a^2c^2x - \frac{b^2c^2x}{2} + \frac{c^3a^2}{3d} - \frac{b^2c^3}{6d} + \frac{b^2\left(2d^2x^2f^2+4cd\,f^2x+2c^2f^2-\right.}{16f^3}$
derivativedivides	$\frac{\frac{d^2a^2(fx+e)^3}{3f^2} + \frac{2d^2ab\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2} + \frac{d^2b^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2} - \frac{(fx+e)^3}{6}\right.}{}}{}$
default	$\frac{\frac{d^2a^2(fx+e)^3}{3f^2} + \frac{2d^2ab\left((fx+e)^2\cosh(fx+e)-2(fx+e)\sinh(fx+e)+2\cosh(fx+e)\right)}{f^2} + \frac{d^2b^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2} - \frac{(fx+e)^3}{6}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*d^2/f^2*a^2*(f*x+e)^3+2*d^2/f^2*a*b*((f*x+e)^2*cosh(f*x+e)-2*(f*x+
e)*sinh(f*x+e)+2*cosh(f*x+e))+d^2/f^2*b^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f
*x+e)-1/6*(f*x+e)^3-1/2*(f*x+e)*cosh(f*x+e)^2+1/4*cosh(f*x+e)*sinh(f*x+e)+1
/4*f*x+1/4*e)-d^2/f^2*e*a^2*(f*x+e)^2-4*d^2/f^2*e*a*b*((f*x+e)*cosh(f*x+e)-
sinh(f*x+e))-2*d^2/f^2*e*b^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+
e)^2-1/4*cosh(f*x+e)^2)+d/f*c*a^2*(f*x+e)^2+4*d/f*c*a*b*((f*x+e)*cosh(f*x+e
)-sinh(f*x+e))+2*d/f*c*b^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)
^2-1/4*cosh(f*x+e)^2)+d^2/f^2*e^2*a^2*(f*x+e)+2*d^2/f^2*e^2*a*b*cosh(f*x+e)
+d^2/f^2*e^2*b^2*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)-2*d/f*e*c*a^2*
(f*x+e)-4*d/f*e*c*a*b*cosh(f*x+e)-2*d/f*e*c*b^2*(1/2*cosh(f*x+e)*sinh(f*x+e
)-1/2*f*x-1/2*e)+a^2*c^2*(f*x+e)+2*c^2*a*b*cosh(f*x+e)+b^2*c^2*(1/2*cosh(f*
x+e)*sinh(f*x+e)-1/2*f*x-1/2*e))
```

Maxima [A]
time = 0.28, size = 339, normalized size = 1.86

$$\frac{1}{3}a^2d^2x^3 + a^2cdx^2 - \frac{1}{8}\left(4x^2 - \frac{(2fxe^{(2e)} - e^{(2e)})e^{(2fx)}}{f^2} + \frac{(2fx+1)e^{(-2fx-2e)}}{f^2}\right)b^2cd - \frac{1}{48}\left(8x^3 - \frac{3(2f^2x^2e^{(2e)} - 2fxe^{(2e)} + e^{(2e)})e^{(2fx)}}{f^3} + \frac{3(2f^2x^2+2fx+1)e^{(-2fx-2e)}}{f^3}\right)b^2d^2 - \frac{1}{8}b^2c^2\left(4x - \frac{e^{(2fx+2e)}}{f} + \frac{e^{(-2fx-2e)}}{f}\right) + a^2c^2x + 2abcd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + abd^2\left(\frac{(f^2x^2e^e - 2fxe^e + 2e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2+2fx+2)e^{(-fx-e)}}{f^3}\right) + \frac{2abc^2\cosh(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*d^2*x^3 + a^2*c*d*x^2 - 1/8*(4*x^2 - (2*f*x*e^(2*e) - e^(2*e))*e^(2
*f*x)/f^2 + (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*b^2*c*d - 1/48*(8*x^3 - 3*(2*
f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*x)/f^3 + 3*(2*f^2*x^2 + 2
*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*b^2*d^2 - 1/8*b^2*c^2*(4*x - e^(2*f*x + 2*e
)/f + e^(-2*f*x - 2*e)/f) + a^2*c^2*x + 2*a*b*c*d*((f*x*e^e - e^e)*e^(f*x)/
f^2 + (f*x + 1)*e^(-f*x - e)/f^2) + a*b*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e
^e)*e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + 2*a*b*c^2*cosh(
f*x + e)/f
```

Fricas [**A**]
time = 0.36, size = 262, normalized size = 1.44

$$\frac{2\,(2\,a^2-b^2)d^2f^3x^3+6\,(2\,a^2-b^2)cdf^3x^2+6\,(2\,a^2-b^2)c^2f^3x-3\,(b^2d^2fx+b^2cdf)\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)^2-3\,(b^2d^2fx+b^2cdf)\sinh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)^2+24\,(abd^2f^2x^2+2\,abcdf^2x+abc^2f^2+2\,abd^2)\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)-3\,(16\,abd^2fx+16\,abcdf-(2\,b^2d^2f^2x^2+4\,b^2cdf^2x+2\,b^2c^2f^2+b^2d^2)\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right))\sinh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)}{12\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*(2*a^2 - b^2)*d^2*f^3*x^3 + 6*(2*a^2 - b^2)*c*d*f^3*x^2 + 6*(2*a^2
- b^2)*c^2*f^3*x - 3*(b^2*d^2*f*x + b^2*c*d*f)*cosh(f*x + cosh(1) + sinh(1)
)^2 - 3*(b^2*d^2*f*x + b^2*c*d*f)*sinh(f*x + cosh(1) + sinh(1))^2 + 24*(a*b
*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2 + 2*a*b*d^2)*cosh(f*x + cosh(1
) + sinh(1)) - 3*(16*a*b*d^2*f*x + 16*a*b*c*d*f - (2*b^2*d^2*f^2*x^2 + 4*b^
2*c*d*f^2*x + 2*b^2*c^2*f^2 + b^2*d^2)*cosh(f*x + cosh(1) + sinh(1)))*sinh(
f*x + cosh(1) + sinh(1)))/f^3
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(177) = 354$.
time = 0.30, size = 456, normalized size = 2.51

$$\begin{cases} a^2c^2x+a^2cdx^2+\frac{a^2d^2x^3}{3}+\frac{2abc^2\cosh\left(e+fx\right)}{f}+\frac{4abcdx\cosh\left(e+fx\right)}{f}-\frac{4abcd\sinh\left(e+fx\right)}{f^2}+\frac{2abd^2x^2\cosh\left(e+fx\right)}{f}-\frac{4abd^2x\sinh\left(e+fx\right)}{f^2}+\frac{4abd^2\cosh\left(e+fx\right)}{f^3}+\frac{b^2c^2x\sinh^2\left(e+fx\right)}{2}-\frac{b^2c^2x\cosh^2\left(e+fx\right)}{2}+\frac{b^2c^2\sinh\left(e+fx\right)\cosh\left(e+fx\right)}{2f}+\frac{b^2cdx^2\sinh^2\left(e+fx\right)}{2}-\frac{b^2cdx^2\cosh^2\left(e+fx\right)}{2}+\frac{b^2cdx\sinh\left(e+fx\right)\cosh\left(e+fx\right)}{f}-\frac{b^2cd\cosh^2\left(e+fx\right)}{2f^2}+\frac{b^2d^2x^3\sinh^2\left(e+fx\right)}{6}-\frac{b^2d^2x^3\cosh^2\left(e+fx\right)}{6}+\frac{b^2d^2x^2\sinh\left(e+fx\right)\cosh\left(e+fx\right)}{2f}-\frac{b^2d^2x\sinh^2\left(e+fx\right)}{4f^2}-\frac{b^2d^2x\cosh^2\left(e+fx\right)}{4f^2}+\frac{b^2d^2\sinh\left(e+fx\right)\cosh\left(e+fx\right)}{4f^3} & \text{for}\: f \neq 0 \\ (a+b\sinh\left(e\right))^2\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*sinh(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**2*x + a**2*c*d*x**2 + a**2*d**2*x**3/3 + 2*a*b*c**2*cosh
(e + f*x)/f + 4*a*b*c*d*x*cosh(e + f*x)/f - 4*a*b*c*d*sinh(e + f*x)/f**2 +
2*a*b*d**2*x**2*cosh(e + f*x)/f - 4*a*b*d**2*x*sinh(e + f*x)/f**2 + 4*a*b*d
**2*cosh(e + f*x)/f**3 + b**2*c**2*x*sinh(e + f*x)**2/2 - b**2*c**2*x*cosh(
e + f*x)**2/2 + b**2*c**2*sinh(e + f*x)*cosh(e + f*x)/(2*f) + b**2*c*d*x**2
*sinh(e + f*x)**2/2 - b**2*c*d*x**2*cosh(e + f*x)**2/2 + b**2*c*d*x*sinh(e
+ f*x)*cosh(e + f*x)/f - b**2*c*d*cosh(e + f*x)**2/(2*f**2) + b**2*d**2*x**
3*sinh(e + f*x)**2/6 - b**2*d**2*x**3*cosh(e + f*x)**2/6 + b**2*d**2*x**2*s
inh(e + f*x)*cosh(e + f*x)/(2*f) - b**2*d**2*x*sinh(e + f*x)**2/(4*f**2) -
b**2*d**2*x*cosh(e + f*x)**2/(4*f**2) + b**2*d**2*sinh(e + f*x)*cosh(e + f*
```

```
x)/(4*f**3), Ne(f, 0)), ((a + b*sinh(e))**2*(c**2*x + c*d*x**2 + d**2*x**3/
3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(170) = 340$.
time = 0.44, size = 344, normalized size = 1.89

$$\frac{1}{3}a^2d^2x^3-\frac{1}{6}b^2d^2x^3+a^2cdx^2-\frac{1}{2}b^2cdx^2+a^2c^2x-\frac{1}{2}b^2c^2x+\frac{(2b^2d^2f^2x^2+4b^2cdf^2x+2b^2c^2f^2-2b^2d^2fx-2b^2cdf+b^2d^2)e^{(2fx+2e)}}{16f^3}+\frac{(abd^2f^2x^2+2abcdf^2x+abc^2f^2-2abd^2fx-2abcdf+2abd^2)e^{(fx+e)}}{f^3}+\frac{(abd^2f^2x^2+2abcdf^2x+abc^2f^2+2abd^2fx+2abcdf+2abd^2)e^{(-fx-e)}}{f^3}-\frac{(2b^2d^2f^2x^2+4b^2cdf^2x+2b^2c^2f^2+2b^2d^2fx+2b^2cdf+b^2d^2)e^{(-2fx-2e)}}{16f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*a^2*d^2*x^3 - 1/6*b^2*d^2*x^3 + a^2*c*d*x^2 - 1/2*b^2*c*d*x^2 + a^2*c^2
*x - 1/2*b^2*c^2*x + 1/16*(2*b^2*d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*
f^2 - 2*b^2*d^2*f*x - 2*b^2*c*d*f + b^2*d^2)*e^(2*f*x + 2*e)/f^3 + (a*b*d^2
*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2 - 2*a*b*d^2*f*x - 2*a*b*c*d*f + 2*
a*b*d^2)*e^(f*x + e)/f^3 + (a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2
 + 2*a*b*d^2*f*x + 2*a*b*c*d*f + 2*a*b*d^2)*e^(-f*x - e)/f^3 - 1/16*(2*b^2*
d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*f^2 + 2*b^2*d^2*f*x + 2*b^2*c*d*f
 + b^2*d^2)*e^(-2*f*x - 2*e)/f^3
```

Mupad [B]
time = 0.55, size = 281, normalized size = 1.54

$$a^2c^2x-\frac{b^2c^2x}{2}+\frac{a^2d^2x^3}{3}-\frac{b^2d^2x^3}{6}+\frac{b^2c^2\sinh(2e+2fx)}{4f}+\frac{b^2d^2\sinh(2e+2fx)}{8f^3}+a^2cdx^2-\frac{b^2cdx^2}{2}+\frac{2abc^2\cosh(e+fx)}{f}+\frac{4abd^2\cosh(e+fx)}{f^3}+\frac{b^2d^2x^2\sinh(2e+2fx)}{4f}-\frac{b^2cd\cosh(2e+2fx)}{4f^2}-\frac{b^2d^2x\cosh(2e+2fx)}{4f^2}-\frac{4abcd\sinh(e+fx)}{f^2}-\frac{4abd^2x\sinh(e+fx)}{f^2}+\frac{2abd^2x^2\cosh(e+fx)}{f}+\frac{b^2cdx\sinh(2e+2fx)}{2f}+\frac{4abcdx\cosh(e+fx)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x))^2*(c + d*x)^2,x)
```

```
[Out] a^2*c^2*x - (b^2*c^2*x)/2 + (a^2*d^2*x^3)/3 - (b^2*d^2*x^3)/6 + (b^2*c^2*si
nh(2*e + 2*f*x))/(4*f) + (b^2*d^2*sinh(2*e + 2*f*x))/(8*f^3) + a^2*c*d*x^2
- (b^2*c*d*x^2)/2 + (2*a*b*c^2*cosh(e + f*x))/f + (4*a*b*d^2*cosh(e + f*x))
/f^3 + (b^2*d^2*x^2*sinh(2*e + 2*f*x))/(4*f) - (b^2*c*d*cosh(2*e + 2*f*x))/
(4*f^2) - (b^2*d^2*x*cosh(2*e + 2*f*x))/(4*f^2) - (4*a*b*c*d*sinh(e + f*x))
/f^2 - (4*a*b*d^2*x*sinh(e + f*x))/f^2 + (2*a*b*d^2*x^2*cosh(e + f*x))/f +
(b^2*c*d*x*sinh(2*e + 2*f*x))/(2*f) + (4*a*b*c*d*x*cosh(e + f*x))/f
```

3.165 $\int (c+dx)(a+b\sinh(e+fx))^2\,dx$

Optimal. Leaf size=116

$$-\frac{1}{2}b^2cx-\frac{1}{4}b^2dx^2+\frac{a^2(c+dx)^2}{2d}+\frac{2ab(c+dx)\cosh(e+fx)}{f}-\frac{2abd\sinh(e+fx)}{f^2}+\frac{b^2(c+dx)\cosh(e+fx)\sin}{2f}$$

```
[Out] -1/2*b^2*c*x-1/4*b^2*d*x^2+1/2*a^2*(d*x+c)^2/d+2*a*b*(d*x+c)*cosh(f*x+e)/f-
2*a*b*d*sinh(f*x+e)/f^2+1/2*b^2*(d*x+c)*cosh(f*x+e)*sinh(f*x+e)/f-1/4*b^2*d
*sinh(f*x+e)^2/f^2
```

Rubi [A]
time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3398, 3377, 2717, 3391}

$$\frac{a^2(c+dx)^2}{2d}+\frac{2ab(c+dx)\cosh(e+fx)}{f}-\frac{2abd\sinh(e+fx)}{f^2}+\frac{b^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{2f}-\frac{1}{2}b^2cx-\frac{b^2d\sinh^2(e+fx)}{4f^2}-\frac{1}{4}b^2dx^2$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] -1/2*(b^2*c*x) - (b^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) + (2*a*b*(c + d*x)
*Cosh[e + f*x])/f - (2*a*b*d*Sinh[e + f*x])/f^2 + (b^2*(c + d*x)*Cosh[e + f
*x]*Sinh[e + f*x])/(2*f) - (b^2*d*Sinh[e + f*x]^2)/(4*f^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (c+dx)(a+b\sinh(e+fx))^2\,dx = \int \left(a^2(c+dx)+2ab(c+dx)\sinh(e+fx)+b^2(c+dx)\sinh^2(e+fx)\right)$$

$$= \frac{a^2(c+dx)^2}{2d}+(2ab)\int (c+dx)\sinh(e+fx)\,dx+b^2\int (c+dx)\sinh^2(e$$

$$= \frac{a^2(c+dx)^2}{2d}+\frac{2ab(c+dx)\cosh(e+fx)}{f}+\frac{b^2(c+dx)\cosh(e+fx)\sin}{2f}$$

$$= -\frac{1}{2}b^2cx-\frac{1}{4}b^2dx^2+\frac{a^2(c+dx)^2}{2d}+\frac{2ab(c+dx)\cosh(e+fx)}{f}-\frac{2abd\,\mathrm{si}}{}$$

Mathematica [A]
time = 0.42, size = 98, normalized size = 0.84

$$-\frac{2(2a^2-b^2)(e+fx)(-2cf+d(e-fx))-16abf(c+dx)\cosh(e+fx)+b^2d\cosh(2(e+fx))+16abd\sinh(e+fx)-2b^2f(c+dx)\sinh(2(e+fx))}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] -1/8*(2*(2*a^2 - b^2)*(e + f*x)*(-2*c*f + d*(e - f*x)) - 16*a*b*f*(c + d*x)
*Cosh[e + f*x] + b^2*d*Cosh[2*(e + f*x)] + 16*a*b*d*Sinh[e + f*x] - 2*b^2*f
*(c + d*x)*Sinh[2*(e + f*x)])/f^2
```

Maple [A]
time = 0.93, size = 208, normalized size = 1.79

method	result
risch	$\frac{d\,a^2x^2}{2}+a^2cx-\frac{b^2d\,x^2}{4}-\frac{b^2cx}{2}+\frac{b^2(2dxf+2cf-d)\mathrm{e}^{2fx+2e}}{16f^2}+\frac{ab(dxf+cf-d)\mathrm{e}^{fx+e}}{f^2}+\frac{ab(dxf+cf+d)\mathrm{e}^{-fx-e}}{f^2}$
derivativedivides	$\frac{\frac{d\,a^2(fx+e)^2}{2f}+\frac{2dab((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}+\frac{d\,b^2\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^2}{4}-\frac{\left(\cosh^2(fx+e)\right)}{4}\right)}{f}-\frac{de\,a^2(}{f}}{}$
default	$\frac{\frac{d\,a^2(fx+e)^2}{2f}+\frac{2dab((fx+e)\cosh(fx+e)-\sinh(fx+e))}{f}+\frac{d\,b^2\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}-\frac{(fx+e)^2}{4}-\frac{\left(\cosh^2(fx+e)\right)}{4}\right)}{f}-\frac{de\,a^2(}{f}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*d/f*a^2*(f*x+e)^2+2*d/f*a*b*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))+d/f*
b^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)-d
/f*e*a^2*(f*x+e)-2*d/f*e*a*b*cosh(f*x+e)-d/f*e*b^2*(1/2*cosh(f*x+e)*sinh(f*
x+e)-1/2*f*x-1/2*e)+a^2*c*(f*x+e)+2*a*b*c*cosh(f*x+e)+b^2*c*(1/2*cosh(f*x+e
)*sinh(f*x+e)-1/2*f*x-1/2*e))
```

Maxima [A]
time = 0.28, size = 173, normalized size = 1.49

$$\frac{1}{2}\,a^2dx^2-\frac{1}{16}\left(4\,x^2-\frac{\left(2\,fxe^{(2\,e)}-e^{(2\,e)}\right)e^{(2\,fx)}}{f^2}+\frac{(2\,fx+1)e^{(-2\,fx-2\,e)}}{f^2}\right)b^2d-\frac{1}{8}\,b^2c\left(4\,x-\frac{e^{(2\,fx+2\,e)}}{f}+\frac{e^{(-2\,fx-2\,e)}}{f}\right)+a^2cx+abd\left(\frac{(fxe^e-e^e)e^{(fx)}}{f^2}+\frac{(fx+1)e^{(-fx-e)}}{f^2}\right)+\frac{2\,abc\cosh\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*d*x^2 - 1/16*(4*x^2 - (2*f*x*e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 + (2*
f*x + 1)*e^(-2*f*x - 2*e)/f^2)*b^2*d - 1/8*b^2*c*(4*x - e^(2*f*x + 2*e)/f +
 e^(-2*f*x - 2*e)/f) + a^2*c*x + a*b*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f*x
+ 1)*e^(-f*x - e)/f^2) + 2*a*b*c*cosh(f*x + e)/f
```

Fricas [A]
time = 0.34, size = 143, normalized size = 1.23

$$\frac{2\,(2\,a^2-b^2)df^2x^2+4\,(2\,a^2-b^2)cf^2x-b^2d\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)^2-b^2d\sinh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)^2+16\,(abdfx+abcf)\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)-4\,(4\,abd-(b^2dfx+b^2cf)\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right))\sinh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)}{8\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^2 - b^2)*d*f^2*x^2 + 4*(2*a^2 - b^2)*c*f^2*x - b^2*d*cosh(f*x +
 cosh(1) + sinh(1))^2 - b^2*d*sinh(f*x + cosh(1) + sinh(1))^2 + 16*(a*b*d*f
*x + a*b*c*f)*cosh(f*x + cosh(1) + sinh(1)) - 4*(4*a*b*d - (b^2*d*f*x + b^2
*c*f)*cosh(f*x + cosh(1) + sinh(1)))*sinh(f*x + cosh(1) + sinh(1)))/f^2
```

Sympy [A]
time = 0.17, size = 219, normalized size = 1.89

$$\begin{cases} a^2cx+\frac{a^2dx^2}{2}+\frac{2abc\cosh\left(e+fx\right)}{f}+\frac{2abdx\cosh\left(e+fx\right)}{f}-\frac{2abd\sinh\left(e+fx\right)}{f^2}+\frac{b^2cx\sinh^2\left(e+fx\right)}{2}-\frac{b^2cx\cosh^2\left(e+fx\right)}{2}+\frac{b^2c\sinh\left(e+fx\right)\cosh\left(e+fx\right)}{2f}+\frac{b^2dx^2\sinh^2\left(e+fx\right)}{4}-\frac{b^2dx^2\cosh^2\left(e+fx\right)}{4}+\frac{b^2dx\sinh\left(e+fx\right)\cosh\left(e+fx\right)}{2f}-\frac{b^2d\cosh^2\left(e+fx\right)}{4f^2} & \text{for}\: f\neq 0 \\ \left(a+b\sinh\left(e\right)\right)^2\left(cx+\frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sinh(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c*x + a**2*d*x**2/2 + 2*a*b*c*cosh(e + f*x)/f + 2*a*b*d*x*c
osh(e + f*x)/f - 2*a*b*d*sinh(e + f*x)/f**2 + b**2*c*x*sinh(e + f*x)**2/2 -
 b**2*c*x*cosh(e + f*x)**2/2 + b**2*c*sinh(e + f*x)*cosh(e + f*x)/(2*f) + b
**2*d*x**2*sinh(e + f*x)**2/4 - b**2*d*x**2*cosh(e + f*x)**2/4 + b**2*d*x*s
inh(e + f*x)*cosh(e + f*x)/(2*f) - b**2*d*cosh(e + f*x)**2/(4*f**2), Ne(f,
0)), ((a + b*sinh(e))**2*(c*x + d*x**2/2), True))
```

Giac [A]
time = 0.43, size = 159, normalized size = 1.37

$$\frac{1}{2}\,a^2dx^2-\frac{1}{4}\,b^2dx^2+a^2cx-\frac{1}{2}\,b^2cx+\frac{(2\,b^2dfx+2\,b^2cf-b^2d)e^{(2\,fx+2\,e)}}{16\,f^2}+\frac{(abdfx+abcf-abd)e^{(fx+e)}}{f^2}+\frac{(abdfx+abcf+abd)e^{(-fx-e)}}{f^2}-\frac{(2\,b^2dfx+2\,b^2cf+b^2d)e^{(-2\,fx-2\,e)}}{16\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*d*x^2 - 1/4*b^2*d*x^2 + a^2*c*x - 1/2*b^2*c*x + 1/16*(2*b^2*d*f*x +
 2*b^2*c*f - b^2*d)*e^(2*f*x + 2*e)/f^2 + (a*b*d*f*x + a*b*c*f - a*b*d)*e^(
f*x + e)/f^2 + (a*b*d*f*x + a*b*c*f + a*b*d)*e^(-f*x - e)/f^2 - 1/16*(2*b^2
*d*f*x + 2*b^2*c*f + b^2*d)*e^(-2*f*x - 2*e)/f^2
```

Mupad [B]
time = 0.15, size = 135, normalized size = 1.16

$$\frac{a^2\,d\,x^2}{2}-\frac{b^2\,d\,x^2}{4}+a^2\,c\,x-\frac{b^2\,c\,x}{2}-\frac{b^2\,d\,{\cosh\left(e+f\,x\right)}^2}{4\,f^2}+\frac{b^2\,c\,\cosh\left(e+f\,x\right)\,\sinh\left(e+f\,x\right)}{2\,f}+\frac{2\,a\,b\,c\,\cosh\left(e+f\,x\right)}{f}-\frac{2\,a\,b\,d\,\sinh\left(e+f\,x\right)}{f^2}+\frac{2\,a\,b\,d\,x\,\cosh\left(e+f\,x\right)}{f}+\frac{b^2\,d\,x\,\cosh\left(e+f\,x\right)\,\sinh\left(e+f\,x\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x))^2*(c + d*x),x)
```

```
[Out] (a^2*d*x^2)/2 - (b^2*d*x^2)/4 + a^2*c*x - (b^2*c*x)/2 - (b^2*d*cosh(e + f*x
)^2)/(4*f^2) + (b^2*c*cosh(e + f*x)*sinh(e + f*x))/(2*f) + (2*a*b*c*cosh(e
+ f*x))/f - (2*a*b*d*sinh(e + f*x))/f^2 + (2*a*b*d*x*cosh(e + f*x))/f + (b^
2*d*x*cosh(e + f*x)*sinh(e + f*x))/(2*f)
```

3.166 $\int \frac{(a+b\sinh(e+fx))^2}{c+dx}\,dx$

Optimal. Leaf size=156

$$\frac{b^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{a^2\log(c+dx)}{d}-\frac{b^2\log(c+dx)}{2d}+\frac{2ab\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\,\text{co}}{}$$

```
[Out] 1/2*b^2*Chi(2*c*f/d+2*f*x)*cosh(-2*e+2*c*f/d)/d+a^2*ln(d*x+c)/d-1/2*b^2*ln(
d*x+c)/d+2*a*b*cosh(-e+c*f/d)*Shi(c*f/d+f*x)/d-1/2*b^2*Shi(2*c*f/d+2*f*x)*s
inh(-2*e+2*c*f/d)/d-2*a*b*Chi(c*f/d+f*x)*sinh(-e+c*f/d)/d
```

Rubi [A]
time = 0.25, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3398, 3384, 3379, 3382, 3393}

$$\frac{a^2\log(c+dx)}{d}+\frac{2ab\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}+\frac{b^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{2d}+\frac{b^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{2d}-\frac{b^2\log(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x])^2/(c + d*x),x]
```

```
[Out] (b^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(2*d) + (a^2*Lo
g[c + d*x])/d - (b^2*Log[c + d*x])/(2*d) + (2*a*b*CoshIntegral[(c*f)/d + f*
x]*Sinh[e - (c*f)/d])/d + (2*a*b*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f
*x])/d + (b^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2*d)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\sinh(e+fx))^2}{c+dx}\,dx &= \int \left(\frac{a^2}{c+dx}+\frac{2ab\sinh(e+fx)}{c+dx}+\frac{b^2\sinh^2(e+fx)}{c+dx}\right)dx \\
&= \frac{a^2\log(c+dx)}{d}+(2ab)\int \frac{\sinh(e+fx)}{c+dx}\,dx+b^2\int \frac{\sinh^2(e+fx)}{c+dx}\,dx \\
&= \frac{a^2\log(c+dx)}{d}-b^2\int \left(\frac{1}{2(c+dx)}-\frac{\cosh(2e+2fx)}{2(c+dx)}\right)dx+\left(2ab\cosh\left(e-\right.\right. \\
&= \frac{a^2\log(c+dx)}{d}-\frac{b^2\log(c+dx)}{2d}+\frac{2ab\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\cosh\left(\right.}{} \\
&= \frac{a^2\log(c+dx)}{d}-\frac{b^2\log(c+dx)}{2d}+\frac{2ab\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\cosh\left(\right.}{} \\
&= \frac{b^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{a^2\log(c+dx)}{d}-\frac{b^2\log(c+dx)}{2d}+\frac{2ab\text{Ch}}{}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 134, normalized size = 0.86

$$\frac{b^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2f(c+dx)}{d}\right)+2a^2\log(c+dx)-b^2\log(c+dx)+4ab\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+4ab\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+b^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2f(c+dx)}{d}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x])^2/(c + d*x),x]
```

```
[Out] (b^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*f*(c + d*x))/d] + 2*a^2*Log[c +
d*x] - b^2*Log[c + d*x] + 4*a*b*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d]
 + 4*a*b*Cosh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] + b^2*Sinh[2*e - (2*c*
f)/d]*SinhIntegral[(2*f*(c + d*x))/d])/(2*d)
```

Maple [A]
time = 8.27, size = 201, normalized size = 1.29

method	result
risch	$-\frac{ab\,\mathrm{e}^{-\frac{cf-de}{d}}\,\mathrm{expIntegral}\left(1,-fx-e-\frac{cf-de}{d}\right)}{d}+\frac{a^{2}\ln(dx+c)}{d}-\frac{b^{2}\ln(dx+c)}{2d}-\frac{b^{2}\mathrm{e}^{-\frac{2(cf-de)}{d}}\,\mathrm{expIntegral}\left(1,-2fx-2e-\frac{2(cf-d}{d}\right.}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e))^2/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -a*b/d*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)+a^2*ln(d*x+c)/d-1/2*b^2*l
n(d*x+c)/d-1/4*b^2/d*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)-1/4
*b^2/d*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)+a*b/d*exp((c*f-d*e)
/d)*Ei(1,f*x+e+(c*f-d*e)/d)
```

Maxima [A]
time = 0.33, size = 152, normalized size = 0.97

$$-\frac{1}{4}\,b^{2}\left(\frac{e^{\left(\frac{2\,cf}{d}-2\,e\right)}E_{1}\left(\frac{2\,(dx+c)f}{d}\right)}{d}+\frac{e^{\left(-\frac{2\,cf}{d}+2\,e\right)}E_{1}\left(-\frac{2\,(dx+c)f}{d}\right)}{d}+\frac{2\,\log\left(dx+c\right)}{d}\right)+ab\left(\frac{e^{\left(\frac{cf}{d}-e\right)}E_{1}\left(\frac{(dx+c)f}{d}\right)}{d}-\frac{e^{\left(-\frac{cf}{d}+e\right)}E_{1}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)+\frac{a^{2}\log\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(e^(2*c*f/d - 2*e)*exp_integral_e(1, 2*(d*x + c)*f/d)/d + e^(-2*c*
f/d + 2*e)*exp_integral_e(1, -2*(d*x + c)*f/d)/d + 2*log(d*x + c)/d) + a*b*
(e^(c*f/d - e)*exp_integral_e(1, (d*x + c)*f/d)/d - e^(-c*f/d + e)*exp_inte
gral_e(1, -(d*x + c)*f/d)/d) + a^2*log(d*x + c)/d
```

Fricas [A]
time = 0.36, size = 255, normalized size = 1.63

$$\frac{4\left(ab\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-ab\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)+\left(b^{2}\mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)+b^{2}\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\right)\cosh\left(-\frac{2\,(cf-d\cosh(1)-d\sinh(1))}{d}\right)+2\left(2\,a^{2}-b^{2}\right)\log\left(dx+c\right)+4\left(ab\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+ab\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\sinh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)+\left(b^{2}\mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)-b^{2}\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\right)\sinh\left(-\frac{2\,(cf-d\cosh(1)-d\sinh(1))}{d}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(4*(a*b*Ei((d*f*x + c*f)/d) - a*b*Ei(-(d*f*x + c*f)/d))*cosh(-(c*f - d*
cosh(1) - d*sinh(1))/d) + (b^2*Ei(2*(d*f*x + c*f)/d) + b^2*Ei(-2*(d*f*x + c
*f)/d))*cosh(-2*(c*f - d*cosh(1) - d*sinh(1))/d) + 2*(2*a^2 - b^2)*log(d*x
+ c) + 4*(a*b*Ei((d*f*x + c*f)/d) + a*b*Ei(-(d*f*x + c*f)/d))*sinh(-(c*f -
d*cosh(1) - d*sinh(1))/d) + (b^2*Ei(2*(d*f*x + c*f)/d) - b^2*Ei(-2*(d*f*x +
 c*f)/d))*sinh(-2*(c*f - d*cosh(1) - d*sinh(1))/d))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\sinh\left(e+fx\right)\right)^{2}}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))**2/(d*x+c),x)
```

```
[Out] Integral((a + b*sinh(e + f*x))**2/(c + d*x), x)
```

Giac [A]
time = 0.43, size = 144, normalized size = 0.92

$$\frac{b^2\mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)e^{\left(2\,e-\frac{2\,cf}{d}\right)}+4\,ab\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}-4\,ab\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+b^2\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(-2\,e+\frac{2\,cf}{d}\right)}+4\,a^2\log\left(dx+c\right)-2\,b^2\log\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(b^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 4*a*b*Ei((d*f*x + c*f)/d
)*e^(e - c*f/d) - 4*a*b*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + b^2*Ei(-2*(d*
f*x + c*f)/d)*e^(-2*e + 2*c*f/d) + 4*a^2*log(d*x + c) - 2*b^2*log(d*x + c))
/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{sinh}\left(e+f\,x\right)\right)}^2}{c+d\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x))^2/(c + d*x),x)
```

```
[Out] int((a + b*sinh(e + f*x))^2/(c + d*x), x)
```

3.167 $\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^2}\,dx$

Optimal. Leaf size=183

$$-\frac{a^2}{d(c+dx)}+\frac{2abf\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d^2}+\frac{b^2f\,\text{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2}-\frac{2ab\sinh(e+fx)}{d(c+dx)}-\frac{b^2}{}$$

```
[Out] -a^2/d/(d*x+c)+2*a*b*f*Chi(c*f/d+f*x)*cosh(-e+c*f/d)/d^2+b^2*f*cosh(-2*e+2*
c*f/d)*Shi(2*c*f/d+2*f*x)/d^2-b^2*f*Chi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/d
^2-2*a*b*f*Shi(c*f/d+f*x)*sinh(-e+c*f/d)/d^2-2*a*b*sinh(f*x+e)/d/(d*x+c)-b^
2*sinh(f*x+e)^2/d/(d*x+c)
```

Rubi [A]
time = 0.25, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3398, 3378, 3384, 3379, 3382, 3394, 12}

$$-\frac{a^2}{d(c+dx)}+\frac{2abf\,\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{2abf\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{2ab\sinh(e+fx)}{d(c+dx)}+\frac{b^2f\,\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2}+\frac{b^2f\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d^2}-\frac{b^2\sinh^2(e+fx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] -(a^2/(d*(c + d*x))) + (2*a*b*f*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*
x])/d^2 + (b^2*f*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/d^2
 - (2*a*b*Sinh[e + f*x])/(d*(c + d*x)) - (b^2*Sinh[e + f*x]^2)/(d*(c + d*x)
) + (2*a*b*f*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d^2 + (b^2*f*Co
sh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^2}\,dx = \int \left(\frac{a^2}{(c+dx)^2} + \frac{2ab\sinh(e+fx)}{(c+dx)^2} + \frac{b^2\sinh^2(e+fx)}{(c+dx)^2}\right)\,dx$$

$$= -\frac{a^2}{d(c+dx)} + (2ab)\int \frac{\sinh(e+fx)}{(c+dx)^2}\,dx + b^2\int \frac{\sinh^2(e+fx)}{(c+dx)^2}\,dx$$

$$= -\frac{a^2}{d(c+dx)} - \frac{2ab\sinh(e+fx)}{d(c+dx)} - \frac{b^2\sinh^2(e+fx)}{d(c+dx)} + \frac{(2abf)\int \frac{\cosh(e+fx)}{c+dx}\,dx}{d}$$

$$= -\frac{a^2}{d(c+dx)} - \frac{2ab\sinh(e+fx)}{d(c+dx)} - \frac{b^2\sinh^2(e+fx)}{d(c+dx)} + \frac{(b^2f)\int \frac{\sinh(2e+2fx)}{c+dx}\,dx}{d}$$

$$= -\frac{a^2}{d(c+dx)} + \frac{2abf\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d^2} - \frac{2ab\sinh(e+fx)}{d(c+dx)} - \frac{b^2\sinh}{d(}$$

$$= -\frac{a^2}{d(c+dx)} + \frac{2abf\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d^2} + \frac{b^2f\text{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(\right.}{d^2}$$

Mathematica [A]
time = 0.40, size = 232, normalized size = 1.27

$$\frac{-2a^2d+b^2d-b^2d\cosh(2(e+fx))+4abf(c+dx)\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)+2b^2f(c+dx)\text{Chi}\left(\frac{2f(c+dx)}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)-4abd\sinh(e+fx)+4abcf\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+4abdfx\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+2b^2cf\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2f(c+dx)}{d}\right)+2b^2dfx\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2f(c+dx)}{d}\right)}{2d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] (-2*a^2*d + b^2*d - b^2*d*Cosh[2*(e + f*x)] + 4*a*b*f*(c + d*x)*Cosh[e - (c
*f)/d]*CoshIntegral[f*(c/d + x)] + 2*b^2*f*(c + d*x)*CoshIntegral[(2*f*(c +
 d*x))/d]*Sinh[2*e - (2*c*f)/d] - 4*a*b*d*Sinh[e + f*x] + 4*a*b*c*f*Sinh[e
- (c*f)/d]*SinhIntegral[f*(c/d + x)] + 4*a*b*d*f*x*Sinh[e - (c*f)/d]*SinhIn
tegral[f*(c/d + x)] + 2*b^2*c*f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c
+ d*x))/d] + 2*b^2*d*f*x*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))
/d])/(2*d^2*(c + d*x))
```

Maple [A]
time = 6.30, size = 319, normalized size = 1.74

method	result
risch	$-\frac{fab\,\mathrm{e}^{fx+e}}{d^2\left(\frac{cf}{d}+fx\right)} - \frac{fab\,\mathrm{e}^{-\frac{cf-de}{d}}\,\text{expIntegral}\left(1,-fx-e-\frac{cf-de}{d}\right)}{d^2} - \frac{a^2}{d(dx+c)} + \frac{b^2}{2(dx+c)d} - \frac{f\,b^2\mathrm{e}^{-2fx-2e}}{4d(dxf+cf)} + \frac{f\,b^2\mathrm{e}^{\frac{2cf-2de}{d}}\,\mathrm{ex}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e))^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*f*a*b*exp(f*x+e)/(c*f/d+f*x)-1/d^2*f*a*b*exp(-(c*f-d*e)/d)*Ei(1,-f*x
-e-(c*f-d*e)/d)-a^2/d/(d*x+c)+1/2*b^2/(d*x+c)/d-1/4*f*b^2*exp(-2*f*x-2*e)/d
/(d*f*x+c*f)+1/2*f*b^2/d^2*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
-1/4*f*b^2/d^2*exp(2*f*x+2*e)/(c*f/d+f*x)-1/2*f*b^2/d^2*exp(-2*(c*f-d*e)/d)
*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)+f*a*b*exp(-f*x-e)/d/(d*f*x+c*f)-f*a*b/d^2*e
xp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)
```

Maxima [**A**]
time = 0.33, size = 185, normalized size = 1.01

$$-\frac{1}{4}b^2\left(\frac{e^{\left(\frac{2cf}{d}-2e\right)}E_2\left(\frac{2(dx+c)f}{d}\right)}{(dx+c)d}+\frac{e^{\left(-\frac{2cf}{d}+2e\right)}E_2\left(-\frac{2(dx+c)f}{d}\right)}{(dx+c)d}-\frac{2}{d^2x+cd}\right)+ab\left(\frac{e^{\left(\frac{cf}{d}-e\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d}-\frac{e^{\left(-\frac{cf}{d}+e\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right)-\frac{a^2}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(e^(2*c*f/d - 2*e)*exp_integral_e(2, 2*(d*x + c)*f/d)/((d*x + c)*d
) + e^(-2*c*f/d + 2*e)*exp_integral_e(2, -2*(d*x + c)*f/d)/((d*x + c)*d) -
2/(d^2*x + c*d)) + a*b*(e^(c*f/d - e)*exp_integral_e(2, (d*x + c)*f/d)/((d*
x + c)*d) - e^(-c*f/d + e)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d))
 - a^2/(d^2*x + c*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(192) = 384$.
time = 0.37, size = 542, normalized size = 2.96

$$-\frac{b^2d\cosh\left(fx+\cosh(1)+\sinh(1)\right)^2+4abd\sinh\left(fx+\cosh(1)+\sinh(1)\right)+\left(b^2d+(b^2dfx+b^2cf)\mathrm{Ei}\left(\frac{2(dfx+cf)}{d}\right)\cosh\left(-\frac{2(cf-d\cosh(1)-d\sinh(1))}{d}\right)\right)\sinh\left(fx+\cosh(1)+\sinh(1)\right)^2+(2a^2-b^2)d-2\left((abdfx+abcf)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+(abdfx+abcf)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)-\left((b^2dfx+b^2cf)\mathrm{Ei}\left(\frac{2(dfx+cf)}{d}\right)\cosh\left(fx+\cosh(1)+\sinh(1)\right)^2-(b^2dfx+b^2cf)\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)\right)\cosh\left(-\frac{2(cf-d\cosh(1)-d\sinh(1))}{d}\right)-2\left((abdfx+abcf)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-(abdfx+abcf)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\sinh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)-\left((b^2dfx+b^2cf)\mathrm{Ei}\left(\frac{2(dfx+cf)}{d}\right)\cosh\left(fx+\cosh(1)+\sinh(1)\right)^2-(b^2dfx+b^2cf)\mathrm{Ei}\left(\frac{2(dfx+cf)}{d}\right)\sinh\left(fx+\cosh(1)+\sinh(1)\right)^2+(b^2dfx+b^2cf)\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)\right)\sinh\left(-\frac{2(cf-d\cosh(1)-d\sinh(1))}{d}\right)}{2\left((d^3x+cd^2)\cosh\left(fx+\cosh(1)+\sinh(1)\right)^2-(d^3x+cd^2)\sinh\left(fx+\cosh(1)+\sinh(1)\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*d*cosh(f*x + cosh(1) + sinh(1))^2 + 4*a*b*d*sinh(f*x + cosh(1) +
sinh(1)) + (b^2*d + (b^2*d*f*x + b^2*c*f)*Ei(2*(d*f*x + c*f)/d)*cosh(-2*(c*
f - d*cosh(1) - d*sinh(1))/d))*sinh(f*x + cosh(1) + sinh(1))^2 + (2*a^2 - b
^2)*d - 2*((a*b*d*f*x + a*b*c*f)*Ei((d*f*x + c*f)/d) + (a*b*d*f*x + a*b*c*f
)*Ei(-(d*f*x + c*f)/d))*cosh(-(c*f - d*cosh(1) - d*sinh(1))/d) - ((b^2*d*f*
x + b^2*c*f)*Ei(2*(d*f*x + c*f)/d)*cosh(f*x + cosh(1) + sinh(1))^2 - (b^2*d
*f*x + b^2*c*f)*Ei(-2*(d*f*x + c*f)/d))*cosh(-2*(c*f - d*cosh(1) - d*sinh(1
))/d) - 2*((a*b*d*f*x + a*b*c*f)*Ei((d*f*x + c*f)/d) - (a*b*d*f*x + a*b*c*f
)*Ei(-(d*f*x + c*f)/d))*sinh(-(c*f - d*cosh(1) - d*sinh(1))/d) - ((b^2*d*f*
x + b^2*c*f)*Ei(2*(d*f*x + c*f)/d)*cosh(f*x + cosh(1) + sinh(1))^2 - (b^2*d
*f*x + b^2*c*f)*Ei(2*(d*f*x + c*f)/d)*sinh(f*x + cosh(1) + sinh(1))^2 + (b^
2*d*f*x + b^2*c*f)*Ei(-2*(d*f*x + c*f)/d))*sinh(-2*(c*f - d*cosh(1) - d*sin
h(1))/d))/((d^3*x + c*d^2)*cosh(f*x + cosh(1) + sinh(1))^2 - (d^3*x + c*d^2
)*sinh(f*x + cosh(1) + sinh(1))^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sinh(e + fx))^2}{(c + dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))**2/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sinh(e + f*x))**2/(c + d*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(186) = 372$.
time = 0.49, size = 1135, normalized size = 6.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*(d*x + c)*b^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(2*((d*x + c
)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(2*(d*e - c*f)/d) -
 2*b^2*d*e*f^2*Ei(2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e +
c*f)/d)*e^(2*(d*e - c*f)/d) + 2*b^2*c*f^3*Ei(2*((d*x + c)*(d*e/(d*x + c) -
c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(2*(d*e - c*f)/d) + 4*(d*x + c)*a*b*(d
*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d
*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) - 4*a*b*d*e*f^2*Ei(((d*x + c
)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) + 4
*a*b*c*f^3*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d
)*e^((d*e - c*f)/d) + 4*(d*x + c)*a*b*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f
^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(
d*e - c*f)/d) - 4*a*b*d*e*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c)
 + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) + 4*a*b*c*f^3*Ei(-((d*x + c)*(d*e/
(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) - 2*(d*x
+ c)*b^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-2*((d*x + c)*(d*e/(d*x
 + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) + 2*b^2*d*e
*f^2*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e
^(-2*(d*e - c*f)/d) - 2*b^2*c*f^3*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*
x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) - b^2*d*f^2*e^(2*(d*x + c)
*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d) - 4*a*b*d*f^2*e^((d*x + c)*(d*e/(d*
x + c) - c*f/(d*x + c) + f)/d) + 4*a*b*d*f^2*e^(-(d*x + c)*(d*e/(d*x + c) -
 c*f/(d*x + c) + f)/d) - b^2*d*f^2*e^(-2*(d*x + c)*(d*e/(d*x + c) - c*f/(d*
x + c) + f)/d) - 4*a^2*d*f^2 + 2*b^2*d*f^2)*d^2/(((d*x + c)*d^4*(d*e/(d*x +
 c) - c*f/(d*x + c) + f) - d^5*e + c*d^4*f)*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\sinh(e+f\,x))^2}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x))^2/(c + d*x)^2,x)
```

```
[Out] int((a + b*sinh(e + f*x))^2/(c + d*x)^2, x)
```

3.168 $\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^3}\,dx$

Optimal. Leaf size=242

$$-\frac{a^2}{2d(c+dx)^2}-\frac{abf\cosh(e+fx)}{d^2(c+dx)}+\frac{b^2f^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{d^3}+\frac{abf^2\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d^3}-$$

```
[Out] -1/2*a^2/d/(d*x+c)^2+b^2*f^2*Chi(2*c*f/d+2*f*x)*cosh(-2*e+2*c*f/d)/d^3-a*b*
f*cosh(f*x+e)/d^2/(d*x+c)+a*b*f^2*cosh(-e+c*f/d)*Shi(c*f/d+f*x)/d^3-b^2*f^2
*Shi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/d^3-a*b*f^2*Chi(c*f/d+f*x)*sinh(-e+c
*f/d)/d^3-a*b*sinh(f*x+e)/d/(d*x+c)^2-b^2*f*cosh(f*x+e)*sinh(f*x+e)/d^2/(d*
x+c)-1/2*b^2*sinh(f*x+e)^2/d/(d*x+c)^2
```

Rubi [A]
time = 0.32, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3398, 3378, 3384, 3379, 3382, 3395, 31, 3393}

$$-\frac{a^2}{2d(c+dx)^2}+\frac{abf^2\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d^3}+\frac{abf^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^3}-\frac{abf\cosh(e+fx)}{d^2(c+dx)}-\frac{ab\sinh(e+fx)}{d(c+dx)^2}+\frac{b^2f^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{d^3}+\frac{b^2f^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d^3}-\frac{b^2f\sinh(e+fx)\cosh(e+fx)}{d^2(c+dx)}-\frac{b^2\sinh^2(e+fx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x])^2/(c + d*x)^3,x]
```

```
[Out] -1/2*a^2/(d*(c + d*x)^2) - (a*b*f*Cosh[e + f*x])/(d^2*(c + d*x)) + (b^2*f^2
*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/d^3 + (a*b*f^2*Cosh
Integral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d^3 - (a*b*Sinh[e + f*x])/(d*(c
+ d*x)^2) - (b^2*f*Cosh[e + f*x]*Sinh[e + f*x])/(d^2*(c + d*x)) - (b^2*Sinh
[e + f*x]^2)/(2*d*(c + d*x)^2) + (a*b*f^2*Cosh[e - (c*f)/d]*SinhIntegral[(c
*f)/d + f*x])/d^3 + (b^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d +
 2*f*x])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^3}\,dx = \int \left(\frac{a^2}{(c+dx)^3}+\frac{2ab\sinh(e+fx)}{(c+dx)^3}+\frac{b^2\sinh^2(e+fx)}{(c+dx)^3}\right)\,dx$$

$$= -\frac{a^2}{2d(c+dx)^2}+(2ab)\int \frac{\sinh(e+fx)}{(c+dx)^3}\,dx+b^2\int \frac{\sinh^2(e+fx)}{(c+dx)^3}\,dx$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{ab\sinh(e+fx)}{d(c+dx)^2}-\frac{b^2f\cosh(e+fx)\sinh(e+fx)}{d^2(c+dx)}-\frac{b^2\sinh^2}{2d(c}$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{abf\cosh(e+fx)}{d^2(c+dx)}+\frac{b^2f^2\log(c+dx)}{d^3}-\frac{ab\sinh(e+fx)}{d(c+dx)^2}-$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{abf\cosh(e+fx)}{d^2(c+dx)}-\frac{ab\sinh(e+fx)}{d(c+dx)^2}-\frac{b^2f\cosh(e+fx)\sin}{d^2(c+dx)}$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{abf\cosh(e+fx)}{d^2(c+dx)}+\frac{abf^2\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d^3}-\frac{ab\,\text{s}}{d}$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{abf\cosh(e+fx)}{d^2(c+dx)}+\frac{b^2f^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{d^3}+$$

Mathematica [A]
time = 0.57, size = 395, normalized size = 1.63

$$\frac{-2a^2d^2+b^2d^2-4abcdf\cosh(e+fx)-4abd^2fx\cosh(e+fx)-b^2d^2\cosh(2(e+fx))+4b^2f^2(c+dx)^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2f(c+dx)}{d}\right)+4abf^2(c+dx)^2\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)-4abd^2\sinh(e+fx)-2b^2cdf\sinh(2(e+fx))-2b^2d^2fx\sinh(2(e+fx))+4abc^2f^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+8abcdf^2x\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+4abd^2f^2x^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+4b^2c^2f^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2f(c+dx)}{d}\right)+8b^2cdf^2x\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2f(c+dx)}{d}\right)+4b^2d^2f^2x^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2f(c+dx)}{d}\right)}{4d^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x])^2/(c + d*x)^3,x]
```

```
[Out] (-2*a^2*d^2 + b^2*d^2 - 4*a*b*c*d*f*Cosh[e + f*x] - 4*a*b*d^2*f*x*Cosh[e +
f*x] - b^2*d^2*Cosh[2*(e + f*x)] + 4*b^2*f^2*(c + d*x)^2*Cosh[2*e - (2*c*f)
/d]*CoshIntegral[(2*f*(c + d*x))/d] + 4*a*b*f^2*(c + d*x)^2*CoshIntegral[f*
(c/d + x)]*Sinh[e - (c*f)/d] - 4*a*b*d^2*Sinh[e + f*x] - 2*b^2*c*d*f*Sinh[2
*(e + f*x)] - 2*b^2*d^2*f*x*Sinh[2*(e + f*x)] + 4*a*b*c^2*f^2*Cosh[e - (c*f
)/d]*SinhIntegral[f*(c/d + x)] + 8*a*b*c*d*f^2*x*Cosh[e - (c*f)/d]*SinhInte
gral[f*(c/d + x)] + 4*a*b*d^2*f^2*x^2*Cosh[e - (c*f)/d]*SinhIntegral[f*(c/d
 + x)] + 4*b^2*c^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d
] + 8*b^2*c*d*f^2*x*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d] +
 4*b^2*d^2*f^2*x^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d])/(
4*d^3*(c + d*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(242) = 484$.
time = 7.54, size = 626, normalized size = 2.59

method	result
risch	$-\frac{ba\,f^2 \mathrm{e}^{fx+e}}{2d^3\left(\frac{cf}{d}+fx\right)^2}-\frac{ba\,f^2 \mathrm{e}^{fx+e}}{2d^3\left(\frac{cf}{d}+fx\right)}-\frac{ba\,f^2 \mathrm{e}^{-\frac{cf-de}{d}}\,\mathrm{expIntegral}\left(1,-fx-e-\frac{cf-de}{d}\right)}{2d^3}-\frac{a^2}{2d(dx+c)^2}+\frac{b^2}{4(dx+c)^2 d}+\frac{f^3 b^2 \mathrm{e}}{4d(d^2x^2f^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e))^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/d^3*a*f^2*exp(f*x+e)/(c*f/d+f*x)^2-1/2*b/d^3*a*f^2*exp(f*x+e)/(c*f/d
+f*x)-1/2*b/d^3*a*f^2*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)-1/2*a^2/d/
(d*x+c)^2+1/4*b^2/(d*x+c)^2/d+1/4*f^3*b^2*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*
c*d*f^2*x+c^2*f^2)*x+1/4*f^3*b^2*exp(-2*f*x-2*e)/d^2/(d^2*f^2*x^2+2*c*d*f^2
*x+c^2*f^2)*c-1/8*f^2*b^2*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^
2)-1/2*f^2*b^2/d^3*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)-1/8*f^2
*b^2/d^3*exp(2*f*x+2*e)/(c*f/d+f*x)^2-1/4*f^2*b^2/d^3*exp(2*f*x+2*e)/(c*f/d
+f*x)-1/2*f^2*b^2/d^3*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)-1/
2*f^3*a*b*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x-1/2*f^3*a*b*exp
(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c+1/2*f^2*a*b*exp(-f*x-e)/d/
(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)+1/2*f^2*a*b/d^3*exp((c*f-d*e)/d)*Ei(1,f*x
+e+(c*f-d*e)/d)
```

Maxima [A]
time = 0.36, size = 207, normalized size = 0.86

$$\frac{1}{4}b^2\left(\frac{1}{d^3x^2+2\,cd^2x+c^2d}-\frac{e^{\left(\frac{2\,cf}{d}-2\,e\right)}E_3\left(\frac{2\,(dx+c)f}{d}\right)}{(dx+c)^2 d}-\frac{e^{\left(-\frac{2\,cf}{d}+2\,e\right)}E_3\left(-\frac{2\,(dx+c)f}{d}\right)}{(dx+c)^2 d}\right)+ab\left(\frac{e^{\left(\frac{cf}{d}-e\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2 d}-\frac{e^{\left(-\frac{cf}{d}+e\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2 d}\right)-\frac{a^2}{2\,(d^3x^2+2\,cd^2x+c^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*(1/(d^3*x^2 + 2*c*d^2*x + c^2*d) - e^(2*c*f/d - 2*e)*exp_integral_e
(3, 2*(d*x + c)*f/d)/((d*x + c)^2*d) - e^(-2*c*f/d + 2*e)*exp_integral_e(3,
 -2*(d*x + c)*f/d)/((d*x + c)^2*d)) + a*b*(e^(c*f/d - e)*exp_integral_e(3,
(d*x + c)*f/d)/((d*x + c)^2*d) - e^(-c*f/d + e)*exp_integral_e(3, -(d*x + c
)*f/d)/((d*x + c)^2*d)) - 1/2*a^2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 834 vs. $2(251) = 502$.
time = 0.37, size = 834, normalized size = 3.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*d^2*cosh(f*x + cosh(1) + sinh(1))^2 + (2*a^2 - b^2)*d^2 + (b^2*d^
2 + 2*(b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*Ei(2*(d*f*x + c*f)/
d)*cosh(-2*(c*f - d*cosh(1) - d*sinh(1))/d))*sinh(f*x + cosh(1) + sinh(1))^
2 + 4*(a*b*d^2*f*x + a*b*c*d*f)*cosh(f*x + cosh(1) + sinh(1)) - 2*((a*b*d^2
*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*Ei((d*f*x + c*f)/d) - (a*b*d^2*f^
2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*cosh(-(c*f - d
*cosh(1) - d*sinh(1))/d) - 2*((b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*
f^2)*Ei(2*(d*f*x + c*f)/d)*cosh(f*x + cosh(1) + sinh(1))^2 + (b^2*d^2*f^2*x
^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d))*cosh(-2*(c*f -
d*cosh(1) - d*sinh(1))/d) + 4*(a*b*d^2 + (b^2*d^2*f*x + b^2*c*d*f)*cosh(f*x
 + cosh(1) + sinh(1)))*sinh(f*x + cosh(1) + sinh(1)) - 2*((a*b*d^2*f^2*x^2
+ 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*Ei((d*f*x + c*f)/d) + (a*b*d^2*f^2*x^2 + 2
*a*b*c*d*f^2*x + a*b*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*sinh(-(c*f - d*cosh(1)
- d*sinh(1))/d) - 2*((b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*Ei(2
*(d*f*x + c*f)/d)*cosh(f*x + cosh(1) + sinh(1))^2 - (b^2*d^2*f^2*x^2 + 2*b^
2*c*d*f^2*x + b^2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d)*sinh(f*x + cosh(1) + sinh(
1))^2 - (b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*Ei(-2*(d*f*x + c*
f)/d))*sinh(-2*(c*f - d*cosh(1) - d*sinh(1))/d))/((d^5*x^2 + 2*c*d^4*x + c^
2*d^3)*cosh(f*x + cosh(1) + sinh(1))^2 - (d^5*x^2 + 2*c*d^4*x + c^2*d^3)*si
nh(f*x + cosh(1) + sinh(1))^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sinh(e + fx))^2}{(c + dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))**2/(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sinh(e + f*x))**2/(c + d*x)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 678 vs. 2(242) = 484.
time = 0.44, size = 678, normalized size = 2.80

$$\frac{4 b^2 d^2 f^2 x^2 \operatorname{Ei}\left(\frac{2(dfx+cf)}{d}\right) e^{\left(2e-\frac{2cf}{d}\right)} + 4 abd^2 f^2 x^2 \operatorname{Ei}\left(\frac{dfx+cf}{d}\right) e^{\left(e-\frac{cf}{d}\right)} - 4 abd^2 f^2 x^2 \operatorname{Ei}\left(-\frac{dfx+cf}{d}\right) e^{\left(-e+\frac{cf}{d}\right)} + 4 b^2 d^2 f^2 x^2 \operatorname{Ei}\left(-\frac{2(dfx+cf)}{d}\right) e^{\left(-2e+\frac{2cf}{d}\right)} + 8 b^2 cdf^2 x \operatorname{Ei}\left(\frac{2(dfx+cf)}{d}\right) e^{\left(2e-\frac{2cf}{d}\right)} + 8 abcdf^2 x \operatorname{Ei}\left(\frac{dfx+cf}{d}\right) e^{\left(e-\frac{cf}{d}\right)} - 8 abcdf^2 x \operatorname{Ei}\left(-\frac{dfx+cf}{d}\right) e^{\left(-e+\frac{cf}{d}\right)} + 8 b^2 cdf^2 x \operatorname{Ei}\left(-\frac{2(dfx+cf)}{d}\right) e^{\left(-2e+\frac{2cf}{d}\right)} + 4 b^2 c^2 f^2 \operatorname{Ei}\left(\frac{2(dfx+cf)}{d}\right) e^{\left(2e-\frac{2cf}{d}\right)} + 4 abc^2 f^2 \operatorname{Ei}\left(\frac{dfx+cf}{d}\right) e^{\left(e-\frac{cf}{d}\right)} - 4 abc^2 f^2 \operatorname{Ei}\left(-\frac{dfx+cf}{d}\right) e^{\left(-e+\frac{cf}{d}\right)} + 4 b^2 c^2 f^2 \operatorname{Ei}\left(-\frac{2(dfx+cf)}{d}\right) e^{\left(-2e+\frac{2cf}{d}\right)} - 2 b^2 d^2 fx e^{(2fx+2e)} - 4 abd^2 fx e^{(fx+e)} - 4 abd^2 fx e^{(-fx-e)} + 2 b^2 d^2 fx e^{(-2fx-2e)} - 2 b^2 cdf e^{(2fx+2e)} - 4 abcdf e^{(fx+e)} - 4 abcdf e^{(-fx-e)} + 2 b^2 cdf e^{(-2fx-2e)} - b^2 d^2 e^{(2fx+2e)} - 4 abd^2 e^{(fx+e)} + 4 abd^2 e^{(-fx-e)} - b^2 d^2 e^{(-2fx-2e)} - 4 a^2 d^2 + 2 b^2 d^2}{8\,(d^5 x^2 + 2 cd^4 x + c^2 d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*b^2*d^2*f^2*x^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 4*a*b*d^2*
f^2*x^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) - 4*a*b*d^2*f^2*x^2*Ei(-(d*f*x +
c*f)/d)*e^(-e + c*f/d) + 4*b^2*d^2*f^2*x^2*Ei(-2*(d*f*x + c*f)/d)*e^(-2*e +
 2*c*f/d) + 8*b^2*c*d*f^2*x*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 8*a*b
*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) - 8*a*b*c*d*f^2*x*Ei(-(d*f*x +
```

```
 c*f)/d)*e^(-e + c*f/d) + 8*b^2*c*d*f^2*x*Ei(-2*(d*f*x + c*f)/d)*e^(-2*e +
2*c*f/d) + 4*b^2*c^2*f^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 4*a*b*c^
2*f^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) - 4*a*b*c^2*f^2*Ei(-(d*f*x + c*f)/d
)*e^(-e + c*f/d) + 4*b^2*c^2*f^2*Ei(-2*(d*f*x + c*f)/d)*e^(-2*e + 2*c*f/d)
- 2*b^2*d^2*f*x*e^(2*f*x + 2*e) - 4*a*b*d^2*f*x*e^(f*x + e) - 4*a*b*d^2*f*x
*e^(-f*x - e) + 2*b^2*d^2*f*x*e^(-2*f*x - 2*e) - 2*b^2*c*d*f*e^(2*f*x + 2*e
) - 4*a*b*c*d*f*e^(f*x + e) - 4*a*b*c*d*f*e^(-f*x - e) + 2*b^2*c*d*f*e^(-2*
f*x - 2*e) - b^2*d^2*e^(2*f*x + 2*e) - 4*a*b*d^2*e^(f*x + e) + 4*a*b*d^2*e^
(-f*x - e) - b^2*d^2*e^(-2*f*x - 2*e) - 4*a^2*d^2 + 2*b^2*d^2)/(d^5*x^2 + 2
*c*d^4*x + c^2*d^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\sinh(e+f\,x))^2}{(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x))^2/(c + d*x)^3,x)
```

```
[Out] int((a + b*sinh(e + f*x))^2/(c + d*x)^3, x)
```

3.169 $\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx$

Optimal. Leaf size=404

$$\frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f}-\frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f}+\frac{3d(c+dx)^2\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+}}\right.}{\sqrt{a^2+b^2}\,f^2}$$

```
[Out] (d*x+c)^3*ln(1+b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/f/(a^2+b^2)^(1/2)-(d*x+c)^
3*ln(1+b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/f/(a^2+b^2)^(1/2)+3*d*(d*x+c)^2*po
lylog(2,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/f^2/(a^2+b^2)^(1/2)-3*d*(d*x+c)^
2*polylog(2,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/f^2/(a^2+b^2)^(1/2)-6*d^2*(d
*x+c)*polylog(3,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/f^3/(a^2+b^2)^(1/2)+6*d^
2*(d*x+c)*polylog(3,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/f^3/(a^2+b^2)^(1/2)+
6*d^3*polylog(4,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/f^4/(a^2+b^2)^(1/2)-6*d^
3*polylog(4,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/f^4/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3403, 2296, 2221, 2611, 6744, 2320, 6724}

$$-\frac{6d^2(c+dx)\text{Li}_3\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^3\sqrt{a^2+b^2}}+\frac{6d^2(c+dx)\text{Li}_3\left(-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f^3\sqrt{a^2+b^2}}+\frac{3d(c+dx)^2\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2\sqrt{a^2+b^2}}-\frac{3d(c+dx)^2\text{Li}_2\left(-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f^2\sqrt{a^2+b^2}}+\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{f\sqrt{a^2+b^2}}-\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{f\sqrt{a^2+b^2}}+\frac{6d^3\text{Li}_4\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^4\sqrt{a^2+b^2}}-\frac{6d^3\text{Li}_4\left(-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f^4\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*Sinh[e + f*x]),x]
```

```
[Out] ((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^
2]*f) - ((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/(Sqrt[
a^2 + b^2]*f) + (3*d*(c + d*x)^2*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2
 + b^2]))])/(Sqrt[a^2 + b^2]*f^2) - (3*d*(c + d*x)^2*PolyLog[2, -((b*E^(e +
 f*x))/(a + Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^2) - (6*d^2*(c + d*x)*Po
lyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^3) +
 (6*d^2*(c + d*x)*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/(Sq
rt[a^2 + b^2]*f^3) + (6*d^3*PolyLog[4, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^
2]))])/(Sqrt[a^2 + b^2]*f^4) - (6*d^3*PolyLog[4, -((b*E^(e + f*x))/(a + Sqr
t[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx = 2\int \frac{e^{e+fx}(c+dx)^3}{-b+2ae^{e+fx}+be^{2(e+fx)}}\,dx$$

$$= \frac{(2b)\int \frac{e^{e+fx}(c+dx)^3}{2a-2\sqrt{a^2+b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2+b^2}} - \frac{(2b)\int \frac{e^{e+fx}(c+dx)^3}{2a+2\sqrt{a^2+b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2+b^2}}$$

$$= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} - \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} - \frac{(3d)\int}{}$$

$$= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} - \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} + \frac{3d(c-}{}$$

$$= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} - \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} + \frac{3d(c-}{}$$

$$= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} - \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} + \frac{3d(c-}{}$$

$$= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} - \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} + \frac{3d(c-}{}$$

Mathematica [A]
time = 0.18, size = 318, normalized size = 0.79

$$\frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)-(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)+\frac{3d\left(f^2(c+dx)^2\text{PolyLog}\left(2,\frac{be^{e+fx}}{-a+\sqrt{a^2+b^2}}\right)-2df(c+dx)\text{PolyLog}\left(3,\frac{be^{e+fx}}{-a+\sqrt{a^2+b^2}}\right)+2d^2\text{PolyLog}\left(4,\frac{be^{e+fx}}{-a+\sqrt{a^2+b^2}}\right)\right)}{f^3}-\frac{3d\left(f^2(c+dx)^2\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)-2df(c+dx)\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)+2d^2\text{PolyLog}\left(4,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)\right)}{f^3}}{\sqrt{a^2+b^2}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*Sinh[e + f*x]),x]
```

```
[Out] ((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])] - (c + d*x)^3*L
og[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])] + (3*d*(f^2*(c + d*x)^2*PolyL
og[2, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*f*(c + d*x)*PolyLog[3,
(b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])] + 2*d^2*PolyLog[4, (b*E^(e + f*x))/
(-a + Sqrt[a^2 + b^2])]))/f^3 - (3*d*(f^2*(c + d*x)^2*PolyLog[2, -((b*E^(e
+ f*x))/(a + Sqrt[a^2 + b^2]))] - 2*d*f*(c + d*x)*PolyLog[3, -((b*E^(e + f*
x))/(a + Sqrt[a^2 + b^2]))] + 2*d^2*PolyLog[4, -((b*E^(e + f*x))/(a + Sqrt[
a^2 + b^2]))]))/f^3)/(Sqrt[a^2 + b^2]*f)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^3}{a+b\sinh(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*sinh(f*x+e)),x)
```

```
[Out] int((d*x+c)^3/(a+b*sinh(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] c^3*log((b*e^(-f*x - e) - a - sqrt(a^2 + b^2))/(b*e^(-f*x - e) - a + sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*f) + integrate(2*d^3*x^3/(b*(e^(f*x + e) - e^(
-f*x - e)) + 2*a) + 6*c*d^2*x^2/(b*(e^(f*x + e) - e^(-f*x - e)) + 2*a) + 6*
c^2*d*x/(b*(e^(f*x + e) - e^(-f*x - e)) + 2*a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1382 vs. 2(370) = 740.
time = 0.36, size = 1382, normalized size = 3.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] (6*b*d^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(f*x + cosh(1) + sinh(1))
+ a*sinh(f*x + cosh(1) + sinh(1)) + (b*cosh(f*x + cosh(1) + sinh(1)) + b*si
nh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b^2))/b) - 6*b*d^3*sqrt((a^2
+ b^2)/b^2)*polylog(4, (a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh
(1) + sinh(1)) - (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) +
sinh(1)))*sqrt((a^2 + b^2)/b^2))/b) + 3*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x +
b*c^2*d*f^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + cosh(1) + sinh(1)) +
 a*sinh(f*x + cosh(1) + sinh(1)) + (b*cosh(f*x + cosh(1) + sinh(1)) + b*sin
h(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(b*d^3*f^
2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(
f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) - (b*cosh(f*x +
cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b^2)
```

```
 - b)/b + 1) - (b*c^3*f^3 - 3*b*c^2*d*f^2*cosh(1) + 3*b*c*d^2*f*cosh(1)^2 -
 b*d^3*cosh(1)^3 - b*d^3*sinh(1)^3 + 3*(b*c*d^2*f - b*d^3*cosh(1))*sinh(1)^
2 - 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(1) + b*d^3*cosh(1)^2)*sinh(1))*sqrt((
a^2 + b^2)/b^2)*log(2*b*cosh(f*x + cosh(1) + sinh(1)) + 2*b*sinh(f*x + cosh
(1) + sinh(1)) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b*c^3*f^3 - 3*b*c^2*d*
f^2*cosh(1) + 3*b*c*d^2*f*cosh(1)^2 - b*d^3*cosh(1)^3 - b*d^3*sinh(1)^3 + 3
*(b*c*d^2*f - b*d^3*cosh(1))*sinh(1)^2 - 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(
1) + b*d^3*cosh(1)^2)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(f*x + cos
h(1) + sinh(1)) + 2*b*sinh(f*x + cosh(1) + sinh(1)) - 2*b*sqrt((a^2 + b^2)/
b^2) + 2*a) + (b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + 3*b*c^
2*d*f^2*cosh(1) - 3*b*c*d^2*f*cosh(1)^2 + b*d^3*cosh(1)^3 + b*d^3*sinh(1)^3
 - 3*(b*c*d^2*f - b*d^3*cosh(1))*sinh(1)^2 + 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*c
osh(1) + b*d^3*cosh(1)^2)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(f*x +
 cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) + (b*cosh(f*x + cosh(
1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b^2) - b)
/b) - (b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + 3*b*c^2*d*f^2*
cosh(1) - 3*b*c*d^2*f*cosh(1)^2 + b*d^3*cosh(1)^3 + b*d^3*sinh(1)^3 - 3*(b*
c*d^2*f - b*d^3*cosh(1))*sinh(1)^2 + 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(1) +
 b*d^3*cosh(1)^2)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(f*x + cosh(1)
 + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) - (b*cosh(f*x + cosh(1) + sin
h(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b^2) - b)/b) - 6*
(b*d^3*f*x + b*c*d^2*f)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(f*x + cosh
(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) + (b*cosh(f*x + cosh(1) +
sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b^2))/b) + 6*(
b*d^3*f*x + b*c*d^2*f)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(f*x + cosh(
1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) - (b*cosh(f*x + cosh(1) + s
inh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b^2))/b))/((a^2
 + b^2)*f^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx)^3}{a + b\sinh{(e + fx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*sinh(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**3/(a + b*sinh(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*sinh(f*x+e)),x, algorithm="giac")
[Out] integrate((d*x + c)^3/(b*sinh(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^3}{a+b\sinh\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*sinh(e + f*x)),x)
[Out] int((c + d*x)^3/(a + b*sinh(e + f*x)), x)
```

3.170 $\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx$

Optimal. Leaf size=296

$$\frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f}-\frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f}+\frac{2d(c+dx)\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b}}\right.}{\sqrt{a^2+b^2}\,f^2}$$

```
[Out] (d*x+c)^2*ln(1+b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/f/(a^2+b^2)^(1/2)-(d*x+c)^
2*ln(1+b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/f/(a^2+b^2)^(1/2)+2*d*(d*x+c)*poly
log(2,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/f^2/(a^2+b^2)^(1/2)-2*d*(d*x+c)*po
lylog(2,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/f^2/(a^2+b^2)^(1/2)-2*d^2*polylo
g(3,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/f^3/(a^2+b^2)^(1/2)+2*d^2*polylog(3,
-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/f^3/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.47, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3403, 2296, 2221, 2611, 2320, 6724}

$$\frac{2d(c+dx)\mathrm{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2\sqrt{a^2+b^2}}-\frac{2d(c+dx)\mathrm{Li}_2\left(-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f^2\sqrt{a^2+b^2}}+\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{f\sqrt{a^2+b^2}}-\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{f\sqrt{a^2+b^2}}-\frac{2d^2\mathrm{Li}_3\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^3\sqrt{a^2+b^2}}+\frac{2d^2\mathrm{Li}_3\left(-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f^3\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*Sinh[e + f*x]),x]
```

```
[Out] ((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^
2]*f) - ((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/(Sqrt[
a^2 + b^2]*f) + (2*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 +
 b^2]))])/(Sqrt[a^2 + b^2]*f^2) - (2*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x
))/(a + Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^2) - (2*d^2*PolyLog[3, -((b*
E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^3) + (2*d^2*PolyLo
g[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx = 2\int \frac{e^{e+fx}(c+dx)^2}{-b+2ae^{e+fx}+be^{2(e+fx)}}\,dx$$

$$= \frac{(2b)\int \frac{e^{e+fx}(c+dx)^2}{2a-2\sqrt{a^2+b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2+b^2}} - \frac{(2b)\int \frac{e^{e+fx}(c+dx)^2}{2a+2\sqrt{a^2+b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2+b^2}}$$

$$= \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} - \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} - \frac{(2d)\int}{}$$

$$= \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} - \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} + \frac{2d(c-}{}$$

$$= \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} - \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} + \frac{2d(c-}{}$$

$$= \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} - \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} + \frac{2d(c-}{}$$

Mathematica [A]
time = 0.11, size = 233, normalized size = 0.79

$$\frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)-(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)+\frac{2d\left(f(c+dx)\text{PolyLog}\left(2,\frac{be^{e+fx}}{-a+\sqrt{a^2+b^2}}\right)-d\text{PolyLog}\left(3,\frac{be^{e+fx}}{-a+\sqrt{a^2+b^2}}\right)\right)}{f^2}-\frac{2d\left(f(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)-d\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)\right)}{f^2}}{\sqrt{a^2+b^2}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*Sinh[e + f*x]),x]
```

```
[Out] ((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])] - (c + d*x)^2*L
og[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])] + (2*d*(f*(c + d*x)*PolyLog[2
, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])] - d*PolyLog[3, (b*E^(e + f*x))/(-
a + Sqrt[a^2 + b^2])]))/f^2 - (2*d*(f*(c + d*x)*PolyLog[2, -((b*E^(e + f*x)
)/(a + Sqrt[a^2 + b^2]))] - d*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 +
b^2]))]))/f^2)/(Sqrt[a^2 + b^2]*f)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{a+b\sinh(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*sinh(f*x+e)),x)
```

```
[Out] int((d*x+c)^2/(a+b*sinh(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

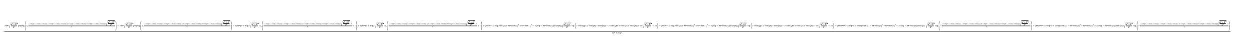
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] c^2*log((b*e^(-f*x - e) - a - sqrt(a^2 + b^2))/(b*e^(-f*x - e) - a + sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*f) + integrate(2*d^2*x^2/(b*(e^(f*x + e) - e^(
-f*x - e)) + 2*a) + 4*c*d*x/(b*(e^(f*x + e) - e^(-f*x - e)) + 2*a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(270) = 540$.
time = 0.40, size = 910, normalized size = 3.07



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(2*b*d^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(f*x + cosh(1) + sinh(1))
 + a*sinh(f*x + cosh(1) + sinh(1)) + (b*cosh(f*x + cosh(1) + sinh(1)) + b*s
inh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b^2))/b) - 2*b*d^2*sqrt((a^2
 + b^2)/b^2)*polylog(3, (a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cos
h(1) + sinh(1)) - (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) +
 sinh(1)))*sqrt((a^2 + b^2)/b^2))/b) - 2*(b*d^2*f*x + b*c*d*f)*sqrt((a^2 +
b^2)/b^2)*dilog((a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + s
inh(1)) + (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)
))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b*d^2*f*x + b*c*d*f)*sqrt((a^2 +
b^2)/b^2)*dilog((a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + s
inh(1)) - (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)
))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b*c^2*f^2 - 2*b*c*d*f*cosh(1) + b*d
^2*cosh(1)^2 + b*d^2*sinh(1)^2 - 2*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*sqrt(
(a^2 + b^2)/b^2)*log(2*b*cosh(f*x + cosh(1) + sinh(1)) + 2*b*sinh(f*x + cos
h(1) + sinh(1)) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b*c^2*f^2 - 2*b*c*d*f
*cosh(1) + b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 - 2*(b*c*d*f - b*d^2*cosh(1))*
sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(f*x + cosh(1) + sinh(1)) + 2*b*
sinh(f*x + cosh(1) + sinh(1)) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b*d^2*f
^2*x^2 + 2*b*c*d*f^2*x + 2*b*c*d*f*cosh(1) - b*d^2*cosh(1)^2 - b*d^2*sinh(1
```

```
)^2 + 2*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(-(a*co
sh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) + (b*cosh(f*x
 + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b
^2) - b)/b) + (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + 2*b*c*d*f*cosh(1) - b*d^2*co
sh(1)^2 - b*d^2*sinh(1)^2 + 2*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*sqrt((a^2
+ b^2)/b^2)*log(-(a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) +
sinh(1)) - (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1
)))*sqrt((a^2 + b^2)/b^2) - b)/b))/((a^2 + b^2)*f^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{a+b\sinh{(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*sinh(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**2/(a + b*sinh(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(b*sinh(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^2}{a+b\sinh\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*sinh(e + f*x)),x)
```

```
[Out] int((c + d*x)^2/(a + b*sinh(e + f*x)), x)
```

3.171 $\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx$

Optimal. Leaf size=187

$$\frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f}-\frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f}+\frac{d\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f^2}-\frac{d\text{PolyI}}{}$$

```
[Out] (d*x+c)*ln(1+b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/f/(a^2+b^2)^(1/2)-(d*x+c)*ln
(1+b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/f/(a^2+b^2)^(1/2)+d*polylog(2,-b*exp(f
*x+e)/(a-(a^2+b^2)^(1/2)))/f^2/(a^2+b^2)^(1/2)-d*polylog(2,-b*exp(f*x+e)/(a
+(a^2+b^2)^(1/2)))/f^2/(a^2+b^2)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3403, 2296, 2221, 2317, 2438}

$$\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{f\sqrt{a^2+b^2}}-\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{f\sqrt{a^2+b^2}}+\frac{d\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2\sqrt{a^2+b^2}}-\frac{d\text{Li}_2\left(-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f^2\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*Sinh[e + f*x]),x]
```

```
[Out] ((c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^2]
*f) - ((c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/(Sqrt[a^2
+ b^2]*f) + (d*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/(Sqrt[
a^2 + b^2]*f^2) - (d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/
(Sqrt[a^2 + b^2]*f^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx = 2\int \frac{e^{e+fx}(c+dx)}{-b+2ae^{e+fx}+be^{2(e+fx)}}\,dx$$

$$= \frac{(2b)\int \frac{e^{e+fx}(c+dx)}{2a-2\sqrt{a^2+b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2+b^2}} - \frac{(2b)\int \frac{e^{e+fx}(c+dx)}{2a+2\sqrt{a^2+b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2+b^2}}$$

$$= \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} - \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} - \frac{d\int \log}{}$$

$$= \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} - \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} - \frac{d\mathrm{Subst}}{}$$

$$= \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} - \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f} + \frac{d\mathrm{Li}_2\left(-\right.}{\sqrt{a}}$$

Mathematica [A]
time = 0.03, size = 142, normalized size = 0.76

$$\frac{f(c+dx)\left(\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)-\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)\right)+d\mathrm{PolyLog}\left(2,\frac{be^{e+fx}}{-a+\sqrt{a^2+b^2}}\right)-d\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*Sinh[e + f*x]),x]
```

```
[Out] (f*(c + d*x)*(Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])] - Log[1 + (b*E
^(e + f*x))/(a + Sqrt[a^2 + b^2])]) + d*PolyLog[2, (b*E^(e + f*x))/(-a + Sq
rt[a^2 + b^2])] - d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/(
Sqrt[a^2 + b^2]*f^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(167) = 334$.
time = 1.03, size = 393, normalized size = 2.10

method	result
risch	$-\frac{2c\,\mathrm{arctanh}\left(\frac{2b\,\mathrm{e}^{fx+e}+2a}{2\sqrt{a^2+b^2}}\right)}{f\sqrt{a^2+b^2}}+\frac{d\ln\left(\frac{-b\,\mathrm{e}^{fx+e}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)x}{f\sqrt{a^2+b^2}}+\frac{d\ln\left(\frac{-b\,\mathrm{e}^{fx+e}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)e}{f^2\sqrt{a^2+b^2}}-\frac{d\ln\left(\frac{b\,\mathrm{e}^{fx+e}+\sqrt{}}{a+\sqrt{}}\right.}{f\sqrt{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*sinh(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(f*x+e)+2*a)/(a^2+b^2)^(1/2))+1/
f*d/(a^2+b^2)^(1/2)*ln((-b*exp(f*x+e)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2
)))*x+1/f^2*d/(a^2+b^2)^(1/2)*ln((-b*exp(f*x+e)+(a^2+b^2)^(1/2)-a)/(-a+(a^2
+b^2)^(1/2)))*e-1/f*d/(a^2+b^2)^(1/2)*ln((b*exp(f*x+e)+(a^2+b^2)^(1/2)+a)/(
a+(a^2+b^2)^(1/2)))*x-1/f^2*d/(a^2+b^2)^(1/2)*ln((b*exp(f*x+e)+(a^2+b^2)^(1
/2)+a)/(a+(a^2+b^2)^(1/2)))*e+1/f^2*d/(a^2+b^2)^(1/2)*dilog((-b*exp(f*x+e)+
(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/f^2*d/(a^2+b^2)^(1/2)*dilog((b*e
xp(f*x+e)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/f^2*d*e/(a^2+b^2)^(1/2)
*arctanh(1/2*(2*b*exp(f*x+e)+2*a)/(a^2+b^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] d*integrate(2*x/(b*(e^(f*x + e) - e^(-f*x - e)) + 2*a), x) + c*log((b*e^(-f
*x - e) - a - sqrt(a^2 + b^2))/(b*e^(-f*x - e) - a + sqrt(a^2 + b^2)))/(sqr
t(a^2 + b^2)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(169) = 338$.
time = 0.35, size = 541, normalized size = 2.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] (b*d*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(
f*x + cosh(1) + sinh(1)) + (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x +
cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - b*d*sqrt((a^2 + b^2
)/b^2)*dilog((a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh
(1)) - (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*
sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b*c*f - b*d*cosh(1) - b*d*sinh(1))*sqr
t((a^2 + b^2)/b^2)*log(2*b*cosh(f*x + cosh(1) + sinh(1)) + 2*b*sinh(f*x + c
osh(1) + sinh(1)) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b*c*f - b*d*cosh(1)
 - b*d*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(f*x + cosh(1) + sinh(1))
 + 2*b*sinh(f*x + cosh(1) + sinh(1)) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (
b*d*f*x + b*d*cosh(1) + b*d*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(f*x
 + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) + (b*cosh(f*x + cos
h(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b^2) -
b)/b) - (b*d*f*x + b*d*cosh(1) + b*d*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(-(a
*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) - (b*cosh(
f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2
)/b^2) - b)/b))/((a^2 + b^2)*f^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sinh(f*x+e)),x)
```

```
[Out] Integral((c + d*x)/(a + b*sinh(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*sinh(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+d\,x}{a+b\sinh\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*sinh(e + f*x)),x)
```

```
[Out] int((c + d*x)/(a + b*sinh(e + f*x)), x)
```

3.172 $\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)(a+b\sinh(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*sinh(f*x+e)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*Sinh[e + f*x])),x]
[Out] Defer[Int][1/((c + d*x)*(a + b*Sinh[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx$$

Mathematica [A]
time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*Sinh[e + f*x])),x]
[Out] Integrate[1/((c + d*x)*(a + b*Sinh[e + f*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\,(a+b\sinh\left(fx+e\right))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*sinh(f*x+e)),x)
[Out] int(1/(d*x+c)/(a+b*sinh(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="maxima")
[Out] integrate(1/((d*x + c)*(b*sinh(f*x + e) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="fricas")
[Out] integral(1/(a*d*x + a*c + (b*d*x + b*c)*sinh(f*x + e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sinh(e+fx))(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sinh(f*x+e)),x)
[Out] Integral(1/((a + b*sinh(e + f*x))*(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="giac")
[Out] integrate(1/((d*x + c)*(b*sinh(f*x + e) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a+b\sinh(e+f\,x))\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sinh(e + f*x))*(c + d*x)),x)
[Out] int(1/((a + b*sinh(e + f*x))*(c + d*x)), x)
```

3.173 $\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+b\sinh(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*Sinh[e + f*x])),x]
[Out] Defer[Int][1/((c + d*x)^2*(a + b*Sinh[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx=\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx$$

Mathematica [A]
time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*Sinh[e + f*x])),x]
[Out] Integrate[1/((c + d*x)^2*(a + b*Sinh[e + f*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\,(a+b\sinh\,(fx+e))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x)
[Out] int(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="maxima")
[Out] integrate(1/((d*x + c)^2*(b*sinh(f*x + e) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="fricas")
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)
*sinh(f*x + e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sinh(e+fx))\,(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*sinh(f*x+e)),x)
[Out] Integral(1/((a + b*sinh(e + f*x))*(c + d*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="giac")
[Out] integrate(1/((d*x + c)^2*(b*sinh(f*x + e) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\sinh\left(e+f\,x\right)\right)\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sinh(e + f*x))*(c + d*x)^2),x)
[Out] int(1/((a + b*sinh(e + f*x))*(c + d*x)^2), x)
```

3.174 $\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx$

Optimal. Leaf size=549

$$-\frac{(c+dx)^2}{(a^2+b^2)\,f}+\frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,f^2}+\frac{a(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,f}+\frac{2d(c+dx)\log\left(1\right.}{(a^2+b}$$

```
[Out] -(d*x+c)^2/(a^2+b^2)/f+2*d*(d*x+c)*ln(1+b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(
a^2+b^2)/f^2+a*(d*x+c)^2*ln(1+b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(
3/2)/f+2*d*(d*x+c)*ln(1+b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/f^2-a*(
d*x+c)^2*ln(1+b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/f+2*d^2*pol
ylog(2,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/f^3+2*a*d*(d*x+c)*polyl
og(2,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/f^2+2*d^2*polylog(2
,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/f^3-2*a*d*(d*x+c)*polylog(2,-
b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/f^2-2*a*d^2*polylog(3,-b*
exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/f^3+2*a*d^2*polylog(3,-b*ex
p(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/f^3-b*(d*x+c)^2*cosh(f*x+e)/(
a^2+b^2)/f/(a+b*sinh(f*x+e))
```

Rubi [A]
time = 0.71, antiderivative size = 549, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3405, 3403, 2296, 2221, 2611, 2320, 6724, 5680, 2317, 2438}

$$\frac{2ad(c+dx)\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2\left(a^2+b^2\right)^{3/2}}-\frac{2ad(c+dx)\text{Li}_2\left(-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f^2\left(a^2+b^2\right)^{3/2}}+\frac{2d(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{f^2\left(a^2+b^2\right)}+\frac{2d(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{f^2\left(a^2+b^2\right)}+\frac{a(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{f\left(a^2+b^2\right)^{3/2}}-\frac{a(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{f\left(a^2+b^2\right)^{3/2}}-\frac{b(c+dx)^2\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}-\frac{(c+dx)^2}{f\left(a^2+b^2\right)}+\frac{2d^2\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^3\left(a^2+b^2\right)}+\frac{2d^2\text{Li}_2\left(-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f^3\left(a^2+b^2\right)}-\frac{2ad^2\text{Li}_3\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^3\left(a^2+b^2\right)^{3/2}}+\frac{2ad^2\text{Li}_3\left(-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f^3\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] -((c + d*x)^2/((a^2 + b^2)*f)) + (2*d*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a
- Sqrt[a^2 + b^2])])/((a^2 + b^2)*f^2) + (a*(c + d*x)^2*Log[1 + (b*E^(e + f
*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*f) + (2*d*(c + d*x)*Log[1 +
 (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*f^2) - (a*(c + d*x)^2
*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*f) + (2
*d^2*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*f^3
) + (2*a*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/
((a^2 + b^2)^(3/2)*f^2) + (2*d^2*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2
 + b^2]))])/((a^2 + b^2)*f^3) - (2*a*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x
))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*f^2) - (2*a*d^2*PolyLog[3, -
((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*f^3) + (2*a*d^
2*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*
f^3) - (b*(c + d*x)^2*Cosh[e + f*x])/((a^2 + b^2)*f*(a + b*Sinh[e + f*x]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx = -\frac{b(c+dx)^2\cosh(e+fx)}{(a^2+b^2)\,f(a+b\sinh(e+fx))} + \frac{a\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx}{a^2+b^2} + \frac{(2bd)\int \frac{(c+dx)\cosh(e+\cdots}{a+b\sinh(e+f\cdots}}{(a^2+b^2)\,f}\cdots$$

$$= -\frac{(c+dx)^2}{(a^2+b^2)\,f} - \frac{b(c+dx)^2\cosh(e+fx)}{(a^2+b^2)\,f(a+b\sinh(e+fx))} + \frac{(2a)\int \frac{e^{e+fx}(c+dx)^2}{-b+2ae^{e+fx}+be^{2(e+fx)}}\,dx}{a^2+b^2}\cdots$$

$$= -\frac{(c+dx)^2}{(a^2+b^2)\,f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,f^2} + \frac{2d(c+dx)\log\left(1+\frac{\cdots}{a+\sqrt{\cdots}}\right.}{(a^2+b^2)\,f^2}\cdots$$

$$= -\frac{(c+dx)^2}{(a^2+b^2)\,f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,f^2} + \frac{a(c+dx)^2\log\left(1+\frac{\cdots}{a-\sqrt{\cdots}}\right.}{(a^2+b^2)^{3/2}\,f\cdots}\cdots$$

$$= -\frac{(c+dx)^2}{(a^2+b^2)\,f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,f^2} + \frac{a(c+dx)^2\log\left(1+\frac{\cdots}{a-\sqrt{\cdots}}\right.}{(a^2+b^2)^{3/2}\,f\cdots}\cdots$$

$$= -\frac{(c+dx)^2}{(a^2+b^2)\,f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,f^2} + \frac{a(c+dx)^2\log\left(1+\frac{\cdots}{a-\sqrt{\cdots}}\right.}{(a^2+b^2)^{3/2}\,f\cdots}\cdots$$

$$= -\frac{(c+dx)^2}{(a^2+b^2)\,f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,f^2} + \frac{a(c+dx)^2\log\left(1+\frac{\cdots}{a-\sqrt{\cdots}}\right.}{(a^2+b^2)^{3/2}\,f\cdots}\cdots$$

Mathematica [A]
time = 1.12, size = 428, normalized size = 0.78

$$\frac{-f^2(c+dx)^2 + 2df(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right) + 2df(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right) + 2d^2\text{PolyLog}\left(2,\frac{be^{e+fx}}{-a+\sqrt{a^2+b^2}}\right) + 2d^2\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right) - \frac{a\left(-f^2(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)+f^2(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)-2df(c+dx)\text{PolyLog}\left(2,\frac{be^{e+fx}}{-a+\sqrt{a^2+b^2}}\right)+2df(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)+2d^2\text{PolyLog}\left(3,\frac{be^{e+fx}}{-a+\sqrt{a^2+b^2}}\right)-2d^2\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)\right)}{\sqrt{a^2+b^2}} - \frac{bf^2(c+dx)^2\cosh(e+fx)}{a+b\sinh(e+fx)}}{(a^2+b^2)\,f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] (-(f^2*(c + d*x)^2) + 2*d*f*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2
 + b^2])] + 2*d*f*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])]
+ 2*d^2*PolyLog[2, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])] + 2*d^2*PolyLog[
2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))] - (a*(-(f^2*(c + d*x)^2*Log[1
+ (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])]) + f^2*(c + d*x)^2*Log[1 + (b*E^(e
 + f*x))/(a + Sqrt[a^2 + b^2])] - 2*d*f*(c + d*x)*PolyLog[2, (b*E^(e + f*x)
)/(-a + Sqrt[a^2 + b^2])] + 2*d*f*(c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a
```

```
 + Sqrt[a^2 + b^2]))] + 2*d^2*PolyLog[3, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b
^2])] - 2*d^2*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a
^2 + b^2] - (b*f^2*(c + d*x)^2*Cosh[e + f*x])/(a + b*Sinh[e + f*x]))/((a^2
+ b^2)*f^3)
```

Maple [F]
time = 1.84, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{(a+b\sinh(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*sinh(f*x+e))^2,x)
[Out] int((d*x+c)^2/(a+b*sinh(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
[Out] 2*a*d^2*f*integrate(x^2*e^(f*x + e)/(a^2*b*f*e^(2*f*x + 2*e) + b^3*f*e^(2*f
*x + 2*e) + 2*a^3*f*e^(f*x + e) + 2*a*b^2*f*e^(f*x + e) - a^2*b*f - b^3*f),
 x) + 4*a*c*d*f*integrate(x*e^(f*x + e)/(a^2*b*f*e^(2*f*x + 2*e) + b^3*f*e^
(2*f*x + 2*e) + 2*a^3*f*e^(f*x + e) + 2*a*b^2*f*e^(f*x + e) - a^2*b*f - b^3
*f), x) + 2*b*c*d*(a*log((b*e^(f*x + e) + a - sqrt(a^2 + b^2))/(b*e^(f*x +
e) + a + sqrt(a^2 + b^2)))/((a^2*b + b^3)*sqrt(a^2 + b^2)*f^2) - 2*(f*x + e
)/((a^2*b + b^3)*f^2) + log(b*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - b)/((a^2*
b + b^3)*f^2)) - 4*a*d^2*integrate(x*e^(f*x + e)/(a^2*b*f*e^(2*f*x + 2*e) +
 b^3*f*e^(2*f*x + 2*e) + 2*a^3*f*e^(f*x + e) + 2*a*b^2*f*e^(f*x + e) - a^2*
b*f - b^3*f), x) + 4*b*d^2*integrate(x/(a^2*b*f*e^(2*f*x + 2*e) + b^3*f*e^(
2*f*x + 2*e) + 2*a^3*f*e^(f*x + e) + 2*a*b^2*f*e^(f*x + e) - a^2*b*f - b^3*
f), x) + c^2*(a*log((b*e^(-f*x - e) - a - sqrt(a^2 + b^2))/(b*e^(-f*x - e)
- a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2)*f) - 2*(a*e^(-f*x - e) + b)/((a^
2*b + b^3 + 2*(a^3 + a*b^2)*e^(-f*x - e) - (a^2*b + b^3)*e^(-2*f*x - 2*e))*
f)) - 2*a*c*d*log((b*e^(f*x + e) + a - sqrt(a^2 + b^2))/(b*e^(f*x + e) + a
+ sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2)*f^2) + 2*(b*d^2*x^2 + 2*b*c*d*x - (a
*d^2*x^2*e^e + 2*a*c*d*x*e^e)*e^(f*x))/(a^2*b*f + b^3*f - (a^2*b*f + b^3*f)
*e^(2*f*x + 2*e) - 2*(a^3*f + a*b^2*f)*e^(f*x + e))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5925 vs. 2(515) = 1030.
time = 0.44, size = 5925, normalized size = 10.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -(2*(a^2*b + b^3)*c^2*f^2 - 4*(a^2*b + b^3)*c*d*f*cosh(1) + 2*(a^2*b + b^3)
*d^2*cosh(1)^2 + 2*(a^2*b + b^3)*d^2*sinh(1)^2 + 2*((a^2*b + b^3)*d^2*f^2*x
^2 + 2*(a^2*b + b^3)*c*d*f^2*x + 2*(a^2*b + b^3)*c*d*f*cosh(1) - (a^2*b + b
^3)*d^2*cosh(1)^2 - (a^2*b + b^3)*d^2*sinh(1)^2 + 2*((a^2*b + b^3)*c*d*f -
(a^2*b + b^3)*d^2*cosh(1))*sinh(1))*cosh(f*x + cosh(1) + sinh(1))^2 + 2*((a
^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*c*d*f^2*x + 2*(a^2*b + b^3)*c*d*f
*cosh(1) - (a^2*b + b^3)*d^2*cosh(1)^2 - (a^2*b + b^3)*d^2*sinh(1)^2 + 2*((
a^2*b + b^3)*c*d*f - (a^2*b + b^3)*d^2*cosh(1))*sinh(1))*sinh(f*x + cosh(1)
 + sinh(1))^2 + 2*(a*b^2*d^2*cosh(f*x + cosh(1) + sinh(1))^2 + a*b^2*d^2*si
nh(f*x + cosh(1) + sinh(1))^2 + 2*a^2*b*d^2*cosh(f*x + cosh(1) + sinh(1)) -
 a*b^2*d^2 + 2*(a*b^2*d^2*cosh(f*x + cosh(1) + sinh(1)) + a^2*b*d^2)*sinh(f
*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(f*x + cos
h(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) + (b*cosh(f*x + cosh(1) +
 sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b^2))/b) - 2*
(a*b^2*d^2*cosh(f*x + cosh(1) + sinh(1))^2 + a*b^2*d^2*sinh(f*x + cosh(1) +
 sinh(1))^2 + 2*a^2*b*d^2*cosh(f*x + cosh(1) + sinh(1)) - a*b^2*d^2 + 2*(a*
b^2*d^2*cosh(f*x + cosh(1) + sinh(1)) + a^2*b*d^2)*sinh(f*x + cosh(1) + sin
h(1)))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(f*x + cosh(1) + sinh(1)) +
a*sinh(f*x + cosh(1) + sinh(1)) - (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh
(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b^2))/b) + 2*((a^3 + a*b^2)*d^2
*f^2*x^2 + 2*(a^3 + a*b^2)*c*d*f^2*x - (a^3 + a*b^2)*c^2*f^2 + 4*(a^3 + a*b
^2)*c*d*f*cosh(1) - 2*(a^3 + a*b^2)*d^2*cosh(1)^2 - 2*(a^3 + a*b^2)*d^2*sin
h(1)^2 + 4*((a^3 + a*b^2)*c*d*f - (a^3 + a*b^2)*d^2*cosh(1))*sinh(1))*cosh(
f*x + cosh(1) + sinh(1)) - 2*((a^2*b + b^3)*d^2*cosh(f*x + cosh(1) + sinh(1
))^2 + (a^2*b + b^3)*d^2*sinh(f*x + cosh(1) + sinh(1))^2 + 2*(a^3 + a*b^2)*
d^2*cosh(f*x + cosh(1) + sinh(1)) - (a^2*b + b^3)*d^2 + 2*((a^2*b + b^3)*d^
2*cosh(f*x + cosh(1) + sinh(1)) + (a^3 + a*b^2)*d^2)*sinh(f*x + cosh(1) + s
inh(1)) - (a*b^2*d^2*f*x + a*b^2*c*d*f - (a*b^2*d^2*f*x + a*b^2*c*d*f)*cosh
(f*x + cosh(1) + sinh(1))^2 - (a*b^2*d^2*f*x + a*b^2*c*d*f)*sinh(f*x + cosh
(1) + sinh(1))^2 - 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*cosh(f*x + cosh(1) + sin
h(1)) - 2*(a^2*b*d^2*f*x + a^2*b*c*d*f + (a*b^2*d^2*f*x + a*b^2*c*d*f)*cosh
(f*x + cosh(1) + sinh(1)))*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/
b^2))*dilog((a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(
1)) + (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*s
qrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*((a^2*b + b^3)*d^2*cosh(f*x + cosh(1)
+ sinh(1))^2 + (a^2*b + b^3)*d^2*sinh(f*x + cosh(1) + sinh(1))^2 + 2*(a^3 +
 a*b^2)*d^2*cosh(f*x + cosh(1) + sinh(1)) - (a^2*b + b^3)*d^2 + 2*((a^2*b +
 b^3)*d^2*cosh(f*x + cosh(1) + sinh(1)) + (a^3 + a*b^2)*d^2)*sinh(f*x + cos
h(1) + sinh(1)) + (a*b^2*d^2*f*x + a*b^2*c*d*f - (a*b^2*d^2*f*x + a*b^2*c*d
*f)*cosh(f*x + cosh(1) + sinh(1))^2 - (a*b^2*d^2*f*x + a*b^2*c*d*f)*sinh(f*
x + cosh(1) + sinh(1))^2 - 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*cosh(f*x + cosh(
```

```
1) + sinh(1)) - 2*(a^2*b*d^2*f*x + a^2*b*c*d*f + (a*b^2*d^2*f*x + a*b^2*c*d
*f)*cosh(f*x + cosh(1) + sinh(1)))*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2
 + b^2)/b^2))*dilog((a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1)
 + sinh(1)) - (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sin
h(1)))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (2*(a^2*b + b^3)*c*d*f - 2*(a^2*
b + b^3)*d^2*cosh(1) - 2*(a^2*b + b^3)*d^2*sinh(1) - 2*((a^2*b + b^3)*c*d*f
 - (a^2*b + b^3)*d^2*cosh(1) - (a^2*b + b^3)*d^2*sinh(1))*cosh(f*x + cosh(1
) + sinh(1))^2 - 2*((a^2*b + b^3)*c*d*f - (a^2*b + b^3)*d^2*cosh(1) - (a^2*
b + b^3)*d^2*sinh(1))*sinh(f*x + cosh(1) + sinh(1))^2 - 4*((a^3 + a*b^2)*c*
d*f - (a^3 + a*b^2)*d^2*cosh(1) - (a^3 + a*b^2)*d^2*sinh(1))*cosh(f*x + cos
h(1) + sinh(1)) - 4*((a^3 + a*b^2)*c*d*f - (a^3 + a*b^2)*d^2*cosh(1) - (a^3
 + a*b^2)*d^2*sinh(1) + ((a^2*b + b^3)*c*d*f - (a^2*b + b^3)*d^2*cosh(1) -
(a^2*b + b^3)*d^2*sinh(1))*cosh(f*x + cosh(1) + sinh(1)))*sinh(f*x + cosh(1
) + sinh(1)) - (a*b^2*c^2*f^2 - 2*a*b^2*c*d*f*cosh(1) + a*b^2*d^2*cosh(1)^2
 + a*b^2*d^2*sinh(1)^2 - (a*b^2*c^2*f^2 - 2*a*b^2*c*d*f*cosh(1) + a*b^2*d^2
*cosh(1)^2 + a*b^2*d^2*sinh(1)^2 - 2*(a*b^2*c*d*f - a*b^2*d^2*cosh(1))*sinh
(1))*cosh(f*x + cosh(1) + sinh(1))^2 - (a*b^2*c^2*f^2 - 2*a*b^2*c*d*f*cosh(
1) + a*b^2*d^2*cosh(1)^2 + a*b^2*d^2*sinh(1)^2 - 2*(a*b^2*c*d*f - a*b^2*d^2
*cosh(1))*sinh(1))*sinh(f*x + cosh(1) + sinh(1))^2 - 2*(a^2*b*c^2*f^2 - 2*a
^2*b*c*d*f*cosh(1) + a^2*b*d^2*cosh(1)^2 + a^2*b*d^2*sinh(1)^2 - 2*(a^2*b*c
*d*f - a^2*b*d^2*cosh(1))*sinh(1))*cosh(f*x + cosh(1) + sinh(1)) - 2*(a*b^2
*c*d*f - a*b^2*d^2*cosh(1))*sinh(1) - 2*(a^2*b*c^2*f^2 - 2*a^2*b*c*d*f*cosh
(1) + a^2*b*d^2*cosh(1)^2 + a^2*b*d^2*sinh(1)^2 + (a*b^2*c^2*f^2 - 2*a*b^2*
c*d*f*cosh(1) + a*b^2*d^2*cosh(1)^2 + a*b^2*d^2*sinh(1)^2 - 2*(a*b^2*c*d*f
- a*b^2*d^2*cosh(1))*sinh(1))*cosh(f*x + cosh(1) + sinh(1)) - 2*(a^2*b*c*d*
f - a^2*b*d^2*cosh(1))*sinh(1))*sinh(f*x + cosh...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*sinh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(b*sinh(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^2}{(a+b\sinh(e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*sinh(e + f*x))^2,x)
[Out] int((c + d*x)^2/(a + b*sinh(e + f*x))^2, x)
```

3.175 $\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx$

Optimal. Leaf size=254

$$\frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}f}-\frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}f}+\frac{d\log(a+b\sinh(e+fx))}{\left(a^2+b^2\right)f^2}+\frac{ad\text{PolyI}}{}$$

```
[Out] d*ln(a+b*sinh(f*x+e))/(a^2+b^2)/f^2+a*(d*x+c)*ln(1+b*exp(f*x+e)/(a-(a^2+b^2
)^(1/2)))/(a^2+b^2)^(3/2)/f-a*(d*x+c)*ln(1+b*exp(f*x+e)/(a+(a^2+b^2)^(1/2))
)/(a^2+b^2)^(3/2)/f+a*d*polylog(2,-b*exp(f*x+e)/(a-(a^2+b^2)^(1/2)))/(a^2+b
^2)^(3/2)/f^2-a*d*polylog(2,-b*exp(f*x+e)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3
/2)/f^2-b*(d*x+c)*cosh(f*x+e)/(a^2+b^2)/f/(a+b*sinh(f*x+e))
```

Rubi [A]
time = 0.32, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3405, 3403, 2296, 2221, 2317, 2438, 2747, 31}

$$\frac{a(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{f\left(a^2+b^2\right)^{3/2}}-\frac{a(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{f\left(a^2+b^2\right)^{3/2}}-\frac{b(c+dx)\cosh(e+fx)}{f\left(a^2+b^2\right)(a+b\sinh(e+fx))}+\frac{ad\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2\left(a^2+b^2\right)^{3/2}}-\frac{ad\text{Li}_2\left(-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{f^2\left(a^2+b^2\right)^{3/2}}+\frac{d\log(a+b\sinh(e+fx))}{f^2\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] (a*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(
3/2)*f) - (a*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/((a^
2 + b^2)^(3/2)*f) + (d*Log[a + b*Sinh[e + f*x]])/((a^2 + b^2)*f^2) + (a*d*P
olyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*f^2
) - (a*d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)
^(3/2)*f^2) - (b*(c + d*x)*Cosh[e + f*x])/((a^2 + b^2)*f*(a + b*Sinh[e + f*
x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx = -\frac{b(c+dx)\cosh(e+fx)}{(a^2+b^2)\,f(a+b\sinh(e+fx))} + \frac{a\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx}{a^2+b^2} + \frac{(bd)\int \frac{\cosh(e+fx)}{a+b\sinh(e+fx)}}{(a^2+b^2)\,f}$$

$$= -\frac{b(c+dx)\cosh(e+fx)}{(a^2+b^2)\,f(a+b\sinh(e+fx))} + \frac{(2a)\int \frac{e^{e+fx}(c+dx)}{-b+2ae^{e+fx}+be^{2(e+fx)}}\,dx}{a^2+b^2} + \frac{d\mathrm{Subst}(\int}{}$$

$$= \frac{d\log(a+b\sinh(e+fx))}{(a^2+b^2)\,f^2} - \frac{b(c+dx)\cosh(e+fx)}{(a^2+b^2)\,f(a+b\sinh(e+fx))} + \frac{(2ab)\int \frac{}{2a-2\sqrt{}}}{(a^2}$$

$$= \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,f} - \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,f} + \frac{d\,l}{}$$

$$= \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,f} - \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,f} + \frac{d\,l}{}$$

$$= \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,f} - \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,f} + \frac{d\,l}{}$$

Mathematica [A]
time = 0.75, size = 194, normalized size = 0.76

$$\frac{d\log(a+b\sinh(e+fx)) + \frac{a\left(f(c+dx)\left(\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)-\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)\right)+d\mathrm{PolyLog}\left(2,\frac{be^{e+fx}}{-a+\sqrt{a^2+b^2}}\right)-d\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)\right)}{\sqrt{a^2+b^2}} - \frac{bf(c+dx)\cosh(e+fx)}{a+b\sinh(e+fx)}}{(a^2+b^2)\,f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] (d*Log[a + b*Sinh[e + f*x]] + (a*(f*(c + d*x)*(Log[1 + (b*E^(e + f*x))/(a -
 Sqrt[a^2 + b^2])] - Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])]) + d*Po
lyLog[2, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])] - d*PolyLog[2, -((b*E^(e +
 f*x))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2] - (b*f*(c + d*x)*Cosh[e +
f*x])/(a + b*Sinh[e + f*x]))/((a^2 + b^2)*f^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(234) = 468$.
time = 2.88, size = 519, normalized size = 2.04

method	result

risch	$\frac{2(dx+c)(a\,e^{fx+e}-b)}{f(a^2+b^2)(b\,e^{2fx+2e}+2a\,e^{fx+e}-b)} - \frac{2d\ln(e^{fx+e})}{f^2(a^2+b^2)} + \frac{d\ln(b\,e^{2fx+2e}+2a\,e^{fx+e}-b)}{f^2(a^2+b^2)} - \frac{2ac\,\operatorname{arctanh}\left(\frac{2b\,e^{fx+e}+2a}{2\sqrt{a^2+b^2}}\right)}{f(a^2+b^2)^{\frac{3}{2}}} + \frac{ad\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*sinh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(d*x+c)*(a*exp(f*x+e)-b)/f/(a^2+b^2)/(b*exp(2*f*x+2*e)+2*a*exp(f*x+e)-b)-
2/f^2/(a^2+b^2)*d*ln(exp(f*x+e))+1/f^2/(a^2+b^2)*d*ln(b*exp(2*f*x+2*e)+2*a*
exp(f*x+e)-b)-2/f/(a^2+b^2)^(3/2)*a*c*arctanh(1/2*(2*b*exp(f*x+e)+2*a)/(a^2
+b^2)^(1/2))+1/f/(a^2+b^2)^(3/2)*a*d*ln((-b*exp(f*x+e)+(a^2+b^2)^(1/2)-a)/(
-a+(a^2+b^2)^(1/2)))*x+1/f^2/(a^2+b^2)^(3/2)*a*d*ln((-b*exp(f*x+e)+(a^2+b^2
)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*e-1/f/(a^2+b^2)^(3/2)*a*d*ln((b*exp(f*x+e)
+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/f^2/(a^2+b^2)^(3/2)*a*d*ln((b*
exp(f*x+e)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*e+1/f^2/(a^2+b^2)^(3/2)*
a*d*dilog((-b*exp(f*x+e)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/f^2/(a^
2+b^2)^(3/2)*a*d*dilog((b*exp(f*x+e)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))
)+2/f^2/(a^2+b^2)^(3/2)*a*d*e*arctanh(1/2*(2*b*exp(f*x+e)+2*a)/(a^2+b^2)^(1
/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (2*a*f*integrate(x*e^(f*x + e)/(a^2*b*f*e^(2*f*x + 2*e) + b^3*f*e^(2*f*x +
2*e) + 2*a^3*f*e^(f*x + e) + 2*a*b^2*f*e^(f*x + e) - a^2*b*f - b^3*f), x) +
 b*(a*log((b*e^(f*x + e) + a - sqrt(a^2 + b^2))/(b*e^(f*x + e) + a + sqrt(a
^2 + b^2)))/((a^2*b + b^3)*sqrt(a^2 + b^2)*f^2) - 2*(f*x + e)/((a^2*b + b^3
)*f^2) + log(b*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - b)/((a^2*b + b^3)*f^2))
- 2*(a*x*e^(f*x + e) - b*x)/(a^2*b*f + b^3*f - (a^2*b*f + b^3*f)*e^(2*f*x +
 2*e) - 2*(a^3*f + a*b^2*f)*e^(f*x + e)) - a*log((b*e^(f*x + e) + a - sqrt(
a^2 + b^2))/(b*e^(f*x + e) + a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2)*f^2))
*d + c*(a*log((b*e^(-f*x - e) - a - sqrt(a^2 + b^2))/(b*e^(-f*x - e) - a +
sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2)*f) - 2*(a*e^(-f*x - e) + b)/((a^2*b +
b^3 + 2*(a^3 + a*b^2)*e^(-f*x - e) - (a^2*b + b^3)*e^(-2*f*x - 2*e))*f))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2245 vs. $2(239) = 478$.
time = 0.41, size = 2245, normalized size = 8.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="fricas")

[Out] -(2*(a^2*b + b^3)*c*f - 2*(a^2*b + b^3)*d*cosh(1) + 2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1))*cosh(f*x + cosh(1) + sinh(1))^2 - 2*(a^2*b + b^3)*d*sinh(1) + 2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1))*sinh(f*x + cosh(1) + sinh(1))^2 - (a*b^2*d*cosh(f*x + cosh(1) + sinh(1))^2 + a*b^2*d*sinh(f*x + cosh(1) + sinh(1))^2 + 2*a^2*b*d*cosh(f*x + cosh(1) + sinh(1)) - a*b^2*d + 2*(a*b^2*d*cosh(f*x + cosh(1) + sinh(1)) + a^2*b*d)*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) + (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (a*b^2*d*cosh(f*x + cosh(1) + sinh(1))^2 + a*b^2*d*sinh(f*x + cosh(1) + sinh(1))^2 + 2*a^2*b*d*cosh(f*x + cosh(1) + sinh(1)) - a*b^2*d + 2*(a*b^2*d*cosh(f*x + cosh(1) + sinh(1)) + a^2*b*d)*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) - (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (a*b^2*d*f*x + a*b^2*d*cosh(1) + a*b^2*d*sinh(1) - (a*b^2*d*f*x + a*b^2*d*cosh(1) + a*b^2*d*sinh(1))*cosh(f*x + cosh(1) + sinh(1))^2 - (a*b^2*d*f*x + a*b^2*d*cosh(1) + a*b^2*d*sinh(1))*sinh(f*x + cosh(1) + sinh(1))^2 - 2*(a^2*b*d*f*x + a^2*b*d*cosh(1) + a^2*b*d*sinh(1))*cosh(f*x + cosh(1) + sinh(1)) - 2*(a^2*b*d*f*x + a^2*b*d*cosh(1) + a^2*b*d*sinh(1) + (a*b^2*d*f*x + a*b^2*d*cosh(1) + a*b^2*d*sinh(1))*cosh(f*x + cosh(1) + sinh(1)))*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) + (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b^2) - b)/b) - (a*b^2*d*f*x + a*b^2*d*cosh(1) + a*b^2*d*sinh(1) - (a*b^2*d*f*x + a*b^2*d*cosh(1) + a*b^2*d*sinh(1))*cosh(f*x + cosh(1) + sinh(1))^2 - (a*b^2*d*f*x + a*b^2*d*cosh(1) + a*b^2*d*sinh(1))*sinh(f*x + cosh(1) + sinh(1))^2 - 2*(a^2*b*d*f*x + a^2*b*d*cosh(1) + a^2*b*d*sinh(1))*cosh(f*x + cosh(1) + sinh(1)) - 2*(a^2*b*d*f*x + a^2*b*d*cosh(1) + a^2*b*d*sinh(1) + (a*b^2*d*f*x + a*b^2*d*cosh(1) + a*b^2*d*sinh(1))*cosh(f*x + cosh(1) + sinh(1)))*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) - (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*((a^3 + a*b^2)*d*f*x - (a^3 + a*b^2)*c*f + 2*(a^3 + a*b^2)*d*cosh(1) + 2*(a^3 + a*b^2)*d*sinh(1))*cosh(f*x + cosh(1) + sinh(1)) - ((a^2*b + b^3)*d*cosh(f*x + cosh(1) + sinh(1))^2 + (a^2*b + b^3)*d*sinh(f*x + cosh(1) + sinh(1))^2 + 2*(a^3 + a*b^2)*d*cosh(f*x + cosh(1) + sinh(1)) - (a^2*b + b^3)*d + 2*((a^2*b + b^3)*d*cosh(f*x + cosh(1) + sinh(1)) + (a^3 + a*b^2)*d)*sinh(f*x + cosh(1) + sinh(1)) + (a*b^2*c*f - a*b^2*d*cosh(1) - a*b^2*d*sinh(1) - (a*b^2*c*f - a*b^2*d*cosh(1) - a*b^2*d*sinh(1))*cosh(f*x + cosh(1) + sinh(1))^2 - (a*b^2*c*f - a*b^2*d*cosh(1) - a*b^2*d*sinh(1))*sinh(f*x + cosh(1) + sinh

```
(1))^2 - 2*(a^2*b*c*f - a^2*b*d*cosh(1) - a^2*b*d*sinh(1))*cosh(f*x + cosh(
1) + sinh(1)) - 2*(a^2*b*c*f - a^2*b*d*cosh(1) - a^2*b*d*sinh(1) + (a*b^2*c
*f - a*b^2*d*cosh(1) - a*b^2*d*sinh(1))*cosh(f*x + cosh(1) + sinh(1)))*sinh
(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^2)/b^2))*log(2*b*cosh(f*x + cosh(1
) + sinh(1)) + 2*b*sinh(f*x + cosh(1) + sinh(1)) + 2*b*sqrt((a^2 + b^2)/b^2
) + 2*a) - ((a^2*b + b^3)*d*cosh(f*x + cosh(1) + sinh(1))^2 + (a^2*b + b^3)
*d*sinh(f*x + cosh(1) + sinh(1))^2 + 2*(a^3 + a*b^2)*d*cosh(f*x + cosh(1) +
 sinh(1)) - (a^2*b + b^3)*d + 2*((a^2*b + b^3)*d*cosh(f*x + cosh(1) + sinh(
1)) + (a^3 + a*b^2)*d)*sinh(f*x + cosh(1) + sinh(1)) - (a*b^2*c*f - a*b^2*d
*cosh(1) - a*b^2*d*sinh(1) - (a*b^2*c*f - a*b^2*d*cosh(1) - a*b^2*d*sinh(1)
)*cosh(f*x + cosh(1) + sinh(1))^2 - (a*b^2*c*f - a*b^2*d*cosh(1) - a*b^2*d*
sinh(1))*sinh(f*x + cosh(1) + sinh(1))^2 - 2*(a^2*b*c*f - a^2*b*d*cosh(1) -
 a^2*b*d*sinh(1))*cosh(f*x + cosh(1) + sinh(1)) - 2*(a^2*b*c*f - a^2*b*d*co
sh(1) - a^2*b*d*sinh(1) + (a*b^2*c*f - a*b^2*d*cosh(1) - a*b^2*d*sinh(1))*c
osh(f*x + cosh(1) + sinh(1)))*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 + b^
2)/b^2))*log(2*b*cosh(f*x + cosh(1) + sinh(1)) + 2*b*sinh(f*x + cosh(1) + s
inh(1)) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*((a^3 + a*b^2)*d*f*x - (a^3
+ a*b^2)*c*f + 2*(a^3 + a*b^2)*d*cosh(1) + 2*(a^3 + a*b^2)*d*sinh(1) + 2*((
a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1))*cos
h(f*x + cosh(1) + sinh(1)))*sinh(f*x + cosh(1) + sinh(1))))/((a^4*b + 2*a^2*
b^3 + b^5)*f^2*cosh(f*x + cosh(1) + sinh(1))^2 + (a^4*b + 2*a^2*b^3 + b^5)*
f^2*sinh(f*x + cosh(1) + sinh(1))^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*f^2*cosh(
f*x + cosh(1) + sinh(1)) - (a^4*b + 2*a^2*b^3 + b^5)*f^2 + 2*((a^4*b + 2*a^
2*b^3 + b^5)*f^2*cosh(f*x + cosh(1) + sinh(1)) + (a^5 + 2*a^3*b^2 + a*b^4)*
f^2)*sinh(f*x + cosh(1) + sinh(1)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sinh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*sinh(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c+d\,x}{{\left(a+b\,\mathrm{sinh}\left(e+f\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*sinh(e + f*x))^2,x)
[Out] int((c + d*x)/(a + b*sinh(e + f*x))^2, x)
```

3.176 $\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)(a+b\sinh(e+fx))^2},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*Sinh[e + f*x])^2),x]
[Out] Defer[Int][1/((c + d*x)*(a + b*Sinh[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2}\,dx$$

Mathematica [A]
time = 33.85, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*Sinh[e + f*x])^2),x]
[Out] Integrate[1/((c + d*x)*(a + b*Sinh[e + f*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(a+b\sinh\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2*(a*e^(f*x + e) - b)/(a^2*b*c*f + b^3*c*f + (a^2*b*d*f + b^3*d*f)*x - ((a
^2*b*d*f + b^3*d*f)*x*e^(2*e) + (a^2*b*c*f + b^3*c*f)*e^(2*e))*e^(2*f*x) -
2*((a^3*d*f + a*b^2*d*f)*x*e^e + (a^3*c*f + a*b^2*c*f)*e^e)*e^(f*x)) + inte
grate(2*(b*d - (a*d*f*x*e^e + (c*f + d)*a*e^e)*e^(f*x))/(a^2*b*c^2*f + b^3*
c^2*f + (a^2*b*d^2*f + b^3*d^2*f)*x^2 + 2*(a^2*b*c*d*f + b^3*c*d*f)*x - ((a
^2*b*d^2*f + b^3*d^2*f)*x^2*e^(2*e) + 2*(a^2*b*c*d*f + b^3*c*d*f)*x*e^(2*e)
 + (a^2*b*c^2*f + b^3*c^2*f)*e^(2*e))*e^(2*f*x) - 2*((a^3*d^2*f + a*b^2*d^2
*f)*x^2*e^e + 2*(a^3*c*d*f + a*b^2*c*d*f)*x*e^e + (a^3*c^2*f + a*b^2*c^2*f)
*e^e)*e^(f*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d*x + a^2*c + (b^2*d*x + b^2*c)*sinh(f*x + e)^2 + 2*(a*b*d*
x + a*b*c)*sinh(f*x + e)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sinh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(b*sinh(f*x + e) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\sinh\left(e+f\,x\right)\right)^2\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sinh(e + f*x))^2*(c + d*x)),x)
```

```
[Out] int(1/((a + b*sinh(e + f*x))^2*(c + d*x)), x)
```

3.177 $\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*Sinh[e + f*x])^2),x]
[Out] Defer[Int][1/((c + d*x)^2*(a + b*Sinh[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2}\,dx=\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2}\,dx$$

Mathematica [A]
time = 35.11, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*Sinh[e + f*x])^2),x]
[Out] Integrate[1/((c + d*x)^2*(a + b*Sinh[e + f*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx+c\right)^{2}\left(a+b\sinh\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2*(a*e^(f*x + e) - b)/(a^2*b*c^2*f + b^3*c^2*f + (a^2*b*d^2*f + b^3*d^2*f)
*x^2 + 2*(a^2*b*c*d*f + b^3*c*d*f)*x - ((a^2*b*d^2*f + b^3*d^2*f)*x^2*e^(2*
e) + 2*(a^2*b*c*d*f + b^3*c*d*f)*x*e^(2*e) + (a^2*b*c^2*f + b^3*c^2*f)*e^(2
*e))*e^(2*f*x) - 2*((a^3*d^2*f + a*b^2*d^2*f)*x^2*e^e + 2*(a^3*c*d*f + a*b^
2*c*d*f)*x*e^e + (a^3*c^2*f + a*b^2*c^2*f)*e^e)*e^(f*x)) + integrate(2*(2*b
*d - (a*d*f*x*e^e + (c*f + 2*d)*a*e^e)*e^(f*x))/(a^2*b*c^3*f + b^3*c^3*f +
(a^2*b*d^3*f + b^3*d^3*f)*x^3 + 3*(a^2*b*c*d^2*f + b^3*c*d^2*f)*x^2 + 3*(a^
2*b*c^2*d*f + b^3*c^2*d*f)*x - ((a^2*b*d^3*f + b^3*d^3*f)*x^3*e^(2*e) + 3*(
a^2*b*c*d^2*f + b^3*c*d^2*f)*x^2*e^(2*e) + 3*(a^2*b*c^2*d*f + b^3*c^2*d*f)*
x*e^(2*e) + (a^2*b*c^3*f + b^3*c^3*f)*e^(2*e))*e^(2*f*x) - 2*((a^3*d^3*f +
a*b^2*d^3*f)*x^3*e^e + 3*(a^3*c*d^2*f + a*b^2*c*d^2*f)*x^2*e^e + 3*(a^3*c^2
*d*f + a*b^2*c^2*d*f)*x*e^e + (a^3*c^3*f + a*b^2*c^3*f)*e^e)*e^(f*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*sinh(f*x + e)^2 + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*sinh
(f*x + e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*sinh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="giac")
[Out] integrate(1/((d*x + c)^2*(b*sinh(f*x + e) + a)^2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\sinh\left(e+f\,x\right)\right)^2\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*sinh(e + f*x))^2*(c + d*x)^2),x)
[Out] int(1/((a + b*sinh(e + f*x))^2*(c + d*x)^2), x)
```

3.178 $\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=544

$$\frac{3a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{5/2}d}-\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d}-\frac{3a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{5/2}d}$$

```
[Out] 3/2*a*f*ln(a+b*sinh(d*x+c))/(a^2+b^2)^2/d^2+3/2*a^2*(f*x+e)*ln(1+b*exp(d*x+
c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(5/2)/d-1/2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d-3/2*a^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a
^2+b^2)^(1/2)))/(a^2+b^2)^(5/2)/d+1/2*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2
)^(1/2)))/(a^2+b^2)^(3/2)/d+3/2*a^2*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^
(1/2)))/(a^2+b^2)^(5/2)/d^2-1/2*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2
)))/(a^2+b^2)^(3/2)/d^2-3/2*a^2*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2
)))/(a^2+b^2)^(5/2)/d^2+1/2*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/
(a^2+b^2)^(3/2)/d^2-1/2*b*(f*x+e)*cosh(d*x+c)/(a^2+b^2)/d/(a+b*sinh(d*x+c))
^2-1/2*f/(a^2+b^2)/d^2/(a+b*sinh(d*x+c))-3/2*a*b*(f*x+e)*cosh(d*x+c)/(a^2+b
^2)^2/d/(a+b*sinh(d*x+c))
```

Rubi [A]

time = 1.46, antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {3406, 3405, 3403, 2296, 2221, 2317, 2438, 2747, 31, 6874, 32}

$$\frac{3a^2f\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2d^2\left(a^2+b^2\right)^{5/2}}-\frac{3a^2f\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{2d^2\left(a^2+b^2\right)^{5/2}}-\frac{f\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2d^2\left(a^2+b^2\right)^{3/2}}+\frac{f\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{2d^2\left(a^2+b^2\right)^{3/2}}-\frac{f}{2d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}+\frac{3af\log(a+b\sinh(c+dx))}{2d^2\left(a^2+b^2\right)^2}+\frac{3a^2(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{2d\left(a^2+b^2\right)^{5/2}}-\frac{3a^2(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{2d\left(a^2+b^2\right)^{5/2}}-\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{2d\left(a^2+b^2\right)^{3/2}}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{2d\left(a^2+b^2\right)^{3/2}}-\frac{3ab(e+fx)\cosh(c+dx)}{2d\left(a^2+b^2\right)^2(a+b\sinh(c+dx))}-\frac{b(e+fx)\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] (3*a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(2*(a^2 +
b^2)^(5/2)*d) - ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/
(2*(a^2 + b^2)^(3/2)*d) - (3*a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2])])/(2*(a^2 + b^2)^(5/2)*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2])])/(2*(a^2 + b^2)^(3/2)*d) + (3*a*f*Log[a + b*Sinh[c
 + d*x]])/(2*(a^2 + b^2)^2*d^2) + (3*a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/(2*(a^2 + b^2)^(5/2)*d^2) - (f*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/(2*(a^2 + b^2)^(3/2)*d^2) - (3*a^2*f*PolyLo
g[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(2*(a^2 + b^2)^(5/2)*d^2) +
 (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(2*(a^2 + b^2)^(3
/2)*d^2) - (b*(e + f*x)*Cosh[c + d*x])/(2*(a^2 + b^2)*d*(a + b*Sinh[c + d*x
])^2) - f/(2*(a^2 + b^2)*d^2*(a + b*Sinh[c + d*x])) - (3*a*b*(e + f*x)*Cosh
[c + d*x])/(2*(a^2 + b^2)^2*d*(a + b*Sinh[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
```

```
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3406

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(-b)*(c + d*x)^m*Cos[e + f*x]*((a + b*Sin[e + f*x])^(n +
 1)/(f*(n + 1)*(a^2 - b^2))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m*(a
+ b*Sin[e + f*x])^(n + 1), x], x] - Dist[b*((n + 2)/((n + 1)*(a^2 - b^2))),
 Int[(c + d*x)^m*Sin[e + f*x]*(a + b*Sin[e + f*x])^(n + 1), x], x] + Dist[b
*d*(m/(f*(n + 1)*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && ILtQ[n, -2] && IGtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx = -\frac{b(e+fx)\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2} + \frac{a\int \frac{e+fx}{(a+b\sinh(c+dx))^2}\,dx}{a^2+b^2} - \frac{b\int \frac{(e+fx)\sinh(c-}{(a+b\sinh(c+d}}{2\left(a^2+b^2\right.}$$

$$= -\frac{b(e+fx)\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2} - \frac{ab(e+fx)\cosh(c+dx)}{\left(a^2+b^2\right)^2 d(a+b\sinh(c+dx))} + \frac{a^2\int}{}$$

$$= -\frac{b(e+fx)\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2} - \frac{f}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))} - \frac{}{(a}$$

$$= \frac{af\log(a+b\sinh(c+dx))}{\left(a^2+b^2\right)^2 d^2} - \frac{b(e+fx)\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2} - \frac{}{2\left(a^2+b^2\right)}$$

$$= \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}d} - \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}d} + \cdots$$

$$= \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}d} - \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d} - \frac{a^2}{}$$

$$= \frac{3a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{5/2}d} - \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d} - \frac{3}{}$$

$$= \frac{3a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{5/2}d} - \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d} - \frac{3}{}$$

$$= \frac{3a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{5/2}d} - \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d} - \frac{3}{}$$

Mathematica [A]
time = 7.08, size = 774, normalized size = 1.42

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] -1/2*((-3*a*Sqrt[-(a^2 + b^2)^2]*f*(c + d*x) + 6*a^2*Sqrt[a^2 + b^2]*f*ArcT
an[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]] - 4*a^2*Sqrt[-a^2 - b^2]*d*e*ArcTa
nh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*b^2*Sqrt[-a^2 - b^2]*d*e*ArcTan
h[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 6*a^2*Sqrt[-a^2 - b^2]*f*ArcTanh[(
a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 4*a^2*Sqrt[-a^2 - b^2]*c*f*ArcTanh[(a
 + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*b^2*Sqrt[-a^2 - b^2]*c*f*ArcTanh[(a
+ b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*a^2*Sqrt[-a^2 - b^2]*f*(c + d*x)*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - b^2*Sqrt[-a^2 - b^2]*f*(c + d*
x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*a^2*Sqrt[-a^2 - b^2]*
f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + b^2*Sqrt[-a^2
- b^2]*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 3*a*Sqr
t[-(a^2 + b^2)^2]*f*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + Sqrt[
-a^2 - b^2]*(2*a^2 - b^2)*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2
])] + Sqrt[-a^2 - b^2]*(-2*a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))]])/(-(a^2 + b^2)^2)^(3/2) + (b*d*(e + f*x)*Cosh[c + d*x])/(
(a^2 + b^2)*(a + b*Sinh[c + d*x])^2) + ((a^2 + b^2)*f + 3*a*b*d*(e + f*x)*C
osh[c + d*x])/((a^2 + b^2)^2*(a + b*Sinh[c + d*x])))/d^2
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1231 vs. $2(480) = 960$.
time = 1.57, size = 1232, normalized size = 2.26

method	result	size
risch	Expression too large to display	1232

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(a+b*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (2*a^2*b*d*f*x*exp(3*d*x+3*c)-b^3*d*f*x*exp(3*d*x+3*c)+6*a^3*d*f*x*exp(2*d*
x+2*c)+2*a^2*b*d*e*exp(3*d*x+3*c)-3*a*b^2*d*f*x*exp(2*d*x+2*c)-b^3*d*e*exp(
3*d*x+3*c)+6*a^3*d*e*exp(2*d*x+2*c)-10*a^2*b*d*f*x*exp(d*x+c)-a^2*b*f*exp(3
*d*x+3*c)-3*a*b^2*d*e*exp(2*d*x+2*c)-b^3*d*f*x*exp(d*x+c)-b^3*f*exp(3*d*x+3
*c)-2*a^3*f*exp(2*d*x+2*c)-10*a^2*b*d*e*exp(d*x+c)+3*a*b^2*d*f*x-2*a*b^2*f*
exp(2*d*x+2*c)-b^3*d*e*exp(d*x+c)+a^2*b*f*exp(d*x+c)+3*d*e*a*b^2+b^3*f*exp(
d*x+c))/d^2/(a^2+b^2)^2/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)^2-1/(a^2+b^2)^(
5/2)/d^2*b^2*f*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-3/(a^2+b
^2)^2/d^2*a*f*ln(exp(d*x+c))+3/2/(a^2+b^2)^2/d^2*a*f*ln(b*exp(2*d*x+2*c)+2*
a*exp(d*x+c)-b)-2/(a^2+b^2)^(5/2)/d*a^2*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/
(a^2+b^2)^(1/2))+2/(a^2+b^2)^(5/2)/d^2*a^2*f*c*arctanh(1/2*(2*b*exp(d*x+c)+
2*a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)^(5/2)/d*a^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)
^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/(a^2+b^2)^(5/2)/d^2*a^2*f*ln((-b*exp(d*
x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/(a^2+b^2)^(5/2)/d*a^2*f*l
n((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/(a^2+b^2)^(5/2)
/d^2*a^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/(a^
2+b^2)^(5/2)/d^2*a^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2
```

```
)^(1/2)))-1/(a^2+b^2)^(5/2)/d^2*a^2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a
)/(a+(a^2+b^2)^(1/2)))+1/(a^2+b^2)^(5/2)/d*b^2*e*arctanh(1/2*(2*b*exp(d*x+c
)+2*a)/(a^2+b^2)^(1/2))-1/2/(a^2+b^2)^(5/2)/d*b^2*f*ln((-b*exp(d*x+c)+(a^2+
b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/2/(a^2+b^2)^(5/2)/d^2*b^2*f*ln((-b*
exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/2/(a^2+b^2)^(5/2)/d
*b^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/2/(a^2+
b^2)^(5/2)/d^2*b^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)
))*c-1/2/(a^2+b^2)^(5/2)/d^2*b^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/
(-a+(a^2+b^2)^(1/2)))+1/2/(a^2+b^2)^(5/2)/d^2*b^2*f*dilog((b*exp(d*x+c)+(a^
2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a^2*d*integrate(x*e^(d*x + c)/(a^4*b*d*e^(2*d*x + 2*c) + 2*a^2*b^3*d
*e^(2*d*x + 2*c) + b^5*d*e^(2*d*x + 2*c) + 2*a^5*d*e^(d*x + c) + 4*a^3*b^2*
d*e^(d*x + c) + 2*a*b^4*d*e^(d*x + c) - a^4*b*d - 2*a^2*b^3*d - b^5*d), x)
- 2*b^2*d*integrate(x*e^(d*x + c)/(a^4*b*d*e^(2*d*x + 2*c) + 2*a^2*b^3*d*e^
(2*d*x + 2*c) + b^5*d*e^(2*d*x + 2*c) + 2*a^5*d*e^(d*x + c) + 4*a^3*b^2*d*e
^(d*x + c) + 2*a*b^4*d*e^(d*x + c) - a^4*b*d - 2*a^2*b^3*d - b^5*d), x) + 3
*a*b*(a*log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqrt
(a^2 + b^2)))/((a^4*b + 2*a^2*b^3 + b^5)*sqrt(a^2 + b^2)*d^2) - 2*(d*x + c)
/((a^4*b + 2*a^2*b^3 + b^5)*d^2) + log(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c)
- b)/((a^4*b + 2*a^2*b^3 + b^5)*d^2)) + 2*(3*a*b^2*d*x - (a^2*b*e^(3*c) + b
^3*e^(3*c) - (2*a^2*b*d*e^(3*c) - b^3*d*e^(3*c))*x)*e^(3*d*x) - (2*a^3*e^(2
*c) + 2*a*b^2*e^(2*c) - 3*(2*a^3*d*e^(2*c) - a*b^2*d*e^(2*c))*x)*e^(2*d*x)
+ (a^2*b*e^c + b^3*e^c - (10*a^2*b*d*e^c + b^3*d*e^c)*x)*e^(d*x))/(a^4*b^2*
d^2 + 2*a^2*b^4*d^2 + b^6*d^2 + (a^4*b^2*d^2*e^(4*c) + 2*a^2*b^4*d^2*e^(4*c
) + b^6*d^2*e^(4*c))*e^(4*d*x) + 4*(a^5*b*d^2*e^(3*c) + 2*a^3*b^3*d^2*e^(3*
c) + a*b^5*d^2*e^(3*c))*e^(3*d*x) + 2*(2*a^6*d^2*e^(2*c) + 3*a^4*b^2*d^2*e^
(2*c) - b^6*d^2*e^(2*c))*e^(2*d*x) - 4*(a^5*b*d^2*e^c + 2*a^3*b^3*d^2*e^c +
 a*b^5*d^2*e^c)*e^(d*x)) - 3*a^2*log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/
(b*e^(d*x + c) + a + sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 +
b^2)*d^2))*f + 1/2*((2*a^2 - b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2)
)/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2
 + b^2)*d) - 2*(3*a*b^2 + (10*a^2*b + b^3)*e^(-d*x - c) + 3*(2*a^3 - a*b^2)
*e^(-2*d*x - 2*c) - (2*a^2*b - b^3)*e^(-3*d*x - 3*c))/((a^4*b^2 + 2*a^2*b^4
 + b^6 + 4*(a^5*b + 2*a^3*b^3 + a*b^5)*e^(-d*x - c) + 2*(2*a^6 + 3*a^4*b^2
- b^6)*e^(-2*d*x - 2*c) - 4*(a^5*b + 2*a^3*b^3 + a*b^5)*e^(-3*d*x - 3*c) +
(a^4*b^2 + 2*a^2*b^4 + b^6)*e^(-4*d*x - 4*c))*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7272 vs. $2(482) = 964$.
time = 0.46, size = 7272, normalized size = 13.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*((a^3*b^2 + a*b^4)*d*f*x + (a^3*b^2 + a*b^4)*c*f)*cosh(d*x + c)^4 +
 6*((a^3*b^2 + a*b^4)*d*f*x + (a^3*b^2 + a*b^4)*c*f)*sinh(d*x + c)^4 + 2*((
10*a^4*b + 11*a^2*b^3 + b^5)*d*f*x - (2*a^4*b + a^2*b^3 - b^5)*d*cosh(1) -
(2*a^4*b + a^2*b^3 - b^5)*d*sinh(1) + (a^4*b + 2*a^2*b^3 + b^5 + 12*(a^4*b
+ a^2*b^3)*c)*f)*cosh(d*x + c)^3 + 2*((10*a^4*b + 11*a^2*b^3 + b^5)*d*f*x -
 (2*a^4*b + a^2*b^3 - b^5)*d*cosh(1) - (2*a^4*b + a^2*b^3 - b^5)*d*sinh(1)
+ (a^4*b + 2*a^2*b^3 + b^5 + 12*(a^4*b + a^2*b^3)*c)*f + 12*((a^3*b^2 + a*b
^4)*d*f*x + (a^3*b^2 + a*b^4)*c*f)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(a^3*
b^2 + a*b^4)*c*f - 6*(a^3*b^2 + a*b^4)*d*cosh(1) + 2*(3*(2*a^5 + a^3*b^2 -
a*b^4)*d*f*x - 3*(2*a^5 + a^3*b^2 - a*b^4)*d*cosh(1) - 3*(2*a^5 + a^3*b^2 -
 a*b^4)*d*sinh(1) + 2*(a^5 + 2*a^3*b^2 + a*b^4 + 3*(2*a^5 + a^3*b^2 - a*b^4
)*c)*f)*cosh(d*x + c)^2 - 6*(a^3*b^2 + a*b^4)*d*sinh(1) + 2*(3*(2*a^5 + a^3
*b^2 - a*b^4)*d*f*x - 3*(2*a^5 + a^3*b^2 - a*b^4)*d*cosh(1) + 18*((a^3*b^2
+ a*b^4)*d*f*x + (a^3*b^2 + a*b^4)*c*f)*cosh(d*x + c)^2 - 3*(2*a^5 + a^3*b^
2 - a*b^4)*d*sinh(1) + 2*(a^5 + 2*a^3*b^2 + a*b^4 + 3*(2*a^5 + a^3*b^2 - a*
b^4)*c)*f + 3*((10*a^4*b + 11*a^2*b^3 + b^5)*d*f*x - (2*a^4*b + a^2*b^3 - b
^5)*d*cosh(1) - (2*a^4*b + a^2*b^3 - b^5)*d*sinh(1) + (a^4*b + 2*a^2*b^3 +
b^5 + 12*(a^4*b + a^2*b^3)*c)*f)*cosh(d*x + c))*sinh(d*x + c)^2 - ((2*a^2*b
^3 - b^5)*f*cosh(d*x + c)^4 + (2*a^2*b^3 - b^5)*f*sinh(d*x + c)^4 + 4*(2*a^
3*b^2 - a*b^4)*f*cosh(d*x + c)^3 + 2*(4*a^4*b - 4*a^2*b^3 + b^5)*f*cosh(d*x
 + c)^2 + 4*((2*a^2*b^3 - b^5)*f*cosh(d*x + c) + (2*a^3*b^2 - a*b^4)*f)*sin
h(d*x + c)^3 - 4*(2*a^3*b^2 - a*b^4)*f*cosh(d*x + c) + 2*(3*(2*a^2*b^3 - b^
5)*f*cosh(d*x + c)^2 + 6*(2*a^3*b^2 - a*b^4)*f*cosh(d*x + c) + (4*a^4*b - 4
*a^2*b^3 + b^5)*f)*sinh(d*x + c)^2 + (2*a^2*b^3 - b^5)*f + 4*((2*a^2*b^3 -
b^5)*f*cosh(d*x + c)^3 + 3*(2*a^3*b^2 - a*b^4)*f*cosh(d*x + c)^2 + (4*a^4*b
 - 4*a^2*b^3 + b^5)*f*cosh(d*x + c) - (2*a^3*b^2 - a*b^4)*f)*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + ((2*a^2*b^3
- b^5)*f*cosh(d*x + c)^4 + (2*a^2*b^3 - b^5)*f*sinh(d*x + c)^4 + 4*(2*a^3*b
^2 - a*b^4)*f*cosh(d*x + c)^3 + 2*(4*a^4*b - 4*a^2*b^3 + b^5)*f*cosh(d*x +
c)^2 + 4*((2*a^2*b^3 - b^5)*f*cosh(d*x + c) + (2*a^3*b^2 - a*b^4)*f)*sinh(d
*x + c)^3 - 4*(2*a^3*b^2 - a*b^4)*f*cosh(d*x + c) + 2*(3*(2*a^2*b^3 - b^5)*
f*cosh(d*x + c)^2 + 6*(2*a^3*b^2 - a*b^4)*f*cosh(d*x + c) + (4*a^4*b - 4*a^
2*b^3 + b^5)*f)*sinh(d*x + c)^2 + (2*a^2*b^3 - b^5)*f + 4*((2*a^2*b^3 - b^5
)*f*cosh(d*x + c)^3 + 3*(2*a^3*b^2 - a*b^4)*f*cosh(d*x + c)^2 + (4*a^4*b -
4*a^2*b^3 + b^5)*f*cosh(d*x + c) - (2*a^3*b^2 - a*b^4)*f)*sinh(d*x + c))*sq
```

```
rt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (((2*a^2*b^3 -
b^5)*d*f*x + (2*a^2*b^3 - b^5)*c*f)*cosh(d*x + c)^4 + ((2*a^2*b^3 - b^5)*d*
f*x + (2*a^2*b^3 - b^5)*c*f)*sinh(d*x + c)^4 + (2*a^2*b^3 - b^5)*d*f*x + 4*
((2*a^3*b^2 - a*b^4)*d*f*x + (2*a^3*b^2 - a*b^4)*c*f)*cosh(d*x + c)^3 + 4*(
(2*a^3*b^2 - a*b^4)*d*f*x + (2*a^3*b^2 - a*b^4)*c*f + ((2*a^2*b^3 - b^5)*d*
f*x + (2*a^2*b^3 - b^5)*c*f)*cosh(d*x + c))*sinh(d*x + c)^3 + (2*a^2*b^3 -
b^5)*c*f + 2*((4*a^4*b - 4*a^2*b^3 + b^5)*d*f*x + (4*a^4*b - 4*a^2*b^3 + b^
5)*c*f)*cosh(d*x + c)^2 + 2*((4*a^4*b - 4*a^2*b^3 + b^5)*d*f*x + (4*a^4*b -
 4*a^2*b^3 + b^5)*c*f + 3*((2*a^2*b^3 - b^5)*d*f*x + (2*a^2*b^3 - b^5)*c*f)
*cosh(d*x + c)^2 + 6*((2*a^3*b^2 - a*b^4)*d*f*x + (2*a^3*b^2 - a*b^4)*c*f)*
cosh(d*x + c))*sinh(d*x + c)^2 - 4*((2*a^3*b^2 - a*b^4)*d*f*x + (2*a^3*b^2
- a*b^4)*c*f)*cosh(d*x + c) - 4*((2*a^3*b^2 - a*b^4)*d*f*x - ((2*a^2*b^3 -
b^5)*d*f*x + (2*a^2*b^3 - b^5)*c*f)*cosh(d*x + c)^3 + (2*a^3*b^2 - a*b^4)*c
*f - 3*((2*a^3*b^2 - a*b^4)*d*f*x + (2*a^3*b^2 - a*b^4)*c*f)*cosh(d*x + c)^
2 - ((4*a^4*b - 4*a^2*b^3 + b^5)*d*f*x + (4*a^4*b - 4*a^2*b^3 + b^5)*c*f)*c
osh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) +
a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
 - b)/b) + (((2*a^2*b^3 - b^5)*d*f*x + (2*a^2*b^3 - b^5)*c*f)*cosh(d*x + c)
^4 + ((2*a^2*b^3 - b^5)*d*f*x + (2*a^2*b^3 - b^5)*c*f)*sinh(d*x + c)^4 + (2
*a^2*b^3 - b^5)*d*f*x + 4*((2*a^3*b^2 - a*b^4)*d*f*x + (2*a^3*b^2 - a*b^4)*
c*f)*cosh(d*x + c)^3 + 4*((2*a^3*b^2 - a*b^4)*d*f*x + (2*a^3*b^2 - a*b^4)*c
*f + ((2*a^2*b^3 - b^5)*d*f*x + (2*a^2*b^3 - b^5)*c*f)*cosh(d*x + c))*sinh(
d*x + c)^3 + (2*a^2*b^3 - b^5)*c*f + 2*((4*a^4*b - 4*a^2*b^3 + b^5)*d*f*x +
 (4*a^4*b - 4*a^2*b^3 + b^5)*c*f)*cosh(d*x + c)^2 + 2*((4*a^4*b - 4*a^2*b^3
 + b^5)*d*f*x + (4*a^4*b - 4*a^2*b^3 + b^5)*c*f + 3*((2*a^2*b^3 - b^5)*d*f*
x + (2*a^2*b^3 - b^5)*c*f)*cosh(d*x + c)^2 + 6*((2*a^3*b^2 - a*b^4)*d*f*x +
 (2*a^3*b^2 - a*b^4)*c*f)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*((2*a^3*b^2 -
a*b^4)*d*f*x + (2*a^3*b^2 - a*b^4)*c*f)*cosh(d*x + c) - 4*((2*a^3*b^2 - a*b
^4)*d*f*x - ((2*a^2*b^3 - b^5)*d*f*x + (2*a^2*b^3 - b^5)*c*f)*cosh(d*x + c)
^3 + (2*a^3*b^2 - a*b^4)*c*f - 3*((2*a^3*b^2 - ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((f*x + e)/(b*sinh(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{{\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(a + b*sinh(c + d*x))^3,x)
[Out] int((e + f*x)/(a + b*sinh(c + d*x))^3, x)
```

3.179 $\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{1}{(e+fx)(a+b\sinh(c+dx))^3},x\right)$$

```
[Out] Unintegrable(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((e + f*x)*(a + b*Sinh[c + d*x])^3),x]
```

```
[Out] Defer[Int][1/((e + f*x)*(a + b*Sinh[c + d*x])^3), x]
```

Rubi steps

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx = \int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx$$

Mathematica [A]
time = 69.67, size = 0, normalized size = 0.00

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((e + f*x)*(a + b*Sinh[c + d*x])^3),x]
```

```
[Out] Integrate[1/((e + f*x)*(a + b*Sinh[c + d*x])^3), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(fx+e)\left(a+b\sinh\left(dx+c\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x)
[Out] int(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
[Out] (3*a*b^2*d*f*x + 3*a*b^2*d*e + (a^2*b*f*e^(3*c) + b^3*f*e^(3*c) + (2*a^2*b*
d*f*e^(3*c) - b^3*d*f*e^(3*c))*x + (2*a^2*b*d*e^(3*c) - b^3*d*e^(3*c))*e)*e
^(3*d*x) + (2*a^3*f*e^(2*c) + 2*a*b^2*f*e^(2*c) + 3*(2*a^3*d*f*e^(2*c) - a*
b^2*d*f*e^(2*c))*x + 3*(2*a^3*d*e^(2*c) - a*b^2*d*e^(2*c))*e)*e^(2*d*x) - (
a^2*b*f*e^c + b^3*f*e^c + (10*a^2*b*d*f*e^c + b^3*d*f*e^c)*x + (10*a^2*b*d*
e^c + b^3*d*e^c)*e)*e^(d*x))/((a^4*b^2*d^2*f^2 + 2*a^2*b^4*d^2*f^2 + b^6*d^
2*f^2)*x^2 + 2*(a^4*b^2*d^2*f + 2*a^2*b^4*d^2*f + b^6*d^2*f)*x*e + (a^4*b^2
*d^2 + 2*a^2*b^4*d^2 + b^6*d^2)*e^2 + ((a^4*b^2*d^2*f^2*e^(4*c) + 2*a^2*b^4
*d^2*f^2*e^(4*c) + b^6*d^2*f^2*e^(4*c))*x^2 + 2*(a^4*b^2*d^2*f*e^(4*c) + 2*
a^2*b^4*d^2*f*e^(4*c) + b^6*d^2*f*e^(4*c))*x*e + (a^4*b^2*d^2*e^(4*c) + 2*a
^2*b^4*d^2*e^(4*c) + b^6*d^2*e^(4*c))*e^2)*e^(4*d*x) + 4*((a^5*b*d^2*f^2*e^
(3*c) + 2*a^3*b^3*d^2*f^2*e^(3*c) + a*b^5*d^2*f^2*e^(3*c))*x^2 + 2*(a^5*b*d
^2*f*e^(3*c) + 2*a^3*b^3*d^2*f*e^(3*c) + a*b^5*d^2*f*e^(3*c))*x*e + (a^5*b*
d^2*e^(3*c) + 2*a^3*b^3*d^2*e^(3*c) + a*b^5*d^2*e^(3*c))*e^2)*e^(3*d*x) + 2
*((2*a^6*d^2*f^2*e^(2*c) + 3*a^4*b^2*d^2*f^2*e^(2*c) - b^6*d^2*f^2*e^(2*c))
*x^2 + 2*(2*a^6*d^2*f*e^(2*c) + 3*a^4*b^2*d^2*f*e^(2*c) - b^6*d^2*f*e^(2*c)
)*x*e + (2*a^6*d^2*e^(2*c) + 3*a^4*b^2*d^2*e^(2*c) - b^6*d^2*e^(2*c))*e^2)*
e^(2*d*x) - 4*((a^5*b*d^2*f^2*e^c + 2*a^3*b^3*d^2*f^2*e^c + a*b^5*d^2*f^2*e
^c)*x^2 + 2*(a^5*b*d^2*f*e^c + 2*a^3*b^3*d^2*f*e^c + a*b^5*d^2*f*e^c)*x*e +
 (a^5*b*d^2*e^c + 2*a^3*b^3*d^2*e^c + a*b^5*d^2*e^c)*e^2)*e^(d*x)) + integr
ate((3*a*b*d*f^2*x + 3*a*b*d*f*e - (3*a^2*d*f*e^(c + 1) + 2*a^2*f^2*e^c + 2
*b^2*f^2*e^c + (2*a^2*d^2*f^2*e^c - b^2*d^2*f^2*e^c)*x^2 + (3*a^2*d*f^2*e^c
 + 2*(2*a^2*d^2*f*e^c - b^2*d^2*f*e^c)*e)*x + (2*a^2*d^2*e^c - b^2*d^2*e^c)
*e^2)*e^(d*x))/((a^4*b*d^2*f^3 + 2*a^2*b^3*d^2*f^3 + b^5*d^2*f^3)*x^3 + 3*(
a^4*b*d^2*f^2 + 2*a^2*b^3*d^2*f^2 + b^5*d^2*f^2)*x^2*e + 3*(a^4*b*d^2*f + 2
*a^2*b^3*d^2*f + b^5*d^2*f)*x*e^2 + (a^4*b*d^2 + 2*a^2*b^3*d^2 + b^5*d^2)*e
^3 - ((a^4*b*d^2*f^3*e^(2*c) + 2*a^2*b^3*d^2*f^3*e^(2*c) + b^5*d^2*f^3*e^(2
*c))*x^3 + 3*(a^4*b*d^2*f^2*e^(2*c) + 2*a^2*b^3*d^2*f^2*e^(2*c) + b^5*d^2*f
^2*e^(2*c))*x^2*e + 3*(a^4*b*d^2*f*e^(2*c) + 2*a^2*b^3*d^2*f*e^(2*c) + b^5*
d^2*f*e^(2*c))*x*e^2 + (a^4*b*d^2*e^(2*c) + 2*a^2*b^3*d^2*e^(2*c) + b^5*d^2
*e^(2*c))*e^3)*e^(2*d*x) - 2*((a^5*d^2*f^3*e^c + 2*a^3*b^2*d^2*f^3*e^c + a*
b^4*d^2*f^3*e^c)*x^3 + 3*(a^5*d^2*f^2*e^c + 2*a^3*b^2*d^2*f^2*e^c + a*b^4*d
^2*f^2*e^c)*x^2*e + 3*(a^5*d^2*f*e^c + 2*a^3*b^2*d^2*f*e^c + a*b^4*d^2*f*e^
c)*x*e^2 + (a^5*d^2*e^c + 2*a^3*b^2*d^2*e^c + a*b^4*d^2*e^c)*e^3)*e^(d*x)),
 x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
[Out] integral(1/(a^3*f*x + a^3*e + (b^3*f*x + b^3*e)*sinh(d*x + c)^3 + 3*(a*b^2*
f*x + a*b^2*e)*sinh(d*x + c)^2 + 3*(a^2*b*f*x + a^2*b*e)*sinh(d*x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x+e)/(a+b*sinh(d*x+c))**3,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
[Out] integrate(1/((f*x + e)*(b*sinh(d*x + c) + a)^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(e+f\,x)\,{\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e + f*x)*(a + b*sinh(c + d*x))^3),x)
[Out] int(1/((e + f*x)*(a + b*sinh(c + d*x))^3), x)
```

3.180 $\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3},x\right)$$

```
[Out] Unintegrable(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((e + f*x)^2*(a + b*Sinh[c + d*x])^3),x]
[Out] Defer[Int][1/((e + f*x)^2*(a + b*Sinh[c + d*x])^3), x]
```

Rubi steps

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx = \int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx$$

Mathematica [A]
time = 65.54, size = 0, normalized size = 0.00

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((e + f*x)^2*(a + b*Sinh[c + d*x])^3),x]
[Out] Integrate[1/((e + f*x)^2*(a + b*Sinh[c + d*x])^3), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(fx+e)^2\,(a+b\sinh(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x)

[Out] int(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")

[Out] (3*a*b^2*d*f*x + 3*a*b^2*d*e + (2*a^2*b*f*e^(3*c) + 2*b^3*f*e^(3*c) + (2*a^2*b*d*f*e^(3*c) - b^3*d*f*e^(3*c))*x + (2*a^2*b*d*e^(3*c) - b^3*d*e^(3*c))*e)*e^(3*d*x) + (4*a^3*f*e^(2*c) + 4*a*b^2*f*e^(2*c) + 3*(2*a^3*d*f*e^(2*c) - a*b^2*d*f*e^(2*c))*x + 3*(2*a^3*d*e^(2*c) - a*b^2*d*e^(2*c))*e)*e^(2*d*x) - (2*a^2*b*f*e^c + 2*b^3*f*e^c + (10*a^2*b*d*f*e^c + b^3*d*f*e^c)*x + (10*a^2*b*d*e^c + b^3*d*e^c)*e)*e^(d*x))/((a^4*b^2*d^2*f^3 + 2*a^2*b^4*d^2*f^3 + b^6*d^2*f^3)*x^3 + 3*(a^4*b^2*d^2*f^2 + 2*a^2*b^4*d^2*f^2 + b^6*d^2*f^2)*x^2*e + 3*(a^4*b^2*d^2*f + 2*a^2*b^4*d^2*f + b^6*d^2*f)*x*e^2 + (a^4*b^2*d^2 + 2*a^2*b^4*d^2 + b^6*d^2)*e^3 + ((a^4*b^2*d^2*f^3*e^(4*c) + 2*a^2*b^4*d^2*f^3*e^(4*c) + b^6*d^2*f^3*e^(4*c))*x^3 + 3*(a^4*b^2*d^2*f^2*e^(4*c) + 2*a^2*b^4*d^2*f^2*e^(4*c) + b^6*d^2*f^2*e^(4*c))*x^2*e + 3*(a^4*b^2*d^2*f*e^(4*c) + 2*a^2*b^4*d^2*f*e^(4*c) + b^6*d^2*f*e^(4*c))*x*e^2 + (a^4*b^2*d^2*e^(4*c) + 2*a^2*b^4*d^2*e^(4*c) + b^6*d^2*e^(4*c))*e^3)*e^(4*d*x) + 4*((a^5*b*d^2*f^3*e^(3*c) + 2*a^3*b^3*d^2*f^3*e^(3*c) + a*b^5*d^2*f^3*e^(3*c))*x^3 + 3*(a^5*b*d^2*f^2*e^(3*c) + 2*a^3*b^3*d^2*f^2*e^(3*c) + a*b^5*d^2*f^2*e^(3*c))*x^2*e + 3*(a^5*b*d^2*f*e^(3*c) + 2*a^3*b^3*d^2*f*e^(3*c) + a*b^5*d^2*f*e^(3*c))*x*e^2 + (a^5*b*d^2*e^(3*c) + 2*a^3*b^3*d^2*e^(3*c) + a*b^5*d^2*e^(3*c))*e^3)*e^(3*d*x) + 2*((2*a^6*d^2*f^3*e^(2*c) + 3*a^4*b^2*d^2*f^3*e^(2*c) - b^6*d^2*f^3*e^(2*c))*x^3 + 3*(2*a^6*d^2*f^2*e^(2*c) + 3*a^4*b^2*d^2*f^2*e^(2*c) - b^6*d^2*f^2*e^(2*c))*x^2*e + 3*(2*a^6*d^2*f*e^(2*c) + 3*a^4*b^2*d^2*f*e^(2*c) - b^6*d^2*f*e^(2*c))*x*e^2 + (2*a^6*d^2*e^(2*c) + 3*a^4*b^2*d^2*e^(2*c) - b^6*d^2*e^(2*c))*e^3)*e^(2*d*x) - 4*((a^5*b*d^2*f^3*e^c + 2*a^3*b^3*d^2*f^3*e^c + a*b^5*d^2*f^3*e^c)*x^3 + 3*(a^5*b*d^2*f^2*e^c + 2*a^3*b^3*d^2*f^2*e^c + a*b^5*d^2*f^2*e^c)*x^2*e + 3*(a^5*b*d^2*f*e^c + 2*a^3*b^3*d^2*f*e^c + a*b^5*d^2*f*e^c)*x*e^2 + (a^5*b*d^2*e^c + 2*a^3*b^3*d^2*e^c + a*b^5*d^2*e^c)*e^3)*e^(d*x)) + integrate((6*a*b*d*f^2*x + 6*a*b*d*f*e - (6*a^2*d*f*e^(c + 1) + 6*a^2*f^2*e^c + 6*b^2*f^2*e^c + (2*a^2*d^2*f^2*e^c - b^2*d^2*f^2*e^c)*x^2 + 2*(3*a^2*d*f^2*e^c + (2*a^2*d^2*f*e^c - b^2*d^2*f*e^c)*e)*x + (2*a^2*d^2*e^c - b^2*d^2*e^c)*e^2)*e^(d*x))/((a^4*b*d^2*f^4 + 2*a^2*b^3*d^2*f^4 + b^5*d^2*f^4)*x^4 + 4*(a^4*b*d^2*f^3 + 2*a^2*b^3*d^2*f^3 + b^5*d^2*f^3)*x^3*e + 6*(a^4*b*d^2*f^2 + 2*a^2*b^3*d^2*f^2 + b^5*d^2*f^2)*x^2*e^2 + 4*(a^4*b*d^2*f + 2*a^2*b^3*d^2*f + b^5*d^2*f)*x*e^3 + (a^4*b*d^2 + 2*a^2*b^3*d^2 + b^5*d^2)*e^4 - ((a^4*b*d^2*f^4*e^(2*c) + 2*a^2*b^3*d^2*f^4*e^(2*c) + b^5*d^2*f^4*e^(2*c))*x^4 + 4*(a^4*b*d^2*f^3*e^(2*c) + 2*a^2*b^3*d^2*f

```
^3*e^(2*c) + b^5*d^2*f^3*e^(2*c))*x^3*e + 6*(a^4*b*d^2*f^2*e^(2*c) + 2*a^2*
b^3*d^2*f^2*e^(2*c) + b^5*d^2*f^2*e^(2*c))*x^2*e^2 + 4*(a^4*b*d^2*f*e^(2*c)
 + 2*a^2*b^3*d^2*f*e^(2*c) + b^5*d^2*f*e^(2*c))*x*e^3 + (a^4*b*d^2*e^(2*c)
+ 2*a^2*b^3*d^2*e^(2*c) + b^5*d^2*e^(2*c))*e^4)*e^(2*d*x) - 2*((a^5*d^2*f^4
*e^c + 2*a^3*b^2*d^2*f^4*e^c + a*b^4*d^2*f^4*e^c)*x^4 + 4*(a^5*d^2*f^3*e^c
+ 2*a^3*b^2*d^2*f^3*e^c + a*b^4*d^2*f^3*e^c)*x^3*e + 6*(a^5*d^2*f^2*e^c + 2
*a^3*b^2*d^2*f^2*e^c + a*b^4*d^2*f^2*e^c)*x^2*e^2 + 4*(a^5*d^2*f*e^c + 2*a^
3*b^2*d^2*f*e^c + a*b^4*d^2*f*e^c)*x*e^3 + (a^5*d^2*e^c + 2*a^3*b^2*d^2*e^c
 + a*b^4*d^2*e^c)*e^4)*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(a^3*f^2*x^2 + 2*a^3*f*x*e + a^3*e^2 + (b^3*f^2*x^2 + 2*b^3*f*x*
e + b^3*e^2)*sinh(d*x + c)^3 + 3*(a*b^2*f^2*x^2 + 2*a*b^2*f*x*e + a*b^2*e^2
)*sinh(d*x + c)^2 + 3*(a^2*b*f^2*x^2 + 2*a^2*b*f*x*e + a^2*b*e^2)*sinh(d*x
+ c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x+e)**2/(a+b*sinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((f*x + e)^2*(b*sinh(d*x + c) + a)^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(e+f\,x\right)}^2\,{\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e + f*x)^2*(a + b*sinh(c + d*x))^3),x)
[Out] int(1/((e + f*x)^2*(a + b*sinh(c + d*x))^3), x)
```

3.181 $\int (c+dx)^m (a+b\sinh(e+fx))^n\,dx$

Optimal. Leaf size=23

$$\text{Int}((c+dx)^m (a+b\sinh(e+fx))^n, x)$$

[Out] Unintegrable((d*x+c)^m*(a+b*sinh(f*x+e))^n,x)

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m (a+b\sinh(e+fx))^n\,dx$$

Verification is not applicable to the result.

[In] Int[(c + d*x)^m*(a + b*Sinh[e + f*x])^n,x]

[Out] Defer[Int][(c + d*x)^m*(a + b*Sinh[e + f*x])^n, x]

Rubi steps

$$\int (c+dx)^m (a+b\sinh(e+fx))^n\,dx = \int (c+dx)^m (a+b\sinh(e+fx))^n\,dx$$

Mathematica [A]
time = 3.04, size = 0, normalized size = 0.00

$$\int (c+dx)^m (a+b\sinh(e+fx))^n\,dx$$

Verification is not applicable to the result.

[In] Integrate[(c + d*x)^m*(a + b*Sinh[e + f*x])^n,x]

[Out] Integrate[(c + d*x)^m*(a + b*Sinh[e + f*x])^n, x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(a+b\sinh\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*sinh(f*x+e))^n,x)
[Out] int((d*x+c)^m*(a+b*sinh(f*x+e))^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*(b*sinh(f*x + e) + a)^n, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e))^n,x, algorithm="fricas")
[Out] integral((d*x + c)^m*(b*sinh(f*x + e) + a)^n, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*sinh(f*x+e))**n,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e))^n,x, algorithm="giac")
[Out] integrate((d*x + c)^m*(b*sinh(f*x + e) + a)^n, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (a+b\sinh(e+f\,x))^n\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x))^n*(c + d*x)^m,x)
[Out] int((a + b*sinh(e + f*x))^n*(c + d*x)^m, x)
```

3.182 $\int (c+dx)^m (a+b\sinh(e+fx))^3 \, dx$

Optimal. Leaf size=543

$$\frac{a^3(c+dx)^{1+m}}{d(1+m)}-\frac{3ab^2(c+dx)^{1+m}}{2d(1+m)}+\frac{3^{-1-m}b^3e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3f(c+dx)}{d}\right)}{8f}+\frac{3\,2^{-3-m}a}{}$$

```
[Out] a^3*(d*x+c)^(1+m)/d/(1+m)-3/2*a*b^2*(d*x+c)^(1+m)/d/(1+m)+1/8*3^(-1-m)*b^3*
exp(3*e-3*c*f/d)*(d*x+c)^m*GAMMA(1+m,-3*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+3
*2^(-3-m)*a*b^2*exp(2*e-2*c*f/d)*(d*x+c)^m*GAMMA(1+m,-2*f*(d*x+c)/d)/f/((-f
*(d*x+c)/d)^m)+3/2*a^2*b*exp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/d)/f/(
(-f*(d*x+c)/d)^m)-3/8*b^3*exp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/d)/f/
((-f*(d*x+c)/d)^m)+3/2*a^2*b*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x+c)/d)
/f/((f*(d*x+c)/d)^m)-3/8*b^3*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x+c)/d)
/f/((f*(d*x+c)/d)^m)-3*2^(-3-m)*a*b^2*exp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA(1+m
,2*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)+1/8*3^(-1-m)*b^3*exp(-3*e+3*c*f/d)*(d*x
+c)^m*GAMMA(1+m,3*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)
```

Rubi [A]

time = 0.56, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3398, 3389, 2212, 3393, 3388}

$$\frac{3a^2be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}+\frac{3a^2be^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+\frac{3ab^22^{-m-3}e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2f(c+dx)}{d}\right)}{f}-\frac{3ab^22^{-m-3}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2f(c+dx)}{d}\right)}{f}+\frac{b^33^{-m-1}e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{3f(c+dx)}{d}\right)}{8f}-\frac{3b^3e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{8f}-\frac{3b^3e^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{8f}+\frac{b^33^{-m-1}e^{\frac{3cf}{d}-3e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{3f(c+dx)}{d}\right)}{8f}+\frac{a^3(c+dx)^{m+1}}{d(m+1)}-\frac{3ab^2(c+dx)^{m+1}}{2d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + b*Sinh[e + f*x])^3,x]
```

```
[Out] (a^3*(c + d*x)^(1 + m))/(d*(1 + m)) - (3*a*b^2*(c + d*x)^(1 + m))/(2*d*(1 +
 m)) + (3^(-1 - m)*b^3*E^(3*e - (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-3*f*(
c + d*x))/d])/(8*f*(-((f*(c + d*x))/d))^m) + (3*2^(-3 - m)*a*b^2*E^(2*e - (
2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c + d*x))/d])/(f*(-((f*(c + d*x))
/d))^m) + (3*a^2*b*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c + d*x))
/d)])/(2*f*(-((f*(c + d*x))/d))^m) - (3*b^3*E^(e - (c*f)/d)*(c + d*x)^m*Gam
ma[1 + m, -((f*(c + d*x))/d)])/(8*f*(-((f*(c + d*x))/d))^m) + (3*a^2*b*E^(-
e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c + d*x))
/d)^m) - (3*b^3*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])
/(8*f*((f*(c + d*x))/d)^m) - (3*2^(-3 - m)*a*b^2*E^(-2*e + (2*c*f)/d)*(c +
d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) + (3^(-1 -
m)*b^3*E^(-3*e + (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (3*f*(c + d*x))/d])/(8
*f*((f*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
```

```
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (c+dx)^m(a+b\sinh(e+fx))^3\,dx = \int \left(a^3(c+dx)^m + 3a^2b(c+dx)^m\sinh(e+fx) + 3ab^2(c+dx)^m\sinh^2\right.$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + (3a^2b)\int (c+dx)^m\sinh(e+fx)\,dx + (3ab^2)\int (c+$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \frac{1}{2}(3a^2b)\int e^{-i(ie+ifx)}(c+dx)^m\,dx - \frac{1}{2}(3a^2b)\int e^{i(i}$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} - \frac{3ab^2(c+dx)^{1+m}}{2d(1+m)} + \frac{3a^2be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)}{2f}$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} - \frac{3ab^2(c+dx)^{1+m}}{2d(1+m)} + \frac{3a^2be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)}{2f}$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} - \frac{3ab^2(c+dx)^{1+m}}{2d(1+m)} + \frac{3^{-1-m}b^3e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c}{}\right.}{8f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2639 vs. $2(543) = 1086$.
time = 18.52, size = 2639, normalized size = 4.86

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + b*Sinh[e + f*x])^3,x]
```

```
[Out] (a^3*(c + d*x)^m*(c*f + d*f*x))/(d*f*(1 + m)) + (3*a^2*b*((f*Cosh[((-c + (d
*e)/f)*f)/d]*(-((c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*(-((f*(c - (d*e)/f
+ (d*(e + f*x))/f))/d))^(-1 - m)*Gamma[1 + m, -((f*(c - (d*e)/f + (d*(e + f
*x))/f))/d)]) + (c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*((f*(c - (d*e)/f +
(d*(e + f*x))/f))/d)^(-1 - m)*Gamma[1 + m, (f*(c - (d*e)/f + (d*(e + f*x))/
f))/d]))/(2*d) + (f*(-((c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*(-((f*(c - (
d*e)/f + (d*(e + f*x))/f))/d))^(-1 - m)*Gamma[1 + m, -((f*(c - (d*e)/f + (d
*(e + f*x))/f))/d)]) - (c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*((f*(c - (d*
e)/f + (d*(e + f*x))/f))/d)^(-1 - m)*Gamma[1 + m, (f*(c - (d*e)/f + (d*(e +
 f*x))/f))/d])*Sinh[((-c + (d*e)/f)*f)/d])/(2*d)))/f + (3*a*b^2*((f*Cosh[((
-c + (d*e)/f)*f)/d]^2*(-1/2*(c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)/(1 + m)
 + (-(2^(-1 - m)*(c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*(-((f*(c - (d*e)/f
 + (d*(e + f*x))/f))/d))^(-1 - m)*Gamma[1 + m, (-2*f*(c - (d*e)/f + (d*(e +
 f*x))/f))/d]) - 2^(-1 - m)*(c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*((f*(c
- (d*e)/f + (d*(e + f*x))/f))/d)^(-1 - m)*Gamma[1 + m, (2*f*(c - (d*e)/f +
(d*(e + f*x))/f))/d])/4))/d + (f*Cosh[((-c + (d*e)/f)*f)/d]*(-(2^(-1 - m)*(
```

```
c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*(-((f*(c - (d*e)/f + (d*(e + f*x))/f
))/d))^(-1 - m)*Gamma[1 + m, (-2*f*(c - (d*e)/f + (d*(e + f*x))/f))/d]) + 2
^(-1 - m)*(c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*((f*(c - (d*e)/f + (d*(e
 + f*x))/f))/d)^(-1 - m)*Gamma[1 + m, (2*f*(c - (d*e)/f + (d*(e + f*x))/f))/
d])*Sinh[((-c + (d*e)/f)*f)/d])/(2*d) + (f*((c - (d*e)/f + (d*(e + f*x))/f)
^(1 + m)/(2*(1 + m)) + (-(2^(-1 - m)*(c - (d*e)/f + (d*(e + f*x))/f)^(1 + m
)*(-((f*(c - (d*e)/f + (d*(e + f*x))/f))/d))^(-1 - m)*Gamma[1 + m, (-2*f*(c
 - (d*e)/f + (d*(e + f*x))/f))/d]) - 2^(-1 - m)*(c - (d*e)/f + (d*(e + f*x)
)/f)^(1 + m)*((f*(c - (d*e)/f + (d*(e + f*x))/f))/d)^(-1 - m)*Gamma[1 + m,
(2*f*(c - (d*e)/f + (d*(e + f*x))/f))/d])/4)*Sinh[((-c + (d*e)/f)*f)/d]^2)/
d))/f + (b^3*((f*Cosh[((-c + (d*e)/f)*f)/d]^3*((-3*(-((c - (d*e)/f + (d*(e
 + f*x))/f)^(1 + m)*(-((f*(c - (d*e)/f + (d*(e + f*x))/f))/d))^(-1 - m)*Gamm
a[1 + m, -((f*(c - (d*e)/f + (d*(e + f*x))/f))/d)]) + (c - (d*e)/f + (d*(e
 + f*x))/f)^(1 + m)*((f*(c - (d*e)/f + (d*(e + f*x))/f))/d)^(-1 - m)*Gamma[1
 + m, (f*(c - (d*e)/f + (d*(e + f*x))/f))/d]))/8 + (-(3^(-1 - m)*(c - (d*e)
/f + (d*(e + f*x))/f)^(1 + m)*(-((f*(c - (d*e)/f + (d*(e + f*x))/f))/d))^(-
1 - m)*Gamma[1 + m, (-3*f*(c - (d*e)/f + (d*(e + f*x))/f))/d]) + 3^(-1 - m)
*(c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*((f*(c - (d*e)/f + (d*(e + f*x))/f
))/d)^(-1 - m)*Gamma[1 + m, (3*f*(c - (d*e)/f + (d*(e + f*x))/f))/d])/8))/d
 + (f*((3*(-((c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*(-((f*(c - (d*e)/f + (
d*(e + f*x))/f))/d))^(-1 - m)*Gamma[1 + m, -((f*(c - (d*e)/f + (d*(e + f*x)
)/f))/d)]) - (c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*((f*(c - (d*e)/f + (d*
(e + f*x))/f))/d)^(-1 - m)*Gamma[1 + m, (f*(c - (d*e)/f + (d*(e + f*x))/f))
/d]))/8 + (-(3^(-1 - m)*(c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*(-((f*(c -
(d*e)/f + (d*(e + f*x))/f))/d))^(-1 - m)*Gamma[1 + m, (-3*f*(c - (d*e)/f +
(d*(e + f*x))/f))/d]) - 3^(-1 - m)*(c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*
((f*(c - (d*e)/f + (d*(e + f*x))/f))/d)^(-1 - m)*Gamma[1 + m, (3*f*(c - (d*
e)/f + (d*(e + f*x))/f))/d])/8)*Sinh[((-c + (d*e)/f)*f)/d]^3)/d + ((c - (d*
e)/f + (d*(e + f*x))/f)^m*Cosh[e - (c*f)/d]^2*((-((f*(c - (d*e)/f + (d*(e +
 f*x))/f))/d))^m*((f*(c - (d*e)/f + (d*(e + f*x))/f))/d)^(2*m)*Gamma[1 + m,
 (-3*f*(c - (d*e)/f + (d*(e + f*x))/f))/d] - (-((f^2*(c - (d*e)/f + (d*(e +
 f*x))/f)^2)/d^2))^m*(3^(1 + m)*((f*(c - (d*e)/f + (d*(e + f*x))/f))/d)^m*G
amma[1 + m, -((f*(c - (d*e)/f + (d*(e + f*x))/f))/d)] + (-((f*(c - (d*e)/f
 + (d*(e + f*x))/f))/d))^m*(-(3^(1 + m)*Gamma[1 + m, (f*(c - (d*e)/f + (d*(e
 + f*x))/f))/d]) + Gamma[1 + m, (3*f*(c - (d*e)/f + (d*(e + f*x))/f))/d])))
*Sinh[e - (c*f)/d])/(8*3^m*(-((f^2*(c - (d*e)/f + (d*(e + f*x))/f)^2)/d^2))
^(2*m)) + ((c - (d*e)/f + (d*(e + f*x))/f)^m*Cosh[e - (c*f)/d]*((-((f*(c -
(d*e)/f + (d*(e + f*x))/f))/d))^m*((f*(c - (d*e)/f + (d*(e + f*x))/f))/d)^(
2*m)*Gamma[1 + m, (-3*f*(c - (d*e)/f + (d*(e + f*x))/f))/d] + (-((f^2*(c -
(d*e)/f + (d*(e + f*x))/f)^2)/d^2))^m*(3^(1 + m)*((f*(c - (d*e)/f + (d*(e +
 f*x))/f))/d)^m*Gamma[1 + m, -((f*(c - (d*e)/f + (d*(e + f*x))/f))/d)] + (-
((f*(c - (d*e)/f + (d*(e + f*x))/f))/d))^m*(3^(1 + m)*Gamma[1 + m, (f*(c -
(d*e)/f + (d*(e + f*x))/f))/d] + Gamma[1 + m, (3*f*(c - (d*e)/f + (d*(e + f
*x))/f))/d])))*Sinh[e - (c*f)/d]^2)/(8*3^m*(-((f^2*(c - (d*e)/f + (d*(e + f
*x))/f)^2)/d^2))^(2*m))))/f
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m (a+b\sinh (fx+e))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*sinh(f*x+e))^3,x)
```

```
[Out] int((d*x+c)^m*(a+b*sinh(f*x+e))^3,x)
```

Maxima [A]
time = 0.12, size = 385, normalized size = 0.71

$$\frac{3}{2}\left(\frac{(dx+c)^{m+1}e^{\left(\frac{cf}{d}-e\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}-\frac{(dx+c)^{m+1}e^{\left(-\frac{cf}{d}+e\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)a^2b-\frac{3}{4}\left(\frac{(dx+c)^{m+1}e^{\left(\frac{2cf}{d}-2e\right)}E_{-m}\left(\frac{2(dx+c)f}{d}\right)}{d}+\frac{(dx+c)^{m+1}e^{\left(-\frac{2cf}{d}+2e\right)}E_{-m}\left(-\frac{2(dx+c)f}{d}\right)}{d}+\frac{2(dx+c)^{m+1}}{d(m+1)}\right)ab^2+\frac{1}{8}\left(\frac{(dx+c)^{m+1}e^{\left(\frac{3cf}{d}-3e\right)}E_{-m}\left(\frac{3(dx+c)f}{d}\right)}{d}-\frac{3(dx+c)^{m+1}e^{\left(\frac{cf}{d}-e\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{3(dx+c)^{m+1}e^{\left(-\frac{cf}{d}+e\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}-\frac{(dx+c)^{m+1}e^{\left(-\frac{3cf}{d}+3e\right)}E_{-m}\left(-\frac{3(dx+c)f}{d}\right)}{d}\right)b^3+\frac{(dx+c)^{m+1}a^3}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 3/2*((d*x + c)^(m + 1)*e^(c*f/d - e)*exp_integral_e(-m, (d*x + c)*f/d)/d -
(d*x + c)^(m + 1)*e^(-c*f/d + e)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a^2*
b - 3/4*((d*x + c)^(m + 1)*e^(2*c*f/d - 2*e)*exp_integral_e(-m, 2*(d*x + c)
*f/d)/d + (d*x + c)^(m + 1)*e^(-2*c*f/d + 2*e)*exp_integral_e(-m, -2*(d*x +
 c)*f/d)/d + 2*(d*x + c)^(m + 1)/(d*(m + 1)))*a*b^2 + 1/8*((d*x + c)^(m + 1
)*e^(3*c*f/d - 3*e)*exp_integral_e(-m, 3*(d*x + c)*f/d)/d - 3*(d*x + c)^(m
+ 1)*e^(c*f/d - e)*exp_integral_e(-m, (d*x + c)*f/d)/d + 3*(d*x + c)^(m + 1
)*e^(-c*f/d + e)*exp_integral_e(-m, -(d*x + c)*f/d)/d - (d*x + c)^(m + 1)*e
^(-3*c*f/d + 3*e)*exp_integral_e(-m, -3*(d*x + c)*f/d)/d)*b^3 + (d*x + c)^(
m + 1)*a^3/(d*(m + 1))
```

Fricas [A]
time = 0.11, size = 899, normalized size = 1.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/24*((b^3*d*m + b^3*d)*cosh((d*m*log(3*f/d) - 3*c*f + 3*d*cosh(1) + 3*d*si
nh(1))/d)*gamma(m + 1, 3*(d*f*x + c*f)/d) - 9*(a*b^2*d*m + a*b^2*d)*cosh((d
*m*log(2*f/d) - 2*c*f + 2*d*cosh(1) + 2*d*sinh(1))/d)*gamma(m + 1, 2*(d*f*x
 + c*f)/d) + 9*((4*a^2*b - b^3)*d*m + (4*a^2*b - b^3)*d)*cosh((d*m*log(f/d)
 - c*f + d*cosh(1) + d*sinh(1))/d)*gamma(m + 1, (d*f*x + c*f)/d) + 9*((4*a^
2*b - b^3)*d*m + (4*a^2*b - b^3)*d)*cosh((d*m*log(-f/d) + c*f - d*cosh(1) -
 d*sinh(1))/d)*gamma(m + 1, -(d*f*x + c*f)/d) + 9*(a*b^2*d*m + a*b^2*d)*cos
h((d*m*log(-2*f/d) + 2*c*f - 2*d*cosh(1) - 2*d*sinh(1))/d)*gamma(m + 1, -2*
```

```
(d*f*x + c*f)/d) + (b^3*d*m + b^3*d)*cosh((d*m*log(-3*f/d) + 3*c*f - 3*d*co
sh(1) - 3*d*sinh(1))/d)*gamma(m + 1, -3*(d*f*x + c*f)/d) - (b^3*d*m + b^3*d
)*gamma(m + 1, 3*(d*f*x + c*f)/d)*sinh((d*m*log(3*f/d) - 3*c*f + 3*d*cosh(1
) + 3*d*sinh(1))/d) + 9*(a*b^2*d*m + a*b^2*d)*gamma(m + 1, 2*(d*f*x + c*f)/
d)*sinh((d*m*log(2*f/d) - 2*c*f + 2*d*cosh(1) + 2*d*sinh(1))/d) - 9*((4*a^2
*b - b^3)*d*m + (4*a^2*b - b^3)*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh((d*m*
log(f/d) - c*f + d*cosh(1) + d*sinh(1))/d) - 9*((4*a^2*b - b^3)*d*m + (4*a^
2*b - b^3)*d)*gamma(m + 1, -(d*f*x + c*f)/d)*sinh((d*m*log(-f/d) + c*f - d*
cosh(1) - d*sinh(1))/d) - 9*(a*b^2*d*m + a*b^2*d)*gamma(m + 1, -2*(d*f*x +
c*f)/d)*sinh((d*m*log(-2*f/d) + 2*c*f - 2*d*cosh(1) - 2*d*sinh(1))/d) - (b^
3*d*m + b^3*d)*gamma(m + 1, -3*(d*f*x + c*f)/d)*sinh((d*m*log(-3*f/d) + 3*c
*f - 3*d*cosh(1) - 3*d*sinh(1))/d) + 12*((2*a^3 - 3*a*b^2)*d*f*x + (2*a^3 -
 3*a*b^2)*c*f)*cosh(m*log(d*x + c)) + 12*((2*a^3 - 3*a*b^2)*d*f*x + (2*a^3
- 3*a*b^2)*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*sinh(f*x+e))**3,x)
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e))^3,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e) + a)^3*(d*x + c)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\sinh(e+f\,x))^3\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x))^3*(c + d*x)^m,x)
[Out] int((a + b*sinh(e + f*x))^3*(c + d*x)^m, x)
```

3.183 $\int (c+dx)^m (a+b\sinh (e+fx))^2 \, dx$

Optimal. Leaf size=281

$$\frac{a^2(c+dx)^{1+m}}{d(1+m)}-\frac{b^2(c+dx)^{1+m}}{2d(1+m)}+\frac{2^{-3-m}b^2e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)}{f}+\frac{abe^{e-\frac{cf}{d}}(c+$$

```
[Out] a^2*(d*x+c)^(1+m)/d/(1+m)-1/2*b^2*(d*x+c)^(1+m)/d/(1+m)+2^(-3-m)*b^2*exp(2*
e-2*c*f/d)*(d*x+c)^m*GAMMA(1+m,-2*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+a*b*exp
(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+a*b*exp(-e
+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)-2^(-3-m)*b^2*e
xp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA(1+m,2*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.26, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3398, 3389, 2212, 3393, 3388}

$$\frac{abe^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{f}+\frac{abe^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{f}+\frac{b^22^{-m-3}e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2f(c+dx)}{d}\right)}{f}-\frac{b^22^{-m-3}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2f(c+dx)}{d}\right)}{f}+\frac{a^2(c+dx)^{m+1}}{d(m+1)}-\frac{b^2(c+dx)^{m+1}}{2d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] (a^2*(c + d*x)^(1 + m))/(d*(1 + m)) - (b^2*(c + d*x)^(1 + m))/(2*d*(1 + m))
 + (2^(-3 - m)*b^2*E^(2*e - (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c +
d*x))/d])/(f*(-((f*(c + d*x))/d))^m) + (a*b*E^(e - (c*f)/d)*(c + d*x)^m*Gam
ma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) + (a*b*E^(-e + (c
*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) -
 (2^(-3 - m)*b^2*E^(-2*e + (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*
x))/d])/(f*((f*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (c+dx)^m (a+b\sinh(e+fx))^2\,dx = \int \left(a^2(c+dx)^m + 2ab(c+dx)^m \sinh(e+fx) + b^2(c+dx)^m \sinh^2(e\right.$$

$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + (2ab)\int (c+dx)^m \sinh(e+fx)\,dx + b^2 \int (c+dx)$$

$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + (ab)\int e^{-i(ie+ifx)}(c+dx)^m\,dx - (ab)\int e^{i(ie+ifx)}(c$$

$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} - \frac{b^2(c+dx)^{1+m}}{2d(1+m)} + \frac{abe^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}}{f}$$

$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} - \frac{b^2(c+dx)^{1+m}}{2d(1+m)} + \frac{abe^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}}{f}$$

$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} - \frac{b^2(c+dx)^{1+m}}{2d(1+m)} + \frac{2^{-3-m}b^2e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+}{d}\right.}{f}$$

Mathematica [A]
time = 6.48, size = 241, normalized size = 0.86

$$\frac{(c+dx)^m\left(\frac{8a^2f(c+dx)}{d(1+m)} - \frac{4b^2f(c+dx)}{d(1+m)} + 8abe^{-e+\frac{cf}{d}}\left(f\left(\frac{c}{d}+x\right)\right)^{-m}\Gamma\left(1+m, f\left(\frac{c}{d}+x\right)\right) + 2^{-m}b^2e^{2e-\frac{2cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m, -\frac{2f(c+dx)}{d}\right) + 8abe^{e-\frac{cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m, -\frac{f(c+dx)}{d}\right) - b^2e^{-2e+\frac{2cf}{d}}\left(\frac{2cf}{d}+2fx\right)^{-m}\Gamma\left(1+m, \frac{2f(c+dx)}{d}\right)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] ((c + d*x)^m*((8*a^2*f*(c + d*x))/(d*(1 + m)) - (4*b^2*f*(c + d*x))/(d*(1 +
 m)) + (8*a*b*E^(-e + (c*f)/d)*Gamma[1 + m, f*(c/d + x)])/(f*(c/d + x))^m +
 (b^2*E^(2*e - (2*c*f)/d)*Gamma[1 + m, (-2*f*(c + d*x))/d])/(2^m*(-((f*(c +
 d*x))/d))^m) + (8*a*b*E^(e - (c*f)/d)*Gamma[1 + m, -((f*(c + d*x))/d)])/(-
((f*(c + d*x))/d))^m - (b^2*E^(-2*e + (2*c*f)/d)*Gamma[1 + m, (2*f*(c + d*x
))/d])/((2*c*f)/d + 2*f*x)^m))/(8*f)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m (a+b\sinh(fx+e))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*sinh(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m*(a+b*sinh(f*x+e))^2,x)
```

Maxima [A]
time = 0.08, size = 212, normalized size = 0.75

$$\left(\frac{(dx+c)^{m+1} e^{\left(\frac{cf}{d}-e\right)} E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d} - \frac{(dx+c)^{m+1} e^{\left(-\frac{cf}{d}+e\right)} E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)ab - \frac{1}{4}\left(\frac{(dx+c)^{m+1} e^{\left(\frac{2cf}{d}-2e\right)} E_{-m}\left(\frac{2(dx+c)f}{d}\right)}{d} + \frac{(dx+c)^{m+1} e^{\left(-\frac{2cf}{d}+2e\right)} E_{-m}\left(-\frac{2(dx+c)f}{d}\right)}{d} + \frac{2(dx+c)^{m+1}}{d(m+1)}\right)b^2 + \frac{(dx+c)^{m+1} a^2}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] ((d*x + c)^(m + 1)*e^(c*f/d - e)*exp_integral_e(-m, (d*x + c)*f/d)/d - (d*x
 + c)^(m + 1)*e^(-c*f/d + e)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a*b - 1/
4*((d*x + c)^(m + 1)*e^(2*c*f/d - 2*e)*exp_integral_e(-m, 2*(d*x + c)*f/d)/
d + (d*x + c)^(m + 1)*e^(-2*c*f/d + 2*e)*exp_integral_e(-m, -2*(d*x + c)*f/
d)/d + 2*(d*x + c)^(m + 1)/(d*(m + 1)))*b^2 + (d*x + c)^(m + 1)*a^2/(d*(m +
 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(279) = 558$.
time = 0.15, size = 563, normalized size = 2.00

$$-\frac{(b^2dm+b^2d)\cosh\left(\frac{dm\log\left(\frac{2f}{d}\right)-2cf+2d\cosh(1)+2d\sinh(1)}{d}\right)\Gamma\left(m+1,\frac{2(dfx+cf)}{d}\right) - 8(abdm+abd)\cosh\left(\frac{dm\log\left(\frac{f}{d}\right)-cf+d\cosh(1)+d\sinh(1)}{d}\right)\Gamma\left(m+1,\frac{dfx+cf}{d}\right) - 8(abdm+abd)\cosh\left(\frac{dm\log\left(-\frac{f}{d}\right)+cf-d\cosh(1)-d\sinh(1)}{d}\right)\Gamma\left(m+1,-\frac{dfx+cf}{d}\right) - (b^2dm+b^2d)\cosh\left(\frac{dm\log\left(-\frac{2f}{d}\right)+2cf-2d\cosh(1)-2d\sinh(1)}{d}\right)\Gamma\left(m+1,-\frac{2(dfx+cf)}{d}\right) - (b^2dm+b^2d)\Gamma\left(m+1,\frac{2(dfx+cf)}{d}\right)\sinh\left(\frac{dm\log\left(\frac{2f}{d}\right)-2cf+2d\cosh(1)+2d\sinh(1)}{d}\right) + 8(abdm+abd)\Gamma\left(m+1,\frac{dfx+cf}{d}\right)\sinh\left(\frac{dm\log\left(\frac{f}{d}\right)-cf+d\cosh(1)+d\sinh(1)}{d}\right) + 8(abdm+abd)\Gamma\left(m+1,-\frac{dfx+cf}{d}\right)\sinh\left(\frac{dm\log\left(-\frac{f}{d}\right)+cf-d\cosh(1)-d\sinh(1)}{d}\right) + (b^2dm+b^2d)\Gamma\left(m+1,-\frac{2(dfx+cf)}{d}\right)\sinh\left(\frac{dm\log\left(-\frac{2f}{d}\right)+2cf-2d\cosh(1)-2d\sinh(1)}{d}\right) - 4\left((2a^2-b^2)dfx+(2a^2-b^2)cf\right)\cosh(m\log(dx+c)) - 4\left((2a^2-b^2)dfx+(2a^2-b^2)cf\right)\sinh(m\log(dx+c))}{8(dfm+df)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/8*((b^2*d*m + b^2*d)*cosh((d*m*log(2*f/d) - 2*c*f + 2*d*cosh(1) + 2*d*si
nh(1))/d)*gamma(m + 1, 2*(d*f*x + c*f)/d) - 8*(a*b*d*m + a*b*d)*cosh((d*m*l
```

```
og(f/d) - c*f + d*cosh(1) + d*sinh(1))/d)*gamma(m + 1, (d*f*x + c*f)/d) - 8
*(a*b*d*m + a*b*d)*cosh((d*m*log(-f/d) + c*f - d*cosh(1) - d*sinh(1))/d)*ga
mma(m + 1, -(d*f*x + c*f)/d) - (b^2*d*m + b^2*d)*cosh((d*m*log(-2*f/d) + 2*
c*f - 2*d*cosh(1) - 2*d*sinh(1))/d)*gamma(m + 1, -2*(d*f*x + c*f)/d) - (b^2
*d*m + b^2*d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) - 2*c*f
+ 2*d*cosh(1) + 2*d*sinh(1))/d) + 8*(a*b*d*m + a*b*d)*gamma(m + 1, (d*f*x +
 c*f)/d)*sinh((d*m*log(f/d) - c*f + d*cosh(1) + d*sinh(1))/d) + 8*(a*b*d*m
+ a*b*d)*gamma(m + 1, -(d*f*x + c*f)/d)*sinh((d*m*log(-f/d) + c*f - d*cosh(
1) - d*sinh(1))/d) + (b^2*d*m + b^2*d)*gamma(m + 1, -2*(d*f*x + c*f)/d)*sin
h((d*m*log(-2*f/d) + 2*c*f - 2*d*cosh(1) - 2*d*sinh(1))/d) - 4*((2*a^2 - b^
2)*d*f*x + (2*a^2 - b^2)*c*f)*cosh(m*log(d*x + c)) - 4*((2*a^2 - b^2)*d*f*x
 + (2*a^2 - b^2)*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*sinh(f*x+e))**2,x)
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e))^2,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e) + a)^2*(d*x + c)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\sinh(e+f\,x))^2\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x))^2*(c + d*x)^m,x)
[Out] int((a + b*sinh(e + f*x))^2*(c + d*x)^m, x)
```

3.184 $\int (c+dx)^m(a+b\sinh(e+fx))\,dx$

Optimal. Leaf size=131

$$\frac{a(c+dx)^{1+m}}{d(1+m)}+\frac{be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{2f}+\frac{be^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m\right.}{2f}$$

```
[Out] a*(d*x+c)^(1+m)/d/(1+m)+1/2*b*exp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/d
)/f/((-f*(d*x+c)/d)^m)+1/2*b*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x+c)/d)
/f/((f*(d*x+c)/d)^m)
```

Rubi [A]

time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3398, 3389, 2212}

$$\frac{be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}+\frac{be^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+\frac{a(c+dx)^{m+1}}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + b*Sinh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^(1 + m))/(d*(1 + m)) + (b*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1
+ m, -((f*(c + d*x))/d)])/(2*f*(-((f*(c + d*x))/d))^m) + (b*E^(-e + (c*f)/d
)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m(a+b\sinh(e+fx))\,dx &= \int \left(a(c+dx)^m+b(c+dx)^m\sinh(e+fx)\right)\,dx\\ &= \frac{a(c+dx)^{1+m}}{d(1+m)}+b\int (c+dx)^m\sinh(e+fx)\,dx\\ &= \frac{a(c+dx)^{1+m}}{d(1+m)}+\frac{1}{2}b\int e^{-i(ie+ifx)}(c+dx)^m\,dx-\frac{1}{2}b\int e^{i(ie+ifx)}(c+d\\ &= \frac{a(c+dx)^{1+m}}{d(1+m)}+\frac{be^{-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{2f}\end{aligned}$$

Mathematica [**A**]
time = 17.29, size = 201, normalized size = 1.53

$$\frac{e^{-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}\left(\cosh\left(\frac{3cf}{d}\right)+\sinh\left(\frac{3cf}{d}\right)\right)\left(2af(c+dx)\left(-\frac{f^2(c+dx)^2}{d^2}\right)^m+bd(1+m)\left(-\frac{f(c+dx)}{d}\right)^m\Gamma\left(1+m,f\left(\frac{c}{d}+x\right)\right)(\cosh(e)-\sinh(e))\left(\cosh\left(\frac{cf}{d}\right)+\sinh\left(\frac{cf}{d}\right)\right)+bd(1+m)\left(f\left(\frac{c}{d}+x\right)\right)^m\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)\left(\cosh\left(e-\frac{cf}{d}\right)+\sinh\left(e-\frac{cf}{d}\right)\right)\right)}{2df(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + b*Sinh[e + f*x]),x]
```

```
[Out] ((c + d*x)^m*(Cosh[(3*c*f)/d] + Sinh[(3*c*f)/d])*(2*a*f*(c + d*x)*(-((f^2*(
c + d*x)^2)/d^2))^m + b*d*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, f*(c/
d + x)]*(Cosh[e] - Sinh[e])*(Cosh[(c*f)/d] + Sinh[(c*f)/d]) + b*d*(1 + m)*(
f*(c/d + x))^m*Gamma[1 + m, -((f*(c + d*x))/d)]*(Cosh[e - (c*f)/d] + Sinh[e
 - (c*f)/d])))/(2*d*E^((3*c*f)/d)*f*(1 + m)*(-((f^2*(c + d*x)^2)/d^2))^m)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(dx+c\right)^{m}\left(a+b\sinh\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*sinh(f*x+e)),x)
```

```
[Out] int((d*x+c)^m*(a+b*sinh(f*x+e)),x)
```

Maxima [**A**]
time = 0.06, size = 103, normalized size = 0.79

$$\frac{1}{2}\left(\frac{(dx+c)^{m+1}e^{\left(\frac{cf}{d}-e\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}-\frac{(dx+c)^{m+1}e^{\left(-\frac{cf}{d}+e\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)b+\frac{(dx+c)^{m+1}a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*((d*x + c)^(m + 1)*e^(c*f/d - e)*exp_integral_e(-m, (d*x + c)*f/d)/d -
(d*x + c)^(m + 1)*e^(-c*f/d + e)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*b +
(d*x + c)^(m + 1)*a/(d*(m + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(127) = 254$.
time = 0.10, size = 271, normalized size = 2.07

$$\frac{(bdm+bd)\cosh\left(\frac{dm\log\left(\frac{f}{d}\right)-cf+d\cosh(1)+d\sinh(1)}{d}\right)\Gamma\left(m+1,\frac{dfx+cf}{d}\right)+(bdm+bd)\cosh\left(\frac{dm\log\left(-\frac{f}{d}\right)+cf-d\cosh(1)-d\sinh(1)}{d}\right)\Gamma\left(m+1,-\frac{dfx+cf}{d}\right)-(bdm+bd)\Gamma\left(m+1,\frac{dfx+cf}{d}\right)\sinh\left(\frac{dm\log\left(\frac{f}{d}\right)-cf+d\cosh(1)+d\sinh(1)}{d}\right)-(bdm+bd)\Gamma\left(m+1,-\frac{dfx+cf}{d}\right)\sinh\left(\frac{dm\log\left(-\frac{f}{d}\right)+cf-d\cosh(1)-d\sinh(1)}{d}\right)+2\,(adfx+acf)\cosh\left(m\log\left(dx+c\right)\right)+2\,(adfx+acf)\sinh\left(m\log\left(dx+c\right)\right)}{2\,(dfm+df)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*((b*d*m + b*d)*cosh((d*m*log(f/d) - c*f + d*cosh(1) + d*sinh(1))/d)*gam
ma(m + 1, (d*f*x + c*f)/d) + (b*d*m + b*d)*cosh((d*m*log(-f/d) + c*f - d*co
sh(1) - d*sinh(1))/d)*gamma(m + 1, -(d*f*x + c*f)/d) - (b*d*m + b*d)*gamma(
m + 1, (d*f*x + c*f)/d)*sinh((d*m*log(f/d) - c*f + d*cosh(1) + d*sinh(1))/d
) - (b*d*m + b*d)*gamma(m + 1, -(d*f*x + c*f)/d)*sinh((d*m*log(-f/d) + c*f
- d*cosh(1) - d*sinh(1))/d) + 2*(a*d*f*x + a*c*f)*cosh(m*log(d*x + c)) + 2*
(a*d*f*x + a*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*sinh(f*x+e)),x)
```

```
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e) + a)*(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{sinh}\left(e+f\,x\right)\right)\,{\left(c+d\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x))*(c + d*x)^m,x)
[Out] int((a + b*sinh(e + f*x))*(c + d*x)^m, x)
```

3.185 $\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(c+dx)^m}{a+b\sinh(e+fx)},x\right)$$

```
[Out] Unintegrable((d*x+c)^m/(a+b*sinh(f*x+e)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + b*Sinh[e + f*x]),x]
[Out] Defer[Int][(c + d*x)^m/(a + b*Sinh[e + f*x]), x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx = \int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx$$

Mathematica [A]
time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + b*Sinh[e + f*x]),x]
[Out] Integrate[(c + d*x)^m/(a + b*Sinh[e + f*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{a+b\sinh(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+b*sinh(f*x+e)),x)
[Out] int((d*x+c)^m/(a+b*sinh(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sinh(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*x + c)^m/(b*sinh(f*x + e) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sinh(f*x+e)),x, algorithm="fricas")
[Out] integral((d*x + c)^m/(b*sinh(f*x + e) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx)^m}{a + b \sinh(e + fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+b*sinh(f*x+e)),x)
[Out] Integral((c + d*x)**m/(a + b*sinh(e + f*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sinh(f*x+e)),x, algorithm="giac")
[Out] integrate((d*x + c)^m/(b*sinh(f*x + e) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c + d\,x)^m}{a + b \sinh(e + f\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + b*sinh(e + f*x)),x)
[Out] int((c + d*x)^m/(a + b*sinh(e + f*x)), x)
```

3.186 $\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(c+dx)^m}{(a+b\sinh(e+fx))^2},x\right)$$

```
[Out] Unintegrable((d*x+c)^m/(a+b*sinh(f*x+e))^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m/(a + b*Sinh[e + f*x])^2, x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx=\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx$$

Mathematica [A]
time = 3.82, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] Integrate[(c + d*x)^m/(a + b*Sinh[e + f*x])^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{(a+b\sinh(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+b*sinh(f*x+e))^2,x)
[Out] int((d*x+c)^m/(a+b*sinh(f*x+e))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m/(b*sinh(f*x + e) + a)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m/(b^2*sinh(f*x + e)^2 + 2*a*b*sinh(f*x + e) + a^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+b*sinh(f*x+e))**2,x)
[Out] Integral((c + d*x)**m/(a + b*sinh(e + f*x))**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sinh(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m/(b*sinh(f*x + e) + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{(a+b\sinh(e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + b*sinh(e + f*x))^2,x)
[Out] int((c + d*x)^m/(a + b*sinh(e + f*x))^2, x)
```

3.187 $\int \frac{(e+fx)^3 \sinh(c+dx)}{a+ia\sinh(c+dx)}\, dx$

Optimal. Leaf size=163

$$\frac{i(e+fx)^3}{ad}-\frac{i(e+fx)^4}{4af}-\frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}-\frac{12if^2(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{12if^3\text{PolyLog}(}{ad^4}$$

```
[Out] I*(f*x+e)^3/a/d-1/4*I*(f*x+e)^4/a/f-6*I*f*(f*x+e)^2*ln(1+I*exp(d*x+c))/a/d^
2-12*I*f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/a/d^3+12*I*f^3*polylog(3,-I*exp
(d*x+c))/a/d^4+I*(f*x+e)^3*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a/d
```

Rubi [A]
time = 0.24, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {5676, 32, 3399, 4269, 3797, 2221, 2611, 2320, 6724}

$$\frac{12if^3\text{Li}_3\left(-ie^{c+dx}\right)}{ad^4}-\frac{12if^2(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{ad^3}-\frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}+\frac{i(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}+\frac{i(e+fx)^3}{ad}-\frac{i(e+fx)^4}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (I*(e + f*x)^3)/(a*d) - ((I/4)*(e + f*x)^4)/(a*f) - ((6*I)*f*(e + f*x)^2*Lo
g[1 + I*E^(c + d*x)])/(a*d^2) - ((12*I)*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c
+ d*x)])/(a*d^3) + ((12*I)*f^3*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^4) + (I*(
e + f*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5676

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e+fx)^3\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx &= i\int \frac{(e+fx)^3}{a+ia\sinh(c+dx)}\,dx-\frac{i\int (e+fx)^3\,dx}{a}\\
&= -\frac{i(e+fx)^4}{4af}+\frac{i\int (e+fx)^3\csc^2\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{idx}{2}\right)\,dx}{2a}\\
&= -\frac{i(e+fx)^4}{4af}+\frac{i(e+fx)^3\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}-\frac{(3if)\int (e+fx)^2\coth\left(\frac{c}{2}-\cdots\right.}{ad}\\
&= \frac{i(e+fx)^3}{ad}-\frac{i(e+fx)^4}{4af}+\frac{i(e+fx)^3\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}+\frac{(6f)\int \frac{e^{2\left(\frac{c}{2}+\frac{dx}{2}\right)}(e\cdots}{1+ie^{2\left(\frac{c}{2}+\cdots\right.}}}{ad}\\
&= \frac{i(e+fx)^3}{ad}-\frac{i(e+fx)^4}{4af}-\frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}+\frac{i(e+fx)^3\tanh\left(\cdots\right.}{ad}\\
&= \frac{i(e+fx)^3}{ad}-\frac{i(e+fx)^4}{4af}-\frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}-\frac{12if^2(e+fx)\mathrm{Li}_2\cdots}{ad^3}\\
&= \frac{i(e+fx)^3}{ad}-\frac{i(e+fx)^4}{4af}-\frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}-\frac{12if^2(e+fx)\mathrm{Li}_2\cdots}{ad^3}\\
&= \frac{i(e+fx)^3}{ad}-\frac{i(e+fx)^4}{4af}-\frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}-\frac{12if^2(e+fx)\mathrm{Li}_2\cdots}{ad^3}
\end{aligned}
$$

Mathematica [A]
time = 2.20, size = 247, normalized size = 1.52

$$
\frac{-ix(4e^3+6e^2fx+4ef^2x^2+f^3x^3)-\frac{8f\left(d^2(-ide^cx(3e^2+3efx+f^2x^2)+3(1+ie^c)(e+fx)^2\log(1+ie^{c+dx}))+6d(1+ie^c)f(e+fx)\mathrm{PolyLog}(2,-ie^{c+dx})-6i(-i+e^c)f^2\mathrm{PolyLog}(3,-ie^{c+dx})\right)}{d^4(-i+e^c)}+\frac{8i(e+fx)^3\sinh\left(\frac{dx}{2}\right)}{d\left(\cosh\left(\frac{c}{2}\right)+i\sinh\left(\frac{c}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}}{4a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-I)*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) - (8*f*(d^2*((-I)*d*E^c
*x*(3*e^2 + 3*e*f*x + f^2*x^2) + 3*(1 + I*E^c)*(e + f*x)^2*Log[1 + I*E^(c +
 d*x)]) + 6*d*(1 + I*E^c)*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)] - (6*I)*
(-I + E^c)*f^2*PolyLog[3, (-I)*E^(c + d*x)]))/(d^4*(-I + E^c)) + ((8*I)*(e
+ f*x)^3*Sinh[(d*x)/2])/(d*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I
*Sinh[(c + d*x)/2])))/(4*a)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 512 vs. 2(142) = 284.
time = 1.55, size = 513, normalized size = 3.15

method	result
risch	$\frac{12 i f^{3} \operatorname{polylog}\left(3,-i e^{d x+c}\right)}{a\, d^{4}}+\frac{6 i e\, f^{2} c^{2}}{a\, d^{3}}+\frac{6 i f^{3} c^{2} \ln \left(1+i e^{d x+c}\right)}{a\, d^{4}}+\frac{6 i e\, f^{2} x^{2}}{a d}+\frac{6 i f^{3} c^{2} \ln \left(e^{d x+c}\right)}{a\, d^{4}}-\frac{6 i \ln \left(e^{d x+c}-i\right) e^{2} f}{a\, d^{2}}-12ie$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -12*I/a/d^3*f^3*polylog(2,-I*exp(d*x+c))*x-6*I/a/d^2*f^3*ln(1+I*exp(d*x+c))
*x^2+6*I/a/d^2*ln(exp(d*x+c))*e^2*f-6*I/a/d^4*f^3*c^2*ln(exp(d*x+c)-I)-6*I/
a/d^3*f^3*c^2*x+12*I*f^3*polylog(3,-I*exp(d*x+c))/a/d^4+6*I/a/d^3*e*f^2*c^2
+6*I/a/d^4*f^3*c^2*ln(1+I*exp(d*x+c))+6*I/a/d*e*f^2*x^2-12*I/a/d^3*e*f^2*po
lylog(2,-I*exp(d*x+c))+6*I/a/d^4*f^3*c^2*ln(exp(d*x+c))-6*I/a/d^2*ln(exp(d*
x+c)-I)*e^2*f-I/a*f^2*e*x^3-3/2*I/a*f*e^2*x^2-2*(f^3*x^3+3*e*f^2*x^2+3*e^2*
f*x+e^3)/d/a/(exp(d*x+c)-I)-1/4*I/a*f^3*x^4-I/a*e^3*x-1/4*I/a/f*e^4-12*I/a/
d^3*e*f^2*ln(1+I*exp(d*x+c))*c+12*I/a/d^2*e*f^2*c*x-12*I/a/d^3*e*f^2*c*ln(e
xp(d*x+c))+12*I/a/d^3*e*f^2*c*ln(exp(d*x+c)-I)+2*I/a/d*f^3*x^3-4*I/a/d^4*f^
3*c^3-12*I/a/d^2*e*f^2*ln(1+I*exp(d*x+c))*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(136) = 272$.
time = 0.36, size = 321, normalized size = 1.97

$$\frac{3}{2} f\left(\frac{-i\, d x^{2}+\left(d x^{2} e^{c}-4\, x e^{c}\right) e^{(d x)}}{i\, a d e^{(d x+c)}+a d}-\frac{4 i \log \left(\left(e^{(d x+c)}-i\right) e^{(-c)}\right)}{a d^{2}}\right) e^{2}-\left(\frac{i\,(d x+c)}{a d}+\frac{2}{\left(a e^{(-d x-c)}+i\, a\right) d}\right) e^{3}-\frac{d f^{3} x^{4}+24\, f^{2} x^{2} e+4\left(d f^{2} e+2\, f^{3}\right) x^{3}+\left(i\, d f^{3} x^{4} e^{c}+4 i\, d f^{2} x^{3} e^{(c+1)}\right) e^{(d x)}}{4\left(a d e^{(d x+c)}-i\, a d\right)}-\frac{12 i\left(d x \log \left(i\, e^{(d x+c)}+1\right)+\operatorname{Li}_2\left(-i\, e^{(d x+c)}\right)\right) f^{2} e}{a d^{3}}-\frac{6 i\left(d^{2} x^{2} \log \left(i\, e^{(d x+c)}+1\right)+2\, d x \operatorname{Li}_2\left(-i\, e^{(d x+c)}\right)-2\, \operatorname{Li}_3\left(-i\, e^{(d x+c)}\right)\right) f^{3}}{a d^{4}}-\frac{2\left(-i\, d^{3} f^{3} x^{3}-3 i\, d^{3} f^{2} x^{2} e\right)}{a d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 3/2*f*((-I*d*x^2 + (d*x^2*e^c - 4*x*e^c)*e^(d*x))/(I*a*d*e^(d*x + c) + a*d)
 - 4*I*log((e^(d*x + c) - I)*e^(-c))/(a*d^2))*e^2 - (I*(d*x + c)/(a*d) + 2/
((a*e^(-d*x - c) + I*a)*d))*e^3 - 1/4*(d*f^3*x^4 + 24*f^2*x^2*e + 4*(d*f^2*
e + 2*f^3)*x^3 + (I*d*f^3*x^4*e^c + 4*I*d*f^2*x^3*e^(c + 1))*e^(d*x))/(a*d*
e^(d*x + c) - I*a*d) - 12*I*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x +
 c)))*f^2*e/(a*d^3) - 6*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*
e^(d*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a*d^4) - 2*(-I*d^3*f^3*x^
3 - 3*I*d^3*f^2*x^2*e)/(a*d^4)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(136) = 272$.
time = 0.42, size = 455, normalized size = 2.79

$$-\frac{d^{4} f^{3} x^{4}-8\, c^{3} f^{3}+48\left(d f^{3} x+d f^{2} e+\left(i\, d f^{3} x+i\, d f^{2} e\right) e^{(d x+c)}\right) \operatorname{Li}_2\left(-i\, e^{(d x+c)}\right)+4\left(d^{4} x+2\, d^{3}\right) e^{3}+6\left(d^{4} f x^{2}-4\, c d^{2} f\right) e^{2}+4\left(d^{4} f^{2} x^{3}+6\, c^{2} d f^{2}\right) e-\left(-i\, d^{4} f^{3} x^{4}+8 i\, d^{3} f^{3} x^{3}+8 i\, c^{3} f^{3}-4 i\, d^{4} x e^{3}-6\left(i\, d^{4} f x^{2}-4 i\, d^{3} f x-4 i\, c d^{2} f\right) e^{2}-4\left(i\, d^{4} f^{2} x^{3}-6 i\, d^{3} f^{2} x^{2}+6 i\, c^{2} d f^{2}\right) e\right) e^{(d x+c)}+24\left(c^{2} f^{3}-2\, c d f^{2} e+d^{2} f e^{2}+\left(i\, c^{2} f^{3}-2 i\, c d f^{2} e+i\, d^{2} f e^{2}\right) e^{(d x+c)}\right) \log \left(e^{(d x+c)}-i\right)+24\left(d^{2} f^{3} x^{2}-c^{2} f^{3}+2\left(d^{2} f^{2} x+c d f^{2}\right) e+\left(i\, d^{2} f^{3} x^{2}-i\, c^{2} f^{3}+2\left(i\, d^{2} f^{2} x+i\, c d f^{2}\right) e\right) e^{(d x+c)}\right) \log \left(i\, e^{(d x+c)}+1\right)+48\left(-i\, f^{3} e^{(d x+c)}-f^{3}\right) \operatorname{polylog}\left(3,-i\, e^{(d x+c)}\right)}{4\left(a d^{4} e^{(d x+c)}-i\, a d^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(d^4*f^3*x^4 - 8*c^3*f^3 + 48*(d*f^3*x + d*f^2*e + (I*d*f^3*x + I*d*f^
2*e)*e^(d*x + c))*dilog(-I*e^(d*x + c)) + 4*(d^4*x + 2*d^3)*e^3 + 6*(d^4*f*
x^2 - 4*c*d^2*f)*e^2 + 4*(d^4*f^2*x^3 + 6*c^2*d*f^2)*e - (-I*d^4*f^3*x^4 +
8*I*d^3*f^3*x^3 + 8*I*c^3*f^3 - 4*I*d^4*x*e^3 - 6*(I*d^4*f*x^2 - 4*I*d^3*f*
x - 4*I*c*d^2*f)*e^2 - 4*(I*d^4*f^2*x^3 - 6*I*d^3*f^2*x^2 + 6*I*c^2*d*f^2)*
e)*e^(d*x + c) + 24*(c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2 + (I*c^2*f^3 - 2*I*c
*d*f^2*e + I*d^2*f*e^2)*e^(d*x + c))*log(e^(d*x + c) - I) + 24*(d^2*f^3*x^2
 - c^2*f^3 + 2*(d^2*f^2*x + c*d*f^2)*e + (I*d^2*f^3*x^2 - I*c^2*f^3 + 2*(I*
d^2*f^2*x + I*c*d*f^2)*e)*e^(d*x + c))*log(I*e^(d*x + c) + 1) + 48*(-I*f^3*
e^(d*x + c) - f^3)*polylog(3, -I*e^(d*x + c)))/(a*d^4*e^(d*x + c) - I*a*d^4
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{-2e^3 - 6e^2fx - 6ef^2x^2 - 2f^3x^3}{ade^ce^{dx} - iad} - \frac{i\left(\int\left(-\frac{ide^3}{e^ce^{dx}-i}\right)dx + \int\frac{6ie^2f}{e^ce^{dx}-i}\,dx + \int\frac{6if^3x^2}{e^ce^{dx}-i}\,dx + \int\left(-\frac{idf^3x^3}{e^ce^{dx}-i}\right)dx + \int\frac{12ief^2x}{e^ce^{dx}-i}\,dx + \int\frac{de^3e^ce^{dx}}{e^ce^{dx}-i}\,dx + \int\left(-\frac{3idef^2x^2}{e^ce^{dx}-i}\right)dx + \int\left(-\frac{3ide^2fx}{e^ce^{dx}-i}\right)dx + \int\frac{df^3x^3e^ce^{dx}}{e^ce^{dx}-i}\,dx + \int\frac{3def^2x^2e^ce^{dx}}{e^ce^{dx}-i}\,dx + \int\frac{3de^2fxe^ce^{dx}}{e^ce^{dx}-i}\,dx\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] (-2*e**3 - 6*e**2*f*x - 6*e*f**2*x**2 - 2*f**3*x**3)/(a*d*exp(c)*exp(d*x) -
 I*a*d) - I*(Integral(-I*d*e**3/(exp(c)*exp(d*x) - I), x) + Integral(6*I*e*
*2*f/(exp(c)*exp(d*x) - I), x) + Integral(6*I*f**3*x**2/(exp(c)*exp(d*x) -
I), x) + Integral(-I*d*f**3*x**3/(exp(c)*exp(d*x) - I), x) + Integral(12*I*
e*f**2*x/(exp(c)*exp(d*x) - I), x) + Integral(d*e**3*exp(c)*exp(d*x)/(exp(c
)*exp(d*x) - I), x) + Integral(-3*I*d*e*f**2*x**2/(exp(c)*exp(d*x) - I), x)
 + Integral(-3*I*d*e**2*f*x/(exp(c)*exp(d*x) - I), x) + Integral(d*f**3*x**
3*exp(c)*exp(d*x)/(exp(c)*exp(d*x) - I), x) + Integral(3*d*e*f**2*x**2*exp(
c)*exp(d*x)/(exp(c)*exp(d*x) - I), x) + Integral(3*d*e**2*f*x*exp(c)*exp(d*
x)/(exp(c)*exp(d*x) - I), x))/(a*d)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sinh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c+d\,x)\,\left(e+f\,x\right)^3}{a+a\,\sinh\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i),x)
[Out] int((sinh(c + d*x)*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i), x)
```

3.188 $\int \frac{(e+fx)^2 \sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{i(e+fx)^2}{ad}-\frac{i(e+fx)^3}{3af}-\frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}-\frac{4if^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\right.}{ad}$$

```
[Out] I*(f*x+e)^2/a/d-1/3*I*(f*x+e)^3/a/f-4*I*f*(f*x+e)*ln(1+I*exp(d*x+c))/a/d^2-
4*I*f^2*polylog(2,-I*exp(d*x+c))/a/d^3+I*(f*x+e)^2*tanh(1/2*c+1/4*I*Pi+1/2*
d*x)/a/d
```

Rubi [A]
time = 0.19, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {5676, 32, 3399, 4269, 3797, 2221, 2317, 2438}

$$-\frac{4if^2\text{Li}_2\left(-ie^{c+dx}\right)}{ad^3}-\frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}+\frac{i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}+\frac{i(e+fx)^2}{ad}-\frac{i(e+fx)^3}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (I*(e + f*x)^2)/(a*d) - ((I/3)*(e + f*x)^3)/(a*f) - ((4*I)*f*(e + f*x)*Log[
1 + I*E^(c + d*x)])/(a*d^2) - ((4*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d
^3) + (I*(e + f*x)^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5676

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = i\int \frac{(e+fx)^2}{a+ia\sinh(c+dx)}\,dx - \frac{i\int (e+fx)^2\,dx}{a}$$

$$= -\frac{i(e+fx)^3}{3af} + \frac{i\int (e+fx)^2\csc^2\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{idx}{2}\right)\,dx}{2a}$$

$$= -\frac{i(e+fx)^3}{3af} + \frac{i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(2if)\int (e+fx)\coth\left(\frac{c}{2}-\frac{i\pi}{4}\right.}{ad}$$

$$= \frac{i(e+fx)^2}{ad} - \frac{i(e+fx)^3}{3af} + \frac{i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad} + \frac{(4f)\int \frac{e^{2\left(\frac{c}{2}+\frac{dx}{2}\right)}(e}{1+ie^{2\left(\frac{c}{2}+\right.}}}{ad}$$

$$= \frac{i(e+fx)^2}{ad} - \frac{i(e+fx)^3}{3af} - \frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2} + \frac{i(e+fx)^2\tanh\left(\frac{c}{2}\right.}{ad}$$

$$= \frac{i(e+fx)^2}{ad} - \frac{i(e+fx)^3}{3af} - \frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2} + \frac{i(e+fx)^2\tanh\left(\frac{c}{2}\right.}{ad}$$

$$= \frac{i(e+fx)^2}{ad} - \frac{i(e+fx)^3}{3af} - \frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2} - \frac{4if^2\text{Li}_2\left(-ie^{c+dx}\right)}{ad^3}$$

Mathematica [A]
time = 1.70, size = 183, normalized size = 1.41

$$\frac{-ix(3e^2+3efx+f^2x^2) + \frac{6f\left(d\left(de^cx(2e+fx)-2(-i+e^c)(e+fx)\log(1+ie^{c+dx})\right)-2(-i+e^c)f\,\text{PolyLog}(2,-ie^{c+dx})\right)}{d^3(-1-ie^c)} + \frac{6i(e+fx)^2\sinh\left(\frac{dx}{2}\right)}{d\left(\cosh\left(\frac{c}{2}\right)+i\sinh\left(\frac{c}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-I)*x*(3*e^2 + 3*e*f*x + f^2*x^2) + (6*f*(d*(d*E^c*x*(2*e + f*x) - 2*(-I
+ E^c)*(e + f*x)*Log[1 + I*E^(c + d*x)]) - 2*(-I + E^c)*f*PolyLog[2, (-I)*E
^(c + d*x)]))/(d^3*(-1 - I*E^c)) + ((6*I)*(e + f*x)^2*Sinh[(d*x)/2])/(d*(Co
sh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])))/(3*a)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(112) = 224.
time = 1.28, size = 281, normalized size = 2.16

method	result
risch	$-\frac{if^2x^3}{3a} - \frac{ifex^2}{a} - \frac{ie^2x}{a} - \frac{ie^3}{3af} - \frac{2(x^2f^2+2efx+e^2)}{da(e^{dx+c}-i)} + \frac{4i\ln(e^{dx+c})ef}{a\,d^2} - \frac{4i\ln(e^{dx+c}-i)ef}{a\,d^2} + \frac{2if^2x^2}{ad} + \frac{4if^2cx}{a\,d^2} + \frac{2if}{a\,c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I/a*f^2*x^3-I/a*f*e*x^2-I/a*e^2*x-1/3*I/a/f*e^3-2*(f^2*x^2+2*e*f*x+e^2
)/d/a/(exp(d*x+c)-I)+4*I/a/d^2*ln(exp(d*x+c))*e*f-4*I/a/d^2*ln(exp(d*x+c)-I
)*e*f+2*I/a/d*f^2*x^2+4*I/a/d^2*f^2*c*x+2*I/a/d^3*f^2*c^2-4*I/a/d^2*f^2*ln(
1+I*exp(d*x+c))*x-4*I/a/d^3*f^2*ln(1+I*exp(d*x+c))*c-4*I*f^2*polylog(2,-I*e
xp(d*x+c))/a/d^3-4*I/a/d^3*f^2*c*ln(exp(d*x+c))+4*I/a/d^3*f^2*c*ln(exp(d*x+
c)-I)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*f^2*((-I*d*x^3*e^(d*x + c) - d*x^3 - 6*x^2)/(a*d*e^(d*x + c) - I*a*d) +
 12*integrate(x/(a*d*e^(d*x + c) - I*a*d), x)) + f*((-I*d*x^2 + (d*x^2*e^c
- 4*x*e^c)*e^(d*x))/(I*a*d*e^(d*x + c) + a*d) - 4*I*log((e^(d*x + c) - I)*e
^(-c))/(a*d^2))*e - (I*(d*x + c)/(a*d) + 2/((a*e^(-d*x - c) + I*a)*d))*e^2
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(107) = 214$.
time = 0.35, size = 263, normalized size = 2.02

$$-\frac{d^3f^2x^3 + 6\,c^2f^2 + 12\,(i\,f^2e^{(dx+c)} + f^2)\mathrm{Li}_2(-i\,e^{(dx+c)}) + 3\,(d^3x + 2\,d^2)e^2 + 3\,(d^3fx^2 - 4\,cdf)e - (-i\,d^3f^2x^3 + 6i\,d^2f^2x^2 - 3i\,d^3xe^2 - 6i\,c^2f^2 - 3\,(i\,d^3fx^2 - 4i\,d^2fx - 4i\,cdf)e)e^{(dx+c)} - 12\,(cf^2 - dfe - (-i\,cf^2 + i\,dfe)e^{(dx+c)})\log(e^{(dx+c)} - i) + 12\,(df^2x + cf^2 + (i\,df^2x + i\,cf^2)e^{(dx+c)})\log(i\,e^{(dx+c)} + 1)}{3\,(ad^3e^{(dx+c)} - i\,ad^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(d^3*f^2*x^3 + 6*c^2*f^2 + 12*(I*f^2*e^(d*x + c) + f^2)*dilog(-I*e^(d*
x + c)) + 3*(d^3*x + 2*d^2)*e^2 + 3*(d^3*f*x^2 - 4*c*d*f)*e - (-I*d^3*f^2*x
^3 + 6*I*d^2*f^2*x^2 - 3*I*d^3*x*e^2 - 6*I*c^2*f^2 - 3*(I*d^3*f*x^2 - 4*I*d
^2*f*x - 4*I*c*d*f)*e)*e^(d*x + c) - 12*(c*f^2 - d*f*e - (-I*c*f^2 + I*d*f*
e)*e^(d*x + c))*log(e^(d*x + c) - I) + 12*(d*f^2*x + c*f^2 + (I*d*f^2*x + I
*c*f^2)*e^(d*x + c))*log(I*e^(d*x + c) + 1))/(a*d^3*e^(d*x + c) - I*a*d^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{-2e^2 - 4efx - 2f^2x^2}{ade^ce^{dx} - iad} - \frac{i\left(\int\left(-\frac{ide^2}{e^ce^{dx} - i}\right)dx + \int\frac{4ief}{e^ce^{dx} - i}\,dx + \int\frac{4if^2x}{e^ce^{dx} - i}\,dx + \int\left(-\frac{idf^2x^2}{e^ce^{dx} - i}\right)dx + \int\frac{de^2e^ce^{dx}}{e^ce^{dx} - i}\,dx + \int\left(-\frac{2idefx}{e^ce^{dx} - i}\right)dx + \int\frac{df^2x^2e^ce^{dx}}{e^ce^{dx} - i}\,dx + \int\frac{2defxe^ce^{dx}}{e^ce^{dx} - i}\,dx\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] (-2*e**2 - 4*e*f*x - 2*f**2*x**2)/(a*d*exp(c)*exp(d*x) - I*a*d) - I*(Integr
al(-I*d*e**2/(exp(c)*exp(d*x) - I), x) + Integral(4*I*e*f/(exp(c)*exp(d*x)
- I), x) + Integral(4*I*f**2*x/(exp(c)*exp(d*x) - I), x) + Integral(-I*d*f*
*2*x**2/(exp(c)*exp(d*x) - I), x) + Integral(d*e**2*exp(c)*exp(d*x)/(exp(c)
*exp(d*x) - I), x) + Integral(-2*I*d*e*f*x/(exp(c)*exp(d*x) - I), x) + Inte
gral(d*f**2*x**2*exp(c)*exp(d*x)/(exp(c)*exp(d*x) - I), x) + Integral(2*d*e
*f*x*exp(c)*exp(d*x)/(exp(c)*exp(d*x) - I), x))/(a*d)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sinh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c+d\,x)\,{\left(e+f\,x\right)}^2}{a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i),x)
```

```
[Out] int((sinh(c + d*x)*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i), x)
```

3.189 $\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=90

$$-\frac{iex}{a}-\frac{ifx^2}{2a}-\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}+\frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

```
[Out] -I*e*x/a-1/2*I*f*x^2/a-2*I*f*ln(cosh(1/2*c+1/4*I*Pi+1/2*d*x))/a/d^2+I*(f*x+
e)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5676, 3399, 4269, 3556}

$$-\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{ad^2}+\frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}-\frac{iex}{a}-\frac{ifx^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-I)*e*x)/a - ((I/2)*f*x^2)/a - ((2*I)*f*Log[Cosh[c/2 + (I/4)*Pi + (d*x)/2
]])/(a*d^2) + (I*(e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5676

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1)/
```

```
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx &= i\int \frac{e+fx}{a+ia\sinh(c+dx)}\,dx - \frac{i\int (e+fx)\,dx}{a} \\
&= -\frac{iex}{a} - \frac{ifx^2}{2a} + \frac{i\int (e+fx)\csc^2\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{idx}{2}\right)\,dx}{2a} \\
&= -\frac{iex}{a} - \frac{ifx^2}{2a} + \frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(if)\int \coth\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\,dx}{ad} \\
&= -\frac{iex}{a} - \frac{ifx^2}{2a} - \frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2} + \frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 239 vs. $2(90) = 180$.
time = 0.41, size = 239, normalized size = 2.66

$$\frac{-2dfx\cosh\left(c+\frac{dx}{2}\right) - i\cosh\left(\frac{dx}{2}\right)\left(d^2x(2e+fx) + 4if\,\mathrm{ArcTan}\left(\mathrm{sech}\left(c+\frac{dx}{2}\right)\sinh\left(\frac{dx}{2}\right)\right) + 2f\log(\cosh(c+dx))\right) + 4ide\sinh\left(\frac{dx}{2}\right) + 2idfx\sinh\left(\frac{dx}{2}\right) + 2d^2ex\sinh\left(c+\frac{dx}{2}\right) + d^2fx^2\sinh\left(c+\frac{dx}{2}\right) + 4if\,\mathrm{ArcTan}\left(\mathrm{sech}\left(c+\frac{dx}{2}\right)\sinh\left(\frac{dx}{2}\right)\right)\sinh\left(c+\frac{dx}{2}\right) + 2f\log(\cosh(c+dx))\sinh\left(c+\frac{dx}{2}\right)}{2ad^2\left(\cosh\left(\frac{c}{2}\right)+i\sinh\left(\frac{c}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (-2*d*f*x*Cosh[c + (d*x)/2] - I*Cosh[(d*x)/2]*(d^2*x*(2*e + f*x) + (4*I)*f*
ArcTan[Sech[c + (d*x)/2]*Sinh[(d*x)/2]] + 2*f*Log[Cosh[c + d*x]]) + (4*I)*d
*e*Sinh[(d*x)/2] + (2*I)*d*f*x*Sinh[(d*x)/2] + 2*d^2*e*x*Sinh[c + (d*x)/2]
+ d^2*f*x^2*Sinh[c + (d*x)/2] + (4*I)*f*ArcTan[Sech[c + (d*x)/2]*Sinh[(d*x)
/2]]*Sinh[c + (d*x)/2] + 2*f*Log[Cosh[c + d*x]]*Sinh[c + (d*x)/2])/(2*a*d^2
*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]))
```

Maple [A]
time = 1.27, size = 86, normalized size = 0.96

method	result	size
risch	$-\frac{i f\,x^{2}}{2a}-\frac{iex}{a}+\frac{2ifx}{ad}+\frac{2ifc}{a\,d^{2}}-\frac{2(fx+e)}{da\left(\mathrm{e}^{dx+c}-i\right)}-\frac{2if\ln\left(\mathrm{e}^{dx+c}-i\right)}{a\,d^{2}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*f*x^2/a-I*e*x/a+2*I*f/a/d*x+2*I*f/a/d^2*c-2*(f*x+e)/d/a/(exp(d*x+c)-
I)-2*I*f/a/d^2*ln(exp(d*x+c)-I)
```

Maxima [A]
time = 0.29, size = 109, normalized size = 1.21

$$\frac{1}{2}\,f\left(\frac{-i\,dx^2+(dx^2e^c-4\,xe^c)e^{(dx)}}{i\,ade^{(dx+c)}+ad}-\frac{4i\,\log\left(\left(e^{(dx+c)}-i\right)e^{(-c)}\right)}{ad^2}\right)-\left(\frac{i\,(dx+c)}{ad}+\frac{2}{(ae^{(-dx-c)}+i\,a)d}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*f*((-I*d*x^2 + (d*x^2*e^c - 4*x*e^c)*e^(d*x))/(I*a*d*e^(d*x + c) + a*d)
 - 4*I*log((e^(d*x + c) - I)*e^(-c))/(a*d^2)) - (I*(d*x + c)/(a*d) + 2/((a*
e^(-d*x - c) + I*a)*d))*e
```

Fricas [A]
time = 0.40, size = 97, normalized size = 1.08

$$-\frac{d^2fx^2+2\,(d^2x+2\,d)e-(-i\,d^2fx^2-2i\,d^2xe+4i\,dfx)e^{(dx+c)}+4\left(i\,fe^{(dx+c)}+f\right)\log\left(e^{(dx+c)}-i\right)}{2\,(ad^2e^{(dx+c)}-i\,ad^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*f*x^2 + 2*(d^2*x + 2*d)*e - (-I*d^2*f*x^2 - 2*I*d^2*x*e + 4*I*d*f
*x)*e^(d*x + c) + 4*(I*f*e^(d*x + c) + f)*log(e^(d*x + c) - I))/(a*d^2*e^(d
*x + c) - I*a*d^2)
```

Sympy [A]
time = 0.16, size = 73, normalized size = 0.81

$$\frac{-2e-2fx}{ade^ce^{dx}-iad}-\frac{ifx^2}{2a}+\frac{x(-ide+2if)}{ad}-\frac{2if\log\left(e^{dx}-ie^{-c}\right)}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] (-2*e - 2*f*x)/(a*d*exp(c)*exp(d*x) - I*a*d) - I*f*x**2/(2*a) + x*(-I*d*e +
 2*I*f)/(a*d) - 2*I*f*log(exp(d*x) - I*exp(-c))/(a*d**2)
```

Giac [A]
time = 0.43, size = 111, normalized size = 1.23

$$-\frac{i\,d^2fx^2e^{(dx+c)}+d^2fx^2+2i\,d^2exe^{(dx+c)}+2\,d^2ex-4i\,dfxe^{(dx+c)}+4i\,fe^{(dx+c)}\log\left(e^{(dx+c)}-i\right)+4\,de+4\,f\log\left(e^{(dx+c)}-i\right)}{2\,(ad^2e^{(dx+c)}-i\,ad^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(I*d^2*f*x^2*e^(d*x + c) + d^2*f*x^2 + 2*I*d^2*e*x*e^(d*x + c) + 2*d^2
*e*x - 4*I*d*f*x*e^(d*x + c) + 4*I*f*e^(d*x + c)*log(e^(d*x + c) - I) + 4*d
*e + 4*f*log(e^(d*x + c) - I))/(a*d^2*e^(d*x + c) - I*a*d^2)
```

Mupad [B]
time = 0.55, size = 74, normalized size = 0.82

$$-\frac{f\,x^2\,1\mathrm{i}}{2\,a}-\frac{2\,\left(e+f\,x\right)}{a\,d\,\left({\mathrm{e}}^{c+d\,x}-\mathrm{i}\right)}+\frac{x\,\left(2\,f-d\,e\right)\,1\mathrm{i}}{a\,d}-\frac{f\,\ln\left({\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c-\mathrm{i}\right)\,2\mathrm{i}}{a\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*(e + f*x))/(a + a*sinh(c + d*x)*1i),x)
```

```
[Out] (x*(2*f - d*e)*1i)/(a*d) - (2*(e + f*x))/(a*d*(exp(c + d*x) - 1i)) - (f*x^2
*1i)/(2*a) - (f*log(exp(d*x)*exp(c) - 1i)*2i)/(a*d^2)
```

3.190 $\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=35

$$-\frac{ix}{a}-\frac{\cosh(c+dx)}{d(a+ia\sinh(c+dx))}$$

```
[Out] -I*x/a-cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))
```

Rubi [A]
time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2814, 2727}

$$-\frac{\cosh(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{ix}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-I)*x)/a - Cosh[c + d*x]/(d*(a + I*a*Sinh[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\frac{ix}{a}+i\int \frac{1}{a+ia\sinh(c+dx)}\,dx\\ &= -\frac{ix}{a}-\frac{\cosh(c+dx)}{d(a+ia\sinh(c+dx))}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 84 vs. 2(35) = 70.
time = 0.10, size = 84, normalized size = 2.40

$$-\frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left((c+dx)\cosh\left(\frac{1}{2}(c+dx)\right)+i(2i+c+dx)\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{ad(-i+\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] -(((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*((c + d*x)*Cosh[(c + d*x)/2] +
 I*(2*I + c + d*x)*Sinh[(c + d*x)/2]))/(a*d*(-I + Sinh[c + d*x])))
```

Maple [A]
time = 0.88, size = 57, normalized size = 1.63

method	result	size
risch	$-\frac{ix}{a}-\frac{2}{da\left(\mathrm{e}^{dx+c}-i\right)}$	28
derivativedivides	$\frac{\frac{4i}{-2i+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$	57
default	$\frac{\frac{4i}{-2i+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a*(1/2*I/(-I+tanh(1/2*d*x+1/2*c))+1/4*I*ln(tanh(1/2*d*x+1/2*c)-1)-1/4*I
*ln(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [A]
time = 0.26, size = 36, normalized size = 1.03

$$-\frac{i\,(dx+c)}{ad}-\frac{2}{(ae^{(-dx-c)}+i\,a)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -I*(d*x + c)/(a*d) - 2/((a*e^(-d*x - c) + I*a)*d)
```

Fricas [A]
time = 0.33, size = 33, normalized size = 0.94

$$\frac{-i\,dxe^{(dx+c)}-dx-2}{ade^{(dx+c)}-i\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (-I*d*x*e^(d*x + c) - d*x - 2)/(a*d*e^(d*x + c) - I*a*d)
```

Sympy [A]
time = 0.08, size = 24, normalized size = 0.69

$$-\frac{2}{ade^{c}e^{dx}-iad}-\frac{ix}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -2/(a*d*exp(c)*exp(d*x) - I*a*d) - I*x/a
```

Giac [A]
time = 0.42, size = 33, normalized size = 0.94

$$-\frac{\frac{i\,(dx+c)}{a}+\frac{2i}{a\left(i\,e^{(dx+c)}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -(I*(d*x + c)/a + 2*I/(a*(I*e^(d*x + c) + 1)))/d
```

Mupad [B]
time = 0.24, size = 27, normalized size = 0.77

$$-\frac{x\,1\mathrm{i}}{a}-\frac{2}{a\,d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a + a*sinh(c + d*x)*1i),x)
```

```
[Out] - (x*1i)/a - 2/(a*d*(exp(c + d*x) - 1i))
```

3.191 $\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sinh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
[Out] Defer[Int][Sinh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 34.86, size = 0, normalized size = 0.00

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
[Out] Integrate[Sinh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(dx+c)}{(fx+e)(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*f*integrate(1/(-I*a*d*f^2*x^2 - 2*I*a*d*f*x*e - I*a*d*e^2 + (a*d*f^2*x^2
*e^c + 2*a*d*f*x*e^(c + 1) + a*d*e^(c + 2))*e^(d*x)), x) - 2/(-I*a*d*f*x -
I*a*d*e + (a*d*f*x*e^c + a*d*e^(c + 1))*e^(d*x)) - I*log(f*x + e)/(a*f)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))*integral(-(d*f*x +
d*e - (-I*d*f*x - I*d*e)*e^(d*x + c) + 2*f)/(-I*a*d*f^2*x^2 - 2*I*a*d*f*x*e
 - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2)*e^(d*x + c)), x) - 2)/
(-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2}{-iade - iadfx + (adee^{c} + adfxe^{c})e^{dx}} - \frac{i\left(\int \left(-\frac{2if}{e^{2}e^{c}e^{dx} - ie^{2} + 2efxe^{c}e^{dx} - 2iefx + f^{2}x^{2}e^{c}e^{dx} - if^{2}x^{2}}\right) dx + \int \left(-\frac{ide}{e^{2}e^{c}e^{dx} - ie^{2} + 2efxe^{c}e^{dx} - 2iefx + f^{2}x^{2}e^{c}e^{dx} - if^{2}x^{2}}\right) dx + \int \left(-\frac{idfx}{e^{2}e^{c}e^{dx} - ie^{2} + 2efxe^{c}e^{dx} - 2iefx + f^{2}x^{2}e^{c}e^{dx} - if^{2}x^{2}}\right) dx + \int \frac{dee^{c}e^{dx}}{e^{2}e^{c}e^{dx} - ie^{2} + 2efxe^{c}e^{dx} - 2iefx + f^{2}x^{2}e^{c}e^{dx} - if^{2}x^{2}}\, dx + \int \frac{dfxe^{c}e^{dx}}{e^{2}e^{c}e^{dx} - ie^{2} + 2efxe^{c}e^{dx} - 2iefx + f^{2}x^{2}e^{c}e^{dx} - if^{2}x^{2}}\, dx\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -2/(-I*a*d*e - I*a*d*f*x + (a*d*e*exp(c) + a*d*f*x*exp(c))*exp(d*x)) - I*(I
ntegral(-2*I*f/(e**2*exp(c)*exp(d*x) - I*e**2 + 2*e*f*x*exp(c)*exp(d*x) - 2
*I*e*f*x + f**2*x**2*exp(c)*exp(d*x) - I*f**2*x**2), x) + Integral(-I*d*e/(
e**2*exp(c)*exp(d*x) - I*e**2 + 2*e*f*x*exp(c)*exp(d*x) - 2*I*e*f*x + f**2*
x**2*exp(c)*exp(d*x) - I*f**2*x**2), x) + Integral(-I*d*f*x/(e**2*exp(c)*ex
p(d*x) - I*e**2 + 2*e*f*x*exp(c)*exp(d*x) - 2*I*e*f*x + f**2*x**2*exp(c)*ex
p(d*x) - I*f**2*x**2), x) + Integral(d*e*exp(c)*exp(d*x)/(e**2*exp(c)*exp(d
*x) - I*e**2 + 2*e*f*x*exp(c)*exp(d*x) - 2*I*e*f*x + f**2*x**2*exp(c)*exp(d
```

```
*x) - I*f**2*x**2), x) + Integral(d*f*x*exp(c)*exp(d*x)/(e**2*exp(c)*exp(d*
x) - I*e**2 + 2*e*f*x*exp(c)*exp(d*x) - 2*I*e*f*x + f**2*x**2*exp(c)*exp(d*
x) - I*f**2*x**2), x))/(a*d)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate(sinh(d*x + c)/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sinh(c+d\,x)}{(e+f\,x)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/((e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
[Out] int(sinh(c + d*x)/((e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.192 $\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sinh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
[Out] Defer[Int][Sinh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 32.80, size = 0, normalized size = 0.00

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
[Out] Integrate[Sinh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(dx+c)}{(fx+e)^2\,(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -4*f*integrate(1/(-I*a*d*f^3*x^3 - 3*I*a*d*f^2*x^2*e - 3*I*a*d*f*x*e^2 - I*
a*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*d*f^2*x^2*e^(c + 1) + 3*a*d*f*x*e^(c + 2)
+ a*d*e^(c + 3))*e^(d*x)), x) + (d*f*x + d*e - (-I*d*f*x*e^c - I*d*e^(c + 1
))*e^(d*x) - 2*f)/(-I*a*d*f^3*x^2 - 2*I*a*d*f^2*x*e - I*a*d*f*e^2 + (a*d*f^
3*x^2*e^c + 2*a*d*f^2*x*e^(c + 1) + a*d*f*e^(c + 2))*e^(d*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((-I*a*d*f^2*x^2 - 2*I*a*d*f*x*e - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*f*x*e +
 a*d*e^2)*e^(d*x + c))*integral(-(d*f*x + d*e - (-I*d*f*x - I*d*e)*e^(d*x +
 c) + 4*f)/(-I*a*d*f^3*x^3 - 3*I*a*d*f^2*x^2*e - 3*I*a*d*f*x*e^2 - I*a*d*e^
3 + (a*d*f^3*x^3 + 3*a*d*f^2*x^2*e + 3*a*d*f*x*e^2 + a*d*e^3)*e^(d*x + c)),
 x) - 2)/(-I*a*d*f^2*x^2 - 2*I*a*d*f*x*e - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d
*f*x*e + a*d*e^2)*e^(d*x + c))
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2}{-iade^2-2iadefx-iadf^2x^2+\left(ade^2e^c+2adefxe^c+adf^2x^2e^c\right)e^{dx}}-\frac{i\left(\int\left(-\frac{4if}{e^3e^ce^{dx}-ie^3+3e^2fxe^ce^{dx}-3ie^2fx+3ef^2x^2e^ce^{dx}-3ief^2x^2+f^3x^3e^ce^{dx}-if^3x^3}\right)dx+\int\left(-\frac{ide}{e^3e^ce^{dx}-ie^3+3e^2fxe^ce^{dx}-3ie^2fx+3ef^2x^2e^ce^{dx}-3ief^2x^2+f^3x^3e^ce^{dx}-if^3x^3}\right)dx+\int\left(-\frac{idfx}{e^3e^ce^{dx}-ie^3+3e^2fxe^ce^{dx}-3ie^2fx+3ef^2x^2e^ce^{dx}-3ief^2x^2+f^3x^3e^ce^{dx}-if^3x^3}\right)dx+\int\frac{dee^ce^{dx}}{e^3e^ce^{dx}-ie^3+3e^2fxe^ce^{dx}-3ie^2fx+3ef^2x^2e^ce^{dx}-3ief^2x^2+f^3x^3e^ce^{dx}-if^3x^3}\,dx+\int\frac{dfxe^ce^{dx}}{e^3e^ce^{dx}-ie^3+3e^2fxe^ce^{dx}-3ie^2fx+3ef^2x^2e^ce^{dx}-3ief^2x^2+f^3x^3e^ce^{dx}-if^3x^3}\,dx\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -2/(-I*a*d*e**2 - 2*I*a*d*e*f*x - I*a*d*f**2*x**2 + (a*d*e**2*exp(c) + 2*a*
d*e*f*x*exp(c) + a*d*f**2*x**2*exp(c))*exp(d*x)) - I*(Integral(-4*I*f/(e**3
*exp(c)*exp(d*x) - I*e**3 + 3*e**2*f*x*exp(c)*exp(d*x) - 3*I*e**2*f*x + 3*e
*f**2*x**2*exp(c)*exp(d*x) - 3*I*e*f**2*x**2 + f**3*x**3*exp(c)*exp(d*x) -
```

```
I*f**3*x**3), x) + Integral(-I*d*e/(e**3*exp(c)*exp(d*x) - I*e**3 + 3*e**2*
f*x*exp(c)*exp(d*x) - 3*I*e**2*f*x + 3*e*f**2*x**2*exp(c)*exp(d*x) - 3*I*e*
f**2*x**2 + f**3*x**3*exp(c)*exp(d*x) - I*f**3*x**3), x) + Integral(-I*d*f*
x/(e**3*exp(c)*exp(d*x) - I*e**3 + 3*e**2*f*x*exp(c)*exp(d*x) - 3*I*e**2*f*
x + 3*e*f**2*x**2*exp(c)*exp(d*x) - 3*I*e*f**2*x**2 + f**3*x**3*exp(c)*exp(
d*x) - I*f**3*x**3), x) + Integral(d*e*exp(c)*exp(d*x)/(e**3*exp(c)*exp(d*x
) - I*e**3 + 3*e**2*f*x*exp(c)*exp(d*x) - 3*I*e**2*f*x + 3*e*f**2*x**2*exp(
c)*exp(d*x) - 3*I*e*f**2*x**2 + f**3*x**3*exp(c)*exp(d*x) - I*f**3*x**3), x
) + Integral(d*f*x*exp(c)*exp(d*x)/(e**3*exp(c)*exp(d*x) - I*e**3 + 3*e**2*
f*x*exp(c)*exp(d*x) - 3*I*e**2*f*x + 3*e*f**2*x**2*exp(c)*exp(d*x) - 3*I*e*
f**2*x**2 + f**3*x**3*exp(c)*exp(d*x) - I*f**3*x**3), x))/(a*d)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate(sinh(d*x + c)/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sinh(c+d\,x)}{{(e+f\,x)}^2\,\left(a+a\,\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
[Out] int(sinh(c + d*x)/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.193 $\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=241

$$-\frac{(e+fx)^3}{ad}+\frac{(e+fx)^4}{4af}-\frac{6if^2(e+fx)\cosh(c+dx)}{ad^3}-\frac{i(e+fx)^3\cosh(c+dx)}{ad}+\frac{6f(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}$$

```
[Out] -(f*x+e)^3/a/d+1/4*(f*x+e)^4/a/f-6*I*f^2*(f*x+e)*cosh(d*x+c)/a/d^3-I*(f*x+e
)^3*cosh(d*x+c)/a/d+6*f*(f*x+e)^2*ln(1+I*exp(d*x+c))/a/d^2+12*f^2*(f*x+e)*p
olylog(2,-I*exp(d*x+c))/a/d^3-12*f^3*polylog(3,-I*exp(d*x+c))/a/d^4+6*I*f^3
*sinh(d*x+c)/a/d^4+3*I*f*(f*x+e)^2*sinh(d*x+c)/a/d^2-(f*x+e)^3*tanh(1/2*c+1
/4*I*Pi+1/2*d*x)/a/d
```

Rubi [A]
time = 0.38, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {5676, 3377, 2717, 32, 3399, 4269, 3797, 2221, 2611, 2320, 6724}

$$-\frac{12f^3\text{Li}_3\left(-ie^{c+dx}\right)}{ad^4}+\frac{6if^3\sinh(c+dx)}{ad^4}+\frac{12f^2(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{ad^3}-\frac{6if^2(e+fx)\cosh(c+dx)}{ad^3}+\frac{6f(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}+\frac{3if(e+fx)^2\sinh(c+dx)}{ad^2}-\frac{i(e+fx)^3\cosh(c+dx)}{ad}-\frac{(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}-\frac{(e+fx)^3}{ad}+\frac{(e+fx)^4}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] -((e + f*x)^3/(a*d)) + (e + f*x)^4/(4*a*f) - ((6*I)*f^2*(e + f*x)*Cosh[c +
d*x])/(a*d^3) - (I*(e + f*x)^3*Cosh[c + d*x])/(a*d) + (6*f*(e + f*x)^2*Log[
1 + I*E^(c + d*x)])/(a*d^2) + (12*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)
])/(a*d^3) - (12*f^3*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^4) + ((6*I)*f^3*Sin
h[c + d*x])/(a*d^4) + ((3*I)*f*(e + f*x)^2*Sinh[c + d*x])/(a*d^2) - ((e + f
*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5676

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = i\int \frac{(e+fx)^3\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx - \frac{i\int (e+fx)^3\sinh(c+dx)\,dx}{a}$$
$$= -\frac{i(e+fx)^3\cosh(c+dx)}{ad} + \frac{\int (e+fx)^3\,dx}{a} + \frac{(3if)\int (e+fx)^2\cosh(c+dx}{ad}$$
$$= \frac{(e+fx)^4}{4af} - \frac{i(e+fx)^3\cosh(c+dx)}{ad} + \frac{3if(e+fx)^2\sinh(c+dx)}{ad^2} - \frac{\int (e+}{}$$
$$= \frac{(e+fx)^4}{4af} - \frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^3\cosh(c+dx)}{ad} + \frac{3if(e}{}$$
$$= -\frac{(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} - \frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^3\cosh(c}{ad}$$
$$= -\frac{(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} - \frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^3\cosh(c}{ad}$$
$$= -\frac{(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} - \frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^3\cosh(c}{ad}$$
$$= -\frac{(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} - \frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^3\cosh(c}{ad}$$
$$= -\frac{(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} - \frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^3\cosh(c}{ad}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 872 vs. $2(241) = 482$.

time = 4.52, size = 872, normalized size = 3.62

Antiderivative was successfully verified.

[In] Integrate[((e + f*x)^3*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]

[Out] (((-8*I)*f*(d^2*((-I)*d*E^c*x*(3*e^2 + 3*e*f*x + f^2*x^2) + 3*(1 + I*E^c)*(e + f*x)^2*Log[1 + I*E^(c + d*x)]) + 6*d*(1 + I*E^c)*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)] - (6*I)*(-I + E^c)*f^2*PolyLog[3, (-I)*E^(c + d*x)]))/(-I + E^c) + ((12*f^3 + 6*d^2*f*(e + f*x)^2 + d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))*Cosh[(d*x)/2] - (2*I)*d*(e + f*x)*(6*f^2 + d^2*(e + f*x)^2)*Cosh[c + (d*x)/2] - (2*I)*d*(e + f*x)*(6*f^2 + d^2*(e + f*x)^2)*Cosh[c + (3*d*x)/2] - 6*d^2*e^2*f*Cosh[2*c + (3*d*x)/2] - 12*f^3*Cosh[2*c + (3*d*x)/2] - 12*d^2*e*f^2*x*Cosh[2*c + (3*d*x)/2] - 6*d^2*f^3*x^2*Cosh[2*c + (3*d*x)/2] - 10*d^3*e^3*Sinh[(d*x)/2] - 12*d*e*f^2*Sinh[(d*x)/2] - 30*d^3*e^2*f*x*Sinh[(d*x)/2] - 12*d*f^3*x*Sinh[(d*x)/2] - 30*d^3*e*f^2*x^2*Sinh[(d*x)/2] - 10*d^3*f^3*x^3*Sinh[(d*x)/2] + (6*I)*d^2*e^2*f*Sinh[c + (d*x)/2] + (12*I)*f^3*Sinh[c + (d*x)/2] + (4*I)*d^4*e^3*x*Sinh[c + (d*x)/2] + (12*I)*d^2*e*f^2*x*Sinh[c + (d*x)/2] + (6*I)*d^4*e^2*f*x^2*Sinh[c + (d*x)/2] + (6*I)*d^2*f^3*x^2*Sinh[c + (d*x)/2] + (4*I)*d^4*e*f^2*x^3*Sinh[c + (d*x)/2] + I*d^4*f^3*x^4*Sinh[c + (d*x)/2] + (6*I)*d^2*e^2*f*Sinh[c + (3*d*x)/2] + (12*I)*f^3*Sinh[c + (3*d*x)/2] + (12*I)*d^2*e*f^2*x*Sinh[c + (3*d*x)/2] + (6*I)*d^2*f^3*x^2*Sinh[c + (3*d*x)/2] + 2*d^3*e^3*Sinh[2*c + (3*d*x)/2] + 12*d*e*f^2*Sinh[2*c + (3*d*x)/2] + 6*d^3*e^2*f*x*Sinh[2*c + (3*d*x)/2] + 12*d*f^3*x*Sinh[2*c + (3*d*x)/2] + 6*d^3*e*f^2*x^2*Sinh[2*c + (3*d*x)/2] + 2*d^3*f^3*x^3*Sinh[2*c + (3*d*x)/2])/((Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])))/(4*a*d^4)

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 698 vs. 2(222) = 444.
time = 3.16, size = 699, normalized size = 2.90

method	result
risch	$\frac{f^2 e x^3}{a}+\frac{3f e^2 x^2}{2a}+\frac{e^3 x}{a}+\frac{f^3 x^4}{4a}+\frac{e^4}{4af}+\frac{4f^3 c^3}{a d^4}-\frac{2f^3 x^3}{ad}-\frac{i(d^3x^3f^3+3d^3e f^2x^2+3d^3e^2fx-3d^2f^3x^2+d^3e^3-6d^2e f^2x-}{2a d^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x+e)^3*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -12*f^3*polylog(3,-I*exp(d*x+c))/a/d^4+1/a*f^2*e*x^3+3/2/a*f*e^2*x^2+1/a*e^3*x+1/4/a*f^3*x^4+1/4/a/f*e^4+4/a/d^4*f^3*c^3-2/a/d*f^3*x^3-1/2*I*(d^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x-3*d^2*f^3*x^2+d^3*e^3-6*d^2*e*f^2*x-3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2-6*f^3)/a/d^4*exp(d*x+c)-2*I*(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e^3)/d/a/(exp(d*x+c)-I)-12/a/d^3*f^2*c*e*ln(exp(d*x+c)-I)+12/a/d

```
^3*f^2*c*e*ln(exp(d*x+c))-12/a/d^2*f^2*e*c*x+12/a/d^2*f^2*e*ln(1+I*exp(d*x+
c))*x+12/a/d^3*f^2*e*ln(1+I*exp(d*x+c))*c-6/a/d^2*f*ln(exp(d*x+c))*e^2+6/a/
d^2*f*ln(exp(d*x+c)-I)*e^2+6/a/d^4*f^3*c^2*ln(exp(d*x+c)-I)+12/a/d^3*f^3*po
lylog(2,-I*exp(d*x+c))*x+6/a/d^2*f^3*ln(1+I*exp(d*x+c))*x^2-6/a/d^4*f^3*ln(
1+I*exp(d*x+c))*c^2-6/a/d^4*f^3*c^2*ln(exp(d*x+c))+6/a/d^3*f^3*c^2*x-6/a/d*
f^2*e*x^2-6/a/d^3*f^2*e*c^2+12/a/d^3*f^2*e*polylog(2,-I*exp(d*x+c))-1/2*I*(
d^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x+3*d^2*f^3*x^2+d^3*e^3+6*d^2*e*f^2
*x+3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2+6*f^3)/a/d^4*exp(-d*x-c)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(221) = 442$.
time = 0.44, size = 697, normalized size = 2.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -3/2*f*(2*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) + (I*d^2*x^2*e^c + I*d*x*
e^c - (-I*d*x*e^(3*c) + I*e^(3*c))*e^(2*d*x) - (d^2*x^2*e^(2*c) - 3*d*x*e^(
2*c) + e^(2*c))*e^(d*x) + (d*x + 1)*e^(-d*x) + I*e^c)/(a*d^2*e^(d*x + 2*c)
- I*a*d^2*e^c) - 4*log((e^(d*x + c) - I)*e^(-c))/(a*d^2))*e^2 + 1/2*(2*(d*x
 + c)/(a*d) + (-5*I*e^(-d*x - c) + 1)/((I*a*e^(-d*x - c) + a*e^(-2*d*x - 2*
c))*d) - I*e^(-d*x - c)/(a*d))*e^3 + 1/4*(-I*d^4*f^3*x^4 + 2*(-2*I*d^4*f^2*
e - 5*I*d^3*f^3)*x^3 - 12*I*d*f^2*e - 12*I*f^3 + 6*(-5*I*d^3*f^2*e - I*d^2*
f^3)*x^2 + 12*(-I*d^2*f^2*e - I*d*f^3)*x + 2*(-I*d^3*f^3*x^3*e^(2*c) + 6*I*
f^3*e^(2*c) - 6*I*d*f^2*e^(2*c + 1) + 3*(I*d^2*f^3*e^(2*c) - I*d^3*f^2*e^(2
*c + 1))*x^2 + 6*(-I*d*f^3*e^(2*c) + I*d^2*f^2*e^(2*c + 1))*x)*e^(2*d*x) +
(d^4*f^3*x^4*e^c + 2*(2*d^4*f^2*e^(c + 1) - d^3*f^3*e^c)*x^3 - 12*d*f^2*e^(
c + 1) + 12*f^3*e^c - 6*(d^3*f^2*e^(c + 1) - d^2*f^3*e^c)*x^2 + 12*(d^2*f^2
*e^(c + 1) - d*f^3*e^c)*x)*e^(d*x))/(a*d^4*e^(d*x + c) - I*a*d^4) + 12*(d*x
*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*f^2*e/(a*d^3) + 6*(d^2*x^2
*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x + c)) - 2*polylog(3, -I*e^(
d*x + c)))*f^3/(a*d^4) - 2*(d^3*f^3*x^3 + 3*d^3*f^2*x^2*e)/(a*d^4)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 825 vs. $2(221) = 442$.
time = 0.41, size = 825, normalized size = 3.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

[Out] -1/4*(2*d^3*f^3*x^3 + 6*d^2*f^3*x^2 + 12*d*f^3*x + 2*d^3*e^3 + 12*f^3 - 48*
((d*f^3*x + d*f^2*e)*e^(2*d*x + 2*c) - (I*d*f^3*x + I*d*f^2*e)*e^(d*x + c))
*dilog(-I*e^(d*x + c)) + 6*(d^3*f*x + d^2*f)*e^2 + 6*(d^3*f^2*x^2 + 2*d^2*f
^2*x + 2*d*f^2)*e + 2*(I*d^3*f^3*x^3 - 3*I*d^2*f^3*x^2 + 6*I*d*f^3*x + I*d^
3*e^3 - 6*I*f^3 + 3*(I*d^3*f*x - I*d^2*f)*e^2 + 3*(I*d^3*f^2*x^2 - 2*I*d^2*
f^2*x + 2*I*d*f^2)*e)*e^(3*d*x + 3*c) - (d^4*f^3*x^4 - 10*d^3*f^3*x^3 + 6*d
^2*f^3*x^2 - 12*d*f^3*x - 4*(2*c^3 - 3)*f^3 + 2*(2*d^4*x - d^3)*e^3 + 6*(d^
4*f*x^2 - 5*d^3*f*x - (4*c - 1)*d^2*f)*e^2 + 2*(2*d^4*f^2*x^3 - 15*d^3*f^2*
x^2 + 6*d^2*f^2*x + 6*(2*c^2 - 1)*d*f^2)*e)*e^(2*d*x + 2*c) - (-I*d^4*f^3*x
^4 - 2*I*d^3*f^3*x^3 - 6*I*d^2*f^3*x^2 - 12*I*d*f^3*x - 4*(-2*I*c^3 + 3*I)*
f^3 - 2*(2*I*d^4*x + 5*I*d^3)*e^3 - 6*(I*d^4*f*x^2 + I*d^3*f*x + (-4*I*c +
I)*d^2*f)*e^2 - 2*(2*I*d^4*f^2*x^3 + 3*I*d^3*f^2*x^2 + 6*I*d^2*f^2*x + 6*(2
*I*c^2 + I)*d*f^2)*e)*e^(d*x + c) - 24*((c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)
*e^(2*d*x + 2*c) - (I*c^2*f^3 - 2*I*c*d*f^2*e + I*d^2*f*e^2)*e^(d*x + c))*l
og(e^(d*x + c) - I) - 24*((d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x + c*d*f^2)*
e)*e^(2*d*x + 2*c) - (I*d^2*f^3*x^2 - I*c^2*f^3 + 2*(I*d^2*f^2*x + I*c*d*f^
2)*e)*e^(d*x + c))*log(I*e^(d*x + c) + 1) + 48*(f^3*e^(2*d*x + 2*c) - I*f^3
*e^(d*x + c))*polylog(3, -I*e^(d*x + c)))/(a*d^4*e^(2*d*x + 2*c) - I*a*d^4*
e^(d*x + c))

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{-2ie^3 - 6ie^2fx - 6ief^2x^2 - 2if^3x^3}{ade^ce^{dx} - iad} - \frac{i\left(\int \frac{ide^3}{e^ce^{2dx} - ie^{dx}}\,dx + \int \frac{idf^3x^3}{e^ce^{2dx} - ie^{dx}}\,dx + \int \frac{de^3e^ce^{dx}}{e^ce^{2dx} - ie^{dx}}\,dx + \int \frac{de^3e^{3c}e^{3dx}}{e^ce^{2dx} - ie^{dx}}\,dx + \int \left(-\frac{12e^2fe^ce^{dx}}{e^ce^{2dx} - ie^{dx}}\right)dx + \int \left(-\frac{12f^3x^2e^ce^{dx}}{e^ce^{2dx} - ie^{dx}}\right)dx + \int \frac{3idef^2x^2}{e^ce^{2dx} - ie^{dx}}\,dx + \int \frac{3ide^2fx}{e^ce^{2dx} - ie^{dx}}\,dx + \int \frac{ide^3e^{2c}e^{2dx}}{e^ce^{2dx} - ie^{dx}}\,dx + \int \frac{df^3x^3e^ce^{dx}}{e^ce^{2dx} - ie^{dx}}\,dx + \int \frac{df^3x^3e^{3c}e^{3dx}}{e^ce^{2dx} - ie^{dx}}\,dx + \int \left(-\frac{24ef^2xe^ce^{dx}}{e^ce^{2dx} - ie^{dx}}\right)dx + \int \frac{idf^3x^3e^{2c}e^{2dx}}{e^ce^{2dx} - ie^{dx}}\,dx + \int \frac{3def^2x^2e^ce^{dx}}{e^ce^{2dx} - ie^{dx}}\,dx + \int \frac{3def^2x^2e^{3c}e^{3dx}}{e^ce^{2dx} - ie^{dx}}\,dx + \int \frac{3de^2fxe^ce^{dx}}{e^ce^{2dx} - ie^{dx}}\,dx + \int \frac{3de^2fxe^{3c}e^{3dx}}{e^ce^{2dx} - ie^{dx}}\,dx + \int \frac{3idef^2x^2e^{2c}e^{2dx}}{e^ce^{2dx} - ie^{dx}}\,dx + \int \frac{3ide^2fxe^{2c}e^{2dx}}{e^ce^{2dx} - ie^{dx}}\,dx\right)e^{-c}}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)**3*sinh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)

[Out] (-2*I*e**3 - 6*I*e**2*f*x - 6*I*e*f**2*x**2 - 2*I*f**3*x**3)/(a*d*exp(c)*ex
p(d*x) - I*a*d) - I*(Integral(I*d*e**3/(exp(c)*exp(2*d*x) - I*exp(d*x)), x)
 + Integral(I*d*f**3*x**3/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(d
*e**3*exp(c)*exp(d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(d*e**
3*exp(3*c)*exp(3*d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(-12*e
**2*f*exp(c)*exp(d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(-12*f
3*x2*exp(c)*exp(d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(3*
I*d*e*f**2*x**2/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(3*I*d*e**2*
f*x/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(I*d*e**3*exp(2*c)*exp(2
*d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(d*f**3*x**3*exp(c)*ex
p(d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(d*f**3*x**3*exp(3*c)
*exp(3*d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(-24*e*f**2*x*ex
p(c)*exp(d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(I*d*f**3*x**3
*exp(2*c)*exp(2*d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(3*d*e*
f**2*x**2*exp(c)*exp(d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(3
*d*e*f**2*x**2*exp(3*c)*exp(3*d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + I
ntegral(3*d*e**2*f*x*exp(c)*exp(d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) +

```
 Integral(3*d*e**2*f*x*exp(3*c)*exp(3*d*x)/(exp(c)*exp(2*d*x) - I*exp(d*x))
, x) + Integral(3*I*d*e*f**2*x**2*exp(2*c)*exp(2*d*x)/(exp(c)*exp(2*d*x) -
I*exp(d*x)), x) + Integral(3*I*d*e**2*f*x*exp(2*c)*exp(2*d*x)/(exp(c)*exp(2
*d*x) - I*exp(d*x)), x))*exp(-c)/(2*a*d)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sinh(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^2\,(e+f\,x)^3}{a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)^2*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i),x)
```

```
[Out] int((sinh(c + d*x)^2*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i), x)
```

3.194 $\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=184

$$-\frac{(e+fx)^2}{ad}+\frac{(e+fx)^3}{3af}-\frac{2if^2\cosh(c+dx)}{ad^3}-\frac{i(e+fx)^2\cosh(c+dx)}{ad}+\frac{4f(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}+\frac{4f^2\mathrm{Po}}{}$$

```
[Out] -(f*x+e)^2/a/d+1/3*(f*x+e)^3/a/f-2*I*f^2*cosh(d*x+c)/a/d^3-I*(f*x+e)^2*cosh
(d*x+c)/a/d+4*f*(f*x+e)*ln(1+I*exp(d*x+c))/a/d^2+4*f^2*polylog(2,-I*exp(d*x
+c))/a/d^3+2*I*f*(f*x+e)*sinh(d*x+c)/a/d^2-(f*x+e)^2*tanh(1/2*c+1/4*I*Pi+1/
2*d*x)/a/d
```

Rubi [A]
time = 0.28, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {5676, 3377, 2718, 32, 3399, 4269, 3797, 2221, 2317, 2438}

$$\frac{4f^2\mathrm{Li}_2\left(-ie^{c+dx}\right)}{ad^3}-\frac{2if^2\cosh(c+dx)}{ad^3}+\frac{4f(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}+\frac{2if(e+fx)\sinh(c+dx)}{ad^2}-\frac{i(e+fx)^2\cosh(c+dx)}{ad}-\frac{(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}-\frac{(e+fx)^2}{ad}+\frac{(e+fx)^3}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] -((e + f*x)^2/(a*d)) + (e + f*x)^3/(3*a*f) - ((2*I)*f^2*Cosh[c + d*x])/(a*d
^3) - (I*(e + f*x)^2*Cosh[c + d*x])/(a*d) + (4*f*(e + f*x)*Log[1 + I*E^(c +
 d*x)])/(a*d^2) + (4*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^3) + ((2*I)*f*(
e + f*x)*Sinh[c + d*x])/(a*d^2) - ((e + f*x)^2*Tanh[c/2 + (I/4)*Pi + (d*x)/
2])/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5676

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = i\int \frac{(e+fx)^2\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx - \frac{i\int (e+fx)^2\sinh(c+dx)\,dx}{a}$$

$$= -\frac{i(e+fx)^2\cosh(c+dx)}{ad} + \frac{\int (e+fx)^2\,dx}{a} + \frac{(2if)\int (e+fx)\cosh(c+dx}{ad}$$

$$= \frac{(e+fx)^3}{3af} - \frac{i(e+fx)^2\cosh(c+dx)}{ad} + \frac{2if(e+fx)\sinh(c+dx)}{ad^2} - \int (e+$$

$$= \frac{(e+fx)^3}{3af} - \frac{2if^2\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2\cosh(c+dx)}{ad} + \frac{2if(e+fx)\,\mathrm{s}}{ad}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} - \frac{2if^2\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2\cosh(c+dx)}{ad} +$$

$$= -\frac{(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} - \frac{2if^2\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2\cosh(c+dx)}{ad} +$$

$$= -\frac{(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} - \frac{2if^2\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2\cosh(c+dx)}{ad} +$$

$$= -\frac{(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} - \frac{2if^2\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2\cosh(c+dx)}{ad} +$$

Mathematica [A]
time = 2.45, size = 249, normalized size = 1.35

$$\frac{x(3e^2+3efx+f^2x^2) + \frac{6f\left(d\left(-\frac{de^c x(2e+fx)}{-i+e^c}+2(e+fx)\log\left(1+ie^{c+dx}\right)\right)+2f\,\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)\right)}{d^3} - \frac{3i\cosh(dx)\left(\left(2f^2+d^2(e+fx)^2\right)\cosh(c)-2df(e+fx)\sinh(c)\right)}{d^3} - \frac{3i\left(-2df(e+fx)\cosh(c)+\left(2f^2+d^2(e+fx)^2\right)\sinh(c)\right)\sinh(dx)}{d^3} - \frac{6(e+fx)^2\sinh\left(\frac{dx}{2}\right)}{d\left(\cosh\left(\frac{c}{2}\right)+i\sinh\left(\frac{c}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2) + (6*f*(d*(-((d*E^c*x*(2*e + f*x))/(-I + E^c
)) + 2*(e + f*x)*Log[1 + I*E^(c + d*x)]) + 2*f*PolyLog[2, (-I)*E^(c + d*x)]
))/d^3 - ((3*I)*Cosh[d*x]*((2*f^2 + d^2*(e + f*x)^2)*Cosh[c] - 2*d*f*(e + f
*x)*Sinh[c]))/d^3 - ((3*I)*(-2*d*f*(e + f*x)*Cosh[c] + (2*f^2 + d^2*(e + f*
x)^2)*Sinh[c])*Sinh[d*x])/d^3 - (6*(e + f*x)^2*Sinh[(d*x)/2])/(d*(Cosh[c/2]
 + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])))/(3*a)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(168) = 336.
time = 1.76, size = 385, normalized size = 2.09

method	result
risch	$\frac{f^2x^3}{3a}+\frac{fe\,x^2}{a}+\frac{e^2x}{a}+\frac{e^3}{3af}-\frac{i(f^2x^2d^2+2d^2efx+d^2e^2-2d\,f^2x-2def+2f^2)\,\mathrm{e}^{dx+c}}{2a\,d^3}-\frac{i(f^2x^2d^2+2d^2efx+d^2e^2+2d\,f^2x+2def}{2a\,d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a*f^2*x^3+1/a*f*e*x^2+1/a*e^2*x+1/3/a/f*e^3-1/2*I*(d^2*f^2*x^2+2*d^2*e*
f*x+d^2*e^2-2*d*f^2*x-2*d*e*f+2*f^2)/a/d^3*exp(d*x+c)-1/2*I*(d^2*f^2*x^2+2*
d^2*e*f*x+d^2*e^2+2*d*f^2*x+2*d*e*f+2*f^2)/a/d^3*exp(-d*x-c)-2*I*(f^2*x^2+2
*e*f*x+e^2)/d/a/(exp(d*x+c)-I)+4/a/d^2*f*ln(exp(d*x+c)-I)*e-4/a/d^2*f*ln(ex
p(d*x+c))*e-2*f^2*x^2/a/d-4/a/d^2*f^2*c*x-2/a/d^3*f^2*c^2+4/a/d^2*f^2*ln(1+
I*exp(d*x+c))*x+4/a/d^3*f^2*ln(1+I*exp(d*x+c))*c+4*f^2*polylog(2,-I*exp(d*x
+c))/a/d^3-4/a/d^3*f^2*c*ln(exp(d*x+c)-I)+4/a/d^3*f^2*c*ln(exp(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/6*f^2*((2*I*d^3*x^3 + 15*I*d^2*x^2 + 6*I*d*x - 3*(-I*d^2*x^2*e^(2*c) + 2
*I*d*x*e^(2*c) - 2*I*e^(2*c))*e^(2*d*x) - (2*d^3*x^3*e^c - 3*d^2*x^2*e^c +
6*d*x*e^c - 6*e^c)*e^(d*x) + 6*I)/(a*d^3*e^(d*x + c) - I*a*d^3) - 24*I*inte
grate(x/(a*d*e^(d*x + c) - I*a*d), x)) - f*(2*x*e^(d*x + c)/(a*d*e^(d*x + c
) - I*a*d) + (I*d^2*x^2*e^c + I*d*x*e^c - (-I*d*x*e^(3*c) + I*e^(3*c))*e^(2
*d*x) - (d^2*x^2*e^(2*c) - 3*d*x*e^(2*c) + e^(2*c))*e^(d*x) + (d*x + 1)*e^(
-d*x) + I*e^c)/(a*d^2*e^(d*x + 2*c) - I*a*d^2*e^c) - 4*log((e^(d*x + c) - I
)*e^(-c))/(a*d^2))*e + 1/2*(2*(d*x + c)/(a*d) + (-5*I*e^(-d*x - c) + 1)/((I
*a*e^(-d*x - c) + a*e^(-2*d*x - 2*c))*d) - I*e^(-d*x - c)/(a*d))*e^2
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(167) = 334$.
time = 0.35, size = 475, normalized size = 2.58

$$-\frac{3\,d^2f^2x^2+6\,df^2x+3\,d^2e^2+6\,f^2-24\,\left(f^2e^{(2\,dx+2\,c)}-i\,f^2e^{(dx+c)}\right)\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)+6\,(d^2fx+df)e+3\,(i\,d^2f^2x^2-2i\,df^2x+i\,d^2e^2+2i\,f^2+2\,(i\,d^2fx-i\,df)e)e^{(3\,dx+3\,c)}-(2\,d^3f^2x^3-15\,d^2f^2x^2+6\,df^2x+6\,(2\,c^2-1)f^2+3\,(2\,d^3x-d^2)e^2+6\,(d^3fx^2-5\,d^2fx-(4\,c-1)df)e)e^{(2\,dx+2\,c)}-(-2i\,d^3f^2x^3-3i\,d^2f^2x^2-6i\,df^2x-6\,(2i\,c^2+i)f^2-3\,(2i\,d^3x+5i\,d^2)e^2-6\,(i\,d^3fx^2+i\,d^2fx+(-4i\,c+i)df)e)e^{(dx+c)}+24\,\left((cf^2-dfe)e^{(2\,dx+2\,c)}+(-i\,cf^2+i\,dfe)e^{(dx+c)}\right)\log\left(e^{(dx+c)}-i\right)-24\,\left((df^2x+cf^2)e^{(2\,dx+2\,c)}-(i\,df^2x+i\,cf^2)e^{(dx+c)}\right)\log\left(i\,e^{(dx+c)}+1\right)}{6\,\left(ad^3e^{(2\,dx+2\,c)}-i\,ad^3e^{(dx+c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/6*(3*d^2*f^2*x^2 + 6*d*f^2*x + 3*d^2*e^2 + 6*f^2 - 24*(f^2*e^(2*d*x + 2*
c) - I*f^2*e^(d*x + c))*dilog(-I*e^(d*x + c)) + 6*(d^2*f*x + d*f)*e + 3*(I*
d^2*f^2*x^2 - 2*I*d*f^2*x + I*d^2*e^2 + 2*I*f^2 + 2*(I*d^2*f*x - I*d*f)*e)*
e^(3*d*x + 3*c) - (2*d^3*f^2*x^3 - 15*d^2*f^2*x^2 + 6*d*f^2*x + 6*(2*c^2 -
1)*f^2 + 3*(2*d^3*x - d^2)*e^2 + 6*(d^3*f*x^2 - 5*d^2*f*x - (4*c - 1)*d*f)*
e)*e^(2*d*x + 2*c) - (-2*I*d^3*f^2*x^3 - 3*I*d^2*f^2*x^2 - 6*I*d*f^2*x - 6*
(2*I*c^2 + I)*f^2 - 3*(2*I*d^3*x + 5*I*d^2)*e^2 - 6*(I*d^3*f*x^2 + I*d^2*f*
x + (-4*I*c + I)*d*f)*e)*e^(d*x + c) + 24*((c*f^2 - d*f*e)*e^(2*d*x + 2*c)
+ (-I*c*f^2 + I*d*f*e)*e^(d*x + c))*log(e^(d*x + c) - I) - 24*((d*f^2*x + c
*f^2)*e^(2*d*x + 2*c) - (I*d*f^2*x + I*c*f^2)*e^(d*x + c))*log(I*e^(d*x + c
) + 1))/(a*d^3*e^(2*d*x + 2*c) - I*a*d^3*e^(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{-2ie^2-4iefx-2if^2x^2}{ade^ce^{dx}-iad}-\frac{i\left(\int\frac{ide^2}{e^ce^{2dx}-ie^{dx}}\,dx+\int\frac{idf^2x^2}{e^ce^{2dx}-ie^{dx}}\,dx+\int\frac{de^2e^ce^{dx}}{e^ce^{2dx}-ie^{dx}}\,dx+\int\frac{de^2e^{3c}e^{3dx}}{e^ce^{2dx}-ie^{dx}}\,dx+\int\left(-\frac{8efe^ce^{dx}}{e^ce^{2dx}-ie^{dx}}\right)dx+\int\left(-\frac{8f^2xe^ce^{dx}}{e^ce^{2dx}-ie^{dx}}\right)dx+\int\frac{2idefx}{e^ce^{2dx}-ie^{dx}}\,dx+\int\frac{ide^2e^{2c}e^{2dx}}{e^ce^{2dx}-ie^{dx}}\,dx+\int\frac{df^2x^2e^ce^{dx}}{e^ce^{2dx}-ie^{dx}}\,dx+\int\frac{df^2x^2e^{3c}e^{3dx}}{e^ce^{2dx}-ie^{dx}}\,dx+\int\frac{idf^2x^2e^{2c}e^{2dx}}{e^ce^{2dx}-ie^{dx}}\,dx+\int\frac{2defxe^ce^{dx}}{e^ce^{2dx}-ie^{dx}}\,dx+\int\frac{2defxe^{3c}e^{3dx}}{e^ce^{2dx}-ie^{dx}}\,dx+\int\frac{2idefxe^{2c}e^{2dx}}{e^ce^{2dx}-ie^{dx}}\,dx\right)e^{-c}}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] (-2*I*e**2 - 4*I*e*f*x - 2*I*f**2*x**2)/(a*d*exp(c)*exp(d*x) - I*a*d) - I*(
Integral(I*d*e**2/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(I*d*f**2*
x**2/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(d*e**2*exp(c)*exp(d*x)
/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(d*e**2*exp(3*c)*exp(3*d*x)
/(exp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(-8*e*f*exp(c)*exp(d*x)/(ex
p(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(-8*f**2*x*exp(c)*exp(d*x)/(exp
(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(2*I*d*e*f*x/(exp(c)*exp(2*d*x)
- I*exp(d*x)), x) + Integral(I*d*e**2*exp(2*c)*exp(2*d*x)/(exp(c)*exp(2*d*x
) - I*exp(d*x)), x) + Integral(d*f**2*x**2*exp(c)*exp(d*x)/(exp(c)*exp(2*d*
x) - I*exp(d*x)), x) + Integral(d*f**2*x**2*exp(3*c)*exp(3*d*x)/(exp(c)*exp
(2*d*x) - I*exp(d*x)), x) + Integral(I*d*f**2*x**2*exp(2*c)*exp(2*d*x)/(exp
(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(2*d*e*f*x*exp(c)*exp(d*x)/(exp(
c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(2*d*e*f*x*exp(3*c)*exp(3*d*x)/(e
xp(c)*exp(2*d*x) - I*exp(d*x)), x) + Integral(2*I*d*e*f*x*exp(2*c)*exp(2*d*
x)/(exp(c)*exp(2*d*x) - I*exp(d*x)), x))*exp(-c)/(2*a*d)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sinh(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c+d\,x)^2\,(e+f\,x)^2}{a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)^2*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i),x)
[Out] int((sinh(c + d*x)^2*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i), x)
```

3.195 $\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=119

$$\frac{ex}{a}+\frac{fx^2}{2a}-\frac{i(e+fx)\cosh(c+dx)}{ad}+\frac{2f\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}+\frac{if\sinh(c+dx)}{ad^2}-\frac{(e+fx)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\right.}{ad}$$

```
[Out] e*x/a+1/2*f*x^2/a-I*(f*x+e)*cosh(d*x+c)/a/d+2*f*ln(cosh(1/2*c+1/4*I*Pi+1/2*
d*x))/a/d^2+I*f*sinh(d*x+c)/a/d^2-(f*x+e)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a/d
```

Rubi [A]
time = 0.13, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {5676, 3377, 2717, 3399, 4269, 3556}

$$\frac{if\sinh(c+dx)}{ad^2}+\frac{2f\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{ad^2}-\frac{i(e+fx)\cosh(c+dx)}{ad}-\frac{(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}+\frac{ex}{a}+\frac{fx^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (e*x)/a + (f*x^2)/(2*a) - (I*(e + f*x)*Cosh[c + d*x])/(a*d) + (2*f*Log[Cosh
[c/2 + (I/4)*Pi + (d*x)/2]])/(a*d^2) + (I*f*Sinh[c + d*x])/(a*d^2) - ((e +
f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5676

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx &= i\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx-\frac{i\int (e+fx)\sinh(c+dx)\,dx}{a}\\ &= -\frac{i(e+fx)\cosh(c+dx)}{ad}+\frac{\int (e+fx)\,dx}{a}+\frac{(if)\int \cosh(c+dx)\,dx}{ad}-\int \frac{}{a+\ldots}\\ &= \frac{ex}{a}+\frac{fx^2}{2a}-\frac{i(e+fx)\cosh(c+dx)}{ad}+\frac{if\sinh(c+dx)}{ad^2}-\frac{\int (e+fx)\csc^2\left(\frac{1}{2}(i\ldots\right.}{2a\ldots}\\ &= \frac{ex}{a}+\frac{fx^2}{2a}-\frac{i(e+fx)\cosh(c+dx)}{ad}+\frac{if\sinh(c+dx)}{ad^2}-\frac{(e+fx)\tanh\left(\frac{c}{2}+\ldots\right.}{ad}\\ &= \frac{ex}{a}+\frac{fx^2}{2a}-\frac{i(e+fx)\cosh(c+dx)}{ad}+\frac{2f\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}+\frac{if\sinh\ldots}{a\ldots}\end{aligned}$$

Mathematica [A]
time = 0.67, size = 238, normalized size = 2.00

$$\frac{\left(-i\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(\sinh\left(\frac{1}{2}(c+dx)\right)\left(i(2i+c+dx)(2de-cf+dfx)-4f\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+2d(e+fx)\cosh(c+dx)+2if\log(\cosh(c+dx))-2f\sinh(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\left(2cde-2icf-c^2f+2d^2ex-2idfx+d^2fx^2+4if\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-2id(e+fx)\cosh(c+dx)+2f\log(\cosh(c+dx))+2if\sinh(c+dx)\right)\right)}{2ad^2(-i+\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (((-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])*(Sinh[(c + d*x)/2]*(I*(2*I +
c + d*x)*(2*d*e - c*f + d*f*x) - 4*f*ArcTan[Tanh[(c + d*x)/2]] + 2*d*(e + f
*x)*Cosh[c + d*x] + (2*I)*f*Log[Cosh[c + d*x]] - 2*f*Sinh[c + d*x]) + Cosh[
(c + d*x)/2]*(2*c*d*e - (2*I)*c*f - c^2*f + 2*d^2*e*x - (2*I)*d*f*x + d^2*f
*x^2 + (4*I)*f*ArcTan[Tanh[(c + d*x)/2]] - (2*I)*d*(e + f*x)*Cosh[c + d*x]
+ 2*f*Log[Cosh[c + d*x]] + (2*I)*f*Sinh[c + d*x])))/(2*a*d^2*(-I + Sinh[c +
 d*x]))
```

Maple [A]
time = 1.62, size = 134, normalized size = 1.13

method	result	size
risch	$\frac{f\,x^{2}}{2a}+\frac{ex}{a}-\frac{i(dxf+de-f)\mathrm{e}^{dx+c}}{2a\,d^{2}}-\frac{i(dxf+de+f)\mathrm{e}^{-dx-c}}{2a\,d^{2}}-\frac{2fx}{ad}-\frac{2fc}{a\,d^{2}}-\frac{2i(fx+e)}{da\left(\mathrm{e}^{dx+c}-i\right)}+\frac{2f\ln\left(\mathrm{e}^{dx+c}-i\right)}{a\,d^{2}}$	134

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f*x^2/a+e*x/a-1/2*I*(d*f*x+d*e-f)/a/d^2*exp(d*x+c)-1/2*I*(d*f*x+d*e+f)/
a/d^2*exp(-d*x-c)-2*f*x/a/d-2*f/a/d^2*c-2*I*(f*x+e)/d/a/(exp(d*x+c)-I)+2*f/
a/d^2*ln(exp(d*x+c)-I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(103) = 206.
time = 0.33, size = 239, normalized size = 2.01

$$-\frac{1}{2}\,f\left(\frac{2\,xe^{(dx+c)}}{ade^{(dx+c)}-i\,ad}+\frac{i\,d^{2}x^{2}e^{c}+i\,dxe^{c}-\left(-i\,dxe^{(3\,c)}+i\,e^{(3\,c)}\right)e^{(2\,dx)}-\left(d^{2}x^{2}e^{(2\,c)}-3\,dxe^{(2\,c)}+e^{(2\,c)}\right)e^{(dx)}+(dx+1)e^{(-dx)}+i\,e^{c}}{ad^{2}e^{(dx+2\,c)}-i\,ad^{2}e^{c}}-\frac{4\,\log\left(\left(e^{(dx+c)}-i\right)e^{(-c)}\right)}{ad^{2}}\right)+\frac{1}{2}\left(\frac{2\,(dx+c)}{ad}+\frac{-5i\,e^{(-dx-c)}+1}{\left(i\,ae^{(-dx-c)}+ae^{(-2\,dx-2\,c)}\right)d}-\frac{i\,e^{(-dx-c)}}{ad}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*f*(2*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) + (I*d^2*x^2*e^c + I*d*x*
e^c - (-I*d*x*e^(3*c) + I*e^(3*c))*e^(2*d*x) - (d^2*x^2*e^(2*c) - 3*d*x*e^(
2*c) + e^(2*c))*e^(d*x) + (d*x + 1)*e^(-d*x) + I*e^c)/(a*d^2*e^(d*x + 2*c)
- I*a*d^2*e^c) - 4*log((e^(d*x + c) - I)*e^(-c))/(a*d^2)) + 1/2*(2*(d*x + c
)/(a*d) + (-5*I*e^(-d*x - c) + 1)/((I*a*e^(-d*x - c) + a*e^(-2*d*x - 2*c))*
d) - I*e^(-d*x - c)/(a*d))*e
```

Fricas [A]
time = 0.44, size = 176, normalized size = 1.48

$$-\frac{dfx+de-(-i\,dfx-i\,de+i\,f)e^{(3\,dx+3\,c)}-(d^{2}fx^{2}-5\,dfx+(2\,d^{2}x-d)e+f)e^{(2\,dx+2\,c)}-(-i\,d^{2}fx^{2}-i\,dfx+(-2i\,d^{2}x-5i\,d)e-i\,f)e^{(dx+c)}-4\left(fe^{(2\,dx+2\,c)}-i\,fe^{(dx+c)}\right)\log\left(e^{(dx+c)}-i\right)+f}{2\left(ad^{2}e^{(2\,dx+2\,c)}-i\,ad^{2}e^{(dx+c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(d*f*x + d*e - (-I*d*f*x - I*d*e + I*f)*e^(3*d*x + 3*c) - (d^2*f*x^2 -
 5*d*f*x + (2*d^2*x - d)*e + f)*e^(2*d*x + 2*c) - (-I*d^2*f*x^2 - I*d*f*x +
 (-2*I*d^2*x - 5*I*d)*e - I*f)*e^(d*x + c) - 4*(f*e^(2*d*x + 2*c) - I*f*e^(
d*x + c))*log(e^(d*x + c) - I) + f)/(a*d^2*e^(2*d*x + 2*c) - I*a*d^2*e^(d*x
 + c))
```

Sympy [A]
time = 0.32, size = 224, normalized size = 1.88

$$\frac{-2ie-2ifx}{ade^ce^{dx}-iad}+\begin{cases}\frac{\left(\left(-2iad^3e-2iad^3fx-2iad^2f\right)e^{-dx}+\left(-2iad^3ee^{2c}-2iad^3fxe^{2c}+2iad^2fe^{2c}\right)e^{dx}\right)e^{-c}}{4a^2d^4} & \text{for } a^2d^4e^c\neq 0\\ \frac{x^2\left(-ife^{2c}+if\right)e^{-c}}{4a}+\frac{x\left(-iee^{2c}+ie\right)e^{-c}}{2a} & \text{otherwise}\end{cases}+\frac{fx^2}{2a}+\frac{x(de-2f)}{ad}+\frac{2f\log\left(e^{dx}-ie^{-c}\right)}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] (-2*I*e - 2*I*f*x)/(a*d*exp(c)*exp(d*x) - I*a*d) + Piecewise((((-2*I*a*d**3
*e - 2*I*a*d**3*f*x - 2*I*a*d**2*f)*exp(-d*x) + (-2*I*a*d**3*e*exp(2*c) - 2
*I*a*d**3*f*x*exp(2*c) + 2*I*a*d**2*f*exp(2*c))*exp(d*x))*exp(-c)/(4*a**2*d
**4), Ne(a**2*d**4*exp(c), 0)), (x**2*(-I*f*exp(2*c) + I*f)*exp(-c)/(4*a) +
 x*(-I*e*exp(2*c) + I*e)*exp(-c)/(2*a), True)) + f*x**2/(2*a) + x*(d*e - 2*
f)/(a*d) + 2*f*log(exp(d*x) - I*exp(-c))/(a*d**2)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(100) = 200.
time = 0.45, size = 251, normalized size = 2.11

$$\frac{d^2fx^2e^{(2dx+2c)}-id^2fx^2e^{(dx+c)}+2d^2exe^{(2dx+2c)}-2id^2exe^{(dx+c)}-idfxe^{(3dx+3c)}-5dfxe^{(2dx+2c)}-idfxe^{(dx+c)}-dfx-idee^{(3dx+3c)}-dee^{(2dx+2c)}-5idee^{(dx+c)}+4fe^{(2dx+2c)}\log\left(e^{(dx+c)}-i\right)-4ife^{(dx+c)}\log\left(e^{(dx+c)}-i\right)-de+ife^{(3dx+3c)}+fe^{(2dx+2c)}-ife^{(dx+c)}-f}{2\left(ad^2e^{(2dx+2c)}-iad^2e^{(dx+c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(d^2*f*x^2*e^(2*d*x + 2*c) - I*d^2*f*x^2*e^(d*x + c) + 2*d^2*e*x*e^(2*d
*x + 2*c) - 2*I*d^2*e*x*e^(d*x + c) - I*d*f*x*e^(3*d*x + 3*c) - 5*d*f*x*e^(
2*d*x + 2*c) - I*d*f*x*e^(d*x + c) - d*f*x - I*d*e*e^(3*d*x + 3*c) - d*e*e^
(2*d*x + 2*c) - 5*I*d*e*e^(d*x + c) + 4*f*e^(2*d*x + 2*c)*log(e^(d*x + c) -
 I) - 4*I*f*e^(d*x + c)*log(e^(d*x + c) - I) - d*e + I*f*e^(3*d*x + 3*c) +
f*e^(2*d*x + 2*c) - I*f*e^(d*x + c) - f)/(a*d^2*e^(2*d*x + 2*c) - I*a*d^2*e
^(d*x + c))
```

Mupad [B]
time = 0.58, size = 143, normalized size = 1.20

$$\frac{f\,x^2}{2\,a}+\mathrm{e}^{c+d\,x}\,\left(\frac{\left(f-d\,e\right)\,1{}\mathrm{i}}{2\,a\,d^2}-\frac{f\,x\,1{}\mathrm{i}}{2\,a\,d}\right)-\mathrm{e}^{-c-d\,x}\,\left(\frac{\left(f+d\,e\right)\,1{}\mathrm{i}}{2\,a\,d^2}+\frac{f\,x\,1{}\mathrm{i}}{2\,a\,d}\right)-\frac{\left(e+f\,x\right)\,2{}\mathrm{i}}{a\,d\,\left({\mathrm{e}}^{c+d\,x}-\mathrm{i}\right)}-\frac{x\,\left(2\,f-d\,e\right)}{a\,d}+\frac{2\,f\,\ln\left({\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c-\mathrm{i}\right)}{a\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)^2*(e + f*x))/(a + a*sinh(c + d*x)*1i),x)
```

```
[Out] exp(c + d*x)*(((f - d*e)*1i)/(2*a*d^2) - (f*x*1i)/(2*a*d)) - exp(- c - d*x)
*(((f + d*e)*1i)/(2*a*d^2) + (f*x*1i)/(2*a*d)) + (f*x^2)/(2*a) - ((e + f*x)
*2i)/(a*d*(exp(c + d*x) - 1i)) - (x*(2*f - d*e))/(a*d) + (2*f*log(exp(d*x)*
exp(c) - 1i))/(a*d^2)
```

3.196 $\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=52

$$\frac{x}{a}-\frac{i\cosh(c+dx)}{ad}-\frac{i\cosh(c+dx)}{ad(1+i\sinh(c+dx))}$$

[Out] x/a-I*cosh(d*x+c)/a/d-I*cosh(d*x+c)/a/d/(1+I*sinh(d*x+c))

Rubi [A]
time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {2825, 12, 2814, 2727}

$$-\frac{i\cosh(c+dx)}{ad}-\frac{i\cosh(c+dx)}{ad(1+i\sinh(c+dx))}+\frac{x}{a}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]

[Out] x/a - (I*Cosh[c + d*x])/(a*d) - (I*Cosh[c + d*x])/(a*d*(1 + I*Sinh[c + d*x]
))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\frac{i\cosh(c+dx)}{ad}+\frac{i\int \frac{a\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx}{a}\\
&= -\frac{i\cosh(c+dx)}{ad}+i\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx\\
&= \frac{x}{a}-\frac{i\cosh(c+dx)}{ad}-\int \frac{1}{a+ia\sinh(c+dx)}\,dx\\
&= \frac{x}{a}-\frac{i\cosh(c+dx)}{ad}-\frac{i\cosh(c+dx)}{d(a+ia\sinh(c+dx))}
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 59, normalized size = 1.13

$$
\frac{\cosh(c+dx)\left(\frac{\sinh^{-1}(\sinh(c+dx))}{\sqrt{\cosh^2(c+dx)}}+\frac{-2-i\sinh(c+dx)}{-i+\sinh(c+dx)}\right)}{ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (Cosh[c + d*x]*(ArcSinh[Sinh[c + d*x]]/Sqrt[Cosh[c + d*x]^2] + (-2 - I*Sinh
[c + d*x])/(-I + Sinh[c + d*x])))/(a*d)
```

Maple [A]
time = 0.91, size = 86, normalized size = 1.65

method	result	size
risch	$\frac{x}{a}-\frac{i e^{dx+c}}{2ad}-\frac{i e^{-dx-c}}{2ad}-\frac{2i}{da\left(e^{dx+c}-i\right)}$	60
derivativedivides	$\frac{\frac{8i}{8\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-8}-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{i}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{ad}$	86
default	$\frac{\frac{8i}{8\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-8}-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{i}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{ad}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a*(1/8*I/(tanh(1/2*d*x+1/2*c)-1)-1/8*ln(tanh(1/2*d*x+1/2*c)-1)-1/8*I/(t
anh(1/2*d*x+1/2*c)+1)+1/8*ln(tanh(1/2*d*x+1/2*c)+1)-1/4/(-I+tanh(1/2*d*x+1/
2*c)))
```

Maxima [A]
time = 0.27, size = 74, normalized size = 1.42

$$\frac{dx+c}{ad}+\frac{-5i\,e^{(-dx-c)}+1}{2\left(i\,ae^{(-dx-c)}+ae^{(-2\,dx-2\,c)}\right)d}-\frac{i\,e^{(-dx-c)}}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(a*d) + 1/2*(-5*I*e^(-d*x - c) + 1)/((I*a*e^(-d*x - c) + a*e^(-2*
d*x - 2*c))*d) - 1/2*I*e^(-d*x - c)/(a*d)
```

Fricas [A]
time = 0.39, size = 69, normalized size = 1.33

$$\frac{(2\,dx-1)e^{(2\,dx+2\,c)}+(-2i\,dx-5i)e^{(dx+c)}-i\,e^{(3\,dx+3\,c)}-1}{2\left(ade^{(2\,dx+2\,c)}-i\,ade^{(dx+c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((2*d*x - 1)*e^(2*d*x + 2*c) + (-2*I*d*x - 5*I)*e^(d*x + c) - I*e^(3*d*
x + 3*c) - 1)/(a*d*e^(2*d*x + 2*c) - I*a*d*e^(d*x + c))
```

Sympy [A]
time = 0.16, size = 99, normalized size = 1.90

$$\begin{cases}\frac{\left(-2iade^{2c}e^{dx}-2iade^{-dx}\right)e^{-c}}{4a^2d^2} & \text{for } a^2d^2e^c\neq 0\\ x\left(\frac{\left(-ie^{2c}+2e^c+i\right)e^{-c}}{2a}-\frac{1}{a}\right) & \text{otherwise}\end{cases}-\frac{2i}{ade^ce^{dx}-iad}+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Piecewise(((-2*I*a*d*exp(2*c)*exp(d*x) - 2*I*a*d*exp(-d*x))*exp(-c)/(4*a**2
*d**2), Ne(a**2*d**2*exp(c), 0)), (x*((-I*exp(2*c) + 2*exp(c) + I)*exp(-c)/
(2*a) - 1/a), True)) - 2*I/(a*d*exp(c)*exp(d*x) - I*a*d) + x/a
```

Giac [A]
time = 0.44, size = 63, normalized size = 1.21

$$\frac{\frac{2\,(dx+c)}{a}-\frac{i\,e^{(dx+c)}}{a}-\frac{\left(5\,e^{(dx+c)}-i\right)e^{(-dx-c)}}{a\left(-i\,e^{(dx+c)}-1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)/a - I*e^(d*x + c)/a - (5*e^(d*x + c) - I)*e^(-d*x - c)/(a*
(-I*e^(d*x + c) - 1)))/d
```

Mupad [B]
time = 0.30, size = 59, normalized size = 1.13

$$\frac{x}{a}-\frac{2\mathrm{i}}{a\,d\,\left({\mathrm{e}}^{c+d\,x}-\mathrm{i}\right)}-\frac{{\mathrm{e}}^{c+d\,x}\,1\mathrm{i}}{2\,a\,d}-\frac{{\mathrm{e}}^{-c-d\,x}\,1\mathrm{i}}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + a*sinh(c + d*x)*1i),x)
```

```
[Out] x/a - 2i/(a*d*(exp(c + d*x) - 1i)) - (exp(c + d*x)*1i)/(2*a*d) - (exp(- c -
 d*x)*1i)/(2*a*d)
```

3.197 $\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sinh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
[Out] Defer[Int][Sinh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 170.22, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
[Out] Integrate[Sinh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(dx+c)}{(fx+e)(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*I*f*integrate(1/(-I*a*d*f^2*x^2 - 2*I*a*d*f*x*e - I*a*d*e^2 + (a*d*f^2*x
^2*e^c + 2*a*d*f*x*e^(c + 1) + a*d*e^(c + 2))*e^(d*x)), x) - 1/2*I*e^(-c +
d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(a*f) + 1/2*I*e^(c - d*e/f)*exp_int
egral_e(1, -(f*x + e)*d/f)/(a*f) - 2*I/(-I*a*d*f*x - I*a*d*e + (a*d*f*x*e^c
 + a*d*e^(c + 1))*e^(d*x)) + log(f*x + e)/(a*f)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))*integral(1/2*(d*f*x
 + d*e + (-I*d*f*x - I*d*e)*e^(3*d*x + 3*c) + (d*f*x + d*e)*e^(2*d*x + 2*c)
 + (-I*d*f*x - I*d*e - 4*I*f)*e^(d*x + c))/((a*d*f^2*x^2 + 2*a*d*f*x*e + a*
d*e^2)*e^(2*d*x + 2*c) - (I*a*d*f^2*x^2 + 2*I*a*d*f*x*e + I*a*d*e^2)*e^(d*x
 + c)), x) - 2*I)/(-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2i}{-iade-iadfx+\left(adee^{c}+adfxe^{c}\right)e^{dx}}-\frac{i\left(\int\frac{ide}{e^{2}e^{c}e^{2dx}-ie^{2}e^{dx}+2efxe^{c}e^{2dx}-2iefxe^{dx}+f^{2}x^{2}e^{c}e^{2dx}-if^{2}x^{2}e^{dx}}\,dx+\int\frac{4fe^{c}e^{dx}}{e^{2}e^{c}e^{2dx}-ie^{2}e^{dx}+2efxe^{c}e^{2dx}-2iefxe^{dx}+f^{2}x^{2}e^{c}e^{2dx}-if^{2}x^{2}e^{dx}}\,dx+\int\frac{idfx}{e^{2}e^{c}e^{2dx}-ie^{2}e^{dx}+2efxe^{c}e^{2dx}-2iefxe^{dx}+f^{2}x^{2}e^{c}e^{2dx}-if^{2}x^{2}e^{dx}}\,dx+\int\frac{dee^{c}e^{dx}}{e^{2}e^{c}e^{2dx}-ie^{2}e^{dx}+2efxe^{c}e^{2dx}-2iefxe^{dx}+f^{2}x^{2}e^{c}e^{2dx}-if^{2}x^{2}e^{dx}}\,dx+\int\frac{dee^{3c}e^{3dx}}{e^{2}e^{c}e^{2dx}-ie^{2}e^{dx}+2efxe^{c}e^{2dx}-2iefxe^{dx}+f^{2}x^{2}e^{c}e^{2dx}-if^{2}x^{2}e^{dx}}\,dx+\int\frac{idee^{2c}e^{2dx}}{e^{2}e^{c}e^{2dx}-ie^{2}e^{dx}+2efxe^{c}e^{2dx}-2iefxe^{dx}+f^{2}x^{2}e^{c}e^{2dx}-if^{2}x^{2}e^{dx}}\,dx+\int\frac{dfxe^{c}e^{dx}}{e^{2}e^{c}e^{2dx}-ie^{2}e^{dx}+2efxe^{c}e^{2dx}-2iefxe^{dx}+f^{2}x^{2}e^{c}e^{2dx}-if^{2}x^{2}e^{dx}}\,dx+\int\frac{dfxe^{3c}e^{3dx}}{e^{2}e^{c}e^{2dx}-ie^{2}e^{dx}+2efxe^{c}e^{2dx}-2iefxe^{dx}+f^{2}x^{2}e^{c}e^{2dx}-if^{2}x^{2}e^{dx}}\,dx+\int\frac{idfxe^{2c}e^{2dx}}{e^{2}e^{c}e^{2dx}-ie^{2}e^{dx}+2efxe^{c}e^{2dx}-2iefxe^{dx}+f^{2}x^{2}e^{c}e^{2dx}-if^{2}x^{2}e^{dx}}\,dx\right)e^{-c}}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -2*I/(-I*a*d*e - I*a*d*f*x + (a*d*e*exp(c) + a*d*f*x*exp(c))*exp(d*x)) - I*
(Integral(I*d*e/(e**2*exp(c)*exp(2*d*x) - I*e**2*exp(d*x) + 2*e*f*x*exp(c)*
exp(2*d*x) - 2*I*e*f*x*exp(d*x) + f**2*x**2*exp(c)*exp(2*d*x) - I*f**2*x**2
*exp(d*x)), x) + Integral(4*f*exp(c)*exp(d*x)/(e**2*exp(c)*exp(2*d*x) - I*e
**2*exp(d*x) + 2*e*f*x*exp(c)*exp(2*d*x) - 2*I*e*f*x*exp(d*x) + f**2*x**2*e
```

```
xp(c)*exp(2*d*x) - I*f**2*x**2*exp(d*x)), x) + Integral(I*d*f*x/(e**2*exp(c
)*exp(2*d*x) - I*e**2*exp(d*x) + 2*e*f*x*exp(c)*exp(2*d*x) - 2*I*e*f*x*exp(
d*x) + f**2*x**2*exp(c)*exp(2*d*x) - I*f**2*x**2*exp(d*x)), x) + Integral(d
*e*exp(c)*exp(d*x)/(e**2*exp(c)*exp(2*d*x) - I*e**2*exp(d*x) + 2*e*f*x*exp(
c)*exp(2*d*x) - 2*I*e*f*x*exp(d*x) + f**2*x**2*exp(c)*exp(2*d*x) - I*f**2*x
**2*exp(d*x)), x) + Integral(d*e*exp(3*c)*exp(3*d*x)/(e**2*exp(c)*exp(2*d*x
) - I*e**2*exp(d*x) + 2*e*f*x*exp(c)*exp(2*d*x) - 2*I*e*f*x*exp(d*x) + f**2
*x**2*exp(c)*exp(2*d*x) - I*f**2*x**2*exp(d*x)), x) + Integral(I*d*e*exp(2*
c)*exp(2*d*x)/(e**2*exp(c)*exp(2*d*x) - I*e**2*exp(d*x) + 2*e*f*x*exp(c)*ex
p(2*d*x) - 2*I*e*f*x*exp(d*x) + f**2*x**2*exp(c)*exp(2*d*x) - I*f**2*x**2*e
xp(d*x)), x) + Integral(d*f*x*exp(c)*exp(d*x)/(e**2*exp(c)*exp(2*d*x) - I*e
**2*exp(d*x) + 2*e*f*x*exp(c)*exp(2*d*x) - 2*I*e*f*x*exp(d*x) + f**2*x**2*e
xp(c)*exp(2*d*x) - I*f**2*x**2*exp(d*x)), x) + Integral(d*f*x*exp(3*c)*exp(
3*d*x)/(e**2*exp(c)*exp(2*d*x) - I*e**2*exp(d*x) + 2*e*f*x*exp(c)*exp(2*d*x
) - 2*I*e*f*x*exp(d*x) + f**2*x**2*exp(c)*exp(2*d*x) - I*f**2*x**2*exp(d*x)
), x) + Integral(I*d*f*x*exp(2*c)*exp(2*d*x)/(e**2*exp(c)*exp(2*d*x) - I*e*
*2*exp(d*x) + 2*e*f*x*exp(c)*exp(2*d*x) - 2*I*e*f*x*exp(d*x) + f**2*x**2*ex
p(c)*exp(2*d*x) - I*f**2*x**2*exp(d*x)), x))*exp(-c)/(2*a*d)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate(sinh(d*x + c)^2/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sinh(c+d\,x)^2}{(e+f\,x)\,\left(a+a\,\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/((e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
[Out] int(sinh(c + d*x)^2/((e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.198 $\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sinh[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Sinh[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 150.47, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Integrate[Sinh[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(dx+c)}{(fx+e)^2\,(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -4*I*f*integrate(1/(-I*a*d*f^3*x^3 - 3*I*a*d*f^2*x^2*e - 3*I*a*d*f*x*e^2 -
I*a*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*d*f^2*x^2*e^(c + 1) + 3*a*d*f*x*e^(c + 2
) + a*d*e^(c + 3))*e^(d*x)), x) - (-I*d*f*x - I*d*e + (d*f*x*e^c + d*e^(c +
 1))*e^(d*x) + 2*I*f)/(-I*a*d*f^3*x^2 - 2*I*a*d*f^2*x*e - I*a*d*f*e^2 + (a*
d*f^3*x^2*e^c + 2*a*d*f^2*x*e^(c + 1) + a*d*f*e^(c + 2))*e^(d*x)) - 1/2*I*e
^(-c + d*e/f)*exp_integral_e(2, (f*x + e)*d/f)/((f*x + e)*a*f) + 1/2*I*e^(c
 - d*e/f)*exp_integral_e(2, -(f*x + e)*d/f)/((f*x + e)*a*f)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] ((-I*a*d*f^2*x^2 - 2*I*a*d*f*x*e - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*f*x*e +
 a*d*e^2)*e^(d*x + c))*integral(1/2*(d*f*x + d*e + (-I*d*f*x - I*d*e)*e^(3*
d*x + 3*c) + (d*f*x + d*e)*e^(2*d*x + 2*c) + (-I*d*f*x - I*d*e - 8*I*f)*e^(
d*x + c))/((a*d*f^3*x^3 + 3*a*d*f^2*x^2*e + 3*a*d*f*x*e^2 + a*d*e^3)*e^(2*d
*x + 2*c) - (I*a*d*f^3*x^3 + 3*I*a*d*f^2*x^2*e + 3*I*a*d*f*x*e^2 + I*a*d*e^
3)*e^(d*x + c)), x) - 2*I)/(-I*a*d*f^2*x^2 - 2*I*a*d*f*x*e - I*a*d*e^2 + (a
*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2)*e^(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^2/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sinh(c+d\,x)^2}{{(e+f\,x)}^2\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

```
[Out] int(sinh(c + d*x)^2/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.199 $\int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=393

$$\frac{3ief^2x}{4ad^2}+\frac{3if^3x^2}{8ad^2}-\frac{i(e+fx)^3}{ad}+\frac{3i(e+fx)^4}{8af}+\frac{6f^2(e+fx)\cosh(c+dx)}{ad^3}+\frac{(e+fx)^3\cosh(c+dx)}{ad}+\frac{6if(e+f$$

```
[Out] 3/4*I*e*f^2*x/a/d^2-I*(f*x+e)^3*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a/d-12*I*f^3*p
olylog(3,-I*exp(d*x+c))/a/d^4+3/8*I*(f*x+e)^4/a/f+6*f^2*(f*x+e)*cosh(d*x+c)
/a/d^3+(f*x+e)^3*cosh(d*x+c)/a/d-I*(f*x+e)^3/a/d+6*I*f*(f*x+e)^2*ln(1+I*exp
(d*x+c))/a/d^2+3/8*I*f^3*sinh(d*x+c)^2/a/d^4-6*f^3*sinh(d*x+c)/a/d^4-3*f*(f
*x+e)^2*sinh(d*x+c)/a/d^2+12*I*f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/a/d^3-3
/4*I*f^2*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/a/d^3-1/2*I*(f*x+e)^3*cosh(d*x+c)*
sinh(d*x+c)/a/d+3/8*I*f^3*x^2/a/d^2+3/4*I*f*(f*x+e)^2*sinh(d*x+c)^2/a/d^2
```

Rubi [A]
time = 0.50, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {5676, 3392, 32, 3391, 3377, 2717, 3399, 4269, 3797, 2221, 2611, 2320, 6724}

$$-\frac{12if^3\text{Li}_3\left(-ie^{c+dx}\right)}{ad^4}+\frac{3if^3\sinh^2(c+dx)}{8ad^4}-\frac{6f^3\sinh(c+dx)}{ad^4}+\frac{12if^2(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{ad^3}+\frac{6f^2(e+fx)\cosh(c+dx)}{ad^3}-\frac{3if^2(e+fx)\sinh(c+dx)\cosh(c+dx)}{4ad^3}+\frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}+\frac{3if(e+fx)^2\sinh^2(c+dx)}{4ad^2}-\frac{3f(e+fx)^2\sinh(c+dx)}{ad^2}+\frac{(e+fx)^3\cosh(c+dx)}{ad}-\frac{i(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}-\frac{i(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2ad}+\frac{3ief^2x}{4ad^2}+\frac{3if^3x^2}{8ad^2}-\frac{i(e+fx)^3}{ad}+\frac{3i(e+fx)^4}{8af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (((3*I)/4)*e*f^2*x)/(a*d^2) + (((3*I)/8)*f^3*x^2)/(a*d^2) - (I*(e + f*x)^3)
/(a*d) + (((3*I)/8)*(e + f*x)^4)/(a*f) + (6*f^2*(e + f*x)*Cosh[c + d*x])/(a
*d^3) + ((e + f*x)^3*Cosh[c + d*x])/(a*d) + ((6*I)*f*(e + f*x)^2*Log[1 + I*
E^(c + d*x)])/(a*d^2) + ((12*I)*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])
/(a*d^3) - ((12*I)*f^3*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^4) - (6*f^3*Sinh[
c + d*x])/(a*d^4) - (3*f*(e + f*x)^2*Sinh[c + d*x])/(a*d^2) - (((3*I)/4)*f^
2*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(a*d^3) - ((I/2)*(e + f*x)^3*Cosh[
c + d*x]*Sinh[c + d*x])/(a*d) + (((3*I)/8)*f^3*Sinh[c + d*x]^2)/(a*d^4) + (
((3*I)/4)*f*(e + f*x)^2*Sinh[c + d*x]^2)/(a*d^2) - (I*(e + f*x)^3*Tanh[c/2
+ (I/4)*Pi + (d*x)/2])/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5676

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = i\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx - \frac{i\int (e+fx)^3 \sinh^2(c+dx)\,dx}{a}$$

$$= -\frac{i(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{2ad} + \frac{3if(e+fx)^2\sinh^2(c+dx)}{4ad^2} + \frac{i\int(\ldots}{}$$

$$= \frac{i(e+fx)^4}{8af} + \frac{(e+fx)^3\cosh(c+dx)}{ad} - \frac{3if^2(e+fx)\cosh(c+dx)\sinh(c+\ldots}{4ad^3}$$

$$= \frac{3ief^2x}{4ad^2} + \frac{3if^3x^2}{8ad^2} + \frac{3i(e+fx)^4}{8af} + \frac{(e+fx)^3\cosh(c+dx)}{ad} - \frac{3f(e+fx)^2\sin\ldots}{ad^2}$$

$$= \frac{3ief^2x}{4ad^2} + \frac{3if^3x^2}{8ad^2} + \frac{3i(e+fx)^4}{8af} + \frac{6f^2(e+fx)\cosh(c+dx)}{ad^3} + \frac{(e+fx)^3\text{co}\ldots}{a\ldots}$$

$$= \frac{3ief^2x}{4ad^2} + \frac{3if^3x^2}{8ad^2} - \frac{i(e+fx)^3}{ad} + \frac{3i(e+fx)^4}{8af} + \frac{6f^2(e+fx)\cosh(c+dx)}{ad^3} - \ldots$$

$$= \frac{3ief^2x}{4ad^2} + \frac{3if^3x^2}{8ad^2} - \frac{i(e+fx)^3}{ad} + \frac{3i(e+fx)^4}{8af} + \frac{6f^2(e+fx)\cosh(c+dx)}{ad^3} - \ldots$$

$$= \frac{3ief^2x}{4ad^2} + \frac{3if^3x^2}{8ad^2} - \frac{i(e+fx)^3}{ad} + \frac{3i(e+fx)^4}{8af} + \frac{6f^2(e+fx)\cosh(c+dx)}{ad^3} - \ldots$$

$$= \frac{3ief^2x}{4ad^2} + \frac{3if^3x^2}{8ad^2} - \frac{i(e+fx)^3}{ad} + \frac{3i(e+fx)^4}{8af} + \frac{6f^2(e+fx)\cosh(c+dx)}{ad^3} - \ldots$$

$$= \frac{3ief^2x}{4ad^2} + \frac{3if^3x^2}{8ad^2} - \frac{i(e+fx)^3}{ad} + \frac{3i(e+fx)^4}{8af} + \frac{6f^2(e+fx)\cosh(c+dx)}{ad^3} - \ldots$$

Mathematica [**A**]
time = 6.98, size = 397, normalized size = 1.01

$$\frac{24ie^3x+36ie^2fx^2+24ief^2x^3+6if^3x^4+\frac{96f^2(e+fx)\cosh(c+dx)}{d^3}+\frac{16(e+fx)^3\cosh(c+dx)}{d}+\frac{3if^3\cosh(2(c+dx))}{d^4}+\frac{6if(e+fx)^2\cosh(2(c+dx))}{d^2}+\frac{32f\left(-ide^cx(3e^2+3efx+f^2x^2)+3(1+ie^c)(e+fx)^2\log(1+ie^{c+dx})\right)}{d^2(-i+e^c)}+\frac{192if^2(e+fx)\text{PolyLog}(2,-ie^{c+dx})}{d^3}-\frac{192if^3\text{PolyLog}(3,-ie^{c+dx})}{d^4}-\frac{32i(e+fx)^3\sinh\left(\frac{dx}{2}\right)}{d\left(\cosh\left(\frac{c}{2}\right)+i\sinh\left(\frac{c}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{96f^3\sinh(c+dx)}{d^4}-\frac{48f(e+fx)^2\sinh(c+dx)}{d^2}-\frac{6if^2(e+fx)\sinh(2(c+dx))}{d^3}-\frac{4i(e+fx)^3\sinh(2(c+dx))}{d}}{16a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((24*I)*e^3*x + (36*I)*e^2*f*x^2 + (24*I)*e*f^2*x^3 + (6*I)*f^3*x^4 + (96*f
^2*(e + f*x)*Cosh[c + d*x])/d^3 + (16*(e + f*x)^3*Cosh[c + d*x])/d + ((3*I)
*f^3*Cosh[2*(c + d*x)])/d^4 + ((6*I)*f*(e + f*x)^2*Cosh[2*(c + d*x)])/d^2 +
 (32*f*((-I)*d*E^c*x*(3*e^2 + 3*e*f*x + f^2*x^2) + 3*(1 + I*E^c)*(e + f*x)^
2*Log[1 + I*E^(c + d*x)]))/(d^2*(-I + E^c)) + ((192*I)*f^2*(e + f*x)*PolyLo
g[2, (-I)*E^(c + d*x)])/d^3 - ((192*I)*f^3*PolyLog[3, (-I)*E^(c + d*x)])/d^
```

```
4 - ((32*I)*(e + f*x)^3*Sinh[(d*x)/2])/(d*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[(
c + d*x)/2] + I*Sinh[(c + d*x)/2])) - (96*f^3*Sinh[c + d*x])/d^4 - (48*f*(e
 + f*x)^2*Sinh[c + d*x])/d^2 - ((6*I)*f^2*(e + f*x)*Sinh[2*(c + d*x)])/d^3
- ((4*I)*(e + f*x)^3*Sinh[2*(c + d*x)])/d)/(16*a)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 939 vs. $2(354) = 708$.
time = 2.79, size = 940, normalized size = 2.39

method	result
risch	$-\frac{12if^3\,\mathrm{polylog}\left(3,-i\mathrm{e}^{dx+c}\right)}{a\,d^4}-\frac{6ie\,f^2c^2}{a\,d^3}-\frac{6if^3c^2\ln\left(1+i\mathrm{e}^{dx+c}\right)}{a\,d^4}+\frac{3ie^4}{8af}+\frac{3if^3x^4}{8a}+\frac{3ie^3x}{2a}+\frac{2f^3x^3+6e\,f^2x^2+6e^2fx+2e^3}{da\left(\mathrm{e}^{dx+c}-i\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*I/a*f^3*x^4+3/2*I/a*e^3*x+3/8*I/a/f*e^4-6*I/a/d^4*f^3*c^2*ln(exp(d*x+c)
)+6*I/a/d^2*ln(exp(d*x+c)-I)*e^2*f+1/2*(d^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e
^2*f*x-3*d^2*f^3*x^2+d^3*e^3-6*d^2*e*f^2*x-3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2-
6*f^3)/a/d^4*exp(d*x+c)+2*(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e^3)/d/a/(exp(d*x+
c)-I)+1/2*(d^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x+3*d^2*f^3*x^2+d^3*e^3+
6*d^2*e*f^2*x+3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2+6*f^3)/a/d^4*exp(-d*x-c)+6*I/
a/d^3*f^3*c^2*x-12*I*f^3*polylog(3,-I*exp(d*x+c))/a/d^4-12*I/a/d^2*e*f^2*c*
x+12*I/a/d^3*e*f^2*c*ln(exp(d*x+c))-12*I/a/d^3*e*f^2*c*ln(exp(d*x+c)-I)+12*
I/a/d^2*e*f^2*ln(1+I*exp(d*x+c))*x+12*I/a/d^3*e*f^2*ln(1+I*exp(d*x+c))*c+12
*I/a/d^3*e*f^2*polylog(2,-I*exp(d*x+c))+6*I/a/d^4*f^3*c^2*ln(exp(d*x+c)-I)-
6*I/a/d*e*f^2*x^2-6*I/a/d^3*e*f^2*c^2-6*I/a/d^4*f^3*c^2*ln(1+I*exp(d*x+c))+
12*I/a/d^3*f^3*polylog(2,-I*exp(d*x+c))*x+6*I/a/d^2*f^3*ln(1+I*exp(d*x+c))*
x^2-6*I/a/d^2*ln(exp(d*x+c))*e^2*f-1/32*I*(4*d^3*f^3*x^3+12*d^3*e*f^2*x^2+1
2*d^3*e^2*f*x-6*d^2*f^3*x^2+4*d^3*e^3-12*d^2*e*f^2*x-6*d^2*e^2*f+6*d*f^3*x+
6*d*e*f^2-3*f^3)/a/d^4*exp(2*d*x+2*c)+1/32*I*(4*d^3*f^3*x^3+12*d^3*e*f^2*x^
2+12*d^3*e^2*f*x+6*d^2*f^3*x^2+4*d^3*e^3+12*d^2*e*f^2*x+6*d^2*e^2*f+6*d*f^3
*x+6*d*e*f^2+3*f^3)/a/d^4*exp(-2*d*x-2*c)+3/2*I/a*f^2*e*x^3+9/4*I/a*f*e^2*x
^2+4*I/a/d^4*f^3*c^3-2*I/a/d*f^3*x^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1045 vs. $2(349) = 698$.
time = 0.37, size = 1045, normalized size = 2.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/32*(4*d^3*f^3*x^3 + 6*d^2*f^3*x^2 + 6*d*f^3*x + 4*d^3*e^3 + 3*f^3 - 384*(
(-I*d*f^3*x - I*d*f^2*e)*e^(3*d*x + 3*c) - (d*f^3*x + d*f^2*e)*e^(2*d*x + 2
*c))*dilog(-I*e^(d*x + c)) + 6*(2*d^3*f*x + d^2*f)*e^2 + 6*(2*d^3*f^2*x^2 +
 2*d^2*f^2*x + d*f^2)*e + (-4*I*d^3*f^3*x^3 + 6*I*d^2*f^3*x^2 - 6*I*d*f^3*x
 - 4*I*d^3*e^3 + 3*I*f^3 - 6*(2*I*d^3*f*x - I*d^2*f)*e^2 - 6*(2*I*d^3*f^2*x
^2 - 2*I*d^2*f^2*x + I*d*f^2)*e)*e^(5*d*x + 5*c) + 3*(4*d^3*f^3*x^3 - 14*d^
2*f^3*x^2 + 30*d*f^3*x + 4*d^3*e^3 - 31*f^3 + 2*(6*d^3*f*x - 7*d^2*f)*e^2 +
 2*(6*d^3*f^2*x^2 - 14*d^2*f^2*x + 15*d*f^2)*e)*e^(4*d*x + 4*c) - 4*(-3*I*d
^4*f^3*x^4 + 20*I*d^3*f^3*x^3 - 12*I*d^2*f^3*x^2 + 24*I*d*f^3*x + 8*(2*I*c^
3 - 3*I)*f^3 + 4*(-3*I*d^4*x + I*d^3)*e^3 + 6*(-3*I*d^4*f*x^2 + 10*I*d^3*f*
x + 2*(4*I*c - I)*d^2*f)*e^2 + 12*(-I*d^4*f^2*x^3 + 5*I*d^3*f^2*x^2 - 2*I*d
^2*f^2*x + 2*(-2*I*c^2 + I)*d*f^2)*e)*e^(3*d*x + 3*c) + 4*(3*d^4*f^3*x^4 +
4*d^3*f^3*x^3 + 12*d^2*f^3*x^2 + 24*d*f^3*x - 8*(2*c^3 - 3)*f^3 + 4*(3*d^4*
x + 5*d^3)*e^3 + 6*(3*d^4*f*x^2 + 2*d^3*f*x - 2*(4*c - 1)*d^2*f)*e^2 + 12*(
d^4*f^2*x^3 + d^3*f^2*x^2 + 2*d^2*f^2*x + 2*(2*c^2 + 1)*d*f^2)*e)*e^(2*d*x
+ 2*c) - 3*(4*I*d^3*f^3*x^3 + 14*I*d^2*f^3*x^2 + 30*I*d*f^3*x + 4*I*d^3*e^3
 + 31*I*f^3 + 2*(6*I*d^3*f*x + 7*I*d^2*f)*e^2 + 2*(6*I*d^3*f^2*x^2 + 14*I*d
^2*f^2*x + 15*I*d*f^2)*e)*e^(d*x + c) - 192*((-I*c^2*f^3 + 2*I*c*d*f^2*e -
I*d^2*f*e^2)*e^(3*d*x + 3*c) - (c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)*e^(2*d*x
 + 2*c))*log(e^(d*x + c) - I) - 192*((-I*d^2*f^3*x^2 + I*c^2*f^3 + 2*(-I*d^
2*f^2*x - I*c*d*f^2)*e)*e^(3*d*x + 3*c) - (d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f
^2*x + c*d*f^2)*e)*e^(2*d*x + 2*c))*log(I*e^(d*x + c) + 1) - 384*(I*f^3*e^(
3*d*x + 3*c) + f^3*e^(2*d*x + 2*c))*polylog(3, -I*e^(d*x + c)))/(a*d^4*e^(3
*d*x + 3*c) - I*a*d^4*e^(2*d*x + 2*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] (2*e**3 + 6*e**2*f*x + 6*e*f**2*x**2 + 2*f**3*x**3)/(a*d*exp(c)*exp(d*x) -
I*a*d) - I*(Integral(-I*d*e**3/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Int
```

```
egral(-I*d*f**3*x**3/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-d*e
**3*exp(c)*exp(d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-4*d*
e**3*exp(3*c)*exp(3*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(
d*e**3*exp(5*c)*exp(5*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integra
l(-3*I*d*e*f**2*x**2/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-3*I
*d*e**2*f*x/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(4*I*d*e**3*ex
p(2*c)*exp(2*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(I*d*e**
3*exp(4*c)*exp(4*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-24
*I*e**2*f*exp(2*c)*exp(2*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Inte
gral(-24*I*f**3*x**2*exp(2*c)*exp(2*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x))
, x) + Integral(-d*f**3*x**3*exp(c)*exp(d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d
*x)), x) + Integral(-4*d*f**3*x**3*exp(3*c)*exp(3*d*x)/(exp(c)*exp(3*d*x) -
 I*exp(2*d*x)), x) + Integral(d*f**3*x**3*exp(5*c)*exp(5*d*x)/(exp(c)*exp(3
*d*x) - I*exp(2*d*x)), x) + Integral(4*I*d*f**3*x**3*exp(2*c)*exp(2*d*x)/(e
xp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(I*d*f**3*x**3*exp(4*c)*exp(
4*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-48*I*e*f**2*x*exp
(2*c)*exp(2*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-3*d*e*f
**2*x**2*exp(c)*exp(d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(
-12*d*e*f**2*x**2*exp(3*c)*exp(3*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x
) + Integral(3*d*e*f**2*x**2*exp(5*c)*exp(5*d*x)/(exp(c)*exp(3*d*x) - I*exp
(2*d*x)), x) + Integral(-3*d*e**2*f*x*exp(c)*exp(d*x)/(exp(c)*exp(3*d*x) -
I*exp(2*d*x)), x) + Integral(-12*d*e**2*f*x*exp(3*c)*exp(3*d*x)/(exp(c)*exp
(3*d*x) - I*exp(2*d*x)), x) + Integral(3*d*e**2*f*x*exp(5*c)*exp(5*d*x)/(ex
p(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(12*I*d*e*f**2*x**2*exp(2*c)*
exp(2*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(3*I*d*e*f**2*x
**2*exp(4*c)*exp(4*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(1
2*I*d*e**2*f*x*exp(2*c)*exp(2*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) +
 Integral(3*I*d*e**2*f*x*exp(4*c)*exp(4*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d
*x)), x))*exp(-2*c)/(4*a*d)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^3*sinh(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\sinh(c+d\,x)}^3\,{\left(e+f\,x\right)}^3}{a+a\,\sinh\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)^3*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i),x)
[Out] int((sinh(c + d*x)^3*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i), x)
```

3.200 $\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=287

$$\frac{if^2x}{4ad^2}-\frac{i(e+fx)^2}{ad}+\frac{i(e+fx)^3}{2af}+\frac{2f^2\cosh(c+dx)}{ad^3}+\frac{(e+fx)^2\cosh(c+dx)}{ad}+\frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}+$$

```
[Out] 1/4*I*f^2*x/a/d^2-I*(f*x+e)^2/a/d+1/2*I*(f*x+e)^3/a/f+2*f^2*cosh(d*x+c)/a/d
^3+(f*x+e)^2*cosh(d*x+c)/a/d+4*I*f*(f*x+e)*ln(1+I*exp(d*x+c))/a/d^2+4*I*f^2
*polylog(2,-I*exp(d*x+c))/a/d^3-2*f*(f*x+e)*sinh(d*x+c)/a/d^2-1/4*I*f^2*cos
h(d*x+c)*sinh(d*x+c)/a/d^3-1/2*I*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/a/d+1/2*
I*f*(f*x+e)*sinh(d*x+c)^2/a/d^2-I*(f*x+e)^2*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a/
d
```

Rubi [A]
time = 0.38, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {5676, 3392, 32, 2715, 8, 3377, 2718, 3399, 4269, 3797, 2221, 2317, 2438}

$$\frac{4if^2\text{Li}_2\left(-ie^{c+dx}\right)}{ad^3}+\frac{2f^2\cosh(c+dx)}{ad^3}-\frac{if^2\sinh(c+dx)\cosh(c+dx)}{4ad^3}+\frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}+\frac{if(e+fx)\sinh^2(c+dx)}{2ad^2}-\frac{2f(e+fx)\sinh(c+dx)}{ad^2}+\frac{(e+fx)^2\cosh(c+dx)}{ad}-\frac{i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}-\frac{i(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2ad}+\frac{if^2x}{4ad^2}-\frac{i(e+fx)^2}{ad}+\frac{i(e+fx)^3}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((I/4)*f^2*x)/(a*d^2) - (I*(e + f*x)^2)/(a*d) + ((I/2)*(e + f*x)^3)/(a*f) +
 (2*f^2*Cosh[c + d*x])/(a*d^3) + ((e + f*x)^2*Cosh[c + d*x])/(a*d) + ((4*I)
*f*(e + f*x)*Log[1 + I*E^(c + d*x)])/(a*d^2) + ((4*I)*f^2*PolyLog[2, (-I)*E
^(c + d*x)])/(a*d^3) - (2*f*(e + f*x)*Sinh[c + d*x])/(a*d^2) - ((I/4)*f^2*C
osh[c + d*x]*Sinh[c + d*x])/(a*d^3) - ((I/2)*(e + f*x)^2*Cosh[c + d*x]*Sinh
[c + d*x])/(a*d) + ((I/2)*f*(e + f*x)*Sinh[c + d*x]^2)/(a*d^2) - (I*(e + f*
x)^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
```

```
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5676

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = i\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx - \frac{i\int (e+fx)^2\sinh^2(c+dx)\,dx}{a}$$

$$= -\frac{i(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{2ad} + \frac{if(e+fx)\sinh^2(c+dx)}{2ad^2} + \frac{i\int(e\cdots}{\cdots}$$

$$= \frac{i(e+fx)^3}{6af} + \frac{(e+fx)^2\cosh(c+dx)}{ad} - \frac{if^2\cosh(c+dx)\sinh(c+dx)}{4ad^3} - \frac{i(e\cdots}{\cdots}$$

$$= \frac{if^2x}{4ad^2} + \frac{i(e+fx)^3}{2af} + \frac{(e+fx)^2\cosh(c+dx)}{ad} - \frac{2f(e+fx)\sinh(c+dx)}{ad^2} - \cdots$$

$$= \frac{if^2x}{4ad^2} + \frac{i(e+fx)^3}{2af} + \frac{2f^2\cosh(c+dx)}{ad^3} + \frac{(e+fx)^2\cosh(c+dx)}{ad} - \frac{2f(e+\cdots}{\cdots}$$

$$= \frac{if^2x}{4ad^2} - \frac{i(e+fx)^2}{ad} + \frac{i(e+fx)^3}{2af} + \frac{2f^2\cosh(c+dx)}{ad^3} + \frac{(e+fx)^2\cosh(c+\cdots}{ad}$$

$$= \frac{if^2x}{4ad^2} - \frac{i(e+fx)^2}{ad} + \frac{i(e+fx)^3}{2af} + \frac{2f^2\cosh(c+dx)}{ad^3} + \frac{(e+fx)^2\cosh(c+\cdots}{ad}$$

$$= \frac{if^2x}{4ad^2} - \frac{i(e+fx)^2}{ad} + \frac{i(e+fx)^3}{2af} + \frac{2f^2\cosh(c+dx)}{ad^3} + \frac{(e+fx)^2\cosh(c+\cdots}{ad}$$

$$= \frac{if^2x}{4ad^2} - \frac{i(e+fx)^2}{ad} + \frac{i(e+fx)^3}{2af} + \frac{2f^2\cosh(c+dx)}{ad^3} + \frac{(e+fx)^2\cosh(c+\cdots}{ad}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 831 vs. $2(287) = 574$.
time = 4.24, size = 831, normalized size = 2.90

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((32*f*(d*((-I)*d*E^c*x*(2*e + f*x) + 2*(1 + I*E^c)*(e + f*x)*Log[1 + I*E^(
c + d*x)]) + 2*(1 + I*E^c)*f*PolyLog[2, (-I)*E^(c + d*x)]))/(-I + E^c) - ((
-8*I)*d*(3*d^2*e^2*x + f^2*x*(2 + d^2*x^2) + e*f*(2 + 3*d^2*x^2))*Cosh[(d*x
)/2] - 8*(2*f^2 + d^2*(e + f*x)^2)*Cosh[c + (d*x)/2] - 6*d^2*e^2*Cosh[c + (
3*d*x)/2] - 15*f^2*Cosh[c + (3*d*x)/2] - 12*d^2*e*f*x*Cosh[c + (3*d*x)/2] -
 6*d^2*f^2*x^2*Cosh[c + (3*d*x)/2] + (14*I)*d*e*f*Cosh[2*c + (3*d*x)/2] + (
14*I)*d*f^2*x*Cosh[2*c + (3*d*x)/2] - (2*I)*d*e*f*Cosh[2*c + (5*d*x)/2] - (
2*I)*d*f^2*x*Cosh[2*c + (5*d*x)/2] - 2*d^2*e^2*Cosh[3*c + (5*d*x)/2] - f^2*
```

```
Cosh[3*c + (5*d*x)/2] - 4*d^2*e*f*x*Cosh[3*c + (5*d*x)/2] - 2*d^2*f^2*x^2*C
osh[3*c + (5*d*x)/2] + (40*I)*d^2*e^2*Sinh[(d*x)/2] + (16*I)*f^2*Sinh[(d*x)
/2] + (80*I)*d^2*e*f*x*Sinh[(d*x)/2] + (40*I)*d^2*f^2*x^2*Sinh[(d*x)/2] + 1
6*d*e*f*Sinh[c + (d*x)/2] + 24*d^3*e^2*x*Sinh[c + (d*x)/2] + 16*d*f^2*x*Sin
h[c + (d*x)/2] + 24*d^3*e*f*x^2*Sinh[c + (d*x)/2] + 8*d^3*f^2*x^3*Sinh[c +
(d*x)/2] + 14*d*e*f*Sinh[c + (3*d*x)/2] + 14*d*f^2*x*Sinh[c + (3*d*x)/2] -
(6*I)*d^2*e^2*Sinh[2*c + (3*d*x)/2] - (15*I)*f^2*Sinh[2*c + (3*d*x)/2] - (1
2*I)*d^2*e*f*x*Sinh[2*c + (3*d*x)/2] - (6*I)*d^2*f^2*x^2*Sinh[2*c + (3*d*x)
/2] + (2*I)*d^2*e^2*Sinh[2*c + (5*d*x)/2] + I*f^2*Sinh[2*c + (5*d*x)/2] + (
4*I)*d^2*e*f*x*Sinh[2*c + (5*d*x)/2] + (2*I)*d^2*f^2*x^2*Sinh[2*c + (5*d*x)
/2] + 2*d*e*f*Sinh[3*c + (5*d*x)/2] + 2*d*f^2*x*Sinh[3*c + (5*d*x)/2])/((Co
sh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])))/(16*a*d^
3)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(257) = 514$.
time = 2.60, size = 520, normalized size = 1.81

method	result
risch	$\frac{4if^2\ln(1+ie^{dx+c})x}{a\,d^2} - \frac{2if^2x^2}{ad} + \frac{4if^2\,\mathrm{polylog}(2,-ie^{dx+c})}{a\,d^3} - \frac{2if^2c^2}{a\,d^3} + \frac{if^2x^3}{2a} + \frac{(f^2x^2d^2+2d^2efx+d^2e^2-2d\,f^2x-2def+2f^2}{2a\,d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 4*I/a/d^2*f^2*ln(1+I*exp(d*x+c))*x-2*I/a/d*f^2*x^2-2*I/a/d^3*f^2*c^2+4*I*f^
2*polylog(2,-I*exp(d*x+c))/a/d^3+1/2*I/a*f^2*x^3+1/2*(d^2*f^2*x^2+2*d^2*e*f
*x+d^2*e^2-2*d*f^2*x-2*d*e*f+2*f^2)/a/d^3*exp(d*x+c)+1/2*(d^2*f^2*x^2+2*d^2
*e*f*x+d^2*e^2+2*d*f^2*x+2*d*e*f+2*f^2)/a/d^3*exp(-d*x-c)+1/2*I/a/f*e^3+2*(
f^2*x^2+2*e*f*x+e^2)/d/a/(exp(d*x+c)-I)+4*I/a/d^3*f^2*c*ln(exp(d*x+c))+4*I/
a/d^3*f^2*ln(1+I*exp(d*x+c))*c+1/16*I*(2*d^2*f^2*x^2+4*d^2*e*f*x+2*d^2*e^2+
2*d*f^2*x+2*d*e*f+f^2)/a/d^3*exp(-2*d*x-2*c)-4*I/a/d^2*f^2*c*x-1/16*I*(2*d^
2*f^2*x^2+4*d^2*e*f*x+2*d^2*e^2-2*d*f^2*x-2*d*e*f+f^2)/a/d^3*exp(2*d*x+2*c)
-4*I/a/d^3*f^2*c*ln(exp(d*x+c)-I)-4*I/a/d^2*ln(exp(d*x+c))*e*f+3/2*I/a*e^2*
x+4*I/a/d^2*ln(exp(d*x+c)-I)*e*f+3/2*I/a*f*e*x^2
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 596 vs. $2(252) = 504$.
time = 0.41, size = 596, normalized size = 2.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/16*(2*d^2*f^2*x^2 + 2*d*f^2*x + 2*d^2*e^2 + f^2 - 64*(-I*f^2*e^(3*d*x + 3
*c) - f^2*e^(2*d*x + 2*c))*dilog(-I*e^(d*x + c)) + 2*(2*d^2*f*x + d*f)*e +
(-2*I*d^2*f^2*x^2 + 2*I*d*f^2*x - 2*I*d^2*e^2 - I*f^2 - 2*(2*I*d^2*f*x - I*
d*f)*e)*e^(5*d*x + 5*c) + (6*d^2*f^2*x^2 - 14*d*f^2*x + 6*d^2*e^2 + 15*f^2
+ 2*(6*d^2*f*x - 7*d*f)*e)*e^(4*d*x + 4*c) - 8*(-I*d^3*f^2*x^3 + 5*I*d^2*f^
2*x^2 - 2*I*d*f^2*x + 2*(-2*I*c^2 + I)*f^2 + (-3*I*d^3*x + I*d^2)*e^2 + (-3
*I*d^3*f*x^2 + 10*I*d^2*f*x + 2*(4*I*c - I)*d*f)*e)*e^(3*d*x + 3*c) + 8*(d^
3*f^2*x^3 + d^2*f^2*x^2 + 2*d*f^2*x + 2*(2*c^2 + 1)*f^2 + (3*d^3*x + 5*d^2)
*e^2 + (3*d^3*f*x^2 + 2*d^2*f*x - 2*(4*c - 1)*d*f)*e)*e^(2*d*x + 2*c) + (-6
*I*d^2*f^2*x^2 - 14*I*d*f^2*x - 6*I*d^2*e^2 - 15*I*f^2 - 2*(6*I*d^2*f*x + 7
*I*d*f)*e)*e^(d*x + c) - 64*((I*c*f^2 - I*d*f*e)*e^(3*d*x + 3*c) + (c*f^2 -
 d*f*e)*e^(2*d*x + 2*c))*log(e^(d*x + c) - I) - 64*((-I*d*f^2*x - I*c*f^2)*
e^(3*d*x + 3*c) - (d*f^2*x + c*f^2)*e^(2*d*x + 2*c))*log(I*e^(d*x + c) + 1)
)/(a*d^3*e^(3*d*x + 3*c) - I*a*d^3*e^(2*d*x + 2*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] (2*e**2 + 4*e*f*x + 2*f**2*x**2)/(a*d*exp(c)*exp(d*x) - I*a*d) - I*(Integra
l(-I*d*e**2/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-I*d*f**2*x**
2/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-d*e**2*exp(c)*exp(d*x)
/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-4*d*e**2*exp(3*c)*exp(3
*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(d*e**2*exp(5*c)*exp
(5*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-2*I*d*e*f*x/(exp
(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(4*I*d*e**2*exp(2*c)*exp(2*d*x
)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(I*d*e**2*exp(4*c)*exp(4
*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-16*I*e*f*exp(2*c)*
exp(2*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-16*I*f**2*x*e
```

```
xp(2*c)*exp(2*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-d*f**
2*x**2*exp(c)*exp(d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(-4
*d*f**2*x**2*exp(3*c)*exp(3*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + I
ntegral(d*f**2*x**2*exp(5*c)*exp(5*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)),
 x) + Integral(4*I*d*f**2*x**2*exp(2*c)*exp(2*d*x)/(exp(c)*exp(3*d*x) - I*e
xp(2*d*x)), x) + Integral(I*d*f**2*x**2*exp(4*c)*exp(4*d*x)/(exp(c)*exp(3*d
*x) - I*exp(2*d*x)), x) + Integral(-2*d*e*f*x*exp(c)*exp(d*x)/(exp(c)*exp(3
*d*x) - I*exp(2*d*x)), x) + Integral(-8*d*e*f*x*exp(3*c)*exp(3*d*x)/(exp(c)
*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(2*d*e*f*x*exp(5*c)*exp(5*d*x)/(e
xp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(8*I*d*e*f*x*exp(2*c)*exp(2*
d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x) + Integral(2*I*d*e*f*x*exp(4*c)
*exp(4*d*x)/(exp(c)*exp(3*d*x) - I*exp(2*d*x)), x))*exp(-2*c)/(4*a*d)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^2*sinh(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^3\,(e+f\,x)^2}{a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)^3*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i),x)
[Out] int((sinh(c + d*x)^3*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i), x)
```

3.201 $\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=175

$$\frac{3iex}{2a}+\frac{3ifx^2}{4a}+\frac{(e+fx)\cosh(c+dx)}{ad}+\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}-\frac{f\sinh(c+dx)}{ad^2}-\frac{i(e+fx)\cosh(c+dx)}{2ad}$$

```
[Out] 3/2*I*e*x/a+3/4*I*f*x^2/a+(f*x+e)*cosh(d*x+c)/a/d+2*I*f*ln(cosh(1/2*c+1/4*I
*Pi+1/2*d*x))/a/d^2-f*sinh(d*x+c)/a/d^2-1/2*I*(f*x+e)*cosh(d*x+c)*sinh(d*x+
c)/a/d+1/4*I*f*sinh(d*x+c)^2/a/d^2-I*(f*x+e)*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a
/d
```

Rubi [A]
time = 0.19, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {5676, 3391, 3377, 2717, 3399, 4269, 3556}

$$\frac{if\sinh^2(c+dx)}{4ad^2}-\frac{f\sinh(c+dx)}{ad^2}+\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{ad^2}+\frac{(e+fx)\cosh(c+dx)}{ad}-\frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}-\frac{i(e+fx)\sinh(c+dx)\cosh(c+dx)}{2ad}+\frac{3iex}{2a}+\frac{3ifx^2}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (((3*I)/2)*e*x)/a + (((3*I)/4)*f*x^2)/a + ((e + f*x)*Cosh[c + d*x])/(a*d) +
 ((2*I)*f*Log[Cosh[c/2 + (I/4)*Pi + (d*x)/2]])/(a*d^2) - (f*Sinh[c + d*x])/
(a*d^2) - ((I/2)*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(a*d) + ((I/4)*f*Si
nh[c + d*x]^2)/(a*d^2) - (I*(e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5676

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx &= i\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx-\frac{i\int (e+fx)\sinh^2(c+dx)\,dx}{a}\\
&=-\frac{i(e+fx)\cosh(c+dx)\sinh(c+dx)}{2ad}+\frac{if\sinh^2(c+dx)}{4ad^2}+\frac{i\int (e+fx)\,dx}{2a}\\
&=\frac{iex}{2a}+\frac{ifx^2}{4a}+\frac{(e+fx)\cosh(c+dx)}{ad}-\frac{i(e+fx)\cosh(c+dx)\sinh(c+dx)}{2ad}\\
&=\frac{3iex}{2a}+\frac{3ifx^2}{4a}+\frac{(e+fx)\cosh(c+dx)}{ad}-\frac{f\sinh(c+dx)}{ad^2}-\frac{i(e+fx)\cosh(}{}\\
&=\frac{3iex}{2a}+\frac{3ifx^2}{4a}+\frac{(e+fx)\cosh(c+dx)}{ad}-\frac{f\sinh(c+dx)}{ad^2}-\frac{i(e+fx)\cosh(}{}\\
&=\frac{3iex}{2a}+\frac{3ifx^2}{4a}+\frac{(e+fx)\cosh(c+dx)}{ad}+\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}-\frac{f}{}\end{aligned}$$

Mathematica [A]

time = 1.13, size = 325, normalized size = 1.86

$$\frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)\left(-8id(e+fx)\cosh(c+dx)+f\cosh(2(c+dx))+2\left(6cde-4icf-3c^2f+6d^2ex-4idfx+3d^2fx^2+8if\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+4f\log(\cosh(c+dx))+4if\sinh(c+dx)-d(e+fx)\sinh(2(c+dx))\right)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\left(8d(e+fx)\cosh(c+dx)+i\left(f\cosh(2(c+dx))+2\left(8ide+6cde-4icf-3c^2f+6d^2ex+4idfx+3d^2fx^2+8if\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+4f\log(\cosh(c+dx))+4if\sinh(c+dx)-d(e+fx)\sinh(2(c+dx))\right)\right)\right)\right)}{8ad^2(-i+\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(Cosh[(c + d*x)/2]*((-8*I)*d*(e
+ f*x)*Cosh[c + d*x] + f*Cosh[2*(c + d*x)] + 2*(6*c*d*e - (4*I)*c*f - 3*c^2
*f + 6*d^2*e*x - (4*I)*d*f*x + 3*d^2*f*x^2 + (8*I)*f*ArcTan[Tanh[(c + d*x)/
2]] + 4*f*Log[Cosh[c + d*x]] + (4*I)*f*Sinh[c + d*x] - d*(e + f*x)*Sinh[2*(
c + d*x)])) + Sinh[(c + d*x)/2]*(8*d*(e + f*x)*Cosh[c + d*x] + I*(f*Cosh[2*
(c + d*x)] + 2*((8*I)*d*e + 6*c*d*e - (4*I)*c*f - 3*c^2*f + 6*d^2*e*x + (4*
I)*d*f*x + 3*d^2*f*x^2 + (8*I)*f*ArcTan[Tanh[(c + d*x)/2]] + 4*f*Log[Cosh[c
 + d*x]] + (4*I)*f*Sinh[c + d*x] - d*(e + f*x)*Sinh[2*(c + d*x)])))))/(8*a*
d^2*(-I + Sinh[c + d*x]))
```

Maple [A]
time = 2.60, size = 197, normalized size = 1.13

method	result
risch	$\frac{3 i f\,x^{2}}{4 a}+\frac{3 i e x}{2 a}-\frac{i \left(2 d x f+2 d e-f\right) {\mathrm e}^{2 d x+2 c}}{16 a\,d^{2}}+\frac{\left(d x f+d e-f\right) {\mathrm e}^{d x+c}}{2 a\,d^{2}}+\frac{\left(d x f+d e+f\right) {\mathrm e}^{-d x-c}}{2 a\,d^{2}}+\frac{i \left(2 d x f+2 d e+f\right) {\mathrm e}^{-2 d x-2 c}}{16 a\,d^{2}}-\frac{2 i f x}{a d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*I*f*x^2/a+3/2*I*e*x/a-1/16*I*(2*d*f*x+2*d*e-f)/a/d^2*exp(2*d*x+2*c)+1/2
*(d*f*x+d*e-f)/a/d^2*exp(d*x+c)+1/2*(d*f*x+d*e+f)/a/d^2*exp(-d*x-c)+1/16*I*
(2*d*f*x+2*d*e+f)/a/d^2*exp(-2*d*x-2*c)-2*I*f/a/d*x-2*I*f/a/d^2*c+2*(f*x+e)
/d/a/(exp(d*x+c)-I)+2*I*f/a/d^2*ln(exp(d*x+c)-I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 0.41, size = 237, normalized size = 1.35

$$\frac{2\,dfx+2\,de+(-2i\,dfx-2i\,de+i\,f)e^{(5\,dx+5\,c)}+(6\,dfx+6\,de-7\,f)e^{(4\,dx+4\,c)}-4\,(-3i\,d^2fx^2+10i\,dfx+2\,(-3i\,d^2x+i\,d)e-2i\,f)e^{(3\,dx+3\,c)}+4\,(3\,d^2fx^2+2\,dfx+2\,(3\,d^2x+5\,d)e+2\,f)e^{(2\,dx+2\,c)}+(-6i\,dfx-6i\,de-7i\,f)e^{(dx+c)}-32\,\left(-i\,fe^{(3\,dx+3\,c)}-fe^{(2\,dx+2\,c)}\right)\log\left(e^{(dx+c)}-i\right)+f}{16\,\left(ad^2e^{(3\,dx+3\,c)}-i\,ad^2e^{(2\,dx+2\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(2*d*f*x + 2*d*e + (-2*I*d*f*x - 2*I*d*e + I*f)*e^(5*d*x + 5*c) + (6*d
*f*x + 6*d*e - 7*f)*e^(4*d*x + 4*c) - 4*(-3*I*d^2*f*x^2 + 10*I*d*f*x + 2*(-
3*I*d^2*x + I*d)*e - 2*I*f)*e^(3*d*x + 3*c) + 4*(3*d^2*f*x^2 + 2*d*f*x + 2*
(3*d^2*x + 5*d)*e + 2*f)*e^(2*d*x + 2*c) + (-6*I*d*f*x - 6*I*d*e - 7*I*f)*e
^(d*x + c) - 32*(-I*f*e^(3*d*x + 3*c) - f*e^(2*d*x + 2*c))*log(e^(d*x + c)
- I) + f)/(a*d^2*e^(3*d*x + 3*c) - I*a*d^2*e^(2*d*x + 2*c))
```

Sympy [A]
time = 0.50, size = 396, normalized size = 2.26

$$\frac{2e+2fx}{ade^{c}e^{dx}-iad}+\begin{cases} \frac{\left(\left(512a^{3}d^{7}ee^{2c}+512a^{3}d^{7}fxe^{2c}+512a^{3}d^{6}fe^{2c}\right)e^{-dx}+\left(512a^{3}d^{7}ee^{4c}+512a^{3}d^{7}fxe^{4c}-512a^{3}d^{6}fe^{4c}\right)e^{dx}+\left(128ia^{3}d^{7}ee^{c}+128ia^{3}d^{7}fxe^{c}+64ia^{3}d^{6}fe^{c}\right)e^{-2dx}+\left(-128ia^{3}d^{7}ee^{5c}-128ia^{3}d^{7}fxe^{5c}+64ia^{3}d^{6}fe^{5c}\right)e^{2dx}\right)e^{-3c}}{1024a^{4}d^{8}} & \text{for}\ a^{4}d^{8}e^{3c}\neq 0 \\ \frac{x^{2}\left(-ife^{4c}+2fe^{3c}-2fe^{c}-if\right)e^{-2c}}{8a}+\frac{x\left(-iee^{4c}+2ee^{3c}-2ee^{c}-ie\right)e^{-2c}}{4a} & \text{otherwise} \end{cases}+\frac{3ifx^{2}}{4a}+\frac{x(3ide-4if)}{2ad}+\frac{2if\log\left(e^{dx}-ie^{-c}\right)}{ad^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] (2*e + 2*f*x)/(a*d*exp(c)*exp(d*x) - I*a*d) + Piecewise((((512*a**3*d**7*e*
exp(2*c) + 512*a**3*d**7*f*x*exp(2*c) + 512*a**3*d**6*f*exp(2*c))*exp(-d*x)
 + (512*a**3*d**7*e*exp(4*c) + 512*a**3*d**7*f*x*exp(4*c) - 512*a**3*d**6*f
*exp(4*c))*exp(d*x) + (128*I*a**3*d**7*e*exp(c) + 128*I*a**3*d**7*f*x*exp(c
) + 64*I*a**3*d**6*f*exp(c))*exp(-2*d*x) + (-128*I*a**3*d**7*e*exp(5*c) - 1
28*I*a**3*d**7*f*x*exp(5*c) + 64*I*a**3*d**6*f*exp(5*c))*exp(2*d*x))*exp(-3
*c)/(1024*a**4*d**8), Ne(a**4*d**8*exp(3*c), 0)), (x**2*(-I*f*exp(4*c) + 2*
f*exp(3*c) - 2*f*exp(c) - I*f)*exp(-2*c)/(8*a) + x*(-I*e*exp(4*c) + 2*e*exp
(3*c) - 2*e*exp(c) - I*e)*exp(-2*c)/(4*a), True)) + 3*I*f*x**2/(4*a) + x*(3
*I*d*e - 4*I*f)/(2*a*d) + 2*I*f*log(exp(d*x) - I*exp(-c))/(a*d**2)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(145) = 290.
time = 0.45, size = 341, normalized size = 1.95

$$\frac{12i\,d^{2}fx^{2}e^{(3\,dx+3\,c)}+12\,d^{2}fx^{2}e^{(2\,dx+2\,c)}+24i\,d^{2}exe^{(3\,dx+3\,c)}+24\,d^{2}exe^{(2\,dx+2\,c)}-2i\,dfxe^{(5\,dx+5\,c)}+6\,dfxe^{(4\,dx+4\,c)}-40i\,dfxe^{(3\,dx+3\,c)}+8\,dfxe^{(2\,dx+2\,c)}-6i\,dfxe^{(dx+c)}+2\,dfx-2i\,dee^{(5\,dx+5\,c)}+6\,dee^{(4\,dx+4\,c)}-8i\,dee^{(3\,dx+3\,c)}+40\,dee^{(2\,dx+2\,c)}-6i\,dee^{(dx+c)}+32i\,fe^{(3\,dx+3\,c)}\log\left(e^{(dx+c)}-i\right)+32\,fe^{(2\,dx+2\,c)}\log\left(e^{(dx+c)}-i\right)+2\,de+i\,fe^{(5\,dx+5\,c)}-7\,fe^{(4\,dx+4\,c)}+8i\,fe^{(3\,dx+3\,c)}+8\,fe^{(2\,dx+2\,c)}-7i\,fe^{(dx+c)}+f}{16\left(ad^{2}e^{(3\,dx+3\,c)}-i\,ad^{2}e^{(2\,dx+2\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(12*I*d^2*f*x^2*e^(3*d*x + 3*c) + 12*d^2*f*x^2*e^(2*d*x + 2*c) + 24*I*
d^2*e*x*e^(3*d*x + 3*c) + 24*d^2*e*x*e^(2*d*x + 2*c) - 2*I*d*f*x*e^(5*d*x +
 5*c) + 6*d*f*x*e^(4*d*x + 4*c) - 40*I*d*f*x*e^(3*d*x + 3*c) + 8*d*f*x*e^(2
*d*x + 2*c) - 6*I*d*f*x*e^(d*x + c) + 2*d*f*x - 2*I*d*e*e^(5*d*x + 5*c) + 6
*d*e*e^(4*d*x + 4*c) - 8*I*d*e*e^(3*d*x + 3*c) + 40*d*e*e^(2*d*x + 2*c) - 6
*I*d*e*e^(d*x + c) + 32*I*f*e^(3*d*x + 3*c)*log(e^(d*x + c) - I) + 32*f*e^(
2*d*x + 2*c)*log(e^(d*x + c) - I) + 2*d*e + I*f*e^(5*d*x + 5*c) - 7*f*e^(4*
```

```
d*x + 4*c) + 8*I*f*e^(3*d*x + 3*c) + 8*f*e^(2*d*x + 2*c) - 7*I*f*e^(d*x + c
) + f)/(a*d^2*e^(3*d*x + 3*c) - I*a*d^2*e^(2*d*x + 2*c))
```

Mupad [B]
time = 0.71, size = 215, normalized size = 1.23

$$e^{-c-dx}\left(\frac{f+de}{2ad^2}+\frac{fx}{2ad}\right)+e^{-2c-2dx}\left(\frac{(f+2de)\ 1i}{16ad^2}+\frac{fx\,1i}{8ad}\right)+e^{2c+2dx}\left(\frac{(f-2de)\ 1i}{16ad^2}-\frac{fx\,1i}{8ad}\right)-e^{c+dx}\left(\frac{f-de}{2ad^2}-\frac{fx}{2ad}\right)+\frac{fx^2\,3i}{4a}+\frac{2(e+fx)}{ad\,(e^{c+dx}-i)}-\frac{x(4f-3de)\ 1i}{2ad}+\frac{f\ln\left(e^{dx}e^{c}-i\right)\ 2i}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)^3*(e + f*x))/(a + a*sinh(c + d*x)*1i),x)
```

```
[Out] exp(- c - d*x)*((f + d*e)/(2*a*d^2) + (f*x)/(2*a*d)) + exp(- 2*c - 2*d*x)*(
((f + 2*d*e)*1i)/(16*a*d^2) + (f*x*1i)/(8*a*d)) + exp(2*c + 2*d*x)*(((f - 2
*d*e)*1i)/(16*a*d^2) - (f*x*1i)/(8*a*d)) - exp(c + d*x)*((f - d*e)/(2*a*d^2
) - (f*x)/(2*a*d)) + (f*x^2*3i)/(4*a) + (2*(e + f*x))/(a*d*(exp(c + d*x) -
1i)) - (x*(4*f - 3*d*e)*1i)/(2*a*d) + (f*log(exp(d*x)*exp(c) - 1i)*2i)/(a*d
^2)
```

3.202 $\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=83

$$\frac{3ix}{2a}+\frac{2\cosh(c+dx)}{ad}-\frac{3i\cosh(c+dx)\sinh(c+dx)}{2ad}-\frac{\cosh(c+dx)\sinh^2(c+dx)}{d(a+ia\sinh(c+dx))}$$

```
[Out] 3/2*I*x/a+2*cosh(d*x+c)/a/d-3/2*I*cosh(d*x+c)*sinh(d*x+c)/a/d-cosh(d*x+c)*s
inh(d*x+c)^2/d/(a+I*a*sinh(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2846, 2813}

$$\frac{2\cosh(c+dx)}{ad}-\frac{\sinh^2(c+dx)\cosh(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{3i\sinh(c+dx)\cosh(c+dx)}{2ad}+\frac{3ix}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (((3*I)/2)*x)/a + (2*Cosh[c + d*x])/(a*d) - (((3*I)/2)*Cosh[c + d*x]*Sinh[c
 + d*x])/(a*d) - (Cosh[c + d*x]*Sinh[c + d*x]^2)/(d*(a + I*a*Sinh[c + d*x])
)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2846

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Dist[d/(a*b), Int[(c + d*S
in[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e
 + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || E
qQ[c, 0])
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{\cosh(c+dx)\sinh^2(c+dx)}{d(a+ia\sinh(c+dx))} + \frac{\int \sinh(c+dx)(2a-3ia\sinh(c+dx))\,dx}{a^2}$$
$$= \frac{3ix}{2a} + \frac{2\cosh(c+dx)}{ad} - \frac{3i\cosh(c+dx)\sinh(c+dx)}{2ad} - \frac{\cosh(c+dx)\sinh^2(c+ }{d(a+ia\sinh(c+dx))}$$

Mathematica [A]
time = 0.13, size = 109, normalized size = 1.31

$$\frac{\cosh(c+dx)\left(3\sinh^{-1}(\sinh(c+dx))\sqrt{1+i\sinh(c+dx)}+\sqrt{1-i\sinh(c+dx)}\left(-4i+\sinh(c+dx)-i\sinh^2(c+dx)\right)\right)}{2ad\sqrt{1-i\sinh(c+dx)}\,(-i+\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (Cosh[c + d*x]*(3*ArcSinh[Sinh[c + d*x]]*Sqrt[1 + I*Sinh[c + d*x]] + Sqrt[1
 - I*Sinh[c + d*x]]*(-4*I + Sinh[c + d*x] - I*Sinh[c + d*x]^2)))/(2*a*d*Sqr
t[1 - I*Sinh[c + d*x]]*(-I + Sinh[c + d*x]))
```

Maple [A]
time = 1.06, size = 123, normalized size = 1.48

method	result
risch	$\frac{3ix}{2a}-\frac{i\mathrm{e}^{2dx+2c}}{8ad}+\frac{\mathrm{e}^{dx+c}}{2ad}+\frac{\mathrm{e}^{-dx-c}}{2ad}+\frac{i\mathrm{e}^{-2dx-2c}}{8ad}+\frac{2}{da\left(\mathrm{e}^{dx+c}-i\right)}$
derivativedivides	$\frac{-\frac{2i}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{3i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{i}{2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{16\left(-\frac{1}{16}-\frac{i}{32}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{i}{2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{3i\ln\left(\tanh\left(\frac{dx}{2}+\right.\right.}{2}}{ad}$
default	$\frac{-\frac{2i}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{3i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{i}{2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{16\left(-\frac{1}{16}-\frac{i}{32}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{i}{2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{3i\ln\left(\tanh\left(\frac{dx}{2}+\right.\right.}{2}}{ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 16/d/a*(-1/8*I/(-I+tanh(1/2*d*x+1/2*c))-3/32*I*ln(tanh(1/2*d*x+1/2*c)-1)-1/
32*I/(tanh(1/2*d*x+1/2*c)-1)^2-(1/16+1/32*I)/(tanh(1/2*d*x+1/2*c)-1)+1/32*I
/(tanh(1/2*d*x+1/2*c)+1)^2+3/32*I*ln(tanh(1/2*d*x+1/2*c)+1)+(1/16-1/32*I)/(
tanh(1/2*d*x+1/2*c)+1))
```

Maxima [A]
time = 0.27, size = 98, normalized size = 1.18

$$\frac{3i\,(dx+c)}{2\,ad} + \frac{3i\,e^{(-dx-c)} + 20\,e^{(-2\,dx-2\,c)} + 1}{8\,(i\,ae^{(-2\,dx-2\,c)} + ae^{(-3\,dx-3\,c)})d} + \frac{i\,\left(-4i\,e^{(-dx-c)} + e^{(-2\,dx-2\,c)}\right)}{8\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 3/2*I*(d*x + c)/(a*d) + 1/8*(3*I*e^(-d*x - c) + 20*e^(-2*d*x - 2*c) + 1)/((
I*a*e^(-2*d*x - 2*c) + a*e^(-3*d*x - 3*c))*d) + 1/8*I*(-4*I*e^(-d*x - c) +
e^(-2*d*x - 2*c))/(a*d)
```

Fricas [A]
time = 0.39, size = 96, normalized size = 1.16

$$-\frac{4(-3i\,dx+i)e^{(3\,dx+3\,c)}-4\,(3\,dx+5)e^{(2\,dx+2\,c)}+i\,e^{(5\,dx+5\,c)}-3\,e^{(4\,dx+4\,c)}+3i\,e^{(dx+c)}-1}{8\left(ade^{(3\,dx+3\,c)}-i\,ade^{(2\,dx+2\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/8*(4*(-3*I*d*x + I)*e^(3*d*x + 3*c) - 4*(3*d*x + 5)*e^(2*d*x + 2*c) + I*
e^(5*d*x + 5*c) - 3*e^(4*d*x + 4*c) + 3*I*e^(d*x + c) - 1)/(a*d*e^(3*d*x +
3*c) - I*a*d*e^(2*d*x + 2*c))
```

Sympy [A]
time = 0.24, size = 175, normalized size = 2.11

$$\begin{cases}\frac{\left(-32ia^3d^3e^{5c}e^{2dx}+128a^3d^3e^{4c}e^{dx}+128a^3d^3e^{2c}e^{-dx}+32ia^3d^3e^{c}e^{-2dx}\right)e^{-3c}}{256a^4d^4} & \text{for}\ a^4d^4e^{3c}\neq 0\\ x\left(\frac{\left(-ie^{4c}+2e^{3c}+6ie^{2c}-2e^{c}-i\right)e^{-2c}}{4a}-\frac{3i}{2a}\right) & \text{otherwise}\end{cases}+\frac{2}{ade^ce^{dx}-iad}+\frac{3ix}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Piecewise(((-32*I*a**3*d**3*exp(5*c)*exp(2*d*x) + 128*a**3*d**3*exp(4*c)*ex
p(d*x) + 128*a**3*d**3*exp(2*c)*exp(-d*x) + 32*I*a**3*d**3*exp(c)*exp(-2*d*
x))*exp(-3*c)/(256*a**4*d**4), Ne(a**4*d**4*exp(3*c), 0)), (x*((-I*exp(4*c)
 + 2*exp(3*c) + 6*I*exp(2*c) - 2*exp(c) - I)*exp(-2*c)/(4*a) - 3*I/(2*a)),
True)) + 2/(a*d*exp(c)*exp(d*x) - I*a*d) + 3*I*x/(2*a)
```

Giac [A]
time = 0.46, size = 87, normalized size = 1.05

$$-\frac{-\frac{12i\,(dx+c)}{a}-\frac{\left(20\,e^{(2\,dx+2\,c)}-3i\,e^{(dx+c)}+1\right)e^{(-2\,dx-2\,c)}}{a\left(e^{(dx+c)}-i\right)}+\frac{i\,ae^{(2\,dx+2\,c)}-4\,ae^{(dx+c)}}{a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/8*(-12*I*(d*x + c)/a - (20*e^(2*d*x + 2*c) - 3*I*e^(d*x + c) + 1)*e^(-2*
d*x - 2*c)/(a*(e^(d*x + c) - I)) + (I*a*e^(2*d*x + 2*c) - 4*a*e^(d*x + c))/
a^2)/d
```

Mupad [B]
time = 0.35, size = 94, normalized size = 1.13

$$\frac{x\,3\mathrm{i}}{2\,a}+\frac{2}{a\,d\,\left({\mathrm{e}}^{c+d\,x}-\mathrm{i}\right)}+\frac{{\mathrm{e}}^{c+d\,x}}{2\,a\,d}+\frac{{\mathrm{e}}^{-c-d\,x}}{2\,a\,d}+\frac{{\mathrm{e}}^{-2\,c-2\,d\,x}\,1\mathrm{i}}{8\,a\,d}-\frac{{\mathrm{e}}^{2\,c+2\,d\,x}\,1\mathrm{i}}{8\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + a*sinh(c + d*x)*1i),x)
```

```
[Out] (x*3i)/(2*a) + 2/(a*d*(exp(c + d*x) - 1i)) + exp(c + d*x)/(2*a*d) + exp(- c
 - d*x)/(2*a*d) + (exp(- 2*c - 2*d*x)*1i)/(8*a*d) - (exp(2*c + 2*d*x)*1i)/(
8*a*d)
```

3.203 $\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=34

$$\mathrm{Int}\left(\frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sinh[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
[Out] Defer[Int][Sinh[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(dx+c)}{(fx+e)(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))*integral(-1/4*(d*f*
x + d*e - (-I*d*f*x - I*d*e)*e^(5*d*x + 5*c) - (d*f*x + d*e)*e^(4*d*x + 4*c
) + 4*(-I*d*f*x - I*d*e)*e^(3*d*x + 3*c) - 4*(d*f*x + d*e + 2*f)*e^(2*d*x +
 2*c) - (I*d*f*x + I*d*e)*e^(d*x + c))/((a*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e^
2)*e^(3*d*x + 3*c) - (I*a*d*f^2*x^2 + 2*I*a*d*f*x*e + I*a*d*e^2)*e^(2*d*x +
 2*c)), x) + 2)/(-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{-iade - iadfx + (adee^{c} + adfxe^{c})e^{dx}} - [illegible]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 2/(-I*a*d*e - I*a*d*f*x + (a*d*e*exp(c) + a*d*f*x*exp(c))*exp(d*x)) - I*(In
tegral(-I*d*e/(e**2*exp(c)*exp(3*d*x) - I*e**2*exp(2*d*x) + 2*e*f*x*exp(c)*
exp(3*d*x) - 2*I*e*f*x*exp(2*d*x) + f**2*x**2*exp(c)*exp(3*d*x) - I*f**2*x*
*2*exp(2*d*x)), x) + Integral(-I*d*f*x/(e**2*exp(c)*exp(3*d*x) - I*e**2*exp
(2*d*x) + 2*e*f*x*exp(c)*exp(3*d*x) - 2*I*e*f*x*exp(2*d*x) + f**2*x**2*exp(
c)*exp(3*d*x) - I*f**2*x**2*exp(2*d*x)), x) + Integral(8*I*f*exp(2*c)*exp(2
*d*x)/(e**2*exp(c)*exp(3*d*x) - I*e**2*exp(2*d*x) + 2*e*f*x*exp(c)*exp(3*d*
x) - 2*I*e*f*x*exp(2*d*x) + f**2*x**2*exp(c)*exp(3*d*x) - I*f**2*x**2*exp(2
*d*x)), x) + Integral(-d*e*exp(c)*exp(d*x)/(e**2*exp(c)*exp(3*d*x) - I*e**2
```

```
*exp(2*d*x) + 2*e*f*x*exp(c)*exp(3*d*x) - 2*I*e*f*x*exp(2*d*x) + f**2*x**2*
exp(c)*exp(3*d*x) - I*f**2*x**2*exp(2*d*x)), x) + Integral(-4*d*e*exp(3*c)*
exp(3*d*x)/(e**2*exp(c)*exp(3*d*x) - I*e**2*exp(2*d*x) + 2*e*f*x*exp(c)*exp
(3*d*x) - 2*I*e*f*x*exp(2*d*x) + f**2*x**2*exp(c)*exp(3*d*x) - I*f**2*x**2*
exp(2*d*x)), x) + Integral(d*e*exp(5*c)*exp(5*d*x)/(e**2*exp(c)*exp(3*d*x)
- I*e**2*exp(2*d*x) + 2*e*f*x*exp(c)*exp(3*d*x) - 2*I*e*f*x*exp(2*d*x) + f*
*2*x**2*exp(c)*exp(3*d*x) - I*f**2*x**2*exp(2*d*x)), x) + Integral(4*I*d*e*
exp(2*c)*exp(2*d*x)/(e**2*exp(c)*exp(3*d*x) - I*e**2*exp(2*d*x) + 2*e*f*x*e
xp(c)*exp(3*d*x) - 2*I*e*f*x*exp(2*d*x) + f**2*x**2*exp(c)*exp(3*d*x) - I*f
**2*x**2*exp(2*d*x)), x) + Integral(I*d*e*exp(4*c)*exp(4*d*x)/(e**2*exp(c)*
exp(3*d*x) - I*e**2*exp(2*d*x) + 2*e*f*x*exp(c)*exp(3*d*x) - 2*I*e*f*x*exp(
2*d*x) + f**2*x**2*exp(c)*exp(3*d*x) - I*f**2*x**2*exp(2*d*x)), x) + Integr
al(-d*f*x*exp(c)*exp(d*x)/(e**2*exp(c)*exp(3*d*x) - I*e**2*exp(2*d*x) + 2*e
*f*x*exp(c)*exp(3*d*x) - 2*I*e*f*x*exp(2*d*x) + f**2*x**2*exp(c)*exp(3*d*x)
 - I*f**2*x**2*exp(2*d*x)), x) + Integral(-4*d*f*x*exp(3*c)*exp(3*d*x)/(e**
2*exp(c)*exp(3*d*x) - I*e**2*exp(2*d*x) + 2*e*f*x*exp(c)*exp(3*d*x) - 2*I*e
*f*x*exp(2*d*x) + f**2*x**2*exp(c)*exp(3*d*x) - I*f**2*x**2*exp(2*d*x)), x)
 + Integral(d*f*x*exp(5*c)*exp(5*d*x)/(e**2*exp(c)*exp(3*d*x) - I*e**2*exp(
2*d*x) + 2*e*f*x*exp(c)*exp(3*d*x) - 2*I*e*f*x*exp(2*d*x) + f**2*x**2*exp(c
)*exp(3*d*x) - I*f**2*x**2*exp(2*d*x)), x) + Integral(4*I*d*f*x*exp(2*c)*ex
p(2*d*x)/(e**2*exp(c)*exp(3*d*x) - I*e**2*exp(2*d*x) + 2*e*f*x*exp(c)*exp(3
*d*x) - 2*I*e*f*x*exp(2*d*x) + f**2*x**2*exp(c)*exp(3*d*x) - I*f**2*x**2*ex
p(2*d*x)), x) + Integral(I*d*f*x*exp(4*c)*exp(4*d*x)/(e**2*exp(c)*exp(3*d*x
) - I*e**2*exp(2*d*x) + 2*e*f*x*exp(c)*exp(3*d*x) - 2*I*e*f*x*exp(2*d*x) +
f**2*x**2*exp(c)*exp(3*d*x) - I*f**2*x**2*exp(2*d*x)), x))*exp(-2*c)/(4*a*d
)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate(sinh(d*x + c)^3/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sinh(c+d\,x)^3}{(e+f\,x)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/((e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
[Out] int(sinh(c + d*x)^3/((e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.204 $\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} dx$$

Verification is not applicable to the result.

```
[In] Int[Sinh[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
[Out] Defer[Int][Sinh[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} dx = \int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} dx$$

Mathematica [A]
time = 161.13, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
[Out] Integrate[Sinh[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(dx+c)}{(fx+e)^2(a+ia\sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] ((-I*a*d*f^2*x^2 - 2*I*a*d*f*x*e - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*f*x*e +
 a*d*e^2)*e^(d*x + c))*integral(-1/4*(d*f*x + d*e - (-I*d*f*x - I*d*e)*e^(5
*d*x + 5*c) - (d*f*x + d*e)*e^(4*d*x + 4*c) + 4*(-I*d*f*x - I*d*e)*e^(3*d*x
 + 3*c) - 4*(d*f*x + d*e + 4*f)*e^(2*d*x + 2*c) - (I*d*f*x + I*d*e)*e^(d*x
+ c))/((a*d*f^3*x^3 + 3*a*d*f^2*x^2*e + 3*a*d*f*x*e^2 + a*d*e^3)*e^(3*d*x +
 3*c) - (I*a*d*f^3*x^3 + 3*I*a*d*f^2*x^2*e + 3*I*a*d*f*x*e^2 + I*a*d*e^3)*e
^(2*d*x + 2*c)), x) + 2)/(-I*a*d*f^2*x^2 - 2*I*a*d*f*x*e - I*a*d*e^2 + (a*d
*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2)*e^(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^3/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sinh(c+d\,x)^3}{{\left(e+f\,x\right)}^2\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

```
[Out] int(sinh(c + d*x)^3/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.205 $\int \frac{(e+fx)^3\mathbf{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=313

$$-\frac{i(e+fx)^3}{ad}-\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}-\frac{3f(e+fx)^2\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}-$$

```
[Out] -I*(f*x+e)^3/a/d-2*(f*x+e)^3*arctanh(exp(d*x+c))/a/d+6*I*f*(f*x+e)^2*ln(1+I
*exp(d*x+c))/a/d^2-3*f*(f*x+e)^2*polylog(2,-exp(d*x+c))/a/d^2+12*I*f^2*(f*x
+e)*polylog(2,-I*exp(d*x+c))/a/d^3+3*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a/d^
2+6*f^2*(f*x+e)*polylog(3,-exp(d*x+c))/a/d^3-12*I*f^3*polylog(3,-I*exp(d*x+
c))/a/d^4-6*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a/d^3-6*f^3*polylog(4,-exp(d*
x+c))/a/d^4+6*f^3*polylog(4,exp(d*x+c))/a/d^4-I*(f*x+e)^3*tanh(1/2*c+1/4*I*
Pi+1/2*d*x)/a/d
```

Rubi [A]
time = 0.36, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {5694, 4267, 2611, 6744, 2320, 6724, 3399, 4269, 3797, 2221}

$$-\frac{12if^3\text{Li}_3\left(-ie^{c+dx}\right)}{ad^4}-\frac{6f^3\text{Li}_4\left(-e^{c+dx}\right)}{ad^4}+\frac{6f^3\text{Li}_4\left(e^{c+dx}\right)}{ad^4}+\frac{12if^2(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{ad^3}+\frac{6f^2(e+fx)\text{Li}_3\left(-e^{c+dx}\right)}{ad^3}-\frac{6f^2(e+fx)\text{Li}_3\left(e^{c+dx}\right)}{ad^3}-\frac{3f(e+fx)^2\text{Li}_2\left(-e^{c+dx}\right)}{ad^2}+\frac{3f(e+fx)^2\text{Li}_2\left(e^{c+dx}\right)}{ad^2}+\frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}-\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{i(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}-\frac{i(e+fx)^3}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-I)*(e + f*x)^3)/(a*d) - (2*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) + ((6
*I)*f*(e + f*x)^2*Log[1 + I*E^(c + d*x)])/(a*d^2) - (3*f*(e + f*x)^2*PolyLo
g[2, -E^(c + d*x)])/(a*d^2) + ((12*I)*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c +
d*x)])/(a*d^3) + (3*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)])/(a*d^2) + (6*f^2
*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) - ((12*I)*f^3*PolyLog[3, (-I)*
E^(c + d*x)])/(a*d^4) - (6*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a*d^3) -
 (6*f^3*PolyLog[4, -E^(c + d*x)])/(a*d^4) + (6*f^3*PolyLog[4, E^(c + d*x)])
/(a*d^4) - (I*(e + f*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5694

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a + b
```

```
*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\left(i\int \frac{(e+fx)^3}{a+ia\sinh(c+dx)}\,dx\right)+\frac{\int (e+fx)^3\text{csch}(c+dx)\,dx}{a}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{i\int (e+fx)^3\csc^2\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{idx}{2}\right)\,dx}{2a}-\frac{(3}{}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{3f(e+fx)^2\text{Li}_2\left(-e^{c+dx}\right)}{ad^2}+\frac{3f(e+fx)^2\text{Li}_2(}{ad^2}$$

$$= -\frac{i(e+fx)^3}{ad}-\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{3f(e+fx)^2\text{Li}_2\left(-e^{c+dx}\right)}{ad^2}+\frac{3f}{}$$

$$= -\frac{i(e+fx)^3}{ad}-\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}-$$

$$= -\frac{i(e+fx)^3}{ad}-\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}-$$

$$= -\frac{i(e+fx)^3}{ad}-\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}-$$

$$= -\frac{i(e+fx)^3}{ad}-\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}-$$

Mathematica [A]

time = 4.41, size = 501, normalized size = 1.60

$$\frac{-2d^3e^3\tanh^{-1}\left(e^{c+dx}\right)+3d^3e^2fx\log\left(1-e^{c+dx}\right)+3d^3ef^2x^2\log\left(1-e^{c+dx}\right)+d^3f^3x^3\log\left(1-e^{c+dx}\right)-3d^3e^2fx\log\left(1+e^{c+dx}\right)-3d^3ef^2x^2\log\left(1+e^{c+dx}\right)-d^3f^3x^3\log\left(1+e^{c+dx}\right)-3d^2f(e+fx)^2\text{PolyLog}(2,-e^{c+dx})+3d^2f(e+fx)^2\text{PolyLog}(2,e^{c+dx})+6def^2\text{PolyLog}(3,-e^{c+dx})+6df^3x\text{PolyLog}(3,-e^{c+dx})+\frac{2f\left(d^2\left(-ide^cx(3e^2+3efx+f^2x^2)+3(1+ie^c)(e+fx)^2\log(1+ie^{c+dx})\right)+6d(1+ie^c)f(e+fx)\text{PolyLog}(2,-ie^{c+dx})-6i(-i+e^c)f^2\text{PolyLog}(3,-ie^{c+dx})\right)}{-i+e^c}-6def^2\text{PolyLog}(3,e^{c+dx})-6df^3x\text{PolyLog}(3,e^{c+dx})-6f^3\text{PolyLog}(4,-e^{c+dx})+6f^3\text{PolyLog}(4,e^{c+dx})-\frac{2id^3(e+fx)^3\sinh\left(\frac{dx}{2}\right)}{\left(\cosh\left(\frac{c}{2}\right)+i\sinh\left(\frac{c}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}}{ad^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (-2*d^3*e^3*ArcTanh[E^(c + d*x)] + 3*d^3*e^2*f*x*Log[1 - E^(c + d*x)] + 3*d
^3*e*f^2*x^2*Log[1 - E^(c + d*x)] + d^3*f^3*x^3*Log[1 - E^(c + d*x)] - 3*d^
3*e^2*f*x*Log[1 + E^(c + d*x)] - 3*d^3*e*f^2*x^2*Log[1 + E^(c + d*x)] - d^3
*f^3*x^3*Log[1 + E^(c + d*x)] - 3*d^2*f*(e + f*x)^2*PolyLog[2, -E^(c + d*x)
] + 3*d^2*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)] + 6*d*e*f^2*PolyLog[3, -E^(
c + d*x)] + 6*d*f^3*x*PolyLog[3, -E^(c + d*x)] + (2*f*(d^2*((-I)*d*E^c*x*(3
*e^2 + 3*e*f*x + f^2*x^2) + 3*(1 + I*E^c)*(e + f*x)^2*Log[1 + I*E^(c + d*x)
]) + 6*d*(1 + I*E^c)*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)] - (6*I)*(-I +
 E^c)*f^2*PolyLog[3, (-I)*E^(c + d*x)]))/(-I + E^c) - 6*d*e*f^2*PolyLog[3,
E^(c + d*x)] - 6*d*f^3*x*PolyLog[3, E^(c + d*x)] - 6*f^3*PolyLog[4, -E^(c +
 d*x)] + 6*f^3*PolyLog[4, E^(c + d*x)] - ((2*I)*d^3*(e + f*x)^3*Sinh[(d*x)/
2])/((Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])))/
(a*d^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1033 vs. 2(288) = 576.
time = 3.34, size = 1034, normalized size = 3.30

method	result
risch	$\frac{e^3\ln\left(e^{dx+c}-1\right)}{ad}-\frac{e^3\ln\left(e^{dx+c}+1\right)}{ad}-\frac{6f^3\,\text{polylog}\left(4,-e^{dx+c}\right)}{a\,d^4}+\frac{6f^3\,\text{polylog}\left(4,e^{dx+c}\right)}{a\,d^4}-\frac{12if^3\,\text{polylog}\left(3,-ie^{dx+c}\right)}{a\,d^4}-\frac{12ie\,f^2c}{a\,d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 6*I/a/d^3*f^3*c^2*x+12*I/a/d^3*e*f^2*polylog(2,-I*exp(d*x+c))+6*I/a/d^4*f^3
*c^2*ln(exp(d*x+c)-I)-6*I/a/d^4*f^3*c^2*ln(exp(d*x+c))-6*I/a/d*e*f^2*x^2-6*
I/a/d^3*e*f^2*c^2+1/a/d*e^3*ln(exp(d*x+c)-1)-1/a/d*e^3*ln(exp(d*x+c)+1)+3/a
/d^2*ln(1-exp(d*x+c))*c*e^2*f-3/a/d*e*f^2*ln(exp(d*x+c)+1)*x^2-6/a/d^2*e*f^
2*polylog(2,-exp(d*x+c))*x+3/a/d*e*f^2*ln(1-exp(d*x+c))*x^2-3/a/d^3*e*f^2*l
n(1-exp(d*x+c))*c^2+6/a/d^2*e*f^2*polylog(2,exp(d*x+c))*x-6*I/a/d^4*f^3*c^2
*ln(1+I*exp(d*x+c))+6*I/a/d^2*f^3*ln(1+I*exp(d*x+c))*x^2+12*I/a/d^3*f^3*pol
ylog(2,-I*exp(d*x+c))*x-6*I/a/d^2*e^2*f*ln(exp(d*x+c))+6*I/a/d^2*e^2*f*ln(e
xp(d*x+c)-I)+12*I/a/d^2*e*f^2*ln(1+I*exp(d*x+c))*x+12*I/a/d^3*e*f^2*c*ln(ex
p(d*x+c))-12*I/a/d^3*e*f^2*c*ln(exp(d*x+c)-I)+12*I/a/d^3*e*f^2*ln(1+I*exp(d
*x+c))*c+3/a/d^3*e*f^2*c^2*ln(exp(d*x+c)-1)-12*I/a/d^2*e*f^2*c*x+2*(f^3*x^3
+3*e*f^2*x^2+3*e^2*f*x+e^3)/d/a/(exp(d*x+c)-I)-3/a/d*ln(exp(d*x+c)+1)*e^2*f
*x+3/a/d*ln(1-exp(d*x+c))*e^2*f*x-3/a/d^2*e^2*f*c*ln(exp(d*x+c)-1)-12*I*f^3
```

```
*polylog(3,-I*exp(d*x+c))/a/d^4-6*f^3*polylog(4,-exp(d*x+c))/a/d^4+6*f^3*po
lylog(4,exp(d*x+c))/a/d^4-3/a/d^2*e^2*f*polylog(2,-exp(d*x+c))+3/a/d^2*e^2*
f*polylog(2,exp(d*x+c))+3/a/d^2*f^3*polylog(2,exp(d*x+c))*x^2-6/a/d^3*f^3*p
olylog(3,exp(d*x+c))*x-1/a/d^4*f^3*c^3*ln(exp(d*x+c)-1)+6/a/d^3*e*f^2*polyl
og(3,-exp(d*x+c))-6/a/d^3*e*f^2*polylog(3,exp(d*x+c))-1/a/d*f^3*ln(exp(d*x+
c)+1)*x^3-3/a/d^2*f^3*polylog(2,-exp(d*x+c))*x^2+6/a/d^3*f^3*polylog(3,-exp
(d*x+c))*x+1/a/d*f^3*ln(1-exp(d*x+c))*x^3+1/a/d^4*f^3*ln(1-exp(d*x+c))*c^3-
2*I/a/d*f^3*x^3+4*I/a/d^4*f^3*c^3
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(285) = 570$.
time = 0.45, size = 579, normalized size = 1.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d) - 2/((a*e^(-d*x
 - c) + I*a)*d))*e^3 - 6*I*f*x*e^2/(a*d) + 2*(f^3*x^3 + 3*f^2*x^2*e + 3*f*x
*e^2)/(a*d*e^(d*x + c) - I*a*d) - 3*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d
*x + c)))*f*e^2/(a*d^2) + 3*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c))
)*f*e^2/(a*d^2) - 3*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c
)) - 2*polylog(3, -e^(d*x + c)))*f^2*e/(a*d^3) + 3*(d^2*x^2*log(-e^(d*x + c
) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*f^2*e/(a*d^3
) + 12*I*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*f^2*e/(a*d^3)
 + 6*I*f*e^2*log(I*e^(d*x + c) + 1)/(a*d^2) - (d^3*x^3*log(e^(d*x + c) + 1)
 + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*polyl
og(4, -e^(d*x + c)))*f^3/(a*d^4) + (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x
^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x
 + c)))*f^3/(a*d^4) + 6*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*
e^(d*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a*d^4) + 2*(-I*d^3*f^3*x^
3 - 3*I*d^3*f^2*x^2*e)/(a*d^4)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1013 vs. $2(285) = 570$.
time = 0.41, size = 1013, normalized size = 3.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*c^3*f^3 - 6*c^2*d*f^2*e + 6*c*d^2*f*e^2 - 2*d^3*e^3 - 12*(d*f^3*x + d*f
^2*e - (-I*d*f^3*x - I*d*f^2*e)*e^(d*x + c))*dilog(-I*e^(d*x + c)) + 3*(-I*
d^2*f^3*x^2 - 2*I*d^2*f^2*x*e - I*d^2*f*e^2 + (d^2*f^3*x^2 + 2*d^2*f^2*x*e
```

```
+ d^2*f*e^2)*e^(d*x + c))*dilog(-e^(d*x + c)) + 3*(I*d^2*f^3*x^2 + 2*I*d^2*
f^2*x*e + I*d^2*f*e^2 - (d^2*f^3*x^2 + 2*d^2*f^2*x*e + d^2*f*e^2)*e^(d*x +
c))*dilog(e^(d*x + c)) + 2*(I*d^3*f^3*x^3 + I*c^3*f^3 + 3*(I*d^3*f*x + I*c*
d^2*f)*e^2 + 3*(I*d^3*f^2*x^2 - I*c^2*d*f^2)*e)*e^(d*x + c) - (I*d^3*f^3*x^
3 + 3*I*d^3*f^2*x^2*e + 3*I*d^3*f*x*e^2 + I*d^3*e^3 - (d^3*f^3*x^3 + 3*d^3*
f^2*x^2*e + 3*d^3*f*x*e^2 + d^3*e^3)*e^(d*x + c))*log(e^(d*x + c) + 1) - 6*
(c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2 - (-I*c^2*f^3 + 2*I*c*d*f^2*e - I*d^2*f*
e^2)*e^(d*x + c))*log(e^(d*x + c) - I) - (I*c^3*f^3 - 3*I*c^2*d*f^2*e + 3*I
*c*d^2*f*e^2 - I*d^3*e^3 - (c^3*f^3 - 3*c^2*d*f^2*e + 3*c*d^2*f*e^2 - d^3*e
^3)*e^(d*x + c))*log(e^(d*x + c) - 1) - 6*(d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f
^2*x + c*d*f^2)*e - (-I*d^2*f^3*x^2 + I*c^2*f^3 + 2*(-I*d^2*f^2*x - I*c*d*f
^2)*e)*e^(d*x + c))*log(I*e^(d*x + c) + 1) - (-I*d^3*f^3*x^3 - I*c^3*f^3 -
3*(I*d^3*f*x + I*c*d^2*f)*e^2 - 3*(I*d^3*f^2*x^2 - I*c^2*d*f^2)*e + (d^3*f^
3*x^3 + c^3*f^3 + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - c^2*d*f^2)*e
)*e^(d*x + c))*log(-e^(d*x + c) + 1) + 6*(f^3*e^(d*x + c) - I*f^3)*polylog(
4, -e^(d*x + c)) - 6*(f^3*e^(d*x + c) - I*f^3)*polylog(4, e^(d*x + c)) + 12
*(I*f^3*e^(d*x + c) + f^3)*polylog(3, -I*e^(d*x + c)) + 6*(I*d*f^3*x + I*d*
f^2*e - (d*f^3*x + d*f^2*e)*e^(d*x + c))*polylog(3, -e^(d*x + c)) + 6*(-I*d
*f^3*x - I*d*f^2*e + (d*f^3*x + d*f^2*e)*e^(d*x + c))*polylog(3, e^(d*x + c
)))/(a*d^4*e^(d*x + c) - I*a*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{e^{3}\operatorname{csch}\left(c+dx\right)}{\sinh\left(c+dx\right)-i}\,dx+\int \frac{f^{3}x^{3}\operatorname{csch}\left(c+dx\right)}{\sinh\left(c+dx\right)-i}\,dx+\int \frac{3ef^{2}x^{2}\operatorname{csch}\left(c+dx\right)}{\sinh\left(c+dx\right)-i}\,dx+\int \frac{3e^{2}fx\operatorname{csch}\left(c+dx\right)}{\sinh\left(c+dx\right)-i}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*(Integral(e**3*csch(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f**3*x**
3*csch(c + d*x)/(sinh(c + d*x) - I), x) + Integral(3*e*f**2*x**2*csch(c + d
*x)/(sinh(c + d*x) - I), x) + Integral(3*e**2*f*x*csch(c + d*x)/(sinh(c + d
*x) - I), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*csch(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^3}{\sinh\left(c+d\,x\right)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(sinh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
[Out] int((e + f*x)^3/(sinh(c + d*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.206 $\int \frac{(e+fx)^2\mathbf{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=224

$$-\frac{i(e+fx)^2}{ad}-\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}-\frac{2f(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}+\frac{4i}{}$$

```
[Out] -I*(f*x+e)^2/a/d-2*(f*x+e)^2*arctanh(exp(d*x+c))/a/d+4*I*f*(f*x+e)*ln(1+I*e
xp(d*x+c))/a/d^2-2*f*(f*x+e)*polylog(2,-exp(d*x+c))/a/d^2+4*I*f^2*polylog(2
,-I*exp(d*x+c))/a/d^3+2*f*(f*x+e)*polylog(2,exp(d*x+c))/a/d^2+2*f^2*polylog
(3,-exp(d*x+c))/a/d^3-2*f^2*polylog(3,exp(d*x+c))/a/d^3-I*(f*x+e)^2*tanh(1/
2*c+1/4*I*Pi+1/2*d*x)/a/d
```

Rubi [A]
time = 0.25, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {5694, 4267, 2611, 2320, 6724, 3399, 4269, 3797, 2221, 2317, 2438}

$$\frac{4if^2\text{Li}_2\left(-ie^{c+dx}\right)}{ad^3}+\frac{2f^2\text{Li}_3\left(-e^{c+dx}\right)}{ad^3}-\frac{2f^2\text{Li}_3\left(e^{c+dx}\right)}{ad^3}-\frac{2f(e+fx)\text{Li}_2\left(-e^{c+dx}\right)}{ad^2}+\frac{2f(e+fx)\text{Li}_2\left(e^{c+dx}\right)}{ad^2}+\frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}-\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}-\frac{i(e+fx)^2}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-I)*(e + f*x)^2)/(a*d) - (2*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) + ((4
*I)*f*(e + f*x)*Log[1 + I*E^(c + d*x)])/(a*d^2) - (2*f*(e + f*x)*PolyLog[2,
 -E^(c + d*x)])/(a*d^2) + ((4*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^3)
+ (2*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^2) + (2*f^2*PolyLog[3, -E^(c
 + d*x)])/(a*d^3) - (2*f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) - (I*(e + f*x)^
2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
```

```
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5694

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a + b
*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\left(i\int \frac{(e+fx)^2}{a+ia\sinh(c+dx)}\,dx\right)+\frac{\int (e+fx)^2\text{csch}(c+dx)\,dx}{a}\\
&= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{i\int (e+fx)^2\csc^2\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{idx}{2}\right)\,dx}{2a}-\frac{(2f)}{}\\
&= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{2f(e+fx)\text{Li}_2\left(-e^{c+dx}\right)}{ad^2}+\frac{2f(e+fx)\text{Li}_2\left(e^{c+}\right.}{ad^2}\\
&= -\frac{i(e+fx)^2}{ad}-\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{2f(e+fx)\text{Li}_2\left(-e^{c+dx}\right)}{ad^2}+\frac{2f(e}{}\\
&= -\frac{i(e+fx)^2}{ad}-\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}-\\
&= -\frac{i(e+fx)^2}{ad}-\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}-\\
&= -\frac{i(e+fx)^2}{ad}-\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}-
\end{aligned}$$

Mathematica [A]
time = 3.76, size = 327, normalized size = 1.46

$$\frac{-2d^2e^2\tanh^{-1}\left(e^{c+dx}\right)+2d^2efx\log\left(1-e^{c+dx}\right)+d^2f^2x^2\log\left(1-e^{c+dx}\right)-2d^2efx\log\left(1+e^{c+dx}\right)-d^2f^2x^2\log\left(1+e^{c+dx}\right)-2df(e+fx)\text{PolyLog}(2,-e^{c+dx})+\frac{2f\left(d\left(-ide^cx(2e+fx)+2(1+ie^c)(e+fx)\log\left(1+ie^{c+dx}\right)\right)+2(1+ie^c)f\text{PolyLog}\left(2,-ie^{c+dx}\right)\right)}{-i+e^c}+2df(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)+2f^2\text{PolyLog}\left(3,-e^{c+dx}\right)-2f^2\text{PolyLog}\left(3,e^{c+dx}\right)-\frac{2id^2(e+fx)^2\sinh\left(\frac{dx}{2}\right)}{\left(\cosh\left(\frac{c}{2}\right)+i\sinh\left(\frac{c}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}}{ad^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (-2*d^2*e^2*ArcTanh[E^(c + d*x)] + 2*d^2*e*f*x*Log[1 - E^(c + d*x)] + d^2*f
^2*x^2*Log[1 - E^(c + d*x)] - 2*d^2*e*f*x*Log[1 + E^(c + d*x)] - d^2*f^2*x^
2*Log[1 + E^(c + d*x)] - 2*d*f*(e + f*x)*PolyLog[2, -E^(c + d*x)] + (2*f*(d
*((-I)*d*E^c*x*(2*e + f*x) + 2*(1 + I*E^c)*(e + f*x)*Log[1 + I*E^(c + d*x)]
) + 2*(1 + I*E^c)*f*PolyLog[2, (-I)*E^(c + d*x)]))/(-I + E^c) + 2*d*f*(e +
f*x)*PolyLog[2, E^(c + d*x)] + 2*f^2*PolyLog[3, -E^(c + d*x)] - 2*f^2*PolyL
og[3, E^(c + d*x)] - ((2*I)*d^2*(e + f*x)^2*Sinh[(d*x)/2])/((Cosh[c/2] + I*
Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])))/(a*d^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 572 vs. 2(204) = 408.
time = 3.06, size = 573, normalized size = 2.56

method	result
risch	$-\frac{e^2\ln(\mathrm{e}^{dx+c}+1)}{ad}+\frac{e^2\ln(\mathrm{e}^{dx+c}-1)}{ad}+\frac{2f^2\,\mathrm{polylog}(3,-\mathrm{e}^{dx+c})}{a\,d^3}-\frac{2f^2\,\mathrm{polylog}(3,\mathrm{e}^{dx+c})}{a\,d^3}+\frac{4if^2\ln(1+i\mathrm{e}^{dx+c})x}{a\,d^2}+\frac{4if^2c\ln(\mathrm{e}}{a\,d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*f^2*polylog(3,-exp(d*x+c))/a/d^3-2*f^2*polylog(3,exp(d*x+c))/a/d^3-1/a/d*
e^2*ln(exp(d*x+c)+1)+1/a/d*e^2*ln(exp(d*x+c)-1)+4*I*f^2*polylog(2,-I*exp(d*
x+c))/a/d^3-2/a/d^2*e*f*c*ln(exp(d*x+c)-1)+2/a/d^2*ln(1-exp(d*x+c))*c*e*f-2
/a/d*ln(exp(d*x+c)+1)*e*f*x+2/a/d*ln(1-exp(d*x+c))*e*f*x-4*I/a/d^2*e*f*ln(e
xp(d*x+c))+4*I/a/d^2*e*f*ln(exp(d*x+c)-I)-4*I/a/d^2*f^2*c*x+4*I/a/d^3*f^2*c
*ln(exp(d*x+c))-4*I/a/d^3*f^2*c*ln(exp(d*x+c)-I)+4*I/a/d^2*f^2*ln(1+I*exp(d
*x+c))*x+4*I/a/d^3*f^2*ln(1+I*exp(d*x+c))*c+2*(f^2*x^2+2*e*f*x+e^2)/d/a/(ex
p(d*x+c)-I)+2/a/d^2*f^2*polylog(2,exp(d*x+c))*x-2/a/d^2*e*f*polylog(2,-exp(
d*x+c))+2/a/d^2*e*f*polylog(2,exp(d*x+c))+1/a/d^3*f^2*c^2*ln(exp(d*x+c)-1)-
1/a/d*f^2*ln(exp(d*x+c)+1)*x^2-2/a/d^2*f^2*polylog(2,-exp(d*x+c))*x+1/a/d*f
^2*ln(1-exp(d*x+c))*x^2-1/a/d^3*f^2*ln(1-exp(d*x+c))*c^2-2*I/a/d*f^2*x^2-2*
I/a/d^3*f^2*c^2
```

Maxima [A]
time = 0.43, size = 351, normalized size = 1.57

$$-\frac{2i\,f^2x^2}{ad}-\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}-\frac{\log\left(e^{(-dx-c)}-1\right)}{ad}-\frac{2}{(ae^{(-dx-c)}+i\,a)d}\right)e^2-\frac{4i\,fxe}{ad}+\frac{2\,(f^2x^2+2\,fxe)}{ade^{(dx+c)}-i\,ad}-\frac{2\,\left(dx\log\left(e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(-e^{(dx+c)}\right)\right)fe}{ad^2}+\frac{2\,\left(dx\log\left(-e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(e^{(dx+c)}\right)\right)fe}{ad^2}+\frac{4i\,fe\log\left(i\,e^{(dx+c)}+1\right)}{ad^2}-\frac{\left(d^2x^2\log\left(e^{(dx+c)}+1\right)+2\,dx\mathrm{Li}_2\left(-e^{(dx+c)}\right)-2\,\mathrm{Li}_3\left(-e^{(dx+c)}\right)\right)f^2}{ad^3}+\frac{\left(d^2x^2\log\left(-e^{(dx+c)}+1\right)+2\,dx\mathrm{Li}_2\left(e^{(dx+c)}\right)-2\,\mathrm{Li}_3\left(e^{(dx+c)}\right)\right)f^2}{ad^3}+\frac{4i\,\left(dx\log\left(i\,e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)\right)f^2}{ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*I*f^2*x^2/(a*d) - (log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(
a*d) - 2/((a*e^(-d*x - c) + I*a)*d))*e^2 - 4*I*f*x*e/(a*d) + 2*(f^2*x^2 + 2
*f*x*e)/(a*d*e^(d*x + c) - I*a*d) - 2*(d*x*log(e^(d*x + c) + 1) + dilog(-e^
```

```
(d*x + c)))*f*e/(a*d^2) + 2*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c))
)*f*e/(a*d^2) + 4*I*f*e*log(I*e^(d*x + c) + 1)/(a*d^2) - (d^2*x^2*log(e^(d*
x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2/(
a*d^3) + (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*poly
log(3, e^(d*x + c)))*f^2/(a*d^3) + 4*I*(d*x*log(I*e^(d*x + c) + 1) + dilog(
-I*e^(d*x + c)))*f^2/(a*d^3)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(200) = 400$.
time = 0.49, size = 571, normalized size = 2.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*c^2*f^2 - 4*c*d*f*e + 2*d^2*e^2 - 4*(-I*f^2*e^(d*x + c) - f^2)*dilog(-I*
e^(d*x + c)) - 2*(-I*d*f^2*x - I*d*f*e + (d*f^2*x + d*f*e)*e^(d*x + c))*dil
og(-e^(d*x + c)) - 2*(I*d*f^2*x + I*d*f*e - (d*f^2*x + d*f*e)*e^(d*x + c))*
dilog(e^(d*x + c)) - 2*(I*d^2*f^2*x^2 - I*c^2*f^2 + 2*(I*d^2*f*x + I*c*d*f)
*e)*e^(d*x + c) + (I*d^2*f^2*x^2 + 2*I*d^2*f*x*e + I*d^2*e^2 - (d^2*f^2*x^2
 + 2*d^2*f*x*e + d^2*e^2)*e^(d*x + c))*log(e^(d*x + c) + 1) - 4*(c*f^2 - d*
f*e + (I*c*f^2 - I*d*f*e)*e^(d*x + c))*log(e^(d*x + c) - I) + (-I*c^2*f^2 +
 2*I*c*d*f*e - I*d^2*e^2 + (c^2*f^2 - 2*c*d*f*e + d^2*e^2)*e^(d*x + c))*log
(e^(d*x + c) - 1) + 4*(d*f^2*x + c*f^2 - (-I*d*f^2*x - I*c*f^2)*e^(d*x + c)
)*log(I*e^(d*x + c) + 1) + (-I*d^2*f^2*x^2 + I*c^2*f^2 - 2*(I*d^2*f*x + I*c
*d*f)*e + (d^2*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)*e)*e^(d*x + c))*log(
-e^(d*x + c) + 1) + 2*(f^2*e^(d*x + c) - I*f^2)*polylog(3, -e^(d*x + c)) -
2*(f^2*e^(d*x + c) - I*f^2)*polylog(3, e^(d*x + c)))/(a*d^3*e^(d*x + c) - I
*a*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{e^2\operatorname{csch}\left(c+dx\right)}{\sinh\left(c+dx\right)-i}\,dx+\int \frac{f^2x^2\operatorname{csch}\left(c+dx\right)}{\sinh\left(c+dx\right)-i}\,dx+\int \frac{2efx\operatorname{csch}\left(c+dx\right)}{\sinh\left(c+dx\right)-i}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*(Integral(e**2*csch(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f**2*x**
2*csch(c + d*x)/(sinh(c + d*x) - I), x) + Integral(2*e*f*x*csch(c + d*x)/(s
inh(c + d*x) - I), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*csch(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^2}{\sinh\left(c+d\,x\right)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(sinh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
```

```
[Out] int((e + f*x)^2/(sinh(c + d*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.207 $\int \frac{(e+fx)\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=126

$$-\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}-\frac{f\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}+\frac{f\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}-\frac{i(e+}{}$$

```
[Out] -2*(f*x+e)*arctanh(exp(d*x+c))/a/d+2*I*f*ln(cosh(1/2*c+1/4*I*Pi+1/2*d*x))/a
/d^2-f*polylog(2,-exp(d*x+c))/a/d^2+f*polylog(2,exp(d*x+c))/a/d^2-I*(f*x+e)
*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a/d
```

Rubi [A]
time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5694, 4267, 2317, 2438, 3399, 4269, 3556}

$$-\frac{f\text{Li}_2\left(-e^{c+dx}\right)}{ad^2}+\frac{f\text{Li}_2\left(e^{c+dx}\right)}{ad^2}+\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{ad^2}-\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (-2*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d) + ((2*I)*f*Log[Cosh[c/2 + (I/4)*P
i + (d*x)/2]])/(a*d^2) - (f*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (f*PolyLog[
2, E^(c + d*x)])/(a*d^2) - (I*(e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*
d)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5694

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a + b
*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\left(i\int \frac{e+fx}{a+ia\sinh(c+dx)}\,dx\right)+\frac{\int (e+fx)\text{csch}(c+dx)\,dx}{a}\\
&= -\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{i\int (e+fx)\csc^2\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{idx}{2}\right)\,dx}{2a}-\frac{f\int \text{lo}\ldots}{\ldots}\\
&= -\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}-\frac{f\text{Subst}\left(\int \frac{\log(1\ldots}{x\ldots}\right.}{a\ldots}\\
&= -\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}-\frac{f\text{Li}_2\left(-e^{c+dx}\right)}{ad^2}\ldots
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 345 vs. 2(126) = 252.
time = 0.79, size = 345, normalized size = 2.74

$$\frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(f(c+dx)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)-2f\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)+if\log(\cosh(c+dx))\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)+de\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)-cf\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)+f\left((c+dx)\left(\log\left(1-e^{-c-dx}\right)-\log\left(1+e^{-c-dx}\right)\right)+\text{PolyLog}(2,-e^{-c-dx})-\text{PolyLog}(2,e^{-c-dx})\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)-2id(e+fx)\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{d^2(a+ia\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(f*(c + d*x)*(Cosh[(c + d*x)/2]
+ I*Sinh[(c + d*x)/2]) - 2*f*ArcTan[Tanh[(c + d*x)/2]]*(Cosh[(c + d*x)/2] +
 I*Sinh[(c + d*x)/2]) + I*f*Log[Cosh[c + d*x]]*(Cosh[(c + d*x)/2] + I*Sinh[
(c + d*x)/2]) + d*e*Log[Tanh[(c + d*x)/2]]*(Cosh[(c + d*x)/2] + I*Sinh[(c +
 d*x)/2]) - c*f*Log[Tanh[(c + d*x)/2]]*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x
)/2]) + f*((c + d*x)*(Log[1 - E^(-c - d*x)] - Log[1 + E^(-c - d*x)]) + Poly
Log[2, -E^(-c - d*x)] - PolyLog[2, E^(-c - d*x)])*(Cosh[(c + d*x)/2] + I*Si
nh[(c + d*x)/2]) - (2*I)*d*(e + f*x)*Sinh[(c + d*x)/2]))/(d^2*(a + I*a*Sinh
[c + d*x]))
```

Maple [A]
time = 3.03, size = 211, normalized size = 1.67

method	result
risch	$\frac{2fx+2e}{da\left(\mathrm{e}^{dx+c}-i\right)}-\frac{2if\ln\left(\mathrm{e}^{dx+c}\right)}{a\,d^{2}}+\frac{f\,\mathrm{polylog}\left(2,\mathrm{e}^{dx+c}\right)}{a\,d^{2}}-\frac{f\,\mathrm{polylog}\left(2,-\mathrm{e}^{dx+c}\right)}{a\,d^{2}}-\frac{e\ln\left(\mathrm{e}^{dx+c}+1\right)}{ad}-\frac{\ln\left(\mathrm{e}^{dx+c}+1\right)fx}{ad}+\frac{\ln(1-\mathrm{e}^{d}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(f*x+e)/d/a/(exp(d*x+c)-I)-2*I/a/d^2*f*ln(exp(d*x+c))+f*polylog(2,exp(d*x
+c))/a/d^2-f*polylog(2,-exp(d*x+c))/a/d^2-1/a/d*e*ln(exp(d*x+c)+1)-1/a/d*ln
(exp(d*x+c)+1)*f*x+1/a/d*ln(1-exp(d*x+c))*f*x+1/a/d^2*ln(1-exp(d*x+c))*c*f+
2*I*f/a/d^2*ln(exp(d*x+c)-I)-1/a/d^2*f*c*ln(exp(d*x+c)-1)+1/a/d*e*ln(exp(d*
x+c)-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*f*(x*e^(d*x + c)/(I*a*d*e^(d*x + c) + a*d) + I*log((e^(d*x + c) - I)*e^(-
c))/(a*d^2) + integrate(1/2*x/(a*e^(d*x + c) + a), x) + integrate(1/2*x/(a*
e^(d*x + c) - a), x)) - (log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1
)/(a*d) - 2/((a*e^(-d*x - c) + I*a)*d))*e
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(106) = 212$.
time = 0.40, size = 217, normalized size = 1.72

$$\frac{-2i\,dfxe^{(dx+c)}-\left(fe^{(dx+c)}-i\,f\right)\mathrm{Li}_2\left(-e^{(dx+c)}\right)+\left(fe^{(dx+c)}-i\,f\right)\mathrm{Li}_2\left(e^{(dx+c)}\right)+2\,de+\left(i\,dfx+i\,de-(dfx+de)e^{(dx+c)}\right)\log\left(e^{(dx+c)}+1\right)-2\left(-i\,fe^{(dx+c)}-f\right)\log\left(e^{(dx+c)}-i\right)+\left(i\,cf-i\,de-(cf-de)e^{(dx+c)}\right)\log\left(e^{(dx+c)}-1\right)+\left(-i\,dfx-i\,cf+(dfx+cf)e^{(dx+c)}\right)\log\left(-e^{(dx+c)}+1\right)}{ad^2e^{(dx+c)}-i\,ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (-2*I*d*f*x*e^(d*x + c) - (f*e^(d*x + c) - I*f)*dilog(-e^(d*x + c)) + (f*e^
(d*x + c) - I*f)*dilog(e^(d*x + c)) + 2*d*e + (I*d*f*x + I*d*e - (d*f*x + d
*e)*e^(d*x + c))*log(e^(d*x + c) + 1) - 2*(-I*f*e^(d*x + c) - f)*log(e^(d*x
 + c) - I) + (I*c*f - I*d*e - (c*f - d*e)*e^(d*x + c))*log(e^(d*x + c) - 1)
 + (-I*d*f*x - I*c*f + (d*f*x + c*f)*e^(d*x + c))*log(-e^(d*x + c) + 1))/(a
*d^2*e^(d*x + c) - I*a*d^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{e\operatorname{csch}(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{fx\operatorname{csch}(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*(Integral(e*csch(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f*x*csch(c
+ d*x)/(sinh(c + d*x) - I), x))/a
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*csch(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e+f\,x}{\sinh\left(c+d\,x\right)\,\left(a+a\,\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(sinh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
```

```
[Out] int((e + f*x)/(sinh(c + d*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.208 $\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=41

$$-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{\cosh(c+dx)}{d(a+ia\sinh(c+dx))}$$

```
[Out] -arctanh(cosh(d*x+c))/a/d+cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2826, 3855, 2727}

$$-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{\cosh(c+dx)}{d(a+ia\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
[Out] -(ArcTanh[Cosh[c + d*x]]/(a*d)) + Cosh[c + d*x]/(d*(a + I*a*Sinh[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\left(i\int \frac{1}{a+ia\sinh(c+dx)}\,dx\right)+\frac{\int \operatorname{csch}(c+dx)\,dx}{a}\\ &= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{\cosh(c+dx)}{d(a+ia\sinh(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 52, normalized size = 1.27

$$-\frac{\operatorname{sech}(c+dx)\left(-1+\tanh^{-1}\left(\sqrt{\cosh^2(c+dx)}\right)\sqrt{\cosh^2(c+dx)}+i\sinh(c+dx)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] -((Sech[c + d*x]*(-1 + ArcTanh[Sqrt[Cosh[c + d*x]^2]]*Sqrt[Cosh[c + d*x]^2]
 + I*Sinh[c + d*x]))/(a*d))
```

Maple [A]
time = 1.01, size = 36, normalized size = 0.88

method	result	size
derivativedivides	$\frac{-\frac{2i}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	36
default	$\frac{-\frac{2i}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	36
risch	$\frac{2}{da\left(e^{dx+c}-i\right)}+\frac{\ln\left(e^{dx+c}-1\right)}{da}-\frac{\ln\left(e^{dx+c}+1\right)}{da}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-2*I/(-I+tanh(1/2*d*x+1/2*c))+ln(tanh(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.26, size = 62, normalized size = 1.51

$$-\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}+\frac{\log\left(e^{(-dx-c)}-1\right)}{ad}+\frac{2}{(ae^{(-dx-c)}+i\,a)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -log(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d) + 2/((a*e^(-d*x
- c) + I*a)*d)
```

Fricas [A]
time = 0.42, size = 57, normalized size = 1.39

$$-\frac{\left(e^{(dx+c)}-i\right)\log\left(e^{(dx+c)}+1\right)-\left(e^{(dx+c)}-i\right)\log\left(e^{(dx+c)}-1\right)-2}{ade^{(dx+c)}-i\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((e^(d*x + c) - I)*log(e^(d*x + c) + 1) - (e^(d*x + c) - I)*log(e^(d*x + c
) - 1) - 2)/(a*d*e^(d*x + c) - I*a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\operatorname{csch}(c+dx)}{\sinh(c+dx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*Integral(csch(c + d*x)/(sinh(c + d*x) - I), x)/a
```

Giac [A]
time = 0.43, size = 48, normalized size = 1.17

$$-\frac{\frac{\log\left(e^{(dx+c)}+1\right)}{a}-\frac{\log\left(e^{(dx+c)}-1\right)}{a}-\frac{2}{a\left(e^{(dx+c)}-i\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -(log(e^(d*x + c) + 1)/a - log(e^(d*x + c) - 1)/a - 2/(a*(e^(d*x + c) - I))
)/d
```

Mupad [B]
time = 0.88, size = 56, normalized size = 1.37

$$-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-a^2\,d^2}}{a\,d}\right)}{\sqrt{-a^2\,d^2}}+\frac{2}{a\,d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
```

```
[Out] 2/(a*d*(exp(c + d*x) - 1i)) - (2*atan((exp(d*x)*exp(c)*(-a^2*d^2)^(1/2))/(a
*d)))/(-a^2*d^2)^(1/2)
```

3.209 $\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=32

$$\operatorname{Int}\left(\frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

[Out] Unintegrable(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

[In] Int[Csch[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]

[Out] Defer[Int][Csch[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 32.36, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

[In] Integrate[Csch[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]

[Out] Integrate[Csch[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(dx+c)}{(fx+e)\left(a+ia\sinh\left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*f*integrate(1/(-I*a*d*f^2*x^2 - 2*I*a*d*f*x*e - I*a*d*e^2 + (a*d*f^2*x^2*
e^c + 2*a*d*f*x*e^(c + 1) + a*d*e^(c + 2))*e^(d*x)), x) + 2/(-I*a*d*f*x - I
*a*d*e + (a*d*f*x*e^c + a*d*e^(c + 1))*e^(d*x)) + 2*integrate(1/2/(a*f*x +
a*e + (a*f*x*e^c + a*e^(c + 1))*e^(d*x)), x) + 2*integrate(-1/2/(a*f*x + a*
e - (a*f*x*e^c + a*e^(c + 1))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))*integral(2*((d*f*x
+ d*e + f)*e^(2*d*x + 2*c) - (I*d*f*x + I*d*e)*e^(d*x + c) - f)/(I*a*d*f^2*
x^2 + 2*I*a*d*f*x*e + I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2)*e^(
3*d*x + 3*c) + (-I*a*d*f^2*x^2 - 2*I*a*d*f*x*e - I*a*d*e^2)*e^(2*d*x + 2*c)
 - (a*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2)*e^(d*x + c)), x) + 2)/(-I*a*d*f*x
- I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i \int \frac{\operatorname{csch}{\left(c + d x \right)}}{e \sinh{\left(c + d x \right)} - i e + f x \sinh{\left(c + d x \right)} - i f x}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*Integral(csch(c + d*x)/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) - I*f*
x), x)/a
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate(csch(d*x + c)/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sinh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+a\,\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
[Out] int(1/(sinh(c + d*x)*(e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.210 $\int \frac{\text{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{\text{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csch[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
[Out] Defer[Int][Csch[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\text{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\text{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 40.58, size = 0, normalized size = 0.00

$$\int \frac{\text{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csch[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
[Out] Integrate[Csch[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\text{csch}(dx+c)}{(fx+e)^2\left(a+ia\sinh(dx+c)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 4*f*integrate(1/(-I*a*d*f^3*x^3 - 3*I*a*d*f^2*x^2*e - 3*I*a*d*f*x*e^2 - I*a
*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*d*f^2*x^2*e^(c + 1) + 3*a*d*f*x*e^(c + 2) +
 a*d*e^(c + 3))*e^(d*x)), x) + 2/(-I*a*d*f^2*x^2 - 2*I*a*d*f*x*e - I*a*d*e^
2 + (a*d*f^2*x^2*e^c + 2*a*d*f*x*e^(c + 1) + a*d*e^(c + 2))*e^(d*x)) + 2*in
tegrate(1/2/(a*f^2*x^2 + 2*a*f*x*e + a*e^2 + (a*f^2*x^2*e^c + 2*a*f*x*e^(c
+ 1) + a*e^(c + 2))*e^(d*x)), x) + 2*integrate(-1/2/(a*f^2*x^2 + 2*a*f*x*e
+ a*e^2 - (a*f^2*x^2*e^c + 2*a*f*x*e^(c + 1) + a*e^(c + 2))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((-I*a*d*f^2*x^2 - 2*I*a*d*f*x*e - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*f*x*e +
 a*d*e^2)*e^(d*x + c))*integral(2*((d*f*x + d*e + 2*f)*e^(2*d*x + 2*c) - (I
*d*f*x + I*d*e)*e^(d*x + c) - 2*f)/(I*a*d*f^3*x^3 + 3*I*a*d*f^2*x^2*e + 3*I
*a*d*f*x*e^2 + I*a*d*e^3 + (a*d*f^3*x^3 + 3*a*d*f^2*x^2*e + 3*a*d*f*x*e^2 +
 a*d*e^3)*e^(3*d*x + 3*c) + (-I*a*d*f^3*x^3 - 3*I*a*d*f^2*x^2*e - 3*I*a*d*f
*x*e^2 - I*a*d*e^3)*e^(2*d*x + 2*c) - (a*d*f^3*x^3 + 3*a*d*f^2*x^2*e + 3*a*
d*f*x*e^2 + a*d*e^3)*e^(d*x + c)), x) + 2)/(-I*a*d*f^2*x^2 - 2*I*a*d*f*x*e
- I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2)*e^(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sinh\left(c+d\,x\right)\,{\left(e+f\,x\right)}^2\,\left(a+a\,\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
[Out] int(1/(sinh(c + d*x)*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.211 $\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=419

$$-\frac{2(e+fx)^3}{ad}+\frac{2i(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^3\coth(c+dx)}{ad}+\frac{6f(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}+\frac{3f(e+$$

```
[Out] -2*(f*x+e)^3/a/d+3*I*f*(f*x+e)^2*polylog(2,-exp(d*x+c))/a/d^2-(f*x+e)^3*cot
h(d*x+c)/a/d+6*f*(f*x+e)^2*ln(1+I*exp(d*x+c))/a/d^2+3*f*(f*x+e)^2*ln(1-exp(
2*d*x+2*c))/a/d^2-3*I*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a/d^2+12*f^2*(f*x+e
)*polylog(2,-I*exp(d*x+c))/a/d^3-6*I*f^2*(f*x+e)*polylog(3,-exp(d*x+c))/a/d
^3+3*f^2*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d^3+6*I*f^3*polylog(4,-exp(d*x
+c))/a/d^4-12*f^3*polylog(3,-I*exp(d*x+c))/a/d^4+6*I*f^2*(f*x+e)*polylog(3,
exp(d*x+c))/a/d^3-3/2*f^3*polylog(3,exp(2*d*x+2*c))/a/d^4-6*I*f^3*polylog(4
,exp(d*x+c))/a/d^4+2*I*(f*x+e)^3*arctanh(exp(d*x+c))/a/d-(f*x+e)^3*tanh(1/2
*c+1/4*I*Pi+1/2*d*x)/a/d
```

Rubi [A]
time = 0.60, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {5694, 4269, 3797, 2221, 2611, 2320, 6724, 4267, 6744, 3399}

$$-\frac{12f^3\text{Li}_3\left(-ie^{c+dx}\right)}{ad^4}-\frac{3f^3\text{Li}_3\left(e^{2(c+dx)}\right)}{2ad^4}+\frac{6if^3\text{Li}_4\left(-e^{c+dx}\right)}{ad^4}-\frac{6if^3\text{Li}_4\left(e^{c+dx}\right)}{ad^4}+\frac{12f^2(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{ad^3}+\frac{3f^2(e+fx)\text{Li}_2\left(e^{2(c+dx)}\right)}{ad^3}-\frac{6if^2(e+fx)\text{Li}_3\left(-e^{c+dx}\right)}{ad^3}+\frac{6if^2(e+fx)\text{Li}_3\left(e^{c+dx}\right)}{ad^3}+\frac{3if(e+fx)^2\text{Li}_2\left(-e^{c+dx}\right)}{ad^2}-\frac{3if(e+fx)^2\text{Li}_2\left(e^{c+dx}\right)}{ad^2}+\frac{6f(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}+\frac{3f(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{ad^2}+\frac{2i(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}-\frac{(e+fx)^3\coth(c+dx)}{ad}-\frac{2(e+fx)^3}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (-2*(e + f*x)^3)/(a*d) + ((2*I)*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) - (
(e + f*x)^3*Coth[c + d*x])/(a*d) + (6*f*(e + f*x)^2*Log[1 + I*E^(c + d*x)])
/(a*d^2) + (3*f*(e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a*d^2) + ((3*I)*f*(e
 + f*x)^2*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (12*f^2*(e + f*x)*PolyLog[2,
(-I)*E^(c + d*x)])/(a*d^3) - ((3*I)*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)])/
(a*d^2) + (3*f^2*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a*d^3) - ((6*I)*f^
2*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) - (12*f^3*PolyLog[3, (-I)*E^(
c + d*x)])/(a*d^4) + ((6*I)*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a*d^3)
- (3*f^3*PolyLog[3, E^(2*(c + d*x))])/(2*a*d^4) + ((6*I)*f^3*PolyLog[4, -E^
(c + d*x)])/(a*d^4) - ((6*I)*f^3*PolyLog[4, E^(c + d*x)])/(a*d^4) - ((e + f
*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5694

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a + b
*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\left(i\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx\right)+\frac{\int (e+fx)^3\text{csch}^2(c+dx)\,dx}{a}$$

$$= -\frac{(e+fx)^3\coth(c+dx)}{ad}-\frac{i\int (e+fx)^3\text{csch}(c+dx)\,dx}{a}+\frac{(3f)\int (e+fx)^2 \cdots}{a\cdots}$$

$$= -\frac{(e+fx)^3}{ad}+\frac{2i(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^3\coth(c+dx)}{ad}-\frac{\int (e\cdots}{\cdots}$$

$$= -\frac{(e+fx)^3}{ad}+\frac{2i(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^3\coth(c+dx)}{ad}+\frac{3f(e\cdots}{\cdots}$$

$$= -\frac{2(e+fx)^3}{ad}+\frac{2i(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^3\coth(c+dx)}{ad}+\frac{3f(\cdots}{\cdots}$$

$$= -\frac{2(e+fx)^3}{ad}+\frac{2i(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^3\coth(c+dx)}{ad}+\frac{6f(\cdots}{\cdots}$$

$$= -\frac{2(e+fx)^3}{ad}+\frac{2i(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^3\coth(c+dx)}{ad}+\frac{6f(\cdots}{\cdots}$$

$$= -\frac{2(e+fx)^3}{ad}+\frac{2i(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^3\coth(c+dx)}{ad}+\frac{6f(\cdots}{\cdots}$$

$$= -\frac{2(e+fx)^3}{ad}+\frac{2i(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^3\coth(c+dx)}{ad}+\frac{6f(\cdots}{\cdots}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1005 vs. $2(419) = 838$.
time = 13.54, size = 1005, normalized size = 2.40

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-2*I)*f*(d^2*((-I)*d*E^c*x*(3*e^2 + 3*e*f*x + f^2*x^2) + 3*(1 + I*E^c)*(e
 + f*x)^2*Log[1 + I*E^(c + d*x)]) + 6*d*(1 + I*E^c)*f*(e + f*x)*PolyLog[2,
(-I)*E^(c + d*x)] - (6*I)*(-I + E^c)*f^2*PolyLog[3, (-I)*E^(c + d*x)]))/(a*
d^4*(-I + E^c)) - (12*d^3*e^2*E^(2*c)*f*x - 12*d^3*e^2*(-1 + E^(2*c))*f*x +
 12*d^3*e*f^2*x^2 + 4*d^3*f^3*x^3 - (4*I)*d^3*e^3*(-1 + E^(2*c))*ArcTanh[E^
(c + d*x)] + 6*d^2*e^2*(-1 + E^(2*c))*f*(2*d*x - Log[1 - E^(2*(c + d*x))])
+ (6*I)*d^2*e^2*(-1 + E^(2*c))*f*(d*x*(Log[1 - E^(c + d*x)] - Log[1 + E^(c
+ d*x)]) - PolyLog[2, -E^(c + d*x)] + PolyLog[2, E^(c + d*x)]) + 6*d*e*(-1
```

```
+ E^(2*c))*f^2*(2*d*x*(d*x - Log[1 - E^(2*(c + d*x))]) - PolyLog[2, E^(2*(c
 + d*x))]) + (6*I)*d*e*(-1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - E^(c + d*x)] - d
^2*x^2*Log[1 + E^(c + d*x)] - 2*d*x*PolyLog[2, -E^(c + d*x)] + 2*d*x*PolyLo
g[2, E^(c + d*x)] + 2*PolyLog[3, -E^(c + d*x)] - 2*PolyLog[3, E^(c + d*x)])
 + (-1 + E^(2*c))*f^3*(2*d^2*x^2*(2*d*x - 3*Log[1 - E^(2*(c + d*x))]) - 6*d
*x*PolyLog[2, E^(2*(c + d*x))] + 3*PolyLog[3, E^(2*(c + d*x))]) + (2*I)*(-1
 + E^(2*c))*f^3*(d^3*x^3*Log[1 - E^(c + d*x)] - d^3*x^3*Log[1 + E^(c + d*x)
] - 3*d^2*x^2*PolyLog[2, -E^(c + d*x)] + 3*d^2*x^2*PolyLog[2, E^(c + d*x)]
+ 6*d*x*PolyLog[3, -E^(c + d*x)] - 6*d*x*PolyLog[3, E^(c + d*x)] - 6*PolyLo
g[4, -E^(c + d*x)] + 6*PolyLog[4, E^(c + d*x)]))/(2*a*d^4*(-1 + E^(2*c))) +
 (Sech[c/2]*Sech[c/2 + (d*x)/2]*(-(e^3*Sinh[(d*x)/2]) - 3*e^2*f*x*Sinh[(d*x
)/2] - 3*e*f^2*x^2*Sinh[(d*x)/2] - f^3*x^3*Sinh[(d*x)/2]))/(2*a*d) + (Csch[
c/2]*Csch[c/2 + (d*x)/2]*(e^3*Sinh[(d*x)/2] + 3*e^2*f*x*Sinh[(d*x)/2] + 3*e
*f^2*x^2*Sinh[(d*x)/2] + f^3*x^3*Sinh[(d*x)/2]))/(2*a*d) - (2*(e^3*Sinh[(d*
x)/2] + 3*e^2*f*x*Sinh[(d*x)/2] + 3*e*f^2*x^2*Sinh[(d*x)/2] + f^3*x^3*Sinh[
(d*x)/2]))/(a*d*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2
 + (d*x)/2]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1534 vs. 2(390) = 780.
time = 3.23, size = 1535, normalized size = 3.66

method	result	size
risch	Expression too large to display	1535

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -6*f^3*polylog(3,-exp(d*x+c))/a/d^4-6*f^3*polylog(3,exp(d*x+c))/a/d^4-3*I/d
/a*e*f^2*ln(1-exp(d*x+c))*x^2-6*I/d^2/a*e*f^2*polylog(2,exp(d*x+c))*x+3*I/d
/a*ln(exp(d*x+c)+1)*e^2*f*x-3*I/d/a*ln(1-exp(d*x+c))*e^2*f*x+3*I/d^3/a*e*f^
2*c^2*ln(1-exp(d*x+c))-3*I/d^3/a*e*f^2*c^2*ln(exp(d*x+c)-1)-6*I*f^3*polylog
(4,exp(d*x+c))/a/d^4+3*I/d/a*e*f^2*ln(exp(d*x+c)+1)*x^2-3*I/d^2/a*ln(1-exp(
d*x+c))*c*e^2*f-12*f^3*polylog(3,-I*exp(d*x+c))/a/d^4+24/d^3/a*e*f^2*c*ln(e
xp(d*x+c))-6/d^3/a*e*f^2*c*ln(exp(d*x+c)-1)-12/d^3/a*e*f^2*c*ln(exp(d*x+c)-
I)+12/d^2/a*e*f^2*ln(1+I*exp(d*x+c))*x+6/d^3/a*e*f^2*ln(1-exp(d*x+c))*c+8/a
/d^4*f^3*c^3+I/d/a*f^3*ln(exp(d*x+c)+1)*x^3+I/d^4/a*f^3*c^3*ln(exp(d*x+c)-1
)+6*I/d^3/a*e*f^2*polylog(3,exp(d*x+c))-6*I/d^3/a*f^3*polylog(3,-exp(d*x+c)
)*x-I/d/a*f^3*ln(1-exp(d*x+c))*x^3-I/d^4/a*f^3*ln(1-exp(d*x+c))*c^3-3*I/d^2
/a*f^3*polylog(2,exp(d*x+c))*x^2+6*I/d^3/a*f^3*polylog(3,exp(d*x+c))*x+6*I*
f^3*polylog(4,-exp(d*x+c))/a/d^4+3*I/d^2/a*e^2*f*c*ln(exp(d*x+c)-1)+6*I/d^2
/a*e*f^2*polylog(2,-exp(d*x+c))*x-24/d^2/a*e*f^2*c*x+6/d^2/a*e*f^2*ln(1-exp
(d*x+c))*x+6/d^2/a*e*f^2*ln(exp(d*x+c)+1)*x-2*I*(f^3*x^3*exp(2*d*x+2*c)+3*e
*f^2*x^2*exp(2*d*x+2*c)+3*e^2*f*x*exp(2*d*x+2*c)-2*f^3*x^3-I*exp(d*x+c)*f^3
*x^3+e^3*exp(2*d*x+2*c)-6*e*f^2*x^2-3*I*exp(d*x+c)*e*f^2*x^2-6*e^2*f*x-3*I*
```

```
exp(d*x+c)*e^2*f*x-2*e^3-I*exp(d*x+c)*e^3)/(exp(2*d*x+2*c)-1)/(exp(d*x+c)-I
)/a/d+12/d^3/a*e*f^2*ln(1+I*exp(d*x+c))*c+3*I/d^2/a*e^2*f*polylog(2,-exp(d*
x+c))-3*I/d^2/a*e^2*f*polylog(2,exp(d*x+c))+3*I/d^2/a*f^3*polylog(2,-exp(d*
x+c))*x^2-6*I/d^3/a*e*f^2*polylog(3,-exp(d*x+c))-I/d/a*e^3*ln(exp(d*x+c)-1)
-12/d^2/a*e^2*f*ln(exp(d*x+c))+3/d^2/a*e^2*f*ln(exp(d*x+c)+1)+3/d^2/a*e^2*f
*ln(exp(d*x+c)-1)+12/d^3/a*f^3*c^2*x+6/d^2/a*e^2*f*ln(exp(d*x+c)-I)+3/d^4/a
*f^3*c^2*ln(exp(d*x+c)-1)+6/d^3/a*e*f^2*polylog(2,-exp(d*x+c))+6/d^3/a*e*f^
2*polylog(2,exp(d*x+c))+12/d^3/a*e*f^2*polylog(2,-I*exp(d*x+c))+6/d^3/a*f^3
*polylog(2,-exp(d*x+c))*x+6/d^3/a*f^3*polylog(2,exp(d*x+c))*x+3/d^2/a*f^3*l
n(exp(d*x+c)+1)*x^2+3/d^2/a*f^3*ln(1-exp(d*x+c))*x^2-3/d^4/a*f^3*ln(1-exp(d
*x+c))*c^2-6/d^4/a*f^3*c^2*ln(1+I*exp(d*x+c))+6/d^2/a*f^3*ln(1+I*exp(d*x+c)
)*x^2+12/d^3/a*f^3*polylog(2,-I*exp(d*x+c))*x+6/d^4/a*f^3*c^2*ln(exp(d*x+c)
-I)-12/d/a*e*f^2*x^2-12/d^3/a*e*f^2*c^2+I/d/a*e^3*ln(exp(d*x+c)+1)-12/a/d^4
*f^3*c^2*ln(exp(d*x+c))-4/d/a*f^3*x^3
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 940 vs. $2(387) = 774$.
time = 0.47, size = 940, normalized size = 2.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -(2*(e^(-d*x - c) - I*e^(-2*d*x - 2*c) + 2*I)/((a*e^(-d*x - c) - I*a*e^(-2*
d*x - 2*c) - a*e^(-3*d*x - 3*c) + I*a)*d) - I*log(e^(-d*x - c) + 1)/(a*d) +
 I*log(e^(-d*x - c) - 1)/(a*d))*e^3 - 12*f*x*e^2/(a*d) - 2*(-2*I*f^3*x^3 -
6*I*f^2*x^2*e - 6*I*f*x*e^2 - (-I*f^3*x^3*e^(2*c) - 3*I*f^2*x^2*e^(2*c + 1)
 - 3*I*f*x*e^(2*c + 2))*e^(2*d*x) + (f^3*x^3*e^c + 3*f^2*x^2*e^(c + 1) + 3*
f*x*e^(c + 2))*e^(d*x))/(a*d*e^(3*d*x + 3*c) - I*a*d*e^(2*d*x + 2*c) - a*d*
e^(d*x + c) + I*a*d) + 12*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c
)))*f^2*e/(a*d^3) + 3*f*e^2*log(e^(d*x + c) + 1)/(a*d^2) + 6*f*e^2*log(e^(d
*x + c) - I)/(a*d^2) + 3*f*e^2*log(e^(d*x + c) - 1)/(a*d^2) + I*(d^3*x^3*lo
g(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d
*x + c)) + 6*polylog(4, -e^(d*x + c)))*f^3/(a*d^4) - I*(d^3*x^3*log(-e^(d*x
 + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) +
 6*polylog(4, e^(d*x + c)))*f^3/(a*d^4) + 6*(d^2*x^2*log(I*e^(d*x + c) + 1)
 + 2*d*x*dilog(-I*e^(d*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a*d^4)
- 3*(-I*d*f*e^2 - 2*f^2*e)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))
/(a*d^3) + 3*(-I*d*f*e^2 + 2*f^2*e)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d
*x + c)))/(a*d^3) + 3*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x +
 c)) - 2*polylog(3, e^(d*x + c)))*(-I*d*f^2*e + f^3)/(a*d^4) - 3*(d^2*x^2*l
og(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)
))*(-I*d*f^2*e - f^3)/(a*d^4) + 1/4*(I*d^4*f^3*x^4 - 4*(-I*d*f^2*e + f^3)*d
```

```
^3*x^3 - 6*(-I*d^2*f*e^2 + 2*d*f^2*e)*d^2*x^2)/(a*d^4) - 1/4*(I*d^4*f^3*x^4
 - 4*(-I*d*f^2*e - f^3)*d^3*x^3 - 6*(-I*d^2*f*e^2 - 2*d*f^2*e)*d^2*x^2)/(a*
d^4) - 2*(d^3*f^3*x^3 + 3*d^3*f^2*x^2*e)/(a*d^4)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2568 vs. $2(387) = 774$.
time = 0.45, size = 2568, normalized size = 6.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] (-4*I*c^3*f^3 + 12*I*c^2*d*f^2*e - 12*I*c*d^2*f*e^2 + 4*I*d^3*e^3 - 12*(-I*
d*f^3*x - I*d*f^2*e - (d*f^3*x + d*f^2*e)*e^(3*d*x + 3*c) + (I*d*f^3*x + I*
d*f^2*e)*e^(2*d*x + 2*c) + (d*f^3*x + d*f^2*e)*e^(d*x + c))*dilog(-I*e^(d*x
 + c)) - 3*(d^2*f^3*x^2 - 2*I*d*f^3*x + d^2*f*e^2 + 2*(d^2*f^2*x - I*d*f^2)
*e + (-I*d^2*f^3*x^2 - 2*d*f^3*x - I*d^2*f*e^2 + 2*(-I*d^2*f^2*x - d*f^2)*e
)*e^(3*d*x + 3*c) - (d^2*f^3*x^2 - 2*I*d*f^3*x + d^2*f*e^2 + 2*(d^2*f^2*x -
 I*d*f^2)*e)*e^(2*d*x + 2*c) + (I*d^2*f^3*x^2 + 2*d*f^3*x + I*d^2*f*e^2 + 2
*(I*d^2*f^2*x + d*f^2)*e)*e^(d*x + c))*dilog(-e^(d*x + c)) + 3*(d^2*f^3*x^2
 + 2*I*d*f^3*x + d^2*f*e^2 + 2*(d^2*f^2*x + I*d*f^2)*e - (I*d^2*f^3*x^2 - 2
*d*f^3*x + I*d^2*f*e^2 + 2*(I*d^2*f^2*x - d*f^2)*e)*e^(3*d*x + 3*c) - (d^2*
f^3*x^2 + 2*I*d*f^3*x + d^2*f*e^2 + 2*(d^2*f^2*x + I*d*f^2)*e)*e^(2*d*x + 2
*c) - (-I*d^2*f^3*x^2 + 2*d*f^3*x - I*d^2*f*e^2 + 2*(-I*d^2*f^2*x + d*f^2)*
e)*e^(d*x + c))*dilog(e^(d*x + c)) - 4*(d^3*f^3*x^3 + c^3*f^3 + 3*(d^3*f*x
+ c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - c^2*d*f^2)*e)*e^(3*d*x + 3*c) - 2*(-I*d^3
*f^3*x^3 - 2*I*c^3*f^3 + I*d^3*e^3 + 3*(-I*d^3*f*x - 2*I*c*d^2*f)*e^2 + 3*(
-I*d^3*f^2*x^2 + 2*I*c^2*d*f^2)*e)*e^(2*d*x + 2*c) + 2*(d^3*f^3*x^3 + 2*c^3
*f^3 - d^3*e^3 + 3*(d^3*f*x + 2*c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - 2*c^2*d*f^2
)*e)*e^(d*x + c) - (d^3*f^3*x^3 - 3*I*d^2*f^3*x^2 + d^3*e^3 + 3*(d^3*f*x -
I*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - 2*I*d^2*f^2*x)*e - (I*d^3*f^3*x^3 + 3*d^2*f
^3*x^2 + I*d^3*e^3 - 3*(-I*d^3*f*x - d^2*f)*e^2 - 3*(-I*d^3*f^2*x^2 - 2*d^2
*f^2*x)*e)*e^(3*d*x + 3*c) - (d^3*f^3*x^3 - 3*I*d^2*f^3*x^2 + d^3*e^3 + 3*(
d^3*f*x - I*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - 2*I*d^2*f^2*x)*e)*e^(2*d*x + 2*c)
 - (-I*d^3*f^3*x^3 - 3*d^2*f^3*x^2 - I*d^3*e^3 - 3*(I*d^3*f*x + d^2*f)*e^2
- 3*(I*d^3*f^2*x^2 + 2*d^2*f^2*x)*e)*e^(d*x + c))*log(e^(d*x + c) + 1) - 6*
(-I*c^2*f^3 + 2*I*c*d*f^2*e - I*d^2*f*e^2 - (c^2*f^3 - 2*c*d*f^2*e + d^2*f*
e^2)*e^(3*d*x + 3*c) + (I*c^2*f^3 - 2*I*c*d*f^2*e + I*d^2*f*e^2)*e^(2*d*x +
 2*c) + (c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)*e^(d*x + c))*log(e^(d*x + c) -
I) - (3*(c - I)*d^2*f*e^2 - 3*(c^2 - 2*I*c)*d*f^2*e + (c^3 - 3*I*c^2)*f^3 -
 d^3*e^3 + (3*(-I*c - 1)*d^2*f*e^2 + 3*(I*c^2 + 2*c)*d*f^2*e - (I*c^3 + 3*c
^2)*f^3 + I*d^3*e^3)*e^(3*d*x + 3*c) - (3*(c - I)*d^2*f*e^2 - 3*(c^2 - 2*I*
c)*d*f^2*e + (c^3 - 3*I*c^2)*f^3 - d^3*e^3)*e^(2*d*x + 2*c) + (3*(I*c + 1)*
```

```
d^2*f*e^2 + 3*(-I*c^2 - 2*c)*d*f^2*e - (-I*c^3 - 3*c^2)*f^3 - I*d^3*e^3)*e^
(d*x + c))*log(e^(d*x + c) - 1) - 6*(-I*d^2*f^3*x^2 + I*c^2*f^3 + 2*(-I*d^2
*f^2*x - I*c*d*f^2)*e - (d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x + c*d*f^2)*e)
*e^(3*d*x + 3*c) + (I*d^2*f^3*x^2 - I*c^2*f^3 + 2*(I*d^2*f^2*x + I*c*d*f^2)
*e)*e^(2*d*x + 2*c) + (d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x + c*d*f^2)*e)*e
^(d*x + c))*log(I*e^(d*x + c) + 1) + (d^3*f^3*x^3 + 3*I*d^2*f^3*x^2 + (c^3
- 3*I*c^2)*f^3 + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 + 2*I*d^2*f^2*x
 - (c^2 - 2*I*c)*d*f^2)*e + (-I*d^3*f^3*x^3 + 3*d^2*f^3*x^2 + (-I*c^3 - 3*c
^2)*f^3 - 3*(I*d^3*f*x + I*c*d^2*f)*e^2 - 3*(I*d^3*f^2*x^2 - 2*d^2*f^2*x +
(-I*c^2 - 2*c)*d*f^2)*e)*e^(3*d*x + 3*c) - (d^3*f^3*x^3 + 3*I*d^2*f^3*x^2 +
 (c^3 - 3*I*c^2)*f^3 + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 + 2*I*d^2
*f^2*x - (c^2 - 2*I*c)*d*f^2)*e)*e^(2*d*x + 2*c) + (I*d^3*f^3*x^3 - 3*d^2*f
^3*x^2 + (I*c^3 + 3*c^2)*f^3 - 3*(-I*d^3*f*x - I*c*d^2*f)*e^2 - 3*(-I*d^3*f
^2*x^2 + 2*d^2*f^2*x + (I*c^2 + 2*c)*d*f^2)*e)*e^(d*x + c))*log(-e^(d*x + c
) + 1) - 6*(-I*f^3*e^(3*d*x + 3*c) - f^3*e^(2*d*x + 2*c) + I*f^3*e^(d*x + c
) + f^3)*polylog(4, -e^(d*x + c)) - 6*(I*f^3*e^(3*d*x + 3*c) + f^3*e^(2*d*x
 + 2*c) - I*f^3*e^(d*x + c) - f^3)*polylog(4, e^(d*x + c)) - 12*(f^3*e^(3*d
*x + 3*c) - I*f^3*e^(2*d*x + 2*c) - f^3*e^(d*x + c) + I*f^3)*polylog(3, -I*
e^(d*x + c)) + 6*(d*f^3*x + d*f^2*e - I*f^3 - (I*d*f^3*x + I*d*f^2*e + f^3)
*e^(3*d*x + 3*c) - (d*f^3*x + d*f^2*e - I*f^3)*e^(2*d*x + 2*c) - (-I*d*f^3*
x - I*d*f^2*e - f^3)*e^(d*x + c))*polylog(3, -e^(d*x + c)) - 6*(d*f^3*x + d
*f^2*e + I*f^3 + (-I*d*f^3*x - I*d*f^2*e + f^3)*e^(3*d*x + 3*c) - (d*f^3*x
+ d*f^2*e + I*f^3)*e^(2*d*x + 2*c) + (I*d*f^3*x + I*d*f^2*e - f^3)*e^(d*x +
 c))*polylog(3, e^(d*x + c)))/(a*d^4*e^(3*d*x + 3*c) - I*a*d^4*e^(2*d*x + 2
*c) - a*d^4*e^(d*x + c) + I*a*d^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^3*csch(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^3}{\sinh\left(c+d\,x\right)^2\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(sinh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
[Out] int((e + f*x)^3/(sinh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.212 $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=296

$$-\frac{2(e+fx)^2}{ad}+\frac{2i(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^2\coth(c+dx)}{ad}+\frac{4f(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}+\frac{2f(e+fx}{}$$

```
[Out] -2*(f*x+e)^2/a/d+2*I*(f*x+e)^2*arctanh(exp(d*x+c))/a/d-(f*x+e)^2*coth(d*x+c
)/a/d+4*f*(f*x+e)*ln(1+I*exp(d*x+c))/a/d^2+2*f*(f*x+e)*ln(1-exp(2*d*x+2*c))
/a/d^2+2*I*f*(f*x+e)*polylog(2,-exp(d*x+c))/a/d^2+4*f^2*polylog(2,-I*exp(d*
x+c))/a/d^3-2*I*f*(f*x+e)*polylog(2,exp(d*x+c))/a/d^2+f^2*polylog(2,exp(2*d
*x+2*c))/a/d^3-2*I*f^2*polylog(3,-exp(d*x+c))/a/d^3+2*I*f^2*polylog(3,exp(d
*x+c))/a/d^3-(f*x+e)^2*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a/d
```

Rubi [A]
time = 0.41, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {5694, 4269, 3797, 2221, 2317, 2438, 4267, 2611, 2320, 6724, 3399}

$$\frac{4f^2\text{Li}_2\left(-ie^{c+dx}\right)}{ad^3}+\frac{f^2\text{Li}_2\left(e^{2(c+dx)}\right)}{ad^3}-\frac{2if^2\text{Li}_3\left(-e^{c+dx}\right)}{ad^3}+\frac{2if^2\text{Li}_3\left(e^{c+dx}\right)}{ad^3}+\frac{2if(e+fx)\text{Li}_2\left(-e^{c+dx}\right)}{ad^2}-\frac{2if(e+fx)\text{Li}_2\left(e^{c+dx}\right)}{ad^2}+\frac{4f(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}+\frac{2f(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad^2}+\frac{2i(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}-\frac{(e+fx)^2\coth(c+dx)}{ad}-\frac{2(e+fx)^2}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (-2*(e + f*x)^2)/(a*d) + ((2*I)*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (
(e + f*x)^2*Coth[c + d*x])/(a*d) + (4*f*(e + f*x)*Log[1 + I*E^(c + d*x)])/(
a*d^2) + (2*f*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a*d^2) + ((2*I)*f*(e + f
*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (4*f^2*PolyLog[2, (-I)*E^(c + d*x)]
)/(a*d^3) - ((2*I)*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^2) + (f^2*Poly
Log[2, E^(2*(c + d*x))])/(a*d^3) - ((2*I)*f^2*PolyLog[3, -E^(c + d*x)])/(a*
d^3) + ((2*I)*f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) - ((e + f*x)^2*Tanh[c/2
+ (I/4)*Pi + (d*x)/2])/(a*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5694

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a + b
*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\left(i\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx\right)+\frac{\int (e+fx)^2\text{csch}^2(c+dx)\,dx}{a}$$

$$= -\frac{(e+fx)^2\coth(c+dx)}{ad}-\frac{i\int (e+fx)^2\text{csch}(c+dx)\,dx}{a}+\frac{(2f)\int (e+fx)\,\text{c}}{ad}$$

$$= -\frac{(e+fx)^2}{ad}+\frac{2i(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^2\coth(c+dx)}{ad}-\frac{\int (e-}{}$$

$$= -\frac{(e+fx)^2}{ad}+\frac{2i(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^2\coth(c+dx)}{ad}+\frac{2f(e}{}$$

$$= -\frac{2(e+fx)^2}{ad}+\frac{2i(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^2\coth(c+dx)}{ad}+\frac{2f(}{}$$

$$= -\frac{2(e+fx)^2}{ad}+\frac{2i(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^2\coth(c+dx)}{ad}+\frac{4f(}{}$$

$$= -\frac{2(e+fx)^2}{ad}+\frac{2i(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^2\coth(c+dx)}{ad}+\frac{4f(}{}$$

$$= -\frac{2(e+fx)^2}{ad}+\frac{2i(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^2\coth(c+dx)}{ad}+\frac{4f(}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 659 vs. 2(296) = 592.
time = 11.07, size = 659, normalized size = 2.23

$$\frac{2f\left(d\left(-\frac{de^cx(2e+fx)}{-i+e^c}+2(e+fx)\log\left(1+ie^{c+dx}\right)\right)+2f\text{PolyLog}\left(2,-ie^{c+dx}\right)\right)}{ad^3}+\frac{-4ee^{2c}fx+4e(-1+e^{2c})fx-2e^{2c}f^2x^2+2(-1+e^{2c})f^2x^2+2ie^2(-1+e^{2c})\tanh^{-1}\left(e^{c+dx}\right)-\frac{2e(-1+e^{2c})f(2dx-\log(1-e^{2(c+dx)}))}{d}+\frac{2ie(-1+e^{2c})f\left(dx(-\log(1-e^{c+dx})+\log(1+e^{c+dx}))+\text{PolyLog}(2,-e^{c+dx})-\text{PolyLog}(2,e^{c+dx})\right)}{d}-\frac{(-1+e^{2c})f^2\left(2dx(dx-\log(1-e^{2(c+dx)}))-\text{PolyLog}(2,e^{2(c+dx)})\right)}{d^2}+\frac{i(-1+e^{2c})f^2\left(-d^2x^2\log(1-e^{c+dx})+d^2x^2\log(1+e^{c+dx})+2dx\text{PolyLog}(2,-e^{c+dx})-2dx\text{PolyLog}(2,e^{c+dx})-2\text{PolyLog}(3,-e^{c+dx})+2\text{PolyLog}(3,e^{c+dx})\right)}{d^2}}{ad(-1+e^{2c})}+\frac{\text{sech}\left(\frac{c}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-e^2\sinh\left(\frac{dx}{2}\right)-2efx\sinh\left(\frac{dx}{2}\right)-f^2x^2\sinh\left(\frac{dx}{2}\right)\right)}{2ad}+\frac{\text{csch}\left(\frac{c}{2}\right)\text{csch}\left(\frac{c}{2}+\frac{dx}{2}\right)\left(e^2\sinh\left(\frac{dx}{2}\right)+2efx\sinh\left(\frac{dx}{2}\right)+f^2x^2\sinh\left(\frac{dx}{2}\right)\right)}{2ad}-\frac{2\left(e^2\sinh\left(\frac{dx}{2}\right)+2efx\sinh\left(\frac{dx}{2}\right)+f^2x^2\sinh\left(\frac{dx}{2}\right)\right)}{ad\left(\cosh\left(\frac{c}{2}\right)+i\sinh\left(\frac{c}{2}\right)\right)\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)+i\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (2*f*(d*(-((d*E^c*x*(2*e + f*x))/(-I + E^c)) + 2*(e + f*x)*Log[1 + I*E^(c +
 d*x)]) + 2*f*PolyLog[2, (-I)*E^(c + d*x)]))/(a*d^3) + (-4*e*E^(2*c)*f*x +
4*e*(-1 + E^(2*c))*f*x - 2*E^(2*c)*f^2*x^2 + 2*(-1 + E^(2*c))*f^2*x^2 + (2*
I)*e^2*(-1 + E^(2*c))*ArcTanh[E^(c + d*x)] - (2*e*(-1 + E^(2*c))*f*(2*d*x -
 Log[1 - E^(2*(c + d*x))]))/d + ((2*I)*e*(-1 + E^(2*c))*f*(d*x*(-Log[1 - E^
(c + d*x)] + Log[1 + E^(c + d*x)]) + PolyLog[2, -E^(c + d*x)] - PolyLog[2,
E^(c + d*x)]))/d - ((-1 + E^(2*c))*f^2*(2*d*x*(d*x - Log[1 - E^(2*(c + d*x)
)]) - PolyLog[2, E^(2*(c + d*x))]))/d^2 + (I*(-1 + E^(2*c))*f^2*(-(d^2*x^2*
Log[1 - E^(c + d*x)]) + d^2*x^2*Log[1 + E^(c + d*x)] + 2*d*x*PolyLog[2, -E^
(c + d*x)] - 2*d*x*PolyLog[2, E^(c + d*x)] - 2*PolyLog[3, -E^(c + d*x)] + 2
*PolyLog[3, E^(c + d*x)]))/d^2)/(a*d*(-1 + E^(2*c))) + (Sech[c/2]*Sech[c/2
+ (d*x)/2]*(-(e^2*Sinh[(d*x)/2]) - 2*e*f*x*Sinh[(d*x)/2] - f^2*x^2*Sinh[(d*
x)/2]))/(2*a*d) + (Csch[c/2]*Csch[c/2 + (d*x)/2]*(e^2*Sinh[(d*x)/2] + 2*e*f
*x*Sinh[(d*x)/2] + f^2*x^2*Sinh[(d*x)/2]))/(2*a*d) - (2*(e^2*Sinh[(d*x)/2]
+ 2*e*f*x*Sinh[(d*x)/2] + f^2*x^2*Sinh[(d*x)/2]))/(a*d*(Cosh[c/2] + I*Sinh[
c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2]))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 846 vs. 2(275) = 550.
time = 3.01, size = 847, normalized size = 2.86

method	result
risch	$\frac{2ef\ln(\mathrm{e}^{dx+c}-1)}{d^2a}-\frac{8f^2cx}{d^2a}+\frac{2f^2\,\mathrm{polylog}(2,-\mathrm{e}^{dx+c})}{a\,d^3}+\frac{2f^2\,\mathrm{polylog}(2,\mathrm{e}^{dx+c})}{a\,d^3}+\frac{4f^2\,\mathrm{polylog}(2,-i\mathrm{e}^{dx+c})}{a\,d^3}+\frac{2iefc\ln(\mathrm{e}^{dx+c}-}{d^2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] I/d/a*f^2*ln(exp(d*x+c)+1)*x^2+I/d^3/a*f^2*ln(1-exp(d*x+c))*c^2+4*f^2*polyl
og(2,-I*exp(d*x+c))/a/d^3+2*f^2*polylog(2,-exp(d*x+c))/a/d^3+2*f^2*polylog(
2,exp(d*x+c))/a/d^3+2/d^2/a*e*f*ln(exp(d*x+c)-1)-8/d^2/a*f^2*c*x-I/d/a*e^2*
ln(exp(d*x+c)-1)-I/d^3/a*f^2*c^2*ln(exp(d*x+c)-1)+2*I/d^2/a*f^2*polylog(2,-
exp(d*x+c))*x-I/d/a*f^2*ln(1-exp(d*x+c))*x^2-2*I*f^2*polylog(3,-exp(d*x+c))
/a/d^3-8/d^2/a*e*f*ln(exp(d*x+c))+2/d^2/a*e*f*ln(exp(d*x+c)+1)-2/d^3/a*f^2*
c*ln(exp(d*x+c)-1)-4/d^3/a*f^2*c*ln(exp(d*x+c)-I)+8/d^3/a*f^2*c*ln(exp(d*x+
c))+I/d/a*e^2*ln(exp(d*x+c)+1)+2/d^2/a*f^2*ln(exp(d*x+c)+1)*x+2/d^2/a*f^2*l
```

```
n(1-exp(d*x+c))*x+2/d^3/a*f^2*ln(1-exp(d*x+c))*c+4/d^2/a*f^2*ln(1+I*exp(d*x
+c))*x+4/d^3/a*f^2*ln(1+I*exp(d*x+c))*c+4/d^2/a*e*f*ln(exp(d*x+c)-I)-2*I*(f
^2*x^2*exp(2*d*x+2*c)+2*e*f*x*exp(2*d*x+2*c)+e^2*exp(2*d*x+2*c)-2*x^2*f^2-I
*exp(d*x+c)*f^2*x^2-4*e*f*x-2*I*exp(d*x+c)*e*f*x-2*e^2-I*exp(d*x+c)*e^2)/(e
xp(2*d*x+2*c)-1)/(exp(d*x+c)-I)/a/d+2*I/d^2/a*e*f*polylog(2,-exp(d*x+c))-2*
I/d^2/a*e*f*polylog(2,exp(d*x+c))-4*f^2*x^2/a/d-2*I/d^2/a*f^2*polylog(2,exp
(d*x+c))*x-4/a/d^3*f^2*c^2+2*I/d^2/a*e*f*c*ln(exp(d*x+c)-1)-2*I/d^2/a*ln(1-
exp(d*x+c))*c*e*f-2*I/d/a*ln(1-exp(d*x+c))*e*f*x+2*I/d/a*ln(exp(d*x+c)+1)*e
*f*x+2*I*f^2*polylog(3,exp(d*x+c))/a/d^3
```

<u>**Maxima**</u> [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(271) = 542$.
time = 0.45, size = 611, normalized size = 2.06

$$-\frac{2\,f^2x^2}{ad}-\left(\frac{2\left(e^{(-dx-c)}-i\,e^{(-2dx-2c)}+2i\right)}{\left(ae^{(-dx-c)}-i\,ae^{(-2dx-2c)}-ae^{(-3dx-3c)}+i\,a\right)d}-\frac{i\log\left(e^{(-dx-c)}+1\right)}{ad}+\frac{i\log\left(e^{(-dx-c)}-1\right)}{ad}\right)e^2-\frac{8\,fxe}{ad}-\frac{2\left(-2i\,f^2x^2-4i\,fxe-\left(-i\,f^2x^2e^{(2c)}-2i\,fxe^{(2c+1)}\right)e^{(2dx)}+\left(f^2x^2e^c+2\,fxe^{(c+1)}\right)e^{(dx)}\right)}{ade^{(3dx+3c)}-i\,ade^{(2dx+2c)}-ade^{(dx+c)}+i\,ad}+\frac{2\,fe\log\left(e^{(dx+c)}+1\right)}{ad^2}+\frac{4\,fe\log\left(e^{(dx+c)}-i\right)}{ad^2}+\frac{2\,fe\log\left(e^{(dx+c)}-1\right)}{ad^2}+\frac{i\left(d^2x^2\log\left(e^{(dx+c)}+1\right)+2\,dx\mathrm{Li}_2\left(-e^{(dx+c)}\right)-2\,\mathrm{Li}_3\left(-e^{(dx+c)}\right)\right)f^2}{ad^3}-\frac{i\left(d^2x^2\log\left(-e^{(dx+c)}+1\right)+2\,dx\mathrm{Li}_2\left(e^{(dx+c)}\right)-2\,\mathrm{Li}_3\left(e^{(dx+c)}\right)\right)f^2}{ad^3}+\frac{4\left(dx\log\left(i\,e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)\right)f^2}{ad^3}-\frac{2\left(-i\,dfe-f^2\right)\left(dx\log\left(e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(-e^{(dx+c)}\right)\right)}{ad^3}+\frac{2\left(-i\,dfe+f^2\right)\left(dx\log\left(-e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(e^{(dx+c)}\right)\right)}{ad^3}+\frac{i\,d^3f^2x^3-3\left(-i\,dfe+f^2\right)d^2x^2}{3\,ad^3}-\frac{i\,d^3f^2x^3-3\left(-i\,dfe-f^2\right)d^2x^2}{3\,ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -2*f^2*x^2/(a*d) - (2*(e^(-d*x - c) - I*e^(-2*d*x - 2*c) + 2*I)/((a*e^(-d*x
 - c) - I*a*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c) + I*a)*d) - I*log(e^(-d*x
 - c) + 1)/(a*d) + I*log(e^(-d*x - c) - 1)/(a*d))*e^2 - 8*f*x*e/(a*d) - 2*(
-2*I*f^2*x^2 - 4*I*f*x*e - (-I*f^2*x^2*e^(2*c) - 2*I*f*x*e^(2*c + 1))*e^(2*
d*x) + (f^2*x^2*e^c + 2*f*x*e^(c + 1))*e^(d*x))/(a*d*e^(3*d*x + 3*c) - I*a*
d*e^(2*d*x + 2*c) - a*d*e^(d*x + c) + I*a*d) + 2*f*e*log(e^(d*x + c) + 1)/(
a*d^2) + 4*f*e*log(e^(d*x + c) - I)/(a*d^2) + 2*f*e*log(e^(d*x + c) - 1)/(a
*d^2) + I*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*pol
ylog(3, -e^(d*x + c)))*f^2/(a*d^3) - I*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d
*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*f^2/(a*d^3) + 4*(d*x*log
(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*f^2/(a*d^3) - 2*(-I*d*f*e - f^
2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a*d^3) + 2*(-I*d*f*e +
 f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a*d^3) + 1/3*(I*d^3
*f^2*x^3 - 3*(-I*d*f*e + f^2)*d^2*x^2)/(a*d^3) - 1/3*(I*d^3*f^2*x^3 - 3*(-I
*d*f*e - f^2)*d^2*x^2)/(a*d^3)
```

<u>**Fricas**</u> [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1381 vs. $2(271) = 542$.
time = 0.37, size = 1381, normalized size = 4.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] (4*I*c^2*f^2 - 8*I*c*d*f*e + 4*I*d^2*e^2 + 4*(f^2*e^(3*d*x + 3*c) - I*f^2*e
^(2*d*x + 2*c) - f^2*e^(d*x + c) + I*f^2)*dilog(-I*e^(d*x + c)) - 2*(d*f^2*
x + d*f*e - I*f^2 + (-I*d*f^2*x - I*d*f*e - f^2)*e^(3*d*x + 3*c) - (d*f^2*x
 + d*f*e - I*f^2)*e^(2*d*x + 2*c) + (I*d*f^2*x + I*d*f*e + f^2)*e^(d*x + c)
)*dilog(-e^(d*x + c)) + 2*(d*f^2*x + d*f*e + I*f^2 - (I*d*f^2*x + I*d*f*e -
 f^2)*e^(3*d*x + 3*c) - (d*f^2*x + d*f*e + I*f^2)*e^(2*d*x + 2*c) - (-I*d*f
^2*x - I*d*f*e + f^2)*e^(d*x + c))*dilog(e^(d*x + c)) - 4*(d^2*f^2*x^2 - c^
2*f^2 + 2*(d^2*f*x + c*d*f)*e)*e^(3*d*x + 3*c) - 2*(-I*d^2*f^2*x^2 + 2*I*c^
2*f^2 + I*d^2*e^2 + 2*(-I*d^2*f*x - 2*I*c*d*f)*e)*e^(2*d*x + 2*c) + 2*(d^2*
f^2*x^2 - 2*c^2*f^2 - d^2*e^2 + 2*(d^2*f*x + 2*c*d*f)*e)*e^(d*x + c) - (d^2
*f^2*x^2 - 2*I*d*f^2*x + d^2*e^2 + 2*(d^2*f*x - I*d*f)*e - (I*d^2*f^2*x^2 +
 2*d*f^2*x + I*d^2*e^2 - 2*(-I*d^2*f*x - d*f)*e)*e^(3*d*x + 3*c) - (d^2*f^2
*x^2 - 2*I*d*f^2*x + d^2*e^2 + 2*(d^2*f*x - I*d*f)*e)*e^(2*d*x + 2*c) - (-I
*d^2*f^2*x^2 - 2*d*f^2*x - I*d^2*e^2 - 2*(I*d^2*f*x + d*f)*e)*e^(d*x + c))*
log(e^(d*x + c) + 1) - 4*(I*c*f^2 - I*d*f*e + (c*f^2 - d*f*e)*e^(3*d*x + 3*
c) + (-I*c*f^2 + I*d*f*e)*e^(2*d*x + 2*c) - (c*f^2 - d*f*e)*e^(d*x + c))*lo
g(e^(d*x + c) - I) - (2*(c - I)*d*f*e - (c^2 - 2*I*c)*f^2 - d^2*e^2 + (2*(-
I*c - 1)*d*f*e - (-I*c^2 - 2*c)*f^2 + I*d^2*e^2)*e^(3*d*x + 3*c) - (2*(c -
I)*d*f*e - (c^2 - 2*I*c)*f^2 - d^2*e^2)*e^(2*d*x + 2*c) + (2*(I*c + 1)*d*f*
e - (I*c^2 + 2*c)*f^2 - I*d^2*e^2)*e^(d*x + c))*log(e^(d*x + c) - 1) - 4*(-
I*d*f^2*x - I*c*f^2 - (d*f^2*x + c*f^2)*e^(3*d*x + 3*c) + (I*d*f^2*x + I*c*
f^2)*e^(2*d*x + 2*c) + (d*f^2*x + c*f^2)*e^(d*x + c))*log(I*e^(d*x + c) + 1
) + (d^2*f^2*x^2 + 2*I*d*f^2*x - (c^2 - 2*I*c)*f^2 + 2*(d^2*f*x + c*d*f)*e
+ (-I*d^2*f^2*x^2 + 2*d*f^2*x + (I*c^2 + 2*c)*f^2 - 2*(I*d^2*f*x + I*c*d*f)
*e)*e^(3*d*x + 3*c) - (d^2*f^2*x^2 + 2*I*d*f^2*x - (c^2 - 2*I*c)*f^2 + 2*(d
^2*f*x + c*d*f)*e)*e^(2*d*x + 2*c) + (I*d^2*f^2*x^2 - 2*d*f^2*x + (-I*c^2 -
 2*c)*f^2 - 2*(-I*d^2*f*x - I*c*d*f)*e)*e^(d*x + c))*log(-e^(d*x + c) + 1)
- 2*(I*f^2*e^(3*d*x + 3*c) + f^2*e^(2*d*x + 2*c) - I*f^2*e^(d*x + c) - f^2)
*polylog(3, -e^(d*x + c)) - 2*(-I*f^2*e^(3*d*x + 3*c) - f^2*e^(2*d*x + 2*c)
 + I*f^2*e^(d*x + c) + f^2)*polylog(3, e^(d*x + c)))/(a*d^3*e^(3*d*x + 3*c)
 - I*a*d^3*e^(2*d*x + 2*c) - a*d^3*e^(d*x + c) + I*a*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{e^2\operatorname{csch}^2(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{f^2x^2\operatorname{csch}^2(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{2efx\operatorname{csch}^2(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*(Integral(e**2*csch(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(f**2*
x**2*csch(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(2*e*f*x*csch(c + d
*x)**2/(sinh(c + d*x) - I), x))/a
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^2*csch(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^2}{\sinh\left(c+d\,x\right)^2\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(sinh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
[Out] int((e + f*x)^2/(sinh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.213 $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=163

$$\frac{2i(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)\coth(c+dx)}{ad}+\frac{2f\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}+\frac{f\log(\sinh(c+dx))}{ad^2}+\frac{ifP}{}$$

```
[Out] 2*I*(f*x+e)*arctanh(exp(d*x+c))/a/d-(f*x+e)*coth(d*x+c)/a/d+2*f*ln(cosh(1/2
*c+1/4*I*Pi+1/2*d*x))/a/d^2+f*ln(sinh(d*x+c))/a/d^2+I*f*polylog(2,-exp(d*x+
c))/a/d^2-I*f*polylog(2,exp(d*x+c))/a/d^2-(f*x+e)*tanh(1/2*c+1/4*I*Pi+1/2*d
*x)/a/d
```

Rubi [A]
time = 0.18, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {5694, 4269, 3556, 4267, 2317, 2438, 3399}

$$\frac{if\operatorname{Li}_2\left(-e^{c+dx}\right)}{ad^2}-\frac{if\operatorname{Li}_2\left(e^{c+dx}\right)}{ad^2}+\frac{f\log(\sinh(c+dx))}{ad^2}+\frac{2f\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{ad^2}+\frac{2i(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}-\frac{(e+fx)\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((2*I)*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d) - ((e + f*x)*Coth[c + d*x])/(a
*d) + (2*f*Log[Cosh[c/2 + (I/4)*Pi + (d*x)/2]])/(a*d^2) + (f*Log[Sinh[c + d
*x]])/(a*d^2) + (I*f*PolyLog[2, -E^(c + d*x)])/(a*d^2) - (I*f*PolyLog[2, E^
(c + d*x)])/(a*d^2) - ((e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5694

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a + b
*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\left(i\int \frac{(e+fx)\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx\right)+\frac{\int (e+fx)\operatorname{csch}^2(c+dx)\,dx}{a}\\
&= -\frac{(e+fx)\coth(c+dx)}{ad}-\frac{i\int (e+fx)\operatorname{csch}(c+dx)\,dx}{a}+\frac{f\int \coth(c+dx)\,dx}{ad}\\
&= \frac{2i(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)\coth(c+dx)}{ad}+\frac{f\log(\sinh(c+dx))}{ad^2}-\\
&= \frac{2i(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)\coth(c+dx)}{ad}+\frac{f\log(\sinh(c+dx))}{ad^2}-\\
&= \frac{2i(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)\coth(c+dx)}{ad}+\frac{2f\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\right.\right.}{ad^2}
\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 454 vs. $2(163)=326$.
time = 3.46, size = 454, normalized size = 2.79

$$\frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(-d(e+fx)\cosh\left(\frac{1}{2}(c+dx)\right)\left(i+\coth\left(\frac{1}{2}(c+dx)\right)\right)+4if\,\mathrm{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)+2f\log(\cosh(c+dx))\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)+2f\log(\sinh(c+dx))\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)+2icf\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)-2if\left((c+dx)\left(\log\left(1-e^{-c-dx}\right)-\log\left(1+e^{-c-dx}\right)\right)+\mathrm{PolyLog}\left(2,-e^{-c-dx}\right)-\mathrm{PolyLog}\left(2,e^{-c-dx}\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)-4d(e+fx)\sinh\left(\frac{1}{2}(c+dx)\right)+2f(c+dx)\left(-i\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)+2de\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\left(-i\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)-id(e+fx)\sinh\left(\frac{1}{2}(c+dx)\right)\left(-i+\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)}{2d^2(a+ia\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(-(d*(e + f*x)*Cosh[(c + d*x)/2]
*(I + Coth[(c + d*x)/2])) + (4*I)*f*ArcTan[Tanh[(c + d*x)/2]]*(Cosh[(c + d*
x)/2] + I*Sinh[(c + d*x)/2]) + 2*f*Log[Cosh[c + d*x]]*(Cosh[(c + d*x)/2] +
I*Sinh[(c + d*x)/2]) + 2*f*Log[Sinh[c + d*x]]*(Cosh[(c + d*x)/2] + I*Sinh[(
c + d*x)/2]) + (2*I)*c*f*Log[Tanh[(c + d*x)/2]]*(Cosh[(c + d*x)/2] + I*Sinh
[(c + d*x)/2]) - (2*I)*f*((c + d*x)*(Log[1 - E^(-c - d*x)] - Log[1 + E^(-c
- d*x)]) + PolyLog[2, -E^(-c - d*x)] - PolyLog[2, E^(-c - d*x)])*(Cosh[(c +
 d*x)/2] + I*Sinh[(c + d*x)/2]) - 4*d*(e + f*x)*Sinh[(c + d*x)/2] + 2*f*(c
+ d*x)*((-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]) + 2*d*e*Log[Tanh[(c + d
*x)/2]]*((-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]) - I*d*(e + f*x)*Sinh[(
c + d*x)/2]*(-I + Tanh[(c + d*x)/2])))/(2*d^2*(a + I*a*Sinh[c + d*x]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(143) = 286$.
time = 3.07, size = 316, normalized size = 1.94

method	result
risch	$-\frac{2i\left(fx\,e^{2dx+2c}+e\,e^{2dx+2c}-2fx-ie^{dx+c}fx-2e-ie^{dx+c}e\right)}{\left(e^{2dx+2c}-1\right)\left(e^{dx+c}-i\right)ad}+\frac{i\ln\left(e^{dx+c}+1\right)fx}{ad}-\frac{i\ln\left(1-e^{dx+c}\right)fx}{ad}-\frac{i\ln\left(1-e^{dx+c}\right)cf}{a\,d^{2}}+\frac{ie\,\mathrm{l}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*(f*x*exp(2*d*x+2*c)+e*exp(2*d*x+2*c)-2*f*x-I*exp(d*x+c)*f*x-2*e-I*exp(
d*x+c)*e)/(exp(2*d*x+2*c)-1)/(exp(d*x+c)-I)/a/d+I/a/d*ln(exp(d*x+c)+1)*f*x-
I/a/d*ln(1-exp(d*x+c))*f*x-I/a/d^2*ln(1-exp(d*x+c))*c*f+I/a/d*e*ln(exp(d*x+
c)+1)-I/a/d*e*ln(exp(d*x+c)-1)+2*f/a/d^2*ln(exp(d*x+c)-I)+1/a/d^2*f*ln(exp(
d*x+c)-1)-4/a/d^2*f*ln(exp(d*x+c))+I*f*polylog(2,-exp(d*x+c))/a/d^2-I*f*pol
ylog(2,exp(d*x+c))/a/d^2+1/a/d^2*f*ln(exp(d*x+c)+1)+I/a/d^2*f*c*ln(exp(d*x+
c)-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(4*I*d*integrate(1/4*x/(a*d*e^(d*x + c) + a*d), x) + 4*I*d*integrate(1/4*x
/(a*d*e^(d*x + c) - a*d), x) + 2*(x*e^(3*d*x + 3*c) - I*x)/(a*d*e^(3*d*x +
3*c) - I*a*d*e^(2*d*x + 2*c) - a*d*e^(d*x + c) + I*a*d) + 2*(d*x + c)/(a*d^
```

2) - 2*log((e^(d*x + c) - I)*e^(-c))/(a*d^2) - log(e^(d*x + c) + 1)/(a*d^2)
 - log(e^(d*x + c) - 1)/(a*d^2))*f - (2*(e^(-d*x - c) - I*e^(-2*d*x - 2*c)
+ 2*I)/((a*e^(-d*x - c) - I*a*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c) + I*a)*
d) - I*log(e^(-d*x - c) + 1)/(a*d) + I*log(e^(-d*x - c) - 1)/(a*d))*e

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(142) = 284$.
time = 0.47, size = 520, normalized size = 3.19

$$\frac{-2icf + \left(ife^{(3dx+3c)} + fe^{(2dx+2c)} - ife^{(dx+c)} - f\right)\mathrm{Li}_2\left(-e^{(dx+c)}\right) + \left(-ife^{(3dx+3c)} - fe^{(2dx+2c)} + ife^{(dx+c)} + f\right)\mathrm{Li}_2\left(e^{(dx+c)}\right) + 4ide - 2\left(2dfx + cf\right)e^{(3dx+3c)} - 2\left(-idfx - icf + ide\right)e^{(2dx+2c)} + 2\left(dfx + cf - de\right)e^{(dx+c)} - \left(dfx + de - \left(idfx + ide + f\right)e^{(3dx+3c)} - \left(dfx + de - if\right)e^{(2dx+2c)} - \left(-idfx - ide - f\right)e^{(dx+c)} - if\right)\log\left(e^{(dx+c)} + 1\right) + 2\left(fe^{(3dx+3c)} - ife^{(2dx+2c)} - fe^{(dx+c)} + if\right)\log\left(e^{(dx+c)} - i\right) - \left((c-i)f - de - \left((ic+1)f - ide\right)e^{(3dx+3c)} - \left((c-i)f - de\right)e^{(2dx+2c)} - \left((-ic-1)f + ide\right)e^{(dx+c)}\right)\log\left(e^{(dx+c)} - 1\right) + \left(dfx + cf + \left(-idfx - icf\right)e^{(3dx+3c)} - \left(dfx + cf\right)e^{(2dx+2c)} + \left(idfx + icf\right)e^{(dx+c)}\right)\log\left(-e^{(dx+c)} + 1\right)}{ad^2e^{(3dx+3c)} - iad^2e^{(2dx+2c)} - ad^2e^{(dx+c)} + iad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")

[Out] (-2*I*c*f + (I*f*e^(3*d*x + 3*c) + f*e^(2*d*x + 2*c) - I*f*e^(d*x + c) - f)
*dilog(-e^(d*x + c)) + (-I*f*e^(3*d*x + 3*c) - f*e^(2*d*x + 2*c) + I*f*e^(d
*x + c) + f)*dilog(e^(d*x + c)) + 4*I*d*e - 2*(2*d*f*x + c*f)*e^(3*d*x + 3*
c) - 2*(-I*d*f*x - I*c*f + I*d*e)*e^(2*d*x + 2*c) + 2*(d*f*x + c*f - d*e)*e
^(d*x + c) - (d*f*x + d*e - (I*d*f*x + I*d*e + f)*e^(3*d*x + 3*c) - (d*f*x
+ d*e - I*f)*e^(2*d*x + 2*c) - (-I*d*f*x - I*d*e - f)*e^(d*x + c) - I*f)*lo
g(e^(d*x + c) + 1) + 2*(f*e^(3*d*x + 3*c) - I*f*e^(2*d*x + 2*c) - f*e^(d*x
+ c) + I*f)*log(e^(d*x + c) - I) - ((c - I)*f - d*e - ((I*c + 1)*f - I*d*e)
*e^(3*d*x + 3*c) - ((c - I)*f - d*e)*e^(2*d*x + 2*c) - ((-I*c - 1)*f + I*d*
e)*e^(d*x + c))*log(e^(d*x + c) - 1) + (d*f*x + c*f + (-I*d*f*x - I*c*f)*e^
(3*d*x + 3*c) - (d*f*x + c*f)*e^(2*d*x + 2*c) + (I*d*f*x + I*c*f)*e^(d*x +
c))*log(-e^(d*x + c) + 1))/(a*d^2*e^(3*d*x + 3*c) - I*a*d^2*e^(2*d*x + 2*c)
 - a*d^2*e^(d*x + c) + I*a*d^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{e\operatorname{csch}^2(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{fx\operatorname{csch}^2(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*csch(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)

[Out] -I*(Integral(e*csch(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(f*x*csch
(c + d*x)**2/(sinh(c + d*x) - I), x))/a

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*csch(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e+f\,x}{{\mathrm{sinh}\left(c+d\,x\right)}^2\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(sinh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

```
[Out] int((e + f*x)/(sinh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.214 $\int \frac{\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=57

$$\frac{i\tanh^{-1}(\cosh(c+dx))}{ad}-\frac{2\coth(c+dx)}{ad}+\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}$$

```
[Out] I*arctanh(cosh(d*x+c))/a/d-2*coth(d*x+c)/a/d+coth(d*x+c)/d/(a+I*a*sinh(d*x+
c))
```

Rubi [A]
time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.208$, Rules used = {2847, 2827, 3852, 8, 3855}

$$-\frac{2\coth(c+dx)}{ad}+\frac{i\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (I*ArcTanh[Cosh[c + d*x]])/(a*d) - (2*Coth[c + d*x])/(a*d) + Coth[c + d*x]/
(d*(a + I*a*Sinh[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx &= \frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{\int \operatorname{csch}^2(c+dx)(-2a+ia\sinh(c+dx))\,dx}{a^2}\\ &= \frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{i\int \operatorname{csch}(c+dx)\,dx}{a}+\frac{2\int \operatorname{csch}^2(c+dx)\,dx}{a}\\ &= \frac{i\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{(2i)\mathrm{Subst}(\int 1\,dx,x,-i\operatorname{cot}}{ad}\\ &= \frac{i\tanh^{-1}(\cosh(c+dx))}{ad}-\frac{2\coth(c+dx)}{ad}+\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 61, normalized size = 1.07

$$-\frac{\operatorname{sech}(c+dx)\left(i-i\tanh^{-1}\left(\sqrt{\cosh^2(c+dx)}\right)\sqrt{\cosh^2(c+dx)}+\operatorname{csch}(c+dx)+2\sinh(c+dx)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
[Out] -((Sech[c + d*x]*(I - I*ArcTanh[Sqrt[Cosh[c + d*x]^2]]*Sqrt[Cosh[c + d*x]^2
] + Csch[c + d*x] + 2*Sinh[c + d*x]))/(a*d))
```

Maple [A]
time = 1.08, size = 63, normalized size = 1.11

method	result	size
derivativedivides	$\frac{-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-2i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}$	63
default	$\frac{-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-2i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}$	63
risch	$-\frac{2i\left(e^{2dx+2c}-2-ie^{dx+c}\right)}{\left(e^{2dx+2c}-1\right)\left(e^{dx+c}-i\right)ad}+\frac{i\ln\left(e^{dx+c}+1\right)}{ad}-\frac{i\ln\left(e^{dx+c}-1\right)}{ad}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a*(-tanh(1/2*d*x+1/2*c)-4/(-I+tanh(1/2*d*x+1/2*c))-1/tanh(1/2*d*x+1/2
*c)-2*I*ln(tanh(1/2*d*x+1/2*c)))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(53) = 106$.
time = 0.26, size = 109, normalized size = 1.91

$$-\frac{2\left(e^{(-dx-c)} - i\,e^{(-2\,dx-2\,c)} + 2i\right)}{\left(ae^{(-dx-c)} - i\,ae^{(-2\,dx-2\,c)} - ae^{(-3\,dx-3\,c)} + i\,a\right)d} + \frac{i\,\log\left(e^{(-dx-c)} + 1\right)}{ad} - \frac{i\,\log\left(e^{(-dx-c)} - 1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*(e^(-d*x - c) - I*e^(-2*d*x - 2*c) + 2*I)/((a*e^(-d*x - c) - I*a*e^(-2*d
*x - 2*c) - a*e^(-3*d*x - 3*c) + I*a)*d) + I*log(e^(-d*x - c) + 1)/(a*d) -
I*log(e^(-d*x - c) - 1)/(a*d)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(53) = 106$.
time = 0.34, size = 146, normalized size = 2.56

$$\frac{\left(i\,e^{(3\,dx+3\,c)} + e^{(2\,dx+2\,c)} - i\,e^{(dx+c)} - 1\right)\log\left(e^{(dx+c)} + 1\right) + \left(-i\,e^{(3\,dx+3\,c)} - e^{(2\,dx+2\,c)} + i\,e^{(dx+c)} + 1\right)\log\left(e^{(dx+c)} - 1\right) - 2i\,e^{(2\,dx+2\,c)} - 2\,e^{(dx+c)} + 4i}{ade^{(3\,dx+3\,c)} - i\,ade^{(2\,dx+2\,c)} - ade^{(dx+c)} + i\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((I*e^(3*d*x + 3*c) + e^(2*d*x + 2*c) - I*e^(d*x + c) - 1)*log(e^(d*x + c)
+ 1) + (-I*e^(3*d*x + 3*c) - e^(2*d*x + 2*c) + I*e^(d*x + c) + 1)*log(e^(d*
x + c) - 1) - 2*I*e^(2*d*x + 2*c) - 2*e^(d*x + c) + 4*I)/(a*d*e^(3*d*x + 3*
c) - I*a*d*e^(2*d*x + 2*c) - a*d*e^(d*x + c) + I*a*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\operatorname{csch}^{2}(c+dx)}{\sinh(c+dx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*Integral(csch(c + d*x)**2/(sinh(c + d*x) - I), x)/a
```

Giac [A]
time = 0.44, size = 90, normalized size = 1.58

$$-\frac{-\frac{i\log\left(e^{(dx+c)}+1\right)}{a}+\frac{i\log\left(e^{(dx+c)}-1\right)}{a}-\frac{2\left(e^{(2\,dx+2\,c)}-i\,e^{(dx+c)}-2\right)}{a\left(i\,e^{(3\,dx+3\,c)}+e^{(2\,dx+2\,c)}-i\,e^{(dx+c)}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -(-I*log(e^(d*x + c) + 1)/a + I*log(e^(d*x + c) - 1)/a - 2*(e^(2*d*x + 2*c)
 - I*e^(d*x + c) - 2)/(a*(I*e^(3*d*x + 3*c) + e^(2*d*x + 2*c) - I*e^(d*x +
c) - 1)))/d
```

Mupad [B]
time = 1.40, size = 122, normalized size = 2.14

$$\frac{\frac{2\,\mathrm{e}^{c+d\,x}}{a\,d}-\frac{4\mathrm{i}}{a\,d}+\frac{\mathrm{e}^{2\,c+2\,d\,x}\,2\mathrm{i}}{a\,d}}{\mathrm{e}^{c+d\,x}+\mathrm{e}^{2\,c+2\,d\,x}\,1\mathrm{i}-\mathrm{e}^{3\,c+3\,d\,x}-\mathrm{i}}-\frac{\ln\left(\mathrm{e}^{c+d\,x}\,2\mathrm{i}-2\mathrm{i}\right)\,1\mathrm{i}}{a\,d}+\frac{\ln\left(\mathrm{e}^{c+d\,x}\,2\mathrm{i}+2\mathrm{i}\right)\,1\mathrm{i}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

```
[Out] ((2*exp(c + d*x))/(a*d) - 4i/(a*d) + (exp(2*c + 2*d*x)*2i)/(a*d))/(exp(c +
d*x) + exp(2*c + 2*d*x)*1i - exp(3*c + 3*d*x) - 1i) - (log(exp(c + d*x)*2i
- 2i)*1i)/(a*d) + (log(exp(c + d*x)*2i + 2i)*1i)/(a*d)
```

3.215 $\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csch[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
[Out] Defer[Int][Csch[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 91.23, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csch[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
[Out] Integrate[Csch[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(dx+c)^2}{(fx+e)\,(a+ia\sinh\,(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -4*I*f*integrate(1/(-2*I*a*d*f^2*x^2 - 4*I*a*d*f*x*e - 2*I*a*d*e^2 + 2*(a*d
*f^2*x^2*e^c + 2*a*d*f*x*e^(c + 1) + a*d*e^(c + 2))*e^(d*x)), x) - 4*(I*e^(
2*d*x + 2*c) + e^(d*x + c) - 2*I)/(2*I*a*d*f*x + 2*I*a*d*e + 2*(a*d*f*x*e^(
3*c) + a*d*e^(3*c + 1))*e^(3*d*x) - 2*(I*a*d*f*x*e^(2*c) + I*a*d*e^(2*c + 1
))*e^(2*d*x) - 2*(a*d*f*x*e^c + a*d*e^(c + 1))*e^(d*x)) - 4*integrate(-1/4*
(I*d*f*x + I*d*e + f)/(a*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2 - (a*d*f^2*x^2*e
^c + 2*a*d*f*x*e^(c + 1) + a*d*e^(c + 2))*e^(d*x)), x) - 4*integrate(1/4*(I
*d*f*x + I*d*e - f)/(a*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2 + (a*d*f^2*x^2*e^c
 + 2*a*d*f*x*e^(c + 1) + a*d*e^(c + 2))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((I*a*d*f*x + I*a*d*e + (a*d*f*x + a*d*e)*e^(3*d*x + 3*c) + (-I*a*d*f*x - I
*a*d*e)*e^(2*d*x + 2*c) - (a*d*f*x + a*d*e)*e^(d*x + c))*integral(-2*((I*d*
f*x + I*d*e + I*f)*e^(2*d*x + 2*c) + (d*f*x + d*e + f)*e^(d*x + c) - 2*I*f)
/(I*a*d*f^2*x^2 + 2*I*a*d*f*x*e + I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*f*x*e +
a*d*e^2)*e^(3*d*x + 3*c) + (-I*a*d*f^2*x^2 - 2*I*a*d*f*x*e - I*a*d*e^2)*e^(
2*d*x + 2*c) - (a*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2)*e^(d*x + c)), x) - 2*I
*e^(2*d*x + 2*c) - 2*e^(d*x + c) + 4*I)/(I*a*d*f*x + I*a*d*e + (a*d*f*x + a
*d*e)*e^(3*d*x + 3*c) + (-I*a*d*f*x - I*a*d*e)*e^(2*d*x + 2*c) - (a*d*f*x +
 a*d*e)*e^(d*x + c))
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\operatorname{csch}^2(c+dx)}{e\sinh(c+dx)-ie+fx\sinh(c+dx)-ifx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*Integral(csch(c + d*x)**2/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) - I
*f*x), x)/a
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\mathrm{sinh}\left(c+d\,x\right)}^2\,\left(e+f\,x\right)\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

```
[Out] int(1/(sinh(c + d*x)^2*(e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.216 $\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=34

$$\operatorname{Int}\left(\frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csch[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
[Out] Defer[Int][Csch[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Csch[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(dx+c)^2}{(fx+e)^2\,(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -4*I*f*integrate(1/(-I*a*d*f^3*x^3 - 3*I*a*d*f^2*x^2*e - 3*I*a*d*f*x*e^2 -
I*a*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*d*f^2*x^2*e^(c + 1) + 3*a*d*f*x*e^(c + 2
) + a*d*e^(c + 3))*e^(d*x)), x) - 4*(I*e^(2*d*x + 2*c) + e^(d*x + c) - 2*I)
/(2*I*a*d*f^2*x^2 + 4*I*a*d*f*x*e + 2*I*a*d*e^2 + 2*(a*d*f^2*x^2*e^(3*c) +
2*a*d*f*x*e^(3*c + 1) + a*d*e^(3*c + 2))*e^(3*d*x) - 2*(I*a*d*f^2*x^2*e^(2*
c) + 2*I*a*d*f*x*e^(2*c + 1) + I*a*d*e^(2*c + 2))*e^(2*d*x) - 2*(a*d*f^2*x^
2*e^c + 2*a*d*f*x*e^(c + 1) + a*d*e^(c + 2))*e^(d*x)) - 4*integrate(-1/4*(I
*d*f*x + I*d*e + 2*f)/(a*d*f^3*x^3 + 3*a*d*f^2*x^2*e + 3*a*d*f*x*e^2 + a*d*
e^3 - (a*d*f^3*x^3*e^c + 3*a*d*f^2*x^2*e^(c + 1) + 3*a*d*f*x*e^(c + 2) + a*
d*e^(c + 3))*e^(d*x)), x) - 4*integrate(1/4*(I*d*f*x + I*d*e - 2*f)/(a*d*f^
3*x^3 + 3*a*d*f^2*x^2*e + 3*a*d*f*x*e^2 + a*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*
d*f^2*x^2*e^(c + 1) + 3*a*d*f*x*e^(c + 2) + a*d*e^(c + 3))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] ((I*a*d*f^2*x^2 + 2*I*a*d*f*x*e + I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*f*x*e +
a*d*e^2)*e^(3*d*x + 3*c) + (-I*a*d*f^2*x^2 - 2*I*a*d*f*x*e - I*a*d*e^2)*e^(
2*d*x + 2*c) - (a*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2)*e^(d*x + c))*integral(
-2*((I*d*f*x + I*d*e + 2*I*f)*e^(2*d*x + 2*c) + (d*f*x + d*e + 2*f)*e^(d*x
+ c) - 4*I*f)/(I*a*d*f^3*x^3 + 3*I*a*d*f^2*x^2*e + 3*I*a*d*f*x*e^2 + I*a*d*
e^3 + (a*d*f^3*x^3 + 3*a*d*f^2*x^2*e + 3*a*d*f*x*e^2 + a*d*e^3)*e^(3*d*x +
3*c) + (-I*a*d*f^3*x^3 - 3*I*a*d*f^2*x^2*e - 3*I*a*d*f*x*e^2 - I*a*d*e^3)*e
^(2*d*x + 2*c) - (a*d*f^3*x^3 + 3*a*d*f^2*x^2*e + 3*a*d*f*x*e^2 + a*d*e^3)*
e^(d*x + c)), x) - 2*I*e^(2*d*x + 2*c) - 2*e^(d*x + c) + 4*I)/(I*a*d*f^2*x^
```

```
2 + 2*I*a*d*f*x*e + I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2)*e^(3*
d*x + 3*c) + (-I*a*d*f^2*x^2 - 2*I*a*d*f*x*e - I*a*d*e^2)*e^(2*d*x + 2*c) -
 (a*d*f^2*x^2 + 2*a*d*f*x*e + a*d*e^2)*e^(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sinh\left(c+d\,x\right)^2\,{\left(e+f\,x\right)}^2\,\left(a+a\,\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
[Out] int(1/(sinh(c + d*x)^2*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.217 $\int \frac{(e+fx)^3\mathrm{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=546

$$\frac{2i(e+fx)^3}{ad}-\frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{3(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{i(e+fx)^3\coth(c+dx)}{ad}-\frac{3f(e+fx)}{2}$$

```
[Out] I*(f*x+e)^3*tanh(1/2*c+1/4*I*Pi+1/2*d*x)/a/d-6*f^2*(f*x+e)*arctanh(exp(d*x+
c))/a/d^3+3*(f*x+e)^3*arctanh(exp(d*x+c))/a/d+12*I*f^3*polylog(3,-I*exp(d*x
+c))/a/d^4-3/2*f*(f*x+e)^2*csch(d*x+c)/a/d^2-1/2*(f*x+e)^3*coth(d*x+c)*csch
(d*x+c)/a/d-6*I*f*(f*x+e)^2*ln(1+I*exp(d*x+c))/a/d^2-3*I*f*(f*x+e)^2*ln(1-e
xp(2*d*x+2*c))/a/d^2-3*f^3*polylog(2,-exp(d*x+c))/a/d^4+9/2*f*(f*x+e)^2*pol
ylog(2,-exp(d*x+c))/a/d^2-12*I*f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/a/d^3+3
*f^3*polylog(2,exp(d*x+c))/a/d^4-9/2*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a/d^
2-3*I*f^2*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d^3-9*f^2*(f*x+e)*polylog(3,-
exp(d*x+c))/a/d^3+3/2*I*f^3*polylog(3,exp(2*d*x+2*c))/a/d^4+9*f^2*(f*x+e)*p
olylog(3,exp(d*x+c))/a/d^3+2*I*(f*x+e)^3/a/d+9*f^3*polylog(4,-exp(d*x+c))/a
/d^4-9*f^3*polylog(4,exp(d*x+c))/a/d^4+I*(f*x+e)^3*coth(d*x+c)/a/d
```

Rubi [A]
time = 0.87, antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {5694, 4271, 4267, 2317, 2438, 2611, 6744, 2320, 6724, 4269, 3797, 2221, 3399}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((2*I)*(e + f*x)^3)/(a*d) - (6*f^2*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d^3)
+ (3*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) + (I*(e + f*x)^3*Coth[c + d*x]
)/(a*d) - (3*f*(e + f*x)^2*Csch[c + d*x])/(2*a*d^2) - ((e + f*x)^3*Coth[c +
 d*x]*Csch[c + d*x])/(2*a*d) - ((6*I)*f*(e + f*x)^2*Log[1 + I*E^(c + d*x)])
/(a*d^2) - ((3*I)*f*(e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a*d^2) - (3*f^3*
PolyLog[2, -E^(c + d*x)])/(a*d^4) + (9*f*(e + f*x)^2*PolyLog[2, -E^(c + d*x
)])/(2*a*d^2) - ((12*I)*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^3)
 + (3*f^3*PolyLog[2, E^(c + d*x)])/(a*d^4) - (9*f*(e + f*x)^2*PolyLog[2, E^
(c + d*x)])/(2*a*d^2) - ((3*I)*f^2*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(
a*d^3) - (9*f^2*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) + ((12*I)*f^3*P
olyLog[3, (-I)*E^(c + d*x)])/(a*d^4) + (9*f^2*(e + f*x)*PolyLog[3, E^(c + d
*x)])/(a*d^3) + (((3*I)/2)*f^3*PolyLog[3, E^(2*(c + d*x))])/(a*d^4) + (9*f^
3*PolyLog[4, -E^(c + d*x)])/(a*d^4) - (9*f^3*PolyLog[4, E^(c + d*x)])/(a*d^
4) + (I*(e + f*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
```

```
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5694

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a + b
*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\left(i\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx\right)+\frac{\int (e+fx)^3\text{csch}^3(c+dx)\,dx}{a}$$
$$= -\frac{3f(e+fx)^2\text{csch}(c+dx)}{2ad^2}-\frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{2ad}-\frac{i\int(e\cdot$$
$$= -\frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{i(e+fx)^3\cot}{ad}$$
$$= \frac{i(e+fx)^3}{ad}-\frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{3(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{i}{}$$
$$= \frac{i(e+fx)^3}{ad}-\frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{3(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{i}{}$$
$$= \frac{2i(e+fx)^3}{ad}-\frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{3(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+$$
$$= \frac{2i(e+fx)^3}{ad}-\frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{3(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+$$
$$= \frac{2i(e+fx)^3}{ad}-\frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{3(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+$$
$$= \frac{2i(e+fx)^3}{ad}-\frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{3(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+$$
$$= \frac{2i(e+fx)^3}{ad}-\frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{3(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2394 vs. 2(546) = 1092.
time = 52.33, size = 2394, normalized size = 4.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
[Out] (-3*e^3*Log[Tanh[(c + d*x)/2]])/(2*a*d) + (3*e*f^2*Log[Tanh[(c + d*x)/2]])/
(a*d^3) - (9*e^2*f*(-(c*Log[Tanh[(c + d*x)/2]]) - I*((I*c + I*d*x)*(Log[1 -
```

```
E^(I*(I*c + I*d*x))] - Log[1 + E^(I*(I*c + I*d*x))]) + I*(PolyLog[2, -E^(I
*(I*c + I*d*x))] - PolyLog[2, E^(I*(I*c + I*d*x))]))))/(2*a*d^2) + (3*f^3*(
-(c*Log[Tanh[(c + d*x)/2]]) - I*((I*c + I*d*x)*(Log[1 - E^(I*(I*c + I*d*x))
] - Log[1 + E^(I*(I*c + I*d*x))]) + I*(PolyLog[2, -E^(I*(I*c + I*d*x))] - P
olyLog[2, E^(I*(I*c + I*d*x))]))))/(a*d^4) - (2*f*(d^2*((-I)*d*E^c*x*(3*e^2
 + 3*e*f*x + f^2*x^2) + 3*(1 + I*E^c)*(e + f*x)^2*Log[1 + I*E^(c + d*x)]) +
 6*d*(1 + I*E^c)*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)] - (6*I)*(-I + E^c
)*f^2*PolyLog[3, (-I)*E^(c + d*x)]))/(a*d^4*(-I + E^c)) + ((I/4)*f^3*Csch[c
]*(2*d^2*x^2*(2*d*E^(2*c)*x - 3*(-1 + E^(2*c))*Log[1 - E^(2*(c + d*x))]) -
6*d*(-1 + E^(2*c))*x*PolyLog[2, E^(2*(c + d*x))] + 3*(-1 + E^(2*c))*PolyLog
[3, E^(2*(c + d*x))]))/(a*d^4*E^c) + (9*e*f^2*(d^2*x^2*ArcTanh[Cosh[c + d*x
] + Sinh[c + d*x]] + d*x*PolyLog[2, -Cosh[c + d*x] - Sinh[c + d*x]] - d*x*P
olyLog[2, Cosh[c + d*x] + Sinh[c + d*x]] - PolyLog[3, -Cosh[c + d*x] - Sinh
[c + d*x]] + PolyLog[3, Cosh[c + d*x] + Sinh[c + d*x]]))/(a*d^3) - (3*f^3*(
d^3*x^3*Log[1 - E^(c + d*x)] - d^3*x^3*Log[1 + E^(c + d*x)] - 3*d^2*x^2*Pol
yLog[2, -E^(c + d*x)] + 3*d^2*x^2*PolyLog[2, E^(c + d*x)] + 6*d*x*PolyLog[3
, -E^(c + d*x)] - 6*d*x*PolyLog[3, E^(c + d*x)] - 6*PolyLog[4, -E^(c + d*x)
] + 6*PolyLog[4, E^(c + d*x)]))/(2*a*d^4) + ((3*I)*e^2*f*Csch[c]*(-(d*x*Cos
h[c]) + Log[Cosh[d*x]*Sinh[c] + Cosh[c]*Sinh[d*x]]*Sinh[c]))/(a*d^2*(-Cosh[
c]^2 + Sinh[c]^2)) + (Csch[c]*Csch[c + d*x]^2*(3*e^2*f*Cosh[(d*x)/2] + 6*e*
f^2*x*Cosh[(d*x)/2] + 3*f^3*x^2*Cosh[(d*x)/2] + 3*e^2*f*Cosh[(3*d*x)/2] + 6
*e*f^2*x*Cosh[(3*d*x)/2] + 3*f^3*x^2*Cosh[(3*d*x)/2] + (5*I)*d*e^3*Cosh[c -
 (d*x)/2] + (15*I)*d*e^2*f*x*Cosh[c - (d*x)/2] + (15*I)*d*e*f^2*x^2*Cosh[c
- (d*x)/2] + (5*I)*d*f^3*x^3*Cosh[c - (d*x)/2] - I*d*e^3*Cosh[c + (d*x)/2]
- (3*I)*d*e^2*f*x*Cosh[c + (d*x)/2] - (3*I)*d*e*f^2*x^2*Cosh[c + (d*x)/2] -
 I*d*f^3*x^3*Cosh[c + (d*x)/2] - 3*e^2*f*Cosh[2*c + (d*x)/2] - 6*e*f^2*x*Co
sh[2*c + (d*x)/2] - 3*f^3*x^2*Cosh[2*c + (d*x)/2] + I*d*e^3*Cosh[c + (3*d*x
)/2] + (3*I)*d*e^2*f*x*Cosh[c + (3*d*x)/2] + (3*I)*d*e*f^2*x^2*Cosh[c + (3*
d*x)/2] + I*d*f^3*x^3*Cosh[c + (3*d*x)/2] - 3*e^2*f*Cosh[2*c + (3*d*x)/2] -
 6*e*f^2*x*Cosh[2*c + (3*d*x)/2] - 3*f^3*x^2*Cosh[2*c + (3*d*x)/2] - (3*I)*
d*e^3*Cosh[3*c + (3*d*x)/2] - (9*I)*d*e^2*f*x*Cosh[3*c + (3*d*x)/2] - (9*I)
*d*e*f^2*x^2*Cosh[3*c + (3*d*x)/2] - (3*I)*d*f^3*x^3*Cosh[3*c + (3*d*x)/2]
- (4*I)*d*e^3*Cosh[c + (5*d*x)/2] - (12*I)*d*e^2*f*x*Cosh[c + (5*d*x)/2] -
(12*I)*d*e*f^2*x^2*Cosh[c + (5*d*x)/2] - (4*I)*d*f^3*x^3*Cosh[c + (5*d*x)/2
] + (2*I)*d*e^3*Cosh[3*c + (5*d*x)/2] + (6*I)*d*e^2*f*x*Cosh[3*c + (5*d*x)/
2] + (6*I)*d*e*f^2*x^2*Cosh[3*c + (5*d*x)/2] + (2*I)*d*f^3*x^3*Cosh[3*c + (
5*d*x)/2] - d*e^3*Sinh[(d*x)/2] - 3*d*e^2*f*x*Sinh[(d*x)/2] - 3*d*e*f^2*x^2
*Sinh[(d*x)/2] - d*f^3*x^3*Sinh[(d*x)/2] - d*e^3*Sinh[(3*d*x)/2] - 3*d*e^2*
f*x*Sinh[(3*d*x)/2] - 3*d*e*f^2*x^2*Sinh[(3*d*x)/2] - d*f^3*x^3*Sinh[(3*d*x
)/2] + (3*I)*e^2*f*Sinh[c - (d*x)/2] + (6*I)*e*f^2*x*Sinh[c - (d*x)/2] + (3
*I)*f^3*x^2*Sinh[c - (d*x)/2] + (3*I)*e^2*f*Sinh[c + (d*x)/2] + (6*I)*e*f^2
*x*Sinh[c + (d*x)/2] + (3*I)*f^3*x^2*Sinh[c + (d*x)/2] - 3*d*e^3*Sinh[2*c +
 (d*x)/2] - 9*d*e^2*f*x*Sinh[2*c + (d*x)/2] - 9*d*e*f^2*x^2*Sinh[2*c + (d*x
)/2] - 3*d*f^3*x^3*Sinh[2*c + (d*x)/2] + (3*I)*e^2*f*Sinh[c + (3*d*x)/2] +
(6*I)*e*f^2*x*Sinh[c + (3*d*x)/2] + (3*I)*f^3*x^2*Sinh[c + (3*d*x)/2] - d*e
```

```
^3*Sinh[2*c + (3*d*x)/2] - 3*d*e^2*f*x*Sinh[2*c + (3*d*x)/2] - 3*d*e*f^2*x^
2*Sinh[2*c + (3*d*x)/2] - d*f^3*x^3*Sinh[2*c + (3*d*x)/2] - (3*I)*e^2*f*Sin
h[3*c + (3*d*x)/2] - (6*I)*e*f^2*x*Sinh[3*c + (3*d*x)/2] - (3*I)*f^3*x^2*Si
nh[3*c + (3*d*x)/2] + 2*d*e^3*Sinh[2*c + (5*d*x)/2] + 6*d*e^2*f*x*Sinh[2*c
+ (5*d*x)/2] + 6*d*e*f^2*x^2*Sinh[2*c + (5*d*x)/2] + 2*d*f^3*x^3*Sinh[2*c +
 (5*d*x)/2]))/(8*a*d^2*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] + I*S
inh[c/2 + (d*x)/2])) + ((3*I)*e*f^2*Csch[c]*Sech[c]*((d^2*x^2)/E^ArcTanh[Ta
nh[c]] - (I*(-(d*x*(-Pi + (2*I)*ArcTanh[Tanh[c]])) - Pi*Log[1 + E^(2*d*x)]
- 2*(I*d*x + I*ArcTanh[Tanh[c]])*Log[1 - E^((2*I)*(I*d*x + I*ArcTanh[Tanh[c
]]))] + Pi*Log[Cosh[d*x]] + (2*I)*ArcTanh[Tanh[c]]*Log[I*Sinh[d*x + ArcTanh
[Tanh[c]]]] + I*PolyLog[2, E^((2*I)*(I*d*x + I*ArcTanh[Tanh[c]]))])*Tanh[c]
)/Sqrt[1 - Tanh[c]^2]))/(a*d^3*Sqrt[Sech[c]^2*(Cosh[c]^2 - Sinh[c]^2)])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2057 vs. $2(504) = 1008$.
time = 3.44, size = 2058, normalized size = 3.77

method	result	size
risch	Expression too large to display	2058

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2/a/d*e^3*ln(exp(d*x+c)-1)+3/2/a/d*e^3*ln(exp(d*x+c)+1)+12*I*f^3*polylog
(3,-I*exp(d*x+c))/a/d^4-9/2/a/d^2*ln(1-exp(d*x+c))*c*e^2*f+9/2/a/d^2*e^2*f*
polylog(2,-exp(d*x+c))-9/2/a/d^2*e^2*f*polylog(2,exp(d*x+c))-3/a/d^4*f^3*c*
ln(exp(d*x+c)-1)+9/a/d^3*e*f^2*polylog(3,exp(d*x+c))-9/a/d^3*e*f^2*polylog(
3,-exp(d*x+c))-3/a/d^3*e*f^2*ln(exp(d*x+c)+1)+3/a/d^3*e*f^2*ln(exp(d*x+c)-1
)-3/a/d^3*f^3*ln(exp(d*x+c)+1)*x+3/a/d^3*f^3*ln(1-exp(d*x+c))*x+3/a/d^4*f^3
*ln(1-exp(d*x+c))*c+3/2/a/d*f^3*ln(exp(d*x+c)+1)*x^3+9/2/a/d^2*f^3*polylog(
2,-exp(d*x+c))*x^2-3/2/a/d^4*f^3*ln(1-exp(d*x+c))*c^3-9/a/d^3*f^3*polylog(3
,-exp(d*x+c))*x-9/2/a/d^2*f^3*polylog(2,exp(d*x+c))*x^2+9/a/d^3*f^3*polylog
(3,exp(d*x+c))*x+3/2/a/d^4*f^3*c^3*ln(exp(d*x+c)-1)+4*I/a/d*f^3*x^3-8*I/a/d
^4*f^3*c^3+6*I/a/d^4*f^3*polylog(3,exp(d*x+c))+6*I/a/d^4*f^3*polylog(3,-exp
(d*x+c))-6*I/a/d^3*f^3*polylog(2,exp(d*x+c))*x-6*I/a/d^2*f^3*ln(1+I*exp(d*x
+c))*x^2-12*I/a/d^3*f^3*polylog(2,-I*exp(d*x+c))*x-12*I/a/d^3*f^3*c^2*x-3*I
/a/d^2*e^2*f*ln(exp(d*x+c)-1)-3*I/a/d^4*f^3*c^2*ln(exp(d*x+c)-1)-3*I/a/d^2*
e^2*f*ln(exp(d*x+c)+1)-3*I/a/d^2*f^3*ln(exp(d*x+c)+1)*x^2-3*I/a/d^2*f^3*ln(
1-exp(d*x+c))*x^2-3*f^3*polylog(2,-exp(d*x+c))/a/d^4+3*f^3*polylog(2,exp(d*
x+c))/a/d^4-9/2/a/d^3*e*f^2*c^2*ln(exp(d*x+c)-1)-9/2/a/d*ln(1-exp(d*x+c))*e
^2*f*x+9/a/d^2*polylog(2,-exp(d*x+c))*e*f^2*x-9/2/a/d*ln(1-exp(d*x+c))*e*f^
2*x^2-9/a/d^2*polylog(2,exp(d*x+c))*e*f^2*x+9/2/a/d*ln(exp(d*x+c)+1)*e^2*f*
x+9/2/a/d*ln(exp(d*x+c)+1)*e*f^2*x^2-6*I/a/d^3*e*f^2*polylog(2,exp(d*x+c))-
12*I/a/d^3*e*f^2*polylog(2,-I*exp(d*x+c))-6*I/a/d^3*e*f^2*polylog(2,-exp(d*
x+c))-6*I/a/d^3*f^3*polylog(2,-exp(d*x+c))*x+9/2/a/d^3*ln(1-exp(d*x+c))*c^2
```

```
*e*f^2+9/2/a/d^2*e^2*f*c*ln(exp(d*x+c)-1)+24*I/a/d^2*c*e*f^2*x-6*I/a/d^2*ln
(exp(d*x+c)+1)*e*f^2*x-6*I/a/d^2*ln(1-exp(d*x+c))*e*f^2*x-12*I/a/d^2*ln(1+I
*exp(d*x+c))*e*f^2*x+9*f^3*polylog(4,-exp(d*x+c))/a/d^4-9*f^3*polylog(4,exp
(d*x+c))/a/d^4+3*I/a/d^4*f^3*ln(1-exp(d*x+c))*c^2+6*I/a/d^4*f^3*c^2*ln(1+I*
exp(d*x+c))+12*I/a/d^2*e^2*f*ln(exp(d*x+c))-6*I/a/d^2*e^2*f*ln(exp(d*x+c)-I
)-6*I/a/d^4*f^3*c^2*ln(exp(d*x+c)-I)+12*I/a/d^4*f^3*c^2*ln(exp(d*x+c))+12*I
/a/d*e*f^2*x^2+12*I/a/d^3*c^2*e*f^2-24*I/a/d^3*e*f^2*c*ln(exp(d*x+c))+12*I/
a/d^3*e*f^2*c*ln(exp(d*x+c)-I)+6*I/a/d^3*e*f^2*c*ln(exp(d*x+c)-1)-6*I/a/d^3
*ln(1-exp(d*x+c))*c*e*f^2-12*I/a/d^3*ln(1+I*exp(d*x+c))*c*e*f^2-3/2/a/d*f^3
*ln(1-exp(d*x+c))*x^3-(6*I*e*f^2*x*exp(d*x+c)+9*d*e*f^2*x^2*exp(4*d*x+4*c)+
9*d*e^2*f*x*exp(4*d*x+4*c)-9*I*d*e^2*f*x*exp(3*d*x+3*c)+4*d*e^3-9*I*d*e*f^2
*x^2*exp(3*d*x+3*c)+3*I*f^3*x^2*exp(d*x+c)+3*I*exp(d*x+c)*e^2*f+3*I*d*e*f^2
*x^2*exp(d*x+c)+3*I*d*e^2*f*x*exp(d*x+c)+12*d*e*f^2*x^2+12*d*e^2*f*x-3*I*d*
e^3*exp(3*d*x+3*c)+6*e*f^2*x*exp(4*d*x+4*c)-5*d*e^3*exp(2*d*x+2*c)-3*f^3*x^
2*exp(2*d*x+2*c)-3*e^2*f*exp(2*d*x+2*c)+3*f^3*x^2*exp(4*d*x+4*c)+3*d*e^3*ex
p(4*d*x+4*c)+3*e^2*f*exp(4*d*x+4*c)+4*d*f^3*x^3+I*d*f^3*x^3*exp(d*x+c)+I*e^
3*d*exp(d*x+c)-3*I*d*f^3*x^3*exp(3*d*x+3*c)-6*I*e*f^2*x*exp(3*d*x+3*c)-15*d
*e*f^2*x^2*exp(2*d*x+2*c)-15*d*e^2*f*x*exp(2*d*x+2*c)-5*d*f^3*x^3*exp(2*d*x
+2*c)-3*I*f^3*x^2*exp(3*d*x+3*c)-6*e*f^2*x*exp(2*d*x+2*c)-3*I*e^2*f*exp(3*d
*x+3*c)+3*d*f^3*x^3*exp(4*d*x+4*c))/(exp(2*d*x+2*c)-1)^2/d^2/(exp(d*x+c)-I)
/a
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1353 vs. $2(500) = 1000$.
time = 0.60, size = 1353, normalized size = 2.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/2*(2*(-I*e^(-d*x - c) - 5*e^(-2*d*x - 2*c) + 3*I*e^(-3*d*x - 3*c) + 3*e^
(-4*d*x - 4*c) + 4)/((a*e^(-d*x - c) - 2*I*a*e^(-2*d*x - 2*c) - 2*a*e^(-3*d
*x - 3*c) + I*a*e^(-4*d*x - 4*c) + a*e^(-5*d*x - 5*c) + I*a)*d) - 3*log(e^(
-d*x - c) + 1)/(a*d) + 3*log(e^(-d*x - c) - 1)/(a*d))*e^3 + 6*I*f*x*e^2/(a*
d) - (4*d*f^3*x^3 + 12*d*f^2*x^2*e + 12*d*f*x*e^2 + 3*(d*f^3*x^3*e^(4*c) +
(f^3*e^(4*c) + 3*d*f^2*e^(4*c + 1))*x^2 + (3*d*f*e^(4*c + 2) + 2*f^2*e^(4*c
 + 1))*x + f*e^(4*c + 2))*e^(4*d*x) - 3*(I*d*f^3*x^3*e^(3*c) + (I*f^3*e^(3*
c) + 3*I*d*f^2*e^(3*c + 1))*x^2 + (3*I*d*f*e^(3*c + 2) + 2*I*f^2*e^(3*c + 1
))*x + I*f*e^(3*c + 2))*e^(3*d*x) - (5*d*f^3*x^3*e^(2*c) + 3*(f^3*e^(2*c) +
 5*d*f^2*e^(2*c + 1))*x^2 + 3*(5*d*f*e^(2*c + 2) + 2*f^2*e^(2*c + 1))*x + 3
*f*e^(2*c + 2))*e^(2*d*x) + (I*d*f^3*x^3*e^c - 3*(-I*d*f^2*e^(c + 1) - I*f^
3*e^c)*x^2 - 3*(-I*d*f*e^(c + 2) - 2*I*f^2*e^(c + 1))*x + 3*I*f*e^(c + 2))*
e^(d*x))/(a*d^2*e^(5*d*x + 5*c) - I*a*d^2*e^(4*d*x + 4*c) - 2*a*d^2*e^(3*d*
```

```
x + 3*c) + 2*I*a*d^2*e^(2*d*x + 2*c) + a*d^2*e^(d*x + c) - I*a*d^2) - 12*I*
(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*f^2*e/(a*d^3) - 6*I*f*
e^2*log(I*e^(d*x + c) + 1)/(a*d^2) + 3/2*(d^3*x^3*log(e^(d*x + c) + 1) + 3*
d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4,
 -e^(d*x + c)))*f^3/(a*d^4) - 3/2*(d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^
2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x
+ c)))*f^3/(a*d^4) - 6*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e
^(d*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a*d^4) - 3*(-I*d*f*e^2 + f
^2*e)*x/(a*d^2) - 3*(-I*d*f*e^2 - f^2*e)*x/(a*d^2) + 3*(-I*d*f*e^2 - f^2*e)
*log(e^(d*x + c) + 1)/(a*d^3) + 3*(-I*d*f*e^2 + f^2*e)*log(e^(d*x + c) - 1)
/(a*d^3) - 3/2*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) -
2*polylog(3, e^(d*x + c)))*(3*d*f^2*e + 2*I*f^3)/(a*d^4) + 3/2*(d^2*x^2*log
(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))
*(3*d*f^2*e - 2*I*f^3)/(a*d^4) + 3/2*(3*d^2*f*e^2 - 4*I*d*f^2*e - 2*f^3)*(d
*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a*d^4) - 3/2*(3*d^2*f*e^2 +
 4*I*d*f^2*e - 2*f^3)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a*d
^4) + 1/8*(3*d^4*f^3*x^4 + 4*(3*d*f^2*e + 2*I*f^3)*d^3*x^3 + 6*(3*d^2*f*e^2
 + 4*I*d*f^2*e - 2*f^3)*d^2*x^2)/(a*d^4) - 1/8*(3*d^4*f^3*x^4 + 4*(3*d*f^2*
e - 2*I*f^3)*d^3*x^3 + 6*(3*d^2*f*e^2 - 4*I*d*f^2*e - 2*f^3)*d^2*x^2)/(a*d^
4) - 2*(-I*d^3*f^3*x^3 - 3*I*d^3*f^2*x^2*e)/(a*d^4)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4261 vs. $2(500) = 1000$.
time = 0.43, size = 4261, normalized size = 7.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/2*(8*c^3*f^3 - 24*c^2*d*f^2*e + 24*c*d^2*f*e^2 - 8*d^3*e^3 - 24*(d*f^3*x
+ d*f^2*e + (I*d*f^3*x + I*d*f^2*e)*e^(5*d*x + 5*c) + (d*f^3*x + d*f^2*e)*e
^(4*d*x + 4*c) + 2*(-I*d*f^3*x - I*d*f^2*e)*e^(3*d*x + 3*c) - 2*(d*f^3*x +
d*f^2*e)*e^(2*d*x + 2*c) + (I*d*f^3*x + I*d*f^2*e)*e^(d*x + c))*dilog(-I*e^
(d*x + c)) - 3*(3*I*d^2*f^3*x^2 + 4*d*f^3*x + 3*I*d^2*f*e^2 - 2*I*f^3 + 2*(
3*I*d^2*f^2*x + 2*d*f^2)*e - (3*d^2*f^3*x^2 - 4*I*d*f^3*x + 3*d^2*f*e^2 - 2
*f^3 + 2*(3*d^2*f^2*x - 2*I*d*f^2)*e)*e^(5*d*x + 5*c) + (3*I*d^2*f^3*x^2 +
4*d*f^3*x + 3*I*d^2*f*e^2 - 2*I*f^3 + 2*(3*I*d^2*f^2*x + 2*d*f^2)*e)*e^(4*d
*x + 4*c) + 2*(3*d^2*f^3*x^2 - 4*I*d*f^3*x + 3*d^2*f*e^2 - 2*f^3 + 2*(3*d^2
*f^2*x - 2*I*d*f^2)*e)*e^(3*d*x + 3*c) + 2*(-3*I*d^2*f^3*x^2 - 4*d*f^3*x -
3*I*d^2*f*e^2 + 2*I*f^3 + 2*(-3*I*d^2*f^2*x - 2*d*f^2)*e)*e^(2*d*x + 2*c) -
 (3*d^2*f^3*x^2 - 4*I*d*f^3*x + 3*d^2*f*e^2 - 2*f^3 + 2*(3*d^2*f^2*x - 2*I*
d*f^2)*e)*e^(d*x + c))*dilog(-e^(d*x + c)) - 3*(-3*I*d^2*f^3*x^2 + 4*d*f^3*
x - 3*I*d^2*f*e^2 + 2*I*f^3 + 2*(-3*I*d^2*f^2*x + 2*d*f^2)*e + (3*d^2*f^3*x
```

```
^2 + 4*I*d*f^3*x + 3*d^2*f*e^2 - 2*f^3 + 2*(3*d^2*f^2*x + 2*I*d*f^2)*e)*e^(
5*d*x + 5*c) + (-3*I*d^2*f^3*x^2 + 4*d*f^3*x - 3*I*d^2*f*e^2 + 2*I*f^3 + 2*
(-3*I*d^2*f^2*x + 2*d*f^2)*e)*e^(4*d*x + 4*c) - 2*(3*d^2*f^3*x^2 + 4*I*d*f^
3*x + 3*d^2*f*e^2 - 2*f^3 + 2*(3*d^2*f^2*x + 2*I*d*f^2)*e)*e^(3*d*x + 3*c)
+ 2*(3*I*d^2*f^3*x^2 - 4*d*f^3*x + 3*I*d^2*f*e^2 - 2*I*f^3 + 2*(3*I*d^2*f^2
*x - 2*d*f^2)*e)*e^(2*d*x + 2*c) + (3*d^2*f^3*x^2 + 4*I*d*f^3*x + 3*d^2*f*e
^2 - 2*f^3 + 2*(3*d^2*f^2*x + 2*I*d*f^2)*e)*e^(d*x + c))*dilog(e^(d*x + c))
 - 8*(-I*d^3*f^3*x^3 - I*c^3*f^3 + 3*(-I*d^3*f*x - I*c*d^2*f)*e^2 + 3*(-I*d
^3*f^2*x^2 + I*c^2*d*f^2)*e)*e^(5*d*x + 5*c) + 2*(d^3*f^3*x^3 - 3*d^2*f^3*x
^2 + 4*c^3*f^3 - 3*d^3*e^3 + 3*(d^3*f*x + (4*c - 1)*d^2*f)*e^2 + 3*(d^3*f^2
*x^2 - 4*c^2*d*f^2 - 2*d^2*f^2*x)*e)*e^(4*d*x + 4*c) - 2*(5*I*d^3*f^3*x^3 -
 3*I*d^2*f^3*x^2 + 8*I*c^3*f^3 - 3*I*d^3*e^3 + 3*(5*I*d^3*f*x + (8*I*c - I)
*d^2*f)*e^2 + 3*(5*I*d^3*f^2*x^2 - 8*I*c^2*d*f^2 - 2*I*d^2*f^2*x)*e)*e^(3*d
*x + 3*c) - 2*(3*d^3*f^3*x^3 - 3*d^2*f^3*x^2 + 8*c^3*f^3 - 5*d^3*e^3 + 3*(3
*d^3*f*x + (8*c - 1)*d^2*f)*e^2 + 3*(3*d^3*f^2*x^2 - 8*c^2*d*f^2 - 2*d^2*f^
2*x)*e)*e^(2*d*x + 2*c) - 2*(-3*I*d^3*f^3*x^3 + 3*I*d^2*f^3*x^2 - 4*I*c^3*f
^3 + I*d^3*e^3 + 3*(-3*I*d^3*f*x + (-4*I*c + I)*d^2*f)*e^2 + 3*(-3*I*d^3*f^
2*x^2 + 4*I*c^2*d*f^2 + 2*I*d^2*f^2*x)*e)*e^(d*x + c) - 3*(I*d^3*f^3*x^3 +
2*d^2*f^3*x^2 - 2*I*d*f^3*x + I*d^3*e^3 + (3*I*d^3*f*x + 2*d^2*f)*e^2 + (3*
I*d^3*f^2*x^2 + 4*d^2*f^2*x - 2*I*d*f^2)*e - (d^3*f^3*x^3 - 2*I*d^2*f^3*x^2
 - 2*d*f^3*x + d^3*e^3 + (3*d^3*f*x - 2*I*d^2*f)*e^2 + (3*d^3*f^2*x^2 - 4*I
*d^2*f^2*x - 2*d*f^2)*e)*e^(5*d*x + 5*c) + (I*d^3*f^3*x^3 + 2*d^2*f^3*x^2 -
 2*I*d*f^3*x + I*d^3*e^3 + (3*I*d^3*f*x + 2*d^2*f)*e^2 + (3*I*d^3*f^2*x^2 +
 4*d^2*f^2*x - 2*I*d*f^2)*e)*e^(4*d*x + 4*c) + 2*(d^3*f^3*x^3 - 2*I*d^2*f^3
*x^2 - 2*d*f^3*x + d^3*e^3 + (3*d^3*f*x - 2*I*d^2*f)*e^2 + (3*d^3*f^2*x^2 -
 4*I*d^2*f^2*x - 2*d*f^2)*e)*e^(3*d*x + 3*c) + 2*(-I*d^3*f^3*x^3 - 2*d^2*f^
3*x^2 + 2*I*d*f^3*x - I*d^3*e^3 + (-3*I*d^3*f*x - 2*d^2*f)*e^2 + (-3*I*d^3*
f^2*x^2 - 4*d^2*f^2*x + 2*I*d*f^2)*e)*e^(2*d*x + 2*c) - (d^3*f^3*x^3 - 2*I*
d^2*f^3*x^2 - 2*d*f^3*x + d^3*e^3 + (3*d^3*f*x - 2*I*d^2*f)*e^2 + (3*d^3*f^
2*x^2 - 4*I*d^2*f^2*x - 2*d*f^2)*e)*e^(d*x + c))*log(e^(d*x + c) + 1) - 12*
(c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2 + (I*c^2*f^3 - 2*I*c*d*f^2*e + I*d^2*f*e
^2)*e^(5*d*x + 5*c) + (c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)*e^(4*d*x + 4*c) +
 2*(-I*c^2*f^3 + 2*I*c*d*f^2*e - I*d^2*f*e^2)*e^(3*d*x + 3*c) - 2*(c^2*f^3
- 2*c*d*f^2*e + d^2*f*e^2)*e^(2*d*x + 2*c) + (I*c^2*f^3 - 2*I*c*d*f^2*e + I
*d^2*f*e^2)*e^(d*x + c))*log(e^(d*x + c) - I) - 3*((3*I*c + 2)*d^2*f*e^2 +
(-3*I*c^2 - 4*c + 2*I)*d*f^2*e + (I*c^3 + 2*c^2 - 2*I*c)*f^3 - I*d^3*e^3 -
((3*c - 2*I)*d^2*f*e^2 - (3*c^2 - 4*I*c - 2)*d*f^2*e + (c^3 - 2*I*c^2 - 2*c
)*f^3 - d^3*e^3)*e^(5*d*x + 5*c) + ((3*I*c + 2)*d^2*f*e^2 + (-3*I*c^2 - 4*c
 + 2*I)*d*f^2*e + (I*c^3 + 2*c^2 - 2*I*c)*f^3 - I*d^3*e^3)*e^(4*d*x + 4*c)
+ 2*((3*c - 2*I)*d^2*f*e^2 - (3*c^2 - 4*I*c - 2)*d*f^2*e + (c^3 - 2*I*c^2 -
 2*c)*f^3 - d^3*e^3)*e^(3*d*x + 3*c) + 2*((-3*I*c - 2)*d^2*f*e^2 + (3*I*c^2
 + 4*c - 2*I)*d*f^2*e + (-I*c^3 - 2*c^2 + 2*I*c)*f^3 + I*d^3*e^3)*e^(2*d*x
+ 2*c) - ((3*c - 2*I)*d^2*f*e^2 - (3*c^2 - 4*I*c - 2)*d*f^2*e + (c^3 - 2*I*
c^2 - 2*c)*f^3 - d^3*e^3)*e^(d*x + c))*log(e^(d*x + c) - 1) - 12*(d^2*f^3*x
^2 - c^2*f^3 + 2*(d^2*f^2*x + c*d*f^2)*e + (I*d^2*f^3*x^2 - I*c^2*f^3 + 2*(
```

```
I*d^2*f^2*x + I*c*d*f^2)*e)*e^(5*d*x + 5*c) + (d^2*f^3*x^2 - c^2*f^3 + 2*(d
^2*f^2*x + c*d*f^2)*e)*e^(4*d*x + 4*c) + 2*(-I*d^2*f^3*x^2 + I*c^2*f^3 + 2*
(-I*d^2*f^2*x - I*c*d*f^2)*e)*e^(3*d*x + 3*c) - 2*(d^2*f^3*x^2 - c^2*f^3 +
2*(d^2*f^2*x + c*d*f^2)*e)*e^(2*d*x + 2*c) + (I*d^2*f^3*x^2 - I*c^2*f^3 + 2
*(I*d^2*f^2*x + I*c*d*f^2)*e)*e^(d*x + c))*log(I*e^(d*x + c) + 1) - 3*(-I*d
^3*f^3*x^3 + 2*d^2*f^3*x^2 + 2*I*d*f^3*x + (-I*c^3 - 2*c^2 + 2*I*c)*f^3 + 3
*(-I*d^3*f*x - I*c*d^2*f)*e^2 + (-3*I*d^3*f^2*x^2 + 4*d^2*f^2*x + (3*I*c^2
+ 4*c)*d*f^2)*e + (d^3*f^3*x^3 + 2*I*d^2*f^3*x^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^3*csch(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^3}{\sinh\left(c+d\,x\right)^3\,\left(a+a\,\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(sinh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
[Out] int((e + f*x)^3/(sinh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)), x)
```

3.218 $\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=368

$$\frac{2i(e+fx)^2}{ad}+\frac{3(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{f^2\tanh^{-1}(\cosh(c+dx))}{ad^3}+\frac{i(e+fx)^2\coth(c+dx)}{ad}-\frac{f(e+fx)\operatorname{csch}}{ad^2}$$

```
[Out] 2*I*(f*x+e)^2/a/d+3*(f*x+e)^2*arctanh(exp(d*x+c))/a/d-f^2*arctanh(cosh(d*x+
c))/a/d^3+I*(f*x+e)^2*coth(d*x+c)/a/d-f*(f*x+e)*csch(d*x+c)/a/d^2-1/2*(f*x+
e)^2*coth(d*x+c)*csch(d*x+c)/a/d-4*I*f*(f*x+e)*ln(1+I*exp(d*x+c))/a/d^2-2*I
*f*(f*x+e)*ln(1-exp(2*d*x+2*c))/a/d^2+3*f*(f*x+e)*polylog(2,-exp(d*x+c))/a/
d^2-4*I*f^2*polylog(2,-I*exp(d*x+c))/a/d^3-3*f*(f*x+e)*polylog(2,exp(d*x+c)
)/a/d^2-I*f^2*polylog(2,exp(2*d*x+2*c))/a/d^3-3*f^2*polylog(3,-exp(d*x+c))/
a/d^3+3*f^2*polylog(3,exp(d*x+c))/a/d^3+I*(f*x+e)^2*tanh(1/2*c+1/4*I*Pi+1/2
*d*x)/a/d
```

Rubi [A]
time = 0.60, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {5694, 4271, 3855, 4267, 2611, 2320, 6724, 4269, 3797, 2221, 2317, 2438, 3399}

$$-\frac{4if^2\text{Li}_2\left(-ie^{c+dx}\right)}{ad^3}-\frac{if^2\text{Li}_2\left(e^{2(c+dx)}\right)}{ad^3}-\frac{3f^2\text{Li}_3\left(-e^{c+dx}\right)}{ad^3}+\frac{3f^2\text{Li}_3\left(e^{c+dx}\right)}{ad^3}-\frac{f^2\tanh^{-1}(\cosh(c+dx))}{ad^3}+\frac{3f(e+fx)\text{Li}_2\left(-e^{c+dx}\right)}{ad^2}-\frac{3f(e+fx)\text{Li}_2\left(e^{c+dx}\right)}{ad^2}-\frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}-\frac{2if(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad^2}-\frac{f(e+fx)\operatorname{csch}(c+dx)}{ad^2}+\frac{3(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}+\frac{i(e+fx)^2\coth(c+dx)}{ad}-\frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}+\frac{2i(e+fx)^2}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((2*I)*(e + f*x)^2)/(a*d) + (3*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (f
^2*ArcTanh[Cosh[c + d*x]])/(a*d^3) + (I*(e + f*x)^2*Coth[c + d*x])/(a*d) -
(f*(e + f*x)*Csch[c + d*x])/(a*d^2) - ((e + f*x)^2*Coth[c + d*x]*Csch[c + d
*x])/(2*a*d) - ((4*I)*f*(e + f*x)*Log[1 + I*E^(c + d*x)])/(a*d^2) - ((2*I)*
f*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a*d^2) + (3*f*(e + f*x)*PolyLog[2, -
E^(c + d*x)])/(a*d^2) - ((4*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^3) -
(3*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^2) - (I*f^2*PolyLog[2, E^(2*(c
 + d*x))])/(a*d^3) - (3*f^2*PolyLog[3, -E^(c + d*x)])/(a*d^3) + (3*f^2*Poly
Log[3, E^(c + d*x)])/(a*d^3) + (I*(e + f*x)^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2
])/(a*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5694

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a + b
*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\left(i\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx\right) + \frac{\int (e+fx)^2\operatorname{csch}^3(c+dx)\,dx}{a}$$

$$= -\frac{f(e+fx)\operatorname{csch}(c+dx)}{ad^2} - \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{i\int(e+$$

$$= \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{f^2\tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{i(e+fx)^2\coth(c+}{ad}$$

$$= \frac{i(e+fx)^2}{ad} + \frac{3(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{f^2\tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{i(e}{}$$

$$= \frac{i(e+fx)^2}{ad} + \frac{3(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{f^2\tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{i(e}{}$$

$$= \frac{2i(e+fx)^2}{ad} + \frac{3(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{f^2\tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{i(e}{}$$

$$= \frac{2i(e+fx)^2}{ad} + \frac{3(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{f^2\tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{i(e}{}$$

$$= \frac{2i(e+fx)^2}{ad} + \frac{3(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{f^2\tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{i(e}{}$$

$$= \frac{2i(e+fx)^2}{ad} + \frac{3(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{f^2\tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{i(e}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1370 vs. $2(368) = 736$.
time = 21.34, size = 1370, normalized size = 3.72

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (2*f*(d*(d*E^c*x*(2*e + f*x) - 2*(-I + E^c)*(e + f*x)*Log[1 + I*E^(c + d*x)
]) - 2*(-I + E^c)*f*PolyLog[2, (-I)*E^(c + d*x)]))/(a*d^3*(-1 - I*E^c)) + (
(8*I)*d*e*E^(2*c)*f*x - (8*I)*d*e*(-1 + E^(2*c))*f*x + (4*I)*d*E^(2*c)*f^2*
x^2 - (4*I)*d*(-1 + E^(2*c))*f^2*x^2 + 6*d*e^2*(-1 + E^(2*c))*ArcTanh[E^(c
+ d*x)] - (4*(-1 + E^(2*c))*f^2*ArcTanh[E^(c + d*x)])/d + (4*I)*e*(-1 + E^(
2*c))*f*(2*d*x - Log[1 - E^(2*(c + d*x))]) + 6*e*(-1 + E^(2*c))*f*(d*x*(-Lo
g[1 - E^(c + d*x)] + Log[1 + E^(c + d*x)]) + PolyLog[2, -E^(c + d*x)] - Pol
yLog[2, E^(c + d*x)]) + ((2*I)*(-1 + E^(2*c))*f^2*(2*d*x*(d*x - Log[1 - E^(
```

```
2*(c + d*x))]) - PolyLog[2, E^(2*(c + d*x))]))/d + (3*(-1 + E^(2*c))*f^2*(-
(d^2*x^2*Log[1 - E^(c + d*x)]) + d^2*x^2*Log[1 + E^(c + d*x)] + 2*d*x*PolyL
og[2, -E^(c + d*x)] - 2*d*x*PolyLog[2, E^(c + d*x)] - 2*PolyLog[3, -E^(c +
d*x)] + 2*PolyLog[3, E^(c + d*x)]))/d)/(2*a*d^2*(-1 + E^(2*c))) + (Csch[c]*
Csch[c + d*x]^2*(2*e*f*Cosh[(d*x)/2] + 2*f^2*x*Cosh[(d*x)/2] + 2*e*f*Cosh[(
3*d*x)/2] + 2*f^2*x*Cosh[(3*d*x)/2] + (5*I)*d*e^2*Cosh[c - (d*x)/2] + (10*I
)*d*e*f*x*Cosh[c - (d*x)/2] + (5*I)*d*f^2*x^2*Cosh[c - (d*x)/2] - I*d*e^2*C
osh[c + (d*x)/2] - (2*I)*d*e*f*x*Cosh[c + (d*x)/2] - I*d*f^2*x^2*Cosh[c + (
d*x)/2] - 2*e*f*Cosh[2*c + (d*x)/2] - 2*f^2*x*Cosh[2*c + (d*x)/2] + I*d*e^2
*Cosh[c + (3*d*x)/2] + (2*I)*d*e*f*x*Cosh[c + (3*d*x)/2] + I*d*f^2*x^2*Cosh
[c + (3*d*x)/2] - 2*e*f*Cosh[2*c + (3*d*x)/2] - 2*f^2*x*Cosh[2*c + (3*d*x)/
2] - (3*I)*d*e^2*Cosh[3*c + (3*d*x)/2] - (6*I)*d*e*f*x*Cosh[3*c + (3*d*x)/2
] - (3*I)*d*f^2*x^2*Cosh[3*c + (3*d*x)/2] - (4*I)*d*e^2*Cosh[c + (5*d*x)/2]
 - (8*I)*d*e*f*x*Cosh[c + (5*d*x)/2] - (4*I)*d*f^2*x^2*Cosh[c + (5*d*x)/2]
+ (2*I)*d*e^2*Cosh[3*c + (5*d*x)/2] + (4*I)*d*e*f*x*Cosh[3*c + (5*d*x)/2] +
 (2*I)*d*f^2*x^2*Cosh[3*c + (5*d*x)/2] - d*e^2*Sinh[(d*x)/2] - 2*d*e*f*x*Si
nh[(d*x)/2] - d*f^2*x^2*Sinh[(d*x)/2] - d*e^2*Sinh[(3*d*x)/2] - 2*d*e*f*x*S
inh[(3*d*x)/2] - d*f^2*x^2*Sinh[(3*d*x)/2] + (2*I)*e*f*Sinh[c - (d*x)/2] +
(2*I)*f^2*x*Sinh[c - (d*x)/2] + (2*I)*e*f*Sinh[c + (d*x)/2] + (2*I)*f^2*x*S
inh[c + (d*x)/2] - 3*d*e^2*Sinh[2*c + (d*x)/2] - 6*d*e*f*x*Sinh[2*c + (d*x)
/2] - 3*d*f^2*x^2*Sinh[2*c + (d*x)/2] + (2*I)*e*f*Sinh[c + (3*d*x)/2] + (2*
I)*f^2*x*Sinh[c + (3*d*x)/2] - d*e^2*Sinh[2*c + (3*d*x)/2] - 2*d*e*f*x*Sinh
[2*c + (3*d*x)/2] - d*f^2*x^2*Sinh[2*c + (3*d*x)/2] - (2*I)*e*f*Sinh[3*c +
(3*d*x)/2] - (2*I)*f^2*x*Sinh[3*c + (3*d*x)/2] + 2*d*e^2*Sinh[2*c + (5*d*x)
/2] + 4*d*e*f*x*Sinh[2*c + (5*d*x)/2] + 2*d*f^2*x^2*Sinh[2*c + (5*d*x)/2]))
/(8*a*d^2*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*
x)/2]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1106 vs. $2(343) = 686$.
time = 3.25, size = 1107, normalized size = 3.01

method	result
risch	$-\frac{2i\ln(1-\mathrm{e}^{dx+c})c f^2}{a d^3}-\frac{3f^2\,\mathrm{polylog}(3,-\mathrm{e}^{dx+c})}{a d^3}+\frac{2if^2c\ln(\mathrm{e}^{dx+c}-1)}{a d^3}-\frac{4if^2\ln(1+i\mathrm{e}^{dx+c})x}{a d^2}-\frac{8if^2c\ln(\mathrm{e}^{dx+c})}{a d^3}-\frac{4if^2\ln(1+}{a d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -3*f^2*polylog(3,-exp(d*x+c))/a/d^3+3*f^2*polylog(3,exp(d*x+c))/a/d^3-8*I/a
/d^3*c*f^2*ln(exp(d*x+c))+4*I/a/d^3*f^2*c*ln(exp(d*x+c)-I)+2*I/a/d^3*f^2*c*
ln(exp(d*x+c)-1)+8*I/a/d^2*c*f^2*x-3/2/a/d*e^2*ln(exp(d*x+c)-1)+3/2/a/d*e^2
*ln(exp(d*x+c)+1)+1/a/d^3*f^2*ln(exp(d*x+c)-1)-1/a/d^3*f^2*ln(exp(d*x+c)+1)
-4*I*f^2*polylog(2,-I*exp(d*x+c))/a/d^3-3/a/d^2*polylog(2,exp(d*x+c))*f^2*x
+3/a/d^2*polylog(2,-exp(d*x+c))*f^2*x+3/2/a/d*ln(exp(d*x+c)+1)*f^2*x^2+3/a/
```

```
d^2*e*f*polylog(2,-exp(d*x+c))-3/a/d^2*e*f*polylog(2,exp(d*x+c))-3/2/a/d*ln
(1-exp(d*x+c))*f^2*x^2+3/a/d*ln(exp(d*x+c)+1)*e*f*x-3/a/d*ln(1-exp(d*x+c))*
e*f*x-3/a/d^2*ln(1-exp(d*x+c))*c*e*f-3/2/a/d^3*f^2*c^2*ln(exp(d*x+c)-1)+3/2
/a/d^3*ln(1-exp(d*x+c))*c^2*f^2-2*I/a/d^3*f^2*polylog(2,exp(d*x+c))-2*I/a/d
^3*f^2*polylog(2,-exp(d*x+c))+4*I/a/d^3*c^2*f^2+4*I/a/d*f^2*x^2-(6*d*e*f*x*
exp(4*d*x+4*c)-2*I*e*f*exp(3*d*x+3*c)+I*d*f^2*x^2*exp(d*x+c)+3*d*f^2*x^2*ex
p(4*d*x+4*c)-6*I*d*e*f*x*exp(3*d*x+3*c)+4*d*e^2+2*I*e*f*exp(d*x+c)+2*I*d*e*
f*x*exp(d*x+c)-3*I*d*f^2*x^2*exp(3*d*x+3*c)-2*I*f^2*x*exp(3*d*x+3*c)-3*I*d*
e^2*exp(3*d*x+3*c)-5*d*e^2*exp(2*d*x+2*c)+8*d*e*f*x+3*d*e^2*exp(4*d*x+4*c)+
2*f^2*x*exp(4*d*x+4*c)+2*e*f*exp(4*d*x+4*c)-2*f^2*x*exp(2*d*x+2*c)-2*e*f*ex
p(2*d*x+2*c)+4*d*f^2*x^2-10*d*e*f*x*exp(2*d*x+2*c)-5*d*f^2*x^2*exp(2*d*x+2*
c)+2*I*f^2*x*exp(d*x+c)+I*d*e^2*exp(d*x+c))/(exp(2*d*x+2*c)-1)^2/d^2/(exp(d
*x+c)-I)/a+3/a/d^2*e*f*c*ln(exp(d*x+c)-1)+8*I/a/d^2*e*f*ln(exp(d*x+c))-2*I/
a/d^2*e*f*ln(exp(d*x+c)+1)-4*I/a/d^2*e*f*ln(exp(d*x+c)-I)-2*I/a/d^2*e*f*ln(
exp(d*x+c)-1)-4*I/a/d^2*ln(1+I*exp(d*x+c))*f^2*x-2*I/a/d^2*ln(exp(d*x+c)+1)
*f^2*x-2*I/a/d^2*ln(1-exp(d*x+c))*f^2*x-4*I/a/d^3*ln(1+I*exp(d*x+c))*c*f^2-
2*I/a/d^3*ln(1-exp(d*x+c))*c*f^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 897 vs. 2(339) = 678.
time = 0.52, size = 897, normalized size = 2.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 2*I*f^2*x^2/(a*d) - 1/2*(2*(-I*e^(-d*x - c) - 5*e^(-2*d*x - 2*c) + 3*I*e^(-
3*d*x - 3*c) + 3*e^(-4*d*x - 4*c) + 4)/((a*e^(-d*x - c) - 2*I*a*e^(-2*d*x -
 2*c) - 2*a*e^(-3*d*x - 3*c) + I*a*e^(-4*d*x - 4*c) + a*e^(-5*d*x - 5*c) +
I*a)*d) - 3*log(e^(-d*x - c) + 1)/(a*d) + 3*log(e^(-d*x - c) - 1)/(a*d))*e^
2 + 4*I*f*x*e/(a*d) - (4*d*f^2*x^2 + 8*d*f*x*e + (3*d*f^2*x^2*e^(4*c) + 2*(
f^2*e^(4*c) + 3*d*f*e^(4*c + 1))*x + 2*f*e^(4*c + 1))*e^(4*d*x) + (-3*I*d*f
^2*x^2*e^(3*c) - 2*(I*f^2*e^(3*c) + 3*I*d*f*e^(3*c + 1))*x - 2*I*f*e^(3*c +
 1))*e^(3*d*x) - (5*d*f^2*x^2*e^(2*c) + 2*(f^2*e^(2*c) + 5*d*f*e^(2*c + 1))
*x + 2*f*e^(2*c + 1))*e^(2*d*x) + (I*d*f^2*x^2*e^c - 2*(-I*d*f*e^(c + 1) -
I*f^2*e^c)*x + 2*I*f*e^(c + 1))*e^(d*x))/(a*d^2*e^(5*d*x + 5*c) - I*a*d^2*e
^(4*d*x + 4*c) - 2*a*d^2*e^(3*d*x + 3*c) + 2*I*a*d^2*e^(2*d*x + 2*c) + a*d^
2*e^(d*x + c) - I*a*d^2) - 4*I*f*e*log(I*e^(d*x + c) + 1)/(a*d^2) + 3/2*(d^
2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d
*x + c)))*f^2/(a*d^3) - 3/2*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^
(d*x + c)) - 2*polylog(3, e^(d*x + c)))*f^2/(a*d^3) - 4*I*(d*x*log(I*e^(d*x
 + c) + 1) + dilog(-I*e^(d*x + c)))*f^2/(a*d^3) + (2*I*d*f*e + f^2)*x/(a*d^
2) + (2*I*d*f*e - f^2)*x/(a*d^2) + (3*d*f*e - 2*I*f^2)*(d*x*log(e^(d*x + c)
```

```
 + 1) + dilog(-e^(d*x + c)))/(a*d^3) - (3*d*f*e + 2*I*f^2)*(d*x*log(-e^(d*x
 + c) + 1) + dilog(e^(d*x + c)))/(a*d^3) - (2*I*d*f*e + f^2)*log(e^(d*x + c
) + 1)/(a*d^3) - (2*I*d*f*e - f^2)*log(e^(d*x + c) - 1)/(a*d^3) + 1/2*(d^3*
f^2*x^3 + (3*d*f*e + 2*I*f^2)*d^2*x^2)/(a*d^3) - 1/2*(d^3*f^2*x^3 + (3*d*f*
e - 2*I*f^2)*d^2*x^2)/(a*d^3)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2238 vs. $2(339) = 678$.
time = 0.38, size = 2238, normalized size = 6.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/2*(8*c^2*f^2 - 16*c*d*f*e + 8*d^2*e^2 + 8*(I*f^2*e^(5*d*x + 5*c) + f^2*e
^(4*d*x + 4*c) - 2*I*f^2*e^(3*d*x + 3*c) - 2*f^2*e^(2*d*x + 2*c) + I*f^2*e^
(d*x + c) + f^2)*dilog(-I*e^(d*x + c)) + 2*(3*I*d*f^2*x + 3*I*d*f*e + 2*f^2
 - (3*d*f^2*x + 3*d*f*e - 2*I*f^2)*e^(5*d*x + 5*c) + (3*I*d*f^2*x + 3*I*d*f
*e + 2*f^2)*e^(4*d*x + 4*c) + 2*(3*d*f^2*x + 3*d*f*e - 2*I*f^2)*e^(3*d*x +
3*c) + 2*(-3*I*d*f^2*x - 3*I*d*f*e - 2*f^2)*e^(2*d*x + 2*c) - (3*d*f^2*x +
3*d*f*e - 2*I*f^2)*e^(d*x + c))*dilog(-e^(d*x + c)) + 2*(-3*I*d*f^2*x - 3*I
*d*f*e + 2*f^2 + (3*d*f^2*x + 3*d*f*e + 2*I*f^2)*e^(5*d*x + 5*c) + (-3*I*d*
f^2*x - 3*I*d*f*e + 2*f^2)*e^(4*d*x + 4*c) - 2*(3*d*f^2*x + 3*d*f*e + 2*I*f
^2)*e^(3*d*x + 3*c) + 2*(3*I*d*f^2*x + 3*I*d*f*e - 2*f^2)*e^(2*d*x + 2*c) +
 (3*d*f^2*x + 3*d*f*e + 2*I*f^2)*e^(d*x + c))*dilog(e^(d*x + c)) + 8*(-I*d^
2*f^2*x^2 + I*c^2*f^2 + 2*(-I*d^2*f*x - I*c*d*f)*e)*e^(5*d*x + 5*c) - 2*(d^
2*f^2*x^2 - 4*c^2*f^2 - 2*d*f^2*x - 3*d^2*e^2 + 2*(d^2*f*x + (4*c - 1)*d*f)
*e)*e^(4*d*x + 4*c) + 2*(5*I*d^2*f^2*x^2 - 8*I*c^2*f^2 - 2*I*d*f^2*x - 3*I*
d^2*e^2 + 2*(5*I*d^2*f*x + (8*I*c - I)*d*f)*e)*e^(3*d*x + 3*c) + 2*(3*d^2*f
^2*x^2 - 8*c^2*f^2 - 2*d*f^2*x - 5*d^2*e^2 + 2*(3*d^2*f*x + (8*c - 1)*d*f)*
e)*e^(2*d*x + 2*c) + 2*(-3*I*d^2*f^2*x^2 + 4*I*c^2*f^2 + 2*I*d*f^2*x + I*d^
2*e^2 + 2*(-3*I*d^2*f*x + (-4*I*c + I)*d*f)*e)*e^(d*x + c) - (-3*I*d^2*f^2*
x^2 - 4*d*f^2*x - 3*I*d^2*e^2 + 2*I*f^2 - 2*(3*I*d^2*f*x + 2*d*f)*e + (3*d^
2*f^2*x^2 - 4*I*d*f^2*x + 3*d^2*e^2 - 2*f^2 + 2*(3*d^2*f*x - 2*I*d*f)*e)*e^
(5*d*x + 5*c) + (-3*I*d^2*f^2*x^2 - 4*d*f^2*x - 3*I*d^2*e^2 + 2*I*f^2 - 2*(
3*I*d^2*f*x + 2*d*f)*e)*e^(4*d*x + 4*c) - 2*(3*d^2*f^2*x^2 - 4*I*d*f^2*x +
3*d^2*e^2 - 2*f^2 + 2*(3*d^2*f*x - 2*I*d*f)*e)*e^(3*d*x + 3*c) - 2*(-3*I*d^
2*f^2*x^2 - 4*d*f^2*x - 3*I*d^2*e^2 + 2*I*f^2 + 2*(-3*I*d^2*f*x - 2*d*f)*e)
*e^(2*d*x + 2*c) + (3*d^2*f^2*x^2 - 4*I*d*f^2*x + 3*d^2*e^2 - 2*f^2 + 2*(3*
d^2*f*x - 2*I*d*f)*e)*e^(d*x + c))*log(e^(d*x + c) + 1) - 8*(c*f^2 - d*f*e
- (-I*c*f^2 + I*d*f*e)*e^(5*d*x + 5*c) + (c*f^2 - d*f*e)*e^(4*d*x + 4*c) -
2*(I*c*f^2 - I*d*f*e)*e^(3*d*x + 3*c) - 2*(c*f^2 - d*f*e)*e^(2*d*x + 2*c) -
 (-I*c*f^2 + I*d*f*e)*e^(d*x + c))*log(e^(d*x + c) - I) + (2*(3*I*c + 2)*d*
```

```
f*e - (3*I*c^2 + 4*c - 2*I)*f^2 - 3*I*d^2*e^2 - (2*(3*c - 2*I)*d*f*e - (3*c
^2 - 4*I*c - 2)*f^2 - 3*d^2*e^2)*e^(5*d*x + 5*c) + (2*(3*I*c + 2)*d*f*e - (
3*I*c^2 + 4*c - 2*I)*f^2 - 3*I*d^2*e^2)*e^(4*d*x + 4*c) + 2*(2*(3*c - 2*I)*
d*f*e - (3*c^2 - 4*I*c - 2)*f^2 - 3*d^2*e^2)*e^(3*d*x + 3*c) + 2*(2*(-3*I*c
 - 2)*d*f*e + (3*I*c^2 + 4*c - 2*I)*f^2 + 3*I*d^2*e^2)*e^(2*d*x + 2*c) - (2
*(3*c - 2*I)*d*f*e - (3*c^2 - 4*I*c - 2)*f^2 - 3*d^2*e^2)*e^(d*x + c))*log(
e^(d*x + c) - 1) + 8*(d*f^2*x + c*f^2 + (I*d*f^2*x + I*c*f^2)*e^(5*d*x + 5*
c) + (d*f^2*x + c*f^2)*e^(4*d*x + 4*c) + 2*(-I*d*f^2*x - I*c*f^2)*e^(3*d*x
+ 3*c) - 2*(d*f^2*x + c*f^2)*e^(2*d*x + 2*c) + (I*d*f^2*x + I*c*f^2)*e^(d*x
 + c))*log(I*e^(d*x + c) + 1) - (3*I*d^2*f^2*x^2 - 4*d*f^2*x + (-3*I*c^2 -
4*c)*f^2 - 6*(-I*d^2*f*x - I*c*d*f)*e - (3*d^2*f^2*x^2 + 4*I*d*f^2*x - (3*c
^2 - 4*I*c)*f^2 + 6*(d^2*f*x + c*d*f)*e)*e^(5*d*x + 5*c) + (3*I*d^2*f^2*x^2
 - 4*d*f^2*x + (-3*I*c^2 - 4*c)*f^2 - 6*(-I*d^2*f*x - I*c*d*f)*e)*e^(4*d*x
+ 4*c) + 2*(3*d^2*f^2*x^2 + 4*I*d*f^2*x - (3*c^2 - 4*I*c)*f^2 + 6*(d^2*f*x
+ c*d*f)*e)*e^(3*d*x + 3*c) - 2*(3*I*d^2*f^2*x^2 - 4*d*f^2*x + (-3*I*c^2 -
4*c)*f^2 + 6*(I*d^2*f*x + I*c*d*f)*e)*e^(2*d*x + 2*c) - (3*d^2*f^2*x^2 + 4*
I*d*f^2*x - (3*c^2 - 4*I*c)*f^2 + 6*(d^2*f*x + c*d*f)*e)*e^(d*x + c))*log(-
e^(d*x + c) + 1) + 6*(f^2*e^(5*d*x + 5*c) - I*f^2*e^(4*d*x + 4*c) - 2*f^2*e
^(3*d*x + 3*c) + 2*I*f^2*e^(2*d*x + 2*c) + f^2*e^(d*x + c) - I*f^2)*polylog
(3, -e^(d*x + c)) - 6*(f^2*e^(5*d*x + 5*c) - I*f^2*e^(4*d*x + 4*c) - 2*f^2*
e^(3*d*x + 3*c) + 2*I*f^2*e^(2*d*x + 2*c) + f^2*e^(d*x + c) - I*f^2)*polylo
g(3, e^(d*x + c)))/(a*d^3*e^(5*d*x + 5*c) - I*a*d^3*e^(4*d*x + 4*c) - 2*a*d
^3*e^(3*d*x + 3*c) + 2*I*a*d^3*e^(2*d*x + 2*c) + a*d^3*e^(d*x + c) - I*a*d^
3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*csch(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^2}{\sinh(c+d\,x)^3\,\left(a+a\sinh(c+d\,x)\;1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(sinh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
[Out] int((e + f*x)^2/(sinh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)), x)
```

3.219 $\int \frac{(e+fx)\mathrm{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=214

$$\frac{3(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{i(e+fx)\coth(c+dx)}{ad}-\frac{f\mathrm{csch}(c+dx)}{2ad^2}-\frac{(e+fx)\coth(c+dx)\mathrm{csch}(c+dx)}{2ad}-\frac{2i$$

```
[Out] 3*(f*x+e)*arctanh(exp(d*x+c))/a/d+I*(f*x+e)*coth(d*x+c)/a/d-1/2*f*csch(d*x+
c)/a/d^2-1/2*(f*x+e)*coth(d*x+c)*csch(d*x+c)/a/d-2*I*f*ln(cosh(1/2*c+1/4*I*
Pi+1/2*d*x))/a/d^2-I*f*ln(sinh(d*x+c))/a/d^2+3/2*f*polylog(2,-exp(d*x+c))/a
/d^2-3/2*f*polylog(2,exp(d*x+c))/a/d^2+I*(f*x+e)*tanh(1/2*c+1/4*I*Pi+1/2*d*
x)/a/d
```

Rubi [A]
time = 0.27, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {5694, 4270, 4267, 2317, 2438, 4269, 3556, 3399}

$$\frac{3f\text{Li}_2\left(-e^{c+dx}\right)}{2ad^2}-\frac{3f\text{Li}_2\left(e^{c+dx}\right)}{2ad^2}-\frac{f\mathrm{csch}(c+dx)}{2ad^2}-\frac{if\log(\sinh(c+dx))}{ad^2}-\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{ad^2}+\frac{3(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}+\frac{i(e+fx)\coth(c+dx)}{ad}-\frac{(e+fx)\coth(c+dx)\mathrm{csch}(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (3*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d) + (I*(e + f*x)*Coth[c + d*x])/(a*d
) - (f*Csch[c + d*x])/(2*a*d^2) - ((e + f*x)*Coth[c + d*x]*Csch[c + d*x])/(
2*a*d) - ((2*I)*f*Log[Cosh[c/2 + (I/4)*Pi + (d*x)/2]])/(a*d^2) - (I*f*Log[S
inh[c + d*x]])/(a*d^2) + (3*f*PolyLog[2, -E^(c + d*x)])/(2*a*d^2) - (3*f*Po
lyLog[2, E^(c + d*x)])/(2*a*d^2) + (I*(e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)
/2])/(a*d)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
```

```
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5694

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a + b
*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rubi steps

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\left(i\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx\right) + \frac{\int (e+fx)\operatorname{csch}^3(c+dx)\,dx}{a}$$

$$= -\frac{f\operatorname{csch}(c+dx)}{2ad^2} - \frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{i\int (e+fx)\operatorname{csch}^2(c \cdots}{a}$$

$$= \frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{i(e+fx)\coth(c+dx)}{ad} - \frac{f\operatorname{csch}(c+dx)}{2ad^2} - \frac{(e+ \cdots}{}$$

$$= \frac{3(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{i(e+fx)\coth(c+dx)}{ad} - \frac{f\operatorname{csch}(c+dx)}{2ad^2} - \frac{(e+ \cdots}{}$$

$$= \frac{3(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{i(e+fx)\coth(c+dx)}{ad} - \frac{f\operatorname{csch}(c+dx)}{2ad^2} - \frac{(e+ \cdots}{}$$

$$= \frac{3(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{i(e+fx)\coth(c+dx)}{ad} - \frac{f\operatorname{csch}(c+dx)}{2ad^2} - \frac{(e+ \cdots}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 541 vs. $2(214) = 428$.
time = 1.76, size = 541, normalized size = 2.53

$$\frac{\left(\cosh\left(\tfrac{1}{2}(c+dx)\right)+i\sinh\left(\tfrac{1}{2}(c+dx)\right)\right)\left(2i(if+2d(e+fx))\cosh\left(\tfrac{1}{2}(c+dx)\right)\left(i+\coth\left(\tfrac{1}{2}(c+dx)\right)\right)-d(e+fx)\left(i+\coth\left(\tfrac{1}{2}(c+dx)\right)\right)\operatorname{csch}\left(\tfrac{1}{2}(c+dx)\right)-8f(c+dx)\left(\cosh\left(\tfrac{1}{2}(c+dx)\right)+i\sinh\left(\tfrac{1}{2}(c+dx)\right)\right)+16f\operatorname{ArcTan}\left(\tanh\left(\tfrac{1}{2}(c+dx)\right)\right)\left(\cosh\left(\tfrac{1}{2}(c+dx)\right)+i\sinh\left(\tfrac{1}{2}(c+dx)\right)\right)-12de\log\left(\tanh\left(\tfrac{1}{2}(c+dx)\right)\right)\left(\cosh\left(\tfrac{1}{2}(c+dx)\right)+i\sinh\left(\tfrac{1}{2}(c+dx)\right)\right)+12cf\log\left(\tanh\left(\tfrac{1}{2}(c+dx)\right)\right)\left(\cosh\left(\tfrac{1}{2}(c+dx)\right)+i\sinh\left(\tfrac{1}{2}(c+dx)\right)\right)-12f\left((c+dx)\left(\log\left(1-e^{-c-dx}\right)-\log\left(1+e^{-c-dx}\right)\right)+\operatorname{PolyLog}\left(2,-e^{-c-dx}\right)-\operatorname{PolyLog}\left(2,e^{-c-dx}\right)\right)\left(\cosh\left(\tfrac{1}{2}(c+dx)\right)+i\sinh\left(\tfrac{1}{2}(c+dx)\right)\right)+16id(e+fx)\sinh\left(\tfrac{1}{2}(c+dx)\right)+8f\log(\cosh(c+dx))\left(-i\cosh\left(\tfrac{1}{2}(c+dx)\right)+\sinh\left(\tfrac{1}{2}(c+dx)\right)\right)+8f\log(\sinh(c+dx))\left(-i\cosh\left(\tfrac{1}{2}(c+dx)\right)+\sinh\left(\tfrac{1}{2}(c+dx)\right)\right)+2(f+2id(e+fx))\left(\cosh\left(\tfrac{1}{2}(c+dx)\right)+i\sinh\left(\tfrac{1}{2}(c+dx)\right)\right)\tanh\left(\tfrac{1}{2}(c+dx)\right)-id(e+fx)\operatorname{sech}\left(\tfrac{1}{2}(c+dx)\right)\left(-i+\tanh\left(\tfrac{1}{2}(c+dx)\right)\right)\right)}{8d^2(a+ia\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*((2*I)*(I*f + 2*d*(e + f*x))*Cos
h[(c + d*x)/2]*(I + Coth[(c + d*x)/2]) - d*(e + f*x)*(I + Coth[(c + d*x)/2]
)*Csch[(c + d*x)/2] - 8*f*(c + d*x)*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2
]) + 16*f*ArcTan[Tanh[(c + d*x)/2]]*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2
]) - 12*d*e*Log[Tanh[(c + d*x)/2]]*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]
) + 12*c*f*Log[Tanh[(c + d*x)/2]]*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])
 - 12*f*((c + d*x)*(Log[1 - E^(-c - d*x)] - Log[1 + E^(-c - d*x)]) + PolyLo
g[2, -E^(-c - d*x)] - PolyLog[2, E^(-c - d*x)])*(Cosh[(c + d*x)/2] + I*Sinh
[(c + d*x)/2]) + (16*I)*d*(e + f*x)*Sinh[(c + d*x)/2] + 8*f*Log[Cosh[c + d*
x]]*((-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]) + 8*f*Log[Sinh[c + d*x]]*(
(-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]) + 2*(f + (2*I)*d*(e + f*x))*(Co
sh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*Tanh[(c + d*x)/2] - I*d*(e + f*x)*Se
ch[(c + d*x)/2]*(-I + Tanh[(c + d*x)/2])))/(8*d^2*(a + I*a*Sinh[c + d*x]))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(185) = 370$.
time = 4.02, size = 423, normalized size = 1.98

method	result
risch	$-\frac{-3ide\,\mathrm{e}^{3dx+3c}-5dfx\,\mathrm{e}^{2dx+2c}+3dfx\,\mathrm{e}^{4dx+4c}+idfx\,\mathrm{e}^{dx+c}-5de\,\mathrm{e}^{2dx+2c}+3de\,\mathrm{e}^{4dx+4c}-i\mathrm{e}^{3dx+3c}f+4dxf+f\,\mathrm{e}^{4dx+4c}+ide\,\mathrm{e}^{dx+c}+i\mathrm{e}^{dx}}{\left(\mathrm{e}^{2dx+2c}-1\right)^{2}d^{2}\left(\mathrm{e}^{dx+c}-i\right)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -(-3*I*d*e*exp(3*d*x+3*c)-5*d*f*x*exp(2*d*x+2*c)+3*d*f*x*exp(4*d*x+4*c)+I*d
*f*x*exp(d*x+c)-5*d*e*exp(2*d*x+2*c)+3*d*e*exp(4*d*x+4*c)-I*exp(3*d*x+3*c)*
f+4*d*x*f+f*exp(4*d*x+4*c)+I*d*e*exp(d*x+c)+I*exp(d*x+c)*f+4*d*e-f*exp(2*d*
x+2*c)-3*I*d*f*x*exp(3*d*x+3*c))/(exp(2*d*x+2*c)-1)^2/d^2/(exp(d*x+c)-I)/a-
I/a/d^2*f*ln(exp(d*x+c)+1)-I/a/d^2*f*ln(exp(d*x+c)-1)+3/2/a/d*e*ln(exp(d*x+
c)+1)-3/2/a/d*e*ln(exp(d*x+c)-1)+3/2/a/d^2*f*c*ln(exp(d*x+c)-1)-2*I*f/a/d^2
*ln(exp(d*x+c)-I)+4*I/a/d^2*f*ln(exp(d*x+c))+3/2/a/d*ln(exp(d*x+c)+1)*f*x-3
/2/a/d*ln(1-exp(d*x+c))*f*x-3/2/a/d^2*ln(1-exp(d*x+c))*c*f+3/2*f*polylog(2,
-exp(d*x+c))/a/d^2-3/2*f*polylog(2,exp(d*x+c))/a/d^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(24*d*integrate(1/16*x/(a*d*e^(d*x + c) + a*d), x) + 24*d*integrate(1/16*x
/(a*d*e^(d*x + c) - a*d), x) + 8*(2*d*x*e^(5*d*x + 5*c) + 2*I*d*x + (I*d*x*
e^(4*c) + I*e^(4*c))*e^(4*d*x) - (d*x*e^(3*c) - e^(3*c))*e^(3*d*x) + (-I*d*
x*e^(2*c) - I*e^(2*c))*e^(2*d*x) + (d*x*e^c - e^c)*e^(d*x))/(8*I*a*d^2*e^(5
*d*x + 5*c) + 8*a*d^2*e^(4*d*x + 4*c) - 16*I*a*d^2*e^(3*d*x + 3*c) - 16*a*d
^2*e^(2*d*x + 2*c) + 8*I*a*d^2*e^(d*x + c) + 8*a*d^2) - 2*I*(d*x + c)/(a*d^
2) + 2*I*log((e^(d*x + c) - I)*e^(-c))/(a*d^2) + I*log(e^(d*x + c) + 1)/(a*
d^2) + I*log(e^(d*x + c) - 1)/(a*d^2))*f - 1/2*(2*(-I*e^(-d*x - c) - 5*e^(-
2*d*x - 2*c) + 3*I*e^(-3*d*x - 3*c) + 3*e^(-4*d*x - 4*c) + 4)/((a*e^(-d*x -
 c) - 2*I*a*e^(-2*d*x - 2*c) - 2*a*e^(-3*d*x - 3*c) + I*a*e^(-4*d*x - 4*c)
+ a*e^(-5*d*x - 5*c) + I*a)*d) - 3*log(e^(-d*x - c) + 1)/(a*d) + 3*log(e^(-
d*x - c) - 1)/(a*d))*e
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 827 vs. $2(185) = 370$.
time = 0.39, size = 827, normalized size = 3.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*c*f + 3*(f*e^(5*d*x + 5*c) - I*f*e^(4*d*x + 4*c) - 2*f*e^(3*d*x + 3*
c) + 2*I*f*e^(2*d*x + 2*c) + f*e^(d*x + c) - I*f)*dilog(-e^(d*x + c)) - 3*(
f*e^(5*d*x + 5*c) - I*f*e^(4*d*x + 4*c) - 2*f*e^(3*d*x + 3*c) + 2*I*f*e^(2*
d*x + 2*c) + f*e^(d*x + c) - I*f)*dilog(e^(d*x + c)) - 8*d*e - 4*(-2*I*d*f*
x - I*c*f)*e^(5*d*x + 5*c) + 2*(d*f*x + (2*c - 1)*f - 3*d*e)*e^(4*d*x + 4*c
) - 2*(5*I*d*f*x + (4*I*c - I)*f - 3*I*d*e)*e^(3*d*x + 3*c) - 2*(3*d*f*x +
(4*c - 1)*f - 5*d*e)*e^(2*d*x + 2*c) - 2*(-3*I*d*f*x + (-2*I*c + I)*f + I*d
*e)*e^(d*x + c) + (-3*I*d*f*x - 3*I*d*e + (3*d*f*x + 3*d*e - 2*I*f)*e^(5*d*
x + 5*c) + (-3*I*d*f*x - 3*I*d*e - 2*f)*e^(4*d*x + 4*c) - 2*(3*d*f*x + 3*d*
e - 2*I*f)*e^(3*d*x + 3*c) - 2*(-3*I*d*f*x - 3*I*d*e - 2*f)*e^(2*d*x + 2*c)
 + (3*d*f*x + 3*d*e - 2*I*f)*e^(d*x + c) - 2*f)*log(e^(d*x + c) + 1) - 4*(I
*f*e^(5*d*x + 5*c) + f*e^(4*d*x + 4*c) - 2*I*f*e^(3*d*x + 3*c) - 2*f*e^(2*d
*x + 2*c) + I*f*e^(d*x + c) + f)*log(e^(d*x + c) - I) + ((-3*I*c - 2)*f + 3
*I*d*e + ((3*c - 2*I)*f - 3*d*e)*e^(5*d*x + 5*c) + ((-3*I*c - 2)*f + 3*I*d*
e)*e^(4*d*x + 4*c) - 2*((3*c - 2*I)*f - 3*d*e)*e^(3*d*x + 3*c) - 2*((-3*I*c
 - 2)*f + 3*I*d*e)*e^(2*d*x + 2*c) + ((3*c - 2*I)*f - 3*d*e)*e^(d*x + c))*l
og(e^(d*x + c) - 1) - 3*(-I*d*f*x - I*c*f + (d*f*x + c*f)*e^(5*d*x + 5*c) +
 (-I*d*f*x - I*c*f)*e^(4*d*x + 4*c) - 2*(d*f*x + c*f)*e^(3*d*x + 3*c) + 2*(
I*d*f*x + I*c*f)*e^(2*d*x + 2*c) + (d*f*x + c*f)*e^(d*x + c))*log(-e^(d*x +
 c) + 1))/(a*d^2*e^(5*d*x + 5*c) - I*a*d^2*e^(4*d*x + 4*c) - 2*a*d^2*e^(3*d
*x + 3*c) + 2*I*a*d^2*e^(2*d*x + 2*c) + a*d^2*e^(d*x + c) - I*a*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{e\operatorname{csch}^{3}(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{fx\operatorname{csch}^{3}(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*(Integral(e*csch(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(f*x*csch
(c + d*x)**3/(sinh(c + d*x) - I), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*csch(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{{\mathrm{sinh}\left(c+d\,x\right)}^3\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(sinh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
[Out] int((e + f*x)/(sinh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)), x)
```

3.220 $\int \frac{\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=87

$$\frac{3\tanh^{-1}(\cosh(c+dx))}{2ad}+\frac{2i\coth(c+dx)}{ad}-\frac{3\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}+\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{d(a+ia\sinh(c+dx))}$$

```
[Out] 3/2*arctanh(cosh(d*x+c))/a/d+2*I*coth(d*x+c)/a/d-3/2*coth(d*x+c)*csch(d*x+c
)/a/d+coth(d*x+c)*csch(d*x+c)/d/(a+I*a*sinh(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2847, 2827, 3853, 3855, 3852, 8}

$$\frac{2i\coth(c+dx)}{ad}+\frac{3\tanh^{-1}(\cosh(c+dx))}{2ad}-\frac{3\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}+\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{d(a+ia\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Cosh[c + d*x]])/(2*a*d) + ((2*I)*Coth[c + d*x])/(a*d) - (3*Coth[
c + d*x]*Csch[c + d*x])/(2*a*d) + (Coth[c + d*x]*Csch[c + d*x])/(d*(a + I*a
*Sinh[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx &= \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{d(a+ia\sinh(c+dx))} - \frac{\int \operatorname{csch}^3(c+dx)(-3a+2ia\sinh(c+dx))\,dx}{a^2} \\ &= \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{d(a+ia\sinh(c+dx))} - \frac{(2i)\int \operatorname{csch}^2(c+dx)\,dx}{a} + \frac{3\int \operatorname{csch}^3(c+dx)\,dx}{a} \\ &= -\frac{3\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} + \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{d(a+ia\sinh(c+dx))} - \frac{3\int \operatorname{csch}(c+dx)\,dx}{2a} \\ &= \frac{3\tanh^{-1}(\cosh(c+dx))}{2ad} + \frac{2i\coth(c+dx)}{ad} - \frac{3\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} + \frac{\coth}{d(c}\end{aligned}$$

Mathematica [A]
time = 0.35, size = 90, normalized size = 1.03

$$\frac{4i\operatorname{csch}(2(c+dx)) - 3\operatorname{sech}(c+dx) + 3\tanh^{-1}\left(\sqrt{\cosh^2(c+dx)}\right)\sqrt{\cosh^2(c+dx)}\operatorname{sech}(c+dx) - \operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx) + 4i\tanh(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((4*I)*Csch[2*(c + d*x)] - 3*Sech[c + d*x] + 3*ArcTanh[Sqrt[Cosh[c + d*x]^2
]]*Sqrt[Cosh[c + d*x]^2]*Sech[c + d*x] - Csch[c + d*x]^2*Sech[c + d*x] + (4
*I)*Tanh[c + d*x])/(2*a*d)
```

Maple [A]
time = 1.17, size = 91, normalized size = 1.05

method	result	size
derivativedivides	$\frac{2i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{8i}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{2i}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-6\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}$	91
default	$\frac{2i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{8i}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{2i}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-6\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}$	91
risch	$-\frac{-3i\,\mathrm{e}^{3dx+3c}-5\,\mathrm{e}^{2dx+2c}+3\,\mathrm{e}^{4dx+4c}+i\,\mathrm{e}^{dx+c}+4}{\left(\mathrm{e}^{2dx+2c}-1\right)^2\left(\mathrm{e}^{dx+c}-i\right)ad}+\frac{3\ln\left(\mathrm{e}^{dx+c}+1\right)}{2da}-\frac{3\ln\left(\mathrm{e}^{dx+c}-1\right)}{2da}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a*(2*I*tanh(1/2*d*x+1/2*c)+1/2*tanh(1/2*d*x+1/2*c)^2+8*I/(-I+tanh(1/2
*d*x+1/2*c))-1/2/tanh(1/2*d*x+1/2*c)^2+2*I/tanh(1/2*d*x+1/2*c)-6*ln(tanh(1/
2*d*x+1/2*c)))
```

Maxima [A]
time = 0.27, size = 156, normalized size = 1.79

$$-\frac{-i\,e^{(-dx-c)}-5\,e^{(-2\,dx-2\,c)}+3i\,e^{(-3\,dx-3\,c)}+3\,e^{(-4\,dx-4\,c)}+4}{\left(ae^{(-dx-c)}-2i\,ae^{(-2\,dx-2\,c)}-2\,ae^{(-3\,dx-3\,c)}+i\,ae^{(-4\,dx-4\,c)}+ae^{(-5\,dx-5\,c)}+i\,a\right)d}+\frac{3\,\log\left(e^{(-dx-c)}+1\right)}{2\,ad}-\frac{3\,\log\left(e^{(-dx-c)}-1\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(-I*e^(-d*x - c) - 5*e^(-2*d*x - 2*c) + 3*I*e^(-3*d*x - 3*c) + 3*e^(-4*d*x
 - 4*c) + 4)/((a*e^(-d*x - c) - 2*I*a*e^(-2*d*x - 2*c) - 2*a*e^(-3*d*x - 3*
c) + I*a*e^(-4*d*x - 4*c) + a*e^(-5*d*x - 5*c) + I*a)*d) + 3/2*log(e^(-d*x
- c) + 1)/(a*d) - 3/2*log(e^(-d*x - c) - 1)/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(79) = 158.
time = 0.39, size = 234, normalized size = 2.69

$$\frac{3\left(e^{(5\,dx+5\,c)}-i\,e^{(4\,dx+4\,c)}-2\,e^{(3\,dx+3\,c)}+2i\,e^{(2\,dx+2\,c)}+e^{(dx+c)}-i\right)\log\left(e^{(dx+c)}+1\right)-3\left(e^{(5\,dx+5\,c)}-i\,e^{(4\,dx+4\,c)}-2\,e^{(3\,dx+3\,c)}+2i\,e^{(2\,dx+2\,c)}+e^{(dx+c)}-i\right)\log\left(e^{(dx+c)}-1\right)-6\,e^{(4\,dx+4\,c)}+6i\,e^{(3\,dx+3\,c)}+10\,e^{(2\,dx+2\,c)}-2i\,e^{(dx+c)}-8}{2\left(ade^{(5\,dx+5\,c)}-i\,ade^{(4\,dx+4\,c)}-2\,ade^{(3\,dx+3\,c)}+2i\,ade^{(2\,dx+2\,c)}+ade^{(dx+c)}-i\,ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(3*(e^(5*d*x + 5*c) - I*e^(4*d*x + 4*c) - 2*e^(3*d*x + 3*c) + 2*I*e^(2*
d*x + 2*c) + e^(d*x + c) - I)*log(e^(d*x + c) + 1) - 3*(e^(5*d*x + 5*c) - I
*e^(4*d*x + 4*c) - 2*e^(3*d*x + 3*c) + 2*I*e^(2*d*x + 2*c) + e^(d*x + c) -
I)*log(e^(d*x + c) - 1) - 6*e^(4*d*x + 4*c) + 6*I*e^(3*d*x + 3*c) + 10*e^(2
*d*x + 2*c) - 2*I*e^(d*x + c) - 8)/(a*d*e^(5*d*x + 5*c) - I*a*d*e^(4*d*x +
```

```
4*c) - 2*a*d*e^(3*d*x + 3*c) + 2*I*a*d*e^(2*d*x + 2*c) + a*d*e^(d*x + c) -
I*a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{\operatorname{csch}^3(c+dx)}{\sinh(c+dx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*Integral(csch(c + d*x)**3/(sinh(c + d*x) - I), x)/a
```

Giac [A]
time = 0.42, size = 97, normalized size = 1.11

$$\frac{\frac{3\log\left(e^{(dx+c)}+1\right)}{a}-\frac{3\log\left(e^{(dx+c)}-1\right)}{a}-\frac{2\left(e^{(3\,dx+3\,c)}-2i\,e^{(2\,dx+2\,c)}+e^{(dx+c)}+2i\right)}{a\left(e^{(2\,dx+2\,c)}-1\right)^2}-\frac{4i}{a\left(i\,e^{(dx+c)}+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(3*log(e^(d*x + c) + 1)/a - 3*log(e^(d*x + c) - 1)/a - 2*(e^(3*d*x + 3*
c) - 2*I*e^(2*d*x + 2*c) + e^(d*x + c) + 2*I)/(a*(e^(2*d*x + 2*c) - 1)^2) -
 4*I/(a*(I*e^(d*x + c) + 1)))/d
```

Mupad [B]
time = 0.61, size = 132, normalized size = 1.52

$$\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-a^2\,d^2}}{a\,d}\right)}{\sqrt{-a^2\,d^2}}-\frac{2}{a\,d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}-\frac{\mathrm{e}^{c+d\,x}}{a\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{2\,\mathrm{e}^{c+d\,x}}{a\,d\,{\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}^2}+\frac{2\mathrm{i}}{a\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
```

```
[Out] (3*atan((exp(d*x)*exp(c)*(-a^2*d^2)^(1/2))/(a*d)))/(-a^2*d^2)^(1/2) - 2/(a*
d*(exp(c + d*x) - 1i)) + 2i/(a*d*(exp(2*c + 2*d*x) - 1)) - exp(c + d*x)/(a*
d*(exp(2*c + 2*d*x) - 1)) - (2*exp(c + d*x))/(a*d*(exp(2*c + 2*d*x) - 1)^2)
```

3.221 $\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=34

$$\operatorname{Int}\left(\frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

[Out] Unintegrable(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

[In] Int[Csch[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]

[Out] Defer[Int][Csch[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

[In] Integrate[Csch[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]

[Out] $Aborted

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(dx+c)^3}{(fx+e)\,(a+ia\sinh\,(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -8*f*integrate(1/(-4*I*a*d*f^2*x^2 - 8*I*a*d*f*x*e - 4*I*a*d*e^2 + 4*(a*d*f
^2*x^2*e^c + 2*a*d*f*x*e^(c + 1) + a*d*e^(c + 2))*e^(d*x)), x) - 8*(4*d*f*x
 + 4*d*e + (3*d*f*x*e^(4*c) - f*e^(4*c) + 3*d*e^(4*c + 1))*e^(4*d*x) + (-3*
I*d*f*x*e^(3*c) + I*f*e^(3*c) - 3*I*d*e^(3*c + 1))*e^(3*d*x) - (5*d*f*x*e^(
2*c) - f*e^(2*c) + 5*d*e^(2*c + 1))*e^(2*d*x) + (I*d*f*x*e^c + I*d*e^(c + 1
) - I*f*e^c)*e^(d*x))/(-8*I*a*d^2*f^2*x^2 - 16*I*a*d^2*f*x*e - 8*I*a*d^2*e^
2 + 8*(a*d^2*f^2*x^2*e^(5*c) + 2*a*d^2*f*x*e^(5*c + 1) + a*d^2*e^(5*c + 2))
*e^(5*d*x) - 8*(I*a*d^2*f^2*x^2*e^(4*c) + 2*I*a*d^2*f*x*e^(4*c + 1) + I*a*d
^2*e^(4*c + 2))*e^(4*d*x) - 16*(a*d^2*f^2*x^2*e^(3*c) + 2*a*d^2*f*x*e^(3*c
+ 1) + a*d^2*e^(3*c + 2))*e^(3*d*x) - 16*(-I*a*d^2*f^2*x^2*e^(2*c) - 2*I*a*
d^2*f*x*e^(2*c + 1) - I*a*d^2*e^(2*c + 2))*e^(2*d*x) + 8*(a*d^2*f^2*x^2*e^c
 + 2*a*d^2*f*x*e^(c + 1) + a*d^2*e^(c + 2))*e^(d*x)) - 8*integrate(1/16*(3*
d^2*f^2*x^2 + 3*d^2*e^2 + 2*I*d*f*e - 2*f^2 + 2*(3*d^2*f*e + I*d*f^2)*x)/(a
*d^2*f^3*x^3 + 3*a*d^2*f^2*x^2*e + 3*a*d^2*f*x*e^2 + a*d^2*e^3 + (a*d^2*f^3
*x^3*e^c + 3*a*d^2*f^2*x^2*e^(c + 1) + 3*a*d^2*f*x*e^(c + 2) + a*d^2*e^(c +
 3))*e^(d*x)), x) - 8*integrate(-1/16*(3*d^2*f^2*x^2 + 3*d^2*e^2 - 2*I*d*f*
e - 2*f^2 + 2*(3*d^2*f*e - I*d*f^2)*x)/(a*d^2*f^3*x^3 + 3*a*d^2*f^2*x^2*e +
 3*a*d^2*f*x*e^2 + a*d^2*e^3 - (a*d^2*f^3*x^3*e^c + 3*a*d^2*f^2*x^2*e^(c +
1) + 3*a*d^2*f*x*e^(c + 2) + a*d^2*e^(c + 3))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(4*d*f*x + 4*d*e + (3*d*f*x + 3*d*e - f)*e^(4*d*x + 4*c) - (3*I*d*f*x + 3*
I*d*e - I*f)*e^(3*d*x + 3*c) - (5*d*f*x + 5*d*e - f)*e^(2*d*x + 2*c) - (-I*
d*f*x - I*d*e + I*f)*e^(d*x + c) - (-I*a*d^2*f^2*x^2 - 2*I*a*d^2*f*x*e - I*
a*d^2*e^2 + (a*d^2*f^2*x^2 + 2*a*d^2*f*x*e + a*d^2*e^2)*e^(5*d*x + 5*c) + (
```

```
-I*a*d^2*f^2*x^2 - 2*I*a*d^2*f*x*e - I*a*d^2*e^2)*e^(4*d*x + 4*c) - 2*(a*d^
2*f^2*x^2 + 2*a*d^2*f*x*e + a*d^2*e^2)*e^(3*d*x + 3*c) - 2*(-I*a*d^2*f^2*x^
2 - 2*I*a*d^2*f*x*e - I*a*d^2*e^2)*e^(2*d*x + 2*c) + (a*d^2*f^2*x^2 + 2*a*d
^2*f*x*e + a*d^2*e^2)*e^(d*x + c))*integral((4*d*f^2*x + 4*d*f*e - (3*d^2*f
^2*x^2 + 2*d*f^2*x + 3*d^2*e^2 - 2*f^2 + 2*(3*d^2*f*x + d*f)*e)*e^(2*d*x +
2*c) + (3*I*d^2*f^2*x^2 + 2*I*d*f^2*x + 3*I*d^2*e^2 - 2*I*f^2 - 2*(-3*I*d^2
*f*x - I*d*f)*e)*e^(d*x + c))/(I*a*d^2*f^3*x^3 + 3*I*a*d^2*f^2*x^2*e + 3*I*
a*d^2*f*x*e^2 + I*a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*f^2*x^2*e + 3*a*d^2*
f*x*e^2 + a*d^2*e^3)*e^(3*d*x + 3*c) + (-I*a*d^2*f^3*x^3 - 3*I*a*d^2*f^2*x^
2*e - 3*I*a*d^2*f*x*e^2 - I*a*d^2*e^3)*e^(2*d*x + 2*c) - (a*d^2*f^3*x^3 + 3
*a*d^2*f^2*x^2*e + 3*a*d^2*f*x*e^2 + a*d^2*e^3)*e^(d*x + c)), x))/(-I*a*d^2
*f^2*x^2 - 2*I*a*d^2*f*x*e - I*a*d^2*e^2 + (a*d^2*f^2*x^2 + 2*a*d^2*f*x*e +
 a*d^2*e^2)*e^(5*d*x + 5*c) + (-I*a*d^2*f^2*x^2 - 2*I*a*d^2*f*x*e - I*a*d^2
*e^2)*e^(4*d*x + 4*c) - 2*(a*d^2*f^2*x^2 + 2*a*d^2*f*x*e + a*d^2*e^2)*e^(3*
d*x + 3*c) - 2*(-I*a*d^2*f^2*x^2 - 2*I*a*d^2*f*x*e - I*a*d^2*e^2)*e^(2*d*x
+ 2*c) + (a*d^2*f^2*x^2 + 2*a*d^2*f*x*e + a*d^2*e^2)*e^(d*x + c))
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\operatorname{csch}^3(c+dx)}{e\sinh(c+dx)-ie+fx\sinh(c+dx)-ifx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
[Out] -I*Integral(csch(c + d*x)**3/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) - I
*f*x), x)/a
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{1}{{\mathrm{sinh}\left(c+d\,x\right)}^3\,\left(e+f\,x\right)\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
[Out] int(1/(sinh(c + d*x)^3*(e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.222 $\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=34

$$\operatorname{Int}\left(\frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csch[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
[Out] Defer[Int][Csch[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.03, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Csch[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(dx+c)^3}{(fx+e)^2\,(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -8*f*integrate(1/(-2*I*a*d*f^3*x^3 - 6*I*a*d*f^2*x^2*e - 6*I*a*d*f*x*e^2 -
2*I*a*d*e^3 + 2*(a*d*f^3*x^3*e^c + 3*a*d*f^2*x^2*e^(c + 1) + 3*a*d*f*x*e^(c
 + 2) + a*d*e^(c + 3))*e^(d*x)), x) - 8*(4*d*f*x + 4*d*e + (3*d*f*x*e^(4*c)
 - 2*f*e^(4*c) + 3*d*e^(4*c + 1))*e^(4*d*x) + (-3*I*d*f*x*e^(3*c) + 2*I*f*e
^(3*c) - 3*I*d*e^(3*c + 1))*e^(3*d*x) - (5*d*f*x*e^(2*c) - 2*f*e^(2*c) + 5*
d*e^(2*c + 1))*e^(2*d*x) + (I*d*f*x*e^c + I*d*e^(c + 1) - 2*I*f*e^c)*e^(d*x
))/(-8*I*a*d^2*f^3*x^3 - 24*I*a*d^2*f^2*x^2*e - 24*I*a*d^2*f*x*e^2 - 8*I*a*
d^2*e^3 + 8*(a*d^2*f^3*x^3*e^(5*c) + 3*a*d^2*f^2*x^2*e^(5*c + 1) + 3*a*d^2*
f*x*e^(5*c + 2) + a*d^2*e^(5*c + 3))*e^(5*d*x) - 8*(I*a*d^2*f^3*x^3*e^(4*c)
 + 3*I*a*d^2*f^2*x^2*e^(4*c + 1) + 3*I*a*d^2*f*x*e^(4*c + 2) + I*a*d^2*e^(4
*c + 3))*e^(4*d*x) - 16*(a*d^2*f^3*x^3*e^(3*c) + 3*a*d^2*f^2*x^2*e^(3*c + 1
) + 3*a*d^2*f*x*e^(3*c + 2) + a*d^2*e^(3*c + 3))*e^(3*d*x) - 16*(-I*a*d^2*f
^3*x^3*e^(2*c) - 3*I*a*d^2*f^2*x^2*e^(2*c + 1) - 3*I*a*d^2*f*x*e^(2*c + 2)
- I*a*d^2*e^(2*c + 3))*e^(2*d*x) + 8*(a*d^2*f^3*x^3*e^c + 3*a*d^2*f^2*x^2*e
^(c + 1) + 3*a*d^2*f*x*e^(c + 2) + a*d^2*e^(c + 3))*e^(d*x)) - 8*integrate(
1/16*(3*d^2*f^2*x^2 + 3*d^2*e^2 + 4*I*d*f*e - 6*f^2 + 2*(3*d^2*f*e + 2*I*d*
f^2)*x)/(a*d^2*f^4*x^4 + 4*a*d^2*f^3*x^3*e + 6*a*d^2*f^2*x^2*e^2 + 4*a*d^2*
f*x*e^3 + a*d^2*e^4 + (a*d^2*f^4*x^4*e^c + 4*a*d^2*f^3*x^3*e^(c + 1) + 6*a*
d^2*f^2*x^2*e^(c + 2) + 4*a*d^2*f*x*e^(c + 3) + a*d^2*e^(c + 4))*e^(d*x)),
x) - 8*integrate(-1/16*(3*d^2*f^2*x^2 + 3*d^2*e^2 - 4*I*d*f*e - 6*f^2 + 2*(
3*d^2*f*e - 2*I*d*f^2)*x)/(a*d^2*f^4*x^4 + 4*a*d^2*f^3*x^3*e + 6*a*d^2*f^2*
x^2*e^2 + 4*a*d^2*f*x*e^3 + a*d^2*e^4 - (a*d^2*f^4*x^4*e^c + 4*a*d^2*f^3*x^
3*e^(c + 1) + 6*a*d^2*f^2*x^2*e^(c + 2) + 4*a*d^2*f*x*e^(c + 3) + a*d^2*e^(
c + 4))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -(4*d*f*x + 4*d*e + (3*d*f*x + 3*d*e - 2*f)*e^(4*d*x + 4*c) - (3*I*d*f*x +
3*I*d*e - 2*I*f)*e^(3*d*x + 3*c) - (5*d*f*x + 5*d*e - 2*f)*e^(2*d*x + 2*c)
- (-I*d*f*x - I*d*e + 2*I*f)*e^(d*x + c) - (-I*a*d^2*f^3*x^3 - 3*I*a*d^2*f^
2*x^2*e - 3*I*a*d^2*f*x*e^2 - I*a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*f^2*x^
2*e + 3*a*d^2*f*x*e^2 + a*d^2*e^3)*e^(5*d*x + 5*c) + (-I*a*d^2*f^3*x^3 - 3*
I*a*d^2*f^2*x^2*e - 3*I*a*d^2*f*x*e^2 - I*a*d^2*e^3)*e^(4*d*x + 4*c) - 2*(a
*d^2*f^3*x^3 + 3*a*d^2*f^2*x^2*e + 3*a*d^2*f*x*e^2 + a*d^2*e^3)*e^(3*d*x +
3*c) - 2*(-I*a*d^2*f^3*x^3 - 3*I*a*d^2*f^2*x^2*e - 3*I*a*d^2*f*x*e^2 - I*a*
d^2*e^3)*e^(2*d*x + 2*c) + (a*d^2*f^3*x^3 + 3*a*d^2*f^2*x^2*e + 3*a*d^2*f*x
*e^2 + a*d^2*e^3)*e^(d*x + c))*integral((8*d*f^2*x + 8*d*f*e - (3*d^2*f^2*x
^2 + 4*d*f^2*x + 3*d^2*e^2 - 6*f^2 + 2*(3*d^2*f*x + 2*d*f)*e)*e^(2*d*x + 2*
c) + (3*I*d^2*f^2*x^2 + 4*I*d*f^2*x + 3*I*d^2*e^2 - 6*I*f^2 - 2*(-3*I*d^2*f
*x - 2*I*d*f)*e)*e^(d*x + c))/(I*a*d^2*f^4*x^4 + 4*I*a*d^2*f^3*x^3*e + 6*I*
a*d^2*f^2*x^2*e^2 + 4*I*a*d^2*f*x*e^3 + I*a*d^2*e^4 + (a*d^2*f^4*x^4 + 4*a*
d^2*f^3*x^3*e + 6*a*d^2*f^2*x^2*e^2 + 4*a*d^2*f*x*e^3 + a*d^2*e^4)*e^(3*d*x
 + 3*c) + (-I*a*d^2*f^4*x^4 - 4*I*a*d^2*f^3*x^3*e - 6*I*a*d^2*f^2*x^2*e^2 -
 4*I*a*d^2*f*x*e^3 - I*a*d^2*e^4)*e^(2*d*x + 2*c) - (a*d^2*f^4*x^4 + 4*a*d^
2*f^3*x^3*e + 6*a*d^2*f^2*x^2*e^2 + 4*a*d^2*f*x*e^3 + a*d^2*e^4)*e^(d*x + c
)), x))/(-I*a*d^2*f^3*x^3 - 3*I*a*d^2*f^2*x^2*e - 3*I*a*d^2*f*x*e^2 - I*a*d
^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*f^2*x^2*e + 3*a*d^2*f*x*e^2 + a*d^2*e^3)*
e^(5*d*x + 5*c) + (-I*a*d^2*f^3*x^3 - 3*I*a*d^2*f^2*x^2*e - 3*I*a*d^2*f*x*e
^2 - I*a*d^2*e^3)*e^(4*d*x + 4*c) - 2*(a*d^2*f^3*x^3 + 3*a*d^2*f^2*x^2*e +
3*a*d^2*f*x*e^2 + a*d^2*e^3)*e^(3*d*x + 3*c) - 2*(-I*a*d^2*f^3*x^3 - 3*I*a*
d^2*f^2*x^2*e - 3*I*a*d^2*f*x*e^2 - I*a*d^2*e^3)*e^(2*d*x + 2*c) + (a*d^2*f
^3*x^3 + 3*a*d^2*f^2*x^2*e + 3*a*d^2*f*x*e^2 + a*d^2*e^3)*e^(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

`[Out] Timed out`

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sinh(c+d\,x)^3\,(e+f\,x)^2\,\left(a+a\sinh(c+d\,x)\,1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
[Out] int(1/(sinh(c + d*x)^3*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.223 $\int \frac{(e+fx)^3 \sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=453

$$\frac{(e+fx)^4}{4bf}-\frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d}+\frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d}-\frac{3af(e+fx)^2\text{PolyLog}}{b\sqrt{a^2+}}$$

```
[Out] 1/4*(f*x+e)^4/b/f-a*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d/(a
^2+b^2)^(1/2)+a*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d/(a^2+b
^2)^(1/2)-3*a*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d^
2/(a^2+b^2)^(1/2)+3*a*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2
)))/b/d^2/(a^2+b^2)^(1/2)+6*a*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b
^2)^(1/2)))/b/d^3/(a^2+b^2)^(1/2)-6*a*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(
a+(a^2+b^2)^(1/2)))/b/d^3/(a^2+b^2)^(1/2)-6*a*f^3*polylog(4,-b*exp(d*x+c)/(
a-(a^2+b^2)^(1/2)))/b/d^4/(a^2+b^2)^(1/2)+6*a*f^3*polylog(4,-b*exp(d*x+c)/(
a+(a^2+b^2)^(1/2)))/b/d^4/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.58, antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5676, 32, 3403, 2296, 2221, 2611, 6744, 2320, 6724}

$$-\frac{6af^3\text{Li}_4\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4\sqrt{a^2+b^2}}+\frac{6af^3\text{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^4\sqrt{a^2+b^2}}+\frac{6af^2(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{6af^2(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{3af(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}}+\frac{3af(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}}-\frac{a(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd\sqrt{a^2+b^2}}+\frac{a(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd\sqrt{a^2+b^2}}+\frac{(e+fx)^4}{4bf}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e + f*x)^4/(4*b*f) - (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2])])/(b*Sqrt[a^2 + b^2]*d) + (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d) - (3*a*f*(e + f*x)^2*PolyLog[2,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^2) + (3*a*
f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt
[a^2 + b^2]*d^2) + (6*a*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (6*a*f^2*(e + f*x)*PolyLog[3, -(
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (6*a*f^3
*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d
^4) + (6*a*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqr
t[a^2 + b^2]*d^4)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5676

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\,dx}{b} - \frac{a\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{(2a)\int \frac{e^{c+dx}(e+fx)^3}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{b}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{(2a)\int \frac{e^{c+dx}(e+fx)^3}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\sqrt{a^2+b^2}} + \frac{(2a)\int \frac{e^{c+dx}(e+fx)^3}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}}\,d}{\sqrt{a^2+b^2}}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d} + \frac{a(e+fx)^3\log\left(1+\frac{be}{a+\sqrt{}}\right.}{b\sqrt{a^2+b^2}\,d}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d} + \frac{a(e+fx)^3\log\left(1+\frac{be}{a+\sqrt{}}\right.}{b\sqrt{a^2+b^2}\,d}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d} + \frac{a(e+fx)^3\log\left(1+\frac{be}{a+\sqrt{}}\right.}{b\sqrt{a^2+b^2}\,d}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d} + \frac{a(e+fx)^3\log\left(1+\frac{be}{a+\sqrt{}}\right.}{b\sqrt{a^2+b^2}\,d}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d} + \frac{a(e+fx)^3\log\left(1+\frac{be}{a+\sqrt{}}\right.}{b\sqrt{a^2+b^2}\,d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1009 vs. 2(453) = 906.
time = 2.73, size = 1009, normalized size = 2.23

$$\frac{x(4e^3+6e^2fx+4ef^2x^2+f^3x^3)}{4b}+\frac{a\left(2d^3e^3\sqrt{(a^2+b^2)e^{2c}}\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-3\sqrt{a^2+b^2}\,d^3e^2e^cfx\log\left(1+\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)-3\sqrt{a^2+b^2}\,d^3ee^cf^2x^2\log\left(1+\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)-\sqrt{a^2+b^2}\,d^3e^cf^3x^3\log\left(1+\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)+3\sqrt{a^2+b^2}\,d^3e^2e^cfx\log\left(1+\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)+3\sqrt{a^2+b^2}\,d^3ee^cf^2x^2\log\left(1+\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)+\sqrt{a^2+b^2}\,d^3e^cf^3x^3\log\left(1+\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)-3\sqrt{a^2+b^2}\,d^2e^cf(e+fx)^2\mathrm{PolyLog}\left(2,-\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)+3\sqrt{a^2+b^2}\,d^2e^cf(e+fx)^2\mathrm{PolyLog}\left(2,-\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)+6\sqrt{a^2+b^2}\,dee^cf^2\mathrm{PolyLog}\left(3,-\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)+6\sqrt{a^2+b^2}\,de^cf^3x\,\mathrm{PolyLog}\left(3,-\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)-6\sqrt{a^2+b^2}\,dee^cf^2\mathrm{PolyLog}\left(3,-\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)-6\sqrt{a^2+b^2}\,de^cf^3x\,\mathrm{PolyLog}\left(3,-\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)-6\sqrt{a^2+b^2}\,e^cf^3\mathrm{PolyLog}\left(4,-\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)+6\sqrt{a^2+b^2}\,e^cf^3\mathrm{PolyLog}\left(4,-\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)\right)}{b\sqrt{a^2+b^2}\,d^4\sqrt{(a^2+b^2)e^{2c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(4*b) + (a*(2*d^3*e^3*Sqrt[
(a^2 + b^2)*E^(2*c)]*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 3*Sqrt[
a^2 + b^2]*d^3*e^2*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b
^2)*E^(2*c)])] - 3*Sqrt[a^2 + b^2]*d^3*e*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*
x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - Sqrt[a^2 + b^2]*d^3*E^c*f^3*x^3*
Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 3*Sqrt[a^2
 + b^2]*d^3*e^2*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)
*E^(2*c)])] + 3*Sqrt[a^2 + b^2]*d^3*e*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))
/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + Sqrt[a^2 + b^2]*d^3*E^c*f^3*x^3*Log
[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*Sqrt[a^2 +
b^2]*d^2*E^c*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^
2 + b^2)*E^(2*c)]))] + 3*Sqrt[a^2 + b^2]*d^2*E^c*f*(e + f*x)^2*PolyLog[2, -
((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[a^2 + b^2
]*d*e*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)]))] + 6*Sqrt[a^2 + b^2]*d*E^c*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E
^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[a^2 + b^2]*d*e*E^c*f^2*PolyLog[3
, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[a^2 +
b^2]*d*E^c*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E
^(2*c)]))] - 6*Sqrt[a^2 + b^2]*E^c*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^
c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[a^2 + b^2]*E^c*f^3*PolyLog[4, -((
b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(b*Sqrt[a^2 + b^2]
*d^4*Sqrt[(a^2 + b^2)*E^(2*c)])
```

Maple [F]
time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \sinh(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*b*d) - (d*x + c)/(b*d))*e^3 + 1/4*(f^3*x^4 + 4
*f^2*x^3*e + 6*f*x^2*e^2)/b - integrate(2*(a*f^3*x^3*e^c + 3*a*f^2*x^2*e^(c
 + 1) + 3*a*f*x*e^(c + 2))*e^(d*x)/(b^2*e^(2*d*x + 2*c) + 2*a*b*e^(d*x + c)
 - b^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1614 vs. $2(416) = 832$.
time = 0.39, size = 1614, normalized size = 3.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*((a^2 + b^2)*d^4*f^3*x^4 + 4*(a^2 + b^2)*d^4*f^2*x^3*cosh(1) + 6*(a^2 +
 b^2)*d^4*f*x^2*cosh(1)^2 + 4*(a^2 + b^2)*d^4*x*cosh(1)^3 + 4*(a^2 + b^2)*d
^4*x*sinh(1)^3 - 24*a*b*f^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x +
c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2))/b) + 24*a*b*f^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) +
a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
)/b) + 6*((a^2 + b^2)*d^4*f*x^2 + 2*(a^2 + b^2)*d^4*x*cosh(1))*sinh(1)^2 -
12*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*f^2*x*cosh(1) + a*b*d^2*f*cosh(1)^2 + a*b*d
^2*f*sinh(1)^2 + 2*(a*b*d^2*f^2*x + a*b*d^2*f*cosh(1))*sinh(1))*sqrt((a^2 +
 b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 12*(a*b*d^2*f^3*x^2 + 2*
a*b*d^2*f^2*x*cosh(1) + a*b*d^2*f*cosh(1)^2 + a*b*d^2*f*sinh(1)^2 + 2*(a*b*
d^2*f^2*x + a*b*d^2*f*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh
(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2) - b)/b + 1) - 4*(a*b*c^3*f^3 - 3*a*b*c^2*d*f^2*cosh(1) + 3*a*b
*c*d^2*f*cosh(1)^2 - a*b*d^3*cosh(1)^3 - a*b*d^3*sinh(1)^3 + 3*(a*b*c*d^2*f
 - a*b*d^3*cosh(1))*sinh(1)^2 - 3*(a*b*c^2*d*f^2 - 2*a*b*c*d^2*f*cosh(1) +
a*b*d^3*cosh(1)^2)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2
*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 4*(a*b*c^3*f^3 - 3*a*
b*c^2*d*f^2*cosh(1) + 3*a*b*c*d^2*f*cosh(1)^2 - a*b*d^3*cosh(1)^3 - a*b*d^3
*sinh(1)^3 + 3*(a*b*c*d^2*f - a*b*d^3*cosh(1))*sinh(1)^2 - 3*(a*b*c^2*d*f^2
 - 2*a*b*c*d^2*f*cosh(1) + a*b*d^3*cosh(1)^2)*sinh(1))*sqrt((a^2 + b^2)/b^2
)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2
*a) - 4*(a*b*d^3*f^3*x^3 + a*b*c^3*f^3 + 3*(a*b*d^3*f*x + a*b*c*d^2*f)*cosh
(1)^2 + 3*(a*b*d^3*f*x + a*b*c*d^2*f)*sinh(1)^2 + 3*(a*b*d^3*f^2*x^2 - a*b*
c^2*d*f^2)*cosh(1) + 3*(a*b*d^3*f^2*x^2 - a*b*c^2*d*f^2 + 2*(a*b*d^3*f*x +
a*b*c*d^2*f)*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c)
```

```
+ a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2) - b)/b) + 4*(a*b*d^3*f^3*x^3 + a*b*c^3*f^3 + 3*(a*b*d^3*f*x + a*b*c*d^2*
f)*cosh(1)^2 + 3*(a*b*d^3*f*x + a*b*c*d^2*f)*sinh(1)^2 + 3*(a*b*d^3*f^2*x^2
 - a*b*c^2*d*f^2)*cosh(1) + 3*(a*b*d^3*f^2*x^2 - a*b*c^2*d*f^2 + 2*(a*b*d^3
*f*x + a*b*c*d^2*f)*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*
x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b) + 24*(a*b*d*f^3*x + a*b*d*f^2*cosh(1) + a*b*d*f^2*sinh(1)
)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 24*(a*b*d*f^3*
x + a*b*d*f^2*cosh(1) + a*b*d*f^2*sinh(1))*sqrt((a^2 + b^2)/b^2)*polylog(3,
 (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 + b^2)/b^2))/b) + 4*((a^2 + b^2)*d^4*f^2*x^3 + 3*(a^2 + b^2)*d^4*f
*x^2*cosh(1) + 3*(a^2 + b^2)*d^4*x*cosh(1)^2)*sinh(1))/((a^2*b + b^3)*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3 \sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)**3*sinh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^3*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)\,{(e+f\,x)}^3}{a+b\sinh(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
[Out] int((sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.224 $\int \frac{(e+fx)^2 \sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=337

$$\frac{(e+fx)^3}{3bf}-\frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d}+\frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d}-\frac{2af(e+fx)\mathrm{PolyLog}\left(\right.}{b\sqrt{a^2+\right.}}$$

```
[Out] 1/3*(f*x+e)^3/b/f-a*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d/(a
^2+b^2)^(1/2)+a*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d/(a^2+b
^2)^(1/2)-2*a*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d^2/
(a^2+b^2)^(1/2)+2*a*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/
b/d^2/(a^2+b^2)^(1/2)+2*a*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/
b/d^3/(a^2+b^2)^(1/2)-2*a*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/
b/d^3/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5676, 32, 3403, 2296, 2221, 2611, 2320, 6724}

$$\frac{2af^2\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{2af^2\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{2af(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}}+\frac{2af(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}}-\frac{a(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd\sqrt{a^2+b^2}}+\frac{a(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd\sqrt{a^2+b^2}}+\frac{(e+fx)^3}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e + f*x)^3/(3*b*f) - (a*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2])])/(b*Sqrt[a^2 + b^2]*d) + (a*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d) - (2*a*f*(e + f*x)*PolyLog[2, -
((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^2) + (2*a*f*
(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2
 + b^2]*d^2) + (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))
])/(b*Sqrt[a^2 + b^2]*d^3) - (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5676

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\,dx}{b} - \frac{a\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{(2a)\int \frac{e^{c+dx}(e+fx)^2}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{b}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{(2a)\int \frac{e^{c+dx}(e+fx)^2}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\sqrt{a^2+b^2}} + \frac{(2a)\int \frac{e^{c+dx}(e+fx)^2}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}}}{\sqrt{a^2+b^2}}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d} + \frac{a(e+fx)^2\log\left(1+\frac{be}{a+\sqrt{}}\right)}{b\sqrt{a^2+b^2}\,d}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d} + \frac{a(e+fx)^2\log\left(1+\frac{be}{a+\sqrt{}}\right)}{b\sqrt{a^2+b^2}\,d}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d} + \frac{a(e+fx)^2\log\left(1+\frac{be}{a+\sqrt{}}\right)}{b\sqrt{a^2+b^2}\,d}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d} + \frac{a(e+fx)^2\log\left(1+\frac{be}{a+\sqrt{}}\right)}{b\sqrt{a^2+b^2}\,d}$$

Mathematica [A]
time = 2.04, size = 610, normalized size = 1.81

$$\frac{x(3e^2+3efx+f^2x^2)}{3b} + \frac{a\left(2d^2e^2\sqrt{(a^2+b^2)e^{2c}}\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) - 2\sqrt{a^2+b^2}d^2ee^cfx\log\left(1+\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right) - \sqrt{a^2+b^2}d^2e^cf^2x^2\log\left(1+\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right) + 2\sqrt{a^2+b^2}d^2ee^cfx\log\left(1+\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right) + \sqrt{a^2+b^2}d^2e^cf^2x^2\log\left(1+\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right) - 2\sqrt{a^2+b^2}de^cf(e+fx)\mathrm{PolyLog}\left(2,-\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right) + 2\sqrt{a^2+b^2}de^cf(e+fx)\mathrm{PolyLog}\left(2,-\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right) + 2\sqrt{a^2+b^2}e^cf^2\mathrm{PolyLog}\left(3,-\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right) - 2\sqrt{a^2+b^2}e^cf^2\mathrm{PolyLog}\left(3,-\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)\right)}{b\sqrt{a^2+b^2}d^3\sqrt{(a^2+b^2)e^{2c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2))/(3*b) + (a*(2*d^2*e^2*Sqrt[(a^2 + b^2)*E^(2
*c)]*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*Sqrt[a^2 + b^2]*d^2*e
*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - S
qrt[a^2 + b^2]*d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2
 + b^2)*E^(2*c)])] + 2*Sqrt[a^2 + b^2]*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*
x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + Sqrt[a^2 + b^2]*d^2*E^c*f^2*x^2*
```

```
Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 2*Sqrt[a^2
 + b^2]*d*E^c*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2
 + b^2)*E^(2*c)]))] + 2*Sqrt[a^2 + b^2]*d*E^c*f*(e + f*x)*PolyLog[2, -((b*E
^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2*Sqrt[a^2 + b^2]*E^c
*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] -
 2*Sqrt[a^2 + b^2]*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^
2 + b^2)*E^(2*c)]))]))/(b*Sqrt[a^2 + b^2]*d^3*Sqrt[(a^2 + b^2)*E^(2*c)])
```

Maple [**F**]
time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \sinh(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*b*d) - (d*x + c)/(b*d))*e^2 + 1/3*(f^2*x^3 + 3
*f*x^2*e)/b - integrate(2*(a*f^2*x^2*e^c + 2*a*f*x*e^(c + 1))*e^(d*x)/(b^2*
e^(2*d*x + 2*c) + 2*a*b*e^(d*x + c) - b^2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 956 vs. 2(308) = 616.
time = 0.36, size = 956, normalized size = 2.84

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*((a^2 + b^2)*d^3*f^2*x^3 + 3*(a^2 + b^2)*d^3*f*x^2*cosh(1) + 3*(a^2 + b
^2)*d^3*x*cosh(1)^2 + 3*(a^2 + b^2)*d^3*x*sinh(1)^2 + 6*a*b*f^2*sqrt((a^2 +
 b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*a*b*f^2*sqrt((a^2 + b^2)/
b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*s
```

```
inh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*(a*b*d*f^2*x + a*b*d*f*cosh(1)
+ a*b*d*f*sinh(1))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*
x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b +
 1) + 6*(a*b*d*f^2*x + a*b*d*f*cosh(1) + a*b*d*f*sinh(1))*sqrt((a^2 + b^2)/
b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 3*(a*b*c^2*f^2 - 2*a*b*c*d*f*c
osh(1) + a*b*d^2*cosh(1)^2 + a*b*d^2*sinh(1)^2 - 2*(a*b*c*d*f - a*b*d^2*cos
h(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
 c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(a*b*c^2*f^2 - 2*a*b*c*d*f*cosh(
1) + a*b*d^2*cosh(1)^2 + a*b*d^2*sinh(1)^2 - 2*(a*b*c*d*f - a*b*d^2*cosh(1)
)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c)
- 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(a*b*d^2*f^2*x^2 - a*b*c^2*f^2 + 2*(
a*b*d^2*f*x + a*b*c*d*f)*cosh(1) + 2*(a*b*d^2*f*x + a*b*c*d*f)*sinh(1))*sqr
t((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 3*(a*b*d^2*f^2*x^2 -
a*b*c^2*f^2 + 2*(a*b*d^2*f*x + a*b*c*d*f)*cosh(1) + 2*(a*b*d^2*f*x + a*b*c*
d*f)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
 - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 3*((
a^2 + b^2)*d^3*f*x^2 + 2*(a^2 + b^2)*d^3*x*cosh(1))*sinh(1))/((a^2*b + b^3)
*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2 \sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*sinh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)\,{(e+f\,x)}^2}{a+b\,\sinh(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
[Out] int((sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.225 $\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=220

$$\frac{ex}{b}+\frac{fx^2}{2b}-\frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d}+\frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d}-\frac{af\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-\ldots}}\right)}{b\sqrt{a^2+b^2}\,d^2}$$

```
[Out] e*x/b+1/2*f*x^2/b-a*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d/(a^2
+b^2)^(1/2)+a*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d/(a^2+b^2)^
(1/2)-a*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d^2/(a^2+b^2)^(1/2
)+a*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d^2/(a^2+b^2)^(1/2)
```

Rubi [A]

time = 0.28, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5676, 3403, 2296, 2221, 2317, 2438}

$$-\frac{af\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}}+\frac{af\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}}-\frac{a(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd\sqrt{a^2+b^2}}+\frac{a(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd\sqrt{a^2+b^2}}+\frac{ex}{b}+\frac{fx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e*x)/b + (f*x^2)/(2*b) - (a*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2])])/(b*Sqrt[a^2 + b^2]*d) + (a*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d) - (a*f*PolyLog[2, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^2) + (a*f*PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5676

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\,dx}{b} - \frac{a\int \frac{e+fx}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{(2a)\int \frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{b}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{(2a)\int \frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\sqrt{a^2+b^2}} + \frac{(2a)\int \frac{e^{c+dx}(e+fx)}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}}\,d}{\sqrt{a^2+b^2}}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d} + \frac{a(e+fx)\log\left(1+\frac{be^{c+d}}{a+\sqrt{a^2}}\right.}{b\sqrt{a^2+b^2}\,d}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d} + \frac{a(e+fx)\log\left(1+\frac{be^{c+d}}{a+\sqrt{a^2}}\right.}{b\sqrt{a^2+b^2}\,d}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d} + \frac{a(e+fx)\log\left(1+\frac{be^{c+d}}{a+\sqrt{a^2}}\right.}{b\sqrt{a^2+b^2}\,d}$$

Mathematica [A]
time = 0.48, size = 163, normalized size = 0.74

$$\frac{x(2e+fx)}{2b} - \frac{a\left(d(e+fx)\left(\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)+f\mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)-f\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)}{b\sqrt{a^2+b^2}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (x*(2*e + f*x))/(2*b) - (a*(d*(e + f*x)*(Log[1 + (b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2])] - Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]) + f*PolyLog[
2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2]))]))/(b*Sqrt[a^2 + b^2]*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(198) = 396$.
time = 1.29, size = 440, normalized size = 2.00

method	result

risch	$\frac{f\,x^{2}}{2b}+\frac{ex}{b}+\frac{2ae\,\mathrm{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^{2}+b^{2}}}\right)}{db\sqrt{a^{2}+b^{2}}}-\frac{af\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^{2}+b^{2}}-a}{-a+\sqrt{a^{2}+b^{2}}}\right)x}{db\sqrt{a^{2}+b^{2}}}-\frac{af\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^{2}+b^{2}}-a}{-a+\sqrt{a^{2}+b^{2}}}\right)c}{d^{2}b\sqrt{a^{2}+b^{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f*x^2/b+e*x/b+2/d*a/b*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a
)/(a^2+b^2)^(1/2))-1/d*a/b*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1
/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d^2*a/b*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c
)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d*a/b*f/(a^2+b^2)^(1/2)*ln((
b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2*a/b*f/(a^2+b^2
)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d^2*a/
b*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(
1/2)))+1/d^2*a/b*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(
a+(a^2+b^2)^(1/2)))-2/d^2*a/b*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+
c)+2*a)/(a^2+b^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(4*a*integrate(x*e^(d*x + c)/(b^2*e^(2*d*x + 2*c) + 2*a*b*e^(d*x + c)
- b^2), x) - x^2/b)*f - (a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^
(-d*x - c) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b*d) - (d*x + c)/(b*d))
*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 532 vs. 2(199) = 398.
time = 0.36, size = 532, normalized size = 2.42

$$\frac{(a^2+b^2)d^2fx^2+2(a^2+b^2)d^2x\cosh(1)+2(a^2+b^2)d^2x\sinh(1)-2abf\sqrt{\frac{a^2+b^2}{b^2}}\,\mathrm{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}+1\right)+2abf\sqrt{\frac{a^2+b^2}{b^2}}\,\mathrm{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}+1\right)-2(abcf-abd\cosh(1)-abd\sinh(1))\sqrt{\frac{a^2+b^2}{b^2}}\log\left(2b\cosh(dx+c)+2b\sinh(dx+c)+2b\sqrt{\frac{a^2+b^2}{b^2}}+2a\right)+2(abcf-abd\cosh(1)-abd\sinh(1))\sqrt{\frac{a^2+b^2}{b^2}}\log\left(2b\cosh(dx+c)+2b\sinh(dx+c)-2b\sqrt{\frac{a^2+b^2}{b^2}}+2a\right)-2(abdfx+abcf)\sqrt{\frac{a^2+b^2}{b^2}}\log\left(-\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}\right)+2(abdfx+abcf)\sqrt{\frac{a^2+b^2}{b^2}}\log\left(-\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}\right)}{2(a^2b+b^3)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + b^2)*d^2*f*x^2 + 2*(a^2 + b^2)*d^2*x*cosh(1) + 2*(a^2 + b^2)*d^
2*x*sinh(1) - 2*a*b*f*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh
(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/
b + 1) + 2*a*b*f*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x
```

```
+ c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1
) - 2*(a*b*c*f - a*b*d*cosh(1) - a*b*d*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2
*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2
*(a*b*c*f - a*b*d*cosh(1) - a*b*d*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*co
sh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(a*b
*d*f*x + a*b*c*f)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x
+ c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) +
2*(a*b*d*f*x + a*b*c*f)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sin
h(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)
/b))/((a^2*b + b^3)*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e + fx)\sinh(c + dx)}{a + b\sinh(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)*sinh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c + d\,x)\,\left(e + f\,x\right)}{a + b\sinh\left(c + d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
[Out] int((sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.226 $\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=54

$$\frac{x}{b}+\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d}$$

```
[Out] x/b+2*a*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/b/d/(a^2+b^2)^(1
/2)
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2814, 2739, 632, 210}

$$\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd\sqrt{a^2+b^2}}+\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] x/b + (2*a*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b*Sqrt[a^2
+ b^2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{x}{b}-\frac{a\int \frac{1}{a+b\sinh(c+dx)}\,dx}{b}\\ &= \frac{x}{b}+\frac{(2ia)\mathrm{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{bd}\\ &= \frac{x}{b}-\frac{(4ia)\mathrm{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx,x,-2ib+2a\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{bd}\\ &= \frac{x}{b}+\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 64, normalized size = 1.19

$$\frac{\frac{c}{d}+x-\frac{2a\,\mathrm{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,d}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]),x]
[Out] (c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(Sqrt[-
a^2 - b^2]*d))/b
```

Maple [A]
time = 0.54, size = 82, normalized size = 1.52

method	result	size
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}}{d}$	82
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}}{d}$	82

risch	$\frac{x}{b}+\frac{a\ln\left(e^{dx+c}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,db}-\frac{a\ln\left(e^{dx+c}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,db}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b*ln(tanh(1/2*d*x+1/2*c)+1)-2*a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*t
anh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-1/b*ln(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [A]
time = 0.49, size = 85, normalized size = 1.57

$$-\frac{a\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,bd}+\frac{dx+c}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*b*d) + (d*x + c)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(51) = 102$.
time = 0.39, size = 186, normalized size = 3.44

$$\frac{(a^2+b^2)dx+\sqrt{a^2+b^2}\,a\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2+b^2+2\,(b^2\cosh(dx+c)+ab)\sinh(dx+c)+2\,\sqrt{a^2+b^2}\,(b\cosh(dx+c)+b\sinh(dx+c)+a)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)-b}\right)}{(a^2b+b^3)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((a^2 + b^2)*d*x + sqrt(a^2 + b^2)*a*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*
x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*
sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/
(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x
+ c) + a)*sinh(d*x + c) - b)))/((a^2*b + b^3)*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(44) = 88$.

time = 36.21, size = 350, normalized size = 6.48

$$\begin{cases} \tilde{\infty}x & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{x}{b} & \text{for } a = 0 \\ \frac{x\sinh(c)}{a+b\sinh(c)} & \text{for } d = 0 \\ \frac{\cosh(c+dx)}{ad} & \text{for } b = 0 \\ \frac{bdx\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{b^2d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-bd\sqrt{-b^2}} - \frac{2b}{b^2d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-bd\sqrt{-b^2}} - \frac{dx\sqrt{-b^2}}{b^2d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-bd\sqrt{-b^2}} & \text{for } a = -\sqrt{-b^2} \\ \frac{bdx\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{b^2d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+bd\sqrt{-b^2}} - \frac{2b}{b^2d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+bd\sqrt{-b^2}} + \frac{dx\sqrt{-b^2}}{b^2d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+bd\sqrt{-b^2}} & \text{for } a = \sqrt{-b^2} \\ \frac{a\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{bd\sqrt{a^2+b^2}} - \frac{a\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{bd\sqrt{a^2+b^2}} + \frac{x}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/b, Eq(a, 0)), (x*sinh
(c)/(a + b*sinh(c)), Eq(d, 0)), (cosh(c + d*x)/(a*d), Eq(b, 0)), (b*d*x*tan
h(c/2 + d*x/2)/(b**2*d*tanh(c/2 + d*x/2) - b*d*sqrt(-b**2)) - 2*b/(b**2*d*t
anh(c/2 + d*x/2) - b*d*sqrt(-b**2)) - d*x*sqrt(-b**2)/(b**2*d*tanh(c/2 + d*
x/2) - b*d*sqrt(-b**2)), Eq(a, -sqrt(-b**2))), (b*d*x*tanh(c/2 + d*x/2)/(b*
*2*d*tanh(c/2 + d*x/2) + b*d*sqrt(-b**2)) - 2*b/(b**2*d*tanh(c/2 + d*x/2) +
 b*d*sqrt(-b**2)) + d*x*sqrt(-b**2)/(b**2*d*tanh(c/2 + d*x/2) + b*d*sqrt(-b
**2)), Eq(a, sqrt(-b**2))), (a*log(tanh(c/2 + d*x/2) - b/a - sqrt(a**2 + b*
*2)/a)/(b*d*sqrt(a**2 + b**2)) - a*log(tanh(c/2 + d*x/2) - b/a + sqrt(a**2
+ b**2)/a)/(b*d*sqrt(a**2 + b**2)) + x/b, True))
```

Giac [A]
time = 0.43, size = 84, normalized size = 1.56

$$-\frac{\frac{a\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b} - \frac{dx+c}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -(a*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c)
+ 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b) - (d*x + c)/b)/d
```

Mupad [B]
time = 0.82, size = 121, normalized size = 2.24

$$\frac{x}{b}-\frac{a\,\ln\left(\frac{2\,a\,\mathrm{e}^{c+d\,x}}{b^2}-\frac{2\,a\,\left(b-a\,\mathrm{e}^{c+d\,x}\right)}{b^2\,\sqrt{a^2+b^2}}\right)}{b\,d\,\sqrt{a^2+b^2}}+\frac{a\,\ln\left(\frac{2\,a\,\mathrm{e}^{c+d\,x}}{b^2}+\frac{2\,a\,\left(b-a\,\mathrm{e}^{c+d\,x}\right)}{b^2\,\sqrt{a^2+b^2}}\right)}{b\,d\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a + b*sinh(c + d*x)),x)
```

```
[Out] x/b - (a*log((2*a*exp(c + d*x))/b^2 - (2*a*(b - a*exp(c + d*x)))/(b^2*(a^2
+ b^2)^(1/2))))/(b*d*(a^2 + b^2)^(1/2)) + (a*log((2*a*exp(c + d*x))/b^2 + (
2*a*(b - a*exp(c + d*x)))/(b^2*(a^2 + b^2)^(1/2))))/(b*d*(a^2 + b^2)^(1/2))
```

3.227 $\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sinh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][Sinh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 6.15, size = 0, normalized size = 0.00

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Integrate[Sinh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(dx+c)}{(fx+e)\,(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*a*integrate(-e^(d*x + c)/(b^2*f*x + b^2*e - (b^2*f*x*e^(2*c) + b^2*e^(2*
c + 1))*e^(2*d*x) - 2*(a*b*f*x*e^c + a*b*e^(c + 1))*e^(d*x)), x) + log(f*x
+ e)/(b*f)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sinh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sinh(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sinh(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))),x)
[Out] int(sinh(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.228 $\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=551

$$-\frac{a(e+fx)^4}{4b^2f}+\frac{6f^2(e+fx)\cosh(c+dx)}{bd^3}+\frac{(e+fx)^3\cosh(c+dx)}{bd}+\frac{a^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d}-\ldots$$

```
[Out] -1/4*a*(f*x+e)^4/b^2/f+6*f^2*(f*x+e)*cosh(d*x+c)/b/d^3+(f*x+e)^3*cosh(d*x+c
)/b/d-6*f^3*sinh(d*x+c)/b/d^4-3*f*(f*x+e)^2*sinh(d*x+c)/b/d^2+a^2*(f*x+e)^3
*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d/(a^2+b^2)^(1/2)-a^2*(f*x+e)^3
*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d/(a^2+b^2)^(1/2)+3*a^2*f*(f*x+
e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d^2/(a^2+b^2)^(1/2)-3
*a^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d^2/(a^2+
b^2)^(1/2)-6*a^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b
^2/d^3/(a^2+b^2)^(1/2)+6*a^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/b^2/d^3/(a^2+b^2)^(1/2)+6*a^2*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^
2+b^2)^(1/2)))/b^2/d^4/(a^2+b^2)^(1/2)-6*a^2*f^3*polylog(4,-b*exp(d*x+c)/(a
+(a^2+b^2)^(1/2)))/b^2/d^4/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.72, antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {5676, 3377, 2717, 32, 3403, 2296, 2221, 2611, 6744, 2320, 6724}

$$\frac{6a^2f^3\text{Li}_4\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^4\sqrt{a^2+b^2}}-\frac{6a^2f^3\text{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^4\sqrt{a^2+b^2}}-\frac{6a^2f^2(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3\sqrt{a^2+b^2}}+\frac{6a^2f^2(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^3\sqrt{a^2+b^2}}+\frac{3a^2f(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2\sqrt{a^2+b^2}}-\frac{3a^2f(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^2\sqrt{a^2+b^2}}+\frac{a^2(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^2d\sqrt{a^2+b^2}}-\frac{a^2(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^2d\sqrt{a^2+b^2}}-\frac{a(e+fx)^4}{4b^2f}-\frac{6f^3\sinh(c+dx)}{bd^4}+\frac{6f^2(e+fx)\cosh(c+dx)}{bd^3}-\frac{3f(e+fx)^2\sinh(c+dx)}{bd^2}+\frac{(e+fx)^3\cosh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/4*(a*(e + f*x)^4)/(b^2*f) + (6*f^2*(e + f*x)*Cosh[c + d*x])/(b*d^3) + ((
e + f*x)^3*Cosh[c + d*x])/(b*d) + (a^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])])/(b^2*Sqrt[a^2 + b^2]*d) - (a^2*(e + f*x)^3*Log[1 +
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*Sqrt[a^2 + b^2]*d) + (3*a^2*f*
(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*Sqrt
[a^2 + b^2]*d^2) - (3*a^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^2) - (6*a^2*f^2*(e + f*x)*PolyLog
[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^3) +
(6*a^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/
(b^2*Sqrt[a^2 + b^2]*d^3) + (6*a^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^4) - (6*a^2*f^3*PolyLog[4, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^4) - (6*f^3*Sinh
[c + d*x])/(b*d^4) - (3*f*(e + f*x)^2*Sinh[c + d*x])/(b*d^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5676

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3 \sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3 \sinh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)^3\cosh(c+dx)}{bd} - \frac{a\int (e+fx)^3\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}\,dx}{b^2} - (3f)\int($$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{(e+fx)^3\cosh(c+dx)}{bd} - \frac{3f(e+fx)^2\sinh(c+dx)}{bd^2} + \frac{(2a^2)}{}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cosh(c+dx)}{bd^3} + \frac{(e+fx)^3\cosh(c+dx)}{bd} - \frac{3f(e}{}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cosh(c+dx)}{bd^3} + \frac{(e+fx)^3\cosh(c+dx)}{bd} + \frac{a^2(e}{}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cosh(c+dx)}{bd^3} + \frac{(e+fx)^3\cosh(c+dx)}{bd} + \frac{a^2(e}{}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cosh(c+dx)}{bd^3} + \frac{(e+fx)^3\cosh(c+dx)}{bd} + \frac{a^2(e}{}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cosh(c+dx)}{bd^3} + \frac{(e+fx)^3\cosh(c+dx)}{bd} + \frac{a^2(e}{}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cosh(c+dx)}{bd^3} + \frac{(e+fx)^3\cosh(c+dx)}{bd} + \frac{a^2(e}{}$$

Mathematica [A]
time = 6.61, size = 1074, normalized size = 1.95

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-(a*d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)) + 4*b*d*(e + f*x)*(
6*f^2 + d^2*(e + f*x)^2)*Cosh[c + d*x] + (4*a^2*(-2*d^3*e^3*Sqrt[(a^2 + b^2
)*E^(2*c)]*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 3*Sqrt[a^2 + b^2]
*d^3*e^2*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c
```

```
)])] + 3*Sqrt[a^2 + b^2]*d^3*e*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c
 - Sqrt[(a^2 + b^2)*E^(2*c)])] + Sqrt[a^2 + b^2]*d^3*E^c*f^3*x^3*Log[1 + (b
*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*Sqrt[a^2 + b^2]*d^
3*e^2*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])
] - 3*Sqrt[a^2 + b^2]*d^3*e*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c +
Sqrt[(a^2 + b^2)*E^(2*c)])] - Sqrt[a^2 + b^2]*d^3*E^c*f^3*x^3*Log[1 + (b*E^
(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 3*Sqrt[a^2 + b^2]*d^2*E
^c*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E
^(2*c)]))] - 3*Sqrt[a^2 + b^2]*d^2*E^c*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c
 + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[a^2 + b^2]*d*e*E^c*
f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] -
6*Sqrt[a^2 + b^2]*d*E^c*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[
(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[a^2 + b^2]*d*e*E^c*f^2*PolyLog[3, -((b*E^(
2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[a^2 + b^2]*d*E^c
*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]
 + 6*Sqrt[a^2 + b^2]*E^c*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(
a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[a^2 + b^2]*E^c*f^3*PolyLog[4, -((b*E^(2*c +
 d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(Sqrt[a^2 + b^2]*Sqrt[(a^2 +
 b^2)*E^(2*c)]) - 12*b*f*(2*f^2 + d^2*(e + f*x)^2)*Sinh[c + d*x])/(4*b^2*d^
4)
```

Maple [**F**]
time = 1.80, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \left(\sinh^2(dx+c)\right)}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a +
 sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2*d) - 2*(d*x + c)*a/(b^2*d) + e^(d*x
 + c)/(b*d) + e^(-d*x - c)/(b*d))*e^3 - 1/4*(a*d^4*f^3*x^4*e^c + 4*a*d^4*f^
2*x^3*e^(c + 1) + 6*a*d^4*f*x^2*e^(c + 2) - 2*(b*d^3*f^3*x^3*e^(2*c) - 6*b*
f^3*e^(2*c) - 3*b*d^2*f*e^(2*c + 2) + 6*b*d*f^2*e^(2*c + 1) - 3*(b*d^2*f^3*
e^(2*c) - b*d^3*f^2*e^(2*c + 1))*x^2 + 3*(2*b*d*f^3*e^(2*c) + b*d^3*f*e^(2*
```

```
c + 2) - 2*b*d^2*f^2*e^(2*c + 1))*x)*e^(d*x) - 2*(b*d^3*f^3*x^3 + 3*b*d^2*f
*e^2 + 6*b*d*f^2*e + 6*b*f^3 + 3*(b*d^3*f^2*e + b*d^2*f^3)*x^2 + 3*(b*d^3*f
*e^2 + 2*b*d^2*f^2*e + 2*b*d*f^3)*x)*e^(-d*x))*e^(-c)/(b^2*d^4) + integrate
(2*(a^2*f^3*x^3*e^c + 3*a^2*f^2*x^2*e^(c + 1) + 3*a^2*f*x*e^(c + 2))*e^(d*x
)/(b^3*e^(2*d*x + 2*c) + 2*a*b^2*e^(d*x + c) - b^3), x)
```

<u>Fricas</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4491 vs. $2(517) = 1034$.

time = 0.56, size = 4491, normalized size = 8.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^2*b + b^3)*d^3*f^3*x^3 + 6*(a^2*b + b^3)*d^2*f^3*x^2 + 2*(a^2*b +
 b^3)*d^3*cosh(1)^3 + 2*(a^2*b + b^3)*d^3*sinh(1)^3 + 12*(a^2*b + b^3)*d*f^
3*x + 12*(a^2*b + b^3)*f^3 + 6*((a^2*b + b^3)*d^3*f*x + (a^2*b + b^3)*d^2*f
)*cosh(1)^2 + 2*((a^2*b + b^3)*d^3*f^3*x^3 - 3*(a^2*b + b^3)*d^2*f^3*x^2 +
(a^2*b + b^3)*d^3*cosh(1)^3 + (a^2*b + b^3)*d^3*sinh(1)^3 + 6*(a^2*b + b^3)
*d*f^3*x - 6*(a^2*b + b^3)*f^3 + 3*((a^2*b + b^3)*d^3*f*x - (a^2*b + b^3)*d
^2*f)*cosh(1)^2 + 3*((a^2*b + b^3)*d^3*f*x + (a^2*b + b^3)*d^3*cosh(1) - (a
^2*b + b^3)*d^2*f)*sinh(1)^2 + 3*((a^2*b + b^3)*d^3*f^2*x^2 - 2*(a^2*b + b^
3)*d^2*f^2*x + 2*(a^2*b + b^3)*d*f^2)*cosh(1) + 3*((a^2*b + b^3)*d^3*f^2*x^
2 - 2*(a^2*b + b^3)*d^2*f^2*x + (a^2*b + b^3)*d^3*cosh(1)^2 + 2*(a^2*b + b^
3)*d*f^2 + 2*((a^2*b + b^3)*d^3*f*x - (a^2*b + b^3)*d^2*f)*cosh(1))*sinh(1)
)*cosh(d*x + c)^2 + 6*((a^2*b + b^3)*d^3*f*x + (a^2*b + b^3)*d^3*cosh(1) +
(a^2*b + b^3)*d^2*f)*sinh(1)^2 + 2*((a^2*b + b^3)*d^3*f^3*x^3 - 3*(a^2*b +
b^3)*d^2*f^3*x^2 + (a^2*b + b^3)*d^3*cosh(1)^3 + (a^2*b + b^3)*d^3*sinh(1)^
3 + 6*(a^2*b + b^3)*d*f^3*x - 6*(a^2*b + b^3)*f^3 + 3*((a^2*b + b^3)*d^3*f*
x - (a^2*b + b^3)*d^2*f)*cosh(1)^2 + 3*((a^2*b + b^3)*d^3*f*x + (a^2*b + b^
3)*d^3*cosh(1) - (a^2*b + b^3)*d^2*f)*sinh(1)^2 + 3*((a^2*b + b^3)*d^3*f^2*
x^2 - 2*(a^2*b + b^3)*d^2*f^2*x + 2*(a^2*b + b^3)*d*f^2)*cosh(1) + 3*((a^2*
b + b^3)*d^3*f^2*x^2 - 2*(a^2*b + b^3)*d^2*f^2*x + (a^2*b + b^3)*d^3*cosh(1
)^2 + 2*(a^2*b + b^3)*d*f^2 + 2*((a^2*b + b^3)*d^3*f*x - (a^2*b + b^3)*d^2*
f)*cosh(1))*sinh(1))*sinh(d*x + c)^2 + 12*((a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2
*f^2*x*cosh(1) + a^2*b*d^2*f*cosh(1)^2 + a^2*b*d^2*f*sinh(1)^2 + 2*(a^2*b*d
^2*f^2*x + a^2*b*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c) + (a^2*b*d^2*f^3*x^2
 + 2*a^2*b*d^2*f^2*x*cosh(1) + a^2*b*d^2*f*cosh(1)^2 + a^2*b*d^2*f*sinh(1)^
2 + 2*(a^2*b*d^2*f^2*x + a^2*b*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 12*((a^2*b*d^2*f^3
*x^2 + 2*a^2*b*d^2*f^2*x*cosh(1) + a^2*b*d^2*f*cosh(1)^2 + a^2*b*d^2*f*sinh
(1)^2 + 2*(a^2*b*d^2*f^2*x + a^2*b*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c) +
(a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*f^2*x*cosh(1) + a^2*b*d^2*f*cosh(1)^2 + a^
```

```
2*b*d^2*f*sinh(1)^2 + 2*(a^2*b*d^2*f^2*x + a^2*b*d^2*f*cosh(1))*sinh(1))*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c)
 - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) +
4*((a^2*b*c^3*f^3 - 3*a^2*b*c^2*d*f^2*cosh(1) + 3*a^2*b*c*d^2*f*cosh(1)^2 -
 a^2*b*d^3*cosh(1)^3 - a^2*b*d^3*sinh(1)^3 + 3*(a^2*b*c*d^2*f - a^2*b*d^3*c
osh(1))*sinh(1)^2 - 3*(a^2*b*c^2*d*f^2 - 2*a^2*b*c*d^2*f*cosh(1) + a^2*b*d^
3*cosh(1)^2)*sinh(1))*cosh(d*x + c) + (a^2*b*c^3*f^3 - 3*a^2*b*c^2*d*f^2*co
sh(1) + 3*a^2*b*c*d^2*f*cosh(1)^2 - a^2*b*d^3*cosh(1)^3 - a^2*b*d^3*sinh(1)
^3 + 3*(a^2*b*c*d^2*f - a^2*b*d^3*cosh(1))*sinh(1)^2 - 3*(a^2*b*c^2*d*f^2 -
 2*a^2*b*c*d^2*f*cosh(1) + a^2*b*d^3*cosh(1)^2)*sinh(1))*sinh(d*x + c))*sqr
t((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^
2 + b^2)/b^2) + 2*a) - 4*((a^2*b*c^3*f^3 - 3*a^2*b*c^2*d*f^2*cosh(1) + 3*a^
2*b*c*d^2*f*cosh(1)^2 - a^2*b*d^3*cosh(1)^3 - a^2*b*d^3*sinh(1)^3 + 3*(a^2*
b*c*d^2*f - a^2*b*d^3*cosh(1))*sinh(1)^2 - 3*(a^2*b*c^2*d*f^2 - 2*a^2*b*c*d
^2*f*cosh(1) + a^2*b*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c) + (a^2*b*c^3*f^3
 - 3*a^2*b*c^2*d*f^2*cosh(1) + 3*a^2*b*c*d^2*f*cosh(1)^2 - a^2*b*d^3*cosh(1
)^3 - a^2*b*d^3*sinh(1)^3 + 3*(a^2*b*c*d^2*f - a^2*b*d^3*cosh(1))*sinh(1)^2
 - 3*(a^2*b*c^2*d*f^2 - 2*a^2*b*c*d^2*f*cosh(1) + a^2*b*d^3*cosh(1)^2)*sinh
(1))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(
d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 4*((a^2*b*d^3*f^3*x^3 + a^2*b
*c^3*f^3 + 3*(a^2*b*d^3*f*x + a^2*b*c*d^2*f)*cosh(1)^2 + 3*(a^2*b*d^3*f*x +
 a^2*b*c*d^2*f)*sinh(1)^2 + 3*(a^2*b*d^3*f^2*x^2 - a^2*b*c^2*d*f^2)*cosh(1)
 + 3*(a^2*b*d^3*f^2*x^2 - a^2*b*c^2*d*f^2 + 2*(a^2*b*d^3*f*x + a^2*b*c*d^2*
f)*cosh(1))*sinh(1))*cosh(d*x + c) + (a^2*b*d^3*f^3*x^3 + a^2*b*c^3*f^3 + 3
*(a^2*b*d^3*f*x + a^2*b*c*d^2*f)*cosh(1)^2 + 3*(a^2*b*d^3*f*x + a^2*b*c*d^2
*f)*sinh(1)^2 + 3*(a^2*b*d^3*f^2*x^2 - a^2*b*c^2*d*f^2)*cosh(1) + 3*(a^2*b*
d^3*f^2*x^2 - a^2*b*c^2*d*f^2 + 2*(a^2*b*d^3*f*x + a^2*b*c*d^2*f)*cosh(1))*
sinh(1))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sin
h(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)
/b) - 4*((a^2*b*d^3*f^3*x^3 + a^2*b*c^3*f^3 + 3*(a^2*b*d^3*f*x + a^2*b*c*d^
2*f)*cosh(1)^2 + 3*(a^2*b*d^3*f*x + a^2*b*c*d^2*f)*sinh(1)^2 + 3*(a^2*b*d^3
*f^2*x^2 - a^2*b*c^2*d*f^2)*cosh(1) + 3*(a^2*b*d^3*f^2*x^2 - a^2*b*c^2*d*f^
2 + 2*(a^2*b*d^3*f*x + a^2*b*c*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c) + (a^
2*b*d^3*f^3*x^3 + a^2*b*c^3*f^3 + 3*(a^2*b*d^3*f*x + a^2*b*c*d^2*f)*cosh(1)
^2 + 3*(a^2*b*d^3*f*x + a^2*b*c*d^2*f)*sinh(1)^2 + 3*(a^2*b*d^3*f^2*x^2 - a
^2*b*c^2*d*f^2)*cosh(1) + 3*(a^2*b*d^3*f^2*x^2 - a^2*b*c^2*d*f^2 + 2*(a^2*b
*d^3*f*x + a^2*b*c*d^2*f)*cosh(1))*sinh(1))*sin...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^3*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
[Out] int((sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.229 $\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=407

$$-\frac{a(e+fx)^3}{3b^2f}+\frac{2f^2\cosh(c+dx)}{bd^3}+\frac{(e+fx)^2\cosh(c+dx)}{bd}+\frac{a^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d}-\frac{a^2(e+fx}{}$$

```
[Out] -1/3*a*(f*x+e)^3/b^2/f+2*f^2*cosh(d*x+c)/b/d^3+(f*x+e)^2*cosh(d*x+c)/b/d-2*
f*(f*x+e)*sinh(d*x+c)/b/d^2+a^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1
/2)))/b^2/d/(a^2+b^2)^(1/2)-a^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/b^2/d/(a^2+b^2)^(1/2)+2*a^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2
+b^2)^(1/2)))/b^2/d^2/(a^2+b^2)^(1/2)-2*a^2*f*(f*x+e)*polylog(2,-b*exp(d*x+
c)/(a+(a^2+b^2)^(1/2)))/b^2/d^2/(a^2+b^2)^(1/2)-2*a^2*f^2*polylog(3,-b*exp(
d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d^3/(a^2+b^2)^(1/2)+2*a^2*f^2*polylog(3,-b*
exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d^3/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.60, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5676, 3377, 2718, 32, 3403, 2296, 2221, 2611, 2320, 6724}

$$-\frac{2a^2f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3\sqrt{a^2+b^2}}+\frac{2a^2f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^3\sqrt{a^2+b^2}}+\frac{2a^2f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2\sqrt{a^2+b^2}}-\frac{2a^2f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^2\sqrt{a^2+b^2}}+\frac{a^2(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^2d\sqrt{a^2+b^2}}-\frac{a^2(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^2d\sqrt{a^2+b^2}}-\frac{a(e+fx)^3}{3b^2f}+\frac{2f^2\cosh(c+dx)}{bd^3}-\frac{2f(e+fx)\sinh(c+dx)}{bd^2}+\frac{(e+fx)^2\cosh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/3*(a*(e + f*x)^3)/(b^2*f) + (2*f^2*Cosh[c + d*x])/(b*d^3) + ((e + f*x)^2
*Cosh[c + d*x])/(b*d) + (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2])])/(b^2*Sqrt[a^2 + b^2]*d) - (a^2*(e + f*x)^2*Log[1 + (b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*Sqrt[a^2 + b^2]*d) + (2*a^2*f*(e + f*x)*
PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*
d^2) - (2*a^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
))])/(b^2*Sqrt[a^2 + b^2]*d^2) - (2*a^2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^3) + (2*a^2*f^2*PolyLog[3, -(
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^3) - (2*f*(
e + f*x)*Sinh[c + d*x])/(b*d^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5676

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)^2\cosh(c+dx)}{bd} - \frac{a\int (e+fx)^2\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}\,dx}{b^2} - \frac{(2f)\int$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{(e+fx)^2\cosh(c+dx)}{bd} - \frac{2f(e+fx)\sinh(c+dx)}{bd^2} + \frac{(2a^2)}{}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cosh(c+dx)}{bd^3} + \frac{(e+fx)^2\cosh(c+dx)}{bd} - \frac{2f(e+fx)}{b}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cosh(c+dx)}{bd^3} + \frac{(e+fx)^2\cosh(c+dx)}{bd} + \frac{a^2(e+fx)^2}{b}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cosh(c+dx)}{bd^3} + \frac{(e+fx)^2\cosh(c+dx)}{bd} + \frac{a^2(e+fx)^2}{b}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cosh(c+dx)}{bd^3} + \frac{(e+fx)^2\cosh(c+dx)}{bd} + \frac{a^2(e+fx)^2}{b}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cosh(c+dx)}{bd^3} + \frac{(e+fx)^2\cosh(c+dx)}{bd} + \frac{a^2(e+fx)^2}{b}$$

Mathematica [A]

time = 5.32, size = 697, normalized size = 1.71

$$\frac{-ax(3e^2+3efx+f^2x^2)+\frac{3a^2\left(-2d^2e^2\sqrt{(a^2+b^2)e^{2c}}\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2\sqrt{a^2+b^2}\,d^2ee^cfx\log\left(1+\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)+\sqrt{a^2+b^2}\,d^2e^cf^2x^2\log\left(1+\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)-2\sqrt{a^2+b^2}\,d^2ee^cfx\log\left(1+\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)-\sqrt{a^2+b^2}\,d^2e^cf^2x^2\log\left(1+\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)+2\sqrt{a^2+b^2}\,de^cf(e+fx)\text{PolyLog}\left(2,-\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)-2\sqrt{a^2+b^2}\,de^cf(e+fx)\text{PolyLog}\left(2,-\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)-2\sqrt{a^2+b^2}\,e^cf^2\text{PolyLog}\left(3,-\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)+2\sqrt{a^2+b^2}\,e^cf^2\text{PolyLog}\left(3,-\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)\right)}{\sqrt{a^2+b^2}\,d^3\sqrt{(a^2+b^2)e^{2c}}}+\frac{3b\cosh(dx)\left((2f^2+d^2(e+fx)^2)\cosh(c)-2df(e+fx)\sinh(c)\right)}{d^3}+\frac{3b\left(-2df(e+fx)\cosh(c)+(2f^2+d^2(e+fx)^2)\sinh(c)\right)\sinh(dx)}{d^3}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-(a*x*(3*e^2 + 3*e*f*x + f^2*x^2)) + (3*a^2*(-2*d^2*e^2*Sqrt[(a^2 + b^2)*E
^(2*c)]*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*Sqrt[a^2 + b^2]*d^
2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])]
+ Sqrt[a^2 + b^2]*d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(
a^2 + b^2)*E^(2*c)])] - 2*Sqrt[a^2 + b^2]*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c +
 d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - Sqrt[a^2 + b^2]*d^2*E^c*f^2*x
^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*Sqrt[
a^2 + b^2]*d*E^c*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(
a^2 + b^2)*E^(2*c)]))] - 2*Sqrt[a^2 + b^2]*d*E^c*f*(e + f*x)*PolyLog[2, -((
b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*Sqrt[a^2 + b^2]*
E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))
] + 2*Sqrt[a^2 + b^2]*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[
(a^2 + b^2)*E^(2*c)]))]))/(Sqrt[a^2 + b^2]*d^3*Sqrt[(a^2 + b^2)*E^(2*c)]) +
 (3*b*Cosh[d*x]*((2*f^2 + d^2*(e + f*x)^2)*Cosh[c] - 2*d*f*(e + f*x)*Sinh[c
]))/d^3 + (3*b*(-2*d*f*(e + f*x)*Cosh[c] + (2*f^2 + d^2*(e + f*x)^2)*Sinh[c
])*Sinh[d*x])/d^3)/(3*b^2)
```

Maple [F]
time = 1.83, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2\left(\sinh^2(dx+c)\right)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a +
 sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2*d) - 2*(d*x + c)*a/(b^2*d) + e^(d*x
```

```
 + c)/(b*d) + e^(-d*x - c)/(b*d))*e^2 - 1/6*(2*a*d^3*f^2*x^3*e^c + 6*a*d^3*
f*x^2*e^(c + 1) - 3*(b*d^2*f^2*x^2*e^(2*c) + 2*b*f^2*e^(2*c) - 2*b*d*f*e^(2
*c + 1) - 2*(b*d*f^2*e^(2*c) - b*d^2*f*e^(2*c + 1))*x)*e^(d*x) - 3*(b*d^2*f
^2*x^2 + 2*b*d*f*e + 2*b*f^2 + 2*(b*d^2*f*e + b*d*f^2)*x)*e^(-d*x))*e^(-c)/
(b^2*d^3) + integrate(2*(a^2*f^2*x^2*e^c + 2*a^2*f*x*e^(c + 1))*e^(d*x)/(b^
3*e^(2*d*x + 2*c) + 2*a*b^2*e^(d*x + c) - b^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2365 vs. $2(380) = 760$.
time = 0.39, size = 2365, normalized size = 5.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(a^2*b + b^3)*d^2*f^2*x^2 + 6*(a^2*b + b^3)*d*f^2*x + 3*(a^2*b + b^3
)*d^2*cosh(1)^2 + 3*(a^2*b + b^3)*d^2*sinh(1)^2 + 6*(a^2*b + b^3)*f^2 + 3*(
(a^2*b + b^3)*d^2*f^2*x^2 - 2*(a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d^2*cos
h(1)^2 + (a^2*b + b^3)*d^2*sinh(1)^2 + 2*(a^2*b + b^3)*f^2 + 2*((a^2*b + b^
3)*d^2*f*x - (a^2*b + b^3)*d*f)*cosh(1) + 2*((a^2*b + b^3)*d^2*f*x + (a^2*b
 + b^3)*d^2*cosh(1) - (a^2*b + b^3)*d*f)*sinh(1))*cosh(d*x + c)^2 + 3*((a^2
*b + b^3)*d^2*f^2*x^2 - 2*(a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d^2*cosh(1)
^2 + (a^2*b + b^3)*d^2*sinh(1)^2 + 2*(a^2*b + b^3)*f^2 + 2*((a^2*b + b^3)*d
^2*f*x - (a^2*b + b^3)*d*f)*cosh(1) + 2*((a^2*b + b^3)*d^2*f*x + (a^2*b + b
^3)*d^2*cosh(1) - (a^2*b + b^3)*d*f)*sinh(1))*sinh(d*x + c)^2 + 12*((a^2*b*
d*f^2*x + a^2*b*d*f*cosh(1) + a^2*b*d*f*sinh(1))*cosh(d*x + c) + (a^2*b*d*f
^2*x + a^2*b*d*f*cosh(1) + a^2*b*d*f*sinh(1))*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sin
h(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 12*((a^2*b*d*f^2*x + a^2*b*
d*f*cosh(1) + a^2*b*d*f*sinh(1))*cosh(d*x + c) + (a^2*b*d*f^2*x + a^2*b*d*f
*cosh(1) + a^2*b*d*f*sinh(1))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a
*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b + 1) - 6*((a^2*b*c^2*f^2 - 2*a^2*b*c*d*f*cosh(1) +
 a^2*b*d^2*cosh(1)^2 + a^2*b*d^2*sinh(1)^2 - 2*(a^2*b*c*d*f - a^2*b*d^2*cos
h(1))*sinh(1))*cosh(d*x + c) + (a^2*b*c^2*f^2 - 2*a^2*b*c*d*f*cosh(1) + a^2
*b*d^2*cosh(1)^2 + a^2*b*d^2*sinh(1)^2 - 2*(a^2*b*c*d*f - a^2*b*d^2*cosh(1)
)*sinh(1))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b
*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 6*((a^2*b*c^2*f^2 - 2*a
^2*b*c*d*f*cosh(1) + a^2*b*d^2*cosh(1)^2 + a^2*b*d^2*sinh(1)^2 - 2*(a^2*b*c
*d*f - a^2*b*d^2*cosh(1))*sinh(1))*cosh(d*x + c) + (a^2*b*c^2*f^2 - 2*a^2*b
*c*d*f*cosh(1) + a^2*b*d^2*cosh(1)^2 + a^2*b*d^2*sinh(1)^2 - 2*(a^2*b*c*d*f
 - a^2*b*d^2*cosh(1))*sinh(1))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b
*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 6*(
(a^2*b*d^2*f^2*x^2 - a^2*b*c^2*f^2 + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*cosh(1
```

```
) + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*sinh(1))*cosh(d*x + c) + (a^2*b*d^2*f^2
*x^2 - a^2*b*c^2*f^2 + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*cosh(1) + 2*(a^2*b*d
^2*f*x + a^2*b*c*d*f)*sinh(1))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a
*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b) - 6*((a^2*b*d^2*f^2*x^2 - a^2*b*c^2*f^2 + 2*(a^2*
b*d^2*f*x + a^2*b*c*d*f)*cosh(1) + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*sinh(1))
*cosh(d*x + c) + (a^2*b*d^2*f^2*x^2 - a^2*b*c^2*f^2 + 2*(a^2*b*d^2*f*x + a^
2*b*c*d*f)*cosh(1) + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*sinh(1))*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 12*(a^2*b*f^2*co
sh(d*x + c) + a^2*b*f^2*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*
cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2))/b) + 12*(a^2*b*f^2*cosh(d*x + c) + a^2*b*f^2*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*((a^2*b + b
^3)*d^2*f*x + (a^2*b + b^3)*d*f)*cosh(1) - 2*((a^3 + a*b^2)*d^3*f^2*x^3 + 3
*(a^3 + a*b^2)*d^3*f*x^2*cosh(1) + 3*(a^3 + a*b^2)*d^3*x*cosh(1)^2 + 3*(a^3
 + a*b^2)*d^3*x*sinh(1)^2 + 3*((a^3 + a*b^2)*d^3*f*x^2 + 2*(a^3 + a*b^2)*d^
3*x*cosh(1))*sinh(1))*cosh(d*x + c) + 6*((a^2*b + b^3)*d^2*f*x + (a^2*b + b
^3)*d^2*cosh(1) + (a^2*b + b^3)*d*f)*sinh(1) - 2*((a^3 + a*b^2)*d^3*f^2*x^3
 + 3*(a^3 + a*b^2)*d^3*f*x^2*cosh(1) + 3*(a^3 + a*b^2)*d^3*x*cosh(1)^2 + 3*
(a^3 + a*b^2)*d^3*x*sinh(1)^2 - 3*((a^2*b + b^3)*d^2*f^2*x^2 - 2*(a^2*b + b
^3)*d*f^2*x + (a^2*b + b^3)*d^2*cosh(1)^2 + (a^2*b + b^3)*d^2*sinh(1)^2 + 2
*(a^2*b + b^3)*f^2 + 2*((a^2*b + b^3)*d^2*f*x - (a^2*b + b^3)*d*f)*cosh(1)
+ 2*((a^2*b + b^3)*d^2*f*x + (a^2*b + b^3)*d^2*cosh(1) - (a^2*b + b^3)*d*f)
*sinh(1))*cosh(d*x + c) + 3*((a^3 + a*b^2)*d^3*f*x^2 + 2*(a^3 + a*b^2)*d^3*
x*cosh(1))*sinh(1))*sinh(d*x + c))/((a^2*b^2 + b^4)*d^3*cosh(d*x + c) + (a^
2*b^2 + b^4)*d^3*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^2\,{(e+f\,x)}^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
[Out] int((sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.230 $\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=264

$$-\frac{aex}{b^2}-\frac{afx^2}{2b^2}+\frac{(e+fx)\cosh(c+dx)}{bd}+\frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d}-\frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right.}{b^2\sqrt{a^2+b^2}\,d}$$

```
[Out] -a*e*x/b^2-1/2*a*f*x^2/b^2+(f*x+e)*cosh(d*x+c)/b/d-f*sinh(d*x+c)/b/d^2+a^2*
(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d/(a^2+b^2)^(1/2)-a^2*(f
*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d/(a^2+b^2)^(1/2)+a^2*f*po
lylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d^2/(a^2+b^2)^(1/2)-a^2*f*po
lylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d^2/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5676, 3377, 2717, 3403, 2296, 2221, 2317, 2438}

$$\frac{a^2 f \text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2\sqrt{a^2+b^2}}-\frac{a^2 f \text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^2\sqrt{a^2+b^2}}+\frac{a^2(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^2d\sqrt{a^2+b^2}}-\frac{a^2(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^2d\sqrt{a^2+b^2}}-\frac{aex}{b^2}-\frac{afx^2}{2b^2}-\frac{f\sinh(c+dx)}{bd^2}+\frac{(e+fx)\cosh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a*e*x)/b^2) - (a*f*x^2)/(2*b^2) + ((e + f*x)*Cosh[c + d*x])/(b*d) + (a^2
*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*Sqrt[a^2 +
b^2]*d) - (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b
^2*Sqrt[a^2 + b^2]*d) + (a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^2) - (a^2*f*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^2) - (f*Sinh[c + d*x])/(b*d^
2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
```

```
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5676

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)\cosh(c+dx)}{bd} - \frac{a\int (e+fx)\,dx}{b^2} + \frac{a^2\int \frac{e+fx}{a+b\sinh(c+dx)}\,dx}{b^2} - \frac{f\int \cosh(c}{bd}$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx)\cosh(c+dx)}{bd} - \frac{f\sinh(c+dx)}{bd^2} + \frac{(2a^2)\int \frac{e^{c+dx}}{-b+2ae^{c+d}}}{b^2}$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx)\cosh(c+dx)}{bd} - \frac{f\sinh(c+dx)}{bd^2} + \frac{(2a^2)\int \frac{e^{c+}}{2a-2\sqrt{a^2}}}{b\sqrt{a^2}}$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx)\cosh(c+dx)}{bd} + \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d}$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx)\cosh(c+dx)}{bd} + \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d}$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx)\cosh(c+dx)}{bd} + \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d}$$

Mathematica [**A**]
time = 1.41, size = 299, normalized size = 1.13

$$\frac{a(c+dx)(cf-d(2e+fx))+2bd(e+fx)\cosh(c+dx)+\frac{2a^2\left(-2de\tanh^{-1}\left(\frac{a+b\cosh(c+dx)+b\sinh(c+dx)}{\sqrt{a^2+b^2}}\right)+2cf\tanh^{-1}\left(\frac{a+b\cosh(c+dx)+b\sinh(c+dx)}{\sqrt{a^2+b^2}}\right)+f(c+dx)\log\left(1+\frac{b(\cosh(c+dx)+\sinh(c+dx))}{a-\sqrt{a^2+b^2}}\right)-f(c+dx)\log\left(1+\frac{b(\cosh(c+dx)+\sinh(c+dx))}{a+\sqrt{a^2+b^2}}\right)+f\,\text{PolyLog}\left(2,\frac{b(\cosh(c+dx)+\sinh(c+dx))}{-a+\sqrt{a^2+b^2}}\right)-f\,\text{PolyLog}\left(2,-\frac{b(\cosh(c+dx)+\sinh(c+dx))}{a+\sqrt{a^2+b^2}}\right)\right)}{\sqrt{a^2+b^2}}-2bf\sinh(c+dx)}{2b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*(c + d*x)*(c*f - d*(2*e + f*x)) + 2*b*d*(e + f*x)*Cosh[c + d*x] + (2*a^2
*(-2*d*e*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] +
 2*c*f*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + f
*(c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2]
)] - f*(c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2
+ b^2])] + f*PolyLog[2, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2
+ b^2])] - f*PolyLog[2, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2
 + b^2]))]))/Sqrt[a^2 + b^2] - 2*b*f*Sinh[c + d*x])/(2*b^2*d^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 509 vs. 2(242) = 484.
time = 1.61, size = 510, normalized size = 1.93

method	result
risch	$-\frac{af\,x^{2}}{2b^{2}}-\frac{aex}{b^{2}}+\frac{(dxf+de-f)\mathrm{e}^{dx+c}}{2d^{2}b}+\frac{(dxf+de+f)\mathrm{e}^{-dx-c}}{2d^{2}b}-\frac{2a^{2}e\,\mathrm{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^{2}+b^{2}}}\right)}{d\,b^{2}\sqrt{a^{2}+b^{2}}}+\frac{a^{2}f\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^{2}}}{-a+\sqrt{a^{2}+}}\right.}{d\,b^{2}\sqrt{a^{2}+b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*f*x^2/b^2-a*e*x/b^2+1/2*(d*f*x+d*e-f)/d^2/b*exp(d*x+c)+1/2*(d*f*x+d*
e+f)/d^2/b*exp(-d*x-c)-2/d*a^2/b^2*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d
*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d*a^2/b^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)
+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d^2*a^2/b^2*f/(a^2+b^2)^(1/2)
*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d*a^2/b^2*f
/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x
-1/d^2*a^2/b^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^
2+b^2)^(1/2)))*c+1/d^2*a^2/b^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+
b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*a^2/b^2*f/(a^2+b^2)^(1/2)*dilog((
b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/d^2*a^2/b^2*f*c/(a^2
+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(4*a^2*integrate(x*e^(d*x + c)/(b^3*e^(2*d*x + 2*c) + 2*a*b^2*e^(d*x +
c) - b^3), x) - (a*d^2*x^2*e^c - (b*d*x*e^(2*c) - b*e^(2*c))*e^(d*x) - (b*d
*x + b)*e^(-d*x))*e^(-c)/(b^2*d^2))*f + 1/2*(2*a^2*log((b*e^(-d*x - c) - a
- sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)
*b^2*d) - 2*(d*x + c)*a/(b^2*d) + e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1078 vs. 2(244) = 488.
time = 0.37, size = 1078, normalized size = 4.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*cosh(1) + ((a^2*b + b^3)*d*f*x +
 (a^2*b + b^3)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1) - (a^2*b + b^3)*f)*cosh(
```

```
d*x + c)^2 + (a^2*b + b^3)*d*sinh(1) + ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)
*d*cosh(1) + (a^2*b + b^3)*d*sinh(1) - (a^2*b + b^3)*f)*sinh(d*x + c)^2 + 2
*(a^2*b*f*cosh(d*x + c) + a^2*b*f*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilo
g((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(a^2*b*f*cosh(d*x + c) + a^2*b*f*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) -
 (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*
((a^2*b*c*f - a^2*b*d*cosh(1) - a^2*b*d*sinh(1))*cosh(d*x + c) + (a^2*b*c*f
 - a^2*b*d*cosh(1) - a^2*b*d*sinh(1))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*
log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a
) - 2*((a^2*b*c*f - a^2*b*d*cosh(1) - a^2*b*d*sinh(1))*cosh(d*x + c) + (a^2
*b*c*f - a^2*b*d*cosh(1) - a^2*b*d*sinh(1))*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2)
 + 2*a) + 2*((a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c) + (a^2*b*d*f*x + a^2*b
*c*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d
*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b)
 - 2*((a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c) + (a^2*b*d*f*x + a^2*b*c*f)*s
inh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
 - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (a^2
*b + b^3)*f - ((a^3 + a*b^2)*d^2*f*x^2 + 2*(a^3 + a*b^2)*d^2*x*cosh(1) + 2*
(a^3 + a*b^2)*d^2*x*sinh(1))*cosh(d*x + c) - ((a^3 + a*b^2)*d^2*f*x^2 + 2*(
a^3 + a*b^2)*d^2*x*cosh(1) + 2*(a^3 + a*b^2)*d^2*x*sinh(1) - 2*((a^2*b + b^
3)*d*f*x + (a^2*b + b^3)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1) - (a^2*b + b^3
)*f)*cosh(d*x + c))*sinh(d*x + c))/((a^2*b^2 + b^4)*d^2*cosh(d*x + c) + (a^
2*b^2 + b^4)*d^2*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^2\,(e+f\,x)}{a+b\sinh\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
[Out] int((sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.231 $\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=71

$$-\frac{ax}{b^2}-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d}+\frac{\cosh(c+dx)}{bd}$$

```
[Out] -a*x/b^2+cosh(d*x+c)/b/d-2*a^2*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^
(1/2))/b^2/d/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2825, 12, 2814, 2739, 632, 210}

$$-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{ax}{b^2}+\frac{\cosh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a*x)/b^2) - (2*a^2*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(
b^2*Sqrt[a^2 + b^2]*d) + Cosh[c + d*x]/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{\cosh(c+dx)}{bd}-\frac{\int \frac{a\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}\\
&= \frac{\cosh(c+dx)}{bd}-\frac{a\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}\\
&= -\frac{ax}{b^2}+\frac{\cosh(c+dx)}{bd}+\frac{a^2\int \frac{1}{a+b\sinh(c+dx)}\,dx}{b^2}\\
&= -\frac{ax}{b^2}+\frac{\cosh(c+dx)}{bd}-\frac{(2ia^2)\,\mathrm{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{b^2d}\\
&= -\frac{ax}{b^2}+\frac{\cosh(c+dx)}{bd}+\frac{(4ia^2)\,\mathrm{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx,x,-2ib+2a\tan\left(\frac{1}{2}(ic+\right.\right.}{b^2d}\\
&= -\frac{ax}{b^2}-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d}+\frac{\cosh(c+dx)}{bd}
\end{aligned}
$$

Mathematica [A]
time = 0.22, size = 74, normalized size = 1.04

$$
\frac{-a\left(c+dx-\frac{2a\,\mathrm{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}\right)+b\cosh(c+dx)}{b^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-(a*(c + d*x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sq
rt[-a^2 - b^2])) + b*Cosh[c + d*x])/(b^2*d)
```

Maple [A]
time = 0.76, size = 121, normalized size = 1.70

method	result
derivativedivides	$\frac{-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}+\frac{2a^2\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}}{d}$
default	$\frac{-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}+\frac{2a^2\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}}{d}$
risch	$-\frac{ax}{b^2}+\frac{e^{dx+c}}{2bd}+\frac{e^{-dx-c}}{2bd}+\frac{a^2\ln\left(e^{dx+c}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d\,b^2}-\frac{a^2\ln\left(e^{dx+c}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d\,b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b/(tanh(1/2*d*x+1/2*c)-1)+a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/b/(tanh
(1/2*d*x+1/2*c)+1)-a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)+2*a^2/b^2/(a^2+b^2)^(1/2
)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [A]
time = 0.47, size = 119, normalized size = 1.68

$$\frac{a^2\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^2d}-\frac{(dx+c)a}{b^2d}+\frac{e^{(dx+c)}}{2\,bd}+\frac{e^{(-dx-c)}}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*b^2*d) - (d*x + c)*a/(b^2*d) + 1/2*e^(d*x + c)
/(b*d) + 1/2*e^(-d*x - c)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(68) = 136.
time = 0.34, size = 331, normalized size = 4.66

$$-\frac{2\,(a^3+ab^2)dx\cosh(dx+c)-a^2b-b^3-(a^2b+b^3)\cosh(dx+c)^2-(a^2b+b^3)\sinh(dx+c)^2-2\,(a^2\cosh(dx+c)+a^2\sinh(dx+c))\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2+b^2+2\,(b^2\cosh(dx+c)+ab)\sinh(dx+c)-2\sqrt{a^2+b^2}\,(b\cosh(dx+c)+b\sinh(dx+c)+a)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)-b}\right)+2\,((a^3+ab^2)dx-(a^2b+b^3)\cosh(dx+c))\sinh(dx+c)}{2\,((a^2b^2+b^4)d\cosh(dx+c)+(a^2b^2+b^4)d\sinh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")

[Out] -1/2*(2*(a^3 + a*b^2)*d*x*cosh(d*x + c) - a^2*b - b^3 - (a^2*b + b^3)*cosh(d*x + c)^2 - (a^2*b + b^3)*sinh(d*x + c)^2 - 2*(a^2*cosh(d*x + c) + a^2*sinh(d*x + c))*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*((a^3 + a*b^2)*d*x - (a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^2*b^2 + b^4)*d*cosh(d*x + c) + (a^2*b^2 + b^4)*d*sinh(d*x + c))

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1748 vs. $2(61) = 122$.
time = 142.62, size = 1748, normalized size = 24.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)

[Out] Piecewise((zoo*x*sinh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-b*d*x*tanh(c/2 + d*x/2)**3/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d + b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**3 - b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)) + b*d*x*tanh(c/2 + d*x/2)/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d + b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**3 - b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)) + 2*b*tanh(c/2 + d*x/2)**2/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d + b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**3 - b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)) - 4*b/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d + b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**3 - b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)) + d*x*sqrt(-b**2)*tanh(c/2 + d*x/2)**2/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d + b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**3 - b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)) - d*x*sqrt(-b**2)/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d + b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**3 - b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)) - 2*sqrt(-b**2)*tanh(c/2 + d*x/2)/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d + b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**3 - b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)), Eq(a, -sqrt(-b**2))), (-b*d*x*tanh(c/2 + d*x/2)**3/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d - b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**3 + b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)) + b*d*x*tanh(c/2 + d*x/2)/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d - b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**3 + b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)) + 2*b*tanh(c/2 + d*x/2)**2/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d - b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**3 + b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)) - 4*b/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d - b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**3 + b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)) - d*x*sqrt(-b**2)*tanh(c/2 + d*x/2)**2/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d - b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**3 + b*d*

```
sqrt(-b**2)*tanh(c/2 + d*x/2)) + d*x*sqrt(-b**2)/(b**2*d*tanh(c/2 + d*x/2)*
*2 - b**2*d - b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**3 + b*d*sqrt(-b**2)*tanh(c
/2 + d*x/2)) + 2*sqrt(-b**2)*tanh(c/2 + d*x/2)/(b**2*d*tanh(c/2 + d*x/2)**2
 - b**2*d - b*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**3 + b*d*sqrt(-b**2)*tanh(c/2
 + d*x/2)), Eq(a, sqrt(-b**2))), ((x*sinh(c + d*x)**2/2 - x*cosh(c + d*x)**
2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))/a, Eq(b, 0)), (x*sinh(c)**2/(a + b
*sinh(c)), Eq(d, 0)), (cosh(c + d*x)/(b*d), Eq(a, 0)), (-a**2*log(tanh(c/2
+ d*x/2) - b/a - sqrt(a**2 + b**2)/a)*tanh(c/2 + d*x/2)**2/(b**2*d*sqrt(a**
2 + b**2)*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt(a**2 + b**2)) + a**2*log(tanh(
c/2 + d*x/2) - b/a - sqrt(a**2 + b**2)/a)/(b**2*d*sqrt(a**2 + b**2)*tanh(c/
2 + d*x/2)**2 - b**2*d*sqrt(a**2 + b**2)) + a**2*log(tanh(c/2 + d*x/2) - b/
a + sqrt(a**2 + b**2)/a)*tanh(c/2 + d*x/2)**2/(b**2*d*sqrt(a**2 + b**2)*tan
h(c/2 + d*x/2)**2 - b**2*d*sqrt(a**2 + b**2)) - a**2*log(tanh(c/2 + d*x/2)
- b/a + sqrt(a**2 + b**2)/a)/(b**2*d*sqrt(a**2 + b**2)*tanh(c/2 + d*x/2)**2
 - b**2*d*sqrt(a**2 + b**2)) - a*d*x*sqrt(a**2 + b**2)*tanh(c/2 + d*x/2)**2
/(b**2*d*sqrt(a**2 + b**2)*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt(a**2 + b**2))
 + a*d*x*sqrt(a**2 + b**2)/(b**2*d*sqrt(a**2 + b**2)*tanh(c/2 + d*x/2)**2 -
 b**2*d*sqrt(a**2 + b**2)) - 2*b*sqrt(a**2 + b**2)/(b**2*d*sqrt(a**2 + b**2
)*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt(a**2 + b**2)), True))
```

Giac [A]
time = 0.44, size = 111, normalized size = 1.56

$$\frac{\frac{2\,a^2\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^2}-\frac{2\,(dx+c)a}{b^2}+\frac{e^{(dx+c)}}{b}+\frac{e^{(-dx-c)}}{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*
x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) - 2*(d*x + c)*a/b^
2 + e^(d*x + c)/b + e^(-d*x - c)/b)/d
```

Mupad [B]
time = 0.37, size = 166, normalized size = 2.34

$$\frac{\mathrm{e}^{c+d\,x}}{2\,b\,d}+\frac{\mathrm{e}^{-c-d\,x}}{2\,b\,d}-\frac{a\,x}{b^2}-\frac{a^2\,\ln\left(-\frac{2\,a^2\,\mathrm{e}^{c+d\,x}}{b^3}-\frac{2\,a^2\,\left(b-a\,\mathrm{e}^{c+d\,x}\right)}{b^3\,\sqrt{a^2+b^2}}\right)}{b^2\,d\,\sqrt{a^2+b^2}}+\frac{a^2\,\ln\left(\frac{2\,a^2\,\left(b-a\,\mathrm{e}^{c+d\,x}\right)}{b^3\,\sqrt{a^2+b^2}}-\frac{2\,a^2\,\mathrm{e}^{c+d\,x}}{b^3}\right)}{b^2\,d\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + b*sinh(c + d*x)),x)
```

```
[Out] exp(c + d*x)/(2*b*d) + exp(- c - d*x)/(2*b*d) - (a*x)/b^2 - (a^2*log(- (2*a
^2*exp(c + d*x))/b^3 - (2*a^2*(b - a*exp(c + d*x)))/(b^3*(a^2 + b^2)^(1/2))
))/(b^2*d*(a^2 + b^2)^(1/2)) + (a^2*log((2*a^2*(b - a*exp(c + d*x)))/(b^3*(
a^2 + b^2)^(1/2)) - (2*a^2*exp(c + d*x))/b^3))/(b^2*d*(a^2 + b^2)^(1/2))
```

3.232 $\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sinh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][Sinh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 91.99, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Integrate[Sinh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(dx+c)}{(fx+e)\,(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*a^2*integrate(-e^(d*x + c)/(b^3*f*x + b^3*e - (b^3*f*x*e^(2*c) + b^3*e^(2
*c + 1))*e^(2*d*x) - 2*(a*b^2*f*x*e^c + a*b^2*e^(c + 1))*e^(d*x)), x) + 1/2
*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b*f) - 1/2*e^(c - d*e/f)*
exp_integral_e(1, -(f*x + e)*d/f)/(b*f) - a*log(f*x + e)/(b^2*f)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sinh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sinh(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))),x)
[Out] int(sinh(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.233 $\int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+b\sinh(c+dx)} dx$

Optimal. Leaf size=712

$$-\frac{3ef^2x}{4bd^2}-\frac{3f^3x^2}{8bd^2}+\frac{a^2(e+fx)^4}{4b^3f}-\frac{(e+fx)^4}{8bf}-\frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3}-\frac{a(e+fx)^3\cosh(c+dx)}{b^2d}-\frac{a^3(e+\ldots}{}$$

```
[Out] -3/4*e*f^2*x/b/d^2-3/8*f^3*x^2/b/d^2+1/4*a^2*(f*x+e)^4/b^3/f-1/8*(f*x+e)^4/
b/f-6*a*f^2*(f*x+e)*cosh(d*x+c)/b^2/d^3-a*(f*x+e)^3*cosh(d*x+c)/b^2/d+6*a*f
^3*sinh(d*x+c)/b^2/d^4+3*a*f*(f*x+e)^2*sinh(d*x+c)/b^2/d^2+3/4*f^2*(f*x+e)*
cosh(d*x+c)*sinh(d*x+c)/b/d^3+1/2*(f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/b/d-3/8
*f^3*sinh(d*x+c)^2/b/d^4-3/4*f*(f*x+e)^2*sinh(d*x+c)^2/b/d^2-a^3*(f*x+e)^3*
ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d/(a^2+b^2)^(1/2)+a^3*(f*x+e)^3*
ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d/(a^2+b^2)^(1/2)-3*a^3*f*(f*x+e
)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^2/(a^2+b^2)^(1/2)+3*
a^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^2/(a^2+b
^2)^(1/2)+6*a^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^
3/d^3/(a^2+b^2)^(1/2)-6*a^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2
)^(1/2)))/b^3/d^3/(a^2+b^2)^(1/2)-6*a^3*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2
+b^2)^(1/2)))/b^3/d^4/(a^2+b^2)^(1/2)+6*a^3*f^3*polylog(4,-b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))/b^3/d^4/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.84, antiderivative size = 712, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {5676, 3392, 32, 3391, 3377, 2717, 3403, 2296, 2221, 2611, 6744, 2320, 6724}

$$\frac{a^2(e+fx)^4}{4b^3f}-\frac{6a^3f^3\text{Li}_4\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^4\sqrt{a^2+b^2}}+\frac{6a^3f^3\text{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^4\sqrt{a^2+b^2}}+\frac{6a^3f^2(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3\sqrt{a^2+b^2}}-\frac{6a^3f^2(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^3\sqrt{a^2+b^2}}-\frac{3a^3f(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2\sqrt{a^2+b^2}}+\frac{3a^3f(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2\sqrt{a^2+b^2}}-\frac{a^3(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^3d\sqrt{a^2+b^2}}+\frac{a^3(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^3d\sqrt{a^2+b^2}}+\frac{6af^3\sinh(c+dx)}{b^2d^4}-\frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3}+\frac{3af(e+fx)^2\sinh(c+dx)}{b^2d^2}-\frac{a(e+fx)^3\cosh(c+dx)}{b^2d}-\frac{3f^3\sinh^2(c+dx)}{8bd^4}+\frac{3f^2(e+fx)\sinh(c+dx)\cosh(c+dx)}{4bd^3}-\frac{3f(e+fx)^2\sinh^2(c+dx)}{4bd^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{3ef^2x}{4bd^2}-\frac{3f^3x^2}{8bd^2}-\frac{(e+fx)^4}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-3*e*f^2*x)/(4*b*d^2) - (3*f^3*x^2)/(8*b*d^2) + (a^2*(e + f*x)^4)/(4*b^3*f
) - (e + f*x)^4/(8*b*f) - (6*a*f^2*(e + f*x)*Cosh[c + d*x])/(b^2*d^3) - (a*
(e + f*x)^3*Cosh[c + d*x])/(b^2*d) - (a^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2])])/(b^3*Sqrt[a^2 + b^2]*d) + (a^3*(e + f*x)^3*Log[1
 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*Sqrt[a^2 + b^2]*d) - (3*a^3
*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*S
qrt[a^2 + b^2]*d^2) + (3*a^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]*d^2) + (6*a^3*f^2*(e + f*x)*Poly
Log[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]*d^3)
 - (6*a^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/(b^3*Sqrt[a^2 + b^2]*d^3) - (6*a^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a -
```

```
 Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]*d^4) + (6*a^3*f^3*PolyLog[4, -((b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]*d^4) + (6*a*f^3
*Sinh[c + d*x])/(b^2*d^4) + (3*a*f*(e + f*x)^2*Sinh[c + d*x])/(b^2*d^2) + (
3*f^2*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^3) + ((e + f*x)^3*Cosh[
c + d*x]*Sinh[c + d*x])/(2*b*d) - (3*f^3*Sinh[c + d*x]^2)/(8*b*d^4) - (3*f*
(e + f*x)^2*Sinh[c + d*x]^2)/(4*b*d^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5676

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{3f(e+fx)^2\sinh^2(c+dx)}{4bd^2} - \frac{a\int(e+$$

$$= -\frac{(e+fx)^4}{8bf} - \frac{a(e+fx)^3\cosh(c+dx)}{b^2d} + \frac{3f^2(e+fx)\cosh(c+dx)\sinh(c+}{4bd^3}$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} - \frac{(e+fx)^4}{8bf} - \frac{a(e+fx)^3\cosh(c+dx)}{b^2d} +$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} - \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3}$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} - \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3}$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} - \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3}$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} - \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3}$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} - \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3}$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} - \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1948 vs.

$2(712) = 1424$.
time = 7.79, size = 1948, normalized size = 2.74

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/2*((-2*a^2 + b^2)*e^3*x)/b^3 - (3*(-2*a^2 + b^2)*e^2*f*x^2)/(4*b^3) - ((
-2*a^2 + b^2)*e*f^2*x^3)/(2*b^3) - ((-2*a^2 + b^2)*f^3*x^4)/(8*b^3) + (a^3*
(2*d^3*e^3*Sqrt[(a^2 + b^2)*E^(2*c)]*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 +
 b^2]] - 3*Sqrt[a^2 + b^2]*d^3*e^2*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c
 - Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*Sqrt[a^2 + b^2]*d^3*e*E^c*f^2*x^2*Log[1
+ (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - Sqrt[a^2 + b^2]*
d^3*E^c*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)
])] + 3*Sqrt[a^2 + b^2]*d^3*e^2*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c +
Sqrt[(a^2 + b^2)*E^(2*c)])] + 3*Sqrt[a^2 + b^2]*d^3*e*E^c*f^2*x^2*Log[1 + (
b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + Sqrt[a^2 + b^2]*d^3
*E^c*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])]
 - 3*Sqrt[a^2 + b^2]*d^2*E^c*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(
a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 3*Sqrt[a^2 + b^2]*d^2*E^c*f*(e + f*x
)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] +
6*Sqrt[a^2 + b^2]*d*e*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[
(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[a^2 + b^2]*d*E^c*f^3*x*PolyLog[3, -((b*E^(
2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[a^2 + b^2]*d*e*E
^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]
 - 6*Sqrt[a^2 + b^2]*d*E^c*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sq
rt[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[a^2 + b^2]*E^c*f^3*PolyLog[4, -((b*E^(2
*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[a^2 + b^2]*E^c*f^
3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(b
^3*Sqrt[a^2 + b^2]*d^4*Sqrt[(a^2 + b^2)*E^(2*c)]) + (-1/2*(a*f^3*x^3*Cosh[c
])/(b^2*d) + (a*f^3*x^3*Sinh[c])/(2*b^2*d) + (d^3*e^3 + 3*d^2*e^2*f + 6*d*e
*f^2 + 6*f^3)*(-1/2*(a*Cosh[c])/(b^2*d^4) + (a*Sinh[c])/(2*b^2*d^4)) + (a*d
^2*e^2*f + 2*a*d*e*f^2 + 2*a*f^3)*((-3*x*Cosh[c])/(2*b^2*d^3) + (3*x*Sinh[c
])/(2*b^2*d^3)) + (a*d*e*f^2 + a*f^3)*((-3*x^2*Cosh[c])/(2*b^2*d^2) + (3*x^
2*Sinh[c])/(2*b^2*d^2)))*(Cosh[d*x] - Sinh[d*x]) + (-1/2*(a*f^3*x^3*Cosh[c]
)/(b^2*d) - (a*f^3*x^3*Sinh[c])/(2*b^2*d) + (d^3*e^3 - 3*d^2*e^2*f + 6*d*e*
f^2 - 6*f^3)*(-1/2*(a*Cosh[c])/(b^2*d^4) - (a*Sinh[c])/(2*b^2*d^4)) - (3*x^
2*(a*d*e*f^2*Cosh[c] - a*f^3*Cosh[c] + a*d*e*f^2*Sinh[c] - a*f^3*Sinh[c]))/
(2*b^2*d^2) - (3*x*(a*d^2*e^2*f*Cosh[c] - 2*a*d*e*f^2*Cosh[c] + 2*a*f^3*Cos
h[c] + a*d^2*e^2*f*Sinh[c] - 2*a*d*e*f^2*Sinh[c] + 2*a*f^3*Sinh[c]))/(2*b^2
*d^3))*(Cosh[d*x] + Sinh[d*x]) + (-1/8*(f^3*x^3*Cosh[2*c])/(b*d) + (f^3*x^3
*Sinh[2*c])/(8*b*d) + (4*d^3*e^3 + 6*d^2*e^2*f + 6*d*e*f^2 + 3*f^3)*(-1/32*
Cosh[2*c]/(b*d^4) + Sinh[2*c]/(32*b*d^4)) + (2*d^2*e^2*f + 2*d*e*f^2 + f^3)
*((-3*x*Cosh[2*c])/(16*b*d^3) + (3*x*Sinh[2*c])/(16*b*d^3)) + (2*d*e*f^2 +
```

```
f^3)*((-3*x^2*Cosh[2*c])/(16*b*d^2) + (3*x^2*Sinh[2*c])/(16*b*d^2)))*(Cosh[
2*d*x] - Sinh[2*d*x]) + ((f^3*x^3*Cosh[2*c])/(8*b*d) + (f^3*x^3*Sinh[2*c])/
(8*b*d) + (4*d^3*e^3 - 6*d^2*e^2*f + 6*d*e*f^2 - 3*f^3)*(Cosh[2*c]/(32*b*d^
4) + Sinh[2*c]/(32*b*d^4)) + (3*x^2*(2*d*e*f^2*Cosh[2*c] - f^3*Cosh[2*c] +
2*d*e*f^2*Sinh[2*c] - f^3*Sinh[2*c]))/(16*b*d^2) + (3*x*(2*d^2*e^2*f*Cosh[2
*c] - 2*d*e*f^2*Cosh[2*c] + f^3*Cosh[2*c] + 2*d^2*e^2*f*Sinh[2*c] - 2*d*e*f
^2*Sinh[2*c] + f^3*Sinh[2*c]))/(16*b*d^3))*(Cosh[2*d*x] + Sinh[2*d*x])
```

Maple [**F**]
time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \left(\sinh^3(dx+c)\right)}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*(8*a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a
+ sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3*d) + (4*a*e^(-d*x - c) - b)*e^(2*d
*x + 2*c)/(b^2*d) - 4*(2*a^2 - b^2)*(d*x + c)/(b^3*d) + (4*a*e^(-d*x - c) +
 b*e^(-2*d*x - 2*c))/(b^2*d))*e^3 + 1/32*(4*(2*a^2*d^4*f^3*e^(2*c) - b^2*d^
4*f^3*e^(2*c))*x^4 + 16*(2*a^2*d^4*f^2*e^(2*c) - b^2*d^4*f^2*e^(2*c))*x^3*e
 + 24*(2*a^2*d^4*f*e^(2*c) - b^2*d^4*f*e^(2*c))*x^2*e^2 + (4*b^2*d^3*f^3*x^
3*e^(4*c) - 3*b^2*f^3*e^(4*c) - 6*b^2*d^2*f*e^(4*c + 2) + 6*b^2*d*f^2*e^(4*
c + 1) - 6*(b^2*d^2*f^3*e^(4*c) - 2*b^2*d^3*f^2*e^(4*c + 1))*x^2 + 6*(b^2*d
*f^3*e^(4*c) + 2*b^2*d^3*f*e^(4*c + 2) - 2*b^2*d^2*f^2*e^(4*c + 1))*x)*e^(2
*d*x) - 16*(a*b*d^3*f^3*x^3*e^(3*c) - 6*a*b*f^3*e^(3*c) - 3*a*b*d^2*f*e^(3*
c + 2) + 6*a*b*d*f^2*e^(3*c + 1) - 3*(a*b*d^2*f^3*e^(3*c) - a*b*d^3*f^2*e^(
3*c + 1))*x^2 + 3*(2*a*b*d*f^3*e^(3*c) + a*b*d^3*f*e^(3*c + 2) - 2*a*b*d^2*
f^2*e^(3*c + 1))*x)*e^(d*x) - 16*(a*b*d^3*f^3*x^3*e^c + 3*a*b*d^2*f*e^(c +
2) + 6*a*b*d*f^2*e^(c + 1) + 6*a*b*f^3*e^c + 3*(a*b*d^3*f^2*e^(c + 1) + a*b
*d^2*f^3*e^c)*x^2 + 3*(a*b*d^3*f*e^(c + 2) + 2*a*b*d^2*f^2*e^(c + 1) + 2*a*
b*d*f^3*e^c)*x)*e^(-d*x) - (4*b^2*d^3*f^3*x^3 + 6*b^2*d^2*f*e^2 + 6*b^2*d*f
^2*e + 3*b^2*f^3 + 6*(2*b^2*d^3*f^2*e + b^2*d^2*f^3)*x^2 + 6*(2*b^2*d^3*f*e
^2 + 2*b^2*d^2*f^2*e + b^2*d*f^3)*x)*e^(-2*d*x))*e^(-2*c)/(b^3*d^4) - integ
```

```
rate(2*(a^3*f^3*x^3*e^c + 3*a^3*f^2*x^2*e^(c + 1) + 3*a^3*f*x*e^(c + 2))*e^
(d*x)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 9525 vs. $2(669) = 1338$.
time = 0.45, size = 9525, normalized size = 13.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/32*(4*(a^2*b^2 + b^4)*d^3*f^3*x^3 + 6*(a^2*b^2 + b^4)*d^2*f^3*x^2 + 4*(a
^2*b^2 + b^4)*d^3*cosh(1)^3 + 4*(a^2*b^2 + b^4)*d^3*sinh(1)^3 + 6*(a^2*b^2
+ b^4)*d*f^3*x - (4*(a^2*b^2 + b^4)*d^3*f^3*x^3 - 6*(a^2*b^2 + b^4)*d^2*f^3
*x^2 + 4*(a^2*b^2 + b^4)*d^3*cosh(1)^3 + 4*(a^2*b^2 + b^4)*d^3*sinh(1)^3 +
6*(a^2*b^2 + b^4)*d*f^3*x - 3*(a^2*b^2 + b^4)*f^3 + 6*(2*(a^2*b^2 + b^4)*d^
3*f*x - (a^2*b^2 + b^4)*d^2*f)*cosh(1)^2 + 6*(2*(a^2*b^2 + b^4)*d^3*f*x + 2
*(a^2*b^2 + b^4)*d^3*cosh(1) - (a^2*b^2 + b^4)*d^2*f)*sinh(1)^2 + 6*(2*(a^2
*b^2 + b^4)*d^3*f^2*x^2 - 2*(a^2*b^2 + b^4)*d^2*f^2*x + (a^2*b^2 + b^4)*d*f
^2)*cosh(1) + 6*(2*(a^2*b^2 + b^4)*d^3*f^2*x^2 - 2*(a^2*b^2 + b^4)*d^2*f^2*
x + 2*(a^2*b^2 + b^4)*d^3*cosh(1)^2 + (a^2*b^2 + b^4)*d*f^2 + 2*(2*(a^2*b^2
 + b^4)*d^3*f*x - (a^2*b^2 + b^4)*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^4
- (4*(a^2*b^2 + b^4)*d^3*f^3*x^3 - 6*(a^2*b^2 + b^4)*d^2*f^3*x^2 + 4*(a^2*b
^2 + b^4)*d^3*cosh(1)^3 + 4*(a^2*b^2 + b^4)*d^3*sinh(1)^3 + 6*(a^2*b^2 + b^
4)*d*f^3*x - 3*(a^2*b^2 + b^4)*f^3 + 6*(2*(a^2*b^2 + b^4)*d^3*f*x - (a^2*b^
2 + b^4)*d^2*f)*cosh(1)^2 + 6*(2*(a^2*b^2 + b^4)*d^3*f*x + 2*(a^2*b^2 + b^4
)*d^3*cosh(1) - (a^2*b^2 + b^4)*d^2*f)*sinh(1)^2 + 6*(2*(a^2*b^2 + b^4)*d^3
*f^2*x^2 - 2*(a^2*b^2 + b^4)*d^2*f^2*x + (a^2*b^2 + b^4)*d*f^2)*cosh(1) + 6
*(2*(a^2*b^2 + b^4)*d^3*f^2*x^2 - 2*(a^2*b^2 + b^4)*d^2*f^2*x + 2*(a^2*b^2
+ b^4)*d^3*cosh(1)^2 + (a^2*b^2 + b^4)*d*f^2 + 2*(2*(a^2*b^2 + b^4)*d^3*f*x
 - (a^2*b^2 + b^4)*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^4 + 3*(a^2*b^2 +
b^4)*f^3 + 16*((a^3*b + a*b^3)*d^3*f^3*x^3 - 3*(a^3*b + a*b^3)*d^2*f^3*x^2
+ (a^3*b + a*b^3)*d^3*cosh(1)^3 + (a^3*b + a*b^3)*d^3*sinh(1)^3 + 6*(a^3*b
+ a*b^3)*d*f^3*x - 6*(a^3*b + a*b^3)*f^3 + 3*((a^3*b + a*b^3)*d^3*f*x - (a^
3*b + a*b^3)*d^2*f)*cosh(1)^2 + 3*((a^3*b + a*b^3)*d^3*f*x + (a^3*b + a*b^3
)*d^3*cosh(1) - (a^3*b + a*b^3)*d^2*f)*sinh(1)^2 + 3*((a^3*b + a*b^3)*d^3*f
^2*x^2 - 2*(a^3*b + a*b^3)*d^2*f^2*x + 2*(a^3*b + a*b^3)*d*f^2)*cosh(1) + 3
*((a^3*b + a*b^3)*d^3*f^2*x^2 - 2*(a^3*b + a*b^3)*d^2*f^2*x + (a^3*b + a*b^
3)*d^3*cosh(1)^2 + 2*(a^3*b + a*b^3)*d*f^2 + 2*((a^3*b + a*b^3)*d^3*f*x - (
a^3*b + a*b^3)*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^3 + 4*(4*(a^3*b + a*b
^3)*d^3*f^3*x^3 - 12*(a^3*b + a*b^3)*d^2*f^3*x^2 + 4*(a^3*b + a*b^3)*d^3*co
sh(1)^3 + 4*(a^3*b + a*b^3)*d^3*sinh(1)^3 + 24*(a^3*b + a*b^3)*d*f^3*x - 24
*(a^3*b + a*b^3)*f^3 + 12*((a^3*b + a*b^3)*d^3*f*x - (a^3*b + a*b^3)*d^2*f)
*cosh(1)^2 + 12*((a^3*b + a*b^3)*d^3*f*x + (a^3*b + a*b^3)*d^3*cosh(1) - (a
```

```
^3*b + a*b^3)*d^2*f)*sinh(1)^2 + 12*((a^3*b + a*b^3)*d^3*f^2*x^2 - 2*(a^3*b
 + a*b^3)*d^2*f^2*x + 2*(a^3*b + a*b^3)*d*f^2)*cosh(1) - (4*(a^2*b^2 + b^4)
*d^3*f^3*x^3 - 6*(a^2*b^2 + b^4)*d^2*f^3*x^2 + 4*(a^2*b^2 + b^4)*d^3*cosh(1
)^3 + 4*(a^2*b^2 + b^4)*d^3*sinh(1)^3 + 6*(a^2*b^2 + b^4)*d*f^3*x - 3*(a^2*
b^2 + b^4)*f^3 + 6*(2*(a^2*b^2 + b^4)*d^3*f*x - (a^2*b^2 + b^4)*d^2*f)*cosh
(1)^2 + 6*(2*(a^2*b^2 + b^4)*d^3*f*x + 2*(a^2*b^2 + b^4)*d^3*cosh(1) - (a^2
*b^2 + b^4)*d^2*f)*sinh(1)^2 + 6*(2*(a^2*b^2 + b^4)*d^3*f^2*x^2 - 2*(a^2*b^
2 + b^4)*d^2*f^2*x + (a^2*b^2 + b^4)*d*f^2)*cosh(1) + 6*(2*(a^2*b^2 + b^4)*
d^3*f^2*x^2 - 2*(a^2*b^2 + b^4)*d^2*f^2*x + 2*(a^2*b^2 + b^4)*d^3*cosh(1)^2
 + (a^2*b^2 + b^4)*d*f^2 + 2*(2*(a^2*b^2 + b^4)*d^3*f*x - (a^2*b^2 + b^4)*d
^2*f)*cosh(1))*sinh(1))*cosh(d*x + c) + 12*((a^3*b + a*b^3)*d^3*f^2*x^2 - 2
*(a^3*b + a*b^3)*d^2*f^2*x + (a^3*b + a*b^3)*d^3*cosh(1)^2 + 2*(a^3*b + a*b
^3)*d*f^2 + 2*((a^3*b + a*b^3)*d^3*f*x - (a^3*b + a*b^3)*d^2*f)*cosh(1))*si
nh(1))*sinh(d*x + c)^3 + 6*(2*(a^2*b^2 + b^4)*d^3*f*x + (a^2*b^2 + b^4)*d^2
*f)*cosh(1)^2 - 4*((2*a^4 + a^2*b^2 - b^4)*d^4*f^3*x^4 + 4*(2*a^4 + a^2*b^2
 - b^4)*d^4*f^2*x^3*cosh(1) + 6*(2*a^4 + a^2*b^2 - b^4)*d^4*f*x^2*cosh(1)^2
 + 4*(2*a^4 + a^2*b^2 - b^4)*d^4*x*cosh(1)^3 + 4*(2*a^4 + a^2*b^2 - b^4)*d^
4*x*sinh(1)^3 + 6*((2*a^4 + a^2*b^2 - b^4)*d^4*f*x^2 + 2*(2*a^4 + a^2*b^2 -
 b^4)*d^4*x*cosh(1))*sinh(1)^2 + 4*((2*a^4 + a^2*b^2 - b^4)*d^4*f^2*x^3 + 3
*(2*a^4 + a^2*b^2 - b^4)*d^4*f*x^2*cosh(1) + 3*(2*a^4 + a^2*b^2 - b^4)*d^4*
x*cosh(1)^2)*sinh(1))*cosh(d*x + c)^2 + 6*(2*(a^2*b^2 + b^4)*d^3*f*x + 2*(a
^2*b^2 + b^4)*d^3*cosh(1) + (a^2*b^2 + b^4)*d^2*f)*sinh(1)^2 - 2*(2*(2*a^4
+ a^2*b^2 - b^4)*d^4*f^3*x^4 + 8*(2*a^4 + a^2*b^2 - b^4)*d^4*f^2*x^3*cosh(1
) + 12*(2*a^4 + a^2*b^2 - b^4)*d^4*f*x^2*cosh(1)^2 + 8*(2*a^4 + a^2*b^2 - b
^4)*d^4*x*cosh(1)^3 + 8*(2*a^4 + a^2*b^2 - b^4)*d^4*x*sinh(1)^3 + 3*(4*(a^2
*b^2 + b^4)*d^3*f^3*x^3 - 6*(a^2*b^2 + b^4)*d^2*f^3*x^2 + 4*(a^2*b^2 + b^4)
*d^3*cosh(1)^3 + 4*(a^2*b^2 + b^4)*d^3*sinh(1)^3 + 6*(a^2*b^2 + b^4)*d*f^3*
x - 3*(a^2*b^2 + b^4)*f^3 + 6*(2*(a^2*b^2 + b^4)*d^3*f*x - (a^2*b^2 + b^4)*
d^2*f)*cosh(1)^2 + 6*(2*(a^2*b^2 + b^4)*d^3*f*x + 2*(a^2*b^2 + b^4)*d^3*cos
h(1) - (a^2*b^2 + b^4)*d^2*f)*sinh(1)^2 + 6*(2*(a^2*b^2 + b^4)*d^3*f^2*x^2
- 2*(a^2*b^2 + b^4)*d^2*f^2*x + (a^2*b^2 + b^4)*d*f^2)*cosh(1) + 6*(2*(a^2*
b^2 + b^4)*d^3*f^2*x^2 - 2*(a^2*b^2 + b^4)*d^2*f^2*x + 2*(a^2*b^2 + b^4)*d^
3*cosh(1)^2 + (a^2*b^2 + b^4)*d*f^2 + 2*(2*(a^2*b^2 + b^4)*d^3*f*x - (a^2*b
^2 + b^4)*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^3*sinh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^3\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
[Out] int((sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.234 $\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=522

$$-\frac{f^2x}{4bd^2}+\frac{a^2(e+fx)^3}{3b^3f}-\frac{(e+fx)^3}{6bf}-\frac{2af^2\cosh(c+dx)}{b^2d^3}-\frac{a(e+fx)^2\cosh(c+dx)}{b^2d}-\frac{a^3(e+fx)^2\log\left(1+\frac{be^{c}}{a-\sqrt{a}}\right.}{b^3\sqrt{a^2+b^2}\,d}$$

```
[Out] -1/4*f^2*x/b/d^2+1/3*a^2*(f*x+e)^3/b^3/f-1/6*(f*x+e)^3/b/f-2*a*f^2*cosh(d*x
+c)/b^2/d^3-a*(f*x+e)^2*cosh(d*x+c)/b^2/d+2*a*f*(f*x+e)*sinh(d*x+c)/b^2/d^2
+1/4*f^2*cosh(d*x+c)*sinh(d*x+c)/b/d^3+1/2*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c
)/b/d-1/2*f*(f*x+e)*sinh(d*x+c)^2/b/d^2-a^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/b^3/d/(a^2+b^2)^(1/2)+a^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))/b^3/d/(a^2+b^2)^(1/2)-2*a^3*f*(f*x+e)*polylog(2,-b*exp(d*
x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^2/(a^2+b^2)^(1/2)+2*a^3*f*(f*x+e)*polylog(2
,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^2/(a^2+b^2)^(1/2)+2*a^3*f^2*polyl
og(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^3/(a^2+b^2)^(1/2)-2*a^3*f^2*p
olylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^3/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.73, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {5676, 3392, 32, 2715, 8, 3377, 2718, 3403, 2296, 2221, 2611, 2320, 6724}

$$\frac{a^2(e+fx)^3}{3b^3f}+\frac{2a^3f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3\sqrt{a^2+b^2}}-\frac{2a^3f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^3\sqrt{a^2+b^2}}-\frac{2a^3f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2\sqrt{a^2+b^2}}+\frac{2a^3f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2\sqrt{a^2+b^2}}-\frac{a^3(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^3d\sqrt{a^2+b^2}}+\frac{a^3(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^3d\sqrt{a^2+b^2}}-\frac{2af^2\cosh(c+dx)}{b^2d^3}+\frac{2af(e+fx)\sinh(c+dx)}{b^2d^2}-\frac{a(e+fx)^2\cosh(c+dx)}{b^2d}+\frac{f^2\sinh(c+dx)\cosh(c+dx)}{4bd^3}-\frac{f(e+fx)\sinh^2(c+dx)}{2bd^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{f^2x}{4bd^2}-\frac{(e+fx)^3}{6bf}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/4*(f^2*x)/(b*d^2) + (a^2*(e + f*x)^3)/(3*b^3*f) - (e + f*x)^3/(6*b*f) -
(2*a*f^2*Cosh[c + d*x])/(b^2*d^3) - (a*(e + f*x)^2*Cosh[c + d*x])/(b^2*d) -
 (a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*Sqrt
[a^2 + b^2]*d) + (a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])])/(b^3*Sqrt[a^2 + b^2]*d) - (2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]*d^2) + (2*a^3*f*(e + f*
x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^
2]*d^2) + (2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/
(b^3*Sqrt[a^2 + b^2]*d^3) - (2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]*d^3) + (2*a*f*(e + f*x)*Sinh[c + d*x
])/(b^2*d^2) + (f^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^3) + ((e + f*x)^2*C
osh[c + d*x]*Sinh[c + d*x])/(2*b*d) - (f*(e + f*x)*Sinh[c + d*x]^2)/(2*b*d^
2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5676

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2 \sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)^2 \cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{f(e+fx)\sinh^2(c+dx)}{2bd^2} - \frac{a\int(e+$$

$$= -\frac{(e+fx)^3}{6bf} - \frac{a(e+fx)^2\cosh(c+dx)}{b^2 d} + \frac{f^2\cosh(c+dx)\sinh(c+dx)}{4bd^3} +$$

$$= -\frac{f^2 x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3 f} - \frac{(e+fx)^3}{6bf} - \frac{a(e+fx)^2\cosh(c+dx)}{b^2 d} + \frac{2af(e+$$

$$= -\frac{f^2 x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3 f} - \frac{(e+fx)^3}{6bf} - \frac{2af^2\cosh(c+dx)}{b^2 d^3} - \frac{a(e+fx)^2\cosh}{b^2 d}$$

$$= -\frac{f^2 x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3 f} - \frac{(e+fx)^3}{6bf} - \frac{2af^2\cosh(c+dx)}{b^2 d^3} - \frac{a(e+fx)^2\cosh}{b^2 d}$$

$$= -\frac{f^2 x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3 f} - \frac{(e+fx)^3}{6bf} - \frac{2af^2\cosh(c+dx)}{b^2 d^3} - \frac{a(e+fx)^2\cosh}{b^2 d}$$

$$= -\frac{f^2 x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3 f} - \frac{(e+fx)^3}{6bf} - \frac{2af^2\cosh(c+dx)}{b^2 d^3} - \frac{a(e+fx)^2\cosh}{b^2 d}$$

$$= -\frac{f^2 x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3 f} - \frac{(e+fx)^3}{6bf} - \frac{2af^2\cosh(c+dx)}{b^2 d^3} - \frac{a(e+fx)^2\cosh}{b^2 d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1589 vs. $2(522) = 1044$.
time = 7.46, size = 1589, normalized size = 3.04

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^3*(2*d^2*e^2*Sqrt[(a^2 + b^2)*E^(2*c)]*ArcTanh[(a + b*E^(c + d*x))/Sqrt[
a^2 + b^2]] - 2*Sqrt[a^2 + b^2]*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*
E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - Sqrt[a^2 + b^2]*d^2*E^c*f^2*x^2*Log[1 +
 (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*Sqrt[a^2 + b^2]
```

```
*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]
)] + Sqrt[a^2 + b^2]*d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqr
t[(a^2 + b^2)*E^(2*c)])] - 2*Sqrt[a^2 + b^2]*d*E^c*f*(e + f*x)*PolyLog[2, -
((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2*Sqrt[a^2 + b^2
]*d*E^c*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2
)*E^(2*c)]))] + 2*Sqrt[a^2 + b^2]*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a
*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*Sqrt[a^2 + b^2]*E^c*f^2*PolyLog[3,
-((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(b^3*Sqrt[a^2 +
 b^2]*d^3*Sqrt[(a^2 + b^2)*E^(2*c)]) + (Cosh[2*c + 2*d*x]/(48*b^3*d^3) - Si
nh[2*c + 2*d*x]/(48*b^3*d^3))*(-6*b^2*d^2*e^2 - 6*b^2*d*e*f - 3*b^2*f^2 - 1
2*b^2*d^2*e*f*x - 6*b^2*d*f^2*x - 6*b^2*d^2*f^2*x^2 - 24*a*b*d^2*e^2*Cosh[c
 + d*x] - 48*a*b*d*e*f*Cosh[c + d*x] - 48*a*b*f^2*Cosh[c + d*x] - 48*a*b*d^
2*e*f*x*Cosh[c + d*x] - 48*a*b*d*f^2*x*Cosh[c + d*x] - 24*a*b*d^2*f^2*x^2*C
osh[c + d*x] + 48*a^2*d^3*e^2*x*Cosh[2*c + 2*d*x] - 24*b^2*d^3*e^2*x*Cosh[2
*c + 2*d*x] + 48*a^2*d^3*e*f*x^2*Cosh[2*c + 2*d*x] - 24*b^2*d^3*e*f*x^2*Cos
h[2*c + 2*d*x] + 16*a^2*d^3*f^2*x^3*Cosh[2*c + 2*d*x] - 8*b^2*d^3*f^2*x^3*C
osh[2*c + 2*d*x] - 24*a*b*d^2*e^2*Cosh[3*c + 3*d*x] + 48*a*b*d*e*f*Cosh[3*c
 + 3*d*x] - 48*a*b*f^2*Cosh[3*c + 3*d*x] - 48*a*b*d^2*e*f*x*Cosh[3*c + 3*d*
x] + 48*a*b*d*f^2*x*Cosh[3*c + 3*d*x] - 24*a*b*d^2*f^2*x^2*Cosh[3*c + 3*d*x
] + 6*b^2*d^2*e^2*Cosh[4*c + 4*d*x] - 6*b^2*d*e*f*Cosh[4*c + 4*d*x] + 3*b^2
*f^2*Cosh[4*c + 4*d*x] + 12*b^2*d^2*e*f*x*Cosh[4*c + 4*d*x] - 6*b^2*d*f^2*x
*Cosh[4*c + 4*d*x] + 6*b^2*d^2*f^2*x^2*Cosh[4*c + 4*d*x] - 24*a*b*d^2*e^2*S
inh[c + d*x] - 48*a*b*d*e*f*Sinh[c + d*x] - 48*a*b*f^2*Sinh[c + d*x] - 48*a
*b*d^2*e*f*x*Sinh[c + d*x] - 48*a*b*d*f^2*x*Sinh[c + d*x] - 24*a*b*d^2*f^2*
x^2*Sinh[c + d*x] + 48*a^2*d^3*e^2*x*Sinh[2*c + 2*d*x] - 24*b^2*d^3*e^2*x*S
inh[2*c + 2*d*x] + 48*a^2*d^3*e*f*x^2*Sinh[2*c + 2*d*x] - 24*b^2*d^3*e*f*x^
2*Sinh[2*c + 2*d*x] + 16*a^2*d^3*f^2*x^3*Sinh[2*c + 2*d*x] - 8*b^2*d^3*f^2*
x^3*Sinh[2*c + 2*d*x] - 24*a*b*d^2*e^2*Sinh[3*c + 3*d*x] + 48*a*b*d*e*f*Sin
h[3*c + 3*d*x] - 48*a*b*f^2*Sinh[3*c + 3*d*x] - 48*a*b*d^2*e*f*x*Sinh[3*c +
 3*d*x] + 48*a*b*d*f^2*x*Sinh[3*c + 3*d*x] - 24*a*b*d^2*f^2*x^2*Sinh[3*c +
3*d*x] + 6*b^2*d^2*e^2*Sinh[4*c + 4*d*x] - 6*b^2*d*e*f*Sinh[4*c + 4*d*x] +
3*b^2*f^2*Sinh[4*c + 4*d*x] + 12*b^2*d^2*e*f*x*Sinh[4*c + 4*d*x] - 6*b^2*d*
f^2*x*Sinh[4*c + 4*d*x] + 6*b^2*d^2*f^2*x^2*Sinh[4*c + 4*d*x])
```

Maple [F]

time = 1.61, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \left(\sinh^3(dx+c)\right)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*(8*a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a
+ sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3*d) + (4*a*e^(-d*x - c) - b)*e^(2*d
*x + 2*c)/(b^2*d) - 4*(2*a^2 - b^2)*(d*x + c)/(b^3*d) + (4*a*e^(-d*x - c) +
 b*e^(-2*d*x - 2*c))/(b^2*d))*e^2 + 1/48*(8*(2*a^2*d^3*f^2*e^(2*c) - b^2*d^
3*f^2*e^(2*c))*x^3 + 24*(2*a^2*d^3*f*e^(2*c) - b^2*d^3*f*e^(2*c))*x^2*e + 3
*(2*b^2*d^2*f^2*x^2*e^(4*c) + b^2*f^2*e^(4*c) - 2*b^2*d*f*e^(4*c + 1) - 2*(
b^2*d*f^2*e^(4*c) - 2*b^2*d^2*f*e^(4*c + 1))*x)*e^(2*d*x) - 24*(a*b*d^2*f^2
*x^2*e^(3*c) + 2*a*b*f^2*e^(3*c) - 2*a*b*d*f*e^(3*c + 1) - 2*(a*b*d*f^2*e^(
3*c) - a*b*d^2*f*e^(3*c + 1))*x)*e^(d*x) - 24*(a*b*d^2*f^2*x^2*e^c + 2*a*b*
d*f*e^(c + 1) + 2*a*b*f^2*e^c + 2*(a*b*d^2*f*e^(c + 1) + a*b*d*f^2*e^c)*x)*
e^(-d*x) - 3*(2*b^2*d^2*f^2*x^2 + 2*b^2*d*f*e + b^2*f^2 + 2*(2*b^2*d^2*f*e
+ b^2*d*f^2)*x)*e^(-2*d*x))*e^(-2*c)/(b^3*d^3) - integrate(2*(a^3*f^2*x^2*e
^c + 2*a^3*f*x*e^(c + 1))*e^(d*x)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c
) - b^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4857 vs. $2(488) = 976$.
time = 0.43, size = 4857, normalized size = 9.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(6*(a^2*b^2 + b^4)*d^2*f^2*x^2 + 6*(a^2*b^2 + b^4)*d*f^2*x + 6*(a^2*b
^2 + b^4)*d^2*cosh(1)^2 - 3*(2*(a^2*b^2 + b^4)*d^2*f^2*x^2 - 2*(a^2*b^2 + b
^4)*d*f^2*x + 2*(a^2*b^2 + b^4)*d^2*cosh(1)^2 + 2*(a^2*b^2 + b^4)*d^2*sinh(
1)^2 + (a^2*b^2 + b^4)*f^2 + 2*(2*(a^2*b^2 + b^4)*d^2*f*x - (a^2*b^2 + b^4)
*d*f)*cosh(1) + 2*(2*(a^2*b^2 + b^4)*d^2*f*x + 2*(a^2*b^2 + b^4)*d^2*cosh(1
) - (a^2*b^2 + b^4)*d*f)*sinh(1))*cosh(d*x + c)^4 + 6*(a^2*b^2 + b^4)*d^2*s
inh(1)^2 - 3*(2*(a^2*b^2 + b^4)*d^2*f^2*x^2 - 2*(a^2*b^2 + b^4)*d*f^2*x + 2
*(a^2*b^2 + b^4)*d^2*cosh(1)^2 + 2*(a^2*b^2 + b^4)*d^2*sinh(1)^2 + (a^2*b^2
 + b^4)*f^2 + 2*(2*(a^2*b^2 + b^4)*d^2*f*x - (a^2*b^2 + b^4)*d*f)*cosh(1) +
 2*(2*(a^2*b^2 + b^4)*d^2*f*x + 2*(a^2*b^2 + b^4)*d^2*cosh(1) - (a^2*b^2 +
b^4)*d*f)*sinh(1))*sinh(d*x + c)^4 + 24*((a^3*b + a*b^3)*d^2*f^2*x^2 - 2*(a
^3*b + a*b^3)*d*f^2*x + (a^3*b + a*b^3)*d^2*cosh(1)^2 + (a^3*b + a*b^3)*d^2
*sinh(1)^2 + 2*(a^3*b + a*b^3)*f^2 + 2*((a^3*b + a*b^3)*d^2*f*x - (a^3*b +
a*b^3)*d*f)*cosh(1) + 2*((a^3*b + a*b^3)*d^2*f*x + (a^3*b + a*b^3)*d^2*cosh
```

(1) - (a^3*b + a*b^3)*d*f)*sinh(1))*cosh(d*x + c)^3 + 12*(2*(a^3*b + a*b^3)
*d^2*f^2*x^2 - 4*(a^3*b + a*b^3)*d*f^2*x + 2*(a^3*b + a*b^3)*d^2*cosh(1)^2
+ 2*(a^3*b + a*b^3)*d^2*sinh(1)^2 + 4*(a^3*b + a*b^3)*f^2 + 4*((a^3*b + a*b
^3)*d^2*f*x - (a^3*b + a*b^3)*d*f)*cosh(1) - (2*(a^2*b^2 + b^4)*d^2*f^2*x^2
 - 2*(a^2*b^2 + b^4)*d*f^2*x + 2*(a^2*b^2 + b^4)*d^2*cosh(1)^2 + 2*(a^2*b^2
 + b^4)*d^2*sinh(1)^2 + (a^2*b^2 + b^4)*f^2 + 2*(2*(a^2*b^2 + b^4)*d^2*f*x
- (a^2*b^2 + b^4)*d*f)*cosh(1) + 2*(2*(a^2*b^2 + b^4)*d^2*f*x + 2*(a^2*b^2
+ b^4)*d^2*cosh(1) - (a^2*b^2 + b^4)*d*f)*sinh(1))*cosh(d*x + c) + 4*((a^3*
b + a*b^3)*d^2*f*x + (a^3*b + a*b^3)*d^2*cosh(1) - (a^3*b + a*b^3)*d*f)*sin
h(1))*sinh(d*x + c)^3 + 3*(a^2*b^2 + b^4)*f^2 - 8*((2*a^4 + a^2*b^2 - b^4)*
d^3*f^2*x^3 + 3*(2*a^4 + a^2*b^2 - b^4)*d^3*f*x^2*cosh(1) + 3*(2*a^4 + a^2*
b^2 - b^4)*d^3*x*cosh(1)^2 + 3*(2*a^4 + a^2*b^2 - b^4)*d^3*x*sinh(1)^2 + 3*
((2*a^4 + a^2*b^2 - b^4)*d^3*f*x^2 + 2*(2*a^4 + a^2*b^2 - b^4)*d^3*x*cosh(1
))*sinh(1))*cosh(d*x + c)^2 - 2*(4*(2*a^4 + a^2*b^2 - b^4)*d^3*f^2*x^3 + 12
*(2*a^4 + a^2*b^2 - b^4)*d^3*f*x^2*cosh(1) + 12*(2*a^4 + a^2*b^2 - b^4)*d^3
*x*cosh(1)^2 + 12*(2*a^4 + a^2*b^2 - b^4)*d^3*x*sinh(1)^2 + 9*(2*(a^2*b^2 +
 b^4)*d^2*f^2*x^2 - 2*(a^2*b^2 + b^4)*d*f^2*x + 2*(a^2*b^2 + b^4)*d^2*cosh(
1)^2 + 2*(a^2*b^2 + b^4)*d^2*sinh(1)^2 + (a^2*b^2 + b^4)*f^2 + 2*(2*(a^2*b^
2 + b^4)*d^2*f*x - (a^2*b^2 + b^4)*d*f)*cosh(1) + 2*(2*(a^2*b^2 + b^4)*d^2*
f*x + 2*(a^2*b^2 + b^4)*d^2*cosh(1) - (a^2*b^2 + b^4)*d*f)*sinh(1))*cosh(d*
x + c)^2 - 36*((a^3*b + a*b^3)*d^2*f^2*x^2 - 2*(a^3*b + a*b^3)*d*f^2*x + (a
^3*b + a*b^3)*d^2*cosh(1)^2 + (a^3*b + a*b^3)*d^2*sinh(1)^2 + 2*(a^3*b + a*
b^3)*f^2 + 2*((a^3*b + a*b^3)*d^2*f*x - (a^3*b + a*b^3)*d*f)*cosh(1) + 2*((
a^3*b + a*b^3)*d^2*f*x + (a^3*b + a*b^3)*d^2*cosh(1) - (a^3*b + a*b^3)*d*f)
*sinh(1))*cosh(d*x + c) + 12*((2*a^4 + a^2*b^2 - b^4)*d^3*f*x^2 + 2*(2*a^4
+ a^2*b^2 - b^4)*d^3*x*cosh(1))*sinh(1))*sinh(d*x + c)^2 + 96*((a^3*b*d*f^2
*x + a^3*b*d*f*cosh(1) + a^3*b*d*f*sinh(1))*cosh(d*x + c)^2 + 2*(a^3*b*d*f^
2*x + a^3*b*d*f*cosh(1) + a^3*b*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c) +
(a^3*b*d*f^2*x + a^3*b*d*f*cosh(1) + a^3*b*d*f*sinh(1))*sinh(d*x + c)^2)*sq
rt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 96*((a^3*b*d*f^
2*x + a^3*b*d*f*cosh(1) + a^3*b*d*f*sinh(1))*cosh(d*x + c)^2 + 2*(a^3*b*d*f
^2*x + a^3*b*d*f*cosh(1) + a^3*b*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c) +
 (a^3*b*d*f^2*x + a^3*b*d*f*cosh(1) + a^3*b*d*f*sinh(1))*sinh(d*x + c)^2)*s
qrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 48*((a^3*b*c^2
*f^2 - 2*a^3*b*c*d*f*cosh(1) + a^3*b*d^2*cosh(1)^2 + a^3*b*d^2*sinh(1)^2 -
2*(a^3*b*c*d*f - a^3*b*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(a^3*b*c^2
*f^2 - 2*a^3*b*c*d*f*cosh(1) + a^3*b*d^2*cosh(1)^2 + a^3*b*d^2*sinh(1)^2 -
2*(a^3*b*c*d*f - a^3*b*d^2*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) +
(a^3*b*c^2*f^2 - 2*a^3*b*c*d*f*cosh(1) + a^3*b*d^2*cosh(1)^2 + a^3*b*d^2*si
nh(1)^2 - 2*(a^3*b*c*d*f - a^3*b*d^2*cosh(1))*sinh(1))*sinh(d*x + c)^2)*sqr
t((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^
2 + b^2)/b^2) + 2*a) + 48*((a^3*b*c^2*f^2 - 2*a^3*b*c*d*f*cosh(1) + a^3*b*d
^2*cosh(1)^2 + a^3*b*d^2*sinh(1)^2 - 2*(a^3*b*c*d*f - a^3*b*d^2*cosh(1))*si

```
nh(1))*cosh(d*x + c)^2 + 2*(a^3*b*c^2*f^2 - 2*a^3*b*c*d*f*cosh(1) + a^3*b*d
^2*cosh(1)^2 + a^3*b*d^2*sinh(1)^2 - 2*(a^3*b*c*d*f - a^3*b*d^2*cosh(1))*si
nh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*c^2*f^2 - 2*a^3*b*c*d*f*cosh(1)
 + a^3*b*d^2*cosh(1)^2 + a^3*b*d^2*sinh(1)^2 - 2*(a^3*b*c*d*f - a^3*b*d^2*c
osh(1))*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x +
c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 48*((a^3*b*d^2*
f^2*x^2 - a^3*b*c^2*f^2 + 2*(a^3*b*d^2*f*x + a^3*b*c*d*f)*cosh(1) + 2*(a^3*
b*d^2*f*x + a^3*b*c*d*f)*sinh(1))*cosh(d*x + c)...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^2*sinh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^3\,{(e+f\,x)}^2}{a+b\,\mathrm{sinh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
[Out] int((sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.235 $\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=335

$$\frac{a^2ex}{b^3}-\frac{ex}{2b}+\frac{a^2fx^2}{2b^3}-\frac{fx^2}{4b}-\frac{a(e+fx)\cosh(c+dx)}{b^2d}-\frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d}+\frac{a^3(e+fx)\log\left(1+\right.}{b^3\sqrt{a^2+\right.}}$$

```
[Out] a^2*e*x/b^3-1/2*e*x/b+1/2*a^2*f*x^2/b^3-1/4*f*x^2/b-a*(f*x+e)*cosh(d*x+c)/b
^2/d+a*f*sinh(d*x+c)/b^2/d^2+1/2*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/b/d-1/4*f*
sinh(d*x+c)^2/b/d^2-a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/
d/(a^2+b^2)^(1/2)+a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d/
(a^2+b^2)^(1/2)-a^3*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^2/
(a^2+b^2)^(1/2)+a^3*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^2/
(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5676, 3391, 3377, 2717, 3403, 2296, 2221, 2317, 2438}

$$\frac{a^2ex}{b^3}+\frac{a^2fx^2}{2b^3}-\frac{a^3f\,\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2\sqrt{a^2+b^2}}+\frac{a^3f\,\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2\sqrt{a^2+b^2}}-\frac{a^3(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^3d\sqrt{a^2+b^2}}+\frac{a^3(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^3d\sqrt{a^2+b^2}}+\frac{af\sinh(c+dx)}{b^2d^2}-\frac{a(e+fx)\cosh(c+dx)}{b^2d}-\frac{f\sinh^2(c+dx)}{4bd^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2bd}-\frac{ex}{2b}-\frac{fx^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*e*x)/b^3 - (e*x)/(2*b) + (a^2*f*x^2)/(2*b^3) - (f*x^2)/(4*b) - (a*(e +
 f*x)*Cosh[c + d*x])/(b^2*d) - (a^3*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2])])/(b^3*Sqrt[a^2 + b^2]*d) + (a^3*(e + f*x)*Log[1 + (b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*Sqrt[a^2 + b^2]*d) - (a^3*f*PolyLog[2
, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]*d^2) + (a
^3*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 +
b^2]*d^2) + (a*f*Sinh[c + d*x])/(b^2*d^2) + ((e + f*x)*Cosh[c + d*x]*Sinh[c
 + d*x])/(2*b*d) - (f*Sinh[c + d*x]^2)/(4*b*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5676

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
```

```
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*(Sinh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{f\sinh^2(c+dx)}{4bd^2} - \frac{a\int (e+fx)\sinh(c-}{b^2}$$

$$= -\frac{ex}{2b} - \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} + \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{2bd}$$

$$= \frac{a^2ex}{b^3} - \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} - \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} + \frac{af\sinh(c+dx)}{b^2d^2} + \frac{(}{}$$

$$= \frac{a^2ex}{b^3} - \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} - \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} + \frac{af\sinh(c+dx)}{b^2d^2} + \frac{(}{}$$

$$= \frac{a^2ex}{b^3} - \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} - \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} - \frac{a^3(e+fx)\log\left(1+\right.}{b^3\sqrt{a^2+}}$$

$$= \frac{a^2ex}{b^3} - \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} - \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} - \frac{a^3(e+fx)\log\left(1+\right.}{b^3\sqrt{a^2+}}$$

$$= \frac{a^2ex}{b^3} - \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} - \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} - \frac{a^3(e+fx)\log\left(1+\right.}{b^3\sqrt{a^2+}}$$

Mathematica [A]
time = 1.64, size = 307, normalized size = 0.92

$$\frac{-2(2a^2-b^2)(c+dx)(cf-d(2e+fx)) - 8abd(e+fx)\cosh(c+dx) - b^2f\cosh(2(c+dx)) + \frac{8a^3\left(2de\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) - 2cf\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) - f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) + f(c+dx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right) - f\,\mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right) + f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)}{\sqrt{a^2+b^2}} + 8abf\sinh(c+dx) + 2b^2d(e+fx)\sinh(2(c+dx))}{8b^3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*(2*a^2 - b^2)*(c + d*x)*(c*f - d*(2*e + f*x)) - 8*a*b*d*(e + f*x)*Cosh[
c + d*x] - b^2*f*Cosh[2*(c + d*x)] + (8*a^3*(2*d*e*ArcTanh[(a + b*E^(c + d*
x))/Sqrt[a^2 + b^2]] - 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] -
```

```
 f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*L
og[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - f*PolyLog[2, (b*E^(c + d*x)
)/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))]))/Sqrt[a^2 + b^2] + 8*a*b*f*Sinh[c + d*x] + 2*b^2*d*(e + f*x)*Sinh[
2*(c + d*x)])/(8*b^3*d^2)
```

Maple [A]
time = 1.59, size = 589, normalized size = 1.76

method	result
risch	$\frac{a^2 f x^2}{2b^3} - \frac{f x^2}{4b} + \frac{a^2 ex}{b^3} - \frac{ex}{2b} + \frac{(2dxf+2de-f)e^{2dx+2c}}{16d^2 b} - \frac{a(dxf+de-f)e^{dx+c}}{2b^2 d^2} - \frac{a(dxf+de+f)e^{-dx-c}}{2b^2 d^2} - \frac{(2dxf+2de+f)}{16d^2 b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*f*x^2/b^3-1/4*f*x^2/b+a^2*e*x/b^3-1/2*e*x/b+1/16*(2*d*f*x+2*d*e-f)/
d^2/b*exp(2*d*x+2*c)-1/2*a*(d*f*x+d*e-f)/b^2/d^2*exp(d*x+c)-1/2*a*(d*f*x+d*
e+f)/b^2/d^2*exp(-d*x-c)-1/16*(2*d*f*x+2*d*e+f)/d^2/b*exp(-2*d*x-2*c)+2/d*a
^3/b^3*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-
1/d*a^3/b^3*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2
+b^2)^(1/2)))*x-1/d^2*a^3/b^3*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)
^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d*a^3/b^3*f/(a^2+b^2)^(1/2)*ln((b*exp(d
*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2*a^3/b^3*f/(a^2+b^2)^(
1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d^2*a^3/b
^3*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^
(1/2)))+1/d^2*a^3/b^3*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)
+a)/(a+(a^2+b^2)^(1/2)))-2/d^2*a^3/b^3*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b
*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(32*a^3*integrate(x*e^(d*x + c)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x
 + c) - b^4), x) - (4*(2*a^2*d^2*e^(2*c) - b^2*d^2*e^(2*c))*x^2 + (2*b^2*d*
x*e^(4*c) - b^2*e^(4*c))*e^(2*d*x) - 8*(a*b*d*x*e^(3*c) - a*b*e^(3*c))*e^(d
*x) - 8*(a*b*d*x*e^c + a*b*e^c)*e^(-d*x) - (2*b^2*d*x + b^2)*e^(-2*d*x))*e^
(-2*c)/(b^3*d^2))*f - 1/8*(8*a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))
/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3*d) + (4*a*e^(
```

```
-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) - 4*(2*a^2 - b^2)*(d*x + c)/(b^3*d)
+ (4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2068 vs. $2(309) = 618$.
time = 0.39, size = 2068, normalized size = 6.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*((2*(a^2*b^2 + b^4)*d*f*x + 2*(a^2*b^2 + b^4)*d*cosh(1) + 2*(a^2*b^2 +
 b^4)*d*sinh(1) - (a^2*b^2 + b^4)*f)*cosh(d*x + c)^4 + (2*(a^2*b^2 + b^4)*d
*f*x + 2*(a^2*b^2 + b^4)*d*cosh(1) + 2*(a^2*b^2 + b^4)*d*sinh(1) - (a^2*b^2
 + b^4)*f)*sinh(d*x + c)^4 - 2*(a^2*b^2 + b^4)*d*f*x - 8*((a^3*b + a*b^3)*d
*f*x + (a^3*b + a*b^3)*d*cosh(1) + (a^3*b + a*b^3)*d*sinh(1) - (a^3*b + a*b
^3)*f)*cosh(d*x + c)^3 - 4*(2*(a^3*b + a*b^3)*d*f*x + 2*(a^3*b + a*b^3)*d*c
osh(1) + 2*(a^3*b + a*b^3)*d*sinh(1) - 2*(a^3*b + a*b^3)*f - (2*(a^2*b^2 +
b^4)*d*f*x + 2*(a^2*b^2 + b^4)*d*cosh(1) + 2*(a^2*b^2 + b^4)*d*sinh(1) - (a
^2*b^2 + b^4)*f)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(a^2*b^2 + b^4)*d*cosh(
1) + 4*((2*a^4 + a^2*b^2 - b^4)*d^2*f*x^2 + 2*(2*a^4 + a^2*b^2 - b^4)*d^2*x
*cosh(1) + 2*(2*a^4 + a^2*b^2 - b^4)*d^2*x*sinh(1))*cosh(d*x + c)^2 - 2*(a^
2*b^2 + b^4)*d*sinh(1) + 2*(2*(2*a^4 + a^2*b^2 - b^4)*d^2*f*x^2 + 4*(2*a^4
+ a^2*b^2 - b^4)*d^2*x*cosh(1) + 4*(2*a^4 + a^2*b^2 - b^4)*d^2*x*sinh(1) +
3*(2*(a^2*b^2 + b^4)*d*f*x + 2*(a^2*b^2 + b^4)*d*cosh(1) + 2*(a^2*b^2 + b^4
)*d*sinh(1) - (a^2*b^2 + b^4)*f)*cosh(d*x + c)^2 - 12*((a^3*b + a*b^3)*d*f*
x + (a^3*b + a*b^3)*d*cosh(1) + (a^3*b + a*b^3)*d*sinh(1) - (a^3*b + a*b^3)
*f)*cosh(d*x + c))*sinh(d*x + c)^2 - 16*(a^3*b*f*cosh(d*x + c)^2 + 2*a^3*b*
f*cosh(d*x + c)*sinh(d*x + c) + a^3*b*f*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b
^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 16*(a^3*b*f*cosh(d*x + c)^2 + 2
*a^3*b*f*cosh(d*x + c)*sinh(d*x + c) + a^3*b*f*sinh(d*x + c)^2)*sqrt((a^2 +
 b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 16*((a^3*b*c*f - a^3*b*d
*cosh(1) - a^3*b*d*sinh(1))*cosh(d*x + c)^2 + 2*(a^3*b*c*f - a^3*b*d*cosh(1
) - a^3*b*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*c*f - a^3*b*d*cos
h(1) - a^3*b*d*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh
(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 16*((a^3
*b*c*f - a^3*b*d*cosh(1) - a^3*b*d*sinh(1))*cosh(d*x + c)^2 + 2*(a^3*b*c*f
- a^3*b*d*cosh(1) - a^3*b*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*c
*f - a^3*b*d*cosh(1) - a^3*b*d*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b
^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) +
 2*a) - 16*((a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x + c)^2 + 2*(a^3*b*d*f*x + a^
3*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d*f*x + a^3*b*c*f)*sinh(d*x +
```

```
c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 16*((a^3*b*
d*f*x + a^3*b*c*f)*cosh(d*x + c)^2 + 2*(a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x +
c)*sinh(d*x + c) + (a^3*b*d*f*x + a^3*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 +
b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (a^2*b^2 + b^4)*f - 8*((a^3*b
+ a*b^3)*d*f*x + (a^3*b + a*b^3)*d*cosh(1) + (a^3*b + a*b^3)*d*sinh(1) + (a
^3*b + a*b^3)*f)*cosh(d*x + c) - 4*(2*(a^3*b + a*b^3)*d*f*x - (2*(a^2*b^2 +
b^4)*d*f*x + 2*(a^2*b^2 + b^4)*d*cosh(1) + 2*(a^2*b^2 + b^4)*d*sinh(1) - (
a^2*b^2 + b^4)*f)*cosh(d*x + c)^3 + 2*(a^3*b + a*b^3)*d*cosh(1) + 6*((a^3*b
+ a*b^3)*d*f*x + (a^3*b + a*b^3)*d*cosh(1) + (a^3*b + a*b^3)*d*sinh(1) - (
a^3*b + a*b^3)*f)*cosh(d*x + c)^2 + 2*(a^3*b + a*b^3)*d*sinh(1) + 2*(a^3*b
+ a*b^3)*f - 2*((2*a^4 + a^2*b^2 - b^4)*d^2*f*x^2 + 2*(2*a^4 + a^2*b^2 - b^
4)*d^2*x*cosh(1) + 2*(2*a^4 + a^2*b^2 - b^4)*d^2*x*sinh(1))*cosh(d*x + c))*
sinh(d*x + c))/((a^2*b^3 + b^5)*d^2*cosh(d*x + c)^2 + 2*(a^2*b^3 + b^5)*d^2
*cosh(d*x + c)*sinh(d*x + c) + (a^2*b^3 + b^5)*d^2*sinh(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)*sinh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\sinh(c+d\,x)}^3\,(e+f\,x)}{a+b\,\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)),x)
[Out] int((sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.236 $\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)} \, dx$

Optimal. Leaf size=107

$$\frac{(2a^2-b^2)\,x}{2b^3}+\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d}-\frac{a\cosh(c+dx)}{b^2d}+\frac{\cosh(c+dx)\sinh(c+dx)}{2bd}$$

```
[Out] 1/2*(2*a^2-b^2)*x/b^3-a*cosh(d*x+c)/b^2/d+1/2*cosh(d*x+c)*sinh(d*x+c)/b/d+2
*a^3*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/b^3/d/(a^2+b^2)^(1/
2)
```

Rubi [A]
time = 0.16, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2872, 3102, 2814, 2739, 632, 210}

$$\frac{x(2a^2-b^2)}{2b^3}+\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^3d\sqrt{a^2+b^2}}-\frac{a\cosh(c+dx)}{b^2d}+\frac{\sinh(c+dx)\cosh(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((2*a^2 - b^2)*x)/(2*b^3) + (2*a^3*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a
^2 + b^2]])/(b^3*Sqrt[a^2 + b^2]*d) - (a*Cosh[c + d*x])/(b^2*d) + (Cosh[c +
 d*x]*Sinh[c + d*x])/(2*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{\int \frac{a+b\sinh(c+dx)+2a\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{2b}$$

$$= -\frac{a\cosh(c+dx)}{b^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{i\int \frac{-iab+i(2a^2-b^2)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx}{2b^2}$$

$$= \frac{(2a^2-b^2)\,x}{2b^3} - \frac{a\cosh(c+dx)}{b^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{a^3\int \frac{1}{a+b\sinh(c+dx)}\,dx}{b^3}$$

$$= \frac{(2a^2-b^2)\,x}{2b^3} - \frac{a\cosh(c+dx)}{b^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} + \frac{(2ia^3)\,\text{Subst}\left(\int \frac{}{a-2\cdots}\right.}{}$$

$$= \frac{(2a^2-b^2)\,x}{2b^3} - \frac{a\cosh(c+dx)}{b^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{(4ia^3)\,\text{Subst}\left(\int \frac{}{-4(\cdots}\right.}{}$$

$$= \frac{(2a^2-b^2)\,x}{2b^3} + \frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d} - \frac{a\cosh(c+dx)}{b^2d} + \frac{\cosh(c+dx)\,\text{si}\cdots}{2bd}$$

Mathematica [A]
time = 0.25, size = 101, normalized size = 0.94

$$\frac{-2\left(-2a^2+b^2\right)(c+dx) - \frac{8a^3\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 4ab\cosh(c+dx) + b^2\sinh(2(c+dx))}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*(-2*a^2 + b^2)*(c + d*x) - (8*a^3*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt
[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 4*a*b*Cosh[c + d*x] + b^2*Sinh[2*(c + d*x
)])/(4*b^3*d)
```

Maple [A]
time = 0.78, size = 191, normalized size = 1.79

method	result
derivativedivides	$\frac{-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2} - \frac{-b+2a}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)} + \frac{\left(2a^2-b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2b^3} - \frac{2a^3\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}} + \frac{}{2b(\tanh\cdots}}{d}$
default	$\frac{-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2} - \frac{-b+2a}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)} + \frac{\left(2a^2-b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2b^3} - \frac{2a^3\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}} + \frac{}{2b(\tanh\cdots}}{d}$

risch	$\frac{x\,a^2}{b^3}-\frac{x}{2b}+\frac{\mathrm{e}^{2dx+2c}}{8bd}-\frac{a\,\mathrm{e}^{dx+c}}{2b^2d}-\frac{a\,\mathrm{e}^{-dx-c}}{2b^2d}-\frac{\mathrm{e}^{-2dx-2c}}{8bd}+\frac{a^3\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d\,b^3}-\frac{a^3\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/b/(tanh(1/2*d*x+1/2*c)+1)^2-1/2*(-b+2*a)/b^2/(tanh(1/2*d*x+1/2*c)
+1)+1/2*(2*a^2-b^2)/b^3*ln(tanh(1/2*d*x+1/2*c)+1)-2*a^3/b^3/(a^2+b^2)^(1/2)
*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))+1/2/b/(tanh(1/2
*d*x+1/2*c)-1)^2-1/2*(-b-2*a)/b^2/(tanh(1/2*d*x+1/2*c)-1)+1/2/b^3*(-2*a^2+b
^2)*ln(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [A]
time = 0.49, size = 164, normalized size = 1.53

$$-\frac{a^3\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^3d}-\frac{\left(4\,ae^{(-dx-c)}-b\right)e^{(2\,dx+2\,c)}}{8\,b^2d}+\frac{\left(2\,a^2-b^2\right)(dx+c)}{2\,b^3d}-\frac{4\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}}{8\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3*d) - 1/8*(4*a*e^(-d*x - c) - b)*e^(2*d*x
+ 2*c)/(b^2*d) + 1/2*(2*a^2 - b^2)*(d*x + c)/(b^3*d) - 1/8*(4*a*e^(-d*x - c
) + b*e^(-2*d*x - 2*c))/(b^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(100) = 200$.
time = 0.38, size = 601, normalized size = 5.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(2*a^4 + a^2*b^2 - b^4)*d*x*cosh(d*x + c)^2 + (a^2*b^2 + b^4)*cosh(d
*x + c)^4 + (a^2*b^2 + b^4)*sinh(d*x + c)^4 - a^2*b^2 - b^4 - 4*(a^3*b + a*
b^3)*cosh(d*x + c)^3 - 4*(a^3*b + a*b^3 - (a^2*b^2 + b^4)*cosh(d*x + c))*si
nh(d*x + c)^3 + 2*(2*(2*a^4 + a^2*b^2 - b^4)*d*x + 3*(a^2*b^2 + b^4)*cosh(d
*x + c)^2 - 6*(a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*(a^3*cosh(
d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*x + c)^2)*sqrt(
a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x
+ c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2
```

```
 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sin
h(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) -
b)) - 4*(a^3*b + a*b^3)*cosh(d*x + c) - 4*(a^3*b + a*b^3 - 2*(2*a^4 + a^2*b
^2 - b^4)*d*x*cosh(d*x + c) - (a^2*b^2 + b^4)*cosh(d*x + c)^3 + 3*(a^3*b +
a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^2*b^3 + b^5)*d*cosh(d*x + c)^2 +
 2*(a^2*b^3 + b^5)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2*b^3 + b^5)*d*sinh(d
*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
[Out] Timed out
```

Giac [A]
time = 0.44, size = 151, normalized size = 1.41

$$-\frac{\frac{8\,a^3\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^3}-\frac{4\,(2\,a^2-b^2)(dx+c)}{b^3}-\frac{be^{(2\,dx+2\,c)}-4\,ae^{(dx+c)}}{b^2}+\frac{(4\,abe^{(dx+c)}+b^2)e^{(-2\,dx-2\,c)}}{b^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] -1/8*(8*a^3*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d
*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3) - 4*(2*a^2 - b^2)
*(d*x + c)/b^3 - (b*e^(2*d*x + 2*c) - 4*a*e^(d*x + c))/b^2 + (4*a*b*e^(d*x
+ c) + b^2)*e^(-2*d*x - 2*c)/b^3)/d
```

Mupad [B]
time = 0.44, size = 212, normalized size = 1.98

$$\frac{x\,(2\,a^2-b^2)}{2\,b^3}-\frac{\mathrm{e}^{-2\,c-2\,d\,x}}{8\,b\,d}+\frac{\mathrm{e}^{2\,c+2\,d\,x}}{8\,b\,d}-\frac{a\,\mathrm{e}^{-c-d\,x}}{2\,b^2\,d}-\frac{a\,\mathrm{e}^{c+d\,x}}{2\,b^2\,d}-\frac{a^3\,\ln\left(\frac{2\,a^3\,\mathrm{e}^{c+d\,x}}{b^4}-\frac{2\,a^3\,(b-a\,\mathrm{e}^{c+d\,x})}{b^4\,\sqrt{a^2+b^2}}\right)}{b^3\,d\,\sqrt{a^2+b^2}}+\frac{a^3\,\ln\left(\frac{2\,a^3\,\mathrm{e}^{c+d\,x}}{b^4}+\frac{2\,a^3\,(b-a\,\mathrm{e}^{c+d\,x})}{b^4\,\sqrt{a^2+b^2}}\right)}{b^3\,d\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + b*sinh(c + d*x)),x)
[Out] (x*(2*a^2 - b^2))/(2*b^3) - exp(- 2*c - 2*d*x)/(8*b*d) + exp(2*c + 2*d*x)/(
8*b*d) - (a*exp(- c - d*x))/(2*b^2*d) - (a*exp(c + d*x))/(2*b^2*d) - (a^3*l
og((2*a^3*exp(c + d*x))/b^4 - (2*a^3*(b - a*exp(c + d*x)))/(b^4*(a^2 + b^2)
^(1/2))))/(b^3*d*(a^2 + b^2)^(1/2)) + (a^3*log((2*a^3*exp(c + d*x))/b^4 + (
2*a^3*(b - a*exp(c + d*x)))/(b^4*(a^2 + b^2)^(1/2))))/(b^3*d*(a^2 + b^2)^(1
/2))
```

3.237 $\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=31

$$\mathrm{Int}\left(\frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sinh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][Sinh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 178.14, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Integrate[Sinh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*a^3*integrate(-e^(d*x + c)/(b^4*f*x + b^4*e - (b^4*f*x*e^(2*c) + b^4*e^(
2*c + 1))*e^(2*d*x) - 2*(a*b^3*f*x*e^c + a*b^3*e^(c + 1))*e^(d*x)), x) - 1/
4*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b*f) - 1/2*a*e^(-c
 + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^2*f) + 1/2*a*e^(c - d*e/f)*ex
p_integral_e(1, -(f*x + e)*d/f)/(b^2*f) - 1/4*e^(2*c - 2*d*e/f)*exp_integra
l_e(1, -2*(f*x + e)*d/f)/(b*f) + 1/2*(2*a^2 - b^2)*log(f*x + e)/(b^3*f)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sinh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate(sinh(d*x + c)^3/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sinh(c+d\,x)^3}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))),x)
[Out] int(sinh(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.238 $\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=605

$$-\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d}+\frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d}-\frac{3f(e$$

```
[Out] -2*(f*x+e)^3*arctanh(exp(d*x+c))/a/d-3*f*(f*x+e)^2*polylog(2,-exp(d*x+c))/a
/d^2+3*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a/d^2+6*f^2*(f*x+e)*polylog(3,-exp
(d*x+c))/a/d^3-6*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a/d^3-6*f^3*polylog(4,-e
xp(d*x+c))/a/d^4+6*f^3*polylog(4,exp(d*x+c))/a/d^4-b*(f*x+e)^3*ln(1+b*exp(d
*x+c)/(a-(a^2+b^2)^(1/2)))/a/d/(a^2+b^2)^(1/2)+b*(f*x+e)^3*ln(1+b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/a/d/(a^2+b^2)^(1/2)-3*b*f*(f*x+e)^2*polylog(2,-b*exp
(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d^2/(a^2+b^2)^(1/2)+3*b*f*(f*x+e)^2*polylog(
2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d^2/(a^2+b^2)^(1/2)+6*b*f^2*(f*x+e)*
polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d^3/(a^2+b^2)^(1/2)-6*b*f^2*
(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d^3/(a^2+b^2)^(1/2)-
6*b*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d^4/(a^2+b^2)^(1/2)+
6*b*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d^4/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.68, antiderivative size = 605, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5694, 4267, 2611, 6744, 2320, 6724, 3403, 2296, 2221}

$$-\frac{6bf^3\text{Li}_4\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^4\sqrt{a^2+b^2}}+\frac{6bf^3\text{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^4\sqrt{a^2+b^2}}+\frac{6bf^2(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}}-\frac{6bf^2(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}}-\frac{3bf(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}}+\frac{3bf(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}}-\frac{b(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{ad\sqrt{a^2+b^2}}+\frac{b(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{ad\sqrt{a^2+b^2}}-\frac{6f^3\text{Li}_4\left(-e^{c+dx}\right)}{ad^4}+\frac{6f^3\text{Li}_4\left(e^{c+dx}\right)}{ad^4}+\frac{6f^2(e+fx)\text{Li}_3\left(-e^{c+dx}\right)}{ad^3}-\frac{6f^2(e+fx)\text{Li}_3\left(e^{c+dx}\right)}{ad^3}-\frac{3f(e+fx)^2\text{Li}_2\left(-e^{c+dx}\right)}{ad^2}+\frac{3f(e+fx)^2\text{Li}_2\left(e^{c+dx}\right)}{ad^2}-\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) - (b*(e + f*x)^3*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) + (b*(e + f*x)^3*Lo
g[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) - (3*f*
(e + f*x)^2*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (3*f*(e + f*x)^2*PolyLog[2,
 E^(c + d*x)])/(a*d^2) - (3*b*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) + (3*b*f*(e + f*x)^2*PolyLog
[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) + (6
*f^2*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) - (6*f^2*(e + f*x)*PolyLog
[3, E^(c + d*x)])/(a*d^3) + (6*b*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^3) - (6*b*f^2*(e + f*x)*Poly
Log[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^3) -
 (6*f^3*PolyLog[4, -E^(c + d*x)])/(a*d^4) + (6*f^3*PolyLog[4, E^(c + d*x)])
/(a*d^4) - (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(
```

```
a*Sqrt[a^2 + b^2]*d^4) + (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
```

```
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5694

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a + b
*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\text{csch}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{(2b)\int \frac{e^{c+dx}(e+fx)^3}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{a} - \frac{(3f)\int (e+fx}{}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{3f(e+fx)^2\text{Li}_2\left(-e^{c+dx}\right)}{ad^2} + \frac{3f(e+fx)^2\text{Li}_2(}{ad^2}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d} + \frac{b(e+}{}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d} + \frac{b(e+}{}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d} + \frac{b(e+}{}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d} + \frac{b(e+}{}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d} + \frac{b(e+}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1264 vs. $2(605) = 1210$.
time = 3.83, size = 1264, normalized size = 2.09

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*d^3*e^3*ArcTanh[E^(c + d*x)] + 3*d^3*e^2*f*x*Log[1 - E^(c + d*x)] + 3*d
^3*e*f^2*x^2*Log[1 - E^(c + d*x)] + d^3*f^3*x^3*Log[1 - E^(c + d*x)] - 3*d^
3*e^2*f*x*Log[1 + E^(c + d*x)] - 3*d^3*e*f^2*x^2*Log[1 + E^(c + d*x)] - d^3
*f^3*x^3*Log[1 + E^(c + d*x)] - 3*d^2*f*(e + f*x)^2*PolyLog[2, -E^(c + d*x)
] + 3*d^2*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)] + 6*d*e*f^2*PolyLog[3, -E^(
c + d*x)] + 6*d*f^3*x*PolyLog[3, -E^(c + d*x)] - 6*d*e*f^2*PolyLog[3, E^(c
```

```
+ d*x)] - 6*d*f^3*x*PolyLog[3, E^(c + d*x)] - 6*f^3*PolyLog[4, -E^(c + d*x)
] + 6*f^3*PolyLog[4, E^(c + d*x)] + (b*(2*d^3*e^3*Sqrt[(a^2 + b^2)*E^(2*c)]
*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 3*Sqrt[a^2 + b^2]*d^3*e^2*E
^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*S
qrt[a^2 + b^2]*d^3*e*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a
^2 + b^2)*E^(2*c)])] - Sqrt[a^2 + b^2]*d^3*E^c*f^3*x^3*Log[1 + (b*E^(2*c +
d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 3*Sqrt[a^2 + b^2]*d^3*e^2*E^c*
f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 3*Sqrt
[a^2 + b^2]*d^3*e*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2
+ b^2)*E^(2*c)])] + Sqrt[a^2 + b^2]*d^3*E^c*f^3*x^3*Log[1 + (b*E^(2*c + d*x
))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*Sqrt[a^2 + b^2]*d^2*E^c*f*(e +
f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))]
 + 3*Sqrt[a^2 + b^2]*d^2*E^c*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(
a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[a^2 + b^2]*d*e*E^c*f^2*PolyLo
g[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[a^2
 + b^2]*d*E^c*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2
)*E^(2*c)]))] - 6*Sqrt[a^2 + b^2]*d*e*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x)
)/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[a^2 + b^2]*d*E^c*f^3*x*Pol
yLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[
a^2 + b^2]*E^c*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)
*E^(2*c)]))] + 6*Sqrt[a^2 + b^2]*E^c*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*
E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(Sqrt[a^2 + b^2]*Sqrt[(a^2 + b^2)*E^(2
*c)]))/(a*d^4)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(b*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a*d) + log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d
*x - c) - 1)/(a*d))*e^3 - 3*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c))
```

```
)*f*e^2/(a*d^2) + 3*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))*f*e^2/
(a*d^2) - 3*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*p
olylog(3, -e^(d*x + c)))*f^2*e/(a*d^3) + 3*(d^2*x^2*log(-e^(d*x + c) + 1) +
 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*f^2*e/(a*d^3) - (d^3
*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3
, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*f^3/(a*d^4) + (d^3*x^3*log(-e
^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x +
c)) + 6*polylog(4, e^(d*x + c)))*f^3/(a*d^4) - integrate(2*(b*f^3*x^3*e^c +
 3*b*f^2*x^2*e^(c + 1) + 3*b*f*x*e^(c + 2))*e^(d*x)/(a*b*e^(2*d*x + 2*c) +
2*a^2*e^(d*x + c) - a*b), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2509 vs. $2(565) = 1130$.
time = 0.40, size = 2509, normalized size = 4.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(6*b^2*f^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x
+ c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*b^
2*f^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) -
 (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*(a^2 + b
^2)*f^3*polylog(4, cosh(d*x + c) + sinh(d*x + c)) + 6*(a^2 + b^2)*f^3*polyl
og(4, -cosh(d*x + c) - sinh(d*x + c)) + 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*
x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x +
b^2*d^2*f*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) +
a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
 - b)/b + 1) - 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cos
h(1)^2 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1
))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b^2*c^3*f
^3 - 3*b^2*c^2*d*f^2*cosh(1) + 3*b^2*c*d^2*f*cosh(1)^2 - b^2*d^3*cosh(1)^3
- b^2*d^3*sinh(1)^3 + 3*(b^2*c*d^2*f - b^2*d^3*cosh(1))*sinh(1)^2 - 3*(b^2*
c^2*d*f^2 - 2*b^2*c*d^2*f*cosh(1) + b^2*d^3*cosh(1)^2)*sinh(1))*sqrt((a^2 +
 b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)
/b^2) + 2*a) - (b^2*c^3*f^3 - 3*b^2*c^2*d*f^2*cosh(1) + 3*b^2*c*d^2*f*cosh(
1)^2 - b^2*d^3*cosh(1)^3 - b^2*d^3*sinh(1)^3 + 3*(b^2*c*d^2*f - b^2*d^3*cos
h(1))*sinh(1)^2 - 3*(b^2*c^2*d*f^2 - 2*b^2*c*d^2*f*cosh(1) + b^2*d^3*cosh(1
)^2)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^2*d^3*f^3*x^3 + b^2*c^3*f^3 + 3*
(b^2*d^3*f*x + b^2*c*d^2*f)*cosh(1)^2 + 3*(b^2*d^3*f*x + b^2*c*d^2*f)*sinh(
1)^2 + 3*(b^2*d^3*f^2*x^2 - b^2*c^2*d*f^2)*cosh(1) + 3*(b^2*d^3*f^2*x^2 - b
^2*c^2*d*f^2 + 2*(b^2*d^3*f*x + b^2*c*d^2*f)*cosh(1))*sinh(1))*sqrt((a^2 +
```

```
b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^2*d^3*f^3*x^3 + b^2*c^3*f^3
 + 3*(b^2*d^3*f*x + b^2*c*d^2*f)*cosh(1)^2 + 3*(b^2*d^3*f*x + b^2*c*d^2*f)*
sinh(1)^2 + 3*(b^2*d^3*f^2*x^2 - b^2*c^2*d*f^2)*cosh(1) + 3*(b^2*d^3*f^2*x^
2 - b^2*c^2*d*f^2 + 2*(b^2*d^3*f*x + b^2*c*d^2*f)*cosh(1))*sinh(1))*sqrt((a
^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 6*(b^2*d*f^3*x + b^2*d*f^
2*cosh(1) + b^2*d*f^2*sinh(1))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x
 + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2))/b) + 6*(b^2*d*f^3*x + b^2*d*f^2*cosh(1) + b^2*d*f^2*sinh(1))*sqrt
((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 3*((a^2 + b^2)*d^2*f
^3*x^2 + 2*(a^2 + b^2)*d^2*f^2*x*cosh(1) + (a^2 + b^2)*d^2*f*cosh(1)^2 + (a
^2 + b^2)*d^2*f*sinh(1)^2 + 2*((a^2 + b^2)*d^2*f^2*x + (a^2 + b^2)*d^2*f*co
sh(1))*sinh(1))*dilog(cosh(d*x + c) + sinh(d*x + c)) + 3*((a^2 + b^2)*d^2*f
^3*x^2 + 2*(a^2 + b^2)*d^2*f^2*x*cosh(1) + (a^2 + b^2)*d^2*f*cosh(1)^2 + (a
^2 + b^2)*d^2*f*sinh(1)^2 + 2*((a^2 + b^2)*d^2*f^2*x + (a^2 + b^2)*d^2*f*co
sh(1))*sinh(1))*dilog(-cosh(d*x + c) - sinh(d*x + c)) + ((a^2 + b^2)*d^3*f^
3*x^3 + 3*(a^2 + b^2)*d^3*f^2*x^2*cosh(1) + 3*(a^2 + b^2)*d^3*f*x*cosh(1)^2
 + (a^2 + b^2)*d^3*cosh(1)^3 + (a^2 + b^2)*d^3*sinh(1)^3 + 3*((a^2 + b^2)*d
^3*f*x + (a^2 + b^2)*d^3*cosh(1))*sinh(1)^2 + 3*((a^2 + b^2)*d^3*f^2*x^2 +
2*(a^2 + b^2)*d^3*f*x*cosh(1) + (a^2 + b^2)*d^3*cosh(1)^2)*sinh(1))*log(cos
h(d*x + c) + sinh(d*x + c) + 1) + ((a^2 + b^2)*c^3*f^3 - 3*(a^2 + b^2)*c^2*
d*f^2*cosh(1) + 3*(a^2 + b^2)*c*d^2*f*cosh(1)^2 - (a^2 + b^2)*d^3*cosh(1)^3
 - (a^2 + b^2)*d^3*sinh(1)^3 + 3*((a^2 + b^2)*c*d^2*f - (a^2 + b^2)*d^3*cos
h(1))*sinh(1)^2 - 3*((a^2 + b^2)*c^2*d*f^2 - 2*(a^2 + b^2)*c*d^2*f*cosh(1)
+ (a^2 + b^2)*d^3*cosh(1)^2)*sinh(1))*log(cosh(d*x + c) + sinh(d*x + c) - 1
) - ((a^2 + b^2)*d^3*f^3*x^3 + (a^2 + b^2)*c^3*f^3 + 3*((a^2 + b^2)*d^3*f*x
 + (a^2 + b^2)*c*d^2*f)*cosh(1)^2 + 3*((a^2 + b^2)*d^3*f*x + (a^2 + b^2)*c*
d^2*f)*sinh(1)^2 + 3*((a^2 + b^2)*d^3*f^2*x^2 - (a^2 + b^2)*c^2*d*f^2)*cosh
(1) + 3*((a^2 + b^2)*d^3*f^2*x^2 - (a^2 + b^2)*c^2*d*f^2 + 2*((a^2 + b^2)*d
^3*f*x + (a^2 + b^2)*c*d^2*f)*cosh(1))*sinh(1))*log(-cosh(d*x + c) - sinh(d
*x + c) + 1) + 6*((a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*f^2*cosh(1) + (a^2 +
b^2)*d*f^2*sinh(1))*polylog(3, cosh(d*x + c) + sinh(d*x + c)) - 6*((a^2 + b
^2)*d*f^3*x + (a^2 + b^2)*d*f^2*cosh(1) + (a^2 + b^2)*d*f^2*sinh(1))*polylo
g(3, -cosh(d*x + c) - sinh(d*x + c)))/((a^3 + a*b^2)*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3 \operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^3}{\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
[Out] int((e + f*x)^3/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.239 $\int \frac{(e+fx)^2 \text{csch}(c+dx)}{a+b\sinh(c+dx)} \, dx$

Optimal. Leaf size=433

$$-\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d}+\frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d}-\frac{2f(e$$

```
[Out] -2*(f*x+e)^2*arctanh(exp(d*x+c))/a/d-2*f*(f*x+e)*polylog(2,-exp(d*x+c))/a/d
^2+2*f*(f*x+e)*polylog(2,exp(d*x+c))/a/d^2+2*f^2*polylog(3,-exp(d*x+c))/a/d
^3-2*f^2*polylog(3,exp(d*x+c))/a/d^3-b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+
b^2)^(1/2)))/a/d/(a^2+b^2)^(1/2)+b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)
^(1/2)))/a/d/(a^2+b^2)^(1/2)-2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+
b^2)^(1/2)))/a/d^2/(a^2+b^2)^(1/2)+2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a
+(a^2+b^2)^(1/2)))/a/d^2/(a^2+b^2)^(1/2)+2*b*f^2*polylog(3,-b*exp(d*x+c)/(a
-(a^2+b^2)^(1/2)))/a/d^3/(a^2+b^2)^(1/2)-2*b*f^2*polylog(3,-b*exp(d*x+c)/(a
+(a^2+b^2)^(1/2)))/a/d^3/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5694, 4267, 2611, 2320, 6724, 3403, 2296, 2221}

$$\frac{2bf^2\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}}-\frac{2bf^2\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}}-\frac{2bf(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}}+\frac{2bf(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}}-\frac{b(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{ad\sqrt{a^2+b^2}}+\frac{b(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{ad\sqrt{a^2+b^2}}+\frac{2f^2\text{Li}_3\left(-e^{c+dx}\right)}{ad^3}-\frac{2f^2\text{Li}_3\left(e^{c+dx}\right)}{ad^3}-\frac{2f(e+fx)\text{Li}_2\left(-e^{c+dx}\right)}{ad^2}+\frac{2f(e+fx)\text{Li}_2\left(e^{c+dx}\right)}{ad^2}-\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (b*(e + f*x)^2*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) + (b*(e + f*x)^2*Lo
g[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) - (2*f*
(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (2*f*(e + f*x)*PolyLog[2, E^(
c + d*x)])/(a*d^2) - (2*b*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) + (2*b*f*(e + f*x)*PolyLog[2, -((b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) + (2*f^2*Pol
yLog[3, -E^(c + d*x)])/(a*d^3) - (2*f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) +
(2*b*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2
+ b^2]*d^3) - (2*b*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/(a*Sqrt[a^2 + b^2]*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5694

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
```

```
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a + b
*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{csch}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{(2b)\int \frac{e^{c+dx}(e+fx)^2}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{a} - \frac{(2f)\int (e+fx)}{}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2f(e+fx)\text{Li}_2\left(-e^{c+dx}\right)}{ad^2} + \frac{2f(e+fx)\text{Li}_2\left(e^{c+}\right.}{ad^2}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d} + \frac{b(e+f}{}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d} + \frac{b(e+f}{}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d} + \frac{b(e+f}{}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d} + \frac{b(e+f}{}$$

Mathematica [A]
time = 2.43, size = 750, normalized size = 1.73

Antiderivative was successfully verified.

[In] Integrate[((e + f*x)^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]

[Out] (-2*d^2*e^2*ArcTanh[E^(c + d*x)] + 2*d^2*e*f*x*Log[1 - E^(c + d*x)] + d^2*f^2*x^2*Log[1 - E^(c + d*x)] - 2*d^2*e*f*x*Log[1 + E^(c + d*x)] - d^2*f^2*x^2*Log[1 + E^(c + d*x)] - 2*d*f*(e + f*x)*PolyLog[2, -E^(c + d*x)] + 2*d*f*(e + f*x)*PolyLog[2, E^(c + d*x)] + 2*f^2*PolyLog[3, -E^(c + d*x)] - 2*f^2*PolyLog[3, E^(c + d*x)] + (b*(2*d^2*e^2*Sqrt[(a^2 + b^2)*E^(2*c)]*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*Sqrt[a^2 + b^2]*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - Sqrt[a^2 + b^2]*d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*Sqrt[a^2 + b^2]*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + Sqrt[a^2 + b^2]*d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 2*Sqrt[a^2 + b^2]*d*E^c*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2*Sqrt[a^2 + b^2]*d*E^c*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2*Sqrt[a^2 + b^2]*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*Sqrt[a^2 + b^2]*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(Sqrt[a^2 + b^2]*Sqrt[(a^2 + b^2)*E^(2*c)]))/(a*d^3)

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)

[Out] int((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")

[Out] -(b*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*d) + log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d))*e^2 - 2*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*f*e/(a*d^2) + 2*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))*f*e/(a*d^2) - (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2/(a*d^3) + (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*f^2/(a*d^3) - integrate(2*(b*f

```
^2*x^2*e^c + 2*b*f*x*e^(c + 1))*e^(d*x)/(a*b*e^(2*d*x + 2*c) + 2*a^2*e^(d*x
 + c) - a*b), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1379 vs. $2(401) = 802$.
time = 0.40, size = 1379, normalized size = 3.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*b^2*f^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x +
 c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*b^2
*f^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) -
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*(a^2 + b^
2)*f^2*polylog(3, cosh(d*x + c) + sinh(d*x + c)) + 2*(a^2 + b^2)*f^2*polylo
g(3, -cosh(d*x + c) - sinh(d*x + c)) - 2*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b
^2*d*f*sinh(1))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x +
 c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1)
 + 2*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*sqrt((a^2 + b^2)/b^2
)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b^2*c^2*f^2 - 2*b^2*c*d*f*cosh(1
) + b^2*d^2*cosh(1)^2 + b^2*d^2*sinh(1)^2 - 2*(b^2*c*d*f - b^2*d^2*cosh(1))
*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) +
 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^2*c^2*f^2 - 2*b^2*c*d*f*cosh(1) + b^
2*d^2*cosh(1)^2 + b^2*d^2*sinh(1)^2 - 2*(b^2*c*d*f - b^2*d^2*cosh(1))*sinh(
1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*s
qrt((a^2 + b^2)/b^2) + 2*a) - (b^2*d^2*f^2*x^2 - b^2*c^2*f^2 + 2*(b^2*d^2*f
*x + b^2*c*d*f)*cosh(1) + 2*(b^2*d^2*f*x + b^2*c*d*f)*sinh(1))*sqrt((a^2 +
b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b^2*d^2*f^2*x^2 - b^2*c^2*f^2
 + 2*(b^2*d^2*f*x + b^2*c*d*f)*cosh(1) + 2*(b^2*d^2*f*x + b^2*c*d*f)*sinh(1
))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*((a^2 + b^2)*
d*f^2*x + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*dilog(cosh(d*x
 + c) + sinh(d*x + c)) - 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(1) +
 (a^2 + b^2)*d*f*sinh(1))*dilog(-cosh(d*x + c) - sinh(d*x + c)) - ((a^2 + b
^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*f*x*cosh(1) + (a^2 + b^2)*d^2*cosh(1)^2
 + (a^2 + b^2)*d^2*sinh(1)^2 + 2*((a^2 + b^2)*d^2*f*x + (a^2 + b^2)*d^2*cos
h(1))*sinh(1))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a^2 + b^2)*c^2*f^
2 - 2*(a^2 + b^2)*c*d*f*cosh(1) + (a^2 + b^2)*d^2*cosh(1)^2 + (a^2 + b^2)*d
^2*sinh(1)^2 - 2*((a^2 + b^2)*c*d*f - (a^2 + b^2)*d^2*cosh(1))*sinh(1))*log
(cosh(d*x + c) + sinh(d*x + c) - 1) + ((a^2 + b^2)*d^2*f^2*x^2 - (a^2 + b^2
)*c^2*f^2 + 2*((a^2 + b^2)*d^2*f*x + (a^2 + b^2)*c*d*f)*cosh(1) + 2*((a^2 +
```

```
b^2)*d^2*f*x + (a^2 + b^2)*c*d*f)*sinh(1))*log(-cosh(d*x + c) - sinh(d*x +
c) + 1))/((a^3 + a*b^2)*d^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2 \operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^2}{\sinh(c+d\,x)\,\left(a+b\sinh(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((e + f*x)^2/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.240 $\int \frac{(e+fx)\mathrm{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=261

$$-\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d}+\frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d}-\frac{f\text{PolyL}}{}$$

```
[Out] -2*(f*x+e)*arctanh(exp(d*x+c))/a/d-f*polylog(2,-exp(d*x+c))/a/d^2+f*polylog
(2,exp(d*x+c))/a/d^2-b*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d/(
a^2+b^2)^(1/2)+b*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d/(a^2+b^
2)^(1/2)-b*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d^2/(a^2+b^2)^(
1/2)+b*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d^2/(a^2+b^2)^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5694, 4267, 2317, 2438, 3403, 2296, 2221}

$$-\frac{bf\,\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}}+\frac{bf\,\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}}-\frac{b(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{ad\sqrt{a^2+b^2}}+\frac{b(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{ad\sqrt{a^2+b^2}}-\frac{f\,\text{Li}_2\left(-e^{c+dx}\right)}{ad^2}+\frac{f\,\text{Li}_2\left(e^{c+dx}\right)}{ad^2}-\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d) - (b*(e + f*x)*Log[1 + (b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) + (b*(e + f*x)*Log[1 +
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) - (f*PolyLog[
2, -E^(c + d*x)])/(a*d^2) + (f*PolyLog[2, E^(c + d*x)])/(a*d^2) - (b*f*Poly
Log[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) +
 (b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b
^2]*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
```

```
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5694

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a + b
*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rubi steps

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\operatorname{csch}(c+dx)\,dx}{a} - \frac{b\int \frac{e+fx}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{(2b)\int \frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{a} - \frac{f\int \log\left(1-e^{c+d\cdots}\right.}{ad}$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{(2b^2)\int \frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{a\sqrt{a^2+b^2}} + \frac{(2b^2)\int \frac{\cdots}{2a+2\sqrt{\cdots}}}{a\sqrt{\cdots}}$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d} + \frac{b(e+fx)\,\mathrm{l}\cdots}{\cdots}$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d} + \frac{b(e+fx)\,\mathrm{l}\cdots}{\cdots}$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d} + \frac{b(e+fx)\,\mathrm{l}\cdots}{\cdots}$$

Mathematica [A]
time = 1.16, size = 306, normalized size = 1.17

$$\frac{de\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - cf\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + f\left((c+dx)\left(\log\left(1-e^{-c-dx}\right)-\log\left(1+e^{-c-dx}\right)\right) + \operatorname{PolyLog}(2,-e^{-c-dx}) - \operatorname{PolyLog}(2,e^{-c-dx})\right) + \frac{b\left(2de\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) - 2cf\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) - f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) + f(c+dx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right) - f\operatorname{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right) + f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)}{\sqrt{a^2+b^2}}}{ad^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (d*e*Log[Tanh[(c + d*x)/2]] - c*f*Log[Tanh[(c + d*x)/2]] + f*((c + d*x)*(Lo
g[1 - E^(-c - d*x)] - Log[1 + E^(-c - d*x)]) + PolyLog[2, -E^(-c - d*x)] -
PolyLog[2, E^(-c - d*x)]) + (b*(2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2
+ b^2]] - 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - f*(c + d*x)*
Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(
c + d*x))/(a + Sqrt[a^2 + b^2])] - f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[
a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/Sqr
t[a^2 + b^2])/(a*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(238) = 476$.
time = 1.58, size = 532, normalized size = 2.04

method	result
risch	$-\frac{e \ln(e^{dx+c}+1)}{ad}+\frac{2eb \operatorname{arctanh}\left(\frac{2b e^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{da\sqrt{a^2+b^2}}+\frac{e\ln(e^{dx+c}-1)}{ad}-\frac{f\ln(e^{dx+c}+1)x}{da}-\frac{f \operatorname{dilog}(e^{dx+c}+1)}{d^2a}-\frac{fb\ln\left(\frac{-b e^{d\ldots}}{\ldots}\right.}{d\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/d*e*ln(exp(d*x+c)+1)+2/d*e/a*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*
x+c)+2*a)/(a^2+b^2)^(1/2))+1/a/d*e*ln(exp(d*x+c)-1)-1/d*f/a*ln(exp(d*x+c)+1
)*x-1/d^2*f/a*dilog(exp(d*x+c)+1)-1/d*f/a*b/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+
c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d^2*f/a*b/(a^2+b^2)^(1/2)*l
n((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d*f/a*b/(a^2+
b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2
*f/a*b/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/
2)))*c-1/d^2*f/a*b/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/
(-a+(a^2+b^2)^(1/2)))+1/d^2*f/a*b/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+
b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2*f*dilog(exp(d*x+c))/a-2/d^2*f*c/a*
b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d^2*f
*c/a*ln(exp(d*x+c)-1)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(b*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a*d) + log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d
*x - c) - 1)/(a*d))*e + 2*f*integrate(2*x/((b*(e^(d*x + c) - e^(-d*x - c))
+ 2*a)*(e^(d*x + c) - e^(-d*x - c))), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 691 vs. 2(237) = 474.
time = 0.42, size = 691, normalized size = 2.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(b^2*f*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - b^2*f
```

```
*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (a^2 + b^2)*
f*dilog(cosh(d*x + c) + sinh(d*x + c)) + (a^2 + b^2)*f*dilog(-cosh(d*x + c)
 - sinh(d*x + c)) + (b^2*c*f - b^2*d*cosh(1) - b^2*d*sinh(1))*sqrt((a^2 + b
^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b
^2) + 2*a) - (b^2*c*f - b^2*d*cosh(1) - b^2*d*sinh(1))*sqrt((a^2 + b^2)/b^2
)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2
*a) + (b^2*d*f*x + b^2*c*f)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a
*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b) - (b^2*d*f*x + b^2*c*f)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c)
 + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2) - b)/b) + ((a^2 + b^2)*d*f*x + (a^2 + b^2)*d*cosh(1) + (a^2 + b^2)*d*si
nh(1))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a^2 + b^2)*c*f - (a^2 + b
^2)*d*cosh(1) - (a^2 + b^2)*d*sinh(1))*log(cosh(d*x + c) + sinh(d*x + c) -
1) - ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*log(-cosh(d*x + c) - sinh(d*x +
c) + 1))/((a^3 + a*b^2)*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)*csch(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\sinh(c+d\,x)\,\left(a+b\sinh(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
[Out] int((e + f*x)/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.241 $\int \frac{\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=64

$$-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d}$$

```
[Out] -arctanh(cosh(d*x+c))/a/d+2*b*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(
1/2))/a/d/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2826, 3855, 2739, 632, 210}

$$\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(ArcTanh[Cosh[c + d*x]]/(a*d)) + (2*b*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sq
rt[a^2 + b^2]])/(a*Sqrt[a^2 + b^2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{\int \operatorname{csch}(c+dx)\,dx}{a}-\frac{b\int \frac{1}{a+b\sinh(c+dx)}\,dx}{a}\\ &= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{(2ib)\operatorname{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{ad}\\ &= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad}-\frac{(4ib)\operatorname{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx,x,-2ib+2a\tan\left(\frac{1}{2}(ic+\right.\right.}{ad}\\ &= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 69, normalized size = 1.08

$$\frac{-\frac{2b\operatorname{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((-2*b*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
 + Log[Tanh[(c + d*x)/2]])/(a*d)
```

Maple [A]
time = 1.00, size = 63, normalized size = 0.98

method	result	s

derivativedivides	$\frac{-\frac{2b\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
default	$\frac{-\frac{2b\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
risch	$\frac{\ln(\mathrm{e}^{dx+c}-1)}{da}+\frac{b\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,da}-\frac{b\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,da}-\frac{\ln(\mathrm{e}^{dx+c}+1)}{da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+
b^2)^(1/2))+1/a*ln(tanh(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.47, size = 112, normalized size = 1.75

$$-\frac{b\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,ad}-\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}+\frac{\log\left(e^{(-dx-c)}-1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -b*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*a*d) - log(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*
x - c) - 1)/(a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(61) = 122.
time = 0.38, size = 223, normalized size = 3.48

$$\frac{\sqrt{a^2+b^2}\,b\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2+b^2+2\,(b^2\cosh(dx+c)+ab)\sinh(dx+c)+2\,\sqrt{a^2+b^2}\,(b\cosh(dx+c)+b\sinh(dx+c)+a)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)-b}\right)-(a^2+b^2)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+1\right)+(a^2+b^2)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1\right)}{(a^3+ab^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*b*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*c
osh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*
sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2
```

```
 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x
 + c) - b)) - (a^2 + b^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + (a^2 + b
^2)*log(cosh(d*x + c) + sinh(d*x + c) - 1))/((a^3 + a*b^2)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A]
time = 0.45, size = 102, normalized size = 1.59

$$-\frac{\frac{b\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a}+\frac{\log\left(e^{(dx+c)}+1\right)}{a}-\frac{\log\left(\left|e^{(dx+c)}-1\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c)
+ 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) + log(e^(d*x + c) + 1)/a -
log(abs(e^(d*x + c) - 1))/a)/d
```

Mupad [B]
time = 0.46, size = 347, normalized size = 5.42

$$\frac{\ln\left(32\,a-32\,a\,e^{d\,x}\,e^c\right)}{a\,d}-\frac{\ln\left(32\,a+32\,a\,e^{d\,x}\,e^c\right)}{a\,d}+\frac{b\,\ln\left(128\,a^5\,e^{d\,x}\,e^c-64\,a^2\,b^3-64\,a^4\,b+32\,a\,b^3\,\sqrt{a^2+b^2}+64\,a^3\,b\,\sqrt{a^2+b^2}+160\,a^3\,b^2\,e^{d\,x}\,e^c-128\,a^4\,e^{d\,x}\,e^c\,\sqrt{a^2+b^2}+32\,a\,b^4\,e^{d\,x}\,e^c-96\,a^2\,b^2\,e^{d\,x}\,e^c\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{d\,a^3+d\,a\,b^2}-\frac{b\,\ln\left(64\,a^4\,b+64\,a^2\,b^3-128\,a^5\,e^{d\,x}\,e^c+32\,a\,b^3\,\sqrt{a^2+b^2}+64\,a^3\,b\,\sqrt{a^2+b^2}-160\,a^3\,b^2\,e^{d\,x}\,e^c-128\,a^4\,e^{d\,x}\,e^c\,\sqrt{a^2+b^2}-32\,a\,b^4\,e^{d\,x}\,e^c-96\,a^2\,b^2\,e^{d\,x}\,e^c\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{d\,a^3+d\,a\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] log(32*a - 32*a*exp(d*x)*exp(c))/(a*d) - log(32*a + 32*a*exp(d*x)*exp(c))/(
a*d) + (b*log(128*a^5*exp(d*x)*exp(c) - 64*a^2*b^3 - 64*a^4*b + 32*a*b^3*(a
^2 + b^2)^(1/2) + 64*a^3*b*(a^2 + b^2)^(1/2) + 160*a^3*b^2*exp(d*x)*exp(c)
- 128*a^4*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) + 32*a*b^4*exp(d*x)*exp(c) - 96
*a^2*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^3*d + a*b
^2*d) - (b*log(64*a^4*b + 64*a^2*b^3 - 128*a^5*exp(d*x)*exp(c) + 32*a*b^3*(
a^2 + b^2)^(1/2) + 64*a^3*b*(a^2 + b^2)^(1/2) - 160*a^3*b^2*exp(d*x)*exp(c)
 - 128*a^4*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) - 32*a*b^4*exp(d*x)*exp(c) - 9
6*a^2*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^3*d + a*
b^2*d)
```

3.242 $\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=29

$$\operatorname{Int}\left(\frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csch[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][Csch[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 3.77, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csch[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Integrate[Csch[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(dx+c)}{(fx+e)\left(a+b\sinh\left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(csch(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(csch(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c + dx)}{(a + b\sinh(c + dx))(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(csch(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sinh{(c+d\,x)}\,\,(e+f\,x)\,\,(a+b\sinh{(c+d\,x)})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
[Out] int(1/(sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.243 $\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=745

$$-\frac{(e+fx)^3}{ad}+\frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d}-\frac{(e+fx)^3\coth(c+dx)}{ad}+\frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d}-\frac{b^2(\cdots}{}$$

```
[Out] -(f*x+e)^3/a/d+2*b*(f*x+e)^3*arctanh(exp(d*x+c))/a^2/d-(f*x+e)^3*coth(d*x+c
)/a/d+3*f*(f*x+e)^2*ln(1-exp(2*d*x+2*c))/a/d^2+3*b*f*(f*x+e)^2*polylog(2,-e
xp(d*x+c))/a^2/d^2-3*b*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a^2/d^2+3*f^2*(f*x
+e)*polylog(2,exp(2*d*x+2*c))/a/d^3-6*b*f^2*(f*x+e)*polylog(3,-exp(d*x+c))/
a^2/d^3+6*b*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a^2/d^3-3/2*f^3*polylog(3,exp
(2*d*x+2*c))/a/d^4+6*b*f^3*polylog(4,-exp(d*x+c))/a^2/d^4-6*b*f^3*polylog(4
,exp(d*x+c))/a^2/d^4+b^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a
^2/d/(a^2+b^2)^(1/2)-b^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a
^2/d/(a^2+b^2)^(1/2)+3*b^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)
^(1/2)))/a^2/d^2/(a^2+b^2)^(1/2)-3*b^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/
(a+(a^2+b^2)^(1/2)))/a^2/d^2/(a^2+b^2)^(1/2)-6*b^2*f^2*(f*x+e)*polylog(3,-b
*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^3/(a^2+b^2)^(1/2)+6*b^2*f^2*(f*x+e)*
polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^3/(a^2+b^2)^(1/2)+6*b^2*
f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^4/(a^2+b^2)^(1/2)-6*
b^2*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^4/(a^2+b^2)^(1/2
)
```

Rubi [A]
time = 0.96, antiderivative size = 745, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {5694, 4269, 3797, 2221, 2611, 2320, 6724, 4267, 6744, 3403, 2296}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((e + f*x)^3/(a*d)) + (2*b*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a^2*d) - ((e
 + f*x)^3*Coth[c + d*x])/(a*d) + (b^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2 + b^2]*d) - (b^2*(e + f*x)^3*Log[1 + (
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2 + b^2]*d) + (3*f*(e +
f*x)^2*Log[1 - E^(2*(c + d*x))])/(a*d^2) + (3*b*f*(e + f*x)^2*PolyLog[2, -E
^(c + d*x)])/(a^2*d^2) - (3*b*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)])/(a^2*d
^2) + (3*b^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/(a^2*Sqrt[a^2 + b^2]*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^2) + (3*f^2*(e + f
```

```
*x)*PolyLog[2, E^(2*(c + d*x))])/(a*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, -E
^(c + d*x)])/(a^2*d^3) + (6*b*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a^2*d
^3) - (6*b^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/(a^2*Sqrt[a^2 + b^2]*d^3) + (6*b^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^3) - (3*f^3*PolyLo
g[3, E^(2*(c + d*x))])/(2*a*d^4) + (6*b*f^3*PolyLog[4, -E^(c + d*x)])/(a^2*
d^4) - (6*b*f^3*PolyLog[4, E^(c + d*x)])/(a^2*d^4) + (6*b^2*f^3*PolyLog[4,
-((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^4) - (6*b
^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2
+ b^2]*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
```

```
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5694

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a + b
*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\text{csch}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^3\coth(c+dx)}{ad} - \frac{b\int (e+fx)^3\text{csch}(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}}{a^2}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{(2b}{}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{3f(}{}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{b^2(e}{}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{b^2(e}{}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{b^2(e}{}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{b^2(e}{}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{b^2(e}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2152 vs. $2(745) = 1490$.
time = 14.15, size = 2152, normalized size = 2.89

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/2*(12*a*d^3*e^2*E^(2*c)*f*x + 12*a*d^3*e*E^(2*c)*f^2*x^2 + 4*a*d^3*E^(2*
c)*f^3*x^3 + 4*b*d^3*e^3*ArcTanh[E^(c + d*x)] - 4*b*d^3*e^3*E^(2*c)*ArcTanh
```

[E^(c + d*x)] - 6*b*d^3*e^2*f*x*Log[1 - E^(c + d*x)] + 6*b*d^3*e^2*E^(2*c)*f*x*Log[1 - E^(c + d*x)] - 6*b*d^3*e*f^2*x^2*Log[1 - E^(c + d*x)] + 6*b*d^3*e*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] - 2*b*d^3*f^3*x^3*Log[1 - E^(c + d*x)] + 2*b*d^3*E^(2*c)*f^3*x^3*Log[1 - E^(c + d*x)] + 6*b*d^3*e^2*f*x*Log[1 + E^(c + d*x)] - 6*b*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(c + d*x)] + 6*b*d^3*e*f^2*x^2*Log[1 + E^(c + d*x)] - 6*b*d^3*e*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] + 2*b*d^3*f^3*x^3*Log[1 + E^(c + d*x)] - 2*b*d^3*E^(2*c)*f^3*x^3*Log[1 + E^(c + d*x)] + 6*a*d^2*e^2*f*Log[1 - E^(2*(c + d*x))] - 6*a*d^2*e^2*E^(2*c)*f*Log[1 - E^(2*(c + d*x))] + 12*a*d^2*e*f^2*x*Log[1 - E^(2*(c + d*x))] - 12*a*d^2*e*E^(2*c)*f^2*x*Log[1 - E^(2*(c + d*x))] + 6*a*d^2*f^3*x^2*Log[1 - E^(2*(c + d*x))] - 6*a*d^2*E^(2*c)*f^3*x^2*Log[1 - E^(2*(c + d*x))] - 6*b*d^2*(-1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, -E^(c + d*x)] + 6*b*d^2*(-1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)] + 6*a*d*e*f^2*PolyLog[2, E^(2*(c + d*x))] - 6*a*d*e*E^(2*c)*f^2*PolyLog[2, E^(2*(c + d*x))] + 6*a*d*f^3*x*PolyLog[2, E^(2*(c + d*x))] - 6*a*d*E^(2*c)*f^3*x*PolyLog[2, E^(2*(c + d*x))] - 12*b*d*e*f^2*PolyLog[3, -E^(c + d*x)] + 12*b*d*e*E^(2*c)*f^2*PolyLog[3, -E^(c + d*x)] - 12*b*d*f^3*x*PolyLog[3, -E^(c + d*x)] + 12*b*d*E^(2*c)*f^3*x*PolyLog[3, -E^(c + d*x)] + 12*b*d*e*f^2*PolyLog[3, E^(c + d*x)] - 12*b*d*e*E^(2*c)*f^2*PolyLog[3, E^(c + d*x)] + 12*b*d*f^3*x*PolyLog[3, E^(c + d*x)] - 12*b*d*E^(2*c)*f^3*x*PolyLog[3, E^(c + d*x)] - 3*a*f^3*PolyLog[3, E^(2*(c + d*x))] + 3*a*E^(2*c)*f^3*PolyLog[3, E^(2*(c + d*x))] + 12*b*f^3*PolyLog[4, -E^(c + d*x)] - 12*b*E^(2*c)*f^3*PolyLog[4, -E^(c + d*x)] - 12*b*f^3*PolyLog[4, E^(c + d*x)] + 12*b*E^(2*c)*f^3*PolyLog[4, E^(c + d*x)])/(a^2*d^4*(-1 + E^(2*c))) + (b^2*(-2*d^3*e^3*Sqrt[(a^2 + b^2)*E^(2*c)]*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 3*Sqrt[a^2 + b^2]*d^3*e^2*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 3*Sqrt[a^2 + b^2]*d^3*e*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + Sqrt[a^2 + b^2]*d^3*E^c*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*Sqrt[a^2 + b^2]*d^3*e^2*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*Sqrt[a^2 + b^2]*d^3*e*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - Sqrt[a^2 + b^2]*d^3*E^c*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 3*Sqrt[a^2 + b^2]*d^2*E^c*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 3*Sqrt[a^2 + b^2]*d^2*E^c*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[a^2 + b^2]*d*e*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[a^2 + b^2]*d*E^c*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[a^2 + b^2]*d*e*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[a^2 + b^2]*d*E^c*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[a^2 + b^2]*E^c*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[a^2 + b^2]*E^c*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(a^2*Sqrt[a^2 + b^2]*d^4*Sqrt[(a^2 + b^2)*E^(2*c)]) + (Sech[c/2]*Sech[c/2 + (d*x)/2]*(-(e^3*Sinh[(d*x)/2]) - 3*e^2*f*x*Sinh[

```
(d*x)/2] - 3*e*f^2*x^2*Sinh[(d*x)/2] - f^3*x^3*Sinh[(d*x)/2]))/(2*a*d) + (C
sch[c/2]*Csch[c/2 + (d*x)/2]*(e^3*Sinh[(d*x)/2] + 3*e^2*f*x*Sinh[(d*x)/2] +
 3*e*f^2*x^2*Sinh[(d*x)/2] + f^3*x^3*Sinh[(d*x)/2]))/(2*a*d)
```

Maple [F]
time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c)^2}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*a^2*d) + b*log(e^(-d*x - c) + 1)/(a^2*d) - b*
log(e^(-d*x - c) - 1)/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d))*e^3 - 6*f*x
*e^2/(a*d) - 2*(f^3*x^3 + 3*f^2*x^2*e + 3*f*x*e^2)/(a*d*e^(2*d*x + 2*c) - a
*d) + 3*f*e^2*log(e^(d*x + c) + 1)/(a*d^2) + 3*f*e^2*log(e^(d*x + c) - 1)/(
a*d^2) + (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*
d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*b*f^3/(a^2*d^4)
- (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*pol
ylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*b*f^3/(a^2*d^4) + 3*(b*d*
f*e^2 + 2*a*f^2*e)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^
3) - 3*(b*d*f*e^2 - 2*a*f^2*e)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x +
c)))/(a^2*d^3) + 3*(b*d*f^2*e + a*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*
x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^2*d^4) - 3*(b*d*f^2*
e - a*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*po
lylog(3, e^(d*x + c)))/(a^2*d^4) - 1/4*(b*d^4*f^3*x^4 + 4*(b*d*f^2*e + a*f^
3)*d^3*x^3 + 6*(b*d^2*f*e^2 + 2*a*d*f^2*e)*d^2*x^2)/(a^2*d^4) + 1/4*(b*d^4*
f^3*x^4 + 4*(b*d*f^2*e - a*f^3)*d^3*x^3 + 6*(b*d^2*f*e^2 - 2*a*d*f^2*e)*d^2
*x^2)/(a^2*d^4) + integrate(2*(b^2*f^3*x^3*e^c + 3*b^2*f^2*x^2*e^(c + 1) +
3*b^2*f*x*e^(c + 2))*e^(d*x)/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(d*x + c) - a
^2*b), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11107 vs. 2(706) = 1412.

time = 0.52, size = 11107, normalized size = 14.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
[Out] (2*(a^3 + a*b^2)*c^3*f^3 - 6*(a^3 + a*b^2)*c^2*d*f^2*cosh(1) + 6*(a^3 + a*b
^2)*c*d^2*f*cosh(1)^2 - 2*(a^3 + a*b^2)*d^3*cosh(1)^3 - 2*(a^3 + a*b^2)*d^3
*sinh(1)^3 - 2*((a^3 + a*b^2)*d^3*f^3*x^3 + (a^3 + a*b^2)*c^3*f^3 + 3*((a^3
 + a*b^2)*d^3*f*x + (a^3 + a*b^2)*c*d^2*f)*cosh(1)^2 + 3*((a^3 + a*b^2)*d^3
*f*x + (a^3 + a*b^2)*c*d^2*f)*sinh(1)^2 + 3*((a^3 + a*b^2)*d^3*f^2*x^2 - (a
^3 + a*b^2)*c^2*d*f^2)*cosh(1) + 3*((a^3 + a*b^2)*d^3*f^2*x^2 - (a^3 + a*b^
2)*c^2*d*f^2 + 2*((a^3 + a*b^2)*d^3*f*x + (a^3 + a*b^2)*c*d^2*f)*cosh(1))*s
inh(1))*cosh(d*x + c)^2 + 6*((a^3 + a*b^2)*c*d^2*f - (a^3 + a*b^2)*d^3*cosh
(1))*sinh(1)^2 - 4*((a^3 + a*b^2)*d^3*f^3*x^3 + (a^3 + a*b^2)*c^3*f^3 + 3*(
(a^3 + a*b^2)*d^3*f*x + (a^3 + a*b^2)*c*d^2*f)*cosh(1)^2 + 3*((a^3 + a*b^2)
*d^3*f*x + (a^3 + a*b^2)*c*d^2*f)*sinh(1)^2 + 3*((a^3 + a*b^2)*d^3*f^2*x^2
- (a^3 + a*b^2)*c^2*d*f^2)*cosh(1) + 3*((a^3 + a*b^2)*d^3*f^2*x^2 - (a^3 +
a*b^2)*c^2*d*f^2 + 2*((a^3 + a*b^2)*d^3*f*x + (a^3 + a*b^2)*c*d^2*f)*cosh(1
))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - 2*((a^3 + a*b^2)*d^3*f^3*x^3 + (a
^3 + a*b^2)*c^3*f^3 + 3*((a^3 + a*b^2)*d^3*f*x + (a^3 + a*b^2)*c*d^2*f)*cos
h(1)^2 + 3*((a^3 + a*b^2)*d^3*f*x + (a^3 + a*b^2)*c*d^2*f)*sinh(1)^2 + 3*((
a^3 + a*b^2)*d^3*f^2*x^2 - (a^3 + a*b^2)*c^2*d*f^2)*cosh(1) + 3*((a^3 + a*b
^2)*d^3*f^2*x^2 - (a^3 + a*b^2)*c^2*d*f^2 + 2*((a^3 + a*b^2)*d^3*f*x + (a^3
 + a*b^2)*c*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 3*(b^3*d^2*f^3*x^2 +
 2*b^3*d^2*f^2*x*cosh(1) + b^3*d^2*f*cosh(1)^2 + b^3*d^2*f*sinh(1)^2 - (b^3
*d^2*f^3*x^2 + 2*b^3*d^2*f^2*x*cosh(1) + b^3*d^2*f*cosh(1)^2 + b^3*d^2*f*si
nh(1)^2 + 2*(b^3*d^2*f^2*x + b^3*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 -
2*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*f^2*x*cosh(1) + b^3*d^2*f*cosh(1)^2 + b^3*d^
2*f*sinh(1)^2 + 2*(b^3*d^2*f^2*x + b^3*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c
)*sinh(d*x + c) - (b^3*d^2*f^3*x^2 + 2*b^3*d^2*f^2*x*cosh(1) + b^3*d^2*f*co
sh(1)^2 + b^3*d^2*f*sinh(1)^2 + 2*(b^3*d^2*f^2*x + b^3*d^2*f*cosh(1))*sinh(
1))*sinh(d*x + c)^2 + 2*(b^3*d^2*f^2*x + b^3*d^2*f*cosh(1))*sinh(1))*sqrt((
a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 3*(b^3*d^2*f^3*x^2
+ 2*b^3*d^2*f^2*x*cosh(1) + b^3*d^2*f*cosh(1)^2 + b^3*d^2*f*sinh(1)^2 - (b^
3*d^2*f^3*x^2 + 2*b^3*d^2*f^2*x*cosh(1) + b^3*d^2*f*cosh(1)^2 + b^3*d^2*f*s
inh(1)^2 + 2*(b^3*d^2*f^2*x + b^3*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 -
 2*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*f^2*x*cosh(1) + b^3*d^2*f*cosh(1)^2 + b^3*d
^2*f*sinh(1)^2 + 2*(b^3*d^2*f^2*x + b^3*d^2*f*cosh(1))*sinh(1))*cosh(d*x +
c)*sinh(d*x + c) - (b^3*d^2*f^3*x^2 + 2*b^3*d^2*f^2*x*cosh(1) + b^3*d^2*f*c
osh(1)^2 + b^3*d^2*f*sinh(1)^2 + 2*(b^3*d^2*f^2*x + b^3*d^2*f*cosh(1))*sinh
(1))*sinh(d*x + c)^2 + 2*(b^3*d^2*f^2*x + b^3*d^2*f*cosh(1))*sinh(1))*sqrt(
(a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c
```

```
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^3*c^3*f^3 - 3*b
^3*c^2*d*f^2*cosh(1) + 3*b^3*c*d^2*f*cosh(1)^2 - b^3*d^3*cosh(1)^3 - b^3*d^
3*sinh(1)^3 - (b^3*c^3*f^3 - 3*b^3*c^2*d*f^2*cosh(1) + 3*b^3*c*d^2*f*cosh(1
)^2 - b^3*d^3*cosh(1)^3 - b^3*d^3*sinh(1)^3 + 3*(b^3*c*d^2*f - b^3*d^3*cosh
(1))*sinh(1)^2 - 3*(b^3*c^2*d*f^2 - 2*b^3*c*d^2*f*cosh(1) + b^3*d^3*cosh(1)
^2)*sinh(1))*cosh(d*x + c)^2 + 3*(b^3*c*d^2*f - b^3*d^3*cosh(1))*sinh(1)^2
- 2*(b^3*c^3*f^3 - 3*b^3*c^2*d*f^2*cosh(1) + 3*b^3*c*d^2*f*cosh(1)^2 - b^3*
d^3*cosh(1)^3 - b^3*d^3*sinh(1)^3 + 3*(b^3*c*d^2*f - b^3*d^3*cosh(1))*sinh(
1)^2 - 3*(b^3*c^2*d*f^2 - 2*b^3*c*d^2*f*cosh(1) + b^3*d^3*cosh(1)^2)*sinh(1
))*cosh(d*x + c)*sinh(d*x + c) - (b^3*c^3*f^3 - 3*b^3*c^2*d*f^2*cosh(1) + 3
*b^3*c*d^2*f*cosh(1)^2 - b^3*d^3*cosh(1)^3 - b^3*d^3*sinh(1)^3 + 3*(b^3*c*d
^2*f - b^3*d^3*cosh(1))*sinh(1)^2 - 3*(b^3*c^2*d*f^2 - 2*b^3*c*d^2*f*cosh(1
) + b^3*d^3*cosh(1)^2)*sinh(1))*sinh(d*x + c)^2 - 3*(b^3*c^2*d*f^2 - 2*b^3*
c*d^2*f*cosh(1) + b^3*d^3*cosh(1)^2)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b
*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^
3*c^3*f^3 - 3*b^3*c^2*d*f^2*cosh(1) + 3*b^3*c*d^2*f*cosh(1)^2 - b^3*d^3*cos
h(1)^3 - b^3*d^3*sinh(1)^3 - (b^3*c^3*f^3 - 3*b^3*c^2*d*f^2*cosh(1) + 3*b^3
*c*d^2*f*cosh(1)^2 - b^3*d^3*cosh(1)^3 - b^3*d^3*sinh(1)^3 + 3*(b^3*c*d^2*f
 - b^3*d^3*cosh(1))*sinh(1)^2 - 3*(b^3*c^2*d*f^2 - 2*b^3*c*d^2*f*cosh(1) +
b^3*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c)^2 + 3*(b^3*c*d^2*f - b^3*d^3*cosh
(1))*sinh(1)^2 - 2*(b^3*c^3*f^3 - 3*b^3*c^2*d*f^2*cosh(1) + 3*b^3*c*d^2*f*c
osh(1)^2 - b^3*d^3*cosh(1)^3 - b^3*d^3*sinh(1)^3 + 3*(b^3*c*d^2*f - b^3*d^3
*cosh(1))*sinh(1)^2 - 3*(b^3*c^2*d*f^2 - 2*b^3*c*d^2*f*cosh(1) + b^3*d^3*co
sh(1)^2)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - (b^3*c^3*f^3 - 3*b^3*c^2*d*
f^2*cosh(1) + 3*b^3*c*d^2*f*cosh(1)^2 - b^3*d^3*cosh(1)^3 - b^3*d^3*sinh(1)
^3 + 3*(b^3*c*d^2*f - b^3*d^3*cosh(1))*sinh(1)^2 - 3*(b^3*c^2*d*f^2 - 2*b^3
*c*d^2*f*cosh(1) + b^3*d^3*cosh(1)^2)*sinh(1))*sinh(d*x + c)^2 - 3*(b^3*c^2
*d*f^2 - 2*b^3*c*d^2*f*cosh(1) + b^3*d^3*cosh(1)^2)*sinh(1))*sqrt((a^2 + b^
2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^3}{\sinh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
[Out] int((e + f*x)^3/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.244 $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=535

$$-\frac{(e+fx)^2}{ad}+\frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d}-\frac{(e+fx)^2\coth(c+dx)}{ad}+\frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d}-\frac{b^2}{}$$

```
[Out] -(f*x+e)^2/a/d+2*b*(f*x+e)^2*arctanh(exp(d*x+c))/a^2/d-(f*x+e)^2*coth(d*x+c
)/a/d+2*f*(f*x+e)*ln(1-exp(2*d*x+2*c))/a/d^2+2*b*f*(f*x+e)*polylog(2,-exp(d
*x+c))/a^2/d^2-2*b*f*(f*x+e)*polylog(2,exp(d*x+c))/a^2/d^2+f^2*polylog(2,ex
p(2*d*x+2*c))/a/d^3-2*b*f^2*polylog(3,-exp(d*x+c))/a^2/d^3+2*b*f^2*polylog(
3,exp(d*x+c))/a^2/d^3+b^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/
a^2/d/(a^2+b^2)^(1/2)-b^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/
a^2/d/(a^2+b^2)^(1/2)+2*b^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^
(1/2)))/a^2/d^2/(a^2+b^2)^(1/2)-2*b^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))/a^2/d^2/(a^2+b^2)^(1/2)-2*b^2*f^2*polylog(3,-b*exp(d*x+c)
/(a-(a^2+b^2)^(1/2)))/a^2/d^3/(a^2+b^2)^(1/2)+2*b^2*f^2*polylog(3,-b*exp(d*
x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^3/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 535, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5694, 4269, 3797, 2221, 2317, 2438, 4267, 2611, 2320, 6724, 3403, 2296}

$$-\frac{2b^2f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3\sqrt{a^2+b^2}}+\frac{2b^2f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^3\sqrt{a^2+b^2}}+\frac{2b^2f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2\sqrt{a^2+b^2}}-\frac{2b^2f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^2\sqrt{a^2+b^2}}+\frac{b^2(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^2d\sqrt{a^2+b^2}}-\frac{b^2(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{a^2d\sqrt{a^2+b^2}}-\frac{2bf^2\text{Li}_3\left(-e^{c+dx}\right)}{a^2d^3}+\frac{2bf^2\text{Li}_3\left(e^{c+dx}\right)}{a^2d^3}+\frac{2bf(e+fx)\text{Li}_2\left(-e^{c+dx}\right)}{a^2d^2}-\frac{2bf(e+fx)\text{Li}_2\left(e^{c+dx}\right)}{a^2d^2}+\frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d}+\frac{f^2\text{Li}_2\left(e^{2(c+dx)}\right)}{ad^3}+\frac{2f(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad^2}-\frac{(e+fx)^2\coth(c+dx)}{ad}-\frac{(e+fx)^2}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((e + f*x)^2/(a*d)) + (2*b*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^2*d) - ((e
 + f*x)^2*Coth[c + d*x])/(a*d) + (b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2 + b^2]*d) - (b^2*(e + f*x)^2*Log[1 + (
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2 + b^2]*d) + (2*f*(e +
f*x)*Log[1 - E^(2*(c + d*x))])/(a*d^2) + (2*b*f*(e + f*x)*PolyLog[2, -E^(c
+ d*x)])/(a^2*d^2) - (2*b*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^2*d^2) +
(2*b^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a
^2*Sqrt[a^2 + b^2]*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^2) + (f^2*PolyLog[2, E^(2*(c
 + d*x))])/(a*d^3) - (2*b*f^2*PolyLog[3, -E^(c + d*x)])/(a^2*d^3) + (2*b*f^
2*PolyLog[3, E^(c + d*x)])/(a^2*d^3) - (2*b^2*f^2*PolyLog[3, -((b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^3) + (2*b^2*f^2*PolyLog
[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5694

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a + b
*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{csch}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^2\coth(c+dx)}{ad} - \frac{b\int (e+fx)^2\text{csch}(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}}{a^2}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{(2b^2)}{}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{2f(e}{}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{b^2(e}{}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{b^2(e}{}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{b^2(e}{}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{b^2(e}{}$$

Mathematica [A]
time = 13.14, size = 1011, normalized size = 1.89

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((-4*a*d^2*e*E^(2*c)*f*x)/(-1 + E^(2*c)) - (2*a*d^2*E^(2*c)*f^2*x^2)/(-1 +
E^(2*c)) + 2*b*d^2*e^2*ArcTanh[E^(c + d*x)] - 2*b*d^2*e*f*x*Log[1 - E^(c +
d*x)] - b*d^2*f^2*x^2*Log[1 - E^(c + d*x)] + 2*b*d^2*e*f*x*Log[1 + E^(c + d
*x)] + b*d^2*f^2*x^2*Log[1 + E^(c + d*x)] + 2*a*d*e*f*Log[1 - E^(2*(c + d*x
))] + 2*a*d*f^2*x*Log[1 - E^(2*(c + d*x))] + 2*b*d*f*(e + f*x)*PolyLog[2, -
E^(c + d*x)] - 2*b*d*f*(e + f*x)*PolyLog[2, E^(c + d*x)] + a*f^2*PolyLog[2,
 E^(2*(c + d*x))] - 2*b*f^2*PolyLog[3, -E^(c + d*x)] + 2*b*f^2*PolyLog[3, E
```

```
^(c + d*x)])/(a^2*d^3) + (b^2*(-2*d^2*e^2*Sqrt[(a^2 + b^2)*E^(2*c)]*ArcTanh
[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*Sqrt[a^2 + b^2]*d^2*e*E^c*f*x*Log
[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + Sqrt[a^2 + b^
2]*d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)])] - 2*Sqrt[a^2 + b^2]*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c +
 Sqrt[(a^2 + b^2)*E^(2*c)])] - Sqrt[a^2 + b^2]*d^2*E^c*f^2*x^2*Log[1 + (b*E
^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*Sqrt[a^2 + b^2]*d*E^
c*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)]))] - 2*Sqrt[a^2 + b^2]*d*E^c*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x)
)/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*Sqrt[a^2 + b^2]*E^c*f^2*PolyLog
[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2*Sqrt[a^2
+ b^2]*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(
2*c)]))]))/(a^2*Sqrt[a^2 + b^2]*d^3*Sqrt[(a^2 + b^2)*E^(2*c)]) + (Sech[c/2]
*Sech[c/2 + (d*x)/2]*(-(e^2*Sinh[(d*x)/2]) - 2*e*f*x*Sinh[(d*x)/2] - f^2*x^
2*Sinh[(d*x)/2]))/(2*a*d) + (Csch[c/2]*Csch[c/2 + (d*x)/2]*(e^2*Sinh[(d*x)/
2] + 2*e*f*x*Sinh[(d*x)/2] + f^2*x^2*Sinh[(d*x)/2]))/(2*a*d)
```

Maple [**F**]
time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^2}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*a^2*d) + b*log(e^(-d*x - c) + 1)/(a^2*d) - b*
log(e^(-d*x - c) - 1)/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d))*e^2 - 4*f*x
*e/(a*d) - 2*(f^2*x^2 + 2*f*x*e)/(a*d*e^(2*d*x + 2*c) - a*d) + 2*f*e*log(e^
(d*x + c) + 1)/(a*d^2) + 2*f*e*log(e^(d*x + c) - 1)/(a*d^2) + (d^2*x^2*log(
e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*
b*f^2/(a^2*d^3) - (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c))
 - 2*polylog(3, e^(d*x + c)))*b*f^2/(a^2*d^3) + 2*(b*d*f*e + a*f^2)*(d*x*lo
g(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) - 2*(b*d*f*e - a*f^2)*(
d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^2*d^3) - 1/3*(b*d^3*f^2*
```

```
x^3 + 3*(b*d*f*e + a*f^2)*d^2*x^2)/(a^2*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*d*
f*e - a*f^2)*d^2*x^2)/(a^2*d^3) + integrate(2*(b^2*f^2*x^2*e^c + 2*b^2*f*x*
e^(c + 1))*e^(d*x)/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(d*x + c) - a^2*b), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5257 vs. $2(505) = 1010$.
time = 0.45, size = 5257, normalized size = 9.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*(a^3 + a*b^2)*c^2*f^2 - 4*(a^3 + a*b^2)*c*d*f*cosh(1) + 2*(a^3 + a*b^2)
*d^2*cosh(1)^2 + 2*(a^3 + a*b^2)*d^2*sinh(1)^2 + 2*((a^3 + a*b^2)*d^2*f^2*x
^2 - (a^3 + a*b^2)*c^2*f^2 + 2*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*c*d*f
)*cosh(1) + 2*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*c*d*f)*sinh(1))*cosh(d
*x + c)^2 + 4*((a^3 + a*b^2)*d^2*f^2*x^2 - (a^3 + a*b^2)*c^2*f^2 + 2*((a^3
+ a*b^2)*d^2*f*x + (a^3 + a*b^2)*c*d*f)*cosh(1) + 2*((a^3 + a*b^2)*d^2*f*x
+ (a^3 + a*b^2)*c*d*f)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + 2*((a^3 + a*b
^2)*d^2*f^2*x^2 - (a^3 + a*b^2)*c^2*f^2 + 2*((a^3 + a*b^2)*d^2*f*x + (a^3 +
 a*b^2)*c*d*f)*cosh(1) + 2*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*c*d*f)*si
nh(1))*sinh(d*x + c)^2 + 2*(b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh(1)
 - (b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh(1))*cosh(d*x + c)^2 - 2*(b
^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c)
 - (b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh(1))*sinh(d*x + c)^2)*sqrt(
(a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b^3*d*f^2*x + b
^3*d*f*cosh(1) + b^3*d*f*sinh(1) - (b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f
*sinh(1))*cosh(d*x + c)^2 - 2*(b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh
(1))*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f
*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a
*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b + 1) - (b^3*c^2*f^2 - 2*b^3*c*d*f*cosh(1) + b^3*d^2*cosh(1)^2 + b^3*
d^2*sinh(1)^2 - (b^3*c^2*f^2 - 2*b^3*c*d*f*cosh(1) + b^3*d^2*cosh(1)^2 + b^
3*d^2*sinh(1)^2 - 2*(b^3*c*d*f - b^3*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^2
- 2*(b^3*c^2*f^2 - 2*b^3*c*d*f*cosh(1) + b^3*d^2*cosh(1)^2 + b^3*d^2*sinh(1
)^2 - 2*(b^3*c*d*f - b^3*d^2*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c)
- (b^3*c^2*f^2 - 2*b^3*c*d*f*cosh(1) + b^3*d^2*cosh(1)^2 + b^3*d^2*sinh(1)^
2 - 2*(b^3*c*d*f - b^3*d^2*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 2*(b^3*c*d*f
 - b^3*d^2*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) +
2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^3*c^2*f^2 - 2*b^3
*c*d*f*cosh(1) + b^3*d^2*cosh(1)^2 + b^3*d^2*sinh(1)^2 - (b^3*c^2*f^2 - 2*b
^3*c*d*f*cosh(1) + b^3*d^2*cosh(1)^2 + b^3*d^2*sinh(1)^2 - 2*(b^3*c*d*f - b
^3*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^2 - 2*(b^3*c^2*f^2 - 2*b^3*c*d*f*cos
```

```
h(1) + b^3*d^2*cosh(1)^2 + b^3*d^2*sinh(1)^2 - 2*(b^3*c*d*f - b^3*d^2*cosh(
1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - (b^3*c^2*f^2 - 2*b^3*c*d*f*cosh(
1) + b^3*d^2*cosh(1)^2 + b^3*d^2*sinh(1)^2 - 2*(b^3*c*d*f - b^3*d^2*cosh(1)
)*sinh(1))*sinh(d*x + c)^2 - 2*(b^3*c*d*f - b^3*d^2*cosh(1))*sinh(1))*sqrt(
(a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2
+ b^2)/b^2) + 2*a) + (b^3*d^2*f^2*x^2 - b^3*c^2*f^2 - (b^3*d^2*f^2*x^2 - b^
3*c^2*f^2 + 2*(b^3*d^2*f*x + b^3*c*d*f)*cosh(1) + 2*(b^3*d^2*f*x + b^3*c*d*
f)*sinh(1))*cosh(d*x + c)^2 - 2*(b^3*d^2*f^2*x^2 - b^3*c^2*f^2 + 2*(b^3*d^2
*f*x + b^3*c*d*f)*cosh(1) + 2*(b^3*d^2*f*x + b^3*c*d*f)*sinh(1))*cosh(d*x +
 c)*sinh(d*x + c) - (b^3*d^2*f^2*x^2 - b^3*c^2*f^2 + 2*(b^3*d^2*f*x + b^3*c
*d*f)*cosh(1) + 2*(b^3*d^2*f*x + b^3*c*d*f)*sinh(1))*sinh(d*x + c)^2 + 2*(b
^3*d^2*f*x + b^3*c*d*f)*cosh(1) + 2*(b^3*d^2*f*x + b^3*c*d*f)*sinh(1))*sqrt
((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^3*d^2*f^2*x^2 - b^3
*c^2*f^2 - (b^3*d^2*f^2*x^2 - b^3*c^2*f^2 + 2*(b^3*d^2*f*x + b^3*c*d*f)*cos
h(1) + 2*(b^3*d^2*f*x + b^3*c*d*f)*sinh(1))*cosh(d*x + c)^2 - 2*(b^3*d^2*f^
2*x^2 - b^3*c^2*f^2 + 2*(b^3*d^2*f*x + b^3*c*d*f)*cosh(1) + 2*(b^3*d^2*f*x
+ b^3*c*d*f)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - (b^3*d^2*f^2*x^2 - b^3*
c^2*f^2 + 2*(b^3*d^2*f*x + b^3*c*d*f)*cosh(1) + 2*(b^3*d^2*f*x + b^3*c*d*f)
*sinh(1))*sinh(d*x + c)^2 + 2*(b^3*d^2*f*x + b^3*c*d*f)*cosh(1) + 2*(b^3*d^
2*f*x + b^3*c*d*f)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a
*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b) + 2*(b^3*f^2*cosh(d*x + c)^2 + 2*b^3*f^2*cosh(d*x + c)*sinh(d*x + c
) + b^3*f^2*sinh(d*x + c)^2 - b^3*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*
cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2))/b) - 2*(b^3*f^2*cosh(d*x + c)^2 + 2*b^3*f^2*cosh(d*x + c)
*sinh(d*x + c) + b^3*f^2*sinh(d*x + c)^2 - b^3*f^2)*sqrt((a^2 + b^2)/b^2)*p
olylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3
)*d*f*cosh(1) + (a^2*b + b^3)*d*f*sinh(1) - (a^3 + a*b^2)*f^2 - ((a^2*b + b
^3)*d*f^2*x + (a^2*b + b^3)*d*f*cosh(1) + (a^2*b + b^3)*d*f*sinh(1) - (a^3
+ a*b^2)*f^2)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*
f*cosh(1) + (a^2*b + b^3)*d*f*sinh(1) - (a^3 + a*b^2)*f^2)*cosh(d*x + c)*si
nh(d*x + c) - ((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*f*cosh(1) + (a^2*b +
 b^3)*d*f*sinh(1) - (a^3 + a*b^2)*f^2)*sinh(d*x...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)**2*csch(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^2}{\sinh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((e + f*x)^2/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.245 $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=306

$$\frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d}-\frac{(e+fx)\coth(c+dx)}{ad}+\frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d}-\frac{b^2(e+fx)\log\left(1\right.}{a^2\sqrt{a^2}}$$

```
[Out] 2*b*(f*x+e)*arctanh(exp(d*x+c))/a^2/d-(f*x+e)*coth(d*x+c)/a/d+f*ln(sinh(d*x
+c))/a/d^2+b*f*polylog(2,-exp(d*x+c))/a^2/d^2-b*f*polylog(2,exp(d*x+c))/a^2
/d^2+b^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d/(a^2+b^2)^(1/
2)-b^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d/(a^2+b^2)^(1/2)
+b^2*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^2/(a^2+b^2)^(1/2)
-b^2*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^2/(a^2+b^2)^(1/2)
```

Rubi [A]

time = 0.39, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5694, 4269, 3556, 4267, 2317, 2438, 3403, 2296, 2221}

$$\frac{b^2f\operatorname{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2\sqrt{a^2+b^2}}-\frac{b^2f\operatorname{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^2\sqrt{a^2+b^2}}+\frac{b^2(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^2d\sqrt{a^2+b^2}}-\frac{b^2(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{a^2d\sqrt{a^2+b^2}}+\frac{bf\operatorname{Li}_2\left(-e^{c+dx}\right)}{a^2d^2}-\frac{bf\operatorname{Li}_2\left(e^{c+dx}\right)}{a^2d^2}+\frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d}+\frac{f\log(\sinh(c+dx))}{ad^2}-\frac{(e+fx)\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*b*(e + f*x)*ArcTanh[E^(c + d*x)])/(a^2*d) - ((e + f*x)*Coth[c + d*x])/(a
*d) + (b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*S
qrt[a^2 + b^2]*d) - (b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2])])/(a^2*Sqrt[a^2 + b^2]*d) + (f*Log[Sinh[c + d*x]])/(a*d^2) + (b*f*Pol
yLog[2, -E^(c + d*x)])/(a^2*d^2) - (b*f*PolyLog[2, E^(c + d*x)])/(a^2*d^2)
+ (b^2*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^
2 + b^2]*d^2) - (b^2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/(a^2*Sqrt[a^2 + b^2]*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5694

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a + b
```

```
*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rubi steps

$$\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{csch}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)\coth(c+dx)}{ad} - \frac{b\int (e+fx)\text{csch}(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{e+fx}{a+b\sinh(c+dx)}\,d}{a^2}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{f\log(\sinh(c+dx))}{ad^2} +$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{f\log(\sinh(c+dx))}{ad^2} +$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{b^2(e+fx)\log\left(1+\right.}{a^2\sqrt{a^2+b}}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{b^2(e+fx)\log\left(1+\right.}{a^2\sqrt{a^2+b}}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{b^2(e+fx)\log\left(1+\right.}{a^2\sqrt{a^2+b}}$$

Mathematica [A]
time = 3.48, size = 405, normalized size = 1.32

$$\frac{-ad(e+fx)\coth\left(\frac{1}{2}(c+dx)\right)+2af\log(\sinh(c+dx))-2bde\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+2bcf\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+2bf\left(-\left((c+dx)\left(\log\left(1-e^{-c-dx}\right)-\log\left(1+e^{-c-dx}\right)\right)\right)-\text{PolyLog}\left(2,-e^{-c-dx}\right)+\text{PolyLog}\left(2,e^{-c-dx}\right)\right)+\frac{2b^2\left(-2de\tanh^{-1}\left(\frac{a+b\cosh(c+dx)+b\sinh(c+dx)}{\sqrt{a^2+b^2}}\right)+2cf\tanh^{-1}\left(\frac{a+b\cosh(c+dx)+b\sinh(c+dx)}{\sqrt{a^2+b^2}}\right)+f(c+dx)\log\left(1+\frac{b(\cosh(c+dx)+\sinh(c+dx))}{a-\sqrt{a^2+b^2}}\right)-f(c+dx)\log\left(1+\frac{b(\cosh(c+dx)+\sinh(c+dx))}{a+\sqrt{a^2+b^2}}\right)+f\text{PolyLog}\left(2,\frac{b(\cosh(c+dx)+\sinh(c+dx))}{-a+\sqrt{a^2+b^2}}\right)-f\text{PolyLog}\left(2,-\frac{b(\cosh(c+dx)+\sinh(c+dx))}{a+\sqrt{a^2+b^2}}\right)\right)}{\sqrt{a^2+b^2}}-ad(e+fx)\tanh\left(\frac{1}{2}(c+dx)\right)}{2a^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-(a*d*(e + f*x)*Coth[(c + d*x)/2]) + 2*a*f*Log[Sinh[c + d*x]] - 2*b*d*e*Lo
g[Tanh[(c + d*x)/2]] + 2*b*c*f*Log[Tanh[(c + d*x)/2]] + 2*b*f*(-((c + d*x)*
(Log[1 - E^(-c - d*x)] - Log[1 + E^(-c - d*x)])) - PolyLog[2, -E^(-c - d*x)
] + PolyLog[2, E^(-c - d*x)]) + (2*b^2*(-2*d*e*ArcTanh[(a + b*Cosh[c + d*x]
 + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a + b*Cosh[c + d*x] +
 b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*(Cosh[c + d*x]
```

```
+ Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*(Cosh[c +
 d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*(Cosh[c +
d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*(Cosh[c
+ d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2] - a*d*(e
 + f*x)*Tanh[(c + d*x)/2])/(2*a^2*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(283) = 566$.
time = 1.57, size = 626, normalized size = 2.05

method	result
risch	$-\frac{2(fx+e)}{da(\mathrm{e}^{2dx+2c}-1)}-\frac{2f\ln(\mathrm{e}^{dx+c})}{a\,d^2}+\frac{f\ln(\mathrm{e}^{dx+c}+1)}{a\,d^2}+\frac{f\ln(\mathrm{e}^{dx+c}-1)}{a\,d^2}+\frac{be\ln(\mathrm{e}^{dx+c}+1)}{a^2d}-\frac{be\ln(\mathrm{e}^{dx+c}-1)}{a^2d}-\frac{2b^2e\,\mathrm{arctanh}\Big(}{a^2d\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(f*x+e)/a/(exp(2*d*x+2*c)-1)-2/a/d^2*f*ln(exp(d*x+c))+1/a/d^2*f*ln(exp
(d*x+c)+1)+1/a/d^2*f*ln(exp(d*x+c)-1)+1/a^2/d*b*e*ln(exp(d*x+c)+1)-1/a^2/d*
b*e*ln(exp(d*x+c)-1)-2/a^2/d*b^2*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x
+c)+2*a)/(a^2+b^2)^(1/2))+1/a^2/d^2*b*f*c*ln(exp(d*x+c)-1)+2/a^2/d^2*b^2*f*
c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/a^2/d
*b*f*ln(exp(d*x+c)+1)*x+1/a^2/d^2*b*f*dilog(exp(d*x+c)+1)+1/a^2/d^2*b*f*dil
og(exp(d*x+c))+1/a^2/d*b^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1
/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/a^2/d^2*b^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d
*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/a^2/d*b^2*f/(a^2+b^2)^(1
/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/a^2/d^2*b^
2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))
)*c+1/a^2/d^2*b^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)
/(-a+(a^2+b^2)^(1/2)))-1/a^2/d^2*b^2*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+
(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] (4*b^2*integrate(1/2*x*e^(d*x + c)/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(d*x +
c) - a^2*b), x) - 4*b*d*integrate(1/4*x/(a^2*d*e^(d*x + c) + a^2*d), x) - 4
*b*d*integrate(1/4*x/(a^2*d*e^(d*x + c) - a^2*d), x) - a*((d*x + c)/(a^2*d^
2) - log(e^(d*x + c) + 1)/(a^2*d^2)) - a*((d*x + c)/(a^2*d^2) - log(e^(d*x
```

```
+ c) - 1)/(a^2*d^2)) - 2*x/(a*d*e^(2*d*x + 2*c) - a*d))*f + (b^2*log((b*e^(
-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(s
qrt(a^2 + b^2)*a^2*d) + b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c
) - 1)/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d))*e
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2027 vs. $2(283) = 566$.
time = 0.37, size = 2027, normalized size = 6.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*(a^3 + a*b^2)*c*f - 2*(a^3 + a*b^2)*d*cosh(1) - 2*((a^3 + a*b^2)*d*f*x +
 (a^3 + a*b^2)*c*f)*cosh(d*x + c)^2 - 2*(a^3 + a*b^2)*d*sinh(1) - 4*((a^3 +
 a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c) - 2*((a^3 +
a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*sinh(d*x + c)^2 + (b^3*f*cosh(d*x + c)^2
+ 2*b^3*f*cosh(d*x + c)*sinh(d*x + c) + b^3*f*sinh(d*x + c)^2 - b^3*f)*sqrt
((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^3*f*cosh(d*x +
 c)^2 + 2*b^3*f*cosh(d*x + c)*sinh(d*x + c) + b^3*f*sinh(d*x + c)^2 - b^3*f
)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^3*c*f -
b^3*d*cosh(1) - b^3*d*sinh(1) - (b^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1))*c
osh(d*x + c)^2 - 2*(b^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1))*cosh(d*x + c)*
sinh(d*x + c) - (b^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1))*sinh(d*x + c)^2)*
sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt(
(a^2 + b^2)/b^2) + 2*a) + (b^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1) - (b^3*c
*f - b^3*d*cosh(1) - b^3*d*sinh(1))*cosh(d*x + c)^2 - 2*(b^3*c*f - b^3*d*co
sh(1) - b^3*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - (b^3*c*f - b^3*d*cosh(
1) - b^3*d*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x
 + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^3*d*f*x +
 b^3*c*f - (b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^2 - 2*(b^3*d*f*x + b^3*c*f)*
cosh(d*x + c)*sinh(d*x + c) - (b^3*d*f*x + b^3*c*f)*sinh(d*x + c)^2)*sqrt((
a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b^3*d*f*x + b^3*c*f - (
b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^2 - 2*(b^3*d*f*x + b^3*c*f)*cosh(d*x + c
)*sinh(d*x + c) - (b^3*d*f*x + b^3*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b
^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - ((a^2*b + b^3)*f*cosh(d*x + c)^2 + 2
*(a^2*b + b^3)*f*cosh(d*x + c)*sinh(d*x + c) + (a^2*b + b^3)*f*sinh(d*x + c
)^2 - (a^2*b + b^3)*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) + ((a^2*b + b^3
)*f*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*f*cosh(d*x + c)*sinh(d*x + c) + (a^2*
b + b^3)*f*sinh(d*x + c)^2 - (a^2*b + b^3)*f)*dilog(-cosh(d*x + c) - sinh(d
```

```
*x + c)) - ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*cosh(1) - ((a^2*b + b^3)*
d*f*x + (a^2*b + b^3)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1) + (a^3 + a*b^2)*f
)*cosh(d*x + c)^2 + (a^2*b + b^3)*d*sinh(1) - 2*((a^2*b + b^3)*d*f*x + (a^2
*b + b^3)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1) + (a^3 + a*b^2)*f)*cosh(d*x +
 c)*sinh(d*x + c) - ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*cosh(1) + (a^2*b
 + b^3)*d*sinh(1) + (a^3 + a*b^2)*f)*sinh(d*x + c)^2 + (a^3 + a*b^2)*f)*log
(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a^2*b + b^3)*d*cosh(1) - ((a^2*b +
b^3)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1) - (a^3 + a*b^2 + (a^2*b + b^3)*c)*
f)*cosh(d*x + c)^2 + (a^2*b + b^3)*d*sinh(1) - 2*((a^2*b + b^3)*d*cosh(1) +
 (a^2*b + b^3)*d*sinh(1) - (a^3 + a*b^2 + (a^2*b + b^3)*c)*f)*cosh(d*x + c)
*sinh(d*x + c) - ((a^2*b + b^3)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1) - (a^3
+ a*b^2 + (a^2*b + b^3)*c)*f)*sinh(d*x + c)^2 - (a^3 + a*b^2 + (a^2*b + b^3
)*c)*f)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + ((a^2*b + b^3)*d*f*x + (a^
2*b + b^3)*c*f - ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*c*f)*cosh(d*x + c)^2
- 2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*c*f)*cosh(d*x + c)*sinh(d*x + c) -
 ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*c*f)*sinh(d*x + c)^2)*log(-cosh(d*x +
 c) - sinh(d*x + c) + 1))/((a^4 + a^2*b^2)*d^2*cosh(d*x + c)^2 + 2*(a^4 + a
^2*b^2)*d^2*cosh(d*x + c)*sinh(d*x + c) + (a^4 + a^2*b^2)*d^2*sinh(d*x + c)
^2 - (a^4 + a^2*b^2)*d^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*csch(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{{\sinh\left(c+d\,x\right)}^2\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
[Out] int((e + f*x)/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.246 $\int \frac{\mathrm{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d}-\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d}-\frac{\coth(c+dx)}{ad}$$

```
[Out] b*arctanh(cosh(d*x+c))/a^2/d-coth(d*x+c)/a/d-2*b^2*arctanh((b-a*tanh(1/2*d*
x+1/2*c))/(a^2+b^2)^(1/2))/a^2/d/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2881, 12, 2826, 3855, 2739, 632, 210}

$$-\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d\sqrt{a^2+b^2}}+\frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d}-\frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Cosh[c + d*x]])/(a^2*d) - (2*b^2*ArcTanh[(b - a*Tanh[(c + d*x)/2
])/Sqrt[a^2 + b^2]])/(a^2*Sqrt[a^2 + b^2]*d) - Coth[c + d*x]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\coth(c+dx)}{ad} - \frac{\int \frac{b\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{\coth(c+dx)}{ad} - \frac{b\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{\coth(c+dx)}{ad} - \frac{b\int \operatorname{csch}(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{1}{a+b\sinh(c+dx)}\,dx}{a^2}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{\coth(c+dx)}{ad} - \frac{(2ib^2)\,\text{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(\right.\right.}{a^2d}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{\coth(c+dx)}{ad} + \frac{(4ib^2)\,\text{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx, x, -2ib\right.}{a^2d}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d} - \frac{\coth(c+dx)}{ad}$$

Mathematica [A]
time = 0.54, size = 100, normalized size = 1.25

$$-\frac{a\coth\left(\frac{1}{2}(c+dx)\right) + 2b\left(-\frac{2b\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right) + a\tanh\left(\frac{1}{2}(c+dx)\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/2*(a*Coth[(c + d*x)/2] + 2*b*((-2*b*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqr
t[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + Log[Tanh[(c + d*x)/2]]) + a*Tanh[(c + d*
x)/2])/(a^2*d)
```

Maple [A]
time = 0.98, size = 97, normalized size = 1.21

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a} - \frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)} - \frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2} + \frac{2b^2\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}}{d}$

default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{2b^2\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}}{d}$
risch	$-\frac{2}{da\left(e^{2dx+2c}-1\right)}+\frac{b^2\ln\left(e^{dx+c}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d\,a^2}-\frac{b^2\ln\left(e^{dx+c}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d\,a^2}-\frac{b\ln(e^{dx+\cdots}}{a^2 d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a*tanh(1/2*d*x+1/2*c)-1/2/a/tanh(1/2*d*x+1/2*c)-1/a^2*b*ln(tanh(1
/2*d*x+1/2*c))+2/a^2*b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*
c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [A]
time = 0.48, size = 137, normalized size = 1.71

$$\frac{b^2\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a^2d}+\frac{b\log\left(e^{(-dx-c)}+1\right)}{a^2d}-\frac{b\log\left(e^{(-dx-c)}-1\right)}{a^2d}+\frac{2}{\left(ae^{(-2\,dx-2\,c)}-a\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] b^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a^2*d) + b*log(e^(-d*x - c) + 1)/(a^2*d) - b*l
og(e^(-d*x - c) - 1)/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 479 vs. 2(77) = 154.
time = 0.37, size = 479, normalized size = 5.99

$$-\frac{2\,a^3+2\,ab^2-\left(b^2\cosh\left(dx+c\right)^2+2\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b^2\sinh\left(dx+c\right)^2-b^2\right)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2+b^2+2\,(b^2\cosh(dx+c)+ab)\sinh(dx+c)-2\sqrt{a^2+b^2}\,(b\cosh(dx+c)+b\sinh(dx+c)+a)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)-b}\right)+\left(a^2b+b^3-(a^2b+b^3)\cosh\left(dx+c\right)^2-2\,(a^2b+b^3)\cosh\left(dx+c\right)\sinh\left(dx+c\right)-(a^2b+b^3)\sinh\left(dx+c\right)^2\right)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+1\right)-\left(a^2b+b^3-(a^2b+b^3)\cosh\left(dx+c\right)^2-2\,(a^2b+b^3)\cosh\left(dx+c\right)\sinh\left(dx+c\right)-(a^2b+b^3)\sinh\left(dx+c\right)^2\right)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1\right)}{(a^4+a^2b^2)d\cosh\left(dx+c\right)^2+2\,(a^4+a^2b^2)d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(a^4+a^2b^2)d\sinh\left(dx+c\right)^2-(a^4+a^2b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*a^3 + 2*a*b^2 - (b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c
) + b^2*sinh(d*x + c)^2 - b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b
^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x +
c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x +
 c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b
*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + (a^2*b + b^3 - (a^2*b + b^3)*cosh
(d*x + c)^2 - 2*(a^2*b + b^3)*cosh(d*x + c)*sinh(d*x + c) - (a^2*b + b^3)*s
```

```
inh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a^2*b + b^3 - (a^
2*b + b^3)*cosh(d*x + c)^2 - 2*(a^2*b + b^3)*cosh(d*x + c)*sinh(d*x + c) -
(a^2*b + b^3)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) - 1))/((a^
4 + a^2*b^2)*d*cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x
 + c) + (a^4 + a^2*b^2)*d*sinh(d*x + c)^2 - (a^4 + a^2*b^2)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(csch(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [A]
time = 0.47, size = 123, normalized size = 1.54

$$\frac{\frac{b^2\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a^2}+\frac{b\log\left(e^{(dx+c)}+1\right)}{a^2}-\frac{b\log\left(\left|e^{(dx+c)}-1\right|\right)}{a^2}-\frac{2}{a\left(e^{(2\,dx+2\,c)}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (b^2*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c)
 + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^2) + b*log(e^(d*x + c) + 1)
/a^2 - b*log(abs(e^(d*x + c) - 1))/a^2 - 2/(a*(e^(2*d*x + 2*c) - 1)))/d
```

Mupad [B]
time = 0.39, size = 360, normalized size = 4.50

$$\frac{2}{a\,d-a\,d\,\mathrm{e}^{2\,c+2\,d\,x}}+\frac{b^2\,\ln\left(128\,a^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-64\,a\,b^3-64\,a^3\,b-32\,b^3\,\sqrt{a^2+b^2}+32\,b^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-64\,a^2\,b\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+128\,a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{a^2+b^2}+96\,a\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{d\,a^4+d\,a^2\,b^2}-\frac{b^2\,\ln\left(32\,b^3\,\sqrt{a^2+b^2}-64\,a\,b^3-64\,a^3\,b+128\,a^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+32\,b^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+64\,a^2\,b\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-128\,a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{a^2+b^2}-96\,a\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{d\,a^4+d\,a^2\,b^2}-\frac{b\,\ln\left(32\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-32\right)}{a^2\,d}+\frac{b\,\ln\left(32\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+32\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

```
[Out] 2/(a*d - a*d*exp(2*c + 2*d*x)) + (b^2*log(128*a^4*exp(d*x)*exp(c) - 64*a*b^
3 - 64*a^3*b - 32*b^3*(a^2 + b^2)^(1/2) + 32*b^4*exp(d*x)*exp(c) - 64*a^2*b
*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(d*x)*exp(c) + 128*a^3*exp(d*x)*exp(c)*
(a^2 + b^2)^(1/2) + 96*a*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 + b^2)
^(1/2))/(a^4*d + a^2*b^2*d) - (b^2*log(32*b^3*(a^2 + b^2)^(1/2) - 64*a*b^3
```

```
- 64*a^3*b + 128*a^4*exp(d*x)*exp(c) + 32*b^4*exp(d*x)*exp(c) + 64*a^2*b*(a
^2 + b^2)^(1/2) + 160*a^2*b^2*exp(d*x)*exp(c) - 128*a^3*exp(d*x)*exp(c)*(a^
2 + b^2)^(1/2) - 96*a*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1
/2))/(a^4*d + a^2*b^2*d) - (b*log(32*exp(d*x)*exp(c) - 32))/(a^2*d) + (b*lo
g(32*exp(d*x)*exp(c) + 32))/(a^2*d)
```

3.247 $\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csch[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][Csch[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 77.44, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csch[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Integrate[Csch[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(dx+c)^2}{(fx+e)\,(a+b\sinh\,(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 4*b^2*integrate(-1/2*e^(d*x + c)/(a^2*b*f*x + a^2*b*e - (a^2*b*f*x*e^(2*c)
+ a^2*b*e^(2*c + 1))*e^(2*d*x) - 2*(a^3*f*x*e^c + a^3*e^(c + 1))*e^(d*x)),
x) + 2/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e^(2*c + 1))*e^(2*d*x)) -
4*integrate(-1/4*(b*d*f*x + b*d*e + a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*f*x*e + a
^2*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*f*x*e^(c + 1) + a^2*d*e^(c + 2))*e^
(d*x)), x) - 4*integrate(1/4*(b*d*f*x + b*d*e - a*f)/(a^2*d*f^2*x^2 + 2*a^2
*d*f*x*e + a^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*d*f*x*e^(c + 1) + a^2*d*e
^(c + 2))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(csch(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\sinh(c+dx))\,(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(csch(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sinh\left(c+d\,x\right)^2\,\left(e+f\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(1/(sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.248 $\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1053

$$\frac{b(e+fx)^3}{a^2d}-\frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{2b^2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^3d}+\frac{b(e+f$$

```
[Out] -3*f^3*polylog(2,-exp(d*x+c))/a/d^4+3*f^3*polylog(2,exp(d*x+c))/a/d^4+3/2*f
*(f*x+e)^2*polylog(2,-exp(d*x+c))/a/d^2-3/2*f*(f*x+e)^2*polylog(2,exp(d*x+c
))/a/d^2-3*f^2*(f*x+e)*polylog(3,-exp(d*x+c))/a/d^3-2*b^2*(f*x+e)^3*arctanh
(exp(d*x+c))/a^3/d+3/2*b*f^3*polylog(3,exp(2*d*x+2*c))/a^2/d^4-6*b^2*f^3*po
lylog(4,-exp(d*x+c))/a^3/d^4+6*b^2*f^3*polylog(4,exp(d*x+c))/a^3/d^4-6*f^2*
(f*x+e)*arctanh(exp(d*x+c))/a/d^3-3/2*f*(f*x+e)^2*csch(d*x+c)/a/d^2-1/2*(f*
x+e)^3*coth(d*x+c)*csch(d*x+c)/a/d+3*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a/d^
3+b*(f*x+e)^3/a^2/d+b*(f*x+e)^3*coth(d*x+c)/a^2/d-3*b^2*f*(f*x+e)^2*polylog
(2,-exp(d*x+c))/a^3/d^2+3*b^2*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a^3/d^2-3*b
*f^2*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a^2/d^3+6*b^2*f^2*(f*x+e)*polylog(3,
-exp(d*x+c))/a^3/d^3-6*b^2*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a^3/d^3-3*b^3*
f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^2/(a^2+b^2)^
(1/2)+3*b^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^
2/(a^2+b^2)^(1/2)+6*b^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1
/2)))/a^3/d^3/(a^2+b^2)^(1/2)-6*b^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))/a^3/d^3/(a^2+b^2)^(1/2)+(f*x+e)^3*arctanh(exp(d*x+c))/a/d
+3*f^3*polylog(4,-exp(d*x+c))/a/d^4-3*f^3*polylog(4,exp(d*x+c))/a/d^4-3*b*f
*(f*x+e)^2*ln(1-exp(2*d*x+2*c))/a^2/d^2-b^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/a^3/d/(a^2+b^2)^(1/2)+b^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))/a^3/d/(a^2+b^2)^(1/2)-6*b^3*f^3*polylog(4,-b*exp(d*x+c)/(
a-(a^2+b^2)^(1/2)))/a^3/d^4/(a^2+b^2)^(1/2)+6*b^3*f^3*polylog(4,-b*exp(d*x+
c)/(a+(a^2+b^2)^(1/2)))/a^3/d^4/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 1.28, antiderivative size = 1053, normalized size of antiderivative = 1.00, number of steps used = 45, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5694, 4271, 4267, 2317, 2438, 2611, 6744, 2320, 6724, 4269, 3797, 2221, 3403, 2296}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*(e + f*x)^3)/(a^2*d) - (6*f^2*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d^3) +
((e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) - (2*b^2*(e + f*x)^3*ArcTanh[E^(c
```

```
+ d*x)])/(a^3*d) + (b*(e + f*x)^3*Coth[c + d*x])/(a^2*d) - (3*f*(e + f*x)^2
*Csch[c + d*x])/(2*a*d^2) - ((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x])/(2*a*
d) - (b^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*
Sqrt[a^2 + b^2]*d) + (b^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2])])/(a^3*Sqrt[a^2 + b^2]*d) - (3*b*f*(e + f*x)^2*Log[1 - E^(2*(c + d
*x))])/(a^2*d^2) - (3*f^3*PolyLog[2, -E^(c + d*x)])/(a*d^4) + (3*f*(e + f*x
)^2*PolyLog[2, -E^(c + d*x)])/(2*a*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -
E^(c + d*x)])/(a^3*d^2) + (3*f^3*PolyLog[2, E^(c + d*x)])/(a*d^4) - (3*f*(e
 + f*x)^2*PolyLog[2, E^(c + d*x)])/(2*a*d^2) + (3*b^2*f*(e + f*x)^2*PolyLog
[2, E^(c + d*x)])/(a^3*d^2) - (3*b^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b^2]*d^2) + (3*b^3*f*(e + f*x
)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b
^2]*d^2) - (3*b*f^2*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a^2*d^3) - (3*f
^2*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) + (6*b^2*f^2*(e + f*x)*PolyL
og[3, -E^(c + d*x)])/(a^3*d^3) + (3*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/
(a*d^3) - (6*b^2*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a^3*d^3) + (6*b^3*
f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*Sq
rt[a^2 + b^2]*d^3) - (6*b^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b^2]*d^3) + (3*b*f^3*PolyLog[3, E^(2*(
c + d*x))])/(2*a^2*d^4) + (3*f^3*PolyLog[4, -E^(c + d*x)])/(a*d^4) - (6*b^2
*f^3*PolyLog[4, -E^(c + d*x)])/(a^3*d^4) - (3*f^3*PolyLog[4, E^(c + d*x)])/
(a*d^4) + (6*b^2*f^3*PolyLog[4, E^(c + d*x)])/(a^3*d^4) - (6*b^3*f^3*PolyLo
g[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b^2]*d^4) +
 (6*b^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*Sqrt
[a^2 + b^2]*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5694

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a + b
*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\text{csch}^3(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{3f(e+fx)^2\text{csch}(c+dx)}{2ad^2} - \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{2ad} - \frac{\int (e+$$

$$= -\frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{b(e+fx)^3\,\text{cot}}{a^2d}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b}{}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b}{}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b}{}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b}{}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b}{}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b}{}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3202 vs. $2(1053) = 2106$.
time = 29.93, size = 3202, normalized size = 3.04

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

[Out] (12*a*b*d^3*e^2*E^(2*c)*f*x + 12*a*b*d^3*e*E^(2*c)*f^2*x^2 + 4*a*b*d^3*E^(2*c)*f^3*x^3 - 2*a^2*d^3*e^3*ArcTanh[E^(c + d*x)] + 4*b^2*d^3*e^3*ArcTanh[E^(c + d*x)] + 2*a^2*d^3*e^3*E^(2*c)*ArcTanh[E^(c + d*x)] - 4*b^2*d^3*e^3*E^(2*c)*ArcTanh[E^(c + d*x)] + 12*a^2*d*e*f^2*ArcTanh[E^(c + d*x)] - 12*a^2*d*e*E^(2*c)*f^2*ArcTanh[E^(c + d*x)] + 3*a^2*d^3*e^2*f*x*Log[1 - E^(c + d*x)] - 6*b^2*d^3*e^2*f*x*Log[1 - E^(c + d*x)] - 3*a^2*d^3*e^2*E^(2*c)*f*x*Log[1 - E^(c + d*x)] + 6*b^2*d^3*e^2*E^(2*c)*f*x*Log[1 - E^(c + d*x)] - 6*a^2*d*f^3*x*Log[1 - E^(c + d*x)] + 6*a^2*d*E^(2*c)*f^3*x*Log[1 - E^(c + d*x)] + 3*a^2*d^3*e*f^2*x^2*Log[1 - E^(c + d*x)] - 6*b^2*d^3*e*f^2*x^2*Log[1 - E^(c + d*x)] - 3*a^2*d^3*e*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + 6*b^2*d^3*e*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + a^2*d^3*f^3*x^3*Log[1 - E^(c + d*x)] - 2*b^2*d^3*f^3*x^3*Log[1 - E^(c + d*x)] - a^2*d^3*E^(2*c)*f^3*x^3*Log[1 - E^(c + d*x)] + 2*b^2*d^3*E^(2*c)*f^3*x^3*Log[1 - E^(c + d*x)] - 3*a^2*d^3*e^2*f*x*Log[1 + E^(c + d*x)] + 6*b^2*d^3*e^2*f*x*Log[1 + E^(c + d*x)] + 3*a^2*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(c + d*x)] - 6*b^2*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(c + d*x)] + 6*a^2*d*f^3*x*Log[1 + E^(c + d*x)] - 6*a^2*d*E^(2*c)*f^3*x*Log[1 + E^(c + d*x)] - 3*a^2*d^3*e*f^2*x^2*Log[1 + E^(c + d*x)] + 6*b^2*d^3*e*f^2*x^2*Log[1 + E^(c + d*x)] + 3*a^2*d^3*e*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] - 6*b^2*d^3*e*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] - a^2*d^3*f^3*x^3*Log[1 + E^(c + d*x)] + 2*b^2*d^3*f^3*x^3*Log[1 + E^(c + d*x)] + a^2*d^3*E^(2*c)*f^3*x^3*Log[1 + E^(c + d*x)] - 2*b^2*d^3*E^(2*c)*f^3*x^3*Log[1 + E^(c + d*x)] + 6*a*b*d^2*e^2*f*Log[1 - E^(2*(c + d*x))] - 6*a*b*d^2*e^2*E^(2*c)*f*Log[1 - E^(2*(c + d*x))] + 12*a*b*d^2*e*f^2*x*Log[1 - E^(2*(c + d*x))] - 12*a*b*d^2*e*E^(2*c)*f^2*x*Log[1 - E^(2*(c + d*x))] + 6*a*b*d^2*f^3*x^2*Log[1 - E^(2*(c + d*x))] - 6*a*b*d^2*E^(2*c)*f^3*x^2*Log[1 - E^(2*(c + d*x))] + 3*(-1 + E^(2*c))*f*(-2*b^2*d^2*(e + f*x)^2 + a^2*(-2*f^2 + d^2*(e + f*x)^2))*PolyLog[2, -E^(c + d*x)] - 3*(-1 + E^(2*c))*f*(-2*b^2*d^2*(e + f*x)^2 + a^2*(-2*f^2 + d^2*(e + f*x)^2))*PolyLog[2, E^(c + d*x)] + 6*a*b*d*e*f^2*PolyLog[2, E^(2*(c + d*x))] - 6*a*b*d*e*E^(2*c)*f^2*PolyLog[2, E^(2*(c + d*x))] + 6*a*b*d*f^3*x*PolyLog[2, E^(2*(c + d*x))] - 6*a*b*d*E^(2*c)*f^3*x*PolyLog[2, E^(2*(c + d*x))] + 6*a^2*d*e*f^2*PolyLog[3, -E^(c + d*x)] - 12*b^2*d*e*f^2*PolyLog[3, -E^(c + d*x)] - 6*a^2*d*e*E^(2*c)*f^2*PolyLog[3, -E^(c + d*x)] + 12*b^2*d*e*E^(2*c)*f^2*PolyLog[3, -E^(c + d*x)] + 6*a^2*d*f^3*x*PolyLog[3, -E^(c + d*x)] - 12*b^2*d*f^3*x*PolyLog[3, -E^(c + d*x)] - 6*a^2*d*E^(2*c)*f^3*x*PolyLog[3, -E^(c + d*x)] + 12*b^2*d*E^(2*c)*f^3*x*PolyLog[3, -E^(c + d*x)] - 6*a^2*d*e*f^2*PolyLog[3, E^(c + d*x)] + 12*b^2*d*e*f^2*PolyLog[3, E^(c + d*x)] + 6*a^2*d*e*E^(2*c)*f^2*PolyLog[3, E^(c + d*x)] - 12*b^2*d*e*E^(2*c)*f^2*PolyLog[3, E^(c + d*x)] - 6*a^2*d*f^3*x*PolyLog[3, E^(c + d*x)] + 12*b^2*d*f^3*x*PolyLog[3, E^(c + d*x)] + 6*a^2*d*E^(2*c)*f^3*x*PolyLog[3, E^(c + d*x)] - 12*b^2*d*E^(2*c)*f^3*x*PolyLog[3, E^(c + d*x)] - 3*a*b*f^3*PolyLog[3, E^(2*(c + d*x))] + 3*a*b*E^(2*c)*f^3*PolyLog[3, E^(2*(c + d*x))] - 6*a^2*f^3*PolyLog[4, -E^(c + d*x)] + 12*b^2*f^3*PolyLog[4, -E^(c + d*x)] + 6*a^2*E^(2*c)*f^3*PolyLog[4, -E^(c + d*x)] - 12*b^2*E^(2*c)*f^3*PolyLog[4, -E^(c + d*x)] + 6*a^2*f^3*PolyLog[4, E^(c + d*x)] - 12*b^2*f^3*PolyLog[4, E^(c + d*x)] - 6*a^2*E^(2*c)*f^3*PolyLog[4, E^(c + d*x)] + 12*b

```
^2*E^(2*c)*f^3*PolyLog[4, E^(c + d*x)])/(2*a^3*d^4*(-1 + E^(2*c))) + (b^3*(
2*d^3*e^3*Sqrt[(a^2 + b^2)*E^(2*c)]*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 +
b^2]] - 3*Sqrt[a^2 + b^2]*d^3*e^2*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c
- Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*Sqrt[a^2 + b^2]*d^3*e*E^c*f^2*x^2*Log[1 +
 (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - Sqrt[a^2 + b^2]*d
^3*E^c*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]
)] + 3*Sqrt[a^2 + b^2]*d^3*e^2*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + S
qrt[(a^2 + b^2)*E^(2*c)])] + 3*Sqrt[a^2 + b^2]*d^3*e*E^c*f^2*x^2*Log[1 + (b
*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + Sqrt[a^2 + b^2]*d^3*
E^c*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])]
- 3*Sqrt[a^2 + b^2]*d^2*E^c*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a
*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 3*Sqrt[a^2 + b^2]*d^2*E^c*f*(e + f*x)
^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6
*Sqrt[a^2 + b^2]*d*e*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(
a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[a^2 + b^2]*d*E^c*f^3*x*PolyLog[3, -((b*E^(2
*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[a^2 + b^2]*d*e*E^
c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]
- 6*Sqrt[a^2 + b^2]*d*E^c*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqr
t[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[a^2 + b^2]*E^c*f^3*PolyLog[4, -((b*E^(2*
c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[a^2 + b^2]*E^c*f^3
*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(a^
3*Sqrt[a^2 + b^2]*d^4*Sqrt[(a^2 + b^2)*E^(2*c)]...
```

Maple [F]
time = 1.72, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c)^3}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a
+ sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3*d) - 2*(a*e^(-d*x - c) + 2*b*e^(-2
*d*x - 2*c) + a*e^(-3*d*x - 3*c) - 2*b)/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-
4*d*x - 4*c) - a^2)*d) - (a^2 - 2*b^2)*log(e^(-d*x - c) + 1)/(a^3*d) + (a^2
```

```
 - 2*b^2)*log(e^(-d*x - c) - 1)/(a^3*d))*e^3 - (2*b*d*f^3*x^3 + 6*b*d*f^2*x
^2*e + 6*b*d*f*x*e^2 + (a*d*f^3*x^3*e^(3*c) + 3*(a*f^3*e^(3*c) + a*d*f^2*e^
(3*c + 1))*x^2 + 3*a*f*e^(3*c + 2) + 3*(a*d*f*e^(3*c + 2) + 2*a*f^2*e^(3*c
+ 1))*x)*e^(3*d*x) - 2*(b*d*f^3*x^3*e^(2*c) + 3*b*d*f^2*x^2*e^(2*c + 1) + 3
*b*d*f*x*e^(2*c + 2))*e^(2*d*x) + (a*d*f^3*x^3*e^c + 3*(a*d*f^2*e^(c + 1) -
 a*f^3*e^c)*x^2 - 3*a*f*e^(c + 2) + 3*(a*d*f*e^(c + 2) - 2*a*f^2*e^(c + 1))
*x)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2
) + 3*(b*d*f*e^2 + a*f^2*e)*x/(a^2*d^2) + 3*(b*d*f*e^2 - a*f^2*e)*x/(a^2*d^
2) - 3*(b*d*f*e^2 + a*f^2*e)*log(e^(d*x + c) + 1)/(a^2*d^3) - 3*(b*d*f*e^2
- a*f^2*e)*log(e^(d*x + c) - 1)/(a^2*d^3) + 1/2*(d^3*x^3*log(e^(d*x + c) +
1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*pol
ylog(4, -e^(d*x + c)))*(a^2*f^3 - 2*b^2*f^3)/(a^3*d^4) - 1/2*(d^3*x^3*log(-
e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x +
 c)) + 6*polylog(4, e^(d*x + c)))*(a^2*f^3 - 2*b^2*f^3)/(a^3*d^4) - 3/2*(2*
a*b*f^3 - (a^2*d*f^2 - 2*b^2*d*f^2)*e)*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*
x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^3*d^4) - 3/2*(2*a*b*
f^3 + (a^2*d*f^2 - 2*b^2*d*f^2)*e)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*d
ilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))/(a^3*d^4) - 3/2*(4*a*b*d*f^2
*e + 2*a^2*f^3 - (a^2*d^2*f - 2*b^2*d^2*f)*e^2)*(d*x*log(e^(d*x + c) + 1) +
 dilog(-e^(d*x + c)))/(a^3*d^4) - 3/2*(4*a*b*d*f^2*e - 2*a^2*f^3 + (a^2*d^2
*f - 2*b^2*d^2*f)*e^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^
3*d^4) + 1/8*((a^2*f^3 - 2*b^2*f^3)*d^4*x^4 + 4*(2*a*b*f^3 + (a^2*d*f^2 - 2
*b^2*d*f^2)*e)*d^3*x^3 + 6*(4*a*b*d*f^2*e - 2*a^2*f^3 + (a^2*d^2*f - 2*b^2*
d^2*f)*e^2)*d^2*x^2)/(a^3*d^4) - 1/8*((a^2*f^3 - 2*b^2*f^3)*d^4*x^4 - 4*(2*
a*b*f^3 - (a^2*d*f^2 - 2*b^2*d*f^2)*e)*d^3*x^3 - 6*(4*a*b*d*f^2*e + 2*a^2*f
^3 - (a^2*d^2*f - 2*b^2*d^2*f)*e^2)*d^2*x^2)/(a^3*d^4) - integrate(2*(b^3*f
^3*x^3*e^c + 3*b^3*f^2*x^2*e^(c + 1) + 3*b^3*f*x*e^(c + 2))*e^(d*x)/(a^3*b*
e^(2*d*x + 2*c) + 2*a^4*e^(d*x + c) - a^3*b), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 32529 vs. $2(1000) = 2000$.
time = 0.76, size = 32529, normalized size = 30.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*(a^3*b + a*b^3)*c^3*f^3 - 12*(a^3*b + a*b^3)*c^2*d*f^2*cosh(1) + 12*
(a^3*b + a*b^3)*c*d^2*f*cosh(1)^2 - 4*(a^3*b + a*b^3)*d^3*cosh(1)^3 - 4*(a^
3*b + a*b^3)*d^3*sinh(1)^3 + 4*((a^3*b + a*b^3)*d^3*f^3*x^3 + (a^3*b + a*b^
3)*c^3*f^3 + 3*((a^3*b + a*b^3)*d^3*f*x + (a^3*b + a*b^3)*c*d^2*f)*cosh(1)^
2 + 3*((a^3*b + a*b^3)*d^3*f*x + (a^3*b + a*b^3)*c*d^2*f)*sinh(1)^2 + 3*((a
^3*b + a*b^3)*d^3*f^2*x^2 - (a^3*b + a*b^3)*c^2*d*f^2)*cosh(1) + 3*((a^3*b
+ a*b^3)*d^3*f^2*x^2 - (a^3*b + a*b^3)*c^2*d*f^2 + 2*((a^3*b + a*b^3)*d^3*f
```

```
*x + (a^3*b + a*b^3)*c*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^4 + 4*((a^3*b
 + a*b^3)*d^3*f^3*x^3 + (a^3*b + a*b^3)*c^3*f^3 + 3*((a^3*b + a*b^3)*d^3*f*
x + (a^3*b + a*b^3)*c*d^2*f)*cosh(1)^2 + 3*((a^3*b + a*b^3)*d^3*f*x + (a^3*
b + a*b^3)*c*d^2*f)*sinh(1)^2 + 3*((a^3*b + a*b^3)*d^3*f^2*x^2 - (a^3*b + a
*b^3)*c^2*d*f^2)*cosh(1) + 3*((a^3*b + a*b^3)*d^3*f^2*x^2 - (a^3*b + a*b^3)
*c^2*d*f^2 + 2*((a^3*b + a*b^3)*d^3*f*x + (a^3*b + a*b^3)*c*d^2*f)*cosh(1))
*sinh(1))*sinh(d*x + c)^4 - 2*((a^4 + a^2*b^2)*d^3*f^3*x^3 + 3*(a^4 + a^2*b
^2)*d^2*f^3*x^2 + (a^4 + a^2*b^2)*d^3*cosh(1)^3 + (a^4 + a^2*b^2)*d^3*sinh(
1)^3 + 3*((a^4 + a^2*b^2)*d^3*f*x + (a^4 + a^2*b^2)*d^2*f)*cosh(1)^2 + 3*((
a^4 + a^2*b^2)*d^3*f*x + (a^4 + a^2*b^2)*d^3*cosh(1) + (a^4 + a^2*b^2)*d^2*
f)*sinh(1)^2 + 3*((a^4 + a^2*b^2)*d^3*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*f^2*x
)*cosh(1) + 3*((a^4 + a^2*b^2)*d^3*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*f^2*x +
(a^4 + a^2*b^2)*d^3*cosh(1)^2 + 2*((a^4 + a^2*b^2)*d^3*f*x + (a^4 + a^2*b^2
)*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^3 - 2*((a^4 + a^2*b^2)*d^3*f^3*x^3
 + 3*(a^4 + a^2*b^2)*d^2*f^3*x^2 + (a^4 + a^2*b^2)*d^3*cosh(1)^3 + (a^4 + a
^2*b^2)*d^3*sinh(1)^3 + 3*((a^4 + a^2*b^2)*d^3*f*x + (a^4 + a^2*b^2)*d^2*f)
*cosh(1)^2 + 3*((a^4 + a^2*b^2)*d^3*f*x + (a^4 + a^2*b^2)*d^3*cosh(1) + (a^
4 + a^2*b^2)*d^2*f)*sinh(1)^2 + 3*((a^4 + a^2*b^2)*d^3*f^2*x^2 + 2*(a^4 + a
^2*b^2)*d^2*f^2*x)*cosh(1) - 8*((a^3*b + a*b^3)*d^3*f^3*x^3 + (a^3*b + a*b^
3)*c^3*f^3 + 3*((a^3*b + a*b^3)*d^3*f*x + (a^3*b + a*b^3)*c*d^2*f)*cosh(1)^
2 + 3*((a^3*b + a*b^3)*d^3*f*x + (a^3*b + a*b^3)*c*d^2*f)*sinh(1)^2 + 3*((a
^3*b + a*b^3)*d^3*f^2*x^2 - (a^3*b + a*b^3)*c^2*d*f^2)*cosh(1) + 3*((a^3*b
+ a*b^3)*d^3*f^2*x^2 - (a^3*b + a*b^3)*c^2*d*f^2 + 2*((a^3*b + a*b^3)*d^3*f
*x + (a^3*b + a*b^3)*c*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c) + 3*((a^4 + a
^2*b^2)*d^3*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*f^2*x + (a^4 + a^2*b^2)*d^3*cos
h(1)^2 + 2*((a^4 + a^2*b^2)*d^3*f*x + (a^4 + a^2*b^2)*d^2*f)*cosh(1))*sinh(
1))*sinh(d*x + c)^3 - 4*((a^3*b + a*b^3)*d^3*f^3*x^3 + 2*(a^3*b + a*b^3)*c^
3*f^3 - (a^3*b + a*b^3)*d^3*cosh(1)^3 - (a^3*b + a*b^3)*d^3*sinh(1)^3 + 3*(
(a^3*b + a*b^3)*d^3*f*x + 2*(a^3*b + a*b^3)*c*d^2*f)*cosh(1)^2 + 3*((a^3*b
+ a*b^3)*d^3*f*x + 2*(a^3*b + a*b^3)*c*d^2*f - (a^3*b + a*b^3)*d^3*cosh(1))
*sinh(1)^2 + 3*((a^3*b + a*b^3)*d^3*f^2*x^2 - 2*(a^3*b + a*b^3)*c^2*d*f^2)*
cosh(1) + 3*((a^3*b + a*b^3)*d^3*f^2*x^2 - 2*(a^3*b + a*b^3)*c^2*d*f^2 - (a
^3*b + a*b^3)*d^3*cosh(1)^2 + 2*((a^3*b + a*b^3)*d^3*f*x + 2*(a^3*b + a*b^3
)*c*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 12*((a^3*b + a*b^3)*c*d^2*f
- (a^3*b + a*b^3)*d^3*cosh(1))*sinh(1)^2 - 2*(2*(a^3*b + a*b^3)*d^3*f^3*x^3
 + 4*(a^3*b + a*b^3)*c^3*f^3 - 2*(a^3*b + a*b^3)*d^3*cosh(1)^3 - 2*(a^3*b +
 a*b^3)*d^3*sinh(1)^3 + 6*((a^3*b + a*b^3)*d^3*f*x + 2*(a^3*b + a*b^3)*c*d^
2*f)*cosh(1)^2 - 12*((a^3*b + a*b^3)*d^3*f^3*x^3 + (a^3*b + a*b^3)*c^3*f^3
+ 3*((a^3*b + a*b^3)*d^3*f*x + (a^3*b + a*b^3)*c*d^2*f)*cosh(1)^2 + 3*((a^3
*b + a*b^3)*d^3*f*x + (a^3*b + a*b^3)*c*d^2*f)*sinh(1)^2 + 3*((a^3*b + a*b^
3)*d^3*f^2*x^2 - (a^3*b + a*b^3)*c^2*d*f^2)*cosh(1) + 3*((a^3*b + a*b^3)*d^
3*f^2*x^2 - (a^3*b + a*b^3)*c^2*d*f^2 + 2*((a^3*b + a*b^3)*d^3*f*x + (a^3*b
 + a*b^3)*c*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 6*((a^3*b + a*b^3)*d
^3*f*x + 2*(a^3*b + a*b^3)*c*d^2*f - (a^3*b + a*b^3)*d^3*cosh(1))*sinh(1)^2
 + 6*((a^3*b + a*b^3)*d^3*f^2*x^2 - 2*(a^3*b + a*b^3)*c^2*d*f^2)*cosh(1) +
```

```
3*((a^4 + a^2*b^2)*d^3*f^3*x^3 + 3*(a^4 + a^2*b^2)*d^2*f^3*x^2 + (a^4 + a^2
*b^2)*d^3*cosh(1)^3 + (a^4 + a^2*b^2)*d^3*sinh(1)^3 + 3*((a^4 + a^2*b^2)*d^
3*f*x + (a^4 + a^2*b^2)*d^2*f)*cosh(1)^2 + 3*((a^4 + a^2*b^2)*d^3*f*x + (a^
4 + a^2*b^2)*d^3*cosh(1) + (a^4 + a^2*b^2)*d^2*f)*sinh(1)^2 + 3*((a^4 + a^2
*b^2)*d^3*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*f^2*x)*cosh(1) + 3*((a^4 + a^2*b^
2)*d^3*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*f^2*x + (a^4 + a^2*b^2)*d^3*cosh(1)^
2 + 2*((a^4 + a^2*b^2)*d^3*f*x + (a^4 + a^2*b^2)*d^2*f)*cosh(1))*sinh(1))*c
osh(d*x + c) + 6*((a^3*b + a*b^3)*d^3*f^2*x^2 - 2*(a^3*b + a*b^3)*c^2*d*f^2
 - (a^3*b + a*b^3)*d^3*cosh(1)^2 + 2*((a^3*b + a*b^3)*d^3*f*x + 2*(a^3*b +
a*b^3)*c*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 6*(b^4*d^2*f^3*x^2 + 2*
b^4*d^2*f^2*x*cosh(1) + b^4*d^2*f*cosh(1)^2 + b^4*d^2*f*sinh(1)^2 + (b^4*d^
2*f^3*x^2 + 2*b^4*d^2*f^2*x*cosh(1) + b^4*d^2*f*cosh(1)^2 + b^4*d^2*f*sinh(
1)^2 + 2*(b^4*d^2*f^2*x + b^4*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^4 + 4*(
b^4*d^2*f^3*x^2 + 2*b^4*d^2*f^2*x*cosh(1) + b^4*d^2*f*cosh(1)^2 + b^4*d^2*f
*sinh(1)^2 + 2*(b^4*d^2*f^2*x + b^4*d^2*f*cosh(...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^3}{{\mathrm{sinh}\left(c+d\,x\right)}^3\,\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((e + f*x)^3/(sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

3.249 $\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=725

$$\frac{b(e+fx)^2}{a^2d}+\frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^3d}-\frac{f^2\tanh^{-1}(\cosh(c+dx))}{ad^3}+\frac{b(e+fx)^2}{}$$

```
[Out] b*(f*x+e)^2/a^2/d+(f*x+e)^2*arctanh(exp(d*x+c))/a/d-2*b^2*(f*x+e)^2*arctanh
(exp(d*x+c))/a^3/d-f^2*arctanh(cosh(d*x+c))/a/d^3+b*(f*x+e)^2*coth(d*x+c)/a
^2/d-f*(f*x+e)*csch(d*x+c)/a/d^2-1/2*(f*x+e)^2*coth(d*x+c)*csch(d*x+c)/a/d-
2*b*f*(f*x+e)*ln(1-exp(2*d*x+2*c))/a^2/d^2+f*(f*x+e)*polylog(2,-exp(d*x+c))
/a/d^2-2*b^2*f*(f*x+e)*polylog(2,-exp(d*x+c))/a^3/d^2-f*(f*x+e)*polylog(2,e
xp(d*x+c))/a/d^2+2*b^2*f*(f*x+e)*polylog(2,exp(d*x+c))/a^3/d^2-b*f^2*polylo
g(2,exp(2*d*x+2*c))/a^2/d^3-f^2*polylog(3,-exp(d*x+c))/a/d^3+2*b^2*f^2*poly
log(3,-exp(d*x+c))/a^3/d^3+f^2*polylog(3,exp(d*x+c))/a/d^3-2*b^2*f^2*polylo
g(3,exp(d*x+c))/a^3/d^3-b^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2))
)/a^3/d/(a^2+b^2)^(1/2)+b^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2))
)/a^3/d/(a^2+b^2)^(1/2)-2*b^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))/a^3/d^2/(a^2+b^2)^(1/2)+2*b^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(
a+(a^2+b^2)^(1/2)))/a^3/d^2/(a^2+b^2)^(1/2)+2*b^3*f^2*polylog(3,-b*exp(d*x+
c)/(a-(a^2+b^2)^(1/2)))/a^3/d^3/(a^2+b^2)^(1/2)-2*b^3*f^2*polylog(3,-b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^3/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.93, antiderivative size = 725, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5694, 4271, 3855, 4267, 2611, 2320, 6724, 4269, 3797, 2221, 2317, 2438, 3403, 2296}

$$\frac{2b^2f^2\text{Li}_3\left(-e^{c+dx}\right)}{a^3d^3}-\frac{2b^2f^2\text{Li}_3\left(e^{c+dx}\right)}{a^3d^3}-\frac{2b^2f(e+fx)\text{Li}_2\left(-e^{c+dx}\right)}{a^3d^2}+\frac{2b^2f(e+fx)\text{Li}_2\left(e^{c+dx}\right)}{a^3d^2}-\frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^3d}-\frac{bf^2\text{Li}_2\left(e^{2(c+dx)}\right)}{a^2d^3}-\frac{2bf(e+fx)\log\left(1-e^{2(c+dx)}\right)}{a^2d^2}+\frac{b(e+fx)^2\coth(c+dx)}{a^2d}+\frac{b(e+fx)^2}{a^2d}+\frac{2b^3f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^3\sqrt{a^2+b^2}}-\frac{2b^3f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^3\sqrt{a^2+b^2}}-\frac{2b^3f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2\sqrt{a^2+b^2}}+\frac{2b^3f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2\sqrt{a^2+b^2}}-\frac{b^3(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^3d\sqrt{a^2+b^2}}+\frac{b^3(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{a^3d\sqrt{a^2+b^2}}-\frac{f^2\text{Li}_3\left(-e^{c+dx}\right)}{ad^3}+\frac{f^2\text{Li}_3\left(e^{c+dx}\right)}{ad^3}-\frac{f^2\tanh^{-1}(\cosh(c+dx))}{ad^3}+\frac{f(e+fx)\text{Li}_2\left(-e^{c+dx}\right)}{ad^2}-\frac{f(e+fx)\text{Li}_2\left(e^{c+dx}\right)}{ad^2}-\frac{f(e+fx)\text{csch}(c+dx)}{ad^2}+\frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*(e + f*x)^2)/(a^2*d) + ((e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (2*b^2
*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^3*d) - (f^2*ArcTanh[Cosh[c + d*x]])/(
a*d^3) + (b*(e + f*x)^2*Coth[c + d*x])/(a^2*d) - (f*(e + f*x)*Csch[c + d*x]
)/(a*d^2) - ((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x])/(2*a*d) - (b^3*(e + f
*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*Sqrt[a^2 + b^2]*
d) + (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*
Sqrt[a^2 + b^2]*d) - (2*b*f*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a^2*d^2) +
 (f*(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) - (2*b^2*f*(e + f*x)*PolyLo
g[2, -E^(c + d*x)])/(a^3*d^2) - (f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^
```

```
2) + (2*b^2*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^3*d^2) - (2*b^3*f*(e +
f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 +
b^2]*d^2) + (2*b^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/(a^3*Sqrt[a^2 + b^2]*d^2) - (b*f^2*PolyLog[2, E^(2*(c + d*x))])/(
a^2*d^3) - (f^2*PolyLog[3, -E^(c + d*x)])/(a*d^3) + (2*b^2*f^2*PolyLog[3, -
E^(c + d*x)])/(a^3*d^3) + (f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) - (2*b^2*f^
2*PolyLog[3, E^(c + d*x)])/(a^3*d^3) + (2*b^3*f^2*PolyLog[3, -((b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b^2]*d^3) - (2*b^3*f^2*PolyLog
[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b^2]*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
```

```
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5694

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a + b
*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{csch}^3(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{f(e+fx)\text{csch}(c+dx)}{ad^2} - \frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2ad} - \frac{\int (e+f$$

$$= \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{f^2\tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{b(e+fx)^2\coth(c-}{a^2d}$$

$$= \frac{b(e+fx)^2}{a^2d} + \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} - f$$

$$= \frac{b(e+fx)^2}{a^2d} + \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} - f$$

$$= \frac{b(e+fx)^2}{a^2d} + \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} - f$$

$$= \frac{b(e+fx)^2}{a^2d} + \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} - f$$

$$= \frac{b(e+fx)^2}{a^2d} + \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} - f$$

$$= \frac{b(e+fx)^2}{a^2d} + \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} - f$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1775 vs. $2(725) = 1450$.
time = 18.87, size = 1775, normalized size = 2.45

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (8*a*b*d^2*e*E^(2*c)*f*x + 4*a*b*d^2*E^(2*c)*f^2*x^2 - 2*a^2*d^2*e^2*ArcTan
h[E^(c + d*x)] + 4*b^2*d^2*e^2*ArcTanh[E^(c + d*x)] + 2*a^2*d^2*e^2*E^(2*c)
*ArcTanh[E^(c + d*x)] - 4*b^2*d^2*e^2*E^(2*c)*ArcTanh[E^(c + d*x)] + 4*a^2*
f^2*ArcTanh[E^(c + d*x)] - 4*a^2*E^(2*c)*f^2*ArcTanh[E^(c + d*x)] + 2*a^2*d
```

```
^2*e*f*x*Log[1 - E^(c + d*x)] - 4*b^2*d^2*e*f*x*Log[1 - E^(c + d*x)] - 2*a^
2*d^2*e*E^(2*c)*f*x*Log[1 - E^(c + d*x)] + 4*b^2*d^2*e*E^(2*c)*f*x*Log[1 -
E^(c + d*x)] + a^2*d^2*f^2*x^2*Log[1 - E^(c + d*x)] - 2*b^2*d^2*f^2*x^2*Log
[1 - E^(c + d*x)] - a^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + 2*b^2*d^
2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] - 2*a^2*d^2*e*f*x*Log[1 + E^(c + d*x
)] + 4*b^2*d^2*e*f*x*Log[1 + E^(c + d*x)] + 2*a^2*d^2*e*E^(2*c)*f*x*Log[1 +
 E^(c + d*x)] - 4*b^2*d^2*e*E^(2*c)*f*x*Log[1 + E^(c + d*x)] - a^2*d^2*f^2*
x^2*Log[1 + E^(c + d*x)] + 2*b^2*d^2*f^2*x^2*Log[1 + E^(c + d*x)] + a^2*d^2
*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] - 2*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 + E
^(c + d*x)] + 4*a*b*d*e*f*Log[1 - E^(2*(c + d*x))] - 4*a*b*d*e*E^(2*c)*f*Lo
g[1 - E^(2*(c + d*x))] + 4*a*b*d*f^2*x*Log[1 - E^(2*(c + d*x))] - 4*a*b*d*E
^(2*c)*f^2*x*Log[1 - E^(2*(c + d*x))] + 2*(a^2 - 2*b^2)*d*(-1 + E^(2*c))*f*
(e + f*x)*PolyLog[2, -E^(c + d*x)] - 2*(a^2 - 2*b^2)*d*(-1 + E^(2*c))*f*(e
+ f*x)*PolyLog[2, E^(c + d*x)] + 2*a*b*f^2*PolyLog[2, E^(2*(c + d*x))] - 2*
a*b*E^(2*c)*f^2*PolyLog[2, E^(2*(c + d*x))] + 2*a^2*f^2*PolyLog[3, -E^(c +
d*x)] - 4*b^2*f^2*PolyLog[3, -E^(c + d*x)] - 2*a^2*E^(2*c)*f^2*PolyLog[3, -
E^(c + d*x)] + 4*b^2*E^(2*c)*f^2*PolyLog[3, -E^(c + d*x)] - 2*a^2*f^2*PolyL
og[3, E^(c + d*x)] + 4*b^2*f^2*PolyLog[3, E^(c + d*x)] + 2*a^2*E^(2*c)*f^2*
PolyLog[3, E^(c + d*x)] - 4*b^2*E^(2*c)*f^2*PolyLog[3, E^(c + d*x)])/(2*a^3
*d^3*(-1 + E^(2*c))) + (b^3*(2*d^2*e^2*Sqrt[(a^2 + b^2)*E^(2*c)]*ArcTanh[(a
 + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*Sqrt[a^2 + b^2]*d^2*e*E^c*f*x*Log[1
+ (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - Sqrt[a^2 + b^2]*
d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)
])] + 2*Sqrt[a^2 + b^2]*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sq
rt[(a^2 + b^2)*E^(2*c)])] + Sqrt[a^2 + b^2]*d^2*E^c*f^2*x^2*Log[1 + (b*E^(2
*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 2*Sqrt[a^2 + b^2]*d*E^c*f
*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)
]))] + 2*Sqrt[a^2 + b^2]*d*E^c*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(
a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2*Sqrt[a^2 + b^2]*E^c*f^2*PolyLog[3,
 -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*Sqrt[a^2 + b
^2]*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c
)]))]))/(a^3*Sqrt[a^2 + b^2]*d^3*Sqrt[(a^2 + b^2)*E^(2*c)]) + (Csch[c]*Csch
[c + d*x]^2*(2*b*d*e^2*Cosh[c] + 4*b*d*e*f*x*Cosh[c] + 2*b*d*f^2*x^2*Cosh[c
] + 2*a*e*f*Cosh[d*x] + 2*a*f^2*x*Cosh[d*x] - 2*a*e*f*Cosh[2*c + d*x] - 2*a
*f^2*x*Cosh[2*c + d*x] - 2*b*d*e^2*Cosh[c + 2*d*x] - 4*b*d*e*f*x*Cosh[c + 2
*d*x] - 2*b*d*f^2*x^2*Cosh[c + 2*d*x] + a*d*e^2*Sinh[d*x] + 2*a*d*e*f*x*Sin
h[d*x] + a*d*f^2*x^2*Sinh[d*x] - a*d*e^2*Sinh[2*c + d*x] - 2*a*d*e*f*x*Sinh
[2*c + d*x] - a*d*f^2*x^2*Sinh[2*c + d*x]))/(4*a^2*d^2)
```

Maple [F]

time = 1.98, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^3}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a
+ sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3*d) - 2*(a*e^(-d*x - c) + 2*b*e^(-2
*d*x - 2*c) + a*e^(-3*d*x - 3*c) - 2*b)/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-
4*d*x - 4*c) - a^2)*d) - (a^2 - 2*b^2)*log(e^(-d*x - c) + 1)/(a^3*d) + (a^2
 - 2*b^2)*log(e^(-d*x - c) - 1)/(a^3*d))*e^2 - (2*b*d*f^2*x^2 + 4*b*d*f*x*e
 + (a*d*f^2*x^2*e^(3*c) + 2*a*f*e^(3*c + 1) + 2*(a*f^2*e^(3*c) + a*d*f*e^(3
*c + 1))*x)*e^(3*d*x) - 2*(b*d*f^2*x^2*e^(2*c) + 2*b*d*f*x*e^(2*c + 1))*e^(
2*d*x) + (a*d*f^2*x^2*e^c - 2*a*f*e^(c + 1) + 2*(a*d*f*e^(c + 1) - a*f^2*e^
c)*x)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d
^2) + (2*b*d*f*e + a*f^2)*x/(a^2*d^2) + (2*b*d*f*e - a*f^2)*x/(a^2*d^2) - (
2*b*d*f*e + a*f^2)*log(e^(d*x + c) + 1)/(a^2*d^3) - (2*b*d*f*e - a*f^2)*log
(e^(d*x + c) - 1)/(a^2*d^3) + 1/2*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dil
og(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*(a^2*f^2 - 2*b^2*f^2)/(a^3*d
^3) - 1/2*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*pol
ylog(3, e^(d*x + c)))*(a^2*f^2 - 2*b^2*f^2)/(a^3*d^3) - (2*a*b*f^2 - (a^2*d
*f - 2*b^2*d*f)*e)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^3*d^
3) - (2*a*b*f^2 + (a^2*d*f - 2*b^2*d*f)*e)*(d*x*log(-e^(d*x + c) + 1) + dil
og(e^(d*x + c)))/(a^3*d^3) + 1/6*((a^2*f^2 - 2*b^2*f^2)*d^3*x^3 + 3*(2*a*b*
f^2 + (a^2*d*f - 2*b^2*d*f)*e)*d^2*x^2)/(a^3*d^3) - 1/6*((a^2*f^2 - 2*b^2*f
^2)*d^3*x^3 - 3*(2*a*b*f^2 - (a^2*d*f - 2*b^2*d*f)*e)*d^2*x^2)/(a^3*d^3) -
integrate(2*(b^3*f^2*x^2*e^c + 2*b^3*f*x*e^(c + 1))*e^(d*x)/(a^3*b*e^(2*d*x
 + 2*c) + 2*a^4*e^(d*x + c) - a^3*b), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14973 vs. 2(691) = 1382.
time = 0.54, size = 14973, normalized size = 20.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(a^3*b + a*b^3)*c^2*f^2 - 8*(a^3*b + a*b^3)*c*d*f*cosh(1) + 4*(a^3*
b + a*b^3)*d^2*cosh(1)^2 - 4*((a^3*b + a*b^3)*d^2*f^2*x^2 - (a^3*b + a*b^3)
```

*c^2*f^2 + 2*((a^3*b + a*b^3)*d^2*f*x + (a^3*b + a*b^3)*c*d*f)*cosh(1) + 2*
((a^3*b + a*b^3)*d^2*f*x + (a^3*b + a*b^3)*c*d*f)*sinh(1))*cosh(d*x + c)^4
+ 4*(a^3*b + a*b^3)*d^2*sinh(1)^2 - 4*((a^3*b + a*b^3)*d^2*f^2*x^2 - (a^3*b
 + a*b^3)*c^2*f^2 + 2*((a^3*b + a*b^3)*d^2*f*x + (a^3*b + a*b^3)*c*d*f)*cos
h(1) + 2*((a^3*b + a*b^3)*d^2*f*x + (a^3*b + a*b^3)*c*d*f)*sinh(1))*sinh(d*
x + c)^4 + 2*((a^4 + a^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d*f^2*x + (a^
4 + a^2*b^2)*d^2*cosh(1)^2 + (a^4 + a^2*b^2)*d^2*sinh(1)^2 + 2*((a^4 + a^2*
b^2)*d^2*f*x + (a^4 + a^2*b^2)*d*f)*cosh(1) + 2*((a^4 + a^2*b^2)*d^2*f*x +
(a^4 + a^2*b^2)*d^2*cosh(1) + (a^4 + a^2*b^2)*d*f)*sinh(1))*cosh(d*x + c)^3
 + 2*((a^4 + a^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*
b^2)*d^2*cosh(1)^2 + (a^4 + a^2*b^2)*d^2*sinh(1)^2 + 2*((a^4 + a^2*b^2)*d^2
*f*x + (a^4 + a^2*b^2)*d*f)*cosh(1) - 8*((a^3*b + a*b^3)*d^2*f^2*x^2 - (a^3
*b + a*b^3)*c^2*f^2 + 2*((a^3*b + a*b^3)*d^2*f*x + (a^3*b + a*b^3)*c*d*f)*c
osh(1) + 2*((a^3*b + a*b^3)*d^2*f*x + (a^3*b + a*b^3)*c*d*f)*sinh(1))*cosh(
d*x + c) + 2*((a^4 + a^2*b^2)*d^2*f*x + (a^4 + a^2*b^2)*d^2*cosh(1) + (a^4
+ a^2*b^2)*d*f)*sinh(1))*sinh(d*x + c)^3 + 4*((a^3*b + a*b^3)*d^2*f^2*x^2 -
 2*(a^3*b + a*b^3)*c^2*f^2 - (a^3*b + a*b^3)*d^2*cosh(1)^2 - (a^3*b + a*b^3
)*d^2*sinh(1)^2 + 2*((a^3*b + a*b^3)*d^2*f*x + 2*(a^3*b + a*b^3)*c*d*f)*cos
h(1) + 2*((a^3*b + a*b^3)*d^2*f*x + 2*(a^3*b + a*b^3)*c*d*f - (a^3*b + a*b^
3)*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(2*(a^3*b + a*b^3)*d^2*f^2*x^2
 - 4*(a^3*b + a*b^3)*c^2*f^2 - 2*(a^3*b + a*b^3)*d^2*cosh(1)^2 - 2*(a^3*b +
 a*b^3)*d^2*sinh(1)^2 - 12*((a^3*b + a*b^3)*d^2*f^2*x^2 - (a^3*b + a*b^3)*c
^2*f^2 + 2*((a^3*b + a*b^3)*d^2*f*x + (a^3*b + a*b^3)*c*d*f)*cosh(1) + 2*((
a^3*b + a*b^3)*d^2*f*x + (a^3*b + a*b^3)*c*d*f)*sinh(1))*cosh(d*x + c)^2 +
4*((a^3*b + a*b^3)*d^2*f*x + 2*(a^3*b + a*b^3)*c*d*f)*cosh(1) + 3*((a^4 + a
^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*d^2*cosh(
1)^2 + (a^4 + a^2*b^2)*d^2*sinh(1)^2 + 2*((a^4 + a^2*b^2)*d^2*f*x + (a^4 +
a^2*b^2)*d*f)*cosh(1) + 2*((a^4 + a^2*b^2)*d^2*f*x + (a^4 + a^2*b^2)*d^2*co
sh(1) + (a^4 + a^2*b^2)*d*f)*sinh(1))*cosh(d*x + c) + 4*((a^3*b + a*b^3)*d^
2*f*x + 2*(a^3*b + a*b^3)*c*d*f - (a^3*b + a*b^3)*d^2*cosh(1))*sinh(1))*sin
h(d*x + c)^2 + 4*(b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1) + (b^4*d*
f^2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1))*cosh(d*x + c)^4 + 4*(b^4*d*f^2*x
 + b^4*d*f*cosh(1) + b^4*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*
d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1))*sinh(d*x + c)^4 - 2*(b^4*d*f^2
*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1))*cosh(d*x + c)^2 - 2*(b^4*d*f^2*x +
b^4*d*f*cosh(1) + b^4*d*f*sinh(1) - 3*(b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*
d*f*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*d*f^2*x + b^4*d*f*c
osh(1) + b^4*d*f*sinh(1))*cosh(d*x + c)^3 - (b^4*d*f^2*x + b^4*d*f*cosh(1)
+ b^4*d*f*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilo
g((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 4*(b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*
d*f*sinh(1) + (b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1))*cosh(d*x +
c)^4 + 4*(b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1))*cosh(d*x + c)*si
nh(d*x + c)^3 + (b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1))*sinh(d*x
+ c)^4 - 2*(b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1))*cosh(d*x + c)^

```
2 - 2*(b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1) - 3*(b^4*d*f^2*x + b
^4*d*f*cosh(1) + b^4*d*f*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^
4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1))*cosh(d*x + c)^3 - (b^4*d*f^2
*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b^4*c^2*f^2 - 2
*b^4*c*d*f*cosh(1) + b^4*d^2*cosh(1)^2 + b^4*d^2*sinh(1)^2 + (b^4*c^2*f^2 -
 2*b^4*c*d*f*cosh(1) + b^4*d^2*cosh(1)^2 + b^4*d^2*sinh(1)^2 - 2*(b^4*c*d*f
 - b^4*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^4 + 4*(b^4*c^2*f^2 - 2*b^4*c*d*f
*cosh(1) + b^4*d^2*cosh(1)^2 + b^4*d^2*sinh(1)^2 - 2*(b^4*c*d*f - b^4*d^2*c
osh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*c^2*f^2 - 2*b^4*c*d*f
*cosh(1) + b^4*d^2*cosh(1)^2 + b^4*d^2*sinh(1)^2 - 2*(b^4*c*d*f - b^4*d^2*c
osh(1))*sinh(1))*sinh(d*x + c)^4 - 2*(b^4*c^2*f^2 - 2*b^4*c*d*f*cosh(1) + b
^4*d^2*cosh(1)^2 + b^4*d^2*sinh(1)^2 - 2*(b^4*c*d*f - b^4*d^2*cosh(1))*sinh
(1))*cosh(d*x + c)^2 - 2*(b^4*c^2*f^2 - 2*b^4*c*d*f*cosh(1) + b^4*d^2*cosh(
1)^2 + b^4*d^2*sinh(1)^2 - 3*(b^4*c^2*f^2 - 2*b^4*c*d*f*cosh(1) + b^4*d^2*c
osh(1)^2 + b^4*d^2*sinh(1)^2 - 2*(b^4*c*d*f - b^4*d^2*cosh(1))*sinh(1))*cos
h(d*x + c)^2 - 2*(b^4*c*d*f - b^4*d^2*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 2
*(b^4*c*d*f - b^4*d^2*cosh(1))*sinh(1) + 4*((b^4*c^2*f^2 - 2*b^4*c*d*f*cosh
(1) + b^4*d^2*cosh(1)^2 + b^4*d^2*sinh(1)^2 - 2...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^2}{\sinh\left(c+d\,x\right)^3\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
[Out] int((e + f*x)^2/(sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

3.250 $\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=420

$$\frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d}+\frac{b(e+fx)\coth(c+dx)}{a^2d}-\frac{f\,\text{csch}(c+dx)}{2ad^2}-\frac{(e+fx)\,\text{c}}{}$$

```
[Out] (f*x+e)*arctanh(exp(d*x+c))/a/d-2*b^2*(f*x+e)*arctanh(exp(d*x+c))/a^3/d+b*(
f*x+e)*coth(d*x+c)/a^2/d-1/2*f*csch(d*x+c)/a/d^2-1/2*(f*x+e)*coth(d*x+c)*cs
ch(d*x+c)/a/d-b*f*ln(sinh(d*x+c))/a^2/d^2+1/2*f*polylog(2,-exp(d*x+c))/a/d^
2-b^2*f*polylog(2,-exp(d*x+c))/a^3/d^2-1/2*f*polylog(2,exp(d*x+c))/a/d^2+b^
2*f*polylog(2,exp(d*x+c))/a^3/d^2-b^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))/a^3/d/(a^2+b^2)^(1/2)+b^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^
(1/2)))/a^3/d/(a^2+b^2)^(1/2)-b^3*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1
/2)))/a^3/d^2/(a^2+b^2)^(1/2)+b^3*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/a^3/d^2/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.52, antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5694, 4270, 4267, 2317, 2438, 4269, 3556, 3403, 2296, 2221}

$$-\frac{b^2 f\,\text{Li}_2\left(-e^{c+dx}\right)}{a^3d^2}+\frac{b^2 f\,\text{Li}_2\left(e^{c+dx}\right)}{a^3d^2}-\frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d}-\frac{bf\log(\sinh(c+dx))}{a^2d^2}+\frac{b(e+fx)\coth(c+dx)}{a^2d}-\frac{b^3 f\,\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2\sqrt{a^2+b^2}}+\frac{b^3 f\,\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2\sqrt{a^2+b^2}}-\frac{b^3(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^3d\sqrt{a^2+b^2}}+\frac{b^3(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{a^3d\sqrt{a^2+b^2}}+\frac{f\,\text{Li}_2\left(-e^{c+dx}\right)}{2ad^2}-\frac{f\,\text{Li}_2\left(e^{c+dx}\right)}{2ad^2}-\frac{f\,\text{csch}(c+dx)}{2ad^2}+\frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((e + f*x)*ArcTanh[E^(c + d*x)])/(a*d) - (2*b^2*(e + f*x)*ArcTanh[E^(c + d*
x)])/(a^3*d) + (b*(e + f*x)*Coth[c + d*x])/(a^2*d) - (f*Csch[c + d*x])/(2*a
*d^2) - ((e + f*x)*Coth[c + d*x]*Csch[c + d*x])/(2*a*d) - (b^3*(e + f*x)*Lo
g[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*Sqrt[a^2 + b^2]*d) + (b^
3*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*Sqrt[a^2 +
 b^2]*d) - (b*f*Log[Sinh[c + d*x]])/(a^2*d^2) + (f*PolyLog[2, -E^(c + d*x)]
)/(2*a*d^2) - (b^2*f*PolyLog[2, -E^(c + d*x)])/(a^3*d^2) - (f*PolyLog[2, E^
(c + d*x)])/(2*a*d^2) + (b^2*f*PolyLog[2, E^(c + d*x)])/(a^3*d^2) - (b^3*f*
PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b^2]*
d^2) + (b^3*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*Sq
rt[a^2 + b^2]*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
```

```
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5694

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*(Csch[c + d*x]^(n - 1)/(a + b
*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rubi steps

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\operatorname{csch}^3(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{f\operatorname{csch}(c+dx)}{2ad^2} - \frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{\int (e+fx)\operatorname{csch}(c+}{2a}$$

$$= \frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{b(e+fx)\coth(c+dx)}{a^2d} - \frac{f\operatorname{csch}(c+dx)}{2ad^2} - \frac{(e+f}{}$$

$$= \frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} + \frac{b(e+fx)\coth(c+}{a^2d}$$

$$= \frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} + \frac{b(e+fx)\coth(c+}{a^2d}$$

$$= \frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} + \frac{b(e+fx)\coth(c+}{a^2d}$$

$$= \frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} + \frac{b(e+fx)\coth(c+}{a^2d}$$

$$= \frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} + \frac{b(e+fx)\coth(c+}{a^2d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.13, size = 736, normalized size = 1.75

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((2*b*d*e*Cosh[(c + d*x)/2] - a*f*Cosh[(c + d*x)/2] - 2*b*c*f*Cosh[(c + d*x
)/2] + 2*b*f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(4*a^2*d^2) +
((-(d*e) + c*f - f*(c + d*x))*Csch[(c + d*x)/2]^2)/(8*a*d^2) - (b*f*Log[Sin
h[c + d*x]])/(a^2*d^2) - (e*Log[Tanh[(c + d*x)/2]])/(2*a*d) + (b^2*e*Log[Ta
nh[(c + d*x)/2]])/(a^3*d) + (c*f*Log[Tanh[(c + d*x)/2]])/(2*a*d^2) - (b^2*c
*f*Log[Tanh[(c + d*x)/2]])/(a^3*d^2) + ((I/2)*f*(I*(c + d*x)*(Log[1 - E^(-c
 - d*x)] - Log[1 + E^(-c - d*x)]) + I*(PolyLog[2, -E^(-c - d*x)] - PolyLog[
2, E^(-c - d*x)])))/(a*d^2) - (I*b^2*f*(I*(c + d*x)*(Log[1 - E^(-c - d*x)]
```

```
- Log[1 + E^(-c - d*x)]) + I*(PolyLog[2, -E^(-c - d*x)] - PolyLog[2, E^(-c
- d*x)])))/(a^3*d^2) + (b^3*(2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b
^2]] - 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - f*(c + d*x)*Log
[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2])] - f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2
 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^3*S
qrt[a^2 + b^2]*d^2) + ((-(d*e) + c*f - f*(c + d*x))*Sech[(c + d*x)/2]^2)/(8
*a*d^2) + (Sech[(c + d*x)/2]*(2*b*d*e*Sinh[(c + d*x)/2] + a*f*Sinh[(c + d*x
)/2] - 2*b*c*f*Sinh[(c + d*x)/2] + 2*b*f*(c + d*x)*Sinh[(c + d*x)/2]))/(4*a
^2*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(386) = 772$.

time = 1.74, size = 861, normalized size = 2.05

method	result
risch	$-\frac{adfx\,\mathrm{e}^{3dx+3c}+ade\,\mathrm{e}^{3dx+3c}-2bdfx\,\mathrm{e}^{2dx+2c}+adfx\,\mathrm{e}^{dx+c}+af\,\mathrm{e}^{3dx+3c}-2bde\,\mathrm{e}^{2dx+2c}+ade\,\mathrm{e}^{dx+c}+2bdfx-af\,\mathrm{e}^{dx+c}+2bed}{d^2a^2\left(\mathrm{e}^{2dx+2c}-1\right)^2}+\frac{f\ln(\mathrm{e}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -(a*d*f*x*exp(3*d*x+3*c)+a*d*e*exp(3*d*x+3*c)-2*b*d*f*x*exp(2*d*x+2*c)+a*d*
f*x*exp(d*x+c)+a*f*exp(3*d*x+3*c)-2*b*d*e*exp(2*d*x+2*c)+a*d*e*exp(d*x+c)+2
*b*d*f*x-a*f*exp(d*x+c)+2*b*e*d)/d^2/a^2/(exp(2*d*x+2*c)-1)^2+1/2/d*f/a*ln(
exp(d*x+c)+1)*x-1/2/a/d*e*ln(exp(d*x+c)-1)+1/2/d^2*f*c/a*ln(exp(d*x+c)-1)-2
/d^2/a^3*b^3*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)
^(1/2))+2/d/a^3*b^3*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2
+b^2)^(1/2))-1/d^2/a^3*b^3*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1
/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d^2/a^3*b^3*f/(a^2+b^2)^(1/2)*ln((b*exp(d*
x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d/a^3*b^2*f*ln(exp(d*x+c)+
1)*x-1/d^2/a^3*b^3*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a
)/(-a+(a^2+b^2)^(1/2)))+1/d^2/a^3*b^3*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)
+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d/a^3*b^2*e*ln(exp(d*x+c)+1)+1/d
/a^3*b^2*e*ln(exp(d*x+c)-1)-1/d^2/a^3*b^2*f*c*ln(exp(d*x+c)-1)-1/d^2/a^3*b^
2*f*dilog(exp(d*x+c)+1)-1/d^2/a^3*b^2*f*dilog(exp(d*x+c))+1/2/a/d*e*ln(exp(
d*x+c)+1)-1/d^2/a^2*b*f*ln(exp(d*x+c)+1)-1/d^2/a^2*b*f*ln(exp(d*x+c)-1)+2/d
^2/a^2*b*f*ln(exp(d*x+c))+1/2/d^2/a*f*dilog(exp(d*x+c)+1)+1/2/d^2/a*f*dilog
(exp(d*x+c))-1/d/a^3*b^3*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2
)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d/a^3*b^3*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)
+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")

[Out] -(8*b^3*integrate(1/4*x*e^(d*x + c)/(a^3*b*e^(2*d*x + 2*c) + 2*a^4*e^(d*x + c) - a^3*b), x) + 8*a^2*d*integrate(1/16*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 16*b^2*d*integrate(1/16*x/(a^3*d*e^(d*x + c) + a^3*d), x) + 8*a^2*d*integrate(1/16*x/(a^3*d*e^(d*x + c) - a^3*d), x) - 16*b^2*d*integrate(1/16*x/(a^3*d*e^(d*x + c) - a^3*d), x) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) + 1)/(a^3*d^2)) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) - 1)/(a^3*d^2)) - (2*b*d*x*e^(2*d*x + 2*c) - 2*b*d*x - (a*d*x*e^(3*c) + a*e^(3*c))*e^(3*d*x) - (a*d*x*e^c - a*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2))*f - 1/2*(2*b^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3*d) - 2*(a*e^(-d*x - c) + 2*b*e^(-2*d*x - 2*c) + a*e^(-3*d*x - 3*c) - 2*b)/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - (a^2 - 2*b^2)*log(e^(-d*x - c) + 1)/(a^3*d) + (a^2 - 2*b^2)*log(e^(-d*x - c) - 1)/(a^3*d))*e

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5442 vs. $2(386) = 772$.
time = 0.42, size = 5442, normalized size = 12.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")

[Out] 1/2*(4*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*c*f)*cosh(d*x + c)^4 + 4*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*c*f)*sinh(d*x + c)^4 - 2*((a^4 + a^2*b^2)*d*f*x + (a^4 + a^2*b^2)*d*cosh(1) + (a^4 + a^2*b^2)*d*sinh(1) + (a^4 + a^2*b^2)*f)*cosh(d*x + c)^3 - 2*((a^4 + a^2*b^2)*d*f*x + (a^4 + a^2*b^2)*d*cosh(1) + (a^4 + a^2*b^2)*d*sinh(1) + (a^4 + a^2*b^2)*f - 8*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*c*f)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^3*b + a*b^3)*c*f - 4*(a^3*b + a*b^3)*d*cosh(1) - 4*((a^3*b + a*b^3)*d*f*x + 2*(a^3*b + a*b^3)*c*f - (a^3*b + a*b^3)*d*cosh(1) - (a^3*b + a*b^3)*d*sinh(1))*cosh(d*x + c)^2 - 4*(a^3*b + a*b^3)*d*sinh(1) - 2*(2*(a^3*b + a*b^3)*d*f*x + 4*(a^3*b + a*b^3)*c*f - 2*(a^3*b + a*b^3)*d*cosh(1) - 12*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*c*f)*cosh(d*x + c)^2 - 2*(a^3*b + a*b^3)*d*sinh(1) + 3*((a^4 + a^2*b^2)*d*f*x + (a^4 + a^2*b^2)*d*cosh(1) + (a^4 + a^2*b^2)*d*sinh(1) + (a^4 + a^2*b^2)*f)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(b^4*f*cosh(d*x + c)^4 + 4*b^4*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^4*f*sinh(d*x + c)^4 - 2*b^4*f*cosh(d*x + c)^2 + b^4*f + 2*(3*b^4*f*cosh(d*x + c)^2 - b^4*f)*sinh(d*x + c)^2 + 4*(b^4*f*cosh(d*x + c)^3 - b^4*f*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b^4*f*cosh(d*x + c)^4 + 4*b^4*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^4*f*sinh(d*x

+ c)^4 - 2*b^4*f*cosh(d*x + c)^2 + b^4*f + 2*(3*b^4*f*cosh(d*x + c)^2 - b^4
*f)*sinh(d*x + c)^2 + 4*(b^4*f*cosh(d*x + c)^3 - b^4*f*cosh(d*x + c))*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) -
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(
b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1) + (b^4*c*f - b^4*d*cosh(1) - b^4*d*
sinh(1))*cosh(d*x + c)^4 + 4*(b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*cosh
(d*x + c)*sinh(d*x + c)^3 + (b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*sinh(
d*x + c)^4 - 2*(b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*cosh(d*x + c)^2 -
2*(b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1) - 3*(b^4*c*f - b^4*d*cosh(1) - b
^4*d*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*c*f - b^4*d*cosh(1
) - b^4*d*sinh(1))*cosh(d*x + c)^3 - (b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(
1))*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x +
c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(b^4*c*f - b^
4*d*cosh(1) - b^4*d*sinh(1) + (b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*cos
h(d*x + c)^4 + 4*(b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*cosh(d*x + c)*si
nh(d*x + c)^3 + (b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*sinh(d*x + c)^4 -
2*(b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*cosh(d*x + c)^2 - 2*(b^4*c*f -
b^4*d*cosh(1) - b^4*d*sinh(1) - 3*(b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1)
)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*c*f - b^4*d*cosh(1) - b^4*d*si
nh(1))*cosh(d*x + c)^3 - (b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*cosh(d*x
+ c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sin
h(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(b^4*d*f*x + b^4*c*f + (b
^4*d*f*x + b^4*c*f)*cosh(d*x + c)^4 + 4*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)
*sinh(d*x + c)^3 + (b^4*d*f*x + b^4*c*f)*sinh(d*x + c)^4 - 2*(b^4*d*f*x + b
^4*c*f)*cosh(d*x + c)^2 - 2*(b^4*d*f*x + b^4*c*f - 3*(b^4*d*f*x + b^4*c*f)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^3
- (b^4*d*f*x + b^4*c*f)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2
)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(b^4*d*f*x + b^4*c*f + (b^4*d*f*x +
b^4*c*f)*cosh(d*x + c)^4 + 4*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)*sinh(d*x +
c)^3 + (b^4*d*f*x + b^4*c*f)*sinh(d*x + c)^4 - 2*(b^4*d*f*x + b^4*c*f)*cos
h(d*x + c)^2 - 2*(b^4*d*f*x + b^4*c*f - 3*(b^4*d*f*x + b^4*c*f)*cosh(d*x +
c)^2)*sinh(d*x + c)^2 + 4*((b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^3 - (b^4*d*f
*x + b^4*c*f)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*c
osh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((
a^2 + b^2)/b^2) - b)/b) - 2*((a^4 + a^2*b^2)*d*f*x + (a^4 + a^2*b^2)*d*cosh
(1) + (a^4 + a^2*b^2)*d*sinh(1) - (a^4 + a^2*b^2)*f)*cosh(d*x + c) - ((a^4
- a^2*b^2 - 2*b^4)*f*cosh(d*x + c)^4 + 4*(a^4 - a^2*b^2 - 2*b^4)*f*cosh(d*x
+ c)*sinh(d*x + c)^3 + (a^4 - a^2*b^2 - 2*b^4)*f*sinh(d*x + c)^4 - 2*(a^4
- a^2*b^2 - 2*b^4)*f*cosh(d*x + c)^2 + 2*(3*(a^4 - a^2*b^2 - 2*b^4)*f*cosh(
d*x + c)^2 - (a^4 - a^2*b^2 - 2*b^4)*f)*sinh(d*x + c)^2 + (a^4 - a^2*b^2 -
2*b^4)*f + 4*((a^4 - a^2*b^2 - 2*b^4)*f*cosh(d*x + c)^3 - (a^4 - a^2*b^2 -
2*b^4)*f*cosh(d*x + c))*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c))
+ ((a^4 - a^2*b^2 - 2*b^4)*f*cosh(d*x + c)^4 + 4*(a^4 - a^2*b^2 - 2*b^4)*f
*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 - a^2*b^2 - 2*b^4)*f*sinh(d*x + c)^4

```
- 2*(a^4 - a^2*b^2 - 2*b^4)*f*cosh(d*x + c)^2 + 2*(3*(a^4 - a^2*b^2 - 2*b^4
)*f*cosh(d*x + c)^2 - (a^4 - a^2*b^2 - 2*b^4)*f)*sinh(d*x + c)^2 + (a^4 - a
^2*b^2 - 2*b^4)*f + 4*((a^4 - a^2*b^2 - 2*b^4)*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**3/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)*csch(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\sinh(c+d\,x)^3\,(a+b\sinh(c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
[Out] int((e + f*x)/(sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

3.251 $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=113

$$\frac{(a^2-2b^2)\tanh^{-1}(\cosh(c+dx))}{2a^3d}+\frac{2b^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}\,d}+\frac{b\coth(c+dx)}{a^2d}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

```
[Out] 1/2*(a^2-2*b^2)*arctanh(cosh(d*x+c))/a^3/d+b*coth(d*x+c)/a^2/d-1/2*coth(d*x
+c)*csch(d*x+c)/a/d+2*b^3*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2)
)/a^3/d/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2881, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{b\coth(c+dx)}{a^2d}+\frac{(a^2-2b^2)\tanh^{-1}(\cosh(c+dx))}{2a^3d}+\frac{2b^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3d\sqrt{a^2+b^2}}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((a^2 - 2*b^2)*ArcTanh[Cosh[c + d*x]])/(2*a^3*d) + (2*b^3*ArcTanh[(b - a*Ta
nh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^3*Sqrt[a^2 + b^2]*d) + (b*Coth[c + d*
x])/(a^2*d) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} + \frac{i\int \frac{\operatorname{csch}^2(c+dx)(2ib+ia\sinh(c+dx)+ib\sinh^2(c+dx))}{a+b\sinh(c+dx)}\,dx}{2a}$$

$$= \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{\int \frac{\operatorname{csch}(c+dx)(a^2-2b^2+ab\sinh(c+dx))}{a+b\sinh(c+dx)}\,dx}{2a^2}$$

$$= \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{b^3\int \frac{1}{a+b\sinh(c+dx)}\,dx}{a^3} - \frac{(a^2-2b^2)}{}$$

$$= \frac{(a^2-2b^2)\tanh^{-1}(\cosh(c+dx))}{2a^3d} + \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

$$= \frac{(a^2-2b^2)\tanh^{-1}(\cosh(c+dx))}{2a^3d} + \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

$$= \frac{(a^2-2b^2)\tanh^{-1}(\cosh(c+dx))}{2a^3d} + \frac{2b^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}\,d} + \frac{b\coth(c+}{a^2d}$$

Mathematica [A]
time = 1.36, size = 145, normalized size = 1.28

$$-\frac{\frac{16b^3\operatorname{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 4ab\coth\left(\frac{1}{2}(c+dx)\right) + a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right) + 4(a^2-2b^2)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right) - 4ab\tanh\left(\frac{1}{2}(c+dx)\right)}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/8*((16*b^3*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2
 - b^2] - 4*a*b*Coth[(c + d*x)/2] + a^2*Csch[(c + d*x)/2]^2 + 4*(a^2 - 2*b^
2)*Log[Tanh[(c + d*x)/2]] + a^2*Sech[(c + d*x)/2]^2 - 4*a*b*Tanh[(c + d*x)/
2])/(a^3*d)
```

Maple [A]
time = 1.19, size = 142, normalized size = 1.26

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2} - \frac{2b^3\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} - \frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2} + \frac{\left(-2a^2+4b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}}{d}$
default	$\frac{\frac{\frac{a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2} - \frac{2b^3\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} - \frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2} + \frac{\left(-2a^2+4b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}}{d}$

risch	$-\frac{a\,\mathrm{e}^{3dx+3c}-2b\,\mathrm{e}^{2dx+2c}+a\,\mathrm{e}^{dx+c}+2b}{a^2 d\left(\mathrm{e}^{2dx+2c}-1\right)^2}+\frac{b^3\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d\,a^3}-\frac{b^3\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}-a^2-}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a^2*(1/2*a*tanh(1/2*d*x+1/2*c)^2+2*b*tanh(1/2*d*x+1/2*c))-2/a^3*b^
3/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)
)-1/8/a/tanh(1/2*d*x+1/2*c)^2+1/4/a^3*(-2*a^2+4*b^2)*ln(tanh(1/2*d*x+1/2*c)
)+1/2*b/a^2/tanh(1/2*d*x+1/2*c))
```

Maxima [**A**]
time = 0.48, size = 211, normalized size = 1.87

$$-\frac{b^3\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a^3 d}+\frac{ae^{(-dx-c)}+2\,be^{(-2\,dx-2\,c)}+ae^{(-3\,dx-3\,c)}-2\,b}{\left(2\,a^2e^{(-2\,dx-2\,c)}-a^2e^{(-4\,dx-4\,c)}-a^2\right)d}+\frac{\left(a^2-2\,b^2\right)\log\left(e^{(-dx-c)}+1\right)}{2\,a^3 d}-\frac{\left(a^2-2\,b^2\right)\log\left(e^{(-dx-c)}-1\right)}{2\,a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -b^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3*d) + (a*e^(-d*x - c) + 2*b*e^(-2*d*x - 2*
c) + a*e^(-3*d*x - 3*c) - 2*b)/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4
*c) - a^2)*d) + 1/2*(a^2 - 2*b^2)*log(e^(-d*x - c) + 1)/(a^3*d) - 1/2*(a^2
- 2*b^2)*log(e^(-d*x - c) - 1)/(a^3*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1203 vs. $2(106) = 212$.
time = 0.43, size = 1203, normalized size = 10.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^3*b + 4*a*b^3 + 2*(a^4 + a^2*b^2)*cosh(d*x + c)^3 + 2*(a^4 + a^2*
b^2)*sinh(d*x + c)^3 - 4*(a^3*b + a*b^3)*cosh(d*x + c)^2 - 2*(2*a^3*b + 2*a
*b^3 - 3*(a^4 + a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(b^3*cosh(d*x +
 c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh(d*x + c)^4 - 2*b^3*c
osh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^2 + 4*
(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)*lo
g((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2
+ b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*co
sh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
```

+ 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*(a^4
+ a^2*b^2)*cosh(d*x + c) - ((a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)^4 + 4*(a^
4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 - a^2*b^2 - 2*b^4
)*sinh(d*x + c)^4 + a^4 - a^2*b^2 - 2*b^4 - 2*(a^4 - a^2*b^2 - 2*b^4)*cosh(
d*x + c)^2 - 2*(a^4 - a^2*b^2 - 2*b^4 - 3*(a^4 - a^2*b^2 - 2*b^4)*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 + 4*((a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)^3 - (a^4
 - a^2*b^2 - 2*b^4)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(
d*x + c) + 1) + ((a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)^4 + 4*(a^4 - a^2*b^2
 - 2*b^4)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 - a^2*b^2 - 2*b^4)*sinh(d*x
+ c)^4 + a^4 - a^2*b^2 - 2*b^4 - 2*(a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)^2
- 2*(a^4 - a^2*b^2 - 2*b^4 - 3*(a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)^2)*sin
h(d*x + c)^2 + 4*((a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)^3 - (a^4 - a^2*b^2
- 2*b^4)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) -
1) + 2*(a^4 + a^2*b^2 + 3*(a^4 + a^2*b^2)*cosh(d*x + c)^2 - 4*(a^3*b + a*b^
3)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a^
5 + a^3*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^5 + a^3*b^2)*d*sinh(d*x +
 c)^4 - 2*(a^5 + a^3*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^5 + a^3*b^2)*d*cosh(d
*x + c)^2 - (a^5 + a^3*b^2)*d)*sinh(d*x + c)^2 + (a^5 + a^3*b^2)*d + 4*((a^
5 + a^3*b^2)*d*cosh(d*x + c)^3 - (a^5 + a^3*b^2)*d*cosh(d*x + c))*sinh(d*x
+ c))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)),x)

[Out] Integral(csch(c + d*x)**3/(a + b*sinh(c + d*x)), x)

Giac [A]
time = 0.43, size = 176, normalized size = 1.56

$$-\frac{\frac{2\,b^3\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a^3}-\frac{(a^2-2\,b^2)\log\left(e^{(dx+c)}+1\right)}{a^3}+\frac{(a^2-2\,b^2)\log\left(\left|e^{(dx+c)}-1\right|\right)}{a^3}+\frac{2\,\left(ae^{(3\,dx+3\,c)}-2\,be^{(2\,dx+2\,c)}+ae^{(dx+c)}+2\,b\right)}{a^2\left(e^{(2\,dx+2\,c)}-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")

[Out] -1/2*(2*b^3*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d
*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3) - (a^2 - 2*b^2)*l
og(e^(d*x + c) + 1)/a^3 + (a^2 - 2*b^2)*log(abs(e^(d*x + c) - 1))/a^3 + 2*(

a*e^(3*d*x + 3*c) - 2*b*e^(2*d*x + 2*c) + a*e^(d*x + c) + 2*b)/(a^2*(e^(2*d
*x + 2*c) - 1)^2))/d

Mupad [B]
time = 0.75, size = 776, normalized size = 6.87

$$\frac{e^{c+dx}}{ad-a\,d\,e^{2c+2dx}}-\frac{2\,e^{c+dx}}{ad-2\,a\,d\,e^{2c+2dx}+a\,d\,e^{4c+4dx}}-\frac{2\,b}{a^2\,d-a^2\,d\,e^{2c+2dx}}-\frac{\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2-4\,a^4\,e^{dx}\,e^c-24\,b^4\,e^{dx}\,e^c+20\,a^2\,b^2\,e^{dx}\,e^c\right)}{2\,a\,d}+\frac{\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2+4\,a^4\,e^{dx}\,e^c+24\,b^4\,e^{dx}\,e^c-20\,a^2\,b^2\,e^{dx}\,e^c\right)}{2\,a\,d}-\frac{b^3\,\ln\left(16\,a^5\,b-48\,a\,b^5-24\,b^5\,\sqrt{a^2+b^2}-32\,a^3\,b^3-40\,a^2\,b^3\,\sqrt{a^2+b^2}-32\,a^6\,e^{dx}\,e^c+24\,b^6\,e^{dx}\,e^c+16\,a^4\,b\,\sqrt{a^2+b^2}+112\,a^2\,b^4\,e^{dx}\,e^c+56\,a^4\,b^2\,e^{dx}\,e^c-32\,a^5\,e^{dx}\,e^c\,\sqrt{a^2+b^2}+72\,a\,b^4\,e^{dx}\,e^c\,\sqrt{a^2+b^2}+72\,a^3\,b^2\,e^{dx}\,e^c\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{d\,a^5+d\,a^3\,b^2}+\frac{b^3\,\ln\left(24\,b^5\,\sqrt{a^2+b^2}-48\,a\,b^5+16\,a^5\,b-32\,a^3\,b^3+40\,a^2\,b^3\,\sqrt{a^2+b^2}-32\,a^6\,e^{dx}\,e^c+24\,b^6\,e^{dx}\,e^c-16\,a^4\,b\,\sqrt{a^2+b^2}+112\,a^2\,b^4\,e^{dx}\,e^c+56\,a^4\,b^2\,e^{dx}\,e^c+32\,a^5\,e^{dx}\,e^c\,\sqrt{a^2+b^2}-72\,a\,b^4\,e^{dx}\,e^c\,\sqrt{a^2+b^2}-72\,a^3\,b^2\,e^{dx}\,e^c\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{d\,a^5+d\,a^3\,b^2}+\frac{b^2\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2-4\,a^4\,e^{dx}\,e^c-24\,b^4\,e^{dx}\,e^c+20\,a^2\,b^2\,e^{dx}\,e^c\right)}{a^3\,d}-\frac{b^2\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2+4\,a^4\,e^{dx}\,e^c+24\,b^4\,e^{dx}\,e^c-20\,a^2\,b^2\,e^{dx}\,e^c\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

```
[Out] exp(c + d*x)/(a*d - a*d*exp(2*c + 2*d*x)) - (2*exp(c + d*x))/(a*d - 2*a*d*e
xp(2*c + 2*d*x) + a*d*exp(4*c + 4*d*x)) - (2*b)/(a^2*d - a^2*d*exp(2*c + 2*
d*x)) - log(4*a^4 + 24*b^4 - 20*a^2*b^2 - 4*a^4*exp(d*x)*exp(c) - 24*b^4*ex
p(d*x)*exp(c) + 20*a^2*b^2*exp(d*x)*exp(c))/(2*a*d) + log(4*a^4 + 24*b^4 -
20*a^2*b^2 + 4*a^4*exp(d*x)*exp(c) + 24*b^4*exp(d*x)*exp(c) - 20*a^2*b^2*ex
p(d*x)*exp(c))/(2*a*d) - (b^3*log(16*a^5*b - 48*a*b^5 - 24*b^5*(a^2 + b^2)^
(1/2) - 32*a^3*b^3 - 40*a^2*b^3*(a^2 + b^2)^(1/2) - 32*a^6*exp(d*x)*exp(c)
+ 24*b^6*exp(d*x)*exp(c) + 16*a^4*b*(a^2 + b^2)^(1/2) + 112*a^2*b^4*exp(d*x
)*exp(c) + 56*a^4*b^2*exp(d*x)*exp(c) - 32*a^5*exp(d*x)*exp(c)*(a^2 + b^2)^
(1/2) + 72*a*b^4*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) + 72*a^3*b^2*exp(d*x)*ex
p(c)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^5*d + a^3*b^2*d) + (b^3*log(2
4*b^5*(a^2 + b^2)^(1/2) - 48*a*b^5 + 16*a^5*b - 32*a^3*b^3 + 40*a^2*b^3*(a^
2 + b^2)^(1/2) - 32*a^6*exp(d*x)*exp(c) + 24*b^6*exp(d*x)*exp(c) - 16*a^4*b
*(a^2 + b^2)^(1/2) + 112*a^2*b^4*exp(d*x)*exp(c) + 56*a^4*b^2*exp(d*x)*exp(
c) + 32*a^5*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) - 72*a*b^4*exp(d*x)*exp(c)*(a
^2 + b^2)^(1/2) - 72*a^3*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 + b^2)
^(1/2))/(a^5*d + a^3*b^2*d) + (b^2*log(4*a^4 + 24*b^4 - 20*a^2*b^2 - 4*a^4*
exp(d*x)*exp(c) - 24*b^4*exp(d*x)*exp(c) + 20*a^2*b^2*exp(d*x)*exp(c)))/(a^
3*d) - (b^2*log(4*a^4 + 24*b^4 - 20*a^2*b^2 + 4*a^4*exp(d*x)*exp(c) + 24*b^
4*exp(d*x)*exp(c) - 20*a^2*b^2*exp(d*x)*exp(c)))/(a^3*d)
```

3.252 $\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csch[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][Csch[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 124.51, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csch[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Integrate[Csch[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(dx+c)^3}{(fx+e)\,(a+b\sinh\,(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -8*b^3*integrate(-1/4*e^(d*x + c)/(a^3*b*f*x + a^3*b*e - (a^3*b*f*x*e^(2*c)
 + a^3*b*e^(2*c + 1))*e^(2*d*x) - 2*(a^4*f*x*e^c + a^4*e^(c + 1))*e^(d*x)),
 x) - (2*b*d*f*x + 2*b*d*e + (a*d*f*x*e^(3*c) - a*f*e^(3*c) + a*d*e^(3*c +
1))*e^(3*d*x) - 2*(b*d*f*x*e^(2*c) + b*d*e^(2*c + 1))*e^(2*d*x) + (a*d*f*x*
e^c + a*d*e^(c + 1) + a*f*e^c)*e^(d*x))/(a^2*d^2*f^2*x^2 + 2*a^2*d^2*f*x*e
+ a^2*d^2*e^2 + (a^2*d^2*f^2*x^2*e^(4*c) + 2*a^2*d^2*f*x*e^(4*c + 1) + a^2*
d^2*e^(4*c + 2))*e^(4*d*x) - 2*(a^2*d^2*f^2*x^2*e^(2*c) + 2*a^2*d^2*f*x*e^(
2*c + 1) + a^2*d^2*e^(2*c + 2))*e^(2*d*x)) - 8*integrate(1/16*(2*a*b*d*f*e
+ 2*a^2*f^2 - (a^2*d^2*f^2 - 2*b^2*d^2*f^2)*x^2 + 2*(a*b*d*f^2 - (a^2*d^2*f
 - 2*b^2*d^2*f)*e)*x - (a^2*d^2 - 2*b^2*d^2)*e^2)/(a^3*d^2*f^3*x^3 + 3*a^3*
d^2*f^2*x^2*e + 3*a^3*d^2*f*x*e^2 + a^3*d^2*e^3 - (a^3*d^2*f^3*x^3*e^c + 3*
a^3*d^2*f^2*x^2*e^(c + 1) + 3*a^3*d^2*f*x*e^(c + 2) + a^3*d^2*e^(c + 3))*e^
(d*x)), x) - 8*integrate(1/16*(2*a*b*d*f*e - 2*a^2*f^2 + (a^2*d^2*f^2 - 2*b
^2*d^2*f^2)*x^2 + 2*(a*b*d*f^2 + (a^2*d^2*f - 2*b^2*d^2*f)*e)*x + (a^2*d^2
- 2*b^2*d^2)*e^2)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*f^2*x^2*e + 3*a^3*d^2*f*x*e^
2 + a^3*d^2*e^3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*f^2*x^2*e^(c + 1) + 3*a^
3*d^2*f*x*e^(c + 2) + a^3*d^2*e^(c + 3))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(csch(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{3}(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(csch(c + d*x)**3/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sinh\left(c+d\,x\right)^3\,\left(e+f\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(1/(sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.253 $\int \frac{(e+fx)^3 \cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=139

$$\frac{i(e+fx)^4}{4af}-\frac{2i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad}-\frac{6if(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2}+\frac{12if^2(e+fx)\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^3}$$

```
[Out] 1/4*I*(f*x+e)^4/a/f-2*I*(f*x+e)^3*ln(1+I*exp(d*x+c))/a/d-6*I*f*(f*x+e)^2*po
lylog(2,-I*exp(d*x+c))/a/d^2+12*I*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/a/d^
3-12*I*f^3*polylog(4,-I*exp(d*x+c))/a/d^4
```

Rubi [A]
time = 0.15, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {5678, 2221, 2611, 6744, 2320, 6724}

$$-\frac{12if^3\text{Li}_4\left(-ie^{c+dx}\right)}{ad^4}+\frac{12if^2(e+fx)\text{Li}_3\left(-ie^{c+dx}\right)}{ad^3}-\frac{6if(e+fx)^2\text{Li}_2\left(-ie^{c+dx}\right)}{ad^2}-\frac{2i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad}+\frac{i(e+fx)^4}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((I/4)*(e + f*x)^4)/(a*f) - ((2*I)*(e + f*x)^3*Log[1 + I*E^(c + d*x)])/(a*d
) - ((6*I)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) + ((12*I)*f^
2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^3) - ((12*I)*f^3*PolyLog[4,
(-I)*E^(c + d*x)])/(a*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5678

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + Dist[2, Int[(e + f*x)^m*(E^(c + d*x)/(a + b*E^(c + d*x))), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[a^2 + b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i(e+fx)^4}{4af}+2\int \frac{e^{c+dx}(e+fx)^3}{a+iae^{c+dx}}\,dx$$
$$= \frac{i(e+fx)^4}{4af}-\frac{2i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad}+\frac{(6if)\int(e+fx)^2\log\left(1+ie^{c+\cdots}\right.}{ad}$$
$$= \frac{i(e+fx)^4}{4af}-\frac{2i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad}-\frac{6if(e+fx)^2\mathrm{Li}_2\left(-ie^{c+dx}\right)}{ad^2}+\frac{(\cdots}{\cdots}$$
$$= \frac{i(e+fx)^4}{4af}-\frac{2i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad}-\frac{6if(e+fx)^2\mathrm{Li}_2\left(-ie^{c+dx}\right)}{ad^2}+\frac{1\cdots}{\cdots}$$
$$= \frac{i(e+fx)^4}{4af}-\frac{2i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad}-\frac{6if(e+fx)^2\mathrm{Li}_2\left(-ie^{c+dx}\right)}{ad^2}+\frac{1\cdots}{\cdots}$$
$$= \frac{i(e+fx)^4}{4af}-\frac{2i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad}-\frac{6if(e+fx)^2\mathrm{Li}_2\left(-ie^{c+dx}\right)}{ad^2}+\frac{1\cdots}{\cdots}$$

Mathematica [A]

time = 0.06, size = 118, normalized size = 0.85

$$\frac{i\left(\frac{(e+fx)^4}{f}-\frac{8(e+fx)^3\log(1+ie^{c+dx})}{d}-\frac{24f\left(d^2(e+fx)^2\text{PolyLog}(2,-ie^{c+dx})-2df(e+fx)\text{PolyLog}(3,-ie^{c+dx})+2f^2\text{PolyLog}(4,-ie^{c+dx})\right)}{d^4}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((I/4)*((e + f*x)^4/f - (8*(e + f*x)^3*Log[1 + I*E^(c + d*x)])/d - (24*f*(d
^2*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)] - 2*d*f*(e + f*x)*PolyLog[3, (-
I)*E^(c + d*x)] + 2*f^2*PolyLog[4, (-I)*E^(c + d*x)]))/d^4))/a
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 646 vs. 2(124) = 248.
time = 2.99, size = 647, normalized size = 4.65

method	result
risch	$\frac{i f^3 x^4}{4a}-\frac{12 i f^3 \text{polylog}(4,-i e^{dx+c})}{a\, d^4}+\frac{2 i \ln(e^{dx+c}) e^3}{da}-\frac{2 i \ln(e^{dx+c}-i) e^3}{da}+\frac{3 i f^3 c^4}{2 d^4 a}+\frac{3 i f\, e^2 x^2}{2a}+\frac{i f^2 e\, x^3}{a}+\frac{2 i f^3 c^3 \ln(e^{dx+}}{d^4 a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -12*I*f^3*polylog(4,-I*exp(d*x+c))/a/d^4+2*I/d^4/a*f^3*c^3*ln(exp(d*x+c)-I)
+2*I/d^3/a*f^3*c^3*x-6*I/d^2/a*f^3*polylog(2,-I*exp(d*x+c))*x^2-2*I/d^4/a*f
^3*c^3*ln(1+I*exp(d*x+c))+12*I/d^3/a*f^3*polylog(3,-I*exp(d*x+c))*x-2*I/d/a
*f^3*ln(1+I*exp(d*x+c))*x^3+12*I/d^3/a*f^2*e*polylog(3,-I*exp(d*x+c))+3*I/d
^2/a*f*e^2*c^2-4*I/d^3/a*f^2*e*c^3-2*I/d^4/a*f^3*c^3*ln(exp(d*x+c))-6*I/d^2
/a*f*e^2*polylog(2,-I*exp(d*x+c))+1/4*I/a*f^3*x^4-I/a*e^3*x-1/4*I/a/f*e^4+2
*I/d/a*ln(exp(d*x+c))*e^3-2*I/d/a*ln(exp(d*x+c)-I)*e^3+3/2*I/d^4/a*f^3*c^4+
3/2*I/a*f*e^2*x^2+I/a*f^2*e*x^3-6*I/d^2/a*f*e^2*ln(1+I*exp(d*x+c))*c+6*I/d^
3/a*f^2*c^2*e*ln(1+I*exp(d*x+c))+6*I/d^3/a*f^2*c^2*e*ln(exp(d*x+c))-6*I/d^2
/a*f^2*e*c^2*x+6*I/d/a*f*e^2*c*x-6*I/d^2/a*f*c*e^2*ln(exp(d*x+c))-6*I/d^3/a
*f^2*c^2*e*ln(exp(d*x+c)-I)+6*I/d^2/a*f*c*e^2*ln(exp(d*x+c)-I)-6*I/d/a*f^2*
e*ln(1+I*exp(d*x+c))*x^2-12*I/d^2/a*f^2*e*polylog(2,-I*exp(d*x+c))*x-6*I/d/
a*f*e^2*ln(1+I*exp(d*x+c))*x
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(118) = 236.
time = 0.33, size = 263, normalized size = 1.89

$$-\frac{i\,(f^3x^4+4\,f^2x^3e+6\,fx^2e^2)}{4\,a}-\frac{6i\,\left(dx\log\left(i\,e^{(dx+c)}+1\right)+\text{Li}_2(-i\,e^{(dx+c)})\right)fe^2}{ad^2}-\frac{6i\,\left(d^2x^2\log\left(i\,e^{(dx+c)}+1\right)+2\,dx\text{Li}_2(-i\,e^{(dx+c)})-2\,\text{Li}_3(-i\,e^{(dx+c)})\right)f^2e}{ad^3}-\frac{i\,e^3\log\left(i\,a\sinh\left(dx+c\right)+a\right)}{ad}-\frac{2i\,\left(d^3x^3\log\left(i\,e^{(dx+c)}+1\right)+3\,d^2x^2\text{Li}_2(-i\,e^{(dx+c)})-6\,dx\text{Li}_3(-i\,e^{(dx+c)})+6\,\text{Li}_4(-i\,e^{(dx+c)})\right)f^3}{ad^4}+\frac{i\,d^4f^3x^4+4i\,d^4f^2x^3e+6i\,d^4fx^2e^2}{2\,ad^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*I*(f^3*x^4 + 4*f^2*x^3*e + 6*f*x^2*e^2)/a - 6*I*(d*x*log(I*e^(d*x + c)
 + 1) + dilog(-I*e^(d*x + c)))*f*e^2/(a*d^2) - 6*I*(d^2*x^2*log(I*e^(d*x +
c) + 1) + 2*d*x*dilog(-I*e^(d*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^2*e
/(a*d^3) - I*e^3*log(I*a*sinh(d*x + c) + a)/(a*d) - 2*I*(d^3*x^3*log(I*e^(d
*x + c) + 1) + 3*d^2*x^2*dilog(-I*e^(d*x + c)) - 6*d*x*polylog(3, -I*e^(d*x
 + c)) + 6*polylog(4, -I*e^(d*x + c)))*f^3/(a*d^4) + 1/2*(I*d^4*f^3*x^4 + 4
*I*d^4*f^2*x^3*e + 6*I*d^4*f*x^2*e^2)/(a*d^4)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(118) = 236$.
time = 0.35, size = 302, normalized size = 2.17

$$\frac{i d^4 f^3 x^4 - 2i c^4 f^3 - 48i f^3 \mathrm{polylog}\left(4, -i e^{(dx+c)}\right) - 24\left(i d^2 f^3 x^2 + 2i d^2 f^2 x e + i d^2 f e^2\right) \mathrm{Li}_2\left(-i e^{(dx+c)}\right) - 4\left(-i d^4 x - 2i c d^3\right) e^3 - 6\left(-i d^4 f x^2 + 2i c^2 d^2 f\right) e^2 - 4\left(-i d^4 f^2 x^3 - 2i c^3 d f^2\right) e - 8\left(-i c^3 f^3 + 3i c^2 d f^2 e - 3i c d^2 f e^2 + i d^3 e^3\right) \log\left(e^{(dx+c)} - i\right) - 8\left(i d^3 f^3 x^3 + i c^3 f^3 + 3\left(i d^3 f x + i c d^2 f\right) e^2 + 3\left(i d^3 f^2 x^2 - i c^2 d f^2\right) e\right) \log\left(i e^{(dx+c)} + 1\right) - 48\left(-i d f^3 x - i d f^2 e\right) \mathrm{polylog}\left(3, -i e^{(dx+c)}\right)}{4 a d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(I*d^4*f^3*x^4 - 2*I*c^4*f^3 - 48*I*f^3*polylog(4, -I*e^(d*x + c)) - 24
*(I*d^2*f^3*x^2 + 2*I*d^2*f^2*x*e + I*d^2*f*e^2)*dilog(-I*e^(d*x + c)) - 4*
(-I*d^4*x - 2*I*c*d^3)*e^3 - 6*(-I*d^4*f*x^2 + 2*I*c^2*d^2*f)*e^2 - 4*(-I*d
^4*f^2*x^3 - 2*I*c^3*d*f^2)*e - 8*(-I*c^3*f^3 + 3*I*c^2*d*f^2*e - 3*I*c*d^2
*f*e^2 + I*d^3*e^3)*log(e^(d*x + c) - I) - 8*(I*d^3*f^3*x^3 + I*c^3*f^3 + 3
*(I*d^3*f*x + I*c*d^2*f)*e^2 + 3*(I*d^3*f^2*x^2 - I*c^2*d*f^2)*e)*log(I*e^(
d*x + c) + 1) - 48*(-I*d*f^3*x - I*d*f^2*e)*polylog(3, -I*e^(d*x + c)))/(a*
d^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{e^3 \cosh(c+dx)}{\sinh(c+dx)-i}\, dx + \int \frac{f^3 x^3 \cosh(c+dx)}{\sinh(c+dx)-i}\, dx + \int \frac{3 e f^2 x^2 \cosh(c+dx)}{\sinh(c+dx)-i}\, dx + \int \frac{3 e^2 f x \cosh(c+dx)}{\sinh(c+dx)-i}\, dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*(Integral(e**3*cosh(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f**3*x**
3*cosh(c + d*x)/(sinh(c + d*x) - I), x) + Integral(3*e*f**2*x**2*cosh(c + d
*x)/(sinh(c + d*x) - I), x) + Integral(3*e**2*f*x*cosh(c + d*x)/(sinh(c + d
*x) - I), x))/a
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^3*cosh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)^3}{a+a\,\sinh\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i),x)
[Out] int((cosh(c + d*x)*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i), x)
```

3.254 $\int \frac{(e+fx)^2 \cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=106

$$\frac{i(e+fx)^3}{3af}-\frac{2i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad}-\frac{4if(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2}+\frac{4if^2\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^3}$$

```
[Out] 1/3*I*(f*x+e)^3/a/f-2*I*(f*x+e)^2*ln(1+I*exp(d*x+c))/a/d-4*I*f*(f*x+e)*poly
log(2,-I*exp(d*x+c))/a/d^2+4*I*f^2*polylog(3,-I*exp(d*x+c))/a/d^3
```

Rubi [A]
time = 0.13, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {5678, 2221, 2611, 2320, 6724}

$$\frac{4if^2\text{Li}_3\left(-ie^{c+dx}\right)}{ad^3}-\frac{4if(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{ad^2}-\frac{2i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad}+\frac{i(e+fx)^3}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((I/3)*(e + f*x)^3)/(a*f) - ((2*I)*(e + f*x)^2*Log[1 + I*E^(c + d*x)])/(a*d
) - ((4*I)*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) + ((4*I)*f^2*P
olyLog[3, (-I)*E^(c + d*x)])/(a*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
f, g, n}, x] && GtQ[m, 0]
```

Rule 5678

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + Dist[2, Int[(e + f*x)^m*(E^(c + d*x)/(a + b*E^(c + d*x))), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[a^2 + b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)^2\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx &= \frac{i(e+fx)^3}{3af}+2\int \frac{e^{c+dx}(e+fx)^2}{a+iae^{c+dx}}\,dx\\ &= \frac{i(e+fx)^3}{3af}-\frac{2i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad}+\frac{(4if)\int(e+fx)\log\left(1+ie^{c+dx}\right)}{ad}\\ &= \frac{i(e+fx)^3}{3af}-\frac{2i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad}-\frac{4if(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{ad^2}+\frac{(4i\\ &= \frac{i(e+fx)^3}{3af}-\frac{2i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad}-\frac{4if(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{ad^2}+\frac{(4i}{}\\ &= \frac{i(e+fx)^3}{3af}-\frac{2i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad}-\frac{4if(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{ad^2}+\frac{4if}{}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 94, normalized size = 0.89

$$\frac{i\left(d^2(e+fx)^2\left(d(e+fx)-6f\log\left(1+ie^{c+dx}\right)\right)-12df^2(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)+12f^3\text{PolyLog}\left(3,-ie^{c+dx}\right)\right)}{3ad^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((I/3)*(d^2*(e + f*x)^2*(d*(e + f*x) - 6*f*Log[1 + I*E^(c + d*x)]) - 12*d*f
^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)] + 12*f^3*PolyLog[3, (-I)*E^(c + d
*x)]))/(a*d^3*f)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(94) = 188$.
time = 1.56, size = 405, normalized size = 3.82

method	result
risch	$\frac{ife\,x^2}{a} - \frac{4ifce\ln(e^{dx+c})}{d^2a} - \frac{2if^2c^2x}{d^2a} - \frac{2if^2\ln(1+ie^{dx+c})x^2}{da} - \frac{4ife\ln(1+ie^{dx+c})x}{da} + \frac{if^2x^3}{3a} - \frac{2i\ln(e^{dx+c}-i)e^2}{da} - \frac{ie^3}{3af}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] I/a*f*e*x^2-4*I/d^2/a*f*e*polylog(2,-I*exp(d*x+c))-4*I/d^2/a*f*c*e*ln(exp(d
*x+c))-2*I/d^2/a*f^2*c^2*x-2*I/d/a*f^2*ln(1+I*exp(d*x+c))*x^2-4*I/d/a*f*e*l
n(1+I*exp(d*x+c))*x+1/3*I/a*f^2*x^3-2*I/d/a*ln(exp(d*x+c)-I)*e^2-1/3*I/a/f*
e^3+2*I/d/a*ln(exp(d*x+c))*e^2-I/a*e^2*x-2*I/d^3/a*f^2*c^2*ln(exp(d*x+c)-I)
+2*I/d^3/a*f^2*c^2*ln(exp(d*x+c))+4*I/d/a*f*e*c*x+2*I/d^3/a*f^2*ln(1+I*exp(
d*x+c))*c^2+4*I*f^2*polylog(3,-I*exp(d*x+c))/a/d^3-4/3*I/d^3/a*f^2*c^3+4*I/
d^2/a*f*c*e*ln(exp(d*x+c)-I)+2*I/d^2/a*f*e*c^2-4*I/d^2/a*f^2*polylog(2,-I*e
xp(d*x+c))*x-4*I/d^2/a*f*e*ln(1+I*exp(d*x+c))*c
```

Maxima [A]
time = 0.33, size = 166, normalized size = 1.57

$$-\frac{i\,f^2x^3+3i\,fx^2e}{3\,a}-\frac{4i\left(dx\log\left(i\,e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)\right)fe}{ad^2}-\frac{i\,e^2\log\left(i\,a\sinh\left(dx+c\right)+a\right)}{ad}-\frac{2i\left(d^2x^2\log\left(i\,e^{(dx+c)}+1\right)+2\,dx\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)-2\,\mathrm{Li}_3\left(-i\,e^{(dx+c)}\right)\right)f^2}{ad^3}-\frac{2\left(-i\,d^3f^2x^3-3i\,d^3fx^2e\right)}{3\,ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3*(I*f^2*x^3 + 3*I*f*x^2*e)/a - 4*I*(d*x*log(I*e^(d*x + c) + 1) + dilog(
-I*e^(d*x + c)))*f*e/(a*d^2) - I*e^2*log(I*a*sinh(d*x + c) + a)/(a*d) - 2*I
*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x + c)) - 2*polylog(
3, -I*e^(d*x + c)))*f^2/(a*d^3) - 2/3*(-I*d^3*f^2*x^3 - 3*I*d^3*f*x^2*e)/(a
*d^3)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(89) = 178$.
time = 0.35, size = 190, normalized size = 1.79

$$\frac{i\,d^3f^2x^3+2i\,c^3f^2+12i\,f^2\mathrm{polylog}\left(3,-i\,e^{(dx+c)}\right)-12\left(i\,df^2x+i\,dfe\right)\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)-3\left(-i\,d^3x-2i\,cd^2\right)e^2-3\left(-i\,d^3fx^2+2i\,c^2df\right)e-6\left(i\,c^2f^2-2i\,cdfe+i\,d^2e^2\right)\log\left(e^{(dx+c)}-i\right)-6\left(i\,d^2f^2x^2-i\,c^2f^2+2\left(i\,d^2fx+i\,cdf\right)e\right)\log\left(i\,e^{(dx+c)}+1\right)}{3\,ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(I*d^3*f^2*x^3 + 2*I*c^3*f^2 + 12*I*f^2*polylog(3, -I*e^(d*x + c)) - 12
*(I*d*f^2*x + I*d*f*e)*dilog(-I*e^(d*x + c)) - 3*(-I*d^3*x - 2*I*c*d^2)*e^2
```

```
 - 3*(-I*d^3*f*x^2 + 2*I*c^2*d*f)*e - 6*(I*c^2*f^2 - 2*I*c*d*f*e + I*d^2*e^
2)*log(e^(d*x + c) - I) - 6*(I*d^2*f^2*x^2 - I*c^2*f^2 + 2*(I*d^2*f*x + I*c
*d*f)*e)*log(I*e^(d*x + c) + 1))/(a*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{e^{2}\cosh \left(c+dx\right)}{\sinh \left(c+dx\right)-i}\, dx+\int \frac{f^{2}x^{2}\cosh \left(c+dx\right)}{\sinh \left(c+dx\right)-i}\, dx+\int \frac{2efx\cosh \left(c+dx\right)}{\sinh \left(c+dx\right)-i}\, dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*(Integral(e**2*cosh(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f**2*x**
2*cosh(c + d*x)/(sinh(c + d*x) - I), x) + Integral(2*e*f*x*cosh(c + d*x)/(s
inh(c + d*x) - I), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{cosh}\left(c+d\,x\right)\,{\left(e+f\,x\right)}^2}{a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i),x)
```

```
[Out] int((cosh(c + d*x)*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i), x)
```

3.255 $\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{i(e+fx)^2}{2af}-\frac{2i(e+fx)\log\left(1+ie^{c+dx}\right)}{ad}-\frac{2if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2}$$

```
[Out] 1/2*I*(f*x+e)^2/a/f-2*I*(f*x+e)*ln(1+I*exp(d*x+c))/a/d-2*I*f*polylog(2,-I*e
xp(d*x+c))/a/d^2
```

Rubi [A]
time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5678, 2221, 2317, 2438}

$$-\frac{2if\text{Li}_2\left(-ie^{c+dx}\right)}{ad^2}-\frac{2i(e+fx)\log\left(1+ie^{c+dx}\right)}{ad}+\frac{i(e+fx)^2}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((I/2)*(e + f*x)^2)/(a*f) - ((2*I)*(e + f*x)*Log[1 + I*E^(c + d*x)])/(a*d)
- ((2*I)*f*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5678

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
```

```
 x] + Dist[2, Int[(e + f*x)^m*(E^(c + d*x)/(a + b*E^(c + d*x))), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx &= \frac{i(e+fx)^2}{2af}+2\int \frac{e^{c+dx}(e+fx)}{a+iae^{c+dx}}\,dx \\
&= \frac{i(e+fx)^2}{2af}-\frac{2i(e+fx)\log\left(1+ie^{c+dx}\right)}{ad}+\frac{(2if)\int \log\left(1+ie^{c+dx}\right)\,dx}{ad} \\
&= \frac{i(e+fx)^2}{2af}-\frac{2i(e+fx)\log\left(1+ie^{c+dx}\right)}{ad}+\frac{(2if)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx,x,e^{c+dx}\right)}{ad^2} \\
&= \frac{i(e+fx)^2}{2af}-\frac{2i(e+fx)\log\left(1+ie^{c+dx}\right)}{ad}-\frac{2if\text{Li}_2\left(-ie^{c+dx}\right)}{ad^2}
\end{aligned}
$$

Mathematica [**A**]
time = 0.02, size = 66, normalized size = 0.90

$$\frac{i\left(d(e+fx)\left(d(e+fx)-4f\log\left(1+ie^{c+dx}\right)\right)-4f^2\text{PolyLog}\left(2,-ie^{c+dx}\right)\right)}{2ad^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
[Out] ((I/2)*(d*(e + f*x)*(d*(e + f*x) - 4*f*Log[1 + I*E^(c + d*x)]) - 4*f^2*Poly
Log[2, (-I)*E^(c + d*x)]))/(a*d^2*f)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(64) = 128.
time = 1.52, size = 188, normalized size = 2.58

method	result
risch	$\frac{ifx^2}{2a}-\frac{iex}{a}-\frac{2i\ln(e^{dx+c}-i)e}{da}+\frac{2i\ln(e^{dx+c})e}{da}+\frac{2ifcx}{da}+\frac{ifc^2}{d^2a}-\frac{2if\ln(1+ie^{dx+c})x}{da}-\frac{2if\ln(1+ie^{dx+c})c}{d^2a}-\frac{2if\,\text{polylo}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/2*I*f*x^2/a-I*e*x/a-2*I/d/a*ln(exp(d*x+c)-I)*e+2*I/d/a*ln(exp(d*x+c))*e+2
*I/d/a*f*c*x+I/d^2/a*f*c^2-2*I/d/a*f*ln(1+I*exp(d*x+c))*x-2*I/d^2/a*f*ln(1+
I*exp(d*x+c))*c-2*I*f*polylog(2,-I*exp(d*x+c))/a/d^2+2*I/d^2/a*f*c*ln(exp(d
*x+c)-I)-2*I/d^2/a*f*c*ln(exp(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*f*(-I*x^2/a + 4*integrate(x/(a*e^(d*x + c) - I*a), x)) - I*e*log(I*a*si
nh(d*x + c) + a)/(a*d)
```

Fricas [A]
time = 0.35, size = 95, normalized size = 1.30

$$\frac{i\,d^2fx^2-2i\,c^2f-4i\,f\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)-2\left(-i\,d^2x-2i\,cd\right)e-4\left(-i\,cf+i\,de\right)\log\left(e^{(dx+c)}-i\right)-4\left(i\,dfx+i\,cf\right)\log\left(i\,e^{(dx+c)}+1\right)}{2\,ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*d^2*f*x^2 - 2*I*c^2*f - 4*I*f*dilog(-I*e^(d*x + c)) - 2*(-I*d^2*x -
2*I*c*d)*e - 4*(-I*c*f + I*d*e)*log(e^(d*x + c) - I) - 4*(I*d*f*x + I*c*f)*
log(I*e^(d*x + c) + 1))/(a*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int\frac{e\cosh\left(c+dx\right)}{\sinh\left(c+dx\right)-i}\,dx+\int\frac{fx\cosh\left(c+dx\right)}{\sinh\left(c+dx\right)-i}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*(Integral(e*cosh(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f*x*cosh(c
+ d*x)/(sinh(c + d*x) - I), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)}{a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(e + f*x))/(a + a*sinh(c + d*x)*1i),x)
[Out] int((cosh(c + d*x)*(e + f*x))/(a + a*sinh(c + d*x)*1i), x)
```

3.256 $\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=23

$$-\frac{i\log(i-\sinh(c+dx))}{ad}$$

```
[Out] -I*ln(I-sinh(d*x+c))/a/d
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2746, 31}

$$-\frac{i\log(-\sinh(c+dx)+i)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-I)*Log[I - Sinh[c + d*x]])/(a*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\frac{i\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, ia\sinh(c+dx)\right)}{ad}\\ &= -\frac{i\log(i-\sinh(c+dx))}{ad}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$-\frac{i\log(i-\sinh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-I)*Log[I - Sinh[c + d*x]])/(a*d)
```

Maple [A]
time = 0.72, size = 23, normalized size = 1.00

method	result	size
derivativedivides	$-\frac{i\ln(a+ia\sinh(dx+c))}{da}$	23
default	$-\frac{i\ln(a+ia\sinh(dx+c))}{da}$	23
risch	$\frac{ix}{a}+\frac{2ic}{ad}-\frac{2i\ln(\mathrm{e}^{dx+c}-i)}{ad}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -I/d*ln(a+I*a*sinh(d*x+c))/a
```

Maxima [A]
time = 0.26, size = 20, normalized size = 0.87

$$-\frac{i\,\log\left(i\,a\sinh\left(dx+c\right)+a\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -I*log(I*a*sinh(d*x + c) + a)/(a*d)
```

Fricas [A]
time = 0.34, size = 23, normalized size = 1.00

$$\frac{i\,dx-2i\,\log\left(e^{(dx+c)}-i\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (I*d*x - 2*I*log(e^(d*x + c) - I))/(a*d)
```

Sympy [A]
time = 0.10, size = 22, normalized size = 0.96

$$\frac{ix}{a}-\frac{2i\log\left(e^{dx}-ie^{-c}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] I*x/a - 2*I*log(exp(d*x) - I*exp(-c))/(a*d)
```

Giac [A]
time = 0.50, size = 32, normalized size = 1.39

$$-\frac{-\frac{i\,(dx+c)}{a}+\frac{2i\,\log\left(i\,e^{(dx+c)}+1\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -(-I*(d*x + c)/a + 2*I*log(I*e^(d*x + c) + 1)/a)/d
```

Mupad [B]
time = 0.24, size = 19, normalized size = 0.83

$$-\frac{\ln\left(\sinh(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + a*sinh(c + d*x)*1i),x)
```

```
[Out] -(log(sinh(c + d*x) - 1i)*1i)/(a*d)
```

3.257 $\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cosh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Cosh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 19.99, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cosh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Integrate[Cosh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(dx+c\right)}{\left(fx+e\right)\left(a+ia\sinh\left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -I*log(f*x + e)/(a*f) + 2*integrate(1/(-I*a*f*x - I*a*e + (a*f*x*e^c + a*e^
(c + 1))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((-I*e^(d*x + c) + 1)/(-I*a*f*x - I*a*e + (a*f*x + a*e)*e^(d*x + c)
), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{\cosh\left(c+dx\right)}{e\sinh\left(c+dx\right)-ie+fx\sinh\left(c+dx\right)-ifx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*Integral(cosh(c + d*x)/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) - I*f*
x), x)/a
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh(c+d\,x)}{(e+f\,x)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/((e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
[Out] int(cosh(c + d*x)/((e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.258 $\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cosh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
[Out] Defer[Int][Cosh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 26.23, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cosh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
[Out] Integrate[Cosh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)}{(fx+e)^2\,(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] I/(a*f^2*x + a*f*e) + 2*integrate(1/(-I*a*f^2*x^2 - 2*I*a*f*x*e - I*a*e^2 +
 (a*f^2*x^2*e^c + 2*a*f*x*e^(c + 1) + a*e^(c + 2))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((-I*e^(d*x + c) + 1)/(-I*a*f^2*x^2 - 2*I*a*f*x*e - I*a*e^2 + (a*f^
2*x^2 + 2*a*f*x*e + a*e^2)*e^(d*x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{\cosh\left(c+dx\right)}{e^{2}\sinh\left(c+dx\right)-ie^{2}+2efx\sinh\left(c+dx\right)-2iefx+f^{2}x^{2}\sinh\left(c+dx\right)-if^{2}x^{2}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*Integral(cosh(c + d*x)/(e**2*sinh(c + d*x) - I*e**2 + 2*e*f*x*sinh(c + d
*x) - 2*I*e*f*x + f**2*x**2*sinh(c + d*x) - I*f**2*x**2), x)/a
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh(c+d\,x)}{{\left(e+f\,x\right)}^2\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
[Out] int(cosh(c + d*x)/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.259 $\int \frac{(e+fx)^3 \cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=108

$$\frac{(e+fx)^4}{4af}-\frac{6if^2(e+fx)\cosh(c+dx)}{ad^3}-\frac{i(e+fx)^3\cosh(c+dx)}{ad}+\frac{6if^3\sinh(c+dx)}{ad^4}+\frac{3if(e+fx)^2\sinh(c+dx)}{ad^2}$$

```
[Out] 1/4*(f*x+e)^4/a/f-6*I*f^2*(f*x+e)*cosh(d*x+c)/a/d^3-I*(f*x+e)^3*cosh(d*x+c)
/a/d+6*I*f^3*sinh(d*x+c)/a/d^4+3*I*f*(f*x+e)^2*sinh(d*x+c)/a/d^2
```

Rubi [A]
time = 0.12, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {5682, 32, 3377, 2717}

$$\frac{6if^3\sinh(c+dx)}{ad^4}-\frac{6if^2(e+fx)\cosh(c+dx)}{ad^3}+\frac{3if(e+fx)^2\sinh(c+dx)}{ad^2}-\frac{i(e+fx)^3\cosh(c+dx)}{ad}+\frac{(e+fx)^4}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (e + f*x)^4/(4*a*f) - ((6*I)*f^2*(e + f*x)*Cosh[c + d*x])/(a*d^3) - (I*(e +
 f*x)^3*Cosh[c + d*x])/(a*d) + ((6*I)*f^3*Sinh[c + d*x])/(a*d^4) + ((3*I)*f
*(e + f*x)^2*Sinh[c + d*x])/(a*d^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5682

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cosh[c
+ d*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Si
nh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ
```

[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int (e+fx)^3\sinh(c+dx)\,dx}{a} + \frac{\int (e+fx)^3\,dx}{a}$$
$$= \frac{(e+fx)^4}{4af} - \frac{i(e+fx)^3\cosh(c+dx)}{ad} + \frac{(3if)\int (e+fx)^2\cosh(c+dx)\,dx}{ad}$$
$$= \frac{(e+fx)^4}{4af} - \frac{i(e+fx)^3\cosh(c+dx)}{ad} + \frac{3if(e+fx)^2\sinh(c+dx)}{ad^2} - \frac{(6if^2 \cdots}{\cdots}$$
$$= \frac{(e+fx)^4}{4af} - \frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^3\cosh(c+dx)}{ad} + \frac{3if(\cdots}{\cdots}$$
$$= \frac{(e+fx)^4}{4af} - \frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^3\cosh(c+dx)}{ad} + \frac{6if^3 \cdots}{\cdots}$$

Mathematica [A]
time = 0.36, size = 106, normalized size = 0.98

$$\frac{d^4x(4e^3+6e^2fx+4ef^2x^2+f^3x^3)-4id(e+fx)\left(6f^2+d^2(e+fx)^2\right)\cosh(c+dx)+12if\left(2f^2+d^2(e+fx)^2\right)\sinh(c+dx)}{4ad^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
[Out] (d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) - (4*I)*d*(e + f*x)*(6*f
^2 + d^2*(e + f*x)^2)*Cosh[c + d*x] + (12*I)*f*(2*f^2 + d^2*(e + f*x)^2)*Si
nh[c + d*x])/(4*a*d^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 447 vs. 2(102) = 204.
time = 1.10, size = 448, normalized size = 4.15

method	result
risch	$\frac{f^3x^4}{4a}+\frac{f^2e\,x^3}{a}+\frac{3f\,e^2x^2}{2a}+\frac{e^3x}{a}+\frac{e^4}{4af}-\frac{i(f^3x^3d^3+3d^3e\,f^2x^2+3d^3e^2fx-3d^2f^3x^2+d^3e^3-6d^2e\,f^2x-3d^2e^2f+\cdots}{2a\,d^4}$
derivativedivides	$-\frac{3if^2c^2ed\cosh(dx+c)+ie^3d^3\cosh(dx+c)-6if^2ced((dx+c)\cosh(dx+c)-\sinh(dx+c))+if^3\left((dx+c)^3\cosh(dx+c)-3(\cdots\right.}{\cdots}$
default	$-\frac{3if^2c^2ed\cosh(dx+c)+ie^3d^3\cosh(dx+c)-6if^2ced((dx+c)\cosh(dx+c)-\sinh(dx+c))+if^3\left((dx+c)^3\cosh(dx+c)-3(\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^4/a*(3*I*f^2*c^2*e*d*cosh(d*x+c)+I*e^3*d^3*cosh(d*x+c)-6*I*f^2*c*e*d*(
(d*x+c)*cosh(d*x+c)-sinh(d*x+c))+I*f^3*((d*x+c)^3*cosh(d*x+c)-3*(d*x+c)^2*s
inh(d*x+c)+6*(d*x+c)*cosh(d*x+c)-6*sinh(d*x+c))+3*I*f*e^2*d^2*((d*x+c)*cosh
(d*x+c)-sinh(d*x+c))-3*I*c*f^3*((d*x+c)^2*cosh(d*x+c)-2*(d*x+c)*sinh(d*x+c)
+2*cosh(d*x+c))-I*c^3*f^3*cosh(d*x+c)-3*I*f*c*e^2*d^2*cosh(d*x+c)+3*I*f^2*e
*d*((d*x+c)^2*cosh(d*x+c)-2*(d*x+c)*sinh(d*x+c)+2*cosh(d*x+c))+3*I*c^2*f^3*
((d*x+c)*cosh(d*x+c)-sinh(d*x+c))+c^3*f^3*(d*x+c)-3*f^2*c^2*e*d*(d*x+c)-3/2
*c^2*f^3*(d*x+c)^2+3*f*c*e^2*d^2*(d*x+c)+3*c*d*e*f^2*(d*x+c)^2+c*f^3*(d*x+c
)^3-e^3*d^3*(d*x+c)-3/2*d^2*e^2*f*(d*x+c)^2-f^2*e*d*(d*x+c)^3-1/4*f^3*(d*x+
c)^4)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(102) = 204$.
time = 0.40, size = 372, normalized size = 3.44

$$\frac{3}{2}f\left(\frac{2\,xe^{(dx+c)}}{ade^{(dx+c)}-i\,ad}-\frac{i\,d^2x^2e^c+i\,dxe^c-\left(-i\,dxe^{(3\,c)}+i\,e^{(3\,c)}\right)e^{(2\,dx)}-\left(d^2x^2e^{(2\,c)}-3\,dxe^{(2\,c)}+e^{(2\,c)}\right)e^{(dx)}+(dx+1)e^{(-dx)}+i\,e^c}{ad^2e^{(dx+2\,c)}-i\,ad^2e^c}\right)e^2+\frac{1}{2}\left(\frac{2\,(dx+c)}{ad}-\frac{i\,e^{(dx+c)}}{ad}-\frac{i\,e^{(-dx-c)}}{ad}\right)e^3+\frac{\left(d^4x^4e^c+2\left(-i\,d^3x^3e^{(2\,c)}+3i\,d^2x^2e^{(2\,c)}-6i\,dxe^{(2\,c)}+6i\,e^{(2\,c)}\right)e^{(dx)}+2\left(-i\,d^3x^3-3i\,d^2x^2-6i\,dx-6i\right)e^{(-dx)}\right)f^3e^{(-c)}}{4\,ad^4}+\frac{\left(2\,d^3x^3e^c+3\left(-i\,d^2x^2e^{(2\,c)}+2i\,dxe^{(2\,c)}-2i\,e^{(2\,c)}\right)e^{(dx)}+3\left(-i\,d^2x^2-2i\,dx-2i\right)e^{(-dx)}\right)f^2e^{(-c+1)}}{2\,ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 3/2*f*(2*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) - (I*d^2*x^2*e^c + I*d*x*e
^c - (-I*d*x*e^(3*c) + I*e^(3*c))*e^(2*d*x) - (d^2*x^2*e^(2*c) - 3*d*x*e^(2
*c) + e^(2*c))*e^(d*x) + (d*x + 1)*e^(-d*x) + I*e^c)/(a*d^2*e^(d*x + 2*c) -
 I*a*d^2*e^c))*e^2 + 1/2*(2*(d*x + c)/(a*d) - I*e^(d*x + c)/(a*d) - I*e^(-d
*x - c)/(a*d))*e^3 + 1/4*(d^4*x^4*e^c + 2*(-I*d^3*x^3*e^(2*c) + 3*I*d^2*x^2
*e^(2*c) - 6*I*d*x*e^(2*c) + 6*I*e^(2*c))*e^(d*x) + 2*(-I*d^3*x^3 - 3*I*d^2
*x^2 - 6*I*d*x - 6*I)*e^(-d*x))*f^3*e^(-c)/(a*d^4) + 1/2*(2*d^3*x^3*e^c + 3
*(-I*d^2*x^2*e^(2*c) + 2*I*d*x*e^(2*c) - 2*I*e^(2*c))*e^(d*x) + 3*(-I*d^2*x
^2 - 2*I*d*x - 2*I)*e^(-d*x))*f^2*e^(-c + 1)/(a*d^3)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(102) = 204$.
time = 0.36, size = 260, normalized size = 2.41

$$\frac{\left(-2i\,d^3f^3x^3-6i\,d^2f^3x^2-12i\,df^3x-2i\,d^3e^3-12i\,f^3-6\,(i\,d^3fx+i\,d^2f)e^2-6\,(i\,d^3f^2x^2+2i\,d^2f^2x+2i\,df^2)e-2\,(i\,d^3f^3x^3-3i\,d^2f^3x^2+6i\,df^3x+i\,d^3e^3-6i\,f^3+3\,(i\,d^3fx-i\,d^2f)e^2+3\,(i\,d^3f^2x^2-2i\,d^2f^2x+2i\,df^2)e)e^{(2\,dx+2\,c)}+(d^4f^3x^4+4\,d^4f^2x^3e+6\,d^4fx^2e^2+4\,d^4xe^3)e^{(dx+c)}\right)e^{(-dx-c)}}{4\,ad^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/4*(-2*I*d^3*f^3*x^3 - 6*I*d^2*f^3*x^2 - 12*I*d*f^3*x - 2*I*d^3*e^3 - 12*I
*f^3 - 6*(I*d^3*f*x + I*d^2*f)*e^2 - 6*(I*d^3*f^2*x^2 + 2*I*d^2*f^2*x + 2*I
*d*f^2)*e - 2*(I*d^3*f^3*x^3 - 3*I*d^2*f^3*x^2 + 6*I*d*f^3*x + I*d^3*e^3 -
6*I*f^3 + 3*(I*d^3*f*x - I*d^2*f)*e^2 + 3*(I*d^3*f^2*x^2 - 2*I*d^2*f^2*x +
```

```
2*I*d*f^2)*e)*e^(2*d*x + 2*c) + (d^4*f^3*x^4 + 4*d^4*f^2*x^3*e + 6*d^4*f*x^
2*e^2 + 4*d^4*x*e^3)*e^(d*x + c))*e^(-d*x - c)/(a*d^4)
```

Sympy [A]
time = 0.39, size = 518, normalized size = 4.80

$$\begin{cases} \frac{((-2iad^7e^3-6iad^7e^2fx-6iad^7ef^2x^2-2iad^7f^3x^3-6iad^6e^2f-12iad^6ef^2x-6iad^6f^3x^2-12iad^5ef^2-12iad^5f^3x-12iad^4f^3)e^{-dx}+(-2iad^7e^3e^{2c}-6iad^7e^2fxe^{2c}-6iad^7ef^2x^2e^{2c}-2iad^7f^3x^3e^{2c}+6iad^6e^2fe^{2c}+12iad^6ef^2xe^{2c}+6iad^6f^3x^2e^{2c}-12iad^5ef^2e^{2c}-12iad^5f^3xe^{2c}+12iad^4f^3e^{2c})e^{dx})e^{-c}}{4a^2d^8} & \text{for } a^2d^8e^c \neq 0 \\ \frac{x^4(-if^3e^{2c}+if^3)e^{-c}}{8a}+\frac{x^3(-ief^2e^{2c}+ief^2)e^{-c}}{2a}+\frac{x^2(-3ie^2fe^{2c}+3ie^2f)e^{-c}}{4a}+\frac{x(-ie^3e^{2c}+ie^3)e^{-c}}{2a} & \text{otherwise} \end{cases} + \frac{e^3x}{a}+\frac{3e^2fx^2}{2a}+\frac{ef^2x^3}{a}+\frac{f^3x^4}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Piecewise((((-2*I*a*d**7*e**3 - 6*I*a*d**7*e**2*f*x - 6*I*a*d**7*e*f**2*x**
2 - 2*I*a*d**7*f**3*x**3 - 6*I*a*d**6*e**2*f - 12*I*a*d**6*e*f**2*x - 6*I*a
*d**6*f**3*x**2 - 12*I*a*d**5*e*f**2 - 12*I*a*d**5*f**3*x - 12*I*a*d**4*f**
3)*exp(-d*x) + (-2*I*a*d**7*e**3*exp(2*c) - 6*I*a*d**7*e**2*f*x*exp(2*c) -
6*I*a*d**7*e*f**2*x**2*exp(2*c) - 2*I*a*d**7*f**3*x**3*exp(2*c) + 6*I*a*d**
6*e**2*f*exp(2*c) + 12*I*a*d**6*e*f**2*x*exp(2*c) + 6*I*a*d**6*f**3*x**2*ex
p(2*c) - 12*I*a*d**5*e*f**2*exp(2*c) - 12*I*a*d**5*f**3*x*exp(2*c) + 12*I*a
*d**4*f**3*exp(2*c))*exp(d*x))*exp(-c)/(4*a**2*d**8), Ne(a**2*d**8*exp(c),
0)), (x**4*(-I*f**3*exp(2*c) + I*f**3)*exp(-c)/(8*a) + x**3*(-I*e*f**2*exp(
2*c) + I*e*f**2)*exp(-c)/(2*a) + x**2*(-3*I*e**2*f*exp(2*c) + 3*I*e**2*f)*e
xp(-c)/(4*a) + x*(-I*e**3*exp(2*c) + I*e**3)*exp(-c)/(2*a), True)) + e**3*x
/a + 3*e**2*f*x**2/(2*a) + e*f**2*x**3/a + f**3*x**4/(4*a)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(98) = 196$.
time = 0.46, size = 355, normalized size = 3.29

$$\frac{\left(d^4f^3x^4e^{(dx+c)}+4\,d^4ef^2x^3e^{(dx+c)}-2i\,d^3f^3x^3e^{(2\,dx+2\,c)}+6\,d^4e^2fx^2e^{(dx+c)}-2i\,d^3f^3x^3-6i\,d^3ef^2x^2e^{(2\,dx+2\,c)}+4\,d^4e^3xe^{(dx+c)}-6i\,d^3ef^2x^2-6i\,d^3e^2fxe^{(2\,dx+2\,c)}+6i\,d^2f^3x^2e^{(2\,dx+2\,c)}-6i\,d^3e^2fx-6i\,d^2f^3x^2-2i\,d^3e^3e^{(2\,dx+2\,c)}+12i\,d^2ef^2xe^{(2\,dx+2\,c)}-2i\,d^3e^3-12i\,d^2ef^2x+6i\,d^2e^2fe^{(2\,dx+2\,c)}-12i\,df^3xe^{(2\,dx+2\,c)}-6i\,d^2e^2f-12i\,df^3x-12i\,def^2e^{(2\,dx+2\,c)}-12i\,def^2+12i\,f^3e^{(2\,dx+2\,c)}-12i\,f^3\right)e^{(-dx-c)}}{4\,ad^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(d^4*f^3*x^4*e^(d*x + c) + 4*d^4*e*f^2*x^3*e^(d*x + c) - 2*I*d^3*f^3*x^
3*e^(2*d*x + 2*c) + 6*d^4*e^2*f*x^2*e^(d*x + c) - 2*I*d^3*f^3*x^3 - 6*I*d^3
*e*f^2*x^2*e^(2*d*x + 2*c) + 4*d^4*e^3*x*e^(d*x + c) - 6*I*d^3*e*f^2*x^2 -
6*I*d^3*e^2*f*x*e^(2*d*x + 2*c) + 6*I*d^2*f^3*x^2*e^(2*d*x + 2*c) - 6*I*d^3
*e^2*f*x - 6*I*d^2*f^3*x^2 - 2*I*d^3*e^3*e^(2*d*x + 2*c) + 12*I*d^2*e*f^2*x
*e^(2*d*x + 2*c) - 2*I*d^3*e^3 - 12*I*d^2*e*f^2*x + 6*I*d^2*e^2*f*e^(2*d*x
+ 2*c) - 12*I*d*f^3*x*e^(2*d*x + 2*c) - 6*I*d^2*e^2*f - 12*I*d*f^3*x - 12*I
*d*e*f^2*e^(2*d*x + 2*c) - 12*I*d*e*f^2 + 12*I*f^3*e^(2*d*x + 2*c) - 12*I*f
^3)*e^(-d*x - c)/(a*d^4)
```

Mupad [B]
time = 0.72, size = 269, normalized size = 2.49

$$\mathrm{e}^{c+d\,x}\left(\frac{\left(-d^3\,e^3+3\,d^2\,e^2\,f-6\,d\,e\,f^2+6\,f^3\right)\,1{}\mathrm{i}}{2\,a\,d^4}-\frac{f^3\,x^3\,1{}\mathrm{i}}{2\,a\,d}+\frac{f^2\,x^2\,\left(f-d\,e\right)\,3{}\mathrm{i}}{2\,a\,d^2}-\frac{f\,x\,\left(d^2\,e^2-2\,d\,e\,f+2\,f^2\right)\,3{}\mathrm{i}}{2\,a\,d^3}\right)-\mathrm{e}^{-c-d\,x}\left(\frac{\left(d^3\,e^3+3\,d^2\,e^2\,f+6\,d\,e\,f^2+6\,f^3\right)\,1{}\mathrm{i}}{2\,a\,d^4}+\frac{f^3\,x^3\,1{}\mathrm{i}}{2\,a\,d}+\frac{f^2\,x^2\,\left(f+d\,e\right)\,3{}\mathrm{i}}{2\,a\,d^2}+\frac{f\,x\,\left(d^2\,e^2+2\,d\,e\,f+2\,f^2\right)\,3{}\mathrm{i}}{2\,a\,d^3}\right)+\frac{e^3\,x}{a}+\frac{f^3\,x^4}{4\,a}+\frac{3\,e^2\,f\,x^2}{2\,a}+\frac{e\,f^2\,x^3}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i),x)
```

```
[Out] exp(c + d*x)*(((6*f^3 - d^3*e^3 + 3*d^2*e^2*f - 6*d*e*f^2)*1i)/(2*a*d^4) -
(f^3*x^3*1i)/(2*a*d) + (f^2*x^2*(f - d*e)*3i)/(2*a*d^2) - (f*x*(2*f^2 + d^2
*e^2 - 2*d*e*f)*3i)/(2*a*d^3)) - exp(- c - d*x)*(((6*f^3 + d^3*e^3 + 3*d^2*
e^2*f + 6*d*e*f^2)*1i)/(2*a*d^4) + (f^3*x^3*1i)/(2*a*d) + (f^2*x^2*(f + d*e
)*3i)/(2*a*d^2) + (f*x*(2*f^2 + d^2*e^2 + 2*d*e*f)*3i)/(2*a*d^3)) + (e^3*x)
/a + (f^3*x^4)/(4*a) + (3*e^2*f*x^2)/(2*a) + (e*f^2*x^3)/a
```

3.260 $\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{(e+fx)^3}{3af}-\frac{2if^2\cosh(c+dx)}{ad^3}-\frac{i(e+fx)^2\cosh(c+dx)}{ad}+\frac{2if(e+fx)\sinh(c+dx)}{ad^2}$$

```
[Out] 1/3*(f*x+e)^3/a/f-2*I*f^2*cosh(d*x+c)/a/d^3-I*(f*x+e)^2*cosh(d*x+c)/a/d+2*I
*f*(f*x+e)*sinh(d*x+c)/a/d^2
```

Rubi [A]
time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.129$, Rules used = {5682, 32, 3377, 2718}

$$-\frac{2if^2\cosh(c+dx)}{ad^3}+\frac{2if(e+fx)\sinh(c+dx)}{ad^2}-\frac{i(e+fx)^2\cosh(c+dx)}{ad}+\frac{(e+fx)^3}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (e + f*x)^3/(3*a*f) - ((2*I)*f^2*Cosh[c + d*x])/(a*d^3) - (I*(e + f*x)^2*Co
sh[c + d*x])/(a*d) + ((2*I)*f*(e + f*x)*Sinh[c + d*x])/(a*d^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5682

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cosh[c
+ d*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Si
nh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ
```

[a^2 + b^2, 0]

Rubi steps

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int (e+fx)^2\sinh(c+dx)\,dx}{a}+\frac{\int (e+fx)^2\,dx}{a}$$
$$=\frac{(e+fx)^3}{3af}-\frac{i(e+fx)^2\cosh(c+dx)}{ad}+\frac{(2if)\int (e+fx)\cosh(c+dx)\,dx}{ad}$$
$$=\frac{(e+fx)^3}{3af}-\frac{i(e+fx)^2\cosh(c+dx)}{ad}+\frac{2if(e+fx)\sinh(c+dx)}{ad^2}-\frac{(2if^2)\ldots}{}$$
$$=\frac{(e+fx)^3}{3af}-\frac{2if^2\cosh(c+dx)}{ad^3}-\frac{i(e+fx)^2\cosh(c+dx)}{ad}+\frac{2if(e+fx)\sin\ldots}{ad^2}$$

Mathematica [A]
time = 0.27, size = 78, normalized size = 0.95

$$\frac{d^3x(3e^2+3efx+f^2x^2)-3i(2f^2+d^2(e+fx)^2)\cosh(c+dx)+6idf(e+fx)\sinh(c+dx)}{3ad^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
[Out] (d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2) - (3*I)*(2*f^2 + d^2*(e + f*x)^2)*Cosh[c
 + d*x] + (6*I)*d*f*(e + f*x)*Sinh[c + d*x])/(3*a*d^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(77) = 154.
time = 1.10, size = 223, normalized size = 2.72

method	result
risch	$\frac{f^2x^3}{3a}+\frac{fex^2}{a}+\frac{e^2x}{a}+\frac{e^3}{3af}-\frac{i(f^2x^2d^2+2d^2efx+d^2e^2-2df^2x-2def+2f^2)e^{dx+c}}{2a\,d^3}-\frac{i(f^2x^2d^2+2d^2efx+d^2e^2+\ldots}{2a\ldots}$
derivativedivides	$-\frac{ic^2f^2\cosh(dx+c)-2ifced\cosh(dx+c)-2icf^2((dx+c)\cosh(dx+c)-\sinh(dx+c))+ie^2d^2\cosh(dx+c)+2ifed((dx+c)\cos\ldots}{}$
default	$-\frac{ic^2f^2\cosh(dx+c)-2ifced\cosh(dx+c)-2icf^2((dx+c)\cosh(dx+c)-\sinh(dx+c))+ie^2d^2\cosh(dx+c)+2ifed((dx+c)\cos\ldots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -1/d^3/a*(I*c^2*f^2*cosh(d*x+c)-2*I*f*c*e*d*cosh(d*x+c)-2*I*c*f^2*((d*x+c)*
cosh(d*x+c)-sinh(d*x+c))+I*e^2*d^2*cosh(d*x+c)+2*I*f*e*d*((d*x+c)*cosh(d*x+
```

```
c)-sinh(d*x+c))+I*f^2*((d*x+c)^2*cosh(d*x+c)-2*(d*x+c)*sinh(d*x+c)+2*cosh(d
*x+c))-c^2*f^2*(d*x+c)+2*f*c*e*d*(d*x+c)+c*f^2*(d*x+c)^2-e^2*d^2*(d*x+c)-d*
e*f*(d*x+c)^2-1/3*f^2*(d*x+c)^3)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(77) = 154$.
time = 0.37, size = 270, normalized size = 3.29

$$f\left(\frac{2\,xe^{(dx+c)}}{ade^{(dx+c)}-i\,ad}-\frac{i\,d^2x^2e^c+i\,dxe^c-\left(-i\,dxe^{(3\,c)}+i\,e^{(3\,c)}\right)e^{(2\,dx)}-\left(d^2x^2e^{(2\,c)}-3\,dxe^{(2\,c)}+e^{(2\,c)}\right)e^{(dx)}+(dx+1)e^{(-dx)}+i\,e^c}{ad^2e^{(dx+2\,c)}-i\,ad^2e^c}\right)e+\frac{1}{2}\left(\frac{2\,(dx+c)}{ad}-\frac{i\,e^{(dx+c)}}{ad}-\frac{i\,e^{(-dx-c)}}{ad}\right)e^2+\frac{\left(2\,d^3x^3e^c+3\left(-i\,d^2x^2e^{(2\,c)}+2i\,dxe^{(2\,c)}-2i\,e^{(2\,c)}\right)e^{(dx)}+3\left(-i\,d^2x^2-2i\,dx-2i\right)e^{(-dx)}\right)f^2e^{(-c)}}{6\,ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] f*(2*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) - (I*d^2*x^2*e^c + I*d*x*e^c -
 (-I*d*x*e^(3*c) + I*e^(3*c))*e^(2*d*x) - (d^2*x^2*e^(2*c) - 3*d*x*e^(2*c)
+ e^(2*c))*e^(d*x) + (d*x + 1)*e^(-d*x) + I*e^c)/(a*d^2*e^(d*x + 2*c) - I*a
*d^2*e^c))*e + 1/2*(2*(d*x + c)/(a*d) - I*e^(d*x + c)/(a*d) - I*e^(-d*x - c
)/(a*d))*e^2 + 1/6*(2*d^3*x^3*e^c + 3*(-I*d^2*x^2*e^(2*c) + 2*I*d*x*e^(2*c)
 - 2*I*e^(2*c))*e^(d*x) + 3*(-I*d^2*x^2 - 2*I*d*x - 2*I)*e^(-d*x))*f^2*e^(-
c)/(a*d^3)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(77) = 154$.
time = 0.35, size = 160, normalized size = 1.95

$$\frac{\left(-3i\,d^2f^2x^2-6i\,df^2x-3i\,d^2e^2-6i\,f^2-6\left(i\,d^2fx+i\,df\right)e-3\left(i\,d^2f^2x^2-2i\,df^2x+i\,d^2e^2+2i\,f^2+2\left(i\,d^2fx-i\,df\right)e\right)e^{(2\,dx+2\,c)}+2\left(d^3f^2x^3+3\,d^3fx^2e+3\,d^3xe^2\right)e^{(dx+c)}\right)e^{(-dx-c)}}{6\,ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/6*(-3*I*d^2*f^2*x^2 - 6*I*d*f^2*x - 3*I*d^2*e^2 - 6*I*f^2 - 6*(I*d^2*f*x
+ I*d*f)*e - 3*(I*d^2*f^2*x^2 - 2*I*d*f^2*x + I*d^2*e^2 + 2*I*f^2 + 2*(I*d^
2*f*x - I*d*f)*e)*e^(2*d*x + 2*c) + 2*(d^3*f^2*x^3 + 3*d^3*f*x^2*e + 3*d^3*
x*e^2)*e^(d*x + c))*e^(-d*x - c)/(a*d^3)
```

Sympy [A]
time = 0.30, size = 318, normalized size = 3.88

$$\begin{cases}\frac{\left(\left(-2iad^5e^2-4iad^5efx-2iad^5f^2x^2-4iad^4ef-4iad^4f^2x-4iad^3f^2\right)e^{-dx}+\left(-2iad^5e^2e^{2c}-4iad^5efxe^{2c}-2iad^5f^2x^2e^{2c}+4iad^4efe^{2c}+4iad^4f^2xe^{2c}-4iad^3f^2e^{2c}\right)e^{dx}\right)e^{-c}}{4a^2d^6} & \text{for}\: a^2d^6e^c \neq 0 \\ \frac{x^3\left(-if^2e^{2c}+if^2\right)e^{-c}}{6a}+\frac{x^2\left(-iefe^{2c}+ief\right)e^{-c}}{2a}+\frac{x\left(-ie^2e^{2c}+ie^2\right)e^{-c}}{2a} & \text{otherwise}\end{cases}+\frac{e^2x}{a}+\frac{efx^2}{a}+\frac{f^2x^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Piecewise((((-2*I*a*d**5*e**2 - 4*I*a*d**5*e*f*x - 2*I*a*d**5*f**2*x**2 - 4
*I*a*d**4*e*f - 4*I*a*d**4*f**2*x - 4*I*a*d**3*f**2)*exp(-d*x) + (-2*I*a*d*
*5*e**2*exp(2*c) - 4*I*a*d**5*e*f*x*exp(2*c) - 2*I*a*d**5*f**2*x**2*exp(2*c
) + 4*I*a*d**4*e*f*exp(2*c) + 4*I*a*d**4*f**2*x*exp(2*c) - 4*I*a*d**3*f**2*
exp(2*c))*exp(d*x))*exp(-c)/(4*a**2*d**6), Ne(a**2*d**6*exp(c), 0)), (x**3*
(-I*f**2*exp(2*c) + I*f**2)*exp(-c)/(6*a) + x**2*(-I*e*f*exp(2*c) + I*e*f)*
exp(-c)/(2*a) + x*(-I*e**2*exp(2*c) + I*e**2)*exp(-c)/(2*a), True)) + e**2*
x/a + e*f*x**2/a + f**2*x**3/(3*a)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(74) = 148$.
time = 0.46, size = 208, normalized size = 2.54

$$\frac{\left(2\,d^3f^2x^3e^{(dx+c)} + 6\,d^3efx^2e^{(dx+c)} - 3i\,d^2f^2x^2e^{(2\,dx+2\,c)} + 6\,d^3e^2xe^{(dx+c)} - 3i\,d^2f^2x^2 - 6i\,d^2efxe^{(2\,dx+2\,c)} - 6i\,d^2efx - 3i\,d^2e^2e^{(2\,dx+2\,c)} + 6i\,df^2xe^{(2\,dx+2\,c)} - 3i\,d^2e^2 - 6i\,df^2x + 6i\,defe^{(2\,dx+2\,c)} - 6i\,def - 6i\,f^2e^{(2\,dx+2\,c)} - 6i\,f^2\right)e^{(-dx-c)}}{6\,ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(2*d^3*f^2*x^3*e^(d*x + c) + 6*d^3*e*f*x^2*e^(d*x + c) - 3*I*d^2*f^2*x^
2*e^(2*d*x + 2*c) + 6*d^3*e^2*x*e^(d*x + c) - 3*I*d^2*f^2*x^2 - 6*I*d^2*e*f
*x*e^(2*d*x + 2*c) - 6*I*d^2*e*f*x - 3*I*d^2*e^2*e^(2*d*x + 2*c) + 6*I*d*f^
2*x*e^(2*d*x + 2*c) - 3*I*d^2*e^2 - 6*I*d*f^2*x + 6*I*d*e*f*e^(2*d*x + 2*c)
 - 6*I*d*e*f - 6*I*f^2*e^(2*d*x + 2*c) - 6*I*f^2)*e^(-d*x - c)/(a*d^3)
```

Mupad [**B**]
time = 0.52, size = 167, normalized size = 2.04

$$\frac{e^2\,x}{a} - \mathrm{e}^{-c-d\,x}\,\left(\frac{\left(d^2\,e^2 + 2\,d\,e\,f + 2\,f^2\right)\,1\mathrm{i}}{2\,a\,d^3} + \frac{f^2\,x^2\,1\mathrm{i}}{2\,a\,d} + \frac{f\,x\,\left(f + d\,e\right)\,1\mathrm{i}}{a\,d^2}\right) - \mathrm{e}^{c+d\,x}\,\left(\frac{\left(d^2\,e^2 - 2\,d\,e\,f + 2\,f^2\right)\,1\mathrm{i}}{2\,a\,d^3} + \frac{f^2\,x^2\,1\mathrm{i}}{2\,a\,d} - \frac{f\,x\,\left(f - d\,e\right)\,1\mathrm{i}}{a\,d^2}\right) + \frac{f^2\,x^3}{3\,a} + \frac{e\,f\,x^2}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i),x)
```

```
[Out] (e^2*x)/a - exp(- c - d*x)*(((2*f^2 + d^2*e^2 + 2*d*e*f)*1i)/(2*a*d^3) + (f
^2*x^2*1i)/(2*a*d) + (f*x*(f + d*e)*1i)/(a*d^2)) - exp(c + d*x)*(((2*f^2 +
d^2*e^2 - 2*d*e*f)*1i)/(2*a*d^3) + (f^2*x^2*1i)/(2*a*d) - (f*x*(f - d*e)*1i
)/(a*d^2)) + (f^2*x^3)/(3*a) + (e*f*x^2)/a
```

3.261 $\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=56

$$\frac{ex}{a}+\frac{fx^2}{2a}-\frac{i(e+fx)\cosh(c+dx)}{ad}+\frac{if\sinh(c+dx)}{ad^2}$$

```
[Out] e*x/a+1/2*f*x^2/a-I*(f*x+e)*cosh(d*x+c)/a/d+I*f*sinh(d*x+c)/a/d^2
```

Rubi [A]
time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {5682, 3377, 2717}

$$\frac{if\sinh(c+dx)}{ad^2}-\frac{i(e+fx)\cosh(c+dx)}{ad}+\frac{ex}{a}+\frac{fx^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (e*x)/a + (f*x^2)/(2*a) - (I*(e + f*x)*Cosh[c + d*x])/(a*d) + (I*f*Sinh[c +
 d*x])/(a*d^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5682

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cosh[c
+ d*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Si
nh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ
[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int (e+fx)\sinh(c+dx)\,dx}{a}+\frac{\int (e+fx)\,dx}{a}$$
$$= \frac{ex}{a}+\frac{fx^2}{2a}-\frac{i(e+fx)\cosh(c+dx)}{ad}+\frac{(if)\int \cosh(c+dx)\,dx}{ad}$$
$$= \frac{ex}{a}+\frac{fx^2}{2a}-\frac{i(e+fx)\cosh(c+dx)}{ad}+\frac{if\sinh(c+dx)}{ad^2}$$

Mathematica [A]
time = 0.38, size = 57, normalized size = 1.02

$$-\frac{(c+dx)(-2de+cf-dfx)+2id(e+fx)\cosh(c+dx)-2if\sinh(c+dx)}{2ad^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] -1/2*((c + d*x)*(-2*d*e + c*f - d*f*x) + (2*I)*d*(e + f*x)*Cosh[c + d*x] -
(2*I)*f*Sinh[c + d*x])/(a*d^2)
```

Maple [A]
time = 1.05, size = 84, normalized size = 1.50

method	result	size
risch	$\frac{f\,x^2}{2a}+\frac{ex}{a}-\frac{i(dxf+de-f)\mathrm{e}^{dx+c}}{2a\,d^2}-\frac{i(dxf+de+f)\mathrm{e}^{-dx-c}}{2a\,d^2}$	70
derivativedivides	$-\frac{-ifc\cosh(dx+c)+ied\cosh(dx+c)+if((dx+c)\cosh(dx+c)-\sinh(dx+c))+fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}}{d^2a}$	84
default	$-\frac{-ifc\cosh(dx+c)+ied\cosh(dx+c)+if((dx+c)\cosh(dx+c)-\sinh(dx+c))+fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}}{d^2a}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2/a*(-I*f*c*cosh(d*x+c)+I*e*d*cosh(d*x+c)+I*f*((d*x+c)*cosh(d*x+c)-sin
h(d*x+c))+f*c*(d*x+c)-e*d*(d*x+c)-1/2*f*(d*x+c)^2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(55) = 110.
time = 0.32, size = 189, normalized size = 3.38

$$\frac{1}{2}\,f\left(\frac{2\,xe^{(dx+c)}}{ade^{(dx+c)}-i\,ad}-\frac{i\,d^2x^2e^c+i\,dxe^c-\left(-i\,dxe^{(3\,c)}+i\,e^{(3\,c)}\right)e^{(2\,dx)}-\left(d^2x^2e^{(2\,c)}-3\,dxe^{(2\,c)}+e^{(2\,c)}\right)e^{(dx)}+(dx+1)e^{(-dx)}+i\,e^c}{ad^2e^{(dx+2\,c)}-i\,ad^2e^c}\right)+\frac{1}{2}\left(\frac{2\,(dx+c)}{ad}-\frac{i\,e^{(dx+c)}}{ad}-\frac{i\,e^{(-dx-c)}}{ad}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*f*(2*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) - (I*d^2*x^2*e^c + I*d*x*e
^c - (-I*d*x*e^(3*c) + I*e^(3*c))*e^(2*d*x) - (d^2*x^2*e^(2*c) - 3*d*x*e^(2
*c) + e^(2*c))*e^(d*x) + (d*x + 1)*e^(-d*x) + I*e^c)/(a*d^2*e^(d*x + 2*c) -
 I*a*d^2*e^c)) + 1/2*(2*(d*x + c)/(a*d) - I*e^(d*x + c)/(a*d) - I*e^(-d*x -
 c)/(a*d))*e
```

Fricas [A]
time = 0.38, size = 79, normalized size = 1.41

$$\frac{\left(-i\,dfx-i\,de+(-i\,dfx-i\,de+i\,f)e^{(2\,dx+2\,c)}+(d^2fx^2+2\,d^2xe)e^{(dx+c)}-i\,f\right)e^{(-dx-c)}}{2\,ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*f*x - I*d*e + (-I*d*f*x - I*d*e + I*f)*e^(2*d*x + 2*c) + (d^2*f*x
^2 + 2*d^2*x*e)*e^(d*x + c) - I*f)*e^(-d*x - c)/(a*d^2)
```

Sympy [A]
time = 0.20, size = 167, normalized size = 2.98

$$\begin{cases} \frac{\left(\left(-2iad^3e-2iad^3fx-2iad^2f\right)e^{-dx}+\left(-2iad^3ee^{2c}-2iad^3fxe^{2c}+2iad^2fe^{2c}\right)e^{dx}\right)e^{-c}}{4a^2d^4} & \text{for } a^2d^4e^c\neq 0 \\ \frac{x^2\left(-ife^{2c}+if\right)e^{-c}}{4a}+\frac{x\left(-iee^{2c}+ie\right)e^{-c}}{2a} & \text{otherwise} \end{cases}+\frac{ex}{a}+\frac{fx^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Piecewise((((-2*I*a*d**3*e - 2*I*a*d**3*f*x - 2*I*a*d**2*f)*exp(-d*x) + (-2
*I*a*d**3*e*exp(2*c) - 2*I*a*d**3*f*x*exp(2*c) + 2*I*a*d**2*f*exp(2*c))*exp
(d*x))*exp(-c)/(4*a**2*d**4), Ne(a**2*d**4*exp(c), 0)), (x**2*(-I*f*exp(2*c
) + I*f)*exp(-c)/(4*a) + x*(-I*e*exp(2*c) + I*e)*exp(-c)/(2*a), True)) + e*
x/a + f*x**2/(2*a)
```

Giac [A]
time = 0.43, size = 96, normalized size = 1.71

$$\frac{\left(d^2fx^2e^{(dx+c)}+2\,d^2exe^{(dx+c)}-i\,dfxe^{(2\,dx+2\,c)}-i\,dfx-i\,dee^{(2\,dx+2\,c)}-i\,de+i\,fe^{(2\,dx+2\,c)}-i\,f\right)e^{(-dx-c)}}{2\,ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(d^2*f*x^2*e^(d*x + c) + 2*d^2*e*x*e^(d*x + c) - I*d*f*x*e^(2*d*x + 2*c
) - I*d*f*x - I*d*e*e^(2*d*x + 2*c) - I*d*e + I*f*e^(2*d*x + 2*c) - I*f)*e^
(-d*x - c)/(a*d^2)
```

Mupad [B]
time = 0.38, size = 87, normalized size = 1.55

$$\frac{f\,x^2}{2\,a}+\mathrm{e}^{c+d\,x}\,\left(\frac{\left(f-d\,e\right)\,1\mathrm{i}}{2\,a\,d^2}-\frac{f\,x\,1\mathrm{i}}{2\,a\,d}\right)-\mathrm{e}^{-c-d\,x}\,\left(\frac{\left(f+d\,e\right)\,1\mathrm{i}}{2\,a\,d^2}+\frac{f\,x\,1\mathrm{i}}{2\,a\,d}\right)+\frac{e\,x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*(e + f*x))/(a + a*sinh(c + d*x)*1i),x)
```

```
[Out] exp(c + d*x)*(((f - d*e)*1i)/(2*a*d^2) - (f*x*1i)/(2*a*d)) - exp(- c - d*x)
*(((f + d*e)*1i)/(2*a*d^2) + (f*x*1i)/(2*a*d)) + (f*x^2)/(2*a) + (e*x)/a
```

3.262 $\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=22

$$\frac{x}{a}-\frac{i\cosh(c+dx)}{ad}$$

[Out] x/a-I*cosh(d*x+c)/a/d

Rubi [A]
time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {2761, 8}

$$\frac{x}{a}-\frac{i\cosh(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]

[Out] x/a - (I*Cosh[c + d*x])/(a*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\frac{i\cosh(c+dx)}{ad}+\frac{\int 1\,dx}{a}\\ &= \frac{x}{a}-\frac{i\cosh(c+dx)}{ad}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 139 vs. 2(22) = 44.
time = 0.12, size = 139, normalized size = 6.32

$$\frac{\cosh^3(c+dx)\left(-2\mathrm{ArcSin}\left(\frac{\sqrt{1-i\sinh(c+dx)}}{\sqrt{2}}\right)\sqrt{1-i\sinh(c+dx)}+\sqrt{1+i\sinh(c+dx)}-i\sqrt{1+i\sinh(c+dx)}\sinh(c+dx)\right)}{ad\sqrt{1+i\sinh(c+dx)}\,(-i+\sinh(c+dx))(i+\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (Cosh[c + d*x]^3*(-2*ArcSin[Sqrt[1 - I*Sinh[c + d*x]]/Sqrt[2]]*Sqrt[1 - I*S
inh[c + d*x]] + Sqrt[1 + I*Sinh[c + d*x]] - I*Sqrt[1 + I*Sinh[c + d*x]]*Sin
h[c + d*x]))/(a*d*Sqrt[1 + I*Sinh[c + d*x]]*(-I + Sinh[c + d*x])*(I + Sinh[
c + d*x])^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(21) = 42$.
time = 1.11, size = 70, normalized size = 3.18

method	result	size
risch	$\frac{x}{a}-\frac{i\mathrm{e}^{dx+c}}{2ad}-\frac{i\mathrm{e}^{-dx-c}}{2ad}$	40
derivativedivides	$\frac{\frac{2i}{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2}-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{i}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$	70
default	$\frac{\frac{2i}{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2}-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{i}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a*(1/2*I/(tanh(1/2*d*x+1/2*c)-1)-1/2*ln(tanh(1/2*d*x+1/2*c)-1)-1/2*I/(t
anh(1/2*d*x+1/2*c)+1)+1/2*ln(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(20) = 40$.
time = 0.27, size = 44, normalized size = 2.00

$$\frac{dx+c}{ad}-\frac{i\,e^{(dx+c)}}{2\,ad}-\frac{i\,e^{(-dx-c)}}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(a*d) - 1/2*I*e^(d*x + c)/(a*d) - 1/2*I*e^(-d*x - c)/(a*d)
```

Fricas [A]
time = 0.33, size = 40, normalized size = 1.82

$$\frac{\left(2\,dxe^{(dx+c)}-i\,e^{(2\,dx+2\,c)}-i\right)e^{(-dx-c)}}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d*x*e^(d*x + c) - I*e^(2*d*x + 2*c) - I)*e^(-d*x - c)/(a*d)
```

Sympy [A]
time = 0.12, size = 78, normalized size = 3.55

$$\begin{cases} \frac{(-2iade^{2c}e^{dx}-2iade^{-dx})e^{-c}}{4a^2d^2} & \text{for } a^2d^2e^c \neq 0 \\ x\left(\frac{(-ie^{2c}+2e^c+i)e^{-c}}{2a} - \frac{1}{a}\right) & \text{otherwise} \end{cases} + \frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Piecewise(((-2*I*a*d*exp(2*c)*exp(d*x) - 2*I*a*d*exp(-d*x))*exp(-c)/(4*a**2
*d**2), Ne(a**2*d**2*exp(c), 0)), (x*((-I*exp(2*c) + 2*exp(c) + I)*exp(-c)/
(2*a) - 1/a), True)) + x/a
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(20) = 40$.
time = 0.45, size = 41, normalized size = 1.86

$$\frac{\frac{2\,(dx+c)}{a} - \frac{i\,e^{(dx+c)}}{a} - \frac{i\,e^{(-dx-c)}}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)/a - I*e^(d*x + c)/a - I*e^(-d*x - c)/a)/d
```

Mupad [B]
time = 0.21, size = 36, normalized size = 1.64

$$\frac{x}{a} - \frac{\frac{\mathrm{e}^{c+d\,x}\,1{}\mathrm{i}}{2} + \frac{\mathrm{e}^{-c-d\,x}\,1{}\mathrm{i}}{2}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/(a + a*sinh(c + d*x)*1i),x)
```

```
[Out] x/a - ((exp(c + d*x)*1i)/2 + (exp(- c - d*x)*1i)/2)/(a*d)
```

3.263 $\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))} dx$

Optimal. Leaf size=76

$$\frac{\log(e+fx)}{af}-\frac{i\text{Chi}\left(\frac{de}{f}+dx\right)\sinh\left(c-\frac{de}{f}\right)}{af}-\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{af}$$

```
[Out] ln(f*x+e)/a/f-I*cosh(c-d*e/f)*Shi(d*e/f+d*x)/a/f-I*Chi(d*e/f+d*x)*sinh(c-d*
e/f)/a/f
```

Rubi [A]
time = 0.15, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {5682, 31, 3384, 3379, 3382}

$$-\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{af}-\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{af}+\frac{\log(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Log[e + f*x]/(a*f) - (I*CoshIntegral[(d*e)/f + d*x]*Sinh[c - (d*e)/f])/(a*f
) - (I*Cosh[c - (d*e)/f]*SinhIntegral[(d*e)/f + d*x])/(a*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5682

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cosh[c
+ d*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Si
nh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ
[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\sinh(c+dx)}{e+fx}\,dx}{a} + \frac{\int \frac{1}{e+fx}\,dx}{a}$$

$$= \frac{\log(e+fx)}{af} - \frac{\left(i\cosh\left(c-\frac{de}{f}\right)\right)\int \frac{\sinh\left(\frac{de}{f}+dx\right)}{e+fx}\,dx}{a} - \frac{\left(i\sinh\left(c-\frac{de}{f}\right)\right.}{}$$

$$= \frac{\log(e+fx)}{af} - \frac{i\mathrm{Chi}\left(\frac{de}{f}+dx\right)\sinh\left(c-\frac{de}{f}\right)}{af} - \frac{i\cosh\left(c-\frac{de}{f}\right)\mathrm{Shi}\left(\right.}{af}$$

Mathematica [A]
time = 0.18, size = 62, normalized size = 0.82

$$\frac{\log(e+fx) - i\mathrm{Chi}\left(d\left(\frac{e}{f}+x\right)\right)\sinh\left(c-\frac{de}{f}\right) - i\cosh\left(c-\frac{de}{f}\right)\mathrm{Shi}\left(d\left(\frac{e}{f}+x\right)\right)}{af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] (Log[e + f*x] - I*CoshIntegral[d*(e/f + x)]*Sinh[c - (d*e)/f] - I*Cosh[c -
(d*e)/f]*SinhIntegral[d*(e/f + x)])/(a*f)
```

Maple [A]
time = 1.62, size = 103, normalized size = 1.36

method	result	size
risch	$\frac{\ln(fx+e)}{af} + \frac{i e^{\frac{cf-de}{f}}\,\mathrm{expIntegral}\left(1,-dx-c-\frac{-cf+de}{f}\right)}{2af} - \frac{i e^{-\frac{cf-de}{f}}\,\mathrm{expIntegral}\left(1,dx+c-\frac{cf-de}{f}\right)}{2af}$	103

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)

[Out] ln(f*x+e)/a/f+1/2*I/a/f*exp((c*f-d*e)/f)*Ei(1,-d*x-c-(-c*f+d*e)/f)-1/2*I/a/f*exp(-(c*f-d*e)/f)*Ei(1,d*x+c-(c*f-d*e)/f)

Maxima [A]
time = 0.37, size = 81, normalized size = 1.07

$$-\frac{i\,e^{\left(-c+\frac{de}{f}\right)}E_1\left(\frac{(fx+e)d}{f}\right)}{2\,af}+\frac{i\,e^{\left(c-\frac{de}{f}\right)}E_1\left(-\frac{(fx+e)d}{f}\right)}{2\,af}+\frac{\log\left(fx+e\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")

[Out] -1/2*I*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(a*f) + 1/2*I*e^(c - d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(a*f) + log(f*x + e)/(a*f)

Fricas [A]
time = 0.34, size = 84, normalized size = 1.11

$$\frac{-i\,\mathrm{Ei}\left(\frac{dfx+de}{f}\right)e^{\left(\frac{cf-de}{f}\right)}+i\,\mathrm{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(-\frac{cf-de}{f}\right)}+2\,\log\left(\frac{fx+e}{f}\right)}{2\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")

[Out] 1/2*(-I*Ei((d*f*x + d*e)/f)*e^((c*f - d*e)/f) + I*Ei(-(d*f*x + d*e)/f)*e^(-(c*f - d*e)/f) + 2*log((f*x + e)/f))/(a*f)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\cosh^2\left(c+dx\right)}{e\sinh\left(c+dx\right)-ie+fx\sinh\left(c+dx\right)-ifx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)

[Out] -I*Integral(cosh(c + d*x)**2/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) - I*f*x), x)/a

Giac [A]
time = 0.45, size = 76, normalized size = 1.00

$$-\frac{\left(i\,\mathrm{Ei}\left(\frac{dfx+de}{f}\right)e^{\left(2\,c-\frac{de}{f}\right)}-i\,\mathrm{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(\frac{de}{f}\right)}-2\,e^{c}\log\left(i\,fx+i\,e\right)\right)e^{(-c)}}{2\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(I*Ei((d*f*x + d*e)/f)*e^(2*c - d*e/f) - I*Ei(-(d*f*x + d*e)/f)*e^(d*e
/f) - 2*e^c*log(I*f*x + I*e))*e^(-c)/(a*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^2}{(e+f\,x)\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/((e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

```
[Out] int(cosh(c + d*x)^2/((e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.264 $\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=103

$$-\frac{1}{af(e+fx)}-\frac{id\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{af^2}+\frac{i\sinh(c+dx)}{af(e+fx)}-\frac{id\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{af^2}$$

```
[Out] -1/a/f/(f*x+e)-I*d*Chi(d*e/f+d*x)*cosh(c-d*e/f)/a/f^2-I*d*Shi(d*e/f+d*x)*si
nh(c-d*e/f)/a/f^2+I*sinh(d*x+c)/a/f/(f*x+e)
```

Rubi [A]
time = 0.16, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$,
Rules used = {5682, 32, 3378, 3384, 3379, 3382}

$$-\frac{id\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{af^2}-\frac{id\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{af^2}+\frac{i\sinh(c+dx)}{af(e+fx)}-\frac{1}{af(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] -(1/(a*f*(e + f*x))) - (I*d*Cosh[c - (d*e)/f]*CoshIntegral[(d*e)/f + d*x])/
(a*f^2) + (I*Sinh[c + d*x])/(a*f*(e + f*x)) - (I*d*Sinh[c - (d*e)/f]*SinhIn
tegral[(d*e)/f + d*x])/(a*f^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5682

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cosh[c
+ d*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Si
nh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ
[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx &= -\frac{i\int \frac{\sinh(c+dx)}{(e+fx)^2}\,dx}{a}+\frac{\int \frac{1}{(e+fx)^2}\,dx}{a}\\ &= -\frac{1}{af(e+fx)}+\frac{i\sinh(c+dx)}{af(e+fx)}-\frac{(id)\int \frac{\cosh(c+dx)}{e+fx}\,dx}{af}\\ &= -\frac{1}{af(e+fx)}+\frac{i\sinh(c+dx)}{af(e+fx)}-\frac{\left(id\cosh\left(c-\frac{de}{f}\right)\right)\int \frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx}\,d}{af}\\ &= -\frac{1}{af(e+fx)}-\frac{id\cosh\left(c-\frac{de}{f}\right)\operatorname{Chi}\left(\frac{de}{f}+dx\right)}{af^2}+\frac{i\sinh(c+dx)}{af(e+fx)}-\end{aligned}$$

Mathematica [A]
time = 0.30, size = 85, normalized size = 0.83

$$-\frac{i\left(d(e+fx)\cosh\left(c-\frac{de}{f}\right)\operatorname{Chi}\left(d\left(\frac{e}{f}+x\right)\right)-f(i+\sinh(c+dx))+d(e+fx)\sinh\left(c-\frac{de}{f}\right)\operatorname{Shi}\left(d\left(\frac{e}{f}+x\right)\right)\right)}{af^2(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

[Out] ((-I)*(d*(e + f*x)*Cosh[c - (d*e)/f]*CoshIntegral[d*(e/f + x)] - f*(I + Sinh[c + d*x]) + d*(e + f*x)*Sinh[c - (d*e)/f]*SinhIntegral[d*(e/f + x)]))/(a*f^2*(e + f*x))

Maple [A]
time = 1.52, size = 164, normalized size = 1.59

method	result
risch	$-\frac{1}{af(fx+e)}+\frac{id\,\mathrm{e}^{dx+c}}{2a\,f^2\left(\frac{de}{f}+dx\right)}+\frac{id\,\mathrm{e}^{\frac{cf-de}{f}}\,\mathrm{expIntegral}\left(1,-dx-c-\frac{-cf+de}{f}\right)}{2a\,f^2}-\frac{id\,\mathrm{e}^{-dx-c}}{2af(dxf+de)}+\frac{id\,\mathrm{e}^{-\frac{cf-de}{f}}\,\mathrm{expIntegral}\left(1,dx\right.}{2a\,f^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -1/a/f/(f*x+e)+1/2*I*d/a/f^2*exp(d*x+c)/(d*e/f+d*x)+1/2*I*d/a/f^2*exp((c*f-d*e)/f)*Ei(1,-d*x-c-(-c*f+d*e)/f)-1/2*I/a*d*exp(-d*x-c)/f/(d*f*x+d*e)+1/2*I/a*d/f^2*exp(-(c*f-d*e)/f)*Ei(1,d*x+c-(c*f-d*e)/f)

Maxima [A]
time = 0.40, size = 99, normalized size = 0.96

$$-\frac{1}{af^2x+afe}-\frac{i\,e^{\left(-c+\frac{de}{f}\right)}E_2\left(\frac{(fx+e)d}{f}\right)}{2\,(fx+e)af}+\frac{i\,e^{\left(c-\frac{de}{f}\right)}E_2\left(-\frac{(fx+e)d}{f}\right)}{2\,(fx+e)af}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")

[Out] -1/(a*f^2*x + a*f*e) - 1/2*I*e^(-c + d*e/f)*exp_integral_e(2, (f*x + e)*d/f)/((f*x + e)*a*f) + 1/2*I*e^(c - d*e/f)*exp_integral_e(2, -(f*x + e)*d/f)/((f*x + e)*a*f)

Fricas [A]
time = 0.36, size = 141, normalized size = 1.37

$$\frac{\left((-i\,dfx-i\,de)\mathrm{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(dx+c-\frac{cf-de}{f}\right)}+i\,fe^{(2\,dx+2\,c)}+\left((-i\,dfx-i\,de)\mathrm{Ei}\left(\frac{dfx+de}{f}\right)e^{\left(\frac{cf-de}{f}\right)}-2\,f\right)e^{(dx+c)}-i\,f\right)e^{(-dx-c)}}{2\,(af^3x+af^2e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")

[Out] 1/2*((-I*d*f*x - I*d*e)*Ei(-(d*f*x + d*e)/f)*e^(d*x + c - (c*f - d*e)/f) + I*f*e^(2*d*x + 2*c) + ((-I*d*f*x - I*d*e)*Ei((d*f*x + d*e)/f)*e^((c*f - d*e)/f) - 2*f)*e^(d*x + c) - I*f)*e^(-d*x - c)/(a*f^3*x + a*f^2*e)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{\cosh^2(c+dx)}{e^2\sinh(c+dx)-ie^2+2efx\sinh(c+dx)-2iefx+f^2x^2\sinh(c+dx)-if^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*Integral(cosh(c + d*x)**2/(e**2*sinh(c + d*x) - I*e**2 + 2*e*f*x*sinh(c
+ d*x) - 2*I*e*f*x + f**2*x**2*sinh(c + d*x) - I*f**2*x**2), x)/a
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 572 vs. 2(97) = 194.
time = 0.49, size = 572, normalized size = 5.55

$$-\frac{\left(i(fx+e)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)d^2\operatorname{Ei}\left(-\frac{(fx+e)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)+de-cf}{f}\right)e^{\left(\frac{de-cf}{f}\right)}+id^3e\operatorname{Ei}\left(-\frac{(fx+e)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)+de-cf}{f}\right)e^{\left(\frac{de-cf}{f}\right)}-icd^2f\operatorname{Ei}\left(-\frac{(fx+e)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)+de-cf}{f}\right)e^{\left(\frac{de-cf}{f}\right)}+i(fx+e)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)d^2\operatorname{Ei}\left(\frac{(fx+e)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)+de-cf}{f}\right)e^{\left(-\frac{de-cf}{f}\right)}+id^3e\operatorname{Ei}\left(\frac{(fx+e)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)+de-cf}{f}\right)e^{\left(-\frac{de-cf}{f}\right)}-icd^2f\operatorname{Ei}\left(\frac{(fx+e)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)+de-cf}{f}\right)e^{\left(-\frac{de-cf}{f}\right)}-id^2fe^{\left(\frac{(fx+e)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)}{f}\right)}+id^2fe^{\left(-\frac{(fx+e)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)}{f}\right)}+2d^2f\right)f^2}{2\left((fx+e)a\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)f^4+adef^4-acf^5\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(I*(f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e))*d^2*Ei(-((f*x + e)*(d
 - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^((d*e - c*f)/f) + I*d^3
*e*Ei(-((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^((d
*e - c*f)/f) - I*c*d^2*f*Ei(-((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e))
 + d*e - c*f)/f)*e^((d*e - c*f)/f) + I*(f*x + e)*(d - d*e/(f*x + e) + c*f/(
f*x + e))*d^2*Ei(((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f
)/f)*e^(-(d*e - c*f)/f) + I*d^3*e*Ei(((f*x + e)*(d - d*e/(f*x + e) + c*f/(f
*x + e)) + d*e - c*f)/f)*e^(-(d*e - c*f)/f) - I*c*d^2*f*Ei(((f*x + e)*(d -
d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^(-(d*e - c*f)/f) - I*d^2*f
*e^((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e))/f) + I*d^2*f*e^(-(f*x + e
)*(d - d*e/(f*x + e) + c*f/(f*x + e))/f) + 2*d^2*f)*f^2/(((f*x + e)*a*(d -
d*e/(f*x + e) + c*f/(f*x + e))*f^4 + a*d*e*f^4 - a*c*f^5)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^2}{(e+f\,x)^2\,\left(a+a\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

```
[Out] int(cosh(c + d*x)^2/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.265 $\int \frac{(e+fx)^3 \cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=231

$$-\frac{3if^3x}{8ad^3}-\frac{i(e+fx)^3}{4ad}-\frac{6f^3\cosh(c+dx)}{ad^4}-\frac{3f(e+fx)^2\cosh(c+dx)}{ad^2}+\frac{6f^2(e+fx)\sinh(c+dx)}{ad^3}+\frac{(e+fx)^3\sinh(c+dx)}{ad}$$

```
[Out] -3/8*I*f^3*x/a/d^3-1/4*I*(f*x+e)^3/a/d-6*f^3*cosh(d*x+c)/a/d^4-3*f*(f*x+e)^
2*cosh(d*x+c)/a/d^2+6*f^2*(f*x+e)*sinh(d*x+c)/a/d^3+(f*x+e)^3*sinh(d*x+c)/a
/d+3/8*I*f^3*cosh(d*x+c)*sinh(d*x+c)/a/d^4+3/4*I*f*(f*x+e)^2*cosh(d*x+c)*si
nh(d*x+c)/a/d^2-3/4*I*f^2*(f*x+e)*sinh(d*x+c)^2/a/d^3-1/2*I*(f*x+e)^3*sinh(
d*x+c)^2/a/d
```

Rubi [A]

time = 0.18, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {5682, 3377, 2718, 5554, 3392, 32, 2715, 8}

$$-\frac{6f^3\cosh(c+dx)}{ad^4}+\frac{3if^3\sinh(c+dx)\cosh(c+dx)}{8ad^4}-\frac{3if^2(e+fx)\sinh^2(c+dx)}{4ad^3}+\frac{6f^2(e+fx)\sinh(c+dx)}{ad^3}-\frac{3f(e+fx)^2\cosh(c+dx)}{ad^2}+\frac{3if(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{4ad^2}-\frac{i(e+fx)^3\sinh^2(c+dx)}{2ad}+\frac{(e+fx)^3\sinh(c+dx)}{ad}-\frac{3if^3x}{8ad^3}-\frac{i(e+fx)^3}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (((-3*I)/8)*f^3*x)/(a*d^3) - ((I/4)*(e + f*x)^3)/(a*d) - (6*f^3*Cosh[c + d*
x])/(a*d^4) - (3*f*(e + f*x)^2*Cosh[c + d*x])/(a*d^2) + (6*f^2*(e + f*x)*Si
nh[c + d*x])/(a*d^3) + ((e + f*x)^3*Sinh[c + d*x])/(a*d) + (((3*I)/8)*f^3*C
osh[c + d*x]*Sinh[c + d*x])/(a*d^4) + (((3*I)/4)*f*(e + f*x)^2*Cosh[c + d*x
]*Sinh[c + d*x])/(a*d^2) - (((3*I)/4)*f^2*(e + f*x)*Sinh[c + d*x]^2)/(a*d^3
) - ((I/2)*(e + f*x)^3*Sinh[c + d*x]^2)/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cosh[c
+ d*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Si
nh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ
[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)\,dx}{a} + \frac{\int (e+fx)^3\cosh(c+dx)\,dx}{a}$$
$$= \frac{(e+fx)^3\sinh(c+dx)}{ad} - \frac{i(e+fx)^3\sinh^2(c+dx)}{2ad} + \frac{(3if)\int (e+fx)^2\sinh^2}{2ad}$$
$$= -\frac{3f(e+fx)^2\cosh(c+dx)}{ad^2} + \frac{(e+fx)^3\sinh(c+dx)}{ad} + \frac{3if(e+fx)^2\cosh(}{4a}$$
$$= -\frac{i(e+fx)^3}{4ad} - \frac{3f(e+fx)^2\cosh(c+dx)}{ad^2} + \frac{6f^2(e+fx)\sinh(c+dx)}{ad^3} + (e$$
$$= -\frac{3if^3x}{8ad^3} - \frac{i(e+fx)^3}{4ad} - \frac{6f^3\cosh(c+dx)}{ad^4} - \frac{3f(e+fx)^2\cosh(c+dx)}{ad^2} + 6$$

Mathematica [A]
time = 0.71, size = 134, normalized size = 0.58

$$\frac{-96f(2f^2+d^2(e+fx)^2)\cosh(c+dx) - 4id(e+fx)(3f^2+2d^2(e+fx)^2)\cosh(2(c+dx)) + 4(8d(e+fx)(6f^2+d^2(e+fx)^2) + 3if(f^2+2d^2(e+fx)^2)\cosh(c+dx))\sinh(c+dx)}{32ad^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (-96*f*(2*f^2 + d^2*(e + f*x)^2)*Cosh[c + d*x] - (4*I)*d*(e + f*x)*(3*f^2 +
 2*d^2*(e + f*x)^2)*Cosh[2*(c + d*x)] + 4*(8*d*(e + f*x)*(6*f^2 + d^2*(e +
f*x)^2) + (3*I)*f*(f^2 + 2*d^2*(e + f*x)^2)*Cosh[c + d*x])*Sinh[c + d*x])/(
32*a*d^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 725 vs. 2(213) = 426.
time = 1.21, size = 726, normalized size = 3.14 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^4/a*(3/2*I*f^2*c^2*e*d*cosh(d*x+c)^2-1/2*I*c^3*f^3*cosh(d*x+c)^2+1/2*I
*e^3*d^3*cosh(d*x+c)^2-3*I*c*f^3*(1/2*(d*x+c)^2*cosh(d*x+c)^2-1/2*(d*x+c)*c
osh(d*x+c)*sinh(d*x+c)-1/4*(d*x+c)^2+1/4*cosh(d*x+c)^2)+I*f^3*(1/2*(d*x+c)^
3*cosh(d*x+c)^2-3/4*(d*x+c)^2*cosh(d*x+c)*sinh(d*x+c)-1/4*(d*x+c)^3+3/4*(d*
x+c)*cosh(d*x+c)^2-3/8*cosh(d*x+c)*sinh(d*x+c)-3/8*d*x-3/8*c)+3*I*f*e^2*d^2
*(1/2*(d*x+c)*cosh(d*x+c)^2-1/4*cosh(d*x+c)*sinh(d*x+c)-1/4*d*x-1/4*c)+3*I*
f^2*e*d*(1/2*(d*x+c)^2*cosh(d*x+c)^2-1/2*(d*x+c)*cosh(d*x+c)*sinh(d*x+c)-1/
4*(d*x+c)^2+1/4*cosh(d*x+c)^2)+3*I*c^2*f^3*(1/2*(d*x+c)*cosh(d*x+c)^2-1/4*c
osh(d*x+c)*sinh(d*x+c)-1/4*d*x-1/4*c)-6*I*f^2*c*e*d*(1/2*(d*x+c)*cosh(d*x+c
)^2-1/4*cosh(d*x+c)*sinh(d*x+c)-1/4*d*x-1/4*c)-3/2*I*f*c*e^2*d^2*cosh(d*x+c
)^2+c^3*f^3*sinh(d*x+c)-3*f^2*c^2*e*d*sinh(d*x+c)-3*c^2*f^3*((d*x+c)*sinh(d
```

```
*x+c)-cosh(d*x+c))+3*f*c*e^2*d^2*sinh(d*x+c)+6*c*d*e*f^2*((d*x+c)*sinh(d*x+
c)-cosh(d*x+c))+3*c*f^3*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh
(d*x+c))-e^3*d^3*sinh(d*x+c)-3*d^2*e^2*f*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-
3*d*e*f^2*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-f^3*(
(d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(
d*x+c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 0.34, size = 401, normalized size = 1.74

$$\frac{\left(-4i\,d^3f^3x^3-6i\,d^2f^3x^2-6i\,df^3x-4i\,d^3e^3-3i\,f^3-6\,(2i\,d^3fx+i\,d^2f)e^2-6\,(2i\,d^3f^2x^2+2i\,d^2f^2x+i\,df^2)e+(-4i\,d^3f^3x^3+6i\,d^2f^3x^2-6i\,df^3x-4i\,d^3e^3+3i\,f^3-6\,(2i\,d^3fx-i\,d^2f)e^2-6\,(2i\,d^3f^2x^2-2i\,d^2f^2x+i\,df^2)e)e^{(4\,dx+4\,c)}+16\,(d^3f^3x^3-3\,d^2f^3x^2+6\,df^3x+d^3e^3-6\,f^3+3\,(d^3fx-d^2f)e^2+3\,(d^3f^2x^2-2\,d^2f^2x+2\,df^2)e)e^{(3\,dx+3\,c)}-16\,(d^3f^3x^3+3\,d^2f^3x^2+6\,df^3x+d^3e^3+6\,f^3+3\,(d^3fx+d^2f)e^2+3\,(d^3f^2x^2+2\,d^2f^2x+2\,df^2)e)e^{(dx+c)}\right)e^{(-2\,dx-2\,c)}}{32\,ad^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/32*(-4*I*d^3*f^3*x^3 - 6*I*d^2*f^3*x^2 - 6*I*d*f^3*x - 4*I*d^3*e^3 - 3*I*
f^3 - 6*(2*I*d^3*f*x + I*d^2*f)*e^2 - 6*(2*I*d^3*f^2*x^2 + 2*I*d^2*f^2*x +
I*d*f^2)*e + (-4*I*d^3*f^3*x^3 + 6*I*d^2*f^3*x^2 - 6*I*d*f^3*x - 4*I*d^3*e^
3 + 3*I*f^3 - 6*(2*I*d^3*f*x - I*d^2*f)*e^2 - 6*(2*I*d^3*f^2*x^2 - 2*I*d^2*
f^2*x + I*d*f^2)*e)*e^(4*d*x + 4*c) + 16*(d^3*f^3*x^3 - 3*d^2*f^3*x^2 + 6*d
*f^3*x + d^3*e^3 - 6*f^3 + 3*(d^3*f*x - d^2*f)*e^2 + 3*(d^3*f^2*x^2 - 2*d^2
*f^2*x + 2*d*f^2)*e)*e^(3*d*x + 3*c) - 16*(d^3*f^3*x^3 + 3*d^2*f^3*x^2 + 6*
d*f^3*x + d^3*e^3 + 6*f^3 + 3*(d^3*f*x + d^2*f)*e^2 + 3*(d^3*f^2*x^2 + 2*d^
2*f^2*x + 2*d*f^2)*e)*e^(d*x + c))*e^(-2*d*x - 2*c)/(a*d^4)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1040 vs. 2(214) = 428.
time = 0.65, size = 1040, normalized size = 4.50

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

[Out] Piecewise((((-2048*a**3*d**15*e**3*exp(2*c) - 6144*a**3*d**15*e**2*f*x*exp(2*c) - 6144*a**3*d**15*e*f**2*x**2*exp(2*c) - 2048*a**3*d**15*f**3*x**3*exp(2*c) - 6144*a**3*d**14*e**2*f*exp(2*c) - 12288*a**3*d**14*e*f**2*x*exp(2*c) - 6144*a**3*d**14*f**3*x**2*exp(2*c) - 12288*a**3*d**13*e*f**2*exp(2*c) - 12288*a**3*d**13*f**3*x*exp(2*c) - 12288*a**3*d**12*f**3*exp(2*c))*exp(-d*x) + (2048*a**3*d**15*e**3*exp(4*c) + 6144*a**3*d**15*e**2*f*x*exp(4*c) + 6144*a**3*d**15*e*f**2*x**2*exp(4*c) + 2048*a**3*d**15*f**3*x**3*exp(4*c) - 6144*a**3*d**14*e**2*f*exp(4*c) - 12288*a**3*d**14*e*f**2*x*exp(4*c) - 6144*a**3*d**14*f**3*x**2*exp(4*c) + 12288*a**3*d**13*e*f**2*exp(4*c) + 12288*a**3*d**13*f**3*x*exp(4*c) - 12288*a**3*d**12*f**3*exp(4*c))*exp(d*x) + (-512*I*a**3*d**15*e**3*exp(c) - 1536*I*a**3*d**15*e**2*f*x*exp(c) - 1536*I*a**3*d**15*e*f**2*x**2*exp(c) - 512*I*a**3*d**15*f**3*x**3*exp(c) - 768*I*a**3*d**14*e**2*f*exp(c) - 1536*I*a**3*d**14*e*f**2*x*exp(c) - 768*I*a**3*d**14*f**3*x**2*exp(c) - 768*I*a**3*d**13*e*f**2*exp(c) - 768*I*a**3*d**13*f**3*x*exp(c) - 384*I*a**3*d**12*f**3*exp(c))*exp(-2*d*x) + (-512*I*a**3*d**15*e**3*exp(5*c) - 1536*I*a**3*d**15*e**2*f*x*exp(5*c) - 1536*I*a**3*d**15*e*f**2*x**2*exp(5*c) - 512*I*a**3*d**15*f**3*x**3*exp(5*c) + 768*I*a**3*d**14*e**2*f*exp(5*c) + 1536*I*a**3*d**14*e*f**2*x*exp(5*c) + 768*I*a**3*d**14*f**3*x**2*exp(5*c) - 768*I*a**3*d**13*e*f**2*exp(5*c) - 768*I*a**3*d**13*f**3*x*exp(5*c) + 384*I*a**3*d**12*f**3*exp(5*c))*exp(2*d*x))*exp(-3*c)/(4096*a**4*d**16), Ne(a**4*d**16*exp(3*c), 0)), (x**4*(-I*f**3*exp(4*c) + 2*f**3*exp(3*c) + 2*f**3*exp(c) + I*f**3)*exp(-2*c)/(16*a) + x**3*(-I*e*f**2*exp(4*c) + 2*e*f**2*exp(3*c) + 2*e*f**2*exp(c) + I*e*f**2)*exp(-2*c)/(4*a) + x**2*(-3*I*e**2*f*exp(4*c) + 6*e**2*f*exp(3*c) + 6*e**2*f*exp(c) + 3*I*e**2*f)*exp(-2*c)/(8*a) + x*(-I*e**3*exp(4*c) + 2*e**3*exp(3*c) + 2*e**3*exp(c) + I*e**3)*exp(-2*c)/(4*a), True))

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(207) = 414$.
time = 0.46, size = 618, normalized size = 2.68

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^3*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")

[Out] -1/32*(4*I*d^3*f^3*x^3*e^(4*d*x + 4*c) - 16*d^3*f^3*x^3*e^(3*d*x + 3*c) + 16*d^3*f^3*x^3*e^(d*x + c) + 4*I*d^3*f^3*x^3 + 12*I*d^3*e*f^2*x^2*e^(4*d*x + 4*c) - 48*d^3*e*f^2*x^2*e^(3*d*x + 3*c) + 48*d^3*e*f^2*x^2*e^(d*x + c) + 12*I*d^3*e*f^2*x^2 + 12*I*d^3*e^2*f*x*e^(4*d*x + 4*c) - 6*I*d^2*f^3*x^2*e^(4*d*x + 4*c) - 48*d^3*e^2*f*x*e^(3*d*x + 3*c) + 48*d^2*f^3*x^2*e^(3*d*x + 3*c) + 48*d^3*e^2*f*x*e^(d*x + c) + 48*d^2*f^3*x^2*e^(d*x + c) + 12*I*d^3*e^2*f*x + 6*I*d^2*f^3*x^2 + 4*I*d^3*e^3*e^(4*d*x + 4*c) - 12*I*d^2*e*f^2*x*e^(4*d*x + 4*c) - 16*d^3*e^3*e^(3*d*x + 3*c) + 96*d^2*e*f^2*x*e^(3*d*x + 3*c) + 16*d^3*e^3*e^(d*x + c) + 96*d^2*e*f^2*x*e^(d*x + c) + 4*I*d^3*e^3 + 12*I*

```
d^2*e*f^2*x - 6*I*d^2*e^2*f*e^(4*d*x + 4*c) + 6*I*d*f^3*x*e^(4*d*x + 4*c) +
 48*d^2*e^2*f*e^(3*d*x + 3*c) - 96*d*f^3*x*e^(3*d*x + 3*c) + 48*d^2*e^2*f*e
^(d*x + c) + 96*d*f^3*x*e^(d*x + c) + 6*I*d^2*e^2*f + 6*I*d*f^3*x + 6*I*d*e
*f^2*e^(4*d*x + 4*c) - 96*d*e*f^2*e^(3*d*x + 3*c) + 96*d*e*f^2*e^(d*x + c)
+ 6*I*d*e*f^2 - 3*I*f^3*e^(4*d*x + 4*c) + 96*f^3*e^(3*d*x + 3*c) + 96*f^3*e
^(d*x + c) + 3*I*f^3)*e^(-2*d*x - 2*c)/(a*d^4)
```

Mupad [B]
time = 1.28, size = 449, normalized size = 1.94

$$-\mathrm{e}^{c+d\,x}\,\left(\frac{-d^3\,e^3+3\,d^2\,e^2\,f-6\,d\,e\,f^2+6\,f^3}{2\,a\,d^4}-\frac{f^3\,x^3}{2\,a\,d}+\frac{3\,f^2\,x^2\,\left(f-d\,e\right)}{2\,a\,d^2}-\frac{3\,f\,x\,\left(d^2\,e^2-2\,d\,e\,f+2\,f^2\right)}{2\,a\,d^3}\right)-\mathrm{e}^{-2\,c-2\,d\,x}\,\left(\frac{\left(4\,d^3\,e^3+6\,d^2\,e^2\,f+6\,d\,e\,f^2+3\,f^3\right)\,1{}\mathrm{i}}{32\,a\,d^4}+\frac{f^3\,x^3\,1{}\mathrm{i}}{8\,a\,d}+\frac{f\,x\,\left(2\,d^2\,e^2+2\,d\,e\,f+f^2\right)\,3{}\mathrm{i}}{16\,a\,d^3}+\frac{f^2\,x^2\,\left(f+2\,d\,e\right)\,3{}\mathrm{i}}{16\,a\,d^2}\right)+\mathrm{e}^{2\,c+2\,d\,x}\,\left(\frac{\left(-4\,d^3\,e^3+6\,d^2\,e^2\,f-6\,d\,e\,f^2+3\,f^3\right)\,1{}\mathrm{i}}{32\,a\,d^4}-\frac{f^3\,x^3\,1{}\mathrm{i}}{8\,a\,d}-\frac{f\,x\,\left(2\,d^2\,e^2-2\,d\,e\,f+f^2\right)\,3{}\mathrm{i}}{16\,a\,d^3}+\frac{f^2\,x^2\,\left(f-2\,d\,e\right)\,3{}\mathrm{i}}{16\,a\,d^2}\right)-\mathrm{e}^{-c-d\,x}\,\left(\frac{d^3\,e^3+3\,d^2\,e^2\,f+6\,d\,e\,f^2+6\,f^3}{2\,a\,d^4}+\frac{f^3\,x^3}{2\,a\,d}+\frac{3\,f^2\,x^2\,\left(f+d\,e\right)}{2\,a\,d^2}+\frac{3\,f\,x\,\left(d^2\,e^2+2\,d\,e\,f+2\,f^2\right)}{2\,a\,d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^3*(e + f*x)^3)/(a + a*sinh(c + d*x)*1i),x)
```

```
[Out] exp(2*c + 2*d*x)*(((3*f^3 - 4*d^3*e^3 + 6*d^2*e^2*f - 6*d*e*f^2)*1i)/(32*a*
d^4) - (f^3*x^3*1i)/(8*a*d) - (f*x*(f^2 + 2*d^2*e^2 - 2*d*e*f)*3i)/(16*a*d^
3) + (f^2*x^2*(f - 2*d*e)*3i)/(16*a*d^2)) - exp(- 2*c - 2*d*x)*(((3*f^3 + 4
*d^3*e^3 + 6*d^2*e^2*f + 6*d*e*f^2)*1i)/(32*a*d^4) + (f^3*x^3*1i)/(8*a*d) +
 (f*x*(f^2 + 2*d^2*e^2 + 2*d*e*f)*3i)/(16*a*d^3) + (f^2*x^2*(f + 2*d*e)*3i)
/(16*a*d^2)) - exp(c + d*x)*((6*f^3 - d^3*e^3 + 3*d^2*e^2*f - 6*d*e*f^2)/(2
*a*d^4) - (f^3*x^3)/(2*a*d) + (3*f^2*x^2*(f - d*e))/(2*a*d^2) - (3*f*x*(2*f
^2 + d^2*e^2 - 2*d*e*f))/(2*a*d^3)) - exp(- c - d*x)*((6*f^3 + d^3*e^3 + 3*
d^2*e^2*f + 6*d*e*f^2)/(2*a*d^4) + (f^3*x^3)/(2*a*d) + (3*f^2*x^2*(f + d*e)
)/(2*a*d^2) + (3*f*x*(2*f^2 + d^2*e^2 + 2*d*e*f))/(2*a*d^3))
```

3.266 $\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=171

$$-\frac{iefx}{2ad}-\frac{if^2x^2}{4ad}-\frac{2f(e+fx)\cosh(c+dx)}{ad^2}+\frac{2f^2\sinh(c+dx)}{ad^3}+\frac{(e+fx)^2\sinh(c+dx)}{ad}+\frac{if(e+fx)\cosh(c+d}{2ad^2}$$

```
[Out] -1/2*I*e*f*x/a/d-1/4*I*f^2*x^2/a/d-2*f*(f*x+e)*cosh(d*x+c)/a/d^2+2*f^2*sinh
(d*x+c)/a/d^3+(f*x+e)^2*sinh(d*x+c)/a/d+1/2*I*f*(f*x+e)*cosh(d*x+c)*sinh(d*
x+c)/a/d^2-1/4*I*f^2*sinh(d*x+c)^2/a/d^3-1/2*I*(f*x+e)^2*sinh(d*x+c)^2/a/d
```

Rubi [A]
time = 0.14, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {5682, 3377, 2717, 5554, 3391}

$$-\frac{if^2\sinh^2(c+dx)}{4ad^3}+\frac{2f^2\sinh(c+dx)}{ad^3}-\frac{2f(e+fx)\cosh(c+dx)}{ad^2}+\frac{if(e+fx)\sinh(c+dx)\cosh(c+dx)}{2ad^2}-\frac{i(e+fx)^2\sinh^2(c+dx)}{2ad}+\frac{(e+fx)^2\sinh(c+dx)}{ad}-\frac{iefx}{2ad}-\frac{if^2x^2}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-1/2*I)*e*f*x)/(a*d) - ((I/4)*f^2*x^2)/(a*d) - (2*f*(e + f*x)*Cosh[c + d*
x])/(a*d^2) + (2*f^2*Sinh[c + d*x])/(a*d^3) + ((e + f*x)^2*Sinh[c + d*x])/(
a*d) + ((I/2)*f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(a*d^2) - ((I/4)*f^2
*Sinh[c + d*x]^2)/(a*d^3) - ((I/2)*(e + f*x)^2*Sinh[c + d*x]^2)/(a*d)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cosh[c
+ d*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Si
nh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ
[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\frac{i\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)\,dx}{a}+\frac{\int (e+fx)^2\cosh(c+dx)\,dx}{a}\\ &= \frac{(e+fx)^2\sinh(c+dx)}{ad}-\frac{i(e+fx)^2\sinh^2(c+dx)}{2ad}+\frac{(if)\int (e+fx)\sinh^2}{ad}\\ &= -\frac{2f(e+fx)\cosh(c+dx)}{ad^2}+\frac{(e+fx)^2\sinh(c+dx)}{ad}+\frac{if(e+fx)\cosh(c}{2a}\\ &= -\frac{iefx}{2ad}-\frac{if^2x^2}{4ad}-\frac{2f(e+fx)\cosh(c+dx)}{ad^2}+\frac{2f^2\sinh(c+dx)}{ad^3}+\frac{(e+fx}{}\end{aligned}$$

Mathematica [A]
time = 0.52, size = 99, normalized size = 0.58

$$\frac{-32df(e+fx)\cosh(c+dx)-2i(f^2+2d^2(e+fx)^2)\cosh(2(c+dx))+8(2(2f^2+d^2(e+fx)^2)+idf(e+fx)\cosh(c+dx))\sinh(c+dx)}{16ad^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
[Out] (-32*d*f*(e + f*x)*Cosh[c + d*x] - (2*I)*(f^2 + 2*d^2*(e + f*x)^2)*Cosh[2*(
c + d*x)] + 8*(2*(2*f^2 + d^2*(e + f*x)^2) + I*d*f*(e + f*x)*Cosh[c + d*x])
*Sinh[c + d*x])/(16*a*d^3)
```

Maple [A]
time = 2.63, size = 241, normalized size = 1.41

method	result
risch	$-\frac{i(2f^2x^2d^2+4d^2efx+2d^2e^2-2d f^2x-2def+f^2)e^{2dx+2c}}{16a\,d^3}+\frac{(f^2x^2d^2+2d^2efx+d^2e^2-2d f^2x-2def+2f^2)e^{dx+c}}{2a\,d^3}-\frac{(f^2x^2d^2+2d}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*I*(2*d^2*f^2*x^2+4*d^2*e*f*x+2*d^2*e^2-2*d*f^2*x-2*d*e*f+f^2)/a/d^3*e
xp(2*d*x+2*c)+1/2*(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2-2*d*f^2*x-2*d*e*f+2*f^2)
/a/d^3*exp(d*x+c)-1/2*(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2+2*d*f^2*x+2*d*e*f+2*
f^2)/a/d^3*exp(-d*x-c)-1/16*I*(2*d^2*f^2*x^2+4*d^2*e*f*x+2*d^2*e^2+2*d*f^2*
x+2*d*e*f+f^2)/a/d^3*exp(-2*d*x-2*c)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**A**]
time = 0.35, size = 227, normalized size = 1.33

$$\frac{\left(-2 i d^2 f^2 x^2 - 2 i d f^2 x - 2 i d^2 e^2 - i f^2 - 2 \left(2 i d^2 f x + i d f\right) e + \left(-2 i d^2 f^2 x^2 + 2 i d f^2 x - 2 i d^2 e^2 - i f^2 - 2 \left(2 i d^2 f x - i d f\right) e\right) e^{(4 d x + 4 c)} + 8 \left(d^2 f^2 x^2 - 2 d f^2 x + d^2 e^2 + 2 f^2 + 2 \left(d^2 f x - d f\right) e\right) e^{(3 d x + 3 c)} - 8 \left(d^2 f^2 x^2 + 2 d f^2 x + d^2 e^2 + 2 f^2 + 2 \left(d^2 f x + d f\right) e\right) e^{(d x + c)}\right) e^{(-2 d x - 2 c)}}{16 a d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/16*(-2*I*d^2*f^2*x^2 - 2*I*d*f^2*x - 2*I*d^2*e^2 - I*f^2 - 2*(2*I*d^2*f*x
 + I*d*f)*e + (-2*I*d^2*f^2*x^2 + 2*I*d*f^2*x - 2*I*d^2*e^2 - I*f^2 - 2*(2*
I*d^2*f*x - I*d*f)*e)*e^(4*d*x + 4*c) + 8*(d^2*f^2*x^2 - 2*d*f^2*x + d^2*e^
2 + 2*f^2 + 2*(d^2*f*x - d*f)*e)*e^(3*d*x + 3*c) - 8*(d^2*f^2*x^2 + 2*d*f^2
*x + d^2*e^2 + 2*f^2 + 2*(d^2*f*x + d*f)*e)*e^(d*x + c))*e^(-2*d*x - 2*c)/(
a*d^3)
```

Sympy [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(150) = 300$.
time = 0.46, size = 631, normalized size = 3.69

$$\begin{cases} \frac{\left(\left(-512 a^3 d^{11} e^2 e^{2c} - 1024 a^3 d^{11} e f x e^{2c} - 512 a^3 d^{11} f^2 x^2 e^{2c} - 1024 a^3 d^{10} e f e^{2c} - 1024 a^3 d^{10} f^2 x e^{2c} - 1024 a^3 d^9 f^2 e^{2c}\right) e^{-dx} + \left(512 a^3 d^{11} e^2 e^{4c} + 1024 a^3 d^{11} e f x e^{4c} + 512 a^3 d^{11} f^2 x^2 e^{4c} - 1024 a^3 d^{10} e f e^{4c} - 1024 a^3 d^{10} f^2 x e^{4c} + 1024 a^3 d^9 f^2 e^{4c}\right) e^{dx} + \left(-128 i a^3 d^{11} e^2 e^{c} - 256 i a^3 d^{11} e f x e^{c} - 128 i a^3 d^{11} f^2 x^2 e^{c} - 128 i a^3 d^{10} e f e^{c} - 128 i a^3 d^{10} f^2 x e^{c} - 64 i a^3 d^9 f^2 e^{c}\right) e^{-2dx} + \left(-128 i a^3 d^{11} e^2 e^{5c} - 256 i a^3 d^{11} e f x e^{5c} - 128 i a^3 d^{11} f^2 x^2 e^{5c} + 128 i a^3 d^{10} e f e^{5c} + 128 i a^3 d^{10} f^2 x e^{5c} - 64 i a^3 d^9 f^2 e^{5c}\right) e^{2dx}\right) e^{-3c}}{1024 a^4 d^{12}} & \text{for}\: a^4 d^{12} e^{3c} \neq 0 \\ \frac{x^3 \left(-i f^2 e^{4c} + 2 f^2 e^{3c} + 2 f^2 e^{c} + i f^2\right) e^{-2c}}{12 a} + \frac{x^2 \left(-i e f e^{4c} + 2 e f e^{3c} + 2 e f e^{c} + i e f\right) e^{-2c}}{4 a} + \frac{x \left(-i e^2 e^{4c} + 2 e^2 e^{3c} + 2 e^2 e^{c} + i e^2\right) e^{-2c}}{4 a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Piecewise((((-512*a**3*d**11*e**2*exp(2*c) - 1024*a**3*d**11*e*f*x*exp(2*c)
 - 512*a**3*d**11*f**2*x**2*exp(2*c) - 1024*a**3*d**10*e*f*exp(2*c) - 1024*
a**3*d**10*f**2*x*exp(2*c) - 1024*a**3*d**9*f**2*exp(2*c))*exp(-d*x) + (512
*a**3*d**11*e**2*exp(4*c) + 1024*a**3*d**11*e*f*x*exp(4*c) + 512*a**3*d**11
*f**2*x**2*exp(4*c) - 1024*a**3*d**10*e*f*exp(4*c) - 1024*a**3*d**10*f**2*x
*exp(4*c) + 1024*a**3*d**9*f**2*exp(4*c))*exp(d*x) + (-128*I*a**3*d**11*e**
2*exp(c) - 256*I*a**3*d**11*e*f*x*exp(c) - 128*I*a**3*d**11*f**2*x**2*exp(c
) - 128*I*a**3*d**10*e*f*exp(c) - 128*I*a**3*d**10*f**2*x*exp(c) - 64*I*a**
3*d**9*f**2*exp(c))*exp(-2*d*x) + (-128*I*a**3*d**11*e**2*exp(5*c) - 256*I*
a**3*d**11*e*f*x*exp(5*c) - 128*I*a**3*d**11*f**2*x**2*exp(5*c) + 128*I*a**
3*d**10*e*f*exp(5*c) + 128*I*a**3*d**10*f**2*x*exp(5*c) - 64*I*a**3*d**9*f*
*2*exp(5*c))*exp(2*d*x))*exp(-3*c)/(1024*a**4*d**12), Ne(a**4*d**12*exp(3*c
), 0)), (x**3*(-I*f**2*exp(4*c) + 2*f**2*exp(3*c) + 2*f**2*exp(c) + I*f**2)
*exp(-2*c)/(12*a) + x**2*(-I*e*f*exp(4*c) + 2*e*f*exp(3*c) + 2*e*f*exp(c) +
 I*e*f)*exp(-2*c)/(4*a) + x*(-I*e**2*exp(4*c) + 2*e**2*exp(3*c) + 2*e**2*ex
p(c) + I*e**2)*exp(-2*c)/(4*a), True))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(151) = 302$.
time = 0.45, size = 338, normalized size = 1.98

$$-\frac{\left(2i\,d^2f^2x^2e^{(4\,dx+4\,c)} - 8\,d^2f^2x^2e^{(3\,dx+3\,c)} + 8\,d^2f^2x^2e^{(dx+c)} + 2i\,d^2f^2x^2 + 4i\,d^2efxe^{(4\,dx+4\,c)} - 16\,d^2efxe^{(3\,dx+3\,c)} + 16\,d^2efxe^{(dx+c)} + 4i\,d^2efx + 2i\,d^2e^2e^{(4\,dx+4\,c)} - 2i\,df^2xe^{(4\,dx+4\,c)} - 8\,d^2e^2e^{(3\,dx+3\,c)} + 16\,df^2xe^{(3\,dx+3\,c)} + 8\,d^2e^2e^{(dx+c)} + 16\,df^2xe^{(dx+c)} + 2i\,d^2e^2 + 2i\,df^2x - 2i\,defe^{(4\,dx+4\,c)} + 16\,defe^{(3\,dx+3\,c)} + 16\,defe^{(dx+c)} + 2i\,def + i\,f^2e^{(4\,dx+4\,c)} - 16\,f^2e^{(3\,dx+3\,c)} + 16\,f^2e^{(dx+c)} + i\,f^2\right)e^{(-2\,dx-2\,c)}}{16\,ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(2*I*d^2*f^2*x^2*e^(4*d*x + 4*c) - 8*d^2*f^2*x^2*e^(3*d*x + 3*c) + 8*
d^2*f^2*x^2*e^(d*x + c) + 2*I*d^2*f^2*x^2 + 4*I*d^2*e*f*x*e^(4*d*x + 4*c) -
 16*d^2*e*f*x*e^(3*d*x + 3*c) + 16*d^2*e*f*x*e^(d*x + c) + 4*I*d^2*e*f*x +
2*I*d^2*e^2*e^(4*d*x + 4*c) - 2*I*d*f^2*x*e^(4*d*x + 4*c) - 8*d^2*e^2*e^(3*
d*x + 3*c) + 16*d*f^2*x*e^(3*d*x + 3*c) + 8*d^2*e^2*e^(d*x + c) + 16*d*f^2*
x*e^(d*x + c) + 2*I*d^2*e^2 + 2*I*d*f^2*x - 2*I*d*e*f*e^(4*d*x + 4*c) + 16*
d*e*f*e^(3*d*x + 3*c) + 16*d*e*f*e^(d*x + c) + 2*I*d*e*f + I*f^2*e^(4*d*x +
 4*c) - 16*f^2*e^(3*d*x + 3*c) + 16*f^2*e^(d*x + c) + I*f^2)*e^(-2*d*x - 2*
c)/(a*d^3)
```

Mupad [**B**]
time = 0.94, size = 271, normalized size = 1.58

$$\mathrm{e}^{c+d\,x}\,\left(\frac{d^2\,e^2-2\,d\,e\,f+2\,f^2}{2\,a\,d^3}+\frac{f^2\,x^2}{2\,a\,d}-\frac{f\,x\,\left(f-d\,e\right)}{a\,d^2}\right)-\mathrm{e}^{-2\,c-2\,d\,x}\,\left(\frac{\left(2\,d^2\,e^2+2\,d\,e\,f+f^2\right)\,1{}\mathrm{i}}{16\,a\,d^3}+\frac{f^2\,x^2\,1{}\mathrm{i}}{8\,a\,d}+\frac{f\,x\,\left(f+2\,d\,e\right)\,1{}\mathrm{i}}{8\,a\,d^2}\right)-\mathrm{e}^{2\,c+2\,d\,x}\,\left(\frac{\left(2\,d^2\,e^2-2\,d\,e\,f+f^2\right)\,1{}\mathrm{i}}{16\,a\,d^3}+\frac{f^2\,x^2\,1{}\mathrm{i}}{8\,a\,d}-\frac{f\,x\,\left(f-2\,d\,e\right)\,1{}\mathrm{i}}{8\,a\,d^2}\right)-\mathrm{e}^{-c-d\,x}\,\left(\frac{d^2\,e^2+2\,d\,e\,f+2\,f^2}{2\,a\,d^3}+\frac{f^2\,x^2}{2\,a\,d}+\frac{f\,x\,\left(f+d\,e\right)}{a\,d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^3*(e + f*x)^2)/(a + a*sinh(c + d*x)*1i),x)
```

```
[Out] exp(c + d*x)*((2*f^2 + d^2*e^2 - 2*d*e*f)/(2*a*d^3) + (f^2*x^2)/(2*a*d) - (
f*x*(f - d*e))/(a*d^2)) - exp(- 2*c - 2*d*x)*(((f^2 + 2*d^2*e^2 + 2*d*e*f)*
1i)/(16*a*d^3) + (f^2*x^2*1i)/(8*a*d) + (f*x*(f + 2*d*e)*1i)/(8*a*d^2)) - e
xp(2*c + 2*d*x)*(((f^2 + 2*d^2*e^2 - 2*d*e*f)*1i)/(16*a*d^3) + (f^2*x^2*1i)
/(8*a*d) - (f*x*(f - 2*d*e)*1i)/(8*a*d^2)) - exp(- c - d*x)*((2*f^2 + d^2*e
^2 + 2*d*e*f)/(2*a*d^3) + (f^2*x^2)/(2*a*d) + (f*x*(f + d*e))/(a*d^2))
```

3.267 $\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=98

$$-\frac{ifx}{4ad}-\frac{f\cosh(c+dx)}{ad^2}+\frac{(e+fx)\sinh(c+dx)}{ad}+\frac{if\cosh(c+dx)\sinh(c+dx)}{4ad^2}-\frac{i(e+fx)\sinh^2(c+dx)}{2ad}$$

```
[Out] -1/4*I*f*x/a/d-f*cosh(d*x+c)/a/d^2+(f*x+e)*sinh(d*x+c)/a/d+1/4*I*f*cosh(d*x
+c)*sinh(d*x+c)/a/d^2-1/2*I*(f*x+e)*sinh(d*x+c)^2/a/d
```

Rubi [A]
time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {5682, 3377, 2718, 5554, 2715, 8}

$$-\frac{f\cosh(c+dx)}{ad^2}+\frac{if\sinh(c+dx)\cosh(c+dx)}{4ad^2}-\frac{i(e+fx)\sinh^2(c+dx)}{2ad}+\frac{(e+fx)\sinh(c+dx)}{ad}-\frac{ifx}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-1/4*I)*f*x)/(a*d) - (f*Cosh[c + d*x])/(a*d^2) + ((e + f*x)*Sinh[c + d*x]
)/(a*d) + ((I/4)*f*Cosh[c + d*x]*Sinh[c + d*x])/(a*d^2) - ((I/2)*(e + f*x)*
Sinh[c + d*x]^2)/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5682

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cosh[c
+ d*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Si
nh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ
[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\frac{i\int (e+fx)\cosh(c+dx)\sinh(c+dx)\,dx}{a}+\frac{\int (e+fx)\cosh(c+dx)\,dx}{a}\\ &= \frac{(e+fx)\sinh(c+dx)}{ad}-\frac{i(e+fx)\sinh^2(c+dx)}{2ad}+\frac{(if)\int \sinh^2(c+dx)\,dx}{2ad}-\\ &= -\frac{f\cosh(c+dx)}{ad^2}+\frac{(e+fx)\sinh(c+dx)}{ad}+\frac{if\cosh(c+dx)\sinh(c+dx)}{4ad^2}-\\ &= -\frac{ifx}{4ad}-\frac{f\cosh(c+dx)}{ad^2}+\frac{(e+fx)\sinh(c+dx)}{ad}+\frac{if\cosh(c+dx)\sinh(c+}{4ad^2}\end{aligned}$$

Mathematica [A]
time = 0.73, size = 60, normalized size = 0.61

$$\frac{if\cosh(c+dx)(4i+\sinh(c+dx))+d(e+fx)(-i\cosh(2(c+dx))+4\sinh(c+dx))}{4ad^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (I*f*Cosh[c + d*x]*(4*I + Sinh[c + d*x]) + d*(e + f*x)*((-I)*Cosh[2*(c + d*
x)] + 4*Sinh[c + d*x]))/(4*a*d^2)
```

Maple [A]
time = 1.20, size = 120, normalized size = 1.22

method	result

risch	$-\frac{i(2dxf+2de-f)\mathrm{e}^{2dx+2c}}{16a\,d^2}+\frac{(dxf+de-f)\mathrm{e}^{dx+c}}{2a\,d^2}-\frac{(dxf+de+f)\mathrm{e}^{-dx-c}}{2a\,d^2}-\frac{i(2dxf+2de+f)\mathrm{e}^{-2dx-2c}}{16a\,d^2}$
derivativedivides	$-\frac{-\frac{ifc\left(\cosh^2(dx+c)\right)}{2}+\frac{ied\left(\cosh^2(dx+c)\right)}{2}+if\left(\frac{(dx+c)\left(\cosh^2(dx+c)\right)}{2}-\frac{\cosh(dx+c)\sinh(dx+c)}{4}-\frac{dx}{4}-\frac{c}{4}\right)+fc\sinh(dx+c)-\mathrm{s}}{d^2a}$
default	$-\frac{-\frac{ifc\left(\cosh^2(dx+c)\right)}{2}+\frac{ied\left(\cosh^2(dx+c)\right)}{2}+if\left(\frac{(dx+c)\left(\cosh^2(dx+c)\right)}{2}-\frac{\cosh(dx+c)\sinh(dx+c)}{4}-\frac{dx}{4}-\frac{c}{4}\right)+fc\sinh(dx+c)-\mathrm{s}}{d^2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2/a*(-1/2*I*f*c*cosh(d*x+c)^2+1/2*I*e*d*cosh(d*x+c)^2+I*f*(1/2*(d*x+c)
*cosh(d*x+c)^2-1/4*cosh(d*x+c)*sinh(d*x+c)-1/4*d*x-1/4*c)+f*c*sinh(d*x+c)-s
inh(d*x+c)*d*e-f*((d*x+c)*sinh(d*x+c)-cosh(d*x+c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 0.34, size = 96, normalized size = 0.98

$$\frac{\left(-2i\,dfx-2i\,de+(-2i\,dfx-2i\,de+i\,f)e^{(4\,dx+4\,c)}+8\,(dfx+de-f)e^{(3\,dx+3\,c)}-8\,(dfx+de+f)e^{(dx+c)}-i\,f\right)e^{(-2\,dx-2\,c)}}{16\,ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(-2*I*d*f*x - 2*I*d*e + (-2*I*d*f*x - 2*I*d*e + I*f)*e^(4*d*x + 4*c) +
 8*(d*f*x + d*e - f)*e^(3*d*x + 3*c) - 8*(d*f*x + d*e + f)*e^(d*x + c) - I*
f)*e^(-2*d*x - 2*c)/(a*d^2)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(82) = 164.
time = 0.33, size = 321, normalized size = 3.28

$$\begin{cases} \frac{\left(\left(-512a^3d^7ee^{2c}-512a^3d^7fxe^{2c}-512a^3d^6fe^{2c}\right)e^{-dx}+\left(512a^3d^7ee^{4c}+512a^3d^7fxe^{4c}-512a^3d^6fe^{4c}\right)e^{dx}+\left(-128ia^3d^7ee^{c}-128ia^3d^7fxe^{c}-64ia^3d^6fe^{c}\right)e^{-2dx}+\left(-128ia^3d^7ee^{5c}-128ia^3d^7fxe^{5c}+64ia^3d^6fe^{5c}\right)e^{2dx}\right)e^{-3c}}{1024a^4d^8} & \text{for}\ a^4d^8e^{3c}\neq 0 \\ \frac{x^2\left(-ife^{4c}+2fe^{3c}+2fe^{c}+if\right)e^{-2c}}{8a}+\frac{x\left(-iee^{4c}+2ee^{3c}+2ee^{c}+ie\right)e^{-2c}}{4a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Piecewise((((-512*a**3*d**7*e*exp(2*c) - 512*a**3*d**7*f*x*exp(2*c) - 512*a
**3*d**6*f*exp(2*c))*exp(-d*x) + (512*a**3*d**7*e*exp(4*c) + 512*a**3*d**7*
f*x*exp(4*c) - 512*a**3*d**6*f*exp(4*c))*exp(d*x) + (-128*I*a**3*d**7*e*exp
(c) - 128*I*a**3*d**7*f*x*exp(c) - 64*I*a**3*d**6*f*exp(c))*exp(-2*d*x) + (
-128*I*a**3*d**7*e*exp(5*c) - 128*I*a**3*d**7*f*x*exp(5*c) + 64*I*a**3*d**6
*f*exp(5*c))*exp(2*d*x))*exp(-3*c)/(1024*a**4*d**8), Ne(a**4*d**8*exp(3*c),
 0)), (x**2*(-I*f*exp(4*c) + 2*f*exp(3*c) + 2*f*exp(c) + I*f)*exp(-2*c)/(8*
a) + x*(-I*e*exp(4*c) + 2*e*exp(3*c) + 2*e*exp(c) + I*e)*exp(-2*c)/(4*a), T
rue))
```

Giac [A]
time = 0.45, size = 138, normalized size = 1.41

$$-\frac{\left(2i\,dfxe^{(4\,dx+4\,c)}-8\,dfxe^{(3\,dx+3\,c)}+8\,dfxe^{(dx+c)}+2i\,dfx+2i\,dee^{(4\,dx+4\,c)}-8\,dee^{(3\,dx+3\,c)}+8\,dee^{(dx+c)}+2i\,de-i\,fe^{(4\,dx+4\,c)}+8\,fe^{(3\,dx+3\,c)}+8\,fe^{(dx+c)}+i\,f\right)e^{(-2\,dx-2\,c)}}{16\,ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(2*I*d*f*x*e^(4*d*x + 4*c) - 8*d*f*x*e^(3*d*x + 3*c) + 8*d*f*x*e^(d*x
 + c) + 2*I*d*f*x + 2*I*d*e*e^(4*d*x + 4*c) - 8*d*e*e^(3*d*x + 3*c) + 8*d*e
*e^(d*x + c) + 2*I*d*e - I*f*e^(4*d*x + 4*c) + 8*f*e^(3*d*x + 3*c) + 8*f*e^
(d*x + c) + I*f)*e^(-2*d*x - 2*c)/(a*d^2)
```

Mupad [B]
time = 0.54, size = 144, normalized size = 1.47

$$-\mathrm{e}^{-c-d\,x}\,\left(\frac{f+d\,e}{2\,a\,d^2}+\frac{f\,x}{2\,a\,d}\right)-\mathrm{e}^{-2\,c-2\,d\,x}\,\left(\frac{\left(f+2\,d\,e\right)\,1{}\mathrm{i}}{16\,a\,d^2}+\frac{f\,x\,1{}\mathrm{i}}{8\,a\,d}\right)+\mathrm{e}^{2\,c+2\,d\,x}\,\left(\frac{\left(f-2\,d\,e\right)\,1{}\mathrm{i}}{16\,a\,d^2}-\frac{f\,x\,1{}\mathrm{i}}{8\,a\,d}\right)-\mathrm{e}^{c+d\,x}\,\left(\frac{f-d\,e}{2\,a\,d^2}-\frac{f\,x}{2\,a\,d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^3*(e + f*x))/(a + a*sinh(c + d*x)*1i),x)
```

```
[Out] exp(2*c + 2*d*x)*(((f - 2*d*e)*1i)/(16*a*d^2) - (f*x*1i)/(8*a*d)) - exp(- 2
*c - 2*d*x)*(((f + 2*d*e)*1i)/(16*a*d^2) + (f*x*1i)/(8*a*d)) - exp(- c - d*
x)*((f + d*e)/(2*a*d^2) + (f*x)/(2*a*d)) - exp(c + d*x)*((f - d*e)/(2*a*d^2
) - (f*x)/(2*a*d))
```

3.268 $\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{\sinh(c+dx)}{ad} - \frac{i\sinh^2(c+dx)}{2ad}$$

```
[Out] sinh(d*x+c)/a/d-1/2*I*sinh(d*x+c)^2/a/d
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {2746}

$$\frac{\sinh(c+dx)}{ad} - \frac{i\sinh^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] Sinh[c + d*x]/(a*d) - ((I/2)*Sinh[c + d*x]^2)/(a*d)
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\frac{i\text{Subst}\left(\int (a-x)\,dx, x, ia\sinh(c+dx)\right)}{a^3d}\\ &= \frac{\sinh(c+dx)}{ad} - \frac{i\sinh^2(c+dx)}{2ad}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 28, normalized size = 0.82

$$\frac{(2-i\sinh(c+dx))\sinh(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
[Out] ((2 - I*Sinh[c + d*x])*Sinh[c + d*x])/(2*a*d)
```

Maple [A]
time = 0.66, size = 30, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{i\left(\frac{\left(\sinh^2(dx+c)\right)}{2}+i\sinh(dx+c)\right)}{da}$	30
default	$-\frac{i\left(\frac{\left(\sinh^2(dx+c)\right)}{2}+i\sinh(dx+c)\right)}{da}$	30
risch	$-\frac{i\mathrm{e}^{2dx+2c}}{8ad}+\frac{\mathrm{e}^{dx+c}}{2ad}-\frac{\mathrm{e}^{-dx-c}}{2ad}-\frac{i\mathrm{e}^{-2dx-2c}}{8ad}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -I/d/a*(1/2*sinh(d*x+c)^2+I*sinh(d*x+c))
```

Maxima [A]
time = 0.26, size = 60, normalized size = 1.76

$$-\frac{i\left(4i\,e^{(-dx-c)}+1\right)e^{(2\,dx+2\,c)}}{8\,ad}-\frac{i\left(-4i\,e^{(-dx-c)}+e^{(-2\,dx-2\,c)}\right)}{8\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
[Out] -1/8*I*(4*I*e^(-d*x - c) + 1)*e^(2*d*x + 2*c)/(a*d) - 1/8*I*(-4*I*e^(-d*x -
 c) + e^(-2*d*x - 2*c))/(a*d)
```

Fricas [A]
time = 0.34, size = 49, normalized size = 1.44

$$\frac{\left(-i\,e^{(4\,dx+4\,c)}+4\,e^{(3\,dx+3\,c)}-4\,e^{(dx+c)}-i\right)e^{(-2\,dx-2\,c)}}{8\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
[Out] 1/8*(-I*e^(4*d*x + 4*c) + 4*e^(3*d*x + 3*c) - 4*e^(d*x + c) - I)*e^(-2*d*x
- 2*c)/(a*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(24) = 48.

time = 0.18, size = 133, normalized size = 3.91

$$\begin{cases} \frac{(-32ia^3d^3e^{5c}e^{2dx}+128a^3d^3e^{4c}e^{dx}-128a^3d^3e^{2c}e^{-dx}-32ia^3d^3e^{c}e^{-2dx})e^{-3c}}{256a^4d^4} & \text{for } a^4d^4e^{3c} \neq 0 \\ \frac{x(-ie^{4c}+2e^{3c}+2e^{c}+i)e^{-2c}}{4a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Piecewise(((-32*I*a**3*d**3*exp(5*c)*exp(2*d*x) + 128*a**3*d**3*exp(4*c)*ex
p(d*x) - 128*a**3*d**3*exp(2*c)*exp(-d*x) - 32*I*a**3*d**3*exp(c)*exp(-2*d*
x))*exp(-3*c)/(256*a**4*d**4), Ne(a**4*d**4*exp(3*c), 0)), (x*(-I*exp(4*c)
+ 2*exp(3*c) + 2*exp(c) + I)*exp(-2*c)/(4*a), True))
```

Giac [A]
time = 0.45, size = 55, normalized size = 1.62

$$-\frac{\frac{(4e^{(dx+c)}+i)e^{(-2dx-2c)}}{a} + \frac{iae^{(2dx+2c)}-4ae^{(dx+c)}}{a^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/8*((4*e^(d*x + c) + I)*e^(-2*d*x - 2*c)/a + (I*a*e^(2*d*x + 2*c) - 4*a*e
^(d*x + c))/a^2)/d
```

Mupad [B]
time = 0.29, size = 29, normalized size = 0.85

$$\frac{4\sinh(c+dx) - \cosh(2c+2dx)\ 1\mathrm{i}}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + a*sinh(c + d*x)*1i),x)
```

```
[Out] (4*sinh(c + d*x) - cosh(2*c + 2*d*x)*1i)/(4*a*d)
```

3.269 $\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=131

$$\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{af}-\frac{i\text{Chi}\left(\frac{2de}{f}+2dx\right)\sinh\left(2c-\frac{2de}{f}\right)}{2af}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{af}-\frac{i\cosh\left(2c-\right.}{}$$

[Out] Chi(d*e/f+d*x)*cosh(c-d*e/f)/a/f-1/2*I*cosh(2*c-2*d*e/f)*Shi(2*d*e/f+2*d*x)
/a/f-1/2*I*Chi(2*d*e/f+2*d*x)*sinh(2*c-2*d*e/f)/a/f+Shi(d*e/f+d*x)*sinh(c-d
*e/f)/a/f

Rubi [A]
time = 0.23, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$,
Rules used = {5682, 3384, 3379, 3382, 5556, 12}

$$-\frac{i\sinh\left(2c-\frac{2de}{f}\right)\text{Chi}\left(\frac{2de}{f}+2dx\right)}{2af}+\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{af}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{af}-\frac{i\cosh\left(2c-\frac{2de}{f}\right)\text{Shi}\left(\frac{2de}{f}+2dx\right)}{2af}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]

[Out] (Cosh[c - (d*e)/f]*CoshIntegral[(d*e)/f + d*x])/(a*f) - ((I/2)*CoshIntegral
[(2*d*e)/f + 2*d*x]*Sinh[2*c - (2*d*e)/f])/(a*f) + (Sinh[c - (d*e)/f]*SinhI
ntegral[(d*e)/f + d*x])/(a*f) - ((I/2)*Cosh[2*c - (2*d*e)/f]*SinhIntegral[(
2*d*e)/f + 2*d*x])/(a*f)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5682

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cosh[c
+ d*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Si
nh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ
[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{e+fx}\,dx}{a} + \frac{\int \frac{\cosh(c+dx)}{e+fx}\,dx}{a}$$

$$= -\frac{i\int \frac{\sinh(2c+2dx)}{2(e+fx)}\,dx}{a} + \frac{\cosh\left(c-\frac{de}{f}\right)\int \frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx}\,dx}{a} + \frac{\sinh\left(c-\frac{de}{f}\right.}{}$$

$$= \frac{\cosh\left(c-\frac{de}{f}\right)\mathrm{Chi}\left(\frac{de}{f}+dx\right)}{af} + \frac{\sinh\left(c-\frac{de}{f}\right)\mathrm{Shi}\left(\frac{de}{f}+dx\right)}{af} - \frac{i\int \frac{\sin}{}}{}$$

$$= \frac{\cosh\left(c-\frac{de}{f}\right)\mathrm{Chi}\left(\frac{de}{f}+dx\right)}{af} + \frac{\sinh\left(c-\frac{de}{f}\right)\mathrm{Shi}\left(\frac{de}{f}+dx\right)}{af} - \frac{\left(i\cos\right.}{}$$

$$= \frac{\cosh\left(c-\frac{de}{f}\right)\mathrm{Chi}\left(\frac{de}{f}+dx\right)}{af} - \frac{i\mathrm{Chi}\left(\frac{2de}{f}+2dx\right)\sinh\left(2c-\frac{2de}{f}\right)}{2af} +$$

Mathematica [A]

time = 0.24, size = 112, normalized size = 0.85

$$\frac{2\cosh\left(c-\frac{de}{f}\right)\mathrm{Chi}\left(d\left(\frac{e}{f}+x\right)\right) - i\left(\mathrm{Chi}\left(\frac{2d(e+fx)}{f}\right)\sinh\left(2c-\frac{2de}{f}\right) + 2i\sinh\left(c-\frac{de}{f}\right)\mathrm{Shi}\left(d\left(\frac{e}{f}+x\right)\right) + \cosh\left(2c-\frac{2de}{f}\right)\mathrm{Shi}\left(\frac{2d(e+fx)}{f}\right)\right)}{2af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] (2*Cosh[c - (d*e)/f]*CoshIntegral[d*(e/f + x)] - I*(CoshIntegral[(2*d*(e + f*x))/f]*Sinh[2*c - (2*d*e)/f] + (2*I)*Sinh[c - (d*e)/f]*SinhIntegral[d*(e/f + x)] + Cosh[2*c - (2*d*e)/f]*SinhIntegral[(2*d*(e + f*x))/f]))/(2*a*f)
```

Maple [A]
time = 2.43, size = 180, normalized size = 1.37

method	result
risch	$-\frac{\mathrm{e}^{-\frac{cf-de}{f}}\operatorname{expIntegral}\left(1,dx+c-\frac{cf-de}{f}\right)}{2af}-\frac{\mathrm{e}^{\frac{cf-de}{f}}\operatorname{expIntegral}\left(1,-dx-c-\frac{-cf+de}{f}\right)}{2af}+\frac{i\mathrm{e}^{\frac{2cf-2de}{f}}\operatorname{expIntegral}\left(1,-2dx-2c-\frac{2(-}{}\right.}{4af}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/f*exp(-(c*f-d*e)/f)*Ei(1,d*x+c-(c*f-d*e)/f)-1/2/a/f*exp((c*f-d*e)/f)*Ei(1,-d*x-c-(-c*f+d*e)/f)+1/4*I/a/f*exp(2*(c*f-d*e)/f)*Ei(1,-2*d*x-2*c-2*(-c*f+d*e)/f)-1/4*I/a/f*exp(-2*(c*f-d*e)/f)*Ei(1,2*d*x+2*c-2*(c*f-d*e)/f)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.
```

Fricas [A]
time = 0.34, size = 135, normalized size = 1.03

$$\frac{-i\operatorname{Ei}\left(\frac{2(dfx+de)}{f}\right)e^{\left(\frac{2(cf-de)}{f}\right)}+2\operatorname{Ei}\left(\frac{dfx+de}{f}\right)e^{\left(\frac{cf-de}{f}\right)}+2\operatorname{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(-\frac{cf-de}{f}\right)}+i\operatorname{Ei}\left(-\frac{2(dfx+de)}{f}\right)e^{\left(-\frac{2(cf-de)}{f}\right)}}{4af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(-I*Ei(2*(d*f*x + d*e)/f)*e^(2*(c*f - d*e)/f) + 2*Ei((d*f*x + d*e)/f)*e^((c*f - d*e)/f) + 2*Ei(-(d*f*x + d*e)/f)*e^(-(c*f - d*e)/f) + I*Ei(-2*(d*f*x + d*e)/f)*e^(-2*(c*f - d*e)/f))/(a*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\cosh^{3}(c+dx)}{e\sinh(c+dx)-ie+fx\sinh(c+dx)-ifx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*Integral(cosh(c + d*x)**3/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) - I
*f*x), x)/a
```

Giac [A]
time = 0.48, size = 144, normalized size = 1.10

$$-\frac{\left(i\,\mathrm{Ei}\left(\frac{2\,(dfx+de)}{f}\right)e^{\left(4\,c-\frac{2\,de}{f}\right)}-2\,\mathrm{Ei}\left(\frac{dfx+de}{f}\right)e^{\left(3\,c-\frac{de}{f}\right)}-2\,\mathrm{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(c+\frac{de}{f}\right)}-i\,\mathrm{Ei}\left(-\frac{2\,(dfx+de)}{f}\right)e^{\left(\frac{2\,de}{f}\right)}+3i\,e^{(2\,c)}\log\left(fx+e\right)-3i\,e^{(2\,c)}\log\left(i\,fx+i\,e\right)\right)e^{(-2\,c)}}{4\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(I*Ei(2*(d*f*x + d*e)/f)*e^(4*c - 2*d*e/f) - 2*Ei((d*f*x + d*e)/f)*e^(
3*c - d*e/f) - 2*Ei(-(d*f*x + d*e)/f)*e^(c + d*e/f) - I*Ei(-2*(d*f*x + d*e)
/f)*e^(2*d*e/f) + 3*I*e^(2*c)*log(f*x + e) - 3*I*e^(2*c)*log(I*f*x + I*e))*
e^(-2*c)/(a*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{cosh}\left(c+d\,x\right)}^3}{\left(e+f\,x\right)\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/((e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

```
[Out] int(cosh(c + d*x)^3/((e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.270 $\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=180

$$-\frac{\cosh(c+dx)}{af(e+fx)}-\frac{id\cosh\left(2c-\frac{2de}{f}\right)\mathrm{Chi}\left(\frac{2de}{f}+2dx\right)}{af^2}+\frac{d\mathrm{Chi}\left(\frac{de}{f}+dx\right)\sinh\left(c-\frac{de}{f}\right)}{af^2}+\frac{i\sinh(2c+2dx)}{2af(e+fx)}+\frac{d\cos}{}$$

```
[Out] -I*d*Chi(2*d*e/f+2*d*x)*cosh(2*c-2*d*e/f)/a/f^2-cosh(d*x+c)/a/f/(f*x+e)+d*c
osh(c-d*e/f)*Shi(d*e/f+d*x)/a/f^2-I*d*Shi(2*d*e/f+2*d*x)*sinh(2*c-2*d*e/f)/
a/f^2+d*Chi(d*e/f+d*x)*sinh(c-d*e/f)/a/f^2+1/2*I*sinh(2*d*x+2*c)/a/f/(f*x+e
)
```

Rubi [A]

time = 0.27, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {5682, 3378, 3384, 3379, 3382, 5556, 12}

$$\frac{d\sinh\left(c-\frac{de}{f}\right)\mathrm{Chi}\left(\frac{de}{f}+dx\right)}{af^2}-\frac{id\cosh\left(2c-\frac{2de}{f}\right)\mathrm{Chi}\left(\frac{2de}{f}+2dx\right)}{af^2}-\frac{id\sinh\left(2c-\frac{2de}{f}\right)\mathrm{Shi}\left(\frac{2de}{f}+2dx\right)}{af^2}+\frac{d\cosh\left(c-\frac{de}{f}\right)\mathrm{Shi}\left(\frac{de}{f}+dx\right)}{af^2}+\frac{i\sinh(2c+2dx)}{2af(e+fx)}-\frac{\cosh(c+dx)}{af(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] -(Cosh[c + d*x]/(a*f*(e + f*x))) - (I*d*Cosh[2*c - (2*d*e)/f]*CoshIntegral[
(2*d*e)/f + 2*d*x])/(a*f^2) + (d*CoshIntegral[(d*e)/f + d*x]*Sinh[c - (d*e)
/f])/(a*f^2) + ((I/2)*Sinh[2*c + 2*d*x])/(a*f*(e + f*x)) + (d*Cosh[c - (d*e
)/f]*SinhIntegral[(d*e)/f + d*x])/(a*f^2) - (I*d*Sinh[2*c - (2*d*e)/f]*Sinh
Integral[(2*d*e)/f + 2*d*x])/(a*f^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5682

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cosh[c
+ d*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Si
nh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ
[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}\,dx}{a} + \frac{\int \frac{\cosh(c+dx)}{(e+fx)^2}\,dx}{a}$$

$$= -\frac{\cosh(c+dx)}{af(e+fx)} - \frac{i\int \frac{\sinh(2c+2dx)}{2(e+fx)^2}\,dx}{a} + \frac{d\int \frac{\sinh(c+dx)}{e+fx}\,dx}{af}$$

$$= -\frac{\cosh(c+dx)}{af(e+fx)} - \frac{i\int \frac{\sinh(2c+2dx)}{(e+fx)^2}\,dx}{2a} + \frac{\left(d\cosh\left(c-\frac{de}{f}\right)\right)\int \frac{\sinh\left(\frac{de}{f}+dx\right.}{e+fx}}{af}$$

$$= -\frac{\cosh(c+dx)}{af(e+fx)} + \frac{d\text{Chi}\left(\frac{de}{f}+dx\right)\sinh\left(c-\frac{de}{f}\right)}{af^2} + \frac{i\sinh(2c+2dx)}{2af(e+fx)} +$$

$$= -\frac{\cosh(c+dx)}{af(e+fx)} + \frac{d\text{Chi}\left(\frac{de}{f}+dx\right)\sinh\left(c-\frac{de}{f}\right)}{af^2} + \frac{i\sinh(2c+2dx)}{2af(e+fx)} +$$

$$= -\frac{\cosh(c+dx)}{af(e+fx)} - \frac{id\cosh\left(2c-\frac{2de}{f}\right)\text{Chi}\left(\frac{2de}{f}+2dx\right)}{af^2} + \frac{d\text{Chi}\left(\frac{de}{f}+d\right.}{}$$

Mathematica [A]
time = 0.38, size = 212, normalized size = 1.18

$$\frac{-2f\cosh(c+dx) - 2id(e+fx)\cosh\left(2c-\frac{2de}{f}\right)\text{Chi}\left(\frac{2d(e+fx)}{f}\right) + 2d(e+fx)\text{Chi}\left(d\left(\frac{e}{f}+x\right)\right)\sinh\left(c-\frac{de}{f}\right) + if\sinh(2(c+dx)) + 2de\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(d\left(\frac{e}{f}+x\right)\right) + 2dfx\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(d\left(\frac{e}{f}+x\right)\right) - 2ide\sinh\left(2c-\frac{2de}{f}\right)\text{Shi}\left(\frac{2d(e+fx)}{f}\right) - 2idfx\sinh\left(2c-\frac{2de}{f}\right)\text{Shi}\left(\frac{2d(e+fx)}{f}\right)}{2af^2(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] (-2*f*Cosh[c + d*x] - (2*I)*d*(e + f*x)*Cosh[2*c - (2*d*e)/f]*CoshIntegral[
(2*d*(e + f*x))/f] + 2*d*(e + f*x)*CoshIntegral[d*(e/f + x)]*Sinh[c - (d*e)
/f] + I*f*Sinh[2*(c + d*x)] + 2*d*e*Cosh[c - (d*e)/f]*SinhIntegral[d*(e/f +
 x)] + 2*d*f*x*Cosh[c - (d*e)/f]*SinhIntegral[d*(e/f + x)] - (2*I)*d*e*Sinh
[2*c - (2*d*e)/f]*SinhIntegral[(2*d*(e + f*x))/f] - (2*I)*d*f*x*Sinh[2*c -
(2*d*e)/f]*SinhIntegral[(2*d*(e + f*x))/f])/(2*a*f^2*(e + f*x))
```

Maple [A]
time = 2.47, size = 299, normalized size = 1.66

method	result
risch	$-\frac{d\,\mathrm{e}^{-dx-c}}{2af(dxf+de)} + \frac{d\,\mathrm{e}^{-\frac{cf-de}{f}}\operatorname{expIntegral}\left(1,dx+c-\frac{cf-de}{f}\right)}{2a\,f^2} - \frac{d\,\mathrm{e}^{dx+c}}{2a\,f^2\left(\frac{de}{f}+dx\right)} - \frac{d\,\mathrm{e}^{\frac{cf-de}{f}}\operatorname{expIntegral}\left(1,-dx-c-\frac{-cf+de}{f}\right)}{2a\,f^2} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d/a*exp(-d*x-c)/f/(d*f*x+d*e)+1/2*d/a/f^2*exp(-(c*f-d*e)/f)*Ei(1,d*x+c
-(c*f-d*e)/f)-1/2*d/a/f^2*exp(d*x+c)/(d*e/f+d*x)-1/2*d/a/f^2*exp((c*f-d*e)/
f)*Ei(1,-d*x-c-(-c*f+d*e)/f)+1/4*I*d/a/f^2*exp(2*d*x+2*c)/(d*e/f+d*x)+1/2*I
*d/a/f^2*exp(2*(c*f-d*e)/f)*Ei(1,-2*d*x-2*c-2*(-c*f+d*e)/f)-1/4*I/a*d*exp(-
2*d*x-2*c)/f/(d*f*x+d*e)+1/2*I/a*d/f^2*exp(-2*(c*f-d*e)/f)*Ei(1,2*d*x+2*c-2
*(c*f-d*e)/f)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 0.38, size = 257, normalized size = 1.43

$$-\frac{\left(2\,(dfx+de)\mathrm{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(2\,dx+2\,c-\frac{cf-de}{f}\right)}+2\,(i\,dfx+i\,de)\mathrm{Ei}\left(-\frac{2\,(dfx+de)}{f}\right)e^{\left(2\,dx+2\,c-\frac{2\,(cf-de)}{f}\right)}-i\,fe^{(4\,dx+4\,c)}+2\,fe^{(3\,dx+3\,c)}+2\left((i\,dfx+i\,de)\mathrm{Ei}\left(\frac{2\,(dfx+de)}{f}\right)e^{\left(\frac{2\,(cf-de)}{f}\right)}-(dfx+de)\mathrm{Ei}\left(\frac{dfx+de}{f}\right)e^{\left(\frac{cf-de}{f}\right)}\right)e^{(2\,dx+2\,c)}+2\,fe^{(dx+c)}+i\,f\right)e^{(-2\,dx-2\,c)}}{4\,(af^3x+af^2e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/4*(2*(d*f*x + d*e)*Ei(-(d*f*x + d*e)/f)*e^(2*d*x + 2*c - (c*f - d*e)/f)
+ 2*(I*d*f*x + I*d*e)*Ei(-2*(d*f*x + d*e)/f)*e^(2*d*x + 2*c - 2*(c*f - d*e)
/f) - I*f*e^(4*d*x + 4*c) + 2*f*e^(3*d*x + 3*c) + 2*((I*d*f*x + I*d*e)*Ei(2
*(d*f*x + d*e)/f)*e^(2*(c*f - d*e)/f) - (d*f*x + d*e)*Ei((d*f*x + d*e)/f)*e
^((c*f - d*e)/f))*e^(2*d*x + 2*c) + 2*f*e^(d*x + c) + I*f)*e^(-2*d*x - 2*c)
/(a*f^3*x + a*f^2*e)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1080 vs. 2(172) = 344.
time = 0.55, size = 1080, normalized size = 6.00

$$-\frac{2i\left(fx+e\right)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)d^{2}\mathrm{Ei}\left(-\frac{2\left(\left(fx+e\right)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)+de-cf\right)}{f}\right)e^{\left(\frac{2\left(de-cf\right)}{f}\right)}+2i\,d^{3}e\,\mathrm{Ei}\left(-\frac{2\left(\left(fx+e\right)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)+de-cf\right)}{f}\right)e^{\left(\frac{2\left(de-cf\right)}{f}\right)}-2i\,cd^{2}f\,\mathrm{Ei}\left(-\frac{2\left(\left(fx+e\right)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)+de-cf\right)}{f}\right)e^{\left(\frac{2\left(de-cf\right)}{f}\right)}+2\left(fx+e\right)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)d^{2}\mathrm{Ei}\left(-\frac{\left(fx+e\right)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)+de-cf}{f}\right)e^{\left(\frac{de-cf}{f}\right)}+2d^{3}e\,\mathrm{Ei}\left(-\frac{\left(fx+e\right)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)+de-cf}{f}\right)e^{\left(\frac{de-cf}{f}\right)}-2cd^{2}f\,\mathrm{Ei}\left(-\frac{\left(fx+e\right)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)+de-cf}{f}\right)e^{\left(\frac{de-cf}{f}\right)}-2\left(fx+e\right)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)d^{2}\mathrm{Ei}\left(\frac{\left(fx+e\right)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)+de-cf}{f}\right)e^{\left(-\frac{de-cf}{f}\right)}-2d^{3}e\,\mathrm{Ei}\left(\frac{\left(fx+e\right)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)+de-cf}{f}\right)e^{\left(-\frac{de-cf}{f}\right)}+2cd^{2}f\,\mathrm{Ei}\left(\frac{\left(fx+e\right)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)+de-cf}{f}\right)e^{\left(-\frac{de-cf}{f}\right)}+2i\left(fx+e\right)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)d^{2}\mathrm{Ei}\left(\frac{2\left(\left(fx+e\right)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)+de-cf\right)}{f}\right)e^{\left(-\frac{2\left(de-cf\right)}{f}\right)}+2i\,d^{3}e\,\mathrm{Ei}\left(\frac{2\left(\left(fx+e\right)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)+de-cf\right)}{f}\right)e^{\left(-\frac{2\left(de-cf\right)}{f}\right)}-2i\,cd^{2}f\,\mathrm{Ei}\left(\frac{2\left(\left(fx+e\right)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)+de-cf\right)}{f}\right)e^{\left(-\frac{2\left(de-cf\right)}{f}\right)}-i\,d^{2}fe^{\left(\frac{2\left(fx+e\right)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)}{f}\right)}+2d^{2}fe^{\left(\frac{\left(fx+e\right)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)}{f}\right)}+2d^{2}fe^{\left(-\frac{\left(fx+e\right)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)}{f}\right)}+i\,d^{2}fe^{\left(-\frac{2\left(fx+e\right)\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)}{f}\right)}}{4\left(\left(fx+e\right)a\left(d-\frac{de}{fx+e}+\frac{cf}{fx+e}\right)f^{4}+adef^{4}-acf^{5}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(2*I*(f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e))*d^2*Ei(-2*((f*x + e
)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^(2*(d*e - c*f)/f) +
 2*I*d^3*e*Ei(-2*((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f
)/f)*e^(2*(d*e - c*f)/f) - 2*I*c*d^2*f*Ei(-2*((f*x + e)*(d - d*e/(f*x + e)
+ c*f/(f*x + e)) + d*e - c*f)/f)*e^(2*(d*e - c*f)/f) + 2*(f*x + e)*(d - d*e
/(f*x + e) + c*f/(f*x + e))*d^2*Ei(-((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*
x + e)) + d*e - c*f)/f)*e^((d*e - c*f)/f) + 2*d^3*e*Ei(-((f*x + e)*(d - d*e
/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^((d*e - c*f)/f) - 2*c*d^2*f*E
i(-((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^((d*e -
 c*f)/f) - 2*(f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e))*d^2*Ei(((f*x + e
)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^(-(d*e - c*f)/f) -
2*d^3*e*Ei(((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e
^(-(d*e - c*f)/f) + 2*c*d^2*f*Ei(((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x +
 e)) + d*e - c*f)/f)*e^(-(d*e - c*f)/f) + 2*I*(f*x + e)*(d - d*e/(f*x + e)
+ c*f/(f*x + e))*d^2*Ei(2*((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) +
d*e - c*f)/f)*e^(-2*(d*e - c*f)/f) + 2*I*d^3*e*Ei(2*((f*x + e)*(d - d*e/(f*
x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^(-2*(d*e - c*f)/f) - 2*I*c*d^2*f*
Ei(2*((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e)) + d*e - c*f)/f)*e^(-2*(
d*e - c*f)/f) - I*d^2*f*e^(2*(f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e))/
f) + 2*d^2*f*e^((f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e))/f) + 2*d^2*f*
e^(-(f*x + e)*(d - d*e/(f*x + e) + c*f/(f*x + e))/f) + I*d^2*f*e^(-2*(f*x +
 e)*(d - d*e/(f*x + e) + c*f/(f*x + e))/f))*f^2/(((f*x + e)*a*(d - d*e/(f*x
 + e) + c*f/(f*x + e))*f^4 + a*d*e*f^4 - a*c*f^5)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{cosh}\left(c+d\,x\right)}^3}{{\left(e+f\,x\right)}^2\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

```
[Out] int(cosh(c + d*x)^3/((e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.271 $\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=463

$$-\frac{3if(e+fx)^2}{2ad^2}-\frac{6f^2(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{ad^3}+\frac{(e+fx)^3\text{ArcTan}\left(e^{c+dx}\right)}{ad}+\frac{3if^2(e+fx)\log\left(1+e^{2(c+dx)}\right)}{ad^3}+\frac{3}{}$$

```
[Out] 3*I*f^3*polylog(4,I*exp(d*x+c))/a/d^4-6*f^2*(f*x+e)*arctan(exp(d*x+c))/a/d^
3+(f*x+e)^3*arctan(exp(d*x+c))/a/d+3*I*f^2*(f*x+e)*ln(1+exp(2*d*x+2*c))/a/d
^3+3*I*f^3*polylog(2,-I*exp(d*x+c))/a/d^4-3*I*f^3*polylog(2,I*exp(d*x+c))/a
/d^4+3/2*I*f^3*polylog(2,-exp(2*d*x+2*c))/a/d^4-3/2*I*f*(f*x+e)^2/a/d^2-3*I
*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/a/d^3-3*I*f^3*polylog(4,-I*exp(d*x+c))
/a/d^4+3/2*I*f*(f*x+e)^2*polylog(2,I*exp(d*x+c))/a/d^2+1/2*I*(f*x+e)^3*sech
(d*x+c)^2/a/d+3*I*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/a/d^3+3/2*f*(f*x+e)^
2*sech(d*x+c)/a/d^2-3/2*I*f*(f*x+e)^2*polylog(2,-I*exp(d*x+c))/a/d^2-3/2*I*
f*(f*x+e)^2*tanh(d*x+c)/a/d^2+1/2*(f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/a/d
```

Rubi [A]
time = 0.35, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {5690, 4271, 4265, 2317, 2438, 2611, 6744, 2320, 6724, 5559, 4269, 3799, 2221}

$$-\frac{6f^2(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{ad^3}+\frac{(e+fx)^3\text{ArcTan}\left(e^{c+dx}\right)}{ad}+\frac{3if^3\text{Li}_2\left(-ie^{c+dx}\right)}{ad^4}-\frac{3if^3\text{Li}_2\left(ie^{c+dx}\right)}{ad^4}+\frac{3if^3\text{Li}_2\left(-e^{2(c+dx)}\right)}{2ad^4}-\frac{3if^3\text{Li}_4\left(-ie^{c+dx}\right)}{ad^4}+\frac{3if^3\text{Li}_4\left(ie^{c+dx}\right)}{ad^4}+\frac{3if^2(e+fx)\text{Li}_3\left(-ie^{c+dx}\right)}{ad^3}-\frac{3if^2(e+fx)\text{Li}_3\left(ie^{c+dx}\right)}{ad^3}+\frac{3if^2(e+fx)\log\left(e^{2(c+dx)}+1\right)}{ad^3}-\frac{3if(e+fx)^2\text{Li}_2\left(-ie^{c+dx}\right)}{2ad^2}+\frac{3if(e+fx)^2\text{Li}_2\left(ie^{c+dx}\right)}{2ad^2}-\frac{3if(e+fx)^2\tanh(c+dx)}{2ad^2}+\frac{3f(e+fx)^2\text{sech}(c+dx)}{2ad^2}+\frac{i(e+fx)^3\text{sech}^2(c+dx)}{2ad}+\frac{(e+fx)^3\tanh(c+dx)\text{sech}(c+dx)}{2ad}-\frac{3if(e+fx)^2}{2ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (((-3*I)/2)*f*(e + f*x)^2)/(a*d^2) - (6*f^2*(e + f*x)*ArcTan[E^(c + d*x)])/
(a*d^3) + ((e + f*x)^3*ArcTan[E^(c + d*x)])/(a*d) + ((3*I)*f^2*(e + f*x)*Lo
g[1 + E^(2*(c + d*x))])/(a*d^3) + ((3*I)*f^3*PolyLog[2, (-I)*E^(c + d*x)])/
(a*d^4) - (((3*I)/2)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) -
((3*I)*f^3*PolyLog[2, I*E^(c + d*x)])/(a*d^4) + (((3*I)/2)*f*(e + f*x)^2*Po
lyLog[2, I*E^(c + d*x)])/(a*d^2) + (((3*I)/2)*f^3*PolyLog[2, -E^(2*(c + d*x
))])/(a*d^4) + ((3*I)*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^3) -
 ((3*I)*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(a*d^3) - ((3*I)*f^3*PolyL
og[4, (-I)*E^(c + d*x)])/(a*d^4) + ((3*I)*f^3*PolyLog[4, I*E^(c + d*x)])/(a
*d^4) + (3*f*(e + f*x)^2*Sech[c + d*x])/(2*a*d^2) + ((I/2)*(e + f*x)^3*Sech
[c + d*x]^2)/(a*d) - (((3*I)/2)*f*(e + f*x)^2*Tanh[c + d*x])/(a*d^2) + ((e
+ f*x)^3*Sech[c + d*x]*Tanh[c + d*x])/(2*a*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5690

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sech[c
 + d*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)*T
anh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && Eq
Q[a^2 + b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int (e+fx)^3\text{sech}^2(c+dx)\tanh(c+dx)\,dx}{a} + \frac{\int (e+fx)^3\text{sech}^3(c+dx)\,dx}{a}$$

$$= \frac{3f(e+fx)^2\text{sech}(c+dx)}{2ad^2} + \frac{i(e+fx)^3\text{sech}^2(c+dx)}{2ad} + \frac{(e+fx)^3\text{sech}(c+dx}{2ad}$$

$$= -\frac{6f^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{3f(e+fx)^2\text{sech}(c}{2ad^2}$$

$$= -\frac{3if(e+fx)^2}{2ad^2} - \frac{6f^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{ad} - \frac{3i}{}$$

$$= -\frac{3if(e+fx)^2}{2ad^2} - \frac{6f^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{3i}{}$$

$$= -\frac{3if(e+fx)^2}{2ad^2} - \frac{6f^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{3i}{}$$

$$= -\frac{3if(e+fx)^2}{2ad^2} - \frac{6f^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{3i}{}$$

Mathematica [A]
time = 7.97, size = 866, normalized size = 1.87

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (-(((-4*I)*d^4*e^3*E^c*x + (48*I)*d^2*e*E^c*f^2*x - (6*I)*d^4*e^2*E^c*f*x^2
 + (24*I)*d^2*E^c*f^3*x^2 - (4*I)*d^4*e*E^c*f^2*x^3 - I*d^4*E^c*f^3*x^4 + 4
*d^3*e^3*Log[I - E^(c + d*x)] + (4*I)*d^3*e^3*E^c*Log[I - E^(c + d*x)] - 48
*d*e*f^2*Log[I - E^(c + d*x)] - (48*I)*d*e*E^c*f^2*Log[I - E^(c + d*x)] + 1
2*d^3*e^2*f*x*Log[1 + I*E^(c + d*x)] + (12*I)*d^3*e^2*E^c*f*x*Log[1 + I*E^(
c + d*x)] - 48*d*f^3*x*Log[1 + I*E^(c + d*x)] - (48*I)*d*E^c*f^3*x*Log[1 +
I*E^(c + d*x)] + 12*d^3*e*f^2*x^2*Log[1 + I*E^(c + d*x)] + (12*I)*d^3*e*E^c
*f^2*x^2*Log[1 + I*E^(c + d*x)] + 4*d^3*f^3*x^3*Log[1 + I*E^(c + d*x)] + (4
*I)*d^3*E^c*f^3*x^3*Log[1 + I*E^(c + d*x)] + 12*(1 + I*E^c)*f*(-4*f^2 + d^2
*(e + f*x)^2)*PolyLog[2, (-I)*E^(c + d*x)] - (24*I)*d*(-I + E^c)*f^2*(e + f
*x)*PolyLog[3, (-I)*E^(c + d*x)] + 24*f^3*PolyLog[4, (-I)*E^(c + d*x)] + (2
4*I)*E^c*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(d^4*(-I + E^c))) + ((-I)*d^3*(d
*E^c*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) - 4*(I + E^c)*(e + f*x)^
3*Log[1 - I*E^(c + d*x)]) + (12*I)*d^2*(I + E^c)*f*(e + f*x)^2*PolyLog[2, I
*E^(c + d*x)] + 24*d*(1 - I*E^c)*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)] +
(24*I)*(I + E^c)*f^3*PolyLog[4, I*E^(c + d*x)])/(d^4*(I + E^c)) + x*(4*e^3
+ 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*Sech[c] + ((4*I)*(e + f*x)^3)/(d*(Cosh
```

```
[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2) - ((24*I)*f*(e + f*x)^2*Sinh[(d*x)/
2])/(d^2*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]
)))/(8*a)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1151 vs. 2(416) = 832.
time = 3.25, size = 1152, normalized size = 2.49

method	result	size
risch	Expression too large to display	1152

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*I/a/d^3*e*f^2*c^2*ln(exp(d*x+c)+I)+3*I*f^3*polylog(4,I*exp(d*x+c))/a/d^
4+1/2*I/a/d*f^3*ln(1-I*exp(d*x+c))*x^3+3/2*I/a/d^2*f^3*polylog(2,I*exp(d*x+
c))*x^2-3*I/a/d^3*f^3*polylog(3,I*exp(d*x+c))*x-6*I/a/d^3*f^3*c*x-1/2*I/a/d
^4*f^3*c^3*ln(1+I*exp(d*x+c))+1/2*I/a/d^4*f^3*c^3*ln(1-I*exp(d*x+c))-1/2*I/
a/d*f^3*ln(1+I*exp(d*x+c))*x^3-3/2*I/a/d^2*f^3*polylog(2,-I*exp(d*x+c))*x^2
+6*I/a/d^3*e*f^2*ln(exp(d*x+c)-I)-3/2*I/a/d^2*e^2*f*c*ln(exp(d*x+c)+I)+3/2*
I/a/d*ln(1-I*exp(d*x+c))*e^2*f*x+3/2*I/a/d^2*ln(1-I*exp(d*x+c))*c*e^2*f+3/2
*I/a/d*ln(1-I*exp(d*x+c))*e*f^2*x^2-3/2*I/a/d*ln(1+I*exp(d*x+c))*e*f^2*x^2+
1/2*I/a/d*e^3*ln(exp(d*x+c)+I)-1/2*I/a/d*e^3*ln(exp(d*x+c)-I)+6*I/a/d^4*f^3
*c*ln(1+I*exp(d*x+c))+6*I/a/d^4*f^3*c*ln(exp(d*x+c))+6*I/a/d^3*f^3*ln(1+I*e
xp(d*x+c))*x+3*I/a/d^3*f^3*polylog(3,-I*exp(d*x+c))*x+3/2*I/a/d^2*e^2*f*c*l
n(exp(d*x+c)-I)-3*I/a/d^2*polylog(2,-I*exp(d*x+c))*e*f^2*x+3*I/a/d^2*polylo
g(2,I*exp(d*x+c))*e*f^2*x-3/2*I/a/d*ln(1+I*exp(d*x+c))*e^2*f*x-3/2*I/a/d^2*
ln(1+I*exp(d*x+c))*c*e^2*f-3/2*I/a/d^3*e*f^2*c^2*ln(exp(d*x+c)-I)-3/2*I/a/d
^3*ln(1-I*exp(d*x+c))*c^2*e*f^2+3/2*I/a/d^3*ln(1+I*exp(d*x+c))*c^2*e*f^2-3*
I*f^3*polylog(4,-I*exp(d*x+c))/a/d^4+6*I/a/d^4*f^3*polylog(2,-I*exp(d*x+c))
-3*I/a/d^2*f^3*x^2-3*I/a/d^4*f^3*c^2-6*I/a/d^4*f^3*c*ln(exp(d*x+c)-I)-1/2*I
/a/d^4*f^3*c^3*ln(exp(d*x+c)+I)+1/2*I/a/d^4*f^3*c^3*ln(exp(d*x+c)-I)-3/2*I/
a/d^2*e^2*f*polylog(2,-I*exp(d*x+c))+3/2*I/a/d^2*e^2*f*polylog(2,I*exp(d*x+
c))-6*I/a/d^3*e*f^2*ln(exp(d*x+c))+3*I/a/d^3*e*f^2*polylog(3,-I*exp(d*x+c))
-3*I/a/d^3*e*f^2*polylog(3,I*exp(d*x+c))+(d*f^3*x^3*exp(d*x+c)+3*d*e*f^2*x^
2*exp(d*x+c)+3*d*e^2*f*x*exp(d*x+c)+d*e^3*exp(d*x+c)+3*f^3*x^2*exp(d*x+c)-3
*I*f^3*x^2+6*e*f^2*x*exp(d*x+c)-6*I*e*f^2*x+3*e^2*f*exp(d*x+c)-3*I*e^2*f)/(
exp(d*x+c)-I)^2/d^2/a
```

Maxima [A]
time = 0.44, size = 691, normalized size = 1.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(4*e^(-d*x - c)/((4*I*a*e^(-d*x - c) + 2*a*e^(-2*d*x - 2*c) - 2*a)*d)
+ I*log(e^(-d*x - c) + I)/(a*d) - I*log(I*e^(-d*x - c) + 1)/(a*d))*e^3 - 6*
I*f^2*x*e/(a*d^2) + (-3*I*f^3*x^2 - 6*I*f^2*x*e - 3*I*f*e^2 + (d*f^3*x^3*e^
c + 3*(d*f^2*e^(c + 1) + f^3*e^c)*x^2 + 3*(d*f*e^(c + 2) + 2*f^2*e^(c + 1))
*x + 3*f*e^(c + 2))*e^(d*x))/(a*d^2*e^(2*d*x + 2*c) - 2*I*a*d^2*e^(d*x + c)
 - a*d^2) + 3/2*I*(d*x*log(-I*e^(d*x + c) + 1) + dilog(I*e^(d*x + c)))*f*e^
2/(a*d^2) - 3/2*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x +
 c)) - 2*polylog(3, -I*e^(d*x + c)))*f^2*e/(a*d^3) + 3/2*I*(d^2*x^2*log(-I*
e^(d*x + c) + 1) + 2*d*x*dilog(I*e^(d*x + c)) - 2*polylog(3, I*e^(d*x + c))
)*f^2*e/(a*d^3) + 6*I*f^2*e*log(I*e^(d*x + c) + 1)/(a*d^3) - 1/2*I*(d^3*x^3
*log(I*e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-I*e^(d*x + c)) - 6*d*x*polylog(3
, -I*e^(d*x + c)) + 6*polylog(4, -I*e^(d*x + c)))*f^3/(a*d^4) + 1/2*I*(d^3*
x^3*log(-I*e^(d*x + c) + 1) + 3*d^2*x^2*dilog(I*e^(d*x + c)) - 6*d*x*polylo
g(3, I*e^(d*x + c)) + 6*polylog(4, I*e^(d*x + c)))*f^3/(a*d^4) - 3/2*I*(d^2
*f*e^2 - 4*f^3)*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))/(a*d^4
) - 1/8*(I*d^4*f^3*x^4 + 4*I*d^4*f^2*x^3*e + 6*I*d^4*f*x^2*e^2)/(a*d^4) + 1
/8*(I*d^4*f^3*x^4 + 4*I*d^4*f^2*x^3*e - 6*(-I*d^2*f*e^2 + 4*I*f^3)*d^2*x^2)
/(a*d^4)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1462 vs. $2(402) = 804$.
time = 0.40, size = 1462, normalized size = 3.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(-6*I*c^2*f^3 + 12*I*c*d*f^2*e - 6*I*d^2*f*e^2 - 3*(I*d^2*f^3*x^2 + 2*I
*d^2*f^2*x*e + I*d^2*f*e^2 + (-I*d^2*f^3*x^2 - 2*I*d^2*f^2*x*e - I*d^2*f*e^
2)*e^(2*d*x + 2*c) - 2*(d^2*f^3*x^2 + 2*d^2*f^2*x*e + d^2*f*e^2)*e^(d*x + c
))*dilog(I*e^(d*x + c)) - 3*(-I*d^2*f^3*x^2 - 2*I*d^2*f^2*x*e - I*d^2*f*e^2
 + 4*I*f^3 + (I*d^2*f^3*x^2 + 2*I*d^2*f^2*x*e + I*d^2*f*e^2 - 4*I*f^3)*e^(2
*d*x + 2*c) + 2*(d^2*f^3*x^2 + 2*d^2*f^2*x*e + d^2*f*e^2 - 4*f^3)*e^(d*x +
c))*dilog(-I*e^(d*x + c)) - 6*(I*d^2*f^3*x^2 - I*c^2*f^3 + 2*(I*d^2*f^2*x +
 I*c*d*f^2)*e)*e^(2*d*x + 2*c) + 2*(d^3*f^3*x^3 - 3*d^2*f^3*x^2 + 6*c^2*f^3
 + d^3*e^3 + 3*(d^3*f*x + d^2*f)*e^2 + 3*(d^3*f^2*x^2 - 2*d^2*f^2*x - 4*c*d
*f^2)*e)*e^(d*x + c) + (I*c^3*f^3 - 3*I*c^2*d*f^2*e + 3*I*c*d^2*f*e^2 - I*d
^3*e^3 + (-I*c^3*f^3 + 3*I*c^2*d*f^2*e - 3*I*c*d^2*f*e^2 + I*d^3*e^3)*e^(2*
d*x + 2*c) - 2*(c^3*f^3 - 3*c^2*d*f^2*e + 3*c*d^2*f*e^2 - d^3*e^3)*e^(d*x +
 c))*log(e^(d*x + c) + I) + (-3*I*c*d^2*f*e^2 - 3*(-I*c^2 + 4*I)*d*f^2*e +
(-I*c^3 + 12*I*c)*f^3 + I*d^3*e^3 + (3*I*c*d^2*f*e^2 - 3*(I*c^2 - 4*I)*d*f^
2*e + (I*c^3 - 12*I*c)*f^3 - I*d^3*e^3)*e^(2*d*x + 2*c) + 2*(3*c*d^2*f*e^2
- 3*(c^2 - 4)*d*f^2*e + (c^3 - 12*c)*f^3 - d^3*e^3)*e^(d*x + c))*log(e^(d*x
```

```
 + c) - I) + (I*d^3*f^3*x^3 - 12*I*d*f^3*x + (I*c^3 - 12*I*c)*f^3 - 3*(-I*d
^3*f*x - I*c*d^2*f)*e^2 - 3*(-I*d^3*f^2*x^2 + I*c^2*d*f^2)*e + (-I*d^3*f^3*
x^3 + 12*I*d*f^3*x + (-I*c^3 + 12*I*c)*f^3 - 3*(I*d^3*f*x + I*c*d^2*f)*e^2
- 3*(I*d^3*f^2*x^2 - I*c^2*d*f^2)*e)*e^(2*d*x + 2*c) - 2*(d^3*f^3*x^3 - 12*
d*f^3*x + (c^3 - 12*c)*f^3 + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - c
^2*d*f^2)*e)*e^(d*x + c))*log(I*e^(d*x + c) + 1) + (-I*d^3*f^3*x^3 - I*c^3*
f^3 - 3*(I*d^3*f*x + I*c*d^2*f)*e^2 - 3*(I*d^3*f^2*x^2 - I*c^2*d*f^2)*e + (
I*d^3*f^3*x^3 + I*c^3*f^3 - 3*(-I*d^3*f*x - I*c*d^2*f)*e^2 - 3*(-I*d^3*f^2*
x^2 + I*c^2*d*f^2)*e)*e^(2*d*x + 2*c) + 2*(d^3*f^3*x^3 + c^3*f^3 + 3*(d^3*f
*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - c^2*d*f^2)*e)*e^(d*x + c))*log(-I*e^(d
*x + c) + 1) - 6*(-I*f^3*e^(2*d*x + 2*c) - 2*f^3*e^(d*x + c) + I*f^3)*polyl
og(4, I*e^(d*x + c)) - 6*(I*f^3*e^(2*d*x + 2*c) + 2*f^3*e^(d*x + c) - I*f^3
)*polylog(4, -I*e^(d*x + c)) - 6*(-I*d*f^3*x - I*d*f^2*e + (I*d*f^3*x + I*d
*f^2*e)*e^(2*d*x + 2*c) + 2*(d*f^3*x + d*f^2*e)*e^(d*x + c))*polylog(3, I*e
^(d*x + c)) - 6*(I*d*f^3*x + I*d*f^2*e + (-I*d*f^3*x - I*d*f^2*e)*e^(2*d*x
+ 2*c) - 2*(d*f^3*x + d*f^2*e)*e^(d*x + c))*polylog(3, -I*e^(d*x + c)))/(a*
d^4*e^(2*d*x + 2*c) - 2*I*a*d^4*e^(d*x + c) - a*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{e^3\operatorname{sech}(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{f^3x^3\operatorname{sech}(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{3ef^2x^2\operatorname{sech}(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{3e^2fx\operatorname{sech}(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*(Integral(e**3*sech(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f**3*x**
3*sech(c + d*x)/(sinh(c + d*x) - I), x) + Integral(3*e*f**2*x**2*sech(c + d
*x)/(sinh(c + d*x) - I), x) + Integral(3*e**2*f*x*sech(c + d*x)/(sinh(c + d
*x) - I), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sech(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^3}{\cosh\left(c+d\,x\right)\,\left(a+a\,\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(cosh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
[Out] int((e + f*x)^3/(cosh(c + d*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.272 $\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=268

$$\frac{(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{ad}-\frac{f^2\text{ArcTan}(\sinh(c+dx))}{ad^3}+\frac{if^2\log(\cosh(c+dx))}{ad^3}-\frac{if(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2}$$

```
[Out] (f*x+e)^2*arctan(exp(d*x+c))/a/d-f^2*arctan(sinh(d*x+c))/a/d^3+I*f^2*ln(cos
h(d*x+c))/a/d^3-I*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/a/d^2+I*f*(f*x+e)*poly
log(2,I*exp(d*x+c))/a/d^2+I*f^2*polylog(3,-I*exp(d*x+c))/a/d^3-I*f^2*polylo
g(3,I*exp(d*x+c))/a/d^3+f*(f*x+e)*sech(d*x+c)/a/d^2+1/2*I*(f*x+e)^2*sech(d*
x+c)^2/a/d-I*f*(f*x+e)*tanh(d*x+c)/a/d^2+1/2*(f*x+e)^2*sech(d*x+c)*tanh(d*x
+c)/a/d
```

Rubi [A]
time = 0.19, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {5690, 4271, 3855, 4265, 2611, 2320, 6724, 5559, 4269, 3556}

$$-\frac{f^2\text{ArcTan}(\sinh(c+dx))}{ad^3}+\frac{(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{ad}+\frac{if^2\text{Li}_3\left(-ie^{c+dx}\right)}{ad^3}-\frac{if^2\text{Li}_3\left(ie^{c+dx}\right)}{ad^3}+\frac{if^2\log(\cosh(c+dx))}{ad^3}-\frac{if(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{ad^2}+\frac{if(e+fx)\text{Li}_2\left(ie^{c+dx}\right)}{ad^2}-\frac{if(e+fx)\tanh(c+dx)}{ad^2}+\frac{f(e+fx)\text{sech}(c+dx)}{ad^2}+\frac{i(e+fx)^2\text{sech}^2(c+dx)}{2ad}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((e + f*x)^2*ArcTan[E^(c + d*x)])/(a*d) - (f^2*ArcTan[Sinh[c + d*x]])/(a*d^
3) + (I*f^2*Log[Cosh[c + d*x]])/(a*d^3) - (I*f*(e + f*x)*PolyLog[2, (-I)*E^
(c + d*x)])/(a*d^2) + (I*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + (
I*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^3) - (I*f^2*PolyLog[3, I*E^(c + d*
x)])/(a*d^3) + (f*(e + f*x)*Sech[c + d*x])/(a*d^2) + ((I/2)*(e + f*x)^2*Sec
h[c + d*x]^2)/(a*d) - (I*f*(e + f*x)*Tanh[c + d*x])/(a*d^2) + ((e + f*x)^2*
Sech[c + d*x]*Tanh[c + d*x])/(2*a*d)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
f, g, n}, x] && GtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5690

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sech[c
 + d*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)*T
anh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && Eq
Q[a^2 + b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)^2\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\frac{i\int (e+fx)^2\operatorname{sech}^2(c+dx)\tanh(c+dx)\,dx}{a}+\frac{\int (e+fx)^2\operatorname{sech}^3(c+dx)\,dx}{a}\\ &= \frac{f(e+fx)\operatorname{sech}(c+dx)}{ad^2}+\frac{i(e+fx)^2\operatorname{sech}^2(c+dx)}{2ad}+\frac{(e+fx)^2\operatorname{sech}(c+dx)}{2ad}\\ &= \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad}-\frac{f^2\tan^{-1}(\sinh(c+dx))}{ad^3}+\frac{f(e+fx)\operatorname{sech}(c+dx)}{ad^2}\\ &= \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad}-\frac{f^2\tan^{-1}(\sinh(c+dx))}{ad^3}+\frac{if^2\log(\cosh(c+dx))}{ad^3}\\ &= \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad}-\frac{f^2\tan^{-1}(\sinh(c+dx))}{ad^3}+\frac{if^2\log(\cosh(c+dx))}{ad^3}\\ &= \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad}-\frac{f^2\tan^{-1}(\sinh(c+dx))}{ad^3}+\frac{if^2\log(\cosh(c+dx))}{ad^3}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 599 vs. 2(268) = 536.
time = 7.25, size = 599, normalized size = 2.24

$$-\frac{\frac{-3id^3e^2e^cx+12ide^cf^2x-3id^3ee^cfx^2-id^3e^cf^2x^3+3d^2e^2\log(i-e^{c+dx})+3id^2e^2e^c\log(i-e^{c+dx})-12f^2\log(i-e^{c+dx})-12ie^cf^2\log(i-e^{c+dx})+6d^2efx\log(1+ie^{c+dx})+6id^2ee^cfx\log(1+ie^{c+dx})+3d^2f^2x^2\log(1+ie^{c+dx})+3id^2e^cf^2x^2\log(1+ie^{c+dx})+6d(1+ie^c)f(e+fx)\operatorname{PolyLog}(2,-ie^{c+dx})-6(1+ie^c)f^2\operatorname{PolyLog}(3,-ie^{c+dx})}{d^3(-i+e^c)}+\frac{d^2\left(ide^cx(3e^2+3efx+f^2x^2)+3(1-ie^c)(e+fx)^2\log(1-ie^{c+dx})\right)+6d(1-ie^c)f(e+fx)\operatorname{PolyLog}(2,ie^{c+dx})+6i(i+e^c)f^2\operatorname{PolyLog}(3,ie^{c+dx})}{d^3(i+e^c)}-x(3e^2+3efx+f^2x^2)\operatorname{sech}(c)-\frac{3i(e+fx)^2}{d\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{12if(e+fx)\sinh\left(\frac{dx}{2}\right)}{d^2\left(\cosh\left(\frac{c}{2}\right)+i\sinh\left(\frac{c}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] -1/6*(((-3*I)*d^3*e^2*E^c*x + (12*I)*d*E^c*f^2*x - (3*I)*d^3*e*E^c*f*x^2 -
I*d^3*E^c*f^2*x^3 + 3*d^2*e^2*Log[I - E^(c + d*x)] + (3*I)*d^2*e^2*E^c*Log[
I - E^(c + d*x)] - 12*f^2*Log[I - E^(c + d*x)] - (12*I)*E^c*f^2*Log[I - E^(
c + d*x)] + 6*d^2*e*f*x*Log[1 + I*E^(c + d*x)] + (6*I)*d^2*e*E^c*f*x*Log[1
+ I*E^(c + d*x)] + 3*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] + (3*I)*d^2*E^c*f^2
```

```
*x^2*Log[1 + I*E^(c + d*x)] + 6*d*(1 + I*E^c)*f*(e + f*x)*PolyLog[2, (-I)*E
^(c + d*x)] - 6*(1 + I*E^c)*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(d^3*(-I + E^
c)) + (d^2*(I*d*E^c*x*(3*e^2 + 3*e*f*x + f^2*x^2) + 3*(1 - I*E^c)*(e + f*x)
^2*Log[1 - I*E^(c + d*x)]) + 6*d*(1 - I*E^c)*f*(e + f*x)*PolyLog[2, I*E^(c
+ d*x)] + (6*I)*(I + E^c)*f^2*PolyLog[3, I*E^(c + d*x)])/(d^3*(I + E^c)) -
x*(3*e^2 + 3*e*f*x + f^2*x^2)*Sech[c] - ((3*I)*(e + f*x)^2)/(d*(Cosh[(c + d
*x)/2] + I*Sinh[(c + d*x)/2])^2) + ((12*I)*f*(e + f*x)*Sinh[(d*x)/2])/(d^2*
(Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])))/a
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 612 vs. 2(248) = 496.
time = 3.21, size = 613, normalized size = 2.29

method	result
risch	$\frac{d x^2 f^2 e^{dx+c}+2 d e f x e^{dx+c}+d e^2 e^{dx+c}-2 i f^2 x+2 f^2 x e^{dx+c}-2 i e f+2 e f e^{dx+c}}{\left(e^{dx+c}-i\right)^2 d^2 a}+\frac{i e^2 \ln\left(e^{dx+c}+i\right)}{2 a d}+\frac{i \,\mathrm{polylog}\left(2, i e^{dx+c}\right) f^2 x}{a d^2}-\frac{ief}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x^2*f^2*exp(d*x+c)+2*d*e*f*x*exp(d*x+c)+d*e^2*exp(d*x+c)-2*I*f^2*x+2*f^2
*x*exp(d*x+c)-2*I*e*f+2*e*f*exp(d*x+c))/(exp(d*x+c)-I)^2/d^2/a+1/2*I/a/d*e^
2*ln(exp(d*x+c)+I)-I/a/d^2*e*f*c*ln(exp(d*x+c)+I)+I/a/d^2*e*f*c*ln(exp(d*x+
c)-I)+I/a/d^2*ln(1-I*exp(d*x+c))*c*e*f+I*f^2*polylog(3,-I*exp(d*x+c))/a/d^3
+1/2*I/a/d*ln(1-I*exp(d*x+c))*f^2*x^2-I/a/d^2*e*f*polylog(2,-I*exp(d*x+c))+
I/a/d^2*polylog(2,I*exp(d*x+c))*f^2*x-2*I/a/d^3*f^2*ln(exp(d*x+c))+I/a/d*ln
(1-I*exp(d*x+c))*e*f*x-1/2*I/a/d*ln(1+I*exp(d*x+c))*f^2*x^2-I*f^2*polylog(3
,I*exp(d*x+c))/a/d^3-I/a/d^2*ln(1+I*exp(d*x+c))*c*e*f+2*I/a/d^3*f^2*ln(exp(
d*x+c)-I)-1/2*I/a/d^3*c^2*f^2*ln(exp(d*x+c)-I)-I/a/d*ln(1+I*exp(d*x+c))*e*f
*x-I/a/d^2*polylog(2,-I*exp(d*x+c))*f^2*x-1/2*I/a/d^3*ln(1-I*exp(d*x+c))*c^
2*f^2+I/a/d^2*e*f*polylog(2,I*exp(d*x+c))-1/2*I/a/d*e^2*ln(exp(d*x+c)-I)+1/
2*I/a/d^3*ln(1+I*exp(d*x+c))*c^2*f^2+1/2*I/a/d^3*c^2*f^2*ln(exp(d*x+c)+I)
```

Maxima [A]
time = 0.44, size = 394, normalized size = 1.47

$$-\frac{1}{2}\left(\frac{4 e^{(-dx-c)}}{-2\left(-2 i a e^{(-dx-c)}-a e^{(-2dx-2c)}+a\right)d}+\frac{i \log\left(e^{(-dx-c)}+i\right)}{ad}-\frac{i \log\left(i e^{(-dx-c)}+1\right)}{ad}\right)e^2+\frac{-2 i f^2 x-2 i f e+\left(d f^2 x^2 e^c+2\left(d f e^{(c+1)}+f^2 e^c\right)x+2 f e^{(c+1)}\right)e^{(dx)}}{a d^2 e^{(2dx+2c)}-2 i a d^2 e^{(dx+c)}-a d^2}-\frac{2 i f^2 x}{a d^2}-\frac{i\left(dx \log\left(i e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(-i e^{(dx+c)}\right)\right)fe}{ad^2}+\frac{i\left(dx \log\left(-i e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(i e^{(dx+c)}\right)\right)fe}{ad^2}-\frac{i\left(d^2x^2 \log\left(i e^{(dx+c)}+1\right)+2 dx\mathrm{Li}_2\left(-i e^{(dx+c)}\right)-2\,\mathrm{Li}_3\left(-i e^{(dx+c)}\right)\right)f^2}{2 a d^3}+\frac{i\left(d^2x^2 \log\left(-i e^{(dx+c)}+1\right)+2 dx\mathrm{Li}_2\left(i e^{(dx+c)}\right)-2\,\mathrm{Li}_3\left(i e^{(dx+c)}\right)\right)f^2}{2 a d^3}+\frac{2 i f^2 \log\left(i e^{(dx+c)}+1\right)}{a d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(4*e^(-d*x - c)/((4*I*a*e^(-d*x - c) + 2*a*e^(-2*d*x - 2*c) - 2*a)*d)
+ I*log(e^(-d*x - c) + I)/(a*d) - I*log(I*e^(-d*x - c) + 1)/(a*d))*e^2 + (-
2*I*f^2*x - 2*I*f*e + (d*f^2*x^2*e^c + 2*(d*f*e^(c + 1) + f^2*e^c)*x + 2*f*
e^(c + 1))*e^(d*x))/(a*d^2*e^(2*d*x + 2*c) - 2*I*a*d^2*e^(d*x + c) - a*d^2)
```

```
 - 2*I*f^2*x/(a*d^2) - I*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)
))*f*e/(a*d^2) + I*(d*x*log(-I*e^(d*x + c) + 1) + dilog(I*e^(d*x + c)))*f*e
/(a*d^2) - 1/2*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x +
c)) - 2*polylog(3, -I*e^(d*x + c)))*f^2/(a*d^3) + 1/2*I*(d^2*x^2*log(-I*e^(
d*x + c) + 1) + 2*d*x*dilog(I*e^(d*x + c)) - 2*polylog(3, I*e^(d*x + c)))*f
^2/(a*d^3) + 2*I*f^2*log(I*e^(d*x + c) + 1)/(a*d^3)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 827 vs. $2(242) = 484$.
time = 0.40, size = 827, normalized size = 3.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*I*c*f^2 - 4*I*d*f*e - 2*(I*d*f^2*x + I*d*f*e + (-I*d*f^2*x - I*d*f*e
)*e^(2*d*x + 2*c) - 2*(d*f^2*x + d*f*e)*e^(d*x + c))*dilog(I*e^(d*x + c)) -
 2*(-I*d*f^2*x - I*d*f*e + (I*d*f^2*x + I*d*f*e)*e^(2*d*x + 2*c) + 2*(d*f^2
*x + d*f*e)*e^(d*x + c))*dilog(-I*e^(d*x + c)) - 4*(I*d*f^2*x + I*c*f^2)*e^
(2*d*x + 2*c) + 2*(d^2*f^2*x^2 - 2*d*f^2*x - 4*c*f^2 + d^2*e^2 + 2*(d^2*f*x
 + d*f)*e)*e^(d*x + c) + (-I*c^2*f^2 + 2*I*c*d*f*e - I*d^2*e^2 + (I*c^2*f^2
 - 2*I*c*d*f*e + I*d^2*e^2)*e^(2*d*x + 2*c) + 2*(c^2*f^2 - 2*c*d*f*e + d^2*
e^2)*e^(d*x + c))*log(e^(d*x + c) + I) + (-2*I*c*d*f*e + (I*c^2 - 4*I)*f^2
+ I*d^2*e^2 + (2*I*c*d*f*e + (-I*c^2 + 4*I)*f^2 - I*d^2*e^2)*e^(2*d*x + 2*c
) + 2*(2*c*d*f*e - (c^2 - 4)*f^2 - d^2*e^2)*e^(d*x + c))*log(e^(d*x + c) -
I) + (I*d^2*f^2*x^2 - I*c^2*f^2 - 2*(-I*d^2*f*x - I*c*d*f)*e + (-I*d^2*f^2*
x^2 + I*c^2*f^2 - 2*(I*d^2*f*x + I*c*d*f)*e)*e^(2*d*x + 2*c) - 2*(d^2*f^2*x
^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)*e)*e^(d*x + c))*log(I*e^(d*x + c) + 1) +
 (-I*d^2*f^2*x^2 + I*c^2*f^2 - 2*(I*d^2*f*x + I*c*d*f)*e + (I*d^2*f^2*x^2 -
 I*c^2*f^2 - 2*(-I*d^2*f*x - I*c*d*f)*e)*e^(2*d*x + 2*c) + 2*(d^2*f^2*x^2 -
 c^2*f^2 + 2*(d^2*f*x + c*d*f)*e)*e^(d*x + c))*log(-I*e^(d*x + c) + 1) - 2*
(I*f^2*e^(2*d*x + 2*c) + 2*f^2*e^(d*x + c) - I*f^2)*polylog(3, I*e^(d*x + c
)) - 2*(-I*f^2*e^(2*d*x + 2*c) - 2*f^2*e^(d*x + c) + I*f^2)*polylog(3, -I*e
^(d*x + c)))/(a*d^3*e^(2*d*x + 2*c) - 2*I*a*d^3*e^(d*x + c) - a*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{e^{2}\operatorname{sech}\left(c+dx\right)}{\sinh\left(c+dx\right)-i}\,dx+\int \frac{f^{2}x^{2}\operatorname{sech}\left(c+dx\right)}{\sinh\left(c+dx\right)-i}\,dx+\int \frac{2efx\operatorname{sech}\left(c+dx\right)}{\sinh\left(c+dx\right)-i}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*(Integral(e**2*sech(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f**2*x**
2*sech(c + d*x)/(sinh(c + d*x) - I), x) + Integral(2*e*f*x*sech(c + d*x)/(s
inh(c + d*x) - I), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sech(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^2}{\cosh\left(c+d\,x\right)\,\left(a+a\,\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(cosh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
```

```
[Out] int((e + f*x)^2/(cosh(c + d*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.273 $\int \frac{(e+fx)\mathbf{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{ad}-\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2ad^2}+\frac{if\text{PolyLog}\left(2,ie^{c+dx}\right)}{2ad^2}+\frac{f\text{sech}(c+dx)}{2ad^2}+\frac{i(e+fx)\text{sech}^2(c}{2ad}$$

```
[Out] (f*x+e)*arctan(exp(d*x+c))/a/d-1/2*I*f*polylog(2,-I*exp(d*x+c))/a/d^2+1/2*I
*f*polylog(2,I*exp(d*x+c))/a/d^2+1/2*f*sech(d*x+c)/a/d^2+1/2*I*(f*x+e)*sech
(d*x+c)^2/a/d-1/2*I*f*tanh(d*x+c)/a/d^2+1/2*(f*x+e)*sech(d*x+c)*tanh(d*x+c)
/a/d
```

Rubi [A]
time = 0.10, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {5690, 4270, 4265, 2317, 2438, 5559, 3852, 8}

$$\frac{(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{ad}-\frac{if\text{Li}_2\left(-ie^{c+dx}\right)}{2ad^2}+\frac{if\text{Li}_2\left(ie^{c+dx}\right)}{2ad^2}-\frac{if\tanh(c+dx)}{2ad^2}+\frac{f\text{sech}(c+dx)}{2ad^2}+\frac{i(e+fx)\text{sech}^2(c+dx)}{2ad}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((e + f*x)*ArcTan[E^(c + d*x)])/(a*d) - ((I/2)*f*PolyLog[2, (-I)*E^(c + d*x
)])/(a*d^2) + ((I/2)*f*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + (f*Sech[c + d*x
])/(2*a*d^2) + ((I/2)*(e + f*x)*Sech[c + d*x]^2)/(a*d) - ((I/2)*f*Tanh[c +
d*x])/(a*d^2) + ((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5690

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sech[c
 + d*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)*T
anh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && Eq
Q[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int (e+fx)\text{sech}^2(c+dx)\tanh(c+dx)\,dx}{a} + \frac{\int (e+fx)\text{sech}^3(c+dx)\,dx}{a}$$

$$= \frac{f\text{sech}(c+dx)}{2ad^2} + \frac{i(e+fx)\text{sech}^2(c+dx)}{2ad} + \frac{(e+fx)\text{sech}(c+dx)\tanh(c+d}{2ad}$$

$$= \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{f\text{sech}(c+dx)}{2ad^2} + \frac{i(e+fx)\text{sech}^2(c+dx)}{2ad} + \frac{(e+f}{}$$

$$= \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{f\text{sech}(c+dx)}{2ad^2} + \frac{i(e+fx)\text{sech}^2(c+dx)}{2ad} - \frac{if\tanh}{2}$$

$$= \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad} - \frac{if\text{Li}_2\left(-ie^{c+dx}\right)}{2ad^2} + \frac{if\text{Li}_2\left(ie^{c+dx}\right)}{2ad^2} + \frac{f\text{sech}(c+dx)}{2ad^2}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 710 vs. 2(161) = 322.
time = 2.21, size = 710, normalized size = 4.41

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] -1/4*((-2*I)*d*(e + f*x) + (c + d*x)*(c*f - d*(2*e + f*x))*(Cosh[(c + d*x)/
2] + I*Sinh[(c + d*x)/2])^2 + d*e*(c + d*x - (2*I)*Log[Cosh[(c + d*x)/2] -
I*Sinh[(c + d*x)/2]])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 - c*f*(c
+ d*x - (2*I)*Log[Cosh[(c + d*x)/2] - I*Sinh[(c + d*x)/2]])*(Cosh[(c + d*x)
/2] + I*Sinh[(c + d*x)/2])^2 + d*e*(c + d*x + (2*I)*Log[Cosh[(c + d*x)/2] +
 I*Sinh[(c + d*x)/2]])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 - c*f*(c
 + d*x + (2*I)*Log[Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])*(Cosh[(c + d*x
)/2] + I*Sinh[(c + d*x)/2])^2 + (f*(-2*(-1)^(3/4)*(c + d*x)^2 + Sqrt[2]*(2*
((-2*I)*c + Pi - (2*I)*d*x)*Log[1 + I*E^(-c - d*x)] + Pi*(3*c + 3*d*x - 4*L
og[1 + E^(c + d*x)] + 4*Log[Cosh[(c + d*x)/2]] - 2*Log[-Sin[(Pi - (2*I)*(c
+ d*x))/4]]) + (4*I)*PolyLog[2, (-I)*E^(-c - d*x)]))*(Cosh[(c + d*x)/2] + I
*Sinh[(c + d*x)/2])^2)/(2*Sqrt[2]) + (f*(2*(-1)^(1/4)*(c + d*x)^2 + Sqrt[2]
*(2*((2*I)*c + Pi + (2*I)*d*x)*Log[1 - I*E^(-c - d*x)] - Pi*(c + d*x - 4*Lo
g[1 + E^(c + d*x)] + 4*Log[Cosh[(c + d*x)/2]] + 2*Log[Sin[(Pi + (2*I)*(c +
d*x))/4]]) - (4*I)*PolyLog[2, I*E^(-c - d*x)]))*(Cosh[(c + d*x)/2] + I*Sinh
[(c + d*x)/2])^2)/(2*Sqrt[2]) - 4*f*Sinh[(c + d*x)/2]*((-I)*Cosh[(c + d*x)/
2] + Sinh[(c + d*x)/2]))/(d^2*(a + I*a*Sinh[c + d*x]))
```

Maple [A]
time = 4.04, size = 268, normalized size = 1.66

method	result
risch	$\frac{dfx\,\mathrm{e}^{dx+c}+de\,\mathrm{e}^{dx+c}+f\,\mathrm{e}^{dx+c}-if}{\left(\mathrm{e}^{dx+c}-i\right)^{2}d^{2}a}+\frac{ie\ln\left(\mathrm{e}^{dx+c}+i\right)}{2da}-\frac{ie\ln\left(\mathrm{e}^{dx+c}-i\right)}{2da}+\frac{if\ln\left(1-i\mathrm{e}^{dx+c}\right)x}{2da}+\frac{if\ln\left(1-i\mathrm{e}^{dx+c}\right)c}{2d^{2}a}+\frac{if\,\mathrm{polylog}(2}{2a\,d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)

[Out] (d*f*x*exp(d*x+c)+d*e*exp(d*x+c)+f*exp(d*x+c)-I*f)/(exp(d*x+c)-I)^2/d^2/a+1/2*I/d/a*e*ln(exp(d*x+c)+I)-1/2*I/d/a*e*ln(exp(d*x+c)-I)+1/2*I/d/a*f*ln(1-I*exp(d*x+c))*x+1/2*I/d^2/a*f*ln(1-I*exp(d*x+c))*c+1/2*I*f*polylog(2,I*exp(d*x+c))/a/d^2-1/2*I/d/a*f*ln(1+I*exp(d*x+c))*x-1/2*I/d^2/a*f*ln(1+I*exp(d*x+c))*c-1/2*I*f*polylog(2,-I*exp(d*x+c))/a/d^2-1/2*I/d^2/a*f*c*ln(exp(d*x+c)+I)+1/2*I/d^2/a*f*c*ln(exp(d*x+c)-I)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")

[Out] f*(((d*x*e^c + e^c)*e^(d*x) - I)/(a*d^2*e^(2*d*x + 2*c) - 2*I*a*d^2*e^(d*x + c) - a*d^2) + 2*integrate(1/4*x/(a*e^(d*x + c) + I*a), x) + 2*integrate(1/4*x/(a*e^(d*x + c) - I*a), x)) - 1/2*(4*e^(-d*x - c)/((4*I*a*e^(-d*x - c) + 2*a*e^(-2*d*x - 2*c) - 2*a)*d) + I*log(e^(-d*x - c) + I)/(a*d) - I*log(I*e^(-d*x - c) + 1)/(a*d))*e

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(138) = 276.
time = 0.40, size = 359, normalized size = 2.23

$$\frac{(i\,fe^{(2\,dx+2\,c)}+2\,fe^{(dx+c)}-i\,f)\mathrm{Li}_2\left(i\,e^{(dx+c)}\right)+\left(-i\,fe^{(2\,dx+2\,c)}-2\,fe^{(dx+c)}+i\,f\right)\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)+2\,(dfx+de+f)e^{(dx+c)}+\left(i\,cf-i\,de+(-i\,cf+i\,de)e^{(2\,dx+2\,c)}-2\,(cf-de)e^{(dx+c)}\right)\log\left(e^{(dx+c)}+i\right)+\left(-i\,cf+i\,de+(i\,cf-i\,de)e^{(2\,dx+2\,c)}+2\,(cf-de)e^{(dx+c)}\right)\log\left(e^{(dx+c)}-i\right)+\left(i\,dfx+i\,cf+(-i\,dfx-i\,cf)e^{(2\,dx+2\,c)}-2\,(dfx+cf)e^{(dx+c)}\right)\log\left(i\,e^{(dx+c)}+1\right)+\left(-i\,dfx-i\,cf+(i\,dfx+i\,cf)e^{(2\,dx+2\,c)}+2\,(dfx+cf)e^{(dx+c)}\right)\log\left(-i\,e^{(dx+c)}+1\right)-2i\,f}{2\,\left(ad^2e^{(2\,dx+2\,c)}-2i\,ad^2e^{(dx+c)}-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")

[Out] 1/2*((I*f*e^(2*d*x + 2*c) + 2*f*e^(d*x + c) - I*f)*dilog(I*e^(d*x + c)) + (-I*f*e^(2*d*x + 2*c) - 2*f*e^(d*x + c) + I*f)*dilog(-I*e^(d*x + c)) + 2*(d*f*x + d*e + f)*e^(d*x + c) + (I*c*f - I*d*e + (-I*c*f + I*d*e)*e^(2*d*x + 2*c) - 2*(c*f - d*e)*e^(d*x + c))*log(e^(d*x + c) + I) + (-I*c*f + I*d*e + (I*c*f - I*d*e)*e^(2*d*x + 2*c) + 2*(c*f - d*e)*e^(d*x + c))*log(e^(d*x + c) - I) + (I*d*f*x + I*c*f + (-I*d*f*x - I*c*f)*e^(2*d*x + 2*c) - 2*(d*f*x + c*f)*e^(d*x + c))*log(I*e^(d*x + c) + 1) + (-I*d*f*x - I*c*f + (I*d*f*x + I

```
*c*f)*e^(2*d*x + 2*c) + 2*(d*f*x + c*f)*e^(d*x + c))*log(-I*e^(d*x + c) + 1
) - 2*I*f)/(a*d^2*e^(2*d*x + 2*c) - 2*I*a*d^2*e^(d*x + c) - a*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{e\operatorname{sech}\left(c+dx\right)}{\sinh\left(c+dx\right)-i}\,dx+\int \frac{fx\operatorname{sech}\left(c+dx\right)}{\sinh\left(c+dx\right)-i}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
[Out] -I*(Integral(e*sech(c + d*x)/(sinh(c + d*x) - I), x) + Integral(f*x*sech(c
+ d*x)/(sinh(c + d*x) - I), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)*sech(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e+f\,x}{\mathrm{cosh}\left(c+d\,x\right)\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(cosh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
[Out] int((e + f*x)/(cosh(c + d*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.274 $\int \frac{\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=42

$$\frac{\operatorname{ArcTan}(\sinh(c+dx))}{2ad}+\frac{i}{2d(a+ia\sinh(c+dx))}$$

```
[Out] 1/2*arctan(sinh(d*x+c))/a/d+1/2*I/d/(a+I*a*sinh(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2746, 46, 212}

$$\frac{\operatorname{ArcTan}(\sinh(c+dx))}{2ad}+\frac{i}{2d(a+ia\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ArcTan[Sinh[c + d*x]]/(2*a*d) + (I/2)/(d*(a + I*a*Sinh[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{(ia)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^2}\,dx, x, ia\sinh(c+dx)\right)}{d}$$
$$= -\frac{(ia)\text{Subst}\left(\int \left(\frac{1}{2a(a+x)^2}+\frac{1}{2a(a^2-x^2)}\right)\,dx, x, ia\sinh(c+dx)\right)}{d}$$
$$= \frac{i}{2d(a+ia\sinh(c+dx))} - \frac{i\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, ia\sinh(c+dx)\right)}{2d}$$
$$= \frac{\tan^{-1}(\sinh(c+dx))}{2ad} + \frac{i}{2d(a+ia\sinh(c+dx))}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 0.71

$$\frac{\text{ArcTan}(\sinh(c+dx)) + \frac{1}{-i+\sinh(c+dx)}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (ArcTan[Sinh[c + d*x]] + (-I + Sinh[c + d*x])^(-1))/(2*a*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(36) = 72.
time = 1.22, size = 75, normalized size = 1.79

method	result	size
risch	$\frac{e^{dx+c}}{\left(e^{dx+c}-i\right)^2 da} - \frac{i\ln\left(e^{dx+c}-i\right)}{2ad} + \frac{i\ln\left(e^{dx+c}+i\right)}{2ad}$	64
derivativedivides	$\frac{\frac{i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}{2} - \frac{i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2} - \frac{i\ln\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2} - \frac{1}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{ad}$	75
default	$\frac{\frac{i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}{2} - \frac{i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2} - \frac{i\ln\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2} - \frac{1}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{ad}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a*(1/4*I*ln(tanh(1/2*d*x+1/2*c)+I)-1/2*I/(-I+tanh(1/2*d*x+1/2*c))^2-1/4
*I*ln(-I+tanh(1/2*d*x+1/2*c))-1/2/(-I+tanh(1/2*d*x+1/2*c)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(34) = 68$.
time = 0.26, size = 87, normalized size = 2.07

$$-\frac{2\,e^{(-dx-c)}}{-2\left(-2i\,ae^{(-dx-c)}-ae^{(-2\,dx-2\,c)}+a\right)d}-\frac{i\,\log\left(e^{(-dx-c)}+i\right)}{2\,ad}+\frac{i\,\log\left(i\,e^{(-dx-c)}+1\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*e^(-d*x - c)/((4*I*a*e^(-d*x - c) + 2*a*e^(-2*d*x - 2*c) - 2*a)*d) - 1/2
*I*log(e^(-d*x - c) + I)/(a*d) + 1/2*I*log(I*e^(-d*x - c) + 1)/(a*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(34) = 68$.
time = 0.37, size = 102, normalized size = 2.43

$$\frac{\left(i\,e^{(2\,dx+2\,c)}+2\,e^{(dx+c)}-i\right)\log\left(e^{(dx+c)}+i\right)+\left(-i\,e^{(2\,dx+2\,c)}-2\,e^{(dx+c)}+i\right)\log\left(e^{(dx+c)}-i\right)+2\,e^{(dx+c)}}{2\left(ade^{(2\,dx+2\,c)}-2i\,ade^{(dx+c)}-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((I*e^(2*d*x + 2*c) + 2*e^(d*x + c) - I)*log(e^(d*x + c) + I) + (-I*e^(
2*d*x + 2*c) - 2*e^(d*x + c) + I)*log(e^(d*x + c) - I) + 2*e^(d*x + c))/(a*
d*e^(2*d*x + 2*c) - 2*I*a*d*e^(d*x + c) - a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\operatorname{sech}\left(c+dx\right)}{\sinh\left(c+dx\right)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*Integral(sech(c + d*x)/(sinh(c + d*x) - I), x)/a
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(34) = 68$.
time = 0.44, size = 104, normalized size = 2.48

$$-\frac{\frac{i\,\log\left(e^{(dx+c)}-e^{(-dx-c)}-2i\right)}{a}-\frac{i\,\log\left(i\,e^{(dx+c)}-i\,e^{(-dx-c)}-2\right)}{a}+\frac{-i\,e^{(dx+c)}+i\,e^{(-dx-c)}-6}{a\left(e^{(dx+c)}-e^{(-dx-c)}-2i\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(I*log(e^(d*x + c) - e^(-d*x - c) - 2*I)/a - I*log(I*e^(d*x + c) - I*e
^(-d*x - c) - 2)/a + (-I*e^(d*x + c) + I*e^(-d*x - c) - 6)/(a*(e^(d*x + c)
- e^(-d*x - c) - 2*I)))/d
```

Mupad [B]
time = 1.10, size = 74, normalized size = 1.76

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{a^2\,d^2}}{a\,d}\right)}{\sqrt{a^2\,d^2}}+\frac{1}{a\,d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}-\frac{1\mathrm{i}}{a\,d\,{\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(a + a*sinh(c + d*x)*1i)),x)
```

```
[Out] atan((exp(d*x)*exp(c)*(a^2*d^2)^(1/2))/(a*d))/(a^2*d^2)^(1/2) + 1/(a*d*(exp
(c + d*x) - 1i)) - 1i/(a*d*(exp(c + d*x)*1i + 1)^2)
```

3.275 $\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=32

$$\operatorname{Int}\left(\frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sech[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
[Out] Defer[Int][Sech[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 45.48, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sech[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
[Out] Integrate[Sech[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(dx+c)}{(fx+e)\left(a+ia\sinh\left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((d*f*x*e^c + d*e^(c + 1) - f*e^c)*e^(d*x) + I*f)/(a*d^2*f^2*x^2 + 2*a*d^2
*f*x*e + a*d^2*e^2 - (a*d^2*f^2*x^2*e^(2*c) + 2*a*d^2*f*x*e^(2*c + 1) + a*d
^2*e^(2*c + 2))*e^(2*d*x) + 2*(I*a*d^2*f^2*x^2*e^c + 2*I*a*d^2*f*x*e^(c + 1
) + I*a*d^2*e^(c + 2))*e^(d*x)) + 2*integrate((d^2*f^2*x^2 + 2*d^2*f*x*e +
d^2*e^2 - 4*f^2)/(-4*I*a*d^2*f^3*x^3 - 12*I*a*d^2*f^2*x^2*e - 12*I*a*d^2*f*
x*e^2 - 4*I*a*d^2*e^3 + 4*(a*d^2*f^3*x^3*e^c + 3*a*d^2*f^2*x^2*e^(c + 1) +
3*a*d^2*f*x*e^(c + 2) + a*d^2*e^(c + 3))*e^(d*x)), x) + 2*integrate(1/(4*I*
a*f*x + 4*I*a*e + 4*(a*f*x*e^c + a*e^(c + 1))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((d*f*x + d*e - f)*e^(d*x + c) - (a*d^2*f^2*x^2 + 2*a*d^2*f*x*e + a*d^2*e^
2 - (a*d^2*f^2*x^2 + 2*a*d^2*f*x*e + a*d^2*e^2)*e^(2*d*x + 2*c) + 2*(I*a*d^
2*f^2*x^2 + 2*I*a*d^2*f*x*e + I*a*d^2*e^2)*e^(d*x + c))*integral((-2*I*f^2
+ (d^2*f^2*x^2 + 2*d^2*f*x*e + d^2*e^2 - 2*f^2)*e^(d*x + c))/(a*d^2*f^3*x^3
 + 3*a*d^2*f^2*x^2*e + 3*a*d^2*f*x*e^2 + a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d
^2*f^2*x^2*e + 3*a*d^2*f*x*e^2 + a*d^2*e^3)*e^(2*d*x + 2*c)), x) + I*f)/(a*
d^2*f^2*x^2 + 2*a*d^2*f*x*e + a*d^2*e^2 - (a*d^2*f^2*x^2 + 2*a*d^2*f*x*e +
a*d^2*e^2)*e^(2*d*x + 2*c) + 2*(I*a*d^2*f^2*x^2 + 2*I*a*d^2*f*x*e + I*a*d^2
*e^2)*e^(d*x + c))
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int \frac{\operatorname{sech}{\left(c+dx \right)}}{e\sinh{\left(c+dx \right)}-ie+fx\sinh{\left(c+dx \right)}-ifx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*Integral(sech(c + d*x)/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) - I*f*
x), x)/a
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sech(d*x + c)/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cosh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
```

```
[Out] int(1/(cosh(c + d*x)*(e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.276 $\int \frac{\text{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{\text{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sech[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
[Out] Defer[Int][Sech[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\text{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\text{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Sech[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\text{sech}(dx+c)}{(fx+e)^2\,(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((d*f*x*e^c + d*e^(c + 1) - 2*f*e^c)*e^(d*x) + 2*I*f)/(a*d^2*f^3*x^3 + 3*a
*d^2*f^2*x^2*e + 3*a*d^2*f*x*e^2 + a*d^2*e^3 - (a*d^2*f^3*x^3*e^(2*c) + 3*a
*d^2*f^2*x^2*e^(2*c + 1) + 3*a*d^2*f*x*e^(2*c + 2) + a*d^2*e^(2*c + 3))*e^(
2*d*x) + 2*(I*a*d^2*f^3*x^3*e^c + 3*I*a*d^2*f^2*x^2*e^(c + 1) + 3*I*a*d^2*f
*x*e^(c + 2) + I*a*d^2*e^(c + 3))*e^(d*x)) + 2*integrate((d^2*f^2*x^2 + 2*d
^2*f*x*e + d^2*e^2 - 12*f^2)/(-4*I*a*d^2*f^4*x^4 - 16*I*a*d^2*f^3*x^3*e - 2
4*I*a*d^2*f^2*x^2*e^2 - 16*I*a*d^2*f*x*e^3 - 4*I*a*d^2*e^4 + 4*(a*d^2*f^4*x
^4*e^c + 4*a*d^2*f^3*x^3*e^(c + 1) + 6*a*d^2*f^2*x^2*e^(c + 2) + 4*a*d^2*f*
x*e^(c + 3) + a*d^2*e^(c + 4))*e^(d*x)), x) + 2*integrate(1/(4*I*a*f^2*x^2
+ 8*I*a*f*x*e + 4*I*a*e^2 + 4*(a*f^2*x^2*e^c + 2*a*f*x*e^(c + 1) + a*e^(c +
 2))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((d*f*x + d*e - 2*f)*e^(d*x + c) - (a*d^2*f^3*x^3 + 3*a*d^2*f^2*x^2*e + 3*
a*d^2*f*x*e^2 + a*d^2*e^3 - (a*d^2*f^3*x^3 + 3*a*d^2*f^2*x^2*e + 3*a*d^2*f*
x*e^2 + a*d^2*e^3)*e^(2*d*x + 2*c) + 2*(I*a*d^2*f^3*x^3 + 3*I*a*d^2*f^2*x^2
*e + 3*I*a*d^2*f*x*e^2 + I*a*d^2*e^3)*e^(d*x + c))*integral((-6*I*f^2 + (d^
2*f^2*x^2 + 2*d^2*f*x*e + d^2*e^2 - 6*f^2)*e^(d*x + c))/(a*d^2*f^4*x^4 + 4*
a*d^2*f^3*x^3*e + 6*a*d^2*f^2*x^2*e^2 + 4*a*d^2*f*x*e^3 + a*d^2*e^4 + (a*d^
2*f^4*x^4 + 4*a*d^2*f^3*x^3*e + 6*a*d^2*f^2*x^2*e^2 + 4*a*d^2*f*x*e^3 + a*d
^2*e^4)*e^(2*d*x + 2*c)), x) + 2*I*f)/(a*d^2*f^3*x^3 + 3*a*d^2*f^2*x^2*e +
3*a*d^2*f*x*e^2 + a*d^2*e^3 - (a*d^2*f^3*x^3 + 3*a*d^2*f^2*x^2*e + 3*a*d^2*
f*x*e^2 + a*d^2*e^3)*e^(2*d*x + 2*c) + 2*(I*a*d^2*f^3*x^3 + 3*I*a*d^2*f^2*x
^2*e + 3*I*a*d^2*f*x*e^2 + I*a*d^2*e^3)*e^(d*x + c))
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\operatorname{sech}(c+dx)}{e^2\sinh(c+dx)-ie^2+2efx\sinh(c+dx)-2iefx+f^2x^2\sinh(c+dx)-if^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*Integral(sech(c + d*x)/(e**2*sinh(c + d*x) - I*e**2 + 2*e*f*x*sinh(c + d
*x) - 2*I*e*f*x + f**2*x**2*sinh(c + d*x) - I*f**2*x**2), x)/a
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sech(d*x + c)/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cosh\left(c+d\,x\right)\,{\left(e+f\,x\right)}^2\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

```
[Out] int(1/(cosh(c + d*x)*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.277 $\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=450

$$\frac{2(e+fx)^3}{3ad}-\frac{if(e+fx)^2\operatorname{ArcTan}\left(e^{c+dx}\right)}{ad^2}+\frac{if^3\operatorname{ArcTan}(\sinh(c+dx))}{ad^4}-\frac{2f(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{ad^2}+\frac{f^3\log(\cdots$$

```
[Out] 2/3*(f*x+e)^3/a/d+I*f^3*arctan(sinh(d*x+c))/a/d^4-1/2*I*f*(f*x+e)^2*sech(d*
x+c)*tanh(d*x+c)/a/d^2-2*f*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/a/d^2+f^3*ln(cosh
(d*x+c))/a/d^4-f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/a/d^3+f^2*(f*x+e)*polyl
og(2,I*exp(d*x+c))/a/d^3-2*f^2*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/a/d^3+f^3
*polylog(3,-I*exp(d*x+c))/a/d^4-f^3*polylog(3,I*exp(d*x+c))/a/d^4+f^3*polyl
og(3,-exp(2*d*x+2*c))/a/d^4+1/3*I*(f*x+e)^3*sech(d*x+c)^3/a/d+1/2*f*(f*x+e)
^2*sech(d*x+c)^2/a/d^2-I*f*(f*x+e)^2*arctan(exp(d*x+c))/a/d^2-f^2*(f*x+e)*t
anh(d*x+c)/a/d^3+2/3*(f*x+e)^3*tanh(d*x+c)/a/d-I*f^2*(f*x+e)*sech(d*x+c)/a/
d^3+1/3*(f*x+e)^3*sech(d*x+c)^2*tanh(d*x+c)/a/d
```

Rubi [A]
time = 0.42, antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {5690, 4271, 4269, 3556, 3799, 2221, 2611, 2320, 6724, 5559, 3855, 4265}

$$\frac{if^3\operatorname{ArcTan}(\sinh(c+dx))}{ad^4}-\frac{if(e+fx)^2\operatorname{ArcTan}\left(e^{c+dx}\right)}{ad^2}+\frac{f^3\operatorname{Li}_3\left(-ie^{c+dx}\right)}{ad^4}-\frac{f^3\operatorname{Li}_3\left(ie^{c+dx}\right)}{ad^4}+\frac{f^3\operatorname{Li}_3\left(-e^{2(c+dx)}\right)}{ad^4}+\frac{f^3\log(\cosh(c+dx))}{ad^4}-\frac{f^2(e+fx)\operatorname{Li}_2\left(-ie^{c+dx}\right)}{ad^3}+\frac{f^2(e+fx)\operatorname{Li}_2\left(ie^{c+dx}\right)}{ad^3}-\frac{2f^2(e+fx)\operatorname{Li}_2\left(-e^{2(c+dx)}\right)}{ad^3}-\frac{f^2(e+fx)\tanh(c+dx)}{ad^3}-\frac{if^2(e+fx)\operatorname{sech}(c+dx)}{ad^3}-\frac{2f(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{ad^2}+\frac{f(e+fx)^2\operatorname{sech}^2(c+dx)}{2ad^2}-\frac{if(e+fx)^2\tanh(c+dx)\operatorname{sech}(c+dx)}{2ad^2}+\frac{2(e+fx)^3\tanh(c+dx)}{3ad}+\frac{i(e+fx)^3\operatorname{sech}^3(c+dx)}{3ad}+\frac{(e+fx)^3\tanh(c+dx)\operatorname{sech}^2(c+dx)}{3ad}+\frac{2(e+fx)^3}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (2*(e + f*x)^3)/(3*a*d) - (I*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/(a*d^2) + (
I*f^3*ArcTan[Sinh[c + d*x]])/(a*d^4) - (2*f*(e + f*x)^2*Log[1 + E^(2*(c + d
*x))])/(a*d^2) + (f^3*Log[Cosh[c + d*x]])/(a*d^4) - (f^2*(e + f*x)*PolyLog[
2, (-I)*E^(c + d*x)])/(a*d^3) + (f^2*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(
a*d^3) - (2*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(a*d^3) + (f^3*Poly
Log[3, (-I)*E^(c + d*x)])/(a*d^4) - (f^3*PolyLog[3, I*E^(c + d*x)])/(a*d^4)
 + (f^3*PolyLog[3, -E^(2*(c + d*x))])/(a*d^4) - (I*f^2*(e + f*x)*Sech[c + d
*x])/(a*d^3) + (f*(e + f*x)^2*Sech[c + d*x]^2)/(2*a*d^2) + ((I/3)*(e + f*x)
^3*Sech[c + d*x]^3)/(a*d) - (f^2*(e + f*x)*Tanh[c + d*x])/(a*d^3) + (2*(e +
 f*x)^3*Tanh[c + d*x])/(3*a*d) - ((I/2)*f*(e + f*x)^2*Sech[c + d*x]*Tanh[c
+ d*x])/(a*d^2) + ((e + f*x)^3*Sech[c + d*x]^2*Tanh[c + d*x])/(3*a*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5690

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sech[c
 + d*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)*T
anh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && Eq
Q[a^2 + b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int (e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)\,dx}{a} + \frac{\int (e+fx)^3\text{sech}^4(c+dx)\,dx}{a}$$

$$= \frac{f(e+fx)^2\text{sech}^2(c+dx)}{2ad^2} + \frac{i(e+fx)^3\text{sech}^3(c+dx)}{3ad} + \frac{(e+fx)^3\text{sech}^2(c+}{3ad}$$

$$= -\frac{if^2(e+fx)\text{sech}(c+dx)}{ad^3} + \frac{f(e+fx)^2\text{sech}^2(c+dx)}{2ad^2} + \frac{i(e+fx)^3\text{sech}^3(c}{3ad}$$

$$= \frac{2(e+fx)^3}{3ad} - \frac{if(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{if^3\tan^{-1}(\sinh(c+dx))}{ad^4} + \frac{f^3\text{lo}}{}$$

$$= \frac{2(e+fx)^3}{3ad} - \frac{if(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{if^3\tan^{-1}(\sinh(c+dx))}{ad^4} - \frac{2f(e}{}$$

$$= \frac{2(e+fx)^3}{3ad} - \frac{if(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{if^3\tan^{-1}(\sinh(c+dx))}{ad^4} - \frac{2f(e}{}$$

$$= \frac{2(e+fx)^3}{3ad} - \frac{if(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{if^3\tan^{-1}(\sinh(c+dx))}{ad^4} - \frac{2f(e}{}$$

$$= \frac{2(e+fx)^3}{3ad} - \frac{if(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{if^3\tan^{-1}(\sinh(c+dx))}{ad^4} - \frac{2f(e}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1127 vs. $2(450) = 900$.
time = 10.08, size = 1127, normalized size = 2.50

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (f*(15*d^3*e^2*E^c*x - 12*d*E^c*f^2*x + 15*d^3*e*E^c*f*x^2 + 5*d^3*E^c*f^2*
x^3 + (15*I)*d^2*e^2*Log[I - E^(c + d*x)] - 15*d^2*e^2*E^c*Log[I - E^(c + d
*x)] - (12*I)*f^2*Log[I - E^(c + d*x)] + 12*E^c*f^2*Log[I - E^(c + d*x)] +
(30*I)*d^2*e*f*x*Log[1 + I*E^(c + d*x)] - 30*d^2*e*E^c*f*x*Log[1 + I*E^(c +
 d*x)] + (15*I)*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - 15*d^2*E^c*f^2*x^2*Log
[1 + I*E^(c + d*x)] - 30*d*(-I + E^c)*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*
x)] + 30*(-I + E^c)*f^2*PolyLog[3, (-I)*E^(c + d*x)]))/(6*a*d^4*(-I + E^c))
 - ((I/2)*f*(d^2*(I*d*E^c*x*(3*e^2 + 3*e*f*x + f^2*x^2) + 3*(1 - I*E^c)*(e
+ f*x)^2*Log[1 - I*E^(c + d*x)]) + 6*d*(1 - I*E^c)*f*(e + f*x)*PolyLog[2, I
*E^(c + d*x)] + (6*I)*(I + E^c)*f^2*PolyLog[3, I*E^(c + d*x)]))/(a*d^4*(I +
 E^c)) + (e^3*Sinh[(d*x)/2] + 3*e^2*f*x*Sinh[(d*x)/2] + 3*e*f^2*x^2*Sinh[(d
*x)/2] + f^3*x^3*Sinh[(d*x)/2])/(2*a*d*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2
+ (d*x)/2] - I*Sinh[c/2 + (d*x)/2])) + (e^3*Sinh[(d*x)/2] + 3*e^2*f*x*Sinh[
```

(d*x)/2] + 3*e*f^2*x^2*Sinh[(d*x)/2] + f^3*x^3*Sinh[(d*x)/2])/(3*a*d*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2])^3) + (I*d*e^3*Cosh[c/2] + 3*e^2*f*Cosh[c/2] + (3*I)*d*e^2*f*x*Cosh[c/2] + 6*e*f^2*x*Cosh[c/2] + (3*I)*d*e*f^2*x^2*Cosh[c/2] + 3*f^3*x^2*Cosh[c/2] + I*d*f^3*x^3*Cosh[c/2] + d*e^3*Sinh[c/2] + (3*I)*e^2*f*Sinh[c/2] + 3*d*e^2*f*x*Sinh[c/2] + (6*I)*e*f^2*x*Sinh[c/2] + 3*d*e*f^2*x^2*Sinh[c/2] + (3*I)*f^3*x^2*Sinh[c/2] + d*f^3*x^3*Sinh[c/2])/(6*a*d^2*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2])^2) + (5*d^2*e^3*Sinh[(d*x)/2] - 12*e*f^2*Sinh[(d*x)/2] + 15*d^2*e^2*f*x*Sinh[(d*x)/2] - 12*f^3*x*Sinh[(d*x)/2] + 15*d^2*e*f^2*x^2*Sinh[(d*x)/2] + 5*d^2*f^3*x^3*Sinh[(d*x)/2])/(6*a*d^3*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2]))

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1000 vs. $2(424) = 848$.
time = 5.18, size = 1001, normalized size = 2.22

method	result
risch	$-\frac{8f^2ec\ln(\mathrm{e}^{dx+c})}{a\,d^3}-\frac{5f^2\ln(1+i\mathrm{e}^{dx+c})ce}{a\,d^3}+\frac{8f^2cex}{a\,d^2}-\frac{5f^3\,\mathrm{polylog}(2,-i\mathrm{e}^{dx+c})x}{a\,d^3}-\frac{5f^3c^2\ln(\mathrm{e}^{dx+c}-i)}{2a\,d^4}-\frac{3f^3c^2\ln(\mathrm{e}^{dx+c}+i)}{2a\,d^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x+e)^3*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -5/a/d^3*f^2*ln(1+I*exp(d*x+c))*c*e-3/a/d^2*f^2*ln(1-I*exp(d*x+c))*e*x+3/a/d^3*f^2*e*c*ln(exp(d*x+c)+I)+5/a/d^3*f^2*e*c*ln(exp(d*x+c)-I)-8/a/d^3*f^2*e*c*ln(exp(d*x+c))-3/a/d^3*f^2*ln(1-I*exp(d*x+c))*c*e+8/a/d^2*f^2*c*e*x-5/a/d^2*f^2*ln(1+I*exp(d*x+c))*e*x+1/3*I*(6*I*e*f^2*exp(2*d*x+2*c)+8*d^2*f^3*x^3*exp(d*x+c)-3*d*f^3*x^2*exp(3*d*x+3*c)+6*I*f^3*x*exp(2*d*x+2*c)+24*d^2*e*f^2*x^2*exp(d*x+c)-6*d*e*f^2*x*exp(3*d*x+3*c)-4*I*d^2*f^3*x^3+6*I*f^2*e+24*d^2*e^2*f*x*exp(d*x+c)-3*d*e^2*f*exp(3*d*x+3*c)-3*d*f^3*x^2*exp(d*x+c)-6*f^3*x*exp(3*d*x+3*c)-4*I*d^2*e^3-12*I*d^2*e^2*f*x+8*d^2*e^3*exp(d*x+c)-6*d*e*f^2*x*exp(d*x+c)-6*e*f^2*exp(3*d*x+3*c)+6*I*f^3*x-3*d*e^2*f*exp(d*x+c)-6*f^3*x*exp(d*x+c)-12*I*d^2*e*f^2*x^2-6*e*f^2*exp(d*x+c))/(exp(d*x+c)+I)/(exp(d*x+c)-I)^3/d^3/a+5*f^3*polylog(3,-I*exp(d*x+c))/a/d^4+3*f^3*polylog(3,I*exp(d*x+c))/a/d^4-5/2/a/d^4*f^3*c^2*ln(exp(d*x+c)-I)-3/2/a/d^4*f^3*c^2*ln(exp(d*x+c)+I)+4/a/d^2*f*e^2*ln(exp(d*x+c))+4/a/d^4*f^3*c^2*ln(exp(d*x+c))+3/2/a/d^4*f^3*ln(1-I*exp(d*x+c))*c^2+4/a/d*f^2*e*x^2+4/a/d^3*f^2*c^2*e-3/2/a/d^2*f^3*ln(1-I*exp(d*x+c))*x^2-3/a/d^3*f^3*polylog(2,I*exp(d*x+c))*x-5/2/a/d^2*f^3*ln(1+I*exp(d*x+c))*x^2-5/a/d^3*f^3*polylog(2,-I*exp(d*x+c))*x+5/2/a/d^4*f^3*ln(1+I*exp(d*x+c))*c^2-3/a/d^3*f^2*e*polylog(2,I*exp(d*x+c))-5/a/d^3*f^2*e*polylog(2,-I*exp(d*x+c))-5/2/a/d^2*f*e^2*ln(exp(d*x+c)-I)-3/2/a/d^2*f*e^2*ln(exp(d*x+c)+I)-8/3/a/d^4*f^3*c^3+4/3/a/d*f^3*x^3-2/a/d^4*f^3*ln(exp(d*x+c))+2/a/d^4*f^3*ln(exp(d*x+c)-I)-4/d^3/a*f^3*c^2*x

Maxima [A]

time = 0.51, size = 749, normalized size = 1.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/2*f*(24*(4*I*d*x*e^(4*d*x + 4*c) + (8*d*x*e^(3*c) + e^(3*c))*e^(3*d*x) +
e^(d*x + c))/(12*I*a*d^2*e^(4*d*x + 4*c) + 24*a*d^2*e^(3*d*x + 3*c) + 24*a*
d^2*e^(d*x + c) - 12*I*a*d^2) - 3*log((e^(d*x + c) + I)*e^(-c))/(a*d^2) - 5
*log(-I*(I*e^(d*x + c) + 1)*e^(-c))/(a*d^2))*e^2 + 4/3*(2*e^(-d*x - c)/((2*
a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) - I*a*e^(-4*d*x - 4*c) + I*a)*d) + I/
((2*a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) - I*a*e^(-4*d*x - 4*c) + I*a)*d))
*e^3 + (4*I*d^2*f^3*x^3 + 12*I*d^2*f^2*x^2*e - 6*I*f^3*x - 6*I*f^2*e + 3*(d
*f^3*x^2*e^(3*c) + 2*f^2*e^(3*c + 1) + 2*(f^3*e^(3*c) + d*f^2*e^(3*c + 1))*
x)*e^(3*d*x) - 6*(I*f^3*x*e^(2*c) + I*f^2*e^(2*c + 1))*e^(2*d*x) - (8*d^2*f
^3*x^3*e^c + 3*(8*d^2*f^2*e^(c + 1) - d*f^3*e^c)*x^2 - 6*f^2*e^(c + 1) - 6*
(d*f^2*e^(c + 1) + f^3*e^c)*x)*e^(d*x))/(3*I*a*d^3*e^(4*d*x + 4*c) + 6*a*d^
3*e^(3*d*x + 3*c) + 6*a*d^3*e^(d*x + c) - 3*I*a*d^3) - 2*f^3*x/(a*d^3) - 5*
(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*f^2*e/(a*d^3) - 3*(d*x
*log(-I*e^(d*x + c) + 1) + dilog(I*e^(d*x + c)))*f^2*e/(a*d^3) - 5/2*(d^2*x
^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x + c)) - 2*polylog(3, -I*e
^(d*x + c)))*f^3/(a*d^4) - 3/2*(d^2*x^2*log(-I*e^(d*x + c) + 1) + 2*d*x*dil
og(I*e^(d*x + c)) - 2*polylog(3, I*e^(d*x + c)))*f^3/(a*d^4) + 2*f^3*log(e^
(d*x + c) - I)/(a*d^4) + 4/3*(d^3*f^3*x^3 + 3*d^3*f^2*x^2*e)/(a*d^4)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1424 vs. 2(425) = 850.
time = 0.37, size = 1424, normalized size = 3.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/6*(24*c*d^2*f*e^2 - 12*(2*c^2 - 1)*d*f^2*e + 4*(2*c^3 - 3*c)*f^3 - 8*d^3
*e^3 - 18*(d*f^3*x + d*f^2*e - (d*f^3*x + d*f^2*e)*e^(4*d*x + 4*c) - 2*(-I*
d*f^3*x - I*d*f^2*e)*e^(3*d*x + 3*c) - 2*(-I*d*f^3*x - I*d*f^2*e)*e^(d*x +
c))*dilog(I*e^(d*x + c)) - 30*(d*f^3*x + d*f^2*e - (d*f^3*x + d*f^2*e)*e^(4
*d*x + 4*c) - 2*(-I*d*f^3*x - I*d*f^2*e)*e^(3*d*x + 3*c) - 2*(-I*d*f^3*x -
I*d*f^2*e)*e^(d*x + c))*dilog(-I*e^(d*x + c)) - 4*(2*d^3*f^3*x^3 - 3*d*f^3*
x + (2*c^3 - 3*c)*f^3 + 6*(d^3*f*x + c*d^2*f)*e^2 + 6*(d^3*f^2*x^2 - c^2*d*
f^2)*e)*e^(4*d*x + 4*c) + 2*(8*I*d^3*f^3*x^3 + 3*I*d^2*f^3*x^2 - 6*I*d*f^3*
```

```
x + 4*(2*I*c^3 - 3*I*c)*f^3 + 3*(8*I*d^3*f*x + (8*I*c + I)*d^2*f)*e^2 + 6*(
4*I*d^3*f^2*x^2 + I*d^2*f^2*x + (-4*I*c^2 + I)*d*f^2)*e)*e^(3*d*x + 3*c) +
12*(d*f^3*x + d*f^2*e)*e^(2*d*x + 2*c) + 2*(3*I*d^2*f^3*x^2 - 6*I*d*f^3*x +
 3*(8*I*c + I)*d^2*f*e^2 + 4*(2*I*c^3 - 3*I*c)*f^3 - 8*I*d^3*e^3 + 6*(I*d^2
*f^2*x + (-4*I*c^2 + I)*d*f^2)*e)*e^(d*x + c) - 9*(c^2*f^3 - 2*c*d*f^2*e +
d^2*f*e^2 - (c^2*f^3 - 2*c*d*f^2*e + d^2*f*e^2)*e^(4*d*x + 4*c) - 2*(-I*c^2
*f^3 + 2*I*c*d*f^2*e - I*d^2*f*e^2)*e^(3*d*x + 3*c) - 2*(-I*c^2*f^3 + 2*I*c
*d*f^2*e - I*d^2*f*e^2)*e^(d*x + c))*log(e^(d*x + c) + I) + 3*(10*c*d*f^2*e
 - (5*c^2 - 4)*f^3 - 5*d^2*f*e^2 - (10*c*d*f^2*e - (5*c^2 - 4)*f^3 - 5*d^2*
f*e^2)*e^(4*d*x + 4*c) + 2*(10*I*c*d*f^2*e + (-5*I*c^2 + 4*I)*f^3 - 5*I*d^2
*f*e^2)*e^(3*d*x + 3*c) + 2*(10*I*c*d*f^2*e + (-5*I*c^2 + 4*I)*f^3 - 5*I*d^
2*f*e^2)*e^(d*x + c))*log(e^(d*x + c) - I) - 15*(d^2*f^3*x^2 - c^2*f^3 + 2*
(d^2*f^2*x + c*d*f^2)*e - (d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x + c*d*f^2)*
e)*e^(4*d*x + 4*c) - 2*(-I*d^2*f^3*x^2 + I*c^2*f^3 + 2*(-I*d^2*f^2*x - I*c*
d*f^2)*e)*e^(3*d*x + 3*c) - 2*(-I*d^2*f^3*x^2 + I*c^2*f^3 + 2*(-I*d^2*f^2*x
 - I*c*d*f^2)*e)*e^(d*x + c))*log(I*e^(d*x + c) + 1) - 9*(d^2*f^3*x^2 - c^2
*f^3 + 2*(d^2*f^2*x + c*d*f^2)*e - (d^2*f^3*x^2 - c^2*f^3 + 2*(d^2*f^2*x +
c*d*f^2)*e)*e^(4*d*x + 4*c) - 2*(-I*d^2*f^3*x^2 + I*c^2*f^3 + 2*(-I*d^2*f^2
*x - I*c*d*f^2)*e)*e^(3*d*x + 3*c) - 2*(-I*d^2*f^3*x^2 + I*c^2*f^3 + 2*(-I*
d^2*f^2*x - I*c*d*f^2)*e)*e^(d*x + c))*log(-I*e^(d*x + c) + 1) - 18*(f^3*e^
(4*d*x + 4*c) - 2*I*f^3*e^(3*d*x + 3*c) - 2*I*f^3*e^(d*x + c) - f^3)*polylo
g(3, I*e^(d*x + c)) - 30*(f^3*e^(4*d*x + 4*c) - 2*I*f^3*e^(3*d*x + 3*c) - 2
*I*f^3*e^(d*x + c) - f^3)*polylog(3, -I*e^(d*x + c)))/(a*d^4*e^(4*d*x + 4*c
) - 2*I*a*d^4*e^(3*d*x + 3*c) - 2*I*a*d^4*e^(d*x + c) - a*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{e^{3}\operatorname{sech}^{2}\left(c+dx\right)}{\sinh\left(c+dx\right)-i}\,dx+\int \frac{f^{3}x^{3}\operatorname{sech}^{2}\left(c+dx\right)}{\sinh\left(c+dx\right)-i}\,dx+\int \frac{3ef^{2}x^{2}\operatorname{sech}^{2}\left(c+dx\right)}{\sinh\left(c+dx\right)-i}\,dx+\int \frac{3e^{2}fx\operatorname{sech}^{2}\left(c+dx\right)}{\sinh\left(c+dx\right)-i}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sech(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*(Integral(e**3*sech(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(f**3*
x**3*sech(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(3*e*f**2*x**2*sech
(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(3*e**2*f*x*sech(c + d*x)**2
/(sinh(c + d*x) - I), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sech(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^3}{\cosh\left(c+d\,x\right)^2\,\left(a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(cosh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

```
[Out] int((e + f*x)^3/(cosh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.278 $\int \frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=325

$$\frac{2(e+fx)^2}{3ad}-\frac{2if(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{3ad^2}-\frac{4f(e+fx)\log\left(1+e^{2(c+dx)}\right)}{3ad^2}-\frac{f^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{3ad^3}+\frac{f^2\text{PolyLo}}{3a}$$

```
[Out] 2/3*(f*x+e)^2/a/d-2/3*I*f*(f*x+e)*arctan(exp(d*x+c))/a/d^2-4/3*f*(f*x+e)*ln
(1+exp(2*d*x+2*c))/a/d^2-1/3*f^2*polylog(2,-I*exp(d*x+c))/a/d^3+1/3*f^2*pol
ylog(2,I*exp(d*x+c))/a/d^3-2/3*f^2*polylog(2,-exp(2*d*x+2*c))/a/d^3-1/3*I*f
^2*sech(d*x+c)/a/d^3+1/3*f*(f*x+e)*sech(d*x+c)^2/a/d^2+1/3*I*(f*x+e)^2*sech
(d*x+c)^3/a/d-1/3*f^2*tanh(d*x+c)/a/d^3+2/3*(f*x+e)^2*tanh(d*x+c)/a/d-1/3*I
*f*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/a/d^2+1/3*(f*x+e)^2*sech(d*x+c)^2*tanh(d
*x+c)/a/d
```

Rubi [A]
time = 0.27, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {5690, 4271, 3852, 8, 4269, 3799, 2221, 2317, 2438, 5559, 4270, 4265}

$$-\frac{2if(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{3ad^2}-\frac{f^2\text{Li}_2\left(-ie^{c+dx}\right)}{3ad^3}+\frac{f^2\text{Li}_2\left(ie^{c+dx}\right)}{3ad^3}-\frac{2f^2\text{Li}_2\left(-e^{2(c+dx)}\right)}{3ad^3}-\frac{f^2\tanh(c+dx)}{3ad^3}-\frac{if^2\text{sech}(c+dx)}{3ad^3}-\frac{4f(e+fx)\log\left(e^{2(c+dx)}+1\right)}{3ad^2}+\frac{f(e+fx)\text{sech}^2(c+dx)}{3ad^2}-\frac{if(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{3ad^2}+\frac{2(e+fx)^2\tanh(c+dx)}{3ad}+\frac{i(e+fx)^2\text{sech}^3(c+dx)}{3ad}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}^2(c+dx)}{3ad}+\frac{2(e+fx)^2}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (2*(e + f*x)^2)/(3*a*d) - (((2*I)/3)*f*(e + f*x)*ArcTan[E^(c + d*x)])/(a*d^
2) - (4*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(3*a*d^2) - (f^2*PolyLog[2, (
-I)*E^(c + d*x)])/(3*a*d^3) + (f^2*PolyLog[2, I*E^(c + d*x)])/(3*a*d^3) - (
2*f^2*PolyLog[2, -E^(2*(c + d*x))])/(3*a*d^3) - ((I/3)*f^2*Sech[c + d*x])/(
a*d^3) + (f*(e + f*x)*Sech[c + d*x]^2)/(3*a*d^2) + ((I/3)*(e + f*x)^2*Sech[
c + d*x]^3)/(a*d) - (f^2*Tanh[c + d*x])/(3*a*d^3) + (2*(e + f*x)^2*Tanh[c +
 d*x])/(3*a*d) - ((I/3)*f*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(a*d^2) +
((e + f*x)^2*Sech[c + d*x]^2*Tanh[c + d*x])/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5690

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sech[c
 + d*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)*T
anh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && Eq
Q[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int (e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)\,dx}{a}+\frac{\int (e+fx)^2\text{sech}^4(c+dx)\,dx}{a}$$

$$= \frac{f(e+fx)\text{sech}^2(c+dx)}{3ad^2}+\frac{i(e+fx)^2\text{sech}^3(c+dx)}{3ad}+\frac{(e+fx)^2\text{sech}^2(c+dx}{3ad}$$

$$= -\frac{if^2\text{sech}(c+dx)}{3ad^3}+\frac{f(e+fx)\text{sech}^2(c+dx)}{3ad^2}+\frac{i(e+fx)^2\text{sech}^3(c+dx)}{3ad}+\frac{2}{}$$

$$= \frac{2(e+fx)^2}{3ad}-\frac{2if(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{3ad^2}-\frac{if^2\text{sech}(c+dx)}{3ad^3}+\frac{f(e+fx)\text{sech}}{3ad^2}$$

$$= \frac{2(e+fx)^2}{3ad}-\frac{2if(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{3ad^2}-\frac{4f(e+fx)\log\left(1+e^{2(c+dx)}\right)}{3ad^2}-\frac{i}{}$$

$$= \frac{2(e+fx)^2}{3ad}-\frac{2if(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{3ad^2}-\frac{4f(e+fx)\log\left(1+e^{2(c+dx)}\right)}{3ad^2}-\frac{f}{}$$

$$= \frac{2(e+fx)^2}{3ad}-\frac{2if(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{3ad^2}-\frac{4f(e+fx)\log\left(1+e^{2(c+dx)}\right)}{3ad^2}-\frac{f}{}$$

Mathematica [A]
time = 8.42, size = 576, normalized size = 1.77

$$\frac{\frac{20if\left(d(e+fx)\left(ie^c\log\left(1-ie^{-c-dx}\right)+\log\left(1+ie^{c+dx}\right)\right)-ie^cf\text{PolyLog}\left(2,ie^{-c-dx}\right)+f\text{PolyLog}\left(2,-ie^{c+dx}\right)\right)}{-i+e^c}+6f\left(d\left(\frac{de^cx(2e+fx)}{i+e^c}-2(e+fx)\log\left(1-ie^{c+dx}\right)\right)-2f\text{PolyLog}\left(2,ie^{c+dx}\right)\right)+\frac{-2if^2\cosh(c)+2df(e+fx)\cosh(dx)-2id^2e^2\cosh(c+dx)+4if^2\cosh(c+dx)-4id^2efx\cosh(c+dx)-2id^2f^2x^2\cosh(c+dx)+2def\cosh(2c+dx)+2df^2x\cosh(2c+dx)+4id^2e^2\cosh(c+2dx)-2if^2\cosh(c+2dx)+8id^2efx\cosh(c+2dx)+4id^2f^2x^2\cosh(c+2dx)+8d^2e^2\sinh(dx)-2f^2\sinh(dx)+16d^2efx\sinh(dx)+8d^2f^2x^2\sinh(dx)+d^2e^2\sinh(2(c+dx))-2f^2\sinh(2(c+dx))+2d^2efx\sinh(2(c+dx))+d^2f^2x^2\sinh(2(c+dx))+2f^2\sinh(2c+dx)}{\left(\cosh\left(\frac{c}{2}\right)-i\sinh\left(\frac{c}{2}\right)\right)\left(\cosh\left(\frac{c}{2}\right)+i\sinh\left(\frac{c}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)-i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^3}}{12ad^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (((20*I)*f*(d*(e + f*x)*(I*E^c*Log[1 - I*E^(-c - d*x)] + Log[1 + I*E^(c + d
*x)]) - I*E^c*f*PolyLog[2, I*E^(-c - d*x)] + f*PolyLog[2, (-I)*E^(c + d*x)]
))/(-I + E^c) + 6*f*(d*((d*E^c*x*(2*e + f*x))/(I + E^c) - 2*(e + f*x)*Log[1
 - I*E^(c + d*x)]) - 2*f*PolyLog[2, I*E^(c + d*x)]) + ((-2*I)*f^2*Cosh[c] +
 2*d*f*(e + f*x)*Cosh[d*x] - (2*I)*d^2*e^2*Cosh[c + d*x] + (4*I)*f^2*Cosh[c
 + d*x] - (4*I)*d^2*e*f*x*Cosh[c + d*x] - (2*I)*d^2*f^2*x^2*Cosh[c + d*x] +
 2*d*e*f*Cosh[2*c + d*x] + 2*d*f^2*x*Cosh[2*c + d*x] + (4*I)*d^2*e^2*Cosh[c
 + 2*d*x] - (2*I)*f^2*Cosh[c + 2*d*x] + (8*I)*d^2*e*f*x*Cosh[c + 2*d*x] + (
4*I)*d^2*f^2*x^2*Cosh[c + 2*d*x] + 8*d^2*e^2*Sinh[d*x] - 2*f^2*Sinh[d*x] +
16*d^2*e*f*x*Sinh[d*x] + 8*d^2*f^2*x^2*Sinh[d*x] + d^2*e^2*Sinh[2*(c + d*x)
] - 2*f^2*Sinh[2*(c + d*x)] + 2*d^2*e*f*x*Sinh[2*(c + d*x)] + d^2*f^2*x^2*S
inh[2*(c + d*x)] + 2*f^2*Sinh[2*c + d*x])/((Cosh[c/2] - I*Sinh[c/2])*(Cosh[
c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] - I*Sinh[(c + d*x)/2])*(Cosh[(c + d*
x)/2] + I*Sinh[(c + d*x)/2])^3))/(12*a*d^3)
```

Maple [A]
time = 3.05, size = 509, normalized size = 1.57

method	result
risch	$\frac{2i\left(-2id^2f^2x^2+4d^2x^2f^2e^{dx+c}-d\,f^2x\,e^{3dx+3c}-4id^2efx+8d^2efx\,e^{dx+c}-def\,e^{3dx+3c}-2id^2e^2+if^2e^{2dx+2c}+4d^2e^2e^{dx+c}-d\,f^2x\,e^{dx}\cdots\right.}{3\left(e^{dx+c}+i\right)\left(e^{dx+c}-i\right)^3d^3a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*(-2*I*d^2*f^2*x^2+4*d^2*x^2*f^2*exp(d*x+c)-d*f^2*x*exp(3*d*x+3*c)-4*I
*d^2*e*f*x+8*d^2*e*f*x*exp(d*x+c)-d*e*f*exp(3*d*x+3*c)-2*I*d^2*e^2+I*f^2*ex
p(2*d*x+2*c)+4*d^2*e^2*exp(d*x+c)-d*f^2*x*exp(d*x+c)-f^2*exp(3*d*x+3*c)-d*e
*f*exp(d*x+c)+I*f^2-f^2*exp(d*x+c))/(exp(d*x+c)+I)/(exp(d*x+c)-I)^3/d^3/a-5
/3/d^2/a*e*f*ln(exp(d*x+c)-I)-1/a/d^2*f*e*ln(exp(d*x+c)+I)+8/3/d^2/a*e*f*ln
(exp(d*x+c))+5/3/d^3/a*f^2*c*ln(exp(d*x+c)-I)+1/a/d^3*f^2*c*ln(exp(d*x+c)+I
)-8/3/d^3/a*f^2*c*ln(exp(d*x+c))+4/3*f^2*x^2/a/d+8/3/d^2/a*f^2*c*x+4/3/a/d^
3*f^2*c^2-1/a/d^2*f^2*ln(1-I*exp(d*x+c))*x-1/a/d^3*f^2*ln(1-I*exp(d*x+c))*c
-f^2*polylog(2,I*exp(d*x+c))/a/d^3-5/3/d^2/a*f^2*ln(1+I*exp(d*x+c))*x-5/3/d
^3/a*f^2*ln(1+I*exp(d*x+c))*c-5/3*f^2*polylog(2,-I*exp(d*x+c))/a/d^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 4*f^2*((2*I*d^2*x^2 + (d*x*e^(3*c) + e^(3*c))*e^(3*d*x) - (4*d^2*x^2*e^c -
d*x*e^c - e^c)*e^(d*x) - I*e^(2*d*x + 2*c) - I)/(6*I*a*d^3*e^(4*d*x + 4*c)
+ 12*a*d^3*e^(3*d*x + 3*c) + 12*a*d^3*e^(d*x + c) - 6*I*a*d^3) + I*integrat
e(1/4*x/(a*d*e^(d*x + c) + I*a*d), x) - 5*I*integrate(1/12*x/(a*d*e^(d*x +
c) - I*a*d), x)) + 1/3*f*(24*(4*I*d*x*e^(4*d*x + 4*c) + (8*d*x*e^(3*c) + e^
(3*c))*e^(3*d*x) + e^(d*x + c))/(12*I*a*d^2*e^(4*d*x + 4*c) + 24*a*d^2*e^(3
*d*x + 3*c) + 24*a*d^2*e^(d*x + c) - 12*I*a*d^2) - 3*log((e^(d*x + c) + I)*
e^(-c))/(a*d^2) - 5*log(-I*(I*e^(d*x + c) + 1)*e^(-c))/(a*d^2))*e + 4/3*(2*
e^(-d*x - c)/((2*a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) - I*a*e^(-4*d*x - 4*
c) + I*a)*d) + I/((2*a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) - I*a*e^(-4*d*x
- 4*c) + I*a)*d))*e^2
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 724 vs. $2(289) = 578$.
time = 0.37, size = 724, normalized size = 2.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/3*(8*c*d*f*e - 2*(2*c^2 - 1)*f^2 - 4*d^2*e^2 + 2*f^2*e^(2*d*x + 2*c) + 3
*(f^2*e^(4*d*x + 4*c) - 2*I*f^2*e^(3*d*x + 3*c) - 2*I*f^2*e^(d*x + c) - f^2
)*dilog(I*e^(d*x + c)) + 5*(f^2*e^(4*d*x + 4*c) - 2*I*f^2*e^(3*d*x + 3*c) -
 2*I*f^2*e^(d*x + c) - f^2)*dilog(-I*e^(d*x + c)) - 4*(d^2*f^2*x^2 - c^2*f^
2 + 2*(d^2*f*x + c*d*f)*e)*e^(4*d*x + 4*c) + 2*(4*I*d^2*f^2*x^2 + I*d*f^2*x
 + (-4*I*c^2 + I)*f^2 + (8*I*d^2*f*x + (8*I*c + I)*d*f)*e)*e^(3*d*x + 3*c)
+ 2*(I*d*f^2*x + (8*I*c + I)*d*f*e + (-4*I*c^2 + I)*f^2 - 4*I*d^2*e^2)*e^(d
*x + c) + 3*(c*f^2 - d*f*e - (c*f^2 - d*f*e)*e^(4*d*x + 4*c) + 2*(I*c*f^2 -
 I*d*f*e)*e^(3*d*x + 3*c) + 2*(I*c*f^2 - I*d*f*e)*e^(d*x + c))*log(e^(d*x +
 c) + I) + 5*(c*f^2 - d*f*e - (c*f^2 - d*f*e)*e^(4*d*x + 4*c) + 2*(I*c*f^2
- I*d*f*e)*e^(3*d*x + 3*c) + 2*(I*c*f^2 - I*d*f*e)*e^(d*x + c))*log(e^(d*x
+ c) - I) - 5*(d*f^2*x + c*f^2 - (d*f^2*x + c*f^2)*e^(4*d*x + 4*c) - 2*(-I*
d*f^2*x - I*c*f^2)*e^(3*d*x + 3*c) - 2*(-I*d*f^2*x - I*c*f^2)*e^(d*x + c))*
log(I*e^(d*x + c) + 1) - 3*(d*f^2*x + c*f^2 - (d*f^2*x + c*f^2)*e^(4*d*x +
4*c) - 2*(-I*d*f^2*x - I*c*f^2)*e^(3*d*x + 3*c) - 2*(-I*d*f^2*x - I*c*f^2)*
e^(d*x + c))*log(-I*e^(d*x + c) + 1))/(a*d^3*e^(4*d*x + 4*c) - 2*I*a*d^3*e^
(3*d*x + 3*c) - 2*I*a*d^3*e^(d*x + c) - a*d^3)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{e^2 \operatorname{sech}^2(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{f^2x^2 \operatorname{sech}^2(c+dx)}{\sinh(c+dx)-i}\,dx + \int \frac{2efx \operatorname{sech}^2(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sech(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*(Integral(e**2*sech(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(f**2*
x**2*sech(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(2*e*f*x*sech(c + d
*x)**2/(sinh(c + d*x) - I), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sech(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^2}{\cosh(c+d\,x)^2\,\left(a+a\sinh(c+d\,x)\,1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(cosh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

```
[Out] int((e + f*x)^2/(cosh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.279 $\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=158

$$-\frac{if\text{ArcTan}(\sinh(c+dx))}{6ad^2}-\frac{2f\log(\cosh(c+dx))}{3ad^2}+\frac{f\text{sech}^2(c+dx)}{6ad^2}+\frac{i(e+fx)\text{sech}^3(c+dx)}{3ad}+\frac{2(e+fx)\tanh}{3ad}$$

```
[Out] -1/6*I*f*arctan(sinh(d*x+c))/a/d^2-2/3*f*ln(cosh(d*x+c))/a/d^2+1/6*f*sech(d
*x+c)^2/a/d^2+1/3*I*(f*x+e)*sech(d*x+c)^3/a/d+2/3*(f*x+e)*tanh(d*x+c)/a/d-1
/6*I*f*sech(d*x+c)*tanh(d*x+c)/a/d^2+1/3*(f*x+e)*sech(d*x+c)^2*tanh(d*x+c)/
a/d
```

Rubi [A]
time = 0.12, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {5690, 4270, 4269, 3556, 5559, 3853, 3855}

$$-\frac{if\text{ArcTan}(\sinh(c+dx))}{6ad^2}+\frac{f\text{sech}^2(c+dx)}{6ad^2}-\frac{2f\log(\cosh(c+dx))}{3ad^2}-\frac{if\tanh(c+dx)\text{sech}(c+dx)}{6ad^2}+\frac{2(e+fx)\tanh(c+dx)}{3ad}+\frac{i(e+fx)\text{sech}^3(c+dx)}{3ad}+\frac{(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-1/6*I)*f*ArcTan[Sinh[c + d*x]])/(a*d^2) - (2*f*Log[Cosh[c + d*x]])/(3*a*
d^2) + (f*Sech[c + d*x]^2)/(6*a*d^2) + ((I/3)*(e + f*x)*Sech[c + d*x]^3)/(a
*d) + (2*(e + f*x)*Tanh[c + d*x])/(3*a*d) - ((I/6)*f*Sech[c + d*x]*Tanh[c +
 d*x])/(a*d^2) + ((e + f*x)*Sech[c + d*x]^2*Tanh[c + d*x])/(3*a*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5690

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sech[c
 + d*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)*T
anh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && Eq
Q[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)\mathrm{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\frac{i\int (e+fx)\mathrm{sech}^3(c+dx)\tanh(c+dx)\,dx}{a}+\frac{\int (e+fx)\mathrm{sech}^4(c+dx)\,dx}{a}\\ &= \frac{f\mathrm{sech}^2(c+dx)}{6ad^2}+\frac{i(e+fx)\mathrm{sech}^3(c+dx)}{3ad}+\frac{(e+fx)\mathrm{sech}^2(c+dx)\tanh(c+\\ &= \frac{f\mathrm{sech}^2(c+dx)}{6ad^2}+\frac{i(e+fx)\mathrm{sech}^3(c+dx)}{3ad}+\frac{2(e+fx)\tanh(c+dx)}{3ad}-\frac{if\mathrm{se}\\ &= -\frac{if\tan^{-1}(\sinh(c+dx))}{6ad^2}-\frac{2f\log(\cosh(c+dx))}{3ad^2}+\frac{f\mathrm{sech}^2(c+dx)}{6ad^2}+\frac{i(e+\end{aligned}$$

Mathematica [A]

time = 0.75, size = 194, normalized size = 1.23

$$\frac{2d(e+fx)(\cosh(2(c+dx))-2i\sinh(c+dx))+\cosh(c+dx)\left(-de-if+cf-2f\mathrm{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+4if\log(\cosh(c+dx))-i\left(de-cf+2f\mathrm{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-4if\log(\cosh(c+dx))\right)\sinh(c+dx)\right)}{6ad^2\left(\cosh\left(\frac{1}{2}(c+dx)\right)-i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)(-i+\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (2*d*(e + f*x)*(Cosh[2*(c + d*x)] - (2*I)*Sinh[c + d*x]) + Cosh[c + d*x]*(-
(d*e) - I*f + c*f - 2*f*ArcTan[Tanh[(c + d*x)/2]] + (4*I)*f*Log[Cosh[c + d*
x]] - I*(d*e - c*f + 2*f*ArcTan[Tanh[(c + d*x)/2]] - (4*I)*f*Log[Cosh[c + d
*x]])*Sinh[c + d*x]))/(6*a*d^2*(Cosh[(c + d*x)/2] - I*Sinh[(c + d*x)/2])*(C
osh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(-I + Sinh[c + d*x]))
```

Maple [A]
time = 3.48, size = 143, normalized size = 0.91

method	result	size
risch	$\frac{4fx}{3ad}+\frac{4fc}{3a\,d^{2}}-\frac{i\left(-8dfx\,{\mathrm e}^{dx+c}+f\,{\mathrm e}^{3dx+3c}-8de\,{\mathrm e}^{dx+c}+f\,{\mathrm e}^{dx+c}+4idxf+4ide\right)}{3\left({\mathrm e}^{dx+c}+i\right)\left({\mathrm e}^{dx+c}-i\right)^{3}d^{2}a}-\frac{f\ln\left({\mathrm e}^{dx+c}+i\right)}{2a\,d^{2}}-\frac{5f\ln\left({\mathrm e}^{dx+c}-i\right)}{6a\,d^{2}}$	143

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*f*x/a/d+4/3*f/a/d^2*c-1/3*I*(-8*d*f*x*exp(d*x+c)+f*exp(3*d*x+3*c)-8*d*e
*exp(d*x+c)+f*exp(d*x+c)+4*I*d*x*f+4*I*d*e)/(exp(d*x+c)+I)/(exp(d*x+c)-I)^3
/d^2/a-1/2*f/a/d^2*ln(exp(d*x+c)+I)-5/6*f/a/d^2*ln(exp(d*x+c)-I)
```

Maxima [A]
time = 0.29, size = 252, normalized size = 1.59

$$\frac{1}{6}\,f\left(\frac{24\left(4i\,dxe^{(4\,dx+4\,c)}+\left(8\,dxe^{(3\,c)}+e^{(3\,c)}\right)e^{(3\,dx)}+e^{(dx+c)}\right)}{12i\,ad^2e^{(4\,dx+4\,c)}+24\,ad^2e^{(3\,dx+3\,c)}+24\,ad^2e^{(dx+c)}-12i\,ad^2}-\frac{3\,\log\left(\left(e^{(dx+c)}+i\right)e^{(-c)}\right)}{ad^2}-\frac{5\,\log\left(-i\left(i\,e^{(dx+c)}+1\right)e^{(-c)}\right)}{ad^2}\right)+\frac{4}{3}\left(\frac{2\,e^{(-dx-c)}}{(2\,ae^{(-dx-c)}+2\,ae^{(-3\,dx-3\,c)}-i\,ae^{(-4\,dx-4\,c)}+i\,a)d}+\frac{i}{(2\,ae^{(-dx-c)}+2\,ae^{(-3\,dx-3\,c)}-i\,ae^{(-4\,dx-4\,c)}+i\,a)d}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*f*(24*(4*I*d*x*e^(4*d*x + 4*c) + (8*d*x*e^(3*c) + e^(3*c))*e^(3*d*x) +
e^(d*x + c))/(12*I*a*d^2*e^(4*d*x + 4*c) + 24*a*d^2*e^(3*d*x + 3*c) + 24*a*
d^2*e^(d*x + c) - 12*I*a*d^2) - 3*log((e^(d*x + c) + I)*e^(-c))/(a*d^2) - 5
*log(-I*(I*e^(d*x + c) + 1)*e^(-c))/(a*d^2)) + 4/3*(2*e^(-d*x - c)/((2*a*e^
(-d*x - c) + 2*a*e^(-3*d*x - 3*c) - I*a*e^(-4*d*x - 4*c) + I*a)*d) + I/((2*
a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) - I*a*e^(-4*d*x - 4*c) + I*a)*d))*e
```

Fricas [A]
time = 0.37, size = 203, normalized size = 1.28

$$\frac{8\,dfxe^{(4\,dx+4\,c)}+8\,de-2\,(8i\,dfx+i\,f)e^{(3\,dx+3\,c)}-2\,(-8i\,de+i\,f)e^{(dx+c)}-3\left(fe^{(4\,dx+4\,c)}-2i\,fe^{(3\,dx+3\,c)}-2i\,fe^{(dx+c)}-f\right)\log\left(e^{(dx+c)}+i\right)-5\left(fe^{(4\,dx+4\,c)}-2i\,fe^{(3\,dx+3\,c)}-2i\,fe^{(dx+c)}-f\right)\log\left(e^{(dx+c)}-i\right)}{6\left(ad^2e^{(4\,dx+4\,c)}-2i\,ad^2e^{(3\,dx+3\,c)}-2i\,ad^2e^{(dx+c)}-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(8*d*f*x*e^(4*d*x + 4*c) + 8*d*e - 2*(8*I*d*f*x + I*f)*e^(3*d*x + 3*c)
- 2*(-8*I*d*e + I*f)*e^(d*x + c) - 3*(f*e^(4*d*x + 4*c) - 2*I*f*e^(3*d*x +
3*c) - 2*I*f*e^(d*x + c) - f)*log(e^(d*x + c) + I) - 5*(f*e^(4*d*x + 4*c) -
 2*I*f*e^(3*d*x + 3*c) - 2*I*f*e^(d*x + c) - f)*log(e^(d*x + c) - I))/(a*d^
2*e^(4*d*x + 4*c) - 2*I*a*d^2*e^(3*d*x + 3*c) - 2*I*a*d^2*e^(d*x + c) - a*d
^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{e\operatorname{sech}^{2}(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{fx\operatorname{sech}^{2}(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*(Integral(e*sech(c + d*x)**2/(sinh(c + d*x) - I), x) + Integral(f*x*sech
(c + d*x)**2/(sinh(c + d*x) - I), x))/a
```

Giac [A]
time = 0.43, size = 260, normalized size = 1.65

$$\frac{8\,dfxe^{(4\,dx+4\,c)}-16i\,dfxe^{(3\,dx+3\,c)}+16i\,dee^{(dx+c)}-3\,fe^{(4\,dx+4\,c)}\log\left(e^{(dx+c)}+i\right)+6i\,fe^{(3\,dx+3\,c)}\log\left(e^{(dx+c)}+i\right)+6i\,fe^{(dx+c)}\log\left(e^{(dx+c)}+i\right)-5\,fe^{(4\,dx+4\,c)}\log\left(e^{(dx+c)}-i\right)+10i\,fe^{(3\,dx+3\,c)}\log\left(e^{(dx+c)}-i\right)+10i\,fe^{(dx+c)}\log\left(e^{(dx+c)}-i\right)+8\,de-2i\,fe^{(3\,dx+3\,c)}-2i\,fe^{(dx+c)}+3\,f\log\left(e^{(dx+c)}+i\right)+5\,f\log\left(e^{(dx+c)}-i\right)}{6\left(ad^{2}e^{(4\,dx+4\,c)}-2i\,ad^{2}e^{(3\,dx+3\,c)}-2i\,ad^{2}e^{(dx+c)}-ad^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(8*d*f*x*e^(4*d*x + 4*c) - 16*I*d*f*x*e^(3*d*x + 3*c) + 16*I*d*e*e^(d*x
 + c) - 3*f*e^(4*d*x + 4*c)*log(e^(d*x + c) + I) + 6*I*f*e^(3*d*x + 3*c)*lo
g(e^(d*x + c) + I) + 6*I*f*e^(d*x + c)*log(e^(d*x + c) + I) - 5*f*e^(4*d*x
+ 4*c)*log(e^(d*x + c) - I) + 10*I*f*e^(3*d*x + 3*c)*log(e^(d*x + c) - I) +
 10*I*f*e^(d*x + c)*log(e^(d*x + c) - I) + 8*d*e - 2*I*f*e^(3*d*x + 3*c) -
2*I*f*e^(d*x + c) + 3*f*log(e^(d*x + c) + I) + 5*f*log(e^(d*x + c) - I))/(a
*d^2*e^(4*d*x + 4*c) - 2*I*a*d^2*e^(3*d*x + 3*c) - 2*I*a*d^2*e^(d*x + c) -
a*d^2)
```

Mupad [B]
time = 2.48, size = 205, normalized size = 1.30

$$\frac{4\,f\,x}{3\,a\,d}-\frac{f+3\,d\,e+3\,d\,f\,x}{3\,a\,d^2\,\left(1-{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{c+d\,x}\,2{}\mathrm{i}\right)}-\frac{5\,f\,\ln\left(f+f\,{\mathrm{e}}^{c+d\,x}\,1{}\mathrm{i}\right)}{6\,a\,d^2}-\frac{\left(e+f\,x\right)\,2{}\mathrm{i}}{3\,a\,d\,\left(3\,{\mathrm{e}}^{c+d\,x}+{\mathrm{e}}^{2\,c+2\,d\,x}\,3{}\mathrm{i}-{\mathrm{e}}^{3\,c+3\,d\,x}-\mathrm{i}\right)}-\frac{\left(e+f\,x\right)\,1{}\mathrm{i}}{2\,a\,d\,\left({\mathrm{e}}^{c+d\,x}+1{}\mathrm{i}\right)}-\frac{f\,\ln\left(-1+{\mathrm{e}}^{c+d\,x}\,1{}\mathrm{i}\right)}{2\,a\,d^2}+\frac{\left(3\,d\,e-2\,f+3\,d\,f\,x\right)\,1{}\mathrm{i}}{6\,a\,d^2\,\left({\mathrm{e}}^{c+d\,x}-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(cosh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

```
[Out] (4*f*x)/(3*a*d) - (f + 3*d*e + 3*d*f*x)/(3*a*d^2*(exp(c + d*x)*2i - exp(2*c
 + 2*d*x) + 1)) - (5*f*log(f + f*exp(c + d*x)*1i))/(6*a*d^2) - ((e + f*x)*2
i)/(3*a*d*(3*exp(c + d*x) + exp(2*c + 2*d*x)*3i - exp(3*c + 3*d*x) - 1i)) -
 ((e + f*x)*1i)/(2*a*d*(exp(c + d*x) + 1i)) - (f*log(exp(c + d*x)*1i - 1))/
(2*a*d^2) + ((3*d*e - 2*f + 3*d*f*x)*1i)/(6*a*d^2*(exp(c + d*x) - 1i))
```

3.280 $\int \frac{\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=47

$$\frac{i\operatorname{sech}(c+dx)}{3d(a+ia\sinh(c+dx))}+\frac{2\tanh(c+dx)}{3ad}$$

```
[Out] 1/3*I*sech(d*x+c)/d/(a+I*a*sinh(d*x+c))+2/3*tanh(d*x+c)/a/d
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2751, 3852, 8}

$$\frac{2\tanh(c+dx)}{3ad}+\frac{i\operatorname{sech}(c+dx)}{3d(a+ia\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((I/3)*Sech[c + d*x])/(d*(a + I*a*Sinh[c + d*x])) + (2*Tanh[c + d*x])/(3*a*
d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i\operatorname{sech}(c+dx)}{3d(a+ia\sinh(c+dx))}+\frac{2\int \operatorname{sech}^2(c+dx)\,dx}{3a}$$
$$= \frac{i\operatorname{sech}(c+dx)}{3d(a+ia\sinh(c+dx))}+\frac{(2i)\text{Subst}(\int 1\,dx,x,-i\tanh(c+dx))}{3ad}$$
$$= \frac{i\operatorname{sech}(c+dx)}{3d(a+ia\sinh(c+dx))}+\frac{2\tanh(c+dx)}{3ad}$$

Mathematica [A]
time = 0.04, size = 47, normalized size = 1.00

$$\frac{\operatorname{sech}(c+dx)(\cosh(2(c+dx))-2i\sinh(c+dx))}{3ad(-i+\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (Sech[c + d*x]*(Cosh[2*(c + d*x)] - (2*I)*Sinh[c + d*x]))/(3*a*d*(-I + Sinh
[c + d*x]))
```

Maple [A]
time = 1.34, size = 75, normalized size = 1.60

method	result	size
risch	$\frac{4i\left(2\,e^{dx+c}-i\right)}{3\left(e^{dx+c}-i\right)^{3}\left(e^{dx+c}+i\right)ad}$	43
derivativedivides	$\frac{\frac{2}{4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4i}-\frac{2}{3\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{3}{2\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{ad}$	75
default	$\frac{\frac{2}{4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4i}-\frac{2}{3\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{3}{2\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{ad}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a*(1/4/(tanh(1/2*d*x+1/2*c)+I)-1/3/(-I+tanh(1/2*d*x+1/2*c))^3+1/2*I/(-I
+tanh(1/2*d*x+1/2*c))^2+3/4/(-I+tanh(1/2*d*x+1/2*c)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(39) = 78.
time = 0.27, size = 104, normalized size = 2.21

$$\frac{8\,e^{(-dx-c)}}{3\,(2\,ae^{(-dx-c)}+2\,ae^{(-3\,dx-3\,c)}-i\,ae^{(-4\,dx-4\,c)}+i\,a)d}+\frac{4i}{3\,(2\,ae^{(-dx-c)}+2\,ae^{(-3\,dx-3\,c)}-i\,ae^{(-4\,dx-4\,c)}+i\,a)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 8/3*e^(-d*x - c)/((2*a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) - I*a*e^(-4*d*x
- 4*c) + I*a)*d) + 4/3*I/((2*a*e^(-d*x - c) + 2*a*e^(-3*d*x - 3*c) - I*a*e^
(-4*d*x - 4*c) + I*a)*d)
```

Fricas [A]
time = 0.33, size = 54, normalized size = 1.15

$$-\frac{4\left(-2i\,e^{(dx+c)}-1\right)}{3\left(ade^{(4\,dx+4\,c)}-2i\,ade^{(3\,dx+3\,c)}-2i\,ade^{(dx+c)}-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -4/3*(-2*I*e^(d*x + c) - 1)/(a*d*e^(4*d*x + 4*c) - 2*I*a*d*e^(3*d*x + 3*c)
- 2*I*a*d*e^(d*x + c) - a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\operatorname{sech}^{2}(c+dx)}{\sinh(c+dx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*Integral(sech(c + d*x)**2/(sinh(c + d*x) - I), x)/a
```

Giac [A]
time = 0.43, size = 59, normalized size = 1.26

$$\frac{\frac{3}{a\left(i\,e^{(dx+c)}-1\right)}-\frac{-3i\,e^{(2\,dx+2\,c)}-12\,e^{(dx+c)}+5i}{a\left(e^{(dx+c)}-i\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3/(a*(I*e^(d*x + c) - 1)) - (-3*I*e^(2*d*x + 2*c) - 12*e^(d*x + c) + 5
*I)/(a*(e^(d*x + c) - I)^3))/d
```

Mupad [B]
time = 0.48, size = 43, normalized size = 0.91

$$\frac{4\left(1+\mathrm{e}^{c+d\,x}\,2\mathrm{i}\right)\,\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)^{2}}{3\,a\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*(a + a*sinh(c + d*x)*1i)),x)
```

```
[Out] (4*(exp(c + d*x)*2i + 1)*(exp(c + d*x) + 1i)^2)/(3*a*d*(exp(2*c + 2*d*x) +
1)^3)
```

3.281 $\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=34

$$\operatorname{Int}\left(\frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

[Out] Unintegrable(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

[In] Int[Sech[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]

[Out] Defer[Int][Sech[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 102.11, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

[In] Integrate[Sech[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]

[Out] Integrate[Sech[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(dx+c)^2}{(fx+e)\,(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -4*I*f*integrate(1/(8*I*a*d*f^2*x^2 + 16*I*a*d*f*x*e + 8*I*a*d*e^2 + 8*(a*d
*f^2*x^2*e^c + 2*a*d*f*x*e^(c + 1) + a*d*e^(c + 2))*e^(d*x)), x) - 1/3*(4*d
^2*f^2*x^2 + 8*d^2*f*x*e + 4*d^2*e^2 - 2*f^2*e^(2*d*x + 2*c) - 2*f^2 + (I*d
*f^2*x*e^(3*c) - 2*I*f^2*e^(3*c) + I*d*f*e^(3*c + 1))*e^(3*d*x) + (8*I*d^2*
f^2*x^2*e^c + 8*I*d^2*e^(c + 2) + I*d*f*e^(c + 1) - 2*I*f^2*e^c + (16*I*d^2
*f*e^(c + 1) + I*d*f^2*e^c)*x)*e^(d*x))/(a*d^3*f^3*x^3 + 3*a*d^3*f^2*x^2*e
+ 3*a*d^3*f*x*e^2 + a*d^3*e^3 - (a*d^3*f^3*x^3*e^(4*c) + 3*a*d^3*f^2*x^2*e^
(4*c + 1) + 3*a*d^3*f*x*e^(4*c + 2) + a*d^3*e^(4*c + 3))*e^(4*d*x) + 2*(I*a
*d^3*f^3*x^3*e^(3*c) + 3*I*a*d^3*f^2*x^2*e^(3*c + 1) + 3*I*a*d^3*f*x*e^(3*c
 + 2) + I*a*d^3*e^(3*c + 3))*e^(3*d*x) + 2*(I*a*d^3*f^3*x^3*e^c + 3*I*a*d^3
*f^2*x^2*e^(c + 1) + 3*I*a*d^3*f*x*e^(c + 2) + I*a*d^3*e^(c + 3))*e^(d*x))
- 4*integrate(1/24*(5*d^2*f^3*x^2 + 10*d^2*f^2*x*e + 5*d^2*f*e^2 - 12*f^3)/
(a*d^3*f^4*x^4 + 4*a*d^3*f^3*x^3*e + 6*a*d^3*f^2*x^2*e^2 + 4*a*d^3*f*x*e^3
+ a*d^3*e^4 - (-I*a*d^3*f^4*x^4*e^c - 4*I*a*d^3*f^3*x^3*e^(c + 1) - 6*I*a*d
^3*f^2*x^2*e^(c + 2) - 4*I*a*d^3*f*x*e^(c + 3) - I*a*d^3*e^(c + 4))*e^(d*x)
), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(4*d^2*f^2*x^2 + 8*d^2*f*x*e + 4*d^2*e^2 - 2*f^2*e^(2*d*x + 2*c) - 2*f
^2 + (I*d*f^2*x + I*d*f*e - 2*I*f^2)*e^(3*d*x + 3*c) + (8*I*d^2*f^2*x^2 + I
*d*f^2*x + 8*I*d^2*e^2 - 2*I*f^2 + (16*I*d^2*f*x + I*d*f)*e)*e^(d*x + c) -
3*(a*d^3*f^3*x^3 + 3*a*d^3*f^2*x^2*e + 3*a*d^3*f*x*e^2 + a*d^3*e^3 - (a*d^3
*f^3*x^3 + 3*a*d^3*f^2*x^2*e + 3*a*d^3*f*x*e^2 + a*d^3*e^3)*e^(4*d*x + 4*c)
 + 2*(I*a*d^3*f^3*x^3 + 3*I*a*d^3*f^2*x^2*e + 3*I*a*d^3*f*x*e^2 + I*a*d^3*e
^3)*e^(3*d*x + 3*c) + 2*(I*a*d^3*f^3*x^3 + 3*I*a*d^3*f^2*x^2*e + 3*I*a*d^3*
```

```
f*x*e^2 + I*a*d^3*e^3)*e^(d*x + c))*integral(-1/3*(4*d^2*f^3*x^2 + 8*d^2*f^
2*x*e + 4*d^2*f*e^2 - 6*f^3 - (I*d^2*f^3*x^2 + 2*I*d^2*f^2*x*e + I*d^2*f*e^
2 - 6*I*f^3)*e^(d*x + c))/(a*d^3*f^4*x^4 + 4*a*d^3*f^3*x^3*e + 6*a*d^3*f^2*
x^2*e^2 + 4*a*d^3*f*x*e^3 + a*d^3*e^4 + (a*d^3*f^4*x^4 + 4*a*d^3*f^3*x^3*e
+ 6*a*d^3*f^2*x^2*e^2 + 4*a*d^3*f*x*e^3 + a*d^3*e^4)*e^(2*d*x + 2*c)), x))/
(a*d^3*f^3*x^3 + 3*a*d^3*f^2*x^2*e + 3*a*d^3*f*x*e^2 + a*d^3*e^3 - (a*d^3*f
^3*x^3 + 3*a*d^3*f^2*x^2*e + 3*a*d^3*f*x*e^2 + a*d^3*e^3)*e^(4*d*x + 4*c) +
 2*(I*a*d^3*f^3*x^3 + 3*I*a*d^3*f^2*x^2*e + 3*I*a*d^3*f*x*e^2 + I*a*d^3*e^3
)*e^(3*d*x + 3*c) + 2*(I*a*d^3*f^3*x^3 + 3*I*a*d^3*f^2*x^2*e + 3*I*a*d^3*f*
x*e^2 + I*a*d^3*e^3)*e^(d*x + c))
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\operatorname{sech}^2(c+dx)}{e\sinh(c+dx)-ie+fx\sinh(c+dx)-ifx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
[Out] -I*Integral(sech(c + d*x)**2/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) - I
*f*x), x)/a
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate(sech(d*x + c)^2/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{1}{{\cosh\left(c+d\,x\right)}^2\,\left(e+f\,x\right)\,\left(a+a\,\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*(e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
[Out] int(1/(cosh(c + d*x)^2*(e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.282 $\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=34

$$\operatorname{Int}\left(\frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sech[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
[Out] Defer[Int][Sech[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Sech[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(dx+c)^2}{(fx+e)^2\,(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -4*I*f*integrate(1/(4*I*a*d*f^3*x^3 + 12*I*a*d*f^2*x^2*e + 12*I*a*d*f*x*e^2
 + 4*I*a*d*e^3 + 4*(a*d*f^3*x^3*e^c + 3*a*d*f^2*x^2*e^(c + 1) + 3*a*d*f*x*e
^(c + 2) + a*d*e^(c + 3))*e^(d*x)), x) - 2/3*(2*d^2*f^2*x^2 + 4*d^2*f*x*e +
 2*d^2*e^2 - 3*f^2*e^(2*d*x + 2*c) - 3*f^2 + (I*d*f^2*x*e^(3*c) - 3*I*f^2*e
^(3*c) + I*d*f*e^(3*c + 1))*e^(3*d*x) + (4*I*d^2*f^2*x^2*e^c + 4*I*d^2*e^(c
 + 2) + I*d*f*e^(c + 1) - 3*I*f^2*e^c + (8*I*d^2*f*e^(c + 1) + I*d*f^2*e^c)
*x)*e^(d*x))/(a*d^3*f^4*x^4 + 4*a*d^3*f^3*x^3*e + 6*a*d^3*f^2*x^2*e^2 + 4*a
*d^3*f*x*e^3 + a*d^3*e^4 - (a*d^3*f^4*x^4*e^(4*c) + 4*a*d^3*f^3*x^3*e^(4*c
+ 1) + 6*a*d^3*f^2*x^2*e^(4*c + 2) + 4*a*d^3*f*x*e^(4*c + 3) + a*d^3*e^(4*c
 + 4))*e^(4*d*x) + 2*(I*a*d^3*f^4*x^4*e^(3*c) + 4*I*a*d^3*f^3*x^3*e^(3*c +
1) + 6*I*a*d^3*f^2*x^2*e^(3*c + 2) + 4*I*a*d^3*f*x*e^(3*c + 3) + I*a*d^3*e^
(3*c + 4))*e^(3*d*x) + 2*(I*a*d^3*f^4*x^4*e^c + 4*I*a*d^3*f^3*x^3*e^(c + 1)
 + 6*I*a*d^3*f^2*x^2*e^(c + 2) + 4*I*a*d^3*f*x*e^(c + 3) + I*a*d^3*e^(c + 4
))*e^(d*x)) - 4*integrate(1/12*(5*d^2*f^3*x^2 + 10*d^2*f^2*x*e + 5*d^2*f*e^
2 - 24*f^3)/(a*d^3*f^5*x^5 + 5*a*d^3*f^4*x^4*e + 10*a*d^3*f^3*x^3*e^2 + 10*
a*d^3*f^2*x^2*e^3 + 5*a*d^3*f*x*e^4 + a*d^3*e^5 - (-I*a*d^3*f^5*x^5*e^c - 5
*I*a*d^3*f^4*x^4*e^(c + 1) - 10*I*a*d^3*f^3*x^3*e^(c + 2) - 10*I*a*d^3*f^2*
x^2*e^(c + 3) - 5*I*a*d^3*f*x*e^(c + 4) - I*a*d^3*e^(c + 5))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/3*(4*d^2*f^2*x^2 + 8*d^2*f*x*e + 4*d^2*e^2 - 6*f^2*e^(2*d*x + 2*c) - 6*f
^2 - 2*(-I*d*f^2*x - I*d*f*e + 3*I*f^2)*e^(3*d*x + 3*c) - 2*(-4*I*d^2*f^2*x
^2 - I*d*f^2*x - 4*I*d^2*e^2 + 3*I*f^2 + (-8*I*d^2*f*x - I*d*f)*e)*e^(d*x +
```

```
 c) - 3*(a*d^3*f^4*x^4 + 4*a*d^3*f^3*x^3*e + 6*a*d^3*f^2*x^2*e^2 + 4*a*d^3*
f*x*e^3 + a*d^3*e^4 - (a*d^3*f^4*x^4 + 4*a*d^3*f^3*x^3*e + 6*a*d^3*f^2*x^2*
e^2 + 4*a*d^3*f*x*e^3 + a*d^3*e^4)*e^(4*d*x + 4*c) + 2*(I*a*d^3*f^4*x^4 + 4
*I*a*d^3*f^3*x^3*e + 6*I*a*d^3*f^2*x^2*e^2 + 4*I*a*d^3*f*x*e^3 + I*a*d^3*e^
4)*e^(3*d*x + 3*c) + 2*(I*a*d^3*f^4*x^4 + 4*I*a*d^3*f^3*x^3*e + 6*I*a*d^3*f
^2*x^2*e^2 + 4*I*a*d^3*f*x*e^3 + I*a*d^3*e^4)*e^(d*x + c))*integral(-2/3*(4
*d^2*f^3*x^2 + 8*d^2*f^2*x*e + 4*d^2*f*e^2 - 12*f^3 + (-I*d^2*f^3*x^2 - 2*I
*d^2*f^2*x*e - I*d^2*f*e^2 + 12*I*f^3)*e^(d*x + c))/(a*d^3*f^5*x^5 + 5*a*d^
3*f^4*x^4*e + 10*a*d^3*f^3*x^3*e^2 + 10*a*d^3*f^2*x^2*e^3 + 5*a*d^3*f*x*e^4
 + a*d^3*e^5 + (a*d^3*f^5*x^5 + 5*a*d^3*f^4*x^4*e + 10*a*d^3*f^3*x^3*e^2 +
10*a*d^3*f^2*x^2*e^3 + 5*a*d^3*f*x*e^4 + a*d^3*e^5)*e^(2*d*x + 2*c)), x))/(
a*d^3*f^4*x^4 + 4*a*d^3*f^3*x^3*e + 6*a*d^3*f^2*x^2*e^2 + 4*a*d^3*f*x*e^3 +
 a*d^3*e^4 - (a*d^3*f^4*x^4 + 4*a*d^3*f^3*x^3*e + 6*a*d^3*f^2*x^2*e^2 + 4*a
*d^3*f*x*e^3 + a*d^3*e^4)*e^(4*d*x + 4*c) + 2*(I*a*d^3*f^4*x^4 + 4*I*a*d^3*
f^3*x^3*e + 6*I*a*d^3*f^2*x^2*e^2 + 4*I*a*d^3*f*x*e^3 + I*a*d^3*e^4)*e^(3*d
*x + 3*c) + 2*(I*a*d^3*f^4*x^4 + 4*I*a*d^3*f^3*x^3*e + 6*I*a*d^3*f^2*x^2*e^
2 + 4*I*a*d^3*f*x*e^3 + I*a*d^3*e^4)*e^(d*x + c))
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\operatorname{sech}^2{(c+dx)}}{e^2\sinh{(c+dx)}-ie^2+2efx\sinh{(c+dx)}-2iefx+f^2x^2\sinh{(c+dx)}-if^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
[Out] -I*Integral(sech(c + d*x)**2/(e**2*sinh(c + d*x) - I*e**2 + 2*e*f*x*sinh(c
+ d*x) - 2*I*e*f*x + f**2*x**2*sinh(c + d*x) - I*f**2*x**2), x)/a
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\mathrm{cosh}\left(c+d\,x\right)}^2\,{\left(e+f\,x\right)}^2\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
[Out] int(1/(cosh(c + d*x)^2*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.283 $\int \frac{(e+fx)^3\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=667

$$-\frac{if(e+fx)^2}{2ad^2}-\frac{5f^2(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{ad^3}+\frac{3(e+fx)^3\text{ArcTan}\left(e^{c+dx}\right)}{4ad}+\frac{if^2(e+fx)\log\left(1+e^{2(c+dx)}\right)}{ad^3}+5if$$

```
[Out] 9/4*I*f^3*polylog(4,I*exp(d*x+c))/a/d^4-5*f^2*(f*x+e)*arctan(exp(d*x+c))/a/
d^3+3/4*(f*x+e)^3*arctan(exp(d*x+c))/a/d+1/4*I*(f*x+e)^3*sech(d*x+c)^4/a/d+
I*f^2*(f*x+e)*ln(1+exp(2*d*x+2*c))/a/d^3+5/2*I*f^3*polylog(2,-I*exp(d*x+c))
/a/d^4-5/2*I*f^3*polylog(2,I*exp(d*x+c))/a/d^4+1/2*I*f^3*polylog(2,-exp(2*d
*x+2*c))/a/d^4-1/2*I*f*(f*x+e)^2/a/d^2-9/4*I*f^2*(f*x+e)*polylog(3,I*exp(d*
x+c))/a/d^3-9/4*I*f^3*polylog(4,-I*exp(d*x+c))/a/d^4+9/8*I*f*(f*x+e)^2*poly
log(2,I*exp(d*x+c))/a/d^2-1/4*I*f*(f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/a/d^2
-1/4*f^3*sech(d*x+c)/a/d^4+9/8*f*(f*x+e)^2*sech(d*x+c)/a/d^2+1/4*I*f^3*tanh
(d*x+c)/a/d^4+1/4*f*(f*x+e)^2*sech(d*x+c)^3/a/d^2+9/4*I*f^2*(f*x+e)*polylog
(3,-I*exp(d*x+c))/a/d^3-9/8*I*f*(f*x+e)^2*polylog(2,-I*exp(d*x+c))/a/d^2-1/
4*I*f^2*(f*x+e)*sech(d*x+c)^2/a/d^3-1/4*f^2*(f*x+e)*sech(d*x+c)*tanh(d*x+c)
/a/d^3+3/8*(f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/a/d-1/2*I*f*(f*x+e)^2*tanh(d*x
+c)/a/d^2+1/4*(f*x+e)^3*sech(d*x+c)^3*tanh(d*x+c)/a/d
```

Rubi [A]
time = 0.52, antiderivative size = 667, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 16, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.516$, Rules used = {5690, 4271, 4270, 4265, 2317, 2438, 2611, 6744, 2320, 6724, 5559, 3852, 8, 4269, 3799, 2221}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-1/2*I)*f*(e + f*x)^2)/(a*d^2) - (5*f^2*(e + f*x)*ArcTan[E^(c + d*x)])/(a
*d^3) + (3*(e + f*x)^3*ArcTan[E^(c + d*x)])/(4*a*d) + (I*f^2*(e + f*x)*Log[
1 + E^(2*(c + d*x))])/(a*d^3) + (((5*I)/2)*f^3*PolyLog[2, (-I)*E^(c + d*x)]
)/(a*d^4) - (((9*I)/8)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2)
- (((5*I)/2)*f^3*PolyLog[2, I*E^(c + d*x)])/(a*d^4) + (((9*I)/8)*f*(e + f*x
)^2*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + ((I/2)*f^3*PolyLog[2, -E^(2*(c + d
*x))])/(a*d^4) + (((9*I)/4)*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(a*
d^3) - (((9*I)/4)*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(a*d^3) - (((9*I
)/4)*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(a*d^4) + (((9*I)/4)*f^3*PolyLog[4,
I*E^(c + d*x)])/(a*d^4) - (f^3*Sech[c + d*x])/(4*a*d^4) + (9*f*(e + f*x)^2*
Sech[c + d*x])/(8*a*d^2) - ((I/4)*f^2*(e + f*x)*Sech[c + d*x]^2)/(a*d^3) +
(f*(e + f*x)^2*Sech[c + d*x]^3)/(4*a*d^2) + ((I/4)*(e + f*x)^3*Sech[c + d*x
```

```
]^4)/(a*d) + ((I/4)*f^3*Tanh[c + d*x])/(a*d^4) - ((I/2)*f*(e + f*x)^2*Tanh[
c + d*x])/(a*d^2) - (f^2*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(4*a*d^3) +
 (3*(e + f*x)^3*Sech[c + d*x]*Tanh[c + d*x])/(8*a*d) - ((I/4)*f*(e + f*x)^2
*Sech[c + d*x]^2*Tanh[c + d*x])/(a*d^2) + ((e + f*x)^3*Sech[c + d*x]^3*Tanh
[c + d*x])/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5690

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sech[c
 + d*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)*T
anh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && Eq
Q[a^2 + b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int (e+fx)^3\text{sech}^4(c+dx)\tanh(c+dx)\,dx}{a} + \frac{\int (e+fx)^3\text{sech}^5(c+dx)\,dx}{a}$$

$$= \frac{f(e+fx)^2\text{sech}^3(c+dx)}{4ad^2} + \frac{i(e+fx)^3\text{sech}^4(c+dx)}{4ad} + \frac{(e+fx)^3\text{sech}^3(c+d}{4ad}$$

$$= -\frac{f^3\text{sech}(c+dx)}{4ad^4} + \frac{9f(e+fx)^2\text{sech}(c+dx)}{8ad^2} - \frac{if^2(e+fx)\text{sech}^2(c+dx)}{4ad^3} +$$

$$= -\frac{5f^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{3(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{4ad} - \frac{f^3\text{sech}(c+dx)}{4ad^4} +$$

$$= -\frac{if(e+fx)^2}{2ad^2} - \frac{5f^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{3(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{4ad} - 9i$$

$$= -\frac{if(e+fx)^2}{2ad^2} - \frac{5f^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{3(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{4ad} + if$$

$$= -\frac{if(e+fx)^2}{2ad^2} - \frac{5f^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{3(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{4ad} + if$$

$$= -\frac{if(e+fx)^2}{2ad^2} - \frac{5f^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{3(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{4ad} + if$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2072 vs. 2(667) = 1334.
time = 11.34, size = 2072, normalized size = 3.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] -1/32*((-12*I)*d^4*e^3*E^c*x + (112*I)*d^2*e*E^c*f^2*x - (18*I)*d^4*e^2*E^c
*f*x^2 + (56*I)*d^2*E^c*f^3*x^2 - (12*I)*d^4*e*E^c*f^2*x^3 - (3*I)*d^4*E^c*
f^3*x^4 + 12*d^3*e^3*Log[I - E^(c + d*x)] + (12*I)*d^3*e^3*E^c*Log[I - E^(c
 + d*x)] - 112*d*e*f^2*Log[I - E^(c + d*x)] - (112*I)*d*e*E^c*f^2*Log[I - E
^(c + d*x)] + 36*d^3*e^2*f*x*Log[1 + I*E^(c + d*x)] + (36*I)*d^3*e^2*E^c*f*
x*Log[1 + I*E^(c + d*x)] - 112*d*f^3*x*Log[1 + I*E^(c + d*x)] - (112*I)*d*E
^c*f^3*x*Log[1 + I*E^(c + d*x)] + 36*d^3*e*f^2*x^2*Log[1 + I*E^(c + d*x)] +
 (36*I)*d^3*e*E^c*f^2*x^2*Log[1 + I*E^(c + d*x)] + 12*d^3*f^3*x^3*Log[1 + I
*E^(c + d*x)] + (12*I)*d^3*E^c*f^3*x^3*Log[1 + I*E^(c + d*x)] + 4*(1 + I*E^
c)*f*(-28*f^2 + 9*d^2*(e + f*x)^2)*PolyLog[2, (-I)*E^(c + d*x)] - (72*I)*d*
(-I + E^c)*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)] + 72*f^3*PolyLog[4, (
-I)*E^(c + d*x)] + (72*I)*E^c*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(a*d^4*(-I
+ E^c)) + (3*((-4*I)*d^4*e^3*E^c*x + (16*I)*d^2*e*E^c*f^2*x - (6*I)*d^4*e^2
*E^c*f*x^2 + (8*I)*d^2*E^c*f^3*x^2 - (4*I)*d^4*e*E^c*f^2*x^3 - I*d^4*E^c*f^
```

```
3*x^4 - 12*d^3*e^2*f*x*Log[1 - I*E^(c + d*x)] + (12*I)*d^3*e^2*E^c*f*x*Log[
1 - I*E^(c + d*x)] + 16*d*f^3*x*Log[1 - I*E^(c + d*x)] - (16*I)*d*E^c*f^3*x
*Log[1 - I*E^(c + d*x)] - 12*d^3*e*f^2*x^2*Log[1 - I*E^(c + d*x)] + (12*I)*
d^3*e*E^c*f^2*x^2*Log[1 - I*E^(c + d*x)] - 4*d^3*f^3*x^3*Log[1 - I*E^(c + d
*x)] + (4*I)*d^3*E^c*f^3*x^3*Log[1 - I*E^(c + d*x)] - 4*d^3*e^3*Log[I + E^(
c + d*x)] + (4*I)*d^3*e^3*E^c*Log[I + E^(c + d*x)] + 16*d*e*f^2*Log[I + E^(
c + d*x)] - (16*I)*d*e*E^c*f^2*Log[I + E^(c + d*x)] + (4*I)*(I + E^c)*f*(-4
*f^2 + 3*d^2*(e + f*x)^2)*PolyLog[2, I*E^(c + d*x)] + 24*d*(1 - I*E^c)*f^2*
(e + f*x)*PolyLog[3, I*E^(c + d*x)] - 24*f^3*PolyLog[4, I*E^(c + d*x)] + (2
4*I)*E^c*f^3*PolyLog[4, I*E^(c + d*x)]))/(32*a*d^4*(I + E^c)) + ((3*e^3*x*C
osh[c])/(4*a) + (3*e^3*x*Sinh[c])/(4*a))/(1 + Cosh[2*c] + Sinh[2*c]) + ((9*
e^2*f*x^2*Cosh[c])/(8*a) + (9*e^2*f*x^2*Sinh[c])/(8*a))/(1 + Cosh[2*c] + Si
nh[2*c]) + ((3*e*f^2*x^3*Cosh[c])/(4*a) + (3*e*f^2*x^3*Sinh[c])/(4*a))/(1 +
 Cosh[2*c] + Sinh[2*c]) + ((3*f^3*x^4*Cosh[c])/(16*a) + (3*f^3*x^4*Sinh[c])
/(16*a))/(1 + Cosh[2*c] + Sinh[2*c]) - ((I/8)*(e^3 + 3*e^2*f*x + 3*e*f^2*x^
2 + f^3*x^3))/(a*d*(Cosh[c/2 + (d*x)/2] - I*Sinh[c/2 + (d*x)/2])^2) + (((3*
I)/4)*(e^2*f*Sinh[(d*x)/2] + 2*e*f^2*x*Sinh[(d*x)/2] + f^3*x^2*Sinh[(d*x)/2
]))/(a*d^2*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] - I*Sinh[c/2 + (d
*x)/2])) + ((I/8)*(e^3 + 3*e^2*f*x + 3*e*f^2*x^2 + f^3*x^3))/(a*d*(Cosh[c/2
 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2])^4) - ((I/4)*(e^2*f*Sinh[(d*x)/2] + 2*e
*f^2*x*Sinh[(d*x)/2] + f^3*x^2*Sinh[(d*x)/2]))/(a*d^2*(Cosh[c/2] + I*Sinh[c
/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2])^3) + ((2*I)*d^2*e^3*Cosh
[c/2] + d*e^2*f*Cosh[c/2] - (2*I)*e*f^2*Cosh[c/2] + (6*I)*d^2*e^2*f*x*Cosh[
c/2] + 2*d*e*f^2*x*Cosh[c/2] - (2*I)*f^3*x*Cosh[c/2] + (6*I)*d^2*e*f^2*x^2*
Cosh[c/2] + d*f^3*x^2*Cosh[c/2] + (2*I)*d^2*f^3*x^3*Cosh[c/2] - 2*d^2*e^3*S
inh[c/2] - I*d*e^2*f*Sinh[c/2] + 2*e*f^2*Sinh[c/2] - 6*d^2*e^2*f*x*Sinh[c/2
] - (2*I)*d*e*f^2*x*Sinh[c/2] + 2*f^3*x*Sinh[c/2] - 6*d^2*e*f^2*x^2*Sinh[c/
2] - I*d*f^3*x^2*Sinh[c/2] - 2*d^2*f^3*x^3*Sinh[c/2])/(8*a*d^3*(Cosh[c/2] +
 I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2])^2) - ((I/4)*(7*
d^2*e^2*f*Sinh[(d*x)/2] - 2*f^3*Sinh[(d*x)/2] + 14*d^2*e*f^2*x*Sinh[(d*x)/2
] + 7*d^2*f^3*x^2*Sinh[(d*x)/2]))/(a*d^4*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c/
2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2025 vs. $2(589) = 1178$.
time = 4.16, size = 2026, normalized size = 3.04

method	result	size
risch	Expression too large to display	2026

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 9/8*I/a/d*ln(1-I*exp(d*x+c))*e*f^2*x^2+9/4*I/a/d^2*polylog(2,I*exp(d*x+c))*
e*f^2*x+9/8*I/a/d^3*ln(1+I*exp(d*x+c))*c^2*e*f^2-9/8*I/a/d^3*ln(1-I*exp(d*x
```

```
+c))*c^2*e*f^2+9/4*I*f^3*polylog(4,I*exp(d*x+c))/a/d^4-9/8*I/a/d^2*e^2*f*po
lylog(2,-I*exp(d*x+c))+9/8*I/a/d^2*e^2*f*polylog(2,I*exp(d*x+c))+2*I/a/d^4*
f^3*c*ln(exp(d*x+c))-7/2*I/a/d^4*f^3*c*ln(exp(d*x+c)-I)+3/2*I/a/d^4*f^3*c*l
n(exp(d*x+c)+I)+3/8*I/a/d^4*f^3*c^3*ln(exp(d*x+c)-I)-3/2*I/a/d^4*f^3*c*ln(1
-I*exp(d*x+c))+7/2*I/a/d^4*f^3*c*ln(1+I*exp(d*x+c))-2*I/a/d^3*f^3*c*x-3/2*I
/a/d^3*f^3*ln(1-I*exp(d*x+c))*x-3/8*I/a/d^4*f^3*c^3*ln(exp(d*x+c)+I)-9/4*I*
f^3*polylog(4,-I*exp(d*x+c))/a/d^4-9/8*I/a/d^3*f^2*c^2*e*ln(exp(d*x+c)-I)-9
/4*I/a/d^2*polylog(2,-I*exp(d*x+c))*e*f^2*x+9/8*I/a/d^2*e^2*f*c*ln(exp(d*x+
c)-I)-9/8*I/a/d^2*e^2*f*c*ln(exp(d*x+c)+I)+3/8*I/a/d^4*f^3*c^3*ln(1-I*exp(d
*x+c))-3/8*I/a/d^4*f^3*c^3*ln(1+I*exp(d*x+c))-9/4*I/a/d^3*f^3*polylog(3,I*e
xp(d*x+c))*x+3/8*I/a/d*f^3*ln(1-I*exp(d*x+c))*x^3+9/8*I/a/d^2*f^3*polylog(2
,I*exp(d*x+c))*x^2-3/8*I/a/d*f^3*ln(1+I*exp(d*x+c))*x^3-9/8*I/a/d^2*f^3*pol
ylog(2,-I*exp(d*x+c))*x^2+9/4*I/a/d^3*f^3*polylog(3,-I*exp(d*x+c))*x+7/2*I/
a/d^3*f^3*ln(1+I*exp(d*x+c))*x+9/4*I/a/d^3*e*f^2*polylog(3,-I*exp(d*x+c))-9
/4*I/a/d^3*e*f^2*polylog(3,I*exp(d*x+c))-2*I/a/d^3*e*f^2*ln(exp(d*x+c))-3/2
*I/a/d^3*e*f^2*ln(exp(d*x+c)+I)+7/2*I/a/d^3*e*f^2*ln(exp(d*x+c)-I)+9/8*I/a/
d^3*f^2*c^2*e*ln(exp(d*x+c)+I)-9/8*I/a/d*ln(1+I*exp(d*x+c))*e^2*f*x-9/8*I/a
/d^2*ln(1+I*exp(d*x+c))*c*e^2*f+9/8*I/a/d*ln(1-I*exp(d*x+c))*e^2*f*x+9/8*I/
a/d^2*ln(1-I*exp(d*x+c))*c*e^2*f-9/8*I/a/d*ln(1+I*exp(d*x+c))*e*f^2*x^2+1/4
*(-2*f^3*exp(d*x+c)-36*I*d^2*e*f^2*x*exp(4*d*x+4*c)+4*I*f^3*exp(2*d*x+2*c)+
3*d^3*e^3*exp(5*d*x+5*c)+2*d^3*e^3*exp(3*d*x+3*c)-18*I*d^3*e*f^2*x^2*exp(4*
d*x+4*c)+2*I*f^3-18*I*d^3*e^2*f*x*exp(4*d*x+4*c)-8*I*d^2*e*f^2*x+2*I*f^3*ex
p(4*d*x+4*c)+3*d^3*e^3*exp(d*x+c)-4*I*d^2*e^2*f-4*I*d^2*f^3*x^2+9*d^3*e^2*f
*x*exp(d*x+c)-2*d^2*e*f^2*x*exp(d*x+c)+9*d^3*e*f^2*x^2*exp(d*x+c)-44*I*d^2*
e*f^2*x*exp(2*d*x+2*c)+18*I*d^3*e*f^2*x^2*exp(2*d*x+2*c)+18*I*d^3*e^2*f*x*e
xp(2*d*x+2*c)-2*f^3*exp(5*d*x+5*c)-4*f^3*exp(3*d*x+3*c)+2*d^3*f^3*x^3*exp(3
*d*x+3*c)-2*d*f^3*x*exp(5*d*x+5*c)-4*d*f^3*x*exp(3*d*x+3*c)-2*d*e*f^2*exp(5
*d*x+5*c)+18*d^2*e*f^2*x*exp(5*d*x+5*c)+16*d^2*e*f^2*x*exp(3*d*x+3*c)-6*I*d
^3*f^3*x^3*exp(4*d*x+4*c)+6*I*d^3*f^3*x^3*exp(2*d*x+2*c)-22*I*d^2*e^2*f*exp
(2*d*x+2*c)-18*I*d^2*f^3*x^2*exp(4*d*x+4*c)-18*I*d^2*e^2*f*exp(4*d*x+4*c)-2
2*I*d^2*f^3*x^2*exp(2*d*x+2*c)+9*d^3*e*f^2*x^2*exp(5*d*x+5*c)+9*d^3*e^2*f*x
*exp(5*d*x+5*c)+6*d^3*e*f^2*x^2*exp(3*d*x+3*c)+6*d^3*e^2*f*x*exp(3*d*x+3*c)
+3*d^3*f^3*x^3*exp(5*d*x+5*c)-6*I*d^3*e^3*exp(4*d*x+4*c)+6*I*d^3*e^3*exp(2*
d*x+2*c)-4*d*e*f^2*exp(3*d*x+3*c)+8*d^2*e^2*f*exp(3*d*x+3*c)+9*d^2*f^3*x^2*
exp(5*d*x+5*c)+8*d^2*f^3*x^2*exp(3*d*x+3*c)+9*d^2*e^2*f*exp(5*d*x+5*c)-2*d*
f^3*x*exp(d*x+c)-2*d*e*f^2*exp(d*x+c)-d^2*e^2*f*exp(d*x+c)-d^2*f^3*x^2*exp(
d*x+c)+3*d^3*f^3*x^3*exp(d*x+c))/(exp(d*x+c)+I)^2/(exp(d*x+c)-I)^4/d^4/a-I/
a/d^2*f^3*x^2-I/a/d^4*f^3*c^2-3/8*I/a/d*e^3*ln(exp(d*x+c)-I)+3/8*I/a/d*e^3*
ln(exp(d*x+c)+I)+7/2*I/a/d^4*f^3*polylog(2,-I*exp(d*x+c))-3/2*I/a/d^4*f^3*p
olylog(2,I*exp(d*x+c))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3847 vs. $2(577) = 1154$.
time = 0.44, size = 3847, normalized size = 5.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/8*(16*I*c*d*f^2*e - 4*(2*I*c^2 - I)*f^3 - 8*I*d^2*f*e^2 - 3*(3*I*d^2*f^3*
x^2 + 6*I*d^2*f^2*x*e + 3*I*d^2*f*e^2 - 4*I*f^3 + (-3*I*d^2*f^3*x^2 - 6*I*d
^2*f^2*x*e - 3*I*d^2*f*e^2 + 4*I*f^3)*e^(6*d*x + 6*c) - 2*(3*d^2*f^3*x^2 +
6*d^2*f^2*x*e + 3*d^2*f*e^2 - 4*f^3)*e^(5*d*x + 5*c) + (-3*I*d^2*f^3*x^2 -
6*I*d^2*f^2*x*e - 3*I*d^2*f*e^2 + 4*I*f^3)*e^(4*d*x + 4*c) - 4*(3*d^2*f^3*x
^2 + 6*d^2*f^2*x*e + 3*d^2*f*e^2 - 4*f^3)*e^(3*d*x + 3*c) + (3*I*d^2*f^3*x^
2 + 6*I*d^2*f^2*x*e + 3*I*d^2*f*e^2 - 4*I*f^3)*e^(2*d*x + 2*c) - 2*(3*d^2*f
^3*x^2 + 6*d^2*f^2*x*e + 3*d^2*f*e^2 - 4*f^3)*e^(d*x + c))*dilog(I*e^(d*x +
 c)) + (9*I*d^2*f^3*x^2 + 18*I*d^2*f^2*x*e + 9*I*d^2*f*e^2 - 28*I*f^3 + (-9
*I*d^2*f^3*x^2 - 18*I*d^2*f^2*x*e - 9*I*d^2*f*e^2 + 28*I*f^3)*e^(6*d*x + 6*
c) - 2*(9*d^2*f^3*x^2 + 18*d^2*f^2*x*e + 9*d^2*f*e^2 - 28*f^3)*e^(5*d*x + 5
*c) + (-9*I*d^2*f^3*x^2 - 18*I*d^2*f^2*x*e - 9*I*d^2*f*e^2 + 28*I*f^3)*e^(4
*d*x + 4*c) - 4*(9*d^2*f^3*x^2 + 18*d^2*f^2*x*e + 9*d^2*f*e^2 - 28*f^3)*e^(
3*d*x + 3*c) + (9*I*d^2*f^3*x^2 + 18*I*d^2*f^2*x*e + 9*I*d^2*f*e^2 - 28*I*f
^3)*e^(2*d*x + 2*c) - 2*(9*d^2*f^3*x^2 + 18*d^2*f^2*x*e + 9*d^2*f*e^2 - 28*
f^3)*e^(d*x + c))*dilog(-I*e^(d*x + c)) - 8*(I*d^2*f^3*x^2 - I*c^2*f^3 + 2*
(I*d^2*f^2*x + I*c*d*f^2)*e)*e^(6*d*x + 6*c) + 2*(3*d^3*f^3*x^3 + d^2*f^3*x
^2 - 2*d*f^3*x + 2*(4*c^2 - 1)*f^3 + 3*d^3*e^3 + 9*(d^3*f*x + d^2*f)*e^2 +
(9*d^3*f^2*x^2 + 2*d^2*f^2*x - 2*(8*c + 1)*d*f^2)*e)*e^(5*d*x + 5*c) - 4*(3
*I*d^3*f^3*x^3 + 11*I*d^2*f^3*x^2 + (-2*I*c^2 - I)*f^3 + 3*I*d^3*e^3 + 9*(I
*d^3*f*x + I*d^2*f)*e^2 + (9*I*d^3*f^2*x^2 + 22*I*d^2*f^2*x + 4*I*c*d*f^2)*
e)*e^(4*d*x + 4*c) + 4*(d^3*f^3*x^3 - 4*d^2*f^3*x^2 - 2*d*f^3*x + 2*(4*c^2
- 1)*f^3 + d^3*e^3 + (3*d^3*f*x + 4*d^2*f)*e^2 + (3*d^3*f^2*x^2 - 8*d^2*f^2
*x - 2*(8*c + 1)*d*f^2)*e)*e^(3*d*x + 3*c) - 4*(-3*I*d^3*f^3*x^3 + 9*I*d^2*
f^3*x^2 + 2*(I*c^2 - I)*f^3 - 3*I*d^3*e^3 + (-9*I*d^3*f*x + 11*I*d^2*f)*e^2
 + (-9*I*d^3*f^2*x^2 + 18*I*d^2*f^2*x - 4*I*c*d*f^2)*e)*e^(2*d*x + 2*c) + 2
*(3*d^3*f^3*x^3 - 9*d^2*f^3*x^2 - 2*d*f^3*x + 2*(4*c^2 - 1)*f^3 + 3*d^3*e^3
 + (9*d^3*f*x - d^2*f)*e^2 + (9*d^3*f^2*x^2 - 18*d^2*f^2*x - 2*(8*c + 1)*d*
```

```
f^2)*e)*e^(d*x + c) - 3*(-3*I*c*d^2*f*e^2 + (3*I*c^2 - 4*I)*d*f^2*e + (-I*c
^3 + 4*I*c)*f^3 + I*d^3*e^3 + (3*I*c*d^2*f*e^2 + (-3*I*c^2 + 4*I)*d*f^2*e +
 (I*c^3 - 4*I*c)*f^3 - I*d^3*e^3)*e^(6*d*x + 6*c) + 2*(3*c*d^2*f*e^2 - (3*c
^2 - 4)*d*f^2*e + (c^3 - 4*c)*f^3 - d^3*e^3)*e^(5*d*x + 5*c) + (3*I*c*d^2*f
*e^2 + (-3*I*c^2 + 4*I)*d*f^2*e + (I*c^3 - 4*I*c)*f^3 - I*d^3*e^3)*e^(4*d*x
 + 4*c) + 4*(3*c*d^2*f*e^2 - (3*c^2 - 4)*d*f^2*e + (c^3 - 4*c)*f^3 - d^3*e^
3)*e^(3*d*x + 3*c) + (-3*I*c*d^2*f*e^2 + (3*I*c^2 - 4*I)*d*f^2*e + (-I*c^3
+ 4*I*c)*f^3 + I*d^3*e^3)*e^(2*d*x + 2*c) + 2*(3*c*d^2*f*e^2 - (3*c^2 - 4)*
d*f^2*e + (c^3 - 4*c)*f^3 - d^3*e^3)*e^(d*x + c))*log(e^(d*x + c) + I) + (-
9*I*c*d^2*f*e^2 + (9*I*c^2 - 28*I)*d*f^2*e + (-3*I*c^3 + 28*I*c)*f^3 + 3*I*
d^3*e^3 + (9*I*c*d^2*f*e^2 + (-9*I*c^2 + 28*I)*d*f^2*e + (3*I*c^3 - 28*I*c)
*f^3 - 3*I*d^3*e^3)*e^(6*d*x + 6*c) + 2*(9*c*d^2*f*e^2 - (9*c^2 - 28)*d*f^2
*e + (3*c^3 - 28*c)*f^3 - 3*d^3*e^3)*e^(5*d*x + 5*c) + (9*I*c*d^2*f*e^2 + (
-9*I*c^2 + 28*I)*d*f^2*e + (3*I*c^3 - 28*I*c)*f^3 - 3*I*d^3*e^3)*e^(4*d*x +
 4*c) + 4*(9*c*d^2*f*e^2 - (9*c^2 - 28)*d*f^2*e + (3*c^3 - 28*c)*f^3 - 3*d^
3*e^3)*e^(3*d*x + 3*c) + (-9*I*c*d^2*f*e^2 + (9*I*c^2 - 28*I)*d*f^2*e + (-3
*I*c^3 + 28*I*c)*f^3 + 3*I*d^3*e^3)*e^(2*d*x + 2*c) + 2*(9*c*d^2*f*e^2 - (9
*c^2 - 28)*d*f^2*e + (3*c^3 - 28*c)*f^3 - 3*d^3*e^3)*e^(d*x + c))*log(e^(d*
x + c) - I) + (3*I*d^3*f^3*x^3 - 28*I*d*f^3*x + (3*I*c^3 - 28*I*c)*f^3 - 9*
(-I*d^3*f*x - I*c*d^2*f)*e^2 - 9*(-I*d^3*f^2*x^2 + I*c^2*d*f^2)*e + (-3*I*d
^3*f^3*x^3 + 28*I*d*f^3*x + (-3*I*c^3 + 28*I*c)*f^3 - 9*(I*d^3*f*x + I*c*d^
2*f)*e^2 - 9*(I*d^3*f^2*x^2 - I*c^2*d*f^2)*e)*e^(6*d*x + 6*c) - 2*(3*d^3*f^
3*x^3 - 28*d*f^3*x + (3*c^3 - 28*c)*f^3 + 9*(d^3*f*x + c*d^2*f)*e^2 + 9*(d^
3*f^2*x^2 - c^2*d*f^2)*e)*e^(5*d*x + 5*c) + (-3*I*d^3*f^3*x^3 + 28*I*d*f^3*
x + (-3*I*c^3 + 28*I*c)*f^3 - 9*(I*d^3*f*x + I*c*d^2*f)*e^2 - 9*(I*d^3*f^2*
x^2 - I*c^2*d*f^2)*e)*e^(4*d*x + 4*c) - 4*(3*d^3*f^3*x^3 - 28*d*f^3*x + (3*
c^3 - 28*c)*f^3 + 9*(d^3*f*x + c*d^2*f)*e^2 + 9*(d^3*f^2*x^2 - c^2*d*f^2)*e
)*e^(3*d*x + 3*c) + (3*I*d^3*f^3*x^3 - 28*I*d*f^3*x + (3*I*c^3 - 28*I*c)*f^
3 - 9*(-I*d^3*f*x - I*c*d^2*f)*e^2 - 9*(-I*d^3*f^2*x^2 + I*c^2*d*f^2)*e)*e^
(2*d*x + 2*c) - 2*(3*d^3*f^3*x^3 - 28*d*f^3*x + (3*c^3 - 28*c)*f^3 + 9*(d^3
*f*x + c*d^2*f)*e^2 + 9*(d^3*f^2*x^2 - c^2*d*f^2)*e)*e^(d*x + c))*log(I*e^(
d*x + c) + 1) - 3*(I*d^3*f^3*x^3 - 4*I*d*f^3*x + (I*c^3 - 4*I*c)*f^3 + 3*(I
*d^3*f*x + I*c*d^2*f)*e^2 + 3*(I*d^3*f^2*x^2 - I*c^2*d*f^2)*e + (-I*d^3*f^3
*x^3 + 4*I*d*f^3*x + (-I*c^3 + 4*I*c)*f^3 + 3*(-I*d^3*f*x - I*c*d^2*f)*e^2
+ 3*(-I*d^3*f^2*x^2 + I*c^2*d*f^2)*e)*e^(6*d*x + 6*c) - 2*(d^3*f^3*x^3 - 4*
d*f^3*x + (c^3 - 4*c)*f^3 + 3*(d^3*f*x + c*d^2*f)*e^2 + 3*(d^3*f^2*x^2 - c^
2*d*f^2)*e)*e^(5*d*x + 5*c) + (-I*d^3*f^3*x^3 + 4*I*d*f^3*x + (-I*c^3 + 4*I
*c)*f^3 + 3*(-I*d^3*f*x - I*c*d^2*f)*e^2 + 3*(-I*d^3*f^2*x^2 + I*c^2*d*f^2)
*e)*e^(4*d*x + 4*c) - 4*(d^3*f^3*x^3 - 4*d*f^3*...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{e^3\operatorname{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{f^3x^3\operatorname{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{3ef^2x^2\operatorname{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{3e^2fx\operatorname{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sech(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*(Integral(e**3*sech(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(f**3*
x**3*sech(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(3*e*f**2*x**2*sech
(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(3*e**2*f*x*sech(c + d*x)**3
/(sinh(c + d*x) - I), x))/a
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sech(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^3}{\cosh\left(c+d\,x\right)^3\,\left(a+a\,\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(cosh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
```

```
[Out] int((e + f*x)^3/(cosh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)), x)
```

3.284 $\int \frac{(e+fx)^2 \operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=423

$$\frac{3(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{4ad}-\frac{5f^2\text{ArcTan}(\sinh(c+dx))}{6ad^3}+\frac{if^2\log(\cosh(c+dx))}{3ad^3}-\frac{3if(e+fx)\text{PolyLog}\left(2,-ie^{c-}\right.}{4ad^2}$$

```
[Out] 3/4*(f*x+e)^2*arctan(exp(d*x+c))/a/d-5/6*f^2*arctan(sinh(d*x+c))/a/d^3-1/3*
I*f*(f*x+e)*tanh(d*x+c)/a/d^2+1/4*I*(f*x+e)^2*sech(d*x+c)^4/a/d-1/12*I*f^2*
sech(d*x+c)^2/a/d^3+3/4*I*f*(f*x+e)*polylog(2,I*exp(d*x+c))/a/d^2+3/4*I*f^2
*polylog(3,-I*exp(d*x+c))/a/d^3+3/4*f*(f*x+e)*sech(d*x+c)/a/d^2-3/4*I*f^2*p
olylog(3,I*exp(d*x+c))/a/d^3+1/6*f*(f*x+e)*sech(d*x+c)^3/a/d^2-1/6*I*f*(f*x
+e)*sech(d*x+c)^2*tanh(d*x+c)/a/d^2+1/3*I*f^2*ln(cosh(d*x+c))/a/d^3-1/12*f^
2*sech(d*x+c)*tanh(d*x+c)/a/d^3+3/8*(f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/a/d-3
/4*I*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/a/d^2+1/4*(f*x+e)^2*sech(d*x+c)^3*t
anh(d*x+c)/a/d
```

Rubi [A]
time = 0.29, antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {5690, 4271, 3853, 3855, 4265, 2611, 2320, 6724, 5559, 4270, 4269, 3556}

$$-\frac{5f^2\text{ArcTan}(\sinh(c+dx))}{6ad^3}+\frac{3(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{4ad}+\frac{3if^2\text{Li}_3\left(-ie^{c+dx}\right)}{4ad^3}-\frac{3if^2\text{Li}_3\left(ie^{c+dx}\right)}{4ad^3}-\frac{if^2\text{sech}^2(c+dx)}{12ad^3}+\frac{if^2\log(\cosh(c+dx))}{3ad^3}-\frac{f^2\tanh(c+dx)\text{sech}(c+dx)}{12ad^3}-\frac{3if(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{4ad^2}+\frac{3if(e+fx)\text{Li}_2\left(ie^{c+dx}\right)}{4ad^2}-\frac{if(e+fx)\tanh(c+dx)}{3ad^2}+\frac{f(e+fx)\text{sech}^3(c+dx)}{6ad^2}+\frac{3f(e+fx)\text{sech}(c+dx)}{4ad^2}-\frac{if(e+fx)\tanh(c+dx)\text{sech}^2(c+dx)}{6ad^2}+\frac{i(e+fx)^2\text{sech}^4(c+dx)}{4ad}+\frac{(e+fx)^2\tanh(c+dx)\text{sech}^3(c+dx)}{4ad}+\frac{3(e+fx)^2\tanh(c+dx)\text{sech}(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (3*(e + f*x)^2*ArcTan[E^(c + d*x)])/(4*a*d) - (5*f^2*ArcTan[Sinh[c + d*x]])
/(6*a*d^3) + ((I/3)*f^2*Log[Cosh[c + d*x]])/(a*d^3) - (((3*I)/4)*f*(e + f*x
)*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) + (((3*I)/4)*f*(e + f*x)*PolyLog[2,
 I*E^(c + d*x)])/(a*d^2) + (((3*I)/4)*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(a*
d^3) - (((3*I)/4)*f^2*PolyLog[3, I*E^(c + d*x)])/(a*d^3) + (3*f*(e + f*x)*S
ech[c + d*x])/(4*a*d^2) - ((I/12)*f^2*Sech[c + d*x]^2)/(a*d^3) + (f*(e + f*
x)*Sech[c + d*x]^3)/(6*a*d^2) + ((I/4)*(e + f*x)^2*Sech[c + d*x]^4)/(a*d) -
 ((I/3)*f*(e + f*x)*Tanh[c + d*x])/(a*d^2) - (f^2*Sech[c + d*x]*Tanh[c + d*
x])/(12*a*d^3) + (3*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(8*a*d) - ((I/
6)*f*(e + f*x)*Sech[c + d*x]^2*Tanh[c + d*x])/(a*d^2) + ((e + f*x)^2*Sech[c
 + d*x]^3*Tanh[c + d*x])/(4*a*d)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
```

```
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5690

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sech[c
 + d*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)*T
anh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && Eq
Q[a^2 + b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int (e+fx)^2\text{sech}^4(c+dx)\tanh(c+dx)\,dx}{a}+\frac{\int (e+fx)^2\text{sech}^5(c+dx)\,dx}{a}$$

$$= \frac{f(e+fx)\text{sech}^3(c+dx)}{6ad^2}+\frac{i(e+fx)^2\text{sech}^4(c+dx)}{4ad}+\frac{(e+fx)^2\text{sech}^3(c+}{4ad}$$

$$= \frac{3f(e+fx)\text{sech}(c+dx)}{4ad^2}-\frac{if^2\text{sech}^2(c+dx)}{12ad^3}+\frac{f(e+fx)\text{sech}^3(c+dx)}{6ad^2}+$$

$$= \frac{3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{4ad}-\frac{5f^2\tan^{-1}(\sinh(c+dx))}{6ad^3}+\frac{3f(e+fx)\text{sech}(c+}{4ad^2}$$

$$= \frac{3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{4ad}-\frac{5f^2\tan^{-1}(\sinh(c+dx))}{6ad^3}+\frac{if^2\log(\cosh(c+dx}{3ad^3}$$

$$= \frac{3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{4ad}-\frac{5f^2\tan^{-1}(\sinh(c+dx))}{6ad^3}+\frac{if^2\log(\cosh(c+dx}{3ad^3}$$

$$= \frac{3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{4ad}-\frac{5f^2\tan^{-1}(\sinh(c+dx))}{6ad^3}+\frac{if^2\log(\cosh(c+dx}{3ad^3}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1396 vs. 2(423) = 846.
time = 11.13, size = 1396, normalized size = 3.30

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] -1/24*((-9*I)*d^3*e^2*E^c*x + (28*I)*d*E^c*f^2*x - (9*I)*d^3*e*E^c*f*x^2 -
(3*I)*d^3*E^c*f^2*x^3 + 9*d^2*e^2*Log[I - E^(c + d*x)] + (9*I)*d^2*e^2*E^c*
Log[I - E^(c + d*x)] - 28*f^2*Log[I - E^(c + d*x)] - (28*I)*E^c*f^2*Log[I -
 E^(c + d*x)] + 18*d^2*e*f*x*Log[1 + I*E^(c + d*x)] + (18*I)*d^2*e*E^c*f*x*
Log[1 + I*E^(c + d*x)] + 9*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] + (9*I)*d^2*E
^c*f^2*x^2*Log[1 + I*E^(c + d*x)] + 18*d*(1 + I*E^c)*f*(e + f*x)*PolyLog[2,
 (-I)*E^(c + d*x)] - 18*(1 + I*E^c)*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^
3*(-I + E^c)) + ((-3*I)*d^3*e^2*E^c*x + (4*I)*d*E^c*f^2*x - (3*I)*d^3*e*E^c
*f*x^2 - I*d^3*E^c*f^2*x^3 - 6*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + (6*I)*d^2
*e*E^c*f*x*Log[1 - I*E^(c + d*x)] - 3*d^2*f^2*x^2*Log[1 - I*E^(c + d*x)] +
(3*I)*d^2*E^c*f^2*x^2*Log[1 - I*E^(c + d*x)] - 3*d^2*e^2*Log[I + E^(c + d*x
)] + (3*I)*d^2*e^2*E^c*Log[I + E^(c + d*x)] + 4*f^2*Log[I + E^(c + d*x)] -
(4*I)*E^c*f^2*Log[I + E^(c + d*x)] + (6*I)*d*(I + E^c)*f*(e + f*x)*PolyLog[
2, I*E^(c + d*x)] + 6*(1 - I*E^c)*f^2*PolyLog[3, I*E^(c + d*x)])/(8*a*d^3*(
I + E^c)) + ((3*e^2*x*Cosh[c])/(4*a) + (3*e^2*x*Sinh[c])/(4*a))/(1 + Cosh[2
*c] + Sinh[2*c]) + ((3*e*f*x^2*Cosh[c])/(4*a) + (3*e*f*x^2*Sinh[c])/(4*a))/
```

```
(1 + Cosh[2*c] + Sinh[2*c]) + ((f^2*x^3*Cosh[c])/(4*a) + (f^2*x^3*Sinh[c])/
(4*a))/(1 + Cosh[2*c] + Sinh[2*c]) - ((I/8)*(e^2 + 2*e*f*x + f^2*x^2))/(a*d
*(Cosh[c/2 + (d*x)/2] - I*Sinh[c/2 + (d*x)/2])^2) + ((I/2)*(e*f*Sinh[(d*x)/
2] + f^2*x*Sinh[(d*x)/2]))/(a*d^2*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2 + (d*
x)/2] - I*Sinh[c/2 + (d*x)/2])) + ((I/8)*(e^2 + 2*e*f*x + f^2*x^2))/(a*d*(C
osh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2])^4) - ((I/6)*(e*f*Sinh[(d*x)/2]
+ f^2*x*Sinh[(d*x)/2]))/(a*d^2*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c/2 + (d*x)/
2] + I*Sinh[c/2 + (d*x)/2])^3) + ((3*I)*d^2*e^2*Cosh[c/2] + d*e*f*Cosh[c/2]
 - I*f^2*Cosh[c/2] + (6*I)*d^2*e*f*x*Cosh[c/2] + d*f^2*x*Cosh[c/2] + (3*I)*
d^2*f^2*x^2*Cosh[c/2] - 3*d^2*e^2*Sinh[c/2] - I*d*e*f*Sinh[c/2] + f^2*Sinh[
c/2] - 6*d^2*e*f*x*Sinh[c/2] - I*d*f^2*x*Sinh[c/2] - 3*d^2*f^2*x^2*Sinh[c/2
])/(12*a*d^3*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 +
(d*x)/2])^2) - (((7*I)/6)*(e*f*Sinh[(d*x)/2] + f^2*x*Sinh[(d*x)/2]))/(a*d^2
*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1043 vs. $2(373) = 746$.
time = 4.26, size = 1044, normalized size = 2.47

method	result
risch	$\frac{9d^2x^2f^2\mathrm{e}^{dx+c}+9d^2e^2\mathrm{e}^{5dx+5c}+6d^2e^2\mathrm{e}^{3dx+3c}+16def\,\mathrm{e}^{3dx+3c}-44idef\,\mathrm{e}^{2dx+2c}+18id^2f^2x^2\mathrm{e}^{2dx+2c}-18id^2f^2x^2\mathrm{e}^{4dx+4c}-36idf^2x\,\mathrm{e}^{4dx}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(9*d^2*x^2*f^2*exp(d*x+c)+9*d^2*e^2*exp(5*d*x+5*c)+6*d^2*e^2*exp(3*d*x
+3*c)+16*d*e*f*exp(3*d*x+3*c)-36*I*d^2*e*f*x*exp(4*d*x+4*c)-44*I*d*e*f*exp(
2*d*x+2*c)+18*I*d^2*f^2*x^2*exp(2*d*x+2*c)-18*I*d^2*f^2*x^2*exp(4*d*x+4*c)+
9*d^2*e^2*exp(d*x+c)-2*d*f^2*x*exp(d*x+c)-2*d*e*f*exp(d*x+c)+36*I*d^2*e*f*x
*exp(2*d*x+2*c)+18*d^2*e*f*x*exp(d*x+c)+12*d^2*e*f*x*exp(3*d*x+3*c)+18*d^2*
e*f*x*exp(5*d*x+5*c)-36*I*d*f^2*x*exp(4*d*x+4*c)-36*I*d*e*f*exp(4*d*x+4*c)-
8*I*d*e*f-8*I*d*f^2*x+16*d*f^2*x*exp(3*d*x+3*c)-44*I*d*f^2*x*exp(2*d*x+2*c)
-2*f^2*exp(d*x+c)-4*f^2*exp(3*d*x+3*c)+18*d*f^2*x*exp(5*d*x+5*c)-2*f^2*exp(
5*d*x+5*c)+18*d*e*f*exp(5*d*x+5*c)+6*d^2*f^2*x^2*exp(3*d*x+3*c)+9*d^2*f^2*x
^2*exp(5*d*x+5*c)+18*I*d^2*e^2*exp(2*d*x+2*c)-18*I*d^2*e^2*exp(4*d*x+4*c))/
(exp(d*x+c)+I)^2/(exp(d*x+c)-I)^4/d^3/a-3/4*I*f^2*polylog(3,I*exp(d*x+c))/a
/d^3+3/4*I*f^2*polylog(3,-I*exp(d*x+c))/a/d^3-1/2*I/a/d^3*f^2*ln(exp(d*x+c)
+I)+3/4*I/a/d^2*polylog(2,I*exp(d*x+c))*f^2*x-3/4*I/a/d^2*f*c*e*ln(exp(d*x+
c)+I)+3/4*I/a/d^2*e*f*polylog(2,I*exp(d*x+c))+3/8*I/a/d^3*ln(1+I*exp(d*x+c)
)*c^2*f^2-2/3*I/a/d^3*f^2*ln(exp(d*x+c))-3/8*I/a/d*e^2*ln(exp(d*x+c)-I)-3/4
*I/a/d^2*ln(1+I*exp(d*x+c))*c*e*f-3/8*I/a/d*ln(1+I*exp(d*x+c))*f^2*x^2-3/8*
I/a/d^3*ln(1-I*exp(d*x+c))*c^2*f^2-3/4*I/a/d*ln(1+I*exp(d*x+c))*e*f*x-3/4*I
/a/d^2*polylog(2,-I*exp(d*x+c))*f^2*x-3/8*I/a/d^3*f^2*c^2*ln(exp(d*x+c)-I)+
3/4*I/a/d^2*f*c*e*ln(exp(d*x+c)-I)+3/8*I/a/d^3*f^2*c^2*ln(exp(d*x+c)+I)+7/6
```

```
*I/a/d^3*f^2*ln(exp(d*x+c)-I)-3/4*I/a/d^2*e*f*polylog(2,-I*exp(d*x+c))+3/4*
I/a/d^2*ln(1-I*exp(d*x+c))*c*e*f+3/4*I/a/d*ln(1-I*exp(d*x+c))*e*f*x+3/8*I/a
/d*ln(1-I*exp(d*x+c))*f^2*x^2+3/8*I/a/d*e^2*ln(exp(d*x+c)+I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2117 vs. 2(368) = 736.
time = 0.43, size = 2117, normalized size = 5.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/24*(16*I*c*f^2 - 16*I*d*f*e - 18*(I*d*f^2*x + I*d*f*e + (-I*d*f^2*x - I*d
*f*e)*e^(6*d*x + 6*c) - 2*(d*f^2*x + d*f*e)*e^(5*d*x + 5*c) + (-I*d*f^2*x -
 I*d*f*e)*e^(4*d*x + 4*c) - 4*(d*f^2*x + d*f*e)*e^(3*d*x + 3*c) + (I*d*f^2*
x + I*d*f*e)*e^(2*d*x + 2*c) - 2*(d*f^2*x + d*f*e)*e^(d*x + c))*dilog(I*e^(
d*x + c)) - 18*(-I*d*f^2*x - I*d*f*e + (I*d*f^2*x + I*d*f*e)*e^(6*d*x + 6*c
) + 2*(d*f^2*x + d*f*e)*e^(5*d*x + 5*c) + (I*d*f^2*x + I*d*f*e)*e^(4*d*x +
4*c) + 4*(d*f^2*x + d*f*e)*e^(3*d*x + 3*c) + (-I*d*f^2*x - I*d*f*e)*e^(2*d*
x + 2*c) + 2*(d*f^2*x + d*f*e)*e^(d*x + c))*dilog(-I*e^(d*x + c)) - 16*(I*d
*f^2*x + I*c*f^2)*e^(6*d*x + 6*c) + 2*(9*d^2*f^2*x^2 + 2*d*f^2*x - 2*(8*c +
 1)*f^2 + 9*d^2*e^2 + 18*(d^2*f*x + d*f)*e)*e^(5*d*x + 5*c) - 4*(9*I*d^2*f^
2*x^2 + 22*I*d*f^2*x + 4*I*c*f^2 + 9*I*d^2*e^2 + 18*(I*d^2*f*x + I*d*f)*e)*
e^(4*d*x + 4*c) + 4*(3*d^2*f^2*x^2 - 8*d*f^2*x - 2*(8*c + 1)*f^2 + 3*d^2*e^
2 + 2*(3*d^2*f*x + 4*d*f)*e)*e^(3*d*x + 3*c) - 4*(-9*I*d^2*f^2*x^2 + 18*I*d
*f^2*x - 4*I*c*f^2 - 9*I*d^2*e^2 + 2*(-9*I*d^2*f*x + 11*I*d*f)*e)*e^(2*d*x
+ 2*c) + 2*(9*d^2*f^2*x^2 - 18*d*f^2*x - 2*(8*c + 1)*f^2 + 9*d^2*e^2 + 2*(9
*d^2*f*x - d*f)*e)*e^(d*x + c) - 3*(-6*I*c*d*f*e + (3*I*c^2 - 4*I)*f^2 + 3*
I*d^2*e^2 + (6*I*c*d*f*e + (-3*I*c^2 + 4*I)*f^2 - 3*I*d^2*e^2)*e^(6*d*x + 6
*c) + 2*(6*c*d*f*e - (3*c^2 - 4)*f^2 - 3*d^2*e^2)*e^(5*d*x + 5*c) + (6*I*c*
d*f*e + (-3*I*c^2 + 4*I)*f^2 - 3*I*d^2*e^2)*e^(4*d*x + 4*c) + 4*(6*c*d*f*e
- (3*c^2 - 4)*f^2 - 3*d^2*e^2)*e^(3*d*x + 3*c) + (-6*I*c*d*f*e + (3*I*c^2 -
```

```
 4*I)*f^2 + 3*I*d^2*e^2)*e^(2*d*x + 2*c) + 2*(6*c*d*f*e - (3*c^2 - 4)*f^2 -
 3*d^2*e^2)*e^(d*x + c))*log(e^(d*x + c) + I) + (-18*I*c*d*f*e + (9*I*c^2 -
 28*I)*f^2 + 9*I*d^2*e^2 + (18*I*c*d*f*e + (-9*I*c^2 + 28*I)*f^2 - 9*I*d^2*
e^2)*e^(6*d*x + 6*c) + 2*(18*c*d*f*e - (9*c^2 - 28)*f^2 - 9*d^2*e^2)*e^(5*d
*x + 5*c) + (18*I*c*d*f*e + (-9*I*c^2 + 28*I)*f^2 - 9*I*d^2*e^2)*e^(4*d*x +
 4*c) + 4*(18*c*d*f*e - (9*c^2 - 28)*f^2 - 9*d^2*e^2)*e^(3*d*x + 3*c) + (-1
8*I*c*d*f*e + (9*I*c^2 - 28*I)*f^2 + 9*I*d^2*e^2)*e^(2*d*x + 2*c) + 2*(18*c
*d*f*e - (9*c^2 - 28)*f^2 - 9*d^2*e^2)*e^(d*x + c))*log(e^(d*x + c) - I) -
9*(-I*d^2*f^2*x^2 + I*c^2*f^2 + 2*(-I*d^2*f*x - I*c*d*f)*e + (I*d^2*f^2*x^2
 - I*c^2*f^2 + 2*(I*d^2*f*x + I*c*d*f)*e)*e^(6*d*x + 6*c) + 2*(d^2*f^2*x^2
- c^2*f^2 + 2*(d^2*f*x + c*d*f)*e)*e^(5*d*x + 5*c) + (I*d^2*f^2*x^2 - I*c^2
*f^2 + 2*(I*d^2*f*x + I*c*d*f)*e)*e^(4*d*x + 4*c) + 4*(d^2*f^2*x^2 - c^2*f^
2 + 2*(d^2*f*x + c*d*f)*e)*e^(3*d*x + 3*c) + (-I*d^2*f^2*x^2 + I*c^2*f^2 +
2*(-I*d^2*f*x - I*c*d*f)*e)*e^(2*d*x + 2*c) + 2*(d^2*f^2*x^2 - c^2*f^2 + 2*
(d^2*f*x + c*d*f)*e)*e^(d*x + c))*log(I*e^(d*x + c) + 1) - 9*(I*d^2*f^2*x^2
 - I*c^2*f^2 + 2*(I*d^2*f*x + I*c*d*f)*e + (-I*d^2*f^2*x^2 + I*c^2*f^2 + 2*
(-I*d^2*f*x - I*c*d*f)*e)*e^(6*d*x + 6*c) - 2*(d^2*f^2*x^2 - c^2*f^2 + 2*(d
^2*f*x + c*d*f)*e)*e^(5*d*x + 5*c) + (-I*d^2*f^2*x^2 + I*c^2*f^2 + 2*(-I*d^
2*f*x - I*c*d*f)*e)*e^(4*d*x + 4*c) - 4*(d^2*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x
 + c*d*f)*e)*e^(3*d*x + 3*c) + (I*d^2*f^2*x^2 - I*c^2*f^2 + 2*(I*d^2*f*x +
I*c*d*f)*e)*e^(2*d*x + 2*c) - 2*(d^2*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f
)*e)*e^(d*x + c))*log(-I*e^(d*x + c) + 1) - 18*(I*f^2*e^(6*d*x + 6*c) + 2*f
^2*e^(5*d*x + 5*c) + I*f^2*e^(4*d*x + 4*c) + 4*f^2*e^(3*d*x + 3*c) - I*f^2*
e^(2*d*x + 2*c) + 2*f^2*e^(d*x + c) - I*f^2)*polylog(3, I*e^(d*x + c)) - 18
*(-I*f^2*e^(6*d*x + 6*c) - 2*f^2*e^(5*d*x + 5*c) - I*f^2*e^(4*d*x + 4*c) -
4*f^2*e^(3*d*x + 3*c) + I*f^2*e^(2*d*x + 2*c) - 2*f^2*e^(d*x + c) + I*f^2)*
polylog(3, -I*e^(d*x + c)))/(a*d^3*e^(6*d*x + 6*c) - 2*I*a*d^3*e^(5*d*x + 5
*c) + a*d^3*e^(4*d*x + 4*c) - 4*I*a*d^3*e^(3*d*x + 3*c) - a*d^3*e^(2*d*x +
2*c) - 2*I*a*d^3*e^(d*x + c) - a*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{e^2\operatorname{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{f^2x^2\operatorname{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{2efx\operatorname{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sech(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*(Integral(e**2*sech(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(f**2*
x**2*sech(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(2*e*f*x*sech(c + d
*x)**3/(sinh(c + d*x) - I), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^2*sech(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^2}{\cosh\left(c+d\,x\right)^3\,\left(a+a\,\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(cosh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
[Out] int((e + f*x)^2/(cosh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)), x)
```

3.285 $\int \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=233

$$\frac{3(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{4ad}-\frac{3if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{8ad^2}+\frac{3if\text{PolyLog}\left(2,ie^{c+dx}\right)}{8ad^2}+\frac{3f\text{sech}(c+dx)}{8ad^2}+\frac{f\text{sech}^3(c+d}{12ad^2}$$

```
[Out] 3/4*(f*x+e)*arctan(exp(d*x+c))/a/d-3/8*I*f*polylog(2,-I*exp(d*x+c))/a/d^2+3
/8*I*f*polylog(2,I*exp(d*x+c))/a/d^2+3/8*f*sech(d*x+c)/a/d^2+1/12*f*sech(d*
x+c)^3/a/d^2+1/4*I*(f*x+e)*sech(d*x+c)^4/a/d-1/4*I*f*tanh(d*x+c)/a/d^2+3/8*
(f*x+e)*sech(d*x+c)*tanh(d*x+c)/a/d+1/4*(f*x+e)*sech(d*x+c)^3*tanh(d*x+c)/a
/d+1/12*I*f*tanh(d*x+c)^3/a/d^2
```

Rubi [A]
time = 0.14, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {5690, 4270, 4265, 2317, 2438, 5559, 3852}

$$\frac{3(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{4ad}-\frac{3if\text{Li}_2\left(-ie^{c+dx}\right)}{8ad^2}+\frac{3if\text{Li}_2\left(ie^{c+dx}\right)}{8ad^2}+\frac{if\tanh^3(c+dx)}{12ad^2}-\frac{if\tanh(c+dx)}{4ad^2}+\frac{f\text{sech}^3(c+dx)}{12ad^2}+\frac{3f\text{sech}(c+dx)}{8ad^2}+\frac{i(e+fx)\text{sech}^4(c+dx)}{4ad}+\frac{(e+fx)\tanh(c+dx)\text{sech}^3(c+dx)}{4ad}+\frac{3(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (3*(e + f*x)*ArcTan[E^(c + d*x)])/(4*a*d) - (((3*I)/8)*f*PolyLog[2, (-I)*E^
(c + d*x)])/(a*d^2) + (((3*I)/8)*f*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + (3*
f*Sech[c + d*x])/(8*a*d^2) + (f*Sech[c + d*x]^3)/(12*a*d^2) + ((I/4)*(e + f
*x)*Sech[c + d*x]^4)/(a*d) - ((I/4)*f*Tanh[c + d*x])/(a*d^2) + (3*(e + f*x)
*Sech[c + d*x]*Tanh[c + d*x])/(8*a*d) + ((e + f*x)*Sech[c + d*x]^3*Tanh[c +
 d*x])/(4*a*d) + ((I/12)*f*Tanh[c + d*x]^3)/(a*d^2)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5690

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sech[c
 + d*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)*T
anh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && Eq
Q[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int(e+fx)\text{sech}^4(c+dx)\tanh(c+dx)\,dx}{a} + \frac{\int(e+fx)\text{sech}^5(c+dx)\,dx}{a}$$

$$= \frac{f\text{sech}^3(c+dx)}{12ad^2} + \frac{i(e+fx)\text{sech}^4(c+dx)}{4ad} + \frac{(e+fx)\text{sech}^3(c+dx)\tanh(c+}{4ad}$$

$$= \frac{3f\text{sech}(c+dx)}{8ad^2} + \frac{f\text{sech}^3(c+dx)}{12ad^2} + \frac{i(e+fx)\text{sech}^4(c+dx)}{4ad} + \frac{3(e+fx)\text{sech}}{}$$

$$= \frac{3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{4ad} + \frac{3f\text{sech}(c+dx)}{8ad^2} + \frac{f\text{sech}^3(c+dx)}{12ad^2} + \frac{i(e+fx)\text{sech}}{4ad}$$

$$= \frac{3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{4ad} + \frac{3f\text{sech}(c+dx)}{8ad^2} + \frac{f\text{sech}^3(c+dx)}{12ad^2} + \frac{i(e+fx)\text{sech}}{4ad}$$

$$= \frac{3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{4ad} - \frac{3if\text{Li}_2\left(-ie^{c+dx}\right)}{8ad^2} + \frac{3if\text{Li}_2\left(ie^{c+dx}\right)}{8ad^2} + \frac{3f\text{sech}(c+}{8ad^2}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 929 vs. 2(233) = 466.
time = 5.02, size = 929, normalized size = 3.99

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (2*(f + (6*I)*d*(e + f*x)) + ((6*I)*d*(e + f*x))/(Cosh[(c + d*x)/2] + I*Sin
h[(c + d*x)/2])^2 - 9*(c + d*x)*(c*f - d*(2*e + f*x))*(Cosh[(c + d*x)/2] +
I*Sinh[(c + d*x)/2])^2 - 9*d*e*(c + d*x - (2*I)*Log[Cosh[(c + d*x)/2] - I*S
inh[(c + d*x)/2]])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 + 9*c*f*(c +
 d*x - (2*I)*Log[Cosh[(c + d*x)/2] - I*Sinh[(c + d*x)/2]])*(Cosh[(c + d*x)/
2] + I*Sinh[(c + d*x)/2])^2 - 9*d*e*(c + d*x + (2*I)*Log[Cosh[(c + d*x)/2]
+ I*Sinh[(c + d*x)/2]])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 + 9*c*f
*(c + d*x + (2*I)*Log[Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])*(Cosh[(c +
d*x)/2] + I*Sinh[(c + d*x)/2])^2 - (9*f*(-2*(-1)^(3/4)*(c + d*x)^2 + Sqrt[2
]*(2*((-2*I)*c + Pi - (2*I)*d*x)*Log[1 + I*E^(-c - d*x)] + Pi*(3*c + 3*d*x
- 4*Log[1 + E^(c + d*x)] + 4*Log[Cosh[(c + d*x)/2]] - 2*Log[-Sin[(Pi - (2*I
)*(c + d*x))/4]]) + (4*I)*PolyLog[2, (-I)*E^(-c - d*x)]))*(Cosh[(c + d*x)/2
] + I*Sinh[(c + d*x)/2])^2)/(2*Sqrt[2]) - (9*f*(2*(-1)^(1/4)*(c + d*x)^2 +
Sqrt[2]*(2*((2*I)*c + Pi + (2*I)*d*x)*Log[1 - I*E^(-c - d*x)] - Pi*(c + d*x
 - 4*Log[1 + E^(c + d*x)] + 4*Log[Cosh[(c + d*x)/2]] + 2*Log[Sin[(Pi + (2*I
)*(c + d*x))/4]]) - (4*I)*PolyLog[2, I*E^(-c - d*x)]))*(Cosh[(c + d*x)/2] +
 I*Sinh[(c + d*x)/2])^2)/(2*Sqrt[2]) - ((6*I)*d*(e + f*x)*(Cosh[(c + d*x)/2
] + I*Sinh[(c + d*x)/2])^2)/(Cosh[(c + d*x)/2] - I*Sinh[(c + d*x)/2])^2 - (
```

```
(4*I)*f*Sinh[(c + d*x)/2])/(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]) + ((12
*I)*f*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2*Sinh[(c + d*x)/2])/(Cosh[
(c + d*x)/2] - I*Sinh[(c + d*x)/2]) + 28*f*Sinh[(c + d*x)/2]*((-I)*Cosh[(c
+ d*x)/2] + Sinh[(c + d*x)/2]))/(48*d^2*(a + I*a*Sinh[c + d*x]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(203) = 406$.
time = 4.92, size = 445, normalized size = 1.91

method	result
risch	$\frac{9dfx\,e^{5dx+5c}+9dfx\,e^{dx+c}-18ide\,e^{4dx+4c}-4if-22if\,e^{2dx+2c}+9f\,e^{5dx+5c}+8f\,e^{3dx+3c}-f\,e^{dx+c}+9de\,e^{5dx+5c}+9de\,e^{dx+c}-18idfx\,e}{12\left(e^{dx+c}+i\right)^{2}\left(e^{dx+c}-i\right)^{4}d^{2}a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(9*d*f*x*exp(5*d*x+5*c)+9*d*f*x*exp(d*x+c)-18*I*d*e*exp(4*d*x+4*c)-4*I
*f-22*I*f*exp(2*d*x+2*c)+9*f*exp(5*d*x+5*c)+8*f*exp(3*d*x+3*c)-f*exp(d*x+c)
+9*d*e*exp(5*d*x+5*c)+9*d*e*exp(d*x+c)-18*I*d*f*x*exp(4*d*x+4*c)+18*I*d*e*e
xp(2*d*x+2*c)+6*d*e*exp(3*d*x+3*c)+6*d*f*x*exp(3*d*x+3*c)-18*I*f*exp(4*d*x+
4*c)+18*I*d*f*x*exp(2*d*x+2*c))/(exp(d*x+c)+I)^2/(exp(d*x+c)-I)^4/d^2/a+3/8
*I/d/a*e*ln(exp(d*x+c)+I)-3/8*I/d/a*e*ln(exp(d*x+c)-I)+3/8*I/d/a*f*ln(1-I*e
xp(d*x+c))*x+3/8*I/d^2/a*f*ln(1-I*exp(d*x+c))*c+3/8*I*f*polylog(2,I*exp(d*x
+c))/a/d^2-3/8*I/d/a*f*ln(1+I*exp(d*x+c))*x-3/8*I/d^2/a*f*ln(1+I*exp(d*x+c)
)*c-3/8*I*f*polylog(2,-I*exp(d*x+c))/a/d^2-3/8*I/d^2/a*f*c*ln(exp(d*x+c)+I)
+3/8*I/d^2/a*f*c*ln(exp(d*x+c)-I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 936 vs. $2(201) = 402$.
time = 0.37, size = 936, normalized size = 4.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(9*(-I*f*e^(6*d*x + 6*c) - 2*f*e^(5*d*x + 5*c) - I*f*e^(4*d*x + 4*c)
- 4*f*e^(3*d*x + 3*c) + I*f*e^(2*d*x + 2*c) - 2*f*e^(d*x + c) + I*f)*dilog(
I*e^(d*x + c)) + 9*(I*f*e^(6*d*x + 6*c) + 2*f*e^(5*d*x + 5*c) + I*f*e^(4*d*
x + 4*c) + 4*f*e^(3*d*x + 3*c) - I*f*e^(2*d*x + 2*c) + 2*f*e^(d*x + c) - I*
f)*dilog(-I*e^(d*x + c)) - 18*(d*f*x + d*e + f)*e^(5*d*x + 5*c) + 36*(I*d*f
*x + I*d*e + I*f)*e^(4*d*x + 4*c) - 4*(3*d*f*x + 3*d*e + 4*f)*e^(3*d*x + 3*
c) + 4*(-9*I*d*f*x - 9*I*d*e + 11*I*f)*e^(2*d*x + 2*c) - 2*(9*d*f*x + 9*d*e
 - f)*e^(d*x + c) + 9*(-I*c*f + I*d*e + (I*c*f - I*d*e)*e^(6*d*x + 6*c) + 2
*(c*f - d*e)*e^(5*d*x + 5*c) + (I*c*f - I*d*e)*e^(4*d*x + 4*c) + 4*(c*f - d
*e)*e^(3*d*x + 3*c) + (-I*c*f + I*d*e)*e^(2*d*x + 2*c) + 2*(c*f - d*e)*e^(d
*x + c))*log(e^(d*x + c) + I) + 9*(I*c*f - I*d*e + (-I*c*f + I*d*e)*e^(6*d*
x + 6*c) - 2*(c*f - d*e)*e^(5*d*x + 5*c) + (-I*c*f + I*d*e)*e^(4*d*x + 4*c)
 - 4*(c*f - d*e)*e^(3*d*x + 3*c) + (I*c*f - I*d*e)*e^(2*d*x + 2*c) - 2*(c*f
 - d*e)*e^(d*x + c))*log(e^(d*x + c) - I) + 9*(-I*d*f*x - I*c*f + (I*d*f*x
+ I*c*f)*e^(6*d*x + 6*c) + 2*(d*f*x + c*f)*e^(5*d*x + 5*c) + (I*d*f*x + I*c
*f)*e^(4*d*x + 4*c) + 4*(d*f*x + c*f)*e^(3*d*x + 3*c) + (-I*d*f*x - I*c*f)*
e^(2*d*x + 2*c) + 2*(d*f*x + c*f)*e^(d*x + c))*log(I*e^(d*x + c) + 1) + 9*(
I*d*f*x + I*c*f + (-I*d*f*x - I*c*f)*e^(6*d*x + 6*c) - 2*(d*f*x + c*f)*e^(5
*d*x + 5*c) + (-I*d*f*x - I*c*f)*e^(4*d*x + 4*c) - 4*(d*f*x + c*f)*e^(3*d*x
 + 3*c) + (I*d*f*x + I*c*f)*e^(2*d*x + 2*c) - 2*(d*f*x + c*f)*e^(d*x + c))*
log(-I*e^(d*x + c) + 1) + 8*I*f)/(a*d^2*e^(6*d*x + 6*c) - 2*I*a*d^2*e^(5*d*
x + 5*c) + a*d^2*e^(4*d*x + 4*c) - 4*I*a*d^2*e^(3*d*x + 3*c) - a*d^2*e^(2*d
*x + 2*c) - 2*I*a*d^2*e^(d*x + c) - a*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\left(\int \frac{e\operatorname{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx+\int \frac{fx\operatorname{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*(Integral(e*sech(c + d*x)**3/(sinh(c + d*x) - I), x) + Integral(f*x*sech
(c + d*x)**3/(sinh(c + d*x) - I), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sech(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+fx}{\cosh(c+dx)^3\ (a+a\sinh(c+dx)\ 1\mathrm{i})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(cosh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)
[Out] int((e + f*x)/(cosh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)), x)
```

3.286 $\int \frac{\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{3\text{ArcTan}(\sinh(c+dx))}{8ad}-\frac{i}{8d(a-ia\sinh(c+dx))}+\frac{ia}{8d(a+ia\sinh(c+dx))^2}+\frac{i}{4d(a+ia\sinh(c+dx))}$$

```
[Out] 3/8*arctan(sinh(d*x+c))/a/d-1/8*I/d/(a-I*a*sinh(d*x+c))+1/8*I*a/d/(a+I*a*si
nh(d*x+c))^2+1/4*I/d/(a+I*a*sinh(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2746, 46, 212}

$$\frac{3\text{ArcTan}(\sinh(c+dx))}{8ad}+\frac{ia}{8d(a+ia\sinh(c+dx))^2}-\frac{i}{8d(a-ia\sinh(c+dx))}+\frac{i}{4d(a+ia\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (3*ArcTan[Sinh[c + d*x]])/(8*a*d) - (I/8)/(d*(a - I*a*Sinh[c + d*x])) + ((I
/8)*a)/(d*(a + I*a*Sinh[c + d*x])^2) + (I/4)/(d*(a + I*a*Sinh[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{(ia^3)\operatorname{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^3}\,dx, x, ia\sinh(c+dx)\right)}{d}$$

$$= -\frac{(ia^3)\operatorname{Subst}\left(\int \left(\frac{1}{8a^3(a-x)^2}+\frac{1}{4a^2(a+x)^3}+\frac{1}{4a^3(a+x)^2}+\frac{3}{8a^3(a^2-x^2)}\right)\,dx, x, ia\sinh(c\right.}{d}$$

$$= -\frac{i}{8d(a-ia\sinh(c+dx))}+\frac{ia}{8d(a+ia\sinh(c+dx))^2}+\frac{i}{4d(a+ia\sinh(c+dx)}$$

$$= \frac{3\tan^{-1}(\sinh(c+dx))}{8ad}-\frac{i}{8d(a-ia\sinh(c+dx))}+\frac{ia}{8d(a+ia\sinh(c+dx))^2}+$$

Mathematica [A]
time = 0.09, size = 101, normalized size = 1.11

$$\frac{\operatorname{sech}^2(c+dx)\,(2-3i\operatorname{ArcTan}(\sinh(c+dx))+3(-i+\operatorname{ArcTan}(\sinh(c+dx)))\sinh(c+dx)+(3-3i\operatorname{ArcTan}(\sinh(c+dx)))\sinh^2(c+dx)+3\operatorname{ArcTan}(\sinh(c+dx))\sinh^3(c+dx))}{8ad(-i+\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (Sech[c + d*x]^2*(2 - (3*I)*ArcTan[Sinh[c + d*x]] + 3*(-I + ArcTan[Sinh[c +
 d*x]])*Sinh[c + d*x] + (3 - (3*I)*ArcTan[Sinh[c + d*x]])*Sinh[c + d*x]^2 +
 3*ArcTan[Sinh[c + d*x]]*Sinh[c + d*x]^3))/(8*a*d*(-I + Sinh[c + d*x]))
```

Maple [A]
time = 1.55, size = 141, normalized size = 1.55

method	result
risch	$\frac{2\,\mathrm{e}^{3dx+3c}+6i\mathrm{e}^{2dx+2c}+3\,\mathrm{e}^{dx+c}+3\,\mathrm{e}^{5dx+5c}-6i\mathrm{e}^{4dx+4c}}{4\left(\mathrm{e}^{dx+c}+i\right)^2\left(\mathrm{e}^{dx+c}-i\right)^4 da}-\frac{3i\ln\left(\mathrm{e}^{dx+c}-i\right)}{8ad}+\frac{3i\ln\left(\mathrm{e}^{dx+c}+i\right)}{8ad}$
derivativedivides	$\frac{\frac{i}{4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}+\frac{3i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}{8}-\frac{1}{4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}+\frac{i}{2\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{3i\ln\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{}{2(-i}}{ad}$
default	$\frac{\frac{i}{4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}+\frac{3i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}{8}-\frac{1}{4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}+\frac{i}{2\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{3i\ln\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{}{2(-i}}{ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a*(1/8*I/(tanh(1/2*d*x+1/2*c)+I)^2+3/16*I*ln(tanh(1/2*d*x+1/2*c)+I)-1/8
/(tanh(1/2*d*x+1/2*c)+I)+1/4*I/(-I+tanh(1/2*d*x+1/2*c))^4-3/16*I*ln(-I+tanh
(1/2*d*x+1/2*c))-3/4*I/(-I+tanh(1/2*d*x+1/2*c))^2+1/2/(-I+tanh(1/2*d*x+1/2*
c))^3-1/2/(-I+tanh(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Exception raised: RuntimeError}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(71) = 142.
time = 0.43, size = 287, normalized size = 3.15

$$-\frac{3\left(-i\,e^{(6\,dx+6\,c)}-2\,e^{(5\,dx+5\,c)}-i\,e^{(4\,dx+4\,c)}-4\,e^{(3\,dx+3\,c)}+i\,e^{(2\,dx+2\,c)}-2\,e^{(dx+c)}+i\right)\log\left(e^{(dx+c)}+i\right)+3\left(i\,e^{(6\,dx+6\,c)}+2\,e^{(5\,dx+5\,c)}+i\,e^{(4\,dx+4\,c)}+4\,e^{(3\,dx+3\,c)}-i\,e^{(2\,dx+2\,c)}+2\,e^{(dx+c)}-i\right)\log\left(e^{(dx+c)}-i\right)-6\,e^{(5\,dx+5\,c)}+12i\,e^{(4\,dx+4\,c)}-4\,e^{(3\,dx+3\,c)}-12i\,e^{(2\,dx+2\,c)}-6\,e^{(dx+c)}}{8\left(ade^{(6\,dx+6\,c)}-2i\,ade^{(5\,dx+5\,c)}+ade^{(4\,dx+4\,c)}-4i\,ade^{(3\,dx+3\,c)}-ade^{(2\,dx+2\,c)}-2i\,ade^{(dx+c)}-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/8*(3*(-I*e^(6*d*x + 6*c) - 2*e^(5*d*x + 5*c) - I*e^(4*d*x + 4*c) - 4*e^(
3*d*x + 3*c) + I*e^(2*d*x + 2*c) - 2*e^(d*x + c) + I)*log(e^(d*x + c) + I)
+ 3*(I*e^(6*d*x + 6*c) + 2*e^(5*d*x + 5*c) + I*e^(4*d*x + 4*c) + 4*e^(3*d*x
 + 3*c) - I*e^(2*d*x + 2*c) + 2*e^(d*x + c) - I)*log(e^(d*x + c) - I) - 6*e
^(5*d*x + 5*c) + 12*I*e^(4*d*x + 4*c) - 4*e^(3*d*x + 3*c) - 12*I*e^(2*d*x +
 2*c) - 6*e^(d*x + c))/(a*d*e^(6*d*x + 6*c) - 2*I*a*d*e^(5*d*x + 5*c) + a*d
*e^(4*d*x + 4*c) - 4*I*a*d*e^(3*d*x + 3*c) - a*d*e^(2*d*x + 2*c) - 2*I*a*d*
e^(d*x + c) - a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\operatorname{sech}^3(c+dx)}{\sinh(c+dx)-i}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*Integral(sech(c + d*x)**3/(sinh(c + d*x) - I), x)/a
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(71) = 142.
time = 0.43, size = 177, normalized size = 1.95

$$-\frac{-\frac{6i\,\log\left(-i\,e^{(dx+c)}+i\,e^{(-dx-c)}+2\right)}{a}+\frac{6i\,\log\left(-i\,e^{(dx+c)}+i\,e^{(-dx-c)}-2\right)}{a}-\frac{2\left(3\,e^{(dx+c)}-3\,e^{(-dx-c)}+10i\right)}{a\left(i\,e^{(dx+c)}-i\,e^{(-dx-c)}-2\right)}+\frac{-9i\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-52\,e^{(dx+c)}+52\,e^{(-dx-c)}+84i}{a\left(e^{(dx+c)}-e^{(-dx-c)}-2i\right)^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")

[Out] -1/32*(-6*I*log(-I*e^(d*x + c) + I*e^(-d*x - c) + 2)/a + 6*I*log(-I*e^(d*x + c) + I*e^(-d*x - c) - 2)/a - 2*(3*e^(d*x + c) - 3*e^(-d*x - c) + 10*I)/(a*(I*e^(d*x + c) - I*e^(-d*x - c) - 2)) + (-9*I*(e^(d*x + c) - e^(-d*x - c))^2 - 52*e^(d*x + c) + 52*e^(-d*x - c) + 84*I)/(a*(e^(d*x + c) - e^(-d*x - c) - 2*I)^2))/d

Mupad [B]
time = 0.99, size = 137, normalized size = 1.51

$$\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{a^2\,d^2}}{a\,d}\right)}{4\,\sqrt{a^2\,d^2}}+\frac{1}{2\,a\,d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}+\frac{1}{4\,a\,d\,\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)}-\frac{1\mathrm{i}}{4\,a\,d\,{\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)}^2}-\frac{1\mathrm{i}}{a\,d\,{\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)}^3}+\frac{1\mathrm{i}}{2\,a\,d\,{\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^3*(a + a*sinh(c + d*x)*1i)),x)

[Out] (3*atan((exp(d*x)*exp(c)*(a^2*d^2)^(1/2))/(a*d)))/(4*(a^2*d^2)^(1/2)) + 1/(2*a*d*(exp(c + d*x) - 1i)) + 1/(4*a*d*(exp(c + d*x) + 1i)) - 1i/(4*a*d*(exp(c + d*x) + 1i)^2) - 1i/(a*d*(exp(c + d*x)*1i + 1)^3) + 1i/(2*a*d*(exp(c + d*x)*1i + 1)^4)

3.287 $\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sech[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
[Out] Defer[Int][Sech[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 94.55, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sech[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
[Out] Integrate[Sech[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(dx+c)^3}{(fx+e)\,(a+ia\sinh\,(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(4*I*d^2*f^3*x^2 + 8*I*d^2*f^2*x*e + 4*I*d^2*f*e^2 - 6*I*f^3 + (9*d^3
*f^3*x^3 - 9*d^2*f^3*x^2 - 2*d*f^3*x + 9*d^3*e^3 + 6*f^3 + 9*(3*d^3*f*x - d
^2*f)*e^2 + (27*d^3*f^2*x^2 - 18*d^2*f^2*x - 2*d*f^2)*e)*e^(5*d*x + 5*c) -
6*(3*I*d^3*f^3*x^3 - 3*I*d^2*f^3*x^2 + 3*I*d^3*e^3 + I*f^3 + 3*(3*I*d^3*f*x
 - I*d^2*f)*e^2 + 3*(3*I*d^3*f^2*x^2 - 2*I*d^2*f^2*x)*e)*e^(4*d*x + 4*c) +
2*(3*d^3*f^3*x^3 - 4*d^2*f^3*x^2 - 2*d*f^3*x + 3*d^3*e^3 + 6*f^3 + (9*d^3*f
*x - 4*d^2*f)*e^2 + (9*d^3*f^2*x^2 - 8*d^2*f^2*x - 2*d*f^2)*e)*e^(3*d*x + 3
*c) - 2*(-9*I*d^3*f^3*x^3 - 11*I*d^2*f^3*x^2 - 9*I*d^3*e^3 + 6*I*f^3 + (-27
*I*d^3*f*x - 11*I*d^2*f)*e^2 + (-27*I*d^3*f^2*x^2 - 22*I*d^2*f^2*x)*e)*e^(2
*d*x + 2*c) + (9*d^3*f^3*x^3 + d^2*f^3*x^2 - 2*d*f^3*x + 9*d^3*e^3 + 6*f^3
+ (27*d^3*f*x + d^2*f)*e^2 + (27*d^3*f^2*x^2 + 2*d^2*f^2*x - 2*d*f^2)*e)*e^
(d*x + c) - 12*(a*d^4*f^4*x^4 + 4*a*d^4*f^3*x^3*e + 6*a*d^4*f^2*x^2*e^2 + 4
*a*d^4*f*x*e^3 + a*d^4*e^4 - (a*d^4*f^4*x^4 + 4*a*d^4*f^3*x^3*e + 6*a*d^4*f
^2*x^2*e^2 + 4*a*d^4*f*x*e^3 + a*d^4*e^4)*e^(6*d*x + 6*c) + 2*(I*a*d^4*f^4*
x^4 + 4*I*a*d^4*f^3*x^3*e + 6*I*a*d^4*f^2*x^2*e^2 + 4*I*a*d^4*f*x*e^3 + I*a
*d^4*e^4)*e^(5*d*x + 5*c) - (a*d^4*f^4*x^4 + 4*a*d^4*f^3*x^3*e + 6*a*d^4*f^
2*x^2*e^2 + 4*a*d^4*f*x*e^3 + a*d^4*e^4)*e^(4*d*x + 4*c) + 4*(I*a*d^4*f^4*x
^4 + 4*I*a*d^4*f^3*x^3*e + 6*I*a*d^4*f^2*x^2*e^2 + 4*I*a*d^4*f*x*e^3 + I*a*
d^4*e^4)*e^(3*d*x + 3*c) + (a*d^4*f^4*x^4 + 4*a*d^4*f^3*x^3*e + 6*a*d^4*f^2
*x^2*e^2 + 4*a*d^4*f*x*e^3 + a*d^4*e^4)*e^(2*d*x + 2*c) + 2*(I*a*d^4*f^4*x^
4 + 4*I*a*d^4*f^3*x^3*e + 6*I*a*d^4*f^2*x^2*e^2 + 4*I*a*d^4*f*x*e^3 + I*a*d
```

```
^4*e^4)*e^(d*x + c))*integral(1/12*(-8*I*d^2*f^4*x^2 - 16*I*d^2*f^3*x*e - 8
*I*d^2*f^2*e^2 + 24*I*f^4 + (9*d^4*f^4*x^4 - 20*d^2*f^4*x^2 + 36*d^4*f*x*e^
3 + 9*d^4*e^4 + 24*f^4 + 2*(27*d^4*f^2*x^2 - 10*d^2*f^2)*e^2 + 4*(9*d^4*f^3
*x^3 - 10*d^2*f^3*x)*e)*e^(d*x + c))/(a*d^4*f^5*x^5 + 5*a*d^4*f^4*x^4*e + 1
0*a*d^4*f^3*x^3*e^2 + 10*a*d^4*f^2*x^2*e^3 + 5*a*d^4*f*x*e^4 + a*d^4*e^5 +
(a*d^4*f^5*x^5 + 5*a*d^4*f^4*x^4*e + 10*a*d^4*f^3*x^3*e^2 + 10*a*d^4*f^2*x^
2*e^3 + 5*a*d^4*f*x*e^4 + a*d^4*e^5)*e^(2*d*x + 2*c)), x))/(a*d^4*f^4*x^4 +
 4*a*d^4*f^3*x^3*e + 6*a*d^4*f^2*x^2*e^2 + 4*a*d^4*f*x*e^3 + a*d^4*e^4 - (a
*d^4*f^4*x^4 + 4*a*d^4*f^3*x^3*e + 6*a*d^4*f^2*x^2*e^2 + 4*a*d^4*f*x*e^3 +
a*d^4*e^4)*e^(6*d*x + 6*c) + 2*(I*a*d^4*f^4*x^4 + 4*I*a*d^4*f^3*x^3*e + 6*I
*a*d^4*f^2*x^2*e^2 + 4*I*a*d^4*f*x*e^3 + I*a*d^4*e^4)*e^(5*d*x + 5*c) - (a*
d^4*f^4*x^4 + 4*a*d^4*f^3*x^3*e + 6*a*d^4*f^2*x^2*e^2 + 4*a*d^4*f*x*e^3 + a
*d^4*e^4)*e^(4*d*x + 4*c) + 4*(I*a*d^4*f^4*x^4 + 4*I*a*d^4*f^3*x^3*e + 6*I*
a*d^4*f^2*x^2*e^2 + 4*I*a*d^4*f*x*e^3 + I*a*d^4*e^4)*e^(3*d*x + 3*c) + (a*d
^4*f^4*x^4 + 4*a*d^4*f^3*x^3*e + 6*a*d^4*f^2*x^2*e^2 + 4*a*d^4*f*x*e^3 + a*
d^4*e^4)*e^(2*d*x + 2*c) + 2*(I*a*d^4*f^4*x^4 + 4*I*a*d^4*f^3*x^3*e + 6*I*a
*d^4*f^2*x^2*e^2 + 4*I*a*d^4*f*x*e^3 + I*a*d^4*e^4)*e^(d*x + c))
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\operatorname{sech}^{3}(c+dx)}{e\sinh(c+dx)-ie+fx\sinh(c+dx)-ifx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
[Out] -I*Integral(sech(c + d*x)**3/(e*sinh(c + d*x) - I*e + f*x*sinh(c + d*x) - I
*f*x), x)/a
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate(sech(d*x + c)^3/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{1}{\cosh(c+d\,x)^3\,(e+f\,x)\,(a+a\sinh(c+d\,x)\,1\mathrm{i})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*(e + f*x)*(a + a*sinh(c + d*x)*1i)),x)
[Out] int(1/(cosh(c + d*x)^3*(e + f*x)*(a + a*sinh(c + d*x)*1i)), x)
```

3.288 $\int \frac{\text{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{\text{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sech[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
[Out] Defer[Int][Sech[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\text{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\text{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.02, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] Integrate[Sech[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\text{sech}(dx+c)^3}{(fx+e)^2\,(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/12*(8*I*d^2*f^3*x^2 + 16*I*d^2*f^2*x*e + 8*I*d^2*f*e^2 - 24*I*f^3 + 3*(3
*d^3*f^3*x^3 - 6*d^2*f^3*x^2 - 2*d*f^3*x + 3*d^3*e^3 + 8*f^3 + 3*(3*d^3*f*x
 - 2*d^2*f)*e^2 + (9*d^3*f^2*x^2 - 12*d^2*f^2*x - 2*d*f^2)*e)*e^(5*d*x + 5*
c) - 6*(3*I*d^3*f^3*x^3 - 6*I*d^2*f^3*x^2 + 3*I*d^3*e^3 + 4*I*f^3 + 3*(3*I*
d^3*f*x - 2*I*d^2*f)*e^2 + 3*(3*I*d^3*f^2*x^2 - 4*I*d^2*f^2*x)*e)*e^(4*d*x
+ 4*c) + 2*(3*d^3*f^3*x^3 - 8*d^2*f^3*x^2 - 6*d*f^3*x + 3*d^3*e^3 + 24*f^3
+ (9*d^3*f*x - 8*d^2*f)*e^2 + (9*d^3*f^2*x^2 - 16*d^2*f^2*x - 6*d*f^2)*e)*e
^(3*d*x + 3*c) - 2*(-9*I*d^3*f^3*x^3 - 22*I*d^2*f^3*x^2 - 9*I*d^3*e^3 + 24*
I*f^3 + (-27*I*d^3*f*x - 22*I*d^2*f)*e^2 + (-27*I*d^3*f^2*x^2 - 44*I*d^2*f^
2*x)*e)*e^(2*d*x + 2*c) + (9*d^3*f^3*x^3 + 2*d^2*f^3*x^2 - 6*d*f^3*x + 9*d^
3*e^3 + 24*f^3 + (27*d^3*f*x + 2*d^2*f)*e^2 + (27*d^3*f^2*x^2 + 4*d^2*f^2*x
 - 6*d*f^2)*e)*e^(d*x + c) - 12*(a*d^4*f^5*x^5 + 5*a*d^4*f^4*x^4*e + 10*a*d
^4*f^3*x^3*e^2 + 10*a*d^4*f^2*x^2*e^3 + 5*a*d^4*f*x*e^4 + a*d^4*e^5 - (a*d^
4*f^5*x^5 + 5*a*d^4*f^4*x^4*e + 10*a*d^4*f^3*x^3*e^2 + 10*a*d^4*f^2*x^2*e^3
 + 5*a*d^4*f*x*e^4 + a*d^4*e^5)*e^(6*d*x + 6*c) + 2*(I*a*d^4*f^5*x^5 + 5*I*
a*d^4*f^4*x^4*e + 10*I*a*d^4*f^3*x^3*e^2 + 10*I*a*d^4*f^2*x^2*e^3 + 5*I*a*d
^4*f*x*e^4 + I*a*d^4*e^5)*e^(5*d*x + 5*c) - (a*d^4*f^5*x^5 + 5*a*d^4*f^4*x^
4*e + 10*a*d^4*f^3*x^3*e^2 + 10*a*d^4*f^2*x^2*e^3 + 5*a*d^4*f*x*e^4 + a*d^4
*e^5)*e^(4*d*x + 4*c) + 4*(I*a*d^4*f^5*x^5 + 5*I*a*d^4*f^4*x^4*e + 10*I*a*d
```

```
^4*f^3*x^3*e^2 + 10*I*a*d^4*f^2*x^2*e^3 + 5*I*a*d^4*f*x*e^4 + I*a*d^4*e^5)*
e^(3*d*x + 3*c) + (a*d^4*f^5*x^5 + 5*a*d^4*f^4*x^4*e + 10*a*d^4*f^3*x^3*e^2
 + 10*a*d^4*f^2*x^2*e^3 + 5*a*d^4*f*x*e^4 + a*d^4*e^5)*e^(2*d*x + 2*c) + 2*
(I*a*d^4*f^5*x^5 + 5*I*a*d^4*f^4*x^4*e + 10*I*a*d^4*f^3*x^3*e^2 + 10*I*a*d^
4*f^2*x^2*e^3 + 5*I*a*d^4*f*x*e^4 + I*a*d^4*e^5)*e^(d*x + c))*integral(1/4*
(-8*I*d^2*f^4*x^2 - 16*I*d^2*f^3*x*e - 8*I*d^2*f^2*e^2 + 40*I*f^4 + (3*d^4*
f^4*x^4 - 20*d^2*f^4*x^2 + 12*d^4*f*x*e^3 + 3*d^4*e^4 + 40*f^4 + 2*(9*d^4*f
^2*x^2 - 10*d^2*f^2)*e^2 + 4*(3*d^4*f^3*x^3 - 10*d^2*f^3*x)*e)*e^(d*x + c))
/(a*d^4*f^6*x^6 + 6*a*d^4*f^5*x^5*e + 15*a*d^4*f^4*x^4*e^2 + 20*a*d^4*f^3*x
^3*e^3 + 15*a*d^4*f^2*x^2*e^4 + 6*a*d^4*f*x*e^5 + a*d^4*e^6 + (a*d^4*f^6*x^
6 + 6*a*d^4*f^5*x^5*e + 15*a*d^4*f^4*x^4*e^2 + 20*a*d^4*f^3*x^3*e^3 + 15*a*
d^4*f^2*x^2*e^4 + 6*a*d^4*f*x*e^5 + a*d^4*e^6)*e^(2*d*x + 2*c)), x))/(a*d^4
*f^5*x^5 + 5*a*d^4*f^4*x^4*e + 10*a*d^4*f^3*x^3*e^2 + 10*a*d^4*f^2*x^2*e^3
+ 5*a*d^4*f*x*e^4 + a*d^4*e^5 - (a*d^4*f^5*x^5 + 5*a*d^4*f^4*x^4*e + 10*a*d
^4*f^3*x^3*e^2 + 10*a*d^4*f^2*x^2*e^3 + 5*a*d^4*f*x*e^4 + a*d^4*e^5)*e^(6*d
*x + 6*c) + 2*(I*a*d^4*f^5*x^5 + 5*I*a*d^4*f^4*x^4*e + 10*I*a*d^4*f^3*x^3*e
^2 + 10*I*a*d^4*f^2*x^2*e^3 + 5*I*a*d^4*f*x*e^4 + I*a*d^4*e^5)*e^(5*d*x + 5
*c) - (a*d^4*f^5*x^5 + 5*a*d^4*f^4*x^4*e + 10*a*d^4*f^3*x^3*e^2 + 10*a*d^4*
f^2*x^2*e^3 + 5*a*d^4*f*x*e^4 + a*d^4*e^5)*e^(4*d*x + 4*c) + 4*(I*a*d^4*f^5
*x^5 + 5*I*a*d^4*f^4*x^4*e + 10*I*a*d^4*f^3*x^3*e^2 + 10*I*a*d^4*f^2*x^2*e^
3 + 5*I*a*d^4*f*x*e^4 + I*a*d^4*e^5)*e^(3*d*x + 3*c) + (a*d^4*f^5*x^5 + 5*a
*d^4*f^4*x^4*e + 10*a*d^4*f^3*x^3*e^2 + 10*a*d^4*f^2*x^2*e^3 + 5*a*d^4*f*x*
e^4 + a*d^4*e^5)*e^(2*d*x + 2*c) + 2*(I*a*d^4*f^5*x^5 + 5*I*a*d^4*f^4*x^4*e
 + 10*I*a*d^4*f^3*x^3*e^2 + 10*I*a*d^4*f^2*x^2*e^3 + 5*I*a*d^4*f*x*e^4 + I*
a*d^4*e^5)*e^(d*x + c))
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{i\int\frac{\operatorname{sech}^3(c+dx)}{e^2\sinh(c+dx)-ie^2+2efx\sinh(c+dx)-2iefx+f^2x^2\sinh(c+dx)-if^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*Integral(sech(c + d*x)**3/(e**2*sinh(c + d*x) - I*e**2 + 2*e*f*x*sinh(c
+ d*x) - 2*I*e*f*x + f**2*x**2*sinh(c + d*x) - I*f**2*x**2), x)/a
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cosh\left(c+d\,x\right)^3\,{\left(e+f\,x\right)}^2\,\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)),x)
[Out] int(1/(cosh(c + d*x)^3*(e + f*x)^2*(a + a*sinh(c + d*x)*1i)), x)
```

3.289 $\int \frac{(e+fx)^3 \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=356

$$-\frac{(e+fx)^4}{4bf}+\frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd}+\frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd}+\frac{3f(e+fx)^2\text{PolyLog}\left(\right.}{bd^2}$$

```
[Out] -1/4*(f*x+e)^4/b/f+(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d+(f*
x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d+3*f*(f*x+e)^2*polylog(2,-
b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d^2+3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+
c)/(a+(a^2+b^2)^(1/2)))/b/d^2-6*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2
+b^2)^(1/2)))/b/d^3-6*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2
)))/b/d^3+6*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d^4+6*f^3*po
lylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d^4
```

Rubi [A]
time = 0.35, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5680, 2221, 2611, 6744, 2320, 6724}

$$\frac{6f^3\text{Li}_4\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4}+\frac{6f^3\text{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^4}-\frac{6f^2(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3}-\frac{6f^2(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^3}+\frac{3f(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{3f(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^4}{4bf}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/4*(e + f*x)^4/(b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
])])/(b*d) + (3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2]))])/(b*d^2) + (3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[
a^2 + b^2]))])/(b*d^2) - (6*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/(b*d^3) - (6*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/(b*d^3) + (6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2]))])/(b*d^4) + (6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/(b*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(e+fx)^4}{4bf} + \int \frac{e^{c+dx}(e+fx)^3}{a-\sqrt{a^2+b^2}+be^{c+dx}}\,dx + \int \frac{e^{c+dx}(e+fx)^3}{a+\sqrt{a^2+b^2}+be^{c+dx}}\,d$$

$$= -\frac{(e+fx)^4}{4bf} + \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^3\log\left(1+\frac{}{a+\sqrt{}}\right.}{bd}$$

$$= -\frac{(e+fx)^4}{4bf} + \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^3\log\left(1+\frac{}{a+\sqrt{}}\right.}{bd}$$

$$= -\frac{(e+fx)^4}{4bf} + \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^3\log\left(1+\frac{}{a+\sqrt{}}\right.}{bd}$$

$$= -\frac{(e+fx)^4}{4bf} + \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^3\log\left(1+\frac{}{a+\sqrt{}}\right.}{bd}$$

$$= -\frac{(e+fx)^4}{4bf} + \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^3\log\left(1+\frac{}{a+\sqrt{}}\right.}{bd}$$

Mathematica [A]
time = 0.11, size = 329, normalized size = 0.92

$$\frac{-\frac{(e+fx)^4}{f}+\frac{4(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}+\frac{4(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}+\frac{12f\left(d^2(e+fx)^2\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)-2df(e+fx)\text{PolyLog}\left(3,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)+2f^2\text{PolyLog}\left(4,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)\right)}{d^4}+\frac{12f\left(d^2(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)-2df(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)+2f^2\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)}{d^4}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-((e + f*x)^4/f) + (4*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2])])/d + (4*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/
d + (12*f*(d^2*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]
)] - 2*d*f*(e + f*x)*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2
*f^2*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]))/d^4 + (12*f*(d^2*
(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*d*f*(e
 + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 2*f^2*PolyLo
g[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/d^4)/(4*b)
```

Maple [F]
time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3\cosh(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(f^3*x^4 + 4*f^2*x^3*e + 6*f*x^2*e^2)/b + e^3*log(b*sinh(d*x + c) + a)/
(b*d) - integrate(-2*(b*f^3*x^3 + 3*b*f^2*x^2*e + 3*b*f*x*e^2 - (a*f^3*x^3*
e^c + 3*a*f^2*x^2*e^(c + 1) + 3*a*f*x*e^(c + 2))*e^(d*x))/(b^2*e^(2*d*x + 2
*c) + 2*a*b*e^(d*x + c) - b^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1278 vs. $2(335) = 670$.
time = 0.44, size = 1278, normalized size = 3.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(d^4*f^3*x^4 + 4*d^4*f^2*x^3*cosh(1) + 6*d^4*f*x^2*cosh(1)^2 + 4*d^4*x
*cosh(1)^3 + 4*d^4*x*sinh(1)^3 - 24*f^3*polylog(4, (a*cosh(d*x + c) + a*sin
h(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b)
- 24*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*(d^4*f*x^2 + 2*d^4*x*cosh(1
))*sinh(1)^2 - 12*(d^2*f^3*x^2 + 2*d^2*f^2*x*cosh(1) + d^2*f*cosh(1)^2 + d^
2*f*sinh(1)^2 + 2*(d^2*f^2*x + d^2*f*cosh(1))*sinh(1))*dilog((a*cosh(d*x +
c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2) - b)/b + 1) - 12*(d^2*f^3*x^2 + 2*d^2*f^2*x*cosh(1) + d^2*f*cosh(1)^2
 + d^2*f*sinh(1)^2 + 2*(d^2*f^2*x + d^2*f*cosh(1))*sinh(1))*dilog((a*cosh(d
*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2) - b)/b + 1) + 4*(c^3*f^3 - 3*c^2*d*f^2*cosh(1) + 3*c*d^2*f*cosh(
1)^2 - d^3*cosh(1)^3 - d^3*sinh(1)^3 + 3*(c*d^2*f - d^3*cosh(1))*sinh(1)^2
- 3*(c^2*d*f^2 - 2*c*d^2*f*cosh(1) + d^3*cosh(1)^2)*sinh(1))*log(2*b*cosh(d
*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 4*(c^3*f^3
 - 3*c^2*d*f^2*cosh(1) + 3*c*d^2*f*cosh(1)^2 - d^3*cosh(1)^3 - d^3*sinh(1)^
3 + 3*(c*d^2*f - d^3*cosh(1))*sinh(1)^2 - 3*(c^2*d*f^2 - 2*c*d^2*f*cosh(1)
+ d^3*cosh(1)^2)*sinh(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*s
```

```
qrt((a^2 + b^2)/b^2) + 2*a) - 4*(d^3*f^3*x^3 + c^3*f^3 + 3*(d^3*f*x + c*d^2
*f)*cosh(1)^2 + 3*(d^3*f*x + c*d^2*f)*sinh(1)^2 + 3*(d^3*f^2*x^2 - c^2*d*f^
2)*cosh(1) + 3*(d^3*f^2*x^2 - c^2*d*f^2 + 2*(d^3*f*x + c*d^2*f)*cosh(1))*si
nh(1))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 4*(d^3*f^3*x^3 + c^3*f^3 + 3*(d^3
*f*x + c*d^2*f)*cosh(1)^2 + 3*(d^3*f*x + c*d^2*f)*sinh(1)^2 + 3*(d^3*f^2*x^
2 - c^2*d*f^2)*cosh(1) + 3*(d^3*f^2*x^2 - c^2*d*f^2 + 2*(d^3*f*x + c*d^2*f)
*cosh(1))*sinh(1))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 24*(d*f^3*x + d*f^2*c
osh(1) + d*f^2*sinh(1))*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24*(d*f^3*x +
d*f^2*cosh(1) + d*f^2*sinh(1))*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c
) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 4*(d^4*
f^2*x^3 + 3*d^4*f*x^2*cosh(1) + 3*d^4*x*cosh(1)^2)*sinh(1))/(b*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)**3*cosh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^3*cosh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,{(e+f\,x)}^3}{a+b\sinh(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.290 $\int \frac{(e+fx)^2 \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=264

$$-\frac{(e+fx)^3}{3bf}+\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd}+\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd}+\frac{2f(e+fx)\text{PolyLog}\left(2\right.}{bd^2}$$

```
[Out] -1/3*(f*x+e)^3/b/f+(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d+(f*
x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d+2*f*(f*x+e)*polylog(2,-b*
exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d^2+2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(
a+(a^2+b^2)^(1/2)))/b/d^2-2*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2))
)/b/d^3-2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d^3
```

Rubi [A]
time = 0.31, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5680, 2221, 2611, 2320, 6724}

$$-\frac{2f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3}-\frac{2f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^3}+\frac{2f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{2f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^3}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/3*(e + f*x)^3/(b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
])])/(b*d) + (2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2]))])/(b*d^2) + (2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2]))])/(b*d^2) - (2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2]))])/(b*d^3) - (2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/(b*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(e+fx)^3}{3bf} + \int \frac{e^{c+dx}(e+fx)^2}{a-\sqrt{a^2+b^2}+be^{c+dx}}\,dx + \int \frac{e^{c+dx}(e+fx)^2}{a+\sqrt{a^2+b^2}+be^{c+dx}}\,d$$

$$= -\frac{(e+fx)^3}{3bf} + \frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^2\log\left(1+\frac{}{a+\sqrt{}}\right.}{bd}$$

$$= -\frac{(e+fx)^3}{3bf} + \frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^2\log\left(1+\frac{}{a+\sqrt{}}\right.}{bd}$$

$$= -\frac{(e+fx)^3}{3bf} + \frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^2\log\left(1+\frac{}{a+\sqrt{}}\right.}{bd}$$

$$= -\frac{(e+fx)^3}{3bf} + \frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^2\log\left(1+\frac{}{a+\sqrt{}}\right.}{bd}$$

Mathematica [A]
time = 0.11, size = 244, normalized size = 0.92

$$\frac{-\frac{(e+fx)^3}{f}+\frac{3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}+\frac{3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d}+\frac{6f\left(d(e+fx)\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)-f\text{PolyLog}\left(3,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)\right)}{d^3}+\frac{6f\left(d(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)-f\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)}{d^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-((e + f*x)^3/f) + (3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2])])/d + (3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/
d + (6*f*(d*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] -
f*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]))/d^3 + (6*f*(d*(e + f
*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - f*PolyLog[3, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/d^3)/(3*b)
```

Maple [F]
time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2\cosh(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*(f^2*x^3 + 3*f*x^2*e)/b + e^2*log(b*sinh(d*x + c) + a)/(b*d) - integrat
e(-2*(b*f^2*x^2 + 2*b*f*x*e - (a*f^2*x^2*e^c + 2*a*f*x*e^(c + 1))*e^(d*x))/
(b^2*e^(2*d*x + 2*c) + 2*a*b*e^(d*x + c) - b^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 738 vs. 2(247) = 494.
time = 0.36, size = 738, normalized size = 2.80

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(d^3*f^2*x^3 + 3*d^3*f*x^2*cosh(1) + 3*d^3*x*cosh(1)^2 + 3*d^3*x*sinh(
1)^2 + 6*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*f^2*polylog(3, (a*cosh(
d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2))/b) - 6*(d*f^2*x + d*f*cosh(1) + d*f*sinh(1))*dilog((a*cosh(d*x
 + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2) - b)/b + 1) - 6*(d*f^2*x + d*f*cosh(1) + d*f*sinh(1))*dilog((a*cos
h(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2) - b)/b + 1) - 3*(c^2*f^2 - 2*c*d*f*cosh(1) + d^2*cosh(1)^2 +
d^2*sinh(1)^2 - 2*(c*d*f - d^2*cosh(1))*sinh(1))*log(2*b*cosh(d*x + c) + 2*
b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(c^2*f^2 - 2*c*d*f*c
osh(1) + d^2*cosh(1)^2 + d^2*sinh(1)^2 - 2*(c*d*f - d^2*cosh(1))*sinh(1))*l
og(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
 - 3*(d^2*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)*cosh(1) + 2*(d^2*f*x + c*
d*f)*sinh(1))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 3*(d^2*f^2*x^2 - c^2*f^2 +
 2*(d^2*f*x + c*d*f)*cosh(1) + 2*(d^2*f*x + c*d*f)*sinh(1))*log(-(a*cosh(d*
x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b) + 3*(d^3*f*x^2 + 2*d^3*x*cosh(1))*sinh(1))/(b*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2 \cosh{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*cosh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{cosh}\left(c+d\,x\right)\,{\left(e+f\,x\right)}^2}{a+b\,\mathrm{sinh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.291 $\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=170

$$-\frac{(e+fx)^2}{2bf}+\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd}+\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd}+\frac{f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}$$

```
[Out] -1/2*(f*x+e)^2/b/f+(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/d+(f*x+
e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d+f*polylog(2,-b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/b/d^2+f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/d^
2
```

Rubi [A]

time = 0.17, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5680, 2221, 2317, 2438}

$$\frac{f\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/2*(e + f*x)^2/(b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])
/(b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) +
 (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(e+fx)^2}{2bf}+\int \frac{e^{c+dx}(e+fx)}{a-\sqrt{a^2+b^2}+be^{c+dx}}\,dx+\int \frac{e^{c+dx}(e+fx)}{a+\sqrt{a^2+b^2}+be^{c+dx}}\,dx$$

$$= -\frac{(e+fx)^2}{2bf}+\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd}+\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-}}\right.}{bd}$$

$$= -\frac{(e+fx)^2}{2bf}+\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd}+\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-}}\right.}{bd}$$

$$= -\frac{(e+fx)^2}{2bf}+\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd}+\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-}}\right.}{bd}$$

Mathematica [A]
time = 0.03, size = 157, normalized size = 0.92

$$\frac{-d(e+fx)\left(de+dfx-2f\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-2f\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)+2f^2\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)+2f^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{2bd^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
[Out] (-(d*(e + f*x)*(d*e + d*f*x - 2*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])] - 2*f*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])) + 2*f^2*PolyLo
g[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f^2*PolyLog[2, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))])/(2*b*d^2*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(156) = 312$.
time = 1.53, size = 412, normalized size = 2.42

method	result
risch	$-\frac{f\,x^{2}}{2b}+\frac{ex}{b}+\frac{2fc\ln\left(\mathrm{e}^{dx+c}\right)}{d^{2}b}-\frac{fc\ln\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-b\right)}{d^{2}b}-\frac{2e\ln\left(\mathrm{e}^{dx+c}\right)}{db}+\frac{e\ln\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-b\right)}{db}-\frac{2cfx}{db}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*f*x^2/b+e*x/b+2/d^2/b*f*c*ln(exp(d*x+c))-1/d^2/b*f*c*ln(b*exp(2*d*x+2*
c)+2*a*exp(d*x+c)-b)-2/d/b*e*ln(exp(d*x+c))+1/d/b*e*ln(b*exp(2*d*x+2*c)+2*a
*exp(d*x+c)-b)-2/d/b*c*f*x-1/d^2/b*f*c^2+1/d/b*f*ln((-b*exp(d*x+c)+(a^2+b^2
)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d^2/b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1
/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d/b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/
(a+(a^2+b^2)^(1/2)))*x+1/d^2/b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^
2+b^2)^(1/2)))*c+1/d^2/b*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b
^2)^(1/2)))+1/d^2/b*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)
^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*f*(x^2/b - integrate(4*(a*x*e^(d*x + c) - b*x)/(b^2*e^(2*d*x + 2*c) + 2
*a*b*e^(d*x + c) - b^2), x)) + e*log(b*sinh(d*x + c) + a)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 401 vs. 2(157) = 314.
time = 0.37, size = 401, normalized size = 2.36

$$-\frac{d^{2} f x^{2}+2 d^{2} x \cosh(1)+2 d^{2} x \sinh(1)-2 f \operatorname{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^{2}+b^{2}}{b^{2}}}-b}{b}+1\right)-2 f \operatorname{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^{2}+b^{2}}{b^{2}}}-b}{b}+1\right)+2(cf-d\cosh(1)-d\sinh(1))\log\left(2b\cosh(dx+c)+2b\sinh(dx+c)+2b\sqrt{\frac{a^{2}+b^{2}}{b^{2}}}+2a\right)+2(cf-d\cosh(1)-d\sinh(1))\log\left(2b\cosh(dx+c)+2b\sinh(dx+c)-2b\sqrt{\frac{a^{2}+b^{2}}{b^{2}}}+2a\right)-2(dfx+cf)\log\left(-\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^{2}+b^{2}}{b^{2}}}-b}{b}\right)-2(dfx+cf)\log\left(-\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^{2}+b^{2}}{b^{2}}}-b}{b}\right)}{2bd^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*f*x^2 + 2*d^2*x*cosh(1) + 2*d^2*x*sinh(1) - 2*f*dilog((a*cosh(d*x
 + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2) - b)/b + 1) - 2*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*co
sh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(c*f -
 d*cosh(1) - d*sinh(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqr
t((a^2 + b^2)/b^2) + 2*a) + 2*(c*f - d*cosh(1) - d*sinh(1))*log(2*b*cosh(d*
```

```
x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(d*f*x +
c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(d*f*x + c*f)*log(-(a*cosh(d*x +
c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2) - b)/b))/(b*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)*cosh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.292 $\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=18

$$\frac{\log(a+b\sinh(c+dx))}{bd}$$

```
[Out] ln(a+b*sinh(d*x+c))/b/d
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2747, 31}

$$\frac{\log(a+b\sinh(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Log[a + b*Sinh[c + d*x]]/(b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh(c+dx)\right)}{bd}\\ &= \frac{\log(a+b\sinh(c+dx))}{bd}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$\frac{\log(a+b\sinh(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Log[a + b*Sinh[c + d*x]]/(b*d)
```

Maple [A]
time = 0.43, size = 19, normalized size = 1.06

method	result	size
derivativedivides	$\frac{\ln(a+b\sinh(dx+c))}{bd}$	19
default	$\frac{\ln(a+b\sinh(dx+c))}{bd}$	19
risch	$-\frac{x}{b}-\frac{2c}{bd}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{bd}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a+b*sinh(d*x+c))/b/d
```

Maxima [A]
time = 0.27, size = 18, normalized size = 1.00

$$\frac{\log\left(b\sinh\left(dx+c\right)+a\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] log(b*sinh(d*x + c) + a)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(18) = 36$.
time = 0.35, size = 44, normalized size = 2.44

$$-\frac{dx-\log\left(\frac{2\left(b\sinh(dx+c)+a\right)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(d*x - log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))))/(b*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(14) = 28$.

time = 0.39, size = 41, normalized size = 2.28

$$\begin{cases} \frac{x\cosh(c)}{a} & \text{for } b = 0 \wedge d = 0 \\ \frac{x\cosh(c)}{a+b\sinh(c)} & \text{for } d = 0 \\ \frac{\sinh(c+dx)}{ad} & \text{for } b = 0 \\ \frac{\log\left(\frac{a}{b}+\sinh(c+dx)\right)}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Piecewise((x*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (x*cosh(c)/(a + b*sinh(c)), E
q(d, 0)), (sinh(c + d*x)/(a*d), Eq(b, 0)), (log(a/b + sinh(c + d*x))/(b*d),
 True))
```

Giac [A]
time = 0.42, size = 33, normalized size = 1.83

$$\frac{\log\left(\left|b\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 2\,a\right|\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(b*d)
```

Mupad [B]
time = 0.08, size = 18, normalized size = 1.00

$$\frac{\ln\left(a + b\sinh(c + d\,x)\right)}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*sinh(c + d*x)),x)
```

```
[Out] log(a + b*sinh(c + d*x))/(b*d)
```

3.293 $\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cosh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Cosh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 8.76, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cosh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Integrate[Cosh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] log(f*x + e)/(b*f) - 1/2*integrate(-4*(a*e^(d*x + c) - b)/(b^2*f*x + b^2*e
- (b^2*f*x*e^(2*c) + b^2*e^(2*c + 1))*e^(2*d*x) - 2*(a*b*f*x*e^c + a*b*e^(c
 + 1))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(cosh(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))),x)
[Out] int(cosh(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.294 $\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=527

$$-\frac{a(e+fx)^4}{4b^2f}+\frac{6f^2(e+fx)\cosh(c+dx)}{bd^3}+\frac{(e+fx)^3\cosh(c+dx)}{bd}+\frac{\sqrt{a^2+b^2}\,(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{b^2d}$$

```
[Out] -1/4*a*(f*x+e)^4/b^2/f+6*f^2*(f*x+e)*cosh(d*x+c)/b/d^3+(f*x+e)^3*cosh(d*x+c
)/b/d-6*f^3*sinh(d*x+c)/b/d^4-3*f*(f*x+e)^2*sinh(d*x+c)/b/d^2+(f*x+e)^3*ln(
1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^2/d-(f*x+e)^3*ln(1+b*
exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^2/d+3*f*(f*x+e)^2*polylog
(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^2/d^2-3*f*(f*x+e)^2
*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^2/d^2-6*f^2
*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^2/d
^3+6*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/
2)/b^2/d^3+6*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/
2)/b^2/d^4-6*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/
2)/b^2/d^4
```

Rubi [A]

time = 0.66, antiderivative size = 527, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {5684, 32, 3377, 2717, 3403, 2296, 2221, 2611, 6744, 2320, 6724}

$$\frac{6f^3\sqrt{a^2+b^2}\,\text{Li}_4\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^4}-\frac{6f^3\sqrt{a^2+b^2}\,\text{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^4}-\frac{6f^2\sqrt{a^2+b^2}\,(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3}+\frac{6f^2\sqrt{a^2+b^2}\,(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^3}+\frac{3f\sqrt{a^2+b^2}\,(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2}-\frac{3f\sqrt{a^2+b^2}\,(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^2}+\frac{\sqrt{a^2+b^2}\,(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^2d}-\frac{\sqrt{a^2+b^2}\,(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^2d}-\frac{a(e+fx)^4}{4b^2f}-\frac{6f^3\sinh(c+dx)}{bd^4}+\frac{6f^2(e+fx)\cosh(c+dx)}{bd^3}-\frac{3f(e+fx)^2\sinh(c+dx)}{bd^2}+\frac{(e+fx)^3\cosh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/4*(a*(e + f*x)^4)/(b^2*f) + (6*f^2*(e + f*x)*Cosh[c + d*x])/(b*d^3) + ((
e + f*x)^3*Cosh[c + d*x])/(b*d) + (Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*d) - (Sqrt[a^2 + b^2]*(e + f*x)^3*
Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*d) + (3*Sqrt[a^2 + b^2
]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*
d^2) - (3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/(b^2*d^2) - (6*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3,
-((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^3) + (6*Sqrt[a^2 + b^2]*f
^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^3
) + (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/(b^2*d^4) - (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2]))])/(b^2*d^4) - (6*f^3*Sinh[c + d*x])/(b*d^4) - (3*f*(e + f
*x)^2*Sinh[c + d*x])/(b*d^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\int (e+fx)^3\,dx}{b^2} + \frac{\int (e+fx)^3\sinh(c+dx)\,dx}{b} + \frac{(a^2+b^2)\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}}{b^2}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{(e+fx)^3\cosh(c+dx)}{bd} + \frac{(2(a^2+b^2))\int \frac{e^{c+dx}(e+fx)^3}{-b+2ae^{c+dx}+be^{2(c+dx)}}}{b^2}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{(e+fx)^3\cosh(c+dx)}{bd} - \frac{3f(e+fx)^2\sinh(c+dx)}{bd^2} + \frac{(2\sqrt{}}{}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cosh(c+dx)}{bd^3} + \frac{(e+fx)^3\cosh(c+dx)}{bd} + \frac{\sqrt{a^2}}{}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cosh(c+dx)}{bd^3} + \frac{(e+fx)^3\cosh(c+dx)}{bd} + \frac{\sqrt{a^2}}{}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cosh(c+dx)}{bd^3} + \frac{(e+fx)^3\cosh(c+dx)}{bd} + \frac{\sqrt{a^2}}{}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cosh(c+dx)}{bd^3} + \frac{(e+fx)^3\cosh(c+dx)}{bd} + \frac{\sqrt{a^2}}{}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cosh(c+dx)}{bd^3} + \frac{(e+fx)^3\cosh(c+dx)}{bd} + \frac{\sqrt{a^2}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1055 vs. $2(527) = 1054$.
time = 9.07, size = 1055, normalized size = 2.00

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/4*(a*d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) - 4*b*d*(e + f*x)
*(6*f^2 + d^2*(e + f*x)^2)*Cosh[c + d*x] + (4*Sqrt[-a^2 - b^2]*(2*d^3*Sqrt[
(a^2 + b^2)*E^(2*c)]*(e + f*x)^3*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2
]] - 3*d^2*f*(e + f*x)^2*(d*x*(2*Sqrt[(a^2 + b^2)*E^(2*c)]*ArcTan[(a + b*E^
(c + d*x))/Sqrt[-a^2 - b^2]] + Sqrt[-a^2 - b^2]*E^c*(-Log[1 + (b*E^(2*c + d
*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + Log[1 + (b*E^(2*c + d*x))/(a*E^
```

```
c + Sqrt[(a^2 + b^2)*E^(2*c)])])) - Sqrt[-a^2 - b^2]*E^c*PolyLog[2, -((b*E^
(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + Sqrt[-a^2 - b^2]*E^c*P
olyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]) + 3*d*
f^2*(e + f*x)*(d^2*x^2*(2*Sqrt[(a^2 + b^2)*E^(2*c)]*ArcTan[(a + b*E^(c + d*
x))/Sqrt[-a^2 - b^2]] + Sqrt[-a^2 - b^2]*E^c*(-Log[1 + (b*E^(2*c + d*x))/(a
*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqr
t[(a^2 + b^2)*E^(2*c)])])) - 2*Sqrt[-a^2 - b^2]*E^c*PolyLog[3, -((b*E^(2*c
+ d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2*Sqrt[-a^2 - b^2]*E^c*Poly
Log[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]) - f^3*(d^
3*x^3*(2*Sqrt[(a^2 + b^2)*E^(2*c)]*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b
^2]] + Sqrt[-a^2 - b^2]*E^c*(-Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2
+ b^2)*E^(2*c)])] + Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(
2*c)])])) - 6*Sqrt[-a^2 - b^2]*E^c*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c -
Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[-a^2 - b^2]*E^c*PolyLog[4, -((b*E^(2*
c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])))/Sqrt[(a^2 + b^2)*E^(2*c)
] + 12*b*f*(2*f^2 + d^2*(e + f*x)^2)*Sinh[c + d*x])/(b^2*d^4)
```

Maple [F]
time = 2.16, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \left(\cosh^2(dx+c)\right)}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) - e^(-d*x - c)/(b*d) - 2*sq
rt(a^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) -
a + sqrt(a^2 + b^2)))/(b^2*d))*e^3 - 1/4*(a*d^4*f^3*x^4*e^c + 4*a*d^4*f^2*x
^3*e^(c + 1) + 6*a*d^4*f*x^2*e^(c + 2) - 2*(b*d^3*f^3*x^3*e^(2*c) - 6*b*f^3
*e^(2*c) - 3*b*d^2*f*e^(2*c + 2) + 6*b*d*f^2*e^(2*c + 1) - 3*(b*d^2*f^3*e^(
2*c) - b*d^3*f^2*e^(2*c + 1))*x^2 + 3*(2*b*d*f^3*e^(2*c) + b*d^3*f*e^(2*c +
 2) - 2*b*d^2*f^2*e^(2*c + 1))*x)*e^(d*x) - 2*(b*d^3*f^3*x^3 + 3*b*d^2*f*e^
2 + 6*b*d*f^2*e + 6*b*f^3 + 3*(b*d^3*f^2*e + b*d^2*f^3)*x^2 + 3*(b*d^3*f*e^
2 + 2*b*d^2*f^2*e + 2*b*d*f^3)*x)*e^(-d*x))*e^(-c)/(b^2*d^4) + integrate(2*
((a^2*f^3*e^c + b^2*f^3*e^c)*x^3 + 3*(a^2*f^2*e^c + b^2*f^2*e^c)*x^2*e + 3*
```

```
(a^2*f*e^c + b^2*f*e^c)*x*e^2)*e^(d*x)/(b^3*e^(2*d*x + 2*c) + 2*a*b^2*e^(d*
x + c) - b^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3291 vs. $2(493) = 986$.
time = 0.42, size = 3291, normalized size = 6.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*d^3*f^3*x^3 + 6*b*d^2*f^3*x^2 + 2*b*d^3*cosh(1)^3 + 2*b*d^3*sinh(1
)^3 + 12*b*d*f^3*x + 12*b*f^3 + 6*(b*d^3*f*x + b*d^2*f)*cosh(1)^2 + 2*(b*d^
3*f^3*x^3 - 3*b*d^2*f^3*x^2 + b*d^3*cosh(1)^3 + b*d^3*sinh(1)^3 + 6*b*d*f^3
*x - 6*b*f^3 + 3*(b*d^3*f*x - b*d^2*f)*cosh(1)^2 + 3*(b*d^3*f*x + b*d^3*cos
h(1) - b*d^2*f)*sinh(1)^2 + 3*(b*d^3*f^2*x^2 - 2*b*d^2*f^2*x + 2*b*d*f^2)*c
osh(1) + 3*(b*d^3*f^2*x^2 - 2*b*d^2*f^2*x + b*d^3*cosh(1)^2 + 2*b*d*f^2 + 2
*(b*d^3*f*x - b*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 6*(b*d^3*f*x + b
*d^3*cosh(1) + b*d^2*f)*sinh(1)^2 + 2*(b*d^3*f^3*x^3 - 3*b*d^2*f^3*x^2 + b*
d^3*cosh(1)^3 + b*d^3*sinh(1)^3 + 6*b*d*f^3*x - 6*b*f^3 + 3*(b*d^3*f*x - b*
d^2*f)*cosh(1)^2 + 3*(b*d^3*f*x + b*d^3*cosh(1) - b*d^2*f)*sinh(1)^2 + 3*(b
*d^3*f^2*x^2 - 2*b*d^2*f^2*x + 2*b*d*f^2)*cosh(1) + 3*(b*d^3*f^2*x^2 - 2*b*
d^2*f^2*x + b*d^3*cosh(1)^2 + 2*b*d*f^2 + 2*(b*d^3*f*x - b*d^2*f)*cosh(1))*
sinh(1))*sinh(d*x + c)^2 + 12*((b*d^2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d
^2*f*cosh(1)^2 + b*d^2*f*sinh(1)^2 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh
(1))*cosh(d*x + c) + (b*d^2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^2*f*cosh(
1)^2 + b*d^2*f*sinh(1)^2 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh(1))*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) +
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 12*
((b*d^2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^2*f*cosh(1)^2 + b*d^2*f*sinh(
1)^2 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c) + (b*d^2*f^
3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^2*f*cosh(1)^2 + b*d^2*f*sinh(1)^2 + 2*(
b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2
)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 4*((b*c^3*f^3 - 3*b*c^2*d*f^2*cos
h(1) + 3*b*c*d^2*f*cosh(1)^2 - b*d^3*cosh(1)^3 - b*d^3*sinh(1)^3 + 3*(b*c*d
^2*f - b*d^3*cosh(1))*sinh(1)^2 - 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(1) + b*
d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c) + (b*c^3*f^3 - 3*b*c^2*d*f^2*cosh(1)
+ 3*b*c*d^2*f*cosh(1)^2 - b*d^3*cosh(1)^3 - b*d^3*sinh(1)^3 + 3*(b*c*d^2*f
- b*d^3*cosh(1))*sinh(1)^2 - 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(1) + b*d^3*c
osh(1)^2)*sinh(1))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x +
c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 4*((b*c^3*f^3 -
 3*b*c^2*d*f^2*cosh(1) + 3*b*c*d^2*f*cosh(1)^2 - b*d^3*cosh(1)^3 - b*d^3*si
nh(1)^3 + 3*(b*c*d^2*f - b*d^3*cosh(1))*sinh(1)^2 - 3*(b*c^2*d*f^2 - 2*b*c*
```

```
d^2*f*cosh(1) + b*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c) + (b*c^3*f^3 - 3*b*
c^2*d*f^2*cosh(1) + 3*b*c*d^2*f*cosh(1)^2 - b*d^3*cosh(1)^3 - b*d^3*sinh(1)
^3 + 3*(b*c*d^2*f - b*d^3*cosh(1))*sinh(1)^2 - 3*(b*c^2*d*f^2 - 2*b*c*d^2*f
*cosh(1) + b*d^3*cosh(1)^2)*sinh(1))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*l
og(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
 + 4*((b*d^3*f^3*x^3 + b*c^3*f^3 + 3*(b*d^3*f*x + b*c*d^2*f)*cosh(1)^2 + 3*
(b*d^3*f*x + b*c*d^2*f)*sinh(1)^2 + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2)*cosh(1)
 + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2 + 2*(b*d^3*f*x + b*c*d^2*f)*cosh(1))*sinh
(1))*cosh(d*x + c) + (b*d^3*f^3*x^3 + b*c^3*f^3 + 3*(b*d^3*f*x + b*c*d^2*f)
*cosh(1)^2 + 3*(b*d^3*f*x + b*c*d^2*f)*sinh(1)^2 + 3*(b*d^3*f^2*x^2 - b*c^2
*d*f^2)*cosh(1) + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2 + 2*(b*d^3*f*x + b*c*d^2*f
)*cosh(1))*sinh(1))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x +
 c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2
)/b^2) - b)/b) - 4*((b*d^3*f^3*x^3 + b*c^3*f^3 + 3*(b*d^3*f*x + b*c*d^2*f)*
cosh(1)^2 + 3*(b*d^3*f*x + b*c*d^2*f)*sinh(1)^2 + 3*(b*d^3*f^2*x^2 - b*c^2*
d*f^2)*cosh(1) + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2 + 2*(b*d^3*f*x + b*c*d^2*f)
*cosh(1))*sinh(1))*cosh(d*x + c) + (b*d^3*f^3*x^3 + b*c^3*f^3 + 3*(b*d^3*f*
x + b*c*d^2*f)*cosh(1)^2 + 3*(b*d^3*f*x + b*c*d^2*f)*sinh(1)^2 + 3*(b*d^3*f
^2*x^2 - b*c^2*d*f^2)*cosh(1) + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2 + 2*(b*d^3*f
*x + b*c*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(
-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 + b^2)/b^2) - b)/b) + 24*(b*f^3*cosh(d*x + c) + b*f^3*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 24*(b*f^3*cos
h(d*x + c) + b*f^3*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(
d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2))/b) - 24*((b*d*f^3*x + b*d*f^2*cosh(1) + b*d*f^2*sinh(1))*cosh(
d*x + c) + (b*d*f^3*x + b*d*f^2*cosh(1) + b*d*f^2*sinh(1))*sinh(d*x + c))*s
qrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24*((b*d*f^3*x +
b*d*f^2*cosh(1) + b*d*f^2*sinh(1))*cosh(d*x + c) + (b*d*f^3*x + b*d*f^2*cos
h(1) + b*d*f^2*sinh(1))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*
cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2))/b) + 6*(b*d^3*f^2*x^2 + 2*b*d...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^3*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.295 $\int \frac{(e+fx)^2 \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=389

$$-\frac{a(e+fx)^3}{3b^2f}+\frac{2f^2\cosh(c+dx)}{bd^3}+\frac{(e+fx)^2\cosh(c+dx)}{bd}+\frac{\sqrt{a^2+b^2}\,(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{\sqrt{}}{}$$

```
[Out] -1/3*a*(f*x+e)^3/b^2/f+2*f^2*cosh(d*x+c)/b/d^3+(f*x+e)^2*cosh(d*x+c)/b/d-2*
f*(f*x+e)*sinh(d*x+c)/b/d^2+(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2))
)*(a^2+b^2)^(1/2)/b^2/d-(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a
^2+b^2)^(1/2)/b^2/d+2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2))
)*(a^2+b^2)^(1/2)/b^2/d^2-2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^
(1/2)))*(a^2+b^2)^(1/2)/b^2/d^2-2*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^
(1/2)))*(a^2+b^2)^(1/2)/b^2/d^3+2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^
(1/2)))*(a^2+b^2)^(1/2)/b^2/d^3
```

Rubi [A]
time = 0.57, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5684, 32, 3377, 2718, 3403, 2296, 2221, 2611, 2320, 6724}

$$-\frac{2f^2\sqrt{a^2+b^2}\,\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3}+\frac{2f^2\sqrt{a^2+b^2}\,\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^3}+\frac{2f\sqrt{a^2+b^2}\,(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2}-\frac{2f\sqrt{a^2+b^2}\,(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^2}+\frac{\sqrt{a^2+b^2}\,(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^2d}-\frac{\sqrt{a^2+b^2}\,(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^2d}-\frac{a(e+fx)^3}{3b^2f}+\frac{2f^2\cosh(c+dx)}{bd^3}-\frac{2f(e+fx)\sinh(c+dx)}{bd^2}+\frac{(e+fx)^2\cosh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/3*(a*(e + f*x)^3)/(b^2*f) + (2*f^2*Cosh[c + d*x])/(b*d^3) + ((e + f*x)^2
*Cosh[c + d*x])/(b*d) + (Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2])])/(b^2*d) - (Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*d) + (2*Sqrt[a^2 + b^2]*f*(e + f
*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^2) - (2*Sq
rt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
))])/(b^2*d^2) - (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - S
qrt[a^2 + b^2]))])/(b^2*d^3) + (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^3) - (2*f*(e + f*x)*Sinh[c + d*x])/
(b*d^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\int (e+fx)^2\,dx}{b^2} + \frac{\int (e+fx)^2\sinh(c+dx)\,dx}{b} + \frac{(a^2+b^2)\int \frac{(e+fx)^2}{a+b\sinh(c+dx}}{b^2}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{(e+fx)^2\cosh(c+dx)}{bd} + \frac{(2(a^2+b^2))\int \frac{e^{c+dx}(e+fx)^2}{-b+2ae^{c+dx}+be^{2(c+dx)}}}{b^2}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{(e+fx)^2\cosh(c+dx)}{bd} - \frac{2f(e+fx)\sinh(c+dx)}{bd^2} + \frac{\left(2\sqrt{\right.}}{}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cosh(c+dx)}{bd^3} + \frac{(e+fx)^2\cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}\,(e}{}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cosh(c+dx)}{bd^3} + \frac{(e+fx)^2\cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}\,(e}{}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cosh(c+dx)}{bd^3} + \frac{(e+fx)^2\cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}\,(e}{}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cosh(c+dx)}{bd^3} + \frac{(e+fx)^2\cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}\,(e}{}$$

Mathematica [A]

time = 4.72, size = 723, normalized size = 1.86

$$\frac{-ax(3e^2+3efx+f^2x^2) + \frac{3(a^2+b^2)\left(\frac{2d^2e^2\text{ArcTan}\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{2d^2ee^cfx\log\left(1+\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}} + \frac{d^2e^cf^2x^2\log\left(1+\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}} - \frac{2d^2ee^cfx\log\left(1+\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}} - \frac{d^2e^cf^2x^2\log\left(1+\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}} + \frac{2de^cf(e+fx)\text{PolyLog}\left(2,-\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}} - \frac{2de^cf(e+fx)\text{PolyLog}\left(2,-\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}} - \frac{2e^cf^2\text{PolyLog}\left(3,-\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}} + \frac{2e^cf^2\text{PolyLog}\left(3,-\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}}\right)}{d^3} + \frac{3b\cosh(dx)\left((2f^2+d^2(e+fx)^2)\cosh(c)-2df(e+fx)\sinh(c)\right)}{d^3} + \frac{3b\left(-2df(e+fx)\cosh(c)+(2f^2+d^2(e+fx)^2)\sinh(c)\right)\sinh(dx)}{d^3}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-(a*x*(3*e^2 + 3*e*f*x + f^2*x^2)) + (3*(a^2 + b^2)*((2*d^2*e^2*ArcTan[(a
+ b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + (2*d^2*e*E^c*f*x*Log
[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^
2)*E^(2*c)] + (d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2
 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*d^2*e*E^c*f*x*Log[1 + (b
*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2
*c)] - (d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)
*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] + (2*d*E^c*f*(e + f*x)*PolyLog[2, -(
(b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E
^(2*c)] - (2*d*E^c*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt
[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*E^c*f^2*PolyLog[3,
 -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2
)*E^(2*c)] + (2*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 +
 b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)]))/d^3 + (3*b*Cosh[d*x]*((2*f^2
 + d^2*(e + f*x)^2)*Cosh[c] - 2*d*f*(e + f*x)*Sinh[c]))/d^3 + (3*b*(-2*d*f*
(e + f*x)*Cosh[c] + (2*f^2 + d^2*(e + f*x)^2)*Sinh[c])*Sinh[d*x])/d^3)/(3*b
^2)
```

Maple [F]
time = 2.08, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \left(\cosh^2(dx+c)\right)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) - e^(-d*x - c)/(b*d) - 2*sq
rt(a^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) -
a + sqrt(a^2 + b^2)))/(b^2*d))*e^2 - 1/6*(2*a*d^3*f^2*x^3*e^c + 6*a*d^3*f*x
^2*e^(c + 1) - 3*(b*d^2*f^2*x^2*e^(2*c) + 2*b*f^2*e^(2*c) - 2*b*d*f*e^(2*c
+ 1) - 2*(b*d*f^2*e^(2*c) - b*d^2*f*e^(2*c + 1))*x)*e^(d*x) - 3*(b*d^2*f^2*
```

```
x^2 + 2*b*d*f*e + 2*b*f^2 + 2*(b*d^2*f*e + b*d*f^2)*x)*e^(-d*x))*e^(-c)/(b^
2*d^3) + integrate(2*((a^2*f^2*e^c + b^2*f^2*e^c)*x^2 + 2*(a^2*f*e^c + b^2*
f*e^c)*x*e)*e^(d*x)/(b^3*e^(2*d*x + 2*c) + 2*a*b^2*e^(d*x + c) - b^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1741 vs. $2(362) = 724$.
time = 0.40, size = 1741, normalized size = 4.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*b*d^2*f^2*x^2 + 6*b*d*f^2*x + 3*b*d^2*cosh(1)^2 + 3*b*d^2*sinh(1)^2
+ 6*b*f^2 + 3*(b*d^2*f^2*x^2 - 2*b*d*f^2*x + b*d^2*cosh(1)^2 + b*d^2*sinh(1
)^2 + 2*b*f^2 + 2*(b*d^2*f*x - b*d*f)*cosh(1) + 2*(b*d^2*f*x + b*d^2*cosh(1
) - b*d*f)*sinh(1))*cosh(d*x + c)^2 + 3*(b*d^2*f^2*x^2 - 2*b*d*f^2*x + b*d^
2*cosh(1)^2 + b*d^2*sinh(1)^2 + 2*b*f^2 + 2*(b*d^2*f*x - b*d*f)*cosh(1) + 2
*(b*d^2*f*x + b*d^2*cosh(1) - b*d*f)*sinh(1))*sinh(d*x + c)^2 + 12*((b*d*f^
2*x + b*d*f*cosh(1) + b*d*f*sinh(1))*cosh(d*x + c) + (b*d*f^2*x + b*d*f*cos
h(1) + b*d*f*sinh(1))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*
x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b + 1) - 12*((b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1))*cos
h(d*x + c) + (b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1))*sinh(d*x + c))*sqr
t((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x +
 c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 6*((b*c^2*f^2 -
2*b*c*d*f*cosh(1) + b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 - 2*(b*c*d*f - b*d^2*
cosh(1))*sinh(1))*cosh(d*x + c) + (b*c^2*f^2 - 2*b*c*d*f*cosh(1) + b*d^2*co
sh(1)^2 + b*d^2*sinh(1)^2 - 2*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*sinh(d*x +
 c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*
sqrt((a^2 + b^2)/b^2) + 2*a) + 6*((b*c^2*f^2 - 2*b*c*d*f*cosh(1) + b*d^2*co
sh(1)^2 + b*d^2*sinh(1)^2 - 2*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*cosh(d*x +
 c) + (b*c^2*f^2 - 2*b*c*d*f*cosh(1) + b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 -
2*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*l
og(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
 + 6*((b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*cosh(1) + 2*(b*d
^2*f*x + b*c*d*f)*sinh(1))*cosh(d*x + c) + (b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(
b*d^2*f*x + b*c*d*f)*cosh(1) + 2*(b*d^2*f*x + b*c*d*f)*sinh(1))*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 6*((b*d^2*f^2*
x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*cosh(1) + 2*(b*d^2*f*x + b*c*d*f)
*sinh(1))*cosh(d*x + c) + (b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d
*f)*cosh(1) + 2*(b*d^2*f*x + b*c*d*f)*sinh(1))*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sin
h(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 12*(b*f^2*cosh(d*x + c) + b*f^2
```

```
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(
d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) +
12*(b*f^2*cosh(d*x + c) + b*f^2*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylo
g(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2))/b) + 6*(b*d^2*f*x + b*d*f)*cosh(1) - 2*(a*d^3*f^2
*x^3 + 3*a*d^3*f*x^2*cosh(1) + 3*a*d^3*x*cosh(1)^2 + 3*a*d^3*x*sinh(1)^2 +
3*(a*d^3*f*x^2 + 2*a*d^3*x*cosh(1))*sinh(1))*cosh(d*x + c) + 6*(b*d^2*f*x +
 b*d^2*cosh(1) + b*d*f)*sinh(1) - 2*(a*d^3*f^2*x^3 + 3*a*d^3*f*x^2*cosh(1)
+ 3*a*d^3*x*cosh(1)^2 + 3*a*d^3*x*sinh(1)^2 - 3*(b*d^2*f^2*x^2 - 2*b*d*f^2*
x + b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 + 2*b*f^2 + 2*(b*d^2*f*x - b*d*f)*cos
h(1) + 2*(b*d^2*f*x + b*d^2*cosh(1) - b*d*f)*sinh(1))*cosh(d*x + c) + 3*(a*
d^3*f*x^2 + 2*a*d^3*x*cosh(1))*sinh(1))*sinh(d*x + c))/(b^2*d^3*cosh(d*x +
c) + b^2*d^3*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^2*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^2\,{(e+f\,x)}^2}{a+b\,\mathrm{sinh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.296 $\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=252

$$-\frac{aex}{b^2}-\frac{afx^2}{2b^2}+\frac{(e+fx)\cosh(c+dx)}{bd}+\frac{\sqrt{a^2+b^2}\,(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{\sqrt{a^2+b^2}\,(e+fx)\log}{b^2d}$$

[Out] -a*e*x/b^2-1/2*a*f*x^2/b^2+(f*x+e)*cosh(d*x+c)/b/d-f*sinh(d*x+c)/b/d^2+(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^2/d-(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^2/d+f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^2/d^2-f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^2/d^2

Rubi [A]
time = 0.33, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5684, 3377, 2717, 3403, 2296, 2221, 2317, 2438}

$$\frac{f\sqrt{a^2+b^2}\,\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2}-\frac{f\sqrt{a^2+b^2}\,\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^2}+\frac{\sqrt{a^2+b^2}\,(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^2d}-\frac{\sqrt{a^2+b^2}\,(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^2d}-\frac{aex}{b^2}-\frac{afx^2}{2b^2}-\frac{f\sinh(c+dx)}{bd^2}+\frac{(e+fx)\cosh(c+dx)}{bd}$$

Antiderivative was successfully verified.

[In] Int[((e + f*x)*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]

[Out] -((a*e*x)/b^2) - (a*f*x^2)/(2*b^2) + ((e + f*x)*Cosh[c + d*x])/(b*d) + (Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*d) - (Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*d) + (Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^2) - (Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^2) - (f*Sinh[c + d*x])/(b*d^2)

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\int(e+fx)\,dx}{b^2} + \frac{\int(e+fx)\sinh(c+dx)\,dx}{b} + \frac{(a^2+b^2)\int\frac{e+fx}{a+b\sinh(c+dx)}\,dx}{b^2}$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx)\cosh(c+dx)}{bd} + \frac{(2(a^2+b^2))\int\frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c+dx)}}}{b^2}$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx)\cosh(c+dx)}{bd} - \frac{f\sinh(c+dx)}{bd^2} + \frac{\left(2\sqrt{a^2+b^2}\right)}{}$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx)\cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}\,(e+fx)\log\left(1+\frac{b}{a-\sqrt{}}\right.}{b^2d}$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx)\cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}\,(e+fx)\log\left(1+\frac{b}{a-\sqrt{}}\right.}{b^2d}$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx)\cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}\,(e+fx)\log\left(1+\frac{b}{a-\sqrt{}}\right.}{b^2d}$$

Mathematica [A]
time = 1.22, size = 258, normalized size = 1.02

$$\frac{a(c+dx)(cf-d(2e+fx))+2bd(e+fx)\cosh(c+dx)+2\sqrt{a^2+b^2}\left(-2de\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2cf\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-f(c+dx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)+f\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)-f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)-2bf\sinh(c+dx)}{2b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*(c + d*x)*(c*f - d*(2*e + f*x)) + 2*b*d*(e + f*x)*Cosh[c + d*x] + 2*Sqrt
[a^2 + b^2]*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*Ar
cTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - f*P
olyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) - 2*b*f*Sinh[c + d*x])
/(2*b^2*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 900 vs. 2(230) = 460.
time = 3.32, size = 901, normalized size = 3.58

method	result

risch	$-\frac{af\,x^2}{2b^2}-\frac{aex}{b^2}+\frac{(dxf+de-f)\mathrm{e}^{dx+c}}{2d^2b}+\frac{(dxf+de+f)\mathrm{e}^{-dx-c}}{2d^2b}-\frac{2a^2e\operatorname{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{d\,b^2\sqrt{a^2+b^2}}-\frac{2e\operatorname{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*f*x^2/b^2-a*e*x/b^2+1/2*(d*f*x+d*e-f)/d^2/b*exp(d*x+c)+1/2*(d*f*x+d*
e+f)/d^2/b*exp(-d*x-c)-2/d/b^2*a^2*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d
*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+
c)+2*a)/(a^2+b^2)^(1/2))+1/d/b^2*a^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a
^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d^2/b^2*a^2*f/(a^2+b^2)^(1/2)*ln
((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d/b^2*a^2*f/(a
^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/
d^2/b^2*a^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b
^2)^(1/2)))*c+1/d^2/b^2*a^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2
)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2/b^2*a^2*f/(a^2+b^2)^(1/2)*dilog((b*e
xp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/d*f/(a^2+b^2)^(1/2)*ln(
(-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d^2*f/(a^2+b^2)
^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d*f/(
a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1
/d^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/
2)))*c+1/d^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+
(a^2+b^2)^(1/2)))-1/d^2*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/
2)+a)/(a+(a^2+b^2)^(1/2)))+2/d^2/b^2*a^2*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2
*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d^2*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(
2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(4*(a^2*e^c + b^2*e^c)*integrate(x*e^(d*x)/(b^3*e^(2*d*x + 2*c) + 2*a*b
^2*e^(d*x + c) - b^3), x) - (a*d^2*x^2*e^c - (b*d*x*e^(2*c) - b*e^(2*c))*e^
(d*x) - (b*d*x + b)*e^(-d*x))*e^(-c)/(b^2*d^2))*f - 1/2*(2*(d*x + c)*a/(b^2
*d) - e^(d*x + c)/(b*d) - e^(-d*x - c)/(b*d) - 2*sqrt(a^2 + b^2)*log((b*e^(
-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b
^2*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(232) = 464$.

time = 0.43, size = 814, normalized size = 3.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*f*x + b*d*cosh(1) + (b*d*f*x + b*d*cosh(1) + b*d*sinh(1) - b*f)*co
sh(d*x + c)^2 + b*d*sinh(1) + (b*d*f*x + b*d*cosh(1) + b*d*sinh(1) - b*f)*s
inh(d*x + c)^2 + 2*(b*f*cosh(d*x + c) + b*f*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b*f*cosh(d*x + c) + b*f*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c)
 - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) +
2*((b*c*f - b*d*cosh(1) - b*d*sinh(1))*cosh(d*x + c) + (b*c*f - b*d*cosh(1)
 - b*d*sinh(1))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c)
+ 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*((b*c*f - b*d*co
sh(1) - b*d*sinh(1))*cosh(d*x + c) + (b*c*f - b*d*cosh(1) - b*d*sinh(1))*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c
) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*((b*d*f*x + b*c*f)*cosh(d*x + c) +
 (b*d*f*x + b*c*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x +
c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2) - b)/b) - 2*((b*d*f*x + b*c*f)*cosh(d*x + c) + (b*d*f*x + b*c*f)*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) -
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + b*f - (
a*d^2*f*x^2 + 2*a*d^2*x*cosh(1) + 2*a*d^2*x*sinh(1))*cosh(d*x + c) - (a*d^2
*f*x^2 + 2*a*d^2*x*cosh(1) + 2*a*d^2*x*sinh(1) - 2*(b*d*f*x + b*d*cosh(1) +
 b*d*sinh(1) - b*f)*cosh(d*x + c))*sinh(d*x + c))/(b^2*d^2*cosh(d*x + c) +
b^2*d^2*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^2\,(e+f\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.297 $\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=68

$$-\frac{ax}{b^2}-\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\cosh(c+dx)}{bd}$$

```
[Out] -a*x/b^2+cosh(d*x+c)/b/d-2*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2
))*(a^2+b^2)^(1/2)/b^2/d
```

Rubi [A]
time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2774, 2814, 2739, 632, 210}

$$-\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{ax}{b^2}+\frac{\cosh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a*x)/b^2) - (2*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^
2 + b^2]])/(b^2*d) + Cosh[c + d*x]/(b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{\cosh(c+dx)}{bd}+\frac{i\int \frac{-ib+ia\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}\\
&= -\frac{ax}{b^2}+\frac{\cosh(c+dx)}{bd}+\frac{(a^2+b^2)\int \frac{1}{a+b\sinh(c+dx)}\,dx}{b^2}\\
&= -\frac{ax}{b^2}+\frac{\cosh(c+dx)}{bd}-\frac{(2i(a^2+b^2))\,\text{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{b^2d}\\
&= -\frac{ax}{b^2}+\frac{\cosh(c+dx)}{bd}+\frac{(4i(a^2+b^2))\,\text{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx,x,-2ib+2a\tan\left(\frac{1}{2}\right.\right.}{b^2d}\\
&= -\frac{ax}{b^2}-\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\cosh(c+dx)}{bd}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.03, size = 458, normalized size = 6.74

$$
\frac{\cosh(c+dx)\left(-2\sqrt{a-ib}\,\sqrt{a+ib}\,\tanh^{-1}\left(\frac{\sqrt{-\frac{b(i+\sinh(c+dx))}{a-ib}}}{\sqrt{-\frac{b(-i+\sinh(c+dx))}{a+ib}}}\right)\sqrt{1+i\sinh(c+dx)}+2(a-ib)\tanh^{-1}\left(\frac{\sqrt{a-ib}\,\sqrt{-\frac{b(i+\sinh(c+dx))}{a-ib}}}{\sqrt{a+ib}\,\sqrt{-\frac{b(-i+\sinh(c+dx))}{a+ib}}}\right)\sqrt{1+i\sinh(c+dx)}+\sqrt{a+ib}\,\sqrt{-\frac{b(-i+\sinh(c+dx))}{a+ib}}\left(-2(-1)^{3/4}\sqrt{b}\,\text{ArcSin}\left(\frac{\sqrt[4]{-1}\,\sqrt{a-ib}\,\sqrt{-\frac{b(i+\sinh(c+dx))}{a-ib}}}{\sqrt{2}\,\sqrt{b}}\right)+\sqrt{a-ib}\,\sqrt{1+i\sinh(c+dx)}\,\sqrt{-\frac{b(i+\sinh(c+dx))}{a-ib}}\right)\right)}{\sqrt{a-ib}\,\sqrt{a+ib}\,bd\sqrt{1+i\sinh(c+dx)}\,\sqrt{-\frac{b(-i+\sinh(c+dx))}{a+ib}}\,\sqrt{-\frac{b(i+\sinh(c+dx))}{a-ib}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (Cosh[c + d*x]*(-2*Sqrt[a - I*b]*Sqrt[a + I*b]*ArcTanh[Sqrt[-((b*(I + Sinh[
c + d*x]))/(a - I*b))]/Sqrt[-((b*(-I + Sinh[c + d*x]))/(a + I*b))]]*Sqrt[1
+ I*Sinh[c + d*x]] + 2*(a - I*b)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh
[c + d*x]))/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[c + d*x]))/(a
+ I*b))])]*Sqrt[1 + I*Sinh[c + d*x]] + Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[c
```

```
 + d*x]))/(a + I*b))]*(-2*(-1)^(3/4)*Sqrt[b]*ArcSin[((-1)^(1/4)*Sqrt[a - I*
b]*Sqrt[-((b*(I + Sinh[c + d*x]))/(a - I*b))])/(Sqrt[2]*Sqrt[b])] + Sqrt[a
 - I*b]*Sqrt[1 + I*Sinh[c + d*x]]*Sqrt[-((b*(I + Sinh[c + d*x]))/(a - I*b))]
)))/(Sqrt[a - I*b]*Sqrt[a + I*b]*b*d*Sqrt[1 + I*Sinh[c + d*x]]*Sqrt[-((b*(-
I + Sinh[c + d*x]))/(a + I*b))]*Sqrt[-((b*(I + Sinh[c + d*x]))/(a - I*b))])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(63) = 126$.
time = 1.21, size = 129, normalized size = 1.90

method	result
risch	$-\frac{ax}{b^2}+\frac{e^{dx+c}}{2bd}+\frac{e^{-dx-c}}{2bd}+\frac{\sqrt{a^2+b^2}\,\ln\left(e^{dx+c}-\frac{-a+\sqrt{a^2+b^2}}{b}\right)}{d\,b^2}-\frac{\sqrt{a^2+b^2}\,\ln\left(e^{dx+c}+\frac{a+\sqrt{a^2}}{b}\right.}{d\,b^2}$
derivativedivides	$\frac{-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{2\left(-a^2-b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\right.}{2\sqrt{a^2}}\right.}{b^2\sqrt{a^2+b^2}}}{d}$
default	$\frac{-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{2\left(-a^2-b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\right.}{2\sqrt{a^2}}\right.}{b^2\sqrt{a^2+b^2}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b/(tanh(1/2*d*x+1/2*c)-1)+a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/b/(tanh
(1/2*d*x+1/2*c)+1)-a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-2/b^2*(-a^2-b^2)/(a^2+b^
2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [A]
time = 0.48, size = 116, normalized size = 1.71

$$-\frac{(dx+c)a}{b^2d}+\frac{e^{(dx+c)}}{2\,bd}+\frac{e^{(-dx-c)}}{2\,bd}+\frac{\sqrt{a^2+b^2}\,\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(d*x + c)*a/(b^2*d) + 1/2*e^(d*x + c)/(b*d) + 1/2*e^(-d*x - c)/(b*d) + sqr
t(a^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a
 + sqrt(a^2 + b^2)))/(b^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(65) = 130$.
time = 0.35, size = 259, normalized size = 3.81

$$-\frac{2\,adx\cosh\left(dx+c\right)-b\cosh\left(dx+c\right)^2-b\sinh\left(dx+c\right)^2-2\,\sqrt{a^2+b^2}\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2+b^2+2\,(b^2\cosh(dx+c)+ab)\sinh(dx+c)-2\,\sqrt{a^2+b^2}\,(b\cosh(dx+c)+b\sinh(dx+c)+a)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)-b}\right)+2\left(adx-b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)-b}{2\left(b^2d\cosh\left(dx+c\right)+b^2d\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*x*cosh(d*x + c) - b*cosh(d*x + c)^2 - b*sinh(d*x + c)^2 - 2*sqr
t(a^2 + b^2)*(cosh(d*x + c) + sinh(d*x + c))*log((b^2*cosh(d*x + c)^2 + b^2
*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c)
 + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c
) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*c
osh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*(a*d*x - b*cosh(d*x + c))*sinh(d*
x + c) - b)/(b^2*d*cosh(d*x + c) + b^2*d*sinh(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(58) = 116$.
time = 86.39, size = 503, normalized size = 7.40

$$\begin{cases} \frac{\tilde{\infty} x \cosh^2(c)}{\sinh(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{-\frac{x\sinh^2(c+dx)}{2} + \frac{x\cosh^2(c+dx)}{2} + \frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{a} & \text{for } b = 0 \\ \frac{x\cosh^2(c)}{a+b\sinh(c)} & \text{for } d = 0 \\ \frac{\frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-d} - \frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-d} - \frac{2}{d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-d}}{b} & \text{for } a = 0 \\ -\frac{adx\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{b^2d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2d} + \frac{adx}{b^2d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2d} - \frac{2b}{b^2d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2d} - \frac{\sqrt{a^2+b^2}\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{b^2d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2d} + \frac{\sqrt{a^2+b^2}\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{b^2d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2d} + \frac{\sqrt{a^2+b^2}\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{b^2d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2d} - \frac{\sqrt{a^2+b^2}\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{b^2d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*cosh(c)**2/sinh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-x*
sinh(c + d*x)**2/2 + x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*
d))/a, Eq(b, 0)), (x*cosh(c)**2/(a + b*sinh(c)), Eq(d, 0)), ((log(tanh(c/2
+ d*x/2))*tanh(c/2 + d*x/2)**2/(d*tanh(c/2 + d*x/2)**2 - d) - log(tanh(c/2
+ d*x/2))/(d*tanh(c/2 + d*x/2)**2 - d) - 2/(d*tanh(c/2 + d*x/2)**2 - d))/b,
 Eq(a, 0)), (-a*d*x*tanh(c/2 + d*x/2)**2/(b**2*d*tanh(c/2 + d*x/2)**2 - b**
2*d) + a*d*x/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d) - 2*b/(b**2*d*tanh(c/2
+ d*x/2)**2 - b**2*d) - sqrt(a**2 + b**2)*log(tanh(c/2 + d*x/2) - b/a - sqr
t(a**2 + b**2)/a)*tanh(c/2 + d*x/2)**2/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*
d) + sqrt(a**2 + b**2)*log(tanh(c/2 + d*x/2) - b/a - sqrt(a**2 + b**2)/a)/(
b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d) + sqrt(a**2 + b**2)*log(tanh(c/2 + d*
x/2) - b/a + sqrt(a**2 + b**2)/a)*tanh(c/2 + d*x/2)**2/(b**2*d*tanh(c/2 + d
*x/2)**2 - b**2*d) - sqrt(a**2 + b**2)*log(tanh(c/2 + d*x/2) - b/a + sqrt(a
**2 + b**2)/a)/(b**2*d*tanh(c/2 + d*x/2)**2 - b**2*d), True))
```

Giac [A]
time = 0.45, size = 110, normalized size = 1.62

$$-\frac{\frac{2\,(dx+c)a}{b^2} - \frac{e^{(dx+c)}}{b} - \frac{e^{(-dx-c)}}{b} - \frac{2\,\sqrt{a^2+b^2}\,\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)*a/b^2 - e^(d*x + c)/b - e^(-d*x - c)/b - 2*sqrt(a^2 + b^2
)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c) +
2*a + 2*sqrt(a^2 + b^2)))/b^2)/d
```

Mupad [B]
time = 0.42, size = 121, normalized size = 1.78

$$\frac{e^{c+dx}}{2\,b\,d}-\frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-b^4\,d^2}}{b^2\,d\,\sqrt{a^2+b^2}}+\frac{e^{d\,x}\,e^{c}\,\sqrt{-b^4\,d^2}}{b\,d\,\sqrt{a^2+b^2}}\right)\,\sqrt{a^2+b^2}}{\sqrt{-b^4\,d^2}}+\frac{e^{-c-d\,x}}{2\,b\,d}-\frac{a\,x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/(a + b*sinh(c + d*x)),x)
```

```
[Out] exp(c + d*x)/(2*b*d) - (2*atan((a*(-b^4*d^2)^(1/2))/(b^2*d*(a^2 + b^2)^(1/2
)) + (exp(d*x)*exp(c)*(-b^4*d^2)^(1/2))/(b*d*(a^2 + b^2)^(1/2)))*(a^2 + b^2
)^(1/2))/(-b^4*d^2)^(1/2) + exp(- c - d*x)/(2*b*d) - (a*x)/b^2
```

3.298 $\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=31

$$\mathrm{Int}\left(\frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cosh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][Cosh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 19.52, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cosh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Integrate[Cosh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*(a^2*e^c + b^2*e^c)*integrate(-e^(d*x)/(b^3*f*x + b^3*e - (b^3*f*x*e^(2*c
) + b^3*e^(2*c + 1))*e^(2*d*x) - 2*(a*b^2*f*x*e^c + a*b^2*e^(c + 1))*e^(d*x
)), x) + 1/2*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b*f) - 1/2*e^
(c - d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b*f) - a*log(f*x + e)/(b^2*f
)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))),x)
[Out] int(cosh(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.299 $\int \frac{(e+fx)^3 \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=642

$$\frac{3f^3x}{8bd^3}+\frac{(e+fx)^3}{4bd}-\frac{(a^2+b^2)(e+fx)^4}{4b^3f}+\frac{6af^3\cosh(c+dx)}{b^2d^4}+\frac{3af(e+fx)^2\cosh(c+dx)}{b^2d^2}+\frac{(a^2+b^2)(e+fx)}{}$$

```
[Out] 3/8*f^3*x/b/d^3+1/4*(f*x+e)^3/b/d-1/4*(a^2+b^2)*(f*x+e)^4/b^3/f+6*a*f^3*cos
h(d*x+c)/b^2/d^4+3*a*f*(f*x+e)^2*cosh(d*x+c)/b^2/d^2+(a^2+b^2)*(f*x+e)^3*ln
(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d+(a^2+b^2)*(f*x+e)^3*ln(1+b*exp(d
*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d+3*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d
*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^2+3*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp
(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^2-6*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*e
xp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^3-6*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b
*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^3+6*(a^2+b^2)*f^3*polylog(4,-b*exp(d
*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^4+6*(a^2+b^2)*f^3*polylog(4,-b*exp(d*x+c)/
(a+(a^2+b^2)^(1/2)))/b^3/d^4-6*a*f^2*(f*x+e)*sinh(d*x+c)/b^2/d^3-a*(f*x+e)^
3*sinh(d*x+c)/b^2/d-3/8*f^3*cosh(d*x+c)*sinh(d*x+c)/b/d^4-3/4*f*(f*x+e)^2*c
osh(d*x+c)*sinh(d*x+c)/b/d^2+3/4*f^2*(f*x+e)*sinh(d*x+c)^2/b/d^3+1/2*(f*x+e
)^3*sinh(d*x+c)^2/b/d
```

Rubi [A]
time = 0.56, antiderivative size = 642, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5684, 3377, 2718, 5554, 3392, 32, 2715, 8, 5680, 2221, 2611, 6744, 2320, 6724}

$$\frac{6f^3(a^2+b^2)\text{Li}_4\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^4}+\frac{6f^3(a^2+b^2)\text{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^4}-\frac{6f^2(a^2+b^2)(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}-\frac{6f^2(a^2+b^2)(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^3}+\frac{3f(a^2+b^2)(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{3f(a^2+b^2)(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{(a^2+b^2)(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^3d}+\frac{(a^2+b^2)(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^3d}-\frac{(a^2+b^2)(e+fx)^4}{4b^3f}+\frac{6af^3\cosh(c+dx)}{b^2d^4}-\frac{6af^2(e+fx)\sinh(c+dx)}{b^2d^3}+\frac{3af(e+fx)^2\cosh(c+dx)}{b^2d^2}-\frac{a(e+fx)^3\sinh(c+dx)}{b^2d}-\frac{3f^3\sinh(c+dx)\cosh(c+dx)}{8bd^4}+\frac{3f^2(e+fx)\sinh^2(c+dx)}{4bd^3}-\frac{3f(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{4bd^2}+\frac{(e+fx)^3\sinh^2(c+dx)}{2bd}+\frac{3f^3x}{8bd^3}+\frac{(e+fx)^3}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (3*f^3*x)/(8*b*d^3) + (e + f*x)^3/(4*b*d) - ((a^2 + b^2)*(e + f*x)^4)/(4*b^
3*f) + (6*a*f^3*Cosh[c + d*x])/(b^2*d^4) + (3*a*f*(e + f*x)^2*Cosh[c + d*x]
)/(b^2*d^2) + ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2])])/(b^3*d) + ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2])])/(b^3*d) + (3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^2) + (3*(a^2 + b^2)*f*(e + f*x
)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^2) - (6*(a
^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))
])/(b^3*d^3) - (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2]))])/(b^3*d^3) + (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^4) + (6*(a^2 + b^2)*f^3*PolyLog[4,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^4) - (6*a*f^2*(e + f*x)*S
```

```
inh[c + d*x])/(b^2*d^3) - (a*(e + f*x)^3*Sinh[c + d*x])/(b^2*d) - (3*f^3*Co
sh[c + d*x]*Sinh[c + d*x])/(8*b*d^4) - (3*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[
c + d*x])/(4*b*d^2) + (3*f^2*(e + f*x)*Sinh[c + d*x]^2)/(4*b*d^3) + ((e + f
*x)^3*Sinh[c + d*x]^2)/(2*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\int (e+fx)^3\cosh(c+dx)\,dx}{b^2} + \frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)\,dx}{b}$$

$$= -\frac{(a^2+b^2)(e+fx)^4}{4b^3f} - \frac{a(e+fx)^3\sinh(c+dx)}{b^2d} + \frac{(e+fx)^3\sinh^2(c+dx)}{2bd}$$

$$= -\frac{(a^2+b^2)(e+fx)^4}{4b^3f} + \frac{3af(e+fx)^2\cosh(c+dx)}{b^2d^2} + \frac{(a^2+b^2)(e+fx)^3\log}{b^3}$$

$$= \frac{(e+fx)^3}{4bd} - \frac{(a^2+b^2)(e+fx)^4}{4b^3f} + \frac{3af(e+fx)^2\cosh(c+dx)}{b^2d^2} + \frac{(a^2+b^2)(}{}$$

$$= \frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} - \frac{(a^2+b^2)(e+fx)^4}{4b^3f} + \frac{6af^3\cosh(c+dx)}{b^2d^4} + \frac{3af(e+f}{}$$

$$= \frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} - \frac{(a^2+b^2)(e+fx)^4}{4b^3f} + \frac{6af^3\cosh(c+dx)}{b^2d^4} + \frac{3af(e+f}{}$$

$$= \frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} - \frac{(a^2+b^2)(e+fx)^4}{4b^3f} + \frac{6af^3\cosh(c+dx)}{b^2d^4} + \frac{3af(e+f}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 2833 vs. $2(642) = 1284$.
time = 16.60, size = 2833, normalized size = 4.41

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]

[Out] -1/2*((a^2 + b^2)*(4*e^3*E^(2*c)*x + 6*e^2*E^(2*c)*f*x^2 + 4*e*E^(2*c)*f^2*x^3 + E^(2*c)*f^3*x^4 + (4*a*Sqrt[a^2 + b^2]*e^3*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (4*a*Sqrt[-(a^2 + b^2)^2]*e^3*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (4*a*Sqrt[-(a^2 + b^2)^2]*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (4*a*Sqrt[-(a^2 + b^2)^2]*e^3*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (2*e^3*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (2*e^3*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (2*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (2*E^(2*c)*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (2*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (2*E^(2*c)*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*(-1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (12*e*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*e*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (12*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*E^(2*c)*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (12*e*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*e*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (12*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*E^(2*c)*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^4 - (12*E^(2*c)*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^4 + (12*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^4 - (12*E^(2*c)*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^4))/(b^3*(-1 + E^(2*c))) + ((a^2 + b^2)*e^3*x*(1 + Cosh[2*c] + Sinh[2*c]))/(b^3*(-1 + Cosh[2*c] + Sinh[2*c])) + (3*(a^2 + b^2)*e^2*f*x^2*(1 + Cosh[2*c] + Sinh[2*c]))/(2*b^3*(-1 + Cosh[2*c] + Sinh[2*c])) + ((a^2 + b^2)*e*f^2*x^3*(1 + Cosh[2*c] + Sinh[2*c]))/(b^3*(-1 + Cosh[2*c] + Sinh[2*c])) +

```
((a^2 + b^2)*f^3*x^4*(1 + Cosh[2*c] + Sinh[2*c]))/(4*b^3*(-1 + Cosh[2*c] +
Sinh[2*c])) + ((a*f^3*x^3*Cosh[c])/(2*b^2*d) - (a*f^3*x^3*Sinh[c])/(2*b^2*d
) + (d^3*e^3 + 3*d^2*e^2*f + 6*d*e*f^2 + 6*f^3)*((a*Cosh[c])/(2*b^2*d^4) -
(a*Sinh[c])/(2*b^2*d^4)) + (a*d^2*e^2*f + 2*a*d*e*f^2 + 2*a*f^3)*((3*x*Cosh
[c])/(2*b^2*d^3) - (3*x*Sinh[c])/(2*b^2*d^3)) + (a*d*e*f^2 + a*f^3)*((3*x^2
*Cosh[c])/(2*b^2*d^2) - (3*x^2*Sinh[c])/(2*b^2*d^2)))*(Cosh[d*x] - Sinh[d*x
]) + (-1/2*(a*f^3*x^3*Cosh[c])/(b^2*d) - (a*f^3*x^3*Sinh[c])/(2*b^2*d) + (d
^3*e^3 - 3*d^2*e^2*f + 6*d*e*f^2 - 6*f^3)*(-1/2*(a*Cosh[c])/(b^2*d^4) - (a*
Sinh[c])/(2*b^2*d^4)) - (3*x^2*(a*d*e*f^2*Cosh[c] - a*f^3*Cosh[c] + a*d*e*f
^2*Sinh[c] - a*f^3*Sinh[c]))/(2*b^2*d^2) - (3*x*(a*d^2*e^2*f*Cosh[c] - 2*a*
d*e*f^2*Cosh[c] + 2*a*f^3*Cosh[c] + a*d^2*e^2*f*Sinh[c] - 2*a*d*e*f^2*Sinh[
c] + 2*a*f^3*Sinh[c]))/(2*b^2*d^3))*(Cosh[d*x] + Sinh[d*x]) + ((f^3*x^3*Cos
h[2*c])/(8*b*d) - (f^3*x^3*Sinh[2*c])/(8*b*d) + (4*d^3*e^3 + 6*d^2*e^2*f +
6*d*e*f^2 + 3*f^3)*(Cosh[2*c]/(32*b*d^4) - Sinh[2*c]/(32*b*d^4)) + (2*d^2*e
^2*f + 2*d*e*f^2 + f^3)*((3*x*Cosh[2*c])/(16*b*d^3) - (3*x*Sinh[2*c])/(16*b
*d^3)) + (2*d*e*f^2 + f^3)*((3*x^2*Cosh[2*c])/(16*b*d^2) - (3*x^2*Sinh[2*c]
)/(16*b*d^2)))*(Cosh[2*d*x] - Sinh[2*d*x]) + ((f^3*x^3*Cosh[2*c])/(8*b*d) +
 (f^3*x^3*Sinh[2*c])/(8*b*d) + (4*d^3*e^3 - 6*d^2*e^2*f + 6*d*e*f^2 - 3*f^3
)*(Cosh[2*c]/(32*b*d^4) + Sinh[2*c]/(32*b*d^4)) + (3*x^2*(2*d*e*f^2*Cosh[2*
c] - f^3*Cosh[2*c] + 2*d*e*f^2*Sinh[2*c] - f^3*Sinh[2*c]))/(16*b*d^2) + (3*
x*(2*d^2*e^2*f*Cosh[2*c] - 2*d*e*f^2*Cosh[2*c] + f^3*Cosh[2*c] + 2*d^2*e^2*
f*Sinh[2*c] - 2*d*e*f^2*Sinh[2*c] + f^3*Sinh[2*c]))/(16*b*d^3))*(Cosh[2*d*x
] + Sinh[2*d*x])
```

Maple [F]
time = 1.45, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \left(\cosh^3 (dx+c)\right)}{a+b\sinh (dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*((4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) - 8*(a^2 + b^2)*(d*x +
 c)/(b^3*d) - (4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d) - 8*(a^2 + b^
2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d))*e^3 + 1/32*(8*(
```

```
a^2*d^4*f^3*e^(2*c) + b^2*d^4*f^3*e^(2*c))*x^4 + 32*(a^2*d^4*f^2*e^(2*c) +
b^2*d^4*f^2*e^(2*c))*x^3*e + 48*(a^2*d^4*f*e^(2*c) + b^2*d^4*f*e^(2*c))*x^2
*e^2 + (4*b^2*d^3*f^3*x^3*e^(4*c) - 3*b^2*f^3*e^(4*c) - 6*b^2*d^2*f*e^(4*c
+ 2) + 6*b^2*d*f^2*e^(4*c + 1) - 6*(b^2*d^2*f^3*e^(4*c) - 2*b^2*d^3*f^2*e^(
4*c + 1))*x^2 + 6*(b^2*d*f^3*e^(4*c) + 2*b^2*d^3*f*e^(4*c + 2) - 2*b^2*d^2*
f^2*e^(4*c + 1))*x)*e^(2*d*x) - 16*(a*b*d^3*f^3*x^3*e^(3*c) - 6*a*b*f^3*e^(
3*c) - 3*a*b*d^2*f*e^(3*c + 2) + 6*a*b*d*f^2*e^(3*c + 1) - 3*(a*b*d^2*f^3*e
^(3*c) - a*b*d^3*f^2*e^(3*c + 1))*x^2 + 3*(2*a*b*d*f^3*e^(3*c) + a*b*d^3*f*
e^(3*c + 2) - 2*a*b*d^2*f^2*e^(3*c + 1))*x)*e^(d*x) + 16*(a*b*d^3*f^3*x^3*e
^c + 3*a*b*d^2*f*e^(c + 2) + 6*a*b*d*f^2*e^(c + 1) + 6*a*b*f^3*e^c + 3*(a*b
*d^3*f^2*e^(c + 1) + a*b*d^2*f^3*e^c)*x^2 + 3*(a*b*d^3*f*e^(c + 2) + 2*a*b*
d^2*f^2*e^(c + 1) + 2*a*b*d*f^3*e^c)*x)*e^(-d*x) + (4*b^2*d^3*f^3*x^3 + 6*b
^2*d^2*f*e^2 + 6*b^2*d*f^2*e + 3*b^2*f^3 + 6*(2*b^2*d^3*f^2*e + b^2*d^2*f^3
)*x^2 + 6*(2*b^2*d^3*f*e^2 + 2*b^2*d^2*f^2*e + b^2*d*f^3)*x)*e^(-2*d*x))*e^
(-2*c)/(b^3*d^4) - integrate(-2*((a^2*b*f^3 + b^3*f^3)*x^3 + 3*(a^2*b*f^2 +
 b^3*f^2)*x^2*e + 3*(a^2*b*f + b^3*f)*x*e^2 - ((a^3*f^3*e^c + a*b^2*f^3*e^c
)*x^3 + 3*(a^3*f^2*e^c + a*b^2*f^2*e^c)*x^2*e + 3*(a^3*f*e^c + a*b^2*f*e^c)
*x*e^2)*e^(d*x))/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 8057 vs. $2(616) = 1232$.
time = 0.51, size = 8057, normalized size = 12.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/32*(4*b^2*d^3*f^3*x^3 + 6*b^2*d^2*f^3*x^2 + 4*b^2*d^3*cosh(1)^3 + 4*b^2*d
^3*sinh(1)^3 + 6*b^2*d*f^3*x + 3*b^2*f^3 + (4*b^2*d^3*f^3*x^3 - 6*b^2*d^2*f
^3*x^2 + 4*b^2*d^3*cosh(1)^3 + 4*b^2*d^3*sinh(1)^3 + 6*b^2*d*f^3*x - 3*b^2*
f^3 + 6*(2*b^2*d^3*f*x - b^2*d^2*f)*cosh(1)^2 + 6*(2*b^2*d^3*f*x + 2*b^2*d^
3*cosh(1) - b^2*d^2*f)*sinh(1)^2 + 6*(2*b^2*d^3*f^2*x^2 - 2*b^2*d^2*f^2*x +
 b^2*d*f^2)*cosh(1) + 6*(2*b^2*d^3*f^2*x^2 - 2*b^2*d^2*f^2*x + 2*b^2*d^3*co
sh(1)^2 + b^2*d*f^2 + 2*(2*b^2*d^3*f*x - b^2*d^2*f)*cosh(1))*sinh(1))*cosh(
d*x + c)^4 + (4*b^2*d^3*f^3*x^3 - 6*b^2*d^2*f^3*x^2 + 4*b^2*d^3*cosh(1)^3 +
 4*b^2*d^3*sinh(1)^3 + 6*b^2*d*f^3*x - 3*b^2*f^3 + 6*(2*b^2*d^3*f*x - b^2*d
^2*f)*cosh(1)^2 + 6*(2*b^2*d^3*f*x + 2*b^2*d^3*cosh(1) - b^2*d^2*f)*sinh(1)
^2 + 6*(2*b^2*d^3*f^2*x^2 - 2*b^2*d^2*f^2*x + b^2*d*f^2)*cosh(1) + 6*(2*b^2
*d^3*f^2*x^2 - 2*b^2*d^2*f^2*x + 2*b^2*d^3*cosh(1)^2 + b^2*d*f^2 + 2*(2*b^2
*d^3*f*x - b^2*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^4 - 16*(a*b*d^3*f^3*x
^3 - 3*a*b*d^2*f^3*x^2 + a*b*d^3*cosh(1)^3 + a*b*d^3*sinh(1)^3 + 6*a*b*d*f^
3*x - 6*a*b*f^3 + 3*(a*b*d^3*f*x - a*b*d^2*f)*cosh(1)^2 + 3*(a*b*d^3*f*x +
a*b*d^3*cosh(1) - a*b*d^2*f)*sinh(1)^2 + 3*(a*b*d^3*f^2*x^2 - 2*a*b*d^2*f^2
*x + 2*a*b*d*f^2)*cosh(1) + 3*(a*b*d^3*f^2*x^2 - 2*a*b*d^2*f^2*x + a*b*d^3*
```

cosh(1)^2 + 2*a*b*d*f^2 + 2*(a*b*d^3*f*x - a*b*d^2*f)*cosh(1))*sinh(1))*cos
h(d*x + c)^3 - 4*(4*a*b*d^3*f^3*x^3 - 12*a*b*d^2*f^3*x^2 + 4*a*b*d^3*cosh(1
)^3 + 4*a*b*d^3*sinh(1)^3 + 24*a*b*d*f^3*x - 24*a*b*f^3 + 12*(a*b*d^3*f*x -
a*b*d^2*f)*cosh(1)^2 + 12*(a*b*d^3*f*x + a*b*d^3*cosh(1) - a*b*d^2*f)*sinh
(1)^2 + 12*(a*b*d^3*f^2*x^2 - 2*a*b*d^2*f^2*x + 2*a*b*d*f^2)*cosh(1) - (4*b
^2*d^3*f^3*x^3 - 6*b^2*d^2*f^3*x^2 + 4*b^2*d^3*cosh(1)^3 + 4*b^2*d^3*sinh(1
)^3 + 6*b^2*d*f^3*x - 3*b^2*f^3 + 6*(2*b^2*d^3*f*x - b^2*d^2*f)*cosh(1)^2 +
6*(2*b^2*d^3*f*x + 2*b^2*d^3*cosh(1) - b^2*d^2*f)*sinh(1)^2 + 6*(2*b^2*d^3
*f^2*x^2 - 2*b^2*d^2*f^2*x + b^2*d*f^2)*cosh(1) + 6*(2*b^2*d^3*f^2*x^2 - 2*
b^2*d^2*f^2*x + 2*b^2*d^3*cosh(1)^2 + b^2*d*f^2 + 2*(2*b^2*d^3*f*x - b^2*d^
2*f)*cosh(1))*sinh(1))*cosh(d*x + c) + 12*(a*b*d^3*f^2*x^2 - 2*a*b*d^2*f^2*
x + a*b*d^3*cosh(1)^2 + 2*a*b*d*f^2 + 2*(a*b*d^3*f*x - a*b*d^2*f)*cosh(1))*
sinh(1))*sinh(d*x + c)^3 + 6*(2*b^2*d^3*f*x + b^2*d^2*f)*cosh(1)^2 - 8*((a^
2 + b^2)*d^4*f^3*x^4 - 2*(a^2 + b^2)*c^4*f^3 + 4*((a^2 + b^2)*d^4*x + 2*(a^
2 + b^2)*c*d^3)*cosh(1)^3 + 4*((a^2 + b^2)*d^4*x + 2*(a^2 + b^2)*c*d^3)*sin
h(1)^3 + 6*((a^2 + b^2)*d^4*f*x^2 - 2*(a^2 + b^2)*c^2*d^2*f)*cosh(1)^2 + 6*
((a^2 + b^2)*d^4*f*x^2 - 2*(a^2 + b^2)*c^2*d^2*f + 2*((a^2 + b^2)*d^4*x + 2
*(a^2 + b^2)*c*d^3)*cosh(1))*sinh(1)^2 + 4*((a^2 + b^2)*d^4*f^2*x^3 + 2*(a^
2 + b^2)*c^3*d*f^2)*cosh(1) + 4*((a^2 + b^2)*d^4*f^2*x^3 + 2*(a^2 + b^2)*c^
3*d*f^2 + 3*((a^2 + b^2)*d^4*x + 2*(a^2 + b^2)*c*d^3)*cosh(1)^2 + 3*((a^2 +
b^2)*d^4*f*x^2 - 2*(a^2 + b^2)*c^2*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^
2 + 6*(2*b^2*d^3*f*x + 2*b^2*d^3*cosh(1) + b^2*d^2*f)*sinh(1)^2 - 2*(4*(a^2
+ b^2)*d^4*f^3*x^4 - 8*(a^2 + b^2)*c^4*f^3 + 16*((a^2 + b^2)*d^4*x + 2*(a^
2 + b^2)*c*d^3)*cosh(1)^3 + 16*((a^2 + b^2)*d^4*x + 2*(a^2 + b^2)*c*d^3)*si
nh(1)^3 + 24*((a^2 + b^2)*d^4*f*x^2 - 2*(a^2 + b^2)*c^2*d^2*f)*cosh(1)^2 -
3*(4*b^2*d^3*f^3*x^3 - 6*b^2*d^2*f^3*x^2 + 4*b^2*d^3*cosh(1)^3 + 4*b^2*d^3*
sinh(1)^3 + 6*b^2*d*f^3*x - 3*b^2*f^3 + 6*(2*b^2*d^3*f*x - b^2*d^2*f)*cosh(
1)^2 + 6*(2*b^2*d^3*f*x + 2*b^2*d^3*cosh(1) - b^2*d^2*f)*sinh(1)^2 + 6*(2*b
^2*d^3*f^2*x^2 - 2*b^2*d^2*f^2*x + b^2*d*f^2)*cosh(1) + 6*(2*b^2*d^3*f^2*x^
2 - 2*b^2*d^2*f^2*x + 2*b^2*d^3*cosh(1)^2 + b^2*d*f^2 + 2*(2*b^2*d^3*f*x -
b^2*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 24*((a^2 + b^2)*d^4*f*x^2 -
2*(a^2 + b^2)*c^2*d^2*f + 2*((a^2 + b^2)*d^4*x + 2*(a^2 + b^2)*c*d^3)*cosh(
1))*sinh(1)^2 + 16*((a^2 + b^2)*d^4*f^2*x^3 + 2*(a^2 + b^2)*c^3*d*f^2)*cosh
(1) + 24*(a*b*d^3*f^3*x^3 - 3*a*b*d^2*f^3*x^2 + a*b*d^3*cosh(1)^3 + a*b*d^3
*sinh(1)^3 + 6*a*b*d*f^3*x - 6*a*b*f^3 + 3*(a*b*d^3*f*x - a*b*d^2*f)*cosh(1
)^2 + 3*(a*b*d^3*f*x + a*b*d^3*cosh(1) - a*b*d^2*f)*sinh(1)^2 + 3*(a*b*d^3*
f^2*x^2 - 2*a*b*d^2*f^2*x + 2*a*b*d*f^2)*cosh(1) + 3*(a*b*d^3*f^2*x^2 - 2*a
*b*d^2*f^2*x + a*b*d^3*cosh(1)^2 + 2*a*b*d*f^2 + 2*(a*b*d^3*f*x - a*b*d^2*f
)*cosh(1))*sinh(1))*cosh(d*x + c) + 16*((a^2 + b^2)*d^4*f^2*x^3 + 2*(a^2 +
b^2)*c^3*d*f^2 + 3*((a^2 + b^2)*d^4*x + 2*(a^2 + b^2)*c*d^3)*cosh(1)^2 + 3*
((a^2 + b^2)*d^4*f*x^2 - 2*(a^2 + b^2)*c^2*d^2*f)*cosh(1))*sinh(1))*sinh(d*
x + c)^2 + 6*(2*b^2*d^3*f^2*x^2 + 2*b^2*d^2*f^2*x + b^2*d*f^2)*cosh(1) + 16
*(a*b*d^3*f^3*x^3 + 3*a*b*d^2*f^3*x^2 + a*b*d^3*cosh(1)^3 + a*b*d^3*sinh(1)
^3 + 6*a*b*d*f^3*x + 6*a*b*f^3 + 3*(a*b*d^3*f*x + a*b*d^2*f)*cosh(1)^2 + 3*
(a*b*d^3*f*x + a*b*d^3*cosh(1) + a*b*d^2*f)*sinh(1)^2 + 3*(a*b*d^3*f^2*x^2

```
+ 2*a*b*d^2*f^2*x + 2*a*b*d*f^2)*cosh(1) + 3*(a*b*d^3*f^2*x^2 + 2*a*b*d^2*f
^2*x + a*b*d^3*cosh(1)^2 + 2*a*b*d*f^2 + 2*(a*b*d^3*f*x + a*b*d^2*f)*cosh(1
))*sinh(1))*cosh(d*x + c) + 96*(((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^
2*f^2*x*cosh(1) + (a^2 + b^2)*d^2*f*cosh(1)^2 + (a^2 + b^2)*d^2*f*sinh(1)^2
 + 2*((a^2 + b^2)*d^2*f^2*x + (a^2 + b^2)*d^2*f...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^3*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^3\,{(e+f\,x)}^3}{a+b\,\sinh\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.300 $\int \frac{(e+fx)^2 \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=477

$$\frac{efx}{2bd}+\frac{f^2x^2}{4bd}-\frac{(a^2+b^2)(e+fx)^3}{3b^3f}+\frac{2af(e+fx)\cosh(c+dx)}{b^2d^2}+\frac{(a^2+b^2)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d}+\cdots$$

```
[Out] 1/2*e*f*x/b/d+1/4*f^2*x^2/b/d-1/3*(a^2+b^2)*(f*x+e)^3/b^3/f+2*a*f*(f*x+e)*c
osh(d*x+c)/b^2/d^2+(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)
))/b^3/d+(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d+2
*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^2+2
*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^2-2
*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^3-2*(a^2+
b^2)*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^3-2*a*f^2*sinh(
d*x+c)/b^2/d^3-a*(f*x+e)^2*sinh(d*x+c)/b^2/d-1/2*f*(f*x+e)*cosh(d*x+c)*sinh
(d*x+c)/b/d^2+1/4*f^2*sinh(d*x+c)^2/b/d^3+1/2*(f*x+e)^2*sinh(d*x+c)^2/b/d
```

Rubi [A]

time = 0.48, antiderivative size = 477, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5684, 3377, 2717, 5554, 3391, 5680, 2221, 2611, 2320, 6724}

$$-\frac{2f^2(a^2+b^2)\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}-\frac{2f^2(a^2+b^2)\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^3}+\frac{2f(a^2+b^2)(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{2f(a^2+b^2)(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{(a^2+b^2)(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^3d}+\frac{(a^2+b^2)(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^3d}-\frac{(a^2+b^2)(e+fx)^3}{3b^3f}-\frac{2af^2\sinh(c+dx)}{b^2d^3}+\frac{2af(e+fx)\cosh(c+dx)}{b^2d^2}-\frac{a(e+fx)^2\sinh(c+dx)}{b^2d}+\frac{f^2\sinh^2(c+dx)}{4bd^3}-\frac{f(e+fx)\sinh(c+dx)\cosh(c+dx)}{2bd^2}+\frac{(e+fx)^2\sinh^2(c+dx)}{2bd}+\frac{efx}{2bd}+\frac{f^2x^2}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e*f*x)/(2*b*d) + (f^2*x^2)/(4*b*d) - ((a^2 + b^2)*(e + f*x)^3)/(3*b^3*f) +
 (2*a*f*(e + f*x)*Cosh[c + d*x])/(b^2*d^2) + ((a^2 + b^2)*(e + f*x)^2*Log[1
 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) + ((a^2 + b^2)*(e + f*x)
^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*d) + (2*(a^2 + b^2)
*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^2
) + (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/(b^3*d^2) - (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2]))])/(b^3*d^3) - (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^3) - (2*a*f^2*Sinh[c + d*x])/(b^2*d^3)
 - (a*(e + f*x)^2*Sinh[c + d*x])/(b^2*d) - (f*(e + f*x)*Cosh[c + d*x]*Sinh[
c + d*x])/(2*b*d^2) + (f^2*Sinh[c + d*x]^2)/(4*b*d^3) + ((e + f*x)^2*Sinh[c
 + d*x]^2)/(2*b*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\int (e+fx)^2\cosh(c+dx)\,dx}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)\,dx}{b}$$

$$= -\frac{(a^2+b^2)(e+fx)^3}{3b^3f}-\frac{a(e+fx)^2\sinh(c+dx)}{b^2d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2bd}$$

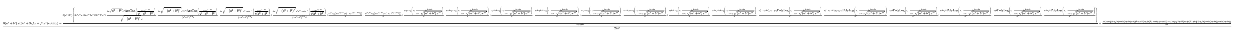
$$= -\frac{(a^2+b^2)(e+fx)^3}{3b^3f}+\frac{2af(e+fx)\cosh(c+dx)}{b^2d^2}+\frac{(a^2+b^2)(e+fx)^2\log}{b^3}$$

$$= \frac{efx}{2bd}+\frac{f^2x^2}{4bd}-\frac{(a^2+b^2)(e+fx)^3}{3b^3f}+\frac{2af(e+fx)\cosh(c+dx)}{b^2d^2}+\frac{(a^2+b^2)}{}$$

$$= \frac{efx}{2bd}+\frac{f^2x^2}{4bd}-\frac{(a^2+b^2)(e+fx)^3}{3b^3f}+\frac{2af(e+fx)\cosh(c+dx)}{b^2d^2}+\frac{(a^2+b^2)}{}$$

$$= \frac{efx}{2bd}+\frac{f^2x^2}{4bd}-\frac{(a^2+b^2)(e+fx)^3}{3b^3f}+\frac{2af(e+fx)\cosh(c+dx)}{b^2d^2}+\frac{(a^2+b^2)}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1253 vs. $2(477) = 954$.
time = 12.97, size = 1253, normalized size = 2.63



Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (8*(a^2 + b^2)*x*(3*e^2 + 3*e*f*x + f^2*x^2)*Coth[c] - (8*(a^2 + b^2)*(6*e^
2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*
e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d)
+ (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^
2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a
 + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^
2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^
2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])
/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6
*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d -
(6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c
)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^
(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a
^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[
(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E
^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/
(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(
2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*(-1 + E^(2*c))*f*(
e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])
)])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E
^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*
x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3,
-((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (6*f^2*Pol
yLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6
*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*
c)]))])/d^3))/(-1 + E^(2*c)) + (3*b*(16*a*d*f*(e + f*x)*Cosh[c + d*x] + b*(
f^2 + 2*d^2*(e + f*x)^2)*Cosh[2*(c + d*x)] - 4*(2*a*(2*f^2 + d^2*(e + f*x)^
2) + b*d*f*(e + f*x)*Cosh[c + d*x])*Sinh[c + d*x]))/d^3)/(24*b^3)
```

Maple [F]
time = 1.56, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \left(\cosh^3(dx+c)\right)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*((4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) - 8*(a^2 + b^2)*(d*x +
 c)/(b^3*d) - (4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d) - 8*(a^2 + b^
2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d))*e^2 + 1/48*(16*
(a^2*d^3*f^2*e^(2*c) + b^2*d^3*f^2*e^(2*c))*x^3 + 48*(a^2*d^3*f*e^(2*c) + b
^2*d^3*f*e^(2*c))*x^2*e + 3*(2*b^2*d^2*f^2*x^2*e^(4*c) + b^2*f^2*e^(4*c) -
2*b^2*d*f*e^(4*c + 1) - 2*(b^2*d*f^2*e^(4*c) - 2*b^2*d^2*f*e^(4*c + 1))*x)*
e^(2*d*x) - 24*(a*b*d^2*f^2*x^2*e^(3*c) + 2*a*b*f^2*e^(3*c) - 2*a*b*d*f*e^(
3*c + 1) - 2*(a*b*d*f^2*e^(3*c) - a*b*d^2*f*e^(3*c + 1))*x)*e^(d*x) + 24*(a
*b*d^2*f^2*x^2*e^c + 2*a*b*d*f*e^(c + 1) + 2*a*b*f^2*e^c + 2*(a*b*d^2*f*e^(
c + 1) + a*b*d*f^2*e^c)*x)*e^(-d*x) + 3*(2*b^2*d^2*f^2*x^2 + 2*b^2*d*f*e +
b^2*f^2 + 2*(2*b^2*d^2*f*e + b^2*d*f^2)*x)*e^(-2*d*x))*e^(-2*c)/(b^3*d^3) -
 integrate(-2*((a^2*b*f^2 + b^3*f^2)*x^2 + 2*(a^2*b*f + b^3*f)*x*e - ((a^3*
f^2*e^c + a*b^2*f^2*e^c)*x^2 + 2*(a^3*f*e^c + a*b^2*f*e^c)*x*e)*e^(d*x))/(b
^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4069 vs. $2(455) = 910$.
time = 0.44, size = 4069, normalized size = 8.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(6*b^2*d^2*f^2*x^2 + 6*b^2*d*f^2*x + 6*b^2*d^2*cosh(1)^2 + 6*b^2*d^2*s
inh(1)^2 + 3*(2*b^2*d^2*f^2*x^2 - 2*b^2*d*f^2*x + 2*b^2*d^2*cosh(1)^2 + 2*b
^2*d^2*sinh(1)^2 + b^2*f^2 + 2*(2*b^2*d^2*f*x - b^2*d*f)*cosh(1) + 2*(2*b^2
*d^2*f*x + 2*b^2*d^2*cosh(1) - b^2*d*f)*sinh(1))*cosh(d*x + c)^4 + 3*(2*b^2
*d^2*f^2*x^2 - 2*b^2*d*f^2*x + 2*b^2*d^2*cosh(1)^2 + 2*b^2*d^2*sinh(1)^2 +
b^2*f^2 + 2*(2*b^2*d^2*f*x - b^2*d*f)*cosh(1) + 2*(2*b^2*d^2*f*x + 2*b^2*d^
2*cosh(1) - b^2*d*f)*sinh(1))*sinh(d*x + c)^4 + 3*b^2*f^2 - 24*(a*b*d^2*f^2
*x^2 - 2*a*b*d*f^2*x + a*b*d^2*cosh(1)^2 + a*b*d^2*sinh(1)^2 + 2*a*b*f^2 +
2*(a*b*d^2*f*x - a*b*d*f)*cosh(1) + 2*(a*b*d^2*f*x + a*b*d^2*cosh(1) - a*b*
d*f)*sinh(1))*cosh(d*x + c)^3 - 12*(2*a*b*d^2*f^2*x^2 - 4*a*b*d*f^2*x + 2*a
```

*b*d^2*cosh(1)^2 + 2*a*b*d^2*sinh(1)^2 + 4*a*b*f^2 + 4*(a*b*d^2*f*x - a*b*d
*f)*cosh(1) - (2*b^2*d^2*f^2*x^2 - 2*b^2*d*f^2*x + 2*b^2*d^2*cosh(1)^2 + 2*
b^2*d^2*sinh(1)^2 + b^2*f^2 + 2*(2*b^2*d^2*f*x - b^2*d*f)*cosh(1) + 2*(2*b^
2*d^2*f*x + 2*b^2*d^2*cosh(1) - b^2*d*f)*sinh(1))*cosh(d*x + c) + 4*(a*b*d^
2*f*x + a*b*d^2*cosh(1) - a*b*d*f)*sinh(1))*sinh(d*x + c)^3 - 16*((a^2 + b^
2)*d^3*f^2*x^3 + 2*(a^2 + b^2)*c^3*f^2 + 3*((a^2 + b^2)*d^3*x + 2*(a^2 + b^
2)*c*d^2)*cosh(1)^2 + 3*((a^2 + b^2)*d^3*x + 2*(a^2 + b^2)*c*d^2)*sinh(1)^2
 + 3*((a^2 + b^2)*d^3*f*x^2 - 2*(a^2 + b^2)*c^2*d*f)*cosh(1) + 3*((a^2 + b^
2)*d^3*f*x^2 - 2*(a^2 + b^2)*c^2*d*f + 2*((a^2 + b^2)*d^3*x + 2*(a^2 + b^2)
*c*d^2)*cosh(1))*sinh(1))*cosh(d*x + c)^2 - 2*(8*(a^2 + b^2)*d^3*f^2*x^3 +
16*(a^2 + b^2)*c^3*f^2 + 24*((a^2 + b^2)*d^3*x + 2*(a^2 + b^2)*c*d^2)*cosh(
1)^2 - 9*(2*b^2*d^2*f^2*x^2 - 2*b^2*d*f^2*x + 2*b^2*d^2*cosh(1)^2 + 2*b^2*d
^2*sinh(1)^2 + b^2*f^2 + 2*(2*b^2*d^2*f*x - b^2*d*f)*cosh(1) + 2*(2*b^2*d^2
*f*x + 2*b^2*d^2*cosh(1) - b^2*d*f)*sinh(1))*cosh(d*x + c)^2 + 24*((a^2 + b
^2)*d^3*x + 2*(a^2 + b^2)*c*d^2)*sinh(1)^2 + 24*((a^2 + b^2)*d^3*f*x^2 - 2*
(a^2 + b^2)*c^2*d*f)*cosh(1) + 36*(a*b*d^2*f^2*x^2 - 2*a*b*d*f^2*x + a*b*d^
2*cosh(1)^2 + a*b*d^2*sinh(1)^2 + 2*a*b*f^2 + 2*(a*b*d^2*f*x - a*b*d*f)*cos
h(1) + 2*(a*b*d^2*f*x + a*b*d^2*cosh(1) - a*b*d*f)*sinh(1))*cosh(d*x + c) +
 24*((a^2 + b^2)*d^3*f*x^2 - 2*(a^2 + b^2)*c^2*d*f + 2*((a^2 + b^2)*d^3*x +
 2*(a^2 + b^2)*c*d^2)*cosh(1))*sinh(1))*sinh(d*x + c)^2 + 6*(2*b^2*d^2*f*x
+ b^2*d*f)*cosh(1) + 24*(a*b*d^2*f^2*x^2 + 2*a*b*d*f^2*x + a*b*d^2*cosh(1)^
2 + a*b*d^2*sinh(1)^2 + 2*a*b*f^2 + 2*(a*b*d^2*f*x + a*b*d*f)*cosh(1) + 2*(
a*b*d^2*f*x + a*b*d^2*cosh(1) + a*b*d*f)*sinh(1))*cosh(d*x + c) + 96*(((a^2
 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*cosh(d
x + c)^2 + 2((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*
d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + ((a^2 + b^2)*d*f^2*x + (a^2 + b^
2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*sinh(d*x + c)^2)*dilog((a*cosh(d*
x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b + 1) + 96*(((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(1)
+ (a^2 + b^2)*d*f*sinh(1))*cosh(d*x + c)^2 + 2*((a^2 + b^2)*d*f^2*x + (a^2
+ b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c) +
 ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*
sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 48*(((a^2 + b^2)*c
^2*f^2 - 2*(a^2 + b^2)*c*d*f*cosh(1) + (a^2 + b^2)*d^2*cosh(1)^2 + (a^2 + b
^2)*d^2*sinh(1)^2 - 2*((a^2 + b^2)*c*d*f - (a^2 + b^2)*d^2*cosh(1))*sinh(1)
)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*c^2*f^2 - 2*(a^2 + b^2)*c*d*f*cosh(1) +
(a^2 + b^2)*d^2*cosh(1)^2 + (a^2 + b^2)*d^2*sinh(1)^2 - 2*((a^2 + b^2)*c*d*
f - (a^2 + b^2)*d^2*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + ((a^2 +
 b^2)*c^2*f^2 - 2*(a^2 + b^2)*c*d*f*cosh(1) + (a^2 + b^2)*d^2*cosh(1)^2 + (
a^2 + b^2)*d^2*sinh(1)^2 - 2*((a^2 + b^2)*c*d*f - (a^2 + b^2)*d^2*cosh(1))*
sinh(1))*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*s
qrt((a^2 + b^2)/b^2) + 2*a) + 48*(((a^2 + b^2)*c^2*f^2 - 2*(a^2 + b^2)*c*d*
f*cosh(1) + (a^2 + b^2)*d^2*cosh(1)^2 + (a^2 + b^2)*d^2*sinh(1)^2 - 2*((a^2
 + b^2)*c*d*f - (a^2 + b^2)*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*((a^2

```
 + b^2)*c^2*f^2 - 2*(a^2 + b^2)*c*d*f*cosh(1) + (a^2 + b^2)*d^2*cosh(1)^2 +
 (a^2 + b^2)*d^2*sinh(1)^2 - 2*((a^2 + b^2)*c*d*f - (a^2 + b^2)*d^2*cosh(1)
)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + ((a^2 + b^2)*c^2*f^2 - 2*(a^2 + b^
2)*c*d*f*cosh(1) + (a^2 + b^2)*d^2*cosh(1)^2 + (a^2 + b^2)*d^2*sinh(1)^2 -
2*((a^2 + b^2)*c*d*f - (a^2 + b^2)*d^2*cosh(1))*sinh(1))*sinh(d*x + c)^2)*l
og(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
 + 48*(((a^2 + b^2)*d^2*f^2*x^2 - (a^2 + b^2)*c^2*f^2 + 2*((a^2 + b^2)*d^2*
f*x + (a^2 + b^2)*c*d*f)*cosh(1) + 2*((a^2 + b^2)*d^2*f*x + (a^2 + b^2)*c*d
*f)*sinh(1))*cosh(d*x + c)^2 + 2*((a^2 + b^2)*d^2*f^2*x^2 - (a^2 + b^2)*c^2
*f^2 + 2*((a^2 + b^2)*d^2*f*x + (a^2 + b^2)*c*d*f)*cosh(1) + 2*((a^2 + b^2)
*d^2*f*x + (a^2 + b^2)*c*d*f)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + ((a^2
+ b^2)*d^2*f^2*x^2 - (a^2 + b^2)*c^2*f^2 + 2*((...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^2*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^3\,(e+f\,x)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.301 $\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=298

$$\frac{fx}{4bd}-\frac{(a^2+b^2)(e+fx)^2}{2b^3f}+\frac{af\cosh(c+dx)}{b^2d^2}+\frac{(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d}+\frac{(a^2+b^2)(e+fx)}{}$$

```
[Out] 1/4*f*x/b/d-1/2*(a^2+b^2)*(f*x+e)^2/b^3/f+a*f*cosh(d*x+c)/b^2/d^2+(a^2+b^2)
*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d+(a^2+b^2)*(f*x+e)*ln(
1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d+(a^2+b^2)*f*polylog(2,-b*exp(d*x+
c)/(a-(a^2+b^2)^(1/2)))/b^3/d^2+(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))/b^3/d^2-a*(f*x+e)*sinh(d*x+c)/b^2/d-1/4*f*cosh(d*x+c)*sinh(d*
x+c)/b/d^2+1/2*(f*x+e)*sinh(d*x+c)^2/b/d
```

Rubi [A]
time = 0.26, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5684, 3377, 2718, 5554, 2715, 8, 5680, 2221, 2317, 2438}

$$\frac{f(a^2+b^2)\,\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{f(a^2+b^2)\,\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{(a^2+b^2)(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^3d}+\frac{(a^2+b^2)(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^3d}-\frac{(a^2+b^2)(e+fx)^2}{2b^3f}+\frac{af\cosh(c+dx)}{b^2d^2}-\frac{a(e+fx)\sinh(c+dx)}{b^2d}-\frac{f\sinh(c+dx)\cosh(c+dx)}{4bd^2}+\frac{(e+fx)\sinh^2(c+dx)}{2bd}+\frac{fx}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (f*x)/(4*b*d) - ((a^2 + b^2)*(e + f*x)^2)/(2*b^3*f) + (a*f*Cosh[c + d*x])/(
b^2*d^2) + ((a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])])/(b^3*d) + ((a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/(b^3*d) + ((a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2]))])/(b^3*d^2) + ((a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2]))])/(b^3*d^2) - (a*(e + f*x)*Sinh[c + d*x])/(b^2*d) - (f
*Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^2) + ((e + f*x)*Sinh[c + d*x]^2)/(2*b*
d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
```

```
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\int (e+fx)\cosh(c+dx)\,dx}{b^2} + \frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)\,dx}{b} +$$

$$= -\frac{(a^2+b^2)(e+fx)^2}{2b^3f} - \frac{a(e+fx)\sinh(c+dx)}{b^2d} + \frac{(e+fx)\sinh^2(c+dx)}{2bd} +$$

$$= -\frac{(a^2+b^2)(e+fx)^2}{2b^3f} + \frac{af\cosh(c+dx)}{b^2d^2} + \frac{(a^2+b^2)(e+fx)\log\left(1+\frac{}{a-\sqrt{}}\right.}{b^3d}$$

$$= \frac{fx}{4bd} - \frac{(a^2+b^2)(e+fx)^2}{2b^3f} + \frac{af\cosh(c+dx)}{b^2d^2} + \frac{(a^2+b^2)(e+fx)\log\left(1+\right.}{b^3d}$$

$$= \frac{fx}{4bd} - \frac{(a^2+b^2)(e+fx)^2}{2b^3f} + \frac{af\cosh(c+dx)}{b^2d^2} + \frac{(a^2+b^2)(e+fx)\log\left(1+\right.}{b^3d}$$

Mathematica [A]
time = 1.12, size = 251, normalized size = 0.84

$$\frac{8abf\cosh(c+dx)+2b^2d(e+fx)\cosh(2(c+dx))+8(a^2+b^2)\left(-\frac{1}{2}f(c+dx)^2+f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+f(c+dx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)+de\log(a+b\sinh(c+dx))-cf\log(a+b\sinh(c+dx))+f\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)+f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)-8abd(e+fx)\sinh(c+dx)-b^2f\sinh(2(c+dx))}{8b^3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (8*a*b*f*Cosh[c + d*x] + 2*b^2*d*(e + f*x)*Cosh[2*(c + d*x)] + 8*(a^2 + b^2
)*(-1/2*(f*(c + d*x)^2) + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d*
e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLog[2, (b
*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2]))]) - 8*a*b*d*(e + f*x)*Sinh[c + d*x] - b^2*f*Sinh[2*(c +
d*x)])/(8*b^3*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 974 vs. 2(278) = 556.
time = 3.31, size = 975, normalized size = 3.27

method	result
risch	$\frac{a^2 f \operatorname{dilog}\left(\frac{b\,e^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right)}{d^2 b^3}+\frac{a^2 e \ln\left(b\,e^{2dx+2c}+2a\,e^{dx+c}-b\right)}{d\,b^3}-\frac{2a^2 e \ln\left(e^{dx+c}\right)}{d\,b^3}+\frac{a^2 f \ln\left(\frac{-b\,e^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right.}{d^2 b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/b^3*a^2*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d/b^3*a^2*e*ln(exp(d*
x+c))+1/d^2/b^3*a^2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(
1/2)))+1/d^2/b^3*a^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2
)^(1/2)))+1/d^2/b^3*a^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2
)^(1/2)))*c+1/d/b^3*a^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^
(1/2)))*x+1/d^2/b^3*a^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^
(1/2)))*c-1/d^2/b^3*a^2*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d^2/b^3
*a^2*f*c*ln(exp(d*x+c))+1/d/b^3*a^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/
(-a+(a^2+b^2)^(1/2)))*x-1/2*a^2*f*x^2/b^3-1/2*f*x^2/b-2/d/b^3*f*a^2*c*x+1/1
6*(2*d*f*x+2*d*e-f)/d^2/b*exp(2*d*x+2*c)+1/16*(2*d*f*x+2*d*e+f)/d^2/b*exp(-
2*d*x-2*c)-1/2*a*(d*f*x+d*e-f)/b^2/d^2*exp(d*x+c)-2/d/b*c*f*x+2/d^2/b*f*c*l
n(exp(d*x+c))-1/d^2/b*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/d/b*f*ln(
(-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d^2/b*f*ln((-b*
exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d/b*f*ln((b*exp(d*x
+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2/b*f*ln((b*exp(d*x+c)+(a
^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+a^2*e*x/b^3-1/d^2/b^3*f*a^2*c^2+e*x
/b-1/d^2/b*f*c^2-2/d/b*e*ln(exp(d*x+c))+1/d/b*e*ln(b*exp(2*d*x+2*c)+2*a*exp
(d*x+c)-b)+1/d^2/b*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1
/2)))+1/d^2/b*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)
))+1/2*a*(d*f*x+d*e+f)/b^2/d^2*exp(-d*x-c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*f*((8*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*x^2 + (2*b^2*d*x*e^(4*c) - b
^2*e^(4*c))*e^(2*d*x) - 8*(a*b*d*x*e^(3*c) - a*b*e^(3*c))*e^(d*x) + 8*(a*b*
d*x*e^c + a*b*e^c)*e^(-d*x) + (2*b^2*d*x + b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d
^2) - 2*integrate(16*((a^3*e^c + a*b^2*e^c)*x*e^(d*x) - (a^2*b + b^3)*x)/(b
^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x)) - 1/8*((4*a*e^(-d*x -
c) - b)*e^(2*d*x + 2*c)/(b^2*d) - 8*(a^2 + b^2)*(d*x + c)/(b^3*d) - (4*a*e^
(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d) - 8*(a^2 + b^2)*log(-2*a*e^(-d*x -
 c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1682 vs. $2(281) = 562$.

time = 0.38, size = 1682, normalized size = 5.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(2*b^2*d*f*x + (2*b^2*d*f*x + 2*b^2*d*cosh(1) + 2*b^2*d*sinh(1) - b^2*
f)*cosh(d*x + c)^4 + (2*b^2*d*f*x + 2*b^2*d*cosh(1) + 2*b^2*d*sinh(1) - b^2
*f)*sinh(d*x + c)^4 + 2*b^2*d*cosh(1) - 8*(a*b*d*f*x + a*b*d*cosh(1) + a*b*
d*sinh(1) - a*b*f)*cosh(d*x + c)^3 + 2*b^2*d*sinh(1) - 4*(2*a*b*d*f*x + 2*a
*b*d*cosh(1) + 2*a*b*d*sinh(1) - 2*a*b*f - (2*b^2*d*f*x + 2*b^2*d*cosh(1) +
 2*b^2*d*sinh(1) - b^2*f)*cosh(d*x + c))*sinh(d*x + c)^3 + b^2*f - 8*((a^2
+ b^2)*d^2*f*x^2 - 2*(a^2 + b^2)*c^2*f + 2*((a^2 + b^2)*d^2*x + 2*(a^2 + b^
2)*c*d)*cosh(1) + 2*((a^2 + b^2)*d^2*x + 2*(a^2 + b^2)*c*d)*sinh(1))*cosh(d
*x + c)^2 - 2*(4*(a^2 + b^2)*d^2*f*x^2 - 8*(a^2 + b^2)*c^2*f - 3*(2*b^2*d*f
*x + 2*b^2*d*cosh(1) + 2*b^2*d*sinh(1) - b^2*f)*cosh(d*x + c)^2 + 8*((a^2 +
 b^2)*d^2*x + 2*(a^2 + b^2)*c*d)*cosh(1) + 12*(a*b*d*f*x + a*b*d*cosh(1) +
a*b*d*sinh(1) - a*b*f)*cosh(d*x + c) + 8*((a^2 + b^2)*d^2*x + 2*(a^2 + b^2)
*c*d)*sinh(1))*sinh(d*x + c)^2 + 8*(a*b*d*f*x + a*b*d*cosh(1) + a*b*d*sinh(
1) + a*b*f)*cosh(d*x + c) + 16*((a^2 + b^2)*f*cosh(d*x + c)^2 + 2*(a^2 + b^
2)*f*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*f*sinh(d*x + c)^2)*dilog((a*
cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2) - b)/b + 1) + 16*((a^2 + b^2)*f*cosh(d*x + c)^2 + 2*(a^2 +
 b^2)*f*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*f*sinh(d*x + c)^2)*dilog(
(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sq
rt((a^2 + b^2)/b^2) - b)/b + 1) - 16*(((a^2 + b^2)*c*f - (a^2 + b^2)*d*cosh
(1) - (a^2 + b^2)*d*sinh(1))*cosh(d*x + c)^2 + 2*((a^2 + b^2)*c*f - (a^2 +
b^2)*d*cosh(1) - (a^2 + b^2)*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + ((a^2
 + b^2)*c*f - (a^2 + b^2)*d*cosh(1) - (a^2 + b^2)*d*sinh(1))*sinh(d*x + c)^
2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) +
2*a) - 16*(((a^2 + b^2)*c*f - (a^2 + b^2)*d*cosh(1) - (a^2 + b^2)*d*sinh(1)
)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*c*f - (a^2 + b^2)*d*cosh(1) - (a^2 + b^2
)*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + ((a^2 + b^2)*c*f - (a^2 + b^2)*d
*cosh(1) - (a^2 + b^2)*d*sinh(1))*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) +
2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 16*(((a^2 + b^2)*d*f
*x + (a^2 + b^2)*c*f)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*
c*f)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*si
nh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 16*(((a^2 + b^2)*d*f*x +
(a^2 + b^2)*c*f)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*
cosh(d*x + c)*sinh(d*x + c) + ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*sinh(d*
x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*si
```

```
nh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 4*(2*a*b*d*f*x + 2*a*b*d*cosh(
1) + (2*b^2*d*f*x + 2*b^2*d*cosh(1) + 2*b^2*d*sinh(1) - b^2*f)*cosh(d*x + c
)^3 + 2*a*b*d*sinh(1) + 2*a*b*f - 6*(a*b*d*f*x + a*b*d*cosh(1) + a*b*d*sinh
(1) - a*b*f)*cosh(d*x + c)^2 - 4*((a^2 + b^2)*d^2*f*x^2 - 2*(a^2 + b^2)*c^2
*f + 2*((a^2 + b^2)*d^2*x + 2*(a^2 + b^2)*c*d)*cosh(1) + 2*((a^2 + b^2)*d^2
*x + 2*(a^2 + b^2)*c*d)*sinh(1))*cosh(d*x + c))*sinh(d*x + c))/(b^3*d^2*cos
h(d*x + c)^2 + 2*b^3*d^2*cosh(d*x + c)*sinh(d*x + c) + b^3*d^2*sinh(d*x + c
)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^3\,(e+f\,x)}{a+b\,\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((cosh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.302 $\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{b^3d}-\frac{a\sinh(c+dx)}{b^2d}+\frac{\sinh^2(c+dx)}{2bd}$$

```
[Out] (a^2+b^2)*ln(a+b*sinh(d*x+c))/b^3/d-a*sinh(d*x+c)/b^2/d+1/2*sinh(d*x+c)^2/b
/d
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2747, 711}

$$\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{b^3d}-\frac{a\sinh(c+dx)}{b^2d}+\frac{\sinh^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/(b^3*d) - (a*Sinh[c + d*x])/(b^2*d)
+ Sinh[c + d*x]^2/(2*b*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{-b^2-x^2}{a+x}\,dx, x, b\sinh(c+dx)\right)}{b^3d}\\ &= -\frac{\text{Subst}\left(\int \left(a-x+\frac{-a^2-b^2}{a+x}\right)\,dx, x, b\sinh(c+dx)\right)}{b^3d}\\ &= \frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{b^3d}-\frac{a\sinh(c+dx)}{b^2d}+\frac{\sinh^2(c+dx)}{2bd}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 53, normalized size = 0.90

$$\frac{b^2\cosh(2(c+dx))+4(a^2+b^2)\log(a+b\sinh(c+dx))-4ab\sinh(c+dx)}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b^2*Cosh[2*(c + d*x)] + 4*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]] - 4*a*b*Sin
h[c + d*x])/(4*b^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(57) = 114.
time = 1.10, size = 190, normalized size = 3.22

method	result
risch	$-\frac{x a^2}{b^3}-\frac{x}{b}+\frac{e^{2dx+2c}}{8bd}-\frac{a\,e^{dx+c}}{2b^2d}+\frac{a\,e^{-dx-c}}{2b^2d}+\frac{e^{-2dx-2c}}{8bd}-\frac{2a^2c}{b^3d}-\frac{2c}{bd}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)a^2}{b^3d}+$
derivativedivides	$\frac{\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-b-2a}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-a^2-b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^3}+\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{b-2a}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d}$
default	$\frac{\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-b-2a}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-a^2-b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^3}+\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{b-2a}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/b/(tanh(1/2*d*x+1/2*c)-1)^2-1/2*(-b-2*a)/b^2/(tanh(1/2*d*x+1/2*c)-
1)+(-a^2-b^2)/b^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/2/b/(tanh(1/2*d*x+1/2*c)+1)^2
-1/2*(b-2*a)/b^2/(tanh(1/2*d*x+1/2*c)+1)+(-a^2-b^2)/b^3*ln(tanh(1/2*d*x+1/2
*c)+1)+2/b^3*(1/2*a^2+1/2*b^2)*ln(a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+
1/2*c)-a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(57) = 114.
time = 0.27, size = 127, normalized size = 2.15

$$-\frac{\left(4\,ae^{(-dx-c)}-b\right)e^{(2\,dx+2\,c)}}{8\,b^2d}+\frac{(a^2+b^2)(dx+c)}{b^3d}+\frac{4\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}}{8\,b^2d}+\frac{(a^2+b^2)\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) + (a^2 + b^2)*(d*x + c)
/(b^3*d) + 1/8*(4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d) + (a^2 + b^2
)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(57) = 114$.
time = 0.40, size = 327, normalized size = 5.54

$$\frac{b^2\cosh(dx+c)^4+b^2\sinh(dx+c)^4-8(a^2+b^2)dx\cosh(dx+c)^2-4ab\cosh(dx+c)^3+4(b^2\cosh(dx+c)-ab)\sinh(dx+c)^3+4ab\cosh(dx+c)+2(3b^2\cosh(dx+c)^2-4(a^2+b^2)dx-6ab\cosh(dx+c))\sinh(dx+c)^2+b^2+8((a^2+b^2)\cosh(dx+c)^2+2(a^2+b^2)\cosh(dx+c)\sinh(dx+c)+(a^2+b^2)\sinh(dx+c)^2)\log\left(\frac{2(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)+4(b^2\cosh(dx+c)^3-4(a^2+b^2)dx\cosh(dx+c)-3ab\cosh(dx+c)^2+ab)\sinh(dx+c)}{8(b^3d\cosh(dx+c)^2+2b^3d\cosh(dx+c)\sinh(dx+c)+b^3d\sinh(dx+c)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*cosh(d*x + c)^4 + b^2*sinh(d*x + c)^4 - 8*(a^2 + b^2)*d*x*cosh(d*x
 + c)^2 - 4*a*b*cosh(d*x + c)^3 + 4*(b^2*cosh(d*x + c) - a*b)*sinh(d*x + c)
^3 + 4*a*b*cosh(d*x + c) + 2*(3*b^2*cosh(d*x + c)^2 - 4*(a^2 + b^2)*d*x - 6
*a*b*cosh(d*x + c))*sinh(d*x + c)^2 + b^2 + 8*((a^2 + b^2)*cosh(d*x + c)^2
+ 2*(a^2 + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*sinh(d*x + c)^2)*
log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(b^2*cosh(
d*x + c)^3 - 4*(a^2 + b^2)*d*x*cosh(d*x + c) - 3*a*b*cosh(d*x + c)^2 + a*b)
*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^2 + 2*b^3*d*cosh(d*x + c)*sinh(d*x + c
) + b^3*d*sinh(d*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 92, normalized size = 1.56

$$\frac{\frac{b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-4\,a\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{b^2}+\frac{8\,(a^2+b^2)\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{b^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*((b*(e^(d*x + c) - e^(-d*x - c))^2 - 4*a*(e^(d*x + c) - e^(-d*x - c)))/
b^2 + 8*(a^2 + b^2)*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/b^3)/d
```

Mupad [B]
time = 0.33, size = 120, normalized size = 2.03

$$\frac{\mathrm{e}^{-2\,c-2\,d\,x}}{8\,b\,d}-\frac{x\,(a^2+b^2)}{b^3}+\frac{\mathrm{e}^{2\,c+2\,d\,x}}{8\,b\,d}+\frac{\ln\left(2\,a\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-b+b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\right)\,(a^2+b^2)}{b^3\,d}+\frac{a\,\mathrm{e}^{-c-d\,x}}{2\,b^2\,d}-\frac{a\,\mathrm{e}^{c+d\,x}}{2\,b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)),x)
```

```
[Out] exp(- 2*c - 2*d*x)/(8*b*d) - (x*(a^2 + b^2))/b^3 + exp(2*c + 2*d*x)/(8*b*d)
 + (log(2*a*exp(d*x)*exp(c) - b + b*exp(2*c)*exp(2*d*x))*(a^2 + b^2))/(b^3*
d) + (a*exp(- c - d*x))/(2*b^2*d) - (a*exp(c + d*x))/(2*b^2*d)
```

3.303 $\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cosh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][Cosh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 63.28, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cosh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Integrate[Cosh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b*f) + 1/2*a*e^(
-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^2*f) + 1/2*a*e^(c - d*e/f)*
exp_integral_e(1, -(f*x + e)*d/f)/(b^2*f) - 1/4*e^(2*c - 2*d*e/f)*exp_integ
ral_e(1, -2*(f*x + e)*d/f)/(b*f) + (a^2 + b^2)*log(f*x + e)/(b^3*f) - 1/8*i
ntegrate(16*(a^2*b + b^3 - (a^3*e^c + a*b^2*e^c)*e^(d*x))/(b^4*f*x + b^4*e
- (b^4*f*x*e^(2*c) + b^4*e^(2*c + 1))*e^(2*d*x) - 2*(a*b^3*f*x*e^c + a*b^3*
e^(c + 1))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^3/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh(c+d\,x)^3}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(cosh(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.304 $\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=786

$$\frac{2a(e+fx)^3\text{ArcTan}\left(e^{c+dx}\right)}{(a^2+b^2)\,d}+\frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d}+\frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d}-\frac{b(e+\cdots}{}$$

```
[Out] 2*a*(f*x+e)^3*arctan(exp(d*x+c))/(a^2+b^2)/d-b*(f*x+e)^3*ln(1+exp(2*d*x+2*c
))/(a^2+b^2)/d+b*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)
/d+b*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d+6*I*a*f^2
*(f*x+e)*polylog(3,-I*exp(d*x+c))/(a^2+b^2)/d^3-6*I*a*f^3*polylog(4,-I*exp(
d*x+c))/(a^2+b^2)/d^4-3/2*b*f*(f*x+e)^2*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2
)/d^2+3*b*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2
)/d^2+3*b*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2
)/d^2-6*I*a*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/(a^2+b^2)/d^3+6*I*a*f^3*pol
ylog(4,I*exp(d*x+c))/(a^2+b^2)/d^4+3/2*b*f^2*(f*x+e)*polylog(3,-exp(2*d*x+2
*c))/(a^2+b^2)/d^3-6*b*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/
2)))/(a^2+b^2)/d^3-6*b*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/
2)))/(a^2+b^2)/d^3-3*I*a*f*(f*x+e)^2*polylog(2,-I*exp(d*x+c))/(a^2+b^2)/d^2
+3*I*a*f*(f*x+e)^2*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^2-3/4*b*f^3*polylog(
4,-exp(2*d*x+2*c))/(a^2+b^2)/d^4+6*b*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))/(a^2+b^2)/d^4+6*b*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)
))/(a^2+b^2)/d^4
```

Rubi [A]
time = 1.04, antiderivative size = 786, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5692, 5680, 2221, 2611, 6744, 2320, 6724, 6874, 4265, 3799}

$$\frac{2a(e+fx)^3\text{ArcTan}\left(e^{c+dx}\right)}{d\left(a^2+b^2\right)}-\frac{6iaf^3\text{Li}_4\left(-ie^{c+dx}\right)}{d^4\left(a^2+b^2\right)}+\frac{6iaf^3\text{Li}_4\left(ie^{c+dx}\right)}{d^4\left(a^2+b^2\right)}+\frac{6bf^3\text{Li}_4\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^4\left(a^2+b^2\right)}+\frac{6bf^3\text{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^4\left(a^2+b^2\right)}-\frac{3bf^3\text{Li}_4\left(-e^{2(c+dx)}\right)}{4d^4\left(a^2+b^2\right)}+\frac{6iaf^2(e+fx)\text{Li}_3\left(-ie^{c+dx}\right)}{d^3\left(a^2+b^2\right)}-\frac{6iaf^2(e+fx)\text{Li}_3\left(ie^{c+dx}\right)}{d^3\left(a^2+b^2\right)}-\frac{6bf^2(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^3\left(a^2+b^2\right)}-\frac{6bf^2(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^3\left(a^2+b^2\right)}+\frac{3bf^2(e+fx)\text{Li}_3\left(-e^{2(c+dx)}\right)}{2d^3\left(a^2+b^2\right)}-\frac{3iaf(e+fx)^2\text{Li}_2\left(-ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)}+\frac{3iaf(e+fx)^2\text{Li}_2\left(ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)}+\frac{3bf(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)}+\frac{3bf(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)}-\frac{3bf(e+fx)^2\text{Li}_2\left(-e^{2(c+dx)}\right)}{2d^2\left(a^2+b^2\right)}+\frac{b(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{d\left(a^2+b^2\right)}+\frac{b(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{d\left(a^2+b^2\right)}-\frac{b(e+fx)^3\log\left(e^{2(c+dx)}+1\right)}{d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a*(e + f*x)^3*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) + (b*(e + f*x)^3*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) + (b*(e + f*x)^
3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) - (b*(e +
 f*x)^3*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)*d) - ((3*I)*a*f*(e + f*x)^2*
PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) + ((3*I)*a*f*(e + f*x)^2*Po
lyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) + (3*b*f*(e + f*x)^2*PolyLog[2,
-((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2) + (3*b*f*(e +
f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d
^2) - (3*b*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)*d^2)
```

```
+ ((6*I)*a*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) -
((6*I)*a*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)*d^3) - (6*b*
f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 +
 b^2)*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2]))])/((a^2 + b^2)*d^3) + (3*b*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x
))])/(2*(a^2 + b^2)*d^3) - ((6*I)*a*f^3*PolyLog[4, (-I)*E^(c + d*x)])/((a^2
 + b^2)*d^4) + ((6*I)*a*f^3*PolyLog[4, I*E^(c + d*x)])/((a^2 + b^2)*d^4) +
(6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)
*d^4) + (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^
2 + b^2)*d^4) - (3*b*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*(a^2 + b^2)*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
```

```
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))\,dx}{a^2+b^2} + \frac{b^2\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}$$

$$= -\frac{b(e+fx)^4}{4(a^2+b^2)f} + \frac{\int \left(a(e+fx)^3\text{sech}(c+dx) - b(e+fx)^3\tanh(c+dx)\right)\,dx}{a^2+b^2}$$

$$= -\frac{b(e+fx)^4}{4(a^2+b^2)f} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^3\log\left(1+\right.}{(a^2+b^2}$$

$$= \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx}{}$$

$$= \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx}{}$$

$$= \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx}{}$$

$$= \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx}{}$$

$$= \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx}{}$$

$$= \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3214 vs. 2(786) = 1572.
time = 22.36, size = 3214, normalized size = 4.09

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/4*(-8*b*d^4*e^3*E^(2*c)*x - 12*b*d^4*e^2*E^(2*c)*f*x^2 - 8*b*d^4*e*E^(2*
c)*f^2*x^3 - 2*b*d^4*E^(2*c)*f^3*x^4 - 8*a*d^3*e^3*ArcTan[E^(c + d*x)] - 8*
```

a*d^3*e^3*E^(2*c)*ArcTan[E^(c + d*x)] - (12*I)*a*d^3*e^2*f*x*Log[1 - I*E^(c + d*x)] - (12*I)*a*d^3*e^2*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] - (12*I)*a*d^3*e*f^2*x^2*Log[1 - I*E^(c + d*x)] - (12*I)*a*d^3*e*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] - (4*I)*a*d^3*f^3*x^3*Log[1 - I*E^(c + d*x)] - (4*I)*a*d^3*E^(2*c)*f^3*x^3*Log[1 - I*E^(c + d*x)] + (12*I)*a*d^3*e^2*f*x*Log[1 + I*E^(c + d*x)] + (12*I)*a*d^3*e^2*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] + (12*I)*a*d^3*e*f^2*x^2*Log[1 + I*E^(c + d*x)] + (12*I)*a*d^3*e*E^(2*c)*f^2*x^2*Log[1 + I*E^(c + d*x)] + (4*I)*a*d^3*f^3*x^3*Log[1 + I*E^(c + d*x)] + (4*I)*a*d^3*E^(2*c)*f^3*x^3*Log[1 + I*E^(c + d*x)] + 4*b*d^3*e^3*Log[1 + E^(2*(c + d*x))] + 4*b*d^3*e^3*E^(2*c)*Log[1 + E^(2*(c + d*x))] + 12*b*d^3*e^2*f*x*Log[1 + E^(2*(c + d*x))] + 12*b*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(2*(c + d*x))] + 12*b*d^3*e*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 12*b*d^3*e*E^(2*c)*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 4*b*d^3*f^3*x^3*Log[1 + E^(2*(c + d*x))] + 4*b*d^3*E^(2*c)*f^3*x^3*Log[1 + E^(2*(c + d*x))] + (12*I)*a*d^2*(1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)] - (12*I)*a*d^2*(1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)] + 6*b*d^2*e^2*f*PolyLog[2, -E^(2*(c + d*x))] + 6*b*d^2*e^2*E^(2*c)*f*PolyLog[2, -E^(2*(c + d*x))] + 12*b*d^2*e*f^2*x*PolyLog[2, -E^(2*(c + d*x))] + 12*b*d^2*e*E^(2*c)*f^2*x*PolyLog[2, -E^(2*(c + d*x))] + 6*b*d^2*f^3*x^2*PolyLog[2, -E^(2*(c + d*x))] + 6*b*d^2*E^(2*c)*f^3*x^2*PolyLog[2, -E^(2*(c + d*x))] - (24*I)*a*d*e*f^2*PolyLog[3, (-I)*E^(c + d*x)] - (24*I)*a*d*e*E^(2*c)*f^2*PolyLog[3, (-I)*E^(c + d*x)] - (24*I)*a*d*f^3*x*PolyLog[3, (-I)*E^(c + d*x)] - (24*I)*a*d*E^(2*c)*f^3*x*PolyLog[3, (-I)*E^(c + d*x)] + (24*I)*a*d*e*f^2*PolyLog[3, I*E^(c + d*x)] + (24*I)*a*d*e*E^(2*c)*f^2*PolyLog[3, I*E^(c + d*x)] + (24*I)*a*d*f^3*x*PolyLog[3, I*E^(c + d*x)] + (24*I)*a*d*E^(2*c)*f^3*x*PolyLog[3, I*E^(c + d*x)] - 6*b*d*e*f^2*PolyLog[3, -E^(2*(c + d*x))] - 6*b*d*e*E^(2*c)*f^2*PolyLog[3, -E^(2*(c + d*x))] - 6*b*d*f^3*x*PolyLog[3, -E^(2*(c + d*x))] - 6*b*d*E^(2*c)*f^3*x*PolyLog[3, -E^(2*(c + d*x))] + (24*I)*a*f^3*PolyLog[4, (-I)*E^(c + d*x)] + (24*I)*a*E^(2*c)*f^3*PolyLog[4, (-I)*E^(c + d*x)] - (24*I)*a*f^3*PolyLog[4, I*E^(c + d*x)] - (24*I)*a*E^(2*c)*f^3*PolyLog[4, I*E^(c + d*x)] + 3*b*f^3*PolyLog[4, -E^(2*(c + d*x))] + 3*b*E^(2*c)*f^3*PolyLog[4, -E^(2*(c + d*x))])/((a^2 + b^2)*d^4*(1 + E^(2*c))) - (b*(4*e^3*E^(2*c)*x + 6*e^2*E^(2*c)*f*x^2 + 4*e*E^(2*c)*f^2*x^3 + E^(2*c)*f^3*x^4 + (4*a*Sqrt[a^2 + b^2]*e^3*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (4*a*Sqrt[-(a^2 + b^2)^2]*e^3*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (4*a*Sqrt[-(a^2 + b^2)^2]*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (4*a*Sqrt[-(a^2 + b^2)^2]*e^3*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (2*e^3*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (2*e^3*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (2*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqr

```
t[(a^2 + b^2)*E^(2*c)])])/d - (2*E^(2*c)*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/
(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*
x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b
*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[
1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c
)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d
 + (2*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])
])/d - (2*E^(2*c)*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^
2)*E^(2*c)])])/d - (6*(-1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c +
 d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e
+ f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])
)])/d^2 - (12*e*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2
)*E^(2*c)]))])/d^3 + (12*e*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^
c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (12*f^3*x*PolyLog[3, -((b*E^(2*c +
d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*E^(2*c)*f^3*x*PolyLo
g[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (12*e
*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/
d^3 + (12*e*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 +
 b^2)*E^(2*c)]))])/d^3 - (12*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c +
Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*E^(2*c)...
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - b*log(-2*a*e^(-d*x - c) + b*e^
(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^2 + b
^2)*d))*e^3 + integrate(4*f^3*x^3/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(
e^(d*x + c) + e^(-d*x - c))) + 12*f^2*x^2*e/((b*(e^(d*x + c) - e^(-d*x - c)
) + 2*a)*(e^(d*x + c) + e^(-d*x - c))) + 12*f*x*e^2/((b*(e^(d*x + c) - e^(-
d*x - c)) + 2*a)*(e^(d*x + c) + e^(-d*x - c))), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2794 vs. 2(734) = 1468.
time = 0.43, size = 2794, normalized size = 3.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (6*b*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*b*f^3*polylog(4, (a*cosh(d*
x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2))/b) + 3*(b*d^2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^2*f*cosh(1)^
2 + b*d^2*f*sinh(1)^2 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh(1))*dilog((a
*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b + 1) + 3*(b*d^2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1) +
b*d^2*f*cosh(1)^2 + b*d^2*f*sinh(1)^2 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*s
inh(1))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sin
h(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(-I*a*d^2*f^3*x^2 + b*d^2
*f^3*x^2 - 2*I*a*d^2*f^2*x*cosh(1) + 2*b*d^2*f^2*x*cosh(1) - I*a*d^2*f*cosh
(1)^2 + b*d^2*f*cosh(1)^2 - I*a*d^2*f*sinh(1)^2 + b*d^2*f*sinh(1)^2 - 2*I*(
a*d^2*f^2*x + a*d^2*f*cosh(1))*sinh(1) + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*
sinh(1))*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) - 3*(I*a*d^2*f^3*x^2 + b*
d^2*f^3*x^2 + 2*I*a*d^2*f^2*x*cosh(1) + 2*b*d^2*f^2*x*cosh(1) + I*a*d^2*f*c
osh(1)^2 + b*d^2*f*cosh(1)^2 + I*a*d^2*f*sinh(1)^2 + b*d^2*f*sinh(1)^2 + 2*
I*(a*d^2*f^2*x + a*d^2*f*cosh(1))*sinh(1) + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1
))*sinh(1))*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) - (b*c^3*f^3 - 3*b*c^
2*d*f^2*cosh(1) + 3*b*c*d^2*f*cosh(1)^2 - b*d^3*cosh(1)^3 - b*d^3*sinh(1)^3
 + 3*(b*c*d^2*f - b*d^3*cosh(1))*sinh(1)^2 - 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*c
osh(1) + b*d^3*cosh(1)^2)*sinh(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c
) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b*c^3*f^3 - 3*b*c^2*d*f^2*cosh(1) +
 3*b*c*d^2*f*cosh(1)^2 - b*d^3*cosh(1)^3 - b*d^3*sinh(1)^3 + 3*(b*c*d^2*f -
 b*d^3*cosh(1))*sinh(1)^2 - 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(1) + b*d^3*co
sh(1)^2)*sinh(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2
 + b^2)/b^2) + 2*a) + (b*d^3*f^3*x^3 + b*c^3*f^3 + 3*(b*d^3*f*x + b*c*d^2*f
)*cosh(1)^2 + 3*(b*d^3*f*x + b*c*d^2*f)*sinh(1)^2 + 3*(b*d^3*f^2*x^2 - b*c^
2*d*f^2)*cosh(1) + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2 + 2*(b*d^3*f*x + b*c*d^2*
f)*cosh(1))*sinh(1))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b*d^3*f^3*x^3 + b*
c^3*f^3 + 3*(b*d^3*f*x + b*c*d^2*f)*cosh(1)^2 + 3*(b*d^3*f*x + b*c*d^2*f)*s
inh(1)^2 + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2)*cosh(1) + 3*(b*d^3*f^2*x^2 - b*c
^2*d*f^2 + 2*(b*d^3*f*x + b*c*d^2*f)*cosh(1))*sinh(1))*log(-(a*cosh(d*x + c
) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2) - b)/b) + (-I*a*c^3*f^3 + b*c^3*f^3 + 3*I*a*c^2*d*f^2*cosh(1) - 3*b*c^
2*d*f^2*cosh(1) - 3*I*a*c*d^2*f*cosh(1)^2 + 3*b*c*d^2*f*cosh(1)^2 + I*a*d^3
```

```
*cosh(1)^3 - b*d^3*cosh(1)^3 + I*a*d^3*sinh(1)^3 - b*d^3*sinh(1)^3 - 3*I*(a
*c*d^2*f - a*d^3*cosh(1))*sinh(1)^2 + 3*(b*c*d^2*f - b*d^3*cosh(1))*sinh(1)
^2 + 3*I*(a*c^2*d*f^2 - 2*a*c*d^2*f*cosh(1) + a*d^3*cosh(1)^2)*sinh(1) - 3*
(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(1) + b*d^3*cosh(1)^2)*sinh(1))*log(cosh(d*x
 + c) + sinh(d*x + c) + I) + (I*a*c^3*f^3 + b*c^3*f^3 - 3*I*a*c^2*d*f^2*cos
h(1) - 3*b*c^2*d*f^2*cosh(1) + 3*I*a*c*d^2*f*cosh(1)^2 + 3*b*c*d^2*f*cosh(1
)^2 - I*a*d^3*cosh(1)^3 - b*d^3*cosh(1)^3 - I*a*d^3*sinh(1)^3 - b*d^3*sinh(
1)^3 + 3*I*(a*c*d^2*f - a*d^3*cosh(1))*sinh(1)^2 + 3*(b*c*d^2*f - b*d^3*cos
h(1))*sinh(1)^2 - 3*I*(a*c^2*d*f^2 - 2*a*c*d^2*f*cosh(1) + a*d^3*cosh(1)^2)
*sinh(1) - 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(1) + b*d^3*cosh(1)^2)*sinh(1))
*log(cosh(d*x + c) + sinh(d*x + c) - I) + (-I*a*d^3*f^3*x^3 - b*d^3*f^3*x^3
 - I*a*c^3*f^3 - b*c^3*f^3 - 3*I*(a*d^3*f*x + a*c*d^2*f)*cosh(1)^2 - 3*(b*d
^3*f*x + b*c*d^2*f)*cosh(1)^2 - 3*I*(a*d^3*f*x + a*c*d^2*f)*sinh(1)^2 - 3*(
b*d^3*f*x + b*c*d^2*f)*sinh(1)^2 - 3*I*(a*d^3*f^2*x^2 - a*c^2*d*f^2)*cosh(1
) - 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2)*cosh(1) - 3*I*(a*d^3*f^2*x^2 - a*c^2*d*
f^2 + 2*(a*d^3*f*x + a*c*d^2*f)*cosh(1))*sinh(1) - 3*(b*d^3*f^2*x^2 - b*c^2
*d*f^2 + 2*(b*d^3*f*x + b*c*d^2*f)*cosh(1))*sinh(1))*log(I*cosh(d*x + c) +
I*sinh(d*x + c) + 1) + (I*a*d^3*f^3*x^3 - b*d^3*f^3*x^3 + I*a*c^3*f^3 - b*c
^3*f^3 + 3*I*(a*d^3*f*x + a*c*d^2*f)*cosh(1)^2 - 3*(b*d^3*f*x + b*c*d^2*f)*
cosh(1)^2 + 3*I*(a*d^3*f*x + a*c*d^2*f)*sinh(1)^2 - 3*(b*d^3*f*x + b*c*d^2*
f)*sinh(1)^2 + 3*I*(a*d^3*f^2*x^2 - a*c^2*d*f^2)*cosh(1) - 3*(b*d^3*f^2*x^2
 - b*c^2*d*f^2)*cosh(1) + 3*I*(a*d^3*f^2*x^2 - a*c^2*d*f^2 + 2*(a*d^3*f*x +
 a*c*d^2*f)*cosh(1))*sinh(1) - 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2 + 2*(b*d^3*f*
x + b*c*d^2*f)*cosh(1))*sinh(1))*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1
) - 6*(-I*a*f^3 + b*f^3)*polylog(4, I*cosh(d*x + c) + I*sinh(d*x + c)) - 6*
(I*a*f^3 + b*f^3)*polylog(4, -I*cosh(d*x + c) - I*sinh(d*x + c)) - 6*(b*d*f
^3*x + b*d*f^2*cosh(1) + b*d*f^2*sinh(1))*polylog(3, (a*cosh(d*x + c) + a*s
inh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b
) - 6*(b*d*f^3*x + b*d*f^2*cosh(1) + b*d*f^2*sinh(1))*polylog(3, (a*cosh(d*
x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2))/b) - 6*(I*a*d*f^3*x - b*d*f^3*x + I*...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sech(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^3}{\cosh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((e + f*x)^3/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.305 $\int \frac{(e+fx)^2\mathbf{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=558

$$\frac{2a(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{(a^2+b^2)\,d}+\frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d}+\frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d}-\frac{b(e}{}$$

```
[Out] 2*a*(f*x+e)^2*arctan(exp(d*x+c))/(a^2+b^2)/d-b*(f*x+e)^2*ln(1+exp(2*d*x+2*c
))/(a^2+b^2)/d+b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)
/d+b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d-2*I*a*f*(
f*x+e)*polylog(2,-I*exp(d*x+c))/(a^2+b^2)/d^2+2*I*a*f*(f*x+e)*polylog(2,I*e
xp(d*x+c))/(a^2+b^2)/d^2-b*f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)/d
^2+2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d^2
+2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d^2+2
*I*a*f^2*polylog(3,-I*exp(d*x+c))/(a^2+b^2)/d^3-2*I*a*f^2*polylog(3,I*exp(d
*x+c))/(a^2+b^2)/d^3+1/2*b*f^2*polylog(3,-exp(2*d*x+2*c))/(a^2+b^2)/d^3-2*b
*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d^3-2*b*f^2*pol
ylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d^3
```

Rubi [A]
time = 0.72, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5692, 5680, 2221, 2611, 2320, 6724, 6874, 4265, 3799}

$$\frac{2a(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{d\,(a^2+b^2)}+\frac{2iaf^2\text{Li}_3\left(-ie^{c+dx}\right)}{d^3\,(a^2+b^2)}-\frac{2iaf^2\text{Li}_3\left(ie^{c+dx}\right)}{d^3\,(a^2+b^2)}-\frac{2bf^2\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^3\,(a^2+b^2)}-\frac{2bf^2\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^3\,(a^2+b^2)}+\frac{bf^2\text{Li}_3\left(-e^{2(c+dx)}\right)}{2d^3\,(a^2+b^2)}-\frac{2iaf(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{d^2\,(a^2+b^2)}+\frac{2iaf(e+fx)\text{Li}_2\left(ie^{c+dx}\right)}{d^2\,(a^2+b^2)}+\frac{2bf(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\,(a^2+b^2)}+\frac{2bf(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2\,(a^2+b^2)}-\frac{bf(e+fx)\text{Li}_2\left(-e^{2(c+dx)}\right)}{d^2\,(a^2+b^2)}+\frac{b(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{d\,(a^2+b^2)}+\frac{b(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{d\,(a^2+b^2)}-\frac{b(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{d\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a*(e + f*x)^2*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) + (b*(e + f*x)^2*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) + (b*(e + f*x)^
2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) - (b*(e +
 f*x)^2*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)*d) - ((2*I)*a*f*(e + f*x)*Po
lyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) + ((2*I)*a*f*(e + f*x)*PolyLo
g[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) + (2*b*f*(e + f*x)*PolyLog[2, -((b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2) + (2*b*f*(e + f*x)*P
olyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2) - (b
*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^2)*d^2) + ((2*I)*a*f^2
*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) - ((2*I)*a*f^2*PolyLog[3,
I*E^(c + d*x)])/((a^2 + b^2)*d^3) - (2*b*f^2*PolyLog[3, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^3) - (2*b*f^2*PolyLog[3, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^3) + (b*f^2*PolyLog[3, -E^(2
*(c + d*x))])/(2*(a^2 + b^2)*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
```

```
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))\,dx}{a^2+b^2} + \frac{b^2\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}$$

$$= -\frac{b(e+fx)^3}{3\left(a^2+b^2\right)f} + \frac{\int \left(a(e+fx)^2\text{sech}(c+dx) - b(e+fx)^2\tanh(c+dx)\right)\,dx}{a^2+b^2} +$$

$$= -\frac{b(e+fx)^3}{3\left(a^2+b^2\right)f} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^2\log\left(1+\right.}{\left(a^2+b^2\right)}$$

$$= \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)}{}$$

$$= \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)}{}$$

$$= \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)}{}$$

$$= \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)}{}$$

$$= \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1639 vs. 2(558) = 1116.
time = 17.10, size = 1639, normalized size = 2.94

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (12*b*d^3*e^2*E^(2*c)*x - 12*b*d^3*e^2*(1 + E^(2*c))*x - 12*b*d^3*e*f*x^2 -
 4*b*d^3*f^2*x^3 + 12*a*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] + 6*b*d^2
*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*a*d*e*(1 + E
^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - PolyLog[
2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) + 6*b*d*e*(1 + E^(2*c))*f
```

```
*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))]) +
(6*I)*a*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Log[1 +
 I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2, I*E
^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d*x)])
 + b*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))]) - 6*
d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))]))/(6*(a^2
 + b^2)*d^3*(1 + E^(2*c))) - (b*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^
(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a
^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c
)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*
a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((
-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b
*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(
c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x
) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^
c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d
*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c
 + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1
 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log
[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*
c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d +
(3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/
d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*
E^(2*c)])])/d - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x)
)/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x
)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2
 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]
))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2
 + b^2)*E^(2*c)]))])/d^3 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + S
qrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c +
d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3))/(3*(a^2 + b^2)*(-1 + E^(
2*c))) + (b*x*(3*e^2 + 3*e*f*x + f^2*x^2)*Csch[c/2]*Sech[c/2]*Sech[c])/(6*(
a^2 + b^2))
```

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - b*log(-2*a*e^(-d*x - c) + b*e^
(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^2 + b
^2)*d))*e^2 + integrate(4*f^2*x^2/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(
e^(d*x + c) + e^(-d*x - c))) + 8*f*x*e/((b*(e^(d*x + c) - e^(-d*x - c)) + 2
*a)*(e^(d*x + c) + e^(-d*x - c))), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1424 vs. 2(521) = 1042.
time = 0.43, size = 1424, normalized size = 2.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*b*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*b*f^2*polylog(3, (a*cosh(d
*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2))/b) - 2*(b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1))*dilog((a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2) - b)/b + 1) - 2*(b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1))*d
ilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(-I*a*d*f^2*x + b*d*f^2*x - I*a*d*
f*cosh(1) + b*d*f*cosh(1) - I*a*d*f*sinh(1) + b*d*f*sinh(1))*dilog(I*cosh(d
*x + c) + I*sinh(d*x + c)) + 2*(I*a*d*f^2*x + b*d*f^2*x + I*a*d*f*cosh(1) +
 b*d*f*cosh(1) + I*a*d*f*sinh(1) + b*d*f*sinh(1))*dilog(-I*cosh(d*x + c) -
I*sinh(d*x + c)) - (b*c^2*f^2 - 2*b*c*d*f*cosh(1) + b*d^2*cosh(1)^2 + b*d^2
*sinh(1)^2 - 2*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*log(2*b*cosh(d*x + c) + 2
*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b*c^2*f^2 - 2*b*c*d*
f*cosh(1) + b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 - 2*(b*c*d*f - b*d^2*cosh(1))
*sinh(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/
b^2) + 2*a) - (b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*cosh(1)
+ 2*(b*d^2*f*x + b*c*d*f)*sinh(1))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
+ (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b*d^
2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*cosh(1) + 2*(b*d^2*f*x + b*
c*d*f)*sinh(1))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (I*a*c^2*f^2 - b*c^2*f^2
 - 2*I*a*c*d*f*cosh(1) + 2*b*c*d*f*cosh(1) + I*a*d^2*cosh(1)^2 - b*d^2*cosh
```

```
(1)^2 + I*a*d^2*sinh(1)^2 - b*d^2*sinh(1)^2 - 2*I*(a*c*d*f - a*d^2*cosh(1))
*sinh(1) + 2*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*log(cosh(d*x + c) + sinh(d*
x + c) + I) - (-I*a*c^2*f^2 - b*c^2*f^2 + 2*I*a*c*d*f*cosh(1) + 2*b*c*d*f*c
osh(1) - I*a*d^2*cosh(1)^2 - b*d^2*cosh(1)^2 - I*a*d^2*sinh(1)^2 - b*d^2*si
nh(1)^2 + 2*I*(a*c*d*f - a*d^2*cosh(1))*sinh(1) + 2*(b*c*d*f - b*d^2*cosh(1
))*sinh(1))*log(cosh(d*x + c) + sinh(d*x + c) - I) - (-I*a*d^2*f^2*x^2 - b*
d^2*f^2*x^2 + I*a*c^2*f^2 + b*c^2*f^2 - 2*I*(a*d^2*f*x + a*c*d*f)*cosh(1) -
 2*(b*d^2*f*x + b*c*d*f)*cosh(1) - 2*I*(a*d^2*f*x + a*c*d*f)*sinh(1) - 2*(b
*d^2*f*x + b*c*d*f)*sinh(1))*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) - (
I*a*d^2*f^2*x^2 - b*d^2*f^2*x^2 - I*a*c^2*f^2 + b*c^2*f^2 + 2*I*(a*d^2*f*x
+ a*c*d*f)*cosh(1) - 2*(b*d^2*f*x + b*c*d*f)*cosh(1) + 2*I*(a*d^2*f*x + a*c
*d*f)*sinh(1) - 2*(b*d^2*f*x + b*c*d*f)*sinh(1))*log(-I*cosh(d*x + c) - I*s
inh(d*x + c) + 1) + 2*(I*a*f^2 - b*f^2)*polylog(3, I*cosh(d*x + c) + I*sinh
(d*x + c)) + 2*(-I*a*f^2 - b*f^2)*polylog(3, -I*cosh(d*x + c) - I*sinh(d*x
+ c)))/((a^2 + b^2)*d^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)**2*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^2*sech(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^2}{\cosh(c+d\,x)\,\left(a+b\sinh(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
[Out] int((e + f*x)^2/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.306 $\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=334

$$\frac{2a(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{(a^2+b^2)\,d}+\frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d}+\frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)\,d}-\frac{b(e+fx)}{}$$

```
[Out] 2*a*(f*x+e)*arctan(exp(d*x+c))/(a^2+b^2)/d-b*(f*x+e)*ln(1+exp(2*d*x+2*c))/(
a^2+b^2)/d+b*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d+b*(
f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d-I*a*f*polylog(2,-
I*exp(d*x+c))/(a^2+b^2)/d^2+I*a*f*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^2-1/2
*b*f*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)/d^2+b*f*polylog(2,-b*exp(d*x+c)/(
a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d^2+b*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^
(1/2)))/(a^2+b^2)/d^2
```

Rubi [A]
time = 0.40, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5692, 5680, 2221, 2317, 2438, 6874, 4265, 3799}

$$\frac{2a(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{d\,(a^2+b^2)}-\frac{iaf\text{Li}_2\left(-ie^{c+dx}\right)}{d^2\,(a^2+b^2)}+\frac{iaf\text{Li}_2\left(ie^{c+dx}\right)}{d^2\,(a^2+b^2)}+\frac{bf\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\,(a^2+b^2)}+\frac{bf\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2\,(a^2+b^2)}-\frac{bf\text{Li}_2\left(-e^{2(c+dx)}\right)}{2d^2\,(a^2+b^2)}+\frac{b(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{d\,(a^2+b^2)}+\frac{b(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{d\,(a^2+b^2)}-\frac{b(e+fx)\log\left(e^{2(c+dx)}+1\right)}{d\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) + (b*(e + f*x)*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) + (b*(e + f*x)*Log[
1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) - (b*(e + f*x)*
Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)*d) - (I*a*f*PolyLog[2, (-I)*E^(c + d
*x)])/((a^2 + b^2)*d^2) + (I*a*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^
2) + (b*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2
)*d^2) + (b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 +
 b^2)*d^2) - (b*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))\,dx}{a^2+b^2} + \frac{b^2\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}$$

$$= -\frac{b(e+fx)^2}{2(a^2+b^2)f} + \frac{\int (a(e+fx)\text{sech}(c+dx) - b(e+fx)\tanh(c+dx))\,dx}{a^2+b^2} + \frac{b^2}{\cdots}$$

$$= -\frac{b(e+fx)^2}{2(a^2+b^2)f} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx)\log\left(1+\frac{\cdots}{a+\sqrt{\cdots}}\right)}{(a^2+b^2)d}$$

$$= \frac{2a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)d} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx)\log\cdots}{(\cdots}$$

$$= \frac{2a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)d} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx)\log\cdots}{(\cdots}$$

$$= \frac{2a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)d} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx)\log\cdots}{(\cdots}$$

$$= \frac{2a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)d} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx)\log\cdots}{(\cdots}$$

Mathematica [A]
time = 1.84, size = 439, normalized size = 1.31

$$\frac{2bcde - 2bc^2f + 2bd^2ex - 2bcdfx + 4adeArcTan(\cosh(c+dx)+\sinh(c+dx)) + 4adfxArcTan(\cosh(c+dx)+\sinh(c+dx)) + 2bcf\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) + 2bdfx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) + 2bcf\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right) + 2bdfx\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right) + 2bde\log(a+b\sinh(c+dx)) - 2bcf\log(a+b\sinh(c+dx)) - 2bde\log(1+\cosh(2(c+dx))+\sinh(2(c+dx))) - 2bdfx\log(1+\cosh(2(c+dx))+\sinh(2(c+dx))) + 2bf\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right) + 2bf\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right) - 2iaf\text{PolyLog}(2,-i(\cosh(c+dx)+\sinh(c+dx))) + 2iaf\text{PolyLog}(2,i(\cosh(c+dx)+\sinh(c+dx))) - bf\text{PolyLog}(2,-\cosh(2(c+dx))-\sinh(2(c+dx)))}{2(a^2+b^2)d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*b*c*d*e - 2*b*c^2*f + 2*b*d^2*e*x - 2*b*c*d*f*x + 4*a*d*e*ArcTan[Cosh[c
+ d*x] + Sinh[c + d*x]] + 4*a*d*f*x*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]] +
 2*b*c*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*b*d*f*x*Log[1 +
 (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*b*c*f*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])] + 2*b*d*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])] + 2*b*d*e*Log[a + b*Sinh[c + d*x]] - 2*b*c*f*Log[a + b*Sinh[c + d*x]]
 - 2*b*d*e*Log[1 + Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]] - 2*b*d*f*x*Log[1
 + Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]] + 2*b*f*PolyLog[2, (b*E^(c + d*x)
```

```
)/(-a + Sqrt[a^2 + b^2])] + 2*b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))] - (2*I)*a*f*PolyLog[2, (-I)*(Cosh[c + d*x] + Sinh[c + d*x])] +
(2*I)*a*f*PolyLog[2, I*(Cosh[c + d*x] + Sinh[c + d*x])] - b*f*PolyLog[2, -C
osh[2*(c + d*x)] - Sinh[2*(c + d*x)]])/(2*(a^2 + b^2)*d^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(313) = 626$.
time = 2.87, size = 954, normalized size = 2.86

method	result
risch	$\frac{2eb\ln(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-b)}{d(2a^2+2b^2)}-\frac{2eb\ln(1+\mathrm{e}^{2dx+2c})}{d(2a^2+2b^2)}+\frac{4ea\arctan(\mathrm{e}^{dx+c})}{d(2a^2+2b^2)}+\frac{2fb\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)x}{d(2a^2+2b^2)}+\frac{2fb\ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*e*b/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d*e/(2*a^2+2*
b^2)*b*ln(1+exp(2*d*x+2*c))+4/d*e/(2*a^2+2*b^2)*a*arctan(exp(d*x+c))+2/d*f*
b/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*
x+2/d^2*f*b/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2
)^(1/2)))*c+2/d*f*b/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a
^2+b^2)^(1/2)))*x+2/d^2*f*b/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+
a)/(a+(a^2+b^2)^(1/2)))*c+2/d^2*f*b/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2
+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+2/d^2*f*b/(2*a^2+2*b^2)*dilog((b*exp(d
*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2/d*f/(2*a^2+2*b^2)*ln(1+I*ex
p(d*x+c))*b*x-2/d^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*b*c+2*I/d^2*f/(2*a^2
+2*b^2)*dilog(1-I*exp(d*x+c))*a+2*I/d*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*
x-2/d*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*b*x-2/d^2*f/(2*a^2+2*b^2)*ln(1-I*e
xp(d*x+c))*b*c+2*I/d^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*c-2/d^2*f/(2*a^
2+2*b^2)*dilog(1+I*exp(d*x+c))*b-2*I/d*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*a
*x-2*I/d^2*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))*a-2/d^2*f/(2*a^2+2*b^2)*di
log(1-I*exp(d*x+c))*b-2*I/d^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*a*c-2/d^2*
f*c*b/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d^2*f*c/(2*a^2+
2*b^2)*b*ln(1+exp(2*d*x+2*c))-4/d^2*f*c/(2*a^2+2*b^2)*a*arctan(exp(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - b*log(-2*a*e^(-d*x - c) + b*e^
(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^2 + b
```

```
^2)*d))*e + 2*f*integrate(2*x/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(e^(d
*x + c) + e^(-d*x - c))), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(308) = 616$.
time = 0.37, size = 632, normalized size = 1.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (b*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + b*f*dilog((a*cosh(d*x + c) + a
*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b + 1) + (I*a*f - b*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + (-I*
a*f - b*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) - (b*c*f - b*d*cosh(1)
 - b*d*sinh(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 +
 b^2)/b^2) + 2*a) - (b*c*f - b*d*cosh(1) - b*d*sinh(1))*log(2*b*cosh(d*x +
c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b*d*f*x + b*c*
f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b*d*f*x + b*c*f)*log(-(a*cosh(d*x +
c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2) - b)/b) + (-I*a*c*f + b*c*f + I*a*d*cosh(1) - b*d*cosh(1) + I*a*d*sin
h(1) - b*d*sinh(1))*log(cosh(d*x + c) + sinh(d*x + c) + I) + (I*a*c*f + b*c
*f - I*a*d*cosh(1) - b*d*cosh(1) - I*a*d*sinh(1) - b*d*sinh(1))*log(cosh(d*
x + c) + sinh(d*x + c) - I) + (-I*a*d*f*x - b*d*f*x - I*a*c*f - b*c*f)*log(
I*cosh(d*x + c) + I*sinh(d*x + c) + 1) + (I*a*d*f*x - b*d*f*x + I*a*c*f - b
*c*f)*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1))/((a^2 + b^2)*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)*sech(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\cosh \left(c+d\,x\right)\,\left(a+b\,\sinh \left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
[Out] int((e + f*x)/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.307 $\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=69

$$\frac{a\text{ArcTan}(\sinh(c+dx))}{(a^2+b^2)\,d}-\frac{b\log(\cosh(c+dx))}{(a^2+b^2)\,d}+\frac{b\log(a+b\sinh(c+dx))}{(a^2+b^2)\,d}$$

```
[Out] a*arctan(sinh(d*x+c))/(a^2+b^2)/d-b*ln(cosh(d*x+c))/(a^2+b^2)/d+b*ln(a+b*si
nh(d*x+c))/(a^2+b^2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2747, 720, 31, 649, 210, 266}

$$\frac{a\text{ArcTan}(\sinh(c+dx))}{d\,(a^2+b^2)}+\frac{b\log(a+b\sinh(c+dx))}{d\,(a^2+b^2)}-\frac{b\log(\cosh(c+dx))}{d\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d) - (b*Log[Cosh[c + d*x]])/((a^2 +
b^2)*d) + (b*Log[a + b*Sinh[c + d*x]])/((a^2 + b^2)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{b\text{Subst}\left(\int \frac{1}{(a+x)(-b^2-x^2)}\,dx, x, b\sinh(c+dx)\right)}{d}\\ &= \frac{b\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh(c+dx)\right)}{(a^2+b^2)\,d} + \frac{b\text{Subst}\left(\int \frac{-a+x}{-b^2-x^2}\,dx, x, b\sinh(c+dx)\right)}{(a^2+b^2)\,d}\\ &= \frac{b\log(a+b\sinh(c+dx))}{(a^2+b^2)\,d} + \frac{b\text{Subst}\left(\int \frac{x}{-b^2-x^2}\,dx, x, b\sinh(c+dx)\right)}{(a^2+b^2)\,d} - \frac{(ab)\text{Subst}(}{}\\ &= \frac{a\tan^{-1}(\sinh(c+dx))}{(a^2+b^2)\,d} - \frac{b\log(\cosh(c+dx))}{(a^2+b^2)\,d} + \frac{b\log(a+b\sinh(c+dx))}{(a^2+b^2)\,d}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 114, normalized size = 1.65

$$-\frac{b\left(\left(-a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\sinh(c+dx)\right)-2\sqrt{-b^2}\,\log(a+b\sinh(c+dx))+\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\sinh(c+dx)\right)\right)}{2\sqrt{-b^2}\,(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/2*(b*((-a + Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[c + d*x]] - 2*Sqrt[-b^2]
*Log[a + b*Sinh[c + d*x]] + (a + Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Sinh[c + d*
x]]))/(Sqrt[-b^2]*(a^2 + b^2)*d)
```

Maple [A]
time = 1.12, size = 88, normalized size = 1.28

method	result
derivativedivides	$\frac{\frac{b\ln\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a^2+b^2}+\frac{-b\ln\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2a\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2+b^2}}{d}$
default	$\frac{\frac{b\ln\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a^2+b^2}+\frac{-b\ln\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2a\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2+b^2}}{d}$
risch	$\frac{2b\,d^2x}{a^2d^2+b^2d^2}+\frac{2bdc}{a^2d^2+b^2d^2}-\frac{2bx}{a^2+b^2}-\frac{2bc}{d(a^2+b^2)}+\frac{i\ln(\mathrm{e}^{dx+c}+i)a}{(a^2+b^2)d}-\frac{\ln(\mathrm{e}^{dx+c}+i)b}{(a^2+b^2)d}-\frac{i\ln(\mathrm{e}^{dx+c}-i)a}{(a^2+b^2)d}-\frac{\ln(\mathrm{e}^{dx}}{(a^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(b/(a^2+b^2)*ln(a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)-a)+2/(a^2+b^2)*(-1/2*b*ln(tanh(1/2*d*x+1/2*c)^2+1)+a*arctan(tanh(1/2*d*x+1/2*c))))

Maxima [A]
time = 0.48, size = 95, normalized size = 1.38

$$-\frac{2\,a\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d}+\frac{b\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^2+b^2)d}-\frac{b\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")

[Out] -2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) - b*log(e^(-2*d*x - 2*c) + 1)/((a^2 + b^2)*d)

Fricas [A]
time = 0.41, size = 92, normalized size = 1.33

$$\frac{2\,a\arctan\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)+b\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)-b\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{(a^2+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")

[Out] (2*a*arctan(cosh(d*x + c) + sinh(d*x + c)) + b*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) - b*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/((a^2 + b^2)*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A]
time = 0.42, size = 121, normalized size = 1.75

$$\frac{\frac{2\,b^2\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^2b+b^3}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)a}{a^2+b^2}-\frac{b\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^2+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^2*b + b^3) + (
pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*a/(a^2 + b^2) - b*lo
g((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2 + b^2))/d
```

Mupad [B]
time = 1.23, size = 129, normalized size = 1.87

$$\frac{b\,\ln\left(2\,a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-4\,b^3-a^2\,b+4\,b^3\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+a^2\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+8\,a\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\right)}{d\,a^2+d\,b^2}-\frac{\ln\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)}{b\,d+a\,d\,1\mathrm{i}}-\frac{\ln\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)\,1\mathrm{i}}{a\,d+b\,d\,1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] (b*log(2*a^3*exp(d*x)*exp(c) - 4*b^3 - a^2*b + 4*b^3*exp(2*c)*exp(2*d*x) +
a^2*b*exp(2*c)*exp(2*d*x) + 8*a*b^2*exp(d*x)*exp(c)))/(a^2*d + b^2*d) - (lo
g(exp(c + d*x)*1i + 1)*1i)/(a*d + b*d*1i) - log(exp(c + d*x) + 1i)/(a*d*1i
+ b*d)
```

3.308 $\int \frac{\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sech[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][Sech[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 14.79, size = 0, normalized size = 0.00

$$\int \frac{\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sech[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Integrate[Sech[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\text{sech}(dx+c)}{(fx+e)\left(a+b\sinh\left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sech(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sech(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sech(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sech(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cosh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
[Out] int(1/(cosh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.309 $\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=780

$$\frac{a(e+fx)^3}{(a^2+b^2)\,d}-\frac{6bf(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2}+\frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}-\frac{b^2(e+fx)^3\log\left(1+\frac{b\ldots}{a+\sqrt{\ldots}}\right.}{(a^2+b^2)^{3/2}\,d}$$

```
[Out] a*(f*x+e)^3/(a^2+b^2)/d-6*b*f*(f*x+e)^2*arctan(exp(d*x+c))/(a^2+b^2)/d^2-3*
a*f*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/(a^2+b^2)/d^2+b^2*(f*x+e)^3*ln(1+b*exp(d
*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d-b^2*(f*x+e)^3*ln(1+b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d+6*I*b*f^2*(f*x+e)*polylog(2,-I*exp
(d*x+c))/(a^2+b^2)/d^3+6*I*b*f^3*polylog(3,I*exp(d*x+c))/(a^2+b^2)/d^4-3*a*
f^2*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)/d^3+3*b^2*f*(f*x+e)^2*poly
log(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-3*b^2*f*(f*x+e
)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-6*I*b*
f^3*polylog(3,-I*exp(d*x+c))/(a^2+b^2)/d^4-6*I*b*f^2*(f*x+e)*polylog(2,I*ex
p(d*x+c))/(a^2+b^2)/d^3+3/2*a*f^3*polylog(3,-exp(2*d*x+2*c))/(a^2+b^2)/d^4-
6*b^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3
/2)/d^3+6*b^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2
+b^2)^(3/2)/d^3+6*b^2*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2
+b^2)^(3/2)/d^4-6*b^2*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2
+b^2)^(3/2)/d^4+b*(f*x+e)^3*sech(d*x+c)/(a^2+b^2)/d+a*(f*x+e)^3*tanh(d*x+c)
/(a^2+b^2)/d
```

Rubi [A]
time = 1.27, antiderivative size = 780, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {5692, 3403, 2296, 2221, 2611, 6744, 2320, 6724, 6874, 4269, 3799, 5559, 4265}

$$-\frac{6bf(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{d^2(a^2+b^2)}-\frac{6ibf^3\text{Li}_3\left(-ie^{c+dx}\right)}{d^4(a^2+b^2)}+\frac{6ibf^3\text{Li}_3\left(ie^{c+dx}\right)}{d^4(a^2+b^2)}+\frac{3af^3\text{Li}_3\left(-e^{2(c+dx)}\right)}{2d^4(a^2+b^2)}+\frac{6b^2f^3\text{Li}_4\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^4(a^2+b^2)^{3/2}}-\frac{6b^2f^3\text{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^4(a^2+b^2)^{3/2}}+\frac{6ibf^2(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{d^3(a^2+b^2)}-\frac{6ibf^2(e+fx)\text{Li}_2\left(ie^{c+dx}\right)}{d^3(a^2+b^2)}-\frac{3af^2(e+fx)\text{Li}_2\left(-e^{2(c+dx)}\right)}{d^3(a^2+b^2)}-\frac{6b^2f^2(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^3(a^2+b^2)^{3/2}}+\frac{6b^2f^2(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^3(a^2+b^2)^{3/2}}+\frac{3b^2f(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2(a^2+b^2)^{3/2}}-\frac{3b^2f(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2(a^2+b^2)^{3/2}}-\frac{3af(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{d^2(a^2+b^2)}+\frac{b^2(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{d(a^2+b^2)^{3/2}}-\frac{b^2(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{d(a^2+b^2)^{3/2}}+\frac{a(e+fx)^3\tanh(c+dx)}{d(a^2+b^2)}+\frac{b(e+fx)^3\text{sech}(c+dx)}{d(a^2+b^2)}+\frac{a(e+fx)^3}{d(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*(e + f*x)^3)/((a^2 + b^2)*d) - (6*b*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/(
(a^2 + b^2)*d^2) + (b^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/((a^2 + b^2)^(3/2)*d) - (b^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (3*a*f*(e + f*x)^2*Log[1 + E
^(2*(c + d*x))])/((a^2 + b^2)*d^2) + ((6*I)*b*f^2*(e + f*x)*PolyLog[2, (-I)
*E^(c + d*x)])/((a^2 + b^2)*d^3) - ((6*I)*b*f^2*(e + f*x)*PolyLog[2, I*E^(c
 + d*x)])/((a^2 + b^2)*d^3) + (3*b^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (3*b^2*f*(e + f*x)^
2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*
```

```
d^2) - (3*a*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^2)*d^3) -
 ((6*I)*b*f^3*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^4) + ((6*I)*b*f^
3*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)*d^4) - (6*b^2*f^2*(e + f*x)*PolyL
og[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) +
(6*b^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/
((a^2 + b^2)^(3/2)*d^3) + (3*a*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*(a^2 +
b^2)*d^4) + (6*b^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]
)/((a^2 + b^2)^(3/2)*d^4) - (6*b^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^4) + (b*(e + f*x)^3*Sech[c + d*x])/(
(a^2 + b^2)*d) + (a*(e + f*x)^3*Tanh[c + d*x])/((a^2 + b^2)*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
```

```
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))\,dx}{a^2+b^2} + \frac{b^2\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{\int \left(a(e+fx)^3\text{sech}^2(c+dx) - b(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)\right)\,dx}{a^2+b^2} +$$

$$= \frac{(2b^3)\int \frac{e^{c+dx}(e+fx)^3}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{(a^2+b^2)^{3/2}} - \frac{(2b^3)\int \frac{e^{c+dx}(e+fx)^3}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{(a^2+b^2)^{3/2}} + \frac{a\int(}{}$$

$$= \frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} - \frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}$$

$$= \frac{a(e+fx)^3}{(a^2+b^2)\,d} - \frac{6bf(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2} + \frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+}}\right)}{(a^2+b^2)^{3/2}\,d}$$

$$= \frac{a(e+fx)^3}{(a^2+b^2)\,d} - \frac{6bf(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2} + \frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+}}\right)}{(a^2+b^2)^{3/2}\,d}$$

$$= \frac{a(e+fx)^3}{(a^2+b^2)\,d} - \frac{6bf(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2} + \frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+}}\right)}{(a^2+b^2)^{3/2}\,d}$$

$$= \frac{a(e+fx)^3}{(a^2+b^2)\,d} - \frac{6bf(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2} + \frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+}}\right)}{(a^2+b^2)^{3/2}\,d}$$

$$= \frac{a(e+fx)^3}{(a^2+b^2)\,d} - \frac{6bf(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2} + \frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+}}\right)}{(a^2+b^2)^{3/2}\,d}$$

Mathematica [A]
time = 14.50, size = 1531, normalized size = 1.96

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/2*(f*(-12*a*d^3*e^2*E^(2*c)*x + 12*a*d^3*e^2*(1 + E^(2*c))*x + 12*a*d^3*
e*f*x^2 + 4*a*d^3*f^2*x^3 + 12*b*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)]
```

```
- 6*a*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*b*d
*e*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) -
 PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*d*e*(1 + E
^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d
*x))]) + (6*I)*b*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^
2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyL
og[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c
 + d*x)]) - a*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x
))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))])
))/((a^2 + b^2)*d^4*(1 + E^(2*c))) - (b^2*(2*d^3*Sqrt[(a^2 + b^2)*E^(2*c)]*
(e + f*x)^3*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]] - 3*d^2*f*(e + f*x
)^2*(d*x*(2*Sqrt[(a^2 + b^2)*E^(2*c)]*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2
- b^2]] + Sqrt[-a^2 - b^2]*E^c*(-Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a
^2 + b^2)*E^(2*c)])] + Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*
E^(2*c)])])) - Sqrt[-a^2 - b^2]*E^c*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c -
 Sqrt[(a^2 + b^2)*E^(2*c)]))] + Sqrt[-a^2 - b^2]*E^c*PolyLog[2, -((b*E^(2*c
 + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]) + 3*d*f^2*(e + f*x)*(d^2*x^
2*(2*Sqrt[(a^2 + b^2)*E^(2*c)]*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]]
 + Sqrt[-a^2 - b^2]*E^c*(-Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^
2)*E^(2*c)])] + Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)
])])) - 2*Sqrt[-a^2 - b^2]*E^c*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt
[(a^2 + b^2)*E^(2*c)]))] + 2*Sqrt[-a^2 - b^2]*E^c*PolyLog[3, -((b*E^(2*c +
d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]) - f^3*(d^3*x^3*(2*Sqrt[(a^2 +
b^2)*E^(2*c)]*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]] + Sqrt[-a^2 - b^
2]*E^c*(-Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + L
og[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])) - 6*Sqrt[-a
^2 - b^2]*E^c*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)]))] + 6*Sqrt[-a^2 - b^2]*E^c*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c + Sq
rt[(a^2 + b^2)*E^(2*c)]))])))/((-a^2 - b^2)^(3/2)*d^4*Sqrt[(a^2 + b^2)*E^(2
*c)]) + (Sech[c]*Sech[c + d*x]*(b*e^3*Cosh[c] + 3*b*e^2*f*x*Cosh[c] + 3*b*e
*f^2*x^2*Cosh[c] + b*f^3*x^3*Cosh[c] + a*e^3*Sinh[d*x] + 3*a*e^2*f*x*Sinh[d
*x] + 3*a*e*f^2*x^2*Sinh[d*x] + a*f^3*x^3*Sinh[d*x]))/((a^2 + b^2)*d)
```

Maple [F]
time = 2.46, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \operatorname{sech}(dx+c)^2}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -6*b*f^3*integrate(x^2*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x
+ 2*c) + a^2*d + b^2*d), x) + 6*a*f^3*integrate(x^2/(a^2*d*e^(2*d*x + 2*c)
+ b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + 12*a*f^2*e*integrate(x/(a^2*
d*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + 3*a*f*(2*(
d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 + b^2)*d^2))*e^
2 - 12*b*f^2*integrate(x*e^(d*x + c + 1)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*e^(
2*d*x + 2*c) + a^2*d + b^2*d), x) + (b^2*log((b*e^(-d*x - c) - a - sqrt(a^2
 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2)*d) + 2*
(b*e^(-d*x - c) + a)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*d))*e^3 -
6*b*f*arctan(e^(d*x + c))*e^2/((a^2 + b^2)*d^2) - 2*(a*f^3*x^3 + 3*a*f^2*x^
2*e + 3*a*f*x*e^2 - (b*f^3*x^3*e^c + 3*b*f^2*x^2*e^(c + 1) + 3*b*f*x*e^(c +
 2))*e^(d*x))/(a^2*d + b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) +
 integrate(-2*(b^2*f^3*x^3*e^c + 3*b^2*f^2*x^2*e^(c + 1) + 3*b^2*f*x*e^(c +
 2))*e^(d*x)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e^(2*c))*e^(2*d*x) - 2*(a^
3*e^c + a*b^2*e^c)*e^(d*x)), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 10547 vs. $2(726) = 1452$.
time = 0.59, size = 10547, normalized size = 13.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*(a^3 + a*b^2)*c^3*f^3 - 6*(a^3 + a*b^2)*c^2*d*f^2*cosh(1) + 6*(a^3 + a*b
^2)*c*d^2*f*cosh(1)^2 - 2*(a^3 + a*b^2)*d^3*cosh(1)^3 - 2*(a^3 + a*b^2)*d^3
*sinh(1)^3 + 2*((a^3 + a*b^2)*d^3*f^3*x^3 + (a^3 + a*b^2)*c^3*f^3 + 3*((a^3
 + a*b^2)*d^3*f*x + (a^3 + a*b^2)*c*d^2*f)*cosh(1)^2 + 3*((a^3 + a*b^2)*d^3
*f*x + (a^3 + a*b^2)*c*d^2*f)*sinh(1)^2 + 3*((a^3 + a*b^2)*d^3*f^2*x^2 - (a
^3 + a*b^2)*c^2*d*f^2)*cosh(1) + 3*((a^3 + a*b^2)*d^3*f^2*x^2 - (a^3 + a*b^
2)*c^2*d*f^2 + 2*((a^3 + a*b^2)*d^3*f*x + (a^3 + a*b^2)*c*d^2*f)*cosh(1))*s
inh(1))*cosh(d*x + c)^2 + 6*((a^3 + a*b^2)*c*d^2*f - (a^3 + a*b^2)*d^3*cosh
(1))*sinh(1)^2 + 2*((a^3 + a*b^2)*d^3*f^3*x^3 + (a^3 + a*b^2)*c^3*f^3 + 3*(
(a^3 + a*b^2)*d^3*f*x + (a^3 + a*b^2)*c*d^2*f)*cosh(1)^2 + 3*((a^3 + a*b^2)
*d^3*f*x + (a^3 + a*b^2)*c*d^2*f)*sinh(1)^2 + 3*((a^3 + a*b^2)*d^3*f^2*x^2
- (a^3 + a*b^2)*c^2*d*f^2)*cosh(1) + 3*((a^3 + a*b^2)*d^3*f^2*x^2 - (a^3 +
a*b^2)*c^2*d*f^2 + 2*((a^3 + a*b^2)*d^3*f*x + (a^3 + a*b^2)*c*d^2*f)*cosh(1
```

))*sinh(1))*sinh(d*x + c)^2 + 3*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*f^2*x*cosh(1)
+ b^3*d^2*f*cosh(1)^2 + b^3*d^2*f*sinh(1)^2 + (b^3*d^2*f^3*x^2 + 2*b^3*d^2*
f^2*x*cosh(1) + b^3*d^2*f*cosh(1)^2 + b^3*d^2*f*sinh(1)^2 + 2*(b^3*d^2*f^2*
x + b^3*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(b^3*d^2*f^3*x^2 + 2*b^
3*d^2*f^2*x*cosh(1) + b^3*d^2*f*cosh(1)^2 + b^3*d^2*f*sinh(1)^2 + 2*(b^3*d^
2*f^2*x + b^3*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (b^3*d^
2*f^3*x^2 + 2*b^3*d^2*f^2*x*cosh(1) + b^3*d^2*f*cosh(1)^2 + b^3*d^2*f*sinh(
1)^2 + 2*(b^3*d^2*f^2*x + b^3*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c)^2 + 2*(
b^3*d^2*f^2*x + b^3*d^2*f*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*dilog((a*
cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2) - b)/b + 1) - 3*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*f^2*x*cosh(1)
 + b^3*d^2*f*cosh(1)^2 + b^3*d^2*f*sinh(1)^2 + (b^3*d^2*f^3*x^2 + 2*b^3*d^2
*f^2*x*cosh(1) + b^3*d^2*f*cosh(1)^2 + b^3*d^2*f*sinh(1)^2 + 2*(b^3*d^2*f^2
*x + b^3*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(b^3*d^2*f^3*x^2 + 2*b
^3*d^2*f^2*x*cosh(1) + b^3*d^2*f*cosh(1)^2 + b^3*d^2*f*sinh(1)^2 + 2*(b^3*d
^2*f^2*x + b^3*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (b^3*d
^2*f^3*x^2 + 2*b^3*d^2*f^2*x*cosh(1) + b^3*d^2*f*cosh(1)^2 + b^3*d^2*f*sinh
(1)^2 + 2*(b^3*d^2*f^2*x + b^3*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c)^2 + 2*
(b^3*d^2*f^2*x + b^3*d^2*f*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*dilog((a
*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b + 1) + (b^3*c^3*f^3 - 3*b^3*c^2*d*f^2*cosh(1) + 3*
b^3*c*d^2*f*cosh(1)^2 - b^3*d^3*cosh(1)^3 - b^3*d^3*sinh(1)^3 + (b^3*c^3*f^
3 - 3*b^3*c^2*d*f^2*cosh(1) + 3*b^3*c*d^2*f*cosh(1)^2 - b^3*d^3*cosh(1)^3 -
 b^3*d^3*sinh(1)^3 + 3*(b^3*c*d^2*f - b^3*d^3*cosh(1))*sinh(1)^2 - 3*(b^3*c
^2*d*f^2 - 2*b^3*c*d^2*f*cosh(1) + b^3*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c
)^2 + 3*(b^3*c*d^2*f - b^3*d^3*cosh(1))*sinh(1)^2 + 2*(b^3*c^3*f^3 - 3*b^3*
c^2*d*f^2*cosh(1) + 3*b^3*c*d^2*f*cosh(1)^2 - b^3*d^3*cosh(1)^3 - b^3*d^3*s
inh(1)^3 + 3*(b^3*c*d^2*f - b^3*d^3*cosh(1))*sinh(1)^2 - 3*(b^3*c^2*d*f^2 -
 2*b^3*c*d^2*f*cosh(1) + b^3*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c)*sinh(d*x
 + c) + (b^3*c^3*f^3 - 3*b^3*c^2*d*f^2*cosh(1) + 3*b^3*c*d^2*f*cosh(1)^2 -
b^3*d^3*cosh(1)^3 - b^3*d^3*sinh(1)^3 + 3*(b^3*c*d^2*f - b^3*d^3*cosh(1))*s
inh(1)^2 - 3*(b^3*c^2*d*f^2 - 2*b^3*c*d^2*f*cosh(1) + b^3*d^3*cosh(1)^2)*si
nh(1))*sinh(d*x + c)^2 - 3*(b^3*c^2*d*f^2 - 2*b^3*c*d^2*f*cosh(1) + b^3*d^3
*cosh(1)^2)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh
(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^3*c^3*f^3 - 3*b^3*c^2*d*f
^2*cosh(1) + 3*b^3*c*d^2*f*cosh(1)^2 - b^3*d^3*cosh(1)^3 - b^3*d^3*sinh(1)^
3 + (b^3*c^3*f^3 - 3*b^3*c^2*d*f^2*cosh(1) + 3*b^3*c*d^2*f*cosh(1)^2 - b^3*
d^3*cosh(1)^3 - b^3*d^3*sinh(1)^3 + 3*(b^3*c*d^2*f - b^3*d^3*cosh(1))*sinh(
1)^2 - 3*(b^3*c^2*d*f^2 - 2*b^3*c*d^2*f*cosh(1) + b^3*d^3*cosh(1)^2)*sinh(1
))*cosh(d*x + c)^2 + 3*(b^3*c*d^2*f - b^3*d^3*cosh(1))*sinh(1)^2 + 2*(b^3*c
^3*f^3 - 3*b^3*c^2*d*f^2*cosh(1) + 3*b^3*c*d^2*f*cosh(1)^2 - b^3*d^3*cosh(1
)^3 - b^3*d^3*sinh(1)^3 + 3*(b^3*c*d^2*f - b^3*d^3*cosh(1))*sinh(1)^2 - 3*(
b^3*c^2*d*f^2 - 2*b^3*c*d^2*f*cosh(1) + b^3*d^3*cosh(1)^2)*sinh(1))*cosh(d*
x + c)*sinh(d*x + c) + (b^3*c^3*f^3 - 3*b^3*c^2*d*f^2*cosh(1) + 3*b^3*c*d^2
*f*cosh(1)^2 - b^3*d^3*cosh(1)^3 - b^3*d^3*sinh(1)^3 + 3*(b^3*c*d^2*f - b^3

```
*d^3*cosh(1))*sinh(1)^2 - 3*(b^3*c^2*d*f^2 - 2*b^3*c*d^2*f*cosh(1) + b^3*d^
3*cosh(1)^2)*sinh(1))*sinh(d*x + c)^2 - 3*(b^3*c^2*d*f^2 - 2*b^3*c*d^2*f*co
sh(1) + b^3*d^3*cosh(1)^2)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x
+ c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^3*d^3*f^3*
x^3 + b^3*c^3*f^3 + 3*(b^3*d^3*f*x + b^3*c*d^2*f)*cosh(1)^2 + (b^3*d^3*f^3*
x^3 + b^3*c^3*f^3 + 3*(b^3*d^3*f*x + b^3*c*d^2*f)*cosh(1)^2 + 3*(b^3*d^3*f*
x + b^3*c*d^2*f)*sinh(1)^2 + 3*(b^3*d^3*f^2*x^2 - b^3*c^2*d*f^2)*cosh(1) +
3*(b^3*d^3*f^2*x^2 - b^3*c^2*d*f^2 + 2*(b^3*d^3*f*x + b^3*c*d^2*f)*cosh(1))
*sinh(1))*cosh(d*x + c)^2 + 3*(b^3*d^3*f*x + b^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3 \operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)**3*sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^3}{\cosh(c+d\,x)^2\,\left(a+b\sinh(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
[Out] int((e + f*x)^3/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.310 $\int \frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=548

$$\frac{a(e+fx)^2}{(a^2+b^2)\,d}-\frac{4bf(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2}+\frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}-\frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+}}{a+\sqrt{a^2}}\right.}{(a^2+b^2)^{3/2}\,d}$$

```
[Out] a*(f*x+e)^2/(a^2+b^2)/d-4*b*f*(f*x+e)*arctan(exp(d*x+c))/(a^2+b^2)/d^2-2*a*
f*(f*x+e)*ln(1+exp(2*d*x+2*c))/(a^2+b^2)/d^2+b^2*(f*x+e)^2*ln(1+b*exp(d*x+c
)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d-b^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a
+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d+2*I*b*f^2*polylog(2,-I*exp(d*x+c))/(a^
2+b^2)/d^3-2*I*b*f^2*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^3-a*f^2*polylog(2,
-exp(2*d*x+2*c))/(a^2+b^2)/d^3+2*b^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(
a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-2*b^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-2*b^2*f^2*polylog(3,-b*exp(d*x+c
)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^3+2*b^2*f^2*polylog(3,-b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^3+b*(f*x+e)^2*sech(d*x+c)/(a^2+b^2
)/d+a*(f*x+e)^2*tanh(d*x+c)/(a^2+b^2)/d
```

Rubi [A]
time = 0.93, antiderivative size = 548, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5692, 3403, 2296, 2221, 2611, 2320, 6724, 6874, 4269, 3799, 2317, 2438, 5559, 4265}

$$-\frac{4bf(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{d^2(a^2+b^2)}-\frac{2b^2f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^3(a^2+b^2)^{3/2}}+\frac{2b^2f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^3(a^2+b^2)^{3/2}}+\frac{2ibf^2\text{Li}_2\left(-ie^{c+dx}\right)}{d^3(a^2+b^2)}-\frac{2ibf^2\text{Li}_2\left(ie^{c+dx}\right)}{d^3(a^2+b^2)}-\frac{af^2\text{Li}_2\left(-e^{2(c+dx)}\right)}{d^3(a^2+b^2)}+\frac{2b^2f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2(a^2+b^2)^{3/2}}-\frac{2b^2f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2(a^2+b^2)^{3/2}}-\frac{2af(e+fx)\log\left(e^{2(c+dx)}+1\right)}{d^2(a^2+b^2)}+\frac{b^2(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{d(a^2+b^2)^{3/2}}-\frac{b^2(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{d(a^2+b^2)^{3/2}}+\frac{a(e+fx)^2\tanh(c+dx)}{d(a^2+b^2)}+\frac{b(e+fx)^2\text{sech}(c+dx)}{d(a^2+b^2)}+\frac{a(e+fx)^2}{d(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*(e + f*x)^2)/((a^2 + b^2)*d) - (4*b*f*(e + f*x)*ArcTan[E^(c + d*x)])/((a
^2 + b^2)*d^2) + (b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])])/((a^2 + b^2)^(3/2)*d) - (b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (2*a*f*(e + f*x)*Log[1 + E^(2*
(c + d*x))])/((a^2 + b^2)*d^2) + ((2*I)*b*f^2*PolyLog[2, (-I)*E^(c + d*x)])
/((a^2 + b^2)*d^3) - ((2*I)*b*f^2*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d
^3) + (2*b^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
)])/((a^2 + b^2)^(3/2)*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (a*f^2*PolyLog[2, -E^
(2*(c + d*x))])/((a^2 + b^2)*d^3) - (2*b^2*f^2*PolyLog[3, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) + (2*b^2*f^2*PolyLog[3, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) + (b*(e +
```

```
f*x)^2*Sech[c + d*x])/((a^2 + b^2)*d) + (a*(e + f*x)^2*Tanh[c + d*x])/((a^
2 + b^2)*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))\,dx}{a^2+b^2} + \frac{b^2\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{\int \left(a(e+fx)^2\operatorname{sech}^2(c+dx) - b(e+fx)^2\operatorname{sech}(c+dx)\tanh(c+dx)\right)\,dx}{a^2+b^2} +$$

$$= \frac{(2b^3)\int \frac{e^{c+dx}(e+fx)^2}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{(a^2+b^2)^{3/2}} - \frac{(2b^3)\int \frac{e^{c+dx}(e+fx)^2}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{(a^2+b^2)^{3/2}} + \frac{a\int(}{}$$

$$= \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} - \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}$$

$$= \frac{a(e+fx)^2}{(a^2+b^2)\,d} - \frac{4bf(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2} + \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+}}\right.}{(a^2+b^2)^{3/2}\,d}$$

$$= \frac{a(e+fx)^2}{(a^2+b^2)\,d} - \frac{4bf(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2} + \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+}}\right.}{(a^2+b^2)^{3/2}\,d}$$

$$= \frac{a(e+fx)^2}{(a^2+b^2)\,d} - \frac{4bf(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2} + \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+}}\right.}{(a^2+b^2)^{3/2}\,d}$$

$$= \frac{a(e+fx)^2}{(a^2+b^2)\,d} - \frac{4bf(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2} + \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+}}\right.}{(a^2+b^2)^{3/2}\,d}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1180 vs. 2(548) = 1096.
time = 9.44, size = 1180, normalized size = 2.15

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f*x)^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]

[Out] (b^2*((2*d^2*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + (2*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] + (d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] - (d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] + (2*d*E^c*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*d*E^c*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] + (2*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)]))/((a^2 + b^2)*d^3) - (2*a*e*f*Sech[c]*(Cosh[c]*Log[Cosh[c]*Cosh[d*x] + Sinh[c]*Sinh[d*x]] - d*x*Sinh[c]))/((a^2 + b^2)*d^2*(Cosh[c]^2 - Sinh[c]^2)) - (4*b*e*f*ArcTan[(Sinh[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]])/((a^2 + b^2)*d^2*Sqrt[Cosh[c]^2 - Sinh[c]^2]) - (a*f^2*Csch[c]*((d^2*x^2)/E^ArcTanh[Coth[c]] - (I*Coth[c]*(-(d*x*(-Pi + (2*I)*ArcTanh[Coth[c]])) - Pi*Log[1 + E^(2*d*x)] - 2*(I*d*x + I*ArcTanh[Coth[c]])*Log[1 - E^((2*I)*(I*d*x + I*ArcTanh[Coth[c]]))] + Pi*Log[Cosh[d*x]] + (2*I)*ArcTanh[Coth[c]]*Log[I*Sinh[d*x + ArcTanh[Coth[c]]]] + I*PolyLog[2, E^((2*I)*(I*d*x + I*ArcTanh[Coth[c]]))]))/Sqrt[1 - Coth[c]^2])*Sech[c])/((a^2 + b^2)*d^3*Sqrt[Csch[c]^2*(-Cosh[c]^2 + Sinh[c]^2)]) - (2*b*f^2*(((-I)*Csch[c]*(I*(d*x + ArcTanh[Coth[c]])*(Log[1 - E^(-(d*x) - ArcTanh[Coth[c]])] - Log[1 + E^(-(d*x) - ArcTanh[Coth[c]])]) + I*(PolyLog[2, -E^(-(d*x) - ArcTanh[Coth[c]])] - PolyLog[2, E^(-(d*x) - ArcTanh[Coth[c]])])))/Sqrt[1 - Coth[c]^2] - (2*ArcTan[(Sinh[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]]*ArcTanh[Coth[c]])/Sqrt[Cosh[c]^2 - Sinh[c]^2]))/((a^2 + b^2)*d^3) + (Sech[c]*Sech[c + d*x]*(b*e^2*Cosh[c] + 2*b*e*f*x*Cosh[c] + b*f^2*x^2*Cosh[c] + a*e^2*Sinh[d*x] + 2*a*e*f*x*Sinh[d*x] + a*f^2*x^2*Sinh[d*x]))/((a^2 + b^2)*d)

Maple [F]
time = 2.13, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c)^2}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)

[Out] int((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*a*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 + b^2
)*d^2))*e - 4*b*f^2*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*
e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + 4*a*f^2*integrate(x/(a^2*d*e^(2*d*x
+ 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + (b^2*log((b*e^(-d*x -
 c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2 +
b^2)^(3/2)*d) + 2*(b*e^(-d*x - c) + a)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x
- 2*c))*d))*e^2 - 4*b*f*arctan(e^(d*x + c))*e/((a^2 + b^2)*d^2) - 2*(a*f^2*
x^2 + 2*a*f*x*e - (b*f^2*x^2*e^c + 2*b*f*x*e^(c + 1))*e^(d*x))/(a^2*d + b^2
*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) + integrate(-2*(b^2*f^2*x^2
*e^c + 2*b^2*f*x*e^(c + 1))*e^(d*x)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e^(
2*c))*e^(2*d*x) - 2*(a^3*e^c + a*b^2*e^c)*e^(d*x)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4630 vs. $2(511) = 1022$.
time = 0.45, size = 4630, normalized size = 8.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*(a^3 + a*b^2)*c^2*f^2 - 4*(a^3 + a*b^2)*c*d*f*cosh(1) + 2*(a^3 + a*b^2)
*d^2*cosh(1)^2 + 2*(a^3 + a*b^2)*d^2*sinh(1)^2 - 2*((a^3 + a*b^2)*d^2*f^2*x
^2 - (a^3 + a*b^2)*c^2*f^2 + 2*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*c*d*f
)*cosh(1) + 2*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*c*d*f)*sinh(1))*cosh(d
*x + c)^2 - 2*((a^3 + a*b^2)*d^2*f^2*x^2 - (a^3 + a*b^2)*c^2*f^2 + 2*((a^3
+ a*b^2)*d^2*f*x + (a^3 + a*b^2)*c*d*f)*cosh(1) + 2*((a^3 + a*b^2)*d^2*f*x
+ (a^3 + a*b^2)*c*d*f)*sinh(1))*sinh(d*x + c)^2 - 2*(b^3*d*f^2*x + b^3*d*f*
cosh(1) + b^3*d*f*sinh(1) + (b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh(1
))*cosh(d*x + c)^2 + 2*(b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh(1))*co
sh(d*x + c)*sinh(d*x + c) + (b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh(1
))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d
*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b
+ 1) + 2*(b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh(1) + (b^3*d*f^2*x +
b^3*d*f*cosh(1) + b^3*d*f*sinh(1))*cosh(d*x + c)^2 + 2*(b^3*d*f^2*x + b^3*d
*f*cosh(1) + b^3*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (b^3*d*f^2*x +
b^3*d*f*cosh(1) + b^3*d*f*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*d
```

```
ilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b^3*c^2*f^2 - 2*b^3*c*d*f*cosh(1) +
 b^3*d^2*cosh(1)^2 + b^3*d^2*sinh(1)^2 + (b^3*c^2*f^2 - 2*b^3*c*d*f*cosh(1)
 + b^3*d^2*cosh(1)^2 + b^3*d^2*sinh(1)^2 - 2*(b^3*c*d*f - b^3*d^2*cosh(1))*
sinh(1))*cosh(d*x + c)^2 + 2*(b^3*c^2*f^2 - 2*b^3*c*d*f*cosh(1) + b^3*d^2*c
osh(1)^2 + b^3*d^2*sinh(1)^2 - 2*(b^3*c*d*f - b^3*d^2*cosh(1))*sinh(1))*cos
h(d*x + c)*sinh(d*x + c) + (b^3*c^2*f^2 - 2*b^3*c*d*f*cosh(1) + b^3*d^2*cos
h(1)^2 + b^3*d^2*sinh(1)^2 - 2*(b^3*c*d*f - b^3*d^2*cosh(1))*sinh(1))*sinh(
d*x + c)^2 - 2*(b^3*c*d*f - b^3*d^2*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)
*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*
a) - (b^3*c^2*f^2 - 2*b^3*c*d*f*cosh(1) + b^3*d^2*cosh(1)^2 + b^3*d^2*sinh(
1)^2 + (b^3*c^2*f^2 - 2*b^3*c*d*f*cosh(1) + b^3*d^2*cosh(1)^2 + b^3*d^2*sin
h(1)^2 - 2*(b^3*c*d*f - b^3*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(b^3*
c^2*f^2 - 2*b^3*c*d*f*cosh(1) + b^3*d^2*cosh(1)^2 + b^3*d^2*sinh(1)^2 - 2*(
b^3*c*d*f - b^3*d^2*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (b^3*c^
2*f^2 - 2*b^3*c*d*f*cosh(1) + b^3*d^2*cosh(1)^2 + b^3*d^2*sinh(1)^2 - 2*(b^
3*c*d*f - b^3*d^2*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 2*(b^3*c*d*f - b^3*d^
2*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(
d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^3*d^2*f^2*x^2 - b^3*c^2*f^
2 + (b^3*d^2*f^2*x^2 - b^3*c^2*f^2 + 2*(b^3*d^2*f*x + b^3*c*d*f)*cosh(1) +
2*(b^3*d^2*f*x + b^3*c*d*f)*sinh(1))*cosh(d*x + c)^2 + 2*(b^3*d^2*f^2*x^2 -
 b^3*c^2*f^2 + 2*(b^3*d^2*f*x + b^3*c*d*f)*cosh(1) + 2*(b^3*d^2*f*x + b^3*c
*d*f)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (b^3*d^2*f^2*x^2 - b^3*c^2*f^2
 + 2*(b^3*d^2*f*x + b^3*c*d*f)*cosh(1) + 2*(b^3*d^2*f*x + b^3*c*d*f)*sinh(1
))*sinh(d*x + c)^2 + 2*(b^3*d^2*f*x + b^3*c*d*f)*cosh(1) + 2*(b^3*d^2*f*x +
 b^3*c*d*f)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d
*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b)
 + (b^3*d^2*f^2*x^2 - b^3*c^2*f^2 + (b^3*d^2*f^2*x^2 - b^3*c^2*f^2 + 2*(b^3
*d^2*f*x + b^3*c*d*f)*cosh(1) + 2*(b^3*d^2*f*x + b^3*c*d*f)*sinh(1))*cosh(d
*x + c)^2 + 2*(b^3*d^2*f^2*x^2 - b^3*c^2*f^2 + 2*(b^3*d^2*f*x + b^3*c*d*f)*
cosh(1) + 2*(b^3*d^2*f*x + b^3*c*d*f)*sinh(1))*cosh(d*x + c)*sinh(d*x + c)
+ (b^3*d^2*f^2*x^2 - b^3*c^2*f^2 + 2*(b^3*d^2*f*x + b^3*c*d*f)*cosh(1) + 2*
(b^3*d^2*f*x + b^3*c*d*f)*sinh(1))*sinh(d*x + c)^2 + 2*(b^3*d^2*f*x + b^3*c
*d*f)*cosh(1) + 2*(b^3*d^2*f*x + b^3*c*d*f)*sinh(1))*sqrt((a^2 + b^2)/b^2)*
log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(b^3*f^2*cosh(d*x + c)^2 + 2*b^3*f^2*c
osh(d*x + c)*sinh(d*x + c) + b^3*f^2*sinh(d*x + c)^2 + b^3*f^2)*sqrt((a^2 +
 b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*(b^3*f^2*cosh(d*x + c)^2
+ 2*b^3*f^2*cosh(d*x + c)*sinh(d*x + c) + b^3*f^2*sinh(d*x + c)^2 + b^3*f^2
)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*((a^2*b + b^
3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*d^2*f*x*cosh(1) + (a^2*b + b^3)*d^2*cosh(1
)^2 + (a^2*b + b^3)*d^2*sinh(1)^2 + 2*((a^2*b + b^3)*d^2*f*x + (a^2*b + b^3
)*d^2*cosh(1))*sinh(1))*cosh(d*x + c) + 2*((a^3 + a*b^2)*f^2 + I*(a^2*b + b
```

```
^3)*f^2 + ((a^3 + a*b^2)*f^2 + I*(a^2*b + b^3)*f^2)*cosh(d*x + c)^2 + 2*((a
^3 + a*b^2)*f^2 + I*(a^2*b + b^3)*f^2)*cosh(d*x + c)*sinh(d*x + c) + ((a^3
+ a*b^2)*f^2 + I*(a^2*b + b^3)*f^2)*sinh(d*x + c)^2)*dilog(I*cosh(d*x + c)
+ I*sinh(d*x + c)) + 2*((a^3 + a*b^2)*f^2 - I*(a^2*b + b^3)*f^2 + ((a^3 + a
*b^2)*f^2 - I*(a^2*b + b^3)*f^2)*cosh(d*x + c)^2 + 2*((a^3 + a*b^2)*f^2 - I
*(a^2*b + b^3)*f^2)*cosh(d*x + c)*sinh(d*x + c)...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^2}{\cosh(c+d\,x)^2\,(a+b\sinh(c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((e + f*x)^2/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.311 $\int \frac{(e+fx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=295

$$-\frac{bf\operatorname{ArcTan}(\sinh(c+dx))}{(a^2+b^2)\,d^2}+\frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}-\frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}-\frac{af\log}{(\cdots}$$

```
[Out] -b*f*arctan(sinh(d*x+c))/(a^2+b^2)/d^2-a*f*ln(cosh(d*x+c))/(a^2+b^2)/d^2+b^
2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d-b^2*(f*x
+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d+b^2*f*polylog(
2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-b^2*f*polylog(2,-b
*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2+b*(f*x+e)*sech(d*x+c)/
(a^2+b^2)/d+a*(f*x+e)*tanh(d*x+c)/(a^2+b^2)/d
```

Rubi [A]

time = 0.51, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {5692, 3403, 2296, 2221, 2317, 2438, 6874, 4269, 3556, 5559, 3855}

$$-\frac{bf\operatorname{ArcTan}(\sinh(c+dx))}{d^2(a^2+b^2)}+\frac{b^2f\operatorname{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2(a^2+b^2)^{3/2}}-\frac{b^2f\operatorname{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2(a^2+b^2)^{3/2}}-\frac{af\log(\cosh(c+dx))}{d^2(a^2+b^2)}+\frac{b^2(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{d(a^2+b^2)^{3/2}}-\frac{b^2(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{d(a^2+b^2)^{3/2}}+\frac{a(e+fx)\tanh(c+dx)}{d(a^2+b^2)}+\frac{b(e+fx)\operatorname{sech}(c+dx)}{d(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((b*f*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d^2)) + (b^2*(e + f*x)*Log[1 + (
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (b^2*(e + f*
x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) -
(a*f*Log[Cosh[c + d*x]])/((a^2 + b^2)*d^2) + (b^2*f*PolyLog[2, -((b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (b^2*f*PolyLog[2,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) + (b*(e
+ f*x)*Sech[c + d*x])/((a^2 + b^2)*d) + (a*(e + f*x)*Tanh[c + d*x])/((a^2 +
 b^2)*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))\,dx}{a^2+b^2} + \frac{b^2\int \frac{e+fx}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{\int \left(a(e+fx)\text{sech}^2(c+dx) - b(e+fx)\text{sech}(c+dx)\tanh(c+dx)\right)\,dx}{a^2+b^2} + \frac{(2b^2}{}$$

$$= \frac{(2b^3)\int \frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{(a^2+b^2)^{3/2}} - \frac{(2b^3)\int \frac{e^{c+dx}(e+fx)}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{(a^2+b^2)^{3/2}} + \frac{a\int (e}{}$$

$$= \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d} - \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d} + \frac{b}{}$$

$$= -\frac{bf\tan^{-1}(\sinh(c+dx))}{(a^2+b^2)d^2} + \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d} - \frac{b^2(e+fx)}{}$$

$$= -\frac{bf\tan^{-1}(\sinh(c+dx))}{(a^2+b^2)d^2} + \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d} - \frac{b^2(e+fx)}{}$$

Mathematica [A]

time = 1.87, size = 284, normalized size = 0.96

$$\frac{-\frac{2bf\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a^2+b^2} - \frac{af\log(\cosh(c+dx))}{a^2+b^2} + \frac{b^2\left(-2de\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) + 2cf\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) + f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) - f(c+dx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right) + f\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right) - f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)}{(a^2+b^2)^{3/2}} + \frac{d(e+fx)\text{sech}(c+dx)(b+a\sinh(c+dx))}{a^2+b^2}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((-2*b*f*ArcTan[Tanh[(c + d*x)/2]])/(a^2 + b^2) - (a*f*Log[Cosh[c + d*x]])/
(a^2 + b^2) + (b^2*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2
*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])
] - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^2 + b^2)^(3
/2) + (d*(e + f*x)*Sech[c + d*x]*(b + a*Sinh[c + d*x]))/(a^2 + b^2))/d^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1927 vs. $2(275) = 550$.
time = 3.11, size = 1928, normalized size = 6.54

method	result	size
risch	Expression too large to display	1928

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(f*x+e)*(-b*exp(d*x+c)+a)/d/(a^2+b^2)/(1+exp(2*d*x+2*c))+2/d^2/(a^2+b^2)
^(3/2)*b^2*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2
+b^2)^(1/2)))*a^2+2/d^2/(a^2+b^2)^(3/2)*b^4*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+
c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/d^2/(a^2+b^2)^(3/2)*b^4*f/(
2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/d
/(a^2+b^2)^(3/2)*b^4*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(
-a+(a^2+b^2)^(1/2)))*x-2/d/(a^2+b^2)^(3/2)*b^4*f/(2*a^2+2*b^2)*ln((b*exp(d*
x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-2/d/(a^2+b^2)^(3/2)*b^2*e/(2
*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2+2/d^2/(a^
2+b^2)^(3/2)*b^4*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^
2)^(1/2))+2/d^2/(a^2+b^2)^(1/2)*b^2*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(
d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d^2/(a^2+b^2)^(3/2)*b^2*f/(2*a^2+2*b^2)*arct
anh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2-2/d^2/(a^2+b^2)^(3/2)*b^2
*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))
)*a^2-1/d^2/(a^2+b^2)^2*a^3*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d^2/(
a^2+b^2)*a*f*ln(exp(d*x+c))-2/d^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*ln(1+exp(2*
d*x+2*c))+2/d^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d
*x+c)-b)-4/d^2/(a^2+b^2)*b^3*f/(2*a^2+2*b^2)*arctan(exp(d*x+c))+2/d^2/(a^2+
b^2)^(5/2)*a^4*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d^2/(a
^2+b^2)^(3/2)*b^2*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a
^2+b^2)^(1/2)))*a^2*c+2/d^2/(a^2+b^2)^(3/2)*b^2*f*c/(2*a^2+2*b^2)*arctanh(1
/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2-1/2/d^2/(a^2+b^2)^2*a*b^2*f*ln
(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d^2/(a^2+b^2)^(1/2)*b^2*f/(2*a^2+2*b^
2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d^2/(a^2+b^2)^(5/2)*
a^2*b^2*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-4/d^2/(a^2+b^2)
*a^2*b*f/(2*a^2+2*b^2)*arctan(exp(d*x+c))+2/d^2/(a^2+b^2)^(3/2)*b^4*f/(2*a^
2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d/(a^2+b^2)^(3
```

```
/2)*b^4*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2
/d/(a^2+b^2)^(1/2)*b^2*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^
2+b^2)^(1/2))+2/d^2/(a^2+b^2)^(3/2)*b^4*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c
)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2/d^2/(a^2+b^2)^(3/2)*b^4*f/(2*a
^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-4/d^2
/(a^2+b^2)^(1/2)*a^2*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+
b^2)^(1/2))+1/d^2/(a^2+b^2)*b^2*f/(2*a^2+2*b^2)*a*ln(b*exp(2*d*x+2*c)+2*a*e
xp(d*x+c)-b)-2/d^2/(a^2+b^2)*b^2*f/(2*a^2+2*b^2)*a*ln(1+exp(2*d*x+2*c))+2/d
/(a^2+b^2)^(3/2)*b^2*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(
-a+(a^2+b^2)^(1/2)))*a^2*x-2/d/(a^2+b^2)^(3/2)*b^2*f/(2*a^2+2*b^2)*ln((b*ex
p(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^2*x+2/d^2/(a^2+b^2)^(3/2
)*b^2*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1
/2)))*a^2*c
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] (4*b^2*integrate(-1/2*x*e^(d*x + c)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e^(
2*c))*e^(2*d*x) - 2*(a^3*e^c + a*b^2*e^c)*e^(d*x)), x) + 2*(b*x*e^(d*x + c)
 - a*x)/(a^2*d + b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) + 2*a*x
/((a^2 + b^2)*d) - 2*b*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) - a*log(e^(2*d
*x + 2*c) + 1)/((a^2 + b^2)*d^2))*f + (b^2*log((b*e^(-d*x - c) - a - sqrt(a
^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2)*d) +
2*(b*e^(-d*x - c) + a)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*d))*e
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1411 vs. 2(277) = 554.
time = 0.42, size = 1411, normalized size = 4.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*(a^3 + a*b^2)*d*f*x*cosh(d*x + c)^2 + 2*(a^3 + a*b^2)*d*f*x*sinh(d*x + c
)^2 - 2*(a^3 + a*b^2)*d*cosh(1) - 2*(a^3 + a*b^2)*d*sinh(1) + (b^3*f*cosh(d
*x + c)^2 + 2*b^3*f*cosh(d*x + c)*sinh(d*x + c) + b^3*f*sinh(d*x + c)^2 + b
^3*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^3*f*
cosh(d*x + c)^2 + 2*b^3*f*cosh(d*x + c)*sinh(d*x + c) + b^3*f*sinh(d*x + c)
^2 + b^3*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c)
```

```
- (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (
b^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1) + (b^3*c*f - b^3*d*cosh(1) - b^3*d*
sinh(1))*cosh(d*x + c)^2 + 2*(b^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1))*cosh
(d*x + c)*sinh(d*x + c) + (b^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1))*sinh(d*
x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) +
 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1
) + (b^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1))*cosh(d*x + c)^2 + 2*(b^3*c*f
- b^3*d*cosh(1) - b^3*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (b^3*c*f - b
^3*d*cosh(1) - b^3*d*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*
b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b
^3*d*f*x + b^3*c*f + (b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^2 + 2*(b^3*d*f*x +
 b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d*f*x + b^3*c*f)*sinh(d*x + c)
^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^3*d*f*x + b
^3*c*f + (b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^2 + 2*(b^3*d*f*x + b^3*c*f)*co
sh(d*x + c)*sinh(d*x + c) + (b^3*d*f*x + b^3*c*f)*sinh(d*x + c)^2)*sqrt((a^
2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*((a^2*b + b^3)*f*cosh(d*
x + c)^2 + 2*(a^2*b + b^3)*f*cosh(d*x + c)*sinh(d*x + c) + (a^2*b + b^3)*f*
sinh(d*x + c)^2 + (a^2*b + b^3)*f)*arctan(cosh(d*x + c) + sinh(d*x + c)) +
2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1))
*cosh(d*x + c) - ((a^3 + a*b^2)*f*cosh(d*x + c)^2 + 2*(a^3 + a*b^2)*f*cosh(
d*x + c)*sinh(d*x + c) + (a^3 + a*b^2)*f*sinh(d*x + c)^2 + (a^3 + a*b^2)*f)
*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 2*(2*(a^3 + a*b^2)*
d*f*x*cosh(d*x + c) + (a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*cosh(1) + (a^2*
b + b^3)*d*sinh(1))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^2*cosh(d*x +
c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2
*a^2*b^2 + b^4)*d^2*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)*sech(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{{\mathrm{cosh}\left(c+d\,x\right)}^2\,\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
[Out] int((e + f*x)/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.312 $\int \frac{\mathrm{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=77

$$-\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}+\frac{\mathrm{sech}(c+dx)(b+a\sinh(c+dx))}{(a^2+b^2)\,d}$$

```
[Out] -2*b^2*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d
+sech(d*x+c)*(b+a*sinh(d*x+c))/(a^2+b^2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2775, 12, 2739, 632, 210}

$$\frac{\mathrm{sech}(c+dx)(a\sinh(c+dx)+b)}{d\,(a^2+b^2)}-\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\,(a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*b^2*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3
/2)*d) + (Sech[c + d*x]*(b + a*Sinh[c + d*x]))/((a^2 + b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{(a^2+b^2)\,d}+\frac{\int \frac{b^2}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}\\ &= \frac{\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{(a^2+b^2)\,d}+\frac{b^2\int \frac{1}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}\\ &= \frac{\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{(a^2+b^2)\,d}-\frac{(2ib^2)\operatorname{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(ic+id\right.\right.}{(a^2+b^2)\,d}\\ &= \frac{\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{(a^2+b^2)\,d}+\frac{(4ib^2)\operatorname{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx,x,-2ib+2a\,\mathrm{t}\right.}{(a^2+b^2)\,d}\\ &= -\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}+\frac{\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{(a^2+b^2)\,d}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 104, normalized size = 1.35

$$-\frac{2b^2\operatorname{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)+b\sqrt{-a^2-b^2}\operatorname{sech}(c+dx)+a\sqrt{-a^2-b^2}\tanh(c+dx)}{(-a^2-b^2)^{3/2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((2*b^2*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] + b*Sqrt[-a^2 -
 b^2]*Sech[c + d*x] + a*Sqrt[-a^2 - b^2]*Tanh[c + d*x])/((-a^2 - b^2)^(3/2)
*d))
```

Maple [A]
time = 1.25, size = 90, normalized size = 1.17

method	result
derivativedivides	$\dfrac{-\dfrac{2\left(-a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-b\right)}{\left(a^{2}+b^{2}\right)\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\dfrac{2b^{2}\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}}{d}$
default	$\dfrac{-\dfrac{2\left(-a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-b\right)}{\left(a^{2}+b^{2}\right)\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\dfrac{2b^{2}\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}}{d}$
risch	$-\dfrac{2\left(-b\,\mathrm{e}^{dx+c}+a\right)}{d\left(a^{2}+b^{2}\right)\left(1+\mathrm{e}^{2dx+2c}\right)}+\dfrac{b^{2}\ln\left(\mathrm{e}^{dx+c}+\frac{a\left(a^{2}+b^{2}\right)^{\frac{3}{2}}-a^{4}-2a^{2}b^{2}-b^{4}}{b\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}d}-\dfrac{b^{2}\ln\left(\mathrm{e}^{dx+c}+\frac{a\left(a^{2}+b^{2}\right)^{\frac{3}{2}}+a^{4}+2a^{2}b^{2}+b^{4}}{b\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/(a^2+b^2)*(-a*tanh(1/2*d*x+1/2*c)-b)/(tanh(1/2*d*x+1/2*c)^2+1)+2*b^
2/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)
))
```

Maxima [A]
time = 0.47, size = 115, normalized size = 1.49

$$\frac{b^{2}\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^{2}+b^{2}}}{be^{(-dx-c)}-a+\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}d}+\frac{2\left(be^{(-dx-c)}+a\right)}{\left(a^{2}+b^{2}+\left(a^{2}+b^{2}\right)e^{(-2\,dx-2\,c)}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] b^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a
^2 + b^2)))/((a^2 + b^2)^(3/2)*d) + 2*(b*e^(-d*x - c) + a)/((a^2 + b^2 + (a
^2 + b^2)*e^(-2*d*x - 2*c))*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(74) = 148.
time = 0.34, size = 353, normalized size = 4.58

$$-\frac{2\,a^{3}+2\,ab^{2}-\left(b^{2}\cosh\left(dx+c\right)^{2}+2\,b^{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b^{2}\sinh\left(dx+c\right)^{2}+b^{2}\right)\sqrt{a^{2}+b^{2}}\log\left(\frac{b^{2}\cosh(dx+c)^{2}+b^{2}\sinh(dx+c)^{2}+2\,ab\cosh(dx+c)+2\,a^{2}+b^{2}+2\,\left(b^{2}\cosh(dx+c)+ab\right)\sinh(dx+c)-2\,\sqrt{a^{2}+b^{2}}\,\left(b\cosh(dx+c)+b\sinh(dx+c)+a\right)}{b\cosh(dx+c)^{2}+b\sinh(dx+c)^{2}+2\,a\cosh(dx+c)+2\,\left(b\cosh(dx+c)+a\right)\sinh(dx+c)-b}\right)-2\,\left(a^{2}b+b^{3}\right)\cosh\left(dx+c\right)-2\,\left(a^{2}b+b^{3}\right)\sinh\left(dx+c\right)}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)d\cosh\left(dx+c\right)^{2}+2\,\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)d\sinh\left(dx+c\right)^{2}+\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*a^3 + 2*a*b^2 - (b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c
) + b^2*sinh(d*x + c)^2 + b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b
^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x +
c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x +
 c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b
*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*(a^2*b + b^3)*cosh(d*x + c) - 2
*(a^2*b + b^3)*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 +
2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2
+ b^4)*d*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [A]
time = 0.60, size = 108, normalized size = 1.40

$$\frac{\frac{b^2\log\left(\left|\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right|\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{2\,(be^{(dx+c)}-a)}{(a^2+b^2)(e^{(2\,dx+2\,c)}+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (b^2*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c)
 + 2*a + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(b*e^(d*x + c) - a)/((a^
2 + b^2)*(e^(2*d*x + 2*c) + 1)))/d
```

Mupad [B]
time = 1.38, size = 413, normalized size = 5.36

$$-\frac{\frac{2\,a}{d\,(a^2+b^2)}-\frac{2\,b\,e^{c+d\,x}}{d\,(a^2+b^2)}}{e^{2\,c+2\,d\,x}+1}-\frac{2\,\mathrm{atan}\left(\left(\frac{b^3\,\sqrt{-a^6\,d^2-3\,a^4\,b^2\,d^2-3\,a^2\,b^4\,d^2-b^6\,d^2}}{2}+\frac{a^2\,b\,\sqrt{-a^6\,d^2-3\,a^4\,b^2\,d^2-3\,a^2\,b^4\,d^2-b^6\,d^2}}{2}\right)\left(e^{d\,x}\,e^{c}\left(\frac{2}{d\,\sqrt{b^4}\,(a^2+b^2)^2}+\frac{2\,a\,\left(a^3\,d\,\sqrt{b^4}+a\,b^2\,d\,\sqrt{b^4}\right)}{b^4\,\sqrt{-d^2\,(a^2+b^2)^3}\,(a^2+b^2)\,\sqrt{-a^6\,d^2-3\,a^4\,b^2\,d^2-3\,a^2\,b^4\,d^2-b^6\,d^2}}\right)-\frac{2\,a\,\left(b^3\,d\,\sqrt{b^4}+a^2\,b\,d\,\sqrt{b^4}\right)}{b^4\,\sqrt{-d^2\,(a^2+b^2)^3}\,(a^2+b^2)\,\sqrt{-a^6\,d^2-3\,a^4\,b^2\,d^2-3\,a^2\,b^4\,d^2-b^6\,d^2}}\right)\right)\sqrt{b^4}}{\sqrt{-a^6\,d^2-3\,a^4\,b^2\,d^2-3\,a^2\,b^4\,d^2-b^6\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

```
[Out] - ((2*a)/(d*(a^2 + b^2)) - (2*b*exp(c + d*x))/(d*(a^2 + b^2)))/(exp(2*c + 2
*d*x) + 1) - (2*atan(((b^3*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b^2
*d^2)^(1/2))/2 + (a^2*b*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b^2*d^
2)^(1/2))/2)*(exp(d*x)*exp(c)*(2/(d*(b^4)^(1/2)*(a^2 + b^2)^2) + (2*a*(a^3*
d*(b^4)^(1/2) + a*b^2*d*(b^4)^(1/2)))/(b^4*(-d^2*(a^2 + b^2)^3)^(1/2)*(a^2
+ b^2)*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b^2*d^2)^(1/2))) - (2*a
*(b^3*d*(b^4)^(1/2) + a^2*b*d*(b^4)^(1/2)))/(b^4*(-d^2*(a^2 + b^2)^3)^(1/2)
*(a^2 + b^2)*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b^2*d^2)^(1/2))))
*(b^4)^(1/2))/(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b^2*d^2)^(1/2)
```

3.313 $\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=31

$$\operatorname{Int}\left(\frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sech[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][Sech[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 50.06, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sech[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Integrate[Sech[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(dx+c)^2}{(fx+e)\,(a+b\sinh\,(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 4*b^2*integrate(-1/2*e^(d*x + c)/((a^2*b*f + b^3*f)*x + (a^2*b + b^3)*e - (
(a^2*b*f*e^(2*c) + b^3*f*e^(2*c))*x + (a^2*b*e^(2*c) + b^3*e^(2*c))*e)*e^(2
*d*x) - 2*((a^3*f*e^c + a*b^2*f*e^c)*x + (a^3*e^c + a*b^2*e^c)*e)*e^(d*x)),
 x) + 2*(b*e^(d*x + c) - a)/((a^2*d*f + b^2*d*f)*x + (a^2*d + b^2*d)*e + ((
a^2*d*f*e^(2*c) + b^2*d*f*e^(2*c))*x + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e)*e
^(2*d*x)) + 4*integrate(1/2*(b*f*e^(d*x + c) - a*f)/((a^2*d*f^2 + b^2*d*f^2
)*x^2 + 2*(a^2*d*f + b^2*d*f)*x*e + (a^2*d + b^2*d)*e^2 + ((a^2*d*f^2*e^(2*
c) + b^2*d*f^2*e^(2*c))*x^2 + 2*(a^2*d*f*e^(2*c) + b^2*d*f*e^(2*c))*x*e + (
a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^2)*e^(2*d*x)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sech(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(c + dx)}{(a + b\sinh(c + dx))(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sech(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cosh\left(c+d\,x\right)^2\,\left(e+f\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))),x)
[Out] int(1/(cosh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.314 $\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=928

$$\frac{2ab^2(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{(a^2+b^2)^2 d}+\frac{a(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{(a^2+b^2)\,d}-\frac{af^2\text{ArcTan}(\sinh(c+dx))}{(a^2+b^2)\,d^3}+\frac{b^3(e+fx)^2\log\Big(1-}{(a^2+b^2}$$

```
[Out] -I*a*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/(a^2+b^2)/d^2-2*I*a*b^2*f^2*polylog
(3,I*exp(d*x+c))/(a^2+b^2)^2/d^3+2*a*b^2*(f*x+e)^2*arctan(exp(d*x+c))/(a^2+
b^2)^2/d-b^3*f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^2+I*a*f^2*p
olylog(3,-I*exp(d*x+c))/(a^2+b^2)/d^3+a*f*(f*x+e)*sech(d*x+c)/(a^2+b^2)/d^2
-b*f*(f*x+e)*tanh(d*x+c)/(a^2+b^2)/d^2+1/2*a*(f*x+e)^2*sech(d*x+c)*tanh(d*x
+c)/(a^2+b^2)/d-I*a*f^2*polylog(3,I*exp(d*x+c))/(a^2+b^2)/d^3+a*(f*x+e)^2*a
rctan(exp(d*x+c))/(a^2+b^2)/d+2*b^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a
^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2+2*b^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2-a*f^2*arctan(sinh(d*x+c))/(a^2+b^2)/d^3+I
*a*f*(f*x+e)*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^2+1/2*b^3*f^2*polylog(3,-e
xp(2*d*x+2*c))/(a^2+b^2)^2/d^3+1/2*b*(f*x+e)^2*sech(d*x+c)^2/(a^2+b^2)/d+2*
I*a*b^2*f*(f*x+e)*polylog(2,I*exp(d*x+c))/(a^2+b^2)^2/d^2-b^3*(f*x+e)^2*ln(
1+exp(2*d*x+2*c))/(a^2+b^2)^2/d+b*f^2*ln(cosh(d*x+c))/(a^2+b^2)/d^3+b^3*(f*
x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d+b^3*(f*x+e)^2*l
n(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d-2*b^3*f^2*polylog(3,-b*
exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^3-2*b^3*f^2*polylog(3,-b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^3+2*I*a*b^2*f^2*polylog(3,-I*exp(
d*x+c))/(a^2+b^2)^2/d^3-2*I*a*b^2*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/(a^2+b
^2)^2/d^2
```

Rubi [A]
time = 1.35, antiderivative size = 928, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5692, 5680, 2221, 2611, 2320, 6724, 6874, 4265, 3799, 4271, 3855, 5559, 4269, 3556}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a*b^2*(e + f*x)^2*ArcTan[E^(c + d*x)])/((a^2 + b^2)^2*d) + (a*(e + f*x)^
2*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) - (a*f^2*ArcTan[Sinh[c + d*x]])/((a^
2 + b^2)*d^3) + (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])])/((a^2 + b^2)^2*d) + (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2])])/((a^2 + b^2)^2*d) - (b^3*(e + f*x)^2*Log[1 + E^(2*(c + d*x)
```

```
)])/((a^2 + b^2)^2*d) + (b*f^2*Log[Cosh[c + d*x]])/((a^2 + b^2)*d^3) - ((2*
I)*a*b^2*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - (I
*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) + ((2*I)*a*b
^2*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) + (I*a*f*(e +
 f*x)*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) + (2*b^3*f*(e + f*x)*Pol
yLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) + (2
*b^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^
2 + b^2)^2*d^2) - (b^3*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^
2)^2*d^2) + ((2*I)*a*b^2*f^2*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d
^3) + (I*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) - ((2*I)*a*b
^2*f^2*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)^2*d^3) - (I*a*f^2*PolyLog[3,
 I*E^(c + d*x)])/((a^2 + b^2)*d^3) - (2*b^3*f^2*PolyLog[3, -((b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) - (2*b^3*f^2*PolyLog[3, -((b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) + (b^3*f^2*PolyL
og[3, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^3) + (a*f*(e + f*x)*Sech[c + d*
x])/((a^2 + b^2)*d^2) + (b*(e + f*x)^2*Sech[c + d*x]^2)/(2*(a^2 + b^2)*d) -
 (b*f*(e + f*x)*Tanh[c + d*x])/((a^2 + b^2)*d^2) + (a*(e + f*x)^2*Sech[c +
d*x]*Tanh[c + d*x])/(2*(a^2 + b^2)*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{sech}^3(c+dx)(a-b\sinh(c+dx))\,dx}{a^2+b^2} + \frac{b^2\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{b^2\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))\,dx}{(a^2+b^2)^2} + \frac{b^4\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx}{(a^2+b^2)^2}$$

$$= -\frac{b^3(e+fx)^3}{3(a^2+b^2)^2 f} + \frac{b^2\int (a(e+fx)^2\text{sech}(c+dx) - b(e+fx)^2\tanh(c+dx))}{(a^2+b^2)^2}$$

$$= -\frac{b^3(e+fx)^3}{3(a^2+b^2)^2 f} + \frac{b^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d} + \frac{b^3(e+fx)^2\log\left(\right.}{(a^2+}$$

$$= \frac{2ab^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)^2 d} + \frac{a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d} - \frac{af^2\tan^{-1}(\sinh(}{(a^2+b^2)}$$

$$= \frac{2ab^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)^2 d} + \frac{a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d} - \frac{af^2\tan^{-1}(\sinh(}{(a^2+b^2)}$$

$$= \frac{2ab^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)^2 d} + \frac{a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d} - \frac{af^2\tan^{-1}(\sinh(}{(a^2+b^2)}$$

$$= \frac{2ab^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)^2 d} + \frac{a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d} - \frac{af^2\tan^{-1}(\sinh(}{(a^2+b^2)}$$

$$= \frac{2ab^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)^2 d} + \frac{a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d} - \frac{af^2\tan^{-1}(\sinh(}{(a^2+b^2)}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3368 vs. 2(928) = 1856.
time = 25.77, size = 3368, normalized size = 3.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
[Out] -1/6*(-12*b^3*d^3*e^2*E^(2*c)*x + 12*a^2*b*d*E^(2*c)*f^2*x + 12*b^3*d*E^(2*
c)*f^2*x - 12*b^3*d^3*e*E^(2*c)*f*x^2 - 4*b^3*d^3*E^(2*c)*f^2*x^3 - 6*a^3*d
```

^2*e^2*ArcTan[E^(c + d*x)] - 18*a*b^2*d^2*e^2*ArcTan[E^(c + d*x)] - 6*a^3*d
^2*e^2*E^(2*c)*ArcTan[E^(c + d*x)] - 18*a*b^2*d^2*e^2*E^(2*c)*ArcTan[E^(c +
 d*x)] + 12*a^3*f^2*ArcTan[E^(c + d*x)] + 12*a*b^2*f^2*ArcTan[E^(c + d*x)]
+ 12*a^3*E^(2*c)*f^2*ArcTan[E^(c + d*x)] + 12*a*b^2*E^(2*c)*f^2*ArcTan[E^(c
 + d*x)] - (6*I)*a^3*d^2*e*f*x*Log[1 - I*E^(c + d*x)] - (18*I)*a*b^2*d^2*e*
f*x*Log[1 - I*E^(c + d*x)] - (6*I)*a^3*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d
*x)] - (18*I)*a*b^2*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] - (3*I)*a^3*d^
2*f^2*x^2*Log[1 - I*E^(c + d*x)] - (9*I)*a*b^2*d^2*f^2*x^2*Log[1 - I*E^(c +
 d*x)] - (3*I)*a^3*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] - (9*I)*a*b^2
*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (6*I)*a^3*d^2*e*f*x*Log[1 + I
*E^(c + d*x)] + (18*I)*a*b^2*d^2*e*f*x*Log[1 + I*E^(c + d*x)] + (6*I)*a^3*d
^2*e*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] + (18*I)*a*b^2*d^2*e*E^(2*c)*f*x*Lo
g[1 + I*E^(c + d*x)] + (3*I)*a^3*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] + (9*I)
*a*b^2*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] + (3*I)*a^3*d^2*E^(2*c)*f^2*x^2*L
og[1 + I*E^(c + d*x)] + (9*I)*a*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 + I*E^(c + d*
x)] + 6*b^3*d^2*e^2*Log[1 + E^(2*(c + d*x))] + 6*b^3*d^2*e^2*E^(2*c)*Log[1
+ E^(2*(c + d*x))] - 6*a^2*b*f^2*Log[1 + E^(2*(c + d*x))] - 6*b^3*f^2*Log[1
 + E^(2*(c + d*x))] - 6*a^2*b*E^(2*c)*f^2*Log[1 + E^(2*(c + d*x))] - 6*b^3*
E^(2*c)*f^2*Log[1 + E^(2*(c + d*x))] + 12*b^3*d^2*e*f*x*Log[1 + E^(2*(c + d
*x))] + 12*b^3*d^2*e*E^(2*c)*f*x*Log[1 + E^(2*(c + d*x))] + 6*b^3*d^2*f^2*x
^2*Log[1 + E^(2*(c + d*x))] + 6*b^3*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(2*(c + d
*x))] + (6*I)*a*(a^2 + 3*b^2)*d*(1 + E^(2*c))*f*(e + f*x)*PolyLog[2, (-I)*E
^(c + d*x)] - (6*I)*a*(a^2 + 3*b^2)*d*(1 + E^(2*c))*f*(e + f*x)*PolyLog[2,
I*E^(c + d*x)] + 6*b^3*d*e*f*PolyLog[2, -E^(2*(c + d*x))] + 6*b^3*d*e*E^(2*
c)*f*PolyLog[2, -E^(2*(c + d*x))] + 6*b^3*d*f^2*x*PolyLog[2, -E^(2*(c + d*x
))] + 6*b^3*d*E^(2*c)*f^2*x*PolyLog[2, -E^(2*(c + d*x))] - (6*I)*a^3*f^2*Po
lyLog[3, (-I)*E^(c + d*x)] - (18*I)*a*b^2*f^2*PolyLog[3, (-I)*E^(c + d*x)]
- (6*I)*a^3*E^(2*c)*f^2*PolyLog[3, (-I)*E^(c + d*x)] - (18*I)*a*b^2*E^(2*c)
*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (6*I)*a^3*f^2*PolyLog[3, I*E^(c + d*x)]
 + (18*I)*a*b^2*f^2*PolyLog[3, I*E^(c + d*x)] + (6*I)*a^3*E^(2*c)*f^2*PolyL
og[3, I*E^(c + d*x)] + (18*I)*a*b^2*E^(2*c)*f^2*PolyLog[3, I*E^(c + d*x)] -
 3*b^3*f^2*PolyLog[3, -E^(2*(c + d*x))] - 3*b^3*E^(2*c)*f^2*PolyLog[3, -E^(
2*(c + d*x))]])/((a^2 + b^2)^2*d^3*(1 + E^(2*c))) - (b^3*(6*e^2*E^(2*c)*x +
6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a
+ b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(
a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a
^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*
x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^
2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)
*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3*e^2*E^
(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*Log[1 +
 (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f
*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f
^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d -
(3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2

```
*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^
(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2
 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[
(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a
*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyL
og[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*(
-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2
 + b^2)*E^(2*c)]))])/d^2 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - S
qrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c +
d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (6*f^2*PolyLog[3, -((b*E
^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*P
olyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3))/
(3*(a^2 + b^2)^2*(-1 + E^(2*c))) + (Csch[c]*Sech[c]*Sech[c + d*x]^2*(-6*a^2
*b*e*f - 6*b^3*e*f + 12*b^3*d^2*e^2*x - 6*a^2*b*f^2*x - 6*b^3*f^2*x + 12*b^
3*d^2*e*f*x^2 + 4*b^3*d^2*f^2*x^3 + 6*a^2*b*e*f*Cosh[2*c] + 6*b^3*e*f*Cosh[
2*c] + 6*a^2*b*f^2*x*Cosh[2*c] + 6*b^3*f^2*x*Cosh[2*c] + 6*a^2*b*e*f*Cosh[2
*d*x] + 6*b^3*e*f*Cosh[2*d*x] + 6*a^2*b*f^2*x*Cosh[2*d*x] + 6*b^3*f^2*x*Cos
h[2*d*x] - 3*a^3*d*e^2*Cosh[c - d*x] - 3*a*b^2*d*e^2*Cosh[c - d*x] - 6*a^3*
d*e*f*x*Cosh[c - d*x] - 6*a*b^2*d*e*f*x*Cosh[c - d*x] - 3*a^3*d*f^2*x^2*Cos
h[c - d*x] - 3*a*b^2*d*f^2*x^2*Cosh[c - d*x] + ...
```

Maple [F]
time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c)^3}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] a^3*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*
d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b
^4*d^2), x) + 3*a*b^2*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x +
 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 +
 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*b^3*d^2*f^2*integrate(x^2/(a^4*d^2*e^(2*d
*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d
```

```
^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 4*b^3*d^2*f*e*integrate(x/(a^4*d^2*e^(2
*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4
*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*a^3*d^2*f*integrate(x*e^(d*x + c +
1)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*
d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 6*a*b^2*d^2*f*integra
te(x*e^(d*x + c + 1)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*
c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - a^2
*b*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x + 2*c) + 1
)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) - b^3*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 +
 b^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) - 2*a^
3*f^2*arctan(e^(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3) - 2*a*b^2*f^2*arcta
n(e^(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3) + (b^3*log(-2*a*e^(-d*x - c) +
 b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b^4)*d) - b^3*log(e^(-2*d*x -
2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a^3 + 3*a*b^2)*arctan(e^(-d*x - c)
)/((a^4 + 2*a^2*b^2 + b^4)*d) + (a*e^(-d*x - c) + 2*b*e^(-2*d*x - 2*c) - a*
e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*e^(-2*d*x - 2*c) + (a^2 + b^2
)*e^(-4*d*x - 4*c))*d))*e^2 + (2*b*f^2*x + 2*b*f*e + (a*d*f^2*x^2*e^(3*c) +
 2*a*f*e^(3*c + 1) + 2*(a*f^2*e^(3*c) + a*d*f*e^(3*c + 1))*x)*e^(3*d*x) + 2
*(b*d*f^2*x^2*e^(2*c) + b*f*e^(2*c + 1) + (b*f^2*e^(2*c) + 2*b*d*f*e^(2*c +
 1))*x)*e^(2*d*x) - (a*d*f^2*x^2*e^c - 2*a*f*e^(c + 1) + 2*(a*d*f*e^(c + 1)
 - a*f^2*e^c)*x)*e^(d*x))/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^(4*c) + b^2*d^2*e
^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*x)) - inte
grate(2*(b^4*f^2*x^2 + 2*b^4*f*x*e - (a*b^3*f^2*x^2*e^c + 2*a*b^3*f*x*e^(c
+ 1))*e^(d*x))/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b*e^(2*c) + 2*a^2*b^3*e^(2*c
) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c + 2*a^3*b^2*e^c + a*b^4*e^c)*e^(d*x
)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 15652 vs. $2(871) = 1742$.
time = 0.62, size = 15652, normalized size = 16.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*cosh(d*x + c)^4 + 4*(
(a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*sinh(d*x + c)^4 + 4*(a^2*b + b
^3)*c*f^2 - 4*(a^2*b + b^3)*d*f*cosh(1) - 2*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*
(a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*d^2*cosh(1)^2 + (a^3 + a*b^2)*d^2*sin
h(1)^2 + 2*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*d*f)*cosh(1) + 2*((a^3 +
a*b^2)*d^2*f*x + (a^3 + a*b^2)*d^2*cosh(1) + (a^3 + a*b^2)*d*f)*sinh(1))*co
sh(d*x + c)^3 - 4*(a^2*b + b^3)*d*f*sinh(1) - 2*((a^3 + a*b^2)*d^2*f^2*x^2
+ 2*(a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*d^2*cosh(1)^2 + (a^3 + a*b^2)*d^2
*sinh(1)^2 + 2*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*d*f)*cosh(1) - 8*((a^
```

2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*cosh(d*x + c) + 2*((a^3 + a*b^2)*
d^2*f*x + (a^3 + a*b^2)*d^2*cosh(1) + (a^3 + a*b^2)*d*f)*sinh(1))*sinh(d*x
+ c)^3 - 4*((a^2*b + b^3)*d^2*f^2*x^2 - (a^2*b + b^3)*d*f^2*x + (a^2*b + b^
3)*d^2*cosh(1)^2 + (a^2*b + b^3)*d^2*sinh(1)^2 - 2*(a^2*b + b^3)*c*f^2 + (2
*(a^2*b + b^3)*d^2*f*x + (a^2*b + b^3)*d*f)*cosh(1) + (2*(a^2*b + b^3)*d^2*
f*x + 2*(a^2*b + b^3)*d^2*cosh(1) + (a^2*b + b^3)*d*f)*sinh(1))*cosh(d*x +
c)^2 - 2*(2*(a^2*b + b^3)*d^2*f^2*x^2 - 2*(a^2*b + b^3)*d*f^2*x + 2*(a^2*b
+ b^3)*d^2*cosh(1)^2 + 2*(a^2*b + b^3)*d^2*sinh(1)^2 - 4*(a^2*b + b^3)*c*f^
2 - 12*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*cosh(d*x + c)^2 + 2*(2
*(a^2*b + b^3)*d^2*f*x + (a^2*b + b^3)*d*f)*cosh(1) + 3*((a^3 + a*b^2)*d^2*
f^2*x^2 + 2*(a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*d^2*cosh(1)^2 + (a^3 + a*
b^2)*d^2*sinh(1)^2 + 2*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*d*f)*cosh(1)
+ 2*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*d^2*cosh(1) + (a^3 + a*b^2)*d*f)
*sinh(1))*cosh(d*x + c) + 2*(2*(a^2*b + b^3)*d^2*f*x + 2*(a^2*b + b^3)*d^2*
cosh(1) + (a^2*b + b^3)*d*f)*sinh(1))*sinh(d*x + c)^2 + 2*((a^3 + a*b^2)*d^
2*f^2*x^2 - 2*(a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*d^2*cosh(1)^2 + (a^3 +
a*b^2)*d^2*sinh(1)^2 + 2*((a^3 + a*b^2)*d^2*f*x - (a^3 + a*b^2)*d*f)*cosh(1
) + 2*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*d^2*cosh(1) - (a^3 + a*b^2)*d*
f)*sinh(1))*cosh(d*x + c) - 4*(b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh
(1) + (b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh(1))*cosh(d*x + c)^4 + 4
*(b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x +
c)^3 + (b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh(1))*sinh(d*x + c)^4 +
2*(b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh(1))*cosh(d*x + c)^2 + 2*(b
^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh(1) + 3*(b^3*d*f^2*x + b^3*d*f*c
osh(1) + b^3*d*f*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^3*d*f^2*
x + b^3*d*f*cosh(1) + b^3*d*f*sinh(1))*cosh(d*x + c)^3 + (b^3*d*f^2*x + b^3
*d*f*cosh(1) + b^3*d*f*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh
(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2) - b)/b + 1) - 4*(b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh(
1) + (b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh(1))*cosh(d*x + c)^4 + 4*
(b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x +
c)^3 + (b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh(1))*sinh(d*x + c)^4 +
2*(b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh(1))*cosh(d*x + c)^2 + 2*(b^
3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh(1) + 3*(b^3*d*f^2*x + b^3*d*f*co
sh(1) + b^3*d*f*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^3*d*f^2*x
+ b^3*d*f*cosh(1) + b^3*d*f*sinh(1))*cosh(d*x + c)^3 + (b^3*d*f^2*x + b^3*
d*f*cosh(1) + b^3*d*f*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(
d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2) - b)/b + 1) + 2*(2*b^3*d*f^2*x + 2*b^3*d*f*cosh(1) + 2*b^3*d*f*
sinh(1) - I*(a^3 + 3*a*b^2)*d*f^2*x + (2*b^3*d*f^2*x + 2*b^3*d*f*cosh(1) +
2*b^3*d*f*sinh(1) - I*(a^3 + 3*a*b^2)*d*f^2*x - I*(a^3 + 3*a*b^2)*d*f*cosh(
1) - I*(a^3 + 3*a*b^2)*d*f*sinh(1))*cosh(d*x + c)^4 + 4*(2*b^3*d*f^2*x + 2*
b^3*d*f*cosh(1) + 2*b^3*d*f*sinh(1) - I*(a^3 + 3*a*b^2)*d*f^2*x - I*(a^3 +
3*a*b^2)*d*f*cosh(1) - I*(a^3 + 3*a*b^2)*d*f*sinh(1))*cosh(d*x + c)*sinh(d*
x + c)^3 + (2*b^3*d*f^2*x + 2*b^3*d*f*cosh(1) + 2*b^3*d*f*sinh(1) - I*(a^3

```
+ 3*a*b^2)*d*f^2*x - I*(a^3 + 3*a*b^2)*d*f*cosh(1) - I*(a^3 + 3*a*b^2)*d*f*
sinh(1))*sinh(d*x + c)^4 - I*(a^3 + 3*a*b^2)*d*f*cosh(1) - I*(a^3 + 3*a*b^2
)*d*f*sinh(1) + 2*(2*b^3*d*f^2*x + 2*b^3*d*f*cosh(1) + 2*b^3*d*f*sinh(1) -
I*(a^3 + 3*a*b^2)*d*f^2*x - I*(a^3 + 3*a*b^2)*d*f*cosh(1) - I*(a^3 + 3*a*b^
2)*d*f*sinh(1))*cosh(d*x + c)^2 + 2*(2*b^3*d*f^2*x + 2*b^3*d*f*cosh(1) + 2*
b^3*d*f*sinh(1) - I*(a^3 + 3*a*b^2)*d*f^2*x - I*(a^3 + 3*a*b^2)*d*f*cosh(1)
 - I*(a^3 + 3*a*b^2)*d*f*sinh(1) + 3*(2*b^3*d*f^2*x + 2*b^3*d*f*cosh(1) + 2
*b^3*d*f*sinh(1) - I*(a^3 + 3*a*b^2)*d*f^2*x - I*(a^3 + 3*a*b^2)*d*f*cosh(1
) - I*(a^3 + 3*a*b^2)*d*f*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((2
*b^3*d*f^2*x + 2*b^3*d*f*cosh(1) + 2*b^3*d*f*sinh(1) - I*(a^3 + 3*a*b^2)*d*
f^2*x - I*(a^3 + 3*a*b^2)*d*f*cosh(1) - I*(a^3 + 3*a*b^2)*d*f*sinh(1))*cosh
(d*x + c)^3 + (2*b^3*d*f^2*x + 2*b^3*d*f*cosh(1) + 2*b^3*d*f*sinh(1) - I*(a
^3 + 3*a*b^2)*d*f^2*x - I*(a^3 + 3*a*b^2)*d*f*c...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2 \operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*sech(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^2}{\cosh(c+d\,x)^3\,(a+b\sinh(c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(cosh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((e + f*x)^2/(cosh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

3.315 $\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=560

$$\frac{2ab^2(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d}+\frac{a(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{\left(a^2+b^2\right) d}+\frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2 d}+\frac{b^3(e+fx)\text{lo}}{(}$$

```
[Out] 2*a*b^2*(f*x+e)*arctan(exp(d*x+c))/(a^2+b^2)^2/d+a*(f*x+e)*arctan(exp(d*x+c
))/(a^2+b^2)/d-b^3*(f*x+e)*ln(1+exp(2*d*x+2*c))/(a^2+b^2)^2/d+b^3*(f*x+e)*l
n(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d+b^3*(f*x+e)*ln(1+b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d-I*a*b^2*f*polylog(2,-I*exp(d*x+c)
)/(a^2+b^2)^2/d^2+1/2*I*a*f*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^2+I*a*b^2*f
*polylog(2,I*exp(d*x+c))/(a^2+b^2)^2/d^2-1/2*I*a*f*polylog(2,-I*exp(d*x+c))
/(a^2+b^2)/d^2-1/2*b^3*f*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^2+b^3*f*p
olylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2+b^3*f*polylog(2
,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2+1/2*a*f*sech(d*x+c)/(a^
2+b^2)/d^2+1/2*b*(f*x+e)*sech(d*x+c)^2/(a^2+b^2)/d-1/2*b*f*tanh(d*x+c)/(a^2
+b^2)/d^2+1/2*a*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a^2+b^2)/d
```

Rubi [A]
time = 0.73, antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5692, 5680, 2221, 2317, 2438, 6874, 4265, 3799, 4270, 5559, 3852, 8}

$$\frac{2ab^2(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{d\left(a^2+b^2\right)^2}+\frac{a(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{d\left(a^2+b^2\right)}-\frac{iab^2f\text{Li}_2\left(-ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)^2}+\frac{iab^2f\text{Li}_2\left(ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)^2}-\frac{iaf\text{Li}_2\left(-ie^{c+dx}\right)}{2d^2\left(a^2+b^2\right)}+\frac{iaf\text{Li}_2\left(ie^{c+dx}\right)}{2d^2\left(a^2+b^2\right)}-\frac{bf\tanh(c+dx)}{2d^2\left(a^2+b^2\right)}+\frac{af\text{sech}(c+dx)}{2d^2\left(a^2+b^2\right)}+\frac{b(e+fx)\text{sech}^2(c+dx)}{2d\left(a^2+b^2\right)}+\frac{a(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d\left(a^2+b^2\right)}+\frac{b^3f\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^2}+\frac{b^3f\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^2}-\frac{b^3f\text{Li}_2\left(-e^{2(c+dx)}\right)}{2d^2\left(a^2+b^2\right)^2}+\frac{b^3(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{d\left(a^2+b^2\right)^2}+\frac{b^3(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{d\left(a^2+b^2\right)^2}-\frac{b^3(e+fx)\log\left(e^{2(c+dx)}+1\right)}{d\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a*b^2*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)^2*d) + (a*(e + f*x)*Ar
cTan[E^(c + d*x)])/((a^2 + b^2)*d) + (b^3*(e + f*x)*Log[1 + (b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) + (b^3*(e + f*x)*Log[1 + (b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) - (b^3*(e + f*x)*Log[1 +
 E^(2*(c + d*x))])/((a^2 + b^2)^2*d) - (I*a*b^2*f*PolyLog[2, (-I)*E^(c + d*
x)])/((a^2 + b^2)^2*d^2) - ((I/2)*a*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 +
 b^2)*d^2) + (I*a*b^2*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) + ((
I/2)*a*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) + (b^3*f*PolyLog[2, -
((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) + (b^3*f*Poly
Log[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) - (b^
3*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^2) + (a*f*Sech[c + d*x
])/(2*(a^2 + b^2)*d^2) + (b*(e + f*x)*Sech[c + d*x]^2)/(2*(a^2 + b^2)*d) -
(b*f*Tanh[c + d*x])/(2*(a^2 + b^2)*d^2) + (a*(e + f*x)*Sech[c + d*x]*Tanh[c
 + d*x])/(2*(a^2 + b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))\,dx}{a^2+b^2} + \frac{b^2\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{b^2\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))\,dx}{(a^2+b^2)^2} + \frac{b^4\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx}{(a^2+b^2)^2} +$$

$$= -\frac{b^3(e+fx)^2}{2(a^2+b^2)^2 f} + \frac{b^2\int (a(e+fx)\text{sech}(c+dx) - b(e+fx)\tanh(c+dx))\,dx}{(a^2+b^2)^2}$$

$$= -\frac{b^3(e+fx)^2}{2(a^2+b^2)^2 f} + \frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d} + \frac{b^3(e+fx)\log\left(1+\right.}{(a^2+b^2)}$$

$$= \frac{2ab^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)^2 d} + \frac{a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d} + \frac{b^3(e+fx)\log\left(1+\right.}{(a^2+b^2)}$$

$$= \frac{2ab^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)^2 d} + \frac{a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d} + \frac{b^3(e+fx)\log\left(1+\right.}{(a^2+b^2)}$$

$$= \frac{2ab^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)^2 d} + \frac{a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d} + \frac{b^3(e+fx)\log\left(1+\right.}{(a^2+b^2)}$$

$$= \frac{2ab^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)^2 d} + \frac{a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d} + \frac{b^3(e+fx)\log\left(1+\right.}{(a^2+b^2)}$$

Mathematica [A]
time = 4.96, size = 588, normalized size = 1.05

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*b^3*d*e*(c + d*x) - 2*b^3*c*f*(c + d*x) + 2*a^3*d*e*ArcTan[E^(c + d*x)]
+ 6*a*b^2*d*e*ArcTan[E^(c + d*x)] - 2*a^3*c*f*ArcTan[E^(c + d*x)] - 6*a*b^2
*c*f*ArcTan[E^(c + d*x)] + I*a^3*f*(c + d*x)*Log[1 - I*E^(c + d*x)] + (3*I)
*a*b^2*f*(c + d*x)*Log[1 - I*E^(c + d*x)] - I*a^3*f*(c + d*x)*Log[1 + I*E^(
c + d*x)] - (3*I)*a*b^2*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + 2*b^3*f*(c + d
*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*b^3*f*(c + d*x)*Log[
```

```
1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*b^3*d*e*Log[1 + E^(2*(c + d*
x))] + 2*b^3*c*f*Log[1 + E^(2*(c + d*x))] - 2*b^3*f*(c + d*x)*Log[1 + E^(2*
(c + d*x))] + 2*b^3*d*e*Log[a + b*Sinh[c + d*x]] - 2*b^3*c*f*Log[a + b*Sinh
[c + d*x]] - I*a*(a^2 + 3*b^2)*f*PolyLog[2, (-I)*E^(c + d*x)] + I*a*(a^2 +
3*b^2)*f*PolyLog[2, I*E^(c + d*x)] + 2*b^3*f*PolyLog[2, (b*E^(c + d*x))/(-a
 + Sqrt[a^2 + b^2])] + 2*b^3*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))] - b^3*f*PolyLog[2, -E^(2*(c + d*x))] + (a^2 + b^2)*d*(e + f*x)*Sec
h[c + d*x]^2*(b + a*Sinh[c + d*x]) + (a^2 + b^2)*f*Sech[c + d*x]*(a - b*Sin
h[c + d*x]))/(2*(a^2 + b^2)^2*d^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2050 vs. $2(520) = 1040$.
time = 3.01, size = 2051, normalized size = 3.66

method	result	size
risch	Expression too large to display	2051

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2/(a^2+b^2)^(1/2)*a*b*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*
a)/(a^2+b^2)^(1/2))-6/d^2/(a^2+b^2)*a*b^2*f*c/(2*a^2+2*b^2)*arctan(exp(d*x+
c))+1/d/(a^2+b^2)^(1/2)*e*a*b/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a
)/(a^2+b^2)^(1/2))-2/d^2/(a^2+b^2)*a^3*f*c/(2*a^2+2*b^2)*arctan(exp(d*x+c))
+2/d/(a^2+b^2)*b^3*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a
+(a^2+b^2)^(1/2)))*x-2/d^2/(a^2+b^2)*b^3*f*c/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2
*c)+2*a*exp(d*x+c)-b)+2/d^2/(a^2+b^2)*b^3*f*c/(2*a^2+2*b^2)*ln(1+exp(2*d*x+
2*c))-2/d^2/(a^2+b^2)*b^3*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c+2/d/(a^2+b^2
)*b^3*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2
)))*x-3*I/d^2/(a^2+b^2)*a*b^2*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))+3*I/d^2
/(a^2+b^2)*a*b^2*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))-I/d/(a^2+b^2)*a^3*f/
(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x-I/d^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*ln(1
+I*exp(d*x+c))*c+I/d/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x+I/d
^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c-1/d/(a^2+b^2)^(3/2)*e
*a*b^3/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d/
(a^2+b^2)^(3/2)*e*a^3*b/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2
+b^2)^(1/2))+2/d/(a^2+b^2)*a^3*e/(2*a^2+2*b^2)*arctan(exp(d*x+c))+2/d/(a^2+
b^2)*b^3*e/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d/(a^2+b^2
)*b^3*e/(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))+I/d^2/(a^2+b^2)*a^3*f/(2*a^2+2*b
^2)*dilog(1-I*exp(d*x+c))+2/d^2/(a^2+b^2)*b^3*f/(2*a^2+2*b^2)*dilog((-b*exp
(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+2/d^2/(a^2+b^2)*b^3*f/(2*a
^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2/d^2
/(a^2+b^2)*b^3*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))-2/d^2/(a^2+b^2)*b^3*f/
(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))-3*I/d^2/(a^2+b^2)*a*b^2*f/(2*a^2+2*b^2)
*ln(1+I*exp(d*x+c))*c+3*I/d/(a^2+b^2)*a*b^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+
```

```
c))*x+2/d^2/(a^2+b^2)*b^3*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+
a)/(a+(a^2+b^2)^(1/2)))*c+2/d^2/(a^2+b^2)*b^3*f/(2*a^2+2*b^2)*ln((-b*exp(d*
x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/d/(a^2+b^2)*b^3*f/(2*a^2+
2*b^2)*ln(1+I*exp(d*x+c))*x-2/d^2/(a^2+b^2)*b^3*f/(2*a^2+2*b^2)*ln(1+I*exp(
d*x+c))*c-2/d/(a^2+b^2)*b^3*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x+6/d/(a^2+b
^2)*e*a*b^2/(2*a^2+2*b^2)*arctan(exp(d*x+c))-I/d^2/(a^2+b^2)*a^3*f/(2*a^2+2
*b^2)*dilog(1+I*exp(d*x+c))+1/d^2/(a^2+b^2)^(3/2)*a^3*b*f*c/(2*a^2+2*b^2)*a
rctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d^2/(a^2+b^2)^(3/2)*a*b^
3*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+3*I/d
^2/(a^2+b^2)*a*b^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c-3*I/d/(a^2+b^2)*a*b
^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x+(a*d*f*x*exp(3*d*x+3*c)+a*d*e*exp(3
*d*x+3*c)+2*b*d*f*x*exp(2*d*x+2*c)-a*d*f*x*exp(d*x+c)+a*f*exp(3*d*x+3*c)+2*
b*d*e*exp(2*d*x+2*c)-a*d*e*exp(d*x+c)+b*f*exp(2*d*x+2*c)+a*f*exp(d*x+c)+b*f
)/d^2/(a^2+b^2)/(1+exp(2*d*x+2*c))^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] f*(((a*d*x*e^(3*c) + a*e^(3*c))*e^(3*d*x) + (2*b*d*x*e^(2*c) + b*e^(2*c))*e
^(2*d*x) - (a*d*x*e^c - a*e^c)*e^(d*x) + b)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e
^(4*c) + b^2*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))
*e^(2*d*x)) - 8*integrate(-1/4*(a*b^3*x*e^(d*x + c) - b^4*x)/(a^4*b + 2*a^2
*b^3 + b^5 - (a^4*b*e^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) -
2*(a^5*e^c + 2*a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) + 8*integrate(1/8*(2*b
^3*x + (a^3*e^c + 3*a*b^2*e^c)*x*e^(d*x))/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^(
2*c) + 2*a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x)) + (b^3*log(-2*a*e^(
-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b^4)*d) - b^3*log(e
^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a^3 + 3*a*b^2)*arctan(e
^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (a*e^(-d*x - c) + 2*b*e^(-2*d*x
- 2*c) - a*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*e^(-2*d*x - 2*c) +
 (a^2 + b^2)*e^(-4*d*x - 4*c))*d))*e
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5468 vs. $2(512) = 1024$.
time = 0.44, size = 5468, normalized size = 9.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

[Out] 1/2*(2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*cosh(1) + (a^3 + a*b^2)*d*sin
h(1) + (a^3 + a*b^2)*f)*cosh(d*x + c)^3 + 2*((a^3 + a*b^2)*d*f*x + (a^3 + a
*b^2)*d*cosh(1) + (a^3 + a*b^2)*d*sinh(1) + (a^3 + a*b^2)*f)*sinh(d*x + c)^
3 + 2*(2*(a^2*b + b^3)*d*f*x + 2*(a^2*b + b^3)*d*cosh(1) + 2*(a^2*b + b^3)*
d*sinh(1) + (a^2*b + b^3)*f)*cosh(d*x + c)^2 + 2*(2*(a^2*b + b^3)*d*f*x + 2
*(a^2*b + b^3)*d*cosh(1) + 2*(a^2*b + b^3)*d*sinh(1) + (a^2*b + b^3)*f + 3*
((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*cosh(1) + (a^3 + a*b^2)*d*sinh(1) +
(a^3 + a*b^2)*f)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(a^2*b + b^3)*f - 2*((a
^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*cosh(1) + (a^3 + a*b^2)*d*sinh(1) - (a^
3 + a*b^2)*f)*cosh(d*x + c) + 2*(b^3*f*cosh(d*x + c)^4 + 4*b^3*f*cosh(d*x +
 c)*sinh(d*x + c)^3 + b^3*f*sinh(d*x + c)^4 + 2*b^3*f*cosh(d*x + c)^2 + b^3
f + 2(3*b^3*f*cosh(d*x + c)^2 + b^3*f)*sinh(d*x + c)^2 + 4*(b^3*f*cosh(d*
x + c)^3 + b^3*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*s
inh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
b)/b + 1) + 2*(b^3*f*cosh(d*x + c)^4 + 4*b^3*f*cosh(d*x + c)*sinh(d*x + c)^
3 + b^3*f*sinh(d*x + c)^4 + 2*b^3*f*cosh(d*x + c)^2 + b^3*f + 2*(3*b^3*f*co
sh(d*x + c)^2 + b^3*f)*sinh(d*x + c)^2 + 4*(b^3*f*cosh(d*x + c)^3 + b^3*f*c
osh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - ((2*b^
3*f - I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)^4 + 4*(2*b^3*f - I*(a^3 + 3*a*b^2)
*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*b^3*f - I*(a^3 + 3*a*b^2)*f)*sinh(d*
x + c)^4 + 2*b^3*f + 2*(2*b^3*f - I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)^2 + 2*
(2*b^3*f + 3*(2*b^3*f - I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)^2 - I*(a^3 + 3*a
*b^2)*f)*sinh(d*x + c)^2 - I*(a^3 + 3*a*b^2)*f + 4*((2*b^3*f - I*(a^3 + 3*a
*b^2)*f)*cosh(d*x + c)^3 + (2*b^3*f - I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c))*s
inh(d*x + c))*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) - ((2*b^3*f + I*(a^3
 + 3*a*b^2)*f)*cosh(d*x + c)^4 + 4*(2*b^3*f + I*(a^3 + 3*a*b^2)*f)*cosh(d*x
 + c)*sinh(d*x + c)^3 + (2*b^3*f + I*(a^3 + 3*a*b^2)*f)*sinh(d*x + c)^4 + 2
*b^3*f + 2*(2*b^3*f + I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)^2 + 2*(2*b^3*f + 3
*(2*b^3*f + I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)^2 + I*(a^3 + 3*a*b^2)*f)*sin
h(d*x + c)^2 + I*(a^3 + 3*a*b^2)*f + 4*((2*b^3*f + I*(a^3 + 3*a*b^2)*f)*cos
h(d*x + c)^3 + (2*b^3*f + I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c))*sinh(d*x + c)
)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) - 2*(b^3*c*f - b^3*d*cosh(1) +
(b^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1))*cosh(d*x + c)^4 - b^3*d*sinh(1) +
 4*(b^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3
+ (b^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1))*sinh(d*x + c)^4 + 2*(b^3*c*f -
b^3*d*cosh(1) - b^3*d*sinh(1))*cosh(d*x + c)^2 + 2*(b^3*c*f - b^3*d*cosh(1)
 - b^3*d*sinh(1) + 3*(b^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1))*cosh(d*x + c
)^2)*sinh(d*x + c)^2 + 4*((b^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1))*cosh(d*
x + c)^3 + (b^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1))*cosh(d*x + c))*sinh(d*
x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^
2) + 2*a) - 2*(b^3*c*f - b^3*d*cosh(1) + (b^3*c*f - b^3*d*cosh(1) - b^3*d*s
inh(1))*cosh(d*x + c)^4 - b^3*d*sinh(1) + 4*(b^3*c*f - b^3*d*cosh(1) - b^3*
d*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (b^3*c*f - b^3*d*cosh(1) - b^3*d
*sinh(1))*sinh(d*x + c)^4 + 2*(b^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1))*cos

```
h(d*x + c)^2 + 2*(b^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1) + 3*(b^3*c*f - b^
3*d*cosh(1) - b^3*d*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^3*c*f
 - b^3*d*cosh(1) - b^3*d*sinh(1))*cosh(d*x + c)^3 + (b^3*c*f - b^3*d*cosh(1
) - b^3*d*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*
b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(b^3*d*f*x + b^3*c*f
 + (b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^4 + 4*(b^3*d*f*x + b^3*c*f)*cosh(d*x
 + c)*sinh(d*x + c)^3 + (b^3*d*f*x + b^3*c*f)*sinh(d*x + c)^4 + 2*(b^3*d*f*
x + b^3*c*f)*cosh(d*x + c)^2 + 2*(b^3*d*f*x + b^3*c*f + 3*(b^3*d*f*x + b^3*
c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^3*d*f*x + b^3*c*f)*cosh(d*x +
 c)^3 + (b^3*d*f*x + b^3*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d*
x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b) + 2*(b^3*d*f*x + b^3*c*f + (b^3*d*f*x + b^3*c*f)*cosh(d*x
 + c)^4 + 4*(b^3*d*f*x + b^3*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^3*d*f*
x + b^3*c*f)*sinh(d*x + c)^4 + 2*(b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^2 + 2*
(b^3*d*f*x + b^3*c*f + 3*(b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^2)*sinh(d*x +
c)^2 + 4*((b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^3 + (b^3*d*f*x + b^3*c*f)*cos
h(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (2*b^3*c*f -
2*b^3*d*cosh(1) + (2*b^3*c*f - 2*b^3*d*cosh(1) - 2*b^3*d*sinh(1) - I*(a^3 +
 3*a*b^2)*c*f + I*(a^3 + 3*a*b^2)*d*cosh(1) + I*(a^3 + 3*a*b^2)*d*sinh(1))*
cosh(d*x + c)^4 - 2*b^3*d*sinh(1) + 4*(2*b^3*c*f - 2*b^3*d*cosh(1) - 2*b^3*
d*sinh(1) - I*(a^3 + 3*a*b^2)*c*f + I*(a^3 + 3*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e + fx)\operatorname{sech}^3(c + dx)}{a + b\sinh(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)*sech(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e + f\,x}{\cosh(c + d\,x)^3\,(a + b\sinh(c + d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(cosh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
[Out] int((e + f*x)/(cosh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

3.316 $\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=119

$$\frac{a(a^2+3b^2)\operatorname{ArcTan}(\sinh(c+dx))}{2\,(a^2+b^2)^2\,d}-\frac{b^3\log(\cosh(c+dx))}{(a^2+b^2)^2\,d}+\frac{b^3\log(a+b\sinh(c+dx))}{(a^2+b^2)^2\,d}+\frac{\operatorname{sech}^2(c+dx)(b+a\sin$$

[Out] 1/2*a*(a^2+3*b^2)*arctan(sinh(d*x+c))/(a^2+b^2)^2/d-b^3*ln(cosh(d*x+c))/(a^2+b^2)^2/d+b^3*ln(a+b*sinh(d*x+c))/(a^2+b^2)^2/d+1/2*sech(d*x+c)^2*(b+a*sinh(d*x+c))/(a^2+b^2)/d

Rubi [A]
time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2747, 755, 815, 649, 209, 266}

$$\frac{a(a^2+3b^2)\operatorname{ArcTan}(\sinh(c+dx))}{2d\,(a^2+b^2)^2}+\frac{\operatorname{sech}^2(c+dx)(a\sinh(c+dx)+b)}{2d\,(a^2+b^2)}+\frac{b^3\log(a+b\sinh(c+dx))}{d\,(a^2+b^2)^2}-\frac{b^3\log(\cosh(c+dx))}{d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]),x]

[Out] (a*(a^2 + 3*b^2)*ArcTan[Sinh[c + d*x]])/(2*(a^2 + b^2)^2*d) - (b^3*Log[Cosh[c + d*x]])/((a^2 + b^2)^2*d) + (b^3*Log[a + b*Sinh[c + d*x]])/((a^2 + b^2)^2*d) + (Sech[c + d*x]^2*(b + a*Sinh[c + d*x]))/(2*(a^2 + b^2)*d)

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
```

```
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b^3\text{Subst}\left(\int \frac{1}{(a+x)(-b^2-x^2)^2}\,dx, x, b\sinh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2(a^2+b^2)\,d} - \frac{b\text{Subst}\left(\int \frac{a^2+2b^2+ax}{(a+x)(-b^2-x^2)}\,dx, x, b\sinh(c+dx)\right)}{2(a^2+b^2)\,d}$$
$$= \frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2(a^2+b^2)\,d} - \frac{b\text{Subst}\left(\int \left(-\frac{2b^2}{(a^2+b^2)(a+x)} + \frac{-a^3-3ab^2+2b^2x}{(a^2+b^2)(b^2+x^2)}\right)\right.}{2(a^2+b^2)\,d}$$
$$= \frac{b^3\log(a+b\sinh(c+dx))}{(a^2+b^2)^2\,d} + \frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2(a^2+b^2)\,d} - \frac{b\text{Subst}\left(\int \frac{-a^3-}{}\right.}{}$$
$$= \frac{b^3\log(a+b\sinh(c+dx))}{(a^2+b^2)^2\,d} + \frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2(a^2+b^2)\,d} - \frac{b^3\text{Subst}\left(\int \frac{x}{b^2+}\right.}{(}$$
$$= \frac{a(a^2+3b^2)\tan^{-1}(\sinh(c+dx))}{2(a^2+b^2)^2\,d} - \frac{b^3\log(\cosh(c+dx))}{(a^2+b^2)^2\,d} + \frac{b^3\log(a+b\sinh(c+}{(a^2+b^2)^2\,d}$$

Mathematica [A]
time = 0.14, size = 104, normalized size = 0.87

$$\frac{2a(a^2+3b^2)\,\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + 2b^3(-\log(\cosh(c+dx)) + \log(a+b\sinh(c+dx))) + b(a^2+b^2)\operatorname{sech}^2(c+dx) + a(a^2+b^2)\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a^2+b^2)^2\,d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]),x]

[Out] (2*a*(a^2 + 3*b^2)*ArcTan[Tanh[(c + d*x)/2]] + 2*b^3*(-Log[Cosh[c + d*x]] + Log[a + b*Sinh[c + d*x]]) + b*(a^2 + b^2)*Sech[c + d*x]^2 + a*(a^2 + b^2)*Sech[c + d*x]*Tanh[c + d*x])/(2*(a^2 + b^2)^2*d)

Maple [A]
time = 1.42, size = 205, normalized size = 1.72

method	result
derivativedivides	$\frac{\frac{b^3\ln\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a^4+2a^2b^2+b^4}+\frac{\frac{2\left(\left(-\frac{1}{2}a^3-\frac{1}{2}a\,b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^2b-b^3\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^3+\frac{1}{2}a\,b^2\right)}{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}}{a^4+2a^2b}}{d}$
default	$\frac{\frac{b^3\ln\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a^4+2a^2b^2+b^4}+\frac{\frac{2\left(\left(-\frac{1}{2}a^3-\frac{1}{2}a\,b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^2b-b^3\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^3+\frac{1}{2}a\,b^2\right)}{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}}{a^4+2a^2b}}{d}$
risch	$\frac{2b^3d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{2b^3dc}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{2b^3x}{a^4+2a^2b^2+b^4}-\frac{2b^3c}{d(a^4+2a^2b^2+b^4)}+\frac{e^{dx+c}\left(a\,e^{2dx+2c}+2b\,e^{dx+c}-\right.}{d(a^2+b^2)\left(1+e^{2dx+2c}\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(b^3/(a^4+2*a^2*b^2+b^4)*ln(a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)-a)+2/(a^4+2*a^2*b^2+b^4)*(((-1/2*a^3-1/2*a*b^2)*tanh(1/2*d*x+1/2*c)^3+(-a^2*b-b^3)*tanh(1/2*d*x+1/2*c)^2+(1/2*a^3+1/2*a*b^2)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2+1)^2-1/2*b^3*ln(tanh(1/2*d*x+1/2*c)^2+1)+1/2*(a^3+3*a*b^2)*arctan(tanh(1/2*d*x+1/2*c))))

Maxima [A]
time = 0.48, size = 216, normalized size = 1.82

$$\frac{b^3\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^4+2\,a^2b^2+b^4)d}-\frac{b^3\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^4+2\,a^2b^2+b^4)d}-\frac{(a^3+3\,ab^2)\arctan\left(e^{(-dx-c)}\right)}{(a^4+2\,a^2b^2+b^4)d}+\frac{ae^{(-dx-c)}+2\,be^{(-2\,dx-2\,c)}-ae^{(-3\,dx-3\,c)}}{(a^2+b^2+2\,(a^2+b^2)e^{(-2\,dx-2\,c)}+(a^2+b^2)e^{(-4\,dx-4\,c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")

[Out] b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b^4)*d) - b^3*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a^3 + 3*a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (a*e^(-d*x - c) + 2*b*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 893 vs. 2(115) = 230.
time = 0.44, size = 893, normalized size = 7.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((a^3 + a*b^2)*cosh(d*x + c)^3 + (a^3 + a*b^2)*sinh(d*x + c)^3 + 2*(a^2*b +
 b^3)*cosh(d*x + c)^2 + (2*a^2*b + 2*b^3 + 3*(a^3 + a*b^2)*cosh(d*x + c))*s
inh(d*x + c)^2 + ((a^3 + 3*a*b^2)*cosh(d*x + c)^4 + 4*(a^3 + 3*a*b^2)*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a*b^2)*sinh(d*x + c)^4 + a^3 + 3*a*b^2
+ 2*(a^3 + 3*a*b^2)*cosh(d*x + c)^2 + 2*(a^3 + 3*a*b^2 + 3*(a^3 + 3*a*b^2)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3 + 3*a*b^2)*cosh(d*x + c)^3 + (a^
3 + 3*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x
+ c)) - (a^3 + a*b^2)*cosh(d*x + c) + (b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x
 + c)*sinh(d*x + c)^3 + b^3*sinh(d*x + c)^4 + 2*b^3*cosh(d*x + c)^2 + b^3 +
 2*(3*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 +
 b^3*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x +
c) - sinh(d*x + c))) - (b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x
+ c)^3 + b^3*sinh(d*x + c)^4 + 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(
d*x + c)^2 + b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 + b^3*cosh(d*x +
 c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) -
(a^3 + a*b^2 - 3*(a^3 + a*b^2)*cosh(d*x + c)^2 - 4*(a^2*b + b^3)*cosh(d*x +
 c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^4 + 4*(a^4 + 2
*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d
*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4
+ 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d)*sinh(d*x
+ c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d + 4*((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x
+ c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sech(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(115) = 230.
time = 0.45, size = 282, normalized size = 2.37

$$\frac{\frac{4\,b^4\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^4b+2\,a^2b^3+b^5}-\frac{2\,b^3\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(a^3+3\,ab^2\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\left(b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+2\,a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)+4\,a^2b+8\,b^3\right)}{\left(a^4+2\,a^2b^2+b^4\right)\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(4*b^4*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^4*b + 2*a^2*b^
3 + b^5) - 2*b^3*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^4 + 2*a^2*b^2 +
 b^4) + (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(a^3 + 3*a*
b^2)/(a^4 + 2*a^2*b^2 + b^4) + 2*(b^3*(e^(d*x + c) - e^(-d*x - c))^2 + 2*a^
3*(e^(d*x + c) - e^(-d*x - c)) + 2*a*b^2*(e^(d*x + c) - e^(-d*x - c)) + 4*a
^2*b + 8*b^3)/((a^4 + 2*a^2*b^2 + b^4)*((e^(d*x + c) - e^(-d*x - c))^2 + 4)
))/d
```

Mupad [B]
time = 2.20, size = 381, normalized size = 3.20

$$\frac{\frac{2\,\left(a^2\,b+b^3\right)}{d\,{\left(a^2+b^2\right)}^2}+\frac{{\mathrm{e}}^{c+d\,x}\,\left(a^3+a\,b^2\right)}{d\,{\left(a^2+b^2\right)}^2}}{{\mathrm{e}}^{2\,c+2\,d\,x}+1}-\frac{\frac{2\,b}{d\,\left(a^2+b^2\right)}+\frac{2\,a\,{\mathrm{e}}^{c+d\,x}}{d\,\left(a^2+b^2\right)}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}-\frac{\ln\left({\mathrm{e}}^{c+d\,x}+1{}\mathrm{i}\right)\,\left(2\,b+a\,1{}\mathrm{i}\right)}{2\,\left(-d\,a^2+2{}\mathrm{i}\,d\,a\,b+d\,b^2\right)}-\frac{\ln\left(1+{\mathrm{e}}^{c+d\,x}\,1{}\mathrm{i}\right)\,\left(a+b\,2{}\mathrm{i}\right)}{2\,\left(-1{}\mathrm{i}\,d\,a^2+2\,d\,a\,b+1{}\mathrm{i}\,d\,b^2\right)}+\frac{b^3\,\ln\left(2\,a^7\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c-16\,b^7-9\,a^2\,b^5-6\,a^4\,b^3-a^6\,b+16\,b^7\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+a^6\,b\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+18\,a^3\,b^4\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c+12\,a^5\,b^2\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c+9\,a^2\,b^5\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+6\,a^4\,b^3\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}+32\,a\,b^6\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\right)}{d\,a^4+2\,d\,a^2\,b^2+d\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

```
[Out] ((2*(a^2*b + b^3))/(d*(a^2 + b^2)^2) + (exp(c + d*x)*(a*b^2 + a^3))/(d*(a^2
 + b^2)^2))/(exp(2*c + 2*d*x) + 1) - ((2*b)/(d*(a^2 + b^2)) + (2*a*exp(c +
d*x))/(d*(a^2 + b^2)))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (log(e
xp(c + d*x) + 1i)*(a*1i + 2*b))/(2*(b^2*d - a^2*d + a*b*d*2i)) - (log(exp(c
 + d*x)*1i + 1)*(a + b*2i))/(2*(b^2*d*1i - a^2*d*1i + 2*a*b*d)) + (b^3*log(
2*a^7*exp(d*x)*exp(c) - 16*b^7 - 9*a^2*b^5 - 6*a^4*b^3 - a^6*b + 16*b^7*exp
(2*c)*exp(2*d*x) + a^6*b*exp(2*c)*exp(2*d*x) + 18*a^3*b^4*exp(d*x)*exp(c) +
 12*a^5*b^2*exp(d*x)*exp(c) + 9*a^2*b^5*exp(2*c)*exp(2*d*x) + 6*a^4*b^3*exp
(2*c)*exp(2*d*x) + 32*a*b^6*exp(d*x)*exp(c)))/(a^4*d + b^4*d + 2*a^2*b^2*d)
```

3.317 $\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$

Optimal. Leaf size=31

$$\operatorname{Int}\left(\frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is not applicable to the result.

```
[In] Int[Sech[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][Sech[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Mathematica [A]
time = 100.17, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sech[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Integrate[Sech[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(dx+c)^3}{(fx+e)\,(a+b\sinh\,(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)

[Out] int(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")

[Out] -(b*f - (a*d*f*x*e^(3*c) - a*f*e^(3*c) + a*d*e^(3*c + 1))*e^(3*d*x) - (2*b*d*f*x*e^(2*c) - b*f*e^(2*c) + 2*b*d*e^(2*c + 1))*e^(2*d*x) + (a*d*f*x*e^c + a*d*e^(c + 1) + a*f*e^c)*e^(d*x))/((a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*(a^2*d^2*f + b^2*d^2*f)*x*e + (a^2*d^2 + b^2*d^2)*e^2 + ((a^2*d^2*f^2*e^(4*c) + b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^2*d^2*f*e^(4*c) + b^2*d^2*f*e^(4*c))*x*e + (a^2*d^2*e^(4*c) + b^2*d^2*e^(4*c))*e^2)*e^(4*d*x) + 2*((a^2*d^2*f^2*e^(2*c) + b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^2*d^2*f*e^(2*c) + b^2*d^2*f*e^(2*c))*x*e + (a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^2)*e^(2*d*x)) + 8*integrate(1/8*(2*b^3*d^2*f^2*x^2 + 4*b^3*d^2*f*x*e + 2*b^3*d^2*e^2 - 2*a^2*b*f^2 - 2*b^3*f^2 - (2*a^3*f^2*e^c + 2*a*b^2*f^2*e^c - (a^3*d^2*f^2*e^c + 3*a*b^2*d^2*f^2*e^c)*x^2 - 2*(a^3*d^2*f*e^c + 3*a*b^2*d^2*f*e^c)*x*e - (a^3*d^2*e^c + 3*a*b^2*d^2*e^c)*e^2)*e^(d*x))/((a^4*d^2*f^3 + 2*a^2*b^2*d^2*f^3 + b^4*d^2*f^3)*x^3 + 3*(a^4*d^2*f^2 + 2*a^2*b^2*d^2*f^2 + b^4*d^2*f^2)*x^2*e + 3*(a^4*d^2*f + 2*a^2*b^2*d^2*f + b^4*d^2*f)*x*e^2 + (a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2)*e^3 + ((a^4*d^2*f^3*e^(2*c) + 2*a^2*b^2*d^2*f^3*e^(2*c) + b^4*d^2*f^3*e^(2*c))*x^3 + 3*(a^4*d^2*f^2*e^(2*c) + 2*a^2*b^2*d^2*f^2*e^(2*c) + b^4*d^2*f^2*e^(2*c))*x^2*e + 3*(a^4*d^2*f*e^(2*c) + 2*a^2*b^2*d^2*f*e^(2*c) + b^4*d^2*f*e^(2*c))*x*e^2 + (a^4*d^2*e^(2*c) + 2*a^2*b^2*d^2*e^(2*c) + b^4*d^2*e^(2*c))*e^3)*e^(2*d*x)), x) - 8*integrate(-1/4*(a*b^3*e^(d*x + c) - b^4)/((a^4*b*f + 2*a^2*b^3*f + b^5*f)*x + (a^4*b + 2*a^2*b^3 + b^5)*e - ((a^4*b*f*e^(2*c) + 2*a^2*b^3*f*e^(2*c) + b^5*f*e^(2*c))*x + (a^4*b*e^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e)*e^(2*d*x) - 2*((a^5*f*e^c + 2*a^3*b^2*f*e^c + a*b^4*f*e^c)*x + (a^5*e^c + 2*a^3*b^2*e^c + a*b^4*e^c)*e)*e^(d*x)), x)

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")

[Out] integral(sech(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sech(c + d*x)**3/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cosh(c+d\,x)^3\,\left(e+f\,x\right)\,\left(a+b\sinh(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(1/(cosh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.318 $\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)},x\right)$$

```
[Out] Unintegrable(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
[Out] Defer[Int][(x^m*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]), x]
```

Rubi steps

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Mathematica [A]
time = 8.71, size = 0, normalized size = 0.00

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
[Out] Integrate[(x^m*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]), x]
```

Maple [A]
time = 2.20, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(\cosh^3\left(dx+c\right)\right)}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
[Out] int(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
[Out] integrate(x^m*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
[Out] integral(x^m*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
[Out] Integral(x**m*cosh(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate(x^m*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,{\cosh\left(c+d\,x\right)}^3}{a+b\,\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*cosh(c + d*x)^3)/(a + b*sinh(c + d*x)),x)
[Out] int((x^m*cosh(c + d*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.319 $\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)},x\right)$$

```
[Out] Unintegrable(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Defer[Int][(x^m*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]), x]
```

Rubi steps

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Mathematica [A]
time = 5.60, size = 0, normalized size = 0.00

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Integrate[(x^m*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]), x]
```

Maple [A]
time = 1.90, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(\cosh^2\left(dx+c\right)\right)}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(x^m*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(x^m*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(x**m*cosh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m\,{\cosh\left(c+d\,x\right)}^2}{a+b\,\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*cosh(c + d*x)^2)/(a + b*sinh(c + d*x)),x)
[Out] int((x^m*cosh(c + d*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.320 $\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)},x\right)$$

```
[Out] Unintegrable(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^m*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
[Out] Defer[Int][(x^m*Cosh[c + d*x])/(a + b*Sinh[c + d*x]), x]
```

Rubi steps

$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Mathematica [A]
time = 4.39, size = 0, normalized size = 0.00

$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
[Out] Integrate[(x^m*Cosh[c + d*x])/(a + b*Sinh[c + d*x]), x]
```

Maple [A]
time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{x^m \cosh(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] int(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
[Out] x*e^(2*d*x + m*log(x) + 2*c)/(b*(m + 1)*e^(2*d*x + 2*c) + 2*a*(m + 1)*e^(d*
x + c) - b*(m + 1)) - 1/2*integrate(2*(2*a*d*x*e^(3*d*x + 3*c) - 2*a*(m + 1
)*e^(d*x + c) + b*(m + 1) - (2*b*d*x*e^(2*c) + b*(m + 1)*e^(2*c))*e^(2*d*x)
)*x^m/(b^2*(m + 1)*e^(4*d*x + 4*c) + 4*a*b*(m + 1)*e^(3*d*x + 3*c) - 4*a*b*
(m + 1)*e^(d*x + c) + b^2*(m + 1) + 2*(2*a^2*(m + 1)*e^(2*c) - b^2*(m + 1)*
e^(2*c))*e^(2*d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
[Out] integral(x^m*cosh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \cosh(c + dx)}{a + b \sinh(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Integral(x**m*cosh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \cosh(c+d\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*cosh(c + d*x))/(a + b*sinh(c + d*x)),x)
[Out] int((x^m*cosh(c + d*x))/(a + b*sinh(c + d*x)), x)
```

3.321 $\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\, dx$

Optimal. Leaf size=74

$$-\frac{2f\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\, d^2}-\frac{e+fx}{bd(a+b\sinh(c+dx))}$$

```
[Out] (-f*x-e)/b/d/(a+b*sinh(d*x+c))-2*f*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b
^2)^(1/2))/b/d^2/(a^2+b^2)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5572, 2739, 632, 210}

$$-\frac{2f\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}}-\frac{e+fx}{bd(a+b\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] (-2*f*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b*Sqrt[a^2 + b^2
]*d^2) - (e + f*x)/(b*d*(a + b*Sinh[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 5572

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx &= -\frac{e+fx}{bd(a+b\sinh(c+dx))}+\frac{f\int \frac{1}{a+b\sinh(c+dx)}\,dx}{bd}\\ &= -\frac{e+fx}{bd(a+b\sinh(c+dx))}-\frac{(2if)\text{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{bd^2}\\ &= -\frac{e+fx}{bd(a+b\sinh(c+dx))}+\frac{(4if)\text{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx,x,-2ib+2a\tan\left(\frac{1}{2}(\right.\right.}{bd^2}\\ &= -\frac{2f\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}-\frac{e+fx}{bd(a+b\sinh(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.42, size = 78, normalized size = 1.05

$$\frac{\frac{2f\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-\frac{d(e+fx)}{a+b\sinh(c+dx)}}{bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] ((2*f*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
- (d*(e + f*x))/(a + b*Sinh[c + d*x]))/(b*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(71) = 142$.
time = 7.70, size = 164, normalized size = 2.22

method	result	size
risch	$-\frac{2(fx+e)\mathrm{e}^{dx+c}}{bd\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-b\right)}+\frac{f\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d^2b}-\frac{f\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d^2b}$	164

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(f*x+e)/b/d*exp(d*x+c)/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/(a^2+b^2)^(
1/2)*f/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(1/2)-a^2-b^2)/(a^2+b^2)^(1/2)/b)-1
/(a^2+b^2)^(1/2)*f/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(1/2)+a^2+b^2)/(a^2+b^2
)^(1/2)/b)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(72) = 144$.
time = 0.54, size = 157, normalized size = 2.12

$$-f\left(\frac{2\,xe^{(dx+c)}}{b^2de^{(2\,dx+2\,c)}+2\,abde^{(dx+c)}-b^2d}-\frac{\log\left(\frac{be^{(dx+c)}+a-\sqrt{a^2+b^2}}{be^{(dx+c)}+a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,bd^2}\right)-\frac{2\,e^{(-dx-c+1)}}{(2\,abe^{(-dx-c)}-b^2e^{(-2\,dx-2\,c)}+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -f*(2*x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b^2*d) -
 log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqrt(a^2 +
b^2)))/(sqrt(a^2 + b^2)*b*d^2)) - 2*e^(-d*x - c + 1)/((2*a*b*e^(-d*x - c) -
 b^2*e^(-2*d*x - 2*c) + b^2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(72) = 144$.
time = 0.39, size = 435, normalized size = 5.88

$$\frac{\left(bf\cosh\left(dx+c\right)^2+bf\sinh\left(dx+c\right)^2+2\,af\cosh\left(dx+c\right)-bf+2\left(bf\cosh\left(dx+c\right)+af\right)\sinh\left(dx+c\right)\right)\sqrt{a^2+b^2}\,\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2+b^2+2\,(b^2\cosh(dx+c)+ab)\sinh(dx+c)-2\,\sqrt{a^2+b^2}\,(b\cosh(dx+c)+b\sinh(dx+c)+a)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)-b}\right)-2\left((a^2+b^2)dfx+(a^2+b^2)d\cosh\left(1\right)+(a^2+b^2)d\sinh\left(1\right)\right)\cosh\left(dx+c\right)-2\left((a^2+b^2)dfx+(a^2+b^2)d\cosh\left(1\right)+(a^2+b^2)d\sinh\left(1\right)\right)\sinh\left(dx+c\right)}{(a^2b^2+b^4)d^2\cosh\left(dx+c\right)^2+(a^2b^2+b^4)d^2\sinh\left(dx+c\right)^2+2\left(a^3b+ab^3\right)d^2\cosh\left(dx+c\right)-(a^2b^2+b^4)d^2+2\left((a^2b^2+b^4)d^2\cosh\left(dx+c\right)+(a^3b+ab^3)d^2\right)\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((b*f*cosh(d*x + c)^2 + b*f*sinh(d*x + c)^2 + 2*a*f*cosh(d*x + c) - b*f + 2
*(b*f*cosh(d*x + c) + a*f)*sinh(d*x + c))*sqrt(a^2 + b^2)*log((b^2*cosh(d*x
 + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*
cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b
*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x
+ c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*((a^2 + b^2)*d*f*x +
 (a^2 + b^2)*d*cosh(1) + (a^2 + b^2)*d*sinh(1))*cosh(d*x + c) - 2*((a^2 + b
^2)*d*f*x + (a^2 + b^2)*d*cosh(1) + (a^2 + b^2)*d*sinh(1))*sinh(d*x + c))/(
(a^2*b^2 + b^4)*d^2*cosh(d*x + c)^2 + (a^2*b^2 + b^4)*d^2*sinh(d*x + c)^2 +
```

```
 2*(a^3*b + a*b^3)*d^2*cosh(d*x + c) - (a^2*b^2 + b^4)*d^2 + 2*((a^2*b^2 +
b^4)*d^2*cosh(d*x + c) + (a^3*b + a*b^3)*d^2)*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

Mupad [**B**]
time = 0.56, size = 199, normalized size = 2.69

$$\frac{f\,\ln\left(\frac{2\,f\,\left(b-a\,{\mathrm{e}}^{c+d\,x}\right)}{b^2\,d\,\sqrt{a^2+b^2}}-\frac{2\,f\,{\mathrm{e}}^{c+d\,x}}{b^2\,d}\right)}{b\,d^2\,\sqrt{a^2+b^2}}-\frac{f\,\ln\left(-\frac{2\,f\,{\mathrm{e}}^{c+d\,x}}{b^2\,d}-\frac{2\,f\,\left(b-a\,{\mathrm{e}}^{c+d\,x}\right)}{b^2\,d\,\sqrt{a^2+b^2}}\right)}{b\,d^2\,\sqrt{a^2+b^2}}-\frac{2\,{\mathrm{e}}^{c+d\,x}\,\left(a^2\,e+b^2\,e+a^2\,f\,x+b^2\,f\,x\right)}{d\,\left(a^2\,b+b^3\right)\,\left(2\,a\,{\mathrm{e}}^{c+d\,x}-b+b\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x))^2,x)
[Out] (f*log((2*f*(b - a*exp(c + d*x)))/(b^2*d*(a^2 + b^2)^(1/2)) - (2*f*exp(c +
d*x))/(b^2*d)))/(b*d^2*(a^2 + b^2)^(1/2)) - (f*log(- (2*f*exp(c + d*x))/(b^
2*d) - (2*f*(b - a*exp(c + d*x)))/(b^2*d*(a^2 + b^2)^(1/2))))/(b*d^2*(a^2 +
 b^2)^(1/2)) - (2*exp(c + d*x)*(a^2*e + b^2*e + a^2*f*x + b^2*f*x))/(d*(a^2
*b + b^3)*(2*a*exp(c + d*x) - b + b*exp(2*c + 2*d*x)))
```

3.322 $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=234

$$\frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}-\frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}+\frac{2f^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^3}$$

```
[Out] -(f*x+e)^2/b/d/(a+b*sinh(d*x+c))+2*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)
^(1/2)))/b/d^2/(a^2+b^2)^(1/2)-2*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))/b/d^2/(a^2+b^2)^(1/2)+2*f^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/
2)))/b/d^3/(a^2+b^2)^(1/2)-2*f^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)
))/b/d^3/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5572, 3403, 2296, 2221, 2317, 2438}

$$\frac{2f^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{2f^2\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}+\frac{2f(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd^2\sqrt{a^2+b^2}}-\frac{2f(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd^2\sqrt{a^2+b^2}}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] (2*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*Sqrt[a^2
+ b^2]*d^2) - (2*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
)/(b*Sqrt[a^2 + b^2]*d^2) + (2*f^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (2*f^2*PolyLog[2, -((b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (e + f*x)^2/(b*d*(a + b
*Sinh[c + d*x]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5572

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = -\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))} + \frac{(2f)\int \frac{e+fx}{a+b\sinh(c+dx)}\,dx}{bd}$$

$$= -\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))} + \frac{(4f)\int \frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{bd}$$

$$= -\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))} + \frac{(4f)\int \frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\sqrt{a^2+b^2}\,d} - \frac{(4f)\int \frac{}{2a+2\sqrt{}}}{\sqrt{}}$$

$$= \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2} - \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}$$

$$= \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2} - \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}$$

$$= \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2} - \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}$$

Mathematica [A]
time = 1.14, size = 175, normalized size = 0.75

$$\frac{2f\left(d(e+fx)\left(\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)+f\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)-f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)}{b\sqrt{a^2+b^2}\,d^3}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] (2*f*(d*(e + f*x)*(Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - Log[1 +
 (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]) + f*PolyLog[2, (b*E^(c + d*x))/(-a
 + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)]))/(b*Sqrt[a^2 + b^2]*d^3) - (e + f*x)^2/(b*d*(a + b*Sinh[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. 2(214) = 428.
time = 6.45, size = 491, normalized size = 2.10

method	result

risch	$-\frac{2(x^2f^2+2efx+e^2)\,\mathrm{e}^{dx+c}}{bd(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-b)}-\frac{4fe\,\operatorname{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{d^2b\sqrt{a^2+b^2}}+\frac{2f^2\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)x}{d^2b\sqrt{a^2+b^2}}+\frac{2f^2\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{}}{-a+\sqrt{a}}\right.}{d^3b\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(f^2*x^2+2*e*f*x+e^2)/b/d*exp(d*x+c)/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)
-4/d^2/b*f*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/
2))+2/d^2/b*f^2/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a
^2+b^2)^(1/2)))*x+2/d^3/b*f^2/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(
1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/d^2/b*f^2/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c
)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-2/d^3/b*f^2/(a^2+b^2)^(1/2)*ln(
(b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/d^3/b*f^2/(a^2+b^
2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2/d^
3/b*f^2/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)
^(1/2)))+4/d^3/b*f^2*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^
2+b^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -2*(x^2*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b^2*d) -
 2*integrate(x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b
^2*d), x))*f^2 - 2*f*(2*x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d
*x + c) - b^2*d) - log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c)
 + a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b*d^2))*e - 2*e^(-d*x - c + 2)/((
2*a*b*e^(-d*x - c) - b^2*e^(-2*d*x - 2*c) + b^2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1590 vs. $2(215) = 430$.
time = 0.38, size = 1590, normalized size = 6.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 2*((b^2*f^2*cosh(d*x + c)^2 + b^2*f^2*sinh(d*x + c)^2 + 2*a*b*f^2*cosh(d*x
+ c) - b^2*f^2 + 2*(b^2*f^2*cosh(d*x + c) + a*b*f^2)*sinh(d*x + c))*sqrt((a
```

```
^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^2*f^2*cosh(d*x +
c)^2 + b^2*f^2*sinh(d*x + c)^2 + 2*a*b*f^2*cosh(d*x + c) - b^2*f^2 + 2*(b^2
*f^2*cosh(d*x + c) + a*b*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a
*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b + 1) - (b^2*c*f^2 - b^2*d*f*cosh(1) - b^2*d*f*sinh
(1) - (b^2*c*f^2 - b^2*d*f*cosh(1) - b^2*d*f*sinh(1))*cosh(d*x + c)^2 - (b^
2*c*f^2 - b^2*d*f*cosh(1) - b^2*d*f*sinh(1))*sinh(d*x + c)^2 - 2*(a*b*c*f^2
 - a*b*d*f*cosh(1) - a*b*d*f*sinh(1))*cosh(d*x + c) - 2*(a*b*c*f^2 - a*b*d*
f*cosh(1) - a*b*d*f*sinh(1) + (b^2*c*f^2 - b^2*d*f*cosh(1) - b^2*d*f*sinh(1
))*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c
) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^2*c*f^2 - b^2
*d*f*cosh(1) - b^2*d*f*sinh(1) - (b^2*c*f^2 - b^2*d*f*cosh(1) - b^2*d*f*sin
h(1))*cosh(d*x + c)^2 - (b^2*c*f^2 - b^2*d*f*cosh(1) - b^2*d*f*sinh(1))*sin
h(d*x + c)^2 - 2*(a*b*c*f^2 - a*b*d*f*cosh(1) - a*b*d*f*sinh(1))*cosh(d*x +
 c) - 2*(a*b*c*f^2 - a*b*d*f*cosh(1) - a*b*d*f*sinh(1) + (b^2*c*f^2 - b^2*d
*f*cosh(1) - b^2*d*f*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2
)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2
) + 2*a) - (b^2*d*f^2*x + b^2*c*f^2 - (b^2*d*f^2*x + b^2*c*f^2)*cosh(d*x +
c)^2 - (b^2*d*f^2*x + b^2*c*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*f^2*x + a*b*c*f
^2)*cosh(d*x + c) - 2*(a*b*d*f^2*x + a*b*c*f^2 + (b^2*d*f^2*x + b^2*c*f^2)*
cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) +
 a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2
) - b)/b) + (b^2*d*f^2*x + b^2*c*f^2 - (b^2*d*f^2*x + b^2*c*f^2)*cosh(d*x +
 c)^2 - (b^2*d*f^2*x + b^2*c*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*f^2*x + a*b*c*
f^2)*cosh(d*x + c) - 2*(a*b*d*f^2*x + a*b*c*f^2 + (b^2*d*f^2*x + b^2*c*f^2)
*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c)
+ a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2) - b)/b) - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*f*x*cosh(1) + (a^
2 + b^2)*d^2*cosh(1)^2 + (a^2 + b^2)*d^2*sinh(1)^2 + 2*((a^2 + b^2)*d^2*f*x
 + (a^2 + b^2)*d^2*cosh(1))*sinh(1))*cosh(d*x + c) - ((a^2 + b^2)*d^2*f^2*x
^2 + 2*(a^2 + b^2)*d^2*f*x*cosh(1) + (a^2 + b^2)*d^2*cosh(1)^2 + (a^2 + b^2
)*d^2*sinh(1)^2 + 2*((a^2 + b^2)*d^2*f*x + (a^2 + b^2)*d^2*cosh(1))*sinh(1)
)*sinh(d*x + c))/((a^2*b^2 + b^4)*d^3*cosh(d*x + c)^2 + (a^2*b^2 + b^4)*d^3
*sinh(d*x + c)^2 + 2*(a^3*b + a*b^3)*d^3*cosh(d*x + c) - (a^2*b^2 + b^4)*d^
3 + 2*((a^2*b^2 + b^4)*d^3*cosh(d*x + c) + (a^3*b + a*b^3)*d^3)*sinh(d*x +
c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
```

[Out] Timed out

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)^2}{\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^2,x)
```

```
[Out] int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^2, x)
```

3.323 $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=348

$$\frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}-\frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}+\frac{6f^2(e+fx)\mathrm{PolyLog}\left(2,-\frac{}{a-\sqrt{}}\right.}{b\sqrt{a^2+b^2}\,d^3}$$

```
[Out] -(f*x+e)^3/b/d/(a+b*sinh(d*x+c))+3*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))/b/d^2/(a^2+b^2)^(1/2)-3*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/b/d^2/(a^2+b^2)^(1/2)+6*f^2*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(
a^2+b^2)^(1/2)))/b/d^3/(a^2+b^2)^(1/2)-6*f^2*(f*x+e)*polylog(2,-b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/b/d^3/(a^2+b^2)^(1/2)-6*f^3*polylog(3,-b*exp(d*x+c)/
(a-(a^2+b^2)^(1/2)))/b/d^4/(a^2+b^2)^(1/2)+6*f^3*polylog(3,-b*exp(d*x+c)/(a
+(a^2+b^2)^(1/2)))/b/d^4/(a^2+b^2)^(1/2)
```

Rubi [A]

time = 0.52, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5572, 3403, 2296, 2221, 2611, 2320, 6724}

$$-\frac{6f^3\mathrm{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4\sqrt{a^2+b^2}}+\frac{6f^3\mathrm{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^4\sqrt{a^2+b^2}}+\frac{6f^2(e+fx)\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{6f^2(e+fx)\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}+\frac{3f(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd^2\sqrt{a^2+b^2}}-\frac{3f(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd^2\sqrt{a^2+b^2}}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] (3*f*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*Sqrt[a^
2 + b^2]*d^2) - (3*f*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(b*Sqrt[a^2 + b^2]*d^2) + (6*f^2*(e + f*x)*PolyLog[2, -((b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (6*f^2*(e + f*x)*Poly
Log[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) -
 (6*f^3*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 +
 b^2]*d^4) + (6*f^3*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
b*Sqrt[a^2 + b^2]*d^4) - (e + f*x)^3/(b*d*(a + b*Sinh[c + d*x]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5572

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = -\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))} + \frac{(3f)\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}\,dx}{bd}$$

$$= -\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))} + \frac{(6f)\int \frac{e^{c+dx}(e+fx)^2}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{bd}$$

$$= -\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))} + \frac{(6f)\int \frac{e^{c+dx}(e+fx)^2}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\sqrt{a^2+b^2}\,d} - \frac{(6f)\int \frac{e^{\cdots}}{2a+2\sqrt{\cdots}}}{\sqrt{a^{\cdots}}}$$

$$= \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2} - \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right.\cdots}{b\sqrt{a^2+b^2}\,d^2}$$

$$= \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2} - \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right.\cdots}{b\sqrt{a^2+b^2}\,d^2}$$

$$= \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2} - \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right.\cdots}{b\sqrt{a^2+b^2}\,d^2}$$

$$= \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2} - \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right.\cdots}{b\sqrt{a^2+b^2}\,d^2}$$

<u>Mathematica</u> [A]
time = 4.73, size = 634, normalized size = 1.82

$$\frac{3f\left(\frac{2d^2e^2\text{ArcTan}\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{2d^2ee^cfx\log\left(1+\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}}+\frac{d^2e^cf^2x^2\log\left(1+\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}}-\frac{2d^2ee^cfx\log\left(1+\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}}-\frac{d^2e^cf^2x^2\log\left(1+\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}}+\frac{2de^cf(e+fx)\text{PolyLog}\left(2,-\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}}-\frac{2de^cf(e+fx)\text{PolyLog}\left(2,-\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}}-\frac{2e^cf^2\text{PolyLog}\left(3,-\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}}+\frac{2e^cf^2\text{PolyLog}\left(3,-\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}}\right)}{bd^4}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] (3*f*((2*d^2*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 -
b^2] + (2*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)
*E^(2*c)])]])/Sqrt[(a^2 + b^2)*E^(2*c)] + (d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c
 + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])]])/Sqrt[(a^2 + b^2)*E^(2*c)] -
(2*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c
)])]])/Sqrt[(a^2 + b^2)*E^(2*c)] - (d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x)
)/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])]])/Sqrt[(a^2 + b^2)*E^(2*c)] + (2*d*E^
c*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)]))]])/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*d*E^c*f*(e + f*x)*PolyLog[2, -((b*E
```

```
^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*
c)] - (2*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E
^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] + (2*E^c*f^2*PolyLog[3, -((b*E^(2*c +
 d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)]))/(
b*d^4) - (e + f*x)^3/(b*d*(a + b*Sinh[c + d*x]))
```

Maple [F]
time = 5.20, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \cosh(dx+c)}{(a+b\sinh(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -3*f*(2*x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b^2*d)
 - log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqrt(a^2
+ b^2)))/(sqrt(a^2 + b^2)*b*d^2))*e^2 - 2*(f^3*x^3*e^c + 3*f^2*x^2*e^(c + 1
))*e^(d*x)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b^2*d) - 2*e^(-d*
x - c + 3)/((2*a*b*e^(-d*x - c) - b^2*e^(-2*d*x - 2*c) + b^2)*d) + integrat
e(6*(f^3*x^2*e^c + 2*f^2*x*e^(c + 1))*e^(d*x)/(b^2*d*e^(2*d*x + 2*c) + 2*a*
b*d*e^(d*x + c) - b^2*d), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3516 vs. $2(321) = 642$.
time = 0.39, size = 3516, normalized size = 10.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(6*(b^2*d*f^3*x + b^2*d*f^2*cosh(1) + b^2*d*f^2*sinh(1) - (b^2*d*f^3*x + b
^2*d*f^2*cosh(1) + b^2*d*f^2*sinh(1))*cosh(d*x + c)^2 - (b^2*d*f^3*x + b^2*
d*f^2*cosh(1) + b^2*d*f^2*sinh(1))*sinh(d*x + c)^2 - 2*(a*b*d*f^3*x + a*b*d
*f^2*cosh(1) + a*b*d*f^2*sinh(1))*cosh(d*x + c) - 2*(a*b*d*f^3*x + a*b*d*f^
```

```
2*cosh(1) + a*b*d*f^2*sinh(1) + (b^2*d*f^3*x + b^2*d*f^2*cosh(1) + b^2*d*f^
2*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2) - b)/b + 1) - 6*(b^2*d*f^3*x + b^2*d*f^2*cosh(1) + b^2*d*f^2*
sinh(1) - (b^2*d*f^3*x + b^2*d*f^2*cosh(1) + b^2*d*f^2*sinh(1))*cosh(d*x +
c)^2 - (b^2*d*f^3*x + b^2*d*f^2*cosh(1) + b^2*d*f^2*sinh(1))*sinh(d*x + c)^
2 - 2*(a*b*d*f^3*x + a*b*d*f^2*cosh(1) + a*b*d*f^2*sinh(1))*cosh(d*x + c) -
 2*(a*b*d*f^3*x + a*b*d*f^2*cosh(1) + a*b*d*f^2*sinh(1) + (b^2*d*f^3*x + b^
2*d*f^2*cosh(1) + b^2*d*f^2*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(b^2*c^2*f^3 - 2*b^
2*c*d*f^2*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 - (b^2*c^2*f^
3 - 2*b^2*c*d*f^2*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 - 2*(
b^2*c*d*f^2 - b^2*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 - (b^2*c^2*f^3 -
2*b^2*c*d*f^2*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 - 2*(b^2*
c*d*f^2 - b^2*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 2*(a*b*c^2*f^3 - 2*
a*b*c*d*f^2*cosh(1) + a*b*d^2*f*cosh(1)^2 + a*b*d^2*f*sinh(1)^2 - 2*(a*b*c*
d*f^2 - a*b*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c) - 2*(b^2*c*d*f^2 - b^2*d^
2*f*cosh(1))*sinh(1) - 2*(a*b*c^2*f^3 - 2*a*b*c*d*f^2*cosh(1) + a*b*d^2*f*c
osh(1)^2 + a*b*d^2*f*sinh(1)^2 + (b^2*c^2*f^3 - 2*b^2*c*d*f^2*cosh(1) + b^2
*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 - 2*(b^2*c*d*f^2 - b^2*d^2*f*cosh(1)
)*sinh(1))*cosh(d*x + c) - 2*(a*b*c*d*f^2 - a*b*d^2*f*cosh(1))*sinh(1))*sin
h(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c)
 + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 3*(b^2*c^2*f^3 - 2*b^2*c*d*f^2*cosh(1
) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 - (b^2*c^2*f^3 - 2*b^2*c*d*f^
2*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 - 2*(b^2*c*d*f^2 - b^
2*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 - (b^2*c^2*f^3 - 2*b^2*c*d*f^2*co
sh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 - 2*(b^2*c*d*f^2 - b^2*d^
2*f*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 2*(a*b*c^2*f^3 - 2*a*b*c*d*f^2*cosh
(1) + a*b*d^2*f*cosh(1)^2 + a*b*d^2*f*sinh(1)^2 - 2*(a*b*c*d*f^2 - a*b*d^2*
f*cosh(1))*sinh(1))*cosh(d*x + c) - 2*(b^2*c*d*f^2 - b^2*d^2*f*cosh(1))*sin
h(1) - 2*(a*b*c^2*f^3 - 2*a*b*c*d*f^2*cosh(1) + a*b*d^2*f*cosh(1)^2 + a*b*d
^2*f*sinh(1)^2 + (b^2*c^2*f^3 - 2*b^2*c*d*f^2*cosh(1) + b^2*d^2*f*cosh(1)^2
 + b^2*d^2*f*sinh(1)^2 - 2*(b^2*c*d*f^2 - b^2*d^2*f*cosh(1))*sinh(1))*cosh(
d*x + c) - 2*(a*b*c*d*f^2 - a*b*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2
 + b^2)/b^2) + 2*a) + 3*(b^2*d^2*f^3*x^2 - b^2*c^2*f^3 - (b^2*d^2*f^3*x^2 -
 b^2*c^2*f^3 + 2*(b^2*d^2*f^2*x + b^2*c*d*f^2)*cosh(1) + 2*(b^2*d^2*f^2*x +
 b^2*c*d*f^2)*sinh(1))*cosh(d*x + c)^2 - (b^2*d^2*f^3*x^2 - b^2*c^2*f^3 + 2
*(b^2*d^2*f^2*x + b^2*c*d*f^2)*cosh(1) + 2*(b^2*d^2*f^2*x + b^2*c*d*f^2)*si
nh(1))*sinh(d*x + c)^2 + 2*(b^2*d^2*f^2*x + b^2*c*d*f^2)*cosh(1) - 2*(a*b*d
^2*f^3*x^2 - a*b*c^2*f^3 + 2*(a*b*d^2*f^2*x + a*b*c*d*f^2)*cosh(1) + 2*(a*b
*d^2*f^2*x + a*b*c*d*f^2)*sinh(1))*cosh(d*x + c) + 2*(b^2*d^2*f^2*x + b^2*c
*d*f^2)*sinh(1) - 2*(a*b*d^2*f^3*x^2 - a*b*c^2*f^3 + 2*(a*b*d^2*f^2*x + a*b
*c*d*f^2)*cosh(1) + (b^2*d^2*f^3*x^2 - b^2*c^2*f^3 + 2*(b^2*d^2*f^2*x + b^2
```

```
*c*d*f^2)*cosh(1) + 2*(b^2*d^2*f^2*x + b^2*c*d*f^2)*sinh(1))*cosh(d*x + c)
+ 2*(a*b*d^2*f^2*x + a*b*c*d*f^2)*sinh(1))*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 3*(b^2*d^2*f^3*x^2 - b^2*c^2*f^3 -
(b^2*d^2*f^3*x^2 - b^2*c^2*f^3 + 2*(b^2*d^2*f^2*x + b^2*c*d*f^2)*cosh(1) +
2*(b^2*d^2*f^2*x + b^2*c*d*f^2)*sinh(1))*cosh(d*x + c)^2 - (b^2*d^2*f^3*x^2
 - b^2*c^2*f^3 + 2*(b^2*d^2*f^2*x + b^2*c*d*f^2)*cosh(1) + 2*(b^2*d^2*f^2*x
 + b^2*c*d*f^2)*sinh(1))*sinh(d*x + c)^2 + 2*(b^2*d^2*f^2*x + b^2*c*d*f^2)*
cosh(1) - 2*(a*b*d^2*f^3*x^2 - a*b*c^2*f^3 + 2*(a*b*d^2*f^2*x + a*b*c*d*f^2
)*cosh(1) + 2*(a*b*d^2*f^2*x + a*b*c*d*f^2)*sinh(1))*cosh(d*x + c) + 2*(b^2
*d^2*f^2*x + b^2*c*d*f^2)*sinh(1) - 2*(a*b*d^2*f^3*x^2 - a*b*c^2*f^3 + 2*(a
*b*d^2*f^2*x + a*b*c*d*f^2)*cosh(1) + (b^2*d^2*f^3*x^2 - b^2*c^2*f^3 + 2*(b
^2*d^2*f^2*x + b^2*c*d*f^2)*cosh(1) + 2*(b^2*d^2*f^2*x + b^2*c*d*f^2)*sinh(
1))*cosh(d*x + c) + 2*(a*b*d^2*f^2*x + a*b*c*d*f^2)*sinh(1))*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*
x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 6*(b^2*f^3*cosh(d
*x + c)^2 + b^2*f^3*sinh(d*x + c)^2 + 2*a*b*f^3*cosh(d*x + c) - b^2*f^3 + 2
*(b^2*f^3*cosh(d*x + c) + a*b*f^3)*sinh(d*x + c...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((f*x + e)^3*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,{\left(e+f\,x\right)}^3}{{\left(a+b\,\sinh\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^2,x)
[Out] int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^2, x)
```

3.324 $\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=74

$$-\frac{2f\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}-\frac{e+fx}{bd(a+b\sinh(c+dx))}$$

```
[Out] (-f*x-e)/b/d/(a+b*sinh(d*x+c))-2*f*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b
^2)^(1/2))/b/d^2/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5572, 2739, 632, 210}

$$-\frac{2f\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}}-\frac{e+fx}{bd(a+b\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] (-2*f*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b*Sqrt[a^2 + b^2
]*d^2) - (e + f*x)/(b*d*(a + b*Sinh[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 5572

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx &= -\frac{e+fx}{bd(a+b\sinh(c+dx))}+\frac{f\int \frac{1}{a+b\sinh(c+dx)}\,dx}{bd}\\ &= -\frac{e+fx}{bd(a+b\sinh(c+dx))}-\frac{(2if)\text{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{bd^2}\\ &= -\frac{e+fx}{bd(a+b\sinh(c+dx))}+\frac{(4if)\text{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx,x,-2ib+2a\tan\left(\frac{1}{2}(\right.\right.}{bd^2}\\ &= -\frac{2f\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}-\frac{e+fx}{bd(a+b\sinh(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.38, size = 78, normalized size = 1.05

$$\frac{\frac{2f\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-\frac{d(e+fx)}{a+b\sinh(c+dx)}}{bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] ((2*f*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
- (d*(e + f*x))/(a + b*Sinh[c + d*x]))/(b*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(71) = 142$.
time = 6.46, size = 164, normalized size = 2.22

method	result	size
risch	$-\frac{2(fx+e)e^{dx+c}}{bd\left(b\,e^{2dx+2c}+2a\,e^{dx+c}-b\right)}+\frac{f\ln\left(e^{dx+c}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d^2b}-\frac{f\ln\left(e^{dx+c}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d^2b}$	164

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(f*x+e)/b/d*exp(d*x+c)/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/(a^2+b^2)^(
1/2)*f/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(1/2)-a^2-b^2)/(a^2+b^2)^(1/2)/b)-1
/(a^2+b^2)^(1/2)*f/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(1/2)+a^2+b^2)/(a^2+b^2
)^(1/2)/b)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(72) = 144$.
time = 0.56, size = 157, normalized size = 2.12

$$-f\left(\frac{2\,xe^{(dx+c)}}{b^2de^{(2\,dx+2\,c)}+2\,abde^{(dx+c)}-b^2d}-\frac{\log\left(\frac{be^{(dx+c)}+a-\sqrt{a^2+b^2}}{be^{(dx+c)}+a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,bd^2}\right)-\frac{2\,e^{(-dx-c+1)}}{(2\,abe^{(-dx-c)}-b^2e^{(-2\,dx-2\,c)}+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -f*(2*x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b^2*d) -
 log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqrt(a^2 +
b^2)))/(sqrt(a^2 + b^2)*b*d^2)) - 2*e^(-d*x - c + 1)/((2*a*b*e^(-d*x - c) -
 b^2*e^(-2*d*x - 2*c) + b^2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(72) = 144$.
time = 0.39, size = 435, normalized size = 5.88

$$\frac{\left(bf\cosh\left(dx+c\right)^2+bf\sinh\left(dx+c\right)^2+2\,af\cosh\left(dx+c\right)-bf+2\,\left(bf\cosh\left(dx+c\right)+af\right)\sinh\left(dx+c\right)\right)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2+b^2+2\,(b^2\cosh(dx+c)+ab)\sinh(dx+c)-2\,\sqrt{a^2+b^2}\,(b\cosh(dx+c)+b\sinh(dx+c)+a)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)-b}\right)-2\,\left((a^2+b^2)dfx+(a^2+b^2)d\cosh\left(1\right)+(a^2+b^2)d\sinh\left(1\right)\right)\cosh\left(dx+c\right)-2\,\left((a^2+b^2)dfx+(a^2+b^2)d\cosh\left(1\right)+(a^2+b^2)d\sinh\left(1\right)\right)\sinh\left(dx+c\right)}{(a^2b^2+b^4)d^2\cosh\left(dx+c\right)^2+(a^2b^2+b^4)d^2\sinh\left(dx+c\right)^2+2\,(a^3b+ab^3)d^2\cosh\left(dx+c\right)-(a^2b^2+b^4)d^2+2\,\left((a^2b^2+b^4)d^2\cosh\left(dx+c\right)+(a^3b+ab^3)d^2\right)\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((b*f*cosh(d*x + c)^2 + b*f*sinh(d*x + c)^2 + 2*a*f*cosh(d*x + c) - b*f + 2
*(b*f*cosh(d*x + c) + a*f)*sinh(d*x + c))*sqrt(a^2 + b^2)*log((b^2*cosh(d*x
 + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*
cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b
*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x
+ c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*((a^2 + b^2)*d*f*x +
 (a^2 + b^2)*d*cosh(1) + (a^2 + b^2)*d*sinh(1))*cosh(d*x + c) - 2*((a^2 + b
^2)*d*f*x + (a^2 + b^2)*d*cosh(1) + (a^2 + b^2)*d*sinh(1))*sinh(d*x + c))/(
(a^2*b^2 + b^4)*d^2*cosh(d*x + c)^2 + (a^2*b^2 + b^4)*d^2*sinh(d*x + c)^2 +
```

```
 2*(a^3*b + a*b^3)*d^2*cosh(d*x + c) - (a^2*b^2 + b^4)*d^2 + 2*((a^2*b^2 +
b^4)*d^2*cosh(d*x + c) + (a^3*b + a*b^3)*d^2)*sinh(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

Mupad [B]
time = 0.00, size = 199, normalized size = 2.69

$$\frac{f\,\ln\left(\frac{2\,f\,\left(b-a\,{\mathrm{e}}^{c+d\,x}\right)}{b^2\,d\,\sqrt{a^2+b^2}}-\frac{2\,f\,{\mathrm{e}}^{c+d\,x}}{b^2\,d}\right)}{b\,d^2\,\sqrt{a^2+b^2}}-\frac{f\,\ln\left(-\frac{2\,f\,{\mathrm{e}}^{c+d\,x}}{b^2\,d}-\frac{2\,f\,\left(b-a\,{\mathrm{e}}^{c+d\,x}\right)}{b^2\,d\,\sqrt{a^2+b^2}}\right)}{b\,d^2\,\sqrt{a^2+b^2}}-\frac{2\,{\mathrm{e}}^{c+d\,x}\,\left(a^2\,e+b^2\,e+a^2\,f\,x+b^2\,f\,x\right)}{d\,\left(a^2\,b+b^3\right)\,\left(2\,a\,{\mathrm{e}}^{c+d\,x}-b+b\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x))^2,x)
[Out] (f*log((2*f*(b - a*exp(c + d*x)))/(b^2*d*(a^2 + b^2)^(1/2)) - (2*f*exp(c +
d*x))/(b^2*d)))/(b*d^2*(a^2 + b^2)^(1/2)) - (f*log(- (2*f*exp(c + d*x))/(b^
2*d) - (2*f*(b - a*exp(c + d*x)))/(b^2*d*(a^2 + b^2)^(1/2))))/(b*d^2*(a^2 +
 b^2)^(1/2)) - (2*exp(c + d*x)*(a^2*e + b^2*e + a^2*f*x + b^2*f*x))/(d*(a^2
*b + b^3)*(2*a*exp(c + d*x) - b + b*exp(2*c + 2*d*x)))
```

3.325 $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=234

$$\frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}-\frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}+\frac{2f^2\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^3}$$

```
[Out] -(f*x+e)^2/b/d/(a+b*sinh(d*x+c))+2*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)
^(1/2)))/b/d^2/(a^2+b^2)^(1/2)-2*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))/b/d^2/(a^2+b^2)^(1/2)+2*f^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/
2)))/b/d^3/(a^2+b^2)^(1/2)-2*f^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)
))/b/d^3/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5572, 3403, 2296, 2221, 2317, 2438}

$$\frac{2f^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{2f^2\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}+\frac{2f(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd^2\sqrt{a^2+b^2}}-\frac{2f(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd^2\sqrt{a^2+b^2}}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] (2*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*Sqrt[a^2
+ b^2]*d^2) - (2*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
)/(b*Sqrt[a^2 + b^2]*d^2) + (2*f^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (2*f^2*PolyLog[2, -((b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (e + f*x)^2/(b*d*(a + b
*Sinh[c + d*x]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5572

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = -\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))} + \frac{(2f)\int \frac{e+fx}{a+b\sinh(c+dx)}\,dx}{bd}$$

$$= -\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))} + \frac{(4f)\int \frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{bd}$$

$$= -\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))} + \frac{(4f)\int \frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\sqrt{a^2+b^2}\,d} - \frac{(4f)\int \frac{}{2a+2\sqrt{}}}{\sqrt{}}$$

$$= \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2} - \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}$$

$$= \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2} - \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}$$

$$= \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2} - \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}$$

Mathematica [A]
time = 0.64, size = 175, normalized size = 0.75

$$\frac{2f\left(d(e+fx)\left(\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)+f\mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)-f\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)}{b\sqrt{a^2+b^2}\,d^3} - \frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] (2*f*(d*(e + f*x)*(Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - Log[1 +
 (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]) + f*PolyLog[2, (b*E^(c + d*x))/(-a
 + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)]))/(b*Sqrt[a^2 + b^2]*d^3) - (e + f*x)^2/(b*d*(a + b*Sinh[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(214) = 428$.
time = 6.20, size = 491, normalized size = 2.10

method	result

risch	$-\frac{2(x^2f^2+2efx+e^2)e^{dx+c}}{bd(b\,e^{2dx+2c}+2a\,e^{dx+c}-b)}-\frac{4fe\,\mathrm{arctanh}\left(\frac{2b\,e^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{d^2b\sqrt{a^2+b^2}}+\frac{2f^2\ln\left(\frac{-b\,e^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)x}{d^2b\sqrt{a^2+b^2}}+\frac{2f^2\ln\left(\frac{-b\,e^{dx+c}+\sqrt{}}{-a+\sqrt{a}}\right.}{d^3b\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(f^2*x^2+2*e*f*x+e^2)/b/d*exp(d*x+c)/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)
-4*f/b/d^2*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/
2))+2/d^2/b*f^2/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a
^2+b^2)^(1/2)))*x+2/d^3/b*f^2/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(
1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/d^2/b*f^2/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c
)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-2*f^2/b/d^3/(a^2+b^2)^(1/2)*ln(
(b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2*f^2/b/d^3/(a^2+b^
2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2*f^
2/b/d^3/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)
^(1/2)))+4*f^2/b/d^3*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^
2+b^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -2*(x^2*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b^2*d) -
 2*integrate(x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b
^2*d), x))*f^2 - 2*f*(2*x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d
*x + c) - b^2*d) - log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c)
 + a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b*d^2))*e - 2*e^(-d*x - c + 2)/((
2*a*b*e^(-d*x - c) - b^2*e^(-2*d*x - 2*c) + b^2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1590 vs. $2(215) = 430$.
time = 0.37, size = 1590, normalized size = 6.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 2*((b^2*f^2*cosh(d*x + c)^2 + b^2*f^2*sinh(d*x + c)^2 + 2*a*b*f^2*cosh(d*x
+ c) - b^2*f^2 + 2*(b^2*f^2*cosh(d*x + c) + a*b*f^2)*sinh(d*x + c))*sqrt((a
```

```
^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^2*f^2*cosh(d*x +
c)^2 + b^2*f^2*sinh(d*x + c)^2 + 2*a*b*f^2*cosh(d*x + c) - b^2*f^2 + 2*(b^2
*f^2*cosh(d*x + c) + a*b*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a
*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b + 1) - (b^2*c*f^2 - b^2*d*f*cosh(1) - b^2*d*f*sinh
(1) - (b^2*c*f^2 - b^2*d*f*cosh(1) - b^2*d*f*sinh(1))*cosh(d*x + c)^2 - (b^
2*c*f^2 - b^2*d*f*cosh(1) - b^2*d*f*sinh(1))*sinh(d*x + c)^2 - 2*(a*b*c*f^2
 - a*b*d*f*cosh(1) - a*b*d*f*sinh(1))*cosh(d*x + c) - 2*(a*b*c*f^2 - a*b*d*
f*cosh(1) - a*b*d*f*sinh(1) + (b^2*c*f^2 - b^2*d*f*cosh(1) - b^2*d*f*sinh(1
))*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c
) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^2*c*f^2 - b^2
*d*f*cosh(1) - b^2*d*f*sinh(1) - (b^2*c*f^2 - b^2*d*f*cosh(1) - b^2*d*f*sin
h(1))*cosh(d*x + c)^2 - (b^2*c*f^2 - b^2*d*f*cosh(1) - b^2*d*f*sinh(1))*sin
h(d*x + c)^2 - 2*(a*b*c*f^2 - a*b*d*f*cosh(1) - a*b*d*f*sinh(1))*cosh(d*x +
 c) - 2*(a*b*c*f^2 - a*b*d*f*cosh(1) - a*b*d*f*sinh(1) + (b^2*c*f^2 - b^2*d
*f*cosh(1) - b^2*d*f*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2
)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2
) + 2*a) - (b^2*d*f^2*x + b^2*c*f^2 - (b^2*d*f^2*x + b^2*c*f^2)*cosh(d*x +
c)^2 - (b^2*d*f^2*x + b^2*c*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*f^2*x + a*b*c*f
^2)*cosh(d*x + c) - 2*(a*b*d*f^2*x + a*b*c*f^2 + (b^2*d*f^2*x + b^2*c*f^2)*
cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) +
 a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2
) - b)/b) + (b^2*d*f^2*x + b^2*c*f^2 - (b^2*d*f^2*x + b^2*c*f^2)*cosh(d*x +
 c)^2 - (b^2*d*f^2*x + b^2*c*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*f^2*x + a*b*c*
f^2)*cosh(d*x + c) - 2*(a*b*d*f^2*x + a*b*c*f^2 + (b^2*d*f^2*x + b^2*c*f^2)
*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c)
+ a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2) - b)/b) - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*f*x*cosh(1) + (a^
2 + b^2)*d^2*cosh(1)^2 + (a^2 + b^2)*d^2*sinh(1)^2 + 2*((a^2 + b^2)*d^2*f*x
 + (a^2 + b^2)*d^2*cosh(1))*sinh(1))*cosh(d*x + c) - ((a^2 + b^2)*d^2*f^2*x
^2 + 2*(a^2 + b^2)*d^2*f*x*cosh(1) + (a^2 + b^2)*d^2*cosh(1)^2 + (a^2 + b^2
)*d^2*sinh(1)^2 + 2*((a^2 + b^2)*d^2*f*x + (a^2 + b^2)*d^2*cosh(1))*sinh(1)
)*sinh(d*x + c))/((a^2*b^2 + b^4)*d^3*cosh(d*x + c)^2 + (a^2*b^2 + b^4)*d^3
*sinh(d*x + c)^2 + 2*(a^3*b + a*b^3)*d^3*cosh(d*x + c) - (a^2*b^2 + b^4)*d^
3 + 2*((a^2*b^2 + b^4)*d^3*cosh(d*x + c) + (a^3*b + a*b^3)*d^3)*sinh(d*x +
c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((f*x + e)^2*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)^2}{\left(a+b\sinh\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^2,x)
[Out] int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^2, x)
```

3.326 $\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=348

$$\frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}-\frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}+\frac{6f^2(e+fx)\text{PolyLog}\left(2,-\frac{\quad}{a-\sqrt{\quad}}\right.}{b\sqrt{a^2+b^2}\,d^3}$$

```
[Out] -(f*x+e)^3/b/d/(a+b*sinh(d*x+c))+3*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))/b/d^2/(a^2+b^2)^(1/2)-3*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/b/d^2/(a^2+b^2)^(1/2)+6*f^2*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(
a^2+b^2)^(1/2)))/b/d^3/(a^2+b^2)^(1/2)-6*f^2*(f*x+e)*polylog(2,-b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/b/d^3/(a^2+b^2)^(1/2)-6*f^3*polylog(3,-b*exp(d*x+c)/
(a-(a^2+b^2)^(1/2)))/b/d^4/(a^2+b^2)^(1/2)+6*f^3*polylog(3,-b*exp(d*x+c)/(a
+(a^2+b^2)^(1/2)))/b/d^4/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5572, 3403, 2296, 2221, 2611, 2320, 6724}

$$-\frac{6f^3\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4\sqrt{a^2+b^2}}+\frac{6f^3\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^4\sqrt{a^2+b^2}}+\frac{6f^2(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{6f^2(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}+\frac{3f(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd^2\sqrt{a^2+b^2}}-\frac{3f(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd^2\sqrt{a^2+b^2}}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] (3*f*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*Sqrt[a^
2 + b^2]*d^2) - (3*f*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(b*Sqrt[a^2 + b^2]*d^2) + (6*f^2*(e + f*x)*PolyLog[2, -((b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (6*f^2*(e + f*x)*Poly
Log[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) -
 (6*f^3*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 +
 b^2]*d^4) + (6*f^3*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
b*Sqrt[a^2 + b^2]*d^4) - (e + f*x)^3/(b*d*(a + b*Sinh[c + d*x]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5572

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = -\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))} + \frac{(3f)\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}\,dx}{bd}$$

$$= -\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))} + \frac{(6f)\int \frac{e^{c+dx}(e+fx)^2}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{bd}$$

$$= -\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))} + \frac{(6f)\int \frac{e^{c+dx}(e+fx)^2}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\sqrt{a^2+b^2}\,d} - \frac{(6f)\int \frac{e}{2a+2\sqrt{}}}{\sqrt{a}}$$

$$= \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2} - \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}$$

$$= \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2} - \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}$$

$$= \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2} - \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}$$

$$= \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2} - \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}\,d^2}$$

Mathematica [A]
time = 0.41, size = 634, normalized size = 1.82

$$\frac{3f\left(\frac{2d^2e^2\text{ArcTan}\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{2d^2ee^cfx\log\left(1+\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}}+\frac{d^2e^cf^2x^2\log\left(1+\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}}-\frac{2d^2ee^cfx\log\left(1+\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}}-\frac{d^2e^cf^2x^2\log\left(1+\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}}+\frac{2de^cf(e+fx)\text{PolyLog}\left(2,-\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}}-\frac{2de^cf(e+fx)\text{PolyLog}\left(2,-\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}}-\frac{2e^cf^2\text{PolyLog}\left(3,-\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}}+\frac{2e^cf^2\text{PolyLog}\left(3,-\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)}{\sqrt{(a^2+b^2)e^{2c}}}\right)}{bd^4}-\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]

[Out] (3*f*((2*d^2*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 -
b^2] + (2*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)
*E^(2*c)])]])/Sqrt[(a^2 + b^2)*E^(2*c)] + (d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c
 + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])]])/Sqrt[(a^2 + b^2)*E^(2*c)] -
(2*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c
)])]])/Sqrt[(a^2 + b^2)*E^(2*c)] - (d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x)
)/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])]])/Sqrt[(a^2 + b^2)*E^(2*c)] + (2*d*E^
c*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)]))]])/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*d*E^c*f*(e + f*x)*PolyLog[2, -((b*E
```

```
^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])))])/Sqrt[(a^2 + b^2)*E^(2*
c)] - (2*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E
^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] + (2*E^c*f^2*PolyLog[3, -((b*E^(2*c +
 d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)]))/(
b*d^4) - (e + f*x)^3/(b*d*(a + b*Sinh[c + d*x]))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \cosh(dx+c)}{(a+b\sinh(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -3*f*(2*x*e^(d*x + c)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b^2*d)
 - log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqrt(a^2
+ b^2)))/(sqrt(a^2 + b^2)*b*d^2))*e^2 - 2*(f^3*x^3*e^c + 3*f^2*x^2*e^(c + 1
))*e^(d*x)/(b^2*d*e^(2*d*x + 2*c) + 2*a*b*d*e^(d*x + c) - b^2*d) - 2*e^(-d*
x - c + 3)/((2*a*b*e^(-d*x - c) - b^2*e^(-2*d*x - 2*c) + b^2)*d) + integrat
e(6*(f^3*x^2*e^c + 2*f^2*x*e^(c + 1))*e^(d*x)/(b^2*d*e^(2*d*x + 2*c) + 2*a*
b*d*e^(d*x + c) - b^2*d), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3516 vs. $2(321) = 642$.
time = 0.41, size = 3516, normalized size = 10.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(6*(b^2*d*f^3*x + b^2*d*f^2*cosh(1) + b^2*d*f^2*sinh(1) - (b^2*d*f^3*x + b
^2*d*f^2*cosh(1) + b^2*d*f^2*sinh(1))*cosh(d*x + c)^2 - (b^2*d*f^3*x + b^2*
d*f^2*cosh(1) + b^2*d*f^2*sinh(1))*sinh(d*x + c)^2 - 2*(a*b*d*f^3*x + a*b*d
*f^2*cosh(1) + a*b*d*f^2*sinh(1))*cosh(d*x + c) - 2*(a*b*d*f^3*x + a*b*d*f^
```

2*cosh(1) + a*b*d*f^2*sinh(1) + (b^2*d*f^3*x + b^2*d*f^2*cosh(1) + b^2*d*f^
2*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2) - b)/b + 1) - 6*(b^2*d*f^3*x + b^2*d*f^2*cosh(1) + b^2*d*f^2*
sinh(1) - (b^2*d*f^3*x + b^2*d*f^2*cosh(1) + b^2*d*f^2*sinh(1))*cosh(d*x +
c)^2 - (b^2*d*f^3*x + b^2*d*f^2*cosh(1) + b^2*d*f^2*sinh(1))*sinh(d*x + c)^
2 - 2*(a*b*d*f^3*x + a*b*d*f^2*cosh(1) + a*b*d*f^2*sinh(1))*cosh(d*x + c) -
 2*(a*b*d*f^3*x + a*b*d*f^2*cosh(1) + a*b*d*f^2*sinh(1) + (b^2*d*f^3*x + b^
2*d*f^2*cosh(1) + b^2*d*f^2*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(b^2*c^2*f^3 - 2*b^
2*c*d*f^2*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 - (b^2*c^2*f^
3 - 2*b^2*c*d*f^2*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 - 2*(
b^2*c*d*f^2 - b^2*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 - (b^2*c^2*f^3 -
2*b^2*c*d*f^2*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 - 2*(b^2*
c*d*f^2 - b^2*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 2*(a*b*c^2*f^3 - 2*
a*b*c*d*f^2*cosh(1) + a*b*d^2*f*cosh(1)^2 + a*b*d^2*f*sinh(1)^2 - 2*(a*b*c*
d*f^2 - a*b*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c) - 2*(b^2*c*d*f^2 - b^2*d^
2*f*cosh(1))*sinh(1) - 2*(a*b*c^2*f^3 - 2*a*b*c*d*f^2*cosh(1) + a*b*d^2*f*c
osh(1)^2 + a*b*d^2*f*sinh(1)^2 + (b^2*c^2*f^3 - 2*b^2*c*d*f^2*cosh(1) + b^2
*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 - 2*(b^2*c*d*f^2 - b^2*d^2*f*cosh(1)
)*sinh(1))*cosh(d*x + c) - 2*(a*b*c*d*f^2 - a*b*d^2*f*cosh(1))*sinh(1))*sin
h(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c)
 + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 3*(b^2*c^2*f^3 - 2*b^2*c*d*f^2*cosh(1
) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 - (b^2*c^2*f^3 - 2*b^2*c*d*f^
2*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 - 2*(b^2*c*d*f^2 - b^
2*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 - (b^2*c^2*f^3 - 2*b^2*c*d*f^2*co
sh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 - 2*(b^2*c*d*f^2 - b^2*d^
2*f*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 2*(a*b*c^2*f^3 - 2*a*b*c*d*f^2*cosh
(1) + a*b*d^2*f*cosh(1)^2 + a*b*d^2*f*sinh(1)^2 - 2*(a*b*c*d*f^2 - a*b*d^2*
f*cosh(1))*sinh(1))*cosh(d*x + c) - 2*(b^2*c*d*f^2 - b^2*d^2*f*cosh(1))*sin
h(1) - 2*(a*b*c^2*f^3 - 2*a*b*c*d*f^2*cosh(1) + a*b*d^2*f*cosh(1)^2 + a*b*d
^2*f*sinh(1)^2 + (b^2*c^2*f^3 - 2*b^2*c*d*f^2*cosh(1) + b^2*d^2*f*cosh(1)^2
 + b^2*d^2*f*sinh(1)^2 - 2*(b^2*c*d*f^2 - b^2*d^2*f*cosh(1))*sinh(1))*cosh(
d*x + c) - 2*(a*b*c*d*f^2 - a*b*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2
 + b^2)/b^2) + 2*a) + 3*(b^2*d^2*f^3*x^2 - b^2*c^2*f^3 - (b^2*d^2*f^3*x^2 -
 b^2*c^2*f^3 + 2*(b^2*d^2*f^2*x + b^2*c*d*f^2)*cosh(1) + 2*(b^2*d^2*f^2*x +
 b^2*c*d*f^2)*sinh(1))*cosh(d*x + c)^2 - (b^2*d^2*f^3*x^2 - b^2*c^2*f^3 + 2
*(b^2*d^2*f^2*x + b^2*c*d*f^2)*cosh(1) + 2*(b^2*d^2*f^2*x + b^2*c*d*f^2)*si
nh(1))*sinh(d*x + c)^2 + 2*(b^2*d^2*f^2*x + b^2*c*d*f^2)*cosh(1) - 2*(a*b*d
^2*f^3*x^2 - a*b*c^2*f^3 + 2*(a*b*d^2*f^2*x + a*b*c*d*f^2)*cosh(1) + 2*(a*b
*d^2*f^2*x + a*b*c*d*f^2)*sinh(1))*cosh(d*x + c) + 2*(b^2*d^2*f^2*x + b^2*c
*d*f^2)*sinh(1) - 2*(a*b*d^2*f^3*x^2 - a*b*c^2*f^3 + 2*(a*b*d^2*f^2*x + a*b
*c*d*f^2)*cosh(1) + (b^2*d^2*f^3*x^2 - b^2*c^2*f^3 + 2*(b^2*d^2*f^2*x + b^2

```
*c*d*f^2)*cosh(1) + 2*(b^2*d^2*f^2*x + b^2*c*d*f^2)*sinh(1))*cosh(d*x + c)
+ 2*(a*b*d^2*f^2*x + a*b*c*d*f^2)*sinh(1))*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 3*(b^2*d^2*f^3*x^2 - b^2*c^2*f^3 -
(b^2*d^2*f^3*x^2 - b^2*c^2*f^3 + 2*(b^2*d^2*f^2*x + b^2*c*d*f^2)*cosh(1) +
2*(b^2*d^2*f^2*x + b^2*c*d*f^2)*sinh(1))*cosh(d*x + c)^2 - (b^2*d^2*f^3*x^2
 - b^2*c^2*f^3 + 2*(b^2*d^2*f^2*x + b^2*c*d*f^2)*cosh(1) + 2*(b^2*d^2*f^2*x
 + b^2*c*d*f^2)*sinh(1))*sinh(d*x + c)^2 + 2*(b^2*d^2*f^2*x + b^2*c*d*f^2)*
cosh(1) - 2*(a*b*d^2*f^3*x^2 - a*b*c^2*f^3 + 2*(a*b*d^2*f^2*x + a*b*c*d*f^2
)*cosh(1) + 2*(a*b*d^2*f^2*x + a*b*c*d*f^2)*sinh(1))*cosh(d*x + c) + 2*(b^2
*d^2*f^2*x + b^2*c*d*f^2)*sinh(1) - 2*(a*b*d^2*f^3*x^2 - a*b*c^2*f^3 + 2*(a
*b*d^2*f^2*x + a*b*c*d*f^2)*cosh(1) + (b^2*d^2*f^3*x^2 - b^2*c^2*f^3 + 2*(b
^2*d^2*f^2*x + b^2*c*d*f^2)*cosh(1) + 2*(b^2*d^2*f^2*x + b^2*c*d*f^2)*sinh(
1))*cosh(d*x + c) + 2*(a*b*d^2*f^2*x + a*b*c*d*f^2)*sinh(1))*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*
x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 6*(b^2*f^3*cosh(d
*x + c)^2 + b^2*f^3*sinh(d*x + c)^2 + 2*a*b*f^3*cosh(d*x + c) - b^2*f^3 + 2
*(b^2*f^3*cosh(d*x + c) + a*b*f^3)*sinh(d*x + c...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((f*x + e)^3*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)^3}{\left(a+b\,\sinh\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^2,x)
[Out] int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^2, x)
```

3.327 $\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=112

$$-\frac{af\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d^2}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}-\frac{f\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))}$$

```
[Out] -a*f*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/b/(a^2+b^2)^(3/2)/d
^2+1/2*(-f*x-e)/b/d/(a+b*sinh(d*x+c))^2-1/2*f*cosh(d*x+c)/(a^2+b^2)/d^2/(a+
b*sinh(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5572, 2743, 12, 2739, 632, 210}

$$-\frac{af\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd^2\left(a^2+b^2\right)^{3/2}}-\frac{f\cosh(c+dx)}{2d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] -((a*f*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b*(a^2 + b^2)^(
3/2)*d^2)) - (e + f*x)/(2*b*d*(a + b*Sinh[c + d*x])^2) - (f*Cosh[c + d*x])/
(2*(a^2 + b^2)*d^2*(a + b*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 5572

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = -\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}+\frac{f\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx}{2bd}$$

$$= -\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}-\frac{f\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))}+\frac{f\int \frac{a}{a+b\sinh(c}$$

$$= -\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}-\frac{f\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))}+\frac{(af)\int \frac{}{a+b\sinh}}{2b\left(a^2-\right.}$$

$$= -\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}-\frac{f\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))}-\frac{(iaf)\text{Subst}}{}$$

$$= -\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}-\frac{f\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))}+\frac{(2iaf)\text{Subs}}{}$$

$$= -\frac{af\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d^2}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}-\frac{f\cosh}{2\left(a^2+b^2\right)d^2(a}$$

Mathematica [A]

time = 0.91, size = 112, normalized size = 1.00

$$-\frac{\frac{f\cosh(c+dx)}{(a^2+b^2)(a+b\sinh(c+dx))}+\frac{\frac{2af\operatorname{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}}+\frac{d(e+fx)}{(a+b\sinh(c+dx))^2}}{b}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] -1/2*((f*Cosh[c + d*x])/((a^2 + b^2)*(a + b*Sinh[c + d*x])) + ((2*a*f*ArcTa
n[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) + (d*(e +
 f*x))/(a + b*Sinh[c + d*x])^2)/b)/d^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(106) = 212$.
time = 6.91, size = 308, normalized size = 2.75

method	result
risch	$-\frac{2a^2dfx\,e^{2dx+2c}+2b^2dfx\,e^{2dx+2c}+2a^2de\,e^{2dx+2c}-abf\,e^{3dx+3c}+2b^2de\,e^{2dx+2c}-2a^2f\,e^{2dx+2c}+b^2f\,e^{2dx+2c}+3fa\,e^{dx+c}b-f\,b^2}{b\,d^2\left(b\,e^{2dx+2c}+2a\,e^{dx+c}-b\right)^2(a^2+b^2)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(2*a^2*d*f*x*exp(2*d*x+2*c)+2*b^2*d*f*x*exp(2*d*x+2*c)+2*a^2*d*e*exp(2
*d*x+2*c)-a*b*f*exp(3*d*x+3*c)+2*b^2*d*e*exp(2*d*x+2*c)-2*a^2*f*exp(2*d*x+2
*c)+b^2*f*exp(2*d*x+2*c)+3*f*a*exp(d*x+c)*b-f*b^2)/d^2/(b*exp(2*d*x+2*c)+2*
a*exp(d*x+c)-b)^2/(a^2+b^2)+1/2/(a^2+b^2)^(3/2)*f*a/d^2/b*ln(exp(d*x+c)+(a*
(a^2+b^2)^(3/2)-a^4-2*a^2*b^2-b^4)/b/(a^2+b^2)^(3/2))-1/2/(a^2+b^2)^(3/2)*f
*a/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(3/2)+a^4+2*a^2*b^2+b^4)/b/(a^2+b^2)^(3
/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(106) = 212$.
time = 0.59, size = 413, normalized size = 3.69

$$\frac{1}{2}f\left(\frac{2\left(abe^{(3dx+3c)}-3abe^{(dx+c)}+b^2+\left(2a^2e^{(2c)}-b^2e^{(2c)}-2\left(a^2de^{(2c)}+b^2de^{(2c)}\right)x\right)e^{(2dx)}\right)}{a^2b^3d^2+b^5d^2+(a^2b^3d^2e^{(4c)}+b^5d^2e^{(4c)})e^{(4dx)}+4(a^3b^2d^2e^{(3c)}+ab^4d^2e^{(3c)})e^{(3dx)}+2(2a^4bd^2e^{(2c)}+a^2b^3d^2e^{(2c)}-b^5d^2e^{(2c)})e^{(2dx)}-4(a^3b^2d^2e^c+ab^4d^2e^c)e^{(dx)}}+\frac{a\log\left(\frac{be^{(dx+2c)}+ae^c-\sqrt{a^2+b^2}\,e^c}{be^{(dx+2c)}+ae^c+\sqrt{a^2+b^2}\,e^c}\right)}{(a^2b+b^3)\sqrt{a^2+b^2}\,d^2}\right)-\frac{2e^{(-2dx-2c+1)}}{(4ab^2e^{(-dx-c)}-4ab^2e^{(-3dx-3c)}+b^3e^{(-4dx-4c)}+b^3+2(2a^2b-b^3)e^{(-2dx-2c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*f*(2*(a*b*e^(3*d*x + 3*c) - 3*a*b*e^(d*x + c) + b^2 + (2*a^2*e^(2*c) -
b^2*e^(2*c) - 2*(a^2*d*e^(2*c) + b^2*d*e^(2*c))*x)*e^(2*d*x))/(a^2*b^3*d^2
```

```
+ b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5*d^2*e^(4*c))*e^(4*d*x) + 4*(a^3*b^2*
d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3*d*x) + 2*(2*a^4*b*d^2*e^(2*c) + a^2*b
^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^(2*d*x) - 4*(a^3*b^2*d^2*e^c + a*b^4*d^
2*e^c)*e^(d*x)) + a*log((b*e^(d*x + 2*c) + a*e^c - sqrt(a^2 + b^2)*e^c)/(b*
e^(d*x + 2*c) + a*e^c + sqrt(a^2 + b^2)*e^c))/((a^2*b + b^3)*sqrt(a^2 + b^2
)*d^2)) - 2*e^(-2*d*x - 2*c + 1)/((4*a*b^2*e^(-d*x - c) - 4*a*b^2*e^(-3*d*x
 - 3*c) + b^3*e^(-4*d*x - 4*c) + b^3 + 2*(2*a^2*b - b^3)*e^(-2*d*x - 2*c))*
d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1293 vs. $2(106) = 212$.

time = 0.38, size = 1293, normalized size = 11.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a^3*b + a*b^3)*f*cosh(d*x + c)^3 + 2*(a^3*b + a*b^3)*f*sinh(d*x + c
)^3 - 6*(a^3*b + a*b^3)*f*cosh(d*x + c) - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*f*
x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(1) + 2*(a^4 + 2*a^2*b^2 + b^4)*d*sinh(
1) - (2*a^4 + a^2*b^2 - b^4)*f)*cosh(d*x + c)^2 - 2*(2*(a^4 + 2*a^2*b^2 + b
^4)*d*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(1) - 3*(a^3*b + a*b^3)*f*cosh(
d*x + c) + 2*(a^4 + 2*a^2*b^2 + b^4)*d*sinh(1) - (2*a^4 + a^2*b^2 - b^4)*f)
*sinh(d*x + c)^2 + (a*b^2*f*cosh(d*x + c)^4 + a*b^2*f*sinh(d*x + c)^4 + 4*a
^2*b*f*cosh(d*x + c)^3 - 4*a^2*b*f*cosh(d*x + c) + a*b^2*f + 2*(2*a^3 - a*b
^2)*f*cosh(d*x + c)^2 + 4*(a*b^2*f*cosh(d*x + c) + a^2*b*f)*sinh(d*x + c)^3
 + 2*(3*a*b^2*f*cosh(d*x + c)^2 + 6*a^2*b*f*cosh(d*x + c) + (2*a^3 - a*b^2)
*f)*sinh(d*x + c)^2 + 4*(a*b^2*f*cosh(d*x + c)^3 + 3*a^2*b*f*cosh(d*x + c)^
2 - a^2*b*f + (2*a^3 - a*b^2)*f*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^
2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2
*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*
(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x +
c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*
(a^2*b^2 + b^4)*f + 2*(3*(a^3*b + a*b^3)*f*cosh(d*x + c)^2 - 3*(a^3*b + a*b
^3)*f - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*co
sh(1) + 2*(a^4 + 2*a^2*b^2 + b^4)*d*sinh(1) - (2*a^4 + a^2*b^2 - b^4)*f)*co
sh(d*x + c))*sinh(d*x + c))/((a^4*b^3 + 2*a^2*b^5 + b^7)*d^2*cosh(d*x + c)^
4 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^2*sinh(d*x + c)^4 + 4*(a^5*b^2 + 2*a^3*b^
4 + a*b^6)*d^2*cosh(d*x + c)^3 + 2*(2*a^6*b + 3*a^4*b^3 - b^7)*d^2*cosh(d*x
 + c)^2 - 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2*cosh(d*x + c) + 4*((a^4*b^3 +
 2*a^2*b^5 + b^7)*d^2*cosh(d*x + c) + (a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2)*si
nh(d*x + c)^3 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^2 + 2*(3*(a^4*b^3 + 2*a^2*b^5
 + b^7)*d^2*cosh(d*x + c)^2 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2*cosh(d*x
+ c) + (2*a^6*b + 3*a^4*b^3 - b^7)*d^2)*sinh(d*x + c)^2 + 4*((a^4*b^3 + 2*a
```

```
^2*b^5 + b^7)*d^2*cosh(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2*cos
h(d*x + c)^2 + (2*a^6*b + 3*a^4*b^3 - b^7)*d^2*cosh(d*x + c) - (a^5*b^2 + 2
*a^3*b^4 + a*b^6)*d^2)*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)}{{\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x))^3,x)
[Out] int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x))^3, x)
```

3.328 $\int \frac{(e+fx)^2 \cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=306

$$\frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d^2}-\frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d^2}+\frac{f^2\log(a+b\sinh(c+dx))}{b\left(a^2+b^2\right)d^3}+\frac{af^2\mathrm{P}}{}$$

```
[Out] f^2*ln(a+b*sinh(d*x+c))/b/(a^2+b^2)/d^3+a*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a
^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2-a*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2+a*f^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b
^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3-a*f^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2
)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3-1/2*(f*x+e)^2/b/d/(a+b*sinh(d*x+c))^2-f*(f*
x+e)*cosh(d*x+c)/(a^2+b^2)/d^2/(a+b*sinh(d*x+c))
```

Rubi [A]
time = 0.37, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5572, 3405, 3403, 2296, 2221, 2317, 2438, 2747, 31}

$$\frac{af^2\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}-\frac{af^2\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}+\frac{f^2\log(a+b\sinh(c+dx))}{bd^3\left(a^2+b^2\right)}+\frac{af(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd^2\left(a^2+b^2\right)^{3/2}}-\frac{af(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd^2\left(a^2+b^2\right)^{3/2}}-\frac{f(e+fx)\cosh(c+dx)}{d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] (a*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^
2)^(3/2)*d^2) - (a*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)])/(b*(a^2 + b^2)^(3/2)*d^2) + (f^2*Log[a + b*Sinh[c + d*x]])/(b*(a^2 + b^
2)*d^3) + (a*f^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(
a^2 + b^2)^(3/2)*d^3) - (a*f^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) - (e + f*x)^2/(2*b*d*(a + b*Sinh[c + d*
x])^2) - (f*(e + f*x)*Cosh[c + d*x])/((a^2 + b^2)*d^2*(a + b*Sinh[c + d*x])
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5572

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = -\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2} + \frac{f\int \frac{e+fx}{(a+b\sinh(c+dx))^2}\,dx}{bd}$$

$$= -\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2} - \frac{f(e+fx)\cosh(c+dx)}{(a^2+b^2)\,d^2(a+b\sinh(c+dx))} + \frac{(af)\int \frac{e}{a+b\sin}}{b\,(a^2+} \cdots$$

$$= -\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2} - \frac{f(e+fx)\cosh(c+dx)}{(a^2+b^2)\,d^2(a+b\sinh(c+dx))} + \frac{(2af)\int \frac{}{-b+}}{b\,(a} \cdots$$

$$= \frac{f^2\log(a+b\sinh(c+dx))}{b\,(a^2+b^2)\,d^3} - \frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2} - \frac{f(e+fx)\cosh(}{(a^2+b^2)\,d^2(a+b\,\mathrm{si}} \cdots$$

$$= \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^2} - \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^2} + \cdots$$

$$= \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^2} - \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^2} + \cdots$$

$$= \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^2} - \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^2} + \cdots$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 623 vs. $2(306) = 612$.

time = 13.02, size = 623, normalized size = 2.04

$$\frac{f^2x\coth(c)}{b(a^2+b^2)d^2} + \frac{2e^cf\left(-e^cfx+e^{-c}(-1+e^{2c})fx - \frac{aee^{-c}(-1+e^{2c})\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{ae^{-c}(-1+e^{2c})f\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d} + \frac{1}{2}e^{-c}(-1+e^{2c})f\left(-2x+\frac{2a\,\mathrm{ArcTan}\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,d} + \frac{\log(2ae^{c+dx}+b(-1+e^{2(c+dx)}))}{d}\right) + \frac{a(-1+e^{2c})f\left(dx\left(\log\left(1+\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)-\log\left(1+\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)\right)+\mathrm{PolyLog}\left(2,-\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)-\mathrm{PolyLog}\left(2,-\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)\right)}{2d\sqrt{(a^2+b^2)e^{2c}}}\right)}{b(a^2+b^2)d^2(-1+e^{2c})} - \frac{f^2x\cosh(c)\mathrm{csch}\left(\frac{c}{2}\right)\mathrm{sech}\left(\frac{c}{2}\right)}{2b(a^2+b^2)d^2} - \frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2} + \frac{\mathrm{csch}\left(\frac{c}{2}\right)\mathrm{sech}\left(\frac{c}{2}\right)(aef\cosh(c)+af^2x\cosh(c)+bef\sinh(dx)+bf^2x\sinh(dx))}{2b(a^2+b^2)d^2(a+b\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] (f^2*x*Coth[c])/(b*(a^2 + b^2)*d^2) + (2*E^c*f*(-(E^c*f*x) + ((-1 + E^(2*c)
)*f*x)/E^c - (a*e*(-1 + E^(2*c))*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2
```

```
]])/(Sqrt[a^2 + b^2]*E^c) + (a*(-1 + E^(2*c))*f*ArcTanh[(a + b*E^(c + d*x))
/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d*E^c) + ((-1 + E^(2*c))*f*(-2*x + (2*a
*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-a^2 - b^2]*d) + Log[2
*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))]/d))/(2*E^c) + (a*(-1 + E^(2*c))*
f*(d*x*(Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - Lo
g[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])]) + PolyLog[2,
-((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - PolyLog[2, -((b
*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(2*d*Sqrt[(a^2 + b^
2)*E^(2*c)])))/(b*(a^2 + b^2)*d^2*(-1 + E^(2*c))) - (f^2*x*Cosh[c]*Csch[c/2
]*Sech[c/2])/(2*b*(a^2 + b^2)*d^2) - (e + f*x)^2/(2*b*d*(a + b*Sinh[c + d*x
])^2) + (Csch[c/2]*Sech[c/2]*(a*e*f*Cosh[c] + a*f^2*x*Cosh[c] + b*e*f*Sinh[
d*x] + b*f^2*x*Sinh[d*x]))/(2*b*(a^2 + b^2)*d^2*(a + b*Sinh[c + d*x]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 804 vs. $2(284) = 568$.
time = 6.94, size = 805, normalized size = 2.63

method	result
risch	$-\frac{2\left(a^{2} d\,f^{2} x^{2} {\mathrm e}^{2 d x +2 c}+b^{2} d\,f^{2} x^{2} {\mathrm e}^{2 d x +2 c}+2 a^{2} d e f x \,{\mathrm e}^{2 d x +2 c}-a b\,f^{2} x \,{\mathrm e}^{3 d x +3 c}+2 b^{2} d e f x \,{\mathrm e}^{2 d x +2 c}+a^{2} d\,e^{2} {\mathrm e}^{2 d x +2 c}-2 a^{2} f^{2} x \,{\mathrm e}^{2 d x +2 c}-a b}{b\,d^{2} \left(a^{2}+b^{2}\right) \left(b\,{\mathrm e}^{2 d x +2 c}+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b*(a^2*d*f^2*x^2*exp(2*d*x+2*c)+b^2*d*f^2*x^2*exp(2*d*x+2*c)+2*a^2*d*e*f
*x*exp(2*d*x+2*c)-a*b*f^2*x*exp(3*d*x+3*c)+2*b^2*d*e*f*x*exp(2*d*x+2*c)+a^2
*d*e^2*exp(2*d*x+2*c)-2*a^2*f^2*x*exp(2*d*x+2*c)-a*b*e*f*exp(3*d*x+3*c)+b^2
*d*e^2*exp(2*d*x+2*c)+b^2*f^2*x*exp(2*d*x+2*c)-2*a^2*e*f*exp(2*d*x+2*c)+3*a
*b*f^2*x*exp(d*x+c)+b^2*e*f*exp(2*d*x+2*c)+3*a*b*e*f*exp(d*x+c)-b^2*f^2*x-b
^2*e*f)/d^2/(a^2+b^2)/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)^2-2/(a^2+b^2)/d^3
/b*f^2*ln(exp(d*x+c))+1/(a^2+b^2)/d^3/b*f^2*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x
+c)-b)-2/(a^2+b^2)^(3/2)/d^2/b*f*a*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+
b^2)^(1/2))+1/(a^2+b^2)^(3/2)/d^2/b*f^2*a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))*x+1/(a^2+b^2)^(3/2)/d^3/b*f^2*a*ln((-b*exp(d*x+c)
+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/(a^2+b^2)^(3/2)/d^2/b*f^2*a*l
n((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/(a^2+b^2)^(3/2)
/d^3/b*f^2*a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/(
a^2+b^2)^(3/2)/d^3/b*f^2*a*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2
+b^2)^(1/2)))-1/(a^2+b^2)^(3/2)/d^3/b*f^2*a*dilog((b*exp(d*x+c)+(a^2+b^2)^(
1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/(a^2+b^2)^(3/2)/d^3/b*f^2*a*c*arctanh(1/2*(2
*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (2*a*d*integrate(x*e^(d*x + c)/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*
d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a^2*b^2*d^
2 - b^4*d^2), x) + b*(a*log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x
 + c) + a + sqrt(a^2 + b^2)))/((a^2*b^2 + b^4)*sqrt(a^2 + b^2)*d^3) - 2*(d*
x + c)/((a^2*b^2 + b^4)*d^3) + log(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - b)
/((a^2*b^2 + b^4)*d^3)) + 2*(a*b*x*e^(3*d*x + 3*c) - 3*a*b*x*e^(d*x + c) +
b^2*x - ((a^2*d*e^(2*c) + b^2*d*e^(2*c))*x^2 - (2*a^2*e^(2*c) - b^2*e^(2*c)
)*x)*e^(2*d*x))/(a^2*b^3*d^2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5*d^2*e^(
4*c))*e^(4*d*x) + 4*(a^3*b^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3*d*x) + 2
*(2*a^4*b*d^2*e^(2*c) + a^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^(2*d*x) -
4*(a^3*b^2*d^2*e^c + a*b^4*d^2*e^c)*e^(d*x)) - a*log((b*e^(d*x + c) + a - s
qrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqrt(a^2 + b^2)))/((a^2*b + b^3)*sqrt(
a^2 + b^2)*d^3))*f^2 + f*(2*(a*b*e^(3*d*x + 3*c) - 3*a*b*e^(d*x + c) + b^2
+ (2*a^2*e^(2*c) - b^2*e^(2*c) - 2*(a^2*d*e^(2*c) + b^2*d*e^(2*c))*x)*e^(2*
d*x))/(a^2*b^3*d^2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5*d^2*e^(4*c))*e^(4
*d*x) + 4*(a^3*b^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3*d*x) + 2*(2*a^4*b*
d^2*e^(2*c) + a^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^(2*d*x) - 4*(a^3*b^2
*d^2*e^c + a*b^4*d^2*e^c)*e^(d*x)) + a*log((b*e^(d*x + 2*c) + a*e^c - sqrt(
a^2 + b^2)*e^c)/(b*e^(d*x + 2*c) + a*e^c + sqrt(a^2 + b^2)*e^c))/((a^2*b +
b^3)*sqrt(a^2 + b^2)*d^2))*e - 2*e^(-2*d*x - 2*c + 2)/((4*a*b^2*e^(-d*x - c
) - 4*a*b^2*e^(-3*d*x - 3*c) + b^3*e^(-4*d*x - 4*c) + b^3 + 2*(2*a^2*b - b^
3)*e^(-2*d*x - 2*c))*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6052 vs. $2(286) = 572$.
time = 0.47, size = 6052, normalized size = 19.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(2*((a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*c*f^2)*cosh(d*x + c)^4 + 2*(
(a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*c*f^2)*sinh(d*x + c)^4 + 2*(a^2*b
^2 + b^4)*c*f^2 - 2*(a^2*b^2 + b^4)*d*f*cosh(1) + 2*(3*(a^3*b + a*b^3)*d*f^
2*x + 4*(a^3*b + a*b^3)*c*f^2 - (a^3*b + a*b^3)*d*f*cosh(1) - (a^3*b + a*b^
3)*d*f*sinh(1))*cosh(d*x + c)^3 - 2*(a^2*b^2 + b^4)*d*f*sinh(1) + 2*(3*(a^3
*b + a*b^3)*d*f^2*x + 4*(a^3*b + a*b^3)*c*f^2 - (a^3*b + a*b^3)*d*f*cosh(1)
 - (a^3*b + a*b^3)*d*f*sinh(1) + 4*((a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^
4)*c*f^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*f
^2*x^2 + (2*a^4 + a^2*b^2 - b^4)*d*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d^2*cosh
(1)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2*sinh(1)^2 + 2*(2*a^4 + a^2*b^2 - b^4)*c
```

f^2 + (2(a^4 + 2*a^2*b^2 + b^4)*d^2*f*x - (2*a^4 + a^2*b^2 - b^4)*d*f)*co
sh(1) + (2*(a^4 + 2*a^2*b^2 + b^4)*d^2*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*
cosh(1) - (2*a^4 + a^2*b^2 - b^4)*d*f)*sinh(1))*cosh(d*x + c)^2 + 2*((a^4 +
 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + (2*a^4 + a^2*b^2 - b^4)*d*f^2*x + (a^4 + 2*
a^2*b^2 + b^4)*d^2*cosh(1)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2*sinh(1)^2 + 2*(2
*a^4 + a^2*b^2 - b^4)*c*f^2 + 6*((a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*
c*f^2)*cosh(d*x + c)^2 + (2*(a^4 + 2*a^2*b^2 + b^4)*d^2*f*x - (2*a^4 + a^2*
b^2 - b^4)*d*f)*cosh(1) + 3*(3*(a^3*b + a*b^3)*d*f^2*x + 4*(a^3*b + a*b^3)*
c*f^2 - (a^3*b + a*b^3)*d*f*cosh(1) - (a^3*b + a*b^3)*d*f*sinh(1))*cosh(d*x
 + c) + (2*(a^4 + 2*a^2*b^2 + b^4)*d^2*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*
cosh(1) - (2*a^4 + a^2*b^2 - b^4)*d*f)*sinh(1))*sinh(d*x + c)^2 - (a*b^3*f^
2*cosh(d*x + c)^4 + a*b^3*f^2*sinh(d*x + c)^4 + 4*a^2*b^2*f^2*cosh(d*x + c)
^3 - 4*a^2*b^2*f^2*cosh(d*x + c) + a*b^3*f^2 + 2*(2*a^3*b - a*b^3)*f^2*cosh
(d*x + c)^2 + 4*(a*b^3*f^2*cosh(d*x + c) + a^2*b^2*f^2)*sinh(d*x + c)^3 + 2
*(3*a*b^3*f^2*cosh(d*x + c)^2 + 6*a^2*b^2*f^2*cosh(d*x + c) + (2*a^3*b - a*
b^3)*f^2)*sinh(d*x + c)^2 + 4*(a*b^3*f^2*cosh(d*x + c)^3 + 3*a^2*b^2*f^2*co
sh(d*x + c)^2 - a^2*b^2*f^2 + (2*a^3*b - a*b^3)*f^2*cosh(d*x + c))*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (a*b^3
*f^2*cosh(d*x + c)^4 + a*b^3*f^2*sinh(d*x + c)^4 + 4*a^2*b^2*f^2*cosh(d*x +
 c)^3 - 4*a^2*b^2*f^2*cosh(d*x + c) + a*b^3*f^2 + 2*(2*a^3*b - a*b^3)*f^2*c
osh(d*x + c)^2 + 4*(a*b^3*f^2*cosh(d*x + c) + a^2*b^2*f^2)*sinh(d*x + c)^3
 + 2*(3*a*b^3*f^2*cosh(d*x + c)^2 + 6*a^2*b^2*f^2*cosh(d*x + c) + (2*a^3*b -
 a*b^3)*f^2)*sinh(d*x + c)^2 + 4*(a*b^3*f^2*cosh(d*x + c)^3 + 3*a^2*b^2*f^2
*cosh(d*x + c)^2 - a^2*b^2*f^2 + (2*a^3*b - a*b^3)*f^2*cosh(d*x + c))*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) -
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (a*
b^3*d*f^2*x + a*b^3*c*f^2 + (a*b^3*d*f^2*x + a*b^3*c*f^2)*cosh(d*x + c)^4 +
 (a*b^3*d*f^2*x + a*b^3*c*f^2)*sinh(d*x + c)^4 + 4*(a^2*b^2*d*f^2*x + a^2*b
^2*c*f^2)*cosh(d*x + c)^3 + 4*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2 + (a*b^3*d*f
^2*x + a*b^3*c*f^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d
*f^2*x + (2*a^3*b - a*b^3)*c*f^2)*cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d*
f^2*x + (2*a^3*b - a*b^3)*c*f^2 + 3*(a*b^3*d*f^2*x + a*b^3*c*f^2)*cosh(d*x
+ c)^2 + 6*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c)^2
 - 4*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*cosh(d*x + c) - 4*(a^2*b^2*d*f^2*x +
 a^2*b^2*c*f^2 - (a*b^3*d*f^2*x + a*b^3*c*f^2)*cosh(d*x + c)^3 - 3*(a^2*b^2
*d*f^2*x + a^2*b^2*c*f^2)*cosh(d*x + c)^2 - ((2*a^3*b - a*b^3)*d*f^2*x + (2
*a^3*b - a*b^3)*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*
log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b) + (a*b^3*d*f^2*x + a*b^3*c*f^2 + (a*b^3*d*
f^2*x + a*b^3*c*f^2)*cosh(d*x + c)^4 + (a*b^3*d*f^2*x + a*b^3*c*f^2)*sinh(d
x + c)^4 + 4(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*cosh(d*x + c)^3 + 4*(a^2*b^
2*d*f^2*x + a^2*b^2*c*f^2 + (a*b^3*d*f^2*x + a*b^3*c*f^2)*cosh(d*x + c))*si
nh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d*f^2*x + (2*a^3*b - a*b^3)*c*f^2)*cos
h(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d*f^2*x + (2*a^3*b - a*b^3)*c*f^2 + 3*(

```
a*b^3*d*f^2*x + a*b^3*c*f^2)*cosh(d*x + c)^2 + 6*(a^2*b^2*d*f^2*x + a^2*b^2
*c*f^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2
)*cosh(d*x + c) - 4*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2 - (a*b^3*d*f^2*x + a*b
^3*c*f^2)*cosh(d*x + c)^3 - 3*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*cosh(d*x +
c)^2 - ((2*a^3*b - a*b^3)*d*f^2*x + (2*a^3*b - a*b^3)*c*f^2)*cosh(d*x + c))
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x +
c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*
((a^3*b + a*b^3)*d*f^2*x + 4*(a^3*b + a*b^3)*c*f^2 - 3*(a^3*b + a*b^3)*d*f*
cosh(1) - 3*(a^3*b + a*b^3)*d*f*sinh(1))*cosh(d*x + c) - ((a^2*b^2 + b^4)*f
^2*cosh(d*x + c)^4 + (a^2*b^2 + b^4)*f^2*sinh(d*x + c)^4 + 4*(a^3*b + a*b^3
)*f^2*cosh(d*x + c)^3 + 2*(2*a^4 + a^2*b^2 - b^4)*f^2*cosh(d*x + c)^2 - 4*(
a^3*b + a*b^3)*f^2*cosh(d*x + c) + 4*((a^2*b^2 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)^2}{{\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^3,x)
```

```
[Out] int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^3, x)
```

3.329 $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=631

$$-\frac{3f(e+fx)^2}{2b\left(a^2+b^2\right)d^2}+\frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3}+\frac{3af(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2b\left(a^2+b^2\right)^{3/2}d^2}+\frac{3f^2(e+fx}{}$$

```
[Out] -3/2*f*(f*x+e)^2/b/(a^2+b^2)/d^2+3*f^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))/b/(a^2+b^2)/d^3+3/2*a*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)
^(1/2)))/b/(a^2+b^2)^(3/2)/d^2+3*f^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)
^(1/2)))/b/(a^2+b^2)/d^3-3/2*a*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))/b/(a^2+b^2)^(3/2)/d^2+3*f^3*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/
2)))/b/(a^2+b^2)/d^4+3*a*f^2*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(
1/2)))/b/(a^2+b^2)^(3/2)/d^3+3*f^3*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/
2)))/b/(a^2+b^2)/d^4-3*a*f^2*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))/b/(a^2+b^2)^(3/2)/d^3-3*a*f^3*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(
1/2)))/b/(a^2+b^2)^(3/2)/d^4+3*a*f^3*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))/b/(a^2+b^2)^(3/2)/d^4-1/2*(f*x+e)^3/b/d/(a+b*sinh(d*x+c))^2-3/2*f*(f
*x+e)^2*cosh(d*x+c)/(a^2+b^2)/d^2/(a+b*sinh(d*x+c))
```

Rubi [A]
time = 0.78, antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {5572, 3405, 3403, 2296, 2221, 2611, 2320, 6724, 5680, 2317, 2438}

$$\frac{3f^3\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4\left(a^2+b^2\right)}+\frac{3f^3\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^4\left(a^2+b^2\right)}-\frac{3af^3\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4\left(a^2+b^2\right)^{3/2}}+\frac{3af^3\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^4\left(a^2+b^2\right)^{3/2}}+\frac{3af^2(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}-\frac{3af^2(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}+\frac{3f^2(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd^3\left(a^2+b^2\right)}+\frac{3f^2(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd^3\left(a^2+b^2\right)}+\frac{3af(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{2bd^2\left(a^2+b^2\right)^{3/2}}-\frac{3af(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{2bd^2\left(a^2+b^2\right)^{3/2}}-\frac{3f(e+fx)^2\cosh(c+dx)}{2d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{3f(e+fx)^2}{2bd^2\left(a^2+b^2\right)}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] (-3*f*(e + f*x)^2)/(2*b*(a^2 + b^2)*d^2) + (3*f^2*(e + f*x)*Log[1 + (b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d^3) + (3*a*f*(e + f*x)^2*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(2*b*(a^2 + b^2)^(3/2)*d^2)
+ (3*f^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2
+ b^2)*d^3) - (3*a*f*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(2*b*(a^2 + b^2)^(3/2)*d^2) + (3*f^3*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^4) + (3*a*f^2*(e + f*x)*PolyLog[2, -
((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) + (3*f^
3*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^4)
 - (3*a*f^2*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])
/(b*(a^2 + b^2)^(3/2)*d^3) - (3*a*f^3*PolyLog[3, -((b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^4) + (3*a*f^3*PolyLog[3, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^4) - (e + f*x)^3/(2
```

```
*b*d*(a + b*Sinh[c + d*x])^2) - (3*f*(e + f*x)^2*Cosh[c + d*x])/(2*(a^2 + b
^2)*d^2*(a + b*Sinh[c + d*x]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5572

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = -\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2} + \frac{(3f)\int \frac{(e+fx)^2}{(a+b\sinh(c+dx))^2}\,dx}{2bd}$$

$$= -\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2} - \frac{3f(e+fx)^2\cosh(c+dx)}{2(a^2+b^2)d^2(a+b\sinh(c+dx))} + \frac{(3af)\int \frac{}{a+}}{2b(a^2}$$

$$= -\frac{3f(e+fx)^2}{2b(a^2+b^2)d^2} - \frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2} - \frac{3f(e+fx)^2\cosh(c+dx)}{2(a^2+b^2)d^2(a+b\sinh(c+}$$

$$= -\frac{3f(e+fx)^2}{2b(a^2+b^2)d^2} + \frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^3} + \frac{3f^2(e+fx)\log\Big(}{b(a^2}$$

$$= -\frac{3f(e+fx)^2}{2b(a^2+b^2)d^2} + \frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^3} + \frac{3af(e+fx)^2\log}{2b(a^2}$$

$$= -\frac{3f(e+fx)^2}{2b(a^2+b^2)d^2} + \frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^3} + \frac{3af(e+fx)^2\log}{2b(a^2}$$

$$= -\frac{3f(e+fx)^2}{2b(a^2+b^2)d^2} + \frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^3} + \frac{3af(e+fx)^2\log}{2b(a^2}$$

$$= -\frac{3f(e+fx)^2}{2b(a^2+b^2)d^2} + \frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^3} + \frac{3af(e+fx)^2\log}{2b(a^2}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 5753 vs. $2(631) = 1262$.
time = 21.71, size = 5753, normalized size = 9.12

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] Result too large to show
```

Maple [**F**]

time = 5.24, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \cosh(dx+c)}{(a+b\sinh(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 3*a*d*f^3*integrate(x^2*e^(d*x + c)/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*
e^(2*d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a^2*b
^2*d^2 - b^4*d^2), x) + 3*b*f^2*(a*log((b*e^(d*x + c) + a - sqrt(a^2 + b^2)
)/(b*e^(d*x + c) + a + sqrt(a^2 + b^2)))/((a^2*b^2 + b^4)*sqrt(a^2 + b^2)*d
^3) - 2*(d*x + c)/((a^2*b^2 + b^4)*d^3) + log(b*e^(2*d*x + 2*c) + 2*a*e^(d*
x + c) - b)/((a^2*b^2 + b^4)*d^3))*e + 6*a*d*f^2*integrate(x*e^(d*x + c + 1
)/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + 2*a^3*b*d^2*e^(d
*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a^2*b^2*d^2 - b^4*d^2), x) - 6*a*f^3*in
tegrate(x*e^(d*x + c)/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c
) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a^2*b^2*d^2 - b^4*d
^2), x) + 6*b*f^3*integrate(x/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d
*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a^2*b^2*d^2
 - b^4*d^2), x) + 3/2*f*(2*(a*b*e^(3*d*x + 3*c) - 3*a*b*e^(d*x + c) + b^2 +
 (2*a^2*e^(2*c) - b^2*e^(2*c) - 2*(a^2*d*e^(2*c) + b^2*d*e^(2*c))*x)*e^(2*d
*x))/(a^2*b^3*d^2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5*d^2*e^(4*c))*e^(4*
d*x) + 4*(a^3*b^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3*d*x) + 2*(2*a^4*b*d
^2*e^(2*c) + a^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^(2*d*x) - 4*(a^3*b^2*
d^2*e^c + a*b^4*d^2*e^c)*e^(d*x)) + a*log((b*e^(d*x + 2*c) + a*e^c - sqrt(a
^2 + b^2)*e^c)/(b*e^(d*x + 2*c) + a*e^c + sqrt(a^2 + b^2)*e^c))/((a^2*b + b
^3)*sqrt(a^2 + b^2)*d^2))*e^2 - 3*a*f^2*e*log((a*e + b*e^(d*x + c + 1) - sq
rt(a^2 + b^2)*e)/(a*e + b*e^(d*x + c + 1) + sqrt(a^2 + b^2)*e))/((a^2*b + b
^3)*sqrt(a^2 + b^2)*d^3) + (3*b^2*f^3*x^2 + 6*b^2*f^2*x*e + 3*(a*b*f^3*x^2*
e^(3*c) + 2*a*b*f^2*x*e^(3*c + 1))*e^(3*d*x) - (2*(a^2*d*f^3*e^(2*c) + b^2*
d*f^3*e^(2*c))*x^3 - 3*(2*a^2*f^3*e^(2*c) - b^2*f^3*e^(2*c) - 2*(a^2*d*f^2*
e^(2*c) + b^2*d*f^2*e^(2*c))*e)*x^2 - 6*(2*a^2*f^2*e^(2*c) - b^2*f^2*e^(2*c
))*x*e)*e^(2*d*x) - 9*(a*b*f^3*x^2*e^c + 2*a*b*f^2*x*e^(c + 1))*e^(d*x))/(a
^2*b^3*d^2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5*d^2*e^(4*c))*e^(4*d*x) +
```

```
4*(a^3*b^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3*d*x) + 2*(2*a^4*b*d^2*e^(2
*c) + a^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^(2*d*x) - 4*(a^3*b^2*d^2*e^c
 + a*b^4*d^2*e^c)*e^(d*x)) - 2*e^(-2*d*x - 2*c + 3)/((4*a*b^2*e^(-d*x - c)
- 4*a*b^2*e^(-3*d*x - 3*c) + b^3*e^(-4*d*x - 4*c) + b^3 + 2*(2*a^2*b - b^3)
*e^(-2*d*x - 2*c))*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 16887 vs. $2(584) = 1168$.
time = 0.53, size = 16887, normalized size = 26.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*(a^2*b^2 + b^4)*c^2*f^3 - 12*(a^2*b^2 + b^4)*c*d*f^2*cosh(1) + 6*(a^
2*b^2 + b^4)*d^2*f*cosh(1)^2 + 6*(a^2*b^2 + b^4)*d^2*f*sinh(1)^2 - 6*((a^2*
b^2 + b^4)*d^2*f^3*x^2 - (a^2*b^2 + b^4)*c^2*f^3 + 2*((a^2*b^2 + b^4)*d^2*f
^2*x + (a^2*b^2 + b^4)*c*d*f^2)*cosh(1) + 2*((a^2*b^2 + b^4)*d^2*f^2*x + (a
^2*b^2 + b^4)*c*d*f^2)*sinh(1))*cosh(d*x + c)^4 - 6*((a^2*b^2 + b^4)*d^2*f^
3*x^2 - (a^2*b^2 + b^4)*c^2*f^3 + 2*((a^2*b^2 + b^4)*d^2*f^2*x + (a^2*b^2 +
 b^4)*c*d*f^2)*cosh(1) + 2*((a^2*b^2 + b^4)*d^2*f^2*x + (a^2*b^2 + b^4)*c*d
*f^2)*sinh(1))*sinh(d*x + c)^4 - 6*(3*(a^3*b + a*b^3)*d^2*f^3*x^2 - 4*(a^3*
b + a*b^3)*c^2*f^3 - (a^3*b + a*b^3)*d^2*f*cosh(1)^2 - (a^3*b + a*b^3)*d^2*
f*sinh(1)^2 + 2*(3*(a^3*b + a*b^3)*d^2*f^2*x + 4*(a^3*b + a*b^3)*c*d*f^2)*c
osh(1) + 2*(3*(a^3*b + a*b^3)*d^2*f^2*x + 4*(a^3*b + a*b^3)*c*d*f^2 - (a^3*
b + a*b^3)*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^3 - 6*(3*(a^3*b + a*b^3)*d
^2*f^3*x^2 - 4*(a^3*b + a*b^3)*c^2*f^3 - (a^3*b + a*b^3)*d^2*f*cosh(1)^2 -
(a^3*b + a*b^3)*d^2*f*sinh(1)^2 + 2*(3*(a^3*b + a*b^3)*d^2*f^2*x + 4*(a^3*b
 + a*b^3)*c*d*f^2)*cosh(1) + 4*((a^2*b^2 + b^4)*d^2*f^3*x^2 - (a^2*b^2 + b^
4)*c^2*f^3 + 2*((a^2*b^2 + b^4)*d^2*f^2*x + (a^2*b^2 + b^4)*c*d*f^2)*cosh(1
) + 2*((a^2*b^2 + b^4)*d^2*f^2*x + (a^2*b^2 + b^4)*c*d*f^2)*sinh(1))*cosh(d
*x + c) + 2*(3*(a^3*b + a*b^3)*d^2*f^2*x + 4*(a^3*b + a*b^3)*c*d*f^2 - (a^3
*b + a*b^3)*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c)^3 - 2*(2*(a^4 + 2*a^2*b^2
 + b^4)*d^3*f^3*x^3 + 3*(2*a^4 + a^2*b^2 - b^4)*d^2*f^3*x^2 + 2*(a^4 + 2*a^
2*b^2 + b^4)*d^3*cosh(1)^3 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*sinh(1)^3 - 6*(2
*a^4 + a^2*b^2 - b^4)*c^2*f^3 + 3*(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*f*x - (2*a
^4 + a^2*b^2 - b^4)*d^2*f)*cosh(1)^2 + 3*(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*f*x
 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(1) - (2*a^4 + a^2*b^2 - b^4)*d^2*f)*s
inh(1)^2 + 6*((a^4 + 2*a^2*b^2 + b^4)*d^3*f^2*x^2 + (2*a^4 + a^2*b^2 - b^4)
*d^2*f^2*x + 2*(2*a^4 + a^2*b^2 - b^4)*c*d*f^2)*cosh(1) + 6*((a^4 + 2*a^2*b
^2 + b^4)*d^3*f^2*x^2 + (2*a^4 + a^2*b^2 - b^4)*d^2*f^2*x + (a^4 + 2*a^2*b^
2 + b^4)*d^3*cosh(1)^2 + 2*(2*a^4 + a^2*b^2 - b^4)*c*d*f^2 + (2*(a^4 + 2*a^
2*b^2 + b^4)*d^3*f*x - (2*a^4 + a^2*b^2 - b^4)*d^2*f)*cosh(1))*sinh(1))*cos
h(d*x + c)^2 - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 3*(2*a^4 + a^2*b^
```

```
2 - b^4)*d^2*f^3*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(1)^3 + 2*(a^4 + 2
*a^2*b^2 + b^4)*d^3*sinh(1)^3 - 6*(2*a^4 + a^2*b^2 - b^4)*c^2*f^3 + 3*(2*(a
^4 + 2*a^2*b^2 + b^4)*d^3*f*x - (2*a^4 + a^2*b^2 - b^4)*d^2*f)*cosh(1)^2 +
18*((a^2*b^2 + b^4)*d^2*f^3*x^2 - (a^2*b^2 + b^4)*c^2*f^3 + 2*((a^2*b^2 + b
^4)*d^2*f^2*x + (a^2*b^2 + b^4)*c*d*f^2)*cosh(1) + 2*((a^2*b^2 + b^4)*d^2*f
^2*x + (a^2*b^2 + b^4)*c*d*f^2)*sinh(1))*cosh(d*x + c)^2 + 3*(2*(a^4 + 2*a^
2*b^2 + b^4)*d^3*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(1) - (2*a^4 + a^2
*b^2 - b^4)*d^2*f)*sinh(1)^2 + 6*((a^4 + 2*a^2*b^2 + b^4)*d^3*f^2*x^2 + (2*
a^4 + a^2*b^2 - b^4)*d^2*f^2*x + 2*(2*a^4 + a^2*b^2 - b^4)*c*d*f^2)*cosh(1)
 + 9*(3*(a^3*b + a*b^3)*d^2*f^3*x^2 - 4*(a^3*b + a*b^3)*c^2*f^3 - (a^3*b +
a*b^3)*d^2*f*cosh(1)^2 - (a^3*b + a*b^3)*d^2*f*sinh(1)^2 + 2*(3*(a^3*b + a*
b^3)*d^2*f^2*x + 4*(a^3*b + a*b^3)*c*d*f^2)*cosh(1) + 2*(3*(a^3*b + a*b^3)*
d^2*f^2*x + 4*(a^3*b + a*b^3)*c*d*f^2 - (a^3*b + a*b^3)*d^2*f*cosh(1))*sinh
(1))*cosh(d*x + c) + 6*((a^4 + 2*a^2*b^2 + b^4)*d^3*f^2*x^2 + (2*a^4 + a^2*
b^2 - b^4)*d^2*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(1)^2 + 2*(2*a^4 + a
^2*b^2 - b^4)*c*d*f^2 + (2*(a^4 + 2*a^2*b^2 + b^4)*d^3*f*x - (2*a^4 + a^2*b
^2 - b^4)*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 6*(a*b^3*f^3*cosh(d*x
+ c)^4 + a*b^3*f^3*sinh(d*x + c)^4 + 4*a^2*b^2*f^3*cosh(d*x + c)^3 - 4*a^2*
b^2*f^3*cosh(d*x + c) + a*b^3*f^3 + 2*(2*a^3*b - a*b^3)*f^3*cosh(d*x + c)^2
 + 4*(a*b^3*f^3*cosh(d*x + c) + a^2*b^2*f^3)*sinh(d*x + c)^3 + 2*(3*a*b^3*f
^3*cosh(d*x + c)^2 + 6*a^2*b^2*f^3*cosh(d*x + c) + (2*a^3*b - a*b^3)*f^3)*s
inh(d*x + c)^2 + 4*(a*b^3*f^3*cosh(d*x + c)^3 + 3*a^2*b^2*f^3*cosh(d*x + c)
^2 - a^2*b^2*f^3 + (2*a^3*b - a*b^3)*f^3*cosh(d*x + c))*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*(a*b^3*f^3*cosh(d*
x + c)^4 + a*b^3*f^3*sinh(d*x + c)^4 + 4*a^2*b^2*f^3*cosh(d*x + c)^3 - 4*a^
2*b^2*f^3*cosh(d*x + c) + a*b^3*f^3 + 2*(2*a^3*b - a*b^3)*f^3*cosh(d*x + c)
^2 + 4*(a*b^3*f^3*cosh(d*x + c) + a^2*b^2*f^3)*sinh(d*x + c)^3 + 2*(3*a*b^3
*f^3*cosh(d*x + c)^2 + 6*a^2*b^2*f^3*cosh(d*x + c) + (2*a^3*b - a*b^3)*f^3)
*sinh(d*x + c)^2 + 4*(a*b^3*f^3*cosh(d*x + c)^3 + 3*a^2*b^2*f^3*cosh(d*x +
c)^2 - a^2*b^2*f^3 + (2*a^3*b - a*b^3)*f^3*cosh(d*x + c))*sinh(d*x + c))*sq
rt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*((a^3*b + a*b^3)
*d^2*f^3*x^2 - 4*(a^3*b + a*b^3)*c^2*f^3 - 3*(a^3*b + a*b^3)*d^2*f*cosh(1)^
2 - 3*(a^3*b + a*b^3)*d^2*f*sinh(1)^2 + 2*((a^3*b + a*b^3)*d^2*f^2*x + 4*(a
^3*b + a*b^3)*c*d*f^2)*cosh(1) + 2*((a^3*b + a*b^3)*d^2*f^2*x + 4*(a^3*b +
a*b^3)*c*d*f^2 - 3*(a^3*b + a*b^3)*d^2*f*cosh(1...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)^3}{\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^3,x)
```

```
[Out] int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^3, x)
```

3.330 $\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=112

$$-\frac{af\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d^2}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}-\frac{f\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))}$$

```
[Out] -a*f*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/b/(a^2+b^2)^(3/2)/d
^2+1/2*(-f*x-e)/b/d/(a+b*sinh(d*x+c))^2-1/2*f*cosh(d*x+c)/(a^2+b^2)/d^2/(a+
b*sinh(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5572, 2743, 12, 2739, 632, 210}

$$-\frac{af\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd^2\left(a^2+b^2\right)^{3/2}}-\frac{f\cosh(c+dx)}{2d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] -((a*f*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b*(a^2 + b^2)^(
3/2)*d^2)) - (e + f*x)/(2*b*d*(a + b*Sinh[c + d*x])^2) - (f*Cosh[c + d*x])/
(2*(a^2 + b^2)*d^2*(a + b*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 5572

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx &= -\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}+\frac{f\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx}{2bd}\\
&= -\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}-\frac{f\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))}+\frac{f\int \frac{a}{a+b\sinh(c}\\
&= -\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}-\frac{f\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))}+\frac{(af)\int \frac{}{a+b\sin}}{2b\left(a^2-\right.}\\
&= -\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}-\frac{f\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))}-\frac{(iaf)\text{Subst}}{}\\
&= -\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}-\frac{f\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))}+\frac{(2iaf)\text{Subs}}{}\\
&= -\frac{af\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d^2}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}-\frac{f\cosh}{2\left(a^2+b^2\right)d^2(a}
\end{aligned}$$

Mathematica [A]

time = 0.79, size = 112, normalized size = 1.00

$$-\frac{\frac{f\cosh(c+dx)}{(a^2+b^2)(a+b\sinh(c+dx))}+\frac{\frac{2afArcTan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}}+\frac{d(e+fx)}{(a+b\sinh(c+dx))^2}}{b}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] -1/2*((f*Cosh[c + d*x])/((a^2 + b^2)*(a + b*Sinh[c + d*x])) + ((2*a*f*ArcTa
n[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) + (d*(e +
 f*x))/(a + b*Sinh[c + d*x])^2)/b)/d^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(106) = 212$.
time = 6.15, size = 308, normalized size = 2.75

method	result
risch	$-\frac{2a^2dfx\,e^{2dx+2c}+2b^2dfx\,e^{2dx+2c}+2a^2de\,e^{2dx+2c}-abf\,e^{3dx+3c}+2b^2de\,e^{2dx+2c}-2a^2f\,e^{2dx+2c}+b^2f\,e^{2dx+2c}+3fa\,e^{dx+c}b-f\,b^2}{b\,d^2\left(b\,e^{2dx+2c}+2a\,e^{dx+c}-b\right)^2(a^2+b^2)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(2*a^2*d*f*x*exp(2*d*x+2*c)+2*b^2*d*f*x*exp(2*d*x+2*c)+2*a^2*d*e*exp(2
*d*x+2*c)-a*b*f*exp(3*d*x+3*c)+2*b^2*d*e*exp(2*d*x+2*c)-2*a^2*f*exp(2*d*x+2
*c)+b^2*f*exp(2*d*x+2*c)+3*f*a*exp(d*x+c)*b-f*b^2)/d^2/(b*exp(2*d*x+2*c)+2*
a*exp(d*x+c)-b)^2/(a^2+b^2)+1/2/(a^2+b^2)^(3/2)*f*a/d^2/b*ln(exp(d*x+c)+(a*
(a^2+b^2)^(3/2)-a^4-2*a^2*b^2-b^4)/b/(a^2+b^2)^(3/2))-1/2/(a^2+b^2)^(3/2)*f
*a/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(3/2)+a^4+2*a^2*b^2+b^4)/b/(a^2+b^2)^(3
/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(106) = 212$.
time = 0.59, size = 413, normalized size = 3.69

$$\frac{1}{2}f\left(\frac{2\left(abe^{(3dx+3c)}-3abe^{(dx+c)}+b^2+\left(2a^2e^{(2c)}-b^2e^{(2c)}-2\left(a^2de^{(2c)}+b^2de^{(2c)}\right)x\right)e^{(2dx)}\right)}{a^2b^3d^2+b^5d^2+(a^2b^3d^2e^{(4c)}+b^5d^2e^{(4c)})e^{(4dx)}+4(a^3b^2d^2e^{(3c)}+ab^4d^2e^{(3c)})e^{(3dx)}+2(2a^4bd^2e^{(2c)}+a^2b^3d^2e^{(2c)}-b^5d^2e^{(2c)})e^{(2dx)}-4(a^3b^2d^2e^{c}+ab^4d^2e^{c})e^{(dx)}}+\frac{a\log\left(\frac{be^{(dx+2c)}+ae^{c}-\sqrt{a^2+b^2}e^{c}}{be^{(dx+2c)}+ae^{c}+\sqrt{a^2+b^2}e^{c}}\right)}{(a^2b+b^3)\sqrt{a^2+b^2}d^2}\right)-\frac{2e^{(-2dx-2c+1)}}{(4ab^2e^{(-dx-c)}-4ab^2e^{(-3dx-3c)}+b^3e^{(-4dx-4c)}+b^3+2(2a^2b-b^3)e^{(-2dx-2c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*f*(2*(a*b*e^(3*d*x + 3*c) - 3*a*b*e^(d*x + c) + b^2 + (2*a^2*e^(2*c) -
b^2*e^(2*c) - 2*(a^2*d*e^(2*c) + b^2*d*e^(2*c))*x)*e^(2*d*x))/(a^2*b^3*d^2
```

```
+ b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5*d^2*e^(4*c))*e^(4*d*x) + 4*(a^3*b^2*
d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3*d*x) + 2*(2*a^4*b*d^2*e^(2*c) + a^2*b
^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^(2*d*x) - 4*(a^3*b^2*d^2*e^c + a*b^4*d^
2*e^c)*e^(d*x)) + a*log((b*e^(d*x + 2*c) + a*e^c - sqrt(a^2 + b^2)*e^c)/(b*
e^(d*x + 2*c) + a*e^c + sqrt(a^2 + b^2)*e^c))/((a^2*b + b^3)*sqrt(a^2 + b^2
)*d^2)) - 2*e^(-2*d*x - 2*c + 1)/((4*a*b^2*e^(-d*x - c) - 4*a*b^2*e^(-3*d*x
 - 3*c) + b^3*e^(-4*d*x - 4*c) + b^3 + 2*(2*a^2*b - b^3)*e^(-2*d*x - 2*c))*
d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1293 vs. $2(106) = 212$.

time = 0.37, size = 1293, normalized size = 11.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a^3*b + a*b^3)*f*cosh(d*x + c)^3 + 2*(a^3*b + a*b^3)*f*sinh(d*x + c
)^3 - 6*(a^3*b + a*b^3)*f*cosh(d*x + c) - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*f*
x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(1) + 2*(a^4 + 2*a^2*b^2 + b^4)*d*sinh(
1) - (2*a^4 + a^2*b^2 - b^4)*f)*cosh(d*x + c)^2 - 2*(2*(a^4 + 2*a^2*b^2 + b
^4)*d*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(1) - 3*(a^3*b + a*b^3)*f*cosh(
d*x + c) + 2*(a^4 + 2*a^2*b^2 + b^4)*d*sinh(1) - (2*a^4 + a^2*b^2 - b^4)*f)
*sinh(d*x + c)^2 + (a*b^2*f*cosh(d*x + c)^4 + a*b^2*f*sinh(d*x + c)^4 + 4*a
^2*b*f*cosh(d*x + c)^3 - 4*a^2*b*f*cosh(d*x + c) + a*b^2*f + 2*(2*a^3 - a*b
^2)*f*cosh(d*x + c)^2 + 4*(a*b^2*f*cosh(d*x + c) + a^2*b*f)*sinh(d*x + c)^3
 + 2*(3*a*b^2*f*cosh(d*x + c)^2 + 6*a^2*b*f*cosh(d*x + c) + (2*a^3 - a*b^2)
*f)*sinh(d*x + c)^2 + 4*(a*b^2*f*cosh(d*x + c)^3 + 3*a^2*b*f*cosh(d*x + c)^
2 - a^2*b*f + (2*a^3 - a*b^2)*f*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^
2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2
*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*
(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x +
c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*
(a^2*b^2 + b^4)*f + 2*(3*(a^3*b + a*b^3)*f*cosh(d*x + c)^2 - 3*(a^3*b + a*b
^3)*f - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*co
sh(1) + 2*(a^4 + 2*a^2*b^2 + b^4)*d*sinh(1) - (2*a^4 + a^2*b^2 - b^4)*f)*co
sh(d*x + c))*sinh(d*x + c))/((a^4*b^3 + 2*a^2*b^5 + b^7)*d^2*cosh(d*x + c)^
4 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^2*sinh(d*x + c)^4 + 4*(a^5*b^2 + 2*a^3*b^
4 + a*b^6)*d^2*cosh(d*x + c)^3 + 2*(2*a^6*b + 3*a^4*b^3 - b^7)*d^2*cosh(d*x
 + c)^2 - 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2*cosh(d*x + c) + 4*((a^4*b^3 +
 2*a^2*b^5 + b^7)*d^2*cosh(d*x + c) + (a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2)*si
nh(d*x + c)^3 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^2 + 2*(3*(a^4*b^3 + 2*a^2*b^5
 + b^7)*d^2*cosh(d*x + c)^2 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2*cosh(d*x
+ c) + (2*a^6*b + 3*a^4*b^3 - b^7)*d^2)*sinh(d*x + c)^2 + 4*((a^4*b^3 + 2*a
```

```
^2*b^5 + b^7)*d^2*cosh(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2*cos
h(d*x + c)^2 + (2*a^6*b + 3*a^4*b^3 - b^7)*d^2*cosh(d*x + c) - (a^5*b^2 + 2
*a^3*b^4 + a*b^6)*d^2)*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)\,\left(e+f\,x\right)}{{\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x))^3,x)
[Out] int((cosh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x))^3, x)
```

3.331 $\int \frac{(e+fx)^2 \cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=306

$$\frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d^2}-\frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d^2}+\frac{f^2\log(a+b\sinh(c+dx))}{b\left(a^2+b^2\right)d^3}+\frac{af^2\mathrm{P}}{}$$

```
[Out] f^2*ln(a+b*sinh(d*x+c))/b/(a^2+b^2)/d^3+a*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a
^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2-a*f*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^2+a*f^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b
^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3-a*f^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2
)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3-1/2*(f*x+e)^2/b/d/(a+b*sinh(d*x+c))^2-f*(f*
x+e)*cosh(d*x+c)/(a^2+b^2)/d^2/(a+b*sinh(d*x+c))
```

Rubi [A]
time = 0.36, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5572, 3405, 3403, 2296, 2221, 2317, 2438, 2747, 31}

$$\frac{af^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}-\frac{af^2\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}+\frac{f^2\log(a+b\sinh(c+dx))}{bd^3\left(a^2+b^2\right)}+\frac{af(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd^2\left(a^2+b^2\right)^{3/2}}-\frac{af(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd^2\left(a^2+b^2\right)^{3/2}}-\frac{f(e+fx)\cosh(c+dx)}{d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] (a*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^
2)^(3/2)*d^2) - (a*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)])/(b*(a^2 + b^2)^(3/2)*d^2) + (f^2*Log[a + b*Sinh[c + d*x]])/(b*(a^2 + b^
2)*d^3) + (a*f^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(
a^2 + b^2)^(3/2)*d^3) - (a*f^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) - (e + f*x)^2/(2*b*d*(a + b*Sinh[c + d*
x])^2) - (f*(e + f*x)*Cosh[c + d*x])/((a^2 + b^2)*d^2*(a + b*Sinh[c + d*x])
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5572

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = -\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}+\frac{f\int\frac{e+fx}{(a+b\sinh(c+dx))^2}\,dx}{bd}$$

$$= -\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}-\frac{f(e+fx)\cosh(c+dx)}{(a^2+b^2)\,d^2(a+b\sinh(c+dx))}+\frac{(af)\int\frac{e\cdots}{a+b\sin\cdots}}{b\,(a^2+\cdots}$$

$$= -\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}-\frac{f(e+fx)\cosh(c+dx)}{(a^2+b^2)\,d^2(a+b\sinh(c+dx))}+\frac{(2af)\int\frac{\cdots}{-b+\cdots}}{b\,(a\cdots}$$

$$= \frac{f^2\log(a+b\sinh(c+dx))}{b\,(a^2+b^2)\,d^3}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}-\frac{f(e+fx)\cosh(\cdots}{(a^2+b^2)\,d^2(a+b\,\mathrm{si}\cdots}$$

$$= \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^2}-\frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^2}+\cdots$$

$$= \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^2}-\frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^2}+\cdots$$

$$= \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^2}-\frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d^2}+\cdots$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 623 vs. 2(306) = 612.
time = 7.27, size = 623, normalized size = 2.04

$$\frac{f^2x\coth(c)}{b(a^2+b^2)d^2}+\frac{2e^cf\left(-e^cfx+e^{-c}(-1+e^{2c})fx-\frac{aee^{-c}(-1+e^{2c})\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{ae^{-c}(-1+e^{2c})f\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}+\frac{1}{2}e^{-c}(-1+e^{2c})f\left(-2x+\frac{2a\mathrm{ArcTan}\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,d}+\frac{\log(2ae^{c+dx}+b(-1+e^{2(c+dx)}))}{d}\right)+\frac{a(-1+e^{2c})f\left(dx\left(\log\left(1+\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)-\log\left(1+\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)\right)+\mathrm{PolyLog}\left(2,-\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}\right)-\mathrm{PolyLog}\left(2,-\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)e^{2c}}}\right)\right)}{2d\sqrt{(a^2+b^2)e^{2c}}}\right)}{b(a^2+b^2)d^2(-1+e^{2c})}-\frac{f^2x\cosh(c)\operatorname{csch}\left(\frac{c}{2}\right)\operatorname{sech}\left(\frac{c}{2}\right)}{2b(a^2+b^2)d^2}-\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2}+\frac{\operatorname{csch}\left(\frac{c}{2}\right)\operatorname{sech}\left(\frac{c}{2}\right)(aef\cosh(c)+af^2x\cosh(c)+bef\sinh(dx)+bf^2x\sinh(dx))}{2b(a^2+b^2)d^2(a+b\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] (f^2*x*Coth[c])/(b*(a^2 + b^2)*d^2) + (2*E^c*f*(-(E^c*f*x) + ((-1 + E^(2*c)
)*f*x)/E^c - (a*e*(-1 + E^(2*c))*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2
```

```
]])/(Sqrt[a^2 + b^2]*E^c) + (a*(-1 + E^(2*c))*f*ArcTanh[(a + b*E^(c + d*x))
/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d*E^c) + ((-1 + E^(2*c))*f*(-2*x + (2*a
*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-a^2 - b^2]*d) + Log[2
*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))]/d))/(2*E^c) + (a*(-1 + E^(2*c))*
f*(d*x*(Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - Lo
g[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])]) + PolyLog[2,
-((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - PolyLog[2, -((b
*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(2*d*Sqrt[(a^2 + b^
2)*E^(2*c)])))/(b*(a^2 + b^2)*d^2*(-1 + E^(2*c))) - (f^2*x*Cosh[c]*Csch[c/2
]*Sech[c/2])/(2*b*(a^2 + b^2)*d^2) - (e + f*x)^2/(2*b*d*(a + b*Sinh[c + d*x
])^2) + (Csch[c/2]*Sech[c/2]*(a*e*f*Cosh[c] + a*f^2*x*Cosh[c] + b*e*f*Sinh[
d*x] + b*f^2*x*Sinh[d*x]))/(2*b*(a^2 + b^2)*d^2*(a + b*Sinh[c + d*x]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 804 vs. $2(284) = 568$.
time = 6.32, size = 805, normalized size = 2.63

method	result
risch	$-\frac{2\left(a^2 d f^2 x^2 \mathrm{e}^{2dx+2c}+b^2 d f^2 x^2 \mathrm{e}^{2dx+2c}+2a^2 d e f x\, \mathrm{e}^{2dx+2c}-ab f^2 x\, \mathrm{e}^{3dx+3c}+2b^2 d e f x\, \mathrm{e}^{2dx+2c}+a^2 d e^2 \mathrm{e}^{2dx+2c}-2a^2 f^2 x\, \mathrm{e}^{2dx+2c}-ab\right.}{b d^2 (a^2+b^2)\left(b\, \mathrm{e}^{2dx+2c}+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b*(a^2*d*f^2*x^2*exp(2*d*x+2*c)+b^2*d*f^2*x^2*exp(2*d*x+2*c)+2*a^2*d*e*f
*x*exp(2*d*x+2*c)-a*b*f^2*x*exp(3*d*x+3*c)+2*b^2*d*e*f*x*exp(2*d*x+2*c)+a^2
*d*e^2*exp(2*d*x+2*c)-2*a^2*f^2*x*exp(2*d*x+2*c)-a*b*e*f*exp(3*d*x+3*c)+b^2
*d*e^2*exp(2*d*x+2*c)+b^2*f^2*x*exp(2*d*x+2*c)-2*a^2*e*f*exp(2*d*x+2*c)+3*a
*b*f^2*x*exp(d*x+c)+b^2*e*f*exp(2*d*x+2*c)+3*a*b*e*f*exp(d*x+c)-b^2*f^2*x-b
^2*e*f)/d^2/(a^2+b^2)/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)^2-2/(a^2+b^2)/d^3
/b*f^2*ln(exp(d*x+c))+1/(a^2+b^2)/d^3/b*f^2*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x
+c)-b)-2/(a^2+b^2)^(3/2)/d^2/b*f*a*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+
b^2)^(1/2))+1/(a^2+b^2)^(3/2)/d^2/b*f^2*a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))*x+1/(a^2+b^2)^(3/2)/d^3/b*f^2*a*ln((-b*exp(d*x+c)
+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/(a^2+b^2)^(3/2)/d^2/b*f^2*a*l
n((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/(a^2+b^2)^(3/2)
/d^3/b*f^2*a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/(
a^2+b^2)^(3/2)/d^3/b*f^2*a*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2
+b^2)^(1/2)))-1/(a^2+b^2)^(3/2)/d^3/b*f^2*a*dilog((b*exp(d*x+c)+(a^2+b^2)^(
1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/(a^2+b^2)^(3/2)/d^3/b*f^2*a*c*arctanh(1/2*(2
*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (2*a*d*integrate(x*e^(d*x + c)/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*
d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a^2*b^2*d^
2 - b^4*d^2), x) + b*(a*log((b*e^(d*x + c) + a - sqrt(a^2 + b^2))/(b*e^(d*x
 + c) + a + sqrt(a^2 + b^2)))/((a^2*b^2 + b^4)*sqrt(a^2 + b^2)*d^3) - 2*(d*
x + c)/((a^2*b^2 + b^4)*d^3) + log(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - b)
/((a^2*b^2 + b^4)*d^3)) + 2*(a*b*x*e^(3*d*x + 3*c) - 3*a*b*x*e^(d*x + c) +
b^2*x - ((a^2*d*e^(2*c) + b^2*d*e^(2*c))*x^2 - (2*a^2*e^(2*c) - b^2*e^(2*c)
)*x)*e^(2*d*x))/(a^2*b^3*d^2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5*d^2*e^(
4*c))*e^(4*d*x) + 4*(a^3*b^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3*d*x) + 2
*(2*a^4*b*d^2*e^(2*c) + a^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^(2*d*x) -
4*(a^3*b^2*d^2*e^c + a*b^4*d^2*e^c)*e^(d*x)) - a*log((b*e^(d*x + c) + a - s
qrt(a^2 + b^2))/(b*e^(d*x + c) + a + sqrt(a^2 + b^2)))/((a^2*b + b^3)*sqrt(
a^2 + b^2)*d^3))*f^2 + f*(2*(a*b*e^(3*d*x + 3*c) - 3*a*b*e^(d*x + c) + b^2
+ (2*a^2*e^(2*c) - b^2*e^(2*c) - 2*(a^2*d*e^(2*c) + b^2*d*e^(2*c))*x)*e^(2*
d*x))/(a^2*b^3*d^2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5*d^2*e^(4*c))*e^(4
*d*x) + 4*(a^3*b^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3*d*x) + 2*(2*a^4*b*
d^2*e^(2*c) + a^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^(2*d*x) - 4*(a^3*b^2
*d^2*e^c + a*b^4*d^2*e^c)*e^(d*x)) + a*log((b*e^(d*x + 2*c) + a*e^c - sqrt(
a^2 + b^2)*e^c)/(b*e^(d*x + 2*c) + a*e^c + sqrt(a^2 + b^2)*e^c))/((a^2*b +
b^3)*sqrt(a^2 + b^2)*d^2))*e - 2*e^(-2*d*x - 2*c + 2)/((4*a*b^2*e^(-d*x - c
) - 4*a*b^2*e^(-3*d*x - 3*c) + b^3*e^(-4*d*x - 4*c) + b^3 + 2*(2*a^2*b - b^
3)*e^(-2*d*x - 2*c))*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6052 vs. $2(286) = 572$.
time = 0.42, size = 6052, normalized size = 19.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(2*((a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*c*f^2)*cosh(d*x + c)^4 + 2*(
(a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*c*f^2)*sinh(d*x + c)^4 + 2*(a^2*b
^2 + b^4)*c*f^2 - 2*(a^2*b^2 + b^4)*d*f*cosh(1) + 2*(3*(a^3*b + a*b^3)*d*f^
2*x + 4*(a^3*b + a*b^3)*c*f^2 - (a^3*b + a*b^3)*d*f*cosh(1) - (a^3*b + a*b^
3)*d*f*sinh(1))*cosh(d*x + c)^3 - 2*(a^2*b^2 + b^4)*d*f*sinh(1) + 2*(3*(a^3
*b + a*b^3)*d*f^2*x + 4*(a^3*b + a*b^3)*c*f^2 - (a^3*b + a*b^3)*d*f*cosh(1)
 - (a^3*b + a*b^3)*d*f*sinh(1) + 4*((a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^
4)*c*f^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*f
^2*x^2 + (2*a^4 + a^2*b^2 - b^4)*d*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d^2*cosh
(1)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2*sinh(1)^2 + 2*(2*a^4 + a^2*b^2 - b^4)*c
```

```
*f^2 + (2*(a^4 + 2*a^2*b^2 + b^4)*d^2*f*x - (2*a^4 + a^2*b^2 - b^4)*d*f)*co
sh(1) + (2*(a^4 + 2*a^2*b^2 + b^4)*d^2*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*
cosh(1) - (2*a^4 + a^2*b^2 - b^4)*d*f)*sinh(1))*cosh(d*x + c)^2 + 2*((a^4 +
 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + (2*a^4 + a^2*b^2 - b^4)*d*f^2*x + (a^4 + 2*
a^2*b^2 + b^4)*d^2*cosh(1)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2*sinh(1)^2 + 2*(2
*a^4 + a^2*b^2 - b^4)*c*f^2 + 6*((a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*
c*f^2)*cosh(d*x + c)^2 + (2*(a^4 + 2*a^2*b^2 + b^4)*d^2*f*x - (2*a^4 + a^2*
b^2 - b^4)*d*f)*cosh(1) + 3*(3*(a^3*b + a*b^3)*d*f^2*x + 4*(a^3*b + a*b^3)*
c*f^2 - (a^3*b + a*b^3)*d*f*cosh(1) - (a^3*b + a*b^3)*d*f*sinh(1))*cosh(d*x
 + c) + (2*(a^4 + 2*a^2*b^2 + b^4)*d^2*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*
cosh(1) - (2*a^4 + a^2*b^2 - b^4)*d*f)*sinh(1))*sinh(d*x + c)^2 - (a*b^3*f^
2*cosh(d*x + c)^4 + a*b^3*f^2*sinh(d*x + c)^4 + 4*a^2*b^2*f^2*cosh(d*x + c)
^3 - 4*a^2*b^2*f^2*cosh(d*x + c) + a*b^3*f^2 + 2*(2*a^3*b - a*b^3)*f^2*cosh
(d*x + c)^2 + 4*(a*b^3*f^2*cosh(d*x + c) + a^2*b^2*f^2)*sinh(d*x + c)^3 + 2
*(3*a*b^3*f^2*cosh(d*x + c)^2 + 6*a^2*b^2*f^2*cosh(d*x + c) + (2*a^3*b - a*
b^3)*f^2)*sinh(d*x + c)^2 + 4*(a*b^3*f^2*cosh(d*x + c)^3 + 3*a^2*b^2*f^2*co
sh(d*x + c)^2 - a^2*b^2*f^2 + (2*a^3*b - a*b^3)*f^2*cosh(d*x + c))*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (a*b^3
*f^2*cosh(d*x + c)^4 + a*b^3*f^2*sinh(d*x + c)^4 + 4*a^2*b^2*f^2*cosh(d*x +
 c)^3 - 4*a^2*b^2*f^2*cosh(d*x + c) + a*b^3*f^2 + 2*(2*a^3*b - a*b^3)*f^2*c
osh(d*x + c)^2 + 4*(a*b^3*f^2*cosh(d*x + c) + a^2*b^2*f^2)*sinh(d*x + c)^3
 + 2*(3*a*b^3*f^2*cosh(d*x + c)^2 + 6*a^2*b^2*f^2*cosh(d*x + c) + (2*a^3*b -
 a*b^3)*f^2)*sinh(d*x + c)^2 + 4*(a*b^3*f^2*cosh(d*x + c)^3 + 3*a^2*b^2*f^2
*cosh(d*x + c)^2 - a^2*b^2*f^2 + (2*a^3*b - a*b^3)*f^2*cosh(d*x + c))*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) -
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (a*
b^3*d*f^2*x + a*b^3*c*f^2 + (a*b^3*d*f^2*x + a*b^3*c*f^2)*cosh(d*x + c)^4 +
 (a*b^3*d*f^2*x + a*b^3*c*f^2)*sinh(d*x + c)^4 + 4*(a^2*b^2*d*f^2*x + a^2*b
^2*c*f^2)*cosh(d*x + c)^3 + 4*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2 + (a*b^3*d*f
^2*x + a*b^3*c*f^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d
*f^2*x + (2*a^3*b - a*b^3)*c*f^2)*cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d*
f^2*x + (2*a^3*b - a*b^3)*c*f^2 + 3*(a*b^3*d*f^2*x + a*b^3*c*f^2)*cosh(d*x
+ c)^2 + 6*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c)^2
 - 4*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*cosh(d*x + c) - 4*(a^2*b^2*d*f^2*x +
 a^2*b^2*c*f^2 - (a*b^3*d*f^2*x + a*b^3*c*f^2)*cosh(d*x + c)^3 - 3*(a^2*b^2
*d*f^2*x + a^2*b^2*c*f^2)*cosh(d*x + c)^2 - ((2*a^3*b - a*b^3)*d*f^2*x + (2
*a^3*b - a*b^3)*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*
log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b) + (a*b^3*d*f^2*x + a*b^3*c*f^2 + (a*b^3*d*
f^2*x + a*b^3*c*f^2)*cosh(d*x + c)^4 + (a*b^3*d*f^2*x + a*b^3*c*f^2)*sinh(d
*x + c)^4 + 4*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*cosh(d*x + c)^3 + 4*(a^2*b^
2*d*f^2*x + a^2*b^2*c*f^2 + (a*b^3*d*f^2*x + a*b^3*c*f^2)*cosh(d*x + c))*si
nh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d*f^2*x + (2*a^3*b - a*b^3)*c*f^2)*cos
h(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d*f^2*x + (2*a^3*b - a*b^3)*c*f^2 + 3*(
```

```
a*b^3*d*f^2*x + a*b^3*c*f^2)*cosh(d*x + c)^2 + 6*(a^2*b^2*d*f^2*x + a^2*b^2
*c*f^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2
)*cosh(d*x + c) - 4*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2 - (a*b^3*d*f^2*x + a*b
^3*c*f^2)*cosh(d*x + c)^3 - 3*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*cosh(d*x +
c)^2 - ((2*a^3*b - a*b^3)*d*f^2*x + (2*a^3*b - a*b^3)*c*f^2)*cosh(d*x + c))
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x +
c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*
((a^3*b + a*b^3)*d*f^2*x + 4*(a^3*b + a*b^3)*c*f^2 - 3*(a^3*b + a*b^3)*d*f*
cosh(1) - 3*(a^3*b + a*b^3)*d*f*sinh(1))*cosh(d*x + c) - ((a^2*b^2 + b^4)*f
^2*cosh(d*x + c)^4 + (a^2*b^2 + b^4)*f^2*sinh(d*x + c)^4 + 4*(a^3*b + a*b^3
)*f^2*cosh(d*x + c)^3 + 2*(2*a^4 + a^2*b^2 - b^4)*f^2*cosh(d*x + c)^2 - 4*(
a^3*b + a*b^3)*f^2*cosh(d*x + c) + 4*((a^2*b^2 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,{\left(e+f\,x\right)}^2}{{\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^3,x)
```

```
[Out] int((cosh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x))^3, x)
```

3.332 $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=631

$$-\frac{3f(e+fx)^2}{2b\left(a^2+b^2\right)d^2}+\frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3}+\frac{3af(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2b\left(a^2+b^2\right)^{3/2}d^2}+\frac{3f^2(e+f}{}$$

```
[Out] -3/2*f*(f*x+e)^2/b/(a^2+b^2)/d^2+3*f^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))/b/(a^2+b^2)/d^3+3/2*a*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)
^(1/2)))/b/(a^2+b^2)^(3/2)/d^2+3*f^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)
^(1/2)))/b/(a^2+b^2)/d^3-3/2*a*f*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))/b/(a^2+b^2)^(3/2)/d^2+3*f^3*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/
2)))/b/(a^2+b^2)/d^4+3*a*f^2*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(
1/2)))/b/(a^2+b^2)^(3/2)/d^3+3*f^3*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/
2)))/b/(a^2+b^2)/d^4-3*a*f^2*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))/b/(a^2+b^2)^(3/2)/d^3-3*a*f^3*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(
1/2)))/b/(a^2+b^2)^(3/2)/d^4+3*a*f^3*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))/b/(a^2+b^2)^(3/2)/d^4-1/2*(f*x+e)^3/b/d/(a+b*sinh(d*x+c))^2-3/2*f*(f
*x+e)^2*cosh(d*x+c)/(a^2+b^2)/d^2/(a+b*sinh(d*x+c))
```

Rubi [A]

time = 0.76, antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {5572, 3405, 3403, 2296, 2221, 2611, 2320, 6724, 5680, 2317, 2438}

$$\frac{3f^3\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4\left(a^2+b^2\right)}+\frac{3f^3\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^4\left(a^2+b^2\right)}-\frac{3af^3\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4\left(a^2+b^2\right)^{3/2}}+\frac{3af^3\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^4\left(a^2+b^2\right)^{3/2}}+\frac{3af^2(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}-\frac{3af^2(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}+\frac{3f^2(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd^3\left(a^2+b^2\right)}+\frac{3f^2(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd^3\left(a^2+b^2\right)}+\frac{3af(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{2bd^2\left(a^2+b^2\right)^{3/2}}-\frac{3af(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{2bd^2\left(a^2+b^2\right)^{3/2}}-\frac{3f(e+fx)^2\cosh(c+dx)}{2d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{3f(e+fx)^2}{2bd^2\left(a^2+b^2\right)}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] (-3*f*(e + f*x)^2)/(2*b*(a^2 + b^2)*d^2) + (3*f^2*(e + f*x)*Log[1 + (b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d^3) + (3*a*f*(e + f*x)^2*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(2*b*(a^2 + b^2)^(3/2)*d^2)
+ (3*f^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2
+ b^2)*d^3) - (3*a*f*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(2*b*(a^2 + b^2)^(3/2)*d^2) + (3*f^3*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^4) + (3*a*f^2*(e + f*x)*PolyLog[2, -
((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) + (3*f^
3*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^4)
 - (3*a*f^2*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])
/(b*(a^2 + b^2)^(3/2)*d^3) - (3*a*f^3*PolyLog[3, -((b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^4) + (3*a*f^3*PolyLog[3, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^4) - (e + f*x)^3/(2
```

```
*b*d*(a + b*Sinh[c + d*x])^2) - (3*f*(e + f*x)^2*Cosh[c + d*x])/(2*(a^2 + b
^2)*d^2*(a + b*Sinh[c + d*x]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5572

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sinh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = -\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2} + \frac{(3f)\int \frac{(e+fx)^2}{(a+b\sinh(c+dx))^2}\,dx}{2bd}$$

$$= -\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2} - \frac{3f(e+fx)^2\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))} + \frac{(3af)\int \frac{\cdots}{a+\cdots}}{2b\left(a^2\cdots\right.}$$

$$= -\frac{3f(e+fx)^2}{2b\left(a^2+b^2\right)d^2} - \frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2} - \frac{3f(e+fx)^2\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+\cdots}$$

$$= -\frac{3f(e+fx)^2}{2b\left(a^2+b^2\right)d^2} + \frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3} + \frac{3f^2(e+fx)\log\left(\cdots\right.}{b\left(a^2\cdots\right.}$$

$$= -\frac{3f(e+fx)^2}{2b\left(a^2+b^2\right)d^2} + \frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3} + \frac{3af(e+fx)^2\log\cdots}{2b\left(a^2\cdots\right.}$$

$$= -\frac{3f(e+fx)^2}{2b\left(a^2+b^2\right)d^2} + \frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3} + \frac{3af(e+fx)^2\log\cdots}{2b\left(a^2\cdots\right.}$$

$$= -\frac{3f(e+fx)^2}{2b\left(a^2+b^2\right)d^2} + \frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3} + \frac{3af(e+fx)^2\log\cdots}{2b\left(a^2\cdots\right.}$$

$$= -\frac{3f(e+fx)^2}{2b\left(a^2+b^2\right)d^2} + \frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3} + \frac{3af(e+fx)^2\log\cdots}{2b\left(a^2\cdots\right.}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 5753 vs. $2(631) = 1262$.
time = 7.41, size = 5753, normalized size = 9.12

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] Result too large to show
```

Maple [**F**]

time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \cosh(dx+c)}{(a+b\sinh(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 3*a*d*f^3*integrate(x^2*e^(d*x + c)/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*
e^(2*d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a^2*b
^2*d^2 - b^4*d^2), x) + 3*b*f^2*(a*log((b*e^(d*x + c) + a - sqrt(a^2 + b^2)
)/(b*e^(d*x + c) + a + sqrt(a^2 + b^2)))/((a^2*b^2 + b^4)*sqrt(a^2 + b^2)*d
^3) - 2*(d*x + c)/((a^2*b^2 + b^4)*d^3) + log(b*e^(2*d*x + 2*c) + 2*a*e^(d*
x + c) - b)/((a^2*b^2 + b^4)*d^3))*e + 6*a*d*f^2*integrate(x*e^(d*x + c + 1
)/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + 2*a^3*b*d^2*e^(d
*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a^2*b^2*d^2 - b^4*d^2), x) - 6*a*f^3*in
tegrate(x*e^(d*x + c)/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c
) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a^2*b^2*d^2 - b^4*d
^2), x) + 6*b*f^3*integrate(x/(a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d
*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + 2*a*b^3*d^2*e^(d*x + c) - a^2*b^2*d^2
 - b^4*d^2), x) + 3/2*f*(2*(a*b*e^(3*d*x + 3*c) - 3*a*b*e^(d*x + c) + b^2 +
 (2*a^2*e^(2*c) - b^2*e^(2*c) - 2*(a^2*d*e^(2*c) + b^2*d*e^(2*c))*x)*e^(2*d
*x))/(a^2*b^3*d^2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5*d^2*e^(4*c))*e^(4*
d*x) + 4*(a^3*b^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3*d*x) + 2*(2*a^4*b*d
^2*e^(2*c) + a^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^(2*d*x) - 4*(a^3*b^2*
d^2*e^c + a*b^4*d^2*e^c)*e^(d*x)) + a*log((b*e^(d*x + 2*c) + a*e^c - sqrt(a
^2 + b^2)*e^c)/(b*e^(d*x + 2*c) + a*e^c + sqrt(a^2 + b^2)*e^c))/((a^2*b + b
^3)*sqrt(a^2 + b^2)*d^2))*e^2 - 3*a*f^2*e*log((a*e + b*e^(d*x + c + 1) - sq
rt(a^2 + b^2)*e)/(a*e + b*e^(d*x + c + 1) + sqrt(a^2 + b^2)*e))/((a^2*b + b
^3)*sqrt(a^2 + b^2)*d^3) + (3*b^2*f^3*x^2 + 6*b^2*f^2*x*e + 3*(a*b*f^3*x^2*
e^(3*c) + 2*a*b*f^2*x*e^(3*c + 1))*e^(3*d*x) - (2*(a^2*d*f^3*e^(2*c) + b^2*
d*f^3*e^(2*c))*x^3 - 3*(2*a^2*f^3*e^(2*c) - b^2*f^3*e^(2*c) - 2*(a^2*d*f^2*
e^(2*c) + b^2*d*f^2*e^(2*c))*e)*x^2 - 6*(2*a^2*f^2*e^(2*c) - b^2*f^2*e^(2*c
))*x*e)*e^(2*d*x) - 9*(a*b*f^3*x^2*e^c + 2*a*b*f^2*x*e^(c + 1))*e^(d*x))/(a
^2*b^3*d^2 + b^5*d^2 + (a^2*b^3*d^2*e^(4*c) + b^5*d^2*e^(4*c))*e^(4*d*x) +
```

4*(a^3*b^2*d^2*e^(3*c) + a*b^4*d^2*e^(3*c))*e^(3*d*x) + 2*(2*a^4*b*d^2*e^(2*c) + a^2*b^3*d^2*e^(2*c) - b^5*d^2*e^(2*c))*e^(2*d*x) - 4*(a^3*b^2*d^2*e^c + a*b^4*d^2*e^c)*e^(d*x)) - 2*e^(-2*d*x - 2*c + 3)/((4*a*b^2*e^(-d*x - c) - 4*a*b^2*e^(-3*d*x - 3*c) + b^3*e^(-4*d*x - 4*c) + b^3 + 2*(2*a^2*b - b^3)*e^(-2*d*x - 2*c))*d)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 16887 vs. $2(584) = 1168$.
time = 0.63, size = 16887, normalized size = 26.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*(a^2*b^2 + b^4)*c^2*f^3 - 12*(a^2*b^2 + b^4)*c*d*f^2*cosh(1) + 6*(a^2*b^2 + b^4)*d^2*f*cosh(1)^2 + 6*(a^2*b^2 + b^4)*d^2*f*sinh(1)^2 - 6*((a^2*b^2 + b^4)*d^2*f^3*x^2 - (a^2*b^2 + b^4)*c^2*f^3 + 2*((a^2*b^2 + b^4)*d^2*f^2*x + (a^2*b^2 + b^4)*c*d*f^2)*cosh(1) + 2*((a^2*b^2 + b^4)*d^2*f^2*x + (a^2*b^2 + b^4)*c*d*f^2)*sinh(1))*cosh(d*x + c)^4 - 6*((a^2*b^2 + b^4)*d^2*f^3*x^2 - (a^2*b^2 + b^4)*c^2*f^3 + 2*((a^2*b^2 + b^4)*d^2*f^2*x + (a^2*b^2 + b^4)*c*d*f^2)*cosh(1) + 2*((a^2*b^2 + b^4)*d^2*f^2*x + (a^2*b^2 + b^4)*c*d*f^2)*sinh(1))*sinh(d*x + c)^4 - 6*(3*(a^3*b + a*b^3)*d^2*f^3*x^2 - 4*(a^3*b + a*b^3)*c^2*f^3 - (a^3*b + a*b^3)*d^2*f*cosh(1)^2 - (a^3*b + a*b^3)*d^2*f*sinh(1)^2 + 2*(3*(a^3*b + a*b^3)*d^2*f^2*x + 4*(a^3*b + a*b^3)*c*d*f^2)*cosh(1) + 2*(3*(a^3*b + a*b^3)*d^2*f^2*x + 4*(a^3*b + a*b^3)*c*d*f^2 - (a^3*b + a*b^3)*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^3 - 6*(3*(a^3*b + a*b^3)*d^2*f^3*x^2 - 4*(a^3*b + a*b^3)*c^2*f^3 - (a^3*b + a*b^3)*d^2*f*cosh(1)^2 - (a^3*b + a*b^3)*d^2*f*sinh(1)^2 + 2*(3*(a^3*b + a*b^3)*d^2*f^2*x + 4*(a^3*b + a*b^3)*c*d*f^2)*cosh(1) + 4*((a^2*b^2 + b^4)*d^2*f^3*x^2 - (a^2*b^2 + b^4)*c^2*f^3 + 2*((a^2*b^2 + b^4)*d^2*f^2*x + (a^2*b^2 + b^4)*c*d*f^2)*cosh(1) + 2*((a^2*b^2 + b^4)*d^2*f^2*x + (a^2*b^2 + b^4)*c*d*f^2)*sinh(1))*cosh(d*x + c) + 2*(3*(a^3*b + a*b^3)*d^2*f^2*x + 4*(a^3*b + a*b^3)*c*d*f^2 - (a^3*b + a*b^3)*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c)^3 - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 3*(2*a^4 + a^2*b^2 - b^4)*d^2*f^3*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(1)^3 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*sinh(1)^3 - 6*(2*a^4 + a^2*b^2 - b^4)*c^2*f^3 + 3*(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*f*x - (2*a^4 + a^2*b^2 - b^4)*d^2*f)*cosh(1)^2 + 3*(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(1) - (2*a^4 + a^2*b^2 - b^4)*d^2*f)*sinh(1)^2 + 6*((a^4 + 2*a^2*b^2 + b^4)*d^3*f^2*x^2 + (2*a^4 + a^2*b^2 - b^4)*d^2*f^2*x + 2*(2*a^4 + a^2*b^2 - b^4)*c*d*f^2)*cosh(1) + 6*((a^4 + 2*a^2*b^2 + b^4)*d^3*f^2*x^2 + (2*a^4 + a^2*b^2 - b^4)*d^2*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(1)^2 + 2*(2*a^4 + a^2*b^2 - b^4)*c*d*f^2 + (2*(a^4 + 2*a^2*b^2 + b^4)*d^3*f*x - (2*a^4 + a^2*b^2 - b^4)*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 3*(2*a^4 + a^2*b^
```

```
2 - b^4)*d^2*f^3*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(1)^3 + 2*(a^4 + 2
*a^2*b^2 + b^4)*d^3*sinh(1)^3 - 6*(2*a^4 + a^2*b^2 - b^4)*c^2*f^3 + 3*(2*(a
^4 + 2*a^2*b^2 + b^4)*d^3*f*x - (2*a^4 + a^2*b^2 - b^4)*d^2*f)*cosh(1)^2 +
18*((a^2*b^2 + b^4)*d^2*f^3*x^2 - (a^2*b^2 + b^4)*c^2*f^3 + 2*((a^2*b^2 + b
^4)*d^2*f^2*x + (a^2*b^2 + b^4)*c*d*f^2)*cosh(1) + 2*((a^2*b^2 + b^4)*d^2*f
^2*x + (a^2*b^2 + b^4)*c*d*f^2)*sinh(1))*cosh(d*x + c)^2 + 3*(2*(a^4 + 2*a^
2*b^2 + b^4)*d^3*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(1) - (2*a^4 + a^2
*b^2 - b^4)*d^2*f)*sinh(1)^2 + 6*((a^4 + 2*a^2*b^2 + b^4)*d^3*f^2*x^2 + (2*
a^4 + a^2*b^2 - b^4)*d^2*f^2*x + 2*(2*a^4 + a^2*b^2 - b^4)*c*d*f^2)*cosh(1)
 + 9*(3*(a^3*b + a*b^3)*d^2*f^3*x^2 - 4*(a^3*b + a*b^3)*c^2*f^3 - (a^3*b +
a*b^3)*d^2*f*cosh(1)^2 - (a^3*b + a*b^3)*d^2*f*sinh(1)^2 + 2*(3*(a^3*b + a*
b^3)*d^2*f^2*x + 4*(a^3*b + a*b^3)*c*d*f^2)*cosh(1) + 2*(3*(a^3*b + a*b^3)*
d^2*f^2*x + 4*(a^3*b + a*b^3)*c*d*f^2 - (a^3*b + a*b^3)*d^2*f*cosh(1))*sinh
(1))*cosh(d*x + c) + 6*((a^4 + 2*a^2*b^2 + b^4)*d^3*f^2*x^2 + (2*a^4 + a^2*
b^2 - b^4)*d^2*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(1)^2 + 2*(2*a^4 + a
^2*b^2 - b^4)*c*d*f^2 + (2*(a^4 + 2*a^2*b^2 + b^4)*d^3*f*x - (2*a^4 + a^2*b
^2 - b^4)*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 6*(a*b^3*f^3*cosh(d*x
+ c)^4 + a*b^3*f^3*sinh(d*x + c)^4 + 4*a^2*b^2*f^3*cosh(d*x + c)^3 - 4*a^2*
b^2*f^3*cosh(d*x + c) + a*b^3*f^3 + 2*(2*a^3*b - a*b^3)*f^3*cosh(d*x + c)^2
 + 4*(a*b^3*f^3*cosh(d*x + c) + a^2*b^2*f^3)*sinh(d*x + c)^3 + 2*(3*a*b^3*f
^3*cosh(d*x + c)^2 + 6*a^2*b^2*f^3*cosh(d*x + c) + (2*a^3*b - a*b^3)*f^3)*s
inh(d*x + c)^2 + 4*(a*b^3*f^3*cosh(d*x + c)^3 + 3*a^2*b^2*f^3*cosh(d*x + c)
^2 - a^2*b^2*f^3 + (2*a^3*b - a*b^3)*f^3*cosh(d*x + c))*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*(a*b^3*f^3*cosh(d*
x + c)^4 + a*b^3*f^3*sinh(d*x + c)^4 + 4*a^2*b^2*f^3*cosh(d*x + c)^3 - 4*a^
2*b^2*f^3*cosh(d*x + c) + a*b^3*f^3 + 2*(2*a^3*b - a*b^3)*f^3*cosh(d*x + c)
^2 + 4*(a*b^3*f^3*cosh(d*x + c) + a^2*b^2*f^3)*sinh(d*x + c)^3 + 2*(3*a*b^3
*f^3*cosh(d*x + c)^2 + 6*a^2*b^2*f^3*cosh(d*x + c) + (2*a^3*b - a*b^3)*f^3)
*sinh(d*x + c)^2 + 4*(a*b^3*f^3*cosh(d*x + c)^3 + 3*a^2*b^2*f^3*cosh(d*x +
c)^2 - a^2*b^2*f^3 + (2*a^3*b - a*b^3)*f^3*cosh(d*x + c))*sinh(d*x + c))*sq
rt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*((a^3*b + a*b^3)
*d^2*f^3*x^2 - 4*(a^3*b + a*b^3)*c^2*f^3 - 3*(a^3*b + a*b^3)*d^2*f*cosh(1)^
2 - 3*(a^3*b + a*b^3)*d^2*f*sinh(1)^2 + 2*((a^3*b + a*b^3)*d^2*f^2*x + 4*(a
^3*b + a*b^3)*c*d*f^2)*cosh(1) + 2*((a^3*b + a*b^3)*d^2*f^2*x + 4*(a^3*b +
a*b^3)*c*d*f^2 - 3*(a^3*b + a*b^3)*d^2*f*cosh(1...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((f*x + e)^3*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,{\left(e+f\,x\right)}^3}{{\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^3,x)
[Out] int((cosh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x))^3, x)
```

3.333 $\int \frac{(e+fx)^3 \cosh(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal. Leaf size=448

$$\frac{a(e+fx)^4}{4b^2f} - \frac{6f^3\cosh(c+dx)}{bd^4} - \frac{3f(e+fx)^2\cosh(c+dx)}{bd^2} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d} - \frac{a(e+fx)}{}$$

```
[Out] 1/4*a*(f*x+e)^4/b^2/f-6*f^3*cosh(d*x+c)/b/d^4-3*f*(f*x+e)^2*cosh(d*x+c)/b/d
^2-a*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d-a*(f*x+e)^3*ln(
1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d-3*a*f*(f*x+e)^2*polylog(2,-b*exp(
d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d^2-3*a*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)
/(a+(a^2+b^2)^(1/2)))/b^2/d^2+6*a*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a
^2+b^2)^(1/2)))/b^2/d^3+6*a*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2
)^(1/2)))/b^2/d^3-6*a*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/
d^4-6*a*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d^4+6*f^2*(f*x
+e)*sinh(d*x+c)/b/d^3+(f*x+e)^3*sinh(d*x+c)/b/d
```

Rubi [A]
time = 0.46, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {5698, 3377, 2718, 5680, 2221, 2611, 6744, 2320, 6724}

$$-\frac{6af^3\text{Li}_4\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^4} - \frac{6af^3\text{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^4} + \frac{6af^2(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3} + \frac{6af^2(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^3} - \frac{3af(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2} - \frac{3af(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^2} - \frac{a(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^2d} - \frac{a(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^2d} + \frac{a(e+fx)^4}{4b^2f} - \frac{6f^3\cosh(c+dx)}{bd^4} + \frac{6f^2(e+fx)\sinh(c+dx)}{bd^3} - \frac{3f(e+fx)^2\cosh(c+dx)}{bd^2} + \frac{(e+fx)^3\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*(e + f*x)^4)/(4*b^2*f) - (6*f^3*Cosh[c + d*x])/(b*d^4) - (3*f*(e + f*x)^
2*Cosh[c + d*x])/(b*d^2) - (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2])])/(b^2*d) - (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/(b^2*d) - (3*a*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/(b^2*d^2) - (3*a*f*(e + f*x)^2*PolyLog[2, -((b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^2) + (6*a*f^2*(e + f*x)*PolyLog[3, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^3) + (6*a*f^2*(e + f*x)*Pol
yLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^3) - (6*a*f^3*Pol
yLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^4) - (6*a*f^3*Pol
yLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^4) + (6*f^2*(e +
f*x)*Sinh[c + d*x])/(b*d^3) + ((e + f*x)^3*Sinh[c + d*x])/(b*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\cosh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{(e+fx)^3\sinh(c+dx)}{bd} - \frac{a\int \frac{e^{c+dx}(e+fx)^3}{a-\sqrt{a^2+b^2}+be^{c+dx}} \cdots}{b}$$

$$= \frac{a(e+fx)^4}{4b^2f} - \frac{3f(e+fx)^2\cosh(c+dx)}{bd^2} - \frac{a(e+fx)^3\log\left(1+\cdots\right.}{b^2d\cdots}$$

$$= \frac{a(e+fx)^4}{4b^2f} - \frac{3f(e+fx)^2\cosh(c+dx)}{bd^2} - \frac{a(e+fx)^3\log\left(1+\cdots\right.}{b^2d\cdots}$$

$$= \frac{a(e+fx)^4}{4b^2f} - \frac{6f^3\cosh(c+dx)}{bd^4} - \frac{3f(e+fx)^2\cosh(c+dx)}{bd^2} - \cdots$$

$$= \frac{a(e+fx)^4}{4b^2f} - \frac{6f^3\cosh(c+dx)}{bd^4} - \frac{3f(e+fx)^2\cosh(c+dx)}{bd^2} - \cdots$$

$$= \frac{a(e+fx)^4}{4b^2f} - \frac{6f^3\cosh(c+dx)}{bd^4} - \frac{3f(e+fx)^2\cosh(c+dx)}{bd^2} - \cdots$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 2945 vs. $2(448) = 896$.

time = 15.28, size = 2945, normalized size = 6.57

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] ((a*(4*e^3*E^(2*c)*x + 6*e^2*E^(2*c)*f*x^2 + 4*e*E^(2*c)*f^2*x^3 + E^(2*c)*
f^3*x^4 + (4*a*Sqrt[a^2 + b^2]*e^3*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b
^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (4*a*Sqrt[-(a^2 + b^2)^2]*e^3*E^(2*c)*ArcT
an[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (4*a*Sqrt
[-(a^2 + b^2)^2]*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 -
 b^2)^(3/2)*d) + (4*a*Sqrt[-(a^2 + b^2)^2]*e^3*E^(2*c)*ArcTanh[(a + b*E^(c
+ d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (2*e^3*Log[2*a*E^(c + d*
x) + b*(-1 + E^(2*(c + d*x)))])/d - (2*e^3*E^(2*c)*Log[2*a*E^(c + d*x) + b*
(-1 + E^(2*(c + d*x)))])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c -
Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x
))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1 + (b*E^(2*c
 + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f^2*x^2*Log
[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (2*f^3*x^3
*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (2*E^(
2*c)*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])]
)/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c
)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 +
 b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[
(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/
(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (2*f^3*x^3*Log[1 + (b*E^(2*c + d*
x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (2*E^(2*c)*f^3*x^3*Log[1 + (b
*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*(-1 + E^(2*c))
*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(
2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*
x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (12*e*f^2*PolyLog[3, -((b*
E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*e*E^(2*c)*f
^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^
3 - (12*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)]))])/d^3 + (12*E^(2*c)*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sq
rt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (12*e*f^2*PolyLog[3, -((b*E^(2*c + d*x))/
(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*e*E^(2*c)*f^2*PolyLog[3, -
((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (12*f^3*x*P
olyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 +
(12*E^(2*c)*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*
E^(2*c)]))])/d^3 + (12*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^
2 + b^2)*E^(2*c)]))])/d^4 - (12*E^(2*c)*f^3*PolyLog[4, -((b*E^(2*c + d*x))/
(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^4 + (12*f^3*PolyLog[4, -((b*E^(2*c
```

```
 + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^4 - (12*E^(2*c)*f^3*PolyL
og[4, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^4))/(b^2
*(-1 + E^(2*c))) + Csch[c]*(Cosh[c + d*x]/(4*b^2*d^4) - Sinh[c + d*x]/(4*b^
2*d^4))*(-4*a*d^4*e^3*x*Cosh[d*x] - 6*a*d^4*e^2*f*x^2*Cosh[d*x] - 4*a*d^4*e
*f^2*x^3*Cosh[d*x] - a*d^4*f^3*x^4*Cosh[d*x] - 4*a*d^4*e^3*x*Cosh[2*c + d*x
] - 6*a*d^4*e^2*f*x^2*Cosh[2*c + d*x] - 4*a*d^4*e*f^2*x^3*Cosh[2*c + d*x] -
 a*d^4*f^3*x^4*Cosh[2*c + d*x] - 2*b*d^3*e^3*Cosh[c + 2*d*x] + 6*b*d^2*e^2*
f*Cosh[c + 2*d*x] - 12*b*d*e*f^2*Cosh[c + 2*d*x] + 12*b*f^3*Cosh[c + 2*d*x]
 - 6*b*d^3*e^2*f*x*Cosh[c + 2*d*x] + 12*b*d^2*e*f^2*x*Cosh[c + 2*d*x] - 12*
b*d*f^3*x*Cosh[c + 2*d*x] - 6*b*d^3*e*f^2*x^2*Cosh[c + 2*d*x] + 6*b*d^2*f^3
*x^2*Cosh[c + 2*d*x] - 2*b*d^3*f^3*x^3*Cosh[c + 2*d*x] + 2*b*d^3*e^3*Cosh[3
*c + 2*d*x] - 6*b*d^2*e^2*f*Cosh[3*c + 2*d*x] + 12*b*d*e*f^2*Cosh[3*c + 2*d
*x] - 12*b*f^3*Cosh[3*c + 2*d*x] + 6*b*d^3*e^2*f*x*Cosh[3*c + 2*d*x] - 12*b
*d^2*e*f^2*x*Cosh[3*c + 2*d*x] + 12*b*d*f^3*x*Cosh[3*c + 2*d*x] + 6*b*d^3*e
*f^2*x^2*Cosh[3*c + 2*d*x] - 6*b*d^2*f^3*x^2*Cosh[3*c + 2*d*x] + 2*b*d^3*f^
3*x^3*Cosh[3*c + 2*d*x] - 4*b*d^3*e^3*Sinh[c] - 12*b*d^2*e^2*f*Sinh[c] - 24
*b*d*e*f^2*Sinh[c] - 24*b*f^3*Sinh[c] - 12*b*d^3*e^2*f*x*Sinh[c] - 24*b*d^2
*e*f^2*x*Sinh[c] - 24*b*d*f^3*x*Sinh[c] - 12*b*d^3*e*f^2*x^2*Sinh[c] - 12*b
*d^2*f^3*x^2*Sinh[c] - 4*b*d^3*f^3*x^3*Sinh[c] - 4*a*d^4*e^3*x*Sinh[d*x] -
6*a*d^4*e^2*f*x^2*Sinh[d*x] - 4*a*d^4*e*f^2*x^3*Sinh[d*x] - a*d^4*f^3*x^4*S
inh[d*x] - 4*a*d^4*e^3*x*Sinh[2*c + d*x] - 6*a*d^4*e^2*f*x^2*Sinh[2*c + d*x
] - 4*a*d^4*e*f^2*x^3*Sinh[2*c + d*x] - a*d^4*f^3*x^4*Sinh[2*c + d*x] - 2*b
*d^3*e^3*Sinh[c + 2*d*x] + 6*b*d^2*e^2*f*Sinh[c + 2*d*x] - 12*b*d*e*f^2*Sin
h[c + 2*d*x] + 12*b*f^3*Sinh[c + 2*d*x] - 6*b*d^3*e^2*f*x*Sinh[c + 2*d*x] +
 12*b*d^2*e*f^2*x*Sinh[c + 2*d*x] - 12*b*d*f^3*x*Sinh[c + 2*d*x] - 6*b*d^3*
e*f^2*x^2*Sinh[c + 2*d*x] + 6*b*d^2*f^3*x^2*Sinh[c + 2*d*x] - 2*b*d^3*f^3*x
^3*Sinh[c + 2*d*x] + 2*b*d^3*e^3*Sinh[3*c + 2*d*x] - 6*b*d^2*e^2*f*Sinh[3*c
 + 2*d*x] + 12*b*d*e*f^2*Sinh[3*c + 2*d*x] - 12*b*f^3*Sinh[3*c + 2*d*x] + 6
*b*d^3*e^2*f*x*Sinh[3*c + 2*d*x] - 12*b*d^2*e*f^2*x*Sinh[3*c + 2*d*x] + 12*
b*d*f^3*x*Sinh[3*c + 2*d*x] + 6*b*d^3*e*f^2*x^2...
```

Maple [F]
time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \cosh(dx+c)\sinh(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/2*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) + 2*a*
log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^2*d))*e^3 - 1/4*(a*d^4*f
^3*x^4*e^c + 4*a*d^4*f^2*x^3*e^(c + 1) + 6*a*d^4*f*x^2*e^(c + 2) - 2*(b*d^3
*f^3*x^3*e^(2*c) - 6*b*f^3*e^(2*c) - 3*b*d^2*f*e^(2*c + 2) + 6*b*d*f^2*e^(2
*c + 1) - 3*(b*d^2*f^3*e^(2*c) - b*d^3*f^2*e^(2*c + 1))*x^2 + 3*(2*b*d*f^3*
e^(2*c) + b*d^3*f*e^(2*c + 2) - 2*b*d^2*f^2*e^(2*c + 1))*x)*e^(d*x) + 2*(b*
d^3*f^3*x^3 + 3*b*d^2*f*e^2 + 6*b*d*f^2*e + 6*b*f^3 + 3*(b*d^3*f^2*e + b*d^
2*f^3)*x^2 + 3*(b*d^3*f*e^2 + 2*b*d^2*f^2*e + 2*b*d*f^3)*x)*e^(-d*x))*e^(-c
)/(b^2*d^4) + integrate(-2*(a*b*f^3*x^3 + 3*a*b*f^2*x^2*e + 3*a*b*f*x*e^2 -
 (a^2*f^3*x^3*e^c + 3*a^2*f^2*x^2*e^(c + 1) + 3*a^2*f*x*e^(c + 2))*e^(d*x))
/(b^3*e^(2*d*x + 2*c) + 2*a*b^2*e^(d*x + c) - b^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3363 vs. $2(430) = 860$.
time = 0.49, size = 3363, normalized size = 7.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -1/4*(2*b*d^3*f^3*x^3 + 6*b*d^2*f^3*x^2 + 2*b*d^3*cosh(1)^3 + 2*b*d^3*sinh(
1)^3 + 12*b*d*f^3*x + 12*b*f^3 + 6*(b*d^3*f*x + b*d^2*f)*cosh(1)^2 - 2*(b*d
^3*f^3*x^3 - 3*b*d^2*f^3*x^2 + b*d^3*cosh(1)^3 + b*d^3*sinh(1)^3 + 6*b*d*f^
3*x - 6*b*f^3 + 3*(b*d^3*f*x - b*d^2*f)*cosh(1)^2 + 3*(b*d^3*f*x + b*d^3*co
sh(1) - b*d^2*f)*sinh(1)^2 + 3*(b*d^3*f^2*x^2 - 2*b*d^2*f^2*x + 2*b*d*f^2)*
cosh(1) + 3*(b*d^3*f^2*x^2 - 2*b*d^2*f^2*x + b*d^3*cosh(1)^2 + 2*b*d*f^2 +
2*(b*d^3*f*x - b*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 6*(b*d^3*f*x +
b*d^3*cosh(1) + b*d^2*f)*sinh(1)^2 - 2*(b*d^3*f^3*x^3 - 3*b*d^2*f^3*x^2 + b
*d^3*cosh(1)^3 + b*d^3*sinh(1)^3 + 6*b*d*f^3*x - 6*b*f^3 + 3*(b*d^3*f*x - b
*d^2*f)*cosh(1)^2 + 3*(b*d^3*f*x + b*d^3*cosh(1) - b*d^2*f)*sinh(1)^2 + 3*(
b*d^3*f^2*x^2 - 2*b*d^2*f^2*x + 2*b*d*f^2)*cosh(1) + 3*(b*d^3*f^2*x^2 - 2*b
*d^2*f^2*x + b*d^3*cosh(1)^2 + 2*b*d*f^2 + 2*(b*d^3*f*x - b*d^2*f)*cosh(1))
*sinh(1))*sinh(d*x + c)^2 + 6*(b*d^3*f^2*x^2 + 2*b*d^2*f^2*x + 2*b*d*f^2)*c
osh(1) - (a*d^4*f^3*x^4 - 2*a*c^4*f^3 + 4*(a*d^4*x + 2*a*c*d^3)*cosh(1)^3 +
 4*(a*d^4*x + 2*a*c*d^3)*sinh(1)^3 + 6*(a*d^4*f*x^2 - 2*a*c^2*d^2*f)*cosh(1
)^2 + 6*(a*d^4*f*x^2 - 2*a*c^2*d^2*f + 2*(a*d^4*x + 2*a*c*d^3)*cosh(1))*sin
h(1)^2 + 4*(a*d^4*f^2*x^3 + 2*a*c^3*d*f^2)*cosh(1) + 4*(a*d^4*f^2*x^3 + 2*a
*c^3*d*f^2 + 3*(a*d^4*x + 2*a*c*d^3)*cosh(1)^2 + 3*(a*d^4*f*x^2 - 2*a*c^2*d
^2*f)*cosh(1))*sinh(1))*cosh(d*x + c) + 12*((a*d^2*f^3*x^2 + 2*a*d^2*f^2*x*
```

cosh(1) + a*d^2*f*cosh(1)^2 + a*d^2*f*sinh(1)^2 + 2*(a*d^2*f^2*x + a*d^2*f*
cosh(1))*sinh(1))*cosh(d*x + c) + (a*d^2*f^3*x^2 + 2*a*d^2*f^2*x*cosh(1) +
a*d^2*f*cosh(1)^2 + a*d^2*f*sinh(1)^2 + 2*(a*d^2*f^2*x + a*d^2*f*cosh(1))*s
inh(1))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 12*((a*d^2*f
^3*x^2 + 2*a*d^2*f^2*x*cosh(1) + a*d^2*f*cosh(1)^2 + a*d^2*f*sinh(1)^2 + 2*
(a*d^2*f^2*x + a*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c) + (a*d^2*f^3*x^2 + 2
*a*d^2*f^2*x*cosh(1) + a*d^2*f*cosh(1)^2 + a*d^2*f*sinh(1)^2 + 2*(a*d^2*f^2
*x + a*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*si
nh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b
)/b + 1) - 4*((a*c^3*f^3 - 3*a*c^2*d*f^2*cosh(1) + 3*a*c*d^2*f*cosh(1)^2 -
a*d^3*cosh(1)^3 - a*d^3*sinh(1)^3 + 3*(a*c*d^2*f - a*d^3*cosh(1))*sinh(1)^2
 - 3*(a*c^2*d*f^2 - 2*a*c*d^2*f*cosh(1) + a*d^3*cosh(1)^2)*sinh(1))*cosh(d*
x + c) + (a*c^3*f^3 - 3*a*c^2*d*f^2*cosh(1) + 3*a*c*d^2*f*cosh(1)^2 - a*d^3
*cosh(1)^3 - a*d^3*sinh(1)^3 + 3*(a*c*d^2*f - a*d^3*cosh(1))*sinh(1)^2 - 3*
(a*c^2*d*f^2 - 2*a*c*d^2*f*cosh(1) + a*d^3*cosh(1)^2)*sinh(1))*sinh(d*x + c
))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) +
2*a) - 4*((a*c^3*f^3 - 3*a*c^2*d*f^2*cosh(1) + 3*a*c*d^2*f*cosh(1)^2 - a*d^
3*cosh(1)^3 - a*d^3*sinh(1)^3 + 3*(a*c*d^2*f - a*d^3*cosh(1))*sinh(1)^2 - 3
*(a*c^2*d*f^2 - 2*a*c*d^2*f*cosh(1) + a*d^3*cosh(1)^2)*sinh(1))*cosh(d*x +
c) + (a*c^3*f^3 - 3*a*c^2*d*f^2*cosh(1) + 3*a*c*d^2*f*cosh(1)^2 - a*d^3*cos
h(1)^3 - a*d^3*sinh(1)^3 + 3*(a*c*d^2*f - a*d^3*cosh(1))*sinh(1)^2 - 3*(a*c
^2*d*f^2 - 2*a*c*d^2*f*cosh(1) + a*d^3*cosh(1)^2)*sinh(1))*sinh(d*x + c))*l
og(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
 + 4*((a*d^3*f^3*x^3 + a*c^3*f^3 + 3*(a*d^3*f*x + a*c*d^2*f)*cosh(1)^2 + 3*
(a*d^3*f*x + a*c*d^2*f)*sinh(1)^2 + 3*(a*d^3*f^2*x^2 - a*c^2*d*f^2)*cosh(1)
 + 3*(a*d^3*f^2*x^2 - a*c^2*d*f^2 + 2*(a*d^3*f*x + a*c*d^2*f)*cosh(1))*sinh
(1))*cosh(d*x + c) + (a*d^3*f^3*x^3 + a*c^3*f^3 + 3*(a*d^3*f*x + a*c*d^2*f)
cosh(1)^2 + 3(a*d^3*f*x + a*c*d^2*f)*sinh(1)^2 + 3*(a*d^3*f^2*x^2 - a*c^2
*d*f^2)*cosh(1) + 3*(a*d^3*f^2*x^2 - a*c^2*d*f^2 + 2*(a*d^3*f*x + a*c*d^2*f
)*cosh(1))*sinh(1))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
+ (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 4*((a
*d^3*f^3*x^3 + a*c^3*f^3 + 3*(a*d^3*f*x + a*c*d^2*f)*cosh(1)^2 + 3*(a*d^3*f
*x + a*c*d^2*f)*sinh(1)^2 + 3*(a*d^3*f^2*x^2 - a*c^2*d*f^2)*cosh(1) + 3*(a*
d^3*f^2*x^2 - a*c^2*d*f^2 + 2*(a*d^3*f*x + a*c*d^2*f)*cosh(1))*sinh(1))*cos
h(d*x + c) + (a*d^3*f^3*x^3 + a*c^3*f^3 + 3*(a*d^3*f*x + a*c*d^2*f)*cosh(1)
^2 + 3*(a*d^3*f*x + a*c*d^2*f)*sinh(1)^2 + 3*(a*d^3*f^2*x^2 - a*c^2*d*f^2)*
cosh(1) + 3*(a*d^3*f^2*x^2 - a*c^2*d*f^2 + 2*(a*d^3*f*x + a*c*d^2*f)*cosh(1
))*sinh(1))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 24*(a*f^3*cos
h(d*x + c) + a*f^3*sinh(d*x + c))*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x
+ c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24*(
a*f^3*cosh(d*x + c) + a*f^3*sinh(d*x + c))*polylog(4, (a*cosh(d*x + c) + a*
sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/
b) - 24*((a*d*f^3*x + a*d*f^2*cosh(1) + a*d*f^2*sinh(1))*cosh(d*x + c) + (a

```
*d*f^3*x + a*d*f^2*cosh(1) + a*d*f^2*sinh(1))*sinh(d*x + c))*polylog(3, (a*
cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2))/b) - 24*((a*d*f^3*x + a*d*f^2...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)\,{\left(e+f\,x\right)}^3}{a+b\,\mathrm{sinh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((cosh(c + d*x)*sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.334 $\int \frac{(e+fx)^2 \cosh(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal. Leaf size=330

$$\frac{a(e+fx)^3}{3b^2f} - \frac{2f(e+fx)\cosh(c+dx)}{bd^2} - \frac{a(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d} - \frac{a(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+}}\right.}{b^2d}$$

```
[Out] 1/3*a*(f*x+e)^3/b^2/f-2*f*(f*x+e)*cosh(d*x+c)/b/d^2-a*(f*x+e)^2*ln(1+b*exp(
d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d-a*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2
)^(1/2)))/b^2/d-2*a*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/
b^2/d^2-2*a*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d^2+
2*a*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d^3+2*a*f^2*polylo
g(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d^3+2*f^2*sinh(d*x+c)/b/d^3+(f*x
+e)^2*sinh(d*x+c)/b/d
```

Rubi [A]

time = 0.39, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5698, 3377, 2717, 5680, 2221, 2611, 2320, 6724}

$$\frac{2af^2\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3} + \frac{2af^2\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^3} - \frac{2af(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2} - \frac{2af(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^2} - \frac{a(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^2d} - \frac{a(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^2d} + \frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\sinh(c+dx)}{bd^3} - \frac{2f(e+fx)\cosh(c+dx)}{bd^2} + \frac{(e+fx)^2\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*(e + f*x)^3)/(3*b^2*f) - (2*f*(e + f*x)*Cosh[c + d*x])/(b*d^2) - (a*(e +
 f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*d) - (a*(e + f
*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*d) - (2*a*f*(e +
 f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^2) - (2*
a*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^
2) + (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^
3) + (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^
3) + (2*f^2*Sinh[c + d*x])/(b*d^3) + ((e + f*x)^2*Sinh[c + d*x])/(b*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{a(e+fx)^3}{3b^2f} + \frac{(e+fx)^2\sinh(c+dx)}{bd} - \frac{a\int \frac{e^{c+dx}(e+fx)^2}{a-\sqrt{a^2+b^2}+be^{c+dx}}}{b}$$

$$= \frac{a(e+fx)^3}{3b^2f} - \frac{2f(e+fx)\cosh(c+dx)}{bd^2} - \frac{a(e+fx)^2\log\left(1+\right.}{b^2d}$$

$$= \frac{a(e+fx)^3}{3b^2f} - \frac{2f(e+fx)\cosh(c+dx)}{bd^2} - \frac{a(e+fx)^2\log\left(1+\right.}{b^2d}$$

$$= \frac{a(e+fx)^3}{3b^2f} - \frac{2f(e+fx)\cosh(c+dx)}{bd^2} - \frac{a(e+fx)^2\log\left(1+\right.}{b^2d}$$

$$= \frac{a(e+fx)^3}{3b^2f} - \frac{2f(e+fx)\cosh(c+dx)}{bd^2} - \frac{a(e+fx)^2\log\left(1+\right.}{b^2d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1301 vs. $2(330) = 660$.
time = 13.17, size = 1301, normalized size = 3.94

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x
]

[Out] ((2*a*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[
a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 +
b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x
))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2
*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*
a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b
^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c
 + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)
))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*
```

```
c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 +
b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^
2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^
c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*
E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c +
 d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2
*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log
[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*(-1 + E
^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2
)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c +
 d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*f^2*PolyLog[3, -((b*
E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*
PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 -
 (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))
])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 +
 b^2)*E^(2*c)]))])/d^3))/(3*b^2*(-1 + E^(2*c))) - (a*x*(3*e^2 + 3*e*f*x + f
^2*x^2)*Cosh[c]*Csch[c/2]*Sech[c/2])/(3*b^2) + (2*Cosh[d*x]*(-2*d*e*f*Cosh[
c] - 2*d*f^2*x*Cosh[c] + d^2*e^2*Sinh[c] + 2*f^2*Sinh[c] + 2*d^2*e*f*x*Sinh
[c] + d^2*f^2*x^2*Sinh[c]))/(b*d^3) + (2*(d^2*e^2*Cosh[c] + 2*f^2*Cosh[c] +
 2*d^2*e*f*x*Cosh[c] + d^2*f^2*x^2*Cosh[c] - 2*d*e*f*Sinh[c] - 2*d*f^2*x*Si
nh[c])*Sinh[d*x])/(b*d^3))/2
```

Maple [**F**]
time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \cosh(dx+c)\sinh(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/2*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) + 2*a*
log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^2*d))*e^2 - 1/6*(2*a*d^3
*f^2*x^3*e^c + 6*a*d^3*f*x^2*e^(c + 1) - 3*(b*d^2*f^2*x^2*e^(2*c) + 2*b*f^2
*e^(2*c) - 2*b*d*f*e^(2*c + 1) - 2*(b*d*f^2*e^(2*c) - b*d^2*f*e^(2*c + 1))*
```

```
x)*e^(d*x) + 3*(b*d^2*f^2*x^2 + 2*b*d*f*e + 2*b*f^2 + 2*(b*d^2*f*e + b*d*f^
2)*x)*e^(-d*x))*e^(-c)/(b^2*d^3) + integrate(-2*(a*b*f^2*x^2 + 2*a*b*f*x*e
- (a^2*f^2*x^2*e^c + 2*a^2*f*x*e^(c + 1))*e^(d*x))/(b^3*e^(2*d*x + 2*c) + 2
*a*b^2*e^(d*x + c) - b^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1745 vs. $2(315) = 630$.
time = 0.38, size = 1745, normalized size = 5.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -1/6*(3*b*d^2*f^2*x^2 + 6*b*d*f^2*x + 3*b*d^2*cosh(1)^2 + 3*b*d^2*sinh(1)^2
 + 6*b*f^2 - 3*(b*d^2*f^2*x^2 - 2*b*d*f^2*x + b*d^2*cosh(1)^2 + b*d^2*sinh(
1)^2 + 2*b*f^2 + 2*(b*d^2*f*x - b*d*f)*cosh(1) + 2*(b*d^2*f*x + b*d^2*cosh(
1) - b*d*f)*sinh(1))*cosh(d*x + c)^2 - 3*(b*d^2*f^2*x^2 - 2*b*d*f^2*x + b*d
^2*cosh(1)^2 + b*d^2*sinh(1)^2 + 2*b*f^2 + 2*(b*d^2*f*x - b*d*f)*cosh(1) +
2*(b*d^2*f*x + b*d^2*cosh(1) - b*d*f)*sinh(1))*sinh(d*x + c)^2 + 6*(b*d^2*f
*x + b*d*f)*cosh(1) - 2*(a*d^3*f^2*x^3 + 2*a*c^3*f^2 + 3*(a*d^3*x + 2*a*c*d
^2)*cosh(1)^2 + 3*(a*d^3*x + 2*a*c*d^2)*sinh(1)^2 + 3*(a*d^3*f*x^2 - 2*a*c^
2*d*f)*cosh(1) + 3*(a*d^3*f*x^2 - 2*a*c^2*d*f + 2*(a*d^3*x + 2*a*c*d^2)*cos
h(1))*sinh(1))*cosh(d*x + c) + 12*((a*d*f^2*x + a*d*f*cosh(1) + a*d*f*sinh(
1))*cosh(d*x + c) + (a*d*f^2*x + a*d*f*cosh(1) + a*d*f*sinh(1))*sinh(d*x +
c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 12*((a*d*f^2*x + a*d*f*cosh(1)
+ a*d*f*sinh(1))*cosh(d*x + c) + (a*d*f^2*x + a*d*f*cosh(1) + a*d*f*sinh(1)
)*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 6*((a*c^2*f^2 - 2*
a*c*d*f*cosh(1) + a*d^2*cosh(1)^2 + a*d^2*sinh(1)^2 - 2*(a*c*d*f - a*d^2*co
sh(1))*sinh(1))*cosh(d*x + c) + (a*c^2*f^2 - 2*a*c*d*f*cosh(1) + a*d^2*cosh
(1)^2 + a*d^2*sinh(1)^2 - 2*(a*c*d*f - a*d^2*cosh(1))*sinh(1))*sinh(d*x + c
))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) +
2*a) + 6*((a*c^2*f^2 - 2*a*c*d*f*cosh(1) + a*d^2*cosh(1)^2 + a*d^2*sinh(1)^
2 - 2*(a*c*d*f - a*d^2*cosh(1))*sinh(1))*cosh(d*x + c) + (a*c^2*f^2 - 2*a*c
*d*f*cosh(1) + a*d^2*cosh(1)^2 + a*d^2*sinh(1)^2 - 2*(a*c*d*f - a*d^2*cosh(
1))*sinh(1))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b
*sqrt((a^2 + b^2)/b^2) + 2*a) + 6*((a*d^2*f^2*x^2 - a*c^2*f^2 + 2*(a*d^2*f*
x + a*c*d*f)*cosh(1) + 2*(a*d^2*f*x + a*c*d*f)*sinh(1))*cosh(d*x + c) + (a*
d^2*f^2*x^2 - a*c^2*f^2 + 2*(a*d^2*f*x + a*c*d*f)*cosh(1) + 2*(a*d^2*f*x +
a*c*d*f)*sinh(1))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) +
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 6*((a*d
^2*f^2*x^2 - a*c^2*f^2 + 2*(a*d^2*f*x + a*c*d*f)*cosh(1) + 2*(a*d^2*f*x + a
```

```
*c*d*f)*sinh(1))*cosh(d*x + c) + (a*d^2*f^2*x^2 - a*c^2*f^2 + 2*(a*d^2*f*x
+ a*c*d*f)*cosh(1) + 2*(a*d^2*f*x + a*c*d*f)*sinh(1))*sinh(d*x + c))*log(-(
a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqr
t((a^2 + b^2)/b^2) - b)/b) - 12*(a*f^2*cosh(d*x + c) + a*f^2*sinh(d*x + c))
*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 12*(a*f^2*cosh(d*x + c) + a*f^2*sinh(
d*x + c))*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*(b*d^2*f*x + b*d^2*cosh(1)
 + b*d*f)*sinh(1) - 2*(a*d^3*f^2*x^3 + 2*a*c^3*f^2 + 3*(a*d^3*x + 2*a*c*d^2
)*cosh(1)^2 + 3*(a*d^3*x + 2*a*c*d^2)*sinh(1)^2 + 3*(a*d^3*f*x^2 - 2*a*c^2*
d*f)*cosh(1) + 3*(b*d^2*f^2*x^2 - 2*b*d*f^2*x + b*d^2*cosh(1)^2 + b*d^2*sin
h(1)^2 + 2*b*f^2 + 2*(b*d^2*f*x - b*d*f)*cosh(1) + 2*(b*d^2*f*x + b*d^2*cos
h(1) - b*d*f)*sinh(1))*cosh(d*x + c) + 3*(a*d^3*f*x^2 - 2*a*c^2*d*f + 2*(a*
d^3*x + 2*a*c*d^2)*cosh(1))*sinh(1))*sinh(d*x + c))/(b^2*d^3*cosh(d*x + c)
+ b^2*d^3*sinh(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2 \sinh(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)**2*sinh(c + d*x)*cosh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
[Out] integrate((f*x + e)^2*cosh(d*x + c)*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)\,{(e+f\,x)}^2}{a+b\sinh(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)*sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.335 $\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=212

$$\frac{a(e+fx)^2}{2b^2f}-\frac{f\cosh(c+dx)}{bd^2}-\frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{afP\ldots}{}$$

```
[Out] 1/2*a*(f*x+e)^2/b^2/f-f*cosh(d*x+c)/b/d^2-a*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a
^2+b^2)^(1/2)))/b^2/d-a*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/
d-a*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/d^2-a*f*polylog(2,-b
*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/d^2+(f*x+e)*sinh(d*x+c)/b/d
```

Rubi [A]
time = 0.23, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {5698, 3377, 2718, 5680, 2221, 2317, 2438}

$$-\frac{af\,\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2}-\frac{af\,\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^2}-\frac{a(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^2d}-\frac{a(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^2d}+\frac{a(e+fx)^2}{2b^2f}-\frac{f\cosh(c+dx)}{bd^2}+\frac{(e+fx)\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*(e + f*x)^2)/(2*b^2*f) - (f*Cosh[c + d*x])/(b*d^2) - (a*(e + f*x)*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*d) - (a*(e + f*x)*Log[1 + (b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*d) - (a*f*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^2) - (a*f*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^2) + ((e + f*x)*Sinh[c + d*x])/(b*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{a(e+fx)^2}{2b^2 f} + \frac{(e+fx)\sinh(c+dx)}{bd} - \frac{a\int \frac{e^{c+dx}(e+fx)}{a-\sqrt{a^2+b^2}+be^{c+dx}}\,c}{b}$$

$$= \frac{a(e+fx)^2}{2b^2 f} - \frac{f\cosh(c+dx)}{bd^2} - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{b^2 d}$$

$$= \frac{a(e+fx)^2}{2b^2 f} - \frac{f\cosh(c+dx)}{bd^2} - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{b^2 d}$$

$$= \frac{a(e+fx)^2}{2b^2 f} - \frac{f\cosh(c+dx)}{bd^2} - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{b^2 d}$$

Mathematica [A]
time = 0.67, size = 206, normalized size = 0.97

$$\frac{-bf\cosh(c+dx) - a\left(-\frac{1}{2}f(c+dx)^2 + f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) + f(c+dx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right) + de\log(a+b\sinh(c+dx)) - cf\log(a+b\sinh(c+dx)) + f\mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right) + f\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right) + bd(e+fx)\sinh(c+dx)}{b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-(b*f*Cosh[c + d*x]) - a*(-1/2*(f*(c + d*x)^2) + f*(c + d*x)*Log[1 + (b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c +
 d*x]] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) + b*d*(e + f*x)*Sinh[c + d*x])
/(b^2*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(198) = 396$.
time = 1.29, size = 483, normalized size = 2.28

method	result
risch	$\frac{afx^2}{2b^2} - \frac{aex}{b^2} + \frac{(dxf+de-f)e^{dx+c}}{2d^2b} - \frac{(dxf+de+f)e^{-dx-c}}{2d^2b} - \frac{2afc\ln(e^{dx+c})}{d^2b^2} + \frac{afc\ln(b\,e^{2dx+2c}+2a\,e^{dx+c}-b)}{d^2b^2} + \frac{2ae\ln(e}{db^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/2*a*f*x^2/b^2-a*e*x/b^2+1/2*(d*f*x+d*e-f)/d^2/b*exp(d*x+c)-1/2*(d*f*x+d*e
+f)/d^2/b*exp(-d*x-c)-2/d^2*a/b^2*f*c*ln(exp(d*x+c))+1/d^2*a/b^2*f*c*ln(b*e
xp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d*a/b^2*e*ln(exp(d*x+c))-1/d*a/b^2*e*ln(b
*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d*a/b^2*c*f*x+1/d^2*a/b^2*f*c^2-1/d*a/b
^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d^2*a/b
^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d*a/b^2
*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*a/b^2*f
*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d^2*a/b^2*f*d
ilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2*a/b^2*f*di
log((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] -1/4*f*(2*(a*d^2*x^2*e^c - (b*d*x*e^(2*c) - b*e^(2*c))*e^(d*x) + (b*d*x + b
)*e^(-d*x))*e^(-c)/(b^2*d^2) - integrate(8*(a^2*x*e^(d*x + c) - a*b*x)/(b^3
*e^(2*d*x + 2*c) + 2*a*b^2*e^(d*x + c) - b^3), x)) - 1/2*(2*(d*x + c)*a/(b^
2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) + 2*a*log(-2*a*e^(-d*x - c) +
 b*e^(-2*d*x - 2*c) - b)/(b^2*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 780 vs. 2(200) = 400.
time = 0.37, size = 780, normalized size = 3.68

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] -1/2*(b*d*f*x + b*d*cosh(1) - (b*d*f*x + b*d*cosh(1) + b*d*sinh(1) - b*f)*c
osh(d*x + c)^2 + b*d*sinh(1) - (b*d*f*x + b*d*cosh(1) + b*d*sinh(1) - b*f)*
sinh(d*x + c)^2 + b*f - (a*d^2*f*x^2 - 2*a*c^2*f + 2*(a*d^2*x + 2*a*c*d)*co
sh(1) + 2*(a*d^2*x + 2*a*c*d)*sinh(1))*cosh(d*x + c) + 2*(a*f*cosh(d*x + c)
 + a*f*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*
x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a*f*cosh(d
*x + c) + a*f*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*
```

```
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*((a*
c*f - a*d*cosh(1) - a*d*sinh(1))*cosh(d*x + c) + (a*c*f - a*d*cosh(1) - a*d
*sinh(1))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sq
rt((a^2 + b^2)/b^2) + 2*a) - 2*((a*c*f - a*d*cosh(1) - a*d*sinh(1))*cosh(d*
x + c) + (a*c*f - a*d*cosh(1) - a*d*sinh(1))*sinh(d*x + c))*log(2*b*cosh(d*
x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*((a*d*f*x
 + a*c*f)*cosh(d*x + c) + (a*d*f*x + a*c*f)*sinh(d*x + c))*log(-(a*cosh(d*x
 + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2) - b)/b) + 2*((a*d*f*x + a*c*f)*cosh(d*x + c) + (a*d*f*x + a*c*f)*s
inh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (a*d^2*f*x^2 - 2*a*c^2*f +
 2*(a*d^2*x + 2*a*c*d)*cosh(1) + 2*(b*d*f*x + b*d*cosh(1) + b*d*sinh(1) - b
*f)*cosh(d*x + c) + 2*(a*d^2*x + 2*a*c*d)*sinh(1))*sinh(d*x + c))/(b^2*d^2*
cosh(d*x + c) + b^2*d^2*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)*sinh(c + d*x)*cosh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
[Out] integrate((f*x + e)*cosh(d*x + c)*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)*sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.336 $\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=34

$$-\frac{a\log(a+b\sinh(c+dx))}{b^2d}+\frac{\sinh(c+dx)}{bd}$$

```
[Out] -a*ln(a+b*sinh(d*x+c))/b^2/d+sinh(d*x+c)/b/d
```

Rubi [A]
time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.120$, Rules used = {2912, 12, 45}

$$\frac{\sinh(c+dx)}{bd}-\frac{a\log(a+b\sinh(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a*Log[a + b*Sinh[c + d*x]])/(b^2*d)) + Sinh[c + d*x]/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x}{b(a+x)}\,dx, x, b\sinh(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{a+x}\,dx, x, b\sinh(c+dx)\right)}{b^2d}$$
$$= \frac{\text{Subst}\left(\int \left(1-\frac{a}{a+x}\right)\,dx, x, b\sinh(c+dx)\right)}{b^2d}$$
$$= -\frac{a\log(a+b\sinh(c+dx))}{b^2d} + \frac{\sinh(c+dx)}{bd}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 0.97

$$-\frac{\frac{a\log(a+b\sinh(c+dx))}{b^2} - \frac{\sinh(c+dx)}{b}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
[Out] -(((a*Log[a + b*Sinh[c + d*x]])/b^2 - Sinh[c + d*x]/b)/d)
```

Maple [A]
time = 0.43, size = 33, normalized size = 0.97

method	result	size
derivativedivides	$\frac{\frac{\sinh(dx+c)}{b}-\frac{a\ln(a+b\sinh(dx+c))}{b^2}}{d}$	33
default	$\frac{\frac{\sinh(dx+c)}{b}-\frac{a\ln(a+b\sinh(dx+c))}{b^2}}{d}$	33
risch	$\frac{ax}{b^2}+\frac{e^{dx+c}}{2bd}-\frac{e^{-dx-c}}{2bd}+\frac{2ac}{b^2d}-\frac{a\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{b^2d}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/b*sinh(d*x+c)-a/b^2*ln(a+b*sinh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(34) = 68.
time = 0.27, size = 83, normalized size = 2.44

$$-\frac{(dx+c)a}{b^2d} + \frac{e^{(dx+c)}}{2\,bd} - \frac{e^{(-dx-c)}}{2\,bd} - \frac{a\log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(d*x + c)*a/(b^2*d) + 1/2*e^(d*x + c)/(b*d) - 1/2*e^(-d*x - c)/(b*d) - a*l
og(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(34) = 68$.
time = 0.34, size = 132, normalized size = 3.88

$$\frac{2\,adx\cosh\left(dx+c\right)+b\cosh\left(dx+c\right)^{2}+b\sinh\left(dx+c\right)^{2}-2\left(a\cosh\left(dx+c\right)+a\sinh\left(dx+c\right)\right)\log\left(\frac{2\left(b\sinh(dx+c)+a\right)}{\cosh(dx+c)-\sinh(dx+c)}\right)+2\left(adx+b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)-b}{2\left(b^{2}d\cosh\left(dx+c\right)+b^{2}d\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*x*cosh(d*x + c) + b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 - 2*(a*c
osh(d*x + c) + a*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c)
- sinh(d*x + c))) + 2*(a*d*x + b*cosh(d*x + c))*sinh(d*x + c) - b)/(b^2*d*c
osh(d*x + c) + b^2*d*sinh(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(27) = 54$.
time = 0.43, size = 65, normalized size = 1.91

$$\begin{cases} \frac{x\sinh\left(c\right)\cosh\left(c\right)}{a} & \text{for}\: b=0 \wedge d=0 \\ \frac{\cosh^{2}\left(c+dx\right)}{2ad} & \text{for}\: b=0 \\ \frac{x\sinh\left(c\right)\cosh\left(c\right)}{a+b\sinh\left(c\right)} & \text{for}\: d=0 \\ -\frac{a\log\left(\frac{a}{b}+\sinh\left(c+dx\right)\right)}{b^{2}d}+\frac{\sinh\left(c+dx\right)}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Piecewise((x*sinh(c)*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (cosh(c + d*x)**2/(2*
a*d), Eq(b, 0)), (x*sinh(c)*cosh(c)/(a + b*sinh(c)), Eq(d, 0)), (-a*log(a/b
 + sinh(c + d*x))/(b**2*d) + sinh(c + d*x)/(b*d), True))
```

Giac [A]
time = 0.45, size = 60, normalized size = 1.76

$$\frac{\frac{e^{(dx+c)}-e^{(-dx-c)}}{b}-\frac{2\,a\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{b^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((e^(d*x + c) - e^(-d*x - c))/b - 2*a*log(abs(b*(e^(d*x + c) - e^(-d*x
- c)) + 2*a))/b^2)/d
```

Mupad [B]
time = 0.07, size = 31, normalized size = 0.91

$$-\frac{a\,\ln\left(a+b\,\sinh(c+d\,x)\right)-b\,\sinh(c+d\,x)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*sinh(c + d*x))/(a + b*sinh(c + d*x)),x)
```

```
[Out] -(a*log(a + b*sinh(c + d*x)) - b*sinh(c + d*x))/(b^2*d)
```

3.337 $\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=35

$$\mathrm{Int}\left(\frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cosh[c + d*x]*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Cosh[c + d*x]*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

Rubi steps

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 55.43, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cosh[c + d*x]*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x
]
```

```
[Out] Integrate[(Cosh[c + d*x]*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)\sinh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] -1/2*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b*f) - 1/2*e^(c - d*e
/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b*f) - a*log(f*x + e)/(b^2*f) + 1/4*
integrate(-8*(a^2*e^(d*x + c) - a*b)/(b^3*f*x + b^3*e - (b^3*f*x*e^(2*c) +
b^3*e^(2*c + 1))*e^(2*d*x) - 2*(a*b^2*f*x*e^c + a*b^2*e^(c + 1))*e^(d*x)),
x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] integral(cosh(d*x + c)*sinh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
+ c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] integrate(cosh(d*x + c)*sinh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*sinh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((cosh(c + d*x)*sinh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.338 $\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal. Leaf size=696

$$\frac{3ef^2x}{4bd^2}+\frac{3f^3x^2}{8bd^2}+\frac{a^2(e+fx)^4}{4b^3f}+\frac{(e+fx)^4}{8bf}-\frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3}-\frac{a(e+fx)^3\cosh(c+dx)}{b^2d}-\frac{3f^3\cosh^2}{8bd}$$

```
[Out] 3/4*e*f^2*x/b/d^2+3/8*f^3*x^2/b/d^2+1/4*a^2*(f*x+e)^4/b^3/f+1/8*(f*x+e)^4/b
/f-6*a*f^2*(f*x+e)*cosh(d*x+c)/b^2/d^3-a*(f*x+e)^3*cosh(d*x+c)/b^2/d-3/8*f^
3*cosh(d*x+c)^2/b/d^4-3/4*f*(f*x+e)^2*cosh(d*x+c)^2/b/d^2+6*a*f^3*sinh(d*x+
c)/b^2/d^4+3*a*f*(f*x+e)^2*sinh(d*x+c)/b^2/d^2+3/4*f^2*(f*x+e)*cosh(d*x+c)*
sinh(d*x+c)/b/d^3+1/2*(f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/b/d-a*(f*x+e)^3*ln(
1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^3/d+a*(f*x+e)^3*ln(1+
b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^3/d-3*a*f*(f*x+e)^2*pol
ylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^3/d^2+3*a*f*(f*
x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^3/d^2
+6*a*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/
2)/b^3/d^3-6*a*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^
2+b^2)^(1/2)/b^3/d^3-6*a*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(
a^2+b^2)^(1/2)/b^3/d^4+6*a*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))
*(a^2+b^2)^(1/2)/b^3/d^4
```

Rubi [A]
time = 0.80, antiderivative size = 696, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 14, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5698, 3392, 32, 3391, 5684, 3377, 2717, 3403, 2296, 2221, 2611, 6744, 2320, 6724}

$$\frac{a^2(e+fx)^4}{4b^3f}-\frac{6af^3\sqrt{a^2+b^2}\,\text{Li}_4\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^4}+\frac{6af^3\sqrt{a^2+b^2}\,\text{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^4}+\frac{6af^2\sqrt{a^2+b^2}\,(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}-\frac{6af^2\sqrt{a^2+b^2}\,(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^3}-\frac{3af\sqrt{a^2+b^2}\,(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{3af\sqrt{a^2+b^2}\,(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2}-\frac{a\sqrt{a^2+b^2}\,(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^3d}+\frac{a\sqrt{a^2+b^2}\,(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^3d}+\frac{6af^3\sinh(c+dx)}{b^2d^4}-\frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3}+\frac{3af(e+fx)^2\sinh(c+dx)}{b^2d^2}-\frac{a(e+fx)^3\cosh(c+dx)}{b^2d}-\frac{3f^3\cosh^2(c+dx)}{8bd^4}+\frac{3f^2(e+fx)\sinh(c+dx)\cosh(c+dx)}{4bd^3}-\frac{3f(e+fx)^2\cosh^2(c+dx)}{4bd^2}+\frac{(e+fx)^3\sinh(c+dx)\cosh(c+dx)}{2bd}+\frac{3ef^2x}{4bd^2}+\frac{3f^3x^2}{8bd^2}+\frac{(e+fx)^4}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (3*e*f^2*x)/(4*b*d^2) + (3*f^3*x^2)/(8*b*d^2) + (a^2*(e + f*x)^4)/(4*b^3*f)
 + (e + f*x)^4/(8*b*f) - (6*a*f^2*(e + f*x)*Cosh[c + d*x])/(b^2*d^3) - (a*(
e + f*x)^3*Cosh[c + d*x])/(b^2*d) - (3*f^3*Cosh[c + d*x]^2)/(8*b*d^4) - (3*
f*(e + f*x)^2*Cosh[c + d*x]^2)/(4*b*d^2) - (a*Sqrt[a^2 + b^2]*(e + f*x)^3*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) + (a*Sqrt[a^2 + b^2]
*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*d) - (3*a
*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/(b^3*d^2) + (3*a*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^2) + (6*a*Sqrt[a^2 + b^2]*f^2*(e
 + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^3) - (
```

```
6*a*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))])/(b^3*d^3) - (6*a*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^4) + (6*a*Sqrt[a^2 + b^2]*f^3*PolyLog[4
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^4) + (6*a*f^3*Sinh[c +
d*x])/(b^2*d^4) + (3*a*f*(e + f*x)^2*Sinh[c + d*x])/(b^2*d^2) + (3*f^2*(e +
 f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^3) + ((e + f*x)^3*Cosh[c + d*x]*S
inh[c + d*x])/(2*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
```

```
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\cosh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{3f(e+fx)^2\cosh^2(c+dx)}{4bd^2} + \frac{(e+fx)^3\cosh(c+dx)\sinh(c$$

$$= \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{a(e+fx)^3\cosh(c+dx)}{b^2d} - \frac{3f^3\,\mathrm{co}$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{a(e+fx)^3\cos}{b^2d}$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\,\mathrm{c}}{b^2d}$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\,\mathrm{c}}{b^2d}$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\,\mathrm{c}}{b^2d}$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\,\mathrm{c}}{b^2d}$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\,\mathrm{c}}{b^2d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.58, size = 2963, normalized size = 4.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (e^3*(c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(S
qrt[-a^2 - b^2]*d)))/(4*b) + (3*e^2*f*(x^2 + ((2*I)*a*Pi*ArcTanh[(-b + a*Ta
nh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d^2) + (2*a*(2*((-I)*c
```

```
+ ArcCos[((-I)*a)/b])*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])
/Sqrt[-a^2 - b^2]] + ((-2*I)*c + Pi - (2*I)*d*x)*ArcTanh[((a - I*b)*Tan[((2
*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] - (ArcCos[((-I)*a)/b] + (2*I)
*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]])*L
og[((I*a + b)*(a + I*(b + Sqrt[-a^2 - b^2]))*(-I + Cot[((2*I)*c + Pi + (2*I
)*d*x)/4]))/(b*(I*a + b + I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)
/4]))] - (ArcCos[((-I)*a)/b] - (2*I)*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi +
 (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]])*Log[((I*a + b)*(I*a - b + Sqrt[-a^2 - b^
2])*(I + Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(a - I*b + Sqrt[-a^2 - b^2]
*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))] + (ArcCos[((-I)*a)/b] - (2*I)*ArcTanh
[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] - (2*I)*Ar
cTanh[((a - I*b)*Tan[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]])*Log[
-(((-1)^(3/4)*Sqrt[-a^2 - b^2]*E^(-1/2*c - (d*x)/2))/(Sqrt[2]*Sqrt[(-I)*b]*
Sqrt[a + b*Sinh[c + d*x]]))] + (ArcCos[((-I)*a)/b] + (2*I)*(ArcTanh[((a + I
*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] + ArcTanh[((a - I*
b)*Tan[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]]))*Log[((-1)^(1/4)*S
qrt[-a^2 - b^2]*E^((c + d*x)/2))/(Sqrt[2]*Sqrt[(-I)*b]*Sqrt[a + b*Sinh[c +
d*x]])] + I*(PolyLog[2, ((I*a + Sqrt[-a^2 - b^2])*(I*a + b - I*Sqrt[-a^2 -
b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(I*a + b + I*Sqrt[-a^2 - b^2]*C
ot[((2*I)*c + Pi + (2*I)*d*x)/4]))] - PolyLog[2, ((a + I*Sqrt[-a^2 - b^2])*
(-a + I*b + Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(I*a +
b + I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))])))/(Sqrt[-a^2 -
 b^2]*d^2)))/(8*b) + (e*f^2*(x^3 - (3*a*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2])] - d^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)] + 2*d*x*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*x*PolyL
og[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*PolyLog[3, (b*E^(c + d*
x))/(-a + Sqrt[a^2 + b^2])] + 2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2]))]))/(Sqrt[a^2 + b^2]*d^3)))/(4*b) + (f^3*(x^4 - (4*a*(d^3*x^3*Log[1
 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^3*x^3*Log[1 + (b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2])] + 3*d^2*x^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a
^2 + b^2])] - 3*d^2*x^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
] - 6*d*x*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*d*x*PolyLo
g[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 6*PolyLog[4, (b*E^(c + d*x
))/(-a + Sqrt[a^2 + b^2])] - 6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))]))/(Sqrt[a^2 + b^2]*d^4)))/(16*b) + (e*f^2*(2*(4*a^2 + b^2)*x^3 - (
6*a*(4*a^2 + 3*b^2)*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]
 - d^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2
, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*x*PolyLog[2, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))] - 2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 +
 b^2])] + 2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^
2 + b^2]*d^3) - (24*a*b*Cosh[d*x]*((2 + d^2*x^2)*Cosh[c] - 2*d*x*Sinh[c]))/
d^3 + (3*b^2*Cosh[2*d*x]*(-2*d*x*Cosh[2*c] + (1 + 2*d^2*x^2)*Sinh[2*c]))/d^
3 - (24*a*b*(-2*d*x*Cosh[c] + (2 + d^2*x^2)*Sinh[c])*Sinh[d*x])/d^3 + (3*b^
2*((1 + 2*d^2*x^2)*Cosh[2*c] - 2*d*x*Sinh[2*c])*Sinh[2*d*x])/d^3))/(8*b^3)
+ (f^3*((4*a^2 + b^2)*x^4 - (4*a*(4*a^2 + 3*b^2)*(d^3*x^3*Log[1 + (b*E^(c +
```

```
 d*x))/(a - Sqrt[a^2 + b^2])] - d^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])] + 3*d^2*x^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
- 3*d^2*x^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*x*Po
lyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*d*x*PolyLog[3, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 6*PolyLog[4, (b*E^(c + d*x))/(-a + Sqr
t[a^2 + b^2])] - 6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(
Sqrt[a^2 + b^2]*d^4) - (16*a*b*Cosh[d*x]*(d*x*(6 + d^2*x^2)*Cosh[c] - 3*(2
+ d^2*x^2)*Sinh[c]))/d^4 + (b^2*Cosh[2*d*x]*(-3*(1 + 2*d^2*x^2)*Cosh[2*c] +
 2*d*x*(3 + 2*d^2*x^2)*Sinh[2*c]))/d^4 - (16*a*b*(-3*(2 + d^2*x^2)*Cosh[c]
+ d*x*(6 + d^2*x^2)*Sinh[c])*Sinh[d*x])/d^4 + (b^2*(2*d*x*(3 + 2*d^2*x^2)*C
osh[2*c] - 3*(1 + 2*d^2*x^2)*Sinh[2*c])*Sinh[2*d*x])/d^4))/(16*b^3) + (e^3*
((4*a^2 + b^2)*(c + d*x) - (2*a*(4*a^2 + 3*b^2)*ArcTan[(b - a*Tanh[(c + d*x
)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 4*a*b*Cosh[c + d*x] + b^2*Sinh[
2*(c + d*x)]))/(4*b^3*d) + (3*e^2*f*((4*a^2 + b^2)*(-c + d*x)*(c + d*x) - 8
*a*b*d*x*Cosh[c + d*x] - b^2*Cosh[2*(c + d*x)] - (2*a*(4*a^2 + 3*b^2)*(2*c*
ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + (c + d*x
)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] - (c +
 d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])] +
PolyLog[2, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/...
```

Maple [F]
time = 1.94, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \left(\cosh^2(dx+c)\right) \sinh(dx+c)}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -1/8*((4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) + 8*sqrt(a^2 + b^2)*a*
log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 +
 b^2)))/(b^3*d) - 4*(2*a^2 + b^2)*(d*x + c)/(b^3*d) + (4*a*e^(-d*x - c) + b
*e^(-2*d*x - 2*c))/(b^2*d))*e^3 + 1/32*(4*(2*a^2*d^4*f^3*e^(2*c) + b^2*d^4*
f^3*e^(2*c))*x^4 + 16*(2*a^2*d^4*f^2*e^(2*c) + b^2*d^4*f^2*e^(2*c))*x^3*e +
 24*(2*a^2*d^4*f*e^(2*c) + b^2*d^4*f*e^(2*c))*x^2*e^2 + (4*b^2*d^3*f^3*x^3*
```

```
e^(4*c) - 3*b^2*f^3*e^(4*c) - 6*b^2*d^2*f*e^(4*c + 2) + 6*b^2*d*f^2*e^(4*c
+ 1) - 6*(b^2*d^2*f^3*e^(4*c) - 2*b^2*d^3*f^2*e^(4*c + 1))*x^2 + 6*(b^2*d*f
^3*e^(4*c) + 2*b^2*d^3*f*e^(4*c + 2) - 2*b^2*d^2*f^2*e^(4*c + 1))*x)*e^(2*d
*x) - 16*(a*b*d^3*f^3*x^3*e^(3*c) - 6*a*b*f^3*e^(3*c) - 3*a*b*d^2*f*e^(3*c
+ 2) + 6*a*b*d*f^2*e^(3*c + 1) - 3*(a*b*d^2*f^3*e^(3*c) - a*b*d^3*f^2*e^(3*
c + 1))*x^2 + 3*(2*a*b*d*f^3*e^(3*c) + a*b*d^3*f*e^(3*c + 2) - 2*a*b*d^2*f^
2*e^(3*c + 1))*x)*e^(d*x) - 16*(a*b*d^3*f^3*x^3*e^c + 3*a*b*d^2*f*e^(c + 2)
 + 6*a*b*d*f^2*e^(c + 1) + 6*a*b*f^3*e^c + 3*(a*b*d^3*f^2*e^(c + 1) + a*b*d
^2*f^3*e^c)*x^2 + 3*(a*b*d^3*f*e^(c + 2) + 2*a*b*d^2*f^2*e^(c + 1) + 2*a*b*
d*f^3*e^c)*x)*e^(-d*x) - (4*b^2*d^3*f^3*x^3 + 6*b^2*d^2*f*e^2 + 6*b^2*d*f^2
*e + 3*b^2*f^3 + 6*(2*b^2*d^3*f^2*e + b^2*d^2*f^3)*x^2 + 6*(2*b^2*d^3*f*e^2
 + 2*b^2*d^2*f^2*e + b^2*d*f^3)*x)*e^(-2*d*x))*e^(-2*c)/(b^3*d^4) - integra
te(2*((a^3*f^3*e^c + a*b^2*f^3*e^c)*x^3 + 3*(a^3*f^2*e^c + a*b^2*f^2*e^c)*x
^2*e + 3*(a^3*f*e^c + a*b^2*f*e^c)*x*e^2)*e^(d*x)/(b^4*e^(2*d*x + 2*c) + 2*
a*b^3*e^(d*x + c) - b^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6807 vs. $2(653) = 1306$.
time = 0.45, size = 6807, normalized size = 9.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -1/32*(4*b^2*d^3*f^3*x^3 + 6*b^2*d^2*f^3*x^2 + 4*b^2*d^3*cosh(1)^3 + 4*b^2*
d^3*sinh(1)^3 + 6*b^2*d*f^3*x + 3*b^2*f^3 - (4*b^2*d^3*f^3*x^3 - 6*b^2*d^2*
f^3*x^2 + 4*b^2*d^3*cosh(1)^3 + 4*b^2*d^3*sinh(1)^3 + 6*b^2*d*f^3*x - 3*b^2
*f^3 + 6*(2*b^2*d^3*f*x - b^2*d^2*f)*cosh(1)^2 + 6*(2*b^2*d^3*f*x + 2*b^2*d
^3*cosh(1) - b^2*d^2*f)*sinh(1)^2 + 6*(2*b^2*d^3*f^2*x^2 - 2*b^2*d^2*f^2*x
+ b^2*d*f^2)*cosh(1) + 6*(2*b^2*d^3*f^2*x^2 - 2*b^2*d^2*f^2*x + 2*b^2*d^3*c
osh(1)^2 + b^2*d*f^2 + 2*(2*b^2*d^3*f*x - b^2*d^2*f)*cosh(1))*sinh(1))*cosh
(d*x + c)^4 - (4*b^2*d^3*f^3*x^3 - 6*b^2*d^2*f^3*x^2 + 4*b^2*d^3*cosh(1)^3
+ 4*b^2*d^3*sinh(1)^3 + 6*b^2*d*f^3*x - 3*b^2*f^3 + 6*(2*b^2*d^3*f*x - b^2*
d^2*f)*cosh(1)^2 + 6*(2*b^2*d^3*f*x + 2*b^2*d^3*cosh(1) - b^2*d^2*f)*sinh(1
)^2 + 6*(2*b^2*d^3*f^2*x^2 - 2*b^2*d^2*f^2*x + b^2*d*f^2)*cosh(1) + 6*(2*b^
2*d^3*f^2*x^2 - 2*b^2*d^2*f^2*x + 2*b^2*d^3*cosh(1)^2 + b^2*d*f^2 + 2*(2*b^
2*d^3*f*x - b^2*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^4 + 16*(a*b*d^3*f^3*
x^3 - 3*a*b*d^2*f^3*x^2 + a*b*d^3*cosh(1)^3 + a*b*d^3*sinh(1)^3 + 6*a*b*d*f
^3*x - 6*a*b*f^3 + 3*(a*b*d^3*f*x - a*b*d^2*f)*cosh(1)^2 + 3*(a*b*d^3*f*x +
 a*b*d^3*cosh(1) - a*b*d^2*f)*sinh(1)^2 + 3*(a*b*d^3*f^2*x^2 - 2*a*b*d^2*f^
2*x + 2*a*b*d*f^2)*cosh(1) + 3*(a*b*d^3*f^2*x^2 - 2*a*b*d^2*f^2*x + a*b*d^3
*cosh(1)^2 + 2*a*b*d*f^2 + 2*(a*b*d^3*f*x - a*b*d^2*f)*cosh(1))*sinh(1))*co
sh(d*x + c)^3 + 4*(4*a*b*d^3*f^3*x^3 - 12*a*b*d^2*f^3*x^2 + 4*a*b*d^3*cosh(
```

1)^3 + 4*a*b*d^3*sinh(1)^3 + 24*a*b*d*f^3*x - 24*a*b*f^3 + 12*(a*b*d^3*f*x
- a*b*d^2*f)*cosh(1)^2 + 12*(a*b*d^3*f*x + a*b*d^3*cosh(1) - a*b*d^2*f)*sin
h(1)^2 + 12*(a*b*d^3*f^2*x^2 - 2*a*b*d^2*f^2*x + 2*a*b*d*f^2)*cosh(1) - (4*
b^2*d^3*f^3*x^3 - 6*b^2*d^2*f^3*x^2 + 4*b^2*d^3*cosh(1)^3 + 4*b^2*d^3*sinh(
1)^3 + 6*b^2*d*f^3*x - 3*b^2*f^3 + 6*(2*b^2*d^3*f*x - b^2*d^2*f)*cosh(1)^2
+ 6*(2*b^2*d^3*f*x + 2*b^2*d^3*cosh(1) - b^2*d^2*f)*sinh(1)^2 + 6*(2*b^2*d^
3*f^2*x^2 - 2*b^2*d^2*f^2*x + b^2*d*f^2)*cosh(1) + 6*(2*b^2*d^3*f^2*x^2 - 2
*b^2*d^2*f^2*x + 2*b^2*d^3*cosh(1)^2 + b^2*d*f^2 + 2*(2*b^2*d^3*f*x - b^2*d
^2*f)*cosh(1))*sinh(1))*cosh(d*x + c) + 12*(a*b*d^3*f^2*x^2 - 2*a*b*d^2*f^2
*x + a*b*d^3*cosh(1)^2 + 2*a*b*d*f^2 + 2*(a*b*d^3*f*x - a*b*d^2*f)*cosh(1))
*sinh(1))*sinh(d*x + c)^3 + 6*(2*b^2*d^3*f*x + b^2*d^2*f)*cosh(1)^2 - 4*((2
*a^2 + b^2)*d^4*f^3*x^4 + 4*(2*a^2 + b^2)*d^4*f^2*x^3*cosh(1) + 6*(2*a^2 +
b^2)*d^4*f*x^2*cosh(1)^2 + 4*(2*a^2 + b^2)*d^4*x*cosh(1)^3 + 4*(2*a^2 + b^2
)*d^4*x*sinh(1)^3 + 6*((2*a^2 + b^2)*d^4*f*x^2 + 2*(2*a^2 + b^2)*d^4*x*cosh
(1))*sinh(1)^2 + 4*((2*a^2 + b^2)*d^4*f^2*x^3 + 3*(2*a^2 + b^2)*d^4*f*x^2*c
osh(1) + 3*(2*a^2 + b^2)*d^4*x*cosh(1)^2)*sinh(1))*cosh(d*x + c)^2 + 6*(2*b
^2*d^3*f*x + 2*b^2*d^3*cosh(1) + b^2*d^2*f)*sinh(1)^2 - 2*(2*(2*a^2 + b^2)*
d^4*f^3*x^4 + 8*(2*a^2 + b^2)*d^4*f^2*x^3*cosh(1) + 12*(2*a^2 + b^2)*d^4*f*
x^2*cosh(1)^2 + 8*(2*a^2 + b^2)*d^4*x*cosh(1)^3 + 8*(2*a^2 + b^2)*d^4*x*sin
h(1)^3 + 3*(4*b^2*d^3*f^3*x^3 - 6*b^2*d^2*f^3*x^2 + 4*b^2*d^3*cosh(1)^3 + 4
*b^2*d^3*sinh(1)^3 + 6*b^2*d*f^3*x - 3*b^2*f^3 + 6*(2*b^2*d^3*f*x - b^2*d^2
*f)*cosh(1)^2 + 6*(2*b^2*d^3*f*x + 2*b^2*d^3*cosh(1) - b^2*d^2*f)*sinh(1)^2
 + 6*(2*b^2*d^3*f^2*x^2 - 2*b^2*d^2*f^2*x + b^2*d*f^2)*cosh(1) + 6*(2*b^2*d
^3*f^2*x^2 - 2*b^2*d^2*f^2*x + 2*b^2*d^3*cosh(1)^2 + b^2*d*f^2 + 2*(2*b^2*d
^3*f*x - b^2*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 12*((2*a^2 + b^2)*d
^4*f*x^2 + 2*(2*a^2 + b^2)*d^4*x*cosh(1))*sinh(1)^2 - 24*(a*b*d^3*f^3*x^3 -
 3*a*b*d^2*f^3*x^2 + a*b*d^3*cosh(1)^3 + a*b*d^3*sinh(1)^3 + 6*a*b*d*f^3*x
- 6*a*b*f^3 + 3*(a*b*d^3*f*x - a*b*d^2*f)*cosh(1)^2 + 3*(a*b*d^3*f*x + a*b*
d^3*cosh(1) - a*b*d^2*f)*sinh(1)^2 + 3*(a*b*d^3*f^2*x^2 - 2*a*b*d^2*f^2*x +
 2*a*b*d*f^2)*cosh(1) + 3*(a*b*d^3*f^2*x^2 - 2*a*b*d^2*f^2*x + a*b*d^3*cosh
(1)^2 + 2*a*b*d*f^2 + 2*(a*b*d^3*f*x - a*b*d^2*f)*cosh(1))*sinh(1))*cosh(d*
x + c) + 8*((2*a^2 + b^2)*d^4*f^2*x^3 + 3*(2*a^2 + b^2)*d^4*f*x^2*cosh(1) +
 3*(2*a^2 + b^2)*d^4*x*cosh(1)^2)*sinh(1))*sinh(d*x + c)^2 + 96*((a*b*d^2*f
^3*x^2 + 2*a*b*d^2*f^2*x*cosh(1) + a*b*d^2*f*cosh(1)^2 + a*b*d^2*f*sinh(1)^
2 + 2*(a*b*d^2*f^2*x + a*b*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(a*b
*d^2*f^3*x^2 + 2*a*b*d^2*f^2*x*cosh(1) + a*b*d^2*f*cosh(1)^2 + a*b*d^2*f*si
nh(1)^2 + 2*(a*b*d^2*f^2*x + a*b*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)*sinh
(d*x + c) + (a*b*d^2*f^3*x^2 + 2*a*b*d^2*f^2*x*cosh(1) + a*b*d^2*f*cosh(1)^
2 + a*b*d^2*f*sinh(1)^2 + 2*(a*b*d^2*f^2*x + a*b*d^2*f*cosh(1))*sinh(1))*si
nh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x +
c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1)
- 96*((a*b*d^2*f^3*x^2 + 2*a*b*d^2*f^2*x*cosh(1) + a*b*d^2*f*cosh(1)^2 + a*
b*d^2*f*sinh(1)^2 + 2*(a*b*d^2*f^2*x + a*b*d^2*f*cosh(1))*sinh(1))*cosh(d*x
 + c)^2 + 2*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*f^2*x*cosh(1) + a*b*d^2*f*cosh(1)^
2 + a*b*d^2*f*sinh(1)^2 + 2*(a*b*d^2*f^2*x + a*b*d^2*f*cosh(1))*sinh(1))*co

```
sh(d*x + c)*sinh(d*x + c) + (a*b*d^2*f^3*x^2 + 2*a*b*d^2*f^2*x*cosh(1) + a*
b*d^2*f*cosh(1)^2 + a*b*d^2*f*sinh(1)^2 + 2*(a*b*d^2*f^2*x + a*b*d^2*f*cosh
(1))*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
[Out] integrate((f*x + e)^3*cosh(d*x + c)^2*sinh(d*x + c)/(b*sinh(d*x + c) + a),
x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^2\,\sinh(c+d\,x)\,\left(e+f\,x\right)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.339 $\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=510

$$\frac{f^2x}{4bd^2}+\frac{a^2(e+fx)^3}{3b^3f}+\frac{(e+fx)^3}{6bf}-\frac{2af^2\cosh(c+dx)}{b^2d^3}-\frac{a(e+fx)^2\cosh(c+dx)}{b^2d}-\frac{f(e+fx)\cosh^2(c+dx)}{2bd^2}-\frac{a}{}-$$

[Out] 1/4*f^2*x/b/d^2+1/3*a^2*(f*x+e)^3/b^3/f+1/6*(f*x+e)^3/b/f-2*a*f^2*cosh(d*x+c)/b^2/d^3-a*(f*x+e)^2*cosh(d*x+c)/b^2/d-1/2*f*(f*x+e)*cosh(d*x+c)^2/b/d^2+2*a*f*(f*x+e)*sinh(d*x+c)/b^2/d^2+1/4*f^2*cosh(d*x+c)*sinh(d*x+c)/b/d^3+1/2*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/b/d-a*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^3/d+a*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^3/d-2*a*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^3/d^2+2*a*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^3/d^2+2*a*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^3/d^3-2*a*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^3/d^3

Rubi [A]
time = 0.70, antiderivative size = 510, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 14, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5698, 3392, 32, 2715, 8, 5684, 3377, 2718, 3403, 2296, 2221, 2611, 2320, 6724}

$$\frac{a^2(e+fx)^3}{3b^3f}+\frac{2af^2\sqrt{a^2+b^2}\,\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}-\frac{2af^2\sqrt{a^2+b^2}\,\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^3}-\frac{2af\sqrt{a^2+b^2}\,(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{2af\sqrt{a^2+b^2}\,(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2}-\frac{a\sqrt{a^2+b^2}\,(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^3d}+\frac{a\sqrt{a^2+b^2}\,(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^3d}-\frac{2af^2\cosh(c+dx)}{b^2d^3}+\frac{2af(e+fx)\sinh(c+dx)}{b^2d^2}-\frac{a(e+fx)^2\cosh(c+dx)}{b^2d}+\frac{f^2\sinh(c+dx)\cosh(c+dx)}{4bd^3}-\frac{f(e+fx)\cosh^2(c+dx)}{2bd^2}+\frac{(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2bd}+\frac{f^2x}{4bd^2}+\frac{(e+fx)^3}{6bf}$$

Antiderivative was successfully verified.

[In] Int[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]

[Out] (f^2*x)/(4*b*d^2) + (a^2*(e + f*x)^3)/(3*b^3*f) + (e + f*x)^3/(6*b*f) - (2*a*f^2*Cosh[c + d*x])/(b^2*d^3) - (a*(e + f*x)^2*Cosh[c + d*x])/(b^2*d) - (f*(e + f*x)*Cosh[c + d*x]^2)/(2*b*d^2) - (a*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) + (a*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*d) - (2*a*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^2) + (2*a*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^2) + (2*a*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^3) - (2*a*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^3) + (2*a*f*(e + f*x)*Sinh[c + d*x])/(b^2*d^2) + (f^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^3) + ((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{f(e+fx)\cosh^2(c+dx)}{2bd^2} + \frac{(e+fx)^2\cosh(c+dx)\sinh(c+a}{2bd}$$

$$= \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{a(e+fx)^2\cosh(c+dx)}{b^2d} - \frac{f(e+}{}$$

$$= \frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{a(e+fx)^2\cosh(c+dx)}{b^2d} -$$

$$= \frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2\cosh(c+dx)}{b^2d^3} - \frac{a(e+}{}$$

$$= \frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2\cosh(c+dx)}{b^2d^3} - \frac{a(e+}{}$$

$$= \frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2\cosh(c+dx)}{b^2d^3} - \frac{a(e+}{}$$

$$= \frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2\cosh(c+dx)}{b^2d^3} - \frac{a(e+}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 8.54, size = 2172, normalized size = 4.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (e^2*(c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(S
qrt[-a^2 - b^2]*d)))/(4*b) + (e*f*(x^2 + ((2*I)*a*Pi*ArcTanh[(-b + a*Tanh[(
c + d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d^2) + (2*a*(2*((-I)*c + Ar
```

cCos[((-I)*a)/b])*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqr
t[-a^2 - b^2]] + ((-2*I)*c + Pi - (2*I)*d*x)*ArcTanh[((a - I*b)*Tan[((2*I)*
c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] - (ArcCos[((-I)*a)/b] + (2*I)*Arc
Tanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]])*Log[(
(I*a + b)*(a + I*(b + Sqrt[-a^2 - b^2]))*(-I + Cot[((2*I)*c + Pi + (2*I)*d*
x)/4]))/(b*(I*a + b + I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])
)] - (ArcCos[((-I)*a)/b] - (2*I)*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*
I)*d*x)/4])/Sqrt[-a^2 - b^2]])*Log[((I*a + b)*(I*a - b + Sqrt[-a^2 - b^2])*
(I + Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(a - I*b + Sqrt[-a^2 - b^2]*Cot
[((2*I)*c + Pi + (2*I)*d*x)/4]))] + (ArcCos[((-I)*a)/b] - (2*I)*ArcTanh[((a
 + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] - (2*I)*ArcTan
h[((a - I*b)*Tan[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]])*Log[-(((
-1)^(3/4)*Sqrt[-a^2 - b^2]*E^(-1/2*c - (d*x)/2))/(Sqrt[2]*Sqrt[(-I)*b]*Sqrt
[a + b*Sinh[c + d*x]]))] + (ArcCos[((-I)*a)/b] + (2*I)*(ArcTanh[((a + I*b)*
Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] + ArcTanh[((a - I*b)*T
an[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]]))*Log[((-1)^(1/4)*Sqrt[
-a^2 - b^2]*E^((c + d*x)/2))/(Sqrt[2]*Sqrt[(-I)*b]*Sqrt[a + b*Sinh[c + d*x]
])] + I*(PolyLog[2, ((I*a + Sqrt[-a^2 - b^2])*(I*a + b - I*Sqrt[-a^2 - b^2]
*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(I*a + b + I*Sqrt[-a^2 - b^2]*Cot[(
(2*I)*c + Pi + (2*I)*d*x)/4]))] - PolyLog[2, ((a + I*Sqrt[-a^2 - b^2])*(-a
+ I*b + Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(I*a + b +
I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))])))/(Sqrt[-a^2 - b^2
]*d^2)))/(4*b) + (f^2*(x^3 - (3*a*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2])] - d^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2
*d*x*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*x*PolyLog[2,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*PolyLog[3, (b*E^(c + d*x))/(-
a + Sqrt[a^2 + b^2])] + 2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
))]))/(Sqrt[a^2 + b^2]*d^3)))/(12*b) + (f^2*(2*(4*a^2 + b^2)*x^3 - (6*a*(4*
a^2 + 3*b^2)*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^2*
x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, (b*E^
(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*x*PolyLog[2, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))] - 2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
 + 2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^2 + b^2
]*d^3) - (24*a*b*Cosh[d*x]*((2 + d^2*x^2)*Cosh[c] - 2*d*x*Sinh[c]))/d^3 + (
3*b^2*Cosh[2*d*x]*(-2*d*x*Cosh[2*c] + (1 + 2*d^2*x^2)*Sinh[2*c]))/d^3 - (24
*a*b*(-2*d*x*Cosh[c] + (2 + d^2*x^2)*Sinh[c])*Sinh[d*x])/d^3 + (3*b^2*((1 +
 2*d^2*x^2)*Cosh[2*c] - 2*d*x*Sinh[2*c])*Sinh[2*d*x])/d^3))/(24*b^3) + (e^2
*((4*a^2 + b^2)*(c + d*x) - (2*a*(4*a^2 + 3*b^2)*ArcTan[(b - a*Tanh[(c + d*
x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 4*a*b*Cosh[c + d*x] + b^2*Sinh
[2*(c + d*x)]))/(4*b^3*d) + (e*f*((4*a^2 + b^2)*(-c + d*x)*(c + d*x) - 8*a*
b*d*x*Cosh[c + d*x] - b^2*Cosh[2*(c + d*x)] - (2*a*(4*a^2 + 3*b^2)*(2*c*Arc
Tanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + (c + d*x)*L
og[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] - (c + d*
x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])] + Pol
yLog[2, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])] - PolyL

```
og[2, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[
a^2 + b^2] + 8*a*b*Sinh[c + d*x] + 2*b^2*d*x*Sinh[2*(c + d*x)]))/(4*b^3*d^2
)
```

Maple [F]
time = 1.84, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \left(\cosh^2 (dx+c)\right) \sinh (dx+c)}{a+b\sinh (dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -1/8*((4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) + 8*sqrt(a^2 + b^2)*a*
log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 +
 b^2)))/(b^3*d) - 4*(2*a^2 + b^2)*(d*x + c)/(b^3*d) + (4*a*e^(-d*x - c) + b
*e^(-2*d*x - 2*c))/(b^2*d))*e^2 + 1/48*(8*(2*a^2*d^3*f^2*e^(2*c) + b^2*d^3*
f^2*e^(2*c))*x^3 + 24*(2*a^2*d^3*f*e^(2*c) + b^2*d^3*f*e^(2*c))*x^2*e + 3*(
2*b^2*d^2*f^2*x^2*e^(4*c) + b^2*f^2*e^(4*c) - 2*b^2*d*f*e^(4*c + 1) - 2*(b^
2*d*f^2*e^(4*c) - 2*b^2*d^2*f*e^(4*c + 1))*x)*e^(2*d*x) - 24*(a*b*d^2*f^2*x
^2*e^(3*c) + 2*a*b*f^2*e^(3*c) - 2*a*b*d*f*e^(3*c + 1) - 2*(a*b*d*f^2*e^(3*
c) - a*b*d^2*f*e^(3*c + 1))*x)*e^(d*x) - 24*(a*b*d^2*f^2*x^2*e^c + 2*a*b*d*
f*e^(c + 1) + 2*a*b*f^2*e^c + 2*(a*b*d^2*f*e^(c + 1) + a*b*d*f^2*e^c)*x)*e^
(-d*x) - 3*(2*b^2*d^2*f^2*x^2 + 2*b^2*d*f*e + b^2*f^2 + 2*(2*b^2*d^2*f*e +
b^2*d*f^2)*x)*e^(-2*d*x))*e^(-2*c)/(b^3*d^3) - integrate(2*((a^3*f^2*e^c +
a*b^2*f^2*e^c)*x^2 + 2*(a^3*f*e^c + a*b^2*f*e^c)*x*e)*e^(d*x)/(b^4*e^(2*d*x
 + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3459 vs. $2(476) = 952$.
time = 0.44, size = 3459, normalized size = 6.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")

[Out] -1/48*(6*b^2*d^2*f^2*x^2 + 6*b^2*d*f^2*x + 6*b^2*d^2*cosh(1)^2 + 6*b^2*d^2*
sinh(1)^2 - 3*(2*b^2*d^2*f^2*x^2 - 2*b^2*d*f^2*x + 2*b^2*d^2*cosh(1)^2 + 2*
b^2*d^2*sinh(1)^2 + b^2*f^2 + 2*(2*b^2*d^2*f*x - b^2*d*f)*cosh(1) + 2*(2*b^
2*d^2*f*x + 2*b^2*d^2*cosh(1) - b^2*d*f)*sinh(1))*cosh(d*x + c)^4 - 3*(2*b^
2*d^2*f^2*x^2 - 2*b^2*d*f^2*x + 2*b^2*d^2*cosh(1)^2 + 2*b^2*d^2*sinh(1)^2 +
 b^2*f^2 + 2*(2*b^2*d^2*f*x - b^2*d*f)*cosh(1) + 2*(2*b^2*d^2*f*x + 2*b^2*d
^2*cosh(1) - b^2*d*f)*sinh(1))*sinh(d*x + c)^4 + 3*b^2*f^2 + 24*(a*b*d^2*f^
2*x^2 - 2*a*b*d*f^2*x + a*b*d^2*cosh(1)^2 + a*b*d^2*sinh(1)^2 + 2*a*b*f^2 +
 2*(a*b*d^2*f*x - a*b*d*f)*cosh(1) + 2*(a*b*d^2*f*x + a*b*d^2*cosh(1) - a*b
*d*f)*sinh(1))*cosh(d*x + c)^3 + 12*(2*a*b*d^2*f^2*x^2 - 4*a*b*d*f^2*x + 2*
a*b*d^2*cosh(1)^2 + 2*a*b*d^2*sinh(1)^2 + 4*a*b*f^2 + 4*(a*b*d^2*f*x - a*b*
d*f)*cosh(1) - (2*b^2*d^2*f^2*x^2 - 2*b^2*d*f^2*x + 2*b^2*d^2*cosh(1)^2 + 2
*b^2*d^2*sinh(1)^2 + b^2*f^2 + 2*(2*b^2*d^2*f*x - b^2*d*f)*cosh(1) + 2*(2*b
^2*d^2*f*x + 2*b^2*d^2*cosh(1) - b^2*d*f)*sinh(1))*cosh(d*x + c) + 4*(a*b*d
^2*f*x + a*b*d^2*cosh(1) - a*b*d*f)*sinh(1))*sinh(d*x + c)^3 - 8*((2*a^2 +
b^2)*d^3*f^2*x^3 + 3*(2*a^2 + b^2)*d^3*f*x^2*cosh(1) + 3*(2*a^2 + b^2)*d^3*
x*cosh(1)^2 + 3*(2*a^2 + b^2)*d^3*x*sinh(1)^2 + 3*((2*a^2 + b^2)*d^3*f*x^2
+ 2*(2*a^2 + b^2)*d^3*x*cosh(1))*sinh(1))*cosh(d*x + c)^2 - 2*(4*(2*a^2 + b
^2)*d^3*f^2*x^3 + 12*(2*a^2 + b^2)*d^3*f*x^2*cosh(1) + 12*(2*a^2 + b^2)*d^3
*x*cosh(1)^2 + 12*(2*a^2 + b^2)*d^3*x*sinh(1)^2 + 9*(2*b^2*d^2*f^2*x^2 - 2*
b^2*d*f^2*x + 2*b^2*d^2*cosh(1)^2 + 2*b^2*d^2*sinh(1)^2 + b^2*f^2 + 2*(2*b^
2*d^2*f*x - b^2*d*f)*cosh(1) + 2*(2*b^2*d^2*f*x + 2*b^2*d^2*cosh(1) - b^2*d
*f)*sinh(1))*cosh(d*x + c)^2 - 36*(a*b*d^2*f^2*x^2 - 2*a*b*d*f^2*x + a*b*d^
2*cosh(1)^2 + a*b*d^2*sinh(1)^2 + 2*a*b*f^2 + 2*(a*b*d^2*f*x - a*b*d*f)*cos
h(1) + 2*(a*b*d^2*f*x + a*b*d^2*cosh(1) - a*b*d*f)*sinh(1))*cosh(d*x + c) +
 12*((2*a^2 + b^2)*d^3*f*x^2 + 2*(2*a^2 + b^2)*d^3*x*cosh(1))*sinh(1))*sinh
(d*x + c)^2 + 96*((a*b*d*f^2*x + a*b*d*f*cosh(1) + a*b*d*f*sinh(1))*cosh(d*
x + c)^2 + 2*(a*b*d*f^2*x + a*b*d*f*cosh(1) + a*b*d*f*sinh(1))*cosh(d*x + c
)*sinh(d*x + c) + (a*b*d*f^2*x + a*b*d*f*cosh(1) + a*b*d*f*sinh(1))*sinh(d*
x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) +
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 96*
((a*b*d*f^2*x + a*b*d*f*cosh(1) + a*b*d*f*sinh(1))*cosh(d*x + c)^2 + 2*(a*b
*d*f^2*x + a*b*d*f*cosh(1) + a*b*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c) +
 (a*b*d*f^2*x + a*b*d*f*cosh(1) + a*b*d*f*sinh(1))*sinh(d*x + c)^2)*sqrt((a
^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 48*((a*b*c^2*f^2 - 2
*a*b*c*d*f*cosh(1) + a*b*d^2*cosh(1)^2 + a*b*d^2*sinh(1)^2 - 2*(a*b*c*d*f -
 a*b*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(a*b*c^2*f^2 - 2*a*b*c*d*f*c
osh(1) + a*b*d^2*cosh(1)^2 + a*b*d^2*sinh(1)^2 - 2*(a*b*c*d*f - a*b*d^2*cos
h(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a*b*c^2*f^2 - 2*a*b*c*d*f*cos
h(1) + a*b*d^2*cosh(1)^2 + a*b*d^2*sinh(1)^2 - 2*(a*b*c*d*f - a*b*d^2*cosh(
1))*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) +
```

```
2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 48*((a*b*c^2*f^2 -
2*a*b*c*d*f*cosh(1) + a*b*d^2*cosh(1)^2 + a*b*d^2*sinh(1)^2 - 2*(a*b*c*d*f
- a*b*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(a*b*c^2*f^2 - 2*a*b*c*d*f*
cosh(1) + a*b*d^2*cosh(1)^2 + a*b*d^2*sinh(1)^2 - 2*(a*b*c*d*f - a*b*d^2*co
sh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a*b*c^2*f^2 - 2*a*b*c*d*f*co
sh(1) + a*b*d^2*cosh(1)^2 + a*b*d^2*sinh(1)^2 - 2*(a*b*c*d*f - a*b*d^2*cosh
(1))*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c)
+ 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 48*((a*b*d^2*f^2*x
^2 - a*b*c^2*f^2 + 2*(a*b*d^2*f*x + a*b*c*d*f)*cosh(1) + 2*(a*b*d^2*f*x + a
*b*c*d*f)*sinh(1))*cosh(d*x + c)^2 + 2*(a*b*d^2*f^2*x^2 - a*b*c^2*f^2 + 2*(
a*b*d^2*f*x + a*b*c*d*f)*cosh(1) + 2*(a*b*d^2*f*x + a*b*c*d*f)*sinh(1))*cos
h(d*x + c)*sinh(d*x + c) + (a*b*d^2*f^2*x^2 - a*b*c^2*f^2 + 2*(a*b*d^2*f*x
+ a*b*c*d*f)*cosh(1) + 2*(a*b*d^2*f*x + a*b*c*d*f)*sinh(1))*sinh(d*x + c)^2
)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 48*((a*b*d^2*f^2
*x^2 - a*b*c^2*f^2 + 2*(a*b*d^2*f*x + a*b*c*d*f)*cosh(1) + 2*(a*b*d^2*f*x +
 a*b*c*d*f)*sinh(1))*cosh(d*x + c)^2 + 2*(a*b*d^2*f^2*x^2 - a*b*c^2*f^2 + 2
*(a*b*d^2*f*x + a*b*c*d*f)*cosh(1) + 2*(a*b*d^2*f*x + a*b*c*d*f)*sinh(1))*c
osh(d*x + c)*sinh(d*x + c) + (a*b*d^2*f^2*x^2 - a*b*c^2*f^2 + 2*(a*b*d^2*f*
x + a*b*c*d*f)*cosh(1) + 2*(a*b*d^2*f*x + a*b*c*d*f)*sinh(1))*sinh(d*x + c)
^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 96*(a*b*f^2*co
sh(d*x + c)^2 + 2*a*b*f^2*cosh(d*x + c)*sinh(d*x + c) + a*b*f^2*sinh(d*x +
c)^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) +
 (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)^2*sinh(d*x + c)/(b*sinh(d*x + c) + a),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^2\,\sinh(c+d\,x)\,\left(e+f\,x\right)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.340 $\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=327

$$\frac{a^2ex}{b^3}+\frac{ex}{2b}+\frac{a^2fx^2}{2b^3}+\frac{fx^2}{4b}-\frac{a(e+fx)\cosh(c+dx)}{b^2d}-\frac{f\cosh^2(c+dx)}{4bd^2}-\frac{a\sqrt{a^2+b^2}\,(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+}}\right.}{b^3d}$$

```
[Out] a^2*e*x/b^3+1/2*e*x/b+1/2*a^2*f*x^2/b^3+1/4*f*x^2/b-a*(f*x+e)*cosh(d*x+c)/b
^2/d-1/4*f*cosh(d*x+c)^2/b/d^2+a*f*sinh(d*x+c)/b^2/d^2+1/2*(f*x+e)*cosh(d*x
+c)*sinh(d*x+c)/b/d-a*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b
^2)^(1/2)/b^3/d+a*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^
(1/2)/b^3/d-a*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2
)/b^3/d^2+a*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/
b^3/d^2
```

Rubi [A]

time = 0.39, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5698, 3391, 5684, 3377, 2717, 3403, 2296, 2221, 2317, 2438}

$$\frac{a^2ex}{b^3}+\frac{a^2fx^2}{2b^3}-\frac{af\sqrt{a^2+b^2}\,\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{af\sqrt{a^2+b^2}\,\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2}-\frac{a\sqrt{a^2+b^2}\,(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^3d}+\frac{a\sqrt{a^2+b^2}\,(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^3d}+\frac{af\sinh(c+dx)}{b^2d^2}-\frac{a(e+fx)\cosh(c+dx)}{b^2d}-\frac{f\cosh^2(c+dx)}{4bd^2}+\frac{(e+fx)\sinh(c+dx)\cosh(c+dx)}{2bd}+\frac{ex}{2b}+\frac{fx^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*e*x)/b^3 + (e*x)/(2*b) + (a^2*f*x^2)/(2*b^3) + (f*x^2)/(4*b) - (a*(e +
 f*x)*Cosh[c + d*x])/(b^2*d) - (f*Cosh[c + d*x]^2)/(4*b*d^2) - (a*Sqrt[a^2
+ b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) +
(a*Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
)/(b^3*d) - (a*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2]))])/(b^3*d^2) + (a*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(b^3*d^2) + (a*f*Sinh[c + d*x])/(b^2*d^2) + ((e + f
*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
```

```
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{f\cosh^2(c+dx)}{4bd^2} + \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{2bd} + \frac{a^2\int(\cdots$$

$$= \frac{a^2ex}{b^3} + \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} + \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} - \frac{f\cosh^2\cdots}{4\cdots}$$

$$= \frac{a^2ex}{b^3} + \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} + \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} - \frac{f\cosh^2\cdots}{4\cdots}$$

$$= \frac{a^2ex}{b^3} + \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} + \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} - \frac{f\cosh^2\cdots}{4\cdots}$$

$$= \frac{a^2ex}{b^3} + \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} + \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} - \frac{f\cosh^2\cdots}{4\cdots}$$

$$= \frac{a^2ex}{b^3} + \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} + \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} - \frac{f\cosh^2\cdots}{4\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.14, size = 1551, normalized size = 4.74

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (2*b^2*e*(c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]]
)/(Sqrt[-a^2 - b^2]*d)) + b^2*f*(x^2 + ((2*I)*a*Pi*ArcTanh[(-b + a*Tanh[(c
+ d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d^2) + (2*a*(2*((-I)*c + ArcC
os[((-I)*a)/b])*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[
-a^2 - b^2]] + ((-2*I)*c + Pi - (2*I)*d*x)*ArcTanh[((a - I*b)*Tan[((2*I)*c
+ Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] - (ArcCos[((-I)*a)/b] + (2*I)*ArcTa
nh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]])*Log[((I
*a + b)*(a + I*(b + Sqrt[-a^2 - b^2]))*(-I + Cot[((2*I)*c + Pi + (2*I)*d*x)
/4]))/(b*(I*a + b + I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))]
 - (ArcCos[((-I)*a)/b] - (2*I)*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)
*d*x)/4])/Sqrt[-a^2 - b^2]])*Log[((I*a + b)*(I*a - b + Sqrt[-a^2 - b^2])*(I
 + Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(a - I*b + Sqrt[-a^2 - b^2]*Cot[(
(2*I)*c + Pi + (2*I)*d*x)/4]))] + (ArcCos[((-I)*a)/b] - (2*I)*ArcTanh[((a +
 I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] - (2*I)*ArcTanh[
((a - I*b)*Tan[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]])*Log[-(((-1
)^(3/4)*Sqrt[-a^2 - b^2]*E^(-1/2*c - (d*x)/2))/(Sqrt[2]*Sqrt[(-I)*b]*Sqrt[a
 + b*Sinh[c + d*x]]))] + (ArcCos[((-I)*a)/b] + (2*I)*(ArcTanh[((a + I*b)*Co
t[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] + ArcTanh[((a - I*b)*Tan
[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]]))*Log[((-1)^(1/4)*Sqrt[-a
^2 - b^2]*E^((c + d*x)/2))/(Sqrt[2]*Sqrt[(-I)*b]*Sqrt[a + b*Sinh[c + d*x]])
] + I*(PolyLog[2, ((I*a + Sqrt[-a^2 - b^2])*(I*a + b - I*Sqrt[-a^2 - b^2]*C
ot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(I*a + b + I*Sqrt[-a^2 - b^2]*Cot[((2
*I)*c + Pi + (2*I)*d*x)/4]))] - PolyLog[2, ((a + I*Sqrt[-a^2 - b^2])*(-a +
I*b + Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(I*a + b + I*
Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))])))/(Sqrt[-a^2 - b^2]*
d^2)) + (2*e*((4*a^2 + b^2)*(c + d*x) - (2*a*(4*a^2 + 3*b^2)*ArcTan[(b - a*
Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 4*a*b*Cosh[c + d*x
] + b^2*Sinh[2*(c + d*x)]))/d + (f*((4*a^2 + b^2)*(-c + d*x)*(c + d*x) - 8*
a*b*d*x*Cosh[c + d*x] - b^2*Cosh[2*(c + d*x)] - (2*a*(4*a^2 + 3*b^2)*(2*c*A
rcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + (c + d*x)
*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] - (c +
d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])] + P
olyLog[2, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])] - Pol
yLog[2, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))]))/Sqr
t[a^2 + b^2] + 8*a*b*Sinh[c + d*x] + 2*b^2*d*x*Sinh[2*(c + d*x)]))/d^2)/(8*
b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1011 vs. $2(297) = 594$.
time = 1.58, size = 1012, normalized size = 3.09

method	result

risch	$\frac{a^2 f x^2}{2b^3}+\frac{f x^2}{4b}+\frac{a^2 e x}{b^3}+\frac{e x}{2b}+\frac{(2dxf+2de-f)e^{2dx+2c}}{16d^2b}-\frac{a(dxf+de-f)e^{dx+c}}{2b^2d^2}-\frac{a(dxf+de+f)e^{-dx-c}}{2b^2d^2}-\frac{(2dxf+2de+f)e^{-}}{16d^2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/2*a^2*f*x^2/b^3+1/4*f*x^2/b+a^2*e*x/b^3+1/2*e*x/b+1/16*(2*d*f*x+2*d*e-f)/
d^2/b*exp(2*d*x+2*c)-1/2*a*(d*f*x+d*e-f)/b^2/d^2*exp(d*x+c)-1/2*a*(d*f*x+d*
e+f)/b^2/d^2*exp(-d*x-c)-1/16*(2*d*f*x+2*d*e+f)/d^2/b*exp(-2*d*x-2*c)+2/d*a
^3/b^3*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+
2/d*a/b*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
-1/d*a^3/b^3*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^
2+b^2)^(1/2)))*x-1/d^2*a^3/b^3*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2
)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d*a^3/b^3*f/(a^2+b^2)^(1/2)*ln((b*exp(
d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2*a^3/b^3*f/(a^2+b^2)^
(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d^2*a^3/
b^3*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)
^(1/2)))+1/d^2*a^3/b^3*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2
)+a)/(a+(a^2+b^2)^(1/2)))-1/d*a/b*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+
b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d^2*a/b*f/(a^2+b^2)^(1/2)*ln((-b*ex
p(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d*a/b*f/(a^2+b^2)^(1/
2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2*a/b*f/(
a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1
/d^2*a/b*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2
+b^2)^(1/2)))+1/d^2*a/b*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/
2)+a)/(a+(a^2+b^2)^(1/2)))-2/d^2*a^3/b^3*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2
*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d^2*a/b*f*c/(a^2+b^2)^(1/2)*arctanh(1
/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/16*(32*(a^3*e^c + a*b^2*e^c)*integrate(x*e^(d*x)/(b^4*e^(2*d*x + 2*c) +
2*a*b^3*e^(d*x + c) - b^4), x) - (4*(2*a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*x
^2 + (2*b^2*d*x*e^(4*c) - b^2*e^(4*c))*e^(2*d*x) - 8*(a*b*d*x*e^(3*c) - a*b
*e^(3*c))*e^(d*x) - 8*(a*b*d*x*e^c + a*b*e^c)*e^(-d*x) - (2*b^2*d*x + b^2)*
```

```
e^(-2*d*x))*e^(-2*c)/(b^3*d^2))*f - 1/8*((4*a*e^(-d*x - c) - b)*e^(2*d*x +
2*c)/(b^2*d) + 8*sqrt(a^2 + b^2)*a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2
))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b^3*d) - 4*(2*a^2 + b^2)*(d*x +
 c)/(b^3*d) + (4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1484 vs. $2(301) = 602$.
time = 0.42, size = 1484, normalized size = 4.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -1/16*(2*b^2*d*f*x - (2*b^2*d*f*x + 2*b^2*d*cosh(1) + 2*b^2*d*sinh(1) - b^2
*f)*cosh(d*x + c)^4 - (2*b^2*d*f*x + 2*b^2*d*cosh(1) + 2*b^2*d*sinh(1) - b^
2*f)*sinh(d*x + c)^4 + 2*b^2*d*cosh(1) + 8*(a*b*d*f*x + a*b*d*cosh(1) + a*b
*d*sinh(1) - a*b*f)*cosh(d*x + c)^3 + 2*b^2*d*sinh(1) + 4*(2*a*b*d*f*x + 2*
a*b*d*cosh(1) + 2*a*b*d*sinh(1) - 2*a*b*f - (2*b^2*d*f*x + 2*b^2*d*cosh(1)
+ 2*b^2*d*sinh(1) - b^2*f)*cosh(d*x + c))*sinh(d*x + c)^3 + b^2*f - 4*((2*a
^2 + b^2)*d^2*f*x^2 + 2*(2*a^2 + b^2)*d^2*x*cosh(1) + 2*(2*a^2 + b^2)*d^2*x
*sinh(1))*cosh(d*x + c)^2 - 2*(2*(2*a^2 + b^2)*d^2*f*x^2 + 4*(2*a^2 + b^2)*
d^2*x*cosh(1) + 4*(2*a^2 + b^2)*d^2*x*sinh(1) + 3*(2*b^2*d*f*x + 2*b^2*d*co
sh(1) + 2*b^2*d*sinh(1) - b^2*f)*cosh(d*x + c)^2 - 12*(a*b*d*f*x + a*b*d*co
sh(1) + a*b*d*sinh(1) - a*b*f)*cosh(d*x + c))*sinh(d*x + c)^2 + 16*(a*b*f*c
osh(d*x + c)^2 + 2*a*b*f*cosh(d*x + c)*sinh(d*x + c) + a*b*f*sinh(d*x + c)^
2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 16*(a*b*f*
cosh(d*x + c)^2 + 2*a*b*f*cosh(d*x + c)*sinh(d*x + c) + a*b*f*sinh(d*x + c)
^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 16*((a*b*
c*f - a*b*d*cosh(1) - a*b*d*sinh(1))*cosh(d*x + c)^2 + 2*(a*b*c*f - a*b*d*c
osh(1) - a*b*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a*b*c*f - a*b*d*cosh
(1) - a*b*d*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*
x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 16*((a*b*c*
f - a*b*d*cosh(1) - a*b*d*sinh(1))*cosh(d*x + c)^2 + 2*(a*b*c*f - a*b*d*cos
h(1) - a*b*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a*b*c*f - a*b*d*cosh(1
) - a*b*d*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x
+ c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 16*((a*b*d*f*
x + a*b*c*f)*cosh(d*x + c)^2 + 2*(a*b*d*f*x + a*b*c*f)*cosh(d*x + c)*sinh(d
*x + c) + (a*b*d*f*x + a*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(
-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 + b^2)/b^2) - b)/b) - 16*((a*b*d*f*x + a*b*c*f)*cosh(d*x + c)^2 +
2*(a*b*d*f*x + a*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b*d*f*x + a*b*c*f)
```

```
*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x
+ c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) +
8*(a*b*d*f*x + a*b*d*cosh(1) + a*b*d*sinh(1) + a*b*f)*cosh(d*x + c) + 4*(2*
a*b*d*f*x + 2*a*b*d*cosh(1) - (2*b^2*d*f*x + 2*b^2*d*cosh(1) + 2*b^2*d*sinh
(1) - b^2*f)*cosh(d*x + c)^3 + 2*a*b*d*sinh(1) + 2*a*b*f + 6*(a*b*d*f*x + a
*b*d*cosh(1) + a*b*d*sinh(1) - a*b*f)*cosh(d*x + c)^2 - 2*((2*a^2 + b^2)*d^
2*f*x^2 + 2*(2*a^2 + b^2)*d^2*x*cosh(1) + 2*(2*a^2 + b^2)*d^2*x*sinh(1))*co
sh(d*x + c))*sinh(d*x + c))/(b^3*d^2*cosh(d*x + c)^2 + 2*b^3*d^2*cosh(d*x +
 c)*sinh(d*x + c) + b^3*d^2*sinh(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)^2*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^2\,\sinh(c+d\,x)\,\left(e+f\,x\right)}{a+b\,\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.341 $\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{(2a^2+b^2)\,x}{2b^3}+\frac{2a\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^3d}-\frac{\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}$$

```
[Out] 1/2*(2*a^2+b^2)*x/b^3-1/2*cosh(d*x+c)*(2*a-b*sinh(d*x+c))/b^2/d+2*a*arctanh
((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/b^3/d
```

Rubi [A]
time = 0.14, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.185$, Rules used = {2944, 2814, 2739, 632, 210}

$$\frac{2a\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^3d}+\frac{x(2a^2+b^2)}{2b^3}-\frac{\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((2*a^2 + b^2)*x)/(2*b^3) + (2*a*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[(c + d
*x)/2])/Sqrt[a^2 + b^2]])/(b^3*d) - (Cosh[c + d*x]*(2*a - b*Sinh[c + d*x]))
/(2*b^2*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}+\frac{i\int \frac{iab-i(2a^2+b^2)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx}{2b^2}\\ &= \frac{(2a^2+b^2)\,x}{2b^3}-\frac{\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}-\frac{(a(a^2+b^2))\int \frac{}{a+b\sin}\\ &= \frac{(2a^2+b^2)\,x}{2b^3}-\frac{\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}+\frac{(2ia(a^2+b^2))\,\text{Subst}}{}\\ &= \frac{(2a^2+b^2)\,x}{2b^3}-\frac{\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}-\frac{(4ia(a^2+b^2))\,\text{Subst}}{}\\ &= \frac{(2a^2+b^2)\,x}{2b^3}+\frac{2a\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^3d}-\frac{\cosh(c+dx)(2}{}\end{aligned}$$

Mathematica [A]
time = 0.42, size = 109, normalized size = 1.15

$$\frac{4a^2c+2b^2c+4a^2dx+2b^2dx+8a\sqrt{-a^2-b^2}\,\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)-4ab\cosh(c+dx)+b^2\sinh(2(c+dx))}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

[Out] (4*a^2*c + 2*b^2*c + 4*a^2*d*x + 2*b^2*d*x + 8*a*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] - 4*a*b*Cosh[c + d*x] + b^2*Sinh[2*(c + d*x)])/(4*b^3*d)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(86) = 172$.
time = 0.93, size = 189, normalized size = 1.99

method	result
risch	$\frac{x a^2}{b^3}+\frac{x}{2b}+\frac{e^{2dx+2c}}{8bd}-\frac{a\,e^{dx+c}}{2b^2 d}-\frac{a\,e^{-dx-c}}{2b^2 d}-\frac{e^{-2dx-2c}}{8bd}+\frac{\sqrt{a^2+b^2}\,a\ln\left(e^{dx+c}+\frac{a+\sqrt{a^2+b^2}}{b}\right)}{d\,b^3}-$
derivativedivides	$\frac{\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-b-2a}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-2a^2-b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^3}-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-b+2a}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d}$
default	$\frac{\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-b-2a}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-2a^2-b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^3}-\frac{1}{2b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-b+2a}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(1/2/b/(tanh(1/2*d*x+1/2*c)-1)^2-1/2*(-b-2*a)/b^2/(tanh(1/2*d*x+1/2*c)-1)+1/2/b^3*(-2*a^2-b^2)*ln(tanh(1/2*d*x+1/2*c)-1)-1/2/b/(tanh(1/2*d*x+1/2*c)+1)^2-1/2*(-b+2*a)/b^2/(tanh(1/2*d*x+1/2*c)+1)+1/2*(2*a^2+b^2)/b^3*ln(tanh(1/2*d*x+1/2*c)+1)-2*a*(a^2+b^2)^(1/2)/b^3*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))

Maxima [A]
time = 0.49, size = 160, normalized size = 1.68

$$-\frac{\left(4\,ae^{(-dx-c)}-b\right)e^{(2\,dx+2\,c)}}{8\,b^2 d}-\frac{\sqrt{a^2+b^2}\,a\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{b^3 d}+\frac{(2\,a^2+b^2)(dx+c)}{2\,b^3 d}-\frac{4\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}}{8\,b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")

[Out] -1/8*(4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) - sqrt(a^2 + b^2)*a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b^3*d) + 1/2*(2*a^2 + b^2)*(d*x + c)/(b^3*d) - 1/8*(4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(87) = 174$.

time = 0.34, size = 446, normalized size = 4.69

$$\frac{b^{2}\cosh\left(dx+c\right)^{4}+b^{2}\sinh\left(dx+c\right)^{4}+4\left(2a^{2}+b^{2}\right)dx\cosh\left(dx+c\right)^{2}-4ab\cosh\left(dx+c\right)^{3}+4\left(b^{2}\cosh\left(dx+c\right)-ab\right)\sinh\left(dx+c\right)^{3}-4ab\cosh\left(dx+c\right)+2\left(3b^{2}\cosh\left(dx+c\right)^{2}+2\left(2a^{2}+b^{2}\right)dx-6ab\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{2}+8\left(a\cosh\left(dx+c\right)^{2}+2a\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a\sinh\left(dx+c\right)^{2}\right)\sqrt{a^{2}+b^{2}}\log\left(\frac{b^{2}\cosh(dx+c)^{2}+b^{2}\sinh(dx+c)^{2}+2ab\cosh(dx+c)+2a^{2}+b^{2}+2\left(b^{2}\cosh(dx+c)+ab\right)\sinh(dx+c)+2\sqrt{a^{2}+b^{2}}\left(b\cosh(dx+c)+b\sinh(dx+c)+a\right)}{b\cosh(dx+c)^{2}+b\sinh(dx+c)^{2}+2a\cosh(dx+c)+2\left(b\cosh(dx+c)+a\right)\sinh(dx+c)-b}\right)-b^{2}+4\left(b^{2}\cosh\left(dx+c\right)^{3}+2\left(2a^{2}+b^{2}\right)dx\cosh\left(dx+c\right)-3ab\cosh\left(dx+c\right)^{2}-ab\right)\sinh\left(dx+c\right)}{8\left(b^{3}d\cosh\left(dx+c\right)^{2}+2b^{3}d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b^{3}d\sinh\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/8*(b^2*cosh(d*x + c)^4 + b^2*sinh(d*x + c)^4 + 4*(2*a^2 + b^2)*d*x*cosh(d
*x + c)^2 - 4*a*b*cosh(d*x + c)^3 + 4*(b^2*cosh(d*x + c) - a*b)*sinh(d*x +
c)^3 - 4*a*b*cosh(d*x + c) + 2*(3*b^2*cosh(d*x + c)^2 + 2*(2*a^2 + b^2)*d*x
 - 6*a*b*cosh(d*x + c))*sinh(d*x + c)^2 + 8*(a*cosh(d*x + c)^2 + 2*a*cosh(d
*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2)*sqrt(a^2 + b^2)*log((b^2*cosh(d*
x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2
*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) +
b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x
 + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - b^2 + 4*(b^2*cosh(d*x
 + c)^3 + 2*(2*a^2 + b^2)*d*x*cosh(d*x + c) - 3*a*b*cosh(d*x + c)^2 - a*b)*
sinh(d*x + c))/(b^3*d*cosh(d*x + c)^2 + 2*b^3*d*cosh(d*x + c)*sinh(d*x + c)
 + b^3*d*sinh(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.46, size = 155, normalized size = 1.63

$$\frac{\frac{4\left(2a^{2}+b^{2}\right)(dx+c)}{b^{3}}+\frac{be^{(2dx+2c)}-4ae^{(dx+c)}}{b^{2}}-\frac{\left(4abe^{(dx+c)}+b^{2}\right)e^{(-2dx-2c)}}{b^{3}}-\frac{8\left(a^{3}+ab^{2}\right)\log\left(\left|\frac{2be^{(dx+c)}+2a-2\sqrt{a^{2}+b^{2}}}{2be^{(dx+c)}+2a+2\sqrt{a^{2}+b^{2}}}\right|\right)}{\sqrt{a^{2}+b^{2}}\,b^{3}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(4*(2*a^2 + b^2)*(d*x + c)/b^3 + (b*e^(2*d*x + 2*c) - 4*a*e^(d*x + c))/
b^2 - (4*a*b*e^(d*x + c) + b^2)*e^(-2*d*x - 2*c)/b^3 - 8*(a^3 + a*b^2)*log(
```

```
abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c) + 2*a +
2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3))/d
```

Mupad [B]
time = 0.49, size = 212, normalized size = 2.23

$$\frac{e^{2c+2dx}}{8bd}-\frac{e^{-2c-2dx}}{8bd}+\frac{x(2a^2+b^2)}{2b^3}-\frac{a\,e^{-c-dx}}{2b^2d}-\frac{a\,e^{c+dx}}{2b^2d}-\frac{a\ln\left(\frac{2a\,e^{c+dx}(a^2+b^2)}{b^4}-\frac{2a\sqrt{a^2+b^2}\,(b-a\,e^{c+dx})}{b^4}\right)\sqrt{a^2+b^2}}{b^3d}+\frac{a\ln\left(\frac{2a\sqrt{a^2+b^2}\,(b-a\,e^{c+dx})}{b^4}+\frac{2a\,e^{c+dx}(a^2+b^2)}{b^4}\right)\sqrt{a^2+b^2}}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*sinh(c + d*x))/(a + b*sinh(c + d*x)),x)
```

```
[Out] exp(2*c + 2*d*x)/(8*b*d) - exp(- 2*c - 2*d*x)/(8*b*d) + (x*(2*a^2 + b^2))/(
2*b^3) - (a*exp(- c - d*x))/(2*b^2*d) - (a*exp(c + d*x))/(2*b^2*d) - (a*log
((2*a*exp(c + d*x)*(a^2 + b^2))/b^4 - (2*a*(a^2 + b^2)^(1/2)*(b - a*exp(c +
 d*x)))/b^4)*(a^2 + b^2)^(1/2))/(b^3*d) + (a*log((2*a*(a^2 + b^2)^(1/2)*(b
- a*exp(c + d*x)))/b^4 + (2*a*exp(c + d*x)*(a^2 + b^2))/b^4)*(a^2 + b^2)^(1
/2))/(b^3*d)
```

3.342 $\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cosh[c + d*x]^2*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Cosh[c + d*x]^2*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$$\text{\$Aborted}$$

Verification is not applicable to the result.

```
[In] Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\cosh^2\left(dx+c\right)\right)\sinh\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -2*(a^3*e^c + a*b^2*e^c)*integrate(-e^(d*x)/(b^4*f*x + b^4*e - (b^4*f*x*e^(
2*c) + b^4*e^(2*c + 1))*e^(2*d*x) - 2*(a*b^3*f*x*e^c + a*b^3*e^(c + 1))*e^(
d*x)), x) - 1/4*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b*f)
 - 1/2*a*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^2*f) + 1/2*a*e^
(c - d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^2*f) - 1/4*e^(2*c - 2*d*e/
f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b*f) + 1/2*(2*a^2 + b^2)*log(f*x +
e)/(b^3*f)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(cosh(d*x + c)^2*sinh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate(cosh(d*x + c)^2*sinh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)),
x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh(c+d\,x)^2\,\sinh(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*sinh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((cosh(c + d*x)^2*sinh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.343 $\int \frac{(e+fx)^3 \cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=864

$$-\frac{3af^3x}{8b^2d^3}-\frac{a(e+fx)^3}{4b^2d}+\frac{a(a^2+b^2)\,(e+fx)^4}{4b^4f}-\frac{6a^2f^3\cosh(c+dx)}{b^3d^4}-\frac{40f^3\cosh(c+dx)}{9bd^4}-\frac{3a^2f(e+fx)^2\cosh(}{b^3d^2}$$

[Out] -3/8*a*f^3*x/b^2/d^3+1/4*a*(a^2+b^2)*(f*x+e)^4/b^4/f-6*a^2*f^3*cosh(d*x+c)/b^3/d^4-1/3*f*(f*x+e)^2*cosh(d*x+c)^3/b/d^2+1/3*(f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/b/d-1/4*a*(f*x+e)^3/b^2/d-2/27*f^3*cosh(d*x+c)^3/b/d^4+2/3*(f*x+e)^3*sinh(d*x+c)/b/d-40/9*f^3*cosh(d*x+c)/b/d^4-3*a^2*f*(f*x+e)^2*cosh(d*x+c)/b^3/d^2+6*a^2*f^2*(f*x+e)*sinh(d*x+c)/b^3/d^3+3/8*a*f^3*cosh(d*x+c)*sinh(d*x+c)/b^2/d^4+2/9*f^2*(f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/b/d^3-3/4*a*f^2*(f*x+e)*sinh(d*x+c)^2/b^2/d^3-3*a*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d^2-3*a*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d^2+6*a*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d^3+6*a*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d^3+3/4*a*f*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/b^2/d^2-1/2*a*(f*x+e)^3*sinh(d*x+c)^2/b^2/d-a*(a^2+b^2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d-a*(a^2+b^2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d-6*a*(a^2+b^2)*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d^4-6*a*(a^2+b^2)*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d^4-2*f*(f*x+e)^2*cosh(d*x+c)/b/d^2+40/9*f^2*(f*x+e)*sinh(d*x+c)/b/d^3+a^2*(f*x+e)^3*sinh(d*x+c)/b^3/d

Rubi [A]
time = 0.78, antiderivative size = 864, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 16, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {5698, 3392, 3377, 2718, 3391, 5684, 5554, 32, 2715, 8, 5680, 2221, 2611, 6744, 2320, 6724}

Antiderivative was successfully verified.

[In] Int[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]

[Out] (-3*a*f^3*x)/(8*b^2*d^3) - (a*(e + f*x)^3)/(4*b^2*d) + (a*(a^2 + b^2)*(e + f*x)^4)/(4*b^4*f) - (6*a^2*f^3*Cosh[c + d*x])/(b^3*d^4) - (40*f^3*Cosh[c + d*x])/(9*b*d^4) - (3*a^2*f*(e + f*x)^2*Cosh[c + d*x])/(b^3*d^2) - (2*f*(e + f*x)^2*Cosh[c + d*x])/(b*d^2) - (2*f^3*Cosh[c + d*x]^3)/(27*b*d^4) - (f*(e + f*x)^2*Cosh[c + d*x]^3)/(3*b*d^2) - (a*(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a*(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^4*d) - (3*a*(a^2 + b^2)

```
*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d
^2) - (3*a*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2]))])/(b^4*d^2) + (6*a*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^3) + (6*a*(a^2 + b^2)*f^2*(e +
 f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^3) - (6*
a*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^
4*d^4) - (6*a*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))])/(b^4*d^4) + (6*a^2*f^2*(e + f*x)*Sinh[c + d*x])/(b^3*d^3) + (40*f^
2*(e + f*x)*Sinh[c + d*x])/(9*b*d^3) + (a^2*(e + f*x)^3*Sinh[c + d*x])/(b^3
*d) + (2*(e + f*x)^3*Sinh[c + d*x])/(3*b*d) + (3*a*f^3*Cosh[c + d*x]*Sinh[c
 + d*x])/(8*b^2*d^4) + (3*a*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b
^2*d^2) + (2*f^2*(e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(9*b*d^3) + ((e +
 f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b*d) - (3*a*f^2*(e + f*x)*Sinh[c
+ d*x]^2)/(4*b^2*d^3) - (a*(e + f*x)^3*Sinh[c + d*x]^2)/(2*b^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\cosh^3(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{f(e+fx)^2\cosh^3(c+dx)}{3bd^2} + \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c$$

$$= \frac{a(a^2+b^2)(e+fx)^4}{4b^4f} - \frac{2f^3\cosh^3(c+dx)}{27bd^4} - \frac{f(e+fx)^2\cosh^3}{3bd^2}$$

$$= \frac{a(a^2+b^2)(e+fx)^4}{4b^4f} - \frac{3a^2f(e+fx)^2\cosh(c+dx)}{b^3d^2} - \frac{2f(e+}{}$$

$$= -\frac{a(e+fx)^3}{4b^2d} + \frac{a(a^2+b^2)(e+fx)^4}{4b^4f} - \frac{4f^3\cosh(c+dx)}{9bd^4} - 3$$

$$= -\frac{3af^3x}{8b^2d^3} - \frac{a(e+fx)^3}{4b^2d} + \frac{a(a^2+b^2)(e+fx)^4}{4b^4f} - \frac{6a^2f^3\cosh(}{b^3d^4}$$

$$= -\frac{3af^3x}{8b^2d^3} - \frac{a(e+fx)^3}{4b^2d} + \frac{a(a^2+b^2)(e+fx)^4}{4b^4f} - \frac{6a^2f^3\cosh(}{b^3d^4}$$

$$= -\frac{3af^3x}{8b^2d^3} - \frac{a(e+fx)^3}{4b^2d} + \frac{a(a^2+b^2)(e+fx)^4}{4b^4f} - \frac{6a^2f^3\cosh(}{b^3d^4}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 5041 vs. $2(864) = 1728$.
time = 16.88, size = 5041, normalized size = 5.83

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] Result too large to show
```

Maple [**F**]
time = 1.88, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3\left(\cosh^3(dx+c)\right)\sinh(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -1/24*((3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 + 3*b^2)*e^(-2*d*x - 2*c))*e^(3
*d*x + 3*c)/(b^3*d) + 24*(a^3 + a*b^2)*(d*x + c)/(b^4*d) + (3*a*b*e^(-2*d*x
 - 2*c) + b^2*e^(-3*d*x - 3*c) + 3*(4*a^2 + 3*b^2)*e^(-d*x - c))/(b^3*d) +
24*(a^3 + a*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^4*d))*e
^3 - 1/864*(216*(a^3*d^4*f^3*e^(3*c) + a*b^2*d^4*f^3*e^(3*c))*x^4 + 864*(a^
3*d^4*f^2*e^(3*c) + a*b^2*d^4*f^2*e^(3*c))*x^3*e + 1296*(a^3*d^4*f*e^(3*c)
+ a*b^2*d^4*f*e^(3*c))*x^2*e^2 - 4*(9*b^3*d^3*f^3*x^3*e^(6*c) - 2*b^3*f^3*e
^(6*c) - 9*b^3*d^2*f*e^(6*c + 2) + 6*b^3*d*f^2*e^(6*c + 1) - 9*(b^3*d^2*f^3
*e^(6*c) - 3*b^3*d^3*f^2*e^(6*c + 1))*x^2 + 3*(2*b^3*d*f^3*e^(6*c) + 9*b^3*
d^3*f*e^(6*c + 2) - 6*b^3*d^2*f^2*e^(6*c + 1))*x)*e^(3*d*x) + 27*(4*a*b^2*d
^3*f^3*x^3*e^(5*c) - 3*a*b^2*f^3*e^(5*c) - 6*a*b^2*d^2*f*e^(5*c + 2) + 6*a*
b^2*d*f^2*e^(5*c + 1) - 6*(a*b^2*d^2*f^3*e^(5*c) - 2*a*b^2*d^3*f^2*e^(5*c +
 1))*x^2 + 6*(a*b^2*d*f^3*e^(5*c) + 2*a*b^2*d^3*f*e^(5*c + 2) - 2*a*b^2*d^2
*f^2*e^(5*c + 1))*x)*e^(2*d*x) + 108*(24*a^2*b*f^3*e^(4*c) + 18*b^3*f^3*e^(
4*c) - (4*a^2*b*d^3*f^3*e^(4*c) + 3*b^3*d^3*f^3*e^(4*c))*x^3 + 3*(4*a^2*b*d
^2*f^3*e^(4*c) + 3*b^3*d^2*f^3*e^(4*c) - (4*a^2*b*d^3*f^2*e^(4*c) + 3*b^3*d
^3*f^2*e^(4*c))*e)*x^2 - 3*(8*a^2*b*d*f^3*e^(4*c) + 6*b^3*d*f^3*e^(4*c) + (
4*a^2*b*d^3*f*e^(4*c) + 3*b^3*d^3*f*e^(4*c))*e^2 - 2*(4*a^2*b*d^2*f^2*e^(4*
c) + 3*b^3*d^2*f^2*e^(4*c))*e)*x + 3*(4*a^2*b*d^2*f*e^(4*c) + 3*b^3*d^2*f*e
^(4*c))*e^2 - 6*(4*a^2*b*d*f^2*e^(4*c) + 3*b^3*d*f^2*e^(4*c))*e)*e^(d*x) +
108*(24*a^2*b*f^3*e^(2*c) + 18*b^3*f^3*e^(2*c) + (4*a^2*b*d^3*f^3*e^(2*c) +
 3*b^3*d^3*f^3*e^(2*c))*x^3 + 3*(4*a^2*b*d^2*f^3*e^(2*c) + 3*b^3*d^2*f^3*e^
(2*c) + (4*a^2*b*d^3*f^2*e^(2*c) + 3*b^3*d^3*f^2*e^(2*c))*e)*x^2 + 3*(8*a^2
*b*d*f^3*e^(2*c) + 6*b^3*d*f^3*e^(2*c) + (4*a^2*b*d^3*f*e^(2*c) + 3*b^3*d^3
*f*e^(2*c))*e^2 + 2*(4*a^2*b*d^2*f^2*e^(2*c) + 3*b^3*d^2*f^2*e^(2*c))*e)*x
+ 3*(4*a^2*b*d^2*f*e^(2*c) + 3*b^3*d^2*f*e^(2*c))*e^2 + 6*(4*a^2*b*d*f^2*e^
(2*c) + 3*b^3*d*f^2*e^(2*c))*e)*e^(-d*x) + 27*(4*a*b^2*d^3*f^3*x^3*e^c + 6*
a*b^2*d^2*f*e^(c + 2) + 6*a*b^2*d*f^2*e^(c + 1) + 3*a*b^2*f^3*e^c + 6*(2*a*
b^2*d^3*f^2*e^(c + 1) + a*b^2*d^2*f^3*e^c)*x^2 + 6*(2*a*b^2*d^3*f*e^(c + 2)
 + 2*a*b^2*d^2*f^2*e^(c + 1) + a*b^2*d*f^3*e^c)*x)*e^(-2*d*x) + 4*(9*b^3*d^
3*f^3*x^3 + 9*b^3*d^2*f*e^2 + 6*b^3*d*f^2*e + 2*b^3*f^3 + 9*(3*b^3*d^3*f^2*
```

```
e + b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*f*e^2 + 6*b^3*d^2*f^2*e + 2*b^3*d*f^3)*
x)*e^(-3*d*x))*e^(-3*c)/(b^4*d^4) + integrate(-2*((a^3*b*f^3 + a*b^3*f^3)*x
^3 + 3*(a^3*b*f^2 + a*b^3*f^2)*x^2*e + 3*(a^3*b*f + a*b^3*f)*x*e^2 - ((a^4*
f^3*e^c + a^2*b^2*f^3*e^c)*x^3 + 3*(a^4*f^2*e^c + a^2*b^2*f^2*e^c)*x^2*e +
3*(a^4*f*e^c + a^2*b^2*f*e^c)*x*e^2)*e^(d*x))/(b^5*e^(2*d*x + 2*c) + 2*a*b^
4*e^(d*x + c) - b^5), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 15138 vs. $2(830) = 1660$.
time = 0.55, size = 15138, normalized size = 17.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -1/864*(36*b^3*d^3*f^3*x^3 + 36*b^3*d^2*f^3*x^2 + 36*b^3*d^3*cosh(1)^3 + 36
*b^3*d^3*sinh(1)^3 + 24*b^3*d*f^3*x - 4*(9*b^3*d^3*f^3*x^3 - 9*b^3*d^2*f^3*
x^2 + 9*b^3*d^3*cosh(1)^3 + 9*b^3*d^3*sinh(1)^3 + 6*b^3*d*f^3*x - 2*b^3*f^3
 + 9*(3*b^3*d^3*f*x - b^3*d^2*f)*cosh(1)^2 + 9*(3*b^3*d^3*f*x + 3*b^3*d^3*c
osh(1) - b^3*d^2*f)*sinh(1)^2 + 3*(9*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 2*
b^3*d*f^2)*cosh(1) + 3*(9*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 9*b^3*d^3*cos
h(1)^2 + 2*b^3*d*f^2 + 6*(3*b^3*d^3*f*x - b^3*d^2*f)*cosh(1))*sinh(1))*cosh
(d*x + c)^6 - 4*(9*b^3*d^3*f^3*x^3 - 9*b^3*d^2*f^3*x^2 + 9*b^3*d^3*cosh(1)^
3 + 9*b^3*d^3*sinh(1)^3 + 6*b^3*d*f^3*x - 2*b^3*f^3 + 9*(3*b^3*d^3*f*x - b^
3*d^2*f)*cosh(1)^2 + 9*(3*b^3*d^3*f*x + 3*b^3*d^3*cosh(1) - b^3*d^2*f)*sinh
(1)^2 + 3*(9*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 2*b^3*d*f^2)*cosh(1) + 3*(
9*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 9*b^3*d^3*cosh(1)^2 + 2*b^3*d*f^2 + 6
*(3*b^3*d^3*f*x - b^3*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^6 + 8*b^3*f^3
+ 27*(4*a*b^2*d^3*f^3*x^3 - 6*a*b^2*d^2*f^3*x^2 + 4*a*b^2*d^3*cosh(1)^3 + 4
*a*b^2*d^3*sinh(1)^3 + 6*a*b^2*d*f^3*x - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*f*x -
 a*b^2*d^2*f)*cosh(1)^2 + 6*(2*a*b^2*d^3*f*x + 2*a*b^2*d^3*cosh(1) - a*b^2*
d^2*f)*sinh(1)^2 + 6*(2*a*b^2*d^3*f^2*x^2 - 2*a*b^2*d^2*f^2*x + a*b^2*d*f^2
)*cosh(1) + 6*(2*a*b^2*d^3*f^2*x^2 - 2*a*b^2*d^2*f^2*x + 2*a*b^2*d^3*cosh(1
)^2 + a*b^2*d*f^2 + 2*(2*a*b^2*d^3*f*x - a*b^2*d^2*f)*cosh(1))*sinh(1))*cos
h(d*x + c)^5 + 3*(36*a*b^2*d^3*f^3*x^3 - 54*a*b^2*d^2*f^3*x^2 + 36*a*b^2*d^
3*cosh(1)^3 + 36*a*b^2*d^3*sinh(1)^3 + 54*a*b^2*d*f^3*x - 27*a*b^2*f^3 + 54
*(2*a*b^2*d^3*f*x - a*b^2*d^2*f)*cosh(1)^2 + 54*(2*a*b^2*d^3*f*x + 2*a*b^2*
d^3*cosh(1) - a*b^2*d^2*f)*sinh(1)^2 + 54*(2*a*b^2*d^3*f^2*x^2 - 2*a*b^2*d^
2*f^2*x + a*b^2*d*f^2)*cosh(1) - 8*(9*b^3*d^3*f^3*x^3 - 9*b^3*d^2*f^3*x^2 +
 9*b^3*d^3*cosh(1)^3 + 9*b^3*d^3*sinh(1)^3 + 6*b^3*d*f^3*x - 2*b^3*f^3 + 9*
(3*b^3*d^3*f*x - b^3*d^2*f)*cosh(1)^2 + 9*(3*b^3*d^3*f*x + 3*b^3*d^3*cosh(1
) - b^3*d^2*f)*sinh(1)^2 + 3*(9*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 2*b^3*d
*f^2)*cosh(1) + 3*(9*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 9*b^3*d^3*cosh(1)^
```

```
2 + 2*b^3*d*f^2 + 6*(3*b^3*d^3*f*x - b^3*d^2*f)*cosh(1))*sinh(1))*cosh(d*x
+ c) + 54*(2*a*b^2*d^3*f^2*x^2 - 2*a*b^2*d^2*f^2*x + 2*a*b^2*d^3*cosh(1)^2
+ a*b^2*d*f^2 + 2*(2*a*b^2*d^3*f*x - a*b^2*d^2*f)*cosh(1))*sinh(1))*sinh(d*
x + c)^5 - 108*((4*a^2*b + 3*b^3)*d^3*f^3*x^3 - 3*(4*a^2*b + 3*b^3)*d^2*f^3
*x^2 + (4*a^2*b + 3*b^3)*d^3*cosh(1)^3 + (4*a^2*b + 3*b^3)*d^3*sinh(1)^3 +
6*(4*a^2*b + 3*b^3)*d*f^3*x - 6*(4*a^2*b + 3*b^3)*f^3 + 3*((4*a^2*b + 3*b^3
)*d^3*f*x - (4*a^2*b + 3*b^3)*d^2*f)*cosh(1)^2 + 3*((4*a^2*b + 3*b^3)*d^3*f
*x + (4*a^2*b + 3*b^3)*d^3*cosh(1) - (4*a^2*b + 3*b^3)*d^2*f)*sinh(1)^2 + 3
*((4*a^2*b + 3*b^3)*d^3*f^2*x^2 - 2*(4*a^2*b + 3*b^3)*d^2*f^2*x + 2*(4*a^2*
b + 3*b^3)*d*f^2)*cosh(1) + 3*((4*a^2*b + 3*b^3)*d^3*f^2*x^2 - 2*(4*a^2*b +
 3*b^3)*d^2*f^2*x + (4*a^2*b + 3*b^3)*d^3*cosh(1)^2 + 2*(4*a^2*b + 3*b^3)*d
*f^2 + 2*((4*a^2*b + 3*b^3)*d^3*f*x - (4*a^2*b + 3*b^3)*d^2*f)*cosh(1))*sin
h(1))*cosh(d*x + c)^4 - 3*(36*(4*a^2*b + 3*b^3)*d^3*f^3*x^3 - 108*(4*a^2*b
+ 3*b^3)*d^2*f^3*x^2 + 36*(4*a^2*b + 3*b^3)*d^3*cosh(1)^3 + 36*(4*a^2*b + 3
*b^3)*d^3*sinh(1)^3 + 216*(4*a^2*b + 3*b^3)*d*f^3*x - 216*(4*a^2*b + 3*b^3)
*f^3 + 108*((4*a^2*b + 3*b^3)*d^3*f*x - (4*a^2*b + 3*b^3)*d^2*f)*cosh(1)^2
+ 20*(9*b^3*d^3*f^3*x^3 - 9*b^3*d^2*f^3*x^2 + 9*b^3*d^3*cosh(1)^3 + 9*b^3*d
^3*sinh(1)^3 + 6*b^3*d*f^3*x - 2*b^3*f^3 + 9*(3*b^3*d^3*f*x - b^3*d^2*f)*co
sh(1)^2 + 9*(3*b^3*d^3*f*x + 3*b^3*d^3*cosh(1) - b^3*d^2*f)*sinh(1)^2 + 3*(
9*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 2*b^3*d*f^2)*cosh(1) + 3*(9*b^3*d^3*f
^2*x^2 - 6*b^3*d^2*f^2*x + 9*b^3*d^3*cosh(1)^2 + 2*b^3*d*f^2 + 6*(3*b^3*d^3
*f*x - b^3*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 108*((4*a^2*b + 3*b^3
)*d^3*f*x + (4*a^2*b + 3*b^3)*d^3*cosh(1) - (4*a^2*b + 3*b^3)*d^2*f)*sinh(1
)^2 + 108*((4*a^2*b + 3*b^3)*d^3*f^2*x^2 - 2*(4*a^2*b + 3*b^3)*d^2*f^2*x +
2*(4*a^2*b + 3*b^3)*d*f^2)*cosh(1) - 45*(4*a*b^2*d^3*f^3*x^3 - 6*a*b^2*d^2*
f^3*x^2 + 4*a*b^2*d^3*cosh(1)^3 + 4*a*b^2*d^3*sinh(1)^3 + 6*a*b^2*d*f^3*x -
 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*f*x - a*b^2*d^2*f)*cosh(1)^2 + 6*(2*a*b^2*d^3
*f*x + 2*a*b^2*d^3*cosh(1) - a*b^2*d^2*f)*sinh(1)^2 + 6*(2*a*b^2*d^3*f^2*x^
2 - 2*a*b^2*d^2*f^2*x + a*b^2*d*f^2)*cosh(1) + 6*(2*a*b^2*d^3*f^2*x^2 - 2*a
*b^2*d^2*f^2*x + 2*a*b^2*d^3*cosh(1)^2 + a*b^2*d*f^2 + 2*(2*a*b^2*d^3*f*x -
 a*b^2*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c) + 108*((4*a^2*b + 3*b^3)*d^3*
f^2*x^2 - 2*(4*a^2*b + 3*b^3)*d^2*f^2*x + (4*a^2*b + 3*b^3)*d^3*cosh(1)^2 +
 2*(4*a^2*b + 3*b^3)*d*f^2 + 2*((4*a^2*b + 3*b^3)*d^3*f*x - (4*a^2*b + 3*b^
3)*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^4 - 216*((a^3 + a*b^2)*d^4*f^3*x^
4 - 2*(a^3 + a*b^2)*c^4*f^3 + 4*((a^3 + a*b^2)*d^4*x + 2*(a^3 + a*b^2)*c*d^
3)*cosh(1)^3 + 4*((a^3 + a*b^2)*d^4*x + 2*(a^3 + a*b^2)*c*d^3)*sinh(1)^3 +
6*((a^3 + a*b^2)*d^4*f*x^2 - 2*(a^3 + a*b^2)*c^2*d^2*f)*cosh(1)^2 + 6*((a^3
 + a*b^2)*d^4*f*x^2 - 2*(a^3 + a*b^2)*c^2*d^2*f + 2*((a^3 + a*b^2)*d^4*x +
2*(a^3 + a*b^2)*c*d^3)*cosh(1))*sinh(1)^2 + 4*(...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)^3*sinh(d*x + c)/(b*sinh(d*x + c) + a),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^3\,\sinh(c+d\,x)\,\left(e+f\,x\right)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.344 $\int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal. Leaf size=636

$$-\frac{aefx}{2b^2d}-\frac{af^2x^2}{4b^2d}+\frac{a(a^2+b^2)(e+fx)^3}{3b^4f}-\frac{2a^2f(e+fx)\cosh(c+dx)}{b^3d^2}-\frac{4f(e+fx)\cosh(c+dx)}{3bd^2}-\frac{2f(e+fx)\text{c}}{9b}$$

```
[Out] -1/2*a*e*f*x/b^2/d-1/4*a*f^2*x^2/b^2/d+1/3*a*(a^2+b^2)*(f*x+e)^3/b^4/f-2*a^
2*f*(f*x+e)*cosh(d*x+c)/b^3/d^2-4/3*f*(f*x+e)*cosh(d*x+c)/b/d^2-2/9*f*(f*x+
e)*cosh(d*x+c)^3/b/d^2-a*(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)
^(1/2)))/b^4/d-a*(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))
/b^4/d-2*a*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))
/b^4/d^2-2*a*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)
))/b^4/d^2+2*a*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b
^4/d^3+2*a*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d
^3+2*a^2*f^2*sinh(d*x+c)/b^3/d^3+14/9*f^2*sinh(d*x+c)/b/d^3+a^2*(f*x+e)^2*s
inh(d*x+c)/b^3/d+2/3*(f*x+e)^2*sinh(d*x+c)/b/d+1/2*a*f*(f*x+e)*cosh(d*x+c)*
sinh(d*x+c)/b^2/d^2+1/3*(f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/b/d-1/4*a*f^2*s
inh(d*x+c)^2/b^2/d^3-1/2*a*(f*x+e)^2*sinh(d*x+c)^2/b^2/d+2/27*f^2*sinh(d*x+
c)^3/b/d^3
```

Rubi [A]
time = 0.60, antiderivative size = 636, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 13, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$, Rules used = {5698, 3392, 3377, 2717, 2713, 5684, 5554, 3391, 5680, 2221, 2611, 2320, 6724}

$$\frac{2a^2f^2\sinh(c+dx)}{b^3d^3}-\frac{2a^2f(e+fx)\cosh(c+dx)}{b^3d^2}+\frac{a^2(e+fx)^2\sinh(c+dx)}{b^3d}+\frac{2af^2(a^2+b^2)\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^3}+\frac{2af^2(a^2+b^2)\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^3}-\frac{2af(a^2+b^2)(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{2af(a^2+b^2)(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{a(a^2+b^2)(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^4d}-\frac{a(a^2+b^2)(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^4d}+\frac{a(a^2+b^2)(e+fx)^3}{3b^4f}-\frac{af^2\sinh^2(c+dx)}{4b^2d^3}+\frac{af(e+fx)\sinh(c+dx)\cosh(c+dx)}{2b^2d^2}-\frac{a(e+fx)^2\sinh^2(c+dx)}{2b^2d}-\frac{aefx}{2b^2d}-\frac{af^2x^2}{4b^2d}+\frac{2f^2\sinh^3(c+dx)}{27bd^3}+\frac{14f^2\sinh(c+dx)}{9bd^3}-\frac{2f(e+fx)\cosh^3(c+dx)}{9bd^2}-\frac{4f(e+fx)\cosh(c+dx)}{3bd^2}+\frac{2(e+fx)^2\sinh(c+dx)}{3bd}+\frac{(e+fx)^2\sinh(c+dx)\cosh^2(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/2*(a*e*f*x)/(b^2*d) - (a*f^2*x^2)/(4*b^2*d) + (a*(a^2 + b^2)*(e + f*x)^3
)/(3*b^4*f) - (2*a^2*f*(e + f*x)*Cosh[c + d*x])/(b^3*d^2) - (4*f*(e + f*x)*
Cosh[c + d*x])/(3*b*d^2) - (2*f*(e + f*x)*Cosh[c + d*x]^3)/(9*b*d^2) - (a*(
a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^4
*d) - (a*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(b^4*d) - (2*a*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/(b^4*d^2) - (2*a*(a^2 + b^2)*f*(e + f*x)*PolyLog[2,
 -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^2) + (2*a*(a^2 + b^2)*f^
2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^3) + (2*a*(a
^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^
3) + (2*a^2*f^2*Sinh[c + d*x])/(b^3*d^3) + (14*f^2*Sinh[c + d*x])/(9*b*d^3)
 + (a^2*(e + f*x)^2*Sinh[c + d*x])/(b^3*d) + (2*(e + f*x)^2*Sinh[c + d*x])/
```

```
(3*b*d) + (a*f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^2*d^2) + ((e + f
*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b*d) - (a*f^2*Sinh[c + d*x]^2)/(4*b
^2*d^3) - (a*(e + f*x)^2*Sinh[c + d*x]^2)/(2*b^2*d) + (2*f^2*Sinh[c + d*x]^
3)/(27*b*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
```

```
0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh^3(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{2f(e+fx)\cosh^3(c+dx)}{9bd^2} + \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c$$

$$= \frac{a(a^2+b^2)(e+fx)^3}{3b^4f} - \frac{2f(e+fx)\cosh^3(c+dx)}{9bd^2} + \frac{a^2(e+f$$

$$= \frac{a(a^2+b^2)(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx)\cosh(c+dx)}{b^3d^2} - \frac{4f(e+$$

$$= -\frac{aefx}{2b^2d} - \frac{af^2x^2}{4b^2d} + \frac{a(a^2+b^2)(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx)\cosh}{b^3d^2}$$

$$= -\frac{aefx}{2b^2d} - \frac{af^2x^2}{4b^2d} + \frac{a(a^2+b^2)(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx)\cosh}{b^3d^2}$$

$$= -\frac{aefx}{2b^2d} - \frac{af^2x^2}{4b^2d} + \frac{a(a^2+b^2)(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx)\cosh}{b^3d^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2455 vs. $2(636) = 1272$.
time = 8.93, size = 2455, normalized size = 3.86

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (f^2*(-12*a*d*x*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^
(2*c)]))] - 12*a*d*x*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^
2)*E^(2*c)]))] + (2*a*d^3*E^c*x^3 - 6*b*Cosh[d*x] + 6*b*E^(2*c)*Cosh[d*x] -
 6*b*d*x*Cosh[d*x] - 6*b*d*E^(2*c)*x*Cosh[d*x] - 3*b*d^2*x^2*Cosh[d*x] + 3*
b*d^2*E^(2*c)*x^2*Cosh[d*x] - 6*a*d^2*E^c*x^2*Log[1 + (b*E^(2*c + d*x))/(a*
E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - 6*a*d^2*E^c*x^2*Log[1 + (b*E^(2*c + d*x
))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 12*a*E^c*PolyLog[3, -((b*E^(2*c +
 d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 12*a*E^c*PolyLog[3, -((b*E^(
2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*b*Sinh[d*x] + 6*b*E^(
2*c)*Sinh[d*x] + 6*b*d*x*Sinh[d*x] - 6*b*d*E^(2*c)*x*Sinh[d*x] + 3*b*d^2*x^
2*Sinh[d*x] + 3*b*d^2*E^(2*c)*x^2*Sinh[d*x])/E^c))/(12*b^2*d^3) + (f^2*(144
*a^3*d^3*E^(3*c)*x^3 + 72*a*b^2*d^3*E^(3*c)*x^3 - 432*a^2*b*E^(2*c)*Cosh[d*
x] - 108*b^3*E^(2*c)*Cosh[d*x] + 432*a^2*b*E^(4*c)*Cosh[d*x] + 108*b^3*E^(4
*c)*Cosh[d*x] - 432*a^2*b*d*E^(2*c)*x*Cosh[d*x] - 108*b^3*d*E^(2*c)*x*Cosh[
d*x] - 432*a^2*b*d*E^(4*c)*x*Cosh[d*x] - 108*b^3*d*E^(4*c)*x*Cosh[d*x] - 21
6*a^2*b*d^2*E^(2*c)*x^2*Cosh[d*x] - 54*b^3*d^2*E^(2*c)*x^2*Cosh[d*x] + 216*
a^2*b*d^2*E^(4*c)*x^2*Cosh[d*x] + 54*b^3*d^2*E^(4*c)*x^2*Cosh[d*x] - 27*a*b
^2*E^c*Cosh[2*d*x] - 27*a*b^2*E^(5*c)*Cosh[2*d*x] - 54*a*b^2*d*E^c*x*Cosh[2
*d*x] + 54*a*b^2*d*E^(5*c)*x*Cosh[2*d*x] - 54*a*b^2*d^2*E^c*x^2*Cosh[2*d*x]
 - 54*a*b^2*d^2*E^(5*c)*x^2*Cosh[2*d*x] - 4*b^3*Cosh[3*d*x] + 4*b^3*E^(6*c)
*Cosh[3*d*x] - 12*b^3*d*x*Cosh[3*d*x] - 12*b^3*d*E^(6*c)*x*Cosh[3*d*x] - 18
*b^3*d^2*x^2*Cosh[3*d*x] + 18*b^3*d^2*E^(6*c)*x^2*Cosh[3*d*x] - 432*a^3*d^2
*E^(3*c)*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])]
 - 216*a*b^2*d^2*E^(3*c)*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 +
 b^2)*E^(2*c)])] - 432*a^3*d^2*E^(3*c)*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c
 + Sqrt[(a^2 + b^2)*E^(2*c)])] - 216*a*b^2*d^2*E^(3*c)*x^2*Log[1 + (b*E^(2*
c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 432*a*(2*a^2 + b^2)*d*E^(3
*c)*x*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))]
- 432*a*(2*a^2 + b^2)*d*E^(3*c)*x*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + S
qrt[(a^2 + b^2)*E^(2*c)]))] + 864*a^3*E^(3*c)*PolyLog[3, -((b*E^(2*c + d*x)
)/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 432*a*b^2*E^(3*c)*PolyLog[3, -((b
*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 864*a^3*E^(3*c)*Pol
yLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 432*a*b
^2*E^(3*c)*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)
]))] + 432*a^2*b*E^(2*c)*Sinh[d*x] + 108*b^3*E^(2*c)*Sinh[d*x] + 432*a^2*b*
E^(4*c)*Sinh[d*x] + 108*b^3*E^(4*c)*Sinh[d*x] + 432*a^2*b*d*E^(2*c)*x*Sinh[
d*x] + 108*b^3*d*E^(2*c)*x*Sinh[d*x] - 432*a^2*b*d*E^(4*c)*x*Sinh[d*x] - 10
8*b^3*d*E^(4*c)*x*Sinh[d*x] + 216*a^2*b*d^2*E^(2*c)*x^2*Sinh[d*x] + 54*b^3*
d^2*E^(2*c)*x^2*Sinh[d*x] + 216*a^2*b*d^2*E^(4*c)*x^2*Sinh[d*x] + 54*b^3*d^
2*E^(4*c)*x^2*Sinh[d*x] + 27*a*b^2*E^c*Sinh[2*d*x] - 27*a*b^2*E^(5*c)*Sinh[
2*d*x] + 54*a*b^2*d*E^c*x*Sinh[2*d*x] + 54*a*b^2*d*E^(5*c)*x*Sinh[2*d*x] +
54*a*b^2*d^2*E^c*x^2*Sinh[2*d*x] - 54*a*b^2*d^2*E^(5*c)*x^2*Sinh[2*d*x] + 4
*b^3*Sinh[3*d*x] + 4*b^3*E^(6*c)*Sinh[3*d*x] + 12*b^3*d*x*Sinh[3*d*x] - 12*
b^3*d*E^(6*c)*x*Sinh[3*d*x] + 18*b^3*d^2*x^2*Sinh[3*d*x] + 18*b^3*d^2*E^(6*
c)*x^2*Sinh[3*d*x]))/(432*b^4*d^3*E^(3*c)) - (e^2*((a*Log[a + b*Sinh[c + d*
```

```
x]])/b^2 - Sinh[c + d*x]/b))/(2*d) + (e*f*(-(b*Cosh[c + d*x]) - a*(-1/2*(c
+ d*x)^2 + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + (c +
d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - c*Log[a + b*Sinh[c +
d*x]] + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) + b*d*x*Sinh[c + d*x]))/(b^2*d^2) +
 (e^2*((-2*a*Cosh[2*(c + d*x)])/(b^2*d) - (4*(2*a^3 + a*b^2)*Log[a + b*Sinh
[c + d*x]])/(b^4*d) + (2*(4*a^2 + b^2)*Sinh[c + d*x])/(b^3*d) + (2*Sinh[3*(
c + d*x)])/(3*b*d)))/8 + (e*f*(-18*b*(4*a^2 + b^2)*Cosh[c + d*x] - 18*a*b^2
*d*x*Cosh[2*(c + d*x)] - 2*b^3*Cosh[3*(c + d*x)] - 36*a*(2*a^2 + b^2)*(-1/2
*(c + d*x)^2 + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + (
c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - c*Log[a + b*Sinh[
c + d*x]] + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + PolyLog[2,
 -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) + 18*b*(4*a^2 + b^2)*d*x*Sinh[c
 + d*x] + 9*a*b^2*Sinh[2*(c + d*x)] + 6*b^3*d*x*Sinh[3*(c + d*x)]))/(36*b^4
*d^2)
```

Maple [**F**]
time = 1.88, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \left(\cosh^3 (dx+c)\right) \sinh (dx+c)}{a+b\sinh (dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -1/24*((3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 + 3*b^2)*e^(-2*d*x - 2*c))*e^(3
*d*x + 3*c)/(b^3*d) + 24*(a^3 + a*b^2)*(d*x + c)/(b^4*d) + (3*a*b*e^(-2*d*x
 - 2*c) + b^2*e^(-3*d*x - 3*c) + 3*(4*a^2 + 3*b^2)*e^(-d*x - c))/(b^3*d) +
24*(a^3 + a*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^4*d))*e
^2 - 1/432*(144*(a^3*d^3*f^2*e^(3*c) + a*b^2*d^3*f^2*e^(3*c))*x^3 + 432*(a^
3*d^3*f*e^(3*c) + a*b^2*d^3*f*e^(3*c))*x^2*e - 2*(9*b^3*d^2*f^2*x^2*e^(6*c)
 + 2*b^3*f^2*e^(6*c) - 6*b^3*d*f*e^(6*c + 1) - 6*(b^3*d*f^2*e^(6*c) - 3*b^3
*d^2*f*e^(6*c + 1))*x)*e^(3*d*x) + 27*(2*a*b^2*d^2*f^2*x^2*e^(5*c) + a*b^2*
f^2*e^(5*c) - 2*a*b^2*d*f*e^(5*c + 1) - 2*(a*b^2*d*f^2*e^(5*c) - 2*a*b^2*d^
2*f*e^(5*c + 1))*x)*e^(2*d*x) - 54*(8*a^2*b*f^2*e^(4*c) + 6*b^3*f^2*e^(4*c)
```

```
 + (4*a^2*b*d^2*f^2*e^(4*c) + 3*b^3*d^2*f^2*e^(4*c))*x^2 - 2*(4*a^2*b*d*f^2
*e^(4*c) + 3*b^3*d*f^2*e^(4*c) - (4*a^2*b*d^2*f*e^(4*c) + 3*b^3*d^2*f*e^(4*
c))*e)*x - 2*(4*a^2*b*d*f*e^(4*c) + 3*b^3*d*f*e^(4*c))*e)*e^(d*x) + 54*(8*a
^2*b*f^2*e^(2*c) + 6*b^3*f^2*e^(2*c) + (4*a^2*b*d^2*f^2*e^(2*c) + 3*b^3*d^2
*f^2*e^(2*c))*x^2 + 2*(4*a^2*b*d*f^2*e^(2*c) + 3*b^3*d*f^2*e^(2*c) + (4*a^2
*b*d^2*f*e^(2*c) + 3*b^3*d^2*f*e^(2*c))*e)*x + 2*(4*a^2*b*d*f*e^(2*c) + 3*b
^3*d*f*e^(2*c))*e)*e^(-d*x) + 27*(2*a*b^2*d^2*f^2*x^2*e^c + 2*a*b^2*d*f*e^(
c + 1) + a*b^2*f^2*e^c + 2*(2*a*b^2*d^2*f*e^(c + 1) + a*b^2*d*f^2*e^c)*x)*e
^(-2*d*x) + 2*(9*b^3*d^2*f^2*x^2 + 6*b^3*d*f*e + 2*b^3*f^2 + 6*(3*b^3*d^2*f
*e + b^3*d*f^2)*x)*e^(-3*d*x))*e^(-3*c)/(b^4*d^3) + integrate(-2*((a^3*b*f^
2 + a*b^3*f^2)*x^2 + 2*(a^3*b*f + a*b^3*f)*x*e - ((a^4*f^2*e^c + a^2*b^2*f^
2*e^c)*x^2 + 2*(a^4*f*e^c + a^2*b^2*f*e^c)*x*e)*e^(d*x))/(b^5*e^(2*d*x + 2*
c) + 2*a*b^4*e^(d*x + c) - b^5), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7539 vs. $2(606) = 1212$.
time = 0.43, size = 7539, normalized size = 11.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -1/432*(18*b^3*d^2*f^2*x^2 + 12*b^3*d*f^2*x + 18*b^3*d^2*cosh(1)^2 - 2*(9*b
^3*d^2*f^2*x^2 - 6*b^3*d*f^2*x + 9*b^3*d^2*cosh(1)^2 + 9*b^3*d^2*sinh(1)^2
+ 2*b^3*f^2 + 6*(3*b^3*d^2*f*x - b^3*d*f)*cosh(1) + 6*(3*b^3*d^2*f*x + 3*b^
3*d^2*cosh(1) - b^3*d*f)*sinh(1))*cosh(d*x + c)^6 + 18*b^3*d^2*sinh(1)^2 -
2*(9*b^3*d^2*f^2*x^2 - 6*b^3*d*f^2*x + 9*b^3*d^2*cosh(1)^2 + 9*b^3*d^2*sinh
(1)^2 + 2*b^3*f^2 + 6*(3*b^3*d^2*f*x - b^3*d*f)*cosh(1) + 6*(3*b^3*d^2*f*x
+ 3*b^3*d^2*cosh(1) - b^3*d*f)*sinh(1))*sinh(d*x + c)^6 + 27*(2*a*b^2*d^2*f
^2*x^2 - 2*a*b^2*d*f^2*x + 2*a*b^2*d^2*cosh(1)^2 + 2*a*b^2*d^2*sinh(1)^2 +
a*b^2*f^2 + 2*(2*a*b^2*d^2*f*x - a*b^2*d*f)*cosh(1) + 2*(2*a*b^2*d^2*f*x +
2*a*b^2*d^2*cosh(1) - a*b^2*d*f)*sinh(1))*cosh(d*x + c)^5 + 3*(18*a*b^2*d^2
*f^2*x^2 - 18*a*b^2*d*f^2*x + 18*a*b^2*d^2*cosh(1)^2 + 18*a*b^2*d^2*sinh(1)
^2 + 9*a*b^2*f^2 + 18*(2*a*b^2*d^2*f*x - a*b^2*d*f)*cosh(1) - 4*(9*b^3*d^2*
f^2*x^2 - 6*b^3*d*f^2*x + 9*b^3*d^2*cosh(1)^2 + 9*b^3*d^2*sinh(1)^2 + 2*b^3
*f^2 + 6*(3*b^3*d^2*f*x - b^3*d*f)*cosh(1) + 6*(3*b^3*d^2*f*x + 3*b^3*d^2*c
osh(1) - b^3*d*f)*sinh(1))*cosh(d*x + c) + 18*(2*a*b^2*d^2*f*x + 2*a*b^2*d^
2*cosh(1) - a*b^2*d*f)*sinh(1))*sinh(d*x + c)^5 + 4*b^3*f^2 - 54*((4*a^2*b
+ 3*b^3)*d^2*f^2*x^2 - 2*(4*a^2*b + 3*b^3)*d*f^2*x + (4*a^2*b + 3*b^3)*d^2*
cosh(1)^2 + (4*a^2*b + 3*b^3)*d^2*sinh(1)^2 + 2*(4*a^2*b + 3*b^3)*f^2 + 2*(
(4*a^2*b + 3*b^3)*d^2*f*x - (4*a^2*b + 3*b^3)*d*f)*cosh(1) + 2*((4*a^2*b +
3*b^3)*d^2*f*x + (4*a^2*b + 3*b^3)*d^2*cosh(1) - (4*a^2*b + 3*b^3)*d*f)*sin
h(1))*cosh(d*x + c)^4 - 3*(18*(4*a^2*b + 3*b^3)*d^2*f^2*x^2 - 36*(4*a^2*b +
```

```
 3*b^3)*d*f^2*x + 18*(4*a^2*b + 3*b^3)*d^2*cosh(1)^2 + 18*(4*a^2*b + 3*b^3)
*d^2*sinh(1)^2 + 36*(4*a^2*b + 3*b^3)*f^2 + 10*(9*b^3*d^2*f^2*x^2 - 6*b^3*d
*f^2*x + 9*b^3*d^2*cosh(1)^2 + 9*b^3*d^2*sinh(1)^2 + 2*b^3*f^2 + 6*(3*b^3*d
^2*f*x - b^3*d*f)*cosh(1) + 6*(3*b^3*d^2*f*x + 3*b^3*d^2*cosh(1) - b^3*d*f)
*sinh(1))*cosh(d*x + c)^2 + 36*((4*a^2*b + 3*b^3)*d^2*f*x - (4*a^2*b + 3*b^
3)*d*f)*cosh(1) - 45*(2*a*b^2*d^2*f^2*x^2 - 2*a*b^2*d*f^2*x + 2*a*b^2*d^2*c
osh(1)^2 + 2*a*b^2*d^2*sinh(1)^2 + a*b^2*f^2 + 2*(2*a*b^2*d^2*f*x - a*b^2*d
*f)*cosh(1) + 2*(2*a*b^2*d^2*f*x + 2*a*b^2*d^2*cosh(1) - a*b^2*d*f)*sinh(1)
)*cosh(d*x + c) + 36*((4*a^2*b + 3*b^3)*d^2*f*x + (4*a^2*b + 3*b^3)*d^2*cos
h(1) - (4*a^2*b + 3*b^3)*d*f)*sinh(1))*sinh(d*x + c)^4 - 144*((a^3 + a*b^2)
*d^3*f^2*x^3 + 2*(a^3 + a*b^2)*c^3*f^2 + 3*((a^3 + a*b^2)*d^3*x + 2*(a^3 +
a*b^2)*c*d^2)*cosh(1)^2 + 3*((a^3 + a*b^2)*d^3*x + 2*(a^3 + a*b^2)*c*d^2)*s
inh(1)^2 + 3*((a^3 + a*b^2)*d^3*f*x^2 - 2*(a^3 + a*b^2)*c^2*d*f)*cosh(1) +
3*((a^3 + a*b^2)*d^3*f*x^2 - 2*(a^3 + a*b^2)*c^2*d*f + 2*((a^3 + a*b^2)*d^3
*x + 2*(a^3 + a*b^2)*c*d^2)*cosh(1))*sinh(1))*cosh(d*x + c)^3 - 2*(72*(a^3
+ a*b^2)*d^3*f^2*x^3 + 144*(a^3 + a*b^2)*c^3*f^2 + 20*(9*b^3*d^2*f^2*x^2 -
6*b^3*d*f^2*x + 9*b^3*d^2*cosh(1)^2 + 9*b^3*d^2*sinh(1)^2 + 2*b^3*f^2 + 6*(
3*b^3*d^2*f*x - b^3*d*f)*cosh(1) + 6*(3*b^3*d^2*f*x + 3*b^3*d^2*cosh(1) - b
^3*d*f)*sinh(1))*cosh(d*x + c)^3 + 216*((a^3 + a*b^2)*d^3*x + 2*(a^3 + a*b^
2)*c*d^2)*cosh(1)^2 - 135*(2*a*b^2*d^2*f^2*x^2 - 2*a*b^2*d*f^2*x + 2*a*b^2*
d^2*cosh(1)^2 + 2*a*b^2*d^2*sinh(1)^2 + a*b^2*f^2 + 2*(2*a*b^2*d^2*f*x - a*
b^2*d*f)*cosh(1) + 2*(2*a*b^2*d^2*f*x + 2*a*b^2*d^2*cosh(1) - a*b^2*d*f)*si
nh(1))*cosh(d*x + c)^2 + 216*((a^3 + a*b^2)*d^3*x + 2*(a^3 + a*b^2)*c*d^2)*
sinh(1)^2 + 216*((a^3 + a*b^2)*d^3*f*x^2 - 2*(a^3 + a*b^2)*c^2*d*f)*cosh(1)
 + 108*((4*a^2*b + 3*b^3)*d^2*f^2*x^2 - 2*(4*a^2*b + 3*b^3)*d*f^2*x + (4*a^
2*b + 3*b^3)*d^2*cosh(1)^2 + (4*a^2*b + 3*b^3)*d^2*sinh(1)^2 + 2*(4*a^2*b +
 3*b^3)*f^2 + 2*((4*a^2*b + 3*b^3)*d^2*f*x - (4*a^2*b + 3*b^3)*d*f)*cosh(1)
 + 2*((4*a^2*b + 3*b^3)*d^2*f*x + (4*a^2*b + 3*b^3)*d^2*cosh(1) - (4*a^2*b
+ 3*b^3)*d*f)*sinh(1))*cosh(d*x + c) + 216*((a^3 + a*b^2)*d^3*f*x^2 - 2*(a^
3 + a*b^2)*c^2*d*f + 2*((a^3 + a*b^2)*d^3*x + 2*(a^3 + a*b^2)*c*d^2)*cosh(1
))*sinh(1))*sinh(d*x + c)^3 + 54*((4*a^2*b + 3*b^3)*d^2*f^2*x^2 + 2*(4*a^2*
b + 3*b^3)*d*f^2*x + (4*a^2*b + 3*b^3)*d^2*cosh(1)^2 + (4*a^2*b + 3*b^3)*d^
2*sinh(1)^2 + 2*(4*a^2*b + 3*b^3)*f^2 + 2*((4*a^2*b + 3*b^3)*d^2*f*x + (4*a
^2*b + 3*b^3)*d*f)*cosh(1) + 2*((4*a^2*b + 3*b^3)*d^2*f*x + (4*a^2*b + 3*b^
3)*d^2*cosh(1) + (4*a^2*b + 3*b^3)*d*f)*sinh(1))*cosh(d*x + c)^2 + 6*(9*(4*
a^2*b + 3*b^3)*d^2*f^2*x^2 + 18*(4*a^2*b + 3*b^3)*d*f^2*x + 9*(4*a^2*b + 3*
b^3)*d^2*cosh(1)^2 - 5*(9*b^3*d^2*f^2*x^2 - 6*b^3*d*f^2*x + 9*b^3*d^2*cosh(
1)^2 + 9*b^3*d^2*sinh(1)^2 + 2*b^3*f^2 + 6*(3*b^3*d^2*f*x - b^3*d*f)*cosh(1
) + 6*(3*b^3*d^2*f*x + 3*b^3*d^2*cosh(1) - b^3*d*f)*sinh(1))*cosh(d*x + c)^
4 + 9*(4*a^2*b + 3*b^3)*d^2*sinh(1)^2 + 45*(2*a*b^2*d^2*f^2*x^2 - 2*a*b^2*d
*f^2*x + 2*a*b^2*d^2*cosh(1)^2 + 2*a*b^2*d^2*sinh(1)^2 + a*b^2*f^2 + 2*(2*a
*b^2*d^2*f*x - a*b^2*d*f)*cosh(1) + 2*(2*a*b^2*d^2*f*x + 2*a*b^2*d^2*cosh(1
) - a*b^2*d*f)*sinh(1))*cosh(d*x + c)^3 + 18*(4*a^2*b + 3*b^3)*f^2 - 54*((4
*a^2*b + 3*b^3)*d^2*f^2*x^2 - 2*(4*a^2*b + 3*b^3)*d*f^2*x + (4*a^2*b + 3*b^
3)*d^2*cosh(1)^2 + (4*a^2*b + 3*b^3)*d^2*sinh(1)^2 + 2*(4*a^2*b + 3*b^3)*f^
```

```
2 + 2*((4*a^2*b + 3*b^3)*d^2*f*x - (4*a^2*b + 3...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
[Out] integrate((f*x + e)^2*cosh(d*x + c)^3*sinh(d*x + c)/(b*sinh(d*x + c) + a),
x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^3\,\sinh(c+d\,x)\,\,{(e+f\,x)}^2}{a+b\,\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.345 $\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=400

$$-\frac{afx}{4b^2d}+\frac{a(a^2+b^2)(e+fx)^2}{2b^4f}-\frac{a^2f\cosh(c+dx)}{b^3d^2}-\frac{2f\cosh(c+dx)}{3bd^2}-\frac{f\cosh^3(c+dx)}{9bd^2}-\frac{a(a^2+b^2)(e+fx)\,\mathrm{lo}}{b}$$

```
[Out] -1/4*a*f*x/b^2/d+1/2*a*(a^2+b^2)*(f*x+e)^2/b^4/f-a^2*f*cosh(d*x+c)/b^3/d^2-
2/3*f*cosh(d*x+c)/b/d^2-1/9*f*cosh(d*x+c)^3/b/d^2-a*(a^2+b^2)*(f*x+e)*ln(1+
b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d-a*(a^2+b^2)*(f*x+e)*ln(1+b*exp(d*x+
c)/(a+(a^2+b^2)^(1/2)))/b^4/d-a*(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a-(a^2
+b^2)^(1/2)))/b^4/d^2-a*(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/b^4/d^2+a^2*(f*x+e)*sinh(d*x+c)/b^3/d+2/3*(f*x+e)*sinh(d*x+c)/b/d+1/4
*a*f*cosh(d*x+c)*sinh(d*x+c)/b^2/d^2+1/3*(f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/
b/d-1/2*a*(f*x+e)*sinh(d*x+c)^2/b^2/d
```

Rubi [A]
time = 0.35, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5698, 3391, 3377, 2718, 5684, 5554, 2715, 8, 5680, 2221, 2317, 2438}

$$-\frac{a^2f\cosh(c+dx)}{b^3d^2}+\frac{a^2(e+fx)\sinh(c+dx)}{b^3d}-\frac{af(a^2+b^2)\,\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{af(a^2+b^2)\,\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{a(a^2+b^2)(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^4d}-\frac{a(a^2+b^2)(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^4d}+\frac{a(a^2+b^2)(e+fx)^2}{2b^4f}+\frac{af\sinh(c+dx)\cosh(c+dx)}{4b^2d^2}-\frac{a(e+fx)\sinh^2(c+dx)}{2b^2d}-\frac{afx}{4b^2d}-\frac{f\cosh^3(c+dx)}{9bd^2}-\frac{2f\cosh(c+dx)}{3bd^2}+\frac{2(e+fx)\sinh(c+dx)}{3bd}+\frac{(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/4*(a*f*x)/(b^2*d) + (a*(a^2 + b^2)*(e + f*x)^2)/(2*b^4*f) - (a^2*f*Cosh[
c + d*x])/(b^3*d^2) - (2*f*Cosh[c + d*x])/(3*b*d^2) - (f*Cosh[c + d*x]^3)/(
9*b*d^2) - (a*(a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/(b^4*d) - (a*(a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2])])/(b^4*d) - (a*(a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2]))])/(b^4*d^2) - (a*(a^2 + b^2)*f*PolyLog[2, -((b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^2) + (a^2*(e + f*x)*Sinh[c + d*x])/(b
^3*d) + (2*(e + f*x)*Sinh[c + d*x])/(3*b*d) + (a*f*Cosh[c + d*x]*Sinh[c + d
*x])/(4*b^2*d^2) + ((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b*d) - (a*(
e + f*x)*Sinh[c + d*x]^2)/(2*b^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
```

```
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh^3(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{f\cosh^3(c+dx)}{9bd^2} + \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{3bd} + \frac{a^2 \int$$

$$= \frac{a(a^2+b^2)(e+fx)^2}{2b^4 f} - \frac{f\cosh^3(c+dx)}{9bd^2} + \frac{a^2(e+fx)\sinh(c+}{b^3 d}$$

$$= \frac{a(a^2+b^2)(e+fx)^2}{2b^4 f} - \frac{a^2 f\cosh(c+dx)}{b^3 d^2} - \frac{2f\cosh(c+dx)}{3bd^2} -$$

$$= -\frac{afx}{4b^2 d} + \frac{a(a^2+b^2)(e+fx)^2}{2b^4 f} - \frac{a^2 f\cosh(c+dx)}{b^3 d^2} - \frac{2f\cosh(c}{3bd^2}$$

$$= -\frac{afx}{4b^2 d} + \frac{a(a^2+b^2)(e+fx)^2}{2b^4 f} - \frac{a^2 f\cosh(c+dx)}{b^3 d^2} - \frac{2f\cosh(c}{3bd^2}$$

Mathematica [A]
time = 2.15, size = 551, normalized size = 1.38

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] -1/72*(-36*b^2*d*e*(-(a*Log[a + b*Sinh[c + d*x]]) + b*Sinh[c + d*x]) + 36*b
^2*f*(b*Cosh[c + d*x] + a*(-1/2*(c + d*x)^2 + (c + d*x)*Log[1 + (b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2])] + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])] - c*Log[a + b*Sinh[c + d*x]] + PolyLog[2, (b*E^(c + d*x))/(-a +
 Sqrt[a^2 + b^2])] + PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])
- b*d*x*Sinh[c + d*x]) + 6*d*e*(3*a*b^2*Cosh[2*(c + d*x)] + 6*a*(2*a^2 + b^
2)*Log[a + b*Sinh[c + d*x]] - 3*b*(4*a^2 + b^2)*Sinh[c + d*x] - b^3*Sinh[3*
(c + d*x)]) + f*(18*b*(4*a^2 + b^2)*Cosh[c + d*x] + 18*a*b^2*d*x*Cosh[2*(c
+ d*x)] + 2*b^3*Cosh[3*(c + d*x)] + 36*a*(2*a^2 + b^2)*(-1/2*(c + d*x)^2 +
(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + (c + d*x)*Log[1
+ (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - c*Log[a + b*Sinh[c + d*x]] + Pol
yLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + PolyLog[2, -((b*E^(c + d*
```

```
x))/(a + Sqrt[a^2 + b^2]))]) - 18*b*(4*a^2 + b^2)*d*x*Sinh[c + d*x] - 9*a*b
^2*Sinh[2*(c + d*x)] - 6*b^3*d*x*Sinh[3*(c + d*x)]))/(b^4*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1101 vs. $2(372) = 744$.
time = 1.63, size = 1102, normalized size = 2.76

method	result
risch	$-\frac{a(2dxf+2de+f)\mathrm{e}^{-2dx-2c}}{16b^2d^2} - \frac{af\,\mathrm{dilog}\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right)}{d^2b^2} + \frac{af\,c^2}{d^2b^2} + \frac{2ae\ln\left(\mathrm{e}^{dx+c}\right)}{d\,b^2} - \frac{ae\ln\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-\right.}{d\,b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/16*a*(2*d*f*x+2*d*e+f)/b^2/d^2*exp(-2*d*x-2*c)+1/d^2*a/b^2*f*c^2+2/d*a/b
^2*e*ln(exp(d*x+c))-1/d*a/b^2*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d^2
*a/b^2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2*
a/b^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2/d^2
*a/b^2*f*c*ln(exp(d*x+c))+1/d^2*a/b^2*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c
)-b)-1/d*a/b^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))
*x-1/d^2*a/b^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))
*c-1/d*a/b^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1
/d^2*a/b^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/2
*a*f*x^2/b^2+1/2*a^3*f*x^2/b^4+1/d^2*a^3/b^4*f*c^2-1/d*a^3/b^4*e*ln(b*exp(2
*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d*a^3/b^4*e*ln(exp(d*x+c))-1/d^2*a^3/b^4*f*di
log((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2*a^3/b^4*f*d
ilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/72*(3*d*f*x+
3*d*e-f)/d^2/b*exp(3*d*x+3*c)-1/d^2*a^3/b^4*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1
/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d*a^3/b^4*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/
2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d^2*a^3/b^4*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(
1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d*a^3/b^4*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1
/2)+a)/(a+(a^2+b^2)^(1/2)))*x-2/d^2*a^3/b^4*f*c*ln(exp(d*x+c))-1/16*a*(2*d*
f*x+2*d*e-f)/b^2/d^2*exp(2*d*x+2*c)+2/d*a^3/b^4*f*c*x+2/d*a/b^2*c*f*x-a*e*x
/b^2-1/72*(3*d*f*x+3*d*e+f)/d^2/b*exp(-3*d*x-3*c)-a^3*e*x/b^4-1/8*(4*a^2+3*
b^2)*(d*f*x+d*e+f)/b^3/d^2*exp(-d*x-c)+1/8*(4*a^2*d*f*x+3*b^2*d*f*x+4*a^2*d
*e+3*b^2*d*e-4*a^2*f-3*b^2*f)/b^3/d^2*exp(d*x+c)+1/d^2*a^3/b^4*f*c*ln(b*exp
(2*d*x+2*c)+2*a*exp(d*x+c)-b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/144*f*((72*(a^3*d^2*e^(3*c) + a*b^2*d^2*e^(3*c))*x^2 - 2*(3*b^3*d*x*e^(6
*c) - b^3*e^(6*c))*e^(3*d*x) + 9*(2*a*b^2*d*x*e^(5*c) - a*b^2*e^(5*c))*e^(2
*d*x) + 18*(4*a^2*b*e^(4*c) + 3*b^3*e^(4*c) - (4*a^2*b*d*e^(4*c) + 3*b^3*d*
e^(4*c))*x)*e^(d*x) + 18*(4*a^2*b*e^(2*c) + 3*b^3*e^(2*c) + (4*a^2*b*d*e^(2
*c) + 3*b^3*d*e^(2*c))*x)*e^(-d*x) + 9*(2*a*b^2*d*x*e^c + a*b^2*e^c)*e^(-2*
d*x) + 2*(3*b^3*d*x + b^3)*e^(-3*d*x))*e^(-3*c)/(b^4*d^2) - 9*integrate(32*
((a^4*e^c + a^2*b^2*e^c)*x*e^(d*x) - (a^3*b + a*b^3)*x)/(b^5*e^(2*d*x + 2*c
) + 2*a*b^4*e^(d*x + c) - b^5), x)) - 1/24*((3*a*b*e^(-d*x - c) - b^2 - 3*(
4*a^2 + 3*b^2)*e^(-2*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) + 24*(a^3 + a*b^2)
*(d*x + c)/(b^4*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-3*d*x - 3*c) + 3*(4*
a^2 + 3*b^2)*e^(-d*x - c))/(b^3*d) + 24*(a^3 + a*b^2)*log(-2*a*e^(-d*x - c)
 + b*e^(-2*d*x - 2*c) - b)/(b^4*d))*e
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3011 vs. $2(377) = 754$.
time = 0.39, size = 3011, normalized size = 7.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] 1/144*(2*(3*b^3*d*f*x + 3*b^3*d*cosh(1) + 3*b^3*d*sinh(1) - b^3*f)*cosh(d*x
 + c)^6 + 2*(3*b^3*d*f*x + 3*b^3*d*cosh(1) + 3*b^3*d*sinh(1) - b^3*f)*sinh(
d*x + c)^6 - 6*b^3*d*f*x - 9*(2*a*b^2*d*f*x + 2*a*b^2*d*cosh(1) + 2*a*b^2*d
*sinh(1) - a*b^2*f)*cosh(d*x + c)^5 - 3*(6*a*b^2*d*f*x + 6*a*b^2*d*cosh(1)
+ 6*a*b^2*d*sinh(1) - 3*a*b^2*f - 4*(3*b^3*d*f*x + 3*b^3*d*cosh(1) + 3*b^3*
d*sinh(1) - b^3*f)*cosh(d*x + c))*sinh(d*x + c)^5 - 6*b^3*d*cosh(1) + 18*((
4*a^2*b + 3*b^3)*d*f*x + (4*a^2*b + 3*b^3)*d*cosh(1) + (4*a^2*b + 3*b^3)*d*
sinh(1) - (4*a^2*b + 3*b^3)*f)*cosh(d*x + c)^4 - 6*b^3*d*sinh(1) + 3*(6*(4*
a^2*b + 3*b^3)*d*f*x + 6*(4*a^2*b + 3*b^3)*d*cosh(1) + 10*(3*b^3*d*f*x + 3*
b^3*d*cosh(1) + 3*b^3*d*sinh(1) - b^3*f)*cosh(d*x + c)^2 + 6*(4*a^2*b + 3*b
^3)*d*sinh(1) - 6*(4*a^2*b + 3*b^3)*f - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*cosh(
1) + 2*a*b^2*d*sinh(1) - a*b^2*f)*cosh(d*x + c))*sinh(d*x + c)^4 - 2*b^3*f
+ 72*((a^3 + a*b^2)*d^2*f*x^2 - 2*(a^3 + a*b^2)*c^2*f + 2*((a^3 + a*b^2)*d^
2*x + 2*(a^3 + a*b^2)*c*d)*cosh(1) + 2*((a^3 + a*b^2)*d^2*x + 2*(a^3 + a*b^
2)*c*d)*sinh(1))*cosh(d*x + c)^3 + 2*(36*(a^3 + a*b^2)*d^2*f*x^2 - 72*(a^3
+ a*b^2)*c^2*f + 20*(3*b^3*d*f*x + 3*b^3*d*cosh(1) + 3*b^3*d*sinh(1) - b^3*
f)*cosh(d*x + c)^3 - 45*(2*a*b^2*d*f*x + 2*a*b^2*d*cosh(1) + 2*a*b^2*d*sinh
(1) - a*b^2*f)*cosh(d*x + c)^2 + 72*((a^3 + a*b^2)*d^2*x + 2*(a^3 + a*b^2)*
c*d)*cosh(1) + 36*((4*a^2*b + 3*b^3)*d*f*x + (4*a^2*b + 3*b^3)*d*cosh(1) +
(4*a^2*b + 3*b^3)*d*sinh(1) - (4*a^2*b + 3*b^3)*f)*cosh(d*x + c) + 72*((a^3
```

+ a*b^2)*d^2*x + 2*(a^3 + a*b^2)*c*d)*sinh(1))*sinh(d*x + c)^3 - 18*((4*a^
2*b + 3*b^3)*d*f*x + (4*a^2*b + 3*b^3)*d*cosh(1) + (4*a^2*b + 3*b^3)*d*sinh
(1) + (4*a^2*b + 3*b^3)*f)*cosh(d*x + c)^2 + 6*(5*(3*b^3*d*f*x + 3*b^3*d*co
sh(1) + 3*b^3*d*sinh(1) - b^3*f)*cosh(d*x + c)^4 - 3*(4*a^2*b + 3*b^3)*d*f*
x - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*cosh(1) + 2*a*b^2*d*sinh(1) - a*b^2*f)*co
sh(d*x + c)^3 - 3*(4*a^2*b + 3*b^3)*d*cosh(1) + 18*((4*a^2*b + 3*b^3)*d*f*x
+ (4*a^2*b + 3*b^3)*d*cosh(1) + (4*a^2*b + 3*b^3)*d*sinh(1) - (4*a^2*b + 3
*b^3)*f)*cosh(d*x + c)^2 - 3*(4*a^2*b + 3*b^3)*d*sinh(1) - 3*(4*a^2*b + 3*b
^3)*f + 36*((a^3 + a*b^2)*d^2*f*x^2 - 2*(a^3 + a*b^2)*c^2*f + 2*((a^3 + a*b
^2)*d^2*x + 2*(a^3 + a*b^2)*c*d)*cosh(1) + 2*((a^3 + a*b^2)*d^2*x + 2*(a^3
+ a*b^2)*c*d)*sinh(1))*cosh(d*x + c))*sinh(d*x + c)^2 - 9*(2*a*b^2*d*f*x +
2*a*b^2*d*cosh(1) + 2*a*b^2*d*sinh(1) + a*b^2*f)*cosh(d*x + c) - 144*((a^3
+ a*b^2)*f*cosh(d*x + c)^3 + 3*(a^3 + a*b^2)*f*cosh(d*x + c)^2*sinh(d*x + c
) + 3*(a^3 + a*b^2)*f*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3 + a*b^2)*f*sinh(
d*x + c)^3)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 144*((a^3 + a*b^2)*f*co
sh(d*x + c)^3 + 3*(a^3 + a*b^2)*f*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3 +
a*b^2)*f*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3 + a*b^2)*f*sinh(d*x + c)^3)*d
ilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 144*(((a^3 + a*b^2)*c*f - (a^3 + a*b
^2)*d*cosh(1) - (a^3 + a*b^2)*d*sinh(1))*cosh(d*x + c)^3 + 3*((a^3 + a*b^2)
*c*f - (a^3 + a*b^2)*d*cosh(1) - (a^3 + a*b^2)*d*sinh(1))*cosh(d*x + c)^2*s
inh(d*x + c) + 3*((a^3 + a*b^2)*c*f - (a^3 + a*b^2)*d*cosh(1) - (a^3 + a*b^
2)*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^2 + ((a^3 + a*b^2)*c*f - (a^3 + a
*b^2)*d*cosh(1) - (a^3 + a*b^2)*d*sinh(1))*sinh(d*x + c)^3)*log(2*b*cosh(d*
x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 144*(((a^3
+ a*b^2)*c*f - (a^3 + a*b^2)*d*cosh(1) - (a^3 + a*b^2)*d*sinh(1))*cosh(d*x
+ c)^3 + 3*((a^3 + a*b^2)*c*f - (a^3 + a*b^2)*d*cosh(1) - (a^3 + a*b^2)*d*s
inh(1))*cosh(d*x + c)^2*sinh(d*x + c) + 3*((a^3 + a*b^2)*c*f - (a^3 + a*b^2
)*d*cosh(1) - (a^3 + a*b^2)*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^2 + ((a^
3 + a*b^2)*c*f - (a^3 + a*b^2)*d*cosh(1) - (a^3 + a*b^2)*d*sinh(1))*sinh(d*
x + c)^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/
b^2) + 2*a) - 144*(((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*cosh(d*x + c)^
3 + 3*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*cosh(d*x + c)^2*sinh(d*x +
c) + 3*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c
)^2 + ((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*sinh(d*x + c)^3)*log(-(a*co
sh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b) - 144*(((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*cos
h(d*x + c)^3 + 3*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*cosh(d*x + c)^2*
sinh(d*x + c) + 3*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*cosh(d*x + c)*s
inh(d*x + c)^2 + ((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*sinh(d*x + c)^3)
*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 3*(6*a*b^2*d*f*x - 4*(3*b^3*d*f*x + 3*b
^3*d*cosh(1) + 3*b^3*d*sinh(1) - b^3*f)*cosh(d*x + c)^5 + 6*a*b^2*d*cosh(1)
+ 15*(2*a*b^2*d*f*x + 2*a*b^2*d*cosh(1) + 2*a*b^2*d*sinh(1) - a*b^2*f)*cos

```
h(d*x + c)^4 + 6*a*b^2*d*sinh(1) + 3*a*b^2*f - 24*((4*a^2*b + 3*b^3)*d*f*x
+ (4*a^2*b + 3*b^3)*d*cosh(1) + (4*a^2*b + 3*b^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)^3*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^3\,\sinh(c+d\,x)\,\left(e+f\,x\right)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.346 $\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=85

$$-\frac{a(a^2+b^2)\log(a+b\sinh(c+dx))}{b^4d}+\frac{(a^2+b^2)\sinh(c+dx)}{b^3d}-\frac{a\sinh^2(c+dx)}{2b^2d}+\frac{\sinh^3(c+dx)}{3bd}$$

```
[Out] -a*(a^2+b^2)*ln(a+b*sinh(d*x+c))/b^4/d+(a^2+b^2)*sinh(d*x+c)/b^3/d-1/2*a*si
nh(d*x+c)^2/b^2/d+1/3*sinh(d*x+c)^3/b/d
```

Rubi [A]
time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 786}

$$-\frac{a(a^2+b^2)\log(a+b\sinh(c+dx))}{b^4d}+\frac{(a^2+b^2)\sinh(c+dx)}{b^3d}-\frac{a\sinh^2(c+dx)}{2b^2d}+\frac{\sinh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/(b^4*d)) + ((a^2 + b^2)*Sinh[c +
 d*x])/(b^3*d) - (a*Sinh[c + d*x]^2)/(2*b^2*d) + Sinh[c + d*x]^3/(3*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x(-b^2-x^2)}{b(a+x)}\,dx, x, b\sinh(c+dx)\right)}{b^3d}$$
$$= -\frac{\text{Subst}\left(\int \frac{x(-b^2-x^2)}{a+x}\,dx, x, b\sinh(c+dx)\right)}{b^4d}$$
$$= -\frac{\text{Subst}\left(\int \left(-a^2\left(1+\frac{b^2}{a^2}\right)+ax-x^2+\frac{a(a^2+b^2)}{a+x}\right)\,dx, x, b\sinh(c+dx)\right)}{b^4d}$$
$$= -\frac{a(a^2+b^2)\log(a+b\sinh(c+dx))}{b^4d}+\frac{(a^2+b^2)\sinh(c+dx)}{b^3d}-\frac{a\sinh^2(c}{2b^2}$$

Mathematica [A]
time = 0.23, size = 90, normalized size = 1.06

$$\frac{-3ab^2\cosh(2(c+dx))-12a^3\log(a+b\sinh(c+dx))-12ab^2\log(a+b\sinh(c+dx))+3b(4a^2+3b^2)\sinh(c+dx)+b^3\sinh(3(c+dx))}{12b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-3*a*b^2*Cosh[2*(c + d*x)] - 12*a^3*Log[a + b*Sinh[c + d*x]] - 12*a*b^2*Lo
g[a + b*Sinh[c + d*x]] + 3*b*(4*a^2 + 3*b^2)*Sinh[c + d*x] + b^3*Sinh[3*(c
+ d*x)])/(12*b^4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(81) = 162$.
time = 0.89, size = 246, normalized size = 2.89

method	result
risch	$\frac{a^3x}{b^4}+\frac{ax}{b^2}+\frac{e^{3dx+3c}}{24bd}-\frac{a\,e^{2dx+2c}}{8b^2d}+\frac{e^{dx+c}a^2}{2b^3d}+\frac{3\,e^{dx+c}}{8bd}-\frac{e^{-dx-c}a^2}{2b^3d}-\frac{3\,e^{-dx-c}}{8bd}-\frac{a\,e^{-2dx-2c}}{8b^2d}-\frac{e^{-3dx-3}}{24bd}$
derivativedivides	$-\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a+b}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2a^2+ab+2b^2}{2b^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\left(a^2+b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^4}-\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}$
default	$-\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a+b}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2a^2+ab+2b^2}{2b^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\left(a^2+b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^4}-\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3/b/(tanh(1/2*d*x+1/2*c)-1)^3-1/2*(a+b)/b^2/(tanh(1/2*d*x+1/2*c)-1)
^2-1/2*(2*a^2+a*b+2*b^2)/b^3/(tanh(1/2*d*x+1/2*c)-1)+a*(a^2+b^2)/b^4*ln(tan
```

```
h(1/2*d*x+1/2*c)-1)-1/3/b/(tanh(1/2*d*x+1/2*c)+1)^3-1/2*(-b+a)/b^2/(tanh(1/
2*d*x+1/2*c)+1)^2-1/2*(2*a^2-a*b+2*b^2)/b^3/(tanh(1/2*d*x+1/2*c)+1)+a*(a^2+
b^2)/b^4*ln(tanh(1/2*d*x+1/2*c)+1)-2/b^4*a*(1/2*a^2+1/2*b^2)*ln(a*tanh(1/2*
d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)-a))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(81) = 162$.
time = 0.27, size = 183, normalized size = 2.15

$$-\frac{\left(3\,abe^{(-dx-c)} - b^2 - 3\,(4\,a^2 + 3\,b^2)e^{(-2\,dx-2\,c)}\right)e^{(3\,dx+3\,c)}}{24\,b^3d} - \frac{(a^3 + ab^2)(dx + c)}{b^4d} - \frac{3\,abe^{(-2\,dx-2\,c)} + b^2e^{(-3\,dx-3\,c)} + 3\,(4\,a^2 + 3\,b^2)e^{(-dx-c)}}{24\,b^3d} - \frac{(a^3 + ab^2)\log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/24*(3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 + 3*b^2)*e^(-2*d*x - 2*c))*e^(3*
d*x + 3*c)/(b^3*d) - (a^3 + a*b^2)*(d*x + c)/(b^4*d) - 1/24*(3*a*b*e^(-2*d*
x - 2*c) + b^2*e^(-3*d*x - 3*c) + 3*(4*a^2 + 3*b^2)*e^(-d*x - c))/(b^3*d) -
 (a^3 + a*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^4*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(81) = 162$.
time = 0.39, size = 652, normalized size = 7.67

$$\frac{b^3\cosh(dx+c)^6 + b^3\sinh(dx+c)^6 - 3\,ab^2\cosh(dx+c)^5 + 24\,(a^3+ab^2)dx\cosh(dx+c)^3 + 3\,(2\,b^3\cosh(dx+c) - ab^2)\sinh(dx+c)^5 + 3\,(4\,a^2b+3\,b^3)\cosh(dx+c)^4 + 3\,(5\,b^3\cosh(dx+c)^2 - 5\,ab^2\cosh(dx+c) + 4\,a^2b + 3\,b^3)\sinh(dx+c)^4 - 3\,ab^2\cosh(dx+c) + 2\,(10\,b^3\cosh(dx+c)^3 - 15\,ab^2\cosh(dx+c)^2 + 12\,(a^3+ab^2)dx + 6\,(4\,a^2b+3\,b^3)\cosh(dx+c))\sinh(dx+c)^3 - b^3 - 3\,(4\,a^2b+3\,b^3)\cosh(dx+c)^2 + 3\,(5\,b^3\cosh(dx+c)^4 - 10\,ab^2\cosh(dx+c)^3 + 24\,(a^3+ab^2)dx\cosh(dx+c) - 4\,a^2b - 3\,b^3 + 6\,(4\,a^2b+3\,b^3)\cosh(dx+c)^2)\sinh(dx+c)^2 - 24\,\left((a^3+ab^2)\cosh(dx+c)^3 + 3\,(a^3+ab^2)\cosh(dx+c)^2\sinh(dx+c) + 3\,(a^3+ab^2)\cosh(dx+c)\sinh(dx+c)^2 + (a^3+ab^2)\sinh(dx+c)^3\right)\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right) + 3\,(2\,b^3\cosh(dx+c)^5 - 5\,ab^2\cosh(dx+c)^4 + 24\,(a^3+ab^2)dx\cosh(dx+c)^2 + 4\,(4\,a^2b+3\,b^3)\cosh(dx+c)^3 - ab^2 - 2\,(4\,a^2b+3\,b^3)\cosh(dx+c))\sinh(dx+c)}{24\,\left(b^4d\cosh(dx+c)^3 + 3\,b^4d\cosh(dx+c)^2\sinh(dx+c) + 3\,b^4d\cosh(dx+c)\sinh(dx+c)^2 + b^4d\sinh(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/24*(b^3*cosh(d*x + c)^6 + b^3*sinh(d*x + c)^6 - 3*a*b^2*cosh(d*x + c)^5 +
 24*(a^3 + a*b^2)*d*x*cosh(d*x + c)^3 + 3*(2*b^3*cosh(d*x + c) - a*b^2)*sin
h(d*x + c)^5 + 3*(4*a^2*b + 3*b^3)*cosh(d*x + c)^4 + 3*(5*b^3*cosh(d*x + c)
^2 - 5*a*b^2*cosh(d*x + c) + 4*a^2*b + 3*b^3)*sinh(d*x + c)^4 - 3*a*b^2*cos
h(d*x + c) + 2*(10*b^3*cosh(d*x + c)^3 - 15*a*b^2*cosh(d*x + c)^2 + 12*(a^3
 + a*b^2)*d*x + 6*(4*a^2*b + 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - b^3 -
3*(4*a^2*b + 3*b^3)*cosh(d*x + c)^2 + 3*(5*b^3*cosh(d*x + c)^4 - 10*a*b^2*c
osh(d*x + c)^3 + 24*(a^3 + a*b^2)*d*x*cosh(d*x + c) - 4*a^2*b - 3*b^3 + 6*(
4*a^2*b + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 24*((a^3 + a*b^2)*cosh(
d*x + c)^3 + 3*(a^3 + a*b^2)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3 + a*b^2
)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3 + a*b^2)*sinh(d*x + c)^3)*log(2*(b*s
inh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 3*(2*b^3*cosh(d*x + c)
^5 - 5*a*b^2*cosh(d*x + c)^4 + 24*(a^3 + a*b^2)*d*x*cosh(d*x + c)^2 + 4*(4*
a^2*b + 3*b^3)*cosh(d*x + c)^3 - a*b^2 - 2*(4*a^2*b + 3*b^3)*cosh(d*x + c))
*sinh(d*x + c))/(b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c)^2*sinh(d*x +
 c) + 3*b^4*d*cosh(d*x + c)*sinh(d*x + c)^2 + b^4*d*sinh(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.45, size = 145, normalized size = 1.71

$$\frac{\frac{b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3-3\,ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+12\,a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)+12\,b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{b^3}-\frac{24\,(a^3+ab^2)\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{b^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*((b^2*(e^(d*x + c) - e^(-d*x - c))^3 - 3*a*b*(e^(d*x + c) - e^(-d*x -
c))^2 + 12*a^2*(e^(d*x + c) - e^(-d*x - c)) + 12*b^2*(e^(d*x + c) - e^(-d*x
 - c)))/b^3 - 24*(a^3 + a*b^2)*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a
))/b^4)/d
```

Mupad [**B**]
time = 0.43, size = 180, normalized size = 2.12

$$\frac{x\,(a^3+a\,b^2)}{b^4}-\frac{\mathrm{e}^{-3\,c-3\,d\,x}}{24\,b\,d}+\frac{\mathrm{e}^{3\,c+3\,d\,x}}{24\,b\,d}-\frac{a\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,b^2\,d}-\frac{a\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,b^2\,d}-\frac{\mathrm{e}^{-c-d\,x}\,(4\,a^2+3\,b^2)}{8\,b^3\,d}-\frac{\ln\left(2\,a\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-b+b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\right)\,(a^3+a\,b^2)}{b^4\,d}+\frac{\mathrm{e}^{c+d\,x}\,(4\,a^2+3\,b^2)}{8\,b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^3*sinh(c + d*x))/(a + b*sinh(c + d*x)),x)
```

```
[Out] (x*(a*b^2 + a^3))/b^4 - exp(- 3*c - 3*d*x)/(24*b*d) + exp(3*c + 3*d*x)/(24*
b*d) - (a*exp(- 2*c - 2*d*x))/(8*b^2*d) - (a*exp(2*c + 2*d*x))/(8*b^2*d) -
(exp(- c - d*x)*(4*a^2 + 3*b^2))/(8*b^3*d) - (log(2*a*exp(d*x)*exp(c) - b +
 b*exp(2*c)*exp(2*d*x))*(a*b^2 + a^3))/(b^4*d) + (exp(c + d*x)*(4*a^2 + 3*b
^2))/(8*b^3*d)
```

3.347 $\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cosh[c + d*x]^3*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Cosh[c + d*x]^3*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\cosh^3\left(dx+c\right)\right)\sinh\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/8*e^(-3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b*f) - 1/4*a*e^
(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b^2*f) + 1/4*a*e^(2*c
- 2*d*e/f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b^2*f) - 1/8*e^(3*c - 3*d*e
/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/(b*f) - 1/8*(4*a^2 + 3*b^2)*e^(-c +
 d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^3*f) - 1/8*(4*a^2*e^c + 3*b^2*e
^c)*e^(-d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^3*f) - (a^3 + a*b^2)*lo
g(f*x + e)/(b^4*f) + 1/16*integrate(32*(a^3*b + a*b^3 - (a^4*e^c + a^2*b^2*
e^c)*e^(d*x))/(b^5*f*x + b^5*e - (b^5*f*x*e^(2*c) + b^5*e^(2*c + 1))*e^(2*d
*x) - 2*(a*b^4*f*x*e^c + a*b^4*e^(c + 1))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(cosh(d*x + c)^3*sinh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
[Out] integrate(cosh(d*x + c)^3*sinh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)),
x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh(c+d\,x)^3\,\sinh(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^3*sinh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))),x)
[Out] int((cosh(c + d*x)^3*sinh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.348 $\int \frac{(e+fx)^3 \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1021

$$\frac{2(e+fx)^3\text{ArcTan}\left(e^{c+dx}\right)}{bd}-\frac{2a^2(e+fx)^3\text{ArcTan}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d}-\frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d}-\frac{a(e+fx)^3\text{lo}}{(}$$

```
[Out] -3*I*f*(f*x+e)^2*polylog(2,-I*exp(d*x+c))/b/d^2-6*I*f^2*(f*x+e)*polylog(3,I
*exp(d*x+c))/b/d^3+2*(f*x+e)^3*arctan(exp(d*x+c))/b/d+a*(f*x+e)^3*ln(1+exp(
2*d*x+2*c))/(a^2+b^2)/d-a*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/
(a^2+b^2)/d-a*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d-
6*a*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d^4-6*a*f^3*
polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d^4-2*a^2*(f*x+e)^3*
arctan(exp(d*x+c))/b/(a^2+b^2)/d+3/2*a*f*(f*x+e)^2*polylog(2,-exp(2*d*x+2*c
))/(a^2+b^2)/d^2-3/2*a*f^2*(f*x+e)*polylog(3,-exp(2*d*x+2*c))/(a^2+b^2)/d^3
+6*I*f^3*polylog(4,I*exp(d*x+c))/b/d^4+3*I*a^2*f*(f*x+e)^2*polylog(2,-I*exp
(d*x+c))/b/(a^2+b^2)/d^2+6*I*a^2*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/b/(a^2
+b^2)/d^3+3*I*f*(f*x+e)^2*polylog(2,I*exp(d*x+c))/b/d^2+6*I*f^2*(f*x+e)*pol
ylog(3,-I*exp(d*x+c))/b/d^3-6*I*a^2*f^3*polylog(4,I*exp(d*x+c))/b/(a^2+b^2)
/d^4-3*a*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)
/d^2-3*a*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)
/d^2+6*a*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)
/d^3+6*a*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)
/d^3+3/4*a*f^3*polylog(4,-exp(2*d*x+2*c))/(a^2+b^2)/d^4-6*I*f^3*polylog(4,-
I*exp(d*x+c))/b/d^4+6*I*a^2*f^3*polylog(4,-I*exp(d*x+c))/b/(a^2+b^2)/d^4-3*
I*a^2*f*(f*x+e)^2*polylog(2,I*exp(d*x+c))/b/(a^2+b^2)/d^2-6*I*a^2*f^2*(f*x+
e)*polylog(3,-I*exp(d*x+c))/b/(a^2+b^2)/d^3
```

Rubi [A]
time = 1.08, antiderivative size = 1021, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {5686, 4265, 2611, 6744, 2320, 6724, 5692, 5680, 2221, 6874, 3799}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b*d) - (2*a^2*(e + f*x)^3*ArcTan[E^(c
 + d*x)])/(b*(a^2 + b^2)*d) - (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2])])/((a^2 + b^2)*d) - (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) + (a*(e + f*x)^3*Log[1 + E^(2*(c + d
*x))])/((a^2 + b^2)*d) - ((3*I)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])
```

```
/(b*d^2) + ((3*I)*a^2*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(b*(a^2 +
 b^2)*d^2) + ((3*I)*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b*d^2) - ((3*
I)*a^2*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) - (3*a*
f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 +
 b^2)*d^2) - (3*a*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2]))])/((a^2 + b^2)*d^2) + (3*a*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x
))])/(2*(a^2 + b^2)*d^2) + ((6*I)*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)
])/(b*d^3) - ((6*I)*a^2*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(b*(a^2
 + b^2)*d^3) - ((6*I)*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(b*d^3) + ((
6*I)*a^2*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^3) + (6*
a*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2
 + b^2)*d^3) + (6*a*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))])/((a^2 + b^2)*d^3) - (3*a*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d
*x))])/(2*(a^2 + b^2)*d^3) - ((6*I)*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(b*d^
4) + ((6*I)*a^2*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^4) + ((6
*I)*f^3*PolyLog[4, I*E^(c + d*x)])/(b*d^4) - ((6*I)*a^2*f^3*PolyLog[4, I*E^
(c + d*x)])/(b*(a^2 + b^2)*d^4) - (6*a*f^3*PolyLog[4, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^4) - (6*a*f^3*PolyLog[4, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^4) + (3*a*f^3*PolyLog[4, -E^(2
*(c + d*x))])/(4*(a^2 + b^2)*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5686

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*Sech[c +
d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\text{sech}(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{a\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+dx))\,dx}{b\left(a^2+b^2\right)}$$

$$= \frac{a(e+fx)^4}{4\left(a^2+b^2\right)f} + \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{3if(e+fx)^2\text{Li}_2\left(-ie^{c+dx}\right)}{bd^2} + \frac{3if}{}$$

$$= \frac{a(e+fx)^4}{4\left(a^2+b^2\right)f} + \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{\left(a^2+b^2\right)d}$$

$$= \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^3\log\left(1\right.}{\left(a^2+\right.}$$

$$= \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^3\log\left(1\right.}{\left(a^2+\right.}$$

$$= \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^3\log\left(1\right.}{\left(a^2+\right.}$$

$$= \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^3\log\left(1\right.}{\left(a^2+\right.}$$

$$= \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^3\log\left(1\right.}{\left(a^2+\right.}$$

$$= \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^3\log\left(1\right.}{\left(a^2+\right.}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3214 vs. $2(1021) = 2042$.
time = 19.42, size = 3214, normalized size = 3.15

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

[Out] (-8*a*d^4*e^3*E^(2*c)*x - 12*a*d^4*e^2*E^(2*c)*f*x^2 - 8*a*d^4*e*E^(2*c)*f^2*x^3 - 2*a*d^4*E^(2*c)*f^3*x^4 + 8*b*d^3*e^3*ArcTan[E^(c + d*x)] + 8*b*d^3*e^3*E^(2*c)*ArcTan[E^(c + d*x)] + (12*I)*b*d^3*e^2*f*x*Log[1 - I*E^(c + d*x)] + (12*I)*b*d^3*e^2*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (12*I)*b*d^3*e*f^2*x^2*Log[1 - I*E^(c + d*x)] + (12*I)*b*d^3*e*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (4*I)*b*d^3*f^3*x^3*Log[1 - I*E^(c + d*x)] + (4*I)*b*d^3*E^(2*c)*f^3*x^3*Log[1 - I*E^(c + d*x)] - (12*I)*b*d^3*e^2*f*x*Log[1 + I*E^(c + d*x)] - (12*I)*b*d^3*e^2*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (12*I)*b*d^3*e*f^2*x^2*Log[1 + I*E^(c + d*x)] - (12*I)*b*d^3*e*E^(2*c)*f^2*x^2*Log[1 + I*E^(c + d*x)] - (4*I)*b*d^3*f^3*x^3*Log[1 + I*E^(c + d*x)] - (4*I)*b*d^3*E^(2*c)*f^3*x^3*Log[1 + I*E^(c + d*x)] + 4*a*d^3*e^3*Log[1 + E^(2*(c + d*x))] + 4*a*d^3*e^3*E^(2*c)*Log[1 + E^(2*(c + d*x))] + 12*a*d^3*e^2*f*x*Log[1 + E^(2*(c + d*x))] + 12*a*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(2*(c + d*x))] + 12*a*d^3*e*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 12*a*d^3*e*E^(2*c)*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 4*a*d^3*f^3*x^3*Log[1 + E^(2*(c + d*x))] + 4*a*d^3*E^(2*c)*f^3*x^3*Log[1 + E^(2*(c + d*x))] - (12*I)*b*d^2*(1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)] + (12*I)*b*d^2*(1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)] + 6*a*d^2*e^2*f*PolyLog[2, -E^(2*(c + d*x))] + 6*a*d^2*e^2*E^(2*c)*f*PolyLog[2, -E^(2*(c + d*x))] + 12*a*d^2*e*f^2*x*PolyLog[2, -E^(2*(c + d*x))] + 12*a*d^2*e*E^(2*c)*f^2*x*PolyLog[2, -E^(2*(c + d*x))] + 6*a*d^2*f^3*x^2*PolyLog[2, -E^(2*(c + d*x))] + 6*a*d^2*E^(2*c)*f^3*x^2*PolyLog[2, -E^(2*(c + d*x))] + (24*I)*b*d*e*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (24*I)*b*d*e*E^(2*c)*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (24*I)*b*d*f^3*x*PolyLog[3, (-I)*E^(c + d*x)] + (24*I)*b*d*E^(2*c)*f^3*x*PolyLog[3, (-I)*E^(c + d*x)] - (24*I)*b*d*e*f^2*PolyLog[3, I*E^(c + d*x)] - (24*I)*b*d*e*E^(2*c)*f^2*PolyLog[3, I*E^(c + d*x)] - (24*I)*b*d*f^3*x*PolyLog[3, I*E^(c + d*x)] - (24*I)*b*d*E^(2*c)*f^3*x*PolyLog[3, I*E^(c + d*x)] - 6*a*d*e*f^2*PolyLog[3, -E^(2*(c + d*x))] - 6*a*d*e*E^(2*c)*f^2*PolyLog[3, -E^(2*(c + d*x))] - 6*a*d*f^3*x*PolyLog[3, -E^(2*(c + d*x))] - 6*a*d*E^(2*c)*f^3*x*PolyLog[3, -E^(2*(c + d*x))] - (24*I)*b*f^3*PolyLog[4, (-I)*E^(c + d*x)] - (24*I)*b*E^(2*c)*f^3*PolyLog[4, (-I)*E^(c + d*x)] + (24*I)*b*f^3*PolyLog[4, I*E^(c + d*x)] + (24*I)*b*E^(2*c)*f^3*PolyLog[4, I*E^(c + d*x)] + 3*a*f^3*PolyLog[4, -E^(2*(c + d*x))] + 3*a*E^(2*c)*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*(a^2 + b^2)*d^4*(1 + E^(2*c))) + (a*(4*e^3*E^(2*c)*x + 6*e^2*E^(2*c)*f*x^2 + 4*e*E^(2*c)*f^2*x^3 + E^(2*c)*f^3*x^4 + (4*a*Sqrt[a^2 + b^2]*e^3*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (4*a*Sqrt[-(a^2 + b^2)^2]*e^3*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (4*a*Sqrt[-(a^2 + b^2)^2]*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (4*a*Sqrt[-(a^2 + b^2)^2]*e^3*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (2*e^3*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (2*e^3*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)

```
])])/d - (6*e*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2
+ b^2)*E^(2*c)])])/d + (2*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(
a^2 + b^2)*E^(2*c)])])/d - (2*E^(2*c)*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*
E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))
/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^
(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1 +
 (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f
^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d +
(2*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/
d - (2*E^(2*c)*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*
E^(2*c)])])/d - (6*(-1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*
x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f
*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])
/d^2 - (12*e*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E
^(2*c)]))])/d^3 + (12*e*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c -
 Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (12*f^3*x*PolyLog[3, -((b*E^(2*c + d*x
))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*E^(2*c)*f^3*x*PolyLog[3
, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (12*e*f^
2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3
 + (12*e*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^
2)*E^(2*c)]))])/d^3 - (12*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqr
t[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*E^(2*c)*f^...
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \tanh(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(-2*a*e^(-d*x - c) + b*e^
(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a^2 + b
^2)*d))*e^3 + integrate(2*f^3*x^3*(e^(d*x + c) - e^(-d*x - c))/((b*(e^(d*x
+ c) - e^(-d*x - c)) + 2*a)*(e^(d*x + c) + e^(-d*x - c))) + 6*f^2*x^2*(e^(d
```

```
*x + c) - e^(-d*x - c))*e/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(e^(d*x +
 c) + e^(-d*x - c))) + 6*f*x*(e^(d*x + c) - e^(-d*x - c))*e^2/((b*(e^(d*x +
 c) - e^(-d*x - c)) + 2*a)*(e^(d*x + c) + e^(-d*x - c))), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2787 vs. $2(941) = 1882$.
time = 0.44, size = 2787, normalized size = 2.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(6*a*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*a*f^3*polylog(4, (a*cosh(d
*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2))/b) + 3*(a*d^2*f^3*x^2 + 2*a*d^2*f^2*x*cosh(1) + a*d^2*f*cosh(1)
^2 + a*d^2*f*sinh(1)^2 + 2*(a*d^2*f^2*x + a*d^2*f*cosh(1))*sinh(1))*dilog((
a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqr
t((a^2 + b^2)/b^2) - b)/b + 1) + 3*(a*d^2*f^3*x^2 + 2*a*d^2*f^2*x*cosh(1) +
 a*d^2*f*cosh(1)^2 + a*d^2*f*sinh(1)^2 + 2*(a*d^2*f^2*x + a*d^2*f*cosh(1))*
sinh(1))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(a*d^2*f^3*x^2 + I*b*d^2
*f^3*x^2 + 2*a*d^2*f^2*x*cosh(1) + 2*I*b*d^2*f^2*x*cosh(1) + a*d^2*f*cosh(1
)^2 + I*b*d^2*f*cosh(1)^2 + a*d^2*f*sinh(1)^2 + I*b*d^2*f*sinh(1)^2 + 2*(a*
d^2*f^2*x + a*d^2*f*cosh(1))*sinh(1) + 2*I*(b*d^2*f^2*x + b*d^2*f*cosh(1))*
sinh(1))*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) - 3*(a*d^2*f^3*x^2 - I*b*
d^2*f^3*x^2 + 2*a*d^2*f^2*x*cosh(1) - 2*I*b*d^2*f^2*x*cosh(1) + a*d^2*f*cos
h(1)^2 - I*b*d^2*f*cosh(1)^2 + a*d^2*f*sinh(1)^2 - I*b*d^2*f*sinh(1)^2 + 2*
(a*d^2*f^2*x + a*d^2*f*cosh(1))*sinh(1) - 2*I*(b*d^2*f^2*x + b*d^2*f*cosh(1
))*sinh(1))*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) - (a*c^3*f^3 - 3*a*c^
2*d*f^2*cosh(1) + 3*a*c*d^2*f*cosh(1)^2 - a*d^3*cosh(1)^3 - a*d^3*sinh(1)^3
 + 3*(a*c*d^2*f - a*d^3*cosh(1))*sinh(1)^2 - 3*(a*c^2*d*f^2 - 2*a*c*d^2*f*c
osh(1) + a*d^3*cosh(1)^2)*sinh(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c
) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (a*c^3*f^3 - 3*a*c^2*d*f^2*cosh(1) +
 3*a*c*d^2*f*cosh(1)^2 - a*d^3*cosh(1)^3 - a*d^3*sinh(1)^3 + 3*(a*c*d^2*f -
 a*d^3*cosh(1))*sinh(1)^2 - 3*(a*c^2*d*f^2 - 2*a*c*d^2*f*cosh(1) + a*d^3*co
sh(1)^2)*sinh(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2
 + b^2)/b^2) + 2*a) + (a*d^3*f^3*x^3 + a*c^3*f^3 + 3*(a*d^3*f*x + a*c*d^2*f
)*cosh(1)^2 + 3*(a*d^3*f*x + a*c*d^2*f)*sinh(1)^2 + 3*(a*d^3*f^2*x^2 - a*c^
2*d*f^2)*cosh(1) + 3*(a*d^3*f^2*x^2 - a*c^2*d*f^2 + 2*(a*d^3*f*x + a*c*d^2*
f)*cosh(1))*sinh(1))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (a*d^3*f^3*x^3 + a*
c^3*f^3 + 3*(a*d^3*f*x + a*c*d^2*f)*cosh(1)^2 + 3*(a*d^3*f*x + a*c*d^2*f)*s
inh(1)^2 + 3*(a*d^3*f^2*x^2 - a*c^2*d*f^2)*cosh(1) + 3*(a*d^3*f^2*x^2 - a*c
```

```
^2*d*f^2 + 2*(a*d^3*f*x + a*c*d^2*f)*cosh(1))*sinh(1))*log(-(a*cosh(d*x + c
) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2) - b)/b) + (a*c^3*f^3 + I*b*c^3*f^3 - 3*a*c^2*d*f^2*cosh(1) - 3*I*b*c^2
*d*f^2*cosh(1) + 3*a*c*d^2*f*cosh(1)^2 + 3*I*b*c*d^2*f*cosh(1)^2 - a*d^3*co
sh(1)^3 - I*b*d^3*cosh(1)^3 - a*d^3*sinh(1)^3 - I*b*d^3*sinh(1)^3 + 3*(a*c*
d^2*f - a*d^3*cosh(1))*sinh(1)^2 + 3*I*(b*c*d^2*f - b*d^3*cosh(1))*sinh(1)^
2 - 3*(a*c^2*d*f^2 - 2*a*c*d^2*f*cosh(1) + a*d^3*cosh(1)^2)*sinh(1) - 3*I*(
b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(1) + b*d^3*cosh(1)^2)*sinh(1))*log(cosh(d*x
+ c) + sinh(d*x + c) + I) + (a*c^3*f^3 - I*b*c^3*f^3 - 3*a*c^2*d*f^2*cosh(1
) + 3*I*b*c^2*d*f^2*cosh(1) + 3*a*c*d^2*f*cosh(1)^2 - 3*I*b*c*d^2*f*cosh(1)
^2 - a*d^3*cosh(1)^3 + I*b*d^3*cosh(1)^3 - a*d^3*sinh(1)^3 + I*b*d^3*sinh(1
)^3 + 3*(a*c*d^2*f - a*d^3*cosh(1))*sinh(1)^2 - 3*I*(b*c*d^2*f - b*d^3*cosh
(1))*sinh(1)^2 - 3*(a*c^2*d*f^2 - 2*a*c*d^2*f*cosh(1) + a*d^3*cosh(1)^2)*si
nh(1) + 3*I*(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(1) + b*d^3*cosh(1)^2)*sinh(1))*
log(cosh(d*x + c) + sinh(d*x + c) - I) - (a*d^3*f^3*x^3 - I*b*d^3*f^3*x^3 +
 a*c^3*f^3 - I*b*c^3*f^3 + 3*(a*d^3*f*x + a*c*d^2*f)*cosh(1)^2 - 3*I*(b*d^3
*f*x + b*c*d^2*f)*cosh(1)^2 + 3*(a*d^3*f*x + a*c*d^2*f)*sinh(1)^2 - 3*I*(b*
d^3*f*x + b*c*d^2*f)*sinh(1)^2 + 3*(a*d^3*f^2*x^2 - a*c^2*d*f^2)*cosh(1) -
3*I*(b*d^3*f^2*x^2 - b*c^2*d*f^2)*cosh(1) + 3*(a*d^3*f^2*x^2 - a*c^2*d*f^2
+ 2*(a*d^3*f*x + a*c*d^2*f)*cosh(1))*sinh(1) - 3*I*(b*d^3*f^2*x^2 - b*c^2*d
*f^2 + 2*(b*d^3*f*x + b*c*d^2*f)*cosh(1))*sinh(1))*log(I*cosh(d*x + c) + I*
sinh(d*x + c) + 1) - (a*d^3*f^3*x^3 + I*b*d^3*f^3*x^3 + a*c^3*f^3 + I*b*c^3
*f^3 + 3*(a*d^3*f*x + a*c*d^2*f)*cosh(1)^2 + 3*I*(b*d^3*f*x + b*c*d^2*f)*co
sh(1)^2 + 3*(a*d^3*f*x + a*c*d^2*f)*sinh(1)^2 + 3*I*(b*d^3*f*x + b*c*d^2*f)
*sinh(1)^2 + 3*(a*d^3*f^2*x^2 - a*c^2*d*f^2)*cosh(1) + 3*I*(b*d^3*f^2*x^2 -
 b*c^2*d*f^2)*cosh(1) + 3*(a*d^3*f^2*x^2 - a*c^2*d*f^2 + 2*(a*d^3*f*x + a*c
*d^2*f)*cosh(1))*sinh(1) + 3*I*(b*d^3*f^2*x^2 - b*c^2*d*f^2 + 2*(b*d^3*f*x
+ b*c*d^2*f)*cosh(1))*sinh(1))*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1)
- 6*(a*f^3 + I*b*f^3)*polylog(4, I*cosh(d*x + c) + I*sinh(d*x + c)) - 6*(a*
f^3 - I*b*f^3)*polylog(4, -I*cosh(d*x + c) - I*sinh(d*x + c)) - 6*(a*d*f^3*
x + a*d*f^2*cosh(1) + a*d*f^2*sinh(1))*polylog(3, (a*cosh(d*x + c) + a*sinh
(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) -
 6*(a*d*f^3*x + a*d*f^2*cosh(1) + a*d*f^2*sinh(1))*polylog(3, (a*cosh(d*x +
 c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2
)/b^2))/b) + 6*(a*d*f^3*x + I*b*d*f^3*x + a*d*f...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)**3*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(c+d\,x)\,\left(e+f\,x\right)^3}{a+b\,\mathrm{sinh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((tanh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.349 $\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=716

$$\frac{2(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{bd}-\frac{2a^2(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d}-\frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d}-\frac{a(e+fx)^2\text{lo}}{(}$$

```
[Out] 2*(f*x+e)^2*arctan(exp(d*x+c))/b/d-2*a^2*(f*x+e)^2*arctan(exp(d*x+c))/b/(a^
2+b^2)/d+a*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/(a^2+b^2)/d-a*(f*x+e)^2*ln(1+b*ex
p(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d-a*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))/(a^2+b^2)/d-2*I*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/b/d^2-
2*I*a^2*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b/(a^2+b^2)/d^2+2*I*f*(f*x+e)*pol
ylog(2,I*exp(d*x+c))/b/d^2+2*I*f^2*polylog(3,-I*exp(d*x+c))/b/d^3+a*f*(f*x+
e)*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)/d^2-2*a*f*(f*x+e)*polylog(2,-b*exp(
d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d^2-2*a*f*(f*x+e)*polylog(2,-b*exp(d*
x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d^2-2*I*a^2*f^2*polylog(3,-I*exp(d*x+c)
)/b/(a^2+b^2)/d^3+2*I*a^2*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/b/(a^2+b^2)/d^
2+2*I*a^2*f^2*polylog(3,I*exp(d*x+c))/b/(a^2+b^2)/d^3-2*I*f^2*polylog(3,I*e
xp(d*x+c))/b/d^3-1/2*a*f^2*polylog(3,-exp(2*d*x+2*c))/(a^2+b^2)/d^3+2*a*f^2
*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d^3+2*a*f^2*polylog
(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d^3
```

Rubi [A]
time = 0.79, antiderivative size = 716, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5686, 4265, 2611, 2320, 6724, 5692, 5680, 2221, 6874, 3799}

$$-\frac{2a^2(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{bd\left(a^2+b^2\right)}-\frac{2ia^2f^2\text{Li}_3\left(-ie^{c+dx}\right)}{bd^3\left(a^2+b^2\right)}+\frac{2ia^2f^2\text{Li}_3\left(ie^{c+dx}\right)}{bd^3\left(a^2+b^2\right)}+\frac{2af^2\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^3\left(a^2+b^2\right)}+\frac{2af^2\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^3\left(a^2+b^2\right)}-\frac{af^2\text{Li}_3\left(-e^{2(c+dx)}\right)}{2d^3\left(a^2+b^2\right)}+\frac{2ia^2f(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{bd^2\left(a^2+b^2\right)}-\frac{2ia^2f(e+fx)\text{Li}_2\left(ie^{c+dx}\right)}{bd^2\left(a^2+b^2\right)}-\frac{2af(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)}-\frac{2af(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)}+\frac{af(e+fx)\text{Li}_2\left(-e^{2(c+dx)}\right)}{d^2\left(a^2+b^2\right)}-\frac{a(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{d\left(a^2+b^2\right)}-\frac{a(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{d\left(a^2+b^2\right)}+\frac{a(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{d\left(a^2+b^2\right)}+\frac{2(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{bd}+\frac{2if^2\text{Li}_3\left(-ie^{c+dx}\right)}{bd^3}-\frac{2if^2\text{Li}_3\left(ie^{c+dx}\right)}{bd^3}-\frac{2if(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{bd^2}+\frac{2if(e+fx)\text{Li}_2\left(ie^{c+dx}\right)}{bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b*d) - (2*a^2*(e + f*x)^2*ArcTan[E^(c
+ d*x)])/(b*(a^2 + b^2)*d) - (a*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2])])/((a^2 + b^2)*d) - (a*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) + (a*(e + f*x)^2*Log[1 + E^(2*(c + d
*x))])/((a^2 + b^2)*d) - ((2*I)*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(
b*d^2) + ((2*I)*a^2*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b*(a^2 + b^2
)*d^2) + ((2*I)*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b*d^2) - ((2*I)*a^2
*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) - (2*a*f*(e + f
*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2)
 - (2*a*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
(a^2 + b^2)*d^2) + (a*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^2
```

```
)*d^2) + ((2*I)*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b*d^3) - ((2*I)*a^2*f^2*
PolyLog[3, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^3) - ((2*I)*f^2*PolyLog[3, I
*E^(c + d*x)])/(b*d^3) + ((2*I)*a^2*f^2*PolyLog[3, I*E^(c + d*x)])/(b*(a^2
+ b^2)*d^3) + (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]
)/((a^2 + b^2)*d^3) + (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/((a^2 + b^2)*d^3) - (a*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*(a^2
+ b^2)*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5686

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*Sech[c +
d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{sech}(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{a\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))\,d}{b\left(a^2+b^2\right)}$$

$$= \frac{a(e+fx)^3}{3\left(a^2+b^2\right)f} + \frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2if(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{bd^2} + \frac{2i}{}$$

$$= \frac{a(e+fx)^3}{3\left(a^2+b^2\right)f} + \frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b}}\right.}{\left(a^2+b^2\right)d}$$

$$= \frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^2\log\left(\right.}{\left(a^2\right.}$$

$$= \frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^2\log\left(\right.}{\left(a^2\right.}$$

$$= \frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^2\log\left(\right.}{\left(a^2\right.}$$

$$= \frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^2\log\left(\right.}{\left(a^2\right.}$$

$$= \frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^2\log\left(\right.}{\left(a^2\right.}$$

Mathematica [A]
time = 6.78, size = 872, normalized size = 1.22

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/2*(-4*b*d^2*e^2*ArcTan[E^(c + d*x)] - (4*I)*b*d^2*e*f*x*Log[1 - I*E^(c +
 d*x)] - (2*I)*b*d^2*f^2*x^2*Log[1 - I*E^(c + d*x)] + (4*I)*b*d^2*e*f*x*Log
[1 + I*E^(c + d*x)] + (2*I)*b*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - 2*a*d^2*
```

```
e^2*Log[1 + E^(2*(c + d*x))] - 4*a*d^2*e*f*x*Log[1 + E^(2*(c + d*x))] - 2*a
*d^2*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 2*a*d^2*e^2*Log[b - 2*a*E^(c + d*x)
 - b*E^(2*(c + d*x))] + 4*a*d^2*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sq
rt[(a^2 + b^2)*E^(2*c)])] + 2*a*d^2*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^
c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 4*a*d^2*e*f*x*Log[1 + (b*E^(2*c + d*x))/(
a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*a*d^2*f^2*x^2*Log[1 + (b*E^(2*c + d
*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + (4*I)*b*d*f*(e + f*x)*PolyLog[2
, (-I)*E^(c + d*x)] - (4*I)*b*d*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)] - 2*a
*d*e*f*PolyLog[2, -E^(2*(c + d*x))] - 2*a*d*f^2*x*PolyLog[2, -E^(2*(c + d*x
))] + 4*a*d*e*f*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^
(2*c)]))] + 4*a*d*f^2*x*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 +
 b^2)*E^(2*c)]))] + 4*a*d*e*f*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[
(a^2 + b^2)*E^(2*c)]))] + 4*a*d*f^2*x*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c
 + Sqrt[(a^2 + b^2)*E^(2*c)]))] - (4*I)*b*f^2*PolyLog[3, (-I)*E^(c + d*x)]
+ (4*I)*b*f^2*PolyLog[3, I*E^(c + d*x)] + a*f^2*PolyLog[3, -E^(2*(c + d*x))
] - 4*a*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c
)]))] - 4*a*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^
(2*c)]))])/((a^2 + b^2)*d^3)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \tanh(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(-2*a*e^(-d*x - c) + b*e^
(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a^2 + b
^2)*d))*e^2 + integrate(2*f^2*x^2*(e^(d*x + c) - e^(-d*x - c))/((b*(e^(d*x
+ c) - e^(-d*x - c)) + 2*a)*(e^(d*x + c) + e^(-d*x - c))) + 4*f*x*(e^(d*x +
 c) - e^(-d*x - c))*e/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(e^(d*x + c)
+ e^(-d*x - c))), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1411 vs. $2(659) = 1318$.
time = 0.41, size = 1411, normalized size = 1.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*a*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*a*f^2*polylog(3, (a*cosh(d*
x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2))/b) - 2*(a*d*f^2*x + a*d*f*cosh(1) + a*d*f*sinh(1))*dilog((a*cosh
(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2) - b)/b + 1) - 2*(a*d*f^2*x + a*d*f*cosh(1) + a*d*f*sinh(1))*di
log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a*d*f^2*x + I*b*d*f^2*x + a*d*f*co
sh(1) + I*b*d*f*cosh(1) + a*d*f*sinh(1) + I*b*d*f*sinh(1))*dilog(I*cosh(d*x
 + c) + I*sinh(d*x + c)) + 2*(a*d*f^2*x - I*b*d*f^2*x + a*d*f*cosh(1) - I*b
*d*f*cosh(1) + a*d*f*sinh(1) - I*b*d*f*sinh(1))*dilog(-I*cosh(d*x + c) - I*
sinh(d*x + c)) - (a*c^2*f^2 - 2*a*c*d*f*cosh(1) + a*d^2*cosh(1)^2 + a*d^2*s
inh(1)^2 - 2*(a*c*d*f - a*d^2*cosh(1))*sinh(1))*log(2*b*cosh(d*x + c) + 2*b
*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (a*c^2*f^2 - 2*a*c*d*f*
cosh(1) + a*d^2*cosh(1)^2 + a*d^2*sinh(1)^2 - 2*(a*c*d*f - a*d^2*cosh(1))*s
inh(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^
2) + 2*a) - (a*d^2*f^2*x^2 - a*c^2*f^2 + 2*(a*d^2*f*x + a*c*d*f)*cosh(1) +
2*(a*d^2*f*x + a*c*d*f)*sinh(1))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) +
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (a*d^2*
f^2*x^2 - a*c^2*f^2 + 2*(a*d^2*f*x + a*c*d*f)*cosh(1) + 2*(a*d^2*f*x + a*c*
d*f)*sinh(1))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (a*c^2*f^2 + I*b*c^2*f^2 -
 2*a*c*d*f*cosh(1) - 2*I*b*c*d*f*cosh(1) + a*d^2*cosh(1)^2 + I*b*d^2*cosh(1
)^2 + a*d^2*sinh(1)^2 + I*b*d^2*sinh(1)^2 - 2*(a*c*d*f - a*d^2*cosh(1))*sin
h(1) - 2*I*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*log(cosh(d*x + c) + sinh(d*x
+ c) + I) + (a*c^2*f^2 - I*b*c^2*f^2 - 2*a*c*d*f*cosh(1) + 2*I*b*c*d*f*cosh
(1) + a*d^2*cosh(1)^2 - I*b*d^2*cosh(1)^2 + a*d^2*sinh(1)^2 - I*b*d^2*sinh(
1)^2 - 2*(a*c*d*f - a*d^2*cosh(1))*sinh(1) + 2*I*(b*c*d*f - b*d^2*cosh(1))*
sinh(1))*log(cosh(d*x + c) + sinh(d*x + c) - I) + (a*d^2*f^2*x^2 - I*b*d^2*
f^2*x^2 - a*c^2*f^2 + I*b*c^2*f^2 + 2*(a*d^2*f*x + a*c*d*f)*cosh(1) - 2*I*(
b*d^2*f*x + b*c*d*f)*cosh(1) + 2*(a*d^2*f*x + a*c*d*f)*sinh(1) - 2*I*(b*d^2
*f*x + b*c*d*f)*sinh(1))*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) + (a*d^
2*f^2*x^2 + I*b*d^2*f^2*x^2 - a*c^2*f^2 - I*b*c^2*f^2 + 2*(a*d^2*f*x + a*c*
d*f)*cosh(1) + 2*I*(b*d^2*f*x + b*c*d*f)*cosh(1) + 2*(a*d^2*f*x + a*c*d*f)*
sinh(1) + 2*I*(b*d^2*f*x + b*c*d*f)*sinh(1))*log(-I*cosh(d*x + c) - I*sinh(
d*x + c) + 1) - 2*(a*f^2 + I*b*f^2)*polylog(3, I*cosh(d*x + c) + I*sinh(d*x
```

```
 + c)) - 2*(a*f^2 - I*b*f^2)*polylog(3, -I*cosh(d*x + c) - I*sinh(d*x + c))
)/((a^2 + b^2)*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2 \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)**2*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(c+d\,x)\,{(e+f\,x)}^2}{a+b\sinh(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
[Out] int((tanh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.350 $\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=421

$$\frac{2(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{bd}-\frac{2a^2(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d}-\frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d}-\frac{a(e+fx)\log\left(\right.}{\left(a^2\right.}$$

```
[Out] 2*(f*x+e)*arctan(exp(d*x+c))/b/d-2*a^2*(f*x+e)*arctan(exp(d*x+c))/b/(a^2+b^
2)/d+a*(f*x+e)*ln(1+exp(2*d*x+2*c))/(a^2+b^2)/d-a*(f*x+e)*ln(1+b*exp(d*x+c)
/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d-a*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^
(1/2)))/(a^2+b^2)/d-I*f*polylog(2,-I*exp(d*x+c))/b/d^2+I*a^2*f*polylog(2,-I
*exp(d*x+c))/b/(a^2+b^2)/d^2+I*f*polylog(2,I*exp(d*x+c))/b/d^2-I*a^2*f*poly
log(2,I*exp(d*x+c))/b/(a^2+b^2)/d^2+1/2*a*f*polylog(2,-exp(2*d*x+2*c))/(a^2
+b^2)/d^2-a*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)/d^2-a*
f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)/d^2
```

Rubi [A]
time = 0.44, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5686, 4265, 2317, 2438, 5692, 5680, 2221, 6874, 3799}

$$-\frac{2a^2(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{bd\left(a^2+b^2\right)}+\frac{ia^2f\text{Li}_2\left(-ie^{c+dx}\right)}{bd^2\left(a^2+b^2\right)}-\frac{ia^2f\text{Li}_2\left(ie^{c+dx}\right)}{bd^2\left(a^2+b^2\right)}-\frac{af\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)}-\frac{af\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)}+\frac{af\text{Li}_2\left(-e^{2(c+dx)}\right)}{2d^2\left(a^2+b^2\right)}-\frac{a(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{d\left(a^2+b^2\right)}-\frac{a(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{d\left(a^2+b^2\right)}+\frac{a(e+fx)\log\left(e^{2(c+dx)}+1\right)}{d\left(a^2+b^2\right)}+\frac{2(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{bd}-\frac{if\text{Li}_2\left(-ie^{c+dx}\right)}{bd^2}+\frac{if\text{Li}_2\left(ie^{c+dx}\right)}{bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*(e + f*x)*ArcTan[E^(c + d*x)])/(b*d) - (2*a^2*(e + f*x)*ArcTan[E^(c + d*
x)])/(b*(a^2 + b^2)*d) - (a*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2])])/((a^2 + b^2)*d) - (a*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])])/((a^2 + b^2)*d) + (a*(e + f*x)*Log[1 + E^(2*(c + d*x))])/((a
^2 + b^2)*d) - (I*f*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + (I*a^2*f*PolyLo
g[2, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) + (I*f*PolyLog[2, I*E^(c + d*x)
])/(b*d^2) - (I*a^2*f*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) - (a*f
*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2) -
(a*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2
) + (a*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5686

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*Sech[c +
d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
```

```
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{sech}(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{a\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))\,dx}{b\left(a^2+b^2\right)}$$

$$= \frac{a(e+fx)^2}{2\left(a^2+b^2\right)f} + \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{a\int (a(e+fx)\text{sech}(c+dx) - b(e}{b\left(a^2+b^2\right)}$$

$$= \frac{a(e+fx)^2}{2\left(a^2+b^2\right)f} + \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d}$$

$$= \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)\log\left(1+\right.}{\left(a^2+b^2\right.}$$

$$= \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)\log\left(1+\right.}{\left(a^2+b^2\right.}$$

$$= \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)\log\left(1+\right.}{\left(a^2+b^2\right.}$$

$$= \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)\log\left(1+\right.}{\left(a^2+b^2\right.}$$

Mathematica [A]
time = 1.98, size = 438, normalized size = 1.04

$$\frac{-2acde + 2ac^2f - 2ad^2ex + 2acdfx + 4bde\text{ArcTan}(\cosh(c+dx)+\sinh(c+dx)) + 4bdfx\text{ArcTan}(\cosh(c+dx)+\sinh(c+dx)) - 2acf\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) - 2adfx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) - 2acf\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right) - 2adfx\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right) - 2ade\log(a+b\sinh(c+dx)) + 2acf\log(a+b\sinh(c+dx)) + 2ade\log(1+\cosh(2(c+dx))+\sinh(2(c+dx))) + 2adfx\log(1+\cosh(2(c+dx))+\sinh(2(c+dx))) - 2af\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right) - 2af\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right) - 2ibf\text{PolyLog}(2,-i(\cosh(c+dx)+\sinh(c+dx))) + 2ibf\text{PolyLog}(2,i(\cosh(c+dx)+\sinh(c+dx))) + af\text{PolyLog}(2,-\cosh(2(c+dx))-\sinh(2(c+dx)))}{2\left(a^2+b^2\right)d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*a*c*d*e + 2*a*c^2*f - 2*a*d^2*e*x + 2*a*c*d*f*x + 4*b*d*e*ArcTan[Cosh[c
 + d*x] + Sinh[c + d*x]] + 4*b*d*f*x*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]]
- 2*a*c*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*a*d*f*x*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*a*c*f*Log[1 + (b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2])] - 2*a*d*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2])] - 2*a*d*e*Log[a + b*Sinh[c + d*x]] + 2*a*c*f*Log[a + b*Sinh[c + d*x]
] + 2*a*d*e*Log[1 + Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]] + 2*a*d*f*x*Log[
1 + Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]] - 2*a*f*PolyLog[2, (b*E^(c + d*x
))/(-a + Sqrt[a^2 + b^2])] - 2*a*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2]))] - (2*I)*b*f*PolyLog[2, (-I)*(Cosh[c + d*x] + Sinh[c + d*x])] +
 (2*I)*b*f*PolyLog[2, I*(Cosh[c + d*x] + Sinh[c + d*x])] + a*f*PolyLog[2, -
Cosh[2*(c + d*x)] - Sinh[2*(c + d*x)]])/(2*(a^2 + b^2)*d^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1286 vs. $2(393) = 786$.
time = 3.09, size = 1287, normalized size = 3.06

method	result	size
risch	Expression too large to display	1287

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*a*x+2/d^2*f/(2*a^2+2*b^2)*ln(1+I*exp
(d*x+c))*a*c-2/d*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^
2+b^2)^(1/2)))*a*x-2/d^2*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a
)/(a+(a^2+b^2)^(1/2)))*a*c+2*I/d^2*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))*b-
2*I/d^2*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))*b+2/d*e/(2*a^2+2*b^2)*(a^2+b^
2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d^2*f/(2*a^2+2
*b^2)*dilog(1-I*exp(d*x+c))*a-2/d^2*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a
^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a-2/d^2*f/(2*a^2+2*b^2)*dilog((b*exp
(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a+2/d^2*f/(2*a^2+2*b^2)*dil
og(1+I*exp(d*x+c))*a-2/d*e/(2*a^2+2*b^2)*a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+
c)-b)+2/d^2*f*c*b^2/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+
c)+2*a)/(a^2+b^2)^(1/2))+2/d^2*f*c/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/
2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2-2*I/d^2*f/(2*a^2+2*b^2)*ln(1+I*
exp(d*x+c))*b*c+2*I/d*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*b*x+2*I/d^2*f/(2*a
^2+2*b^2)*ln(1-I*exp(d*x+c))*b*c-2*I/d*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*b
*x+2/d*e/(2*a^2+2*b^2)*a*ln(1+exp(2*d*x+2*c))+4/d*e/(2*a^2+2*b^2)*b*arctan(
exp(d*x+c))+2/d*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*x+2/d^2*f/(2*a^2+2*b^2
)*ln(1-I*exp(d*x+c))*a*c-2/d*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1
/2)-a)/(-a+(a^2+b^2)^(1/2)))*a*x-2/d^2*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a
```

```
^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a*c-2/d*e*b^2/(2*a^2+2*b^2)/(a^2+b^2
)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d*e/(2*a^2+2*b^
2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2+2/
d^2*f*c/(2*a^2+2*b^2)*a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d^2*f*c/(2*
a^2+2*b^2)*(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2)
)-2/d^2*f*c/(2*a^2+2*b^2)*a*ln(1+exp(2*d*x+2*c))-4/d^2*f*c/(2*a^2+2*b^2)*b*
arctan(exp(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(-2*a*e^(-d*x - c) + b*e^
(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a^2 + b
^2)*d))*e + f*integrate(2*x*(e^(d*x + c) - e^(-d*x - c))/((b*(e^(d*x + c) -
 e^(-d*x - c)) + 2*a)*(e^(d*x + c) + e^(-d*x - c))), x)
```

Fricas [A]
time = 0.42, size = 631, normalized size = 1.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(a*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + a*f*dilog((a*cosh(d*x + c) +
a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
 - b)/b + 1) - (a*f + I*b*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) - (a*
f - I*b*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) - (a*c*f - a*d*cosh(1)
 - a*d*sinh(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 +
 b^2)/b^2) + 2*a) - (a*c*f - a*d*cosh(1) - a*d*sinh(1))*log(2*b*cosh(d*x +
c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (a*d*f*x + a*c*
f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (a*d*f*x + a*c*f)*log(-(a*cosh(d*x +
c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2) - b)/b) + (a*c*f + I*b*c*f - a*d*cosh(1) - I*b*d*cosh(1) - a*d*sinh(1
) - I*b*d*sinh(1))*log(cosh(d*x + c) + sinh(d*x + c) + I) + (a*c*f - I*b*c*
f - a*d*cosh(1) + I*b*d*cosh(1) - a*d*sinh(1) + I*b*d*sinh(1))*log(cosh(d*x
 + c) + sinh(d*x + c) - I) - (a*d*f*x - I*b*d*f*x + a*c*f - I*b*c*f)*log(I*
cosh(d*x + c) + I*sinh(d*x + c) + 1) - (a*d*f*x + I*b*d*f*x + a*c*f + I*b*c
*f)*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1))/((a^2 + b^2)*d^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(c+d\,x)\,\left(e+f\,x\right)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((tanh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.351 $\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=69

$$\frac{b\text{ArcTan}(\sinh(c+dx))}{(a^2+b^2)\,d}+\frac{a\log(\cosh(c+dx))}{(a^2+b^2)\,d}-\frac{a\log(a+b\sinh(c+dx))}{(a^2+b^2)\,d}$$

```
[Out] b*arctan(sinh(d*x+c))/(a^2+b^2)/d+a*ln(cosh(d*x+c))/(a^2+b^2)/d-a*ln(a+b*si
nh(d*x+c))/(a^2+b^2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2800, 815, 649, 209, 266}

$$\frac{b\text{ArcTan}(\sinh(c+dx))}{d\,(a^2+b^2)}-\frac{a\log(a+b\sinh(c+dx))}{d\,(a^2+b^2)}+\frac{a\log(\cosh(c+dx))}{d\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d) + (a*Log[Cosh[c + d*x]])/((a^2 +
b^2)*d) - (a*Log[a + b*Sinh[c + d*x]])/((a^2 + b^2)*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x}{(a+x)(-b^2-x^2)}\,dx, x, b\sinh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a}{(a^2+b^2)(a+x)} + \frac{-b^2-ax}{(a^2+b^2)(b^2+x^2)}\right)\,dx, x, b\sinh(c+dx)\right)}{d}$$
$$= -\frac{a\log(a+b\sinh(c+dx))}{(a^2+b^2)\,d} - \frac{\text{Subst}\left(\int \frac{-b^2-ax}{b^2+x^2}\,dx, x, b\sinh(c+dx)\right)}{(a^2+b^2)\,d}$$
$$= -\frac{a\log(a+b\sinh(c+dx))}{(a^2+b^2)\,d} + \frac{a\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sinh(c+dx)\right)}{(a^2+b^2)\,d} + \frac{b^2\text{Subst}\left(\int\right.}{}$$
$$= \frac{b\tan^{-1}(\sinh(c+dx))}{(a^2+b^2)\,d} + \frac{a\log(\cosh(c+dx))}{(a^2+b^2)\,d} - \frac{a\log(a+b\sinh(c+dx))}{(a^2+b^2)\,d}$$

Mathematica [A]
time = 0.06, size = 51, normalized size = 0.74

$$\frac{2b\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + a(\log(\cosh(c+dx)) - \log(a+b\sinh(c+dx)))}{(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/(a + b*Sinh[c + d*x]),x]
[Out] (2*b*ArcTan[Tanh[(c + d*x)/2]] + a*(Log[Cosh[c + d*x]] - Log[a + b*Sinh[c +
 d*x]]))/((a^2 + b^2)*d)
```

Maple [A]
time = 1.18, size = 97, normalized size = 1.41

method	result
derivativedivides	$\frac{\frac{2a\ln\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+4b\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2+2b^2} - \frac{2a\ln\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{2a^2+2b^2}}{d}$

default	$\frac{\frac{2a\ln\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+4b\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2+2b^2}-\frac{2a\ln\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{2a^2+2b^2}}{d}$
risch	$-\frac{2a\,d^2x}{a^2d^2+b^2d^2}-\frac{2adc}{a^2d^2+b^2d^2}+\frac{2ax}{a^2+b^2}+\frac{2ac}{d(a^2+b^2)}+\frac{i\ln(e^{dx+c}+i)b}{(a^2+b^2)d}+\frac{\ln(e^{dx+c}+i)a}{(a^2+b^2)d}-\frac{i\ln(e^{dx+c}-i)b}{(a^2+b^2)d}+\frac{\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(4/(2*a^2+2*b^2)*(1/2*a*ln(tanh(1/2*d*x+1/2*c)^2+1)+b*arctan(tanh(1/2*d
*x+1/2*c)))-2*a/(2*a^2+2*b^2)*ln(a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1
/2*c)-a))
```

Maxima [A]
time = 0.47, size = 95, normalized size = 1.38

$$-\frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d}-\frac{a\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^2+b^2)d}+\frac{a\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(-2*a*e^(-d*x - c) + b*e^(
-2*d*x - 2*c) - b)/((a^2 + b^2)*d) + a*log(e^(-2*d*x - 2*c) + 1)/((a^2 + b^
2)*d)
```

Fricas [A]
time = 0.34, size = 92, normalized size = 1.33

$$\frac{2\,b\arctan\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)-a\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)+a\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{(a^2+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*b*arctan(cosh(d*x + c) + sinh(d*x + c)) - a*log(2*(b*sinh(d*x + c) + a)/
(cosh(d*x + c) - sinh(d*x + c))) + a*log(2*cosh(d*x + c)/(cosh(d*x + c) - s
inh(d*x + c))))/((a^2 + b^2)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tanh\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A]
time = 0.45, size = 121, normalized size = 1.75

$$-\frac{\frac{2\,ab\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^2b+b^3}-\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)b}{a^2+b^2}-\frac{a\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^2+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*b*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^2*b + b^3) -
(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*b/(a^2 + b^2) - a*l
og((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2 + b^2))/d
```

Mupad [B]
time = 1.27, size = 130, normalized size = 1.88

$$\frac{\ln\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)}{a\,d-b\,d\,1\mathrm{i}}-\frac{a\,\ln\left(8\,a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-b^3-4\,a^2\,b+b^3\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+4\,a^2\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+2\,a\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right)}{d\,a^2+d\,b^2}+\frac{\ln\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)\,1\mathrm{i}}{-b\,d+a\,d\,1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)/(a + b*sinh(c + d*x)),x)
```

```
[Out] log(exp(c + d*x) + 1i)/(a*d - b*d*1i) + (log(exp(c + d*x)*1i + 1)*1i)/(a*d*
1i - b*d) - (a*log(8*a^3*exp(d*x)*exp(c) - b^3 - 4*a^2*b + b^3*exp(2*c)*exp
(2*d*x) + 4*a^2*b*exp(2*c)*exp(2*d*x) + 2*a*b^2*exp(d*x)*exp(c)))/(a^2*d +
b^2*d)
```

3.352 $\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=29

$$\operatorname{Int}\left(\frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Tanh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Tanh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 10.89, size = 0, normalized size = 0.00

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tanh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Integrate[Tanh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(c + dx)}{(a + b \sinh(c + dx))(e + fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tanh(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))),x)
[Out] int(tanh(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.353 $\int \frac{(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=917

$$\frac{(e+fx)^3}{bd}-\frac{a^2(e+fx)^3}{b\left(a^2+b^2\right)d}+\frac{6af(e+fx)^2\operatorname{ArcTan}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d^2}-\frac{ab(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d}+\frac{ab(e+fx)^3\,\mathrm{lo}}{(a}$$

```
[Out] (f*x+e)^3/b/d-a^2*(f*x+e)^3/b/(a^2+b^2)/d+6*a*f*(f*x+e)^2*arctan(exp(d*x+c)
)/(a^2+b^2)/d^2-3*f*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/b/d^2+3*a^2*f*(f*x+e)^2*
ln(1+exp(2*d*x+2*c))/b/(a^2+b^2)/d^2-a*b*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^
2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d+a*b*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/(a^2+b^2)^(3/2)/d+6*I*a*f^2*(f*x+e)*polylog(2,I*exp(d*x+c))/(a^2
+b^2)/d^3+6*I*a*f^3*polylog(3,-I*exp(d*x+c))/(a^2+b^2)/d^4-3*f^2*(f*x+e)*po
lylog(2,-exp(2*d*x+2*c))/b/d^3+3*a^2*f^2*(f*x+e)*polylog(2,-exp(2*d*x+2*c))
/b/(a^2+b^2)/d^3-3*a*b*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/
2)))/(a^2+b^2)^(3/2)/d^2+3*a*b*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+
b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-6*I*a*f^3*polylog(3,I*exp(d*x+c))/(a^2+b^2
)/d^4-6*I*a*f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/(a^2+b^2)/d^3+3/2*f^3*poly
log(3,-exp(2*d*x+2*c))/b/d^4-3/2*a^2*f^3*polylog(3,-exp(2*d*x+2*c))/b/(a^2+
b^2)/d^4+6*a*b*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^
2+b^2)^(3/2)/d^3-6*a*b*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/
2)))/(a^2+b^2)^(3/2)/d^3-6*a*b*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/
2)))/(a^2+b^2)^(3/2)/d^4+6*a*b*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/
2)))/(a^2+b^2)^(3/2)/d^4-a*(f*x+e)^3*sech(d*x+c)/(a^2+b^2)/d+(f*x+e)^3*tanh
(d*x+c)/b/d-a^2*(f*x+e)^3*tanh(d*x+c)/b/(a^2+b^2)/d
```

Rubi [A]
time = 1.38, antiderivative size = 917, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 14, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5702, 4269, 3799, 2221, 2611, 2320, 6724, 5692, 3403, 2296, 6744, 6874, 5559, 4265}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e + f*x)^3/(b*d) - (a^2*(e + f*x)^3)/(b*(a^2 + b^2)*d) + (6*a*f*(e + f*x)^
2*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d^2) - (a*b*(e + f*x)^3*Log[1 + (b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) + (a*b*(e + f*x)^3*L
og[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (3*f
*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b*d^2) + (3*a^2*f*(e + f*x)^2*Log[1
 + E^(2*(c + d*x))])/(b*(a^2 + b^2)*d^2) - ((6*I)*a*f^2*(e + f*x)*PolyLog[2
```

```
, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) + ((6*I)*a*f^2*(e + f*x)*PolyLog[2,
I*E^(c + d*x)])/((a^2 + b^2)*d^3) - (3*a*b*f*(e + f*x)^2*PolyLog[2, -((b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) + (3*a*b*f*(e +
 f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^
(3/2)*d^2) - (3*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(b*d^3) + (3*a^
2*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(b*(a^2 + b^2)*d^3) + ((6*I)*
a*f^3*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^4) - ((6*I)*a*f^3*PolyLo
g[3, I*E^(c + d*x)])/((a^2 + b^2)*d^4) + (6*a*b*f^2*(e + f*x)*PolyLog[3, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) - (6*a*b*f
^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 +
b^2)^(3/2)*d^3) + (3*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*b*d^4) - (3*a^2*f
^3*PolyLog[3, -E^(2*(c + d*x))])/(2*b*(a^2 + b^2)*d^4) - (6*a*b*f^3*PolyLog
[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^4) + (6
*a*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)
^(3/2)*d^4) - (a*(e + f*x)^3*Sech[c + d*x])/((a^2 + b^2)*d) + ((e + f*x)^3*
Tanh[c + d*x])/(b*d) - (a^2*(e + f*x)^3*Tanh[c + d*x])/(b*(a^2 + b^2)*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5702

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\text{sech}^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)^3\tanh(c+dx)}{bd} - \frac{a\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh}{b\,(a^2+b^2)}$$

$$= \frac{(e+fx)^3}{bd} + \frac{(e+fx)^3\tanh(c+dx)}{bd} - a\int \left(a(e+fx)^3\text{sech}^2(c+\right.$$

$$= \frac{(e+fx)^3}{bd} - \frac{3f(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{bd^2} + \frac{(e+fx)^3\tanh(c}{bd}$$

$$= \frac{(e+fx)^3}{bd} - \frac{ab(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} + ab(e+fx)$$

$$= \frac{(e+fx)^3}{bd} - \frac{a^2(e+fx)^3}{b\,(a^2+b^2)\,d} + \frac{6af(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2} - ab(e$$

$$= \frac{(e+fx)^3}{bd} - \frac{a^2(e+fx)^3}{b\,(a^2+b^2)\,d} + \frac{6af(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2} - ab(e$$

$$= \frac{(e+fx)^3}{bd} - \frac{a^2(e+fx)^3}{b\,(a^2+b^2)\,d} + \frac{6af(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2} - ab(e$$

$$= \frac{(e+fx)^3}{bd} - \frac{a^2(e+fx)^3}{b\,(a^2+b^2)\,d} + \frac{6af(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2} - ab(e$$

$$= \frac{(e+fx)^3}{bd} - \frac{a^2(e+fx)^3}{b\,(a^2+b^2)\,d} + \frac{6af(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2} - ab(e$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1928 vs. 2(917) = 1834.
time = 14.52, size = 1928, normalized size = 2.10

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x
]

[Out] (f*(12*b*d^3*e^2*E^(2*c)*x + 12*b*d^3*e*E^(2*c)*f*x^2 + 4*b*d^3*E^(2*c)*f^2
*x^3 + 12*a*d^2*e^2*ArcTan[E^(c + d*x)] + 12*a*d^2*e^2*E^(2*c)*ArcTan[E^(c
+ d*x)] + (12*I)*a*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + (12*I)*a*d^2*e*E^(2*c
)*f*x*Log[1 - I*E^(c + d*x)] + (6*I)*a*d^2*f^2*x^2*Log[1 - I*E^(c + d*x)] +
 (6*I)*a*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] - (12*I)*a*d^2*e*f*x*Lo
g[1 + I*E^(c + d*x)] - (12*I)*a*d^2*e*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] -
(6*I)*a*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - (6*I)*a*d^2*E^(2*c)*f^2*x^2*Lo
g[1 + I*E^(c + d*x)] - 6*b*d^2*e^2*Log[1 + E^(2*(c + d*x))] - 6*b*d^2*e^2*E
^(2*c)*Log[1 + E^(2*(c + d*x))] - 12*b*d^2*e*f*x*Log[1 + E^(2*(c + d*x))] -
 12*b*d^2*e*E^(2*c)*f*x*Log[1 + E^(2*(c + d*x))] - 6*b*d^2*f^2*x^2*Log[1 +
E^(2*(c + d*x))] - 6*b*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(2*(c + d*x))] - (12*I
)*a*d*(1 + E^(2*c))*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)] + (12*I)*a*d*(
1 + E^(2*c))*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)] - 6*b*d*e*f*PolyLog[2, -
E^(2*(c + d*x))] - 6*b*d*e*E^(2*c)*f*PolyLog[2, -E^(2*(c + d*x))] - 6*b*d*f
^2*x*PolyLog[2, -E^(2*(c + d*x))] - 6*b*d*E^(2*c)*f^2*x*PolyLog[2, -E^(2*(c
 + d*x))] + (12*I)*a*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (12*I)*a*E^(2*c)*f^
2*PolyLog[3, (-I)*E^(c + d*x)] - (12*I)*a*f^2*PolyLog[3, I*E^(c + d*x)] - (
12*I)*a*E^(2*c)*f^2*PolyLog[3, I*E^(c + d*x)] + 3*b*f^2*PolyLog[3, -E^(2*(c
 + d*x))] + 3*b*E^(2*c)*f^2*PolyLog[3, -E^(2*(c + d*x))]))/(2*(a^2 + b^2)*d
^4*(1 + E^(2*c))) + (a*b*(2*d^3*e^3*Sqrt[(a^2 + b^2)*E^(2*c)]*ArcTan[(a + b
*E^(c + d*x))/Sqrt[-a^2 - b^2]] + 3*Sqrt[-a^2 - b^2]*d^3*e^2*E^c*f*x*Log[1
+ (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 3*Sqrt[-a^2 - b^
2]*d^3*e*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^
(2*c)])] + Sqrt[-a^2 - b^2]*d^3*E^c*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^
c - Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*Sqrt[-a^2 - b^2]*d^3*e^2*E^c*f*x*Log[1
+ (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*Sqrt[-a^2 - b^
2]*d^3*e*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^
(2*c)])] - Sqrt[-a^2 - b^2]*d^3*E^c*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^
c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 3*Sqrt[-a^2 - b^2]*d^2*E^c*f*(e + f*x)^2*
PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 3*Sq
rt[-a^2 - b^2]*d^2*E^c*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c
+ Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[-a^2 - b^2]*d*e*E^c*f^2*PolyLog[3,
-((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[-a^2 - b
^2]*d*E^c*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^
(2*c)]))] + 6*Sqrt[-a^2 - b^2]*d*e*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(
a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[-a^2 - b^2]*d*E^c*f^3*x*PolyL
og[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[-a
^2 - b^2]*E^c*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*
E^(2*c)]))] - 6*Sqrt[-a^2 - b^2]*E^c*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*
E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/((-a^2 - b^2)^(3/2)*d^4*Sqrt[(a^2 + b^
2)*E^(2*c)]) + (Sech[c]*Sech[c + d*x]*(-(a*e^3*Cosh[c]) - 3*a*e^2*f*x*Cosh[
c] - 3*a*e*f^2*x^2*Cosh[c] - a*f^3*x^3*Cosh[c] + b*e^3*Sinh[d*x] + 3*b*e^2*
```

```
f*x*Sinh[d*x] + 3*b*e*f^2*x^2*Sinh[d*x] + b*f^3*x^3*Sinh[d*x]))/((a^2 + b^2
)*d)
```

Maple [F]
time = 2.28, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \operatorname{sech}(dx+c) \tanh(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] 6*a*f^3*integrate(x^2*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x +
 2*c) + a^2*d + b^2*d), x) + 6*b*f^3*integrate(x^2/(a^2*d*e^(2*d*x + 2*c) +
 b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + 12*b*f^2*e*integrate(x/(a^2*d
*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + 3*b*f*(2*(d
*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 + b^2)*d^2))*e^2
 + 12*a*f^2*integrate(x*e^(d*x + c + 1)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*e^(2
*d*x + 2*c) + a^2*d + b^2*d), x) - (a*b*log((b*e^(-d*x - c) - a - sqrt(a^2
+ b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2)*d) + 2*(
a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*d))*e^3 + 6
*a*f*arctan(e^(d*x + c))*e^2/((a^2 + b^2)*d^2) - 2*(b*f^3*x^3 + 3*b*f^2*x^2
*e + 3*b*f*x*e^2 + (a*f^3*x^3*e^c + 3*a*f^2*x^2*e^(c + 1) + 3*a*f*x*e^(c +
2))*e^(d*x))/(a^2*d + b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) -
integrate(-2*(a*b*f^3*x^3*e^c + 3*a*b*f^2*x^2*e^(c + 1) + 3*a*b*f*x*e^(c +
2))*e^(d*x)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e^(2*c))*e^(2*d*x) - 2*(a^3
*e^c + a*b^2*e^c)*e^(d*x)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 10811 vs. 2(864) = 1728.
time = 0.57, size = 10811, normalized size = 11.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")

[Out] (2*(a^2*b + b^3)*c^3*f^3 - 6*(a^2*b + b^3)*c^2*d*f^2*cosh(1) + 6*(a^2*b + b
^3)*c*d^2*f*cosh(1)^2 - 2*(a^2*b + b^3)*d^3*cosh(1)^3 - 2*(a^2*b + b^3)*d^3
*sinh(1)^3 + 2*((a^2*b + b^3)*d^3*f^3*x^3 + (a^2*b + b^3)*c^3*f^3 + 3*((a^2
*b + b^3)*d^3*f*x + (a^2*b + b^3)*c*d^2*f)*cosh(1)^2 + 3*((a^2*b + b^3)*d^3
*f*x + (a^2*b + b^3)*c*d^2*f)*sinh(1)^2 + 3*((a^2*b + b^3)*d^3*f^2*x^2 - (a
^2*b + b^3)*c^2*d*f^2)*cosh(1) + 3*((a^2*b + b^3)*d^3*f^2*x^2 - (a^2*b + b^
3)*c^2*d*f^2 + 2*((a^2*b + b^3)*d^3*f*x + (a^2*b + b^3)*c*d^2*f)*cosh(1))*s
inh(1))*cosh(d*x + c)^2 + 6*((a^2*b + b^3)*c*d^2*f - (a^2*b + b^3)*d^3*cosh
(1))*sinh(1)^2 + 2*((a^2*b + b^3)*d^3*f^3*x^3 + (a^2*b + b^3)*c^3*f^3 + 3*(
(a^2*b + b^3)*d^3*f*x + (a^2*b + b^3)*c*d^2*f)*cosh(1)^2 + 3*((a^2*b + b^3)
*d^3*f*x + (a^2*b + b^3)*c*d^2*f)*sinh(1)^2 + 3*((a^2*b + b^3)*d^3*f^2*x^2
- (a^2*b + b^3)*c^2*d*f^2)*cosh(1) + 3*((a^2*b + b^3)*d^3*f^2*x^2 - (a^2*b
+ b^3)*c^2*d*f^2 + 2*((a^2*b + b^3)*d^3*f*x + (a^2*b + b^3)*c*d^2*f)*cosh(1
))*sinh(1))*sinh(d*x + c)^2 - 3*(a*b^2*d^2*f^3*x^2 + 2*a*b^2*d^2*f^2*x*cosh
(1) + a*b^2*d^2*f*cosh(1)^2 + a*b^2*d^2*f*sinh(1)^2 + (a*b^2*d^2*f^3*x^2 +
2*a*b^2*d^2*f^2*x*cosh(1) + a*b^2*d^2*f*cosh(1)^2 + a*b^2*d^2*f*sinh(1)^2 +
 2*(a*b^2*d^2*f^2*x + a*b^2*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(a*
b^2*d^2*f^3*x^2 + 2*a*b^2*d^2*f^2*x*cosh(1) + a*b^2*d^2*f*cosh(1)^2 + a*b^2
*d^2*f*sinh(1)^2 + 2*(a*b^2*d^2*f^2*x + a*b^2*d^2*f*cosh(1))*sinh(1))*cosh(
d*x + c)*sinh(d*x + c) + (a*b^2*d^2*f^3*x^2 + 2*a*b^2*d^2*f^2*x*cosh(1) + a
*b^2*d^2*f*cosh(1)^2 + a*b^2*d^2*f*sinh(1)^2 + 2*(a*b^2*d^2*f^2*x + a*b^2*d
^2*f*cosh(1))*sinh(1))*sinh(d*x + c)^2 + 2*(a*b^2*d^2*f^2*x + a*b^2*d^2*f*c
osh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x
+ c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1
) + 3*(a*b^2*d^2*f^3*x^2 + 2*a*b^2*d^2*f^2*x*cosh(1) + a*b^2*d^2*f*cosh(1)^
2 + a*b^2*d^2*f*sinh(1)^2 + (a*b^2*d^2*f^3*x^2 + 2*a*b^2*d^2*f^2*x*cosh(1)
+ a*b^2*d^2*f*cosh(1)^2 + a*b^2*d^2*f*sinh(1)^2 + 2*(a*b^2*d^2*f^2*x + a*b^
2*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(a*b^2*d^2*f^3*x^2 + 2*a*b^2*
d^2*f^2*x*cosh(1) + a*b^2*d^2*f*cosh(1)^2 + a*b^2*d^2*f*sinh(1)^2 + 2*(a*b^
2*d^2*f^2*x + a*b^2*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (
a*b^2*d^2*f^3*x^2 + 2*a*b^2*d^2*f^2*x*cosh(1) + a*b^2*d^2*f*cosh(1)^2 + a*b
^2*d^2*f*sinh(1)^2 + 2*(a*b^2*d^2*f^2*x + a*b^2*d^2*f*cosh(1))*sinh(1))*sin
h(d*x + c)^2 + 2*(a*b^2*d^2*f^2*x + a*b^2*d^2*f*cosh(1))*sinh(1))*sqrt((a^2
 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (a*b^2*c^3*f^3 - 3*a*b
^2*c^2*d*f^2*cosh(1) + 3*a*b^2*c*d^2*f*cosh(1)^2 - a*b^2*d^3*cosh(1)^3 - a*
b^2*d^3*sinh(1)^3 + (a*b^2*c^3*f^3 - 3*a*b^2*c^2*d*f^2*cosh(1) + 3*a*b^2*c*
d^2*f*cosh(1)^2 - a*b^2*d^3*cosh(1)^3 - a*b^2*d^3*sinh(1)^3 + 3*(a*b^2*c*d^
2*f - a*b^2*d^3*cosh(1))*sinh(1)^2 - 3*(a*b^2*c^2*d*f^2 - 2*a*b^2*c*d^2*f*c
osh(1) + a*b^2*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c)^2 + 3*(a*b^2*c*d^2*f -
 a*b^2*d^3*cosh(1))*sinh(1)^2 + 2*(a*b^2*c^3*f^3 - 3*a*b^2*c^2*d*f^2*cosh(1
) + 3*a*b^2*c*d^2*f*cosh(1)^2 - a*b^2*d^3*cosh(1)^3 - a*b^2*d^3*sinh(1)^3 +
```

```
 3*(a*b^2*c*d^2*f - a*b^2*d^3*cosh(1))*sinh(1)^2 - 3*(a*b^2*c^2*d*f^2 - 2*a
*b^2*c*d^2*f*cosh(1) + a*b^2*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c)*sinh(d*x
 + c) + (a*b^2*c^3*f^3 - 3*a*b^2*c^2*d*f^2*cosh(1) + 3*a*b^2*c*d^2*f*cosh(1
)^2 - a*b^2*d^3*cosh(1)^3 - a*b^2*d^3*sinh(1)^3 + 3*(a*b^2*c*d^2*f - a*b^2*
d^3*cosh(1))*sinh(1)^2 - 3*(a*b^2*c^2*d*f^2 - 2*a*b^2*c*d^2*f*cosh(1) + a*b
^2*d^3*cosh(1)^2)*sinh(1))*sinh(d*x + c)^2 - 3*(a*b^2*c^2*d*f^2 - 2*a*b^2*c
*d^2*f*cosh(1) + a*b^2*d^3*cosh(1)^2)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*
b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (a
*b^2*c^3*f^3 - 3*a*b^2*c^2*d*f^2*cosh(1) + 3*a*b^2*c*d^2*f*cosh(1)^2 - a*b^
2*d^3*cosh(1)^3 - a*b^2*d^3*sinh(1)^3 + (a*b^2*c^3*f^3 - 3*a*b^2*c^2*d*f^2*
cosh(1) + 3*a*b^2*c*d^2*f*cosh(1)^2 - a*b^2*d^3*cosh(1)^3 - a*b^2*d^3*sinh(
1)^3 + 3*(a*b^2*c*d^2*f - a*b^2*d^3*cosh(1))*sinh(1)^2 - 3*(a*b^2*c^2*d*f^2
 - 2*a*b^2*c*d^2*f*cosh(1) + a*b^2*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c)^2
+ 3*(a*b^2*c*d^2*f - a*b^2*d^3*cosh(1))*sinh(1)^2 + 2*(a*b^2*c^3*f^3 - 3*a*
b^2*c^2*d*f^2*cosh(1) + 3*a*b^2*c*d^2*f*cosh(1)^2 - a*b^2*d^3*cosh(1)^3 - a
*b^2*d^3*sinh(1)^3 + 3*(a*b^2*c*d^2*f - a*b^2*d^3*cosh(1))*sinh(1)^2 - 3*(a
*b^2*c^2*d*f^2 - 2*a*b^2*c*d^2*f*cosh(1) + a*b^2*d^3*cosh(1)^2)*sinh(1))*co
sh(d*x + c)*sinh(d*x + c) + (a*b^2*c^3*f^3 - 3*a*b^2*c^2*d*f^2*cosh(1) + 3*
a*b^2*c*d^2*f*cosh(1)^2 - a*b^2*d^3*cosh(1)^3 - a*b^2*d^3*sinh(1)^3 + 3*(a*
b^2*c*d^2*f - a*b^2*d^3*cosh(1))*sinh(1)^2 - 3*(a*b^2*c^2*d*f^2 - 2*a*b^2*c
*d^2*f*cosh(1) + a*b^2*d^3*cosh(1)^2)*sinh(1))*sinh(d*x + c)^2 - 3*(a*b^2*c
^2*d*f^2 - 2*a*b^2*c*d^2*f*cosh(1) + a*b^2*d^3*cosh(1)^2)*sinh(1))*sqrt((a^
2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b
^2)/b^2) + 2*a) - (a*b^2*d^3*f^3*x^3 + a*b^2*c^3*f^3 + 3*(a*b^2*d^3*f*x + a
*b^2*c*d^2*f)*cosh(1)^2 + (a*b^2*d^3*f^3*x^3 + ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3 \tanh{(c+dx)} \operatorname{sech}{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*tanh(c + d*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(c+d\,x)\,\left(e+f\,x\right)^3}{\cosh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(c + d*x)*(e + f*x)^3)/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
[Out] int((tanh(c + d*x)*(e + f*x)^3)/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.354 $\int \frac{(e+fx)^2\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=648

$$\frac{(e+fx)^2}{bd}-\frac{a^2(e+fx)^2}{b\left(a^2+b^2\right)d}+\frac{4af(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d^2}-\frac{ab(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d}+\frac{ab(e+fx)^2\log}{(a^2}$$

[Out] (f*x+e)^2/b/d-a^2*(f*x+e)^2/b/(a^2+b^2)/d+4*a*f*(f*x+e)*arctan(exp(d*x+c))/(a^2+b^2)/d^2-2*f*(f*x+e)*ln(1+exp(2*d*x+2*c))/b/d^2+2*a^2*f*(f*x+e)*ln(1+exp(2*d*x+2*c))/b/(a^2+b^2)/d^2-a*b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d+a*b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d-2*I*a*f^2*polylog(2,-I*exp(d*x+c))/(a^2+b^2)/d^3+2*I*a*f^2*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^3-f^2*polylog(2,-exp(2*d*x+2*c))/b/d^3+a^2*f^2*polylog(2,-exp(2*d*x+2*c))/b/(a^2+b^2)/d^3-2*a*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2+2*a*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2+2*a*b*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^3-2*a*b*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^3-a*(f*x+e)^2*sech(d*x+c)/(a^2+b^2)/d+(f*x+e)^2*tanh(d*x+c)/b/d-a^2*(f*x+e)^2*tanh(d*x+c)/b/(a^2+b^2)/d

Rubi [A]
time = 1.06, antiderivative size = 648, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 15, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {5702, 4269, 3799, 2221, 2317, 2438, 5692, 3403, 2296, 2611, 2320, 6724, 6874, 5559, 4265}

$$\frac{4af(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{d^2\left(a^2+b^2\right)}+\frac{a^2f^2\text{Li}_2\left(-e^{2(c+dx)}\right)}{bd^3\left(a^2+b^2\right)}-\frac{2iaf^2\text{Li}_2\left(-ie^{c+dx}\right)}{d^3\left(a^2+b^2\right)}+\frac{2iaf^2\text{Li}_2\left(ie^{c+dx}\right)}{d^3\left(a^2+b^2\right)}+\frac{2abf^2\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^3\left(a^2+b^2\right)^{3/2}}-\frac{2abf^2\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^3\left(a^2+b^2\right)^{3/2}}-\frac{2abf(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^{3/2}}+\frac{2abf(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^{3/2}}+\frac{2a^2f(e+fx)\log\left(e^{2(c+dx)}+1\right)}{bd^2\left(a^2+b^2\right)}-\frac{ab(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{d\left(a^2+b^2\right)^{3/2}}+\frac{ab(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{d\left(a^2+b^2\right)^{3/2}}-\frac{a^2(e+fx)^2\tanh(c+dx)}{bd\left(a^2+b^2\right)}-\frac{a(e+fx)^2\text{sech}(c+dx)}{d\left(a^2+b^2\right)}-\frac{a^2(e+fx)^2}{bd\left(a^2+b^2\right)}-\frac{f^2\text{Li}_2\left(-e^{2(c+dx)}\right)}{bd^3}-\frac{2f(e+fx)\log\left(e^{2(c+dx)}+1\right)}{bd^2}+\frac{(e+fx)^2\tanh(c+dx)}{bd}+\frac{(e+fx)^2}{bd}$$

Antiderivative was successfully verified.

[In] Int[((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]

[Out] (e + f*x)^2/(b*d) - (a^2*(e + f*x)^2)/(b*(a^2 + b^2)*d) + (4*a*f*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d^2) - (a*b*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) + (a*b*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (2*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(b*d^2) + (2*a^2*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(b*(a^2 + b^2)*d^2) - ((2*I)*a*f^2*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) + ((2*I)*a*f^2*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^3) - (2*a*b*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) + (2*a*b*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (f^2*PolyLog[2, -E^(2*(c + d*x))])/(b*d^3) + (a^2*f^2*PolyLog[2, -E^(2*(c + d*x))])/(b*(a^2

```
+ b^2)*d^3) + (2*a*b*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
)])/((a^2 + b^2)^(3/2)*d^3) - (2*a*b*f^2*PolyLog[3, -((b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) - (a*(e + f*x)^2*Sech[c + d*x])
/((a^2 + b^2)*d) + ((e + f*x)^2*Tanh[c + d*x])/(b*d) - (a^2*(e + f*x)^2*Tan
h[c + d*x])/(b*(a^2 + b^2)*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5702

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{sech}^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)^2\tanh(c+dx)}{bd} - \frac{a\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh}{b\,(a^2+b^2)}$$

$$= \frac{(e+fx)^2}{bd} + \frac{(e+fx)^2\tanh(c+dx)}{bd} - \frac{a\int \left(a(e+fx)^2\text{sech}^2(c+\right.}{}$$

$$= \frac{(e+fx)^2}{bd} - \frac{2f(e+fx)\log\left(1+e^{2(c+dx)}\right)}{bd^2} + \frac{(e+fx)^2\tanh(c+}{bd}$$

$$= \frac{(e+fx)^2}{bd} - \frac{ab(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} + \frac{ab(e+fx)^2}{}$$

$$= \frac{(e+fx)^2}{bd} - \frac{a^2(e+fx)^2}{b\,(a^2+b^2)\,d} + \frac{4af(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2} - \frac{ab(e}{}$$

$$= \frac{(e+fx)^2}{bd} - \frac{a^2(e+fx)^2}{b\,(a^2+b^2)\,d} + \frac{4af(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2} - \frac{ab(e}{}$$

$$= \frac{(e+fx)^2}{bd} - \frac{a^2(e+fx)^2}{b\,(a^2+b^2)\,d} + \frac{4af(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2} - \frac{ab(e}{}$$

$$= \frac{(e+fx)^2}{bd} - \frac{a^2(e+fx)^2}{b\,(a^2+b^2)\,d} + \frac{4af(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d^2} - \frac{ab(e}{}$$

Mathematica [A]
time = 9.20, size = 1182, normalized size = 1.82

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] -((a*b*((2*d^2*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-a^2
- b^2] + (2*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^
```

```
2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] + (d^2*E^c*f^2*x^2*Log[1 + (b*E^(2
*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)]
- (2*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2
*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] - (d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*
x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] + (2*d*
E^c*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^
(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*d*E^c*f*(e + f*x)*PolyLog[2, -((b
*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(
2*c)] - (2*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)
*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] + (2*E^c*f^2*PolyLog[3, -((b*E^(2*c
 + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)]))
/((a^2 + b^2)*d^3)) - (2*b*e*f*Sech[c]*(Cosh[c]*Log[Cosh[c]*Cosh[d*x] + Sin
h[c]*Sinh[d*x]] - d*x*Sinh[c]))/((a^2 + b^2)*d^2*(Cosh[c]^2 - Sinh[c]^2)) +
 (4*a*e*f*ArcTan[(Sinh[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]
^2]])/((a^2 + b^2)*d^2*Sqrt[Cosh[c]^2 - Sinh[c]^2]) - (b*f^2*Csch[c]*((d^2*
x^2)/E^ArcTanh[Coth[c]] - (I*Coth[c]*(-(d*x*(-Pi + (2*I)*ArcTanh[Coth[c]]))
 - Pi*Log[1 + E^(2*d*x)] - 2*(I*d*x + I*ArcTanh[Coth[c]])*Log[1 - E^((2*I)*
(I*d*x + I*ArcTanh[Coth[c]]))] + Pi*Log[Cosh[d*x]] + (2*I)*ArcTanh[Coth[c]]
*Log[I*Sinh[d*x + ArcTanh[Coth[c]]]] + I*PolyLog[2, E^((2*I)*(I*d*x + I*Arc
Tanh[Coth[c]]))]))/Sqrt[1 - Coth[c]^2])*Sech[c])/((a^2 + b^2)*d^3*Sqrt[Csch
[c]^2*(-Cosh[c]^2 + Sinh[c]^2)]) + (2*a*f^2*(((-I)*Csch[c]*(I*(d*x + ArcTan
h[Coth[c]])*(Log[1 - E^(-(d*x) - ArcTanh[Coth[c]])] - Log[1 + E^(-(d*x) - A
rcTanh[Coth[c]])]) + I*(PolyLog[2, -E^(-(d*x) - ArcTanh[Coth[c]])] - PolyLo
g[2, E^(-(d*x) - ArcTanh[Coth[c]])])))/Sqrt[1 - Coth[c]^2] - (2*ArcTan[(Sin
h[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]]*ArcTanh[Coth[c]]
)/Sqrt[Cosh[c]^2 - Sinh[c]^2]))/((a^2 + b^2)*d^3) + (Sech[c]*Sech[c + d*x]*
(-(a*e^2*Cosh[c]) - 2*a*e*f*x*Cosh[c] - a*f^2*x^2*Cosh[c] + b*e^2*Sinh[d*x]
 + 2*b*e*f*x*Sinh[d*x] + b*f^2*x^2*Sinh[d*x]))/((a^2 + b^2)*d)
```

Maple [F]
time = 2.52, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c) \tanh(dx+c)}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] 2*b*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 + b^2
)*d^2))*e + 4*a*f^2*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*
e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + 4*b*f^2*integrate(x/(a^2*d*e^(2*d*x
+ 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - (a*b*log((b*e^(-d*x -
 c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2 +
b^2)^(3/2)*d) + 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x
- 2*c))*d))*e^2 + 4*a*f*arctan(e^(d*x + c))*e/((a^2 + b^2)*d^2) - 2*(b*f^2*
x^2 + 2*b*f*x*e + (a*f^2*x^2*e^c + 2*a*f*x*e^(c + 1))*e^(d*x))/(a^2*d + b^2
*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) - integrate(-2*(a*b*f^2*x^2
*e^c + 2*a*b*f*x*e^(c + 1))*e^(d*x)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e^(
2*c))*e^(2*d*x) - 2*(a^3*e^c + a*b^2*e^c)*e^(d*x)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4750 vs. $2(613) = 1226$.
time = 0.46, size = 4750, normalized size = 7.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -(2*(a^2*b + b^3)*c^2*f^2 - 4*(a^2*b + b^3)*c*d*f*cosh(1) + 2*(a^2*b + b^3)
*d^2*cosh(1)^2 + 2*(a^2*b + b^3)*d^2*sinh(1)^2 - 2*((a^2*b + b^3)*d^2*f^2*x
^2 - (a^2*b + b^3)*c^2*f^2 + 2*((a^2*b + b^3)*d^2*f*x + (a^2*b + b^3)*c*d*f
)*cosh(1) + 2*((a^2*b + b^3)*d^2*f*x + (a^2*b + b^3)*c*d*f)*sinh(1))*cosh(d
*x + c)^2 - 2*((a^2*b + b^3)*d^2*f^2*x^2 - (a^2*b + b^3)*c^2*f^2 + 2*((a^2*
b + b^3)*d^2*f*x + (a^2*b + b^3)*c*d*f)*cosh(1) + 2*((a^2*b + b^3)*d^2*f*x
+ (a^2*b + b^3)*c*d*f)*sinh(1))*sinh(d*x + c)^2 + 2*(a*b^2*d*f^2*x + a*b^2*
d*f*cosh(1) + a*b^2*d*f*sinh(1) + (a*b^2*d*f^2*x + a*b^2*d*f*cosh(1) + a*b^
2*d*f*sinh(1))*cosh(d*x + c)^2 + 2*(a*b^2*d*f^2*x + a*b^2*d*f*cosh(1) + a*b
^2*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d*f^2*x + a*b^2*d*f*co
sh(1) + a*b^2*d*f*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*
cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2) - b)/b + 1) - 2*(a*b^2*d*f^2*x + a*b^2*d*f*cosh(1) + a*b^2
*d*f*sinh(1) + (a*b^2*d*f^2*x + a*b^2*d*f*cosh(1) + a*b^2*d*f*sinh(1))*cosh
(d*x + c)^2 + 2*(a*b^2*d*f^2*x + a*b^2*d*f*cosh(1) + a*b^2*d*f*sinh(1))*cos
h(d*x + c)*sinh(d*x + c) + (a*b^2*d*f^2*x + a*b^2*d*f*cosh(1) + a*b^2*d*f*s
inh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*s
inh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
b)/b + 1) - (a*b^2*c^2*f^2 - 2*a*b^2*c*d*f*cosh(1) + a*b^2*d^2*cosh(1)^2 +
a*b^2*d^2*sinh(1)^2 + (a*b^2*c^2*f^2 - 2*a*b^2*c*d*f*cosh(1) + a*b^2*d^2*co
sh(1)^2 + a*b^2*d^2*sinh(1)^2 - 2*(a*b^2*c*d*f - a*b^2*d^2*cosh(1))*sinh(1)
```

)*cosh(d*x + c)^2 + 2*(a*b^2*c^2*f^2 - 2*a*b^2*c*d*f*cosh(1) + a*b^2*d^2*co
sh(1)^2 + a*b^2*d^2*sinh(1)^2 - 2*(a*b^2*c*d*f - a*b^2*d^2*cosh(1))*sinh(1)
)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*c^2*f^2 - 2*a*b^2*c*d*f*cosh(1) + a*
b^2*d^2*cosh(1)^2 + a*b^2*d^2*sinh(1)^2 - 2*(a*b^2*c*d*f - a*b^2*d^2*cosh(1
))*sinh(1))*sinh(d*x + c)^2 - 2*(a*b^2*c*d*f - a*b^2*d^2*cosh(1))*sinh(1))*
sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt(
(a^2 + b^2)/b^2) + 2*a) + (a*b^2*c^2*f^2 - 2*a*b^2*c*d*f*cosh(1) + a*b^2*d^
2*cosh(1)^2 + a*b^2*d^2*sinh(1)^2 + (a*b^2*c^2*f^2 - 2*a*b^2*c*d*f*cosh(1)
+ a*b^2*d^2*cosh(1)^2 + a*b^2*d^2*sinh(1)^2 - 2*(a*b^2*c*d*f - a*b^2*d^2*co
sh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(a*b^2*c^2*f^2 - 2*a*b^2*c*d*f*cosh(1)
+ a*b^2*d^2*cosh(1)^2 + a*b^2*d^2*sinh(1)^2 - 2*(a*b^2*c*d*f - a*b^2*d^2*co
sh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*c^2*f^2 - 2*a*b^2*c*d*
f*cosh(1) + a*b^2*d^2*cosh(1)^2 + a*b^2*d^2*sinh(1)^2 - 2*(a*b^2*c*d*f - a*
b^2*d^2*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 2*(a*b^2*c*d*f - a*b^2*d^2*cosh
(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (a*b^2*d^2*f^2*x^2 - a*b^2*c^2*f^2
+ (a*b^2*d^2*f^2*x^2 - a*b^2*c^2*f^2 + 2*(a*b^2*d^2*f*x + a*b^2*c*d*f)*cosh
(1) + 2*(a*b^2*d^2*f*x + a*b^2*c*d*f)*sinh(1))*cosh(d*x + c)^2 + 2*(a*b^2*d
^2*f^2*x^2 - a*b^2*c^2*f^2 + 2*(a*b^2*d^2*f*x + a*b^2*c*d*f)*cosh(1) + 2*(a
*b^2*d^2*f*x + a*b^2*c*d*f)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d
^2*f^2*x^2 - a*b^2*c^2*f^2 + 2*(a*b^2*d^2*f*x + a*b^2*c*d*f)*cosh(1) + 2*(a
*b^2*d^2*f*x + a*b^2*c*d*f)*sinh(1))*sinh(d*x + c)^2 + 2*(a*b^2*d^2*f*x + a
*b^2*c*d*f)*cosh(1) + 2*(a*b^2*d^2*f*x + a*b^2*c*d*f)*sinh(1))*sqrt((a^2 +
b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (a*b^2*d^2*f^2*x^2 - a*b^2*c^2
*f^2 + (a*b^2*d^2*f^2*x^2 - a*b^2*c^2*f^2 + 2*(a*b^2*d^2*f*x + a*b^2*c*d*f)
cosh(1) + 2(a*b^2*d^2*f*x + a*b^2*c*d*f)*sinh(1))*cosh(d*x + c)^2 + 2*(a*
b^2*d^2*f^2*x^2 - a*b^2*c^2*f^2 + 2*(a*b^2*d^2*f*x + a*b^2*c*d*f)*cosh(1) +
 2*(a*b^2*d^2*f*x + a*b^2*c*d*f)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a*
b^2*d^2*f^2*x^2 - a*b^2*c^2*f^2 + 2*(a*b^2*d^2*f*x + a*b^2*c*d*f)*cosh(1) +
 2*(a*b^2*d^2*f*x + a*b^2*c*d*f)*sinh(1))*sinh(d*x + c)^2 + 2*(a*b^2*d^2*f*
x + a*b^2*c*d*f)*cosh(1) + 2*(a*b^2*d^2*f*x + a*b^2*c*d*f)*sinh(1))*sqrt((a
^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(a*b^2*f^2*cosh(d*x + c
)^2 + 2*a*b^2*f^2*cosh(d*x + c)*sinh(d*x + c) + a*b^2*f^2*sinh(d*x + c)^2 +
 a*b^2*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x
+ c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*(a
*b^2*f^2*cosh(d*x + c)^2 + 2*a*b^2*f^2*cosh(d*x + c)*sinh(d*x + c) + a*b^2*
f^2*sinh(d*x + c)^2 + a*b^2*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d
*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2))/b) + 2*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*f*x*cos
h(1) + (a^3 + a*b^2)*d^2*cosh(1)^2 + (a^3 + a*b^2)*d^2*sinh(1)^2 + 2*((a^3
+ a*b^2)*d^2*f*x + (a^3 + a*b^2)*d^2*cosh(1))*sinh(1))*cosh(d*x + c) + 2*(-
I*(a^3 + a*b^2)*f^2 + (a^2*b + b^3)*f^2 + (-I*(a^3 + a*b^2)*f^2 + (a^2*b +
b^3)*f^2)*cosh(d*x + c)^2 + 2*(-I*(a^3 + a*b^2)*f^2 + (a^2*b + b^3)*f^2)*co

```
sh(d*x + c)*sinh(d*x + c) + (-I*(a^3 + a*b^2)*f...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2 \tanh(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)**2*tanh(c + d*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(c+d\,x)\,\left(e+f\,x\right)^2}{\cosh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(c + d*x)*(e + f*x)^2)/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
[Out] int((tanh(c + d*x)*(e + f*x)^2)/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.355 $\int \frac{(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=335

$$\frac{af\operatorname{ArcTan}(\sinh(c+dx))}{(a^2+b^2)\,d^2}-\frac{ab(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}+\frac{ab(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}-\frac{f\log(}{}$$

```
[Out] a*f*arctan(sinh(d*x+c))/(a^2+b^2)/d^2-f*ln(cosh(d*x+c))/b/d^2+a^2*f*ln(cosh
(d*x+c))/b/(a^2+b^2)/d^2-a*b*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))
/(a^2+b^2)^(3/2)/d+a*b*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+
b^2)^(3/2)/d-a*b*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(
3/2)/d^2+a*b*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)
/d^2-a*(f*x+e)*sech(d*x+c)/(a^2+b^2)/d+(f*x+e)*tanh(d*x+c)/b/d-a^2*(f*x+e)*
tanh(d*x+c)/b/(a^2+b^2)/d
```

Rubi [A]
time = 0.54, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5702, 4269, 3556, 5692, 3403, 2296, 2221, 2317, 2438, 6874, 5559, 3855}

$$\frac{af\operatorname{ArcTan}(\sinh(c+dx))}{d^2(a^2+b^2)}-\frac{abf\operatorname{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2(a^2+b^2)^{3/2}}+\frac{abf\operatorname{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2(a^2+b^2)^{3/2}}+\frac{a^2f\log(\cosh(c+dx))}{bd^2(a^2+b^2)}-\frac{ab(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{d(a^2+b^2)^{3/2}}+\frac{ab(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{d(a^2+b^2)^{3/2}}-\frac{a^2(e+fx)\tanh(c+dx)}{bd(a^2+b^2)}-\frac{a(e+fx)\operatorname{sech}(c+dx)}{d(a^2+b^2)}-\frac{f\log(\cosh(c+dx))}{bd^2}+\frac{(e+fx)\tanh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*f*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d^2) - (a*b*(e + f*x)*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) + (a*b*(e + f*x)*
Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (f*
Log[Cosh[c + d*x]])/(b*d^2) + (a^2*f*Log[Cosh[c + d*x]])/(b*(a^2 + b^2)*d^2
) - (a*b*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^
2)^(3/2)*d^2) + (a*b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/((a^2 + b^2)^(3/2)*d^2) - (a*(e + f*x)*Sech[c + d*x])/((a^2 + b^2)*d) + (
(e + f*x)*Tanh[c + d*x])/(b*d) - (a^2*(e + f*x)*Tanh[c + d*x])/(b*(a^2 + b^
2)*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5702

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{sech}^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)\tanh(c+dx)}{bd} - \frac{a\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+\ldots}{b\,(a^2+b^2)}$$

$$= -\frac{f\log(\cosh(c+dx))}{bd^2} + \frac{(e+fx)\tanh(c+dx)}{bd} - \frac{a\int \big(a(e+fx)\ldots}{}$$

$$= -\frac{f\log(\cosh(c+dx))}{bd^2} + \frac{(e+fx)\tanh(c+dx)}{bd} - \frac{(2ab^2)\int \frac{}{2a-2\sqrt{\ldots}}}{(a^2\ldots}$$

$$= -\frac{ab(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} + \frac{ab(e+fx)\log\left(1+\frac{}{a+\sqrt{\ldots}}\right.}{(a^2+b^2)^{3/2}\,d}$$

$$= \frac{af\tan^{-1}(\sinh(c+dx))}{(a^2+b^2)\,d^2} - \frac{ab(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} + \ldots$$

$$= \frac{af\tan^{-1}(\sinh(c+dx))}{(a^2+b^2)\,d^2} - \frac{ab(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} + \ldots$$

Mathematica [A]
time = 2.04, size = 285, normalized size = 0.85

$$\frac{\frac{2af\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a^2+b^2} - \frac{bf\log(\cosh(c+dx))}{a^2+b^2} + \frac{ab\left(2de\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) - 2cf\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) - f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) + f(c+dx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right) - f\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right) + f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)}{(a^2+b^2)^{3/2}} + \frac{d(e+fx)\text{sech}(c+dx)(-a+b\sinh(c+dx))}{a^2+b^2}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((2*a*f*ArcTan[Tanh[(c + d*x)/2]])/(a^2 + b^2) - (b*f*Log[Cosh[c + d*x]])/(
a^2 + b^2) + (a*b*(2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*c
*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - f*(c + d*x)*Log[1 + (b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])] - f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
+ f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^2 + b^2)^(3/2
) + (d*(e + f*x)*Sech[c + d*x]*(-a + b*Sinh[c + d*x]))/(a^2 + b^2))/d^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1857 vs. $2(315) = 630$.
time = 3.20, size = 1858, normalized size = 5.55

method	result	size
risch	Expression too large to display	1858

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBO
SE)
```

```
[Out] 2/(a^2+b^2)/d^2*b*f*ln(exp(d*x+c))-1/2/(a^2+b^2)^2/d^2*f*b^3*ln(b*exp(2*d*x
+2*c)+2*a*exp(d*x+c)-b)-2/d^2/(a^2+b^2)^(1/2)*a*b*f*c/(2*a^2+2*b^2)*arctanh
(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2*(f*x+e)*(a*exp(d*x+c)+b)/d/(a^
2+b^2)/(1+exp(2*d*x+2*c))-1/(a^2+b^2)^2/d^2*f*b*ln(b*exp(2*d*x+2*c)+2*a*exp
(d*x+c)-b)*a^2-2/(a^2+b^2)/d^2*f*b^3/(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))+1/(
a^2+b^2)/d^2*f*b^3/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+4/(a
^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*arctan(exp(d*x+c))+2/d/(a^2+b^2)^(1/2)*e*a*
b/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^
2)^(5/2)/d^2*f*b*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^3+2/(a
^2+b^2)^(5/2)/d^2*f*b^3*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a
-2/(a^2+b^2)/d^2*a^2*b*f/(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))+2/(a^2+b^2)^(3/
2)/d^2*a*b^3*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^
2)^(1/2)))*c-2/(a^2+b^2)^(3/2)/d*a^3*b*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a
^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-2/(a^2+b^2)^(3/2)/d*a*b^3*f/(2*a^2
+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-2/(a^2
+b^2)^(1/2)/d^2*a*b*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b
^2)^(1/2))-2/(a^2+b^2)^(3/2)/d^2*a^3*b*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)
+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2/(a^2+b^2)^(3/2)/d^2*a*b^3*f/(2*
a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+2/
(a^2+b^2)^(3/2)/d^2*a^3*b*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/
2)+a)/(a+(a^2+b^2)^(1/2)))+2/(a^2+b^2)^(3/2)/d^2*a*b^3*f/(2*a^2+2*b^2)*dilo
g((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2/(a^2+b^2)^(3/2)/d
^2*a*b^3*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-
2/(a^2+b^2)^(3/2)/d^2*a^3*b*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a
)/(a^2+b^2)^(1/2))+2/d/(a^2+b^2)^(3/2)*e*a*b^3/(2*a^2+2*b^2)*arctanh(1/2*(2
*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d/(a^2+b^2)^(3/2)*e*a^3*b/(2*a^2+2*b^
2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/(a^2+b^2)^(3/2)/d^2*
a*b^3*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1
/2)))*c+2/(a^2+b^2)^(3/2)/d^2*a^3*b*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b
^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/(a^2+b^2)/d^2*a^2*b*f/(2*a^2+2*b^2)*l
n(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+4/(a^2+b^2)/d^2*f*b^2/(2*a^2+2*b^2)*a*
arctan(exp(d*x+c))-2/d^2/(a^2+b^2)^(3/2)*a^3*b*f*c/(2*a^2+2*b^2)*arctanh(1/
2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d^2/(a^2+b^2)^(3/2)*a*b^3*f*c/(2*
a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)^(3
/2)/d*a^3*b*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2
)^(1/2)))*x+2/(a^2+b^2)^(3/2)/d*a*b^3*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2
```

```
+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-2/(a^2+b^2)^(3/2)/d^2*a^3*b*f/(2*a^2+
2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] -(2*a*b*integrate(-x*e^(d*x + c)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e^(2*c
))*e^(2*d*x) - 2*(a^3*e^c + a*b^2*e^c)*e^(d*x)), x) + 2*(a*x*e^(d*x + c) +
b*x)/(a^2*d + b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) - 2*b*x/((
a^2 + b^2)*d) - 2*a*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) + b*log(e^(2*d*x
+ 2*c) + 1)/((a^2 + b^2)*d^2))*f - (a*b*log((b*e^(-d*x - c) - a - sqrt(a^2
+ b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2)*d) + 2*(
a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1462 vs. $2(318) = 636$.
time = 0.41, size = 1462, normalized size = 4.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] (2*(a^2*b + b^3)*d*f*x*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*d*f*x*sinh(d*x + c
)^2 - 2*(a^2*b + b^3)*d*cosh(1) - 2*(a^2*b + b^3)*d*sinh(1) - (a*b^2*f*cosh
(d*x + c)^2 + 2*a*b^2*f*cosh(d*x + c)*sinh(d*x + c) + a*b^2*f*sinh(d*x + c)
^2 + a*b^2*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c
) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) +
 (a*b^2*f*cosh(d*x + c)^2 + 2*a*b^2*f*cosh(d*x + c)*sinh(d*x + c) + a*b^2*f
*sinh(d*x + c)^2 + a*b^2*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) +
a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
 - b)/b + 1) - (a*b^2*c*f - a*b^2*d*cosh(1) - a*b^2*d*sinh(1) + (a*b^2*c*f
- a*b^2*d*cosh(1) - a*b^2*d*sinh(1))*cosh(d*x + c)^2 + 2*(a*b^2*c*f - a*b^2
*d*cosh(1) - a*b^2*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*c*f - a*
b^2*d*cosh(1) - a*b^2*d*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log
(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) +
 (a*b^2*c*f - a*b^2*d*cosh(1) - a*b^2*d*sinh(1) + (a*b^2*c*f - a*b^2*d*cosh
(1) - a*b^2*d*sinh(1))*cosh(d*x + c)^2 + 2*(a*b^2*c*f - a*b^2*d*cosh(1) - a
*b^2*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*c*f - a*b^2*d*cosh(1)
```

```
- a*b^2*d*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x
+ c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (a*b^2*d*f*x
+ a*b^2*c*f + (a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*f*x +
a*b^2*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d*f*x + a*b^2*c*f)*sinh(d*x
 + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (a*b^2*d*
f*x + a*b^2*c*f + (a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*f*
x + a*b^2*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d*f*x + a*b^2*c*f)*sinh
(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
- (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*((a
^3 + a*b^2)*f*cosh(d*x + c)^2 + 2*(a^3 + a*b^2)*f*cosh(d*x + c)*sinh(d*x +
c) + (a^3 + a*b^2)*f*sinh(d*x + c)^2 + (a^3 + a*b^2)*f)*arctan(cosh(d*x + c
) + sinh(d*x + c)) - 2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*cosh(1) + (a^
3 + a*b^2)*d*sinh(1))*cosh(d*x + c) - ((a^2*b + b^3)*f*cosh(d*x + c)^2 + 2*
(a^2*b + b^3)*f*cosh(d*x + c)*sinh(d*x + c) + (a^2*b + b^3)*f*sinh(d*x + c)
^2 + (a^2*b + b^3)*f)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)))
+ 2*(2*(a^2*b + b^3)*d*f*x*cosh(d*x + c) - (a^3 + a*b^2)*d*f*x - (a^3 + a*b
^2)*d*cosh(1) - (a^3 + a*b^2)*d*sinh(1))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 +
 b^4)*d^2*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*cosh(d*x + c)*sin
h(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*d^2*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2
 + b^4)*d^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\tanh(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*tanh(c + d*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(c+d\,x)\,\left(e+f\,x\right)}{\cosh\left(c+d\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(c + d*x)*(e + f*x))/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
[Out] int((tanh(c + d*x)*(e + f*x))/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.356 $\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=78

$$\frac{2ab\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}-\frac{\operatorname{sech}(c+dx)(a-b\sinh(c+dx))}{(a^2+b^2)\,d}$$

[Out] 2*a*b*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d-sech(d*x+c)*(a-b*sinh(d*x+c))/(a^2+b^2)/d

Rubi [A]
time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2945, 12, 2739, 632, 210}

$$\frac{2ab\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\,(a^2+b^2)^{3/2}}-\frac{\operatorname{sech}(c+dx)(a-b\sinh(c+dx))}{d\,(a^2+b^2)}$$

Antiderivative was successfully verified.

[In] Int[(Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]

[Out] (2*a*b*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*d) - (Sech[c + d*x]*(a - b*Sinh[c + d*x]))/((a^2 + b^2)*d)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{\operatorname{sech}(c+dx)(a-b\sinh(c+dx))}{(a^2+b^2)\,d} - \frac{\int \frac{ab}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}\\
&= -\frac{\operatorname{sech}(c+dx)(a-b\sinh(c+dx))}{(a^2+b^2)\,d} - \frac{(ab)\int \frac{1}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}\\
&= -\frac{\operatorname{sech}(c+dx)(a-b\sinh(c+dx))}{(a^2+b^2)\,d} + \frac{(2iab)\text{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx, x, \tan\right.}{(a^2+b^2)\,d}\\
&= -\frac{\operatorname{sech}(c+dx)(a-b\sinh(c+dx))}{(a^2+b^2)\,d} - \frac{(4iab)\text{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx, x, -\right.}{(a^2+b^2)}\\
&= \frac{2ab\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} - \frac{\operatorname{sech}(c+dx)(a-b\sinh(c+dx))}{(a^2+b^2)\,d}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 104, normalized size = 1.33

$$-\frac{-2ab\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right) - a\sqrt{-a^2-b^2}\operatorname{sech}(c+dx) + b\sqrt{-a^2-b^2}\tanh(c+dx)}{(-a^2-b^2)^{3/2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((-2*a*b*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] - a*Sqrt[-a^2
- b^2]*Sech[c + d*x] + b*Sqrt[-a^2 - b^2]*Tanh[c + d*x])/((-a^2 - b^2)^(3/2
)*d))
```

Maple [A]
time = 1.19, size = 101, normalized size = 1.29

method	result
derivativedivides	$\frac{-\frac{4ab \,\mathrm{arctanh}\left(\frac{2a \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(2a^2+2b^2\right)\sqrt{a^2+b^2}}+\frac{2b \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2a}{\left(a^2+b^2\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d}$
default	$\frac{-\frac{4ab \,\mathrm{arctanh}\left(\frac{2a \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(2a^2+2b^2\right)\sqrt{a^2+b^2}}+\frac{2b \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2a}{\left(a^2+b^2\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d}$
risch	$-\frac{2\left(a\,\mathrm{e}^{dx+c}+b\right)}{d\left(a^2+b^2\right)\left(1+\mathrm{e}^{2dx+2c}\right)}+\frac{ba\ln\left(\mathrm{e}^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}d}-\frac{ba\ln\left(\mathrm{e}^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-4*a*b/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2
*c)-2*b)/(a^2+b^2)^(1/2))+2/(a^2+b^2)*(b*tanh(1/2*d*x+1/2*c)-a)/(tanh(1/2*d
*x+1/2*c)^2+1))
```

Maxima [A]
time = 0.48, size = 117, normalized size = 1.50

$$-\frac{ab\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}d}-\frac{2\left(ae^{(-dx-c)}-b\right)}{\left(a^2+b^2+\left(a^2+b^2\right)e^{(-2\,dx-2\,c)}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -a*b*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(
a^2 + b^2)))/((a^2 + b^2)^(3/2)*d) - 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 + (
a^2 + b^2)*e^(-2*d*x - 2*c))*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(75) = 150.
time = 0.37, size = 350, normalized size = 4.49

$$-\frac{2\,a^2b+2\,b^3-\left(ab\cosh\left(dx+c\right)^2+2\,ab\cosh\left(dx+c\right)\sinh\left(dx+c\right)+ab\sinh\left(dx+c\right)^2+ab\right)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2+b^2+2\,\left(b^2\cosh(dx+c)+ab\right)\sinh(dx+c)+2\,\sqrt{a^2+b^2}\,\left(b\cosh(dx+c)+b\sinh(dx+c)+a\right)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,\left(b\cosh(dx+c)+a\right)\sinh(dx+c)-b}\right)+2\,\left(a^3+ab^2\right)\cosh\left(dx+c\right)+2\,\left(a^3+ab^2\right)\sinh\left(dx+c\right)}{\left(a^4+2\,a^2b^2+b^4\right)d\cosh\left(dx+c\right)^2+2\,\left(a^4+2\,a^2b^2+b^4\right)d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\left(a^4+2\,a^2b^2+b^4\right)d\sinh\left(dx+c\right)^2+\left(a^4+2\,a^2b^2+b^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*a^2*b + 2*b^3 - (a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c
) + a*b*sinh(d*x + c)^2 + a*b)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b
^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x +
c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x +
 c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b
*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*(a^3 + a*b^2)*cosh(d*x + c) + 2
*(a^3 + a*b^2)*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 +
2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2
+ b^4)*d*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A]
time = 0.46, size = 106, normalized size = 1.36

$$-\frac{\frac{ab\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{2\,(ae^{(dx+c)}+b)}{(a^2+b^2)(e^{(2\,dx+2\,c)}+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -(a*b*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c
) + 2*a + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(a*e^(d*x + c) + b)/((a
^2 + b^2)*(e^(2*d*x + 2*c) + 1)))/d
```

Mupad [B]
time = 0.60, size = 170, normalized size = 2.18

$$\frac{a\,b\,\ln\left(\frac{2\,a\,e^{c+d\,x}}{a^2+b^2}+\frac{2\,a\,(b-a\,e^{c+d\,x})}{(a^2+b^2)^{3/2}}\right)}{d\,(a^2+b^2)^{3/2}}-\frac{a\,b\,\ln\left(\frac{2\,a\,e^{c+d\,x}}{a^2+b^2}-\frac{2\,a\,(b-a\,e^{c+d\,x})}{(a^2+b^2)^{3/2}}\right)}{d\,(a^2+b^2)^{3/2}}-\frac{\frac{2\,b}{d\,(a^2+b^2)}+\frac{2\,a\,e^{c+d\,x}}{d\,(a^2+b^2)}}{e^{2\,c+2\,d\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] (a*b*log((2*a*exp(c + d*x))/(a^2 + b^2) + (2*a*(b - a*exp(c + d*x)))/(a^2 +
 b^2)^(3/2)))/(d*(a^2 + b^2)^(3/2)) - (a*b*log((2*a*exp(c + d*x))/(a^2 + b^
2) - (2*a*(b - a*exp(c + d*x)))/(a^2 + b^2)^(3/2)))/(d*(a^2 + b^2)^(3/2)) -
 ((2*b)/(d*(a^2 + b^2)) + (2*a*exp(c + d*x))/(d*(a^2 + b^2)))/(exp(2*c + 2*
d*x) + 1)
```

3.357 $\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=35

$$\operatorname{Int}\left(\frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sech[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][(Sech[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 46.13, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sech[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x
]
[Out] Integrate[(Sech[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] -2*a*b*integrate(-e^(d*x + c)/((a^2*b*f + b^3*f)*x + (a^2*b + b^3)*e - ((a^
2*b*f*e^(2*c) + b^3*f*e^(2*c))*x + (a^2*b*e^(2*c) + b^3*e^(2*c))*e)*e^(2*d*
x) - 2*((a^3*f*e^c + a*b^2*f*e^c)*x + (a^3*e^c + a*b^2*e^c)*e)*e^(d*x)), x)
 - 2*(a*e^(d*x + c) + b)/((a^2*d*f + b^2*d*f)*x + (a^2*d + b^2*d)*e + ((a^2
*d*f*e^(2*c) + b^2*d*f*e^(2*c))*x + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e)*e^(2
*d*x)) - 2*integrate((a*f*e^(d*x + c) + b*f)/((a^2*d*f^2 + b^2*d*f^2)*x^2 +
 2*(a^2*d*f + b^2*d*f)*x*e + (a^2*d + b^2*d)*e^2 + ((a^2*d*f^2*e^(2*c) + b^
2*d*f^2*e^(2*c))*x^2 + 2*(a^2*d*f*e^(2*c) + b^2*d*f*e^(2*c))*x*e + (a^2*d*e
^(2*c) + b^2*d*e^(2*c))*e^2)*e^(2*d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] integral(sech(d*x + c)*tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
+ c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(c + dx)\operatorname{sech}(c + dx)}{(a + b\sinh(c + dx))(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)*sech(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tanh(c+d\,x)}{\cosh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)/(cosh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(tanh(c + d*x)/(cosh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.358 $\int \frac{(e+fx)^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1176

$$\frac{(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{bd}-\frac{2a^2b(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2d}-\frac{a^2(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d}-\frac{f^2\text{ArcTan}(\sinh(c+}{bd^3}$$

[Out] -2*a^2*b*(f*x+e)^2*arctan(exp(d*x+c))/(a^2+b^2)^2/d+a^2*f^2*arctan(sinh(d*x
+c))/b/(a^2+b^2)/d^3+I*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b/d^2-1/2*a*b^2*f^
2*polylog(3,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^3+a*f*(f*x+e)*tanh(d*x+c)/(a^2+b
^2)/d^2-a^2*(f*x+e)^2*arctan(exp(d*x+c))/b/(a^2+b^2)/d+(f*x+e)^2*arctan(exp
(d*x+c))/b/d-f^2*arctan(sinh(d*x+c))/b/d^3-a*f^2*ln(cosh(d*x+c))/(a^2+b^2)/
d^3+2*I*a^2*b*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/(a^2+b^2)^2/d^2-2*a*b^2*f*
(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2-2*a*b^
2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2+a*
b^2*f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^2+I*a^2*f^2*polylog(
3,I*exp(d*x+c))/b/(a^2+b^2)/d^3-a^2*f*(f*x+e)*sech(d*x+c)/b/(a^2+b^2)/d^2-1
/2*a^2*(f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/b/(a^2+b^2)/d-2*I*a^2*b*f^2*polylo
g(3,-I*exp(d*x+c))/(a^2+b^2)^2/d^3-I*a^2*f^2*polylog(3,-I*exp(d*x+c))/b/(a^
2+b^2)/d^3+a*b^2*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/(a^2+b^2)^2/d-a*b^2*(f*x+e)
^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d-a*b^2*(f*x+e)^2*ln(
1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d+2*a*b^2*f^2*polylog(3,-b*
exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^3+2*a*b^2*f^2*polylog(3,-b*ex
p(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^3+I*f^2*polylog(3,-I*exp(d*x+c)
)/b/d^3+f*(f*x+e)*sech(d*x+c)/b/d^2-1/2*a*(f*x+e)^2*sech(d*x+c)^2/(a^2+b^2)
/d+1/2*(f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/b/d-I*f^2*polylog(3,I*exp(d*x+c))/
b/d^3-I*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/b/d^2+I*a^2*f*(f*x+e)*polylog(2,
-I*exp(d*x+c))/b/(a^2+b^2)/d^2+2*I*a^2*b*f^2*polylog(3,I*exp(d*x+c))/(a^2+b
^2)^2/d^3-2*I*a^2*b*f*(f*x+e)*polylog(2,I*exp(d*x+c))/(a^2+b^2)^2/d^2-I*a^2
f(f*x+e)*polylog(2,I*exp(d*x+c))/b/(a^2+b^2)/d^2

Rubi [A]
time = 1.42, antiderivative size = 1176, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 15, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.441$, Rules used = {5702, 4271, 3855, 4265, 2611, 2320, 6724, 5692, 5680, 2221, 6874, 3799, 5559, 4269, 3556}

Antiderivative was successfully verified.

[In] Int[((e + f*x)^2*Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]

[Out] ((e + f*x)^2*ArcTan[E^(c + d*x)])/(b*d) - (2*a^2*b*(e + f*x)^2*ArcTan[E^(c
+ d*x)])/((a^2 + b^2)^2*d) - (a^2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b*(a^2

```
+ b^2)*d) - (f^2*ArcTan[Sinh[c + d*x]])/(b*d^3) + (a^2*f^2*ArcTan[Sinh[c +
d*x]])/(b*(a^2 + b^2)*d^3) - (a*b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) - (a*b^2*(e + f*x)^2*Log[1 + (b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) + (a*b^2*(e + f*x)^2*Log
[1 + E^(2*(c + d*x))])/((a^2 + b^2)^2*d) - (a*f^2*Log[Cosh[c + d*x]])/((a^2
 + b^2)*d^3) - (I*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + ((2*I
)*a^2*b*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) + (I*
a^2*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) + (I*f*(e
 + f*x)*PolyLog[2, I*E^(c + d*x)])/(b*d^2) - ((2*I)*a^2*b*f*(e + f*x)*PolyL
og[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - (I*a^2*f*(e + f*x)*PolyLog[2, I
*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) - (2*a*b^2*f*(e + f*x)*PolyLog[2, -((b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) - (2*a*b^2*f*(e +
f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d
^2) + (a*b^2*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^2)^2*d^2)
+ (I*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b*d^3) - ((2*I)*a^2*b*f^2*PolyLog[3
, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^3) - (I*a^2*f^2*PolyLog[3, (-I)*E^(c
+ d*x)])/(b*(a^2 + b^2)*d^3) - (I*f^2*PolyLog[3, I*E^(c + d*x)])/(b*d^3) +
((2*I)*a^2*b*f^2*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)^2*d^3) + (I*a^2*f^
2*PolyLog[3, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^3) + (2*a*b^2*f^2*PolyLog[3,
-((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) + (2*a*b^2*f
^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3
) - (a*b^2*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^3) + (f*(e
+ f*x)*Sech[c + d*x])/(b*d^2) - (a^2*f*(e + f*x)*Sech[c + d*x])/(b*(a^2 + b
^2)*d^2) - (a*(e + f*x)^2*Sech[c + d*x]^2)/(2*(a^2 + b^2)*d) + (a*f*(e + f*
x)*Tanh[c + d*x])/((a^2 + b^2)*d^2) + ((e + f*x)^2*Sech[c + d*x]*Tanh[c + d
*x])/(2*b*d) - (a^2*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*b*(a^2 + b^
2)*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
```

```
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5702

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{sech}^3(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{f(e+fx)\text{sech}(c+dx)}{bd^2} + \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{2bd}$$

$$= \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{f^2\tan^{-1}(\sinh(c+dx))}{bd^3} + \frac{f(e+f}{}$$

$$= \frac{ab^2(e+fx)^3}{3\left(a^2+b^2\right)^2 f} + \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{f^2\tan^{-1}(\sinh(c+}{bd^3}$$

$$= \frac{ab^2(e+fx)^3}{3\left(a^2+b^2\right)^2 f} + \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{f^2\tan^{-1}(\sinh(c+}{bd^3}$$

$$= \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{a^2(}{}$$

$$= \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{a^2(}{}$$

$$= \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{a^2(}{}$$

$$= \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{a^2(}{}$$

$$= \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{a^2(}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3264 vs. 2(1176) = 2352.
time = 25.00, size = 3264, normalized size = 2.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (-12*a*b^2*d^3*e^2*E^(2*c)*x + 12*a^3*d*E^(2*c)*f^2*x + 12*a*b^2*d*E^(2*c)*
f^2*x - 12*a*b^2*d^3*e*E^(2*c)*f*x^2 - 4*a*b^2*d^3*E^(2*c)*f^2*x^3 - 6*a^2*
b*d^2*e^2*ArcTan[E^(c + d*x)] + 6*b^3*d^2*e^2*ArcTan[E^(c + d*x)] - 6*a^2*b
*d^2*e^2*E^(2*c)*ArcTan[E^(c + d*x)] + 6*b^3*d^2*e^2*E^(2*c)*ArcTan[E^(c +
d*x)] - 12*a^2*b*f^2*ArcTan[E^(c + d*x)] - 12*b^3*f^2*ArcTan[E^(c + d*x)] -
 12*a^2*b*E^(2*c)*f^2*ArcTan[E^(c + d*x)] - 12*b^3*E^(2*c)*f^2*ArcTan[E^(c
+ d*x)] - (6*I)*a^2*b*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + (6*I)*b^3*d^2*e*f*
x*Log[1 - I*E^(c + d*x)] - (6*I)*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d
*x)] + (6*I)*b^3*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] - (3*I)*a^2*b*d^2
*f^2*x^2*Log[1 - I*E^(c + d*x)] + (3*I)*b^3*d^2*f^2*x^2*Log[1 - I*E^(c + d*
x)] - (3*I)*a^2*b*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (3*I)*b^3*d^
2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (6*I)*a^2*b*d^2*e*f*x*Log[1 + I*
E^(c + d*x)] - (6*I)*b^3*d^2*e*f*x*Log[1 + I*E^(c + d*x)] + (6*I)*a^2*b*d^2
*e*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (6*I)*b^3*d^2*e*E^(2*c)*f*x*Log[1 +
 I*E^(c + d*x)] + (3*I)*a^2*b*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - (3*I)*b^
3*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] + (3*I)*a^2*b*d^2*E^(2*c)*f^2*x^2*Log[
1 + I*E^(c + d*x)] - (3*I)*b^3*d^2*E^(2*c)*f^2*x^2*Log[1 + I*E^(c + d*x)] +
 6*a*b^2*d^2*e^2*Log[1 + E^(2*(c + d*x))] + 6*a*b^2*d^2*e^2*E^(2*c)*Log[1 +
 E^(2*(c + d*x))] - 6*a^3*f^2*Log[1 + E^(2*(c + d*x))] - 6*a*b^2*f^2*Log[1
+ E^(2*(c + d*x))] - 6*a^3*E^(2*c)*f^2*Log[1 + E^(2*(c + d*x))] - 6*a*b^2*E
^(2*c)*f^2*Log[1 + E^(2*(c + d*x))] + 12*a*b^2*d^2*e*f*x*Log[1 + E^(2*(c +
d*x))] + 12*a*b^2*d^2*e*E^(2*c)*f*x*Log[1 + E^(2*(c + d*x))] + 6*a*b^2*d^2*
f^2*x^2*Log[1 + E^(2*(c + d*x))] + 6*a*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(2
*(c + d*x))] + (6*I)*b*(a^2 - b^2)*d*(1 + E^(2*c))*f*(e + f*x)*PolyLog[2, (
-I)*E^(c + d*x)] + (6*I)*b*(-a^2 + b^2)*d*(1 + E^(2*c))*f*(e + f*x)*PolyLog
[2, I*E^(c + d*x)] + 6*a*b^2*d*e*f*PolyLog[2, -E^(2*(c + d*x))] + 6*a*b^2*d
*e*E^(2*c)*f*PolyLog[2, -E^(2*(c + d*x))] + 6*a*b^2*d*f^2*x*PolyLog[2, -E^(
2*(c + d*x))] + 6*a*b^2*d*E^(2*c)*f^2*x*PolyLog[2, -E^(2*(c + d*x))] - (6*I
)*a^2*b*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (6*I)*b^3*f^2*PolyLog[3, (-I)*E^
(c + d*x)] - (6*I)*a^2*b*E^(2*c)*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (6*I)*b
^3*E^(2*c)*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (6*I)*a^2*b*f^2*PolyLog[3, I*
E^(c + d*x)] - (6*I)*b^3*f^2*PolyLog[3, I*E^(c + d*x)] + (6*I)*a^2*b*E^(2*c
)*f^2*PolyLog[3, I*E^(c + d*x)] - (6*I)*b^3*E^(2*c)*f^2*PolyLog[3, I*E^(c +
 d*x)] - 3*a*b^2*f^2*PolyLog[3, -E^(2*(c + d*x))] - 3*a*b^2*E^(2*c)*f^2*Pol
yLog[3, -E^(2*(c + d*x))])/(6*(a^2 + b^2)^2*d^3*(1 + E^(2*c))) + (a*b^2*(6*
e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[-(a^2 + b
^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^
2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d
*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[b - 2*a*E^(c + d
*x) - b*E^(2*(c + d*x))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 +
E^(2*(c + d*x)))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^
2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c
```

```
- Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*
E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c
 + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2
*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1
 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*L
og[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*
c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/
d - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - S
qrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2,
 -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*f^2*Po
lyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (
6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)]))])/d^3 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b
^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c
 + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3))/(3*(a^2 + b^2)^2*(-1 + E^(2*c))) + (
Csch[c]*Sech[c]*Sech[c + d*x]^2*(6*a^3*e*f + 6*a*b^2*e*f - 12*a*b^2*d^2*e^2
*x + 6*a^3*f^2*x + 6*a*b^2*f^2*x - 12*a*b^2*d^2*e*f*x^2 - 4*a*b^2*d^2*f^2*x
^3 - 6*a^3*e*f*Cosh[2*c] - 6*a*b^2*e*f*Cosh[2*c] - 6*a^3*f^2*x*Cosh[2*c] -
6*a*b^2*f^2*x*Cosh[2*c] - 6*a^3*e*f*Cosh[2*d*x] - 6*a*b^2*e*f*Cosh[2*d*x] -
 6*a^3*f^2*x*Cosh[2*d*x] - 6*a*b^2*f^2*x*Cosh[2*d*x] - 3*a^2*b*d*e^2*Cosh[c
 - d*x] - 3*b^3*d*e^2*Cosh[c - d*x] - 6*a^2*b*d*e*f*x*Cosh[c - d*x] - 6*b^3
*d*e*f*x*Cosh[c - d*x] - 3*a^2*b*d*f^2*x^2*Cosh[c - d*x] - 3*b^3*d*f^2*x^2*
Cosh[c - d*x] + 3*a^2*b*d*e^2*Cosh[3*c + d*x] + 3*b^3*d*e^2*Cosh[3*c + d*x]
 + 6*a^2*b*d*e*f*x*Cosh[3*c + d*x] + 6*b^3*d*e*f*x*Cosh[3*c + d*x] + 3*a^2*
b*d*f^2*x^2*Cosh[3*c + d*x] + 3*b^3*d*f^2*x^2*C...
```

Maple [F]
time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c)^2 \tanh(dx+c)}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] int((f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -a^2*b*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b
^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2
+ b^4*d^2), x) + b^3*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x +
2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 +
2*a^2*b^2*d^2 + b^4*d^2), x) - 2*a*b^2*d^2*f^2*integrate(x^2/(a^4*d^2*e^(2*
d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*
d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 4*a*b^2*d^2*f*e*integrate(x/(a^4*d^2*e
^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) +
a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 2*a^2*b*d^2*f*integrate(x*e^(d*x +
 c + 1)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*
e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*b^3*d^2*f*inte
grate(x*e^(d*x + c + 1)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x +
 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) +
a^3*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x + 2*c) +
1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) + a*b^2*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^
2 + b^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) - 2
*a^2*b*f^2*arctan(e^(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3) - 2*b^3*f^2*ar
ctan(e^(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3) - (a*b^2*log(-2*a*e^(-d*x -
 c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b^4)*d) - a*b^2*log(e^(-2
*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a^2*b - b^3)*arctan(e^(-d*x
 - c))/((a^4 + 2*a^2*b^2 + b^4)*d) - (b*e^(-d*x - c) - 2*a*e^(-2*d*x - 2*c)
 - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*e^(-2*d*x - 2*c) + (a^2
+ b^2)*e^(-4*d*x - 4*c))*d))*e^2 - (2*a*f^2*x + 2*a*f*e - (b*d*f^2*x^2*e^(3
*c) + 2*b*f*e^(3*c + 1) + 2*(b*f^2*e^(3*c) + b*d*f*e^(3*c + 1))*x)*e^(3*d*x
) + 2*(a*d*f^2*x^2*e^(2*c) + a*f*e^(2*c + 1) + (a*f^2*e^(2*c) + 2*a*d*f*e^(
2*c + 1))*x)*e^(2*d*x) + (b*d*f^2*x^2*e^c - 2*b*f*e^(c + 1) + 2*(b*d*f*e^(c
 + 1) - b*f^2*e^c)*x)*e^(d*x))/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^(4*c) + b^2*
d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*x)) +
 integrate(2*(a*b^3*f^2*x^2 + 2*a*b^3*f*x*e - (a^2*b^2*f^2*x^2*e^c + 2*a^2*
b^2*f*x*e^(c + 1))*e^(d*x))/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b*e^(2*c) + 2*a
^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c + 2*a^3*b^2*e^c + a*b^
4*e^c)*e^(d*x)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 16498 vs. $2(1099) = 2198$.
time = 0.68, size = 16498, normalized size = 14.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/2*(4*((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*cosh(d*x + c)^4 + 4*((
a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*sinh(d*x + c)^4 + 4*(a^3 + a*b^
```

```
2)*c*f^2 - 4*(a^3 + a*b^2)*d*f*cosh(1) + 2*((a^2*b + b^3)*d^2*f^2*x^2 + 2*(
a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d^2*cosh(1)^2 + (a^2*b + b^3)*d^2*sinh
(1)^2 + 2*((a^2*b + b^3)*d^2*f*x + (a^2*b + b^3)*d*f)*cosh(1) + 2*((a^2*b +
 b^3)*d^2*f*x + (a^2*b + b^3)*d^2*cosh(1) + (a^2*b + b^3)*d*f)*sinh(1))*cos
h(d*x + c)^3 - 4*(a^3 + a*b^2)*d*f*sinh(1) + 2*((a^2*b + b^3)*d^2*f^2*x^2 +
 2*(a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d^2*cosh(1)^2 + (a^2*b + b^3)*d^2*
sinh(1)^2 + 2*((a^2*b + b^3)*d^2*f*x + (a^2*b + b^3)*d*f)*cosh(1) + 8*((a^3
 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*cosh(d*x + c) + 2*((a^2*b + b^3)*d
^2*f*x + (a^2*b + b^3)*d^2*cosh(1) + (a^2*b + b^3)*d*f)*sinh(1))*sinh(d*x +
 c)^3 - 4*((a^3 + a*b^2)*d^2*f^2*x^2 - (a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2
)*d^2*cosh(1)^2 + (a^3 + a*b^2)*d^2*sinh(1)^2 - 2*(a^3 + a*b^2)*c*f^2 + (2*
(a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*d*f)*cosh(1) + (2*(a^3 + a*b^2)*d^2*f
*x + 2*(a^3 + a*b^2)*d^2*cosh(1) + (a^3 + a*b^2)*d*f)*sinh(1))*cosh(d*x + c
)^2 - 2*(2*(a^3 + a*b^2)*d^2*f^2*x^2 - 2*(a^3 + a*b^2)*d*f^2*x + 2*(a^3 + a
*b^2)*d^2*cosh(1)^2 + 2*(a^3 + a*b^2)*d^2*sinh(1)^2 - 4*(a^3 + a*b^2)*c*f^2
 - 12*((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*cosh(d*x + c)^2 + 2*(2*
(a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*d*f)*cosh(1) - 3*((a^2*b + b^3)*d^2*f
^2*x^2 + 2*(a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d^2*cosh(1)^2 + (a^2*b + b
^3)*d^2*sinh(1)^2 + 2*((a^2*b + b^3)*d^2*f*x + (a^2*b + b^3)*d*f)*cosh(1) +
 2*((a^2*b + b^3)*d^2*f*x + (a^2*b + b^3)*d^2*cosh(1) + (a^2*b + b^3)*d*f)*
sinh(1))*cosh(d*x + c) + 2*(2*(a^3 + a*b^2)*d^2*f*x + 2*(a^3 + a*b^2)*d^2*c
osh(1) + (a^3 + a*b^2)*d*f)*sinh(1))*sinh(d*x + c)^2 - 2*((a^2*b + b^3)*d^2
*f^2*x^2 - 2*(a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d^2*cosh(1)^2 + (a^2*b +
 b^3)*d^2*sinh(1)^2 + 2*((a^2*b + b^3)*d^2*f*x - (a^2*b + b^3)*d*f)*cosh(1)
 + 2*((a^2*b + b^3)*d^2*f*x + (a^2*b + b^3)*d^2*cosh(1) - (a^2*b + b^3)*d*f
)*sinh(1))*cosh(d*x + c) - 4*(a*b^2*d*f^2*x + a*b^2*d*f*cosh(1) + a*b^2*d*f
*sinh(1) + (a*b^2*d*f^2*x + a*b^2*d*f*cosh(1) + a*b^2*d*f*sinh(1))*cosh(d*x
 + c)^4 + 4*(a*b^2*d*f^2*x + a*b^2*d*f*cosh(1) + a*b^2*d*f*sinh(1))*cosh(d*
x + c)*sinh(d*x + c)^3 + (a*b^2*d*f^2*x + a*b^2*d*f*cosh(1) + a*b^2*d*f*sin
h(1))*sinh(d*x + c)^4 + 2*(a*b^2*d*f^2*x + a*b^2*d*f*cosh(1) + a*b^2*d*f*si
nh(1))*cosh(d*x + c)^2 + 2*(a*b^2*d*f^2*x + a*b^2*d*f*cosh(1) + a*b^2*d*f*s
inh(1) + 3*(a*b^2*d*f^2*x + a*b^2*d*f*cosh(1) + a*b^2*d*f*sinh(1))*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 + 4*((a*b^2*d*f^2*x + a*b^2*d*f*cosh(1) + a*b^2*d*
f*sinh(1))*cosh(d*x + c)^3 + (a*b^2*d*f^2*x + a*b^2*d*f*cosh(1) + a*b^2*d*f
*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x
 + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b +
1) - 4*(a*b^2*d*f^2*x + a*b^2*d*f*cosh(1) + a*b^2*d*f*sinh(1) + (a*b^2*d*f^
2*x + a*b^2*d*f*cosh(1) + a*b^2*d*f*sinh(1))*cosh(d*x + c)^4 + 4*(a*b^2*d*f
^2*x + a*b^2*d*f*cosh(1) + a*b^2*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3
 + (a*b^2*d*f^2*x + a*b^2*d*f*cosh(1) + a*b^2*d*f*sinh(1))*sinh(d*x + c)^4
+ 2*(a*b^2*d*f^2*x + a*b^2*d*f*cosh(1) + a*b^2*d*f*sinh(1))*cosh(d*x + c)^2
 + 2*(a*b^2*d*f^2*x + a*b^2*d*f*cosh(1) + a*b^2*d*f*sinh(1) + 3*(a*b^2*d*f^
2*x + a*b^2*d*f*cosh(1) + a*b^2*d*f*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)
^2 + 4*((a*b^2*d*f^2*x + a*b^2*d*f*cosh(1) + a*b^2*d*f*sinh(1))*cosh(d*x +
c)^3 + (a*b^2*d*f^2*x + a*b^2*d*f*cosh(1) + a*b^2*d*f*sinh(1))*cosh(d*x + c
```

```
))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(2*a*b^2*d*f^2*
x + 2*a*b^2*d*f*cosh(1) + 2*a*b^2*d*f*sinh(1) - I*(a^2*b - b^3)*d*f^2*x + (
2*a*b^2*d*f^2*x + 2*a*b^2*d*f*cosh(1) + 2*a*b^2*d*f*sinh(1) - I*(a^2*b - b^
3)*d*f^2*x - I*(a^2*b - b^3)*d*f*cosh(1) - I*(a^2*b - b^3)*d*f*sinh(1))*cos
h(d*x + c)^4 + 4*(2*a*b^2*d*f^2*x + 2*a*b^2*d*f*cosh(1) + 2*a*b^2*d*f*sinh(
1) - I*(a^2*b - b^3)*d*f^2*x - I*(a^2*b - b^3)*d*f*cosh(1) - I*(a^2*b - b^3
)*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b^2*d*f^2*x + 2*a*b^2*d
*f*cosh(1) + 2*a*b^2*d*f*sinh(1) - I*(a^2*b - b^3)*d*f^2*x - I*(a^2*b - b^3
)*d*f*cosh(1) - I*(a^2*b - b^3)*d*f*sinh(1))*sinh(d*x + c)^4 - I*(a^2*b - b
^3)*d*f*cosh(1) - I*(a^2*b - b^3)*d*f*sinh(1) + 2*(2*a*b^2*d*f^2*x + 2*a*b^
2*d*f*cosh(1) + 2*a*b^2*d*f*sinh(1) - I*(a^2*b - b^3)*d*f^2*x - I*(a^2*b -
b^3)*d*f*cosh(1) - I*(a^2*b - b^3)*d*f*sinh(1))*cosh(d*x + c)^2 + 2*(2*a*b^
2*d*f^2*x + 2*a*b^2*d*f*cosh(1) + 2*a*b^2*d*f*sinh(1) - I*(a^2*b - b^3)*d*f
^2*x - I*(a^2*b - b^3)*d*f*cosh(1) - I*(a^2*b - b^3)*d*f*sinh(1) + 3*(2*a*b
^2*d*f^2*x + 2*a*b^2*d*f*cosh(1) + 2*a*b^2*d*f*sinh(1) - I*(a^2*b - b^3)*d*
f^2*x - I*(a^2*b - b^3)*d*f*cosh(1) - I*(a^2*b - b^3)*d*f*sinh(1))*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 + 4*((2*a*b^2*d*f^2*x + 2*a*b^2*d*f*cosh(1) + 2*a*
b^2*d*f*sinh(1) - I*(a^2*b - b^3)*d*f^2*x - I*(a^2*b - b^3)*d*f*cosh(1) - I
*(a^2*b - b^3)*d*f*sinh(1))*cosh(d*x + c)^3 + (...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2\tanh(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sech(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*tanh(c + d*x)*sech(c + d*x)**2/(a + b*sinh(c + d*x)),
 x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(c+d\,x)\,\left(e+f\,x\right)^2}{{\cosh\left(c+d\,x\right)}^2\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(c + d*x)*(e + f*x)^2)/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
[Out] int((tanh(c + d*x)*(e + f*x)^2)/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.359 $\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=711

$$\frac{(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{bd}-\frac{2a^2b(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2d}-\frac{a^2(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d}-\frac{ab^2(e+fx)\log\left(1+\frac{}{a-}\right.}{\left(a^2+b^2\right)^2d}$$

```
[Out] (f*x+e)*arctan(exp(d*x+c))/b/d-2*a^2*b*(f*x+e)*arctan(exp(d*x+c))/(a^2+b^2)
^2/d-a^2*(f*x+e)*arctan(exp(d*x+c))/b/(a^2+b^2)/d+a*b^2*(f*x+e)*ln(1+exp(2*
d*x+2*c))/(a^2+b^2)^2/d-a*b^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2))
)/(a^2+b^2)^2/d-a*b^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b
^2)^2/d+I*a^2*b*f*polylog(2,-I*exp(d*x+c))/(a^2+b^2)^2/d^2+1/2*I*a^2*f*poly
log(2,-I*exp(d*x+c))/b/(a^2+b^2)/d^2+1/2*I*f*polylog(2,I*exp(d*x+c))/b/d^2-
1/2*I*f*polylog(2,-I*exp(d*x+c))/b/d^2-I*a^2*b*f*polylog(2,I*exp(d*x+c))/(a
^2+b^2)^2/d^2-1/2*I*a^2*f*polylog(2,I*exp(d*x+c))/b/(a^2+b^2)/d^2+1/2*a*b^2
*f*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^2-a*b^2*f*polylog(2,-b*exp(d*x+
c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2-a*b^2*f*polylog(2,-b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2+1/2*f*sech(d*x+c)/b/d^2-1/2*a^2*f*sech(d*
x+c)/b/(a^2+b^2)/d^2-1/2*a*(f*x+e)*sech(d*x+c)^2/(a^2+b^2)/d+1/2*a*f*tanh(d
*x+c)/(a^2+b^2)/d^2+1/2*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/b/d-1/2*a^2*(f*x+e)
*sech(d*x+c)*tanh(d*x+c)/b/(a^2+b^2)/d
```

Rubi [A]
time = 0.82, antiderivative size = 711, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {5702, 4270, 4265, 2317, 2438, 5692, 5680, 2221, 6874, 3799, 5559, 3852, 8}

$$-\frac{a^2(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{bd\left(a^2+b^2\right)}-\frac{2a^2b(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{d\left(a^2+b^2\right)^2}+\frac{ia^2f\text{Li}_2\left(-ie^{c+dx}\right)}{2bd^2\left(a^2+b^2\right)}+\frac{ia^2bf\text{Li}_2\left(-ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)^2}-\frac{ia^2f\text{Li}_2\left(ie^{c+dx}\right)}{2bd^2\left(a^2+b^2\right)}-\frac{ia^2bf\text{Li}_2\left(ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)^2}-\frac{ab^2f\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^2}-\frac{ab^2f\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^2}+\frac{ab^2f\text{Li}_2\left(-e^{2(c+dx)}\right)}{2d^2\left(a^2+b^2\right)^2}+\frac{af\tanh(c+dx)}{2d^2\left(a^2+b^2\right)}-\frac{a^2f\text{sech}(c+dx)}{2bd^2\left(a^2+b^2\right)}-\frac{ab^2(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{d\left(a^2+b^2\right)^2}-\frac{ab^2(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{d\left(a^2+b^2\right)^2}+\frac{ab^2(e+fx)\log\left(e^{2(c+dx)}+1\right)}{d\left(a^2+b^2\right)^2}-\frac{a(e+fx)\text{sech}^2(c+dx)}{2d\left(a^2+b^2\right)}-\frac{a^2(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2bd\left(a^2+b^2\right)}+\frac{(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{bd}-\frac{if\text{Li}_2\left(-ie^{c+dx}\right)}{2bd^2}+\frac{if\text{Li}_2\left(ie^{c+dx}\right)}{2bd^2}+\frac{f\text{sech}(c+dx)}{2bd^2}+\frac{(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((e + f*x)*ArcTan[E^(c + d*x)])/(b*d) - (2*a^2*b*(e + f*x)*ArcTan[E^(c + d*
x)])/((a^2 + b^2)^2*d) - (a^2*(e + f*x)*ArcTan[E^(c + d*x)])/(b*(a^2 + b^2)
*d) - (a*b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^
2 + b^2)^2*d) - (a*b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/((a^2 + b^2)^2*d) + (a*b^2*(e + f*x)*Log[1 + E^(2*(c + d*x))])/((a^2
+ b^2)^2*d) - ((I/2)*f*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + (I*a^2*b*f*P
olyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) + ((I/2)*a^2*f*PolyLog[2,
(-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) + ((I/2)*f*PolyLog[2, I*E^(c + d*x)]
)/(b*d^2) - (I*a^2*b*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - ((I
/2)*a^2*f*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) - (a*b^2*f*PolyLog
[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) - (a*b^2
```

```
*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2
) + (a*b^2*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^2) + (f*Sech[
c + d*x])/(2*b*d^2) - (a^2*f*Sech[c + d*x])/(2*b*(a^2 + b^2)*d^2) - (a*(e +
 f*x)*Sech[c + d*x]^2)/(2*(a^2 + b^2)*d) + (a*f*Tanh[c + d*x])/(2*(a^2 + b^
2)*d^2) + ((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*b*d) - (a^2*(e + f*x)*
Sech[c + d*x]*Tanh[c + d*x])/(2*b*(a^2 + b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
```

```
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5702

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{sech}^3(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{f\text{sech}(c+dx)}{2bd^2} + \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{2bd} + \frac{\int (e+ \ldots}{\ldots}$$

$$= \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} + \frac{f\text{sech}(c+dx)}{2bd^2} + \frac{(e+fx)\text{sech}(c+d \ldots}{2bd \ldots}$$

$$= \frac{ab^2(e+fx)^2}{2\left(a^2+b^2\right)^2 f} + \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} + \frac{f\text{sech}(c+dx)}{2bd^2} + \frac{(e \ldots}{\ldots}$$

$$= \frac{ab^2(e+fx)^2}{2\left(a^2+b^2\right)^2 f} + \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{ab^2(e+fx)\log\left(1 \ldots\right.}{\left(a^2+b^{\ldots}\right.}$$

$$= \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{a^2(e+ \ldots}{\ldots}$$

$$= \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{a^2(e+ \ldots}{\ldots}$$

$$= \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{a^2(e+ \ldots}{\ldots}$$

$$= \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{a^2(e+ \ldots}{\ldots}$$

Mathematica [A]
time = 6.89, size = 587, normalized size = 0.83

$$\frac{-2ab^2\left(-\frac{1}{2}f(c+dx)^2+f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+f(c+dx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)+de\log(a+b\sinh(c+dx))-cf\log(a+b\sinh(c+dx))+f\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)+f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)+b\left(-2abde(c+dx)+2abcf(c+dx)-abf(c+dx)^2-2a^2de\text{ArcTan}\left(e^{c+dx}\right)+2b^2de\text{ArcTan}\left(e^{c+dx}\right)+2a^2cf\text{ArcTan}\left(e^{c+dx}\right)-2b^2cf\text{ArcTan}\left(e^{c+dx}\right)-ia^2f(c+dx)\log\left(1-ie^{c+dx}\right)+ib^2f(c+dx)\log\left(1-ie^{c+dx}\right)+ia^2f(c+dx)\log\left(1+ie^{c+dx}\right)-ib^2f(c+dx)\log\left(1+ie^{c+dx}\right)+2abde\log\left(1+e^{2(c+dx)}\right)-2abcf\log\left(1+e^{2(c+dx)}\right)+2abf(c+dx)\log\left(1+e^{2(c+dx)}\right)+i\left(a^2-b^2\right)f\text{PolyLog}\left(2,-ie^{c+dx}\right)-i\left(a^2-b^2\right)f\text{PolyLog}\left(2,ie^{c+dx}\right)+abf\text{PolyLog}\left(2,-e^{2(c+dx)}\right)\right)+\left(a^2+b^2\right)f\text{sech}(c+dx)(b+a\sinh(c+dx))+\left(a^2+b^2\right)d(e+fx)\text{sech}^2(c+dx)(-a+b\sinh(c+dx))}{2\left(a^2+b^2\right)^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (-2*a*b^2*(-1/2*(f*(c + d*x)^2) + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
])] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyL
og[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))]) + b*(-2*a*b*d*e*(c + d*x) + 2*a*b*c*f*(c + d*x
) - a*b*f*(c + d*x)^2 - 2*a^2*d*e*ArcTan[E^(c + d*x)] + 2*b^2*d*e*ArcTan[E^
(c + d*x)] + 2*a^2*c*f*ArcTan[E^(c + d*x)] - 2*b^2*c*f*ArcTan[E^(c + d*x)]
- I*a^2*f*(c + d*x)*Log[1 - I*E^(c + d*x)] + I*b^2*f*(c + d*x)*Log[1 - I*E^
(c + d*x)] + I*a^2*f*(c + d*x)*Log[1 + I*E^(c + d*x)] - I*b^2*f*(c + d*x)*L
og[1 + I*E^(c + d*x)] + 2*a*b*d*e*Log[1 + E^(2*(c + d*x))] - 2*a*b*c*f*Log[
1 + E^(2*(c + d*x))] + 2*a*b*f*(c + d*x)*Log[1 + E^(2*(c + d*x))] + I*(a^2
- b^2)*f*PolyLog[2, (-I)*E^(c + d*x)] - I*(a^2 - b^2)*f*PolyLog[2, I*E^(c +
 d*x)] + a*b*f*PolyLog[2, -E^(2*(c + d*x))]) + (a^2 + b^2)*f*Sech[c + d*x]*
(b + a*Sinh[c + d*x]) + (a^2 + b^2)*d*(e + f*x)*Sech[c + d*x]^2*(-a + b*Sin
h[c + d*x]))/(2*(a^2 + b^2)^2*d^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2073 vs. $2(656) = 1312$.
time = 3.83, size = 2074, normalized size = 2.92

method	result	size
risch	Expression too large to display	2074

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVER
BOSE)
```

```
[Out] -I*b/d^2/(a^2+b^2)*a^2*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))-I*b^3/d/(a^2+b
^2)*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x-I*b^3/d^2/(a^2+b^2)*f/(2*a^2+2*b^2
)*ln(1+I*exp(d*x+c))*c+I*b^3/d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c
))*c+2*b^2/d/(a^2+b^2)*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*a*x-2*b^2/d/(a^2+
b^2)*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/
2)))*a*x-2*b^2/d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1
/2)-a)/(-a+(a^2+b^2)^(1/2)))*a*c+I*b^3/d/(a^2+b^2)*f/(2*a^2+2*b^2)*ln(1-I*e
xp(d*x+c))*x+2*b^2/d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*a*c+2*b
^2/d/(a^2+b^2)*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*x+2*b^2/d^2/(a^2+b^2)*f
/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*c-2*b^2/d/(a^2+b^2)*f/(2*a^2+2*b^2)*ln(
(b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a*x+I*b/d^2/(a^2+b^2)
*a^2*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))+2*b^2/d^2/(a^2+b^2)*f*c/(2*a^2+2
*b^2)*a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2*b^2/d^2/(a^2+b^2)*f*c/(2*a^
2+2*b^2)*a*ln(1+exp(2*d*x+2*c))+2*b/d^2/(a^2+b^2)*a^2*f*c/(2*a^2+2*b^2)*arc
tan(exp(d*x+c))+b^4/d^2/(a^2+b^2)^(3/2)*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*
exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2*b^2/d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*ln((b*
```

```
exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a*c-b^2/d/(a^2+b^2)^(3/2
)*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2-b^4
/d/(a^2+b^2)^(3/2)*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^
2)^(1/2))+2*b^2/d/(a^2+b^2)*e/(2*a^2+2*b^2)*a*ln(1+exp(2*d*x+2*c))-2*b^3/d^
2/(a^2+b^2)*f*c/(2*a^2+2*b^2)*arctan(exp(d*x+c))-2*b^2/d^2/(a^2+b^2)*f/(2*a
^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a-2*b
^2/d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-
a+(a^2+b^2)^(1/2)))*a+2*b^2/d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x
+c))*a-2*b^2/d/(a^2+b^2)*e/(2*a^2+2*b^2)*a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+
c)-b)-2*b/d/(a^2+b^2)*e*a^2/(2*a^2+2*b^2)*arctan(exp(d*x+c))+2*b^2/d^2/(a^2
+b^2)*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))*a+I*b^3/d^2/(a^2+b^2)*f/(2*a^2+
2*b^2)*dilog(1-I*exp(d*x+c))-I*b/d/(a^2+b^2)*a^2*f/(2*a^2+2*b^2)*ln(1-I*exp
(d*x+c))*x-I*b/d^2/(a^2+b^2)*a^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c+I*b/d
/(a^2+b^2)*a^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x+I*b/d^2/(a^2+b^2)*a^2*f
/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c+b^2/d^2/(a^2+b^2)^(3/2)*f*c/(2*a^2+2*b^
2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2-1/d^2*f*c*b^2/(2*a
^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
-I*b^3/d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))+2*b^3/d/(a^2+b^2
)*e/(2*a^2+2*b^2)*arctan(exp(d*x+c))+1/d*e*b^2/(2*a^2+2*b^2)/(a^2+b^2)^(1/2
)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-(-b*d*f*x*exp(3*d*x+3*c
)+2*a*d*f*x*exp(2*d*x+2*c)-b*d*e*exp(3*d*x+3*c)+2*a*d*e*exp(2*d*x+2*c)+b*d*
f*x*exp(d*x+c)-b*f*exp(3*d*x+3*c)+a*f*exp(2*d*x+2*c)+b*d*e*exp(d*x+c)-f*b*e
xp(d*x+c)+f*a)/d^2/(a^2+b^2)/(1+exp(2*d*x+2*c))^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] f*(((b*d*x*e^(3*c) + b*e^(3*c))*e^(3*d*x) - (2*a*d*x*e^(2*c) + a*e^(2*c))*e
^(2*d*x) - (b*d*x*e^c - b*e^c)*e^(d*x) - a)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e
^(4*c) + b^2*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))
*e^(2*d*x)) + 4*integrate(-1/2*(a^2*b^2*x*e^(d*x + c) - a*b^3*x)/(a^4*b + 2
*a^2*b^3 + b^5 - (a^4*b*e^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x
) - 2*(a^5*e^c + 2*a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) - 4*integrate(1/4*
(2*a*b^2*x + (a^2*b*e^c - b^3*e^c)*x*e^(d*x))/(a^4 + 2*a^2*b^2 + b^4 + (a^4
*e^(2*c) + 2*a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x)) - (a*b^2*log(-2
*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b^4)*d) - a*b
^2*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a^2*b - b^3)*ar
ctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) - (b*e^(-d*x - c) - 2*a*e^(-
2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*e^(-2*d*x -
2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d))*e
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5790 vs. $2(644) = 1288$.
time = 0.46, size = 5790, normalized size = 8.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] 1/2*(2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*cosh(1) + (a^2*b + b^3)*d*sin
h(1) + (a^2*b + b^3)*f)*cosh(d*x + c)^3 + 2*((a^2*b + b^3)*d*f*x + (a^2*b +
 b^3)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1) + (a^2*b + b^3)*f)*sinh(d*x + c)^
3 - 2*(2*(a^3 + a*b^2)*d*f*x + 2*(a^3 + a*b^2)*d*cosh(1) + 2*(a^3 + a*b^2)*
d*sinh(1) + (a^3 + a*b^2)*f)*cosh(d*x + c)^2 - 2*(2*(a^3 + a*b^2)*d*f*x + 2
*(a^3 + a*b^2)*d*cosh(1) + 2*(a^3 + a*b^2)*d*sinh(1) + (a^3 + a*b^2)*f - 3*
((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1) +
(a^2*b + b^3)*f)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(a^3 + a*b^2)*f - 2*((a
^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1) - (a^
2*b + b^3)*f)*cosh(d*x + c) - 2*(a*b^2*f*cosh(d*x + c)^4 + 4*a*b^2*f*cosh(d
*x + c)*sinh(d*x + c)^3 + a*b^2*f*sinh(d*x + c)^4 + 2*a*b^2*f*cosh(d*x + c)
^2 + a*b^2*f + 2*(3*a*b^2*f*cosh(d*x + c)^2 + a*b^2*f)*sinh(d*x + c)^2 + 4*
(a*b^2*f*cosh(d*x + c)^3 + a*b^2*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*c
osh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((
a^2 + b^2)/b^2) - b)/b + 1) - 2*(a*b^2*f*cosh(d*x + c)^4 + 4*a*b^2*f*cosh(d
*x + c)*sinh(d*x + c)^3 + a*b^2*f*sinh(d*x + c)^4 + 2*a*b^2*f*cosh(d*x + c)
^2 + a*b^2*f + 2*(3*a*b^2*f*cosh(d*x + c)^2 + a*b^2*f)*sinh(d*x + c)^2 + 4*
(a*b^2*f*cosh(d*x + c)^3 + a*b^2*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*c
osh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((
a^2 + b^2)/b^2) - b)/b + 1) + ((2*a*b^2*f - I*(a^2*b - b^3)*f)*cosh(d*x + c
)^4 + 4*(2*a*b^2*f - I*(a^2*b - b^3)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*
a*b^2*f - I*(a^2*b - b^3)*f)*sinh(d*x + c)^4 + 2*a*b^2*f + 2*(2*a*b^2*f - I
*(a^2*b - b^3)*f)*cosh(d*x + c)^2 + 2*(2*a*b^2*f + 3*(2*a*b^2*f - I*(a^2*b
- b^3)*f)*cosh(d*x + c)^2 - I*(a^2*b - b^3)*f)*sinh(d*x + c)^2 - I*(a^2*b -
 b^3)*f + 4*((2*a*b^2*f - I*(a^2*b - b^3)*f)*cosh(d*x + c)^3 + (2*a*b^2*f -
 I*(a^2*b - b^3)*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(I*cosh(d*x + c) + I
*sinh(d*x + c)) + ((2*a*b^2*f + I*(a^2*b - b^3)*f)*cosh(d*x + c)^4 + 4*(2*a
*b^2*f + I*(a^2*b - b^3)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b^2*f + I*
(a^2*b - b^3)*f)*sinh(d*x + c)^4 + 2*a*b^2*f + 2*(2*a*b^2*f + I*(a^2*b - b^
3)*f)*cosh(d*x + c)^2 + 2*(2*a*b^2*f + 3*(2*a*b^2*f + I*(a^2*b - b^3)*f)*co
sh(d*x + c)^2 + I*(a^2*b - b^3)*f)*sinh(d*x + c)^2 + I*(a^2*b - b^3)*f + 4*
((2*a*b^2*f + I*(a^2*b - b^3)*f)*cosh(d*x + c)^3 + (2*a*b^2*f + I*(a^2*b -
b^3)*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(-I*cosh(d*x + c) - I*sinh(d*x +
 c)) + 2*(a*b^2*c*f - a*b^2*d*cosh(1) + (a*b^2*c*f - a*b^2*d*cosh(1) - a*b^
2*d*sinh(1))*cosh(d*x + c)^4 - a*b^2*d*sinh(1) + 4*(a*b^2*c*f - a*b^2*d*cos
```

```
h(1) - a*b^2*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b^2*c*f - a*b^2*
d*cosh(1) - a*b^2*d*sinh(1))*sinh(d*x + c)^4 + 2*(a*b^2*c*f - a*b^2*d*cosh(
1) - a*b^2*d*sinh(1))*cosh(d*x + c)^2 + 2*(a*b^2*c*f - a*b^2*d*cosh(1) - a*
b^2*d*sinh(1) + 3*(a*b^2*c*f - a*b^2*d*cosh(1) - a*b^2*d*sinh(1))*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 + 4*((a*b^2*c*f - a*b^2*d*cosh(1) - a*b^2*d*sinh(1)
)*cosh(d*x + c)^3 + (a*b^2*c*f - a*b^2*d*cosh(1) - a*b^2*d*sinh(1))*cosh(d*
x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt
((a^2 + b^2)/b^2) + 2*a) + 2*(a*b^2*c*f - a*b^2*d*cosh(1) + (a*b^2*c*f - a*
b^2*d*cosh(1) - a*b^2*d*sinh(1))*cosh(d*x + c)^4 - a*b^2*d*sinh(1) + 4*(a*b
^2*c*f - a*b^2*d*cosh(1) - a*b^2*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a*b^2*c*f - a*b^2*d*cosh(1) - a*b^2*d*sinh(1))*sinh(d*x + c)^4 + 2*(a*b^2
*c*f - a*b^2*d*cosh(1) - a*b^2*d*sinh(1))*cosh(d*x + c)^2 + 2*(a*b^2*c*f -
a*b^2*d*cosh(1) - a*b^2*d*sinh(1) + 3*(a*b^2*c*f - a*b^2*d*cosh(1) - a*b^2*
d*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a*b^2*c*f - a*b^2*d*cosh(
1) - a*b^2*d*sinh(1))*cosh(d*x + c)^3 + (a*b^2*c*f - a*b^2*d*cosh(1) - a*b^
2*d*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh
(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(a*b^2*d*f*x + a*b^2*c*f +
 (a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x + c)^4 + 4*(a*b^2*d*f*x + a*b^2*c*f)*co
sh(d*x + c)*sinh(d*x + c)^3 + (a*b^2*d*f*x + a*b^2*c*f)*sinh(d*x + c)^4 + 2
*(a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*f*x + a*b^2*c*f + 3
*(a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a*b^2*d*f
*x + a*b^2*c*f)*cosh(d*x + c)^3 + (a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x + c))*
sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(a*b^2*d*f*x + a*b^2*c*
f + (a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x + c)^4 + 4*(a*b^2*d*f*x + a*b^2*c*f)
*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b^2*d*f*x + a*b^2*c*f)*sinh(d*x + c)^4
+ 2*(a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*f*x + a*b^2*c*f
+ 3*(a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a*b^2*
d*f*x + a*b^2*c*f)*cosh(d*x + c)^3 + (a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x + c
))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (2*a*b^2*c*f - 2*a*b^2
*d*cosh(1) + (2*a*b^2*c*f - 2*a*b^2*d*cosh(1) -...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\tanh(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*tanh(c + d*x)*sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(c+d\,x)\,\left(e+f\,x\right)}{{\cosh\left(c+d\,x\right)}^2\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(c + d*x)*(e + f*x))/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((tanh(c + d*x)*(e + f*x))/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.360 $\int \frac{\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=122

$$-\frac{b(a^2-b^2)\,\text{ArcTan}(\sinh(c+dx))}{2\left(a^2+b^2\right)^2 d}+\frac{ab^2\log(\cosh(c+dx))}{\left(a^2+b^2\right)^2 d}-\frac{ab^2\log(a+b\sinh(c+dx))}{\left(a^2+b^2\right)^2 d}-\frac{\text{sech}^2(c+dx)(a-}{2\left(a^2+\right.}$$

```
[Out] -1/2*b*(a^2-b^2)*arctan(sinh(d*x+c))/(a^2+b^2)^2/d+a*b^2*ln(cosh(d*x+c))/(a
^2+b^2)^2/d-a*b^2*ln(a+b*sinh(d*x+c))/(a^2+b^2)^2/d-1/2*sech(d*x+c)^2*(a-b*
sinh(d*x+c))/(a^2+b^2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2916, 12, 837, 815, 649, 209, 266}

$$-\frac{b(a^2-b^2)\,\text{ArcTan}(\sinh(c+dx))}{2d\left(a^2+b^2\right)^2}-\frac{ab^2\log(a+b\sinh(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{ab^2\log(\cosh(c+dx))}{d\left(a^2+b^2\right)^2}-\frac{\text{sech}^2(c+dx)(a-b\sinh(c+dx))}{2d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/2*(b*(a^2 - b^2)*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)^2*d) + (a*b^2*Log[C
osh[c + d*x]])/((a^2 + b^2)^2*d) - (a*b^2*Log[a + b*Sinh[c + d*x]])/((a^2 +
 b^2)^2*d) - (Sech[c + d*x]^2*(a - b*Sinh[c + d*x]))/(2*(a^2 + b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] && !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b^3\text{Subst}\left(\int \frac{x}{b(a+x)(-b^2-x^2)^2}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= \frac{b^2\text{Subst}\left(\int \frac{x}{(a+x)(-b^2-x^2)^2}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= -\frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2)d} + \frac{\text{Subst}\left(\int \frac{ab^2-b^2x}{(a+x)(-b^2-x^2)}\,dx, x, b\sin\right.}{2(a^2+b^2)d}$$

$$= -\frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2)d} + \frac{\text{Subst}\left(\int \left(-\frac{2ab^2}{(a^2+b^2)(a+x)} + \frac{b^2(-a^2}{(a^2+b^2}\right.\right.}{2(a^2+}$$

$$= -\frac{ab^2\log(a+b\sinh(c+dx))}{(a^2+b^2)^2 d} - \frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2)d} + \frac{b^2}{}$$

$$= -\frac{ab^2\log(a+b\sinh(c+dx))}{(a^2+b^2)^2 d} - \frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2)d} + \frac{(a}{}$$

$$= -\frac{b(a^2-b^2)\tan^{-1}(\sinh(c+dx))}{2(a^2+b^2)^2 d} + \frac{ab^2\log(\cosh(c+dx))}{(a^2+b^2)^2 d} - \frac{ab^2\log(a+}{(a^2}$$

Mathematica [A]
time = 0.16, size = 105, normalized size = 0.86

$$\frac{2b\left((-a^2+b^2)\,\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+ab(\log(\cosh(c+dx))-\log(a+b\sinh(c+dx)))\right)-a(a^2+b^2)\operatorname{sech}^2(c+dx)+b(a^2+b^2)\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a^2+b^2)^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*b*((-a^2 + b^2)*ArcTan[Tanh[(c + d*x)/2]] + a*b*(Log[Cosh[c + d*x]] - Lo
g[a + b*Sinh[c + d*x]])) - a*(a^2 + b^2)*Sech[c + d*x]^2 + b*(a^2 + b^2)*Se
ch[c + d*x]*Tanh[c + d*x])/(2*(a^2 + b^2)^2*d)
```

Maple [A]
time = 1.32, size = 212, normalized size = 1.74

method	result
derivativedivides	$\frac{-\frac{2ab^2\ln\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{2a^4+4a^2b^2+2b^4}-\frac{2\left(\frac{\left(\frac{1}{2}a^2b+\frac{1}{2}b^3\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^3-ab^2\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{1}{2}a^2b\right.}{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}\right.}{}}{d}$

default	$\frac{-\frac{2a\,b^2\ln\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{2a^4+4a^2b^2+2b^4}-\frac{2\left(\frac{\left(\frac{1}{2}a^2b+\frac{1}{2}b^3\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^3-a\,b^2\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{1}{2}a^2b-\right.}{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}\right.}{a^4}}{d}$
risch	$-\frac{2a\,b^2d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{2a\,b^2dc}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{2a\,b^2x}{a^4+2a^2b^2+b^4}+\frac{2a\,b^2c}{d(a^4+2a^2b^2+b^4)}-\frac{e^{dx+c}\left(-b\,e^{2dx+2c}+2a\,e^{dx}\right.}{d(a^2+b^2)\left(1+e^{2dx+2c}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a*b^2/(2*a^4+4*a^2*b^2+2*b^4)*ln(a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1
/2*d*x+1/2*c)-a)-2/(a^4+2*a^2*b^2+b^4)*(((1/2*a^2*b+1/2*b^3)*tanh(1/2*d*x+1
/2*c)^3+(-a^3-a*b^2)*tanh(1/2*d*x+1/2*c)^2+(-1/2*a^2*b-1/2*b^3)*tanh(1/2*d*
x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2+1)^2+1/2*b*(-a*b*ln(tanh(1/2*d*x+1/2*c)^2+
1)+(a^2-b^2)*arctan(tanh(1/2*d*x+1/2*c)))))
```

Maxima [A]
time = 0.49, size = 218, normalized size = 1.79

$$-\frac{ab^2\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^4+2\,a^2b^2+b^4)d}+\frac{ab^2\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^4+2\,a^2b^2+b^4)d}+\frac{(a^2b-b^3)\arctan\left(e^{(-dx-c)}\right)}{(a^4+2\,a^2b^2+b^4)d}+\frac{be^{(-dx-c)}-2\,ae^{(-2\,dx-2\,c)}-be^{(-3\,dx-3\,c)}}{(a^2+b^2+2\,(a^2+b^2)e^{(-2\,dx-2\,c)}+(a^2+b^2)e^{(-4\,dx-4\,c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -a*b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 +
b^4)*d) + a*b^2*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) + (a^
2*b - b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (b*e^(-d*x -
c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)
*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 926 vs. $2(119) = 238$.
time = 0.37, size = 926, normalized size = 7.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] ((a^2*b + b^3)*cosh(d*x + c)^3 + (a^2*b + b^3)*sinh(d*x + c)^3 - 2*(a^3 + a
*b^2)*cosh(d*x + c)^2 - (2*a^3 + 2*a*b^2 - 3*(a^2*b + b^3)*cosh(d*x + c))*s
inh(d*x + c)^2 - ((a^2*b - b^3)*cosh(d*x + c)^4 + 4*(a^2*b - b^3)*cosh(d*x
+ c)*sinh(d*x + c)^3 + (a^2*b - b^3)*sinh(d*x + c)^4 + a^2*b - b^3 + 2*(a^2
```

```
*b - b^3)*cosh(d*x + c)^2 + 2*(a^2*b - b^3 + 3*(a^2*b - b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + 4*((a^2*b - b^3)*cosh(d*x + c)^3 + (a^2*b - b^3)*cosh(
d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - (a^2*b + b
^3)*cosh(d*x + c) - (a*b^2*cosh(d*x + c)^4 + 4*a*b^2*cosh(d*x + c)*sinh(d*x
 + c)^3 + a*b^2*sinh(d*x + c)^4 + 2*a*b^2*cosh(d*x + c)^2 + a*b^2 + 2*(3*a*
b^2*cosh(d*x + c)^2 + a*b^2)*sinh(d*x + c)^2 + 4*(a*b^2*cosh(d*x + c)^3 + a
*b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x +
c) - sinh(d*x + c))) + (a*b^2*cosh(d*x + c)^4 + 4*a*b^2*cosh(d*x + c)*sinh(
d*x + c)^3 + a*b^2*sinh(d*x + c)^4 + 2*a*b^2*cosh(d*x + c)^2 + a*b^2 + 2*(3
*a*b^2*cosh(d*x + c)^2 + a*b^2)*sinh(d*x + c)^2 + 4*(a*b^2*cosh(d*x + c)^3
+ a*b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) -
sinh(d*x + c))) - (a^2*b + b^3 - 3*(a^2*b + b^3)*cosh(d*x + c)^2 + 4*(a^3 +
 a*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x +
 c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 +
2*a^2*b^2 + b^4)*d*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x +
 c)^2 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 +
 b^4)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d + 4*((a^4 + 2*a^2*b^2
+ b^4)*d*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c))*sinh(d*
x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh (c+dx) \operatorname{sech}^2 (c+dx)}{a+b \sinh (c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)*sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(119) = 238.
time = 0.45, size = 286, normalized size = 2.34

$$-\frac{\frac{4\,ab^3 \log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^4b+2\,a^2b^3+b^5}-\frac{2\,ab^2 \log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(a^2b-b^3\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\left(ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-2\,a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)-2\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)+4\,a^3+8\,ab^2\right)}{\left(a^4+2\,a^2b^2+b^4\right)\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(4*a*b^3*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^4*b + 2*a^2
*b^3 + b^5) - 2*a*b^2*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^4 + 2*a^2*
b^2 + b^4) + (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(a^2*b
 - b^3)/(a^4 + 2*a^2*b^2 + b^4) + 2*(a*b^2*(e^(d*x + c) - e^(-d*x - c))^2 -
 2*a^2*b*(e^(d*x + c) - e^(-d*x - c)) - 2*b^3*(e^(d*x + c) - e^(-d*x - c))
```

```
+ 4*a^3 + 8*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*((e^(d*x + c) - e^(-d*x - c))^2
 + 4)))/d
```

Mupad [B]
time = 1.93, size = 337, normalized size = 2.76

$$\frac{\frac{2\,a}{d\,\left(a^2+b^2\right)}-\frac{2\,b\,\mathrm{e}^{c+d\,x}}{d\,\left(a^2+b^2\right)}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1}-\frac{\frac{2\,\left(a^3+a\,b^2\right)}{d\,{\left(a^2+b^2\right)}^2}-\frac{\mathrm{e}^{c+d\,x}\,\left(a^2\,b+b^3\right)}{d\,{\left(a^2+b^2\right)}^2}}{\mathrm{e}^{2\,c+2\,d\,x}+1}+\frac{b\,\ln\left(1+\mathrm{e}^{c+d\,x}\,1{}\mathrm{i}\right)}{2\,\left(-1{}\mathrm{i}\,d\,a^2+2\,d\,a\,b+1{}\mathrm{i}\,d\,b^2\right)}-\frac{a\,b^2\,\ln\left(b^6\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-14\,a^2\,b^4-a^4\,b^2-b^6+28\,a^3\,b^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+14\,a^2\,b^4\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+a^4\,b^2\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+2\,a\,b^5\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+2\,a^5\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\right)}{d\,a^4+2\,d\,a^2\,b^2+d\,b^4}+\frac{b\,\ln\left(\mathrm{e}^{c+d\,x}+1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2\,\left(-d\,a^2+2{}\mathrm{i}\,d\,a\,b+d\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)/(cosh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

```
[Out] ((2*a)/(d*(a^2 + b^2)) - (2*b*exp(c + d*x))/(d*(a^2 + b^2)))/(2*exp(2*c + 2
*d*x) + exp(4*c + 4*d*x) + 1) - ((2*(a*b^2 + a^3))/(d*(a^2 + b^2)^2) - (exp
(c + d*x)*(a^2*b + b^3))/(d*(a^2 + b^2)^2))/(exp(2*c + 2*d*x) + 1) + (b*log
(exp(c + d*x) + 1i)*1i)/(2*(b^2*d - a^2*d + a*b*d*2i)) + (b*log(exp(c + d*x
)*1i + 1))/(2*(b^2*d*1i - a^2*d*1i + 2*a*b*d)) - (a*b^2*log(b^6*exp(2*c)*ex
p(2*d*x) - 14*a^2*b^4 - a^4*b^2 - b^6 + 28*a^3*b^3*exp(d*x)*exp(c) + 14*a^2
*b^4*exp(2*c)*exp(2*d*x) + a^4*b^2*exp(2*c)*exp(2*d*x) + 2*a*b^5*exp(d*x)*e
xp(c) + 2*a^5*b*exp(d*x)*exp(c)))/(a^4*d + b^4*d + 2*a^2*b^2*d)
```

3.361 $\int \frac{\text{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{\text{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sech[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][(Sech[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\text{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\text{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 115.44, size = 0, normalized size = 0.00

$$\int \frac{\text{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sech[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
[Out] Integrate[(Sech[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(dx+c)^2 \tanh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
[Out] int(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
[Out] (a*f + (b*d*f*x*e^(3*c) - b*f*e^(3*c) + b*d*e^(3*c + 1))*e^(3*d*x) - (2*a*d
*f*x*e^(2*c) - a*f*e^(2*c) + 2*a*d*e^(2*c + 1))*e^(2*d*x) - (b*d*f*x*e^c +
b*d*e^(c + 1) + b*f*e^c)*e^(d*x))/((a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*(a^2
*d^2*f + b^2*d^2*f)*x*e + (a^2*d^2 + b^2*d^2)*e^2 + ((a^2*d^2*f^2*e^(4*c) +
 b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^2*d^2*f*e^(4*c) + b^2*d^2*f*e^(4*c))*x*e +
 (a^2*d^2*e^(4*c) + b^2*d^2*e^(4*c))*e^2)*e^(4*d*x) + 2*((a^2*d^2*f^2*e^(2*
c) + b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^2*d^2*f*e^(2*c) + b^2*d^2*f*e^(2*c))*x
*e + (a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^2)*e^(2*d*x)) - 4*integrate(1/4*
(2*a*b^2*d^2*f^2*x^2 + 4*a*b^2*d^2*f*x*e + 2*a*b^2*d^2*e^2 - 2*a^3*f^2 - 2*
a*b^2*f^2 + (2*a^2*b*f^2*e^c + 2*b^3*f^2*e^c + (a^2*b*d^2*f^2*e^c - b^3*d^2
*f^2*e^c)*x^2 + 2*(a^2*b*d^2*f*e^c - b^3*d^2*f*e^c)*x*e + (a^2*b*d^2*e^c -
b^3*d^2*e^c)*e^2)*e^(d*x))/((a^4*d^2*f^3 + 2*a^2*b^2*d^2*f^3 + b^4*d^2*f^3)
*x^3 + 3*(a^4*d^2*f^2 + 2*a^2*b^2*d^2*f^2 + b^4*d^2*f^2)*x^2*e + 3*(a^4*d^2
*f + 2*a^2*b^2*d^2*f + b^4*d^2*f)*x*e^2 + (a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^
2)*e^3 + ((a^4*d^2*f^3*e^(2*c) + 2*a^2*b^2*d^2*f^3*e^(2*c) + b^4*d^2*f^3*e^
(2*c))*x^3 + 3*(a^4*d^2*f^2*e^(2*c) + 2*a^2*b^2*d^2*f^2*e^(2*c) + b^4*d^2*f
^2*e^(2*c))*x^2*e + 3*(a^4*d^2*f*e^(2*c) + 2*a^2*b^2*d^2*f*e^(2*c) + b^4*d^
2*f*e^(2*c))*x*e^2 + (a^4*d^2*e^(2*c) + 2*a^2*b^2*d^2*e^(2*c) + b^4*d^2*e^(
2*c))*e^3)*e^(2*d*x)), x) + 4*integrate(-1/2*(a^2*b^2*e^(d*x + c) - a*b^3)/
((a^4*b*f + 2*a^2*b^3*f + b^5*f)*x + (a^4*b + 2*a^2*b^3 + b^5)*e - ((a^4*b*
f*e^(2*c) + 2*a^2*b^3*f*e^(2*c) + b^5*f*e^(2*c))*x + (a^4*b*e^(2*c) + 2*a^2
*b^3*e^(2*c) + b^5*e^(2*c))*e)*e^(2*d*x) - 2*((a^5*f*e^c + 2*a^3*b^2*f*e^c
+ a*b^4*f*e^c)*x + (a^5*e^c + 2*a^3*b^2*e^c + a*b^4*e^c)*e)*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(sech(d*x + c)^2*tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(c+dx)\operatorname{sech}^2(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)*sech(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)),
x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tanh(c+d\,x)}{\cosh(c+d\,x)^2\,\left(e+f\,x\right)\,\left(a+b\sinh(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)/(cosh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(tanh(c + d*x)/(cosh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.362 $\int \frac{(e+fx)^3 \cosh(c+dx) \sinh^2(c+dx)}{a+b\sinh(c+dx)} \, dx$

Optimal. Leaf size=606

$$\frac{3f^3x}{8bd^3}+\frac{(e+fx)^3}{4bd}-\frac{a^2(e+fx)^4}{4b^3f}+\frac{6af^3\cosh(c+dx)}{b^2d^4}+\frac{3af(e+fx)^2\cosh(c+dx)}{b^2d^2}+\frac{a^2(e+fx)^3\log\left(1+\frac{b}{a-\sqrt{}}\right.}{b^3d}$$

```
[Out] 3/8*f^3*x/b/d^3+1/4*(f*x+e)^3/b/d-1/4*a^2*(f*x+e)^4/b^3/f+6*a*f^3*cosh(d*x+
c)/b^2/d^4+3*a*f*(f*x+e)^2*cosh(d*x+c)/b^2/d^2+a^2*(f*x+e)^3*ln(1+b*exp(d*x
+c)/(a-(a^2+b^2)^(1/2)))/b^3/d+a^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)
^(1/2)))/b^3/d+3*a^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)
))/b^3/d^2+3*a^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b
^3/d^2-6*a^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d
^3-6*a^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^3+6
*a^2*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^4+6*a^2*f^3*pol
ylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^4-6*a*f^2*(f*x+e)*sinh(d*x+
c)/b^2/d^3-a*(f*x+e)^3*sinh(d*x+c)/b^2/d-3/8*f^3*cosh(d*x+c)*sinh(d*x+c)/b/
d^4-3/4*f*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/b/d^2+3/4*f^2*(f*x+e)*sinh(d*x+
c)^2/b/d^3+1/2*(f*x+e)^3*sinh(d*x+c)^2/b/d
```

Rubi [A]

time = 0.61, antiderivative size = 606, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 14, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5698, 5554, 3392, 32, 2715, 8, 3377, 2718, 5680, 2221, 2611, 6744, 2320, 6724}

$$-\frac{a^2(e+fx)^4}{4b^3f}+\frac{6a^2f^3\text{Li}_4\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^4}+\frac{6a^2f^3\text{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^4}-\frac{6a^2f^2(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}-\frac{6a^2f^2(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^3}+\frac{3a^2f(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{3a^2f(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{a^2(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^3d}+\frac{a^2(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^3d}+\frac{6af^3\cosh(c+dx)}{b^2d^4}-\frac{6af^2(e+fx)\sinh(c+dx)}{b^2d^3}+\frac{3af(e+fx)^2\cosh(c+dx)}{b^2d^2}-\frac{a(e+fx)^3\sinh(c+dx)}{b^2d}-\frac{3f^3\sinh(c+dx)\cosh(c+dx)}{8bd^4}+\frac{3f^2(e+fx)\sinh^2(c+dx)}{4bd^3}-\frac{3f(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{4bd^2}+\frac{(e+fx)^3\sinh^2(c+dx)}{2bd}+\frac{3f^3x}{8bd^3}+\frac{(e+fx)^3}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (3*f^3*x)/(8*b*d^3) + (e + f*x)^3/(4*b*d) - (a^2*(e + f*x)^4)/(4*b^3*f) + (
6*a*f^3*Cosh[c + d*x])/(b^2*d^4) + (3*a*f*(e + f*x)^2*Cosh[c + d*x])/(b^2*d
^2) + (a^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3
*d) + (a^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3
*d) + (3*a^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/(b^3*d^2) + (3*a^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/(b^3*d^2) - (6*a^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^3) - (6*a^2*f^2*(e + f*x)*PolyLog[3, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^3) + (6*a^2*f^3*PolyLog[4,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^4) + (6*a^2*f^3*PolyLog[
4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^4) - (6*a*f^2*(e + f*x
)*Sinh[c + d*x])/(b^2*d^3) - (a*(e + f*x)^3*Sinh[c + d*x])/(b^2*d) - (3*f^3
*Cosh[c + d*x]*Sinh[c + d*x])/(8*b*d^4) - (3*f*(e + f*x)^2*Cosh[c + d*x]*Si
```

nh[c + d*x])/(4*b*d^2) + (3*f^2*(e + f*x)*Sinh[c + d*x]^2)/(4*b*d^3) + ((e + f*x)^3*Sinh[c + d*x]^2)/(2*b*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\cosh(c+d}{a+b\sinh(c}}{b}$$

$$= \frac{(e+fx)^3\sinh^2(c+dx)}{2bd} - \frac{a\int (e+fx)^3\cosh(c+dx)\,dx}{b^2} + \frac{a^2}{}$$

$$= -\frac{a^2(e+fx)^4}{4b^3f} - \frac{a(e+fx)^3\sinh(c+dx)}{b^2d} - \frac{3f(e+fx)^2\cosh}{}$$

$$= \frac{(e+fx)^3}{4bd} - \frac{a^2(e+fx)^4}{4b^3f} + \frac{3af(e+fx)^2\cosh(c+dx)}{b^2d^2} + \frac{a^2(}{}$$

$$= \frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} - \frac{a^2(e+fx)^4}{4b^3f} + \frac{3af(e+fx)^2\cosh(c+d}{b^2d^2}$$

$$= \frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} - \frac{a^2(e+fx)^4}{4b^3f} + \frac{6af^3\cosh(c+dx)}{b^2d^4} + \frac{3af}{}$$

$$= \frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} - \frac{a^2(e+fx)^4}{4b^3f} + \frac{6af^3\cosh(c+dx)}{b^2d^4} + \frac{3af}{}$$

$$= \frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} - \frac{a^2(e+fx)^4}{4b^3f} + \frac{6af^3\cosh(c+dx)}{b^2d^4} + \frac{3af}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2518 vs. $2(606) = 1212$.
time = 11.05, size = 2518, normalized size = 4.16

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] ((-4*b^2*e^3*Log[a + b*Sinh[c + d*x]])/d + (6*b^2*e^2*f*(d*x*(d*x - 2*Log[1
 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*Log[1 + (b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2])]) - 2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] -
 2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/d^2 + (4*b^2*e*f^
2*(d^3*x^3 - 3*d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 3*d
^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 6*d*x*PolyLog[2, (b
*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6*d*x*PolyLog[2, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))] + 6*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2
])] + 6*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/d^3 + (b^2*f
^3*(d^4*x^4 - 4*d^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 4*
d^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 12*d^2*x^2*PolyLog
[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 12*d^2*x^2*PolyLog[2, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 24*d*x*PolyLog[3, (b*E^(c + d*x))/(-a
+ Sqrt[a^2 + b^2])] + 24*d*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2]))] - 24*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 24*PolyLog
[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/d^4 + 2*e*f^2*(2*(4*a^2 + b
^2)*x^3*Coth[c] - (2*(4*a^2 + b^2)*(2*d^3*E^(2*c)*x^3 + 3*d^2*x^2*Log[1 + (
b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*d^2*E^(2*c)*x^2*L
og[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 3*d^2*x^2*L
og[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*d^2*E^(2*
c)*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 6*d
*(-1 + E^(2*c))*x*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*
E^(2*c)]))] - 6*d*(-1 + E^(2*c))*x*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c +
Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sq
rt[(a^2 + b^2)*E^(2*c)]))] + 6*E^(2*c)*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^
c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c
+ Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*E^(2*c)*PolyLog[3, -((b*E^(2*c + d*x))/(
a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(d^3*(-1 + E^(2*c))) - (24*a*b*Cosh[
d*x]*(-2*d*x*Cosh[c] + (2 + d^2*x^2)*Sinh[c]))/d^3 + (3*b^2*Cosh[2*d*x]*((1
 + 2*d^2*x^2)*Cosh[2*c] - 2*d*x*Sinh[2*c]))/d^3 - (24*a*b*((2 + d^2*x^2)*Co
sh[c] - 2*d*x*Sinh[c])*Sinh[d*x])/d^3 + (3*b^2*(-2*d*x*Cosh[2*c] + (1 + 2*d
^2*x^2)*Sinh[2*c])*Sinh[2*d*x])/d^3) + f^3*((4*a^2 + b^2)*x^4*Coth[c] - (2*
(4*a^2 + b^2)*(d^4*E^(2*c)*x^4 + 2*d^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c
 - Sqrt[(a^2 + b^2)*E^(2*c)])] - 2*d^3*E^(2*c)*x^3*Log[1 + (b*E^(2*c + d*x)
)/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*d^3*x^3*Log[1 + (b*E^(2*c + d*x)
)/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 2*d^3*E^(2*c)*x^3*Log[1 + (b*E^(2*
c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 6*d^2*(-1 + E^(2*c))*x^2*P
olyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*d^2
*(-1 + E^(2*c))*x^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2
)*E^(2*c)]))] - 12*d*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 +
b^2)*E^(2*c)]))] + 12*d*E^(2*c)*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - S
qrt[(a^2 + b^2)*E^(2*c)]))] - 12*d*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c
```

```
+ Sqrt[(a^2 + b^2)*E^(2*c)]))] + 12*d*E^(2*c)*x*PolyLog[3, -((b*E^(2*c + d*
x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 12*PolyLog[4, -((b*E^(2*c + d*x
))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 12*E^(2*c)*PolyLog[4, -((b*E^(2*
c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 12*PolyLog[4, -((b*E^(2*c
 + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 12*E^(2*c)*PolyLog[4, -((b
*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(d^4*(-1 + E^(2*c))
) - (16*a*b*Cosh[d*x]*(-3*(2 + d^2*x^2)*Cosh[c] + d*x*(6 + d^2*x^2)*Sinh[c]
))/d^4 + (b^2*Cosh[2*d*x]*(2*d*x*(3 + 2*d^2*x^2)*Cosh[2*c] - 3*(1 + 2*d^2*x
^2)*Sinh[2*c]))/d^4 - (16*a*b*(d*x*(6 + d^2*x^2)*Cosh[c] - 3*(2 + d^2*x^2)*
Sinh[c])*Sinh[d*x])/d^4 + (b^2*(-3*(1 + 2*d^2*x^2)*Cosh[2*c] + 2*d*x*(3 + 2
*d^2*x^2)*Sinh[2*c])*Sinh[2*d*x])/d^4) + (4*e^3*(b^2*Cosh[2*(c + d*x)] + (4
*a^2 + b^2)*Log[a + b*Sinh[c + d*x]] - 4*a*b*Sinh[c + d*x]))/d + (6*e^2*f*(
8*a*b*Cosh[c + d*x] + 2*b^2*d*x*Cosh[2*(c + d*x)] + 2*(4*a^2 + b^2)*(-1/2*(
c + d*x)^2 + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + (c
+ d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - c*Log[a + b*Sinh[c
+ d*x]] + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) - 8*a*b*d*x*Sinh[c + d*x] - b^2*S
inh[2*(c + d*x)]))/d^2)/(16*b^3)
```

Maple [F]
time = 2.78, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \cosh(dx+c)\left(\sinh^2(dx+c)\right)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] 1/8*(8*(d*x + c)*a^2/(b^3*d) - (4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*
d) + 8*a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d) + (4*a*e
^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d))*e^3 + 1/32*(8*a^2*d^4*f^3*x^4*e^
(2*c) + 32*a^2*d^4*f^2*x^3*e^(2*c + 1) + 48*a^2*d^4*f*x^2*e^(2*c + 2) + (4*
b^2*d^3*f^3*x^3*e^(4*c) - 3*b^2*f^3*e^(4*c) - 6*b^2*d^2*f*e^(4*c + 2) + 6*b
^2*d*f^2*e^(4*c + 1) - 6*(b^2*d^2*f^3*e^(4*c) - 2*b^2*d^3*f^2*e^(4*c + 1))*
x^2 + 6*(b^2*d*f^3*e^(4*c) + 2*b^2*d^3*f*e^(4*c + 2) - 2*b^2*d^2*f^2*e^(4*c
```

```
 + 1))*x)*e^(2*d*x) - 16*(a*b*d^3*f^3*x^3*e^(3*c) - 6*a*b*f^3*e^(3*c) - 3*a
*b*d^2*f*e^(3*c + 2) + 6*a*b*d*f^2*e^(3*c + 1) - 3*(a*b*d^2*f^3*e^(3*c) - a
*b*d^3*f^2*e^(3*c + 1))*x^2 + 3*(2*a*b*d*f^3*e^(3*c) + a*b*d^3*f*e^(3*c + 2
) - 2*a*b*d^2*f^2*e^(3*c + 1))*x)*e^(d*x) + 16*(a*b*d^3*f^3*x^3*e^c + 3*a*b
*d^2*f*e^(c + 2) + 6*a*b*d*f^2*e^(c + 1) + 6*a*b*f^3*e^c + 3*(a*b*d^3*f^2*e
^(c + 1) + a*b*d^2*f^3*e^c)*x^2 + 3*(a*b*d^3*f*e^(c + 2) + 2*a*b*d^2*f^2*e^
(c + 1) + 2*a*b*d*f^3*e^c)*x)*e^(-d*x) + (4*b^2*d^3*f^3*x^3 + 6*b^2*d^2*f*e
^2 + 6*b^2*d*f^2*e + 3*b^2*f^3 + 6*(2*b^2*d^3*f^2*e + b^2*d^2*f^3)*x^2 + 6*
(2*b^2*d^3*f*e^2 + 2*b^2*d^2*f^2*e + b^2*d*f^3)*x)*e^(-2*d*x))*e^(-2*c)/(b^
3*d^4) - integrate(-2*(a^2*b*f^3*x^3 + 3*a^2*b*f^2*x^2*e + 3*a^2*b*f*x*e^2
- (a^3*f^3*x^3*e^c + 3*a^3*f^2*x^2*e^(c + 1) + 3*a^3*f*x*e^(c + 2))*e^(d*x)
)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7049 vs. $2(580) = 1160$.
time = 0.47, size = 7049, normalized size = 11.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/32*(4*b^2*d^3*f^3*x^3 + 6*b^2*d^2*f^3*x^2 + 4*b^2*d^3*cosh(1)^3 + 4*b^2*d
^3*sinh(1)^3 + 6*b^2*d*f^3*x + 3*b^2*f^3 + (4*b^2*d^3*f^3*x^3 - 6*b^2*d^2*f
^3*x^2 + 4*b^2*d^3*cosh(1)^3 + 4*b^2*d^3*sinh(1)^3 + 6*b^2*d*f^3*x - 3*b^2*
f^3 + 6*(2*b^2*d^3*f*x - b^2*d^2*f)*cosh(1)^2 + 6*(2*b^2*d^3*f*x + 2*b^2*d^
3*cosh(1) - b^2*d^2*f)*sinh(1)^2 + 6*(2*b^2*d^3*f^2*x^2 - 2*b^2*d^2*f^2*x +
 b^2*d*f^2)*cosh(1) + 6*(2*b^2*d^3*f^2*x^2 - 2*b^2*d^2*f^2*x + 2*b^2*d^3*co
sh(1)^2 + b^2*d*f^2 + 2*(2*b^2*d^3*f*x - b^2*d^2*f)*cosh(1))*sinh(1))*cosh(
d*x + c)^4 + (4*b^2*d^3*f^3*x^3 - 6*b^2*d^2*f^3*x^2 + 4*b^2*d^3*cosh(1)^3 +
 4*b^2*d^3*sinh(1)^3 + 6*b^2*d*f^3*x - 3*b^2*f^3 + 6*(2*b^2*d^3*f*x - b^2*d
^2*f)*cosh(1)^2 + 6*(2*b^2*d^3*f*x + 2*b^2*d^3*cosh(1) - b^2*d^2*f)*sinh(1)
^2 + 6*(2*b^2*d^3*f^2*x^2 - 2*b^2*d^2*f^2*x + b^2*d*f^2)*cosh(1) + 6*(2*b^2
*d^3*f^2*x^2 - 2*b^2*d^2*f^2*x + 2*b^2*d^3*cosh(1)^2 + b^2*d*f^2 + 2*(2*b^2
*d^3*f*x - b^2*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^4 - 16*(a*b*d^3*f^3*x
^3 - 3*a*b*d^2*f^3*x^2 + a*b*d^3*cosh(1)^3 + a*b*d^3*sinh(1)^3 + 6*a*b*d*f^
3*x - 6*a*b*f^3 + 3*(a*b*d^3*f*x - a*b*d^2*f)*cosh(1)^2 + 3*(a*b*d^3*f*x +
a*b*d^3*cosh(1) - a*b*d^2*f)*sinh(1)^2 + 3*(a*b*d^3*f^2*x^2 - 2*a*b*d^2*f^2
*x + 2*a*b*d*f^2)*cosh(1) + 3*(a*b*d^3*f^2*x^2 - 2*a*b*d^2*f^2*x + a*b*d^3*
cosh(1)^2 + 2*a*b*d*f^2 + 2*(a*b*d^3*f*x - a*b*d^2*f)*cosh(1))*sinh(1))*cos
h(d*x + c)^3 - 4*(4*a*b*d^3*f^3*x^3 - 12*a*b*d^2*f^3*x^2 + 4*a*b*d^3*cosh(1
)^3 + 4*a*b*d^3*sinh(1)^3 + 24*a*b*d*f^3*x - 24*a*b*f^3 + 12*(a*b*d^3*f*x -
 a*b*d^2*f)*cosh(1)^2 + 12*(a*b*d^3*f*x + a*b*d^3*cosh(1) - a*b*d^2*f)*sinh
(1)^2 + 12*(a*b*d^3*f^2*x^2 - 2*a*b*d^2*f^2*x + 2*a*b*d*f^2)*cosh(1) - (4*b
```

^2*d^3*f^3*x^3 - 6*b^2*d^2*f^3*x^2 + 4*b^2*d^3*cosh(1)^3 + 4*b^2*d^3*sinh(1
)^3 + 6*b^2*d*f^3*x - 3*b^2*f^3 + 6*(2*b^2*d^3*f*x - b^2*d^2*f)*cosh(1)^2 +
6*(2*b^2*d^3*f*x + 2*b^2*d^3*cosh(1) - b^2*d^2*f)*sinh(1)^2 + 6*(2*b^2*d^3
*f^2*x^2 - 2*b^2*d^2*f^2*x + b^2*d*f^2)*cosh(1) + 6*(2*b^2*d^3*f^2*x^2 - 2*
b^2*d^2*f^2*x + 2*b^2*d^3*cosh(1)^2 + b^2*d*f^2 + 2*(2*b^2*d^3*f*x - b^2*d^
2*f)*cosh(1))*sinh(1))*cosh(d*x + c) + 12*(a*b*d^3*f^2*x^2 - 2*a*b*d^2*f^2*
x + a*b*d^3*cosh(1)^2 + 2*a*b*d*f^2 + 2*(a*b*d^3*f*x - a*b*d^2*f)*cosh(1))*
sinh(1))*sinh(d*x + c)^3 + 6*(2*b^2*d^3*f*x + b^2*d^2*f)*cosh(1)^2 - 8*(a^2
*d^4*f^3*x^4 - 2*a^2*c^4*f^3 + 4*(a^2*d^4*x + 2*a^2*c*d^3)*cosh(1)^3 + 4*(a
^2*d^4*x + 2*a^2*c*d^3)*sinh(1)^3 + 6*(a^2*d^4*f*x^2 - 2*a^2*c^2*d^2*f)*cos
h(1)^2 + 6*(a^2*d^4*f*x^2 - 2*a^2*c^2*d^2*f + 2*(a^2*d^4*x + 2*a^2*c*d^3)*c
osh(1))*sinh(1)^2 + 4*(a^2*d^4*f^2*x^3 + 2*a^2*c^3*d*f^2)*cosh(1) + 4*(a^2*
d^4*f^2*x^3 + 2*a^2*c^3*d*f^2 + 3*(a^2*d^4*x + 2*a^2*c*d^3)*cosh(1)^2 + 3*(
a^2*d^4*f*x^2 - 2*a^2*c^2*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 6*(2*b
^2*d^3*f*x + 2*b^2*d^3*cosh(1) + b^2*d^2*f)*sinh(1)^2 - 2*(4*a^2*d^4*f^3*x^
4 - 8*a^2*c^4*f^3 + 16*(a^2*d^4*x + 2*a^2*c*d^3)*cosh(1)^3 + 16*(a^2*d^4*x
+ 2*a^2*c*d^3)*sinh(1)^3 + 24*(a^2*d^4*f*x^2 - 2*a^2*c^2*d^2*f)*cosh(1)^2 -
3*(4*b^2*d^3*f^3*x^3 - 6*b^2*d^2*f^3*x^2 + 4*b^2*d^3*cosh(1)^3 + 4*b^2*d^3
*sinh(1)^3 + 6*b^2*d*f^3*x - 3*b^2*f^3 + 6*(2*b^2*d^3*f*x - b^2*d^2*f)*cosh
(1)^2 + 6*(2*b^2*d^3*f*x + 2*b^2*d^3*cosh(1) - b^2*d^2*f)*sinh(1)^2 + 6*(2*
b^2*d^3*f^2*x^2 - 2*b^2*d^2*f^2*x + b^2*d*f^2)*cosh(1) + 6*(2*b^2*d^3*f^2*x
^2 - 2*b^2*d^2*f^2*x + 2*b^2*d^3*cosh(1)^2 + b^2*d*f^2 + 2*(2*b^2*d^3*f*x -
b^2*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 24*(a^2*d^4*f*x^2 - 2*a^2*c
^2*d^2*f + 2*(a^2*d^4*x + 2*a^2*c*d^3)*cosh(1))*sinh(1)^2 + 16*(a^2*d^4*f^2
*x^3 + 2*a^2*c^3*d*f^2)*cosh(1) + 24*(a*b*d^3*f^3*x^3 - 3*a*b*d^2*f^3*x^2 +
a*b*d^3*cosh(1)^3 + a*b*d^3*sinh(1)^3 + 6*a*b*d*f^3*x - 6*a*b*f^3 + 3*(a*b
*d^3*f*x - a*b*d^2*f)*cosh(1)^2 + 3*(a*b*d^3*f*x + a*b*d^3*cosh(1) - a*b*d^
2*f)*sinh(1)^2 + 3*(a*b*d^3*f^2*x^2 - 2*a*b*d^2*f^2*x + 2*a*b*d*f^2)*cosh(1
) + 3*(a*b*d^3*f^2*x^2 - 2*a*b*d^2*f^2*x + a*b*d^3*cosh(1)^2 + 2*a*b*d*f^2
+ 2*(a*b*d^3*f*x - a*b*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c) + 16*(a^2*d^4
*f^2*x^3 + 2*a^2*c^3*d*f^2 + 3*(a^2*d^4*x + 2*a^2*c*d^3)*cosh(1)^2 + 3*(a^2
*d^4*f*x^2 - 2*a^2*c^2*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^2 + 6*(2*b^2*
d^3*f^2*x^2 + 2*b^2*d^2*f^2*x + b^2*d*f^2)*cosh(1) + 16*(a*b*d^3*f^3*x^3 +
3*a*b*d^2*f^3*x^2 + a*b*d^3*cosh(1)^3 + a*b*d^3*sinh(1)^3 + 6*a*b*d*f^3*x +
6*a*b*f^3 + 3*(a*b*d^3*f*x + a*b*d^2*f)*cosh(1)^2 + 3*(a*b*d^3*f*x + a*b*d
^3*cosh(1) + a*b*d^2*f)*sinh(1)^2 + 3*(a*b*d^3*f^2*x^2 + 2*a*b*d^2*f^2*x +
2*a*b*d*f^2)*cosh(1) + 3*(a*b*d^3*f^2*x^2 + 2*a*b*d^2*f^2*x + a*b*d^3*cosh(
1)^2 + 2*a*b*d*f^2 + 2*(a*b*d^3*f*x + a*b*d^2*f)*cosh(1))*sinh(1))*cosh(d*x
+ c) + 96*((a^2*d^2*f^3*x^2 + 2*a^2*d^2*f^2*x*cosh(1) + a^2*d^2*f*cosh(1)^
2 + a^2*d^2*f*sinh(1)^2 + 2*(a^2*d^2*f^2*x + a^2*d^2*f*cosh(1))*sinh(1))*co
sh(d*x + c)^2 + 2*(a^2*d^2*f^3*x^2 + 2*a^2*d^2*f^2*x*cosh(1) + a^2*d^2*f*co
sh(1)^2 + a^2*d^2*f*sinh(1)^2 + 2*(a^2*d^2*f^2*x + a^2*d^2*f*cosh(1))*sinh(
1))*cosh(d*x + c)*sinh(d*x + c) + (a^2*d^2*f^3*x^2 + 2*a^2*d^2*f^2*x*cosh(1
) + a^2*d^2*f*cosh(1)^2 + a^2*d^2*f*sinh(1)^2 + 2*(a^2*d^2*f^2*x + a^2*d^2*
f*cosh(1))*sinh(1))*sinh(d*x + c)^2)*dilog((a*c...

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)*sinh(d*x + c)^2/(b*sinh(d*x + c) + a),
x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.363 $\int \frac{(e+fx)^2 \cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=449

$$\frac{efx}{2bd}+\frac{f^2x^2}{4bd}-\frac{a^2(e+fx)^3}{3b^3f}+\frac{2af(e+fx)\cosh(c+dx)}{b^2d^2}+\frac{a^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d}+\frac{a^2(e+fx)^2\log}{}$$

```
[Out] 1/2*e*f*x/b/d+1/4*f^2*x^2/b/d-1/3*a^2*(f*x+e)^3/b^3/f+2*a*f*(f*x+e)*cosh(d*
x+c)/b^2/d^2+a^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d+a^2
*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d+2*a^2*f*(f*x+e)*pol
ylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d^2+2*a^2*f*(f*x+e)*polylog(2
,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^2-2*a^2*f^2*polylog(3,-b*exp(d*x+
c)/(a-(a^2+b^2)^(1/2)))/b^3/d^3-2*a^2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))/b^3/d^3-2*a*f^2*sinh(d*x+c)/b^2/d^3-a*(f*x+e)^2*sinh(d*x+c)/b^2
/d-1/2*f*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/b/d^2+1/4*f^2*sinh(d*x+c)^2/b/d^3+
1/2*(f*x+e)^2*sinh(d*x+c)^2/b/d
```

Rubi [A]

time = 0.51, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5698, 5554, 3391, 3377, 2717, 5680, 2221, 2611, 2320, 6724}

$$-\frac{a^2(e+fx)^3}{3b^3f}-\frac{2a^2f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}-\frac{2a^2f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^3}+\frac{2a^2f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{2a^2f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{a^2(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^3d}+\frac{a^2(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^3d}-\frac{2af^2\sinh(c+dx)}{b^2d^3}+\frac{2af(e+fx)\cosh(c+dx)}{b^2d^2}-\frac{a(e+fx)^2\sinh(c+dx)}{b^2d}+\frac{f^2\sinh^2(c+dx)}{4bd^3}-\frac{f(e+fx)\sinh(c+dx)\cosh(c+dx)}{2bd^2}+\frac{(e+fx)^2\sinh^2(c+dx)}{2bd}+\frac{efx}{2bd}+\frac{f^2x^2}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e*f*x)/(2*b*d) + (f^2*x^2)/(4*b*d) - (a^2*(e + f*x)^3)/(3*b^3*f) + (2*a*f*
(e + f*x)*Cosh[c + d*x])/(b^2*d^2) + (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2])])/(b^3*d) + (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2])])/(b^3*d) + (2*a^2*f*(e + f*x)*PolyLog[2, -((b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^2) + (2*a^2*f*(e + f*x)*PolyLog[2
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^2) - (2*a^2*f^2*PolyLog
[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^3) - (2*a^2*f^2*PolyL
og[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^3) - (2*a*f^2*Sinh[
c + d*x])/(b^2*d^3) - (a*(e + f*x)^2*Sinh[c + d*x])/(b^2*d) - (f*(e + f*x)*
Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d^2) + (f^2*Sinh[c + d*x]^2)/(4*b*d^3) +
((e + f*x)^2*Sinh[c + d*x]^2)/(2*b*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)\,dx}{b} - \frac{a\int\frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+}}{b}$$

$$= \frac{(e+fx)^2\sinh^2(c+dx)}{2bd} - \frac{a\int(e+fx)^2\cosh(c+dx)\,dx}{b^2} + \frac{a^2}{}$$

$$= -\frac{a^2(e+fx)^3}{3b^3f} - \frac{a(e+fx)^2\sinh(c+dx)}{b^2d} - \frac{f(e+fx)\cosh(c+}{2b}$$

$$= \frac{efx}{2bd} + \frac{f^2x^2}{4bd} - \frac{a^2(e+fx)^3}{3b^3f} + \frac{2af(e+fx)\cosh(c+dx)}{b^2d^2} + \frac{a^2(}{}$$

$$= \frac{efx}{2bd} + \frac{f^2x^2}{4bd} - \frac{a^2(e+fx)^3}{3b^3f} + \frac{2af(e+fx)\cosh(c+dx)}{b^2d^2} + \frac{a^2(}{}$$

$$= \frac{efx}{2bd} + \frac{f^2x^2}{4bd} - \frac{a^2(e+fx)^3}{3b^3f} + \frac{2af(e+fx)\cosh(c+dx)}{b^2d^2} + \frac{a^2(}{}$$

$$= \frac{efx}{2bd} + \frac{f^2x^2}{4bd} - \frac{a^2(e+fx)^3}{3b^3f} + \frac{2af(e+fx)\cosh(c+dx)}{b^2d^2} + \frac{a^2(}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1324 vs. $2(449) = 898$.
time $= 3.39$, size $= 1324$, normalized size $= 2.95$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] ((-6*b^2*e^2*Log[a + b*Sinh[c + d*x]])/d + (6*b^2*e*f*(d*x*(d*x - 2*Log[1 +
 (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*Log[1 + (b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2])]) - 2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2
*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/d^2 + (2*b^2*f^2*(d
^3*x^3 - 3*d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 3*d^2*x
^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 6*d*x*PolyLog[2, (b*E^(
c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6*d*x*PolyLog[2, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2]))] + 6*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
```

```
+ 6*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/d^3 + f^2*(2*(4*
a^2 + b^2)*x^3*Coth[c] - (2*(4*a^2 + b^2)*(2*d^3*E^(2*c)*x^3 + 3*d^2*x^2*Lo
g[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*d^2*E^(2*c
)*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 3*d^
2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*d^
2*E^(2*c)*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])
] - 6*d*(-1 + E^(2*c))*x*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2
+ b^2)*E^(2*c)]))] - 6*d*(-1 + E^(2*c))*x*PolyLog[2, -((b*E^(2*c + d*x))/(a
*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*PolyLog[3, -((b*E^(2*c + d*x))/(a*E
^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*E^(2*c)*PolyLog[3, -((b*E^(2*c + d*x)
)/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*PolyLog[3, -((b*E^(2*c + d*x))/
(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*E^(2*c)*PolyLog[3, -((b*E^(2*c +
d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(d^3*(-1 + E^(2*c))) - (24*a*
b*Cosh[d*x]*(-2*d*x*Cosh[c] + (2 + d^2*x^2)*Sinh[c]))/d^3 + (3*b^2*Cosh[2*d
*x]*((1 + 2*d^2*x^2)*Cosh[2*c] - 2*d*x*Sinh[2*c]))/d^3 - (24*a*b*((2 + d^2*
x^2)*Cosh[c] - 2*d*x*Sinh[c])*Sinh[d*x])/d^3 + (3*b^2*(-2*d*x*Cosh[2*c] + (
1 + 2*d^2*x^2)*Sinh[2*c])*Sinh[2*d*x])/d^3) + (6*e^2*(b^2*Cosh[2*(c + d*x)]
 + (4*a^2 + b^2)*Log[a + b*Sinh[c + d*x]] - 4*a*b*Sinh[c + d*x]))/d + (6*e*
f*(8*a*b*Cosh[c + d*x] + 2*b^2*d*x*Cosh[2*(c + d*x)] + 2*(4*a^2 + b^2)*(-1/
2*(c + d*x)^2 + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] +
(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - c*Log[a + b*Sinh
[c + d*x]] + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + PolyLog[2
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) - 8*a*b*d*x*Sinh[c + d*x] - b^
2*Sinh[2*(c + d*x)]))/d^2)/(24*b^3)
```

Maple [F]
time = 2.78, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \cosh(dx+c) \left(\sinh^2(dx+c)\right)}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
[Out] int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] 1/8*(8*(d*x + c)*a^2/(b^3*d) - (4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*
d) + 8*a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d) + (4*a*e
^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d))*e^2 + 1/48*(16*a^2*d^3*f^2*x^3*e
^(2*c) + 48*a^2*d^3*f*x^2*e^(2*c + 1) + 3*(2*b^2*d^2*f^2*x^2*e^(4*c) + b^2*
f^2*e^(4*c) - 2*b^2*d*f*e^(4*c + 1) - 2*(b^2*d*f^2*e^(4*c) - 2*b^2*d^2*f*e^
(4*c + 1))*x)*e^(2*d*x) - 24*(a*b*d^2*f^2*x^2*e^(3*c) + 2*a*b*f^2*e^(3*c) -
 2*a*b*d*f*e^(3*c + 1) - 2*(a*b*d*f^2*e^(3*c) - a*b*d^2*f*e^(3*c + 1))*x)*e
^(d*x) + 24*(a*b*d^2*f^2*x^2*e^c + 2*a*b*d*f*e^(c + 1) + 2*a*b*f^2*e^c + 2*
(a*b*d^2*f*e^(c + 1) + a*b*d*f^2*e^c)*x)*e^(-d*x) + 3*(2*b^2*d^2*f^2*x^2 +
2*b^2*d*f*e + b^2*f^2 + 2*(2*b^2*d^2*f*e + b^2*d*f^2)*x)*e^(-2*d*x))*e^(-2*
c)/(b^3*d^3) - integrate(-2*(a^2*b*f^2*x^2 + 2*a^2*b*f*x*e - (a^3*f^2*x^2*e
^c + 2*a^3*f*x*e^(c + 1))*e^(d*x))/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x +
c) - b^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3541 vs. $2(427) = 854$.
time = 0.39, size = 3541, normalized size = 7.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/48*(6*b^2*d^2*f^2*x^2 + 6*b^2*d*f^2*x + 6*b^2*d^2*cosh(1)^2 + 6*b^2*d^2*s
inh(1)^2 + 3*(2*b^2*d^2*f^2*x^2 - 2*b^2*d*f^2*x + 2*b^2*d^2*cosh(1)^2 + 2*b
^2*d^2*sinh(1)^2 + b^2*f^2 + 2*(2*b^2*d^2*f*x - b^2*d*f)*cosh(1) + 2*(2*b^2
*d^2*f*x + 2*b^2*d^2*cosh(1) - b^2*d*f)*sinh(1))*cosh(d*x + c)^4 + 3*(2*b^2
*d^2*f^2*x^2 - 2*b^2*d*f^2*x + 2*b^2*d^2*cosh(1)^2 + 2*b^2*d^2*sinh(1)^2 +
b^2*f^2 + 2*(2*b^2*d^2*f*x - b^2*d*f)*cosh(1) + 2*(2*b^2*d^2*f*x + 2*b^2*d^
2*cosh(1) - b^2*d*f)*sinh(1))*sinh(d*x + c)^4 + 3*b^2*f^2 - 24*(a*b*d^2*f^2
*x^2 - 2*a*b*d*f^2*x + a*b*d^2*cosh(1)^2 + a*b*d^2*sinh(1)^2 + 2*a*b*f^2 +
2*(a*b*d^2*f*x - a*b*d*f)*cosh(1) + 2*(a*b*d^2*f*x + a*b*d^2*cosh(1) - a*b*
d*f)*sinh(1))*cosh(d*x + c)^3 - 12*(2*a*b*d^2*f^2*x^2 - 4*a*b*d*f^2*x + 2*a
*b*d^2*cosh(1)^2 + 2*a*b*d^2*sinh(1)^2 + 4*a*b*f^2 + 4*(a*b*d^2*f*x - a*b*d
*f)*cosh(1) - (2*b^2*d^2*f^2*x^2 - 2*b^2*d*f^2*x + 2*b^2*d^2*cosh(1)^2 + 2*
b^2*d^2*sinh(1)^2 + b^2*f^2 + 2*(2*b^2*d^2*f*x - b^2*d*f)*cosh(1) + 2*(2*b^
2*d^2*f*x + 2*b^2*d^2*cosh(1) - b^2*d*f)*sinh(1))*cosh(d*x + c) + 4*(a*b*d^
2*f*x + a*b*d^2*cosh(1) - a*b*d*f)*sinh(1))*sinh(d*x + c)^3 - 16*(a^2*d^3*f
^2*x^3 + 2*a^2*c^3*f^2 + 3*(a^2*d^3*x + 2*a^2*c*d^2)*cosh(1)^2 + 3*(a^2*d^3
*x + 2*a^2*c*d^2)*sinh(1)^2 + 3*(a^2*d^3*f*x^2 - 2*a^2*c^2*d*f)*cosh(1) + 3
*(a^2*d^3*f*x^2 - 2*a^2*c^2*d*f + 2*(a^2*d^3*x + 2*a^2*c*d^2)*cosh(1))*sinh
(1))*cosh(d*x + c)^2 - 2*(8*a^2*d^3*f^2*x^3 + 16*a^2*c^3*f^2 + 24*(a^2*d^3*
x + 2*a^2*c*d^2)*cosh(1)^2 - 9*(2*b^2*d^2*f^2*x^2 - 2*b^2*d*f^2*x + 2*b^2*d
^2*cosh(1)^2 + 2*b^2*d^2*sinh(1)^2 + b^2*f^2 + 2*(2*b^2*d^2*f*x - b^2*d*f)*
```

cosh(1) + 2*(2*b^2*d^2*f*x + 2*b^2*d^2*cosh(1) - b^2*d*f)*sinh(1))*cosh(d*x + c)^2 + 24*(a^2*d^3*x + 2*a^2*c*d^2)*sinh(1)^2 + 24*(a^2*d^3*f*x^2 - 2*a^2*c^2*d*f)*cosh(1) + 36*(a*b*d^2*f^2*x^2 - 2*a*b*d*f^2*x + a*b*d^2*cosh(1)^2 + a*b*d^2*sinh(1)^2 + 2*a*b*f^2 + 2*(a*b*d^2*f*x - a*b*d*f)*cosh(1) + 2*(a*b*d^2*f*x + a*b*d^2*cosh(1) - a*b*d*f)*sinh(1))*cosh(d*x + c) + 24*(a^2*d^3*f*x^2 - 2*a^2*c^2*d*f + 2*(a^2*d^3*x + 2*a^2*c*d^2)*cosh(1))*sinh(1))*sinh(d*x + c)^2 + 6*(2*b^2*d^2*f*x + b^2*d*f)*cosh(1) + 24*(a*b*d^2*f^2*x^2 + 2*a*b*d*f^2*x + a*b*d^2*cosh(1)^2 + a*b*d^2*sinh(1)^2 + 2*a*b*f^2 + 2*(a*b*d^2*f*x + a*b*d*f)*cosh(1) + 2*(a*b*d^2*f*x + a*b*d^2*cosh(1) + a*b*d*f)*sinh(1))*cosh(d*x + c) + 96*((a^2*d*f^2*x + a^2*d*f*cosh(1) + a^2*d*f*sinh(1))*cosh(d*x + c)^2 + 2*(a^2*d*f^2*x + a^2*d*f*cosh(1) + a^2*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^2*d*f^2*x + a^2*d*f*cosh(1) + a^2*d*f*sinh(1))*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 96*((a^2*d*f^2*x + a^2*d*f*cosh(1) + a^2*d*f*sinh(1))*cosh(d*x + c)^2 + 2*(a^2*d*f^2*x + a^2*d*f*cosh(1) + a^2*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^2*d*f^2*x + a^2*d*f*cosh(1) + a^2*d*f*sinh(1))*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 48*((a^2*c^2*f^2 - 2*a^2*c*d*f*cosh(1) + a^2*d^2*cosh(1)^2 + a^2*d^2*sinh(1)^2 - 2*(a^2*c*d*f - a^2*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(a^2*c^2*f^2 - 2*a^2*c*d*f*cosh(1) + a^2*d^2*cosh(1)^2 + a^2*d^2*sinh(1)^2 - 2*(a^2*c*d*f - a^2*d^2*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^2*c^2*f^2 - 2*a^2*c*d*f*cosh(1) + a^2*d^2*cosh(1)^2 + a^2*d^2*sinh(1)^2 - 2*(a^2*c*d*f - a^2*d^2*cosh(1))*sinh(1))*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 48*((a^2*c^2*f^2 - 2*a^2*c*d*f*cosh(1) + a^2*d^2*cosh(1)^2 + a^2*d^2*sinh(1)^2 - 2*(a^2*c*d*f - a^2*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(a^2*c^2*f^2 - 2*a^2*c*d*f*cosh(1) + a^2*d^2*cosh(1)^2 + a^2*d^2*sinh(1)^2 - 2*(a^2*c*d*f - a^2*d^2*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^2*c^2*f^2 - 2*a^2*c*d*f*cosh(1) + a^2*d^2*cosh(1)^2 + a^2*d^2*sinh(1)^2 - 2*(a^2*c*d*f - a^2*d^2*cosh(1))*sinh(1))*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 48*((a^2*d^2*f^2*x^2 - a^2*c^2*f^2 + 2*(a^2*d^2*f*x + a^2*c*d*f)*cosh(1) + 2*(a^2*d^2*f*x + a^2*c*d*f)*sinh(1))*cosh(d*x + c)^2 + 2*(a^2*d^2*f^2*x^2 - a^2*c^2*f^2 + 2*(a^2*d^2*f*x + a^2*c*d*f)*cosh(1) + 2*(a^2*d^2*f*x + a^2*c*d*f)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^2*d^2*f^2*x^2 - a^2*c^2*f^2 + 2*(a^2*d^2*f*x + a^2*c*d*f)*cosh(1) + 2*(a^2*d^2*f*x + a^2*c*d*f)*sinh(1))*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 48*((a^2*d^2*f^2*x^2 - a^2*c^2*f^2 + 2*(a^2*d^2*f*x + a^2*c*d*f)*cosh(1) + 2*(a^2*d^2*f*x + a^2*c*d*f)*sinh(1))*cosh(d*x + c)^2 + 2*(a^2*d^2*f^2*x^2 - a^2*c^2*f^2 + 2*(a^2*d^2*f*x + a^2*c*d*f)*cosh(1) + 2*(a^2*d^2*f*x + a^2*c*d*f)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^2*d^2*f^2*x^2 - a^2*c^2*f^2 + 2*(a^2*d^2*f*x + a^2*c*d*f)*cosh(1) + 2*(a^2*d^2*f*x + a^2*c*d*f)*sinh(1))*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b

```
^2) - b)/b) - 96*(a^2*f^2*cosh(d*x + c)^2 + 2*a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
[Out] integrate((f*x + e)^2*cosh(d*x + c)*sinh(d*x + c)^2/(b*sinh(d*x + c) + a),
x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^2\,(e+f\,x)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.364 $\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=278

$$\frac{fx}{4bd}-\frac{a^2(e+fx)^2}{2b^3f}+\frac{af\cosh(c+dx)}{b^2d^2}+\frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d}+\frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d}$$

```
[Out] 1/4*f*x/b/d-1/2*a^2*(f*x+e)^2/b^3/f+a*f*cosh(d*x+c)/b^2/d^2+a^2*(f*x+e)*ln(
1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^3/d+a^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))/b^3/d+a^2*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/
b^3/d^2+a^2*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^3/d^2-a*(f*x+e
)*sinh(d*x+c)/b^2/d-1/4*f*cosh(d*x+c)*sinh(d*x+c)/b/d^2+1/2*(f*x+e)*sinh(d*
x+c)^2/b/d
```

Rubi [A]
time = 0.30, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5698, 5554, 2715, 8, 3377, 2718, 5680, 2221, 2317, 2438}

$$-\frac{a^2(e+fx)^2}{2b^3f}+\frac{a^2f\,\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{a^2f\,\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{a^2(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^3d}+\frac{a^2(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^3d}+\frac{af\cosh(c+dx)}{b^2d^2}-\frac{a(e+fx)\sinh(c+dx)}{b^2d}-\frac{f\sinh(c+dx)\cosh(c+dx)}{4bd^2}+\frac{(e+fx)\sinh^2(c+dx)}{2bd}+\frac{fx}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (f*x)/(4*b*d) - (a^2*(e + f*x)^2)/(2*b^3*f) + (a*f*Cosh[c + d*x])/(b^2*d^2)
 + (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) +
 (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*d) + (
a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^2) + (a^
2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^2) - (a*(e
 + f*x)*Sinh[c + d*x])/(b^2*d) - (f*Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^2)
+ ((e + f*x)*Sinh[c + d*x]^2)/(2*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
```

```
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\cosh(c+dx)\,\mathrm{si}}{a+b\sinh(c+d}}{b}$$

$$= \frac{(e+fx)\sinh^2(c+dx)}{2bd} - \frac{a\int (e+fx)\cosh(c+dx)\,dx}{b^2} + \frac{a^2\int}{}$$

$$= -\frac{a^2(e+fx)^2}{2b^3f} - \frac{a(e+fx)\sinh(c+dx)}{b^2d} - \frac{f\cosh(c+dx)\,\mathrm{sinh}}{4bd^2}$$

$$= \frac{fx}{4bd} - \frac{a^2(e+fx)^2}{2b^3f} + \frac{af\cosh(c+dx)}{b^2d^2} + \frac{a^2(e+fx)\log\left(1+\right.}{b^3d}$$

$$= \frac{fx}{4bd} - \frac{a^2(e+fx)^2}{2b^3f} + \frac{af\cosh(c+dx)}{b^2d^2} + \frac{a^2(e+fx)\log\left(1+\right.}{b^3d}$$

$$= \frac{fx}{4bd} - \frac{a^2(e+fx)^2}{2b^3f} + \frac{af\cosh(c+dx)}{b^2d^2} + \frac{a^2(e+fx)\log\left(1+\right.}{b^3d}$$

Mathematica [A]
time = 0.70, size = 422, normalized size = 1.52

$$\frac{-2b^2de\log(a+b\sinh(c+dx)) + b^2f\left(dx\left(dx - 2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) - 2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right) - 2\mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right) - 2\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right) + 2de(b^2\cosh(2(c+dx)) + (4a^2+b^2)\log(a+b\sinh(c+dx)) - 4ab\sinh(c+dx)) + f\left(8ab\cosh(c+dx) + 2b^2dx\cosh(2(c+dx)) + 2(4a^2+b^2)\left(-\frac{1}{2}(c+dx)^2 + (c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) + (c+dx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right) - c\log(a+b\sinh(c+dx)) + \mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right) + \mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right) - 8abdx\sinh(c+dx) - b^2\sinh(2(c+dx))\right)}{8b^3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (-2*b^2*d*e*Log[a + b*Sinh[c + d*x]] + b^2*f*(d*x*(d*x - 2*Log[1 + (b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2])] - 2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2])]) - 2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*PolyLog[
2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) + 2*d*e*(b^2*Cosh[2*(c + d*x)
] + (4*a^2 + b^2)*Log[a + b*Sinh[c + d*x]] - 4*a*b*Sinh[c + d*x]) + f*(8*a*
b*Cosh[c + d*x] + 2*b^2*d*x*Cosh[2*(c + d*x)] + 2*(4*a^2 + b^2)*(-1/2*(c +
```

```
d*x)^2 + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + (c + d*
x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - c*Log[a + b*Sinh[c + d*
x]] + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + PolyLog[2, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) - 8*a*b*d*x*Sinh[c + d*x] - b^2*Sinh[
2*(c + d*x)]))/(8*b^3*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(258) = 516$.
time = 3.74, size = 565, normalized size = 2.03

method	result
risch	$-\frac{a^2 f x^2}{2b^3}+\frac{a^2 e x}{b^3}+\frac{(2dxf+2de-f)e^{2dx+2c}}{16d^2 b}-\frac{a(dxf+de-f)e^{dx+c}}{2b^2 d^2}+\frac{a(dxf+de+f)e^{-dx-c}}{2b^2 d^2}+\frac{(2dxf+2de+f)e^{-2dx-2c}}{16d^2 b}+$ 2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/2*a^2*f*x^2/b^3+a^2*e*x/b^3+1/16*(2*d*f*x+2*d*e-f)/d^2/b*exp(2*d*x+2*c)-
1/2*a*(d*f*x+d*e-f)/b^2/d^2*exp(d*x+c)+1/2*a*(d*f*x+d*e+f)/b^2/d^2*exp(-d*x
-c)+1/16*(2*d*f*x+2*d*e+f)/d^2/b*exp(-2*d*x-2*c)+2/d^2*a^2/b^3*f*c*ln(exp(d
*x+c))-1/d^2*a^2/b^3*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d*a^2/b^3*
e*ln(exp(d*x+c))+1/d*a^2/b^3*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d*a^
2/b^3*c*f*x-1/d^2*a^2/b^3*f*c^2+1/d*a^2/b^3*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(
1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d^2*a^2/b^3*f*ln((-b*exp(d*x+c)+(a^2+b^2)
^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d*a^2/b^3*f*ln((b*exp(d*x+c)+(a^2+b^2)^
(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2*a^2/b^3*f*ln((b*exp(d*x+c)+(a^2+b^2)^
(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d^2*a^2/b^3*f*dilog((b*exp(d*x+c)+(a^2+b^
2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/d^2*a^2/b^3*f*dilog((-b*exp(d*x+c)+(a^2+
b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] 1/16*f*((8*a^2*d^2*x^2*e^(2*c) + (2*b^2*d*x*e^(4*c) - b^2*e^(4*c))*e^(2*d*x
) - 8*(a*b*d*x*e^(3*c) - a*b*e^(3*c))*e^(d*x) + 8*(a*b*d*x*e^c + a*b*e^c)*e
^(-d*x) + (2*b^2*d*x + b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^2) - 2*integrate(16
*(a^3*x*e^(d*x + c) - a^2*b*x)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) -
 b^4), x)) + 1/8*(8*(d*x + c)*a^2/(b^3*d) - (4*a*e^(-d*x - c) - b)*e^(2*d*x
```

```
 + 2*c)/(b^2*d) + 8*a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^
3*d) + (4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1466 vs. $2(261) = 522$.
time = 0.37, size = 1466, normalized size = 5.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] 1/16*(2*b^2*d*f*x + (2*b^2*d*f*x + 2*b^2*d*cosh(1) + 2*b^2*d*sinh(1) - b^2*
f)*cosh(d*x + c)^4 + (2*b^2*d*f*x + 2*b^2*d*cosh(1) + 2*b^2*d*sinh(1) - b^2
*f)*sinh(d*x + c)^4 + 2*b^2*d*cosh(1) - 8*(a*b*d*f*x + a*b*d*cosh(1) + a*b*
d*sinh(1) - a*b*f)*cosh(d*x + c)^3 + 2*b^2*d*sinh(1) - 4*(2*a*b*d*f*x + 2*a
*b*d*cosh(1) + 2*a*b*d*sinh(1) - 2*a*b*f - (2*b^2*d*f*x + 2*b^2*d*cosh(1) +
 2*b^2*d*sinh(1) - b^2*f)*cosh(d*x + c))*sinh(d*x + c)^3 + b^2*f - 8*(a^2*d
^2*f*x^2 - 2*a^2*c^2*f + 2*(a^2*d^2*x + 2*a^2*c*d)*cosh(1) + 2*(a^2*d^2*x +
 2*a^2*c*d)*sinh(1))*cosh(d*x + c)^2 - 2*(4*a^2*d^2*f*x^2 - 8*a^2*c^2*f - 3
*(2*b^2*d*f*x + 2*b^2*d*cosh(1) + 2*b^2*d*sinh(1) - b^2*f)*cosh(d*x + c)^2
+ 8*(a^2*d^2*x + 2*a^2*c*d)*cosh(1) + 12*(a*b*d*f*x + a*b*d*cosh(1) + a*b*d
*sinh(1) - a*b*f)*cosh(d*x + c) + 8*(a^2*d^2*x + 2*a^2*c*d)*sinh(1))*sinh(d
*x + c)^2 + 8*(a*b*d*f*x + a*b*d*cosh(1) + a*b*d*sinh(1) + a*b*f)*cosh(d*x
+ c) + 16*(a^2*f*cosh(d*x + c)^2 + 2*a^2*f*cosh(d*x + c)*sinh(d*x + c) + a^
2*f*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 16*(a^2*f*cosh
(d*x + c)^2 + 2*a^2*f*cosh(d*x + c)*sinh(d*x + c) + a^2*f*sinh(d*x + c)^2)*
dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 16*((a^2*c*f - a^2*d*cosh(1) - a^2*
d*sinh(1))*cosh(d*x + c)^2 + 2*(a^2*c*f - a^2*d*cosh(1) - a^2*d*sinh(1))*co
sh(d*x + c)*sinh(d*x + c) + (a^2*c*f - a^2*d*cosh(1) - a^2*d*sinh(1))*sinh(
d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2
)/b^2) + 2*a) - 16*((a^2*c*f - a^2*d*cosh(1) - a^2*d*sinh(1))*cosh(d*x + c)
^2 + 2*(a^2*c*f - a^2*d*cosh(1) - a^2*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c
) + (a^2*c*f - a^2*d*cosh(1) - a^2*d*sinh(1))*sinh(d*x + c)^2)*log(2*b*cosh
(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 16*((a^2
*d*f*x + a^2*c*f)*cosh(d*x + c)^2 + 2*(a^2*d*f*x + a^2*c*f)*cosh(d*x + c)*s
inh(d*x + c) + (a^2*d*f*x + a^2*c*f)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c)
 + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2) - b)/b) + 16*((a^2*d*f*x + a^2*c*f)*cosh(d*x + c)^2 + 2*(a^2*d*f*x + a^
2*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*d*f*x + a^2*c*f)*sinh(d*x + c)^2)
*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 4*(2*a*b*d*f*x + 2*a*b*d*cosh(1) + (2*b
```

```
^2*d*f*x + 2*b^2*d*cosh(1) + 2*b^2*d*sinh(1) - b^2*f)*cosh(d*x + c)^3 + 2*a
*b*d*sinh(1) + 2*a*b*f - 6*(a*b*d*f*x + a*b*d*cosh(1) + a*b*d*sinh(1) - a*b
*f)*cosh(d*x + c)^2 - 4*(a^2*d^2*f*x^2 - 2*a^2*c^2*f + 2*(a^2*d^2*x + 2*a^2
*c*d)*cosh(1) + 2*(a^2*d^2*x + 2*a^2*c*d)*sinh(1))*cosh(d*x + c))*sinh(d*x
+ c))/(b^3*d^2*cosh(d*x + c)^2 + 2*b^3*d^2*cosh(d*x + c)*sinh(d*x + c) + b^
3*d^2*sinh(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^2\,(e+f\,x)}{a+b\sinh\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.365 $\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{a^2\log(a+b\sinh(c+dx))}{b^3d}-\frac{a\sinh(c+dx)}{b^2d}+\frac{\sinh^2(c+dx)}{2bd}$$

```
[Out] a^2*ln(a+b*sinh(d*x+c))/b^3/d-a*sinh(d*x+c)/b^2/d+1/2*sinh(d*x+c)^2/b/d
```

Rubi [A]
time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {2912, 12, 45}

$$\frac{a^2\log(a+b\sinh(c+dx))}{b^3d}-\frac{a\sinh(c+dx)}{b^2d}+\frac{\sinh^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*Log[a + b*Sinh[c + d*x]])/(b^3*d) - (a*Sinh[c + d*x])/(b^2*d) + Sinh[c
 + d*x]^2/(2*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{b^2(a+x)}\,dx, x, b\sinh(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{a+x}\,dx, x, b\sinh(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \left(-a+x+\frac{a^2}{a+x}\right)\,dx, x, b\sinh(c+dx)\right)}{b^3d}$$
$$= \frac{a^2\log(a+b\sinh(c+dx))}{b^3d} - \frac{a\sinh(c+dx)}{b^2d} + \frac{\sinh^2(c+dx)}{2bd}$$

Mathematica [A]
time = 0.11, size = 49, normalized size = 0.89

$$\frac{b^2\cosh(2(c+dx)) + 4a(a\log(a+b\sinh(c+dx)) - b\sinh(c+dx))}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
[Out] (b^2*Cosh[2*(c + d*x)] + 4*a*(a*Log[a + b*Sinh[c + d*x]] - b*Sinh[c + d*x])
)/(4*b^3*d)
```

Maple [A]
time = 0.57, size = 49, normalized size = 0.89

method	result	size
derivativedivides	$-\frac{\frac{-\frac{b\left(\sinh^2(dx+c)\right)}{2}+a\sinh(dx+c)}{b^2}+\frac{a^2\ln(a+b\sinh(dx+c))}{b^3}}{d}$	49
default	$-\frac{\frac{-\frac{b\left(\sinh^2(dx+c)\right)}{2}+a\sinh(dx+c)}{b^2}+\frac{a^2\ln(a+b\sinh(dx+c))}{b^3}}{d}$	49
risch	$-\frac{x\,a^2}{b^3}+\frac{\mathrm{e}^{2dx+2c}}{8bd}-\frac{a\,\mathrm{e}^{dx+c}}{2b^2d}+\frac{a\,\mathrm{e}^{-dx-c}}{2b^2d}+\frac{\mathrm{e}^{-2dx-2c}}{8bd}-\frac{2a^2c}{b^3d}+\frac{a^2\ln\left(\mathrm{e}^{2dx+2c}+\frac{2a\,\mathrm{e}^{dx+c}}{b}-1\right)}{b^3d}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/b^2*(-1/2*b*sinh(d*x+c)^2+a*sinh(d*x+c))+a^2/b^3*ln(a+b*sinh(d*x+c)
))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(53) = 106$.
time = 0.28, size = 119, normalized size = 2.16

$$\frac{(dx+c)a^2}{b^3d} - \frac{\left(4\,ae^{(-dx-c)} - b\right)e^{(2\,dx+2\,c)}}{8\,b^2d} + \frac{a^2\log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{b^3d} + \frac{4\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)}}{8\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] (d*x + c)*a^2/(b^3*d) - 1/8*(4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d)
+ a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d) + 1/8*(4*a*e^
(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(53) = 106$.
time = 0.36, size = 309, normalized size = 5.62

$$-\frac{8\,a^2dx\cosh(dx+c)^2 - b^2\cosh(dx+c)^4 - b^2\sinh(dx+c)^4 + 4\,ab\cosh(dx+c)^3 - 4\left(b^2\cosh(dx+c) - ab\right)\sinh(dx+c)^3 - 4\,ab\cosh(dx+c) + 2\left(4\,a^2dx - 3\,b^2\cosh(dx+c)^2 + 6\,ab\cosh(dx+c)\right)\sinh(dx+c)^2 - b^2 - 8\left(a^2\cosh(dx+c)^2 + 2\,a^2\cosh(dx+c)\sinh(dx+c) + a^2\sinh(dx+c)^2\right)\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right) + 4\left(4\,a^2dx\cosh(dx+c) - b^2\cosh(dx+c)^3 + 3\,ab\cosh(dx+c)^2 - ab\right)\sinh(dx+c)}{8\left(b^3d\cosh(dx+c)^2 + 2\,b^3d\cosh(dx+c)\sinh(dx+c) + b^3d\sinh(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/8*(8*a^2*d*x*cosh(d*x + c)^2 - b^2*cosh(d*x + c)^4 - b^2*sinh(d*x + c)^4
 + 4*a*b*cosh(d*x + c)^3 - 4*(b^2*cosh(d*x + c) - a*b)*sinh(d*x + c)^3 - 4*
a*b*cosh(d*x + c) + 2*(4*a^2*d*x - 3*b^2*cosh(d*x + c)^2 + 6*a*b*cosh(d*x +
 c))*sinh(d*x + c)^2 - b^2 - 8*(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*s
inh(d*x + c) + a^2*sinh(d*x + c)^2)*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x +
 c) - sinh(d*x + c))) + 4*(4*a^2*d*x*cosh(d*x + c) - b^2*cosh(d*x + c)^3 +
3*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^2 + 2*b^3*
d*cosh(d*x + c)*sinh(d*x + c) + b^3*d*sinh(d*x + c)^2)
```

Sympy [A]
time = 0.50, size = 87, normalized size = 1.58

$$\begin{cases} \frac{x\sinh^2(c)\cosh(c)}{a} & \text{for } b = 0 \wedge d = 0 \\ \frac{\sinh^3(c+dx)}{3ad} & \text{for } b = 0 \\ \frac{x\sinh^2(c)\cosh(c)}{a+b\sinh(c)} & \text{for } d = 0 \\ \frac{a^2\log\left(\frac{a}{b}+\sinh(c+dx)\right)}{b^3d} - \frac{a\sinh(c+dx)}{b^2d} + \frac{\cosh^2(c+dx)}{2bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Piecewise((x*sinh(c)**2*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (sinh(c + d*x)**3/
(3*a*d), Eq(b, 0)), (x*sinh(c)**2*cosh(c)/(a + b*sinh(c)), Eq(d, 0)), (a**2
*log(a/b + sinh(c + d*x))/(b**3*d) - a*sinh(c + d*x)/(b**2*d) + cosh(c + d*
x)**2/(2*b*d), True))
```

Giac [A]
time = 0.43, size = 88, normalized size = 1.60

$$\frac{\frac{8\,a^2\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{b^3}+\frac{b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-4\,a\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{b^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/b^3 + (b*(e^(d*x
+ c) - e^(-d*x - c))^2 - 4*a*(e^(d*x + c) - e^(-d*x - c)))/b^2)/d
```

Mupad [B]
time = 0.11, size = 46, normalized size = 0.84

$$\frac{a^2\,\ln\left(a+b\sinh(c+d\,x)\right)+\frac{b^2\sinh(c+d\,x)^2}{2}-a\,b\sinh(c+d\,x)}{b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*sinh(c + d*x)^2)/(a + b*sinh(c + d*x)),x)
```

```
[Out] (a^2*log(a + b*sinh(c + d*x)) + (b^2*sinh(c + d*x)^2)/2 - a*b*sinh(c + d*x)
)/(b^3*d)
```

3.366 $\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

[Out] Unintegrable(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

[In] Int[(Cosh[c + d*x]*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]

[Out] Defer[Int][(Cosh[c + d*x]*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]

Rubi steps

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

[In] Integrate[(Cosh[c + d*x]*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]

[Out] $Aborted

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)\left(\sinh^2(dx+c)\right)}{(fx+e)\,(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] 1/4*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b*f) + 1/2*a*e^(
-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^2*f) + 1/2*a*e^(c - d*e/f)*
exp_integral_e(1, -(f*x + e)*d/f)/(b^2*f) - 1/4*e^(2*c - 2*d*e/f)*exp_integ
ral_e(1, -2*(f*x + e)*d/f)/(b*f) + a^2*log(f*x + e)/(b^3*f) - 1/8*integrate
(-16*(a^3*e^(d*x + c) - a^2*b)/(b^4*f*x + b^4*e - (b^4*f*x*e^(2*c) + b^4*e^
(2*c + 1))*e^(2*d*x) - 2*(a*b^3*f*x*e^c + a*b^3*e^(c + 1))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(cosh(d*x + c)*sinh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate(cosh(d*x + c)*sinh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a)),
x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*sinh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((cosh(c + d*x)*sinh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.367 $\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=897

$$-\frac{3aef^2x}{4b^2d^2}-\frac{3af^3x^2}{8b^2d^2}-\frac{a^3(e+fx)^4}{4b^4f}-\frac{a(e+fx)^4}{8b^2f}+\frac{6a^2f^2(e+fx)\cosh(c+dx)}{b^3d^3}+\frac{4f^2(e+fx)\cosh(c+dx)}{3bd^3}+\frac{a^2(}{}$$

```
[Out] -1/8*a*(f*x+e)^4/b^2/f-14/9*f^3*sinh(d*x+c)/b/d^4+a^2*(f*x+e)^3*cosh(d*x+c)
/b^3/d-6*a^2*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/
2)/b^4/d^4-1/4*a^3*(f*x+e)^4/b^4/f+1/3*(f*x+e)^3*cosh(d*x+c)^3/b/d-2/27*f^3
*sinh(d*x+c)^3/b/d^4+3*a^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)
^(1/2)))*(a^2+b^2)^(1/2)/b^4/d^2-3*a^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/
(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4/d^2-6*a^2*f^2*(f*x+e)*polylog(3,-b
*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4/d^3+6*a^2*f^2*(f*x+e)*
polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4/d^3-3/4*a*
e*f^2*x/b^2/d^2+6*a^2*f^2*(f*x+e)*cosh(d*x+c)/b^3/d^3+3/4*a*f*(f*x+e)^2*cos
h(d*x+c)^2/b^2/d^2-3*a^2*f*(f*x+e)^2*sinh(d*x+c)/b^3/d^2-1/2*a*(f*x+e)^3*co
sh(d*x+c)*sinh(d*x+c)/b^2/d-1/3*f*(f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/b/d^2
+a^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4/d
-a^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4/d
+6*a^2*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4
/d^4-3/4*a*f^2*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/b^2/d^3-3/8*a*f^3*x^2/b^2/d^
2+3/8*a*f^3*cosh(d*x+c)^2/b^2/d^4+2/9*f^2*(f*x+e)*cosh(d*x+c)^3/b/d^3-6*a^2
*f^3*sinh(d*x+c)/b^3/d^4+4/3*f^2*(f*x+e)*cosh(d*x+c)/b/d^3-2/3*f*(f*x+e)^2*
sinh(d*x+c)/b/d^2
```

Rubi [A]

time = 1.03, antiderivative size = 897, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 16, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5698, 5555, 3392, 3377, 2717, 2713, 32, 3391, 5684, 3403, 2296, 2221, 2611, 6744, 2320, 6724}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-3*a*e*f^2*x)/(4*b^2*d^2) - (3*a*f^3*x^2)/(8*b^2*d^2) - (a^3*(e + f*x)^4)/
(4*b^4*f) - (a*(e + f*x)^4)/(8*b^2*f) + (6*a^2*f^2*(e + f*x)*Cosh[c + d*x])
/(b^3*d^3) + (4*f^2*(e + f*x)*Cosh[c + d*x])/(3*b*d^3) + (a^2*(e + f*x)^3*C
osh[c + d*x])/(b^3*d) + (3*a*f^3*Cosh[c + d*x]^2)/(8*b^2*d^4) + (3*a*f*(e +
 f*x)^2*Cosh[c + d*x]^2)/(4*b^2*d^2) + (2*f^2*(e + f*x)*Cosh[c + d*x]^3)/(9
*b*d^3) + ((e + f*x)^3*Cosh[c + d*x]^3)/(3*b*d) + (a^2*Sqrt[a^2 + b^2]*(e +
```

```
f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a^2*Sqrt
[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^
4*d) + (3*a^2*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2]))])/(b^4*d^2) - (3*a^2*Sqrt[a^2 + b^2]*f*(e + f*x)^2*Pol
yLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^2) - (6*a^2*Sqrt[
a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
)])/(b^4*d^3) + (6*a^2*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^3) + (6*a^2*Sqrt[a^2 + b^2]*f^3*Poly
Log[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^4) - (6*a^2*Sqrt[a
^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^
4) - (6*a^2*f^3*Sinh[c + d*x])/(b^3*d^4) - (14*f^3*Sinh[c + d*x])/(9*b*d^4)
 - (3*a^2*f*(e + f*x)^2*Sinh[c + d*x])/(b^3*d^2) - (2*f*(e + f*x)^2*Sinh[c
+ d*x])/(3*b*d^2) - (3*a*f^2*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^2*
d^3) - (a*(e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^2*d) - (f*(e + f*x)
^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b*d^2) - (2*f^3*Sinh[c + d*x]^3)/(27*b
*d^4)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5555

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\cosh^2(c+dx)\sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\cosh^2(c+}{a+b\sinh(}}{b}$$

$$= \frac{(e+fx)^3\cosh^3(c+dx)}{3bd} - \frac{a\int (e+fx)^3\cosh^2(c+dx)\,dx}{b^2} + \frac{a}{}$$

$$= \frac{3af(e+fx)^2\cosh^2(c+dx)}{4b^2d^2} + \frac{2f^2(e+fx)\cosh^3(c+dx)}{9bd^3} + \frac{(}{}$$

$$= -\frac{a^3(e+fx)^4}{4b^4f} - \frac{a(e+fx)^4}{8b^2f} + \frac{a^2(e+fx)^3\cosh(c+dx)}{b^3d} + \frac{3a}{}$$

$$= -\frac{3aef^2x}{4b^2d^2} - \frac{3af^3x^2}{8b^2d^2} - \frac{a^3(e+fx)^4}{4b^4f} - \frac{a(e+fx)^4}{8b^2f} + \frac{4f^2(e+}{}$$

$$= -\frac{3aef^2x}{4b^2d^2} - \frac{3af^3x^2}{8b^2d^2} - \frac{a^3(e+fx)^4}{4b^4f} - \frac{a(e+fx)^4}{8b^2f} + \frac{6a^2f^2(e+}{}$$

$$= -\frac{3aef^2x}{4b^2d^2} - \frac{3af^3x^2}{8b^2d^2} - \frac{a^3(e+fx)^4}{4b^4f} - \frac{a(e+fx)^4}{8b^2f} + \frac{6a^2f^2(e+}{}$$

$$= -\frac{3aef^2x}{4b^2d^2} - \frac{3af^3x^2}{8b^2d^2} - \frac{a^3(e+fx)^4}{4b^4f} - \frac{a(e+fx)^4}{8b^2f} + \frac{6a^2f^2(e+}{}$$

$$= -\frac{3aef^2x}{4b^2d^2} - \frac{3af^3x^2}{8b^2d^2} - \frac{a^3(e+fx)^4}{4b^4f} - \frac{a(e+fx)^4}{8b^2f} + \frac{6a^2f^2(e+}{}$$

$$= -\frac{3aef^2x}{4b^2d^2} - \frac{3af^3x^2}{8b^2d^2} - \frac{a^3(e+fx)^4}{4b^4f} - \frac{a(e+fx)^4}{8b^2f} + \frac{6a^2f^2(e+}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2729 vs. $2(897) = 1794$.
time = 9.80, size = 2729, normalized size = 3.04

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x
]),x]
```

```
[Out] ((-2*a*(2*a^2 + b^2)*e^3*x)/b^4 - (3*a*(2*a^2 + b^2)*e^2*f*x^2)/b^4 - (2*a*
(2*a^2 + b^2)*e*f^2*x^3)/b^4 - (a*(2*a^2 + b^2)*f^3*x^4)/(2*b^4) - (4*a^2*S
qrt[-a^2 - b^2]*(2*d^3*e^3*Sqrt[(a^2 + b^2)*E^(2*c)]*ArcTan[(a + b*E^(c + d
*x))/Sqrt[-a^2 - b^2]] + 3*Sqrt[-a^2 - b^2]*d^3*e^2*E^c*f*x*Log[1 + (b*E^(2
*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 3*Sqrt[-a^2 - b^2]*d^3*e*
E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])]
 + Sqrt[-a^2 - b^2]*d^3*E^c*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[
(a^2 + b^2)*E^(2*c)])] - 3*Sqrt[-a^2 - b^2]*d^3*e^2*E^c*f*x*Log[1 + (b*E^(2
*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*Sqrt[-a^2 - b^2]*d^3*e*
E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])]
 - Sqrt[-a^2 - b^2]*d^3*E^c*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[
(a^2 + b^2)*E^(2*c)])] + 3*Sqrt[-a^2 - b^2]*d^2*E^c*f*(e + f*x)^2*PolyLog[2
, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 3*Sqrt[-a^2 -
 b^2]*d^2*E^c*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a
^2 + b^2)*E^(2*c)]))] - 6*Sqrt[-a^2 - b^2]*d*e*E^c*f^2*PolyLog[3, -((b*E^(2
*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[-a^2 - b^2]*d*E^c
*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))]
 + 6*Sqrt[-a^2 - b^2]*d*e*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + S
qrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[-a^2 - b^2]*d*E^c*f^3*x*PolyLog[3, -((
b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[-a^2 - b^2]
*E^c*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])
)] - 6*Sqrt[-a^2 - b^2]*E^c*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c + Sqr
t[(a^2 + b^2)*E^(2*c)])])))/(b^4*d^4*Sqrt[(a^2 + b^2)*E^(2*c)]) + ((4*a^2 +
 b^2)*(d^3*e^3 + 3*d^2*e^2*f + 6*d*e*f^2 + 6*f^3)*(Cosh[c]/(2*b^3*d^4) - Si
nh[c]/(2*b^3*d^4)) + (4*a^2*d^2*e^2*f + b^2*d^2*e^2*f + 8*a^2*d*e*f^2 + 2*b
^2*d*e*f^2 + 8*a^2*f^3 + 2*b^2*f^3)*((3*x*Cosh[c])/(2*b^3*d^3) - (3*x*Sinh[
c])/(2*b^3*d^3)) + (4*a^2*d*e*f^2 + b^2*d*e*f^2 + 4*a^2*f^3 + b^2*f^3)*((3*
x^2*Cosh[c])/(2*b^3*d^2) - (3*x^2*Sinh[c])/(2*b^3*d^2)) + (4*a^2 + b^2)*((f
^3*x^3*Cosh[c])/(2*b^3*d) - (f^3*x^3*Sinh[c])/(2*b^3*d)))*(Cosh[d*x] - Sinh
[d*x]) + ((4*a^2 + b^2)*(d^3*e^3 - 3*d^2*e^2*f + 6*d*e*f^2 - 6*f^3)*(Cosh[c
]/(2*b^3*d^4) + Sinh[c]/(2*b^3*d^4)) + (3*x^2*(4*a^2*d*e*f^2*Cosh[c] + b^2*
d*e*f^2*Cosh[c] - 4*a^2*f^3*Cosh[c] - b^2*f^3*Cosh[c] + 4*a^2*d*e*f^2*Sinh[
c] + b^2*d*e*f^2*Sinh[c] - 4*a^2*f^3*Sinh[c] - b^2*f^3*Sinh[c]))/(2*b^3*d^2
) + (3*x*(4*a^2*d^2*e^2*f*Cosh[c] + b^2*d^2*e^2*f*Cosh[c] - 8*a^2*d*e*f^2*C
osh[c] - 2*b^2*d*e*f^2*Cosh[c] + 8*a^2*f^3*Cosh[c] + 2*b^2*f^3*Cosh[c] + 4*
a^2*d^2*e^2*f*Sinh[c] + b^2*d^2*e^2*f*Sinh[c] - 8*a^2*d*e*f^2*Sinh[c] - 2*b
^2*d*e*f^2*Sinh[c] + 8*a^2*f^3*Sinh[c] + 2*b^2*f^3*Sinh[c]))/(2*b^3*d^3) +
(4*a^2 + b^2)*((f^3*x^3*Cosh[c])/(2*b^3*d) + (f^3*x^3*Sinh[c])/(2*b^3*d)))*
(Cosh[d*x] + Sinh[d*x]) + ((a*f^3*x^3*Cosh[2*c])/(2*b^2*d) - (a*f^3*x^3*Sin
h[2*c])/(2*b^2*d) + (4*d^3*e^3 + 6*d^2*e^2*f + 6*d*e*f^2 + 3*f^3)*((a*Cosh[
2*c])/(8*b^2*d^4) - (a*Sinh[2*c])/(8*b^2*d^4)) + (2*a*d^2*e^2*f + 2*a*d*e*f
^2 + a*f^3)*((3*x*Cosh[2*c])/(4*b^2*d^3) - (3*x*Sinh[2*c])/(4*b^2*d^3)) + (
2*a*d*e*f^2 + a*f^3)*((3*x^2*Cosh[2*c])/(4*b^2*d^2) - (3*x^2*Sinh[2*c])/(4*
b^2*d^2)))*(Cosh[2*d*x] - Sinh[2*d*x]) + (-1/2*(a*f^3*x^3*Cosh[2*c])/(b^2*d
) - (a*f^3*x^3*Sinh[2*c])/(2*b^2*d) + (4*d^3*e^3 - 6*d^2*e^2*f + 6*d*e*f^2
```

```
- 3*f^3)*(-1/8*(a*Cosh[2*c])/(b^2*d^4) - (a*Sinh[2*c])/(8*b^2*d^4)) - (3*x^
2*(2*a*d*e*f^2*Cosh[2*c] - a*f^3*Cosh[2*c] + 2*a*d*e*f^2*Sinh[2*c] - a*f^3*
Sinh[2*c]))/(4*b^2*d^2) - (3*x*(2*a*d^2*e^2*f*Cosh[2*c] - 2*a*d*e*f^2*Cosh[
2*c] + a*f^3*Cosh[2*c] + 2*a*d^2*e^2*f*Sinh[2*c] - 2*a*d*e*f^2*Sinh[2*c] +
a*f^3*Sinh[2*c]))/(4*b^2*d^3))*(Cosh[2*d*x] + Sinh[2*d*x]) + ((f^3*x^3*Cosh
[3*c])/(6*b*d) - (f^3*x^3*Sinh[3*c])/(6*b*d) + (9*d^3*e^3 + 9*d^2*e^2*f + 6
*d*e*f^2 + 2*f^3)*(Cosh[3*c]/(54*b*d^4) - Sinh[3*c]/(54*b*d^4)) + (-9*d^2*e
^2*f - 6*d*e*f^2 - 2*f^3)*(-1/18*(x*Cosh[3*c])/(b*d^3) + (x*Sinh[3*c])/(18*
b*d^3)) + (-3*d*e*f^2 - f^3)*(-1/6*(x^2*Cosh[3*c])/(b*d^2) + (x^2*Sinh[3*c]
)/(6*b*d^2)))*(Cosh[3*d*x] - Sinh[3*d*x]) + ((f^3*x^3*Cosh[3*c])/(6*b*d) +
(f^3*x^3*Sinh[3*c])/(6*b*d) + (9*d^3*e^3 - 9*d^2*e^2*f + 6*d*e*f^2 - 2*f^3)
*(Cosh[3*c]/(54*b*d^4) + Sinh[3*c]/(54*b*d^4)) + (x^2*(3*d*e*f^2*Cosh[3*c]
- f^3*Cosh[3*c] + 3*d*e*f^2*Sinh[3*c] - f^3*Sinh[3*c]))/(6*b*d^2) + (x*(9*d
^2*e^2*f*Cosh[3*c] - 6*d*e*f^2*Cosh[3*c] + 2*f^3*Cosh[3*c] + 9*d^2*e^2*f*Si
nh[3*c] - 6*d*e*f^2*Sinh[3*c] + 2*f^3*Sinh[3*c]))/(18*b*d^3))*(Cosh[3*d*x]
+ Sinh[3*d*x]))/4
```

Maple [F]
time = 2.14, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \left(\cosh^2(dx+c)\right) \left(\sinh^2(dx+c)\right)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="maxima")
```

```
[Out] 1/24*(24*sqrt(a^2 + b^2)*a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*
e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b^4*d) - (3*a*b*e^(-d*x - c) - b^2 -
3*(4*a^2 + b^2)*e^(-2*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) - 12*(2*a^3 + a*b
^2)*(d*x + c)/(b^4*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-3*d*x - 3*c) + 3*
(4*a^2 + b^2)*e^(-d*x - c))/(b^3*d))*e^3 - 1/864*(108*(2*a^3*d^4*f^3*e^(3*c
) + a*b^2*d^4*f^3*e^(3*c))*x^4 + 432*(2*a^3*d^4*f^2*e^(3*c) + a*b^2*d^4*f^2
*e^(3*c))*x^3*e + 648*(2*a^3*d^4*f*e^(3*c) + a*b^2*d^4*f*e^(3*c))*x^2*e^2 -
 4*(9*b^3*d^3*f^3*x^3*e^(6*c) - 2*b^3*f^3*e^(6*c) - 9*b^3*d^2*f*e^(6*c + 2)
 + 6*b^3*d*f^2*e^(6*c + 1) - 9*(b^3*d^2*f^3*e^(6*c) - 3*b^3*d^3*f^2*e^(6*c
```

```
+ 1))*x^2 + 3*(2*b^3*d*f^3*e^(6*c) + 9*b^3*d^3*f*e^(6*c + 2) - 6*b^3*d^2*f^
2*e^(6*c + 1))*x)*e^(3*d*x) + 27*(4*a*b^2*d^3*f^3*x^3*e^(5*c) - 3*a*b^2*f^3
*e^(5*c) - 6*a*b^2*d^2*f*e^(5*c + 2) + 6*a*b^2*d*f^2*e^(5*c + 1) - 6*(a*b^2
*d^2*f^3*e^(5*c) - 2*a*b^2*d^3*f^2*e^(5*c + 1))*x^2 + 6*(a*b^2*d*f^3*e^(5*c
) + 2*a*b^2*d^3*f*e^(5*c + 2) - 2*a*b^2*d^2*f^2*e^(5*c + 1))*x)*e^(2*d*x) +
 108*(24*a^2*b*f^3*e^(4*c) + 6*b^3*f^3*e^(4*c) - (4*a^2*b*d^3*f^3*e^(4*c) +
 b^3*d^3*f^3*e^(4*c))*x^3 + 3*(4*a^2*b*d^2*f^3*e^(4*c) + b^3*d^2*f^3*e^(4*c
) - (4*a^2*b*d^3*f^2*e^(4*c) + b^3*d^3*f^2*e^(4*c))*e)*x^2 - 3*(8*a^2*b*d*f
^3*e^(4*c) + 2*b^3*d*f^3*e^(4*c) + (4*a^2*b*d^3*f*e^(4*c) + b^3*d^3*f*e^(4*
c))*e^2 - 2*(4*a^2*b*d^2*f^2*e^(4*c) + b^3*d^2*f^2*e^(4*c))*e)*x + 3*(4*a^2
*b*d^2*f*e^(4*c) + b^3*d^2*f*e^(4*c))*e^2 - 6*(4*a^2*b*d*f^2*e^(4*c) + b^3*
d*f^2*e^(4*c))*e)*e^(d*x) - 108*(24*a^2*b*f^3*e^(2*c) + 6*b^3*f^3*e^(2*c) +
 (4*a^2*b*d^3*f^3*e^(2*c) + b^3*d^3*f^3*e^(2*c))*x^3 + 3*(4*a^2*b*d^2*f^3*e
^(2*c) + b^3*d^2*f^3*e^(2*c) + (4*a^2*b*d^3*f^2*e^(2*c) + b^3*d^3*f^2*e^(2*
c))*e)*x^2 + 3*(8*a^2*b*d*f^3*e^(2*c) + 2*b^3*d*f^3*e^(2*c) + (4*a^2*b*d^3*
f*e^(2*c) + b^3*d^3*f*e^(2*c))*e^2 + 2*(4*a^2*b*d^2*f^2*e^(2*c) + b^3*d^2*f
^2*e^(2*c))*e)*x + 3*(4*a^2*b*d^2*f*e^(2*c) + b^3*d^2*f*e^(2*c))*e^2 + 6*(4
*a^2*b*d*f^2*e^(2*c) + b^3*d*f^2*e^(2*c))*e)*e^(-d*x) - 27*(4*a*b^2*d^3*f^3
*x^3*e^c + 6*a*b^2*d^2*f*e^(c + 2) + 6*a*b^2*d*f^2*e^(c + 1) + 3*a*b^2*f^3*
e^c + 6*(2*a*b^2*d^3*f^2*e^(c + 1) + a*b^2*d^2*f^3*e^c)*x^2 + 6*(2*a*b^2*d^
3*f*e^(c + 2) + 2*a*b^2*d^2*f^2*e^(c + 1) + a*b^2*d*f^3*e^c)*x)*e^(-2*d*x)
- 4*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^2*f*e^2 + 6*b^3*d*f^2*e + 2*b^3*f^3 + 9*(3
*b^3*d^3*f^2*e + b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*f*e^2 + 6*b^3*d^2*f^2*e +
2*b^3*d*f^3)*x)*e^(-3*d*x))*e^(-3*c)/(b^4*d^4) + integrate(2*((a^4*f^3*e^c
+ a^2*b^2*f^3*e^c)*x^3 + 3*(a^4*f^2*e^c + a^2*b^2*f^2*e^c)*x^2*e + 3*(a^4*f
*e^c + a^2*b^2*f*e^c)*x*e^2)*e^(d*x)/(b^5*e^(2*d*x + 2*c) + 2*a*b^4*e^(d*x
+ c) - b^5), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 13024 vs. $2(845) = 1690$.
time = 0.51, size = 13024, normalized size = 14.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="fricas")
```

```
[Out] 1/864*(36*b^3*d^3*f^3*x^3 + 36*b^3*d^2*f^3*x^2 + 36*b^3*d^3*cosh(1)^3 + 36*
b^3*d^3*sinh(1)^3 + 24*b^3*d*f^3*x + 4*(9*b^3*d^3*f^3*x^3 - 9*b^3*d^2*f^3*x
^2 + 9*b^3*d^3*cosh(1)^3 + 9*b^3*d^3*sinh(1)^3 + 6*b^3*d*f^3*x - 2*b^3*f^3
+ 9*(3*b^3*d^3*f*x - b^3*d^2*f)*cosh(1)^2 + 9*(3*b^3*d^3*f*x + 3*b^3*d^3*co
sh(1) - b^3*d^2*f)*sinh(1)^2 + 3*(9*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 2*b
^3*d*f^2)*cosh(1) + 3*(9*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 9*b^3*d^3*cosh
(1)^2 + 2*b^3*d*f^2 + 6*(3*b^3*d^3*f*x - b^3*d^2*f)*cosh(1))*sinh(1))*cosh(
```

d*x + c)^6 + 4*(9*b^3*d^3*f^3*x^3 - 9*b^3*d^2*f^3*x^2 + 9*b^3*d^3*cosh(1)^3
 + 9*b^3*d^3*sinh(1)^3 + 6*b^3*d*f^3*x - 2*b^3*f^3 + 9*(3*b^3*d^3*f*x - b^3
*d^2*f)*cosh(1)^2 + 9*(3*b^3*d^3*f*x + 3*b^3*d^3*cosh(1) - b^3*d^2*f)*sinh(
1)^2 + 3*(9*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 2*b^3*d*f^2)*cosh(1) + 3*(9
*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 9*b^3*d^3*cosh(1)^2 + 2*b^3*d*f^2 + 6*
(3*b^3*d^3*f*x - b^3*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^6 + 8*b^3*f^3 -
 27*(4*a*b^2*d^3*f^3*x^3 - 6*a*b^2*d^2*f^3*x^2 + 4*a*b^2*d^3*cosh(1)^3 + 4*
a*b^2*d^3*sinh(1)^3 + 6*a*b^2*d*f^3*x - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*f*x -
a*b^2*d^2*f)*cosh(1)^2 + 6*(2*a*b^2*d^3*f*x + 2*a*b^2*d^3*cosh(1) - a*b^2*d
^2*f)*sinh(1)^2 + 6*(2*a*b^2*d^3*f^2*x^2 - 2*a*b^2*d^2*f^2*x + a*b^2*d*f^2)
cosh(1) + 6(2*a*b^2*d^3*f^2*x^2 - 2*a*b^2*d^2*f^2*x + 2*a*b^2*d^3*cosh(1)
^2 + a*b^2*d*f^2 + 2*(2*a*b^2*d^3*f*x - a*b^2*d^2*f)*cosh(1))*sinh(1))*cosh
(d*x + c)^5 - 3*(36*a*b^2*d^3*f^3*x^3 - 54*a*b^2*d^2*f^3*x^2 + 36*a*b^2*d^3
*cosh(1)^3 + 36*a*b^2*d^3*sinh(1)^3 + 54*a*b^2*d*f^3*x - 27*a*b^2*f^3 + 54*
(2*a*b^2*d^3*f*x - a*b^2*d^2*f)*cosh(1)^2 + 54*(2*a*b^2*d^3*f*x + 2*a*b^2*d
^3*cosh(1) - a*b^2*d^2*f)*sinh(1)^2 + 54*(2*a*b^2*d^3*f^2*x^2 - 2*a*b^2*d^2
*f^2*x + a*b^2*d*f^2)*cosh(1) - 8*(9*b^3*d^3*f^3*x^3 - 9*b^3*d^2*f^3*x^2 +
9*b^3*d^3*cosh(1)^3 + 9*b^3*d^3*sinh(1)^3 + 6*b^3*d*f^3*x - 2*b^3*f^3 + 9*(
3*b^3*d^3*f*x - b^3*d^2*f)*cosh(1)^2 + 9*(3*b^3*d^3*f*x + 3*b^3*d^3*cosh(1)
 - b^3*d^2*f)*sinh(1)^2 + 3*(9*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 2*b^3*d*
f^2)*cosh(1) + 3*(9*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 9*b^3*d^3*cosh(1)^2
 + 2*b^3*d*f^2 + 6*(3*b^3*d^3*f*x - b^3*d^2*f)*cosh(1))*sinh(1))*cosh(d*x +
 c) + 54*(2*a*b^2*d^3*f^2*x^2 - 2*a*b^2*d^2*f^2*x + 2*a*b^2*d^3*cosh(1)^2 +
 a*b^2*d*f^2 + 2*(2*a*b^2*d^3*f*x - a*b^2*d^2*f)*cosh(1))*sinh(1))*sinh(d*x
 + c)^5 + 108*((4*a^2*b + b^3)*d^3*f^3*x^3 - 3*(4*a^2*b + b^3)*d^2*f^3*x^2
+ (4*a^2*b + b^3)*d^3*cosh(1)^3 + (4*a^2*b + b^3)*d^3*sinh(1)^3 + 6*(4*a^2*
b + b^3)*d*f^3*x - 6*(4*a^2*b + b^3)*f^3 + 3*((4*a^2*b + b^3)*d^3*f*x - (4*
a^2*b + b^3)*d^2*f)*cosh(1)^2 + 3*((4*a^2*b + b^3)*d^3*f*x + (4*a^2*b + b^3
)*d^3*cosh(1) - (4*a^2*b + b^3)*d^2*f)*sinh(1)^2 + 3*((4*a^2*b + b^3)*d^3*f
^2*x^2 - 2*(4*a^2*b + b^3)*d^2*f^2*x + 2*(4*a^2*b + b^3)*d*f^2)*cosh(1) + 3
*((4*a^2*b + b^3)*d^3*f^2*x^2 - 2*(4*a^2*b + b^3)*d^2*f^2*x + (4*a^2*b + b^
3)*d^3*cosh(1)^2 + 2*(4*a^2*b + b^3)*d*f^2 + 2*((4*a^2*b + b^3)*d^3*f*x - (
4*a^2*b + b^3)*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^4 + 3*(36*(4*a^2*b +
b^3)*d^3*f^3*x^3 - 108*(4*a^2*b + b^3)*d^2*f^3*x^2 + 36*(4*a^2*b + b^3)*d^3
cosh(1)^3 + 36(4*a^2*b + b^3)*d^3*sinh(1)^3 + 216*(4*a^2*b + b^3)*d*f^3*x
 - 216*(4*a^2*b + b^3)*f^3 + 108*((4*a^2*b + b^3)*d^3*f*x - (4*a^2*b + b^3)
*d^2*f)*cosh(1)^2 + 20*(9*b^3*d^3*f^3*x^3 - 9*b^3*d^2*f^3*x^2 + 9*b^3*d^3*c
osh(1)^3 + 9*b^3*d^3*sinh(1)^3 + 6*b^3*d*f^3*x - 2*b^3*f^3 + 9*(3*b^3*d^3*f
*x - b^3*d^2*f)*cosh(1)^2 + 9*(3*b^3*d^3*f*x + 3*b^3*d^3*cosh(1) - b^3*d^2*
f)*sinh(1)^2 + 3*(9*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 2*b^3*d*f^2)*cosh(1
) + 3*(9*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 9*b^3*d^3*cosh(1)^2 + 2*b^3*d*
f^2 + 6*(3*b^3*d^3*f*x - b^3*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 108
*((4*a^2*b + b^3)*d^3*f*x + (4*a^2*b + b^3)*d^3*cosh(1) - (4*a^2*b + b^3)*d
^2*f)*sinh(1)^2 + 108*((4*a^2*b + b^3)*d^3*f^2*x^2 - 2*(4*a^2*b + b^3)*d^2*
f^2*x + 2*(4*a^2*b + b^3)*d*f^2)*cosh(1) - 45*(4*a*b^2*d^3*f^3*x^3 - 6*a*b^

```
2*d^2*f^3*x^2 + 4*a*b^2*d^3*cosh(1)^3 + 4*a*b^2*d^3*sinh(1)^3 + 6*a*b^2*d*f
^3*x - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*f*x - a*b^2*d^2*f)*cosh(1)^2 + 6*(2*a*b
^2*d^3*f*x + 2*a*b^2*d^3*cosh(1) - a*b^2*d^2*f)*sinh(1)^2 + 6*(2*a*b^2*d^3*
f^2*x^2 - 2*a*b^2*d^2*f^2*x + a*b^2*d*f^2)*cosh(1) + 6*(2*a*b^2*d^3*f^2*x^2
 - 2*a*b^2*d^2*f^2*x + 2*a*b^2*d^3*cosh(1)^2 + a*b^2*d*f^2 + 2*(2*a*b^2*d^3
*f*x - a*b^2*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c) + 108*((4*a^2*b + b^3)*
d^3*f^2*x^2 - 2*(4*a^2*b + b^3)*d^2*f^2*x + (4*a^2*b + b^3)*d^3*cosh(1)^2 +
 2*(4*a^2*b + b^3)*d*f^2 + 2*((4*a^2*b + b^3)*d^3*f*x - (4*a^2*b + b^3)*d^2
*f)*cosh(1))*sinh(1))*sinh(d*x + c)^4 - 108*((2*a^3 + a*b^2)*d^4*f^3*x^4 +
4*(2*a^3 + a*b^2)*d^4*f^2*x^3*cosh(1) + 6*(2*a^3 + a*b^2)*d^4*f*x^2*cosh(1)
^2 + 4*(2*a^3 + a*b^2)*d^4*x*cosh(1)^3 + 4*(2*a^3 + a*b^2)*d^4*x*sinh(1)^3
+ 6*((2*a^3 + a*b^2)*d^4*f*x^2 + 2*(2*a^3 + a*b^2)*d^4*x*cosh(1))*sinh(1)^2
 + 4*((2*a^3 + a*b^2)*d^4*f^2*x^3 + 3*(2*a^3 + a*b^2)*d^4*f*x^2*cosh(1) + 3
*(2*a^3 + a*b^2)*d^4*x*cosh(1)^2)*sinh(1))*cosh(d*x + c)^3 - 2*(54*(2*a^3 +
 a*b^2)*d^4*f^3*x^4 + 216*(2*a^3 + a*b^2)*d^4*f...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**2*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)^2*sinh(d*x + c)^2/(b*sinh(d*x + c) + a)
, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^2\,\sinh(c+d\,x)^2\,{\left(e+f\,x\right)}^3}{a+b\,\mathrm{sinh}\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.368 $\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=649

$$-\frac{af^2x}{4b^2d^2}-\frac{a^3(e+fx)^3}{3b^4f}-\frac{a(e+fx)^3}{6b^2f}+\frac{2a^2f^2\cosh(c+dx)}{b^3d^3}+\frac{4f^2\cosh(c+dx)}{9bd^3}+\frac{a^2(e+fx)^2\cosh(c+dx)}{b^3d}+af(e$$

```
[Out] -1/4*a*f^2*x/b^2/d^2-1/3*a^3*(f*x+e)^3/b^4/f-1/6*a*(f*x+e)^3/b^2/f+2*a^2*f^
2*cosh(d*x+c)/b^3/d^3+4/9*f^2*cosh(d*x+c)/b/d^3+a^2*(f*x+e)^2*cosh(d*x+c)/b
^3/d+1/2*a*f*(f*x+e)*cosh(d*x+c)^2/b^2/d^2+2/27*f^2*cosh(d*x+c)^3/b/d^3+1/3
*(f*x+e)^2*cosh(d*x+c)^3/b/d-2*a^2*f*(f*x+e)*sinh(d*x+c)/b^3/d^2-4/9*f*(f*x
+e)*sinh(d*x+c)/b/d^2-1/4*a*f^2*cosh(d*x+c)*sinh(d*x+c)/b^2/d^3-1/2*a*(f*x+
e)^2*cosh(d*x+c)*sinh(d*x+c)/b^2/d-2/9*f*(f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/
b/d^2+a^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/
b^4/d-a^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/
b^4/d+2*a^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2
)^(1/2)/b^4/d^2-2*a^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2))
)*(a^2+b^2)^(1/2)/b^4/d^2-2*a^2*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1
/2)))*(a^2+b^2)^(1/2)/b^4/d^3+2*a^2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2
)^(1/2)))*(a^2+b^2)^(1/2)/b^4/d^3
```

Rubi [A]
time = 0.84, antiderivative size = 649, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 16, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5698, 5555, 3391, 3377, 2718, 3392, 32, 2715, 8, 5684, 3403, 2296, 2221, 2611, 2320, 6724}

$$-\frac{a^3(e+fx)^3}{3b^4f}+\frac{2a^2f^2\cosh(c+dx)}{b^3d^3}-\frac{2a^2f(e+fx)\sinh(c+dx)}{b^3d^2}+\frac{a^2(e+fx)^2\cosh(c+dx)}{b^3d}-\frac{2a^2f^2\sqrt{a^2+b^2}\,\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^3}+\frac{2a^2f^2\sqrt{a^2+b^2}\,\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^3}+\frac{2a^2f\sqrt{a^2+b^2}(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{2a^2f\sqrt{a^2+b^2}(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2}+\frac{a^2\sqrt{a^2+b^2}(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^4d}-\frac{a^2\sqrt{a^2+b^2}(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^4d}-\frac{af^2\sinh(c+dx)\cosh(c+dx)}{4b^2d^3}+\frac{af(e+fx)\cosh^2(c+dx)}{2b^2d^2}-\frac{a(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{2b^2d}-\frac{af^2x}{4b^2d^2}-\frac{a(e+fx)^3}{6b^2f}+\frac{2f^2\cosh^3(c+dx)}{27bd^3}+\frac{4f^2\cosh(c+dx)}{9bd^3}-\frac{4f(e+fx)\sinh(c+dx)}{9bd^2}-\frac{2f(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{9bd^2}+\frac{(e+fx)^2\cosh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/4*(a*f^2*x)/(b^2*d^2) - (a^3*(e + f*x)^3)/(3*b^4*f) - (a*(e + f*x)^3)/(6
*b^2*f) + (2*a^2*f^2*Cosh[c + d*x])/(b^3*d^3) + (4*f^2*Cosh[c + d*x])/(9*b*
d^3) + (a^2*(e + f*x)^2*Cosh[c + d*x])/(b^3*d) + (a*f*(e + f*x)*Cosh[c + d*
x]^2)/(2*b^2*d^2) + (2*f^2*Cosh[c + d*x]^3)/(27*b*d^3) + ((e + f*x)^2*Cosh[
c + d*x]^3)/(3*b*d) + (a^2*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a^2*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[
1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^4*d) + (2*a^2*Sqrt[a^2 + b^2
]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^
2) - (2*a^2*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/(b^4*d^2) - (2*a^2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^3) + (2*a^2*Sqrt[a^2 + b^2]*f^2
```

```
*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^3) - (2*a^2*f
*(e + f*x)*Sinh[c + d*x])/(b^3*d^2) - (4*f*(e + f*x)*Sinh[c + d*x])/(9*b*d^
2) - (a*f^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^2*d^3) - (a*(e + f*x)^2*Cosh[
c + d*x]*Sinh[c + d*x])/(2*b^2*d) - (2*f*(e + f*x)*Cosh[c + d*x]^2*Sinh[c +
 d*x])/(9*b*d^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5555

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
```

```
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh^2(c+dx)\sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\cosh^2(c+\ldots}{a+b\sinh(\ldots}}{b}$$

$$= \frac{(e+fx)^2\cosh^3(c+dx)}{3bd} - \frac{a\int (e+fx)^2\cosh^2(c+dx)\,dx}{b^2} + \ldots$$

$$= \frac{af(e+fx)\cosh^2(c+dx)}{2b^2d^2} + \frac{2f^2\cosh^3(c+dx)}{27bd^3} + \frac{(e+fx)^2\ldots}{3\ldots}$$

$$= -\frac{a^3(e+fx)^3}{3b^4f} - \frac{a(e+fx)^3}{6b^2f} + \frac{a^2(e+fx)^2\cosh(c+dx)}{b^3d} + \ldots$$

$$= -\frac{af^2x}{4b^2d^2} - \frac{a^3(e+fx)^3}{3b^4f} - \frac{a(e+fx)^3}{6b^2f} + \frac{4f^2\cosh(c+dx)}{9bd^3} + \ldots$$

$$= -\frac{af^2x}{4b^2d^2} - \frac{a^3(e+fx)^3}{3b^4f} - \frac{a(e+fx)^3}{6b^2f} + \frac{2a^2f^2\cosh(c+dx)}{b^3d^3} + \ldots$$

$$= -\frac{af^2x}{4b^2d^2} - \frac{a^3(e+fx)^3}{3b^4f} - \frac{a(e+fx)^3}{6b^2f} + \frac{2a^2f^2\cosh(c+dx)}{b^3d^3} + \ldots$$

$$= -\frac{af^2x}{4b^2d^2} - \frac{a^3(e+fx)^3}{3b^4f} - \frac{a(e+fx)^3}{6b^2f} + \frac{2a^2f^2\cosh(c+dx)}{b^3d^3} + \ldots$$

$$= -\frac{af^2x}{4b^2d^2} - \frac{a^3(e+fx)^3}{3b^4f} - \frac{a(e+fx)^3}{6b^2f} + \frac{2a^2f^2\cosh(c+dx)}{b^3d^3} + \ldots$$

Mathematica [A]
time = 4.68, size = 991, normalized size = 1.53

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x
]),x]
```

```
[Out] (-216*a*(2*a^2 + b^2)*e^2*x - 216*a*(2*a^2 + b^2)*e*f*x^2 - 72*a*(2*a^2 + b
^2)*f^2*x^3 + (432*a^2*(a^2 + b^2)*((2*d^2*e^2*ArcTan[(a + b*E^(c + d*x))/S
qrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + (2*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d
*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] + (d^2
```

```
*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])]
)/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a
*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] - (d^2*E^c*f^
2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[
(a^2 + b^2)*E^(2*c)] + (2*d*E^c*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/
(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*d*E^c
*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*
c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x
))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] + (2*E^
c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])
/Sqrt[(a^2 + b^2)*E^(2*c)]))/d^3 + (54*b*(4*a^2 + b^2)*(2*f^2 + 2*d*f*(e +
f*x) + d^2*(e + f*x)^2)*(Cosh[c + d*x] - Sinh[c + d*x]))/d^3 + (54*b*(4*a^2
 + b^2)*(2*f^2 - 2*d*f*(e + f*x) + d^2*(e + f*x)^2)*(Cosh[c + d*x] + Sinh[c
 + d*x]))/d^3 + (27*a*b^2*(f^2 + 2*d*f*(e + f*x) + 2*d^2*(e + f*x)^2)*(Cosh
[2*(c + d*x)] - Sinh[2*(c + d*x)]))/d^3 - (27*a*b^2*(f^2 - 2*d*f*(e + f*x)
+ 2*d^2*(e + f*x)^2)*(Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]))/d^3 + (2*b^3*
(2*f^2 + 6*d*f*(e + f*x) + 9*d^2*(e + f*x)^2)*(Cosh[3*(c + d*x)] - Sinh[3*(
c + d*x)]))/d^3 + (2*b^3*(2*f^2 - 6*d*f*(e + f*x) + 9*d^2*(e + f*x)^2)*(Cos
h[3*(c + d*x)] + Sinh[3*(c + d*x)]))/d^3)/(432*b^4)
```

Maple [F]
time = 2.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \left(\cosh^2(dx+c)\right) \left(\sinh^2(dx+c)\right)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="maxima")
```

```
[Out] 1/24*(24*sqrt(a^2 + b^2)*a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*
e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b^4*d) - (3*a*b*e^(-d*x - c) - b^2 -
3*(4*a^2 + b^2)*e^(-2*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) - 12*(2*a^3 + a*b
^2)*(d*x + c)/(b^4*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-3*d*x - 3*c) + 3*
(4*a^2 + b^2)*e^(-d*x - c))/(b^3*d))*e^2 - 1/432*(72*(2*a^3*d^3*f^2*e^(3*c)
 + a*b^2*d^3*f^2*e^(3*c))*x^3 + 216*(2*a^3*d^3*f*e^(3*c) + a*b^2*d^3*f*e^(3
```

```
*c))*x^2*e - 2*(9*b^3*d^2*f^2*x^2*e^(6*c) + 2*b^3*f^2*e^(6*c) - 6*b^3*d*f*e
^(6*c + 1) - 6*(b^3*d*f^2*e^(6*c) - 3*b^3*d^2*f*e^(6*c + 1))*x)*e^(3*d*x) +
 27*(2*a*b^2*d^2*f^2*x^2*e^(5*c) + a*b^2*f^2*e^(5*c) - 2*a*b^2*d*f*e^(5*c +
 1) - 2*(a*b^2*d*f^2*e^(5*c) - 2*a*b^2*d^2*f*e^(5*c + 1))*x)*e^(2*d*x) - 54
*(8*a^2*b*f^2*e^(4*c) + 2*b^3*f^2*e^(4*c) + (4*a^2*b*d^2*f^2*e^(4*c) + b^3*
d^2*f^2*e^(4*c))*x^2 - 2*(4*a^2*b*d*f^2*e^(4*c) + b^3*d*f^2*e^(4*c) - (4*a^
2*b*d^2*f*e^(4*c) + b^3*d^2*f*e^(4*c))*e)*x - 2*(4*a^2*b*d*f*e^(4*c) + b^3*
d*f*e^(4*c))*e)*e^(d*x) - 54*(8*a^2*b*f^2*e^(2*c) + 2*b^3*f^2*e^(2*c) + (4*
a^2*b*d^2*f^2*e^(2*c) + b^3*d^2*f^2*e^(2*c))*x^2 + 2*(4*a^2*b*d*f^2*e^(2*c)
 + b^3*d*f^2*e^(2*c) + (4*a^2*b*d^2*f*e^(2*c) + b^3*d^2*f*e^(2*c))*e)*x + 2
*(4*a^2*b*d*f*e^(2*c) + b^3*d*f*e^(2*c))*e)*e^(-d*x) - 27*(2*a*b^2*d^2*f^2*
x^2*e^c + 2*a*b^2*d*f*e^(c + 1) + a*b^2*f^2*e^c + 2*(2*a*b^2*d^2*f*e^(c + 1
) + a*b^2*d*f^2*e^c)*x)*e^(-2*d*x) - 2*(9*b^3*d^2*f^2*x^2 + 6*b^3*d*f*e + 2
*b^3*f^2 + 6*(3*b^3*d^2*f*e + b^3*d*f^2)*x)*e^(-3*d*x))*e^(-3*c)/(b^4*d^3)
+ integrate(2*((a^4*f^2*e^c + a^2*b^2*f^2*e^c)*x^2 + 2*(a^4*f*e^c + a^2*b^2
*f*e^c)*x*e)*e^(d*x)/(b^5*e^(2*d*x + 2*c) + 2*a*b^4*e^(d*x + c) - b^5), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6483 vs. $2(608) = 1216$.
time = 0.45, size = 6483, normalized size = 9.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="fricas")
```

```
[Out] 1/432*(18*b^3*d^2*f^2*x^2 + 12*b^3*d*f^2*x + 18*b^3*d^2*cosh(1)^2 + 2*(9*b^
3*d^2*f^2*x^2 - 6*b^3*d*f^2*x + 9*b^3*d^2*cosh(1)^2 + 9*b^3*d^2*sinh(1)^2 +
 2*b^3*f^2 + 6*(3*b^3*d^2*f*x - b^3*d*f)*cosh(1) + 6*(3*b^3*d^2*f*x + 3*b^3
*d^2*cosh(1) - b^3*d*f)*sinh(1))*cosh(d*x + c)^6 + 18*b^3*d^2*sinh(1)^2 + 2
*(9*b^3*d^2*f^2*x^2 - 6*b^3*d*f^2*x + 9*b^3*d^2*cosh(1)^2 + 9*b^3*d^2*sinh(
1)^2 + 2*b^3*f^2 + 6*(3*b^3*d^2*f*x - b^3*d*f)*cosh(1) + 6*(3*b^3*d^2*f*x +
 3*b^3*d^2*cosh(1) - b^3*d*f)*sinh(1))*sinh(d*x + c)^6 - 27*(2*a*b^2*d^2*f^
2*x^2 - 2*a*b^2*d*f^2*x + 2*a*b^2*d^2*cosh(1)^2 + 2*a*b^2*d^2*sinh(1)^2 + a
*b^2*f^2 + 2*(2*a*b^2*d^2*f*x - a*b^2*d*f)*cosh(1) + 2*(2*a*b^2*d^2*f*x + 2
*a*b^2*d^2*cosh(1) - a*b^2*d*f)*sinh(1))*cosh(d*x + c)^5 - 3*(18*a*b^2*d^2*
f^2*x^2 - 18*a*b^2*d*f^2*x + 18*a*b^2*d^2*cosh(1)^2 + 18*a*b^2*d^2*sinh(1)^
2 + 9*a*b^2*f^2 + 18*(2*a*b^2*d^2*f*x - a*b^2*d*f)*cosh(1) - 4*(9*b^3*d^2*f
^2*x^2 - 6*b^3*d*f^2*x + 9*b^3*d^2*cosh(1)^2 + 9*b^3*d^2*sinh(1)^2 + 2*b^3*
f^2 + 6*(3*b^3*d^2*f*x - b^3*d*f)*cosh(1) + 6*(3*b^3*d^2*f*x + 3*b^3*d^2*co
sh(1) - b^3*d*f)*sinh(1))*cosh(d*x + c) + 18*(2*a*b^2*d^2*f*x + 2*a*b^2*d^2
*cosh(1) - a*b^2*d*f)*sinh(1))*sinh(d*x + c)^5 + 4*b^3*f^2 + 54*((4*a^2*b +
 b^3)*d^2*f^2*x^2 - 2*(4*a^2*b + b^3)*d*f^2*x + (4*a^2*b + b^3)*d^2*cosh(1)
^2 + (4*a^2*b + b^3)*d^2*sinh(1)^2 + 2*(4*a^2*b + b^3)*f^2 + 2*((4*a^2*b +
```

b^3)*d^2*f*x - (4*a^2*b + b^3)*d*f)*cosh(1) + 2*((4*a^2*b + b^3)*d^2*f*x +
(4*a^2*b + b^3)*d^2*cosh(1) - (4*a^2*b + b^3)*d*f)*sinh(1))*cosh(d*x + c)^4
+ 3*(18*(4*a^2*b + b^3)*d^2*f^2*x^2 - 36*(4*a^2*b + b^3)*d*f^2*x + 18*(4*a
^2*b + b^3)*d^2*cosh(1)^2 + 18*(4*a^2*b + b^3)*d^2*sinh(1)^2 + 36*(4*a^2*b
+ b^3)*f^2 + 10*(9*b^3*d^2*f^2*x^2 - 6*b^3*d*f^2*x + 9*b^3*d^2*cosh(1)^2 +
9*b^3*d^2*sinh(1)^2 + 2*b^3*f^2 + 6*(3*b^3*d^2*f*x - b^3*d*f)*cosh(1) + 6*(
3*b^3*d^2*f*x + 3*b^3*d^2*cosh(1) - b^3*d*f)*sinh(1))*cosh(d*x + c)^2 + 36*
((4*a^2*b + b^3)*d^2*f*x - (4*a^2*b + b^3)*d*f)*cosh(1) - 45*(2*a*b^2*d^2*f
^2*x^2 - 2*a*b^2*d*f^2*x + 2*a*b^2*d^2*cosh(1)^2 + 2*a*b^2*d^2*sinh(1)^2 +
a*b^2*f^2 + 2*(2*a*b^2*d^2*f*x - a*b^2*d*f)*cosh(1) + 2*(2*a*b^2*d^2*f*x +
2*a*b^2*d^2*cosh(1) - a*b^2*d*f)*sinh(1))*cosh(d*x + c) + 36*((4*a^2*b + b^
3)*d^2*f*x + (4*a^2*b + b^3)*d^2*cosh(1) - (4*a^2*b + b^3)*d*f)*sinh(1))*si
nh(d*x + c)^4 - 72*((2*a^3 + a*b^2)*d^3*f^2*x^3 + 3*(2*a^3 + a*b^2)*d^3*f*x
^2*cosh(1) + 3*(2*a^3 + a*b^2)*d^3*x*cosh(1)^2 + 3*(2*a^3 + a*b^2)*d^3*x*si
nh(1)^2 + 3*((2*a^3 + a*b^2)*d^3*f*x^2 + 2*(2*a^3 + a*b^2)*d^3*x*cosh(1))*s
inh(1))*cosh(d*x + c)^3 - 2*(36*(2*a^3 + a*b^2)*d^3*f^2*x^3 + 108*(2*a^3 +
a*b^2)*d^3*f*x^2*cosh(1) + 108*(2*a^3 + a*b^2)*d^3*x*cosh(1)^2 + 108*(2*a^3
+ a*b^2)*d^3*x*sinh(1)^2 - 20*(9*b^3*d^2*f^2*x^2 - 6*b^3*d*f^2*x + 9*b^3*d
^2*cosh(1)^2 + 9*b^3*d^2*sinh(1)^2 + 2*b^3*f^2 + 6*(3*b^3*d^2*f*x - b^3*d*f
)*cosh(1) + 6*(3*b^3*d^2*f*x + 3*b^3*d^2*cosh(1) - b^3*d*f)*sinh(1))*cosh(d
x + c)^3 + 135(2*a*b^2*d^2*f^2*x^2 - 2*a*b^2*d*f^2*x + 2*a*b^2*d^2*cosh(1
)^2 + 2*a*b^2*d^2*sinh(1)^2 + a*b^2*f^2 + 2*(2*a*b^2*d^2*f*x - a*b^2*d*f)*c
osh(1) + 2*(2*a*b^2*d^2*f*x + 2*a*b^2*d^2*cosh(1) - a*b^2*d*f)*sinh(1))*cos
h(d*x + c)^2 - 108*((4*a^2*b + b^3)*d^2*f^2*x^2 - 2*(4*a^2*b + b^3)*d*f^2*x
+ (4*a^2*b + b^3)*d^2*cosh(1)^2 + (4*a^2*b + b^3)*d^2*sinh(1)^2 + 2*(4*a^2
*b + b^3)*f^2 + 2*((4*a^2*b + b^3)*d^2*f*x - (4*a^2*b + b^3)*d*f)*cosh(1) +
2*((4*a^2*b + b^3)*d^2*f*x + (4*a^2*b + b^3)*d^2*cosh(1) - (4*a^2*b + b^3)
*d*f)*sinh(1))*cosh(d*x + c) + 108*((2*a^3 + a*b^2)*d^3*f*x^2 + 2*(2*a^3 +
a*b^2)*d^3*x*cosh(1))*sinh(1))*sinh(d*x + c)^3 + 54*((4*a^2*b + b^3)*d^2*f^
2*x^2 + 2*(4*a^2*b + b^3)*d*f^2*x + (4*a^2*b + b^3)*d^2*cosh(1)^2 + (4*a^2*
b + b^3)*d^2*sinh(1)^2 + 2*(4*a^2*b + b^3)*f^2 + 2*((4*a^2*b + b^3)*d^2*f*x
+ (4*a^2*b + b^3)*d*f)*cosh(1) + 2*((4*a^2*b + b^3)*d^2*f*x + (4*a^2*b + b
^3)*d^2*cosh(1) + (4*a^2*b + b^3)*d*f)*sinh(1))*cosh(d*x + c)^2 + 6*(9*(4*a
^2*b + b^3)*d^2*f^2*x^2 + 18*(4*a^2*b + b^3)*d*f^2*x + 9*(4*a^2*b + b^3)*d^
2*cosh(1)^2 + 5*(9*b^3*d^2*f^2*x^2 - 6*b^3*d*f^2*x + 9*b^3*d^2*cosh(1)^2 +
9*b^3*d^2*sinh(1)^2 + 2*b^3*f^2 + 6*(3*b^3*d^2*f*x - b^3*d*f)*cosh(1) + 6*(
3*b^3*d^2*f*x + 3*b^3*d^2*cosh(1) - b^3*d*f)*sinh(1))*cosh(d*x + c)^4 + 9*(
4*a^2*b + b^3)*d^2*sinh(1)^2 - 45*(2*a*b^2*d^2*f^2*x^2 - 2*a*b^2*d*f^2*x +
2*a*b^2*d^2*cosh(1)^2 + 2*a*b^2*d^2*sinh(1)^2 + a*b^2*f^2 + 2*(2*a*b^2*d^2*
f*x - a*b^2*d*f)*cosh(1) + 2*(2*a*b^2*d^2*f*x + 2*a*b^2*d^2*cosh(1) - a*b^2
*d*f)*sinh(1))*cosh(d*x + c)^3 + 18*(4*a^2*b + b^3)*f^2 + 54*((4*a^2*b + b^
3)*d^2*f^2*x^2 - 2*(4*a^2*b + b^3)*d*f^2*x + (4*a^2*b + b^3)*d^2*cosh(1)^2
+ (4*a^2*b + b^3)*d^2*sinh(1)^2 + 2*(4*a^2*b + b^3)*f^2 + 2*((4*a^2*b + b^3
)*d^2*f*x - (4*a^2*b + b^3)*d*f)*cosh(1) + 2*((4*a^2*b + b^3)*d^2*f*x + (4*
a^2*b + b^3)*d^2*cosh(1) - (4*a^2*b + b^3)*d*f)*sinh(1))*cosh(d*x + c)^2 +

```
18*((4*a^2*b + b^3)*d^2*f*x + (4*a^2*b + b^3)*d*f)*cosh(1) - 36*((2*a^3 + a
*b^2)*d^3*f^2*x^3 + 3*(2*a^3 + a*b^2)*d^3*f*x^2*cosh(1) + 3*(2*a^3 + a*b^2)
*d^3*x*cosh(1)^2 + 3*(2*a^3 + a*b^2)*d^3*x*sinh(1)^2 + 3*((2*a^3 + a*b^2)*d
^3*f*x^2 + 2*(2*a^3 + a*b^2)*d^3*x*cosh(1))*sin...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**2*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)^2*sinh(d*x + c)^2/(b*sinh(d*x + c) + a)
, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^2\,\sinh(c+d\,x)^2\,{(e+f\,x)}^2}{a+b\,\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.369 $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=403

$$-\frac{a^3ex}{b^4}-\frac{aex}{2b^2}-\frac{a^3fx^2}{2b^4}-\frac{afx^2}{4b^2}+\frac{a^2(e+fx)\cosh(c+dx)}{b^3d}+\frac{af\cosh^2(c+dx)}{4b^2d^2}+\frac{(e+fx)\cosh^3(c+dx)}{3bd}+\frac{a^2\sqrt{a^2}}{}$$

```
[Out] -a^3*e*x/b^4-1/2*a*e*x/b^2-1/2*a^3*f*x^2/b^4-1/4*a*f*x^2/b^2+a^2*(f*x+e)*co
sh(d*x+c)/b^3/d+1/4*a*f*cosh(d*x+c)^2/b^2/d^2+1/3*(f*x+e)*cosh(d*x+c)^3/b/d
-a^2*f*sinh(d*x+c)/b^3/d^2-1/3*f*sinh(d*x+c)/b/d^2-1/2*a*(f*x+e)*cosh(d*x+c
)*sinh(d*x+c)/b^2/d-1/9*f*sinh(d*x+c)^3/b/d^2+a^2*(f*x+e)*ln(1+b*exp(d*x+c)
/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4/d-a^2*(f*x+e)*ln(1+b*exp(d*x+c)/(
a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4/d+a^2*f*polylog(2,-b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4/d^2-a^2*f*polylog(2,-b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^4/d^2
```

Rubi [A]
time = 0.50, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 12, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {5698, 5555, 2713, 3391, 5684, 3377, 2717, 3403, 2296, 2221, 2317, 2438}

$$-\frac{a^3ex}{b^4}-\frac{a^3fx^2}{2b^4}-\frac{a^2f\sinh(c+dx)}{b^3d^2}+\frac{a^2(e+fx)\cosh(c+dx)}{b^3d}+\frac{a^2f\sqrt{a^2+b^2}\,\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{a^2f\sqrt{a^2+b^2}\,\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2}+\frac{a^2\sqrt{a^2+b^2}(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^4d}-\frac{a^2\sqrt{a^2+b^2}(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^4d}+\frac{af\cosh^2(c+dx)}{4b^2d^2}-\frac{a(e+fx)\sinh(c+dx)\cosh(c+dx)}{2b^2d}-\frac{aex}{2b^2}-\frac{afx^2}{4b^2}-\frac{f\sinh^3(c+dx)}{9bd^2}-\frac{f\sinh(c+dx)}{3bd^2}+\frac{(e+fx)\cosh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a^3*e*x)/b^4) - (a*e*x)/(2*b^2) - (a^3*f*x^2)/(2*b^4) - (a*f*x^2)/(4*b^2
) + (a^2*(e + f*x)*Cosh[c + d*x])/(b^3*d) + (a*f*Cosh[c + d*x]^2)/(4*b^2*d^
2) + ((e + f*x)*Cosh[c + d*x]^3)/(3*b*d) + (a^2*Sqrt[a^2 + b^2]*(e + f*x)*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a^2*Sqrt[a^2 + b^
2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^4*d) + (a^2
*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b
^4*d^2) - (a^2*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2]))])/(b^4*d^2) - (a^2*f*Sinh[c + d*x])/(b^3*d^2) - (f*Sinh[c + d*x])
/(3*b*d^2) - (a*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^2*d) - (f*Sinh[
c + d*x]^3)/(9*b*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
```

```
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5555

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh^2(c+dx)\sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+}}{b}$$

$$= \frac{(e+fx)\cosh^3(c+dx)}{3bd} - \frac{a\int (e+fx)\cosh^2(c+dx)\,dx}{b^2} + \frac{a^2\int}{}$$

$$= \frac{af\cosh^2(c+dx)}{4b^2d^2} + \frac{(e+fx)\cosh^3(c+dx)}{3bd} - \frac{a(e+fx)\cosh(}{2}$$

$$= -\frac{a^3ex}{b^4} - \frac{aex}{2b^2} - \frac{a^3fx^2}{2b^4} - \frac{afx^2}{4b^2} + \frac{a^2(e+fx)\cosh(c+dx)}{b^3d} +$$

$$= -\frac{a^3ex}{b^4} - \frac{aex}{2b^2} - \frac{a^3fx^2}{2b^4} - \frac{afx^2}{4b^2} + \frac{a^2(e+fx)\cosh(c+dx)}{b^3d} +$$

$$= -\frac{a^3ex}{b^4} - \frac{aex}{2b^2} - \frac{a^3fx^2}{2b^4} - \frac{afx^2}{4b^2} + \frac{a^2(e+fx)\cosh(c+dx)}{b^3d} +$$

$$= -\frac{a^3ex}{b^4} - \frac{aex}{2b^2} - \frac{a^3fx^2}{2b^4} - \frac{afx^2}{4b^2} + \frac{a^2(e+fx)\cosh(c+dx)}{b^3d} +$$

$$= -\frac{a^3ex}{b^4} - \frac{aex}{2b^2} - \frac{a^3fx^2}{2b^4} - \frac{afx^2}{4b^2} + \frac{a^2(e+fx)\cosh(c+dx)}{b^3d} +$$

Mathematica [A]
time = 1.79, size = 676, normalized size = 1.68

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] -1/72*(72*a^3*c*d*e + 36*a*b^2*c*d*e - 36*a^3*c^2*f - 18*a*b^2*c^2*f + 72*a
^3*d^2*e*x + 36*a*b^2*d^2*e*x + 36*a^3*d^2*f*x^2 + 18*a*b^2*d^2*f*x^2 + 144
*a^2*Sqrt[a^2 + b^2]*d*e*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sq
rt[a^2 + b^2]] - 144*a^2*Sqrt[a^2 + b^2]*c*f*ArcTanh[(a + b*Cosh[c + d*x] +
 b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] - 72*a^2*b*d*e*Cosh[c + d*x] - 18*b^3*d*
e*Cosh[c + d*x] - 72*a^2*b*d*f*x*Cosh[c + d*x] - 18*b^3*d*f*x*Cosh[c + d*x]
 - 9*a*b^2*f*Cosh[2*(c + d*x)] - 6*b^3*d*e*Cosh[3*(c + d*x)] - 6*b^3*d*f*x*
Cosh[3*(c + d*x)] - 72*a^2*Sqrt[a^2 + b^2]*c*f*Log[1 + (b*(Cosh[c + d*x] +
```

```
Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] - 72*a^2*Sqrt[a^2 + b^2]*d*f*x*Log[1
 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] + 72*a^2*Sqrt
[a^2 + b^2]*c*f*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 +
 b^2])] + 72*a^2*Sqrt[a^2 + b^2]*d*f*x*Log[1 + (b*(Cosh[c + d*x] + Sinh[c +
 d*x]))/(a + Sqrt[a^2 + b^2])] - 72*a^2*Sqrt[a^2 + b^2]*f*PolyLog[2, (b*(Co
sh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])] + 72*a^2*Sqrt[a^2 + b
^2]*f*PolyLog[2, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]
))] + 72*a^2*b*f*Sinh[c + d*x] + 18*b^3*f*Sinh[c + d*x] + 18*a*b^2*d*e*Sinh
[2*(c + d*x)] + 18*a*b^2*d*f*x*Sinh[2*(c + d*x)] + 2*b^3*f*Sinh[3*(c + d*x)
])/(b^4*d^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1127 vs. $2(367) = 734$.

time = 3.10, size = 1128, normalized size = 2.80

method	result
risch	$\frac{a(2dxf+2de+f)e^{-2dx-2c}}{16b^2d^2} - \frac{2a^4 e \operatorname{arctanh}\left(\frac{2b\,e^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{d\,b^4\sqrt{a^2+b^2}} - \frac{2a^2 e \operatorname{arctanh}\left(\frac{2b\,e^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{d\,b^2\sqrt{a^2+b^2}} + \frac{a^4 f \operatorname{dilog}\left(\frac{-b\,e^{dx+c}+\sqrt{a^2}}{-a+\sqrt{a^2}}\right)}{d^2b^4\sqrt{a^2+b^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/16*a*(2*d*f*x+2*d*e+f)/b^2/d^2*exp(-2*d*x-2*c)-1/4*a*f*x^2/b^2-1/2*a^3*f*
x^2/b^4+1/72*(3*d*f*x+3*d*e-f)/d^2/b*exp(3*d*x+3*c)+1/d*a^4/b^4*f/(a^2+b^2)
^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d^2*a
^4/b^4*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)
^(1/2)))*c-1/d*a^4/b^4*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a
)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*a^4/b^4*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(
a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d*a^2/b^2*f/(a^2+b^2)^(1/2)*ln((
-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d^2*a^2/b^2*f/(a
^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-
1/d*a^2/b^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b
^2)^(1/2)))*x-1/d^2*a^2/b^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1
/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/d^2*a^4/b^4*f*c/(a^2+b^2)^(1/2)*arctanh(1/2
*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d^2*a^2/b^2*f*c/(a^2+b^2)^(1/2)*ar
ctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/16*a*(2*d*f*x+2*d*e-f)/b^
2/d^2*exp(2*d*x+2*c)+1/8*(4*a^2*d*f*x+b^2*d*f*x+4*a^2*d*e+b^2*d*e-4*a^2*f-b
^2*f)/b^3/d^2*exp(d*x+c)-1/2*a*e*x/b^2+1/72*(3*d*f*x+3*d*e+f)/d^2/b*exp(-3*
d*x-3*c)-a^3*e*x/b^4-1/d^2*a^2/b^2*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a
^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/d^2*a^2/b^2*f/(a^2+b^2)^(1/2)*dilog
((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2/d*a^4/b^4*e/(a^2
+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d*a^2/b^2*e
/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d^2*a^
```

```
4/b^4*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^
2)^(1/2)))-1/d^2*a^4/b^4*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1
/2)+a)/(a+(a^2+b^2)^(1/2)))+1/8*(4*a^2+b^2)*(d*f*x+d*e+f)/b^3/d^2*exp(-d*x-
c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] 1/144*(288*(a^4*e^c + a^2*b^2*e^c)*integrate(x*e^(d*x)/(b^5*e^(2*d*x + 2*c)
 + 2*a*b^4*e^(d*x + c) - b^5), x) - (36*(2*a^3*d^2*e^(3*c) + a*b^2*d^2*e^(3
*c))*x^2 - 2*(3*b^3*d*x*e^(6*c) - b^3*e^(6*c))*e^(3*d*x) + 9*(2*a*b^2*d*x*e
^(5*c) - a*b^2*e^(5*c))*e^(2*d*x) + 18*(4*a^2*b*e^(4*c) + b^3*e^(4*c) - (4*
a^2*b*d*e^(4*c) + b^3*d*e^(4*c))*x)*e^(d*x) - 18*(4*a^2*b*e^(2*c) + b^3*e^(
2*c) + (4*a^2*b*d*e^(2*c) + b^3*d*e^(2*c))*x)*e^(-d*x) - 9*(2*a*b^2*d*x*e^c
 + a*b^2*e^c)*e^(-2*d*x) - 2*(3*b^3*d*x + b^3)*e^(-3*d*x))*e^(-3*c)/(b^4*d^
2))*f + 1/24*(24*sqrt(a^2 + b^2)*a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b
^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b^4*d) - (3*a*b*e^(-d*x - c)
- b^2 - 3*(4*a^2 + b^2)*e^(-2*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) - 12*(2*a
^3 + a*b^2)*(d*x + c)/(b^4*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-3*d*x - 3
*c) + 3*(4*a^2 + b^2)*e^(-d*x - c))/(b^3*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2629 vs. $2(372) = 744$.
time = 0.41, size = 2629, normalized size = 6.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/144*(2*(3*b^3*d*f*x + 3*b^3*d*cosh(1) + 3*b^3*d*sinh(1) - b^3*f)*cosh(d*x
 + c)^6 + 2*(3*b^3*d*f*x + 3*b^3*d*cosh(1) + 3*b^3*d*sinh(1) - b^3*f)*sinh(
d*x + c)^6 + 6*b^3*d*f*x - 9*(2*a*b^2*d*f*x + 2*a*b^2*d*cosh(1) + 2*a*b^2*d
*sinh(1) - a*b^2*f)*cosh(d*x + c)^5 - 3*(6*a*b^2*d*f*x + 6*a*b^2*d*cosh(1)
+ 6*a*b^2*d*sinh(1) - 3*a*b^2*f - 4*(3*b^3*d*f*x + 3*b^3*d*cosh(1) + 3*b^3*
d*sinh(1) - b^3*f)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*b^3*d*cosh(1) + 18*((
4*a^2*b + b^3)*d*f*x + (4*a^2*b + b^3)*d*cosh(1) + (4*a^2*b + b^3)*d*sinh(1
) - (4*a^2*b + b^3)*f)*cosh(d*x + c)^4 + 6*b^3*d*sinh(1) + 3*(6*(4*a^2*b +
b^3)*d*f*x + 6*(4*a^2*b + b^3)*d*cosh(1) + 10*(3*b^3*d*f*x + 3*b^3*d*cosh(1
```

) + 3*b^3*d*sinh(1) - b^3*f)*cosh(d*x + c)^2 + 6*(4*a^2*b + b^3)*d*sinh(1)
- 6*(4*a^2*b + b^3)*f - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*cosh(1) + 2*a*b^2*d*s
inh(1) - a*b^2*f)*cosh(d*x + c))*sinh(d*x + c)^4 + 2*b^3*f - 36*((2*a^3 + a
*b^2)*d^2*f*x^2 + 2*(2*a^3 + a*b^2)*d^2*x*cosh(1) + 2*(2*a^3 + a*b^2)*d^2*x
*sinh(1))*cosh(d*x + c)^3 - 2*(18*(2*a^3 + a*b^2)*d^2*f*x^2 + 36*(2*a^3 + a
*b^2)*d^2*x*cosh(1) + 36*(2*a^3 + a*b^2)*d^2*x*sinh(1) - 20*(3*b^3*d*f*x +
3*b^3*d*cosh(1) + 3*b^3*d*sinh(1) - b^3*f)*cosh(d*x + c)^3 + 45*(2*a*b^2*d*
f*x + 2*a*b^2*d*cosh(1) + 2*a*b^2*d*sinh(1) - a*b^2*f)*cosh(d*x + c)^2 - 36
*((4*a^2*b + b^3)*d*f*x + (4*a^2*b + b^3)*d*cosh(1) + (4*a^2*b + b^3)*d*sin
h(1) - (4*a^2*b + b^3)*f)*cosh(d*x + c))*sinh(d*x + c)^3 + 18*((4*a^2*b + b
^3)*d*f*x + (4*a^2*b + b^3)*d*cosh(1) + (4*a^2*b + b^3)*d*sinh(1) + (4*a^2*
b + b^3)*f)*cosh(d*x + c)^2 + 6*(5*(3*b^3*d*f*x + 3*b^3*d*cosh(1) + 3*b^3*d
*sinh(1) - b^3*f)*cosh(d*x + c)^4 + 3*(4*a^2*b + b^3)*d*f*x - 15*(2*a*b^2*d
*f*x + 2*a*b^2*d*cosh(1) + 2*a*b^2*d*sinh(1) - a*b^2*f)*cosh(d*x + c)^3 + 3
*(4*a^2*b + b^3)*d*cosh(1) + 18*((4*a^2*b + b^3)*d*f*x + (4*a^2*b + b^3)*d*
cosh(1) + (4*a^2*b + b^3)*d*sinh(1) - (4*a^2*b + b^3)*f)*cosh(d*x + c)^2 +
3*(4*a^2*b + b^3)*d*sinh(1) + 3*(4*a^2*b + b^3)*f - 18*((2*a^3 + a*b^2)*d^2
*f*x^2 + 2*(2*a^3 + a*b^2)*d^2*x*cosh(1) + 2*(2*a^3 + a*b^2)*d^2*x*sinh(1))
*cosh(d*x + c))*sinh(d*x + c)^2 + 144*(a^2*b*f*cosh(d*x + c)^3 + 3*a^2*b*f*
cosh(d*x + c)^2*sinh(d*x + c) + 3*a^2*b*f*cosh(d*x + c)*sinh(d*x + c)^2 + a
^2*b*f*sinh(d*x + c)^3)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*si
nh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b
)/b + 1) - 144*(a^2*b*f*cosh(d*x + c)^3 + 3*a^2*b*f*cosh(d*x + c)^2*sinh(d*
x + c) + 3*a^2*b*f*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*b*f*sinh(d*x + c)^3)
*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 144*((a^2*b*
c*f - a^2*b*d*cosh(1) - a^2*b*d*sinh(1))*cosh(d*x + c)^3 + 3*(a^2*b*c*f - a
^2*b*d*cosh(1) - a^2*b*d*sinh(1))*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^2*b*
c*f - a^2*b*d*cosh(1) - a^2*b*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^2 + (a
^2*b*c*f - a^2*b*d*cosh(1) - a^2*b*d*sinh(1))*sinh(d*x + c)^3)*sqrt((a^2 +
b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/
b^2) + 2*a) - 144*((a^2*b*c*f - a^2*b*d*cosh(1) - a^2*b*d*sinh(1))*cosh(d*x
 + c)^3 + 3*(a^2*b*c*f - a^2*b*d*cosh(1) - a^2*b*d*sinh(1))*cosh(d*x + c)^2
*sinh(d*x + c) + 3*(a^2*b*c*f - a^2*b*d*cosh(1) - a^2*b*d*sinh(1))*cosh(d*x
 + c)*sinh(d*x + c)^2 + (a^2*b*c*f - a^2*b*d*cosh(1) - a^2*b*d*sinh(1))*sin
h(d*x + c)^3)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 144*((a^2*b*d*f*x + a^2*b*c*f)*cosh
(d*x + c)^3 + 3*(a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^2*sinh(d*x + c) + 3
*(a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2*b*d*f*x + a
^2*b*c*f)*sinh(d*x + c)^3)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*
sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
 b)/b) - 144*((a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^3 + 3*(a^2*b*d*f*x +
a^2*b*c*f)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^2*b*d*f*x + a^2*b*c*f)*cosh
(d*x + c)*sinh(d*x + c)^2 + (a^2*b*d*f*x + a^2*b*c*f)*sinh(d*x + c)^3)*sqrt
((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c

```
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 9*(2*a*b^2*d*f*x + 2*a
*b^2*d*cosh(1) + 2*a*b^2*d*sinh(1) + a*b^2*f)*cosh(d*x + c) + 3*(6*a*b^2*d*
f*x + 4*(3*b^3*d*f*x + 3*b^3*d*cosh(1) + 3*b^3*d*sinh(1) - b^3*f)*cosh(d*x
+ c)^5 + 6*a*b^2*d*cosh(1) - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*cosh(1) + 2*a*b^
2*d*sinh(1) - a*b^2*f)*cosh(d*x + c)^4 + 6*a*b^2*d*sinh(1) + 3*a*b^2*f + 24
*((4*a^2*b + b^3)*d*f*x + (4*a^2*b + b^3)*d*cosh(1) + (4*a^2*b + b^3)*d*sin
h(1) - (4*a^2*b + b^3)*f)*cosh(d*x + c)^3 - 36*((2*a^3 + a*b^2)*d^2*f*x^2 +
 2*(2*a^3 + a*b^2)*d^2*x*cosh(1) + 2*(2*a^3 + a*b^2)*d^2*x*sinh(1))*cosh(d*
x + c)^2 + 12*((4*a^2*b + b^3)*d*f*x + (4*a^2*b + b^3)*d*cosh(1) + (4*a^2*b
 + b^3)*d*sinh(1) + (4*a^2*b + b^3)*f)*cosh(d*x + c))*sinh(d*x + c))/(b^4*d
^2*cosh(d*x + c)^3 + 3*b^4*d^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^4*d^2*co
sh(d*x + c)*sinh(d*x + c)^2 + b^4*d^2*sinh(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**2*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)^2*sinh(d*x + c)^2/(b*sinh(d*x + c) + a),
x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^2\,\sinh(c+d\,x)^2\,(e+f\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.370 $\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=141

$$-\frac{a(2a^2+b^2)\,x}{2b^4}-\frac{2a^2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^4d}+\frac{(3a^2+b^2)\cosh(c+dx)}{3b^3d}-\frac{a\cosh(c+dx)\sinh(c+}{2b^2d}$$

```
[Out] -1/2*a*(2*a^2+b^2)*x/b^4+1/3*(3*a^2+b^2)*cosh(d*x+c)/b^3/d-1/2*a*cosh(d*x+c
)*sinh(d*x+c)/b^2/d+1/3*cosh(d*x+c)*sinh(d*x+c)^2/b/d-2*a^2*arctanh((b-a*ta
nh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/b^4/d
```

Rubi [A]

time = 0.35, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2968, 3129, 3128, 3102, 2814, 2739, 632, 210}

$$-\frac{2a^2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{ax(2a^2+b^2)}{2b^4}+\frac{(3a^2+b^2)\cosh(c+dx)}{3b^3d}-\frac{a\sinh(c+dx)\cosh(c+dx)}{2b^2d}+\frac{\sinh^2(c+dx)\cosh(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/2*(a*(2*a^2 + b^2)*x)/b^4 - (2*a^2*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[(
c + d*x)/2])/Sqrt[a^2 + b^2]])/(b^4*d) + ((3*a^2 + b^2)*Cosh[c + d*x])/(3*b
^3*d) - (a*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^2*d) + (Cosh[c + d*x]*Sinh[c +
 d*x]^2)/(3*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3129

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x]
)^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n
 + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(
a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0
```

]))

Rubi steps

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh^2(c+dx)\left(1+\sinh^2(c+dx)\right)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{\cosh(c+dx)\sinh^2(c+dx)}{3bd} + \frac{\int \frac{\sinh(c+dx)\left(-2a+b\sinh(c+dx)-3a\sinh^2(c+dx)\right)}{a+b\sinh(c+dx)}\,d}{3b}$$

$$= -\frac{a\cosh(c+dx)\sinh(c+dx)}{2b^2d} + \frac{\cosh(c+dx)\sinh^2(c+dx)}{3bd} + \frac{\int \frac{3a^2-a}{}}{}$$

$$= \frac{(3a^2+b^2)\cosh(c+dx)}{3b^3d} - \frac{a\cosh(c+dx)\sinh(c+dx)}{2b^2d} + \frac{\cosh(c+dx)}{3}$$

$$= -\frac{a(2a^2+b^2)\,x}{2b^4} + \frac{(3a^2+b^2)\cosh(c+dx)}{3b^3d} - \frac{a\cosh(c+dx)\sinh(c+d}{2b^2d}$$

$$= -\frac{a(2a^2+b^2)\,x}{2b^4} + \frac{(3a^2+b^2)\cosh(c+dx)}{3b^3d} - \frac{a\cosh(c+dx)\sinh(c+d}{2b^2d}$$

$$= -\frac{a(2a^2+b^2)\,x}{2b^4} + \frac{(3a^2+b^2)\cosh(c+dx)}{3b^3d} - \frac{a\cosh(c+dx)\sinh(c+d}{2b^2d}$$

$$= -\frac{a(2a^2+b^2)\,x}{2b^4} - \frac{2a^2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^4d} + \frac{(3a^2+b}{}$$

Mathematica [A]
time = 0.28, size = 123, normalized size = 0.87

$$\frac{3b(4a^2+b^2)\cosh(c+dx)+b^3\cosh(3(c+dx))-3a\left(2(2a^2+b^2)(c+dx)+8a\sqrt{-a^2-b^2}\,\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)+b^2\sinh(2(c+dx))\right)}{12b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (3*b*(4*a^2 + b^2)*Cosh[c + d*x] + b^3*Cosh[3*(c + d*x)] - 3*a*(2*(2*a^2 +
b^2)*(c + d*x) + 8*a*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt
[-a^2 - b^2]] + b^2*Sinh[2*(c + d*x)]))/(12*b^4*d)
```

Maple [A]
time = 1.16, size = 247, normalized size = 1.75

method	result

risch	$-\frac{a^3x}{b^4}-\frac{ax}{2b^2}+\frac{e^{3dx+3c}}{24bd}-\frac{a\,e^{2dx+2c}}{8b^2d}+\frac{e^{dx+c}a^2}{2b^3d}+\frac{e^{dx+c}}{8bd}+\frac{e^{-dx-c}a^2}{2b^3d}+\frac{e^{-dx-c}}{8bd}+\frac{a\,e^{-2dx-2c}}{8b^2d}+\frac{e^{-3dx-3}}{24bd}$
derivativedivides	$\frac{-\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a+b}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2a^2+ab+b^2}{2b^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\left(2a^2+b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^4}+\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{}$
default	$\frac{-\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a+b}{2b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2a^2+ab+b^2}{2b^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\left(2a^2+b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^4}+\frac{1}{3b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3/b/(tanh(1/2*d*x+1/2*c)-1)^3-1/2*(a+b)/b^2/(tanh(1/2*d*x+1/2*c)-1)
^2-1/2*(2*a^2+a*b+b^2)/b^3/(tanh(1/2*d*x+1/2*c)-1)+1/2*a*(2*a^2+b^2)/b^4*ln
(tanh(1/2*d*x+1/2*c)-1)+1/3/b/(tanh(1/2*d*x+1/2*c)+1)^3-1/2*(b-a)/b^2/(tanh
(1/2*d*x+1/2*c)+1)^2-1/2*(-2*a^2+a*b-b^2)/b^3/(tanh(1/2*d*x+1/2*c)+1)-1/2*a
*(2*a^2+b^2)/b^4*ln(tanh(1/2*d*x+1/2*c)+1)+2*a^2*(a^2+b^2)^(1/2)/b^4*arctan
h(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [A]
time = 0.48, size = 209, normalized size = 1.48

$$\frac{\sqrt{a^2+b^2}\,a^2\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{\left(3\,abe^{(-dx-c)}-b^2-3\,(4\,a^2+b^2)e^{(-2\,dx-2\,c)}\right)e^{(3\,dx+3\,c)}}{24\,b^3d}-\frac{(2\,a^3+ab^2)(dx+c)}{2\,b^4d}+\frac{3\,abe^{(-2\,dx-2\,c)}+b^2e^{(-3\,dx-3\,c)}+3\,(4\,a^2+b^2)e^{(-dx-c)}}{24\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxim
a")
```

```
[Out] sqrt(a^2 + b^2)*a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x -
 c) - a + sqrt(a^2 + b^2)))/(b^4*d) - 1/24*(3*a*b*e^(-d*x - c) - b^2 - 3*(4
*a^2 + b^2)*e^(-2*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) - 1/2*(2*a^3 + a*b^2)
*(d*x + c)/(b^4*d) + 1/24*(3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-3*d*x - 3*c) +
3*(4*a^2 + b^2)*e^(-d*x - c))/(b^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(130) = 260$.
time = 0.38, size = 745, normalized size = 5.28

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="frica
s")
```

```
[Out] 1/24*(b^3*cosh(d*x + c)^6 + b^3*sinh(d*x + c)^6 - 3*a*b^2*cosh(d*x + c)^5 -
 12*(2*a^3 + a*b^2)*d*x*cosh(d*x + c)^3 + 3*(2*b^3*cosh(d*x + c) - a*b^2)*s
inh(d*x + c)^5 + 3*(4*a^2*b + b^3)*cosh(d*x + c)^4 + 3*(5*b^3*cosh(d*x + c)
^2 - 5*a*b^2*cosh(d*x + c) + 4*a^2*b + b^3)*sinh(d*x + c)^4 + 3*a*b^2*cosh(
d*x + c) + 2*(10*b^3*cosh(d*x + c)^3 - 15*a*b^2*cosh(d*x + c)^2 - 6*(2*a^3
+ a*b^2)*d*x + 6*(4*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + b^3 + 3*(
4*a^2*b + b^3)*cosh(d*x + c)^2 + 3*(5*b^3*cosh(d*x + c)^4 - 10*a*b^2*cosh(d
*x + c)^3 - 12*(2*a^3 + a*b^2)*d*x*cosh(d*x + c) + 4*a^2*b + b^3 + 6*(4*a^2
*b + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 24*(a^2*cosh(d*x + c)^3 + 3*a^
2*cosh(d*x + c)^2*sinh(d*x + c) + 3*a^2*cosh(d*x + c)*sinh(d*x + c)^2 + a^2
*sinh(d*x + c)^3)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x +
 c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sin
h(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*
cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c
) + a)*sinh(d*x + c) - b)) + 3*(2*b^3*cosh(d*x + c)^5 - 5*a*b^2*cosh(d*x +
c)^4 - 12*(2*a^3 + a*b^2)*d*x*cosh(d*x + c)^2 + 4*(4*a^2*b + b^3)*cosh(d*x
+ c)^3 + a*b^2 + 2*(4*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c))/(b^4*d*cos
h(d*x + c)^3 + 3*b^4*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^4*d*cosh(d*x + c
)*sinh(d*x + c)^2 + b^4*d*sinh(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.43, size = 211, normalized size = 1.50

$$-\frac{\frac{12\,(2\,a^3+ab^2)(dx+c)}{b^4}-\frac{b^2e^{(3\,dx+3\,c)}-3\,abe^{(2\,dx+2\,c)}+12\,a^2e^{(dx+c)}+3\,b^2e^{(dx+c)}}{b^3}-\frac{\left(3\,ab^2e^{(dx+c)}+b^3+3\,(4\,a^2b+b^3)e^{(2\,dx+2\,c)}\right)e^{(-3\,dx-3\,c)}}{b^4}-\frac{24\,(a^4+a^2b^2)\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac"
)
```

```
[Out] -1/24*(12*(2*a^3 + a*b^2)*(d*x + c)/b^4 - (b^2*e^(3*d*x + 3*c) - 3*a*b*e^(2
*d*x + 2*c) + 12*a^2*e^(d*x + c) + 3*b^2*e^(d*x + c))/b^3 - (3*a*b^2*e^(d*x
```

```
 + c) + b^3 + 3*(4*a^2*b + b^3)*e^(2*d*x + 2*c))*e^(-3*d*x - 3*c)/b^4 - 24*
(a^4 + a^2*b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*
e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4))/d
```

Mupad [B]
time = 0.61, size = 278, normalized size = 1.97

$$\frac{\mathrm{e}^{-3\,c-3\,d\,x}}{24\,b\,d}-\frac{x\,\left(2\,a^3+a\,b^2\right)}{2\,b^4}+\frac{\mathrm{e}^{3\,c+3\,d\,x}}{24\,b\,d}+\frac{a\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,b^2\,d}-\frac{a\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,b^2\,d}+\frac{\mathrm{e}^{c+d\,x}\,\left(4\,a^2+b^2\right)}{8\,b^3\,d}+\frac{\mathrm{e}^{-c-d\,x}\,\left(4\,a^2+b^2\right)}{8\,b^3\,d}-\frac{a^2\,\ln\left(-\frac{2\,a^2\,\sqrt{a^2+b^2}\,\left(b-a\,\mathrm{e}^{c+d\,x}\right)}{b^5}-\frac{2\,a^2\,\mathrm{e}^{c+d\,x}\,\left(a^2+b^2\right)}{b^5}\right)\,\sqrt{a^2+b^2}}{b^4\,d}+\frac{a^2\,\ln\left(\frac{2\,a^2\,\sqrt{a^2+b^2}\,\left(b-a\,\mathrm{e}^{c+d\,x}\right)}{b^5}-\frac{2\,a^2\,\mathrm{e}^{c+d\,x}\,\left(a^2+b^2\right)}{b^5}\right)\,\sqrt{a^2+b^2}}{b^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*sinh(c + d*x)^2)/(a + b*sinh(c + d*x)),x)
```

```
[Out] exp(- 3*c - 3*d*x)/(24*b*d) - (x*(a*b^2 + 2*a^3))/(2*b^4) + exp(3*c + 3*d*x
)/(24*b*d) + (a*exp(- 2*c - 2*d*x))/(8*b^2*d) - (a*exp(2*c + 2*d*x))/(8*b^2
*d) + (exp(c + d*x)*(4*a^2 + b^2))/(8*b^3*d) + (exp(- c - d*x)*(4*a^2 + b^2
))/(8*b^3*d) - (a^2*log(- (2*a^2*(a^2 + b^2)^(1/2)*(b - a*exp(c + d*x)))/b^
5 - (2*a^2*exp(c + d*x)*(a^2 + b^2))/b^5)*(a^2 + b^2)^(1/2))/(b^4*d) + (a^2
*log((2*a^2*(a^2 + b^2)^(1/2)*(b - a*exp(c + d*x)))/b^5 - (2*a^2*exp(c + d*
x)*(a^2 + b^2))/b^5)*(a^2 + b^2)^(1/2))/(b^4*d)
```

3.371 $\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=39

$$\text{Int}\left(\frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

[Out] Unintegrable(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

[In] Int[(Cosh[c + d*x]^2*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]

[Out] Defer[Int][(Cosh[c + d*x]^2*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])), x]

Rubi steps

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

[In] Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]

[Out] $Aborted

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\cosh^2(dx+c)\right)\left(\sinh^2(dx+c)\right)}{(fx+e)\left(a+b\sinh(dx+c)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] 2*(a^4*e^c + a^2*b^2*e^c)*integrate(-e^(d*x)/(b^5*f*x + b^5*e - (b^5*f*x*e^
(2*c) + b^5*e^(2*c + 1))*e^(2*d*x) - 2*(a*b^4*f*x*e^c + a*b^4*e^(c + 1))*e^
(d*x)), x) + 1/8*e^(-3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b*f
) + 1/4*a*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b^2*f) + 1
/4*a*e^(2*c - 2*d*e/f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b^2*f) - 1/8*e^
(3*c - 3*d*e/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/(b*f) + 1/8*(4*a^2 + b^
2)*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^3*f) - 1/8*(4*a^2*e^c
 + b^2*e^c)*e^(-d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^3*f) - 1/2*(2*a
^3 + a*b^2)*log(f*x + e)/(b^4*f)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] integral(cosh(d*x + c)^2*sinh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(
d*x + c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*sinh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate(cosh(d*x + c)^2*sinh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a))
, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh(c+d\,x)^2\,\sinh(c+d\,x)^2}{(e+f\,x)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*sinh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((cosh(c + d*x)^2*sinh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.372 $\int \frac{(e+fx)^3 \cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1123

$$\frac{3a^2f^3x}{8b^3d^3}-\frac{45f^3x}{256bd^3}+\frac{a^2(e+fx)^3}{4b^3d}-\frac{3(e+fx)^3}{32bd}-\frac{a^2(a^2+b^2)\,(e+fx)^4}{4b^5f}+\frac{6a^3f^3\cosh(c+dx)}{b^4d^4}+\frac{40af^3\cosh(c+dx}{9b^2d^4}$$

```
[Out] -a^3*(f*x+e)^3*sinh(d*x+c)/b^4/d-3/32*(f*x+e)^3/b/d-45/256*f^3*x/b/d^3+a^2*
(a^2+b^2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^5/d+a^2*(a^2+b
^2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^5/d+6*a^2*(a^2+b^2)*
f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^5/d^4+6*a^2*(a^2+b^2)*f^
3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^5/d^4+2*a*f*(f*x+e)^2*cosh
(d*x+c)/b^2/d^2-40/9*a*f^2*(f*x+e)*sinh(d*x+c)/b^2/d^3-9/32*f*(f*x+e)^2*cos
h(d*x+c)*sinh(d*x+c)/b/d^2+3*a^3*f*(f*x+e)^2*cosh(d*x+c)/b^4/d^2+1/3*a*f*(f
*x+e)^2*cosh(d*x+c)^3/b^2/d^2-6*a^3*f^2*(f*x+e)*sinh(d*x+c)/b^4/d^3-3/8*a^2
*f^3*cosh(d*x+c)*sinh(d*x+c)/b^3/d^4-1/3*a*(f*x+e)^3*cosh(d*x+c)^2*sinh(d*x
+c)/b^2/d-3/16*f*(f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/b/d^2+3/4*a^2*f^2*(f*x
+e)*sinh(d*x+c)^2/b^3/d^3+1/4*a^2*(f*x+e)^3/b^3/d+1/4*(f*x+e)^3*cosh(d*x+c)
^4/b/d+3*a^2*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/
2)))/b^5/d^2+3*a^2*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/b^5/d^2-6*a^2*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(
a^2+b^2)^(1/2)))/b^5/d^3-6*a^2*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/b^5/d^3-45/256*f^3*cosh(d*x+c)*sinh(d*x+c)/b/d^4+40/
9*a*f^3*cosh(d*x+c)/b^2/d^4+3/8*a^2*f^3*x/b^3/d^3-3/4*a^2*f*(f*x+e)^2*cosh(
d*x+c)*sinh(d*x+c)/b^3/d^2-2/9*a*f^2*(f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/b^2/
d^3-1/4*a^2*(a^2+b^2)*(f*x+e)^4/b^5/f+6*a^3*f^3*cosh(d*x+c)/b^4/d^4+9/32*f^
2*(f*x+e)*cosh(d*x+c)^2/b/d^3+2/27*a*f^3*cosh(d*x+c)^3/b^2/d^4+3/32*f^2*(f*
x+e)*cosh(d*x+c)^4/b/d^3-2/3*a*(f*x+e)^3*sinh(d*x+c)/b^2/d-3/128*f^3*cosh(d
*x+c)^3*sinh(d*x+c)/b/d^4+1/2*a^2*(f*x+e)^3*sinh(d*x+c)^2/b^3/d
```

Rubi [A]
time = 1.08, antiderivative size = 1123, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 17, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.472$, Rules used = {5698, 5555, 3392, 32, 2715, 8, 3377, 2718, 3391, 5684, 5554, 5680, 2221, 2611, 6744, 2320, 6724}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (3*a^2*f^3*x)/(8*b^3*d^3) - (45*f^3*x)/(256*b*d^3) + (a^2*(e + f*x)^3)/(4*b
^3*d) - (3*(e + f*x)^3)/(32*b*d) - (a^2*(a^2 + b^2)*(e + f*x)^4)/(4*b^5*f)
```

```
+ (6*a^3*f^3*Cosh[c + d*x])/(b^4*d^4) + (40*a*f^3*Cosh[c + d*x])/(9*b^2*d^4
) + (3*a^3*f*(e + f*x)^2*Cosh[c + d*x])/(b^4*d^2) + (2*a*f*(e + f*x)^2*Cosh
[c + d*x])/(b^2*d^2) + (9*f^2*(e + f*x)*Cosh[c + d*x]^2)/(32*b*d^3) + (2*a*
f^3*Cosh[c + d*x]^3)/(27*b^2*d^4) + (a*f*(e + f*x)^2*Cosh[c + d*x]^3)/(3*b^
2*d^2) + (3*f^2*(e + f*x)*Cosh[c + d*x]^4)/(32*b*d^3) + ((e + f*x)^3*Cosh[c
 + d*x]^4)/(4*b*d) + (a^2*(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])])/(b^5*d) + (a^2*(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^5*d) + (3*a^2*(a^2 + b^2)*f*(e + f*x)
^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^2) + (3*a^2
*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
]))])/(b^5*d^2) - (6*a^2*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^3) - (6*a^2*(a^2 + b^2)*f^2*(e + f*x)*
PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^3) + (6*a^2*(a
^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^
4) + (6*a^2*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2]))])/(b^5*d^4) - (6*a^3*f^2*(e + f*x)*Sinh[c + d*x])/(b^4*d^3) - (40*a*f^
2*(e + f*x)*Sinh[c + d*x])/(9*b^2*d^3) - (a^3*(e + f*x)^3*Sinh[c + d*x])/(b
^4*d) - (2*a*(e + f*x)^3*Sinh[c + d*x])/(3*b^2*d) - (3*a^2*f^3*Cosh[c + d*x
]*Sinh[c + d*x])/(8*b^3*d^4) - (45*f^3*Cosh[c + d*x]*Sinh[c + d*x])/(256*b*
d^4) - (3*a^2*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^3*d^2) - (9*f
*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(32*b*d^2) - (2*a*f^2*(e + f*x)*C
osh[c + d*x]^2*Sinh[c + d*x])/(9*b^2*d^3) - (a*(e + f*x)^3*Cosh[c + d*x]^2*
Sinh[c + d*x])/(3*b^2*d) - (3*f^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(128*b*d^4
) - (3*f*(e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(16*b*d^2) + (3*a^2*f^2
*(e + f*x)*Sinh[c + d*x]^2)/(4*b^3*d^3) + (a^2*(e + f*x)^3*Sinh[c + d*x]^2)
/(2*b^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
```

```
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5555

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\cosh^3(c+dx)\sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\cosh^3(c+}{a+b\sinh(}}{b}$$

$$= \frac{(e+fx)^3\cosh^4(c+dx)}{4bd} - \frac{a\int (e+fx)^3\cosh^3(c+dx)\,dx}{b^2} + \frac{a}{}$$

$$= \frac{af(e+fx)^2\cosh^3(c+dx)}{3b^2d^2} + \frac{3f^2(e+fx)\cosh^4(c+dx)}{32bd^3} + \frac{(e}{}$$

$$= -\frac{a^2(a^2+b^2)(e+fx)^4}{4b^5f} + \frac{9f^2(e+fx)\cosh^2(c+dx)}{32bd^3} + \frac{2af^3}{}$$

$$= -\frac{3(e+fx)^3}{32bd} - \frac{a^2(a^2+b^2)(e+fx)^4}{4b^5f} + \frac{3a^3f(e+fx)^2\cosh(c}{b^4d^2}$$

$$= -\frac{45f^3x}{256bd^3} + \frac{a^2(e+fx)^3}{4b^3d} - \frac{3(e+fx)^3}{32bd} - \frac{a^2(a^2+b^2)(e+fx)}{4b^5f}$$

$$= \frac{3a^2f^3x}{8b^3d^3} - \frac{45f^3x}{256bd^3} + \frac{a^2(e+fx)^3}{4b^3d} - \frac{3(e+fx)^3}{32bd} - \frac{a^2(a^2+b^2)}{4b^5}$$

$$= \frac{3a^2f^3x}{8b^3d^3} - \frac{45f^3x}{256bd^3} + \frac{a^2(e+fx)^3}{4b^3d} - \frac{3(e+fx)^3}{32bd} - \frac{a^2(a^2+b^2)}{4b^5}$$

$$= \frac{3a^2f^3x}{8b^3d^3} - \frac{45f^3x}{256bd^3} + \frac{a^2(e+fx)^3}{4b^3d} - \frac{3(e+fx)^3}{32bd} - \frac{a^2(a^2+b^2)}{4b^5}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 6348 vs. $2(1123) = 2246$.

time = 16.98, size = 6348, normalized size = 5.65

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x
]),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 2.24, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \left(\cosh^3(dx+c)\right) \left(\sinh^2(dx+c)\right)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="maxima")
```

```
[Out] -1/192*((8*a*b^2*e^(-d*x - c) - 3*b^3 - 12*(2*a^2*b + b^3)*e^(-2*d*x - 2*c)
 + 24*(4*a^3 + 3*a*b^2)*e^(-3*d*x - 3*c))*e^(4*d*x + 4*c)/(b^4*d) - 192*(a^
4 + a^2*b^2)*(d*x + c)/(b^5*d) - (8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e^(-4*d*
x - 4*c) + 24*(4*a^3 + 3*a*b^2)*e^(-d*x - c) + 12*(2*a^2*b + b^3)*e^(-2*d*x
 - 2*c))/(b^4*d) - 192*(a^4 + a^2*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x
- 2*c) - b)/(b^5*d))*e^3 + 1/55296*(13824*(a^4*d^4*f^3*e^(4*c) + a^2*b^2*d^
4*f^3*e^(4*c))*x^4 + 55296*(a^4*d^4*f^2*e^(4*c) + a^2*b^2*d^4*f^2*e^(4*c))*
x^3*e + 82944*(a^4*d^4*f*e^(4*c) + a^2*b^2*d^4*f*e^(4*c))*x^2*e^2 + 27*(32*
b^4*d^3*f^3*x^3*e^(8*c) - 3*b^4*f^3*e^(8*c) - 24*b^4*d^2*f*e^(8*c + 2) + 12
*b^4*d*f^2*e^(8*c + 1) - 24*(b^4*d^2*f^3*e^(8*c) - 4*b^4*d^3*f^2*e^(8*c + 1
))*x^2 + 12*(b^4*d*f^3*e^(8*c) + 8*b^4*d^3*f*e^(8*c + 2) - 4*b^4*d^2*f^2*e^
(8*c + 1))*x)*e^(4*d*x) - 256*(9*a*b^3*d^3*f^3*x^3*e^(7*c) - 2*a*b^3*f^3*e^
(7*c) - 9*a*b^3*d^2*f*e^(7*c + 2) + 6*a*b^3*d*f^2*e^(7*c + 1) - 9*(a*b^3*d^
2*f^3*e^(7*c) - 3*a*b^3*d^3*f^2*e^(7*c + 1))*x^2 + 3*(2*a*b^3*d*f^3*e^(7*c)
 + 9*a*b^3*d^3*f*e^(7*c + 2) - 6*a*b^3*d^2*f^2*e^(7*c + 1))*x)*e^(3*d*x) -
```

```
864*(6*a^2*b^2*f^3*e^(6*c) + 3*b^4*f^3*e^(6*c) - 4*(2*a^2*b^2*d^3*f^3*e^(6*
c) + b^4*d^3*f^3*e^(6*c))*x^3 + 6*(2*a^2*b^2*d^2*f^3*e^(6*c) + b^4*d^2*f^3*
e^(6*c) - 2*(2*a^2*b^2*d^3*f^2*e^(6*c) + b^4*d^3*f^2*e^(6*c))*e)*x^2 - 6*(2
*a^2*b^2*d*f^3*e^(6*c) + b^4*d*f^3*e^(6*c) + 2*(2*a^2*b^2*d^3*f*e^(6*c) + b
^4*d^3*f*e^(6*c))*e^2 - 2*(2*a^2*b^2*d^2*f^2*e^(6*c) + b^4*d^2*f^2*e^(6*c))
*e)*x + 6*(2*a^2*b^2*d^2*f*e^(6*c) + b^4*d^2*f*e^(6*c))*e^2 - 6*(2*a^2*b^2*
d*f^2*e^(6*c) + b^4*d*f^2*e^(6*c))*e)*e^(2*d*x) + 6912*(24*a^3*b*f^3*e^(5*c
) + 18*a*b^3*f^3*e^(5*c) - (4*a^3*b*d^3*f^3*e^(5*c) + 3*a*b^3*d^3*f^3*e^(5*
c))*x^3 + 3*(4*a^3*b*d^2*f^3*e^(5*c) + 3*a*b^3*d^2*f^3*e^(5*c) - (4*a^3*b*d
^3*f^2*e^(5*c) + 3*a*b^3*d^3*f^2*e^(5*c))*e)*x^2 - 3*(8*a^3*b*d*f^3*e^(5*c)
 + 6*a*b^3*d*f^3*e^(5*c) + (4*a^3*b*d^3*f*e^(5*c) + 3*a*b^3*d^3*f*e^(5*c))*
e^2 - 2*(4*a^3*b*d^2*f^2*e^(5*c) + 3*a*b^3*d^2*f^2*e^(5*c))*e)*x + 3*(4*a^3
*b*d^2*f*e^(5*c) + 3*a*b^3*d^2*f*e^(5*c))*e^2 - 6*(4*a^3*b*d*f^2*e^(5*c) +
3*a*b^3*d*f^2*e^(5*c))*e)*e^(d*x) + 6912*(24*a^3*b*f^3*e^(3*c) + 18*a*b^3*f
^3*e^(3*c) + (4*a^3*b*d^3*f^3*e^(3*c) + 3*a*b^3*d^3*f^3*e^(3*c))*x^3 + 3*(4
*a^3*b*d^2*f^3*e^(3*c) + 3*a*b^3*d^2*f^3*e^(3*c) + (4*a^3*b*d^3*f^2*e^(3*c)
 + 3*a*b^3*d^3*f^2*e^(3*c))*e)*x^2 + 3*(8*a^3*b*d*f^3*e^(3*c) + 6*a*b^3*d*f
^3*e^(3*c) + (4*a^3*b*d^3*f*e^(3*c) + 3*a*b^3*d^3*f*e^(3*c))*e^2 + 2*(4*a^3
*b*d^2*f^2*e^(3*c) + 3*a*b^3*d^2*f^2*e^(3*c))*e)*x + 3*(4*a^3*b*d^2*f*e^(3*
c) + 3*a*b^3*d^2*f*e^(3*c))*e^2 + 6*(4*a^3*b*d*f^2*e^(3*c) + 3*a*b^3*d*f^2*
e^(3*c))*e)*e^(-d*x) + 864*(6*a^2*b^2*f^3*e^(2*c) + 3*b^4*f^3*e^(2*c) + 4*(
2*a^2*b^2*d^3*f^3*e^(2*c) + b^4*d^3*f^3*e^(2*c))*x^3 + 6*(2*a^2*b^2*d^2*f^3
*e^(2*c) + b^4*d^2*f^3*e^(2*c) + 2*(2*a^2*b^2*d^3*f^2*e^(2*c) + b^4*d^3*f^2
*e^(2*c))*e)*x^2 + 6*(2*a^2*b^2*d*f^3*e^(2*c) + b^4*d*f^3*e^(2*c) + 2*(2*a^
2*b^2*d^3*f*e^(2*c) + b^4*d^3*f*e^(2*c))*e^2 + 2*(2*a^2*b^2*d^2*f^2*e^(2*c)
 + b^4*d^2*f^2*e^(2*c))*e)*x + 6*(2*a^2*b^2*d^2*f*e^(2*c) + b^4*d^2*f*e^(2*
c))*e^2 + 6*(2*a^2*b^2*d*f^2*e^(2*c) + b^4*d*f^2*e^(2*c))*e)*e^(-2*d*x) + 2
56*(9*a*b^3*d^3*f^3*x^3*e^c + 9*a*b^3*d^2*f*e^(c + 2) + 6*a*b^3*d*f^2*e^(c
+ 1) + 2*a*b^3*f^3*e^c + 9*(3*a*b^3*d^3*f^2*e^(c + 1) + a*b^3*d^2*f^3*e^c)*
x^2 + 3*(9*a*b^3*d^3*f*e^(c + 2) + 6*a*b^3*d^2*f^2*e^(c + 1) + 2*a*b^3*d*f^
3*e^c)*x)*e^(-3*d*x) + 27*(32*b^4*d^3*f^3*x^3 + 24*b^4*d^2*f*e^2 + 12*b^4*d
*f^2*e + 3*b^4*f^3 + 24*(4*b^4*d^3*f^2*e + b^4*d^2*f^3)*x^2 + 12*(8*b^4*d^3
*f*e^2 + 4*b^4*d^2*f^2*e + b^4*d*f^3)*x)*e^(-4*d*x))*e^(-4*c)/(b^5*d^4) - i
ntegrate(-2*((a^4*b*f^3 + a^2*b^3*f^3)*x^3 + 3*(a^4*b*f^2 + a^2*b^3*f^2)*x^
2*e + 3*(a^4*b*f + a^2*b^3*f)*x*e^2 - ((a^5*f^3*e^c + a^3*b^2*f^3*e^c)*x^3
+ 3*(a^5*f^2*e^c + a^3*b^2*f^2*e^c)*x^2*e + 3*(a^5*f*e^c + a^3*b^2*f*e^c)*x
*e^2)*e^(d*x))/(b^6*e^(2*d*x + 2*c) + 2*a*b^5*e^(d*x + c) - b^6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 24245 vs. $2(1077) = 2154$.
time = 0.62, size = 24245, normalized size = 21.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori thm="fricas")

[Out] 1/55296*(864*b^4*d^3*f^3*x^3 + 648*b^4*d^2*f^3*x^2 + 864*b^4*d^3*cosh(1)^3 + 864*b^4*d^3*sinh(1)^3 + 324*b^4*d*f^3*x + 27*(32*b^4*d^3*f^3*x^3 - 24*b^4 *d^2*f^3*x^2 + 32*b^4*d^3*cosh(1)^3 + 32*b^4*d^3*sinh(1)^3 + 12*b^4*d*f^3*x - 3*b^4*f^3 + 24*(4*b^4*d^3*f*x - b^4*d^2*f)*cosh(1)^2 + 24*(4*b^4*d^3*f*x + 4*b^4*d^3*cosh(1) - b^4*d^2*f)*sinh(1)^2 + 12*(8*b^4*d^3*f^2*x^2 - 4*b^4 *d^2*f^2*x + b^4*d*f^2)*cosh(1) + 12*(8*b^4*d^3*f^2*x^2 - 4*b^4*d^2*f^2*x + 8*b^4*d^3*cosh(1)^2 + b^4*d*f^2 + 4*(4*b^4*d^3*f*x - b^4*d^2*f)*cosh(1))*s inh(1))*cosh(d*x + c)^8 + 27*(32*b^4*d^3*f^3*x^3 - 24*b^4*d^2*f^3*x^2 + 32* b^4*d^3*cosh(1)^3 + 32*b^4*d^3*sinh(1)^3 + 12*b^4*d*f^3*x - 3*b^4*f^3 + 24* (4*b^4*d^3*f*x - b^4*d^2*f)*cosh(1)^2 + 24*(4*b^4*d^3*f*x + 4*b^4*d^3*cosh(1) - b^4*d^2*f)*sinh(1)^2 + 12*(8*b^4*d^3*f^2*x^2 - 4*b^4*d^2*f^2*x + b^4*d *f^2)*cosh(1) + 12*(8*b^4*d^3*f^2*x^2 - 4*b^4*d^2*f^2*x + 8*b^4*d^3*cosh(1) ^2 + b^4*d*f^2 + 4*(4*b^4*d^3*f*x - b^4*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^8 - 256*(9*a*b^3*d^3*f^3*x^3 - 9*a*b^3*d^2*f^3*x^2 + 9*a*b^3*d^3*cosh(1)^3 + 9*a*b^3*d^3*sinh(1)^3 + 6*a*b^3*d*f^3*x - 2*a*b^3*f^3 + 9*(3*a*b^3*d^ 3*f*x - a*b^3*d^2*f)*cosh(1)^2 + 9*(3*a*b^3*d^3*f*x + 3*a*b^3*d^3*cosh(1) - a*b^3*d^2*f)*sinh(1)^2 + 3*(9*a*b^3*d^3*f^2*x^2 - 6*a*b^3*d^2*f^2*x + 2*a* b^3*d*f^2)*cosh(1) + 3*(9*a*b^3*d^3*f^2*x^2 - 6*a*b^3*d^2*f^2*x + 9*a*b^3*d ^3*cosh(1)^2 + 2*a*b^3*d*f^2 + 6*(3*a*b^3*d^3*f*x - a*b^3*d^2*f)*cosh(1))*s inh(1))*cosh(d*x + c)^7 - 8*(288*a*b^3*d^3*f^3*x^3 - 288*a*b^3*d^2*f^3*x^2 + 288*a*b^3*d^3*cosh(1)^3 + 288*a*b^3*d^3*sinh(1)^3 + 192*a*b^3*d*f^3*x - 6 4*a*b^3*f^3 + 288*(3*a*b^3*d^3*f*x - a*b^3*d^2*f)*cosh(1)^2 + 288*(3*a*b^3* d^3*f*x + 3*a*b^3*d^3*cosh(1) - a*b^3*d^2*f)*sinh(1)^2 + 96*(9*a*b^3*d^3*f^ 2*x^2 - 6*a*b^3*d^2*f^2*x + 2*a*b^3*d*f^2)*cosh(1) - 27*(32*b^4*d^3*f^3*x^3 - 24*b^4*d^2*f^3*x^2 + 32*b^4*d^3*cosh(1)^3 + 32*b^4*d^3*sinh(1)^3 + 12*b^ 4*d*f^3*x - 3*b^4*f^3 + 24*(4*b^4*d^3*f*x - b^4*d^2*f)*cosh(1)^2 + 24*(4*b^ 4*d^3*f*x + 4*b^4*d^3*cosh(1) - b^4*d^2*f)*sinh(1)^2 + 12*(8*b^4*d^3*f^2*x^ 2 - 4*b^4*d^2*f^2*x + b^4*d*f^2)*cosh(1) + 12*(8*b^4*d^3*f^2*x^2 - 4*b^4*d^ 2*f^2*x + 8*b^4*d^3*cosh(1)^2 + b^4*d*f^2 + 4*(4*b^4*d^3*f*x - b^4*d^2*f)*c osh(1))*sinh(1))*cosh(d*x + c) + 96*(9*a*b^3*d^3*f^2*x^2 - 6*a*b^3*d^2*f^2* x + 9*a*b^3*d^3*cosh(1)^2 + 2*a*b^3*d*f^2 + 6*(3*a*b^3*d^3*f*x - a*b^3*d^2* f)*cosh(1))*sinh(1))*sinh(d*x + c)^7 + 81*b^4*f^3 + 864*(4*(2*a^2*b^2 + b^4)*d^3*f^3*x^3 - 6*(2*a^2*b^2 + b^4)*d^2*f^3*x^2 + 4*(2*a^2*b^2 + b^4)*d^3*c osh(1)^3 + 4*(2*a^2*b^2 + b^4)*d^3*sinh(1)^3 + 6*(2*a^2*b^2 + b^4)*d*f^3*x - 3*(2*a^2*b^2 + b^4)*f^3 + 6*(2*(2*a^2*b^2 + b^4)*d^3*f*x - (2*a^2*b^2 + b ^4)*d^2*f)*cosh(1)^2 + 6*(2*(2*a^2*b^2 + b^4)*d^3*f*x + 2*(2*a^2*b^2 + b^4) *d^3*cosh(1) - (2*a^2*b^2 + b^4)*d^2*f)*sinh(1)^2 + 6*(2*(2*a^2*b^2 + b^4)* d^3*f^2*x^2 - 2*(2*a^2*b^2 + b^4)*d^2*f^2*x + (2*a^2*b^2 + b^4)*d*f^2)*cosh (1) + 6*(2*(2*a^2*b^2 + b^4)*d^3*f^2*x^2 - 2*(2*a^2*b^2 + b^4)*d^2*f^2*x + 2*(2*a^2*b^2 + b^4)*d^3*cosh(1)^2 + (2*a^2*b^2 + b^4)*d*f^2 + 2*(2*(2*a^2*b ^2 + b^4)*d^3*f*x - (2*a^2*b^2 + b^4)*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^6 + 4*(864*(2*a^2*b^2 + b^4)*d^3*f^3*x^3 - 1296*(2*a^2*b^2 + b^4)*d^2*f^3

```
*x^2 + 864*(2*a^2*b^2 + b^4)*d^3*cosh(1)^3 + 864*(2*a^2*b^2 + b^4)*d^3*sinh
(1)^3 + 1296*(2*a^2*b^2 + b^4)*d*f^3*x - 648*(2*a^2*b^2 + b^4)*f^3 + 1296*(
2*(2*a^2*b^2 + b^4)*d^3*f*x - (2*a^2*b^2 + b^4)*d^2*f)*cosh(1)^2 + 189*(32*
b^4*d^3*f^3*x^3 - 24*b^4*d^2*f^3*x^2 + 32*b^4*d^3*cosh(1)^3 + 32*b^4*d^3*si
nh(1)^3 + 12*b^4*d*f^3*x - 3*b^4*f^3 + 24*(4*b^4*d^3*f*x - b^4*d^2*f)*cosh(
1)^2 + 24*(4*b^4*d^3*f*x + 4*b^4*d^3*cosh(1) - b^4*d^2*f)*sinh(1)^2 + 12*(8
*b^4*d^3*f^2*x^2 - 4*b^4*d^2*f^2*x + b^4*d*f^2)*cosh(1) + 12*(8*b^4*d^3*f^2
*x^2 - 4*b^4*d^2*f^2*x + 8*b^4*d^3*cosh(1)^2 + b^4*d*f^2 + 4*(4*b^4*d^3*f*x
 - b^4*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 1296*(2*(2*a^2*b^2 + b^4)
*d^3*f*x + 2*(2*a^2*b^2 + b^4)*d^3*cosh(1) - (2*a^2*b^2 + b^4)*d^2*f)*sinh(
1)^2 + 1296*(2*(2*a^2*b^2 + b^4)*d^3*f^2*x^2 - 2*(2*a^2*b^2 + b^4)*d^2*f^2*
x + (2*a^2*b^2 + b^4)*d*f^2)*cosh(1) - 448*(9*a*b^3*d^3*f^3*x^3 - 9*a*b^3*d
^2*f^3*x^2 + 9*a*b^3*d^3*cosh(1)^3 + 9*a*b^3*d^3*sinh(1)^3 + 6*a*b^3*d*f^3*
x - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*f*x - a*b^3*d^2*f)*cosh(1)^2 + 9*(3*a*b^3*
d^3*f*x + 3*a*b^3*d^3*cosh(1) - a*b^3*d^2*f)*sinh(1)^2 + 3*(9*a*b^3*d^3*f^2
*x^2 - 6*a*b^3*d^2*f^2*x + 2*a*b^3*d*f^2)*cosh(1) + 3*(9*a*b^3*d^3*f^2*x^2
- 6*a*b^3*d^2*f^2*x + 9*a*b^3*d^3*cosh(1)^2 + 2*a*b^3*d*f^2 + 6*(3*a*b^3*d^
3*f*x - a*b^3*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c) + 1296*(2*(2*a^2*b^2 +
 b^4)*d^3*f^2*x^2 - 2*(2*a^2*b^2 + b^4)*d^2*f^2*x + 2*(2*a^2*b^2 + b^4)*d^3
*cosh(1)^2 + (2*a^2*b^2 + b^4)*d*f^2 + 2*(2*(2*a^2*b^2 + b^4)*d^3*f*x - (2*
a^2*b^2 + b^4)*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^6 - 6912*((4*a^3*b +
3*a*b^3)*d^3*f^3*x^3 - 3*(4*a^3*b + 3*a*b^3)*d^2*f^3*x^2 + (4*a^3*b + 3*a*b
^3)*d^3*cosh(1)^3 + (4*a^3*b + 3*a*b^3)*d^3*sinh(1)^3 + 6*(4*a^3*b + 3*a*b^
3)*d*f^3*x - 6*(4*a^3*b + 3*a*b^3)*f^3 + 3*((4*a^3*b + 3*a*b^3)*d^3*f*x - (
4*a^3*b + 3*a*b^3)*d^2*f)*cosh(1)^2 + 3*((4*a^3...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**3*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)^3*sinh(d*x + c)^2/(b*sinh(d*x + c) + a)
, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^3\,\sinh(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.373 $\int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal. Leaf size=819

$$\frac{a^2efx}{2b^3d} - \frac{3efx}{16bd} + \frac{a^2f^2x^2}{4b^3d} - \frac{3f^2x^2}{32bd} - \frac{a^2(a^2+b^2)(e+fx)^3}{3b^5f} + \frac{2a^3f(e+fx)\cosh(c+dx)}{b^4d^2} + \frac{4af(e+fx)\cosh(c+d}{3b^2d^2}$$

```
[Out] 2*a^3*f*(f*x+e)*cosh(d*x+c)/b^4/d^2+2/9*a*f*(f*x+e)*cosh(d*x+c)^3/b^2/d^2-1
/3*a*(f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/b^2/d-1/8*f*(f*x+e)*cosh(d*x+c)^3*
sinh(d*x+c)/b/d^2-a^3*(f*x+e)^2*sinh(d*x+c)/b^4/d-3/32*f^2*x^2/b/d+a^2*(a^2
+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^5/d+a^2*(a^2+b^2)*
(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^5/d-2*a^2*(a^2+b^2)*f^2*
polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^5/d^3-2*a^2*(a^2+b^2)*f^2*po
lylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^5/d^3+4/3*a*f*(f*x+e)*cosh(d*x
+c)/b^2/d^2+1/2*a^2*e*f*x/b^3/d+3/32*f^2*cosh(d*x+c)^2/b/d^3+1/32*f^2*cosh(
d*x+c)^4/b/d^3+1/4*(f*x+e)^2*cosh(d*x+c)^4/b/d+2*a^2*(a^2+b^2)*f*(f*x+e)*po
lylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^5/d^2+2*a^2*(a^2+b^2)*f*(f*x+e
)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^5/d^2-3/16*e*f*x/b/d-14/9*
a*f^2*sinh(d*x+c)/b^2/d^3-1/2*a^2*f*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/b^3/d^2
+1/4*a^2*f^2*x^2/b^3/d-1/3*a^2*(a^2+b^2)*(f*x+e)^3/b^5/f-2*a^3*f^2*sinh(d*x
+c)/b^4/d^3-2/3*a*(f*x+e)^2*sinh(d*x+c)/b^2/d+1/4*a^2*f^2*sinh(d*x+c)^2/b^3
/d^3+1/2*a^2*(f*x+e)^2*sinh(d*x+c)^2/b^3/d-2/27*a*f^2*sinh(d*x+c)^3/b^2/d^3
-3/16*f*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/b/d^2
```

Rubi [A]
time = 0.80, antiderivative size = 819, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 14, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5698, 5555, 3391, 3392, 3377, 2717, 2713, 5684, 5554, 5680, 2221, 2611, 2320, 6724}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*e*f*x)/(2*b^3*d) - (3*e*f*x)/(16*b*d) + (a^2*f^2*x^2)/(4*b^3*d) - (3*f
^2*x^2)/(32*b*d) - (a^2*(a^2 + b^2)*(e + f*x)^3)/(3*b^5*f) + (2*a^3*f*(e +
f*x)*Cosh[c + d*x])/(b^4*d^2) + (4*a*f*(e + f*x)*Cosh[c + d*x])/(3*b^2*d^2)
 + (3*f^2*Cosh[c + d*x]^2)/(32*b*d^3) + (2*a*f*(e + f*x)*Cosh[c + d*x]^3)/(
9*b^2*d^2) + (f^2*Cosh[c + d*x]^4)/(32*b*d^3) + ((e + f*x)^2*Cosh[c + d*x]^
4)/(4*b*d) + (a^2*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2])])/(b^5*d) + (a^2*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2])])/(b^5*d) + (2*a^2*(a^2 + b^2)*f*(e + f*x)*PolyLog
```

```
[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^2) + (2*a^2*(a^2 + b^
2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d
^2) - (2*a^2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/(b^5*d^3) - (2*a^2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))])/(b^5*d^3) - (2*a^3*f^2*Sinh[c + d*x])/(b^4*d^3) - (14
*a*f^2*Sinh[c + d*x])/(9*b^2*d^3) - (a^3*(e + f*x)^2*Sinh[c + d*x])/(b^4*d)
 - (2*a*(e + f*x)^2*Sinh[c + d*x])/(3*b^2*d) - (a^2*f*(e + f*x)*Cosh[c + d*
x]*Sinh[c + d*x])/(2*b^3*d^2) - (3*f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])
/(16*b*d^2) - (a*(e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b^2*d) - (f*
(e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x])/(8*b*d^2) + (a^2*f^2*Sinh[c + d*x]
^2)/(4*b^3*d^3) + (a^2*(e + f*x)^2*Sinh[c + d*x]^2)/(2*b^3*d) - (2*a*f^2*Si
nh[c + d*x]^3)/(27*b^2*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5555

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh^3(c+dx)\sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\cosh^3(c+}{a+b\sinh(}}{b}$$

$$= \frac{(e+fx)^2\cosh^4(c+dx)}{4bd} - \frac{a\int (e+fx)^2\cosh^3(c+dx)\,dx}{b^2} + \frac{a}{}$$

$$= \frac{2af(e+fx)\cosh^3(c+dx)}{9b^2d^2} + \frac{f^2\cosh^4(c+dx)}{32bd^3} + \frac{(e+fx)^2\mathrm{co}}{4}$$

$$= -\frac{a^2(a^2+b^2)(e+fx)^3}{3b^5f} + \frac{3f^2\cosh^2(c+dx)}{32bd^3} + \frac{2af(e+fx)\mathrm{c}}{9b^2}$$

$$= -\frac{3efx}{16bd} - \frac{3f^2x^2}{32bd} - \frac{a^2(a^2+b^2)(e+fx)^3}{3b^5f} + \frac{2a^3f(e+fx)\cosh}{b^4d^2}$$

$$= \frac{a^2efx}{2b^3d} - \frac{3efx}{16bd} + \frac{a^2f^2x^2}{4b^3d} - \frac{3f^2x^2}{32bd} - \frac{a^2(a^2+b^2)(e+fx)^3}{3b^5f} +$$

$$= \frac{a^2efx}{2b^3d} - \frac{3efx}{16bd} + \frac{a^2f^2x^2}{4b^3d} - \frac{3f^2x^2}{32bd} - \frac{a^2(a^2+b^2)(e+fx)^3}{3b^5f} +$$

$$= \frac{a^2efx}{2b^3d} - \frac{3efx}{16bd} + \frac{a^2f^2x^2}{4b^3d} - \frac{3f^2x^2}{32bd} - \frac{a^2(a^2+b^2)(e+fx)^3}{3b^5f} +$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 3947 vs. $2(819) = 1638$.
time = 9.64, size = 3947, normalized size = 4.82

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x
]),x]
```

```
[Out] -1/8*(e^2*Log[a + b*Sinh[c + d*x]])/(b*d) - (e*f*(-1/2*x^2/b + (x*Log[1 + (
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (x*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])])/(b*d) + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 +
b^2])]/(b*d^2) + PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(b*d^
2)))/4 - (f^2*(-1/3*x^3/b + (x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])])/(b*d) + (x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) +
```

(2*x*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (2*x*P
olyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2) - (2*PolyLog[3
, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])])/(b*d^3) - (2*PolyLog[3, -((b*E^(
c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^3)))/8 + (f^2*(2*(4*a^2 + b^2)*x^3*
Coth[c] - (2*(4*a^2 + b^2)*(2*d^3*E^(2*c)*x^3 + 3*d^2*x^2*Log[1 + (b*E^(2*c
 + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*d^2*E^(2*c)*x^2*Log[1 + (
b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 3*d^2*x^2*Log[1 + (
b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*d^2*E^(2*c)*x^2*L
og[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 6*d*(-1 + E
^(2*c))*x*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]
))] - 6*d*(-1 + E^(2*c))*x*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^
2 + b^2)*E^(2*c)]))] - 6*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2
+ b^2)*E^(2*c)]))] + 6*E^(2*c)*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt
[(a^2 + b^2)*E^(2*c)]))] - 6*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(
a^2 + b^2)*E^(2*c)]))] + 6*E^(2*c)*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c +
Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(d^3*(-1 + E^(2*c))) - (24*a*b*Cosh[d*x]*(-2
*d*x*Cosh[c] + (2 + d^2*x^2)*Sinh[c]))/d^3 + (3*b^2*Cosh[2*d*x]*((1 + 2*d^2
*x^2)*Cosh[2*c] - 2*d*x*Sinh[2*c]))/d^3 - (24*a*b*((2 + d^2*x^2)*Cosh[c] -
2*d*x*Sinh[c])*Sinh[d*x])/d^3 + (3*b^2*(-2*d*x*Cosh[2*c] + (1 + 2*d^2*x^2)*
Sinh[2*c])*Sinh[2*d*x])/d^3))/(96*b^3) + (f^2*(-4608*a^4*d^3*E^(4*c)*x^3 -
3456*a^2*b^2*d^3*E^(4*c)*x^3 - 288*b^4*d^3*E^(4*c)*x^3 + 13824*a^3*b*E^(3*c
)*Cosh[d*x] + 6912*a*b^3*E^(3*c)*Cosh[d*x] - 13824*a^3*b*E^(5*c)*Cosh[d*x]
- 6912*a*b^3*E^(5*c)*Cosh[d*x] + 13824*a^3*b*d*E^(3*c)*x*Cosh[d*x] + 6912*a
*b^3*d*E^(3*c)*x*Cosh[d*x] + 13824*a^3*b*d*E^(5*c)*x*Cosh[d*x] + 6912*a*b^3
*d*E^(5*c)*x*Cosh[d*x] + 6912*a^3*b*d^2*E^(3*c)*x^2*Cosh[d*x] + 3456*a*b^3*
d^2*E^(3*c)*x^2*Cosh[d*x] - 6912*a^3*b*d^2*E^(5*c)*x^2*Cosh[d*x] - 3456*a*b
^3*d^2*E^(5*c)*x^2*Cosh[d*x] + 864*a^2*b^2*E^(2*c)*Cosh[2*d*x] + 216*b^4*E^
(2*c)*Cosh[2*d*x] + 864*a^2*b^2*E^(6*c)*Cosh[2*d*x] + 216*b^4*E^(6*c)*Cosh[
2*d*x] + 1728*a^2*b^2*d*E^(2*c)*x*Cosh[2*d*x] + 432*b^4*d*E^(2*c)*x*Cosh[2*
d*x] - 1728*a^2*b^2*d*E^(6*c)*x*Cosh[2*d*x] - 432*b^4*d*E^(6*c)*x*Cosh[2*d*
x] + 1728*a^2*b^2*d^2*E^(2*c)*x^2*Cosh[2*d*x] + 432*b^4*d^2*E^(2*c)*x^2*Cos
h[2*d*x] + 1728*a^2*b^2*d^2*E^(6*c)*x^2*Cosh[2*d*x] + 432*b^4*d^2*E^(6*c)*x
^2*Cosh[2*d*x] + 128*a*b^3*E^c*Cosh[3*d*x] - 128*a*b^3*E^(7*c)*Cosh[3*d*x]
+ 384*a*b^3*d*E^c*x*Cosh[3*d*x] + 384*a*b^3*d*E^(7*c)*x*Cosh[3*d*x] + 576*a
*b^3*d^2*E^c*x^2*Cosh[3*d*x] - 576*a*b^3*d^2*E^(7*c)*x^2*Cosh[3*d*x] + 27*b
^4*Cosh[4*d*x] + 27*b^4*E^(8*c)*Cosh[4*d*x] + 108*b^4*d*x*Cosh[4*d*x] - 108
*b^4*d*E^(8*c)*x*Cosh[4*d*x] + 216*b^4*d^2*x^2*Cosh[4*d*x] + 216*b^4*d^2*E^
(8*c)*x^2*Cosh[4*d*x] + 13824*a^4*d^2*E^(4*c)*x^2*Log[1 + (b*E^(2*c + d*x))
/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 10368*a^2*b^2*d^2*E^(4*c)*x^2*Log[1
 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 864*b^4*d^2*E^(
4*c)*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 1
3824*a^4*d^2*E^(4*c)*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2
)*E^(2*c)])] + 10368*a^2*b^2*d^2*E^(4*c)*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E
^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 864*b^4*d^2*E^(4*c)*x^2*Log[1 + (b*E^(2*
c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 1728*(16*a^4 + 12*a^2*b^2

```
+ b^4)*d*E^(4*c)*x*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)
*E^(2*c)]))] + 1728*(16*a^4 + 12*a^2*b^2 + b^4)*d*E^(4*c)*x*PolyLog[2, -((b
*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 27648*a^4*E^(4*c)*P
olyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 20736
*a^2*b^2*E^(4*c)*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E
^(2*c)]))] - 1728*b^4*E^(4*c)*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[
(a^2 + b^2)*E^(2*c)]))] - 27648*a^4*E^(4*c)*PolyLog[3, -((b*E^(2*c + d*x))/
(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 20736*a^2*b^2*E^(4*c)*PolyLog[3, -(
(b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 1728*b^4*E^(4*c)*
PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 1382
4*a^3*b*E^(3*c)*Sinh[d*x] - 6912*a*b^3*E^(3*c)*Sinh[d*x] - 13824*a^3*b*E^(5
*c)*Sinh[d*x] - 6912*a*b^3*E^(5*c)*Sinh[d*x] - 13824*a^3*b*d*E^(3*c)*x*Sinh
[d*x] - 6912*a*b^3*d*E^(3*c)*x*Sinh[d*x] + 13824*a^3*b*d*E^(5*c)*x*Sinh[d*x
] + 6912*a*b^3*d*E^(5*c)*x*Sinh[d*x] - 6912*a^3*b*d^2*E^(3*c)*x^2*Sinh[d*x]
 - 3456*a*b^3*d^2*E^(3*c)*x^2*Sinh[d*x] - 6912*a^3*b*d^2*E^(5*c)*x^2*Sinh[d
*x] - 3456*a*b^3*d^2*E^(5*c)*x^2*Sinh[d*x] - 86...
```

Maple [F]
time = 2.22, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \left(\cosh^3(dx+c)\right) \left(\sinh^2(dx+c)\right)}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="maxima")
```

```
[Out] -1/192*((8*a*b^2*e^(-d*x - c) - 3*b^3 - 12*(2*a^2*b + b^3)*e^(-2*d*x - 2*c)
 + 24*(4*a^3 + 3*a*b^2)*e^(-3*d*x - 3*c))*e^(4*d*x + 4*c)/(b^4*d) - 192*(a^
4 + a^2*b^2)*(d*x + c)/(b^5*d) - (8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e^(-4*d*
x - 4*c) + 24*(4*a^3 + 3*a*b^2)*e^(-d*x - c) + 12*(2*a^2*b + b^3)*e^(-2*d*x
 - 2*c))/(b^4*d) - 192*(a^4 + a^2*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x
- 2*c) - b)/(b^5*d))*e^2 + 1/13824*(4608*(a^4*d^3*f^2*e^(4*c) + a^2*b^2*d^3
*f^2*e^(4*c))*x^3 + 13824*(a^4*d^3*f*e^(4*c) + a^2*b^2*d^3*f*e^(4*c))*x^2*e
 + 27*(8*b^4*d^2*f^2*x^2*e^(8*c) + b^4*f^2*e^(8*c) - 4*b^4*d*f*e^(8*c + 1)
- 4*(b^4*d*f^2*e^(8*c) - 4*b^4*d^2*f*e^(8*c + 1))*x)*e^(4*d*x) - 64*(9*a*b^
```

```
3*d^2*f^2*x^2*e^(7*c) + 2*a*b^3*f^2*e^(7*c) - 6*a*b^3*d*f*e^(7*c + 1) - 6*(
a*b^3*d*f^2*e^(7*c) - 3*a*b^3*d^2*f*e^(7*c + 1))*x)*e^(3*d*x) + 432*(2*a^2*
b^2*f^2*e^(6*c) + b^4*f^2*e^(6*c) + 2*(2*a^2*b^2*d^2*f^2*e^(6*c) + b^4*d^2*
f^2*e^(6*c))*x^2 - 2*(2*a^2*b^2*d*f^2*e^(6*c) + b^4*d*f^2*e^(6*c) - 2*(2*a^
2*b^2*d^2*f*e^(6*c) + b^4*d^2*f*e^(6*c))*e)*x - 2*(2*a^2*b^2*d*f*e^(6*c) +
b^4*d*f*e^(6*c))*e)*e^(2*d*x) - 1728*(8*a^3*b*f^2*e^(5*c) + 6*a*b^3*f^2*e^(
5*c) + (4*a^3*b*d^2*f^2*e^(5*c) + 3*a*b^3*d^2*f^2*e^(5*c))*x^2 - 2*(4*a^3*b
*d*f^2*e^(5*c) + 3*a*b^3*d*f^2*e^(5*c) - (4*a^3*b*d^2*f*e^(5*c) + 3*a*b^3*d
^2*f*e^(5*c))*e)*x - 2*(4*a^3*b*d*f*e^(5*c) + 3*a*b^3*d*f*e^(5*c))*e)*e^(d*
x) + 1728*(8*a^3*b*f^2*e^(3*c) + 6*a*b^3*f^2*e^(3*c) + (4*a^3*b*d^2*f^2*e^(
3*c) + 3*a*b^3*d^2*f^2*e^(3*c))*x^2 + 2*(4*a^3*b*d*f^2*e^(3*c) + 3*a*b^3*d*
f^2*e^(3*c) + (4*a^3*b*d^2*f*e^(3*c) + 3*a*b^3*d^2*f*e^(3*c))*e)*x + 2*(4*a
^3*b*d*f*e^(3*c) + 3*a*b^3*d*f*e^(3*c))*e)*e^(-d*x) + 432*(2*a^2*b^2*f^2*e^
(2*c) + b^4*f^2*e^(2*c) + 2*(2*a^2*b^2*d^2*f^2*e^(2*c) + b^4*d^2*f^2*e^(2*c
))*x^2 + 2*(2*a^2*b^2*d*f^2*e^(2*c) + b^4*d*f^2*e^(2*c) + 2*(2*a^2*b^2*d^2*
f*e^(2*c) + b^4*d^2*f*e^(2*c))*e)*x + 2*(2*a^2*b^2*d*f*e^(2*c) + b^4*d*f*e^
(2*c))*e)*e^(-2*d*x) + 64*(9*a*b^3*d^2*f^2*x^2*e^c + 6*a*b^3*d*f*e^(c + 1)
+ 2*a*b^3*f^2*e^c + 6*(3*a*b^3*d^2*f*e^(c + 1) + a*b^3*d*f^2*e^c)*x)*e^(-3*
d*x) + 27*(8*b^4*d^2*f^2*x^2 + 4*b^4*d*f*e + b^4*f^2 + 4*(4*b^4*d^2*f*e + b
^4*d*f^2)*x)*e^(-4*d*x))*e^(-4*c)/(b^5*d^3) - integrate(-2*((a^4*b*f^2 + a^
2*b^3*f^2)*x^2 + 2*(a^4*b*f + a^2*b^3*f)*x*e - ((a^5*f^2*e^c + a^3*b^2*f^2*
e^c)*x^2 + 2*(a^5*f*e^c + a^3*b^2*f*e^c)*x*e)*e^(d*x))/(b^6*e^(2*d*x + 2*c)
 + 2*a*b^5*e^(d*x + c) - b^6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11997 vs. $2(779) = 1558$.
time = 0.51, size = 11997, normalized size = 14.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="fricas")
```

```
[Out] 1/13824*(216*b^4*d^2*f^2*x^2 + 27*(8*b^4*d^2*f^2*x^2 - 4*b^4*d*f^2*x + 8*b^
4*d^2*cosh(1)^2 + 8*b^4*d^2*sinh(1)^2 + b^4*f^2 + 4*(4*b^4*d^2*f*x - b^4*d*
f)*cosh(1) + 4*(4*b^4*d^2*f*x + 4*b^4*d^2*cosh(1) - b^4*d*f)*sinh(1))*cosh(
d*x + c)^8 + 27*(8*b^4*d^2*f^2*x^2 - 4*b^4*d*f^2*x + 8*b^4*d^2*cosh(1)^2 +
8*b^4*d^2*sinh(1)^2 + b^4*f^2 + 4*(4*b^4*d^2*f*x - b^4*d*f)*cosh(1) + 4*(4*
b^4*d^2*f*x + 4*b^4*d^2*cosh(1) - b^4*d*f)*sinh(1))*sinh(d*x + c)^8 + 108*b
^4*d*f^2*x + 216*b^4*d^2*cosh(1)^2 - 64*(9*a*b^3*d^2*f^2*x^2 - 6*a*b^3*d*f^
2*x + 9*a*b^3*d^2*cosh(1)^2 + 9*a*b^3*d^2*sinh(1)^2 + 2*a*b^3*f^2 + 6*(3*a*
b^3*d^2*f*x - a*b^3*d*f)*cosh(1) + 6*(3*a*b^3*d^2*f*x + 3*a*b^3*d^2*cosh(1)
 - a*b^3*d*f)*sinh(1))*cosh(d*x + c)^7 + 216*b^4*d^2*sinh(1)^2 - 8*(72*a*b^
3*d^2*f^2*x^2 - 48*a*b^3*d*f^2*x + 72*a*b^3*d^2*cosh(1)^2 + 72*a*b^3*d^2*si
```

nh(1)^2 + 16*a*b^3*f^2 + 48*(3*a*b^3*d^2*f*x - a*b^3*d*f)*cosh(1) - 27*(8*b
^4*d^2*f^2*x^2 - 4*b^4*d*f^2*x + 8*b^4*d^2*cosh(1)^2 + 8*b^4*d^2*sinh(1)^2
+ b^4*f^2 + 4*(4*b^4*d^2*f*x - b^4*d*f)*cosh(1) + 4*(4*b^4*d^2*f*x + 4*b^4*
d^2*cosh(1) - b^4*d*f)*sinh(1))*cosh(d*x + c) + 48*(3*a*b^3*d^2*f*x + 3*a*b
^3*d^2*cosh(1) - a*b^3*d*f)*sinh(1))*sinh(d*x + c)^7 + 432*(2*(2*a^2*b^2 +
b^4)*d^2*f^2*x^2 - 2*(2*a^2*b^2 + b^4)*d*f^2*x + 2*(2*a^2*b^2 + b^4)*d^2*co
sh(1)^2 + 2*(2*a^2*b^2 + b^4)*d^2*sinh(1)^2 + (2*a^2*b^2 + b^4)*f^2 + 2*(2*
(2*a^2*b^2 + b^4)*d^2*f*x - (2*a^2*b^2 + b^4)*d*f)*cosh(1) + 2*(2*(2*a^2*b^
2 + b^4)*d^2*f*x + 2*(2*a^2*b^2 + b^4)*d^2*cosh(1) - (2*a^2*b^2 + b^4)*d*f)
*sinh(1))*cosh(d*x + c)^6 + 4*(216*(2*a^2*b^2 + b^4)*d^2*f^2*x^2 - 216*(2*a
^2*b^2 + b^4)*d*f^2*x + 216*(2*a^2*b^2 + b^4)*d^2*cosh(1)^2 + 216*(2*a^2*b^
2 + b^4)*d^2*sinh(1)^2 + 108*(2*a^2*b^2 + b^4)*f^2 + 189*(8*b^4*d^2*f^2*x^2
 - 4*b^4*d*f^2*x + 8*b^4*d^2*cosh(1)^2 + 8*b^4*d^2*sinh(1)^2 + b^4*f^2 + 4*
(4*b^4*d^2*f*x - b^4*d*f)*cosh(1) + 4*(4*b^4*d^2*f*x + 4*b^4*d^2*cosh(1) -
b^4*d*f)*sinh(1))*cosh(d*x + c)^2 + 216*(2*(2*a^2*b^2 + b^4)*d^2*f*x - (2*a
^2*b^2 + b^4)*d*f)*cosh(1) - 112*(9*a*b^3*d^2*f^2*x^2 - 6*a*b^3*d*f^2*x + 9
*a*b^3*d^2*cosh(1)^2 + 9*a*b^3*d^2*sinh(1)^2 + 2*a*b^3*f^2 + 6*(3*a*b^3*d^2
*f*x - a*b^3*d*f)*cosh(1) + 6*(3*a*b^3*d^2*f*x + 3*a*b^3*d^2*cosh(1) - a*b^
3*d*f)*sinh(1))*cosh(d*x + c) + 216*(2*(2*a^2*b^2 + b^4)*d^2*f*x + 2*(2*a^2
*b^2 + b^4)*d^2*cosh(1) - (2*a^2*b^2 + b^4)*d*f)*sinh(1))*sinh(d*x + c)^6 +
 27*b^4*f^2 - 1728*((4*a^3*b + 3*a*b^3)*d^2*f^2*x^2 - 2*(4*a^3*b + 3*a*b^3)
*d*f^2*x + (4*a^3*b + 3*a*b^3)*d^2*cosh(1)^2 + (4*a^3*b + 3*a*b^3)*d^2*sinh
(1)^2 + 2*(4*a^3*b + 3*a*b^3)*f^2 + 2*((4*a^3*b + 3*a*b^3)*d^2*f*x - (4*a^3
*b + 3*a*b^3)*d*f)*cosh(1) + 2*((4*a^3*b + 3*a*b^3)*d^2*f*x + (4*a^3*b + 3*
a*b^3)*d^2*cosh(1) - (4*a^3*b + 3*a*b^3)*d*f)*sinh(1))*cosh(d*x + c)^5 - 24
(72(4*a^3*b + 3*a*b^3)*d^2*f^2*x^2 - 144*(4*a^3*b + 3*a*b^3)*d*f^2*x + 72
*(4*a^3*b + 3*a*b^3)*d^2*cosh(1)^2 + 72*(4*a^3*b + 3*a*b^3)*d^2*sinh(1)^2 -
 63*(8*b^4*d^2*f^2*x^2 - 4*b^4*d*f^2*x + 8*b^4*d^2*cosh(1)^2 + 8*b^4*d^2*si
nh(1)^2 + b^4*f^2 + 4*(4*b^4*d^2*f*x - b^4*d*f)*cosh(1) + 4*(4*b^4*d^2*f*x
+ 4*b^4*d^2*cosh(1) - b^4*d*f)*sinh(1))*cosh(d*x + c)^3 + 144*(4*a^3*b + 3*
a*b^3)*f^2 + 56*(9*a*b^3*d^2*f^2*x^2 - 6*a*b^3*d*f^2*x + 9*a*b^3*d^2*cosh(1
)^2 + 9*a*b^3*d^2*sinh(1)^2 + 2*a*b^3*f^2 + 6*(3*a*b^3*d^2*f*x - a*b^3*d*f)
cosh(1) + 6(3*a*b^3*d^2*f*x + 3*a*b^3*d^2*cosh(1) - a*b^3*d*f)*sinh(1))*c
osh(d*x + c)^2 + 144*((4*a^3*b + 3*a*b^3)*d^2*f*x - (4*a^3*b + 3*a*b^3)*d*f
)*cosh(1) - 108*(2*(2*a^2*b^2 + b^4)*d^2*f^2*x^2 - 2*(2*a^2*b^2 + b^4)*d*f^
2*x + 2*(2*a^2*b^2 + b^4)*d^2*cosh(1)^2 + 2*(2*a^2*b^2 + b^4)*d^2*sinh(1)^2
 + (2*a^2*b^2 + b^4)*f^2 + 2*(2*(2*a^2*b^2 + b^4)*d^2*f*x - (2*a^2*b^2 + b^
4)*d*f)*cosh(1) + 2*(2*(2*a^2*b^2 + b^4)*d^2*f*x + 2*(2*a^2*b^2 + b^4)*d^2*
cosh(1) - (2*a^2*b^2 + b^4)*d*f)*sinh(1))*cosh(d*x + c) + 144*((4*a^3*b + 3
*a*b^3)*d^2*f*x + (4*a^3*b + 3*a*b^3)*d^2*cosh(1) - (4*a^3*b + 3*a*b^3)*d*f
)*sinh(1))*sinh(d*x + c)^5 - 4608*((a^4 + a^2*b^2)*d^3*f^2*x^3 + 2*(a^4 + a
^2*b^2)*c^3*f^2 + 3*((a^4 + a^2*b^2)*d^3*x + 2*(a^4 + a^2*b^2)*c*d^2)*cosh(
1)^2 + 3*((a^4 + a^2*b^2)*d^3*x + 2*(a^4 + a^2*b^2)*c*d^2)*sinh(1)^2 + 3*((
a^4 + a^2*b^2)*d^3*f*x^2 - 2*(a^4 + a^2*b^2)*c^2*d*f)*cosh(1) + 3*((a^4 + a
^2*b^2)*d^3*f*x^2 - 2*(a^4 + a^2*b^2)*c^2*d*f + 2*((a^4 + a^2*b^2)*d^3*x +

```
2*(a^4 + a^2*b^2)*c*d^2)*cosh(1))*sinh(1))*cosh(d*x + c)^4 - 2*(2304*(a^4 +
 a^2*b^2)*d^3*f^2*x^3 + 4608*(a^4 + a^2*b^2)*c^3*f^2 - 945*(8*b^4*d^2*f^2*x
^2 - 4*b^4*d*f^2*x + 8*b^4*d^2*cosh(1)^2 + 8*b^4*d^2*sinh(1)^2 + b^4*f^2 +
4*(4*b^4*d^2*f*x - b^4*d*f)*cosh(1) + 4*(4*b^4*d^2*f*x + 4*b^4*d^2*cosh(1)
- b^4*d*f)*sinh(1))*cosh(d*x + c)^4 + 1120*(9*a*b^3*d^2*f^2*x^2 - 6*a*b^3*d
*f^2*x + 9*a*b^3*d^2*cosh(1)^2 + 9*a*b^3*d^2*sinh(1)^2 + 2*a*b^3*f^2 + 6*(3
*a*b^3*d^2*f*x - a*b^3*d*f)*cosh(1) + 6*(3*a*b^3*d^2*f*x + 3*a*b^3*d^2*cosh
(1) - a*b^3*d*f)*sinh(1))*cosh(d*x + c)^3 + 6912*((a^4 + a^2*b^2)*d^3*x + 2
*(a^4 + a^2*b^2)*c*d^2)*cosh(1)^2 - 3240*(2*(2*a^2*b^2 + b^4)*d^2*f^2*x^2 -
 2*(2*a^2*b^2 + b^4)*d*f^2*x + 2*(2*a^2*b^2 + b^4)*d^2*cosh(1)^2 + 2*(2*a^2
*b^2 + b^4)*d^2*sinh(1)^2 + (2*a^2*b^2 + b^4)*f...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**3*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="giac")
[Out] integrate((f*x + e)^2*cosh(d*x + c)^3*sinh(d*x + c)^2/(b*sinh(d*x + c) + a)
, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^3\,\sinh(c+d\,x)^2\,{(e+f\,x)}^2}{a+b\,\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.374 $\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=499

$$\frac{a^2fx}{4b^3d}-\frac{3fx}{32bd}-\frac{a^2(a^2+b^2)\,(e+fx)^2}{2b^5f}+\frac{a^3f\cosh(c+dx)}{b^4d^2}+\frac{2af\cosh(c+dx)}{3b^2d^2}+\frac{af\cosh^3(c+dx)}{9b^2d^2}+\frac{(e+fx)\cosh}{4bd}$$

```
[Out] 1/4*a^2*f*x/b^3/d-3/32*f*x/b/d-1/2*a^2*(a^2+b^2)*(f*x+e)^2/b^5/f+a^3*f*cosh
(d*x+c)/b^4/d^2+2/3*a*f*cosh(d*x+c)/b^2/d^2+1/9*a*f*cosh(d*x+c)^3/b^2/d^2+1
/4*(f*x+e)*cosh(d*x+c)^4/b/d+a^2*(a^2+b^2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^
2+b^2)^(1/2)))/b^5/d+a^2*(a^2+b^2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))/b^5/d+a^2*(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b
^5/d^2+a^2*(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^5/d^2
-a^3*(f*x+e)*sinh(d*x+c)/b^4/d-2/3*a*(f*x+e)*sinh(d*x+c)/b^2/d-1/4*a^2*f*co
sh(d*x+c)*sinh(d*x+c)/b^3/d^2-3/32*f*cosh(d*x+c)*sinh(d*x+c)/b/d^2-1/3*a*(f
*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/b^2/d-1/16*f*cosh(d*x+c)^3*sinh(d*x+c)/b/d^
2+1/2*a^2*(f*x+e)*sinh(d*x+c)^2/b^3/d
```

Rubi [A]
time = 0.47, antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$, Rules used = {5698, 5555, 2715, 8, 3391, 3377, 2718, 5684, 5554, 5680, 2221, 2317, 2438}

$$\frac{a^3f\cosh(c+dx)}{b^4d^2}-\frac{a^3(e+fx)\sinh(c+dx)}{b^4d}-\frac{a^2f\sinh(c+dx)\cosh(c+dx)}{4b^3d^2}+\frac{a^2(e+fx)\sinh^2(c+dx)}{2b^3d}+\frac{a^2fx}{4b^3d}+\frac{a^2f(a^2+b^2)\,\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^2}+\frac{a^2f(a^2+b^2)\,\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^2}+\frac{a^2(a^2+b^2)\,(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^5d}+\frac{a^2(a^2+b^2)\,(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^5d}-\frac{a^2(a^2+b^2)\,(e+fx)^2}{2b^5f}+\frac{af\cosh^3(c+dx)}{9b^2d^2}+\frac{2af\cosh(c+dx)}{3b^2d^2}-\frac{2a(e+fx)\sinh(c+dx)}{3b^2d}-\frac{a(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3b^2d}-\frac{f\sinh(c+dx)\cosh^3(c+dx)}{16bd^2}-\frac{3f\sinh(c+dx)\cosh(c+dx)}{32bd^2}+\frac{(e+fx)\cosh^4(c+dx)}{4bd}-\frac{3fx}{32bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*f*x)/(4*b^3*d) - (3*f*x)/(32*b*d) - (a^2*(a^2 + b^2)*(e + f*x)^2)/(2*b
^5*f) + (a^3*f*Cosh[c + d*x])/(b^4*d^2) + (2*a*f*Cosh[c + d*x])/(3*b^2*d^2)
 + (a*f*Cosh[c + d*x]^3)/(9*b^2*d^2) + ((e + f*x)*Cosh[c + d*x]^4)/(4*b*d)
+ (a^2*(a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]
)/(b^5*d) + (a^2*(a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2])])/(b^5*d) + (a^2*(a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2]))])/(b^5*d^2) + (a^2*(a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^2) - (a^3*(e + f*x)*Sinh[c + d*x])/(b^
4*d) - (2*a*(e + f*x)*Sinh[c + d*x])/(3*b^2*d) - (a^2*f*Cosh[c + d*x]*Sinh[
c + d*x])/(4*b^3*d^2) - (3*f*Cosh[c + d*x]*Sinh[c + d*x])/(32*b*d^2) - (a*(
e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b^2*d) - (f*Cosh[c + d*x]^3*Sinh
[c + d*x])/(16*b*d^2) + (a^2*(e + f*x)*Sinh[c + d*x]^2)/(2*b^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5555

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh^3(c+dx)\sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\cosh^3(c+d}{a+b\sinh(c}}{b}$$

$$= \frac{(e+fx)\cosh^4(c+dx)}{4bd} - \frac{a\int (e+fx)\cosh^3(c+dx)\,dx}{b^2} + \frac{a^2}{}$$

$$= \frac{af\cosh^3(c+dx)}{9b^2d^2} + \frac{(e+fx)\cosh^4(c+dx)}{4bd} - \frac{a(e+fx)\cosh}{}$$

$$= -\frac{a^2(a^2+b^2)(e+fx)^2}{2b^5f} + \frac{af\cosh^3(c+dx)}{9b^2d^2} + \frac{(e+fx)\cosh^4}{4bd}$$

$$= -\frac{3fx}{32bd} - \frac{a^2(a^2+b^2)(e+fx)^2}{2b^5f} + \frac{a^3f\cosh(c+dx)}{b^4d^2} + \frac{2af\cos}{3}$$

$$= \frac{a^2fx}{4b^3d} - \frac{3fx}{32bd} - \frac{a^2(a^2+b^2)(e+fx)^2}{2b^5f} + \frac{a^3f\cosh(c+dx)}{b^4d^2} +$$

$$= \frac{a^2fx}{4b^3d} - \frac{3fx}{32bd} - \frac{a^2(a^2+b^2)(e+fx)^2}{2b^5f} + \frac{a^3f\cosh(c+dx)}{b^4d^2} +$$

Mathematica [A]
time = 2.29, size = 848, normalized size = 1.70

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (-144*b^4*d*e*Log[a + b*Sinh[c + d*x]] + 72*b^4*f*(d*x*(d*x - 2*Log[1 + (b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])]) - 2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*Pol
yLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) + 72*b^2*d*e*(b^2*Cosh[2
*(c + d*x)] + (4*a^2 + b^2)*Log[a + b*Sinh[c + d*x]] - 4*a*b*Sinh[c + d*x])
 + 36*b^2*f*(8*a*b*Cosh[c + d*x] + 2*b^2*d*x*Cosh[2*(c + d*x)] + 2*(4*a^2 +
 b^2)*(-1/2*(c + d*x)^2 + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])] + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - c*Log[
a + b*Sinh[c + d*x]] + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] +
 PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) - 8*a*b*d*x*Sinh[c +
```

```
d*x] - b^2*Sinh[2*(c + d*x)]) + 12*d*e*(6*b^2*(4*a^2 + b^2)*Cosh[2*(c + d*
x)] + 3*b^4*Cosh[4*(c + d*x)] + 6*(16*a^4 + 12*a^2*b^2 + b^4)*Log[a + b*Sin
h[c + d*x]] - 48*a*b*(2*a^2 + b^2)*Sinh[c + d*x] - 8*a*b^3*Sinh[3*(c + d*x)
]) + f*(576*a*b*(2*a^2 + b^2)*Cosh[c + d*x] + 72*b^2*(4*a^2 + b^2)*d*x*Cosh
[2*(c + d*x)] + 32*a*b^3*Cosh[3*(c + d*x)] + 36*b^4*d*x*Cosh[4*(c + d*x)] +
 72*(16*a^4 + 12*a^2*b^2 + b^4)*(-1/2*(c + d*x)^2 + (c + d*x)*Log[1 + (b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2])] + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2])] - c*Log[a + b*Sinh[c + d*x]] + PolyLog[2, (b*E^(c + d*x))
/(-a + Sqrt[a^2 + b^2])] + PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
]))]) - 576*a*b*(2*a^2 + b^2)*d*x*Sinh[c + d*x] - 36*b^2*(4*a^2 + b^2)*Sinh
[2*(c + d*x)] - 96*a*b^3*d*x*Sinh[3*(c + d*x)] - 9*b^4*Sinh[4*(c + d*x)]))/
(1152*b^5*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1216 vs. $2(463) = 926$.

time = 3.12, size = 1217, normalized size = 2.44

method	result
risch	$\frac{a^2 f \operatorname{dilog}\left(\frac{b\,e^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right)}{d^2 b^3}+\frac{a^2 f \ln\left(\frac{-b\,e^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right) c}{d^2 b^3}+\frac{a^4 f \ln\left(\frac{-b\,e^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right) c}{d^2 b^5}+\frac{a^4 f \ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/d^2*a^2/b^3*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*
c+1/d^2*a^4/b^5*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))
)*c+1/d*a^4/b^5*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*
x+1/d^2*a^4/b^5*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*
c-1/d^2*a^2/b^3*f*c^2+1/256*(4*d*f*x+4*d*e-f)/d^2/b*exp(4*d*x+4*c)+1/32*(4*
a^2*d*f*x+2*b^2*d*f*x+4*a^2*d*e+2*b^2*d*e-2*a^2*f-b^2*f)/b^3/d^2*exp(2*d*x+
2*c)-1/2*a^2*f*x^2/b^3+1/256*(4*d*f*x+4*d*e+f)/d^2/b*exp(-4*d*x-4*c)-1/d^2*
a^4/b^5*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d^2*a^4/b^5*f*c*ln(exp(
d*x+c))+1/d*a^4/b^5*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1
/2)))*x+1/8*a*(4*a^2+3*b^2)*(d*f*x+d*e+f)/b^4/d^2*exp(-d*x-c)-1/2*a^4*f*x^2
/b^5+2/d^2*a^2/b^3*f*c*ln(exp(d*x+c))+1/d*a^2/b^3*f*ln((-b*exp(d*x+c)+(a^2+
b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d*a^2/b^3*f*ln((b*exp(d*x+c)+(a^2+b
^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2*a^2/b^3*f*ln((b*exp(d*x+c)+(a^2+b
^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d^2*a^2/b^3*f*c*ln(b*exp(2*d*x+2*c)+2
*a*exp(d*x+c)-b)+a^2*e*x/b^3-2/d*a^4/b^5*f*c*x-2/d*a^2/b^3*f*c*x+1/32*(2*a^
2+b^2)*(2*d*f*x+2*d*e+f)/b^3/d^2*exp(-2*d*x-2*c)-1/d^2*a^4/b^5*f*c^2+1/d^2*
a^2/b^3*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/d^2
*a^2/b^3*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/
d^2*a^4/b^5*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1
```

```
/d^2*a^4/b^5*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))
)+1/d*a^2/b^3*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d*a^2/b^3*e*ln(exp(
d*x+c))+1/d*a^4/b^5*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d*a^4/b^5*e*l
n(exp(d*x+c))-1/8*a*(4*a^2*d*f*x+3*b^2*d*f*x+4*a^2*d*e+3*b^2*d*e-4*a^2*f-3*
b^2*f)/b^4/d^2*exp(d*x+c)-1/72*a*(3*d*f*x+3*d*e-f)/b^2/d^2*exp(3*d*x+3*c)+a
^4*e*x/b^5+1/72*a*(3*d*f*x+3*d*e+f)/b^2/d^2*exp(-3*d*x-3*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] 1/2304*f*((1152*(a^4*d^2*e^(4*c) + a^2*b^2*d^2*e^(4*c))*x^2 + 9*(4*b^4*d*x*
e^(8*c) - b^4*e^(8*c))*e^(4*d*x) - 32*(3*a*b^3*d*x*e^(7*c) - a*b^3*e^(7*c))
*e^(3*d*x) - 72*(2*a^2*b^2*e^(6*c) + b^4*e^(6*c) - 2*(2*a^2*b^2*d*e^(6*c) +
 b^4*d*e^(6*c))*x)*e^(2*d*x) + 288*(4*a^3*b*e^(5*c) + 3*a*b^3*e^(5*c) - (4*
a^3*b*d*e^(5*c) + 3*a*b^3*d*e^(5*c))*x)*e^(d*x) + 288*(4*a^3*b*e^(3*c) + 3*
a*b^3*e^(3*c) + (4*a^3*b*d*e^(3*c) + 3*a*b^3*d*e^(3*c))*x)*e^(-d*x) + 72*(2
*a^2*b^2*e^(2*c) + b^4*e^(2*c) + 2*(2*a^2*b^2*d*e^(2*c) + b^4*d*e^(2*c))*x)
*e^(-2*d*x) + 32*(3*a*b^3*d*x*e^c + a*b^3*e^c)*e^(-3*d*x) + 9*(4*b^4*d*x +
b^4)*e^(-4*d*x))*e^(-4*c)/(b^5*d^2) - 72*integrate(64*((a^5*e^c + a^3*b^2*e
^c)*x*e^(d*x) - (a^4*b + a^2*b^3)*x)/(b^6*e^(2*d*x + 2*c) + 2*a*b^5*e^(d*x
+ c) - b^6), x)) - 1/192*((8*a*b^2*e^(-d*x - c) - 3*b^3 - 12*(2*a^2*b + b^3
)*e^(-2*d*x - 2*c) + 24*(4*a^3 + 3*a*b^2)*e^(-3*d*x - 3*c))*e^(4*d*x + 4*c)
/(b^4*d) - 192*(a^4 + a^2*b^2)*(d*x + c)/(b^5*d) - (8*a*b^2*e^(-3*d*x - 3*c
) + 3*b^3*e^(-4*d*x - 4*c) + 24*(4*a^3 + 3*a*b^2)*e^(-d*x - c) + 12*(2*a^2*
b + b^3)*e^(-2*d*x - 2*c))/(b^4*d) - 192*(a^4 + a^2*b^2)*log(-2*a*e^(-d*x -
 c) + b*e^(-2*d*x - 2*c) - b)/(b^5*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4707 vs. $2(469) = 938$.
time = 0.46, size = 4707, normalized size = 9.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/2304*(9*(4*b^4*d*f*x + 4*b^4*d*cosh(1) + 4*b^4*d*sinh(1) - b^4*f)*cosh(d*
x + c)^8 + 9*(4*b^4*d*f*x + 4*b^4*d*cosh(1) + 4*b^4*d*sinh(1) - b^4*f)*sinh
(d*x + c)^8 - 32*(3*a*b^3*d*f*x + 3*a*b^3*d*cosh(1) + 3*a*b^3*d*sinh(1) - a
```

```
*b^3*f)*cosh(d*x + c)^7 - 8*(12*a*b^3*d*f*x + 12*a*b^3*d*cosh(1) + 12*a*b^3
*d*sinh(1) - 4*a*b^3*f - 9*(4*b^4*d*f*x + 4*b^4*d*cosh(1) + 4*b^4*d*sinh(1)
 - b^4*f)*cosh(d*x + c))*sinh(d*x + c)^7 + 36*b^4*d*f*x + 72*(2*(2*a^2*b^2
+ b^4)*d*f*x + 2*(2*a^2*b^2 + b^4)*d*cosh(1) + 2*(2*a^2*b^2 + b^4)*d*sinh(1
) - (2*a^2*b^2 + b^4)*f)*cosh(d*x + c)^6 + 4*(36*(2*a^2*b^2 + b^4)*d*f*x +
36*(2*a^2*b^2 + b^4)*d*cosh(1) + 63*(4*b^4*d*f*x + 4*b^4*d*cosh(1) + 4*b^4*
d*sinh(1) - b^4*f)*cosh(d*x + c)^2 + 36*(2*a^2*b^2 + b^4)*d*sinh(1) - 18*(2
*a^2*b^2 + b^4)*f - 56*(3*a*b^3*d*f*x + 3*a*b^3*d*cosh(1) + 3*a*b^3*d*sinh(
1) - a*b^3*f)*cosh(d*x + c))*sinh(d*x + c)^6 + 36*b^4*d*cosh(1) - 288*((4*a
^3*b + 3*a*b^3)*d*f*x + (4*a^3*b + 3*a*b^3)*d*cosh(1) + (4*a^3*b + 3*a*b^3)
*d*sinh(1) - (4*a^3*b + 3*a*b^3)*f)*cosh(d*x + c)^5 + 36*b^4*d*sinh(1) - 24
*(12*(4*a^3*b + 3*a*b^3)*d*f*x - 21*(4*b^4*d*f*x + 4*b^4*d*cosh(1) + 4*b^4*
d*sinh(1) - b^4*f)*cosh(d*x + c)^3 + 12*(4*a^3*b + 3*a*b^3)*d*cosh(1) + 28*
(3*a*b^3*d*f*x + 3*a*b^3*d*cosh(1) + 3*a*b^3*d*sinh(1) - a*b^3*f)*cosh(d*x
+ c)^2 + 12*(4*a^3*b + 3*a*b^3)*d*sinh(1) - 12*(4*a^3*b + 3*a*b^3)*f - 18*(
2*(2*a^2*b^2 + b^4)*d*f*x + 2*(2*a^2*b^2 + b^4)*d*cosh(1) + 2*(2*a^2*b^2 +
b^4)*d*sinh(1) - (2*a^2*b^2 + b^4)*f)*cosh(d*x + c))*sinh(d*x + c)^5 + 9*b^
4*f - 1152*((a^4 + a^2*b^2)*d^2*f*x^2 - 2*(a^4 + a^2*b^2)*c^2*f + 2*((a^4 +
 a^2*b^2)*d^2*x + 2*(a^4 + a^2*b^2)*c*d)*cosh(1) + 2*((a^4 + a^2*b^2)*d^2*x
 + 2*(a^4 + a^2*b^2)*c*d)*sinh(1))*cosh(d*x + c)^4 - 2*(576*(a^4 + a^2*b^2)
*d^2*f*x^2 - 315*(4*b^4*d*f*x + 4*b^4*d*cosh(1) + 4*b^4*d*sinh(1) - b^4*f)*
cosh(d*x + c)^4 - 1152*(a^4 + a^2*b^2)*c^2*f + 560*(3*a*b^3*d*f*x + 3*a*b^3
*d*cosh(1) + 3*a*b^3*d*sinh(1) - a*b^3*f)*cosh(d*x + c)^3 - 540*(2*(2*a^2*b
^2 + b^4)*d*f*x + 2*(2*a^2*b^2 + b^4)*d*cosh(1) + 2*(2*a^2*b^2 + b^4)*d*sin
h(1) - (2*a^2*b^2 + b^4)*f)*cosh(d*x + c)^2 + 1152*((a^4 + a^2*b^2)*d^2*x +
 2*(a^4 + a^2*b^2)*c*d)*cosh(1) + 720*((4*a^3*b + 3*a*b^3)*d*f*x + (4*a^3*b
 + 3*a*b^3)*d*cosh(1) + (4*a^3*b + 3*a*b^3)*d*sinh(1) - (4*a^3*b + 3*a*b^3)
*f)*cosh(d*x + c) + 1152*((a^4 + a^2*b^2)*d^2*x + 2*(a^4 + a^2*b^2)*c*d)*si
nh(1))*sinh(d*x + c)^4 + 288*((4*a^3*b + 3*a*b^3)*d*f*x + (4*a^3*b + 3*a*b^
3)*d*cosh(1) + (4*a^3*b + 3*a*b^3)*d*sinh(1) + (4*a^3*b + 3*a*b^3)*f)*cosh(
d*x + c)^3 + 8*(63*(4*b^4*d*f*x + 4*b^4*d*cosh(1) + 4*b^4*d*sinh(1) - b^4*f
)*cosh(d*x + c)^5 - 140*(3*a*b^3*d*f*x + 3*a*b^3*d*cosh(1) + 3*a*b^3*d*sinh
(1) - a*b^3*f)*cosh(d*x + c)^4 + 36*(4*a^3*b + 3*a*b^3)*d*f*x + 180*(2*(2*a
^2*b^2 + b^4)*d*f*x + 2*(2*a^2*b^2 + b^4)*d*cosh(1) + 2*(2*a^2*b^2 + b^4)*d
*sinh(1) - (2*a^2*b^2 + b^4)*f)*cosh(d*x + c)^3 + 36*(4*a^3*b + 3*a*b^3)*d*
cosh(1) - 360*((4*a^3*b + 3*a*b^3)*d*f*x + (4*a^3*b + 3*a*b^3)*d*cosh(1) +
(4*a^3*b + 3*a*b^3)*d*sinh(1) - (4*a^3*b + 3*a*b^3)*f)*cosh(d*x + c)^2 + 36
*(4*a^3*b + 3*a*b^3)*d*sinh(1) + 36*(4*a^3*b + 3*a*b^3)*f - 576*((a^4 + a^2
*b^2)*d^2*f*x^2 - 2*(a^4 + a^2*b^2)*c^2*f + 2*((a^4 + a^2*b^2)*d^2*x + 2*(a
^4 + a^2*b^2)*c*d)*cosh(1) + 2*((a^4 + a^2*b^2)*d^2*x + 2*(a^4 + a^2*b^2)*c
*d)*sinh(1))*cosh(d*x + c))*sinh(d*x + c)^3 + 72*(2*(2*a^2*b^2 + b^4)*d*f*x
 + 2*(2*a^2*b^2 + b^4)*d*cosh(1) + 2*(2*a^2*b^2 + b^4)*d*sinh(1) + (2*a^2*b
^2 + b^4)*f)*cosh(d*x + c)^2 + 12*(21*(4*b^4*d*f*x + 4*b^4*d*cosh(1) + 4*b^
4*d*sinh(1) - b^4*f)*cosh(d*x + c)^6 - 56*(3*a*b^3*d*f*x + 3*a*b^3*d*cosh(1
) + 3*a*b^3*d*sinh(1) - a*b^3*f)*cosh(d*x + c)^5 + 90*(2*(2*a^2*b^2 + b^4)*
```

```
d*f*x + 2*(2*a^2*b^2 + b^4)*d*cosh(1) + 2*(2*a^2*b^2 + b^4)*d*sinh(1) - (2*
a^2*b^2 + b^4)*f)*cosh(d*x + c)^4 + 12*(2*a^2*b^2 + b^4)*d*f*x - 240*((4*a^
3*b + 3*a*b^3)*d*f*x + (4*a^3*b + 3*a*b^3)*d*cosh(1) + (4*a^3*b + 3*a*b^3)*
d*sinh(1) - (4*a^3*b + 3*a*b^3)*f)*cosh(d*x + c)^3 + 12*(2*a^2*b^2 + b^4)*d
*cosh(1) - 576*((a^4 + a^2*b^2)*d^2*f*x^2 - 2*(a^4 + a^2*b^2)*c^2*f + 2*((a
^4 + a^2*b^2)*d^2*x + 2*(a^4 + a^2*b^2)*c*d)*cosh(1) + 2*((a^4 + a^2*b^2)*d
^2*x + 2*(a^4 + a^2*b^2)*c*d)*sinh(1))*cosh(d*x + c)^2 + 12*(2*a^2*b^2 + b^
4)*d*sinh(1) + 6*(2*a^2*b^2 + b^4)*f + 72*((4*a^3*b + 3*a*b^3)*d*f*x + (4*a
^3*b + 3*a*b^3)*d*cosh(1) + (4*a^3*b + 3*a*b^3)*d*sinh(1) + (4*a^3*b + 3*a*
b^3)*f)*cosh(d*x + c))*sinh(d*x + c)^2 + 32*(3*a*b^3*d*f*x + 3*a*b^3*d*cosh
(1) + 3*a*b^3*d*sinh(1) + a*b^3*f)*cosh(d*x + c) + 2304*((a^4 + a^2*b^2)*f*
cosh(d*x + c)^4 + 4*(a^4 + a^2*b^2)*f*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^
4 + a^2*b^2)*f*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^4 + a^2*b^2)*f*cosh(d
*x + c)*sinh(d*x + c)^3 + (a^4 + a^2*b^2)*f*sinh(d*x + c)^4)*dilog((a*cosh(
d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2) - b)/b + 1) + 2304*((a^4 + a^2*b^2)*f*cosh(d*x + c)^4 + 4*(a^4
+ a^2*b^2)*f*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^4 + a^2*b^2)*f*cosh(d*x +
 c)^2*sinh(d*x + c)^2 + 4*(a^4 + a^2*b^2)*f*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a^4 + a^2*b^2)*f*sinh(d*x + c)^4)*dilog((a*co...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**3*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)^3*sinh(d*x + c)^2/(b*sinh(d*x + c) + a),
x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{cosh}\left(c+d\,x\right)}^3\,{\mathrm{sinh}\left(c+d\,x\right)}^2\,\left(e+f\,x\right)}{a+b\,\mathrm{sinh}\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.375 $\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=113

$$\frac{a^2(a^2+b^2)\log(a+b\sinh(c+dx))}{b^5d}-\frac{a(a^2+b^2)\sinh(c+dx)}{b^4d}+\frac{(a^2+b^2)\sinh^2(c+dx)}{2b^3d}-\frac{a\sinh^3(c+dx)}{3b^2d}+\frac{\sin}{}$$

```
[Out] a^2*(a^2+b^2)*ln(a+b*sinh(d*x+c))/b^5/d-a*(a^2+b^2)*sinh(d*x+c)/b^4/d+1/2*(
a^2+b^2)*sinh(d*x+c)^2/b^3/d-1/3*a*sinh(d*x+c)^3/b^2/d+1/4*sinh(d*x+c)^4/b/
d
```

Rubi [A]
time = 0.11, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 908}

$$\frac{a^2(a^2+b^2)\log(a+b\sinh(c+dx))}{b^5d}-\frac{a(a^2+b^2)\sinh(c+dx)}{b^4d}+\frac{(a^2+b^2)\sinh^2(c+dx)}{2b^3d}-\frac{a\sinh^3(c+dx)}{3b^2d}+\frac{\sinh^4(c+dx)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/(b^5*d) - (a*(a^2 + b^2)*Sinh[c
+ d*x])/(b^4*d) + ((a^2 + b^2)*Sinh[c + d*x]^2)/(2*b^3*d) - (a*Sinh[c + d*x
]^3)/(3*b^2*d) + Sinh[c + d*x]^4/(4*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2(-b^2-x^2)}{b^2(a+x)}\,dx, x, b\sinh(c+dx)\right)}{b^3d}$$
$$= -\frac{\text{Subst}\left(\int \frac{x^2(-b^2-x^2)}{a+x}\,dx, x, b\sinh(c+dx)\right)}{b^5d}$$
$$= -\frac{\text{Subst}\left(\int \left(a(a^2+b^2)-(a^2+b^2)x+ax^2-x^3-\frac{a^2(a^2+b^2)}{a+x}\right)\,dx, x, b\sin\right.}{b^5d}$$
$$= \frac{a^2(a^2+b^2)\log(a+b\sinh(c+dx))}{b^5d} - \frac{a(a^2+b^2)\sinh(c+dx)}{b^4d} + \frac{(a^2+b}{}$$

Mathematica [A]
time = 0.35, size = 104, normalized size = 0.92

$$\frac{12(2a^2b^2+b^4)\cosh(2(c+dx))+3b^4\cosh(4(c+dx))+8a(12a(a^2+b^2)\log(a+b\sinh(c+dx))-3b(4a^2+3b^2)\sinh(c+dx)-b^3\sinh(3(c+dx)))}{96b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
[Out] (12*(2*a^2*b^2 + b^4)*Cosh[2*(c + d*x)] + 3*b^4*Cosh[4*(c + d*x)] + 8*a*(12
*a*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]] - 3*b*(4*a^2 + 3*b^2)*Sinh[c + d*x]
 - b^3*Sinh[3*(c + d*x)]))/(96*b^5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(107) = 214.
time = 1.12, size = 343, normalized size = 3.04

method	result
risch	$-\frac{a^4x}{b^5}-\frac{x\,a^2}{b^3}+\frac{e^{4dx+4c}}{64bd}-\frac{a\,e^{3dx+3c}}{24b^2d}+\frac{e^{2dx+2c}a^2}{8b^3d}+\frac{e^{2dx+2c}}{16bd}-\frac{a^3e^{dx+c}}{2b^4d}-\frac{3a\,e^{dx+c}}{8b^2d}+\frac{a^3e^{-dx-c}}{2b^4d}+\frac{3a\,e^{-}}{8b}$
derivativedivides	$\frac{\frac{1}{4b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{3b-2a}{6b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{-4a^2+4ab-5b^2}{8b^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-8a^3+4a^2b-8a\,b^2+3b^3}{8b^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a^2\left(a^2+b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{b^5}}{}$
default	$\frac{\frac{1}{4b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{3b-2a}{6b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{-4a^2+4ab-5b^2}{8b^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-8a^3+4a^2b-8a\,b^2+3b^3}{8b^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a^2\left(a^2+b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{b^5}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/b/(tanh(1/2*d*x+1/2*c)+1)^4-1/6*(3*b-2*a)/b^2/(tanh(1/2*d*x+1/2*c)
+1)^3-1/8*(-4*a^2+4*a*b-5*b^2)/b^3/(tanh(1/2*d*x+1/2*c)+1)^2-1/8*(-8*a^3+4*
a^2*b-8*a*b^2+3*b^3)/b^4/(tanh(1/2*d*x+1/2*c)+1)-a^2*(a^2+b^2)/b^5*ln(tanh(
1/2*d*x+1/2*c)+1)+1/4/b/(tanh(1/2*d*x+1/2*c)-1)^4-1/6*(-3*b-2*a)/b^2/(tanh(
1/2*d*x+1/2*c)-1)^3-1/8*(-4*a^2-4*a*b-5*b^2)/b^3/(tanh(1/2*d*x+1/2*c)-1)^2-
1/8*(-8*a^3-4*a^2*b-8*a*b^2-3*b^3)/b^4/(tanh(1/2*d*x+1/2*c)-1)-a^2*(a^2+b^2
)/b^5*ln(tanh(1/2*d*x+1/2*c)-1)+2*a^2/b^5*(1/2*a^2+1/2*b^2)*ln(a*tanh(1/2*d
*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)-a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(107) = 214$.
time $= 0.27$, size $= 234$, normalized size $= 2.07$

$$-\frac{\left(8\,ab^2e^{(-dx-c)} - 3\,b^3 - 12\,(2\,a^2b + b^3)e^{(-2\,dx-2\,c)} + 24\,(4\,a^3 + 3\,ab^2)e^{(-3\,dx-3\,c)}\right)e^{(4\,dx+4\,c)}}{192\,b^4d} + \frac{(a^4 + a^2b^2)(dx + c)}{b^5d} + \frac{8\,ab^2e^{(-3\,dx-3\,c)} + 3\,b^3e^{(-4\,dx-4\,c)} + 24\,(4\,a^3 + 3\,ab^2)e^{(-dx-c)} + 12\,(2\,a^2b + b^3)e^{(-2\,dx-2\,c)}}{192\,b^4d} + \frac{(a^4 + a^2b^2)\log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxim
a")
```

```
[Out] -1/192*(8*a*b^2*e^(-d*x - c) - 3*b^3 - 12*(2*a^2*b + b^3)*e^(-2*d*x - 2*c)
+ 24*(4*a^3 + 3*a*b^2)*e^(-3*d*x - 3*c))*e^(4*d*x + 4*c)/(b^4*d) + (a^4 + a
^2*b^2)*(d*x + c)/(b^5*d) + 1/192*(8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e^(-4*d
*x - 4*c) + 24*(4*a^3 + 3*a*b^2)*e^(-d*x - c) + 12*(2*a^2*b + b^3)*e^(-2*d*
x - 2*c))/(b^4*d) + (a^4 + a^2*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2
*c) - b)/(b^5*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1069 vs. $2(107) = 214$.
time $= 0.38$, size $= 1069$, normalized size $= 9.46$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="frica
s")
```

```
[Out] 1/192*(3*b^4*cosh(d*x + c)^8 + 3*b^4*sinh(d*x + c)^8 - 8*a*b^3*cosh(d*x + c
)^7 + 8*(3*b^4*cosh(d*x + c) - a*b^3)*sinh(d*x + c)^7 - 192*(a^4 + a^2*b^2)
*d*x*cosh(d*x + c)^4 + 12*(2*a^2*b^2 + b^4)*cosh(d*x + c)^6 + 4*(21*b^4*cos
h(d*x + c)^2 - 14*a*b^3*cosh(d*x + c) + 6*a^2*b^2 + 3*b^4)*sinh(d*x + c)^6
- 24*(4*a^3*b + 3*a*b^3)*cosh(d*x + c)^5 + 24*(7*b^4*cosh(d*x + c)^3 - 7*a*
b^3*cosh(d*x + c)^2 - 4*a^3*b - 3*a*b^3 + 3*(2*a^2*b^2 + b^4)*cosh(d*x + c)
)*sinh(d*x + c)^5 + 8*a*b^3*cosh(d*x + c) + 2*(105*b^4*cosh(d*x + c)^4 - 14
0*a*b^3*cosh(d*x + c)^3 - 96*(a^4 + a^2*b^2)*d*x + 90*(2*a^2*b^2 + b^4)*cos
h(d*x + c)^2 - 60*(4*a^3*b + 3*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 3*b^
4 + 24*(4*a^3*b + 3*a*b^3)*cosh(d*x + c)^3 + 8*(21*b^4*cosh(d*x + c)^5 - 35
```

```
*a*b^3*cosh(d*x + c)^4 + 12*a^3*b + 9*a*b^3 - 96*(a^4 + a^2*b^2)*d*x*cosh(d
*x + c) + 30*(2*a^2*b^2 + b^4)*cosh(d*x + c)^3 - 30*(4*a^3*b + 3*a*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^3 + 12*(2*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 12*(
7*b^4*cosh(d*x + c)^6 - 14*a*b^3*cosh(d*x + c)^5 - 96*(a^4 + a^2*b^2)*d*x*c
osh(d*x + c)^2 + 15*(2*a^2*b^2 + b^4)*cosh(d*x + c)^4 + 2*a^2*b^2 + b^4 - 2
0*(4*a^3*b + 3*a*b^3)*cosh(d*x + c)^3 + 6*(4*a^3*b + 3*a*b^3)*cosh(d*x + c)
)*sinh(d*x + c)^2 + 192*((a^4 + a^2*b^2)*cosh(d*x + c)^4 + 4*(a^4 + a^2*b^2
)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^4 + a^2*b^2)*cosh(d*x + c)^2*sinh(d*
x + c)^2 + 4*(a^4 + a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + a^2*b^2
)*sinh(d*x + c)^4)*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x +
c))) + 8*(3*b^4*cosh(d*x + c)^7 - 7*a*b^3*cosh(d*x + c)^6 - 96*(a^4 + a^2*b
^2)*d*x*cosh(d*x + c)^3 + 9*(2*a^2*b^2 + b^4)*cosh(d*x + c)^5 - 15*(4*a^3*b
 + 3*a*b^3)*cosh(d*x + c)^4 + a*b^3 + 9*(4*a^3*b + 3*a*b^3)*cosh(d*x + c)^2
 + 3*(2*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c))/(b^5*d*cosh(d*x + c)^4
 + 4*b^5*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*b^5*d*cosh(d*x + c)^2*sinh(d*x
 + c)^2 + 4*b^5*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^5*d*sinh(d*x + c)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 202, normalized size = 1.79

$$\frac{\frac{3\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^4-8\,ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+24\,a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+24\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-96\,a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)-96\,ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{b^4}+\frac{192\,(a^4+a^2b^2)\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{b^5}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac"
)
```

```
[Out] 1/192*((3*b^3*(e^(d*x + c) - e^(-d*x - c))^4 - 8*a*b^2*(e^(d*x + c) - e^(-d
*x - c))^3 + 24*a^2*b*(e^(d*x + c) - e^(-d*x - c))^2 + 24*b^3*(e^(d*x + c)
- e^(-d*x - c))^2 - 96*a^3*(e^(d*x + c) - e^(-d*x - c)) - 96*a*b^2*(e^(d*x
+ c) - e^(-d*x - c)))/b^4 + 192*(a^4 + a^2*b^2)*log(abs(b*(e^(d*x + c) - e^
(-d*x - c)) + 2*a))/b^5)/d
```

Mupad [B]
time = 0.59, size = 238, normalized size = 2.11

$$\frac{e^{-4\,c-4\,d\,x}}{64\,b\,d}-\frac{x\,\left(a^4+a^2\,b^2\right)}{b^5}+\frac{e^{4\,c+4\,d\,x}}{64\,b\,d}+\frac{a\,e^{-3\,c-3\,d\,x}}{24\,b^2\,d}-\frac{a\,e^{3\,c+3\,d\,x}}{24\,b^2\,d}+\frac{\ln\left(2\,a\,e^{d\,x}\,e^c-b+b\,e^{2\,c}\,e^{2\,d\,x}\right)\,\left(a^4+a^2\,b^2\right)}{b^5\,d}-\frac{e^{c+d\,x}\,\left(4\,a^3+3\,a\,b^2\right)}{8\,b^4\,d}+\frac{e^{-c-d\,x}\,\left(4\,a^3+3\,a\,b^2\right)}{8\,b^4\,d}+\frac{e^{-2\,c-2\,d\,x}\,\left(2\,a^2+b^2\right)}{16\,b^3\,d}+\frac{e^{2\,c+2\,d\,x}\,\left(2\,a^2+b^2\right)}{16\,b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^3*sinh(c + d*x)^2)/(a + b*sinh(c + d*x)),x)
```

```
[Out] exp(- 4*c - 4*d*x)/(64*b*d) - (x*(a^4 + a^2*b^2))/b^5 + exp(4*c + 4*d*x)/(6
4*b*d) + (a*exp(- 3*c - 3*d*x))/(24*b^2*d) - (a*exp(3*c + 3*d*x))/(24*b^2*d
) + (log(2*a*exp(d*x)*exp(c) - b + b*exp(2*c)*exp(2*d*x))*(a^4 + a^2*b^2))/
(b^5*d) - (exp(c + d*x)*(3*a*b^2 + 4*a^3))/(8*b^4*d) + (exp(- c - d*x)*(3*a
*b^2 + 4*a^3))/(8*b^4*d) + (exp(- 2*c - 2*d*x)*(2*a^2 + b^2))/(16*b^3*d) +
(exp(2*c + 2*d*x)*(2*a^2 + b^2))/(16*b^3*d)
```

3.376 $\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=39

$$\mathrm{Int}\left(\frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cosh[c + d*x]^3*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][(Cosh[c + d*x]^3*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]
)),x]
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\cosh^3(dx+c)\right)\left(\sinh^2(dx+c)\right)}{(fx+e)\,(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] 1/16*e^(-4*c + 4*d*e/f)*exp_integral_e(1, 4*(f*x + e)*d/f)/(b*f) + 1/8*a*e^
(-3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b^2*f) + 1/8*a*e^(3*c
- 3*d*e/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/(b^2*f) - 1/16*e^(4*c - 4*d*
e/f)*exp_integral_e(1, -4*(f*x + e)*d/f)/(b*f) + 1/8*(2*a^2 + b^2)*e^(-2*c
+ 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b^3*f) - 1/8*(2*a^2*e^(2*c)
+ b^2*e^(2*c))*e^(-2*d*e/f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b^3*f) + 1
/8*(4*a^3 + 3*a*b^2)*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^4*f
) + 1/8*(4*a^3*e^c + 3*a*b^2*e^c)*e^(-d*e/f)*exp_integral_e(1, -(f*x + e)*d
/f)/(b^4*f) + (a^4 + a^2*b^2)*log(f*x + e)/(b^5*f) - 1/32*integrate(64*(a^4
*b + a^2*b^3 - (a^5*e^c + a^3*b^2*e^c)*e^(d*x))/(b^6*f*x + b^6*e - (b^6*f*x
*e^(2*c) + b^6*e^(2*c + 1))*e^(2*d*x) - 2*(a*b^5*f*x*e^c + a*b^5*e^(c + 1))
*e^(d*x)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] integral(cosh(d*x + c)^3*sinh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(
d*x + c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*sinh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate(cosh(d*x + c)^3*sinh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a))
, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh(c+d\,x)^3\,\sinh(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^3*sinh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((cosh(c + d*x)^3*sinh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.377 $\int \frac{(e+fx)^3 \sinh(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)} \, dx$

Optimal. Leaf size=1218

$$-\frac{(e+fx)^4}{4bf}-\frac{2a(e+fx)^3\text{ArcTan}\left(e^{c+dx}\right)}{b^2d}+\frac{2a^3(e+fx)^3\text{ArcTan}\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d}+\frac{a^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{b\left(a^2+b^2\right)d}$$

```
[Out] 3*a^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)/
d^2+3*a^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b
^2)/d^2-6*a^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a
^2+b^2)/d^3-6*a^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/
b/(a^2+b^2)/d^3-3/4*a^2*f^3*polylog(4,-exp(2*d*x+2*c))/b/(a^2+b^2)/d^4-6*I*
a*f^3*polylog(4,I*exp(d*x+c))/b^2/d^4+3*I*a^3*f*(f*x+e)^2*polylog(2,I*exp(d
*x+c))/b^2/(a^2+b^2)/d^2+6*I*a^3*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/b^2/(
a^2+b^2)/d^3+(f*x+e)^3*ln(1+exp(2*d*x+2*c))/b/d+2*a^3*(f*x+e)^3*arctan(exp(
d*x+c))/b^2/(a^2+b^2)/d-1/4*(f*x+e)^4/b/f+6*a^2*f^3*polylog(4,-b*exp(d*x+c)
/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d^4+6*a^2*f^3*polylog(4,-b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d^4-a^2*(f*x+e)^3*ln(1+exp(2*d*x+2*c))/b/(a^2
+b^2)/d+a^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d+
a^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d-2*a*(f*x
+e)^3*arctan(exp(d*x+c))/b^2/d+3/2*f*(f*x+e)^2*polylog(2,-exp(2*d*x+2*c))/b
/d^2-3/2*f^2*(f*x+e)*polylog(3,-exp(2*d*x+2*c))/b/d^3+3/4*f^3*polylog(4,-ex
p(2*d*x+2*c))/b/d^4-3/2*a^2*f*(f*x+e)^2*polylog(2,-exp(2*d*x+2*c))/b/(a^2+b
^2)/d^2+3/2*a^2*f^2*(f*x+e)*polylog(3,-exp(2*d*x+2*c))/b/(a^2+b^2)/d^3+6*I*
a*f^3*polylog(4,-I*exp(d*x+c))/b^2/d^4-3*I*a*f*(f*x+e)^2*polylog(2,I*exp(d*
x+c))/b^2/d^2-6*I*a*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/b^2/d^3-6*I*a^3*f^
3*polylog(4,-I*exp(d*x+c))/b^2/(a^2+b^2)/d^4+3*I*a*f*(f*x+e)^2*polylog(2,-I
*exp(d*x+c))/b^2/d^2+6*I*a*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/b^2/d^3+6*I*
a^3*f^3*polylog(4,I*exp(d*x+c))/b^2/(a^2+b^2)/d^4-3*I*a^3*f*(f*x+e)^2*polyl
og(2,-I*exp(d*x+c))/b^2/(a^2+b^2)/d^2-6*I*a^3*f^2*(f*x+e)*polylog(3,I*exp(d
*x+c))/b^2/(a^2+b^2)/d^3
```

Rubi [A]
time = 1.30, antiderivative size = 1218, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5700, 3799, 2221, 2611, 6744, 2320, 6724, 5686, 4265, 5692, 5680, 6874}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/4*(e + f*x)^4/(b*f) - (2*a*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b^2*d) + (2
*a^3*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b^2*(a^2 + b^2)*d) + (a^2*(e + f*x)^
```

```
3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d) + (a^2*
(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*
d) + ((e + f*x)^3*Log[1 + E^(2*(c + d*x))])/(b*d) - (a^2*(e + f*x)^3*Log[1
+ E^(2*(c + d*x))])/(b*(a^2 + b^2)*d) + ((3*I)*a*f*(e + f*x)^2*PolyLog[2, (
-I)*E^(c + d*x)])/(b^2*d^2) - ((3*I)*a^3*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c
 + d*x)])/(b^2*(a^2 + b^2)*d^2) - ((3*I)*a*f*(e + f*x)^2*PolyLog[2, I*E^(c
+ d*x)])/(b^2*d^2) + ((3*I)*a^3*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b
^2*(a^2 + b^2)*d^2) + (3*a^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^2) + (3*a^2*f*(e + f*x)^2*PolyLog[2,
 -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^2) + (3*f*(e +
 f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*b*d^2) - (3*a^2*f*(e + f*x)^2*Poly
Log[2, -E^(2*(c + d*x))])/(2*b*(a^2 + b^2)*d^2) - ((6*I)*a*f^2*(e + f*x)*Po
lyLog[3, (-I)*E^(c + d*x)])/(b^2*d^3) + ((6*I)*a^3*f^2*(e + f*x)*PolyLog[3,
 (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) + ((6*I)*a*f^2*(e + f*x)*PolyLog[
3, I*E^(c + d*x)])/(b^2*d^3) - ((6*I)*a^3*f^2*(e + f*x)*PolyLog[3, I*E^(c +
 d*x)])/(b^2*(a^2 + b^2)*d^3) - (6*a^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^3) - (6*a^2*f^2*(e + f*x)*
PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^3) -
 (3*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*b*d^3) + (3*a^2*f^2*(e +
 f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*b*(a^2 + b^2)*d^3) + ((6*I)*a*f^3*Po
lyLog[4, (-I)*E^(c + d*x)])/(b^2*d^4) - ((6*I)*a^3*f^3*PolyLog[4, (-I)*E^(c
 + d*x)])/(b^2*(a^2 + b^2)*d^4) - ((6*I)*a*f^3*PolyLog[4, I*E^(c + d*x)])/(
b^2*d^4) + ((6*I)*a^3*f^3*PolyLog[4, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^4)
+ (6*a^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2
+ b^2)*d^4) + (6*a^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)])/(b*(a^2 + b^2)*d^4) + (3*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*b*d^4) -
(3*a^2*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*b*(a^2 + b^2)*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
```

```
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5686

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*Sech[c +
d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5700

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - Di
st[a/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*(Tanh[c + d*x]^n/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\tanh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{a\int (e+fx)^3\text{sech}(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^3\text{sech}}{a+b\sinh(c}}{b^2}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{(e+fx)^3\log\left(1+\right.}{bd}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{a^2(e+fx)^4}{4b\left(a^2+b^2\right)f} - \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2d} +$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{a^2(e+fx)^4}{4b\left(a^2+b^2\right)f} - \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{a}{}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^3\tan^{-}}{b^2\left(a^2+b^2\right)}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^3\tan^{-}}{b^2\left(a^2+b^2\right)}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^3\tan^{-}}{b^2\left(a^2+b^2\right)}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^3\tan^{-}}{b^2\left(a^2+b^2\right)}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^3\tan^{-}}{b^2\left(a^2+b^2\right)}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^3\tan^{-}}{b^2\left(a^2+b^2\right)}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2891 vs. $2(1218) = 2436$.
time = 19.93, size = 2891, normalized size = 2.37

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x
]

[Out] -1/4*(8*a*d^3*e^3*(1 + E^(2*c))*ArcTan[E^(c + d*x)] + 4*b*d^3*e^3*E^(2*c)*(
2*d*x - Log[1 + E^(2*(c + d*x))]) - 4*b*d^3*e^3*Log[1 + E^(2*(c + d*x))] +
(12*I)*a*d^2*e^2*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E
^(c + d*x)]) - PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) +
6*b*d^2*e^2*E^(2*c)*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2,
-E^(2*(c + d*x))]) - 6*b*d^2*e^2*f*(2*d*x*Log[1 + E^(2*(c + d*x))] + PolyLo
g[2, -E^(2*(c + d*x))]) + (12*I)*a*d*e*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I
*E^(c + d*x)] - d^2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c
 + d*x)] + 2*d*x*PolyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)]
 - 2*PolyLog[3, I*E^(c + d*x)]) - 6*b*d*e*f^2*(2*d^2*x^2*Log[1 + E^(2*(c +
d*x))] + 2*d*x*PolyLog[2, -E^(2*(c + d*x))] - PolyLog[3, -E^(2*(c + d*x))])
 + 2*b*d*e*E^(2*c)*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))]) - 6*
d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))]) + (4*I)*
a*(1 + E^(2*c))*f^3*(d^3*x^3*Log[1 - I*E^(c + d*x)] - d^3*x^3*Log[1 + I*E^(
c + d*x)] - 3*d^2*x^2*PolyLog[2, (-I)*E^(c + d*x)] + 3*d^2*x^2*PolyLog[2, I
*E^(c + d*x)] + 6*d*x*PolyLog[3, (-I)*E^(c + d*x)] - 6*d*x*PolyLog[3, I*E^(
c + d*x)] - 6*PolyLog[4, (-I)*E^(c + d*x)] + 6*PolyLog[4, I*E^(c + d*x)]) +
 b*E^(2*c)*f^3*(2*d^4*x^4 - 4*d^3*x^3*Log[1 + E^(2*(c + d*x))] - 6*d^2*x^2*
PolyLog[2, -E^(2*(c + d*x))] + 6*d*x*PolyLog[3, -E^(2*(c + d*x))] - 3*PolyL
og[4, -E^(2*(c + d*x))]) - b*f^3*(4*d^3*x^3*Log[1 + E^(2*(c + d*x))] + 6*d^
2*x^2*PolyLog[2, -E^(2*(c + d*x))] - 6*d*x*PolyLog[3, -E^(2*(c + d*x))] + 3
*PolyLog[4, -E^(2*(c + d*x))]))/((a^2 + b^2)*d^4*(1 + E^(2*c))) - (a^2*(4*e
^3*E^(2*c)*x + 6*e^2*E^(2*c)*f*x^2 + 4*e*E^(2*c)*f^2*x^3 + E^(2*c)*f^3*x^4
+ (4*a*Sqrt[a^2 + b^2]*e^3*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(S
qrt[-(a^2 + b^2)^2]*d) + (4*a*Sqrt[-(a^2 + b^2)^2]*e^3*E^(2*c)*ArcTan[(a +
b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (4*a*Sqrt[-(a^2 +
 b^2)^2]*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3
/2)*d) + (4*a*Sqrt[-(a^2 + b^2)^2]*e^3*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/
Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (2*e^3*Log[2*a*E^(c + d*x) + b*(
-1 + E^(2*(c + d*x)))])/d - (2*e^3*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^
(2*(c + d*x)))])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^
2 + b^2)*E^(2*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^
c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))
/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f^2*x^2*Log[1 + (b*
E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (2*f^3*x^3*Log[1 +
 (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (2*E^(2*c)*f^3
*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6
*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d
- (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^
(2*c)])])/d + (6*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b
```

```
^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c +
 Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (2*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E
^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (2*E^(2*c)*f^3*x^3*Log[1 + (b*E^(2*c
+ d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*(-1 + E^(2*c))*f*(e +
f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))]
)/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E
^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (12*e*f^2*PolyLog[3, -((b*E^(2*c +
 d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*e*E^(2*c)*f^2*PolyL
og[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (12*
f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])
/d^3 + (12*E^(2*c)*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2
+ b^2)*E^(2*c)]))])/d^3 - (12*e*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c +
 Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*e*E^(2*c)*f^2*PolyLog[3, -((b*E^(2
*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (12*f^3*x*PolyLog[3
, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*E^(2
*c)*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]
))])/d^3 + (12*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)
*E^(2*c)]))])/d^4 - (12*E^(2*c)*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c -
 Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^4 + (12*f^3*PolyLog[4, -((b*E^(2*c + d*x))
/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^4 - (12*E^(2*c)*f^3*PolyLog[4, -(
(b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^4))/(2*b*(a^2 +
b^2)*(-1 + E^(2*c))) + ((4*a^2*e^3*x - 4*b^2*e^3*x + 6*a^2*e^2*f*x^2 - 6*b^
2*e^2*f*x^2 + 4*a^2*e*f^2*x^3 - 4*b^2*e*f^2*x^3 + a^2*f^3*x^4 - b^2*f^3*x^4
 + 4*a^2*e^3*x*Cosh[2*c] + 4*b^2*e^3*x*Cosh[2*c] + 6*a^2*e^2*f*x^2*Cosh[2*c
] + 6*b^2*e^2*f*x^2*Cosh[2*c] + 4*a^2*e*f^2*x^3*Cosh[2*c] + 4*b^2*e*f^2*x^3
*Cosh[2*c] + a^2*f^3*x^4*Cosh[2*c] + b^2*f^3*x^4*Cosh[2*c])*Csch[c]*Sech[c]
)/(8*b*(a^2 + b^2))
```

Maple [**F**]
time = 2.66, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \sinh(dx+c)\tanh(dx+c)}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] int((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] (a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b + b^3)*d) + 2*
a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^2
+ b^2)*d) + (d*x + c)/(b*d))*e^3 + 1/4*(f^3*x^4 + 4*f^2*x^3*e + 6*f*x^2*e^2
)/b - integrate(2*(a^2*b*f^3*x^3 + 3*a^2*b*f^2*x^2*e + 3*a^2*b*f*x*e^2 - (a
^3*f^3*x^3*e^c + 3*a^3*f^2*x^2*e^(c + 1) + 3*a^3*f*x*e^(c + 2))*e^(d*x))/(a
^2*b^2 + b^4 - (a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x) - 2*(a^3*b*e^c + a
*b^3*e^c)*e^(d*x)), x) - integrate(2*(b*f^3*x^3 + 3*b*f^2*x^2*e + 3*b*f*x*e
^2 + (a*f^3*x^3*e^c + 3*a*f^2*x^2*e^(c + 1) + 3*a*f*x*e^(c + 2))*e^(d*x))/(
a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3310 vs. $2(1133) = 2266$.
time = 0.52, size = 3310, normalized size = 2.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -1/4*((a^2 + b^2)*d^4*f^3*x^4 + 4*(a^2 + b^2)*d^4*f^2*x^3*cosh(1) + 6*(a^2
+ b^2)*d^4*f*x^2*cosh(1)^2 + 4*(a^2 + b^2)*d^4*x*cosh(1)^3 + 4*(a^2 + b^2)*
d^4*x*sinh(1)^3 - 24*a^2*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c)
+ (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 24*a^2*f^
3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*((a^2 + b^2)*d^4*f*x^2 + 2*(a^2 +
b^2)*d^4*x*cosh(1))*sinh(1)^2 - 12*(a^2*d^2*f^3*x^2 + 2*a^2*d^2*f^2*x*cosh(
1) + a^2*d^2*f*cosh(1)^2 + a^2*d^2*f*sinh(1)^2 + 2*(a^2*d^2*f^2*x + a^2*d^2
*f*cosh(1))*sinh(1))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 12*(a^2*d^2*f^
3*x^2 + 2*a^2*d^2*f^2*x*cosh(1) + a^2*d^2*f*cosh(1)^2 + a^2*d^2*f*sinh(1)^2
 + 2*(a^2*d^2*f^2*x + a^2*d^2*f*cosh(1))*sinh(1))*dilog((a*cosh(d*x + c) +
a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
 - b)/b + 1) + 12*(I*a*b*d^2*f^3*x^2 - b^2*d^2*f^3*x^2 + 2*I*a*b*d^2*f^2*x*
cosh(1) - 2*b^2*d^2*f^2*x*cosh(1) + I*a*b*d^2*f*cosh(1)^2 - b^2*d^2*f*cosh(
1)^2 + I*a*b*d^2*f*sinh(1)^2 - b^2*d^2*f*sinh(1)^2 + 2*I*(a*b*d^2*f^2*x + a
*b*d^2*f*cosh(1))*sinh(1) - 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*
dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + 12*(-I*a*b*d^2*f^3*x^2 - b^2*d^2
*f^3*x^2 - 2*I*a*b*d^2*f^2*x*cosh(1) - 2*b^2*d^2*f^2*x*cosh(1) - I*a*b*d^2*
f*cosh(1)^2 - b^2*d^2*f*cosh(1)^2 - I*a*b*d^2*f*sinh(1)^2 - b^2*d^2*f*sinh(
1)^2 - 2*I*(a*b*d^2*f^2*x + a*b*d^2*f*cosh(1))*sinh(1) - 2*(b^2*d^2*f^2*x +
 b^2*d^2*f*cosh(1))*sinh(1))*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + 4*
(a^2*c^3*f^3 - 3*a^2*c^2*d*f^2*cosh(1) + 3*a^2*c*d^2*f*cosh(1)^2 - a^2*d^3*
```

cosh(1)^3 - a^2*d^3*sinh(1)^3 + 3*(a^2*c*d^2*f - a^2*d^3*cosh(1))*sinh(1)^2 - 3*(a^2*c^2*d*f^2 - 2*a^2*c*d^2*f*cosh(1) + a^2*d^3*cosh(1)^2)*sinh(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 4*(a^2*c^3*f^3 - 3*a^2*c^2*d*f^2*cosh(1) + 3*a^2*c*d^2*f*cosh(1)^2 - a^2*d^3*cosh(1)^3 - a^2*d^3*sinh(1)^3 + 3*(a^2*c*d^2*f - a^2*d^3*cosh(1))*sinh(1)^2 - 3*(a^2*c^2*d*f^2 - 2*a^2*c*d^2*f*cosh(1) + a^2*d^3*cosh(1)^2)*sinh(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 4*(a^2*d^3*f^3*x^3 + a^2*c^3*f^3 + 3*(a^2*d^3*f*x + a^2*c*d^2*f)*cosh(1)^2 + 3*(a^2*d^3*f*x + a^2*c*d^2*f)*sinh(1)^2 + 3*(a^2*d^3*f^2*x^2 - a^2*c^2*d*f^2)*cosh(1) + 3*(a^2*d^3*f^2*x^2 - a^2*c^2*d*f^2 + 2*(a^2*d^3*f*x + a^2*c*d^2*f)*cosh(1))*sinh(1))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 4*(a^2*d^3*f^3*x^3 + a^2*c^3*f^3 + 3*(a^2*d^3*f*x + a^2*c*d^2*f)*cosh(1)^2 + 3*(a^2*d^3*f*x + a^2*c*d^2*f)*sinh(1)^2 + 3*(a^2*d^3*f^2*x^2 - a^2*c^2*d*f^2)*cosh(1) + 3*(a^2*d^3*f^2*x^2 - a^2*c^2*d*f^2 + 2*(a^2*d^3*f*x + a^2*c*d^2*f)*cosh(1))*sinh(1))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 4*(-I*a*b*c^3*f^3 + b^2*c^3*f^3 + 3*I*a*b*c^2*d*f^2*cosh(1) - 3*b^2*c^2*d*f^2*cosh(1) - 3*I*a*b*c*d^2*f*cosh(1)^2 + 3*b^2*c*d^2*f*cosh(1)^2 + I*a*b*d^3*cosh(1)^3 - b^2*d^3*cosh(1)^3 + I*a*b*d^3*sinh(1)^3 - b^2*d^3*sinh(1)^3 - 3*I*(a*b*c*d^2*f - a*b*d^3*cosh(1))*sinh(1)^2 + 3*(b^2*c*d^2*f - b^2*d^3*cosh(1))*sinh(1)^2 + 3*I*(a*b*c^2*d*f^2 - 2*a*b*c*d^2*f*cosh(1) + a*b*d^3*cosh(1)^2)*sinh(1) - 3*(b^2*c^2*d*f^2 - 2*b^2*c*d^2*f*cosh(1) + b^2*d^3*cosh(1)^2)*sinh(1))*log(cosh(d*x + c) + sinh(d*x + c) + I) + 4*(I*a*b*c^3*f^3 + b^2*c^3*f^3 - 3*I*a*b*c^2*d*f^2*cosh(1) - 3*b^2*c^2*d*f^2*cosh(1) + 3*I*a*b*c*d^2*f*cosh(1)^2 + 3*b^2*c*d^2*f*cosh(1)^2 - I*a*b*d^3*cosh(1)^3 - b^2*d^3*cosh(1)^3 - I*a*b*d^3*sinh(1)^3 - b^2*d^3*sinh(1)^3 + 3*I*(a*b*c*d^2*f - a*b*d^3*cosh(1))*sinh(1)^2 + 3*(b^2*c*d^2*f - b^2*d^3*cosh(1))*sinh(1)^2 - 3*I*(a*b*c^2*d*f^2 - 2*a*b*c*d^2*f*cosh(1) + a*b*d^3*cosh(1)^2)*sinh(1) - 3*(b^2*c^2*d*f^2 - 2*b^2*c*d^2*f*cosh(1) + b^2*d^3*cosh(1)^2)*sinh(1))*log(cosh(d*x + c) + sinh(d*x + c) - I) + 4*(-I*a*b*d^3*f^3*x^3 - b^2*d^3*f^3*x^3 - I*a*b*c^3*f^3 - b^2*c^3*f^3 - 3*I*(a*b*d^3*f*x + a*b*c*d^2*f)*cosh(1)^2 - 3*(b^2*d^3*f*x + b^2*c*d^2*f)*cosh(1)^2 - 3*I*(a*b*d^3*f*x + a*b*c*d^2*f)*sinh(1)^2 - 3*(b^2*d^3*f*x + b^2*c*d^2*f)*sinh(1)^2 - 3*I*(a*b*d^3*f^2*x^2 - a*b*c^2*d*f^2)*cosh(1) - 3*(b^2*d^3*f^2*x^2 - b^2*c^2*d*f^2)*cosh(1) - 3*I*(a*b*d^3*f^2*x^2 - a*b*c^2*d*f^2 + 2*(a*b*d^3*f*x + a*b*c*d^2*f)*cosh(1))*sinh(1) - 3*(b^2*d^3*f^2*x^2 - b^2*c^2*d*f^2 + 2*(b^2*d^3*f*x + b^2*c*d^2*f)*cosh(1))*sinh(1))*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) + 4*(I*a*b*d^3*f^3*x^3 - b^2*d^3*f^3*x^3 + I*a*b*c^3*f^3 - b^2*c^3*f^3 + 3*I*(a*b*d^3*f*x + a*b*c*d^2*f)*cosh(1)^2 - 3*(b^2*d^3*f*x + b^2*c*d^2*f)*cosh(1)^2 + 3*I*(a*b*d^3*f*x + a*b*c*d^2*f)*sinh(1)^2 - 3*(b^2*d^3*f*x + b^2*c*d^2*f)*sinh(1)^2 + 3*I*(a*b*d^3*f^2*x^2 - a*b*c^2*d*f^2)*cosh(1) - 3*(b^2*d^3*f^2*x^2 - b^2*c^2*d*f^2)*cosh(1) + 3*I*(a*b*d^3*f^2*x^2 - a*b*c^2*d*f^2 + 2*(a*b*d^3*f*x + a*b*c*d^2*f)*cosh(1))*sinh(1) - 3*(b^2*d^3*f^2*x^2 - b^2*c^...

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3 \sinh{(c+dx)} \tanh{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*sinh(c + d*x)*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)\,{(e+f\,x)}^3}{a+b\sinh{(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*tanh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((sinh(c + d*x)*tanh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.378 $\int \frac{(e+fx)^2 \sinh(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal. Leaf size=861

$$-\frac{(e+fx)^3}{3bf}-\frac{2a(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{b^2d}+\frac{2a^3(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d}+\frac{a^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{b\left(a^2+b^2\right)d}$$

```
[Out] -1/3*(f*x+e)^3/b/f-2*a*(f*x+e)^2*arctan(exp(d*x+c))/b^2/d+2*a^3*(f*x+e)^2*a
rctan(exp(d*x+c))/b^2/(a^2+b^2)/d+(f*x+e)^2*ln(1+exp(2*d*x+2*c))/b/d-a^2*(f
*x+e)^2*ln(1+exp(2*d*x+2*c))/b/(a^2+b^2)/d+a^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/
(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d+a^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+
b^2)^(1/2)))/b/(a^2+b^2)/d+2*I*a^3*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b^2/(a
^2+b^2)/d^2-2*I*a^3*f^2*polylog(3,I*exp(d*x+c))/b^2/(a^2+b^2)/d^3+2*I*a*f*(
f*x+e)*polylog(2,-I*exp(d*x+c))/b^2/d^2-2*I*a*f*(f*x+e)*polylog(2,I*exp(d*x
+c))/b^2/d^2+f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/b/d^2-a^2*f*(f*x+e)*polyl
og(2,-exp(2*d*x+2*c))/b/(a^2+b^2)/d^2+2*a^2*f*(f*x+e)*polylog(2,-b*exp(d*x+
c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d^2+2*a^2*f*(f*x+e)*polylog(2,-b*exp(d*
x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d^2+2*I*a^3*f^2*polylog(3,-I*exp(d*x+
c))/b^2/(a^2+b^2)/d^3-2*I*a^3*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/b^2/(a^2+b
^2)/d^2-2*I*a*f^2*polylog(3,-I*exp(d*x+c))/b^2/d^3+2*I*a*f^2*polylog(3,I*ex
p(d*x+c))/b^2/d^3-1/2*f^2*polylog(3,-exp(2*d*x+2*c))/b/d^3+1/2*a^2*f^2*poly
log(3,-exp(2*d*x+2*c))/b/(a^2+b^2)/d^3-2*a^2*f^2*polylog(3,-b*exp(d*x+c)/(a
-(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d^3-2*a^2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^
2+b^2)^(1/2)))/b/(a^2+b^2)/d^3
```

Rubi [A]
time = 0.99, antiderivative size = 861, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {5700, 3799, 2221, 2611, 2320, 6724, 5686, 4265, 5692, 5680, 6874}

$$\frac{2(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)a^3}{b^2\left(a^2+b^2\right)d}-\frac{2if(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)a^3}{b^2\left(a^2+b^2\right)d^2}+\frac{2if(e+fx)\text{Li}_2\left(ie^{c+dx}\right)a^3}{b^2\left(a^2+b^2\right)d^2}+\frac{2if^2\text{Li}_3\left(-ie^{c+dx}\right)a^3}{b^2\left(a^2+b^2\right)d^3}-\frac{2if^2\text{Li}_3\left(ie^{c+dx}\right)a^3}{b^2\left(a^2+b^2\right)d^3}+\frac{(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)a^2}{b\left(a^2+b^2\right)d}+\frac{(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)a^2}{b\left(a^2+b^2\right)d}-\frac{(e+fx)^2\log\left(1+e^{2(c+dx)}\right)a^2}{b\left(a^2+b^2\right)d}+\frac{2f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)a^2}{b\left(a^2+b^2\right)d^2}+\frac{2f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)a^2}{b\left(a^2+b^2\right)d^2}-\frac{f(e+fx)\text{Li}_2\left(-e^{2(c+dx)}\right)a^2}{b\left(a^2+b^2\right)d^2}-\frac{2f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)a^2}{b\left(a^2+b^2\right)d^3}-\frac{2f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)a^2}{b\left(a^2+b^2\right)d^3}+\frac{f^2\text{Li}_3\left(-e^{2(c+dx)}\right)a^2}{2b\left(a^2+b^2\right)d^3}-\frac{2(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)a}{b^2d}+\frac{2if(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)a}{b^2d^2}-\frac{2if(e+fx)\text{Li}_2\left(ie^{c+dx}\right)a}{b^2d^2}-\frac{2if^2\text{Li}_3\left(-ie^{c+dx}\right)a}{b^2d^3}+\frac{2if^2\text{Li}_3\left(ie^{c+dx}\right)a}{b^2d^3}-\frac{(e+fx)^3}{3bf}+\frac{(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{bd}+\frac{f(e+fx)\text{Li}_2\left(-e^{2(c+dx)}\right)}{bd^2}-\frac{f^2\text{Li}_3\left(-e^{2(c+dx)}\right)}{2bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/3*(e + f*x)^3/(b*f) - (2*a*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^2*d) + (2
*a^3*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^2*(a^2 + b^2)*d) + (a^2*(e + f*x)^
2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d) + (a^2*
(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*
d) + ((e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b*d) - (a^2*(e + f*x)^2*Log[1
+ E^(2*(c + d*x))])/(b*(a^2 + b^2)*d) + ((2*I)*a*f*(e + f*x)*PolyLog[2, (-I
)*E^(c + d*x)])/(b^2*d^2) - ((2*I)*a^3*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d
*x)])/(b^2*(a^2 + b^2)*d^2) - ((2*I)*a*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)
```

```
])/(b^2*d^2) + ((2*I)*a^3*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b^2*(a^2
+ b^2)*d^2) + (2*a^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2]))])/(b*(a^2 + b^2)*d^2) + (2*a^2*f*(e + f*x)*PolyLog[2, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^2) + (f*(e + f*x)*PolyLog[
2, -E^(2*(c + d*x))])/(b*d^2) - (a^2*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x)
)])/(b*(a^2 + b^2)*d^2) - ((2*I)*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b^2*d
^3) + ((2*I)*a^3*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) +
((2*I)*a*f^2*PolyLog[3, I*E^(c + d*x)])/(b^2*d^3) - ((2*I)*a^3*f^2*PolyLog[
3, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) - (2*a^2*f^2*PolyLog[3, -((b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^3) - (2*a^2*f^2*PolyLog[3
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^3) - (f^2*Pol
yLog[3, -E^(2*(c + d*x))])/(2*b*d^3) + (a^2*f^2*PolyLog[3, -E^(2*(c + d*x))
])/(2*b*(a^2 + b^2)*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5686

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*Sech[c +
d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5700

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - Di
st[a/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*(Tanh[c + d*x]^n/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\tanh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{a\int (e+fx)^2\text{sech}(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^2\text{sech}(c}{a+b\sinh(c+d}}{b^2}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{(e+fx)^2\log\left(1+e\right.}{bd}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{a^2(e+fx)^3}{3b\left(a^2+b^2\right)f} - \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{(e}{}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{a^2(e+fx)^3}{3b\left(a^2+b^2\right)f} - \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{a^2(}{}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^2\tan^{-1}}{b^2\left(a^2+b^2\right)c}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^2\tan^{-1}}{b^2\left(a^2+b^2\right)c}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^2\tan^{-1}}{b^2\left(a^2+b^2\right)c}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^2\tan^{-1}}{b^2\left(a^2+b^2\right)c}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^2\tan^{-1}}{b^2\left(a^2+b^2\right)c}$$

Mathematica [A]
time = 15.24, size = 1202, normalized size = 1.40

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] -1/6*(12*b*d^3*e^2*E^(2*c)*x + 12*b*d^3*e*E^(2*c)*f*x^2 + 4*b*d^3*E^(2*c)*f
^2*x^3 + 12*a*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] - 6*b*d^2*e^2*(1 +
E^(2*c))*Log[1 + E^(2*(c + d*x))] + (12*I)*a*d*e*(1 + E^(2*c))*f*(d*x*(Log[
1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - PolyLog[2, (-I)*E^(c + d*x)]
 + PolyLog[2, I*E^(c + d*x)]) - 6*b*d*e*(1 + E^(2*c))*f*(2*d*x*Log[1 + E^(2
*(c + d*x))] + PolyLog[2, -E^(2*(c + d*x))]) + (6*I)*a*(1 + E^(2*c))*f^2*(d
^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*Poly
Log[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (
-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d*x)]) - 3*b*(1 + E^(2*c))*f^2*(2*
d^2*x^2*Log[1 + E^(2*(c + d*x))] + 2*d*x*PolyLog[2, -E^(2*(c + d*x))] - Pol
yLog[3, -E^(2*(c + d*x))]))/((a^2 + b^2)*d^3*(1 + E^(2*c))) + (a^2*((-2*E^(
2*c)*x*(3*e^2 + 3*e*f*x + f^2*x^2))/(-1 + E^(2*c)) + (3*((2*a*Sqrt[a^2 + b^
2]*d^2*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^
2] + (2*a*Sqrt[-a^2 - b^2]*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b
^2]])/Sqrt[-(a^2 + b^2)^2] + d^2*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c
+ d*x)))] + 2*d^2*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)
*E^(2*c)])] + d^2*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^
2)*E^(2*c)])] + 2*d^2*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 +
b^2)*E^(2*c)])] + d^2*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2
+ b^2)*E^(2*c)])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c -
 Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*
x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*f^2*PolyLog[3, -((b*E^(2*c +
d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*f^2*PolyLog[3, -((b*E^(2*c
+ d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/d^3))/(3*b*(a^2 + b^2)) + (
(3*a^2*e^2*x - 3*b^2*e^2*x + 3*a^2*e*f*x^2 - 3*b^2*e*f*x^2 + a^2*f^2*x^3 -
b^2*f^2*x^3 + 3*a^2*e^2*x*Cosh[2*c] + 3*b^2*e^2*x*Cosh[2*c] + 3*a^2*e*f*x^2
*Cosh[2*c] + 3*b^2*e*f*x^2*Cosh[2*c] + a^2*f^2*x^3*Cosh[2*c] + b^2*f^2*x^3*
Cosh[2*c])*Csch[c]*Sech[c])/(6*b*(a^2 + b^2))
```

Maple [F]
time = 2.52, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \sinh(dx+c) \tanh(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] (a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b + b^3)*d) + 2*
a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^2
+ b^2)*d) + (d*x + c)/(b*d))*e^2 + 1/3*(f^2*x^3 + 3*f*x^2*e)/b - integrate(
2*(a^2*b*f^2*x^2 + 2*a^2*b*f*x*e - (a^3*f^2*x^2*e^c + 2*a^3*f*x*e^(c + 1))*
e^(d*x))/(a^2*b^2 + b^4 - (a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x) - 2*(a^
3*b*e^c + a*b^3*e^c)*e^(d*x)), x) - integrate(2*(b*f^2*x^2 + 2*b*f*x*e + (a
*f^2*x^2*e^c + 2*a*f*x*e^(c + 1))*e^(d*x))/(a^2 + b^2 + (a^2*e^(2*c) + b^2*
e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1694 vs. 2(802) = 1604.
time = 0.41, size = 1694, normalized size = 1.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -1/3*((a^2 + b^2)*d^3*f^2*x^3 + 3*(a^2 + b^2)*d^3*f*x^2*cosh(1) + 3*(a^2 +
b^2)*d^3*x*cosh(1)^2 + 3*(a^2 + b^2)*d^3*x*sinh(1)^2 + 6*a^2*f^2*polylog(3,
 (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 + b^2)/b^2))/b) + 6*a^2*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d
*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6
*(a^2*d*f^2*x + a^2*d*f*cosh(1) + a^2*d*f*sinh(1))*dilog((a*cosh(d*x + c) +
 a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2
) - b)/b + 1) - 6*(a^2*d*f^2*x + a^2*d*f*cosh(1) + a^2*d*f*sinh(1))*dilog((
a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqr
t((a^2 + b^2)/b^2) - b)/b + 1) + 6*(I*a*b*d*f^2*x - b^2*d*f^2*x + I*a*b*d*f
*cosh(1) - b^2*d*f*cosh(1) + I*a*b*d*f*sinh(1) - b^2*d*f*sinh(1))*dilog(I*c
osh(d*x + c) + I*sinh(d*x + c)) + 6*(-I*a*b*d*f^2*x - b^2*d*f^2*x - I*a*b*d
*f*cosh(1) - b^2*d*f*cosh(1) - I*a*b*d*f*sinh(1) - b^2*d*f*sinh(1))*dilog(-
I*cosh(d*x + c) - I*sinh(d*x + c)) - 3*(a^2*c^2*f^2 - 2*a^2*c*d*f*cosh(1) +
 a^2*d^2*cosh(1)^2 + a^2*d^2*sinh(1)^2 - 2*(a^2*c*d*f - a^2*d^2*cosh(1))*si
nh(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2
) + 2*a) - 3*(a^2*c^2*f^2 - 2*a^2*c*d*f*cosh(1) + a^2*d^2*cosh(1)^2 + a^2*d
^2*sinh(1)^2 - 2*(a^2*c*d*f - a^2*d^2*cosh(1))*sinh(1))*log(2*b*cosh(d*x +
```

```
c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(a^2*d^2*f^2*
x^2 - a^2*c^2*f^2 + 2*(a^2*d^2*f*x + a^2*c*d*f)*cosh(1) + 2*(a^2*d^2*f*x +
a^2*c*d*f)*sinh(1))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
 c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 3*(a^2*d^2*f^2*x^2 -
 a^2*c^2*f^2 + 2*(a^2*d^2*f*x + a^2*c*d*f)*cosh(1) + 2*(a^2*d^2*f*x + a^2*c
*d*f)*sinh(1))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 3*(I*a*b*c^2*f^2 - b^2*c^
2*f^2 - 2*I*a*b*c*d*f*cosh(1) + 2*b^2*c*d*f*cosh(1) + I*a*b*d^2*cosh(1)^2 -
 b^2*d^2*cosh(1)^2 + I*a*b*d^2*sinh(1)^2 - b^2*d^2*sinh(1)^2 - 2*I*(a*b*c*d
*f - a*b*d^2*cosh(1))*sinh(1) + 2*(b^2*c*d*f - b^2*d^2*cosh(1))*sinh(1))*lo
g(cosh(d*x + c) + sinh(d*x + c) + I) + 3*(-I*a*b*c^2*f^2 - b^2*c^2*f^2 + 2*
I*a*b*c*d*f*cosh(1) + 2*b^2*c*d*f*cosh(1) - I*a*b*d^2*cosh(1)^2 - b^2*d^2*c
osh(1)^2 - I*a*b*d^2*sinh(1)^2 - b^2*d^2*sinh(1)^2 + 2*I*(a*b*c*d*f - a*b*d
^2*cosh(1))*sinh(1) + 2*(b^2*c*d*f - b^2*d^2*cosh(1))*sinh(1))*log(cosh(d*x
 + c) + sinh(d*x + c) - I) + 3*(-I*a*b*d^2*f^2*x^2 - b^2*d^2*f^2*x^2 + I*a*
b*c^2*f^2 + b^2*c^2*f^2 - 2*I*(a*b*d^2*f*x + a*b*c*d*f)*cosh(1) - 2*(b^2*d^
2*f*x + b^2*c*d*f)*cosh(1) - 2*I*(a*b*d^2*f*x + a*b*c*d*f)*sinh(1) - 2*(b^2
*d^2*f*x + b^2*c*d*f)*sinh(1))*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) +
 3*(I*a*b*d^2*f^2*x^2 - b^2*d^2*f^2*x^2 - I*a*b*c^2*f^2 + b^2*c^2*f^2 + 2*I
*(a*b*d^2*f*x + a*b*c*d*f)*cosh(1) - 2*(b^2*d^2*f*x + b^2*c*d*f)*cosh(1) +
2*I*(a*b*d^2*f*x + a*b*c*d*f)*sinh(1) - 2*(b^2*d^2*f*x + b^2*c*d*f)*sinh(1)
)*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1) + 6*(-I*a*b*f^2 + b^2*f^2)*po
lylog(3, I*cosh(d*x + c) + I*sinh(d*x + c)) + 6*(I*a*b*f^2 + b^2*f^2)*polyl
og(3, -I*cosh(d*x + c) - I*sinh(d*x + c)) + 3*((a^2 + b^2)*d^3*f*x^2 + 2*(a
^2 + b^2)*d^3*x*cosh(1))*sinh(1))/((a^2*b + b^3)*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2 \sinh(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*sinh(c + d*x)*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)\,\left(e+f\,x\right)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*tanh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
[Out] int((sinh(c + d*x)*tanh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.379 $\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=516

$$-\frac{(e+fx)^2}{2bf}-\frac{2a(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{b^2d}+\frac{2a^3(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d}+\frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d}$$

```
[Out] -1/2*(f*x+e)^2/b/f-2*a*(f*x+e)*arctan(exp(d*x+c))/b^2/d+2*a^3*(f*x+e)*arcta
n(exp(d*x+c))/b^2/(a^2+b^2)/d+(f*x+e)*ln(1+exp(2*d*x+2*c))/b/d-a^2*(f*x+e)*
ln(1+exp(2*d*x+2*c))/b/(a^2+b^2)/d+a^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))/b/(a^2+b^2)/d+a^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))
/b/(a^2+b^2)/d+I*a*f*polylog(2,-I*exp(d*x+c))/b^2/d^2-I*a^3*f*polylog(2,-I*
exp(d*x+c))/b^2/(a^2+b^2)/d^2-I*a*f*polylog(2,I*exp(d*x+c))/b^2/d^2+I*a^3*f
*polylog(2,I*exp(d*x+c))/b^2/(a^2+b^2)/d^2+1/2*f*polylog(2,-exp(2*d*x+2*c))
/b/d^2-1/2*a^2*f*polylog(2,-exp(2*d*x+2*c))/b/(a^2+b^2)/d^2+a^2*f*polylog(2
,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d^2+a^2*f*polylog(2,-b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)/d^2
```

Rubi [A]

time = 0.56, antiderivative size = 516, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5700, 3799, 2221, 2317, 2438, 5686, 4265, 5692, 5680, 6874}

$$\frac{a^2f\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2\left(a^2+b^2\right)}+\frac{a^2f\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2\left(a^2+b^2\right)}-\frac{a^2f\text{Li}_2\left(-e^{2(c+dx)}\right)}{2bd^2\left(a^2+b^2\right)}+\frac{a^2(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd\left(a^2+b^2\right)}+\frac{a^2(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd\left(a^2+b^2\right)}-\frac{a^2(e+fx)\log\left(e^{2(c+dx)}+1\right)}{bd\left(a^2+b^2\right)}+\frac{2a^3(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{b^2d\left(a^2+b^2\right)}-\frac{ia^3f\text{Li}_2\left(-ie^{c+dx}\right)}{b^2d^2\left(a^2+b^2\right)}+\frac{ia^3f\text{Li}_2\left(ie^{c+dx}\right)}{b^2d^2\left(a^2+b^2\right)}-\frac{2a(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{b^2d}+\frac{iaf\text{Li}_2\left(-ie^{c+dx}\right)}{b^2d^2}-\frac{iaf\text{Li}_2\left(ie^{c+dx}\right)}{b^2d^2}+\frac{f\text{Li}_2\left(-e^{2(c+dx)}\right)}{2bd^2}+\frac{(e+fx)\log\left(e^{2(c+dx)}+1\right)}{bd}-\frac{(e+fx)^2}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/2*(e + f*x)^2/(b*f) - (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/(b^2*d) + (2*a
^3*(e + f*x)*ArcTan[E^(c + d*x)])/(b^2*(a^2 + b^2)*d) + (a^2*(e + f*x)*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d) + (a^2*(e + f
*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d) + ((e
 + f*x)*Log[1 + E^(2*(c + d*x))])/(b*d) - (a^2*(e + f*x)*Log[1 + E^(2*(c +
d*x))])/(b*(a^2 + b^2)*d) + (I*a*f*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*d^2)
- (I*a^3*f*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) - (I*a*f*Pol
yLog[2, I*E^(c + d*x)])/(b^2*d^2) + (I*a^3*f*PolyLog[2, I*E^(c + d*x)])/(b^
2*(a^2 + b^2)*d^2) + (a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2]))])/(b*(a^2 + b^2)*d^2) + (a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[
a^2 + b^2]))])/(b*(a^2 + b^2)*d^2) + (f*PolyLog[2, -E^(2*(c + d*x))])/(2*b*
d^2) - (a^2*f*PolyLog[2, -E^(2*(c + d*x))])/(2*b*(a^2 + b^2)*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5686

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*Sech[c +
d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5700

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - Di
st[a/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*(Tanh[c + d*x]^n/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\tanh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)^2}{2bf} - \frac{a\int (e+fx)\text{sech}(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)\text{sech}(c+dx}{a+b\sinh(c+dx)}}{b^2}$$

$$= -\frac{(e+fx)^2}{2bf} - \frac{2a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{(e+fx)\log\left(1+e^{2(c}\right.}{bd}$$

$$= -\frac{(e+fx)^2}{2bf} - \frac{a^2(e+fx)^2}{2b\,(a^2+b^2)\,f} - \frac{2a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{(e+}{}$$

$$= -\frac{(e+fx)^2}{2bf} - \frac{a^2(e+fx)^2}{2b\,(a^2+b^2)\,f} - \frac{2a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{a^2(e}{}$$

$$= -\frac{(e+fx)^2}{2bf} - \frac{2a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)\tan^{-1}\left(e^{c}\right.}{b^2\,(a^2+b^2)\,d}$$

$$= -\frac{(e+fx)^2}{2bf} - \frac{2a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)\tan^{-1}\left(e^{c}\right.}{b^2\,(a^2+b^2)\,d}$$

$$= -\frac{(e+fx)^2}{2bf} - \frac{2a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)\tan^{-1}\left(e^{c}\right.}{b^2\,(a^2+b^2)\,d}$$

$$= -\frac{(e+fx)^2}{2bf} - \frac{2a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)\tan^{-1}\left(e^{c}\right.}{b^2\,(a^2+b^2)\,d}$$

Mathematica [A]
time = 1.94, size = 441, normalized size = 0.85

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-(b*d*e*(c + d*x)) + b*c*f*(c + d*x) + (b*f*(c + d*x)^2)/2 - 2*a*d*e*ArcTa
n[Cosh[c + d*x] + Sinh[c + d*x]] + 2*a*c*f*ArcTan[Cosh[c + d*x] + Sinh[c +
d*x]] - 2*a*f*(c + d*x)*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]] + b*f*(c + d*
x)*Log[2*Cosh[c + d*x]*(Cosh[c + d*x] - Sinh[c + d*x])] + b*d*e*Log[1 + Cos
```

```
h[2*(c + d*x)] + Sinh[2*(c + d*x)]] - b*c*f*Log[1 + Cosh[2*(c + d*x)] + Sin
h[2*(c + d*x)]] + (a^2*(-1/2*(f*(c + d*x)^2) + f*(c + d*x)*Log[1 + (b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2])] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*
x]] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/b + I*a*f*PolyLog[2, (-I)*(Cosh[
c + d*x] + Sinh[c + d*x])] - I*a*f*PolyLog[2, I*(Cosh[c + d*x] + Sinh[c + d
*x])] - (b*f*PolyLog[2, -Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]])/2)/((a^2 +
 b^2)*d^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3881 vs. $2(484) = 968$.
time = 5.74, size = 3882, normalized size = 7.52

method	result	size
risch	Expression too large to display	3882

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/2/d^2*b*f*c/(a^2+b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d^2*b*f/(2
*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/
d*b*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))
)*x-1/d^2*b*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2
)^(1/2)))*c+1/2/d*b*f/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a
^2+b^2)^(1/2)))*x+1/2/d^2*b*f/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a
)/(-a+(a^2+b^2)^(1/2)))*c+1/2/d*b*f/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1
/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/2/d^2*b*f/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b
^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d/b*f/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2
+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^2*x+1/d^2/b*f/(a^2+b^2)*ln((b*exp(d*x
+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^2*c-1/d/b*f/(a^2+b^2)^(3/2)*l
n((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^3*x-1/d^2/b*f/(a^
2+b^2)^(3/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^3*c
-2/d^2/b*a^3*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)
^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2/d/b*a^3*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*l
n((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-2/d^2/b*a*f*c/(
2*a^2+2*b^2)*(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/
2))-2/d^2/b*a^3*f*c/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+
c)+2*a)/(a^2+b^2)^(1/2))+2/d/b*a^3*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((b*ex
p(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+2/d^2/b*a^3*f/(2*a^2+2*b
^2)/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))
)*c-2/d^2/b*a^3*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)
^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/d^2/(a^2+b^2)^(1/2)*a*b*f*c/(2*a^2+2*b^
2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d/(a^2+b^2)^(1/2)*e*
```

```
a*b/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/2*f*x
^2/b+2/d/b*e*a^3/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+
2*a)/(a^2+b^2)^(1/2))+2/d/b*e*a/(2*a^2+2*b^2)*(a^2+b^2)^(1/2)*arctanh(1/2*(
2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d/b*f/(a^2+b^2)*ln((-b*exp(d*x+c)+(a
^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a^2*x+1/d^2/b*f/(a^2+b^2)*ln((-b*exp
(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a^2*c-2/d^2*b*a*f/(2*a^2+2
*b^2)/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)
^(1/2)))-2/d^2*b*f*c/(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))-1/d^2*b*f/(a^2+b^2)
^(3/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a+2/d^2*
b*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c+2/d*b*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x
+c))*x+2/d^2*b*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c-1/d*b*f/(2*a^2+2*b^2)*l
n((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d/b*f/(a^2+b^
2)^(3/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a^3*x+1
/d^2/b*f/(a^2+b^2)^(3/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)
^(1/2)))*a^3*c+2*I/d*a*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x+2*I/d^2*a*f/(2*
a^2+2*b^2)*ln(1+I*exp(d*x+c))*c-2*I/d*a*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*
x-1/d^2/b*f*c^2-2/d/b*e*ln(exp(d*x+c))-2*I/d^2*a*f/(2*a^2+2*b^2)*ln(1-I*exp
(d*x+c))*c+2/d*b*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x+1/d^2*b*f*c/(2*a^2+2*
b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d/b*e/(a^2+b^2)^(3/2)*arctanh(
1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^3+1/d^2*b*f/(a^2+b^2)^(3/2)*dil
og((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a+1/d/b*e/(a^2+b
^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)*a^2-1/d^2*b*f/(a^2+b^2)^(3/2)*ln(
(b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a*c+2/d^2*b*a*f/(2*a^
2+2*b^2)/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2
)^(1/2)))+1/d^2/b*f/(a^2+b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(
a^2+b^2)^(1/2)))*a^2+1/d^2/b*f/(a^2+b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2
)+a)/(a+(a^2+b^2)^(1/2)))*a^2+1/d^2/b*f/(a^2+b^2)^(3/2)*dilog((-b*exp(d*x+c
)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a^3-1/d^2/b*f/(a^2+b^2)^(3/2)*di
log((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^3-2/d*b*e/(a^2+
b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a-2/d*b*a*f/(2
*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b
^2)^(1/2)))*x-2/d^2*b*a*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(
a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+2/d*b*a*f/(2*a^2+2*b^2)/(a^2+b^2)
^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+2/d^2*b*a
*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^
2+b^2)^(1/2)))*c+4/d^2*f*c/(2*a^2+2*b^2)*a*arctan(exp(d*x+c))+2*I/d^2*a*f/(
2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))-2*I/d^2*a*f/(2*a^2+2*b^2)*dilog(1-I*exp(
d*x+c))-1/d*b*f/(a^2+b^2)^(3/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2
+b^2)^(1/2)))*a*x+e*x/b+2/d^2/b*a^3*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*dilog((
b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2/b*f*c/(a^2+b^2)*
ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)*a^2+2/d^2*b*f*c/(a^2+b^2)^(3/2)*arcta
nh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a+1/d*b*f/(a^2+b^2)^(3/2)*ln((
-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] 1/2*f*(x^2/b - integrate(-4*(a^3*x*e^(d*x + c) - a^2*b*x)/(a^2*b^2 + b^4 -
(a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x) - 2*(a^3*b*e^c + a*b^3*e^c)*e^(d*
x)), x) - integrate(4*(a*x*e^(d*x + c) + b*x)/(a^2 + b^2 + (a^2*e^(2*c) + b
^2*e^(2*c))*e^(2*d*x)), x)) + (a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*
c) - b)/((a^2*b + b^3)*d) + 2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*lo
g(e^(-2*d*x - 2*c) + 1)/((a^2 + b^2)*d) + (d*x + c)/(b*d))*e
```

Fricas [**A**]
time = 0.40, size = 750, normalized size = 1.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] -1/2*((a^2 + b^2)*d^2*f*x^2 + 2*(a^2 + b^2)*d^2*x*cosh(1) + 2*(a^2 + b^2)*d
^2*x*sinh(1) - 2*a^2*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*a^2*f*dilo
g((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(I*a*b*f - b^2*f)*dilog(I*cosh(d*x +
c) + I*sinh(d*x + c)) + 2*(-I*a*b*f - b^2*f)*dilog(-I*cosh(d*x + c) - I*sin
h(d*x + c)) + 2*(a^2*c*f - a^2*d*cosh(1) - a^2*d*sinh(1))*log(2*b*cosh(d*x
+ c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(a^2*c*f -
a^2*d*cosh(1) - a^2*d*sinh(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) -
2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(a^2*d*f*x + a^2*c*f)*log(-(a*cosh(d*x
 + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2) - b)/b) - 2*(a^2*d*f*x + a^2*c*f)*log(-(a*cosh(d*x + c) + a*sinh(d
*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b)
 + 2*(-I*a*b*c*f + b^2*c*f + I*a*b*d*cosh(1) - b^2*d*cosh(1) + I*a*b*d*sinh
(1) - b^2*d*sinh(1))*log(cosh(d*x + c) + sinh(d*x + c) + I) + 2*(I*a*b*c*f
+ b^2*c*f - I*a*b*d*cosh(1) - b^2*d*cosh(1) - I*a*b*d*sinh(1) - b^2*d*sinh(
1))*log(cosh(d*x + c) + sinh(d*x + c) - I) + 2*(-I*a*b*d*f*x - b^2*d*f*x -
I*a*b*c*f - b^2*c*f)*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) + 2*(I*a*b*
d*f*x - b^2*d*f*x + I*a*b*c*f - b^2*c*f)*log(-I*cosh(d*x + c) - I*sinh(d*x
+ c) + 1))/((a^2*b + b^3)*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e + fx)\sinh(c + dx)\tanh(c + dx)}{a + b\sinh(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*sinh(c + d*x)*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c + d\,x)\,\tanh(c + d\,x)\,(e + f\,x)}{a + b\sinh(c + d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*tanh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((sinh(c + d*x)*tanh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.380 $\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=74

$$-\frac{a\text{ArcTan}(\sinh(c+dx))}{(a^2+b^2)\,d}+\frac{b\log(\cosh(c+dx))}{(a^2+b^2)\,d}+\frac{a^2\log(a+b\sinh(c+dx))}{b\,(a^2+b^2)\,d}$$

```
[Out] -a*arctan(sinh(d*x+c))/(a^2+b^2)/d+b*ln(cosh(d*x+c))/(a^2+b^2)/d+a^2*ln(a+b
*sinh(d*x+c))/b/(a^2+b^2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2916, 12, 1643, 649, 209, 266}

$$-\frac{a\text{ArcTan}(\sinh(c+dx))}{d\,(a^2+b^2)}+\frac{a^2\log(a+b\sinh(c+dx))}{bd\,(a^2+b^2)}+\frac{b\log(\cosh(c+dx))}{d\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d)) + (b*Log[Cosh[c + d*x]])/((a^2
 + b^2)*d) + (a^2*Log[a + b*Sinh[c + d*x]])/(b*(a^2 + b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{b\text{Subst}\left(\int \frac{x^2}{b^2(a+x)(-b^2-x^2)}\,dx, x, b\sinh(c+dx)\right)}{d} \\
&= -\frac{\text{Subst}\left(\int \frac{x^2}{(a+x)(-b^2-x^2)}\,dx, x, b\sinh(c+dx)\right)}{bd} \\
&= -\frac{\text{Subst}\left(\int \left(-\frac{a^2}{(a^2+b^2)(a+x)}+\frac{b^2(a-x)}{(a^2+b^2)(b^2+x^2)}\right)\,dx, x, b\sinh(c+dx)\right)}{bd} \\
&= \frac{a^2\log(a+b\sinh(c+dx))}{b(a^2+b^2)d}-\frac{b\text{Subst}\left(\int \frac{a-x}{b^2+x^2}\,dx, x, b\sinh(c+dx)\right)}{(a^2+b^2)d} \\
&= \frac{a^2\log(a+b\sinh(c+dx))}{b(a^2+b^2)d}+\frac{b\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sinh(c+dx)\right)}{(a^2+b^2)d}-\frac{(ab)
\end{aligned}$$

$$= -\frac{a\tan^{-1}(\sinh(c+dx))}{(a^2+b^2)d}+\frac{b\log(\cosh(c+dx))}{(a^2+b^2)d}+\frac{a^2\log(a+b\sinh(c+dx}{b(a^2+b^2)d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.08, size = 78, normalized size = 1.05

$$\frac{b(ia+b)\log(i-\sinh(c+dx))+b(-ia+b)\log(i+\sinh(c+dx))+2a^2\log(a+b\sinh(c+dx))}{2b(a^2+b^2)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*(I*a + b)*Log[I - Sinh[c + d*x]] + b*((-I)*a + b)*Log[I + Sinh[c + d*x]]
 + 2*a^2*Log[a + b*Sinh[c + d*x]])/(2*b*(a^2 + b^2)*d)
```

Maple [A]
time = 1.54, size = 132, normalized size = 1.78

method	result
derivativedivides	$\frac{\frac{4b\ln\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-8a\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2+4b^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}+\frac{a^2\ln\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{b\left(a^2+b^2\right)}-\frac{\ln(}{}}{d}$
default	$\frac{\frac{4b\ln\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-8a\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2+4b^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}+\frac{a^2\ln\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{b\left(a^2+b^2\right)}-\frac{\ln(}{}}{d}$
risch	$\frac{x}{b}-\frac{2bd^2x}{a^2d^2+b^2d^2}-\frac{2bdc}{a^2d^2+b^2d^2}-\frac{2a^2x}{b(a^2+b^2)}-\frac{2a^2c}{bd(a^2+b^2)}+\frac{i\ln(e^{dx+c}-i)a}{(a^2+b^2)d}+\frac{\ln(e^{dx+c}-i)b}{(a^2+b^2)d}-\frac{i\ln(e^{dx+c}+i)}{(a^2+b^2)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(8/(4*a^2+4*b^2)*(1/2*b*ln(tanh(1/2*d*x+1/2*c)^2+1)-a*arctan(tanh(1/2*d
*x+1/2*c)))-1/b*ln(tanh(1/2*d*x+1/2*c)-1)+a^2/b/(a^2+b^2)*ln(a*tanh(1/2*d*x
+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)-a)-1/b*ln(tanh(1/2*d*x+1/2*c)+1))
```

Maxima [A]
time = 0.48, size = 110, normalized size = 1.49

$$\frac{a^2\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^2b+b^3)d}+\frac{2\,a\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d}+\frac{b\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d}+\frac{dx+c}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b + b^3)*d) + 2*a
*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^2 +
 b^2)*d) + (d*x + c)/(b*d)
```

Fricas [A]
time = 0.38, size = 111, normalized size = 1.50

$$-\frac{(a^2+b^2)dx+2\,ab\arctan\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)-a^2\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)-b^2\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{(a^2b+b^3)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((a^2 + b^2)*d*x + 2*a*b*arctan(cosh(d*x + c) + sinh(d*x + c)) - a^2*log(2
*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) - b^2*log(2*cosh(d*
x + c)/(cosh(d*x + c) - sinh(d*x + c))))/((a^2*b + b^3)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sinh(c + d*x)*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A]
time = 0.43, size = 121, normalized size = 1.64

$$\frac{\frac{2\,a^2\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^2b+b^3}-\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)a}{a^2+b^2}+\frac{b\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^2+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^2*b + b^3) - (
pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*a/(a^2 + b^2) + b*lo
g((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2 + b^2))/d
```

Mupad [B]
time = 1.26, size = 174, normalized size = 2.35

$$\frac{\ln\left(\mathrm{e}^{c+d\,x}+1{}\mathrm{i}\right)}{b\,d+a\,d\,1{}\mathrm{i}}-\frac{x}{b}+\frac{a^2\,\ln\left(a^2\,b^3-b^5-a^4\,b+2\,a^5\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+b^5\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+a^4\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-2\,a^3\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-a^2\,b^3\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+2\,a\,b^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right)}{d\,a^2\,b+d\,b^3}+\frac{\ln\left(1+\mathrm{e}^{c+d\,x}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{a\,d+b\,d\,1{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*tanh(c + d*x))/(a + b*sinh(c + d*x)),x)
```

```
[Out] log(exp(c + d*x) + 1i)/(a*d*1i + b*d) - x/b + (log(exp(c + d*x)*1i + 1)*1i)
/(a*d + b*d*1i) + (a^2*log(a^2*b^3 - b^5 - a^4*b + 2*a^5*exp(d*x)*exp(c) +
b^5*exp(2*c)*exp(2*d*x) + a^4*b*exp(2*c)*exp(2*d*x) - 2*a^3*b^2*exp(d*x)*ex
p(c) - a^2*b^3*exp(2*c)*exp(2*d*x) + 2*a*b^4*exp(d*x)*exp(c)))/(b^3*d + a^2
*b*d)
```

3.381 $\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=35

$$\text{Int}\left(\frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sinh[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Sinh[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

Rubi steps

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 145.45, size = 0, normalized size = 0.00

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sinh[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x
]
```

```
[Out] Integrate[(Sinh[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(dx+c)\tanh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] log(f*x + e)/(b*f) - 1/2*integrate(-4*(a^3*e^(d*x + c) - a^2*b)/((a^2*b^2*f
 + b^4*f)*x + (a^2*b^2 + b^4)*e - ((a^2*b^2*f*e^(2*c) + b^4*f*e^(2*c))*x +
(a^2*b^2*e^(2*c) + b^4*e^(2*c))*e)*e^(2*d*x) - 2*((a^3*b*f*e^c + a*b^3*f*e^
c)*x + (a^3*b*e^c + a*b^3*e^c)*e)*e^(d*x)), x) - 1/2*integrate(4*(a*e^(d*x
+ c) + b)/((a^2*f + b^2*f)*x + (a^2 + b^2)*e + ((a^2*f*e^(2*c) + b^2*f*e^(2
*c))*x + (a^2*e^(2*c) + b^2*e^(2*c))*e)*e^(2*d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] integral(sinh(d*x + c)*tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
+ c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sinh(c + d*x)*tanh(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*tanh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((sinh(c + d*x)*tanh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.382 $\int \frac{(e+fx)^3 \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1118

$$-\frac{a(e+fx)^3}{b^2d}+\frac{a^3(e+fx)^3}{b^2\left(a^2+b^2\right)d}+\frac{6f(e+fx)^2\mathrm{ArcTan}\left(e^{c+dx}\right)}{bd^2}-\frac{6a^2f(e+fx)^2\mathrm{ArcTan}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d^2}+\frac{a^2(e+fx)^3\log\Big(}{(a^2+}$$

[Out] -3*a^3*f*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/b^2/(a^2+b^2)/d^2-6*a^2*f*(f*x+e)^2*arctan(exp(d*x+c))/b/(a^2+b^2)/d^2+6*I*f^2*(f*x+e)*polylog(2,I*exp(d*x+c))/b/d^3-3*a^3*f^2*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/b^2/(a^2+b^2)/d^3-6*I*a^2*f^3*polylog(3,-I*exp(d*x+c))/b/(a^2+b^2)/d^4+6*I*a^2*f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/b/(a^2+b^2)/d^3+a^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d-a^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d+6*a^2*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^4-6*a^2*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^4-a*(f*x+e)^3/b^2/d+3*a*f*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/b^2/d^2+3*a^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-3*a^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2+a^3*(f*x+e)^3/b^2/(a^2+b^2)/d+6*f*(f*x+e)^2*arctan(exp(d*x+c))/b/d^2-3/2*a*f^3*polylog(3,-exp(2*d*x+2*c))/b^2/d^4-a*(f*x+e)^3*tanh(d*x+c)/b^2/d-6*I*f^3*polylog(3,I*exp(d*x+c))/b/d^4-6*a^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^3+6*a^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^3-(f*x+e)^3*sech(d*x+c)/b/d+3*a*f^2*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/b^2/d^3+6*I*f^3*polylog(3,-I*exp(d*x+c))/b/d^4+3/2*a^3*f^3*polylog(3,-exp(2*d*x+2*c))/b^2/(a^2+b^2)/d^4+a^2*(f*x+e)^3*sech(d*x+c)/b/(a^2+b^2)/d+a^3*(f*x+e)^3*tanh(d*x+c)/b^2/(a^2+b^2)/d-6*I*f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/b/d^3+6*I*a^2*f^3*polylog(3,I*exp(d*x+c))/b/(a^2+b^2)/d^4-6*I*a^2*f^2*(f*x+e)*polylog(2,I*exp(d*x+c))/b/(a^2+b^2)/d^3

Rubi [A]
time = 1.62, antiderivative size = 1118, normalized size of antiderivative = 1.00, number of steps used = 45, number of rules used = 15, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {5686, 5559, 4265, 2611, 2320, 6724, 5702, 4269, 3799, 2221, 5692, 3403, 2296, 6744, 6874}

Antiderivative was successfully verified.

[In] Int[((e + f*x)^3*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]

[Out] -((a*(e + f*x)^3)/(b^2*d)) + (a^3*(e + f*x)^3)/(b^2*(a^2 + b^2)*d) + (6*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b*d^2) - (6*a^2*f*(e + f*x)^2*ArcTan[E^(c

```
+ d*x)])/(b*(a^2 + b^2)*d^2) + (a^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (a^2*(e + f*x)^3*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) + (3*a*f*(e + f*x)
^2*Log[1 + E^(2*(c + d*x))])/(b^2*d^2) - (3*a^3*f*(e + f*x)^2*Log[1 + E^(2*
(c + d*x))])/(b^2*(a^2 + b^2)*d^2) - ((6*I)*f^2*(e + f*x)*PolyLog[2, (-I)*E
^(c + d*x)])/(b*d^3) + ((6*I)*a^2*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)
])/(b*(a^2 + b^2)*d^3) + ((6*I)*f^2*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b
*d^3) - ((6*I)*a^2*f^2*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)*
d^3) + (3*a^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2]))])/((a^2 + b^2)^(3/2)*d^2) - (3*a^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) + (3*a*f^2*(e + f*
x)*PolyLog[2, -E^(2*(c + d*x))])/(b^2*d^3) - (3*a^3*f^2*(e + f*x)*PolyLog[2
, -E^(2*(c + d*x))])/(b^2*(a^2 + b^2)*d^3) + ((6*I)*f^3*PolyLog[3, (-I)*E^(
c + d*x)])/(b*d^4) - ((6*I)*a^2*f^3*PolyLog[3, (-I)*E^(c + d*x)])/(b*(a^2 +
 b^2)*d^4) - ((6*I)*f^3*PolyLog[3, I*E^(c + d*x)])/(b*d^4) + ((6*I)*a^2*f^3
*PolyLog[3, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^4) - (6*a^2*f^2*(e + f*x)*Poly
Log[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) +
 (6*a^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])
/((a^2 + b^2)^(3/2)*d^3) - (3*a*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*d^
4) + (3*a^3*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*(a^2 + b^2)*d^4) + (6*
a^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^
(3/2)*d^4) - (6*a^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/((a^2 + b^2)^(3/2)*d^4) - ((e + f*x)^3*Sech[c + d*x])/(b*d) + (a^2*(e +
f*x)^3*Sech[c + d*x])/(b*(a^2 + b^2)*d) - (a*(e + f*x)^3*Tanh[c + d*x])/(b^
2*d) + (a^3*(e + f*x)^3*Tanh[c + d*x])/(b^2*(a^2 + b^2)*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5686

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*Sech[c +
d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5702

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\text{sech}(c+dx)\tanh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)^3\text{sech}(c+dx)}{bd} - \frac{a\int (e+fx)^3\text{sech}^2(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^3\text{sech}}{a+b\sinh(c}}{b^2}$$

$$= \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{(e+fx)^3\text{sech}(c+dx)}{bd} - \frac{a(e+fx)^3\tanh(c+}{b^2d}$$

$$= -\frac{a(e+fx)^3}{b^2d} + \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{6if^2(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{bd^3} +$$

$$= -\frac{a(e+fx)^3}{b^2d} + \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd^2} + \frac{3af(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{b^2d^2}$$

$$= -\frac{a(e+fx)^3}{b^2d} + \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd^2} + \frac{a^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+}}\right.}{(a^2+b^2)^{3/2}\,d}$$

$$= -\frac{a(e+fx)^3}{b^2d} + \frac{a^3(e+fx)^3}{b^2(a^2+b^2)\,d} + \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{6a^2f(e+fx)}{b(a^2}$$

$$= -\frac{a(e+fx)^3}{b^2d} + \frac{a^3(e+fx)^3}{b^2(a^2+b^2)\,d} + \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{6a^2f(e+fx)}{b(a^2}$$

$$= -\frac{a(e+fx)^3}{b^2d} + \frac{a^3(e+fx)^3}{b^2(a^2+b^2)\,d} + \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{6a^2f(e+fx)}{b(a^2}$$

$$= -\frac{a(e+fx)^3}{b^2d} + \frac{a^3(e+fx)^3}{b^2(a^2+b^2)\,d} + \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{6a^2f(e+fx)}{b(a^2}$$

$$= -\frac{a(e+fx)^3}{b^2d} + \frac{a^3(e+fx)^3}{b^2(a^2+b^2)\,d} + \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{6a^2f(e+fx)}{b(a^2}$$

Mathematica [A]
time = 13.55, size = 1614, normalized size = 1.44

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (f*(-12*a*d^3*e^2*E^(2*c)*x + 12*a*d^3*e^2*(1 + E^(2*c))*x + 12*a*d^3*e*f*x
^2 + 4*a*d^3*f^2*x^3 + 12*b*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] - 6*a
*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*b*d*e*(1
 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - Poly
Log[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*d*e*(1 + E^(2*c
))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))]
) + (6*I)*b*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Log
[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2,
 I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d*
x)]) - a*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))])
- 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))])))/(2
*(a^2 + b^2)*d^4*(1 + E^(2*c))) - (a^2*(2*d^3*e^3*Sqrt[(a^2 + b^2)*E^(2*c)]
*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]] + 3*Sqrt[-a^2 - b^2]*d^3*e^2*
E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 3*
Sqrt[-a^2 - b^2]*d^3*e*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[
(a^2 + b^2)*E^(2*c)])] + Sqrt[-a^2 - b^2]*d^3*E^c*f^3*x^3*Log[1 + (b*E^(2*c
 + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*Sqrt[-a^2 - b^2]*d^3*e^2*
E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*
Sqrt[-a^2 - b^2]*d^3*e*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[
(a^2 + b^2)*E^(2*c)])] - Sqrt[-a^2 - b^2]*d^3*E^c*f^3*x^3*Log[1 + (b*E^(2*c
 + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 3*Sqrt[-a^2 - b^2]*d^2*E^c*
f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)]))] - 3*Sqrt[-a^2 - b^2]*d^2*E^c*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c +
 d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[-a^2 - b^2]*d*e*E^c*f
^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6
*Sqrt[-a^2 - b^2]*d*E^c*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[
(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[-a^2 - b^2]*d*e*E^c*f^2*PolyLog[3, -((b*E^
(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[-a^2 - b^2]*d*E
^c*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])
)] + 6*Sqrt[-a^2 - b^2]*E^c*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c - Sqr
t[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[-a^2 - b^2]*E^c*f^3*PolyLog[4, -((b*E^(2
*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/((-a^2 - b^2)^(3/2)*d^4*
Sqrt[(a^2 + b^2)*E^(2*c)]) + (Sech[c]*Sech[c + d*x]*(-(b*e^3*Cosh[c]) - 3*b
*e^2*f*x*Cosh[c] - 3*b*e*f^2*x^2*Cosh[c] - b*f^3*x^3*Cosh[c] - a*e^3*Sinh[d
*x] - 3*a*e^2*f*x*Sinh[d*x] - 3*a*e*f^2*x^2*Sinh[d*x] - a*f^3*x^3*Sinh[d*x]
))/((a^2 + b^2)*d)
```

Maple [F]

time = 2.06, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \left(\tanh^2 (dx+c)\right)}{a+b\sinh (dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 6*b*f^3*integrate(x^2*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x +
 2*c) + a^2*d + b^2*d), x) - 6*a*f^3*integrate(x^2/(a^2*d*e^(2*d*x + 2*c) +
 b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 12*a*f^2*e*integrate(x/(a^2*d
*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 3*a*f*(2*(d
*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 + b^2)*d^2))*e^2
 + 12*b*f^2*integrate(x*e^(d*x + c + 1)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*e^(2
*d*x + 2*c) + a^2*d + b^2*d), x) + (a^2*log((b*e^(-d*x - c) - a - sqrt(a^2
+ b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2)*d) - 2*(
b*e^(-d*x - c) + a)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*d))*e^3 + 6
*b*f*arctan(e^(d*x + c))*e^2/((a^2 + b^2)*d^2) + 2*(a*f^3*x^3 + 3*a*f^2*x^2
*e + 3*a*f*x*e^2 - (b*f^3*x^3*e^c + 3*b*f^2*x^2*e^(c + 1) + 3*b*f*x*e^(c +
2))*e^(d*x))/(a^2*d + b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) +
integrate(-2*(a^2*f^3*x^3*e^c + 3*a^2*f^2*x^2*e^(c + 1) + 3*a^2*f*x*e^(c +
2))*e^(d*x)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e^(2*c))*e^(2*d*x) - 2*(a^3
*e^c + a*b^2*e^c)*e^(d*x)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 10804 vs. 2(1046) = 2092.
time = 0.56, size = 10804, normalized size = 9.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*(a^3 + a*b^2)*c^3*f^3 - 6*(a^3 + a*b^2)*c^2*d*f^2*cosh(1) + 6*(a^3 + a*
b^2)*c*d^2*f*cosh(1)^2 - 2*(a^3 + a*b^2)*d^3*cosh(1)^3 - 2*(a^3 + a*b^2)*d^
3*sinh(1)^3 + 2*((a^3 + a*b^2)*d^3*f^3*x^3 + (a^3 + a*b^2)*c^3*f^3 + 3*((a^
3 + a*b^2)*d^3*f*x + (a^3 + a*b^2)*c*d^2*f)*cosh(1)^2 + 3*((a^3 + a*b^2)*d^
3*f*x + (a^3 + a*b^2)*c*d^2*f)*sinh(1)^2 + 3*((a^3 + a*b^2)*d^3*f^2*x^2 - (
a^3 + a*b^2)*c^2*d*f^2)*cosh(1) + 3*((a^3 + a*b^2)*d^3*f^2*x^2 - (a^3 + a*b
^2)*c^2*d*f^2 + 2*((a^3 + a*b^2)*d^3*f*x + (a^3 + a*b^2)*c*d^2*f)*cosh(1))*
sinh(1))*cosh(d*x + c)^2 + 6*((a^3 + a*b^2)*c*d^2*f - (a^3 + a*b^2)*d^3*cos
h(1))*sinh(1)^2 + 2*((a^3 + a*b^2)*d^3*f^3*x^3 + (a^3 + a*b^2)*c^3*f^3 + 3*
```

```
((a^3 + a*b^2)*d^3*f*x + (a^3 + a*b^2)*c*d^2*f)*cosh(1)^2 + 3*((a^3 + a*b^2
)*d^3*f*x + (a^3 + a*b^2)*c*d^2*f)*sinh(1)^2 + 3*((a^3 + a*b^2)*d^3*f^2*x^2
 - (a^3 + a*b^2)*c^2*d*f^2)*cosh(1) + 3*((a^3 + a*b^2)*d^3*f^2*x^2 - (a^3 +
 a*b^2)*c^2*d*f^2 + 2*((a^3 + a*b^2)*d^3*f*x + (a^3 + a*b^2)*c*d^2*f)*cosh(
1))*sinh(1))*sinh(d*x + c)^2 - 3*(a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*f^2*x*cos
h(1) + a^2*b*d^2*f*cosh(1)^2 + a^2*b*d^2*f*sinh(1)^2 + (a^2*b*d^2*f^3*x^2 +
 2*a^2*b*d^2*f^2*x*cosh(1) + a^2*b*d^2*f*cosh(1)^2 + a^2*b*d^2*f*sinh(1)^2
+ 2*(a^2*b*d^2*f^2*x + a^2*b*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(a
^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*f^2*x*cosh(1) + a^2*b*d^2*f*cosh(1)^2 + a^2*
b*d^2*f*sinh(1)^2 + 2*(a^2*b*d^2*f^2*x + a^2*b*d^2*f*cosh(1))*sinh(1))*cosh
(d*x + c)*sinh(d*x + c) + (a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*f^2*x*cosh(1) +
a^2*b*d^2*f*cosh(1)^2 + a^2*b*d^2*f*sinh(1)^2 + 2*(a^2*b*d^2*f^2*x + a^2*b*
d^2*f*cosh(1))*sinh(1))*sinh(d*x + c)^2 + 2*(a^2*b*d^2*f^2*x + a^2*b*d^2*f*
cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x
 + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b +
1) + 3*(a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*f^2*x*cosh(1) + a^2*b*d^2*f*cosh(1)
^2 + a^2*b*d^2*f*sinh(1)^2 + (a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*f^2*x*cosh(1)
 + a^2*b*d^2*f*cosh(1)^2 + a^2*b*d^2*f*sinh(1)^2 + 2*(a^2*b*d^2*f^2*x + a^2
*b*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(a^2*b*d^2*f^3*x^2 + 2*a^2*b
*d^2*f^2*x*cosh(1) + a^2*b*d^2*f*cosh(1)^2 + a^2*b*d^2*f*sinh(1)^2 + 2*(a^2
*b*d^2*f^2*x + a^2*b*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) +
(a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*f^2*x*cosh(1) + a^2*b*d^2*f*cosh(1)^2 + a^
2*b*d^2*f*sinh(1)^2 + 2*(a^2*b*d^2*f^2*x + a^2*b*d^2*f*cosh(1))*sinh(1))*si
nh(d*x + c)^2 + 2*(a^2*b*d^2*f^2*x + a^2*b*d^2*f*cosh(1))*sinh(1))*sqrt((a^
2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (a^2*b*c^3*f^3 - 3*a^
2*b*c^2*d*f^2*cosh(1) + 3*a^2*b*c*d^2*f*cosh(1)^2 - a^2*b*d^3*cosh(1)^3 - a
^2*b*d^3*sinh(1)^3 + (a^2*b*c^3*f^3 - 3*a^2*b*c^2*d*f^2*cosh(1) + 3*a^2*b*c
*d^2*f*cosh(1)^2 - a^2*b*d^3*cosh(1)^3 - a^2*b*d^3*sinh(1)^3 + 3*(a^2*b*c*d
^2*f - a^2*b*d^3*cosh(1))*sinh(1)^2 - 3*(a^2*b*c^2*d*f^2 - 2*a^2*b*c*d^2*f*
cosh(1) + a^2*b*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c)^2 + 3*(a^2*b*c*d^2*f
- a^2*b*d^3*cosh(1))*sinh(1)^2 + 2*(a^2*b*c^3*f^3 - 3*a^2*b*c^2*d*f^2*cosh(
1) + 3*a^2*b*c*d^2*f*cosh(1)^2 - a^2*b*d^3*cosh(1)^3 - a^2*b*d^3*sinh(1)^3
+ 3*(a^2*b*c*d^2*f - a^2*b*d^3*cosh(1))*sinh(1)^2 - 3*(a^2*b*c^2*d*f^2 - 2*
a^2*b*c*d^2*f*cosh(1) + a^2*b*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c)*sinh(d*
x + c) + (a^2*b*c^3*f^3 - 3*a^2*b*c^2*d*f^2*cosh(1) + 3*a^2*b*c*d^2*f*cosh(
1)^2 - a^2*b*d^3*cosh(1)^3 - a^2*b*d^3*sinh(1)^3 + 3*(a^2*b*c*d^2*f - a^2*b
*d^3*cosh(1))*sinh(1)^2 - 3*(a^2*b*c^2*d*f^2 - 2*a^2*b*c*d^2*f*cosh(1) + a^
2*b*d^3*cosh(1)^2)*sinh(1))*sinh(d*x + c)^2 - 3*(a^2*b*c^2*d*f^2 - 2*a^2*b*
c*d^2*f*cosh(1) + a^2*b*d^3*cosh(1)^2)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2
*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (
a^2*b*c^3*f^3 - 3*a^2*b*c^2*d*f^2*cosh(1) + 3*a^2*b*c*d^2*f*cosh(1)^2 - a^2
*b*d^3*cosh(1)^3 - a^2*b*d^3*sinh(1)^3 + (a^2*b*c^3*f^3 - 3*a^2*b*c^2*d*f^2
*cosh(1) + 3*a^2*b*c*d^2*f*cosh(1)^2 - a^2*b*d^3*cosh(1)^3 - a^2*b*d^3*sinh
(1)^3 + 3*(a^2*b*c*d^2*f - a^2*b*d^3*cosh(1))*sinh(1)^2 - 3*(a^2*b*c^2*d*f^
```

```
2 - 2*a^2*b*c*d^2*f*cosh(1) + a^2*b*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c)^2
 + 3*(a^2*b*c*d^2*f - a^2*b*d^3*cosh(1))*sinh(1)^2 + 2*(a^2*b*c^3*f^3 - 3*a
^2*b*c^2*d*f^2*cosh(1) + 3*a^2*b*c*d^2*f*cosh(1)^2 - a^2*b*d^3*cosh(1)^3 -
a^2*b*d^3*sinh(1)^3 + 3*(a^2*b*c*d^2*f - a^2*b*d^3*cosh(1))*sinh(1)^2 - 3*(
a^2*b*c^2*d*f^2 - 2*a^2*b*c*d^2*f*cosh(1) + a^2*b*d^3*cosh(1)^2)*sinh(1))*c
osh(d*x + c)*sinh(d*x + c) + (a^2*b*c^3*f^3 - 3*a^2*b*c^2*d*f^2*cosh(1) + 3
*a^2*b*c*d^2*f*cosh(1)^2 - a^2*b*d^3*cosh(1)^3 - a^2*b*d^3*sinh(1)^3 + 3*(a
^2*b*c*d^2*f - a^2*b*d^3*cosh(1))*sinh(1)^2 - 3*(a^2*b*c^2*d*f^2 - 2*a^2*b*
c*d^2*f*cosh(1) + a^2*b*d^3*cosh(1)^2)*sinh(1))*sinh(d*x + c)^2 - 3*(a^2*b*
c^2*d*f^2 - 2*a^2*b*c*d^2*f*cosh(1) + a^2*b*d^3*cosh(1)^2)*sinh(1))*sqrt((a
^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 +
b^2)/b^2) + 2*a) - (a^2*b*d^3*f^3*x^3 + a^2*b*c^3*f^3 + 3*(a^2*b*d^3*f*x +
a^2*b*c*d^2*f)*cosh(1)^2 + (a^2*b*d^3*f^3*x^3 +...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((tanh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.383 $\int \frac{(e+fx)^2 \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=772

$$-\frac{a(e+fx)^2}{b^2d}+\frac{a^3(e+fx)^2}{b^2\left(a^2+b^2\right)d}+\frac{4f(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{bd^2}-\frac{4a^2f(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d^2}+\frac{a^2(e+fx)^2\log\Big(}{(a^2+}$$

```
[Out] -a*(f*x+e)^2/b^2/d+a^3*(f*x+e)^2/b^2/(a^2+b^2)/d+4*f*(f*x+e)*arctan(exp(d*x
+c))/b/d^2-4*a^2*f*(f*x+e)*arctan(exp(d*x+c))/b/(a^2+b^2)/d^2+2*a*f*(f*x+e)
*ln(1+exp(2*d*x+2*c))/b^2/d^2-2*a^3*f*(f*x+e)*ln(1+exp(2*d*x+2*c))/b^2/(a^2
+b^2)/d^2+a^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3
/2)/d-a^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/
d-2*I*f^2*polylog(2,-I*exp(d*x+c))/b/d^3+2*I*a^2*f^2*polylog(2,-I*exp(d*x+c
))/b/(a^2+b^2)/d^3+2*I*f^2*polylog(2,I*exp(d*x+c))/b/d^3-2*I*a^2*f^2*polylo
g(2,I*exp(d*x+c))/b/(a^2+b^2)/d^3+a*f^2*polylog(2,-exp(2*d*x+2*c))/b^2/d^3-
a^3*f^2*polylog(2,-exp(2*d*x+2*c))/b^2/(a^2+b^2)/d^3+2*a^2*f*(f*x+e)*polylo
g(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-2*a^2*f*(f*x+e)*
polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-2*a^2*f^2*
polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^3+2*a^2*f^2*
polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^3-(f*x+e)^2*
sech(d*x+c)/b/d+a^2*(f*x+e)^2*sech(d*x+c)/b/(a^2+b^2)/d-a*(f*x+e)^2*tanh(d*
x+c)/b^2/d+a^3*(f*x+e)^2*tanh(d*x+c)/b^2/(a^2+b^2)/d
```

Rubi [A]
time = 1.23, antiderivative size = 772, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 16, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5686, 5559, 4265, 2317, 2438, 5702, 4269, 3799, 2221, 5692, 3403, 2296, 2611, 2320, 6724, 6874}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a*(e + f*x)^2)/(b^2*d)) + (a^3*(e + f*x)^2)/(b^2*(a^2 + b^2)*d) + (4*f*(
e + f*x)*ArcTan[E^(c + d*x)])/(b*d^2) - (4*a^2*f*(e + f*x)*ArcTan[E^(c + d*
x)])/(b*(a^2 + b^2)*d^2) + (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (a^2*(e + f*x)^2*Log[1 + (b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) + (2*a*f*(e + f*x)*Log
[1 + E^(2*(c + d*x))])/(b^2*d^2) - (2*a^3*f*(e + f*x)*Log[1 + E^(2*(c + d*x
))])/(b^2*(a^2 + b^2)*d^2) - ((2*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^
3) + ((2*I)*a^2*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^3) + ((2
*I)*f^2*PolyLog[2, I*E^(c + d*x)])/(b*d^3) - ((2*I)*a^2*f^2*PolyLog[2, I*E^
```

```
(c + d*x)])/(b*(a^2 + b^2)*d^3) + (2*a^2*f*(e + f*x)*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (2*a^2*f*(e + f*x
)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*
d^2) + (a*f^2*PolyLog[2, -E^(2*(c + d*x))])/(b^2*d^3) - (a^3*f^2*PolyLog[2,
 -E^(2*(c + d*x))])/(b^2*(a^2 + b^2)*d^3) - (2*a^2*f^2*PolyLog[3, -((b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) + (2*a^2*f^2*Poly
Log[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) -
 ((e + f*x)^2*Sech[c + d*x])/(b*d) + (a^2*(e + f*x)^2*Sech[c + d*x])/(b*(a^
2 + b^2)*d) - (a*(e + f*x)^2*Tanh[c + d*x])/(b^2*d) + (a^3*(e + f*x)^2*Tanh
[c + d*x])/(b^2*(a^2 + b^2)*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5686

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
```

```
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*Sech[c +
d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5702

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{sech}(c+dx)\tanh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,d}{b}$$

$$= -\frac{(e+fx)^2\text{sech}(c+dx)}{bd} - \frac{a\int (e+fx)^2\text{sech}^2(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^2\text{sec}}{a+b\sinh}}{b^2}$$

$$= \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{(e+fx)^2\text{sech}(c+dx)}{bd} - \frac{a(e+fx)^2\tanh(c+}{b^2 d}$$

$$= -\frac{a(e+fx)^2}{b^2 d} + \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{(e+fx)^2\text{sech}(c+dx)}{bd} - \frac{a(e}{}$$

$$= -\frac{a(e+fx)^2}{b^2 d} + \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd^2} + \frac{2af(e+fx)\log\left(1+e^{2(c+dx)}\right)}{b^2 d^2}$$

$$= -\frac{a(e+fx)^2}{b^2 d} + \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd^2} + \frac{a^2(e+fx)^2\log\left(1+\frac{be^{c+d}}{a-\sqrt{a^2}}\right.}{\left(a^2+b^2\right)^{3/2} d}$$

$$= -\frac{a(e+fx)^2}{b^2 d} + \frac{a^3(e+fx)^2}{b^2\left(a^2+b^2\right)d} + \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{4a^2f(e+fx)}{b\left(a^2\right.}$$

$$= -\frac{a(e+fx)^2}{b^2 d} + \frac{a^3(e+fx)^2}{b^2\left(a^2+b^2\right)d} + \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{4a^2f(e+fx)}{b\left(a^2\right.}$$

$$= -\frac{a(e+fx)^2}{b^2 d} + \frac{a^3(e+fx)^2}{b^2\left(a^2+b^2\right)d} + \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{4a^2f(e+fx)}{b\left(a^2\right.}$$

$$= -\frac{a(e+fx)^2}{b^2 d} + \frac{a^3(e+fx)^2}{b^2\left(a^2+b^2\right)d} + \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{4a^2f(e+fx)}{b\left(a^2\right.}$$

Mathematica [A]
time = 9.41, size = 1183, normalized size = 1.53

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*((2*d^2*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 -
b^2] + (2*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)
*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] + (d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c
 + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] -
(2*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c
)])])/Sqrt[(a^2 + b^2)*E^(2*c)] - (d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x)
)/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] + (2*d*E^
c*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*d*E^c*f*(e + f*x)*PolyLog[2, -((b*E
^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*
c)] - (2*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E
^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] + (2*E^c*f^2*PolyLog[3, -((b*E^(2*c +
 d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)]))/(
(a^2 + b^2)*d^3) + (2*a*e*f*Sech[c]*(Cosh[c]*Log[Cosh[c]*Cosh[d*x] + Sinh[c
]*Sinh[d*x]] - d*x*Sinh[c]))/((a^2 + b^2)*d^2*(Cosh[c]^2 - Sinh[c]^2)) + (4
*b*e*f*ArcTan[(Sinh[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]
])/((a^2 + b^2)*d^2*Sqrt[Cosh[c]^2 - Sinh[c]^2]) + (a*f^2*Csch[c]*((d^2*x^2
)/E^ArcTanh[Coth[c]] - (I*Coth[c]*(-(d*x*(-Pi + (2*I)*ArcTanh[Coth[c]])) -
Pi*Log[1 + E^(2*d*x)] - 2*(I*d*x + I*ArcTanh[Coth[c]])*Log[1 - E^((2*I)*(I*
d*x + I*ArcTanh[Coth[c]]))] + Pi*Log[Cosh[d*x]] + (2*I)*ArcTanh[Coth[c]]*Lo
g[I*Sinh[d*x + ArcTanh[Coth[c]]]] + I*PolyLog[2, E^((2*I)*(I*d*x + I*ArcTan
h[Coth[c]]))]))/Sqrt[1 - Coth[c]^2])*Sech[c])/((a^2 + b^2)*d^3*Sqrt[Csch[c]
^2*(-Cosh[c]^2 + Sinh[c]^2)]) + (2*b*f^2*(((-I)*Csch[c]*(I*(d*x + ArcTanh[C
oth[c]])*(Log[1 - E^(-(d*x) - ArcTanh[Coth[c]])] - Log[1 + E^(-(d*x) - ArcT
anh[Coth[c]])]) + I*(PolyLog[2, -E^(-(d*x) - ArcTanh[Coth[c]])] - PolyLog[2
, E^(-(d*x) - ArcTanh[Coth[c]])])))/Sqrt[1 - Coth[c]^2] - (2*ArcTan[(Sinh[c
] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]]*ArcTanh[Coth[c]])/S
qrt[Cosh[c]^2 - Sinh[c]^2]))/((a^2 + b^2)*d^3) + (Sech[c]*Sech[c + d*x]*(-(
b*e^2*Cosh[c]) - 2*b*e*f*x*Cosh[c] - b*f^2*x^2*Cosh[c] - a*e^2*Sinh[d*x] -
2*a*e*f*x*Sinh[d*x] - a*f^2*x^2*Sinh[d*x]))/((a^2 + b^2)*d)
```

Maple [F]
time = 2.36, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \left(\tanh^2(dx+c)\right)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*a*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 + b^
2)*d^2))*e + 4*b*f^2*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*d
*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 4*a*f^2*integrate(x/(a^2*d*e^(2*d*x
 + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + (a^2*log((b*e^(-d*x
- c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2 +
 b^2)^(3/2)*d) - 2*(b*e^(-d*x - c) + a)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x
 - 2*c))*d))*e^2 + 4*b*f*arctan(e^(d*x + c))*e/((a^2 + b^2)*d^2) + 2*(a*f^2
*x^2 + 2*a*f*x*e - (b*f^2*x^2*e^c + 2*b*f*x*e^(c + 1))*e^(d*x))/(a^2*d + b^
2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) + integrate(-2*(a^2*f^2*x^
2*e^c + 2*a^2*f*x*e^(c + 1))*e^(d*x)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e^
(2*c))*e^(2*d*x) - 2*(a^3*e^c + a*b^2*e^c)*e^(d*x)), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4757 vs. 2(726) = 1452.
time = 0.45, size = 4757, normalized size = 6.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*(a^3 + a*b^2)*c^2*f^2 - 4*(a^3 + a*b^2)*c*d*f*cosh(1) + 2*(a^3 + a*b^2)*
d^2*cosh(1)^2 + 2*(a^3 + a*b^2)*d^2*sinh(1)^2 - 2*((a^3 + a*b^2)*d^2*f^2*x^
2 - (a^3 + a*b^2)*c^2*f^2 + 2*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*c*d*f)
*cosh(1) + 2*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*c*d*f)*sinh(1))*cosh(d*
x + c)^2 - 2*((a^3 + a*b^2)*d^2*f^2*x^2 - (a^3 + a*b^2)*c^2*f^2 + 2*((a^3 +
 a*b^2)*d^2*f*x + (a^3 + a*b^2)*c*d*f)*cosh(1) + 2*((a^3 + a*b^2)*d^2*f*x +
 (a^3 + a*b^2)*c*d*f)*sinh(1))*sinh(d*x + c)^2 + 2*(a^2*b*d*f^2*x + a^2*b*d
*f*cosh(1) + a^2*b*d*f*sinh(1) + (a^2*b*d*f^2*x + a^2*b*d*f*cosh(1) + a^2*b
*d*f*sinh(1))*cosh(d*x + c)^2 + 2*(a^2*b*d*f^2*x + a^2*b*d*f*cosh(1) + a^2*
b*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d*f^2*x + a^2*b*d*f*cos
h(1) + a^2*b*d*f*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*c
osh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((
a^2 + b^2)/b^2) - b)/b + 1) - 2*(a^2*b*d*f^2*x + a^2*b*d*f*cosh(1) + a^2*b*
d*f*sinh(1) + (a^2*b*d*f^2*x + a^2*b*d*f*cosh(1) + a^2*b*d*f*sinh(1))*cosh(
d*x + c)^2 + 2*(a^2*b*d*f^2*x + a^2*b*d*f*cosh(1) + a^2*b*d*f*sinh(1))*cosh
(d*x + c)*sinh(d*x + c) + (a^2*b*d*f^2*x + a^2*b*d*f*cosh(1) + a^2*b*d*f*si
nh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*si
nh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b
)/b + 1) - (a^2*b*c^2*f^2 - 2*a^2*b*c*d*f*cosh(1) + a^2*b*d^2*cosh(1)^2 + a
^2*b*d^2*sinh(1)^2 + (a^2*b*c^2*f^2 - 2*a^2*b*c*d*f*cosh(1) + a^2*b*d^2*cos
h(1)^2 + a^2*b*d^2*sinh(1)^2 - 2*(a^2*b*c*d*f - a^2*b*d^2*cosh(1))*sinh(1))
```

```
*cosh(d*x + c)^2 + 2*(a^2*b*c^2*f^2 - 2*a^2*b*c*d*f*cosh(1) + a^2*b*d^2*cos
h(1)^2 + a^2*b*d^2*sinh(1)^2 - 2*(a^2*b*c*d*f - a^2*b*d^2*cosh(1))*sinh(1))
*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*c^2*f^2 - 2*a^2*b*c*d*f*cosh(1) + a^2
*b*d^2*cosh(1)^2 + a^2*b*d^2*sinh(1)^2 - 2*(a^2*b*c*d*f - a^2*b*d^2*cosh(1)
)*sinh(1))*sinh(d*x + c)^2 - 2*(a^2*b*c*d*f - a^2*b*d^2*cosh(1))*sinh(1))*s
qrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((
a^2 + b^2)/b^2) + 2*a) + (a^2*b*c^2*f^2 - 2*a^2*b*c*d*f*cosh(1) + a^2*b*d^2
*cosh(1)^2 + a^2*b*d^2*sinh(1)^2 + (a^2*b*c^2*f^2 - 2*a^2*b*c*d*f*cosh(1) +
 a^2*b*d^2*cosh(1)^2 + a^2*b*d^2*sinh(1)^2 - 2*(a^2*b*c*d*f - a^2*b*d^2*cos
h(1))*sinh(1))*cosh(d*x + c)^2 + 2*(a^2*b*c^2*f^2 - 2*a^2*b*c*d*f*cosh(1) +
 a^2*b*d^2*cosh(1)^2 + a^2*b*d^2*sinh(1)^2 - 2*(a^2*b*c*d*f - a^2*b*d^2*cos
h(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*c^2*f^2 - 2*a^2*b*c*d*f
*cosh(1) + a^2*b*d^2*cosh(1)^2 + a^2*b*d^2*sinh(1)^2 - 2*(a^2*b*c*d*f - a^2
*b*d^2*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 2*(a^2*b*c*d*f - a^2*b*d^2*cosh(
1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c
) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (a^2*b*d^2*f^2*x^2 - a^2*b*c^2*f^2 +
 (a^2*b*d^2*f^2*x^2 - a^2*b*c^2*f^2 + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*cosh(
1) + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*sinh(1))*cosh(d*x + c)^2 + 2*(a^2*b*d^
2*f^2*x^2 - a^2*b*c^2*f^2 + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*cosh(1) + 2*(a^
2*b*d^2*f*x + a^2*b*c*d*f)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d^
2*f^2*x^2 - a^2*b*c^2*f^2 + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*cosh(1) + 2*(a^
2*b*d^2*f*x + a^2*b*c*d*f)*sinh(1))*sinh(d*x + c)^2 + 2*(a^2*b*d^2*f*x + a^
2*b*c*d*f)*cosh(1) + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*sinh(1))*sqrt((a^2 + b
^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sin
h(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (a^2*b*d^2*f^2*x^2 - a^2*b*c^2*
f^2 + (a^2*b*d^2*f^2*x^2 - a^2*b*c^2*f^2 + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*
cosh(1) + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*sinh(1))*cosh(d*x + c)^2 + 2*(a^2
*b*d^2*f^2*x^2 - a^2*b*c^2*f^2 + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*cosh(1) +
2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^2
*b*d^2*f^2*x^2 - a^2*b*c^2*f^2 + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*cosh(1) +
2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*sinh(1))*sinh(d*x + c)^2 + 2*(a^2*b*d^2*f*x
 + a^2*b*c*d*f)*cosh(1) + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*sinh(1))*sqrt((a^
2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(a^2*b*f^2*cosh(d*x + c)
^2 + 2*a^2*b*f^2*cosh(d*x + c)*sinh(d*x + c) + a^2*b*f^2*sinh(d*x + c)^2 +
a^2*b*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x +
 c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*(a^
2*b*f^2*cosh(d*x + c)^2 + 2*a^2*b*f^2*cosh(d*x + c)*sinh(d*x + c) + a^2*b*f
^2*sinh(d*x + c)^2 + a^2*b*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*
x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2))/b) - 2*((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*d^2*f*x*cosh
(1) + (a^2*b + b^3)*d^2*cosh(1)^2 + (a^2*b + b^3)*d^2*sinh(1)^2 + 2*((a^2*b
 + b^3)*d^2*f*x + (a^2*b + b^3)*d^2*cosh(1))*sinh(1))*cosh(d*x + c) + 2*((a
^3 + a*b^2)*f^2 + I*(a^2*b + b^3)*f^2 + ((a^3 + a*b^2)*f^2 + I*(a^2*b + b^3
)*f^2)*cosh(d*x + c)^2 + 2*((a^3 + a*b^2)*f^2 + I*(a^2*b + b^3)*f^2)*cosh(d
```

```
*x + c)*sinh(d*x + c) + ((a^3 + a*b^2)*f^2 + I*...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2 \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)**2*tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(c+d\,x)^2\,(e+f\,x)^2}{a+b\sinh(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
[Out] int((tanh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.384 $\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=385

$$\frac{f\text{ArcTan}(\sinh(c+dx))}{bd^2}-\frac{a^2f\text{ArcTan}(\sinh(c+dx))}{b\left(a^2+b^2\right)d^2}+\frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d}-\frac{a^2(e+fx)\log\left(1\right.}{\left(a^2+b\right.}$$

```
[Out] f*arctan(sinh(d*x+c))/b/d^2-a^2*f*arctan(sinh(d*x+c))/b/(a^2+b^2)/d^2+a*f*l
n(cosh(d*x+c))/b^2/d^2-a^3*f*ln(cosh(d*x+c))/b^2/(a^2+b^2)/d^2+a^2*(f*x+e)*
ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d-a^2*(f*x+e)*ln(1+b
*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d+a^2*f*polylog(2,-b*exp(d
*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-a^2*f*polylog(2,-b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^(3/2)/d^2-(f*x+e)*sech(d*x+c)/b/d+a^2*(f*x
+e)*sech(d*x+c)/b/(a^2+b^2)/d-a*(f*x+e)*tanh(d*x+c)/b^2/d+a^3*(f*x+e)*tanh(
d*x+c)/b^2/(a^2+b^2)/d
```

Rubi [A]
time = 0.63, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5686, 5559, 3855, 5702, 4269, 3556, 5692, 3403, 2296, 2221, 2317, 2438, 6874}

$$-\frac{a^2f\text{ArcTan}(\sinh(c+dx))}{bd^2\left(a^2+b^2\right)}+\frac{a^2f\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^{3/2}}-\frac{a^2f\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^{3/2}}+\frac{a^2(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{d\left(a^2+b^2\right)^{3/2}}-\frac{a^2(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{d\left(a^2+b^2\right)^{3/2}}+\frac{a^2(e+fx)\text{sech}(c+dx)}{bd\left(a^2+b^2\right)}-\frac{a^3f\log(\cosh(c+dx))}{b^2d^2\left(a^2+b^2\right)}+\frac{a^3(e+fx)\tanh(c+dx)}{b^2d\left(a^2+b^2\right)}+\frac{af\log(\cosh(c+dx))}{b^2d^2}-\frac{a(e+fx)\tanh(c+dx)}{b^2d}+\frac{f\text{ArcTan}(\sinh(c+dx))}{bd^2}-\frac{(e+fx)\text{sech}(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (f*ArcTan[Sinh[c + d*x]])/(b*d^2) - (a^2*f*ArcTan[Sinh[c + d*x]])/(b*(a^2 +
 b^2)*d^2) + (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])
/((a^2 + b^2)^(3/2)*d) - (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/((a^2 + b^2)^(3/2)*d) + (a*f*Log[Cosh[c + d*x]])/(b^2*d^2) - (
a^3*f*Log[Cosh[c + d*x]])/(b^2*(a^2 + b^2)*d^2) + (a^2*f*PolyLog[2, -((b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (a^2*f*PolyLo
g[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (
(e + f*x)*Sech[c + d*x])/(b*d) + (a^2*(e + f*x)*Sech[c + d*x])/(b*(a^2 + b^
2)*d) - (a*(e + f*x)*Tanh[c + d*x])/(b^2*d) + (a^3*(e + f*x)*Tanh[c + d*x])
/(b^2*(a^2 + b^2)*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
```

```
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5686

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*Sech[c +
d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5702

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{sech}(c+dx)\tanh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)\text{sech}(c+dx)}{bd} - \frac{a\int (e+fx)\text{sech}^2(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)\text{sech}^2(c}{a+b\sinh(c+c}}{b^2}$$

$$= \frac{f\tan^{-1}(\sinh(c+dx))}{bd^2} - \frac{(e+fx)\text{sech}(c+dx)}{bd} - \frac{a(e+fx)\tanh(c+dx)}{b^2d}$$

$$= \frac{f\tan^{-1}(\sinh(c+dx))}{bd^2} + \frac{af\log(\cosh(c+dx))}{b^2d^2} - \frac{(e+fx)\text{sech}(c+dx)}{bd} -$$

$$= \frac{f\tan^{-1}(\sinh(c+dx))}{bd^2} + \frac{af\log(\cosh(c+dx))}{b^2d^2} - \frac{(e+fx)\text{sech}(c+dx)}{bd} -$$

$$= \frac{f\tan^{-1}(\sinh(c+dx))}{bd^2} + \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d} - \frac{a^2(e+fx)}{}$$

$$= \frac{f\tan^{-1}(\sinh(c+dx))}{bd^2} - \frac{a^2f\tan^{-1}(\sinh(c+dx))}{b(a^2+b^2)d^2} + \frac{a^2(e+fx)\log\left(1+\right.}{(a^2+b^2)^{3/2}}$$

$$= \frac{f\tan^{-1}(\sinh(c+dx))}{bd^2} - \frac{a^2f\tan^{-1}(\sinh(c+dx))}{b(a^2+b^2)d^2} + \frac{a^2(e+fx)\log\left(1+\right.}{(a^2+b^2)^{3/2}}$$

Mathematica [A]
time = 1.93, size = 284, normalized size = 0.74

$$\frac{\frac{2bf\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a^2+b^2} + \frac{af\log(\cosh(c+dx))}{a^2+b^2} + \frac{a^2\left(-2de\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+2cf\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-f(c+dx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)+f\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)-f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)}{(a^2+b^2)^{3/2}} - \frac{d(e+fx)\text{sech}(c+dx)(b+a\sinh(c+dx))}{a^2+b^2}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((2*b*f*ArcTan[Tanh[(c + d*x)/2]])/(a^2 + b^2) + (a*f*Log[Cosh[c + d*x]])/(
a^2 + b^2) + (a^2*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*
c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
 - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^2 + b^2)^(3/
2) - (d*(e + f*x)*Sech[c + d*x]*(b + a*Sinh[c + d*x]))/(a^2 + b^2))/d^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1927 vs. $2(365) = 730$.

time = 5.55, size = 1928, normalized size = 5.01

method	result	size
risch	Expression too large to display	1928

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^2/(a^2+b^2)^(3/2)*b^2*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))*a^2*c-2/d^2/(a^2+b^2)*a*f*ln(exp(d*x+c))+1/d^2/(a
^2+b^2)^2*a^3*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2*b^2/d/(a^2+b^2)^(3/
2)*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2+2/
d^2/(a^2+b^2)^(1/2)*b^2*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a
^2+b^2)^(1/2))-2/d^2/(a^2+b^2)^(3/2)*b^2*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*e
xp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2-2/d^2/(a^2+b^2)^(3/2)*b^2*f/(2*a^2+2*b^
2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^2-2/d^2/(a
^2+b^2)*a^3*f/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d^2/(a^
2+b^2)^(1/2)*a^2*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^
2)^(1/2))+2/d^2/(a^2+b^2)^(3/2)*a^4*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+
b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+2/d^2/(a^2+b^2)^(3/2)*b^2*f/(2*a^2+2*
b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a^2-2/d/
(a^2+b^2)^(3/2)*a^4*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+
(a^2+b^2)^(1/2)))*x+2/d/(a^2+b^2)^(3/2)*a^4*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+
c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-2/d^2/(a^2+b^2)^(3/2)*a^4*f/(
2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/d
^2/(a^2+b^2)^(3/2)*a^4*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(
a^2+b^2)^(1/2))+2*b^2/d^2/(a^2+b^2)^(3/2)*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*
b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2+1/2/d^2/(a^2+b^2)^2*a*b^2*f*ln(b*exp
(2*d*x+2*c)+2*a*exp(d*x+c)-b)+4/d^2/(a^2+b^2)*b^3*f/(2*a^2+2*b^2)*arctan(ex
p(d*x+c))-2/d^2/(a^2+b^2)^(5/2)*a^4*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2
+b^2)^(1/2))+2*(f*x+e)*(-b*exp(d*x+c)+a)/d/(a^2+b^2)/(1+exp(2*d*x+2*c))+2/d
^2/(a^2+b^2)*b^2*f/(2*a^2+2*b^2)*a*ln(1+exp(2*d*x+2*c))-2/d^2/(a^2+b^2)^(3/
2)*b^4*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+4/
d^2/(a^2+b^2)^(1/2)*a^2*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a
^2+b^2)^(1/2))-2/d^2/(a^2+b^2)^(3/2)*b^2*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(
a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^2*c-2/d/(a^2+b^2)^(3/2)*b^2*f/(2*a
^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^2*x+2/
d^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))-2/d/(a^2+b^2)^(1/2)*
a^2*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d/(
a^2+b^2)^(3/2)*a^4*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^
2)^(1/2))-2/d^2/(a^2+b^2)^(5/2)*a^2*b^2*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/
(a^2+b^2)^(1/2))+4/d^2/(a^2+b^2)*a^2*b*f/(2*a^2+2*b^2)*arctan(exp(d*x+c))-2
```

```
/d^2/(a^2+b^2)^(3/2)*a^4*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2
)+a)/(a+(a^2+b^2)^(1/2)))+2/d^2/(a^2+b^2)^(3/2)*a^4*f/(2*a^2+2*b^2)*dilog((
-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2/(a^2+b^2)*b^2*
f/(2*a^2+2*b^2)*a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d/(a^2+b^2)^(3/2)
*b^2*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/
2)))*a^2*x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] (2*a^2*integrate(-x*e^(d*x + c)/(a^2*b + b^3 - (a^2*b*e^(2*c) + b^3*e^(2*c)
)*e^(2*d*x) - 2*(a^3*e^c + a*b^2*e^c)*e^(d*x)), x) - 2*(b*x*e^(d*x + c) - a
*x)/(a^2*d + b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) - 2*a*x/((a
^2 + b^2)*d) + 2*b*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) + a*log(e^(2*d*x +
 2*c) + 1)/((a^2 + b^2)*d^2))*f + (a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 +
 b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2)*d) - 2*(b
*e^(-d*x - c) + a)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1460 vs. 2(369) = 738.
time = 0.38, size = 1460, normalized size = 3.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*(a^3 + a*b^2)*d*f*x*cosh(d*x + c)^2 + 2*(a^3 + a*b^2)*d*f*x*sinh(d*x +
c)^2 - 2*(a^3 + a*b^2)*d*cosh(1) - 2*(a^3 + a*b^2)*d*sinh(1) - (a^2*b*f*cos
h(d*x + c)^2 + 2*a^2*b*f*cosh(d*x + c)*sinh(d*x + c) + a^2*b*f*sinh(d*x + c
)^2 + a^2*b*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x +
c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1)
+ (a^2*b*f*cosh(d*x + c)^2 + 2*a^2*b*f*cosh(d*x + c)*sinh(d*x + c) + a^2*b*
f*sinh(d*x + c)^2 + a^2*b*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) +
 a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2
) - b)/b + 1) - (a^2*b*c*f - a^2*b*d*cosh(1) - a^2*b*d*sinh(1) + (a^2*b*c*f
 - a^2*b*d*cosh(1) - a^2*b*d*sinh(1))*cosh(d*x + c)^2 + 2*(a^2*b*c*f - a^2*
b*d*cosh(1) - a^2*b*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*c*f - a
^2*b*d*cosh(1) - a^2*b*d*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*lo
g(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
+ (a^2*b*c*f - a^2*b*d*cosh(1) - a^2*b*d*sinh(1) + (a^2*b*c*f - a^2*b*d*cos
```

```
h(1) - a^2*b*d*sinh(1))*cosh(d*x + c)^2 + 2*(a^2*b*c*f - a^2*b*d*cosh(1) -
a^2*b*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*c*f - a^2*b*d*cosh(1)
 - a^2*b*d*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x
 + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (a^2*b*d*f*x
 + a^2*b*c*f + (a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*f*x +
 a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d*f*x + a^2*b*c*f)*sinh(d*
x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (a^2*b*d
*f*x + a^2*b*c*f + (a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*f
*x + a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d*f*x + a^2*b*c*f)*sin
h(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
 - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*((
a^2*b + b^3)*f*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*f*cosh(d*x + c)*sinh(d*x +
 c) + (a^2*b + b^3)*f*sinh(d*x + c)^2 + (a^2*b + b^3)*f)*arctan(cosh(d*x +
c) + sinh(d*x + c)) + 2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*cosh(1) + (a
^2*b + b^3)*d*sinh(1))*cosh(d*x + c) - ((a^3 + a*b^2)*f*cosh(d*x + c)^2 + 2
*(a^3 + a*b^2)*f*cosh(d*x + c)*sinh(d*x + c) + (a^3 + a*b^2)*f*sinh(d*x + c
)^2 + (a^3 + a*b^2)*f)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)))
 + 2*(2*(a^3 + a*b^2)*d*f*x*cosh(d*x + c) + (a^2*b + b^3)*d*f*x + (a^2*b +
b^3)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1))*sinh(d*x + c))/((a^4 + 2*a^2*b^2
+ b^4)*d^2*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*cosh(d*x + c)*si
nh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*d^2*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^
2 + b^4)*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)*tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(c+d\,x)^2\,(e+f\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
[Out] int((tanh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.385 $\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=90

$$-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}-\frac{b\text{sech}(c+dx)}{(a^2+b^2)\,d}-\frac{a\tanh(c+dx)}{(a^2+b^2)\,d}$$

```
[Out] -2*a^2*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d
-b*sech(d*x+c)/(a^2+b^2)/d-a*tanh(d*x+c)/(a^2+b^2)/d
```

Rubi [A]
time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2806, 3852, 8, 2686, 2739, 632, 210}

$$-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\,(a^2+b^2)^{3/2}}-\frac{a\tanh(c+dx)}{d\,(a^2+b^2)}-\frac{b\text{sech}(c+dx)}{d\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*a^2*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3
/2)*d) - (b*Sech[c + d*x])/((a^2 + b^2)*d) - (a*Tanh[c + d*x])/((a^2 + b^2)
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2806

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[b*(g/(a^2 - b^2)), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[a^2*(g^2/(a^2 - b^2)), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\int \text{sech}^2(c+dx)\,dx}{a^2+b^2}+\frac{a^2\int \frac{1}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}+\frac{b\int \text{sech}(c+dx)\tanh(c+dx)\,d}{a^2+b^2}$$

$$= -\frac{(ia)\text{Subst}\left(\int 1\,dx,x,-i\tanh(c+dx)\right)}{(a^2+b^2)\,d}-\frac{(2ia^2)\,\text{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx,x,\tan\left(\right.\right.}{(a^2+b^2)\,d}$$

$$= -\frac{b\text{sech}(c+dx)}{(a^2+b^2)\,d}-\frac{a\tanh(c+dx)}{(a^2+b^2)\,d}+\frac{(4ia^2)\,\text{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx,x,-2ib+2\right.}{(a^2+b^2)\,d}$$

$$= -\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}-\frac{b\text{sech}(c+dx)}{(a^2+b^2)\,d}-\frac{a\tanh(c+dx)}{(a^2+b^2)\,d}$$

Mathematica [A]
time = 0.16, size = 106, normalized size = 1.18

$$-\frac{-b\sqrt{-a^2-b^2}\,\text{sech}(c+dx)+a\left(2a\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)-\sqrt{-a^2-b^2}\,\tanh(c+dx)\right)}{(-a^2-b^2)^{3/2}\,d}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]

[Out] -((-(b*Sqrt[-a^2 - b^2]*Sech[c + d*x]) + a*(2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] - Sqrt[-a^2 - b^2]*Tanh[c + d*x]))/((-a^2 - b^2)^(3/2)*d))

Maple [A]
time = 1.16, size = 103, normalized size = 1.14

method	result
derivativedivides	$\frac{\frac{8a^2 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(4a^2+4b^2)\sqrt{a^2+b^2}}+\frac{-2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{(a^2+b^2)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d}$
default	$\frac{\frac{8a^2 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(4a^2+4b^2)\sqrt{a^2+b^2}}+\frac{-2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{(a^2+b^2)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d}$
risch	$\frac{-2b\,\mathrm{e}^{dx+c}+2a}{d(a^2+b^2)(1+\mathrm{e}^{2dx+2c})}+\frac{a^2\ln\left(\mathrm{e}^{dx+c}+\frac{a(a^2+b^2)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b(a^2+b^2)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}d}-\frac{a^2\ln\left(\mathrm{e}^{dx+c}+\frac{a(a^2+b^2)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b(a^2+b^2)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(8*a^2/(4*a^2+4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))+2/(a^2+b^2)*(-a*tanh(1/2*d*x+1/2*c)-b)/(tanh(1/2*d*x+1/2*c)^2+1))

Maxima [A]
time = 0.47, size = 115, normalized size = 1.28

$$\frac{a^2\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}d}-\frac{2\left(be^{(-dx-c)}+a\right)}{(a^2+b^2+(a^2+b^2)e^{(-2\,dx-2\,c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")

[Out] a^2*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2 + b^2)^(3/2)*d) - 2*(b*e^(-d*x - c) + a)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*d)

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(87) = 174$.
time = 0.38, size = 351, normalized size = 3.90

$$\frac{2\,a^3+2\,ab^2+\left(a^2\cosh\left(dx+c\right)^2+2\,a^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a^2\sinh\left(dx+c\right)^2+a^2\right)\sqrt{a^2+b^2}\,\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2+b^2+2\,(b^2\cosh(dx+c)+ab)\sinh(dx+c)-2\,\sqrt{a^2+b^2}\,(b\cosh(dx+c)+b\sinh(dx+c)+a)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)-b}\right)-2\,(a^2b+b^3)\cosh\left(dx+c\right)-2\,(a^2b+b^3)\sinh\left(dx+c\right)}{(a^4+2\,a^2b^2+b^4)d\cosh\left(dx+c\right)^2+2\,(a^4+2\,a^2b^2+b^4)d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(a^4+2\,a^2b^2+b^4)d\sinh\left(dx+c\right)^2+(a^4+2\,a^2b^2+b^4)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*a^3 + 2*a*b^2 + (a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c)
 + a^2*sinh(d*x + c)^2 + a^2)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^
2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c
) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x +
c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*
cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*(a^2*b + b^3)*cosh(d*x + c) - 2*
(a^2*b + b^3)*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + 2
*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 +
 b^4)*d*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [**A**]
time = 0.48, size = 108, normalized size = 1.20

$$\frac{\frac{a^2\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\,(be^{(dx+c)}-a)}{(a^2+b^2)(e^{(2\,dx+2\,c)}+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (a^2*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c)
 + 2*a + 2*sqrt(a^2 + b^2))))/(a^2 + b^2)^(3/2) - 2*(b*e^(d*x + c) - a)/((a^
2 + b^2)*(e^(2*d*x + 2*c) + 1)))/d
```

Mupad [B]
time = 0.80, size = 422, normalized size = 4.69

$$\frac{\frac{2a}{d(a^2+b^2)}-\frac{2b\,e^{c+dx}}{d(a^2+b^2)}}{e^{2c+2dx}+1}-\frac{2\,\mathrm{atan}\left(\left(e^{dx}\,e^c\left(\frac{2a^2}{b^2d\sqrt{a^4}\,(a^2+b^2)^2}+\frac{2\left(a^3d\sqrt{a^4}+a\,b^2d\sqrt{a^4}\right)}{a\,b^2\sqrt{-d^2(a^2+b^2)^3}\,(a^2+b^2)\sqrt{-a^6d^2-3a^4b^2d^2-3a^2b^4d^2-b^6d^2}}\right)-\frac{2\left(b^3d\sqrt{a^4}+a^2b\,d\sqrt{a^4}\right)}{a\,b^2\sqrt{-d^2(a^2+b^2)^3}\,(a^2+b^2)\sqrt{-a^6d^2-3a^4b^2d^2-3a^2b^4d^2-b^6d^2}}\right)\left(\frac{b^3\sqrt{-a^6d^2-3a^4b^2d^2-3a^2b^4d^2-b^6d^2}}{2}+\frac{a^2b\sqrt{-a^6d^2-3a^4b^2d^2-3a^2b^4d^2-b^6d^2}}{2}\right)\right)\sqrt{a^4}}{\sqrt{-a^6d^2-3a^4b^2d^2-3a^2b^4d^2-b^6d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2/(a + b*sinh(c + d*x)),x)
```

```
[Out] ((2*a)/(d*(a^2 + b^2)) - (2*b*exp(c + d*x))/(d*(a^2 + b^2)))/(exp(2*c + 2*d
*x) + 1) - (2*atan((exp(d*x)*exp(c)*((2*a^2)/(b^2*d*(a^4)^(1/2)*(a^2 + b^2)
^2) + (2*(a^3*d*(a^4)^(1/2) + a*b^2*d*(a^4)^(1/2)))/(a*b^2*(-d^2*(a^2 + b^2
)^3)^(1/2)*(a^2 + b^2)*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b^2*d^2
)^(1/2))) - (2*(b^3*d*(a^4)^(1/2) + a^2*b*d*(a^4)^(1/2)))/(a*b^2*(-d^2*(a^2
 + b^2)^3)^(1/2)*(a^2 + b^2)*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b
^2*d^2)^(1/2)))*((b^3*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b^2*d^2)
^(1/2))/2 + (a^2*b*(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b^2*d^2)^(1
/2))/2))*(a^4)^(1/2))/(- a^6*d^2 - b^6*d^2 - 3*a^2*b^4*d^2 - 3*a^4*b^2*d^2)
^(1/2)
```

3.386 $\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Tanh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][Tanh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Tanh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*a^2*integrate(-e^(d*x + c)/((a^2*b*f + b^3*f)*x + (a^2*b + b^3)*e - ((a^2
*b*f*e^(2*c) + b^3*f*e^(2*c))*x + (a^2*b*e^(2*c) + b^3*e^(2*c))*e)*e^(2*d*x
) - 2*((a^3*f*e^c + a*b^2*f*e^c)*x + (a^3*e^c + a*b^2*e^c)*e)*e^(d*x)), x)
- 2*(b*e^(d*x + c) - a)/((a^2*d*f + b^2*d*f)*x + (a^2*d + b^2*d)*e + ((a^2*
d*f*e^(2*c) + b^2*d*f*e^(2*c))*x + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e)*e^(2*
d*x)) - integrate(2*(b*f*e^(d*x + c) - a*f)/((a^2*d*f^2 + b^2*d*f^2)*x^2 +
2*(a^2*d*f + b^2*d*f)*x*e + (a^2*d + b^2*d)*e^2 + ((a^2*d*f^2*e^(2*c) + b^2
*d*f^2*e^(2*c))*x^2 + 2*(a^2*d*f*e^(2*c) + b^2*d*f*e^(2*c))*x*e + (a^2*d*e^
(2*c) + b^2*d*e^(2*c))*e^2)*e^(2*d*x)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(tanh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tanh(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(tanh(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.387 $\int \frac{(e+fx)^2\mathbf{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1256

$$-\frac{a(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{b^2d}+\frac{2a^3(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2d}+\frac{a^3(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d}+\frac{af^2\text{ArcTan}(\sinh(c}{b^2d^3}$$

```
[Out] 2*a^2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/
d^2+2*a^2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2
)^2/d^2+a^3*(f*x+e)^2*arctan(exp(d*x+c))/b^2/(a^2+b^2)/d+I*a*f*(f*x+e)*poly
log(2,-I*exp(d*x+c))/b^2/d^2-a^2*b*f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/(a^
2+b^2)^2/d^2-f^2*ln(cosh(d*x+c))/b/d^3+2*I*a^3*f^2*polylog(3,-I*exp(d*x+c))
/(a^2+b^2)^2/d^3+I*a^3*f^2*polylog(3,-I*exp(d*x+c))/b^2/(a^2+b^2)/d^3+a^3*f
*(f*x+e)*sech(d*x+c)/b^2/(a^2+b^2)/d^2-a^2*f*(f*x+e)*tanh(d*x+c)/b/(a^2+b^2
)/d^2+1/2*a^3*(f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/b^2/(a^2+b^2)/d-2*I*a^3*f*(
f*x+e)*polylog(2,-I*exp(d*x+c))/(a^2+b^2)^2/d^2-I*a*f*(f*x+e)*polylog(2,I*e
xp(d*x+c))/b^2/d^2-I*a^3*f^2*polylog(3,I*exp(d*x+c))/b^2/(a^2+b^2)/d^3-a*(f
*x+e)^2*arctan(exp(d*x+c))/b^2/d+2*a^3*(f*x+e)^2*arctan(exp(d*x+c))/(a^2+b^
2)^2/d+a*f^2*arctan(sinh(d*x+c))/b^2/d^3+f*(f*x+e)*tanh(d*x+c)/b/d^2-a^2*b*
(f*x+e)^2*ln(1+exp(2*d*x+2*c))/(a^2+b^2)^2/d+a^2*f^2*ln(cosh(d*x+c))/b/(a^2
+b^2)/d^3+a^2*b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^
2/d+a^2*b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d-2*
a^2*b*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^3-2*a^
2*b*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^3-1/2*(f
*x+e)^2*sech(d*x+c)^2/b/d-a^3*f^2*arctan(sinh(d*x+c))/b^2/(a^2+b^2)/d^3+I*a
*f^2*polylog(3,I*exp(d*x+c))/b^2/d^3+1/2*a^2*b*f^2*polylog(3,-exp(2*d*x+2*c
))/(a^2+b^2)^2/d^3-a*f*(f*x+e)*sech(d*x+c)/b^2/d^2+1/2*a^2*(f*x+e)^2*sech(d
*x+c)^2/b/(a^2+b^2)/d-1/2*a*(f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/b^2/d-I*a*f^2
*polylog(3,-I*exp(d*x+c))/b^2/d^3-2*I*a^3*f^2*polylog(3,I*exp(d*x+c))/(a^2+
b^2)^2/d^3+2*I*a^3*f*(f*x+e)*polylog(2,I*exp(d*x+c))/(a^2+b^2)^2/d^2+I*a^3*
f*(f*x+e)*polylog(2,I*exp(d*x+c))/b^2/(a^2+b^2)/d^2-I*a^3*f*(f*x+e)*polylog
(2,-I*exp(d*x+c))/b^2/(a^2+b^2)/d^2
```

Rubi [A]
time = 1.64, antiderivative size = 1256, normalized size of antiderivative = 1.00, number of steps used = 53, number of rules used = 15, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.441$, Rules used = {5702, 5559, 4269, 3556, 4271, 3855, 4265, 2611, 2320, 6724, 5692, 5680, 2221, 6874, 3799}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^2*d)) + (2*a^3*(e + f*x)^2*ArcTan[
E^(c + d*x)])/((a^2 + b^2)^2*d) + (a^3*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^
2*(a^2 + b^2)*d) + (a*f^2*ArcTan[Sinh[c + d*x]])/(b^2*d^3) - (a^3*f^2*ArcTa
n[Sinh[c + d*x]])/(b^2*(a^2 + b^2)*d^3) + (a^2*b*(e + f*x)^2*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) + (a^2*b*(e + f*x)^2*Lo
g[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) - (a^2*b*(e
 + f*x)^2*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)^2*d) - (f^2*Log[Cosh[c + d
*x]])/(b*d^3) + (a^2*f^2*Log[Cosh[c + d*x]])/(b*(a^2 + b^2)*d^3) + (I*a*f*(
e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*d^2) - ((2*I)*a^3*f*(e + f*x)*P
olyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - (I*a^3*f*(e + f*x)*PolyL
og[2, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) - (I*a*f*(e + f*x)*PolyLog[2
, I*E^(c + d*x)])/(b^2*d^2) + ((2*I)*a^3*f*(e + f*x)*PolyLog[2, I*E^(c + d*
x)])/((a^2 + b^2)^2*d^2) + (I*a^3*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b
^2*(a^2 + b^2)*d^2) + (2*a^2*b*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) + (2*a^2*b*f*(e + f*x)*PolyLog[2,
 -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) - (a^2*b*f*
(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^2)^2*d^2) - (I*a*f^2*Poly
Log[3, (-I)*E^(c + d*x)])/(b^2*d^3) + ((2*I)*a^3*f^2*PolyLog[3, (-I)*E^(c +
 d*x)])/((a^2 + b^2)^2*d^3) + (I*a^3*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b^2
*(a^2 + b^2)*d^3) + (I*a*f^2*PolyLog[3, I*E^(c + d*x)])/(b^2*d^3) - ((2*I)*
a^3*f^2*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)^2*d^3) - (I*a^3*f^2*PolyLog
[3, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) - (2*a^2*b*f^2*PolyLog[3, -((b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) - (2*a^2*b*f^2*Poly
Log[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) + (a^
2*b*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^3) - (a*f*(e + f*x
)*Sech[c + d*x])/(b^2*d^2) + (a^3*f*(e + f*x)*Sech[c + d*x])/(b^2*(a^2 + b^
2)*d^2) - ((e + f*x)^2*Sech[c + d*x]^2)/(2*b*d) + (a^2*(e + f*x)^2*Sech[c +
 d*x]^2)/(2*b*(a^2 + b^2)*d) + (f*(e + f*x)*Tanh[c + d*x])/(b*d^2) - (a^2*f
*(e + f*x)*Tanh[c + d*x])/(b*(a^2 + b^2)*d^2) - (a*(e + f*x)^2*Sech[c + d*x
]*Tanh[c + d*x])/(2*b^2*d) + (a^3*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/
(2*b^2*(a^2 + b^2)*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
```

```
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5702

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\text{sech}^2(c+\ldots}{a+b\sinh(\ldots}}{b}$$

$$= -\frac{(e+fx)^2\text{sech}^2(c+dx)}{2bd} - \frac{a\int (e+fx)^2\text{sech}^3(c+dx)\,dx}{b^2} + \ldots$$

$$= -\frac{af(e+fx)\text{sech}(c+dx)}{b^2d^2} - \frac{(e+fx)^2\text{sech}^2(c+dx)}{2bd} + \frac{f(e+\ldots}{}$$

$$= -\frac{a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{af^2\tan^{-1}(\sinh(c+dx))}{b^2d^3} - \frac{f^2\log\ldots}{}$$

$$= -\frac{a^2b(e+fx)^3}{3\left(a^2+b^2\right)^2f} - \frac{a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{af^2\tan^{-1}(\sinh(\ldots}{b^2d^3}$$

$$= -\frac{a^2b(e+fx)^3}{3\left(a^2+b^2\right)^2f} - \frac{a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{af^2\tan^{-1}(\sinh(\ldots}{b^2d^3}$$

$$= -\frac{a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2d} + \frac{a^3(\ldots}{}$$

$$= -\frac{a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2d} + \frac{a^3(\ldots}{}$$

$$= -\frac{a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2d} + \frac{a^3(\ldots}{}$$

$$= -\frac{a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2d} + \frac{a^3(\ldots}{}$$

$$= -\frac{a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2d} + \frac{a^3(\ldots}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than

twice the leaf count of optimal. 2735 vs. 2(1256) = 2512.
time = 20.91, size = 2735, normalized size = 2.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] -1/6*(-12*a^2*b*d^3*e^2*E^(2*c)*x - 12*a^2*b*d*E^(2*c)*f^2*x - 12*b^3*d*E^(
2*c)*f^2*x - 12*a^2*b*d^3*e*E^(2*c)*f*x^2 - 4*a^2*b*d^3*E^(2*c)*f^2*x^3 - 6
*a^3*d^2*e^2*ArcTan[E^(c + d*x)] + 6*a*b^2*d^2*e^2*ArcTan[E^(c + d*x)] - 6*
a^3*d^2*e^2*E^(2*c)*ArcTan[E^(c + d*x)] + 6*a*b^2*d^2*e^2*E^(2*c)*ArcTan[E^
(c + d*x)] - 12*a^3*f^2*ArcTan[E^(c + d*x)] - 12*a*b^2*f^2*ArcTan[E^(c + d*
x)] - 12*a^3*E^(2*c)*f^2*ArcTan[E^(c + d*x)] - 12*a*b^2*E^(2*c)*f^2*ArcTan[
E^(c + d*x)] - (6*I)*a^3*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + (6*I)*a*b^2*d^2
*e*f*x*Log[1 - I*E^(c + d*x)] - (6*I)*a^3*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c
+ d*x)] + (6*I)*a*b^2*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] - (3*I)*a^3*
d^2*f^2*x^2*Log[1 - I*E^(c + d*x)] + (3*I)*a*b^2*d^2*f^2*x^2*Log[1 - I*E^(c
 + d*x)] - (3*I)*a^3*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (3*I)*a*b
^2*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (6*I)*a^3*d^2*e*f*x*Log[1 +
 I*E^(c + d*x)] - (6*I)*a*b^2*d^2*e*f*x*Log[1 + I*E^(c + d*x)] + (6*I)*a^3*
d^2*e*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (6*I)*a*b^2*d^2*e*E^(2*c)*f*x*Lo
g[1 + I*E^(c + d*x)] + (3*I)*a^3*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - (3*I)
*a*b^2*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] + (3*I)*a^3*d^2*E^(2*c)*f^2*x^2*L
og[1 + I*E^(c + d*x)] - (3*I)*a*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 + I*E^(c + d*
x)] + 6*a^2*b*d^2*e^2*Log[1 + E^(2*(c + d*x))] + 6*a^2*b*d^2*e^2*E^(2*c)*Lo
g[1 + E^(2*(c + d*x))] + 6*a^2*b*f^2*Log[1 + E^(2*(c + d*x))] + 6*b^3*f^2*L
og[1 + E^(2*(c + d*x))] + 6*a^2*b*E^(2*c)*f^2*Log[1 + E^(2*(c + d*x))] + 6*
b^3*E^(2*c)*f^2*Log[1 + E^(2*(c + d*x))] + 12*a^2*b*d^2*e*f*x*Log[1 + E^(2*
(c + d*x))] + 12*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 + E^(2*(c + d*x))] + 6*a^2*b
*d^2*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 6*a^2*b*d^2*E^(2*c)*f^2*x^2*Log[1 +
 E^(2*(c + d*x))] + (6*I)*a*(a^2 - b^2)*d*(1 + E^(2*c))*f*(e + f*x)*PolyLog
[2, (-I)*E^(c + d*x)] - (6*I)*a*(a^2 - b^2)*d*(1 + E^(2*c))*f*(e + f*x)*Pol
yLog[2, I*E^(c + d*x)] + 6*a^2*b*d*e*f*PolyLog[2, -E^(2*(c + d*x))] + 6*a^2
*b*d*e*E^(2*c)*f*PolyLog[2, -E^(2*(c + d*x))] + 6*a^2*b*d*f^2*x*PolyLog[2,
-E^(2*(c + d*x))] + 6*a^2*b*d*E^(2*c)*f^2*x*PolyLog[2, -E^(2*(c + d*x))] -
(6*I)*a^3*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (6*I)*a*b^2*f^2*PolyLog[3, (-I
)*E^(c + d*x)] - (6*I)*a^3*E^(2*c)*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (6*I)
*a*b^2*E^(2*c)*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (6*I)*a^3*f^2*PolyLog[3,
I*E^(c + d*x)] - (6*I)*a*b^2*f^2*PolyLog[3, I*E^(c + d*x)] + (6*I)*a^3*E^(2
*c)*f^2*PolyLog[3, I*E^(c + d*x)] - (6*I)*a*b^2*E^(2*c)*f^2*PolyLog[3, I*E^
(c + d*x)] - 3*a^2*b*f^2*PolyLog[3, -E^(2*(c + d*x))] - 3*a^2*b*E^(2*c)*f^2
*PolyLog[3, -E^(2*(c + d*x))])/((a^2 + b^2)^2*d^3*(1 + E^(2*c))) + (a^2*b*(
(-2*E^(2*c)*x*(3*e^2 + 3*e*f*x + f^2*x^2))/(-1 + E^(2*c)) + (3*(d^2*e^2*Log
```

```
[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 2*d^2*e*f*x*Log[1 + (b*E^(2*c +
 d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + d^2*f^2*x^2*Log[1 + (b*E^(2*c
 + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*d^2*e*f*x*Log[1 + (b*E^(2
*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + d^2*f^2*x^2*Log[1 + (b*E^
(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*d*f*(e + f*x)*PolyLog
[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2*d*f*(e +
f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] -
 2*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))]
 - 2*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])
)]))/d^3))/(3*(a^2 + b^2)^2) + (Csch[c]*Sech[c]*Sech[c + d*x]^2*(6*a^2*b*e*
f + 6*b^3*e*f + 12*a^2*b*d^2*e^2*x + 6*a^2*b*f^2*x + 6*b^3*f^2*x + 12*a^2*b
*d^2*e*f*x^2 + 4*a^2*b*d^2*f^2*x^3 - 6*a^2*b*e*f*Cosh[2*c] - 6*b^3*e*f*Cosh
[2*c] - 6*a^2*b*f^2*x*Cosh[2*c] - 6*b^3*f^2*x*Cosh[2*c] - 6*a^2*b*e*f*Cosh[
2*d*x] - 6*b^3*e*f*Cosh[2*d*x] - 6*a^2*b*f^2*x*Cosh[2*d*x] - 6*b^3*f^2*x*Co
sh[2*d*x] + 3*a^3*d*e^2*Cosh[c - d*x] + 3*a*b^2*d*e^2*Cosh[c - d*x] + 6*a^3
*d*e*f*x*Cosh[c - d*x] + 6*a*b^2*d*e*f*x*Cosh[c - d*x] + 3*a^3*d*f^2*x^2*Co
sh[c - d*x] + 3*a*b^2*d*f^2*x^2*Cosh[c - d*x] - 3*a^3*d*e^2*Cosh[3*c + d*x]
 - 3*a*b^2*d*e^2*Cosh[3*c + d*x] - 6*a^3*d*e*f*x*Cosh[3*c + d*x] - 6*a*b^2*
d*e*f*x*Cosh[3*c + d*x] - 3*a^3*d*f^2*x^2*Cosh[3*c + d*x] - 3*a*b^2*d*f^2*x
^2*Cosh[3*c + d*x] + 6*a^2*b*e*f*Cosh[2*c + 2*d*x] + 6*b^3*e*f*Cosh[2*c + 2
*d*x] + 12*a^2*b*d^2*e^2*x*Cosh[2*c + 2*d*x] + 6*a^2*b*f^2*x*Cosh[2*c + 2*d
*x] + 6*b^3*f^2*x*Cosh[2*c + 2*d*x] + 12*a^2*b*d^2*e*f*x^2*Cosh[2*c + 2*d*x
] + 4*a^2*b*d^2*f^2*x^3*Cosh[2*c + 2*d*x] - 6*a^2*b*d*e^2*Sinh[2*c] - 6*b^3
*d*e^2*Sinh[2*c] - 12*a^2*b*d*e*f*x*Sinh[2*c] - 12*b^3*d*e*f*x*Sinh[2*c] -
6*a^2*b*d*f^2*x^2*Sinh[2*c] - 6*b^3*d*f^2*x^2*Sinh[2*c] - 6*a^3*e*f*Sinh[c
- d*x] - 6*a*b^2*e*f*Sinh[c - d*x] - 6*a^3*f^2*x*Sinh[c - d*x] - 6*a*b^2*f^
2*x*Sinh[c - d*x] - 6*a^3*e*f*Sinh[3*c + d*x] - 6*a*b^2*e*f*Sinh[3*c + d*x]
 - 6*a^3*f^2*x*Sinh[3*c + d*x] - 6*a*b^2*f^2*x*Sinh[3*c + d*x]))/(24*(a^2 +
 b^2)^2*d^2)
```

Maple [F]
time = 1.35, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c) \left(\tanh^2 (dx+c)\right)}{a+b\sinh (dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] a^3*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*
d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b
^4*d^2), x) - a*b^2*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2
*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2
*a^2*b^2*d^2 + b^4*d^2), x) + 2*a^2*b*d^2*f^2*integrate(x^2/(a^4*d^2*e^(2*d
*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d
^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 4*a^2*b*d^2*f*e*integrate(x/(a^4*d^2*e^
(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a
^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*a^3*d^2*f*integrate(x*e^(d*x + c
+ 1)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(
2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 2*a*b^2*d^2*f*integ
rate(x*e^(d*x + c + 1)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x +
2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + a
^2*b*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x + 2*c) +
 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) + b^3*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2
 + b^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) + 2*
a^3*f^2*arctan(e^(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3) + 2*a*b^2*f^2*arc
tan(e^(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3) + (a^2*b*log(-2*a*e^(-d*x -
c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b^4)*d) - a^2*b*log(e^(-2*
d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a^3 - a*b^2)*arctan(e^(-d*x
- c))/((a^4 + 2*a^2*b^2 + b^4)*d) - (a*e^(-d*x - c) + 2*b*e^(-2*d*x - 2*c)
- a*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*e^(-2*d*x - 2*c) + (a^2 +
 b^2)*e^(-4*d*x - 4*c))*d))*e^2 - (2*b*f^2*x + 2*b*f*e + (a*d*f^2*x^2*e^(3*
c) + 2*a*f*e^(3*c + 1) + 2*(a*f^2*e^(3*c) + a*d*f*e^(3*c + 1))*x)*e^(3*d*x)
 + 2*(b*d*f^2*x^2*e^(2*c) + b*f*e^(2*c + 1) + (b*f^2*e^(2*c) + 2*b*d*f*e^(2
*c + 1))*x)*e^(2*d*x) - (a*d*f^2*x^2*e^c - 2*a*f*e^(c + 1) + 2*(a*d*f*e^(c
+ 1) - a*f^2*e^c)*x)*e^(d*x))/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^(4*c) + b^2*d
^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*x)) -
integrate(2*(a^2*b^2*f^2*x^2 + 2*a^2*b^2*f*x*e - (a^3*b*f^2*x^2*e^c + 2*a^3
*b*f*x*e^(c + 1))*e^(d*x))/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b*e^(2*c) + 2*a^
2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c + 2*a^3*b^2*e^c + a*b^4
*e^c)*e^(d*x)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 16156 vs. $2(1179) = 2358$.
time = 0.64, size = 16156, normalized size = 12.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/2*(4*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*cosh(d*x + c)^4 + 4*((
a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*sinh(d*x + c)^4 + 4*(a^2*b + b^
3)*c*f^2 - 4*(a^2*b + b^3)*d*f*cosh(1) - 2*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(
a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*d^2*cosh(1)^2 + (a^3 + a*b^2)*d^2*sinh
(1)^2 + 2*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*d*f)*cosh(1) + 2*((a^3 + a
*b^2)*d^2*f*x + (a^3 + a*b^2)*d^2*cosh(1) + (a^3 + a*b^2)*d*f)*sinh(1))*cos
h(d*x + c)^3 - 4*(a^2*b + b^3)*d*f*sinh(1) - 2*((a^3 + a*b^2)*d^2*f^2*x^2 +
 2*(a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*d^2*cosh(1)^2 + (a^3 + a*b^2)*d^2*
sinh(1)^2 + 2*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*d*f)*cosh(1) - 8*((a^2
*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*cosh(d*x + c) + 2*((a^3 + a*b^2)*d
^2*f*x + (a^3 + a*b^2)*d^2*cosh(1) + (a^3 + a*b^2)*d*f)*sinh(1))*sinh(d*x +
 c)^3 - 4*((a^2*b + b^3)*d^2*f^2*x^2 - (a^2*b + b^3)*d*f^2*x + (a^2*b + b^3
)*d^2*cosh(1)^2 + (a^2*b + b^3)*d^2*sinh(1)^2 - 2*(a^2*b + b^3)*c*f^2 + (2*
(a^2*b + b^3)*d^2*f*x + (a^2*b + b^3)*d*f)*cosh(1) + (2*(a^2*b + b^3)*d^2*f
*x + 2*(a^2*b + b^3)*d^2*cosh(1) + (a^2*b + b^3)*d*f)*sinh(1))*cosh(d*x + c
)^2 - 2*(2*(a^2*b + b^3)*d^2*f^2*x^2 - 2*(a^2*b + b^3)*d*f^2*x + 2*(a^2*b +
 b^3)*d^2*cosh(1)^2 + 2*(a^2*b + b^3)*d^2*sinh(1)^2 - 4*(a^2*b + b^3)*c*f^2
 - 12*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*cosh(d*x + c)^2 + 2*(2*
(a^2*b + b^3)*d^2*f*x + (a^2*b + b^3)*d*f)*cosh(1) + 3*((a^3 + a*b^2)*d^2*f
^2*x^2 + 2*(a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*d^2*cosh(1)^2 + (a^3 + a*b
^2)*d^2*sinh(1)^2 + 2*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*d*f)*cosh(1) +
 2*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*d^2*cosh(1) + (a^3 + a*b^2)*d*f)*
sinh(1))*cosh(d*x + c) + 2*(2*(a^2*b + b^3)*d^2*f*x + 2*(a^2*b + b^3)*d^2*c
osh(1) + (a^2*b + b^3)*d*f)*sinh(1))*sinh(d*x + c)^2 + 2*((a^3 + a*b^2)*d^2
*f^2*x^2 - 2*(a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*d^2*cosh(1)^2 + (a^3 + a
*b^2)*d^2*sinh(1)^2 + 2*((a^3 + a*b^2)*d^2*f*x - (a^3 + a*b^2)*d*f)*cosh(1)
 + 2*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*d^2*cosh(1) - (a^3 + a*b^2)*d*f
)*sinh(1))*cosh(d*x + c) + 4*(a^2*b*d*f^2*x + a^2*b*d*f*cosh(1) + a^2*b*d*f
*sinh(1) + (a^2*b*d*f^2*x + a^2*b*d*f*cosh(1) + a^2*b*d*f*sinh(1))*cosh(d*x
 + c)^4 + 4*(a^2*b*d*f^2*x + a^2*b*d*f*cosh(1) + a^2*b*d*f*sinh(1))*cosh(d*
x + c)*sinh(d*x + c)^3 + (a^2*b*d*f^2*x + a^2*b*d*f*cosh(1) + a^2*b*d*f*sin
h(1))*sinh(d*x + c)^4 + 2*(a^2*b*d*f^2*x + a^2*b*d*f*cosh(1) + a^2*b*d*f*si
nh(1))*cosh(d*x + c)^2 + 2*(a^2*b*d*f^2*x + a^2*b*d*f*cosh(1) + a^2*b*d*f*s
inh(1) + 3*(a^2*b*d*f^2*x + a^2*b*d*f*cosh(1) + a^2*b*d*f*sinh(1))*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 + 4*((a^2*b*d*f^2*x + a^2*b*d*f*cosh(1) + a^2*b*d*
f*sinh(1))*cosh(d*x + c)^3 + (a^2*b*d*f^2*x + a^2*b*d*f*cosh(1) + a^2*b*d*f
*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x
 + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b +
1) + 4*(a^2*b*d*f^2*x + a^2*b*d*f*cosh(1) + a^2*b*d*f*sinh(1) + (a^2*b*d*f^
2*x + a^2*b*d*f*cosh(1) + a^2*b*d*f*sinh(1))*cosh(d*x + c)^4 + 4*(a^2*b*d*f
^2*x + a^2*b*d*f*cosh(1) + a^2*b*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3
 + (a^2*b*d*f^2*x + a^2*b*d*f*cosh(1) + a^2*b*d*f*sinh(1))*sinh(d*x + c)^4
+ 2*(a^2*b*d*f^2*x + a^2*b*d*f*cosh(1) + a^2*b*d*f*sinh(1))*cosh(d*x + c)^2
 + 2*(a^2*b*d*f^2*x + a^2*b*d*f*cosh(1) + a^2*b*d*f*sinh(1) + 3*(a^2*b*d*f^
2*x + a^2*b*d*f*cosh(1) + a^2*b*d*f*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)
```

```
^2 + 4*((a^2*b*d*f^2*x + a^2*b*d*f*cosh(1) + a^2*b*d*f*sinh(1))*cosh(d*x +
c)^3 + (a^2*b*d*f^2*x + a^2*b*d*f*cosh(1) + a^2*b*d*f*sinh(1))*cosh(d*x + c
))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(2*a^2*b*d*f^2*
x + 2*a^2*b*d*f*cosh(1) + 2*a^2*b*d*f*sinh(1) - I*(a^3 - a*b^2)*d*f^2*x + (
2*a^2*b*d*f^2*x + 2*a^2*b*d*f*cosh(1) + 2*a^2*b*d*f*sinh(1) - I*(a^3 - a*b^
2)*d*f^2*x - I*(a^3 - a*b^2)*d*f*cosh(1) - I*(a^3 - a*b^2)*d*f*sinh(1))*cos
h(d*x + c)^4 + 4*(2*a^2*b*d*f^2*x + 2*a^2*b*d*f*cosh(1) + 2*a^2*b*d*f*sinh(
1) - I*(a^3 - a*b^2)*d*f^2*x - I*(a^3 - a*b^2)*d*f*cosh(1) - I*(a^3 - a*b^2
)*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2*b*d*f^2*x + 2*a^2*b*d
*f*cosh(1) + 2*a^2*b*d*f*sinh(1) - I*(a^3 - a*b^2)*d*f^2*x - I*(a^3 - a*b^2
)*d*f*cosh(1) - I*(a^3 - a*b^2)*d*f*sinh(1))*sinh(d*x + c)^4 - I*(a^3 - a*b
^2)*d*f*cosh(1) - I*(a^3 - a*b^2)*d*f*sinh(1) + 2*(2*a^2*b*d*f^2*x + 2*a^2*
b*d*f*cosh(1) + 2*a^2*b*d*f*sinh(1) - I*(a^3 - a*b^2)*d*f^2*x - I*(a^3 - a*
b^2)*d*f*cosh(1) - I*(a^3 - a*b^2)*d*f*sinh(1))*cosh(d*x + c)^2 + 2*(2*a^2*
b*d*f^2*x + 2*a^2*b*d*f*cosh(1) + 2*a^2*b*d*f*sinh(1) - I*(a^3 - a*b^2)*d*f
^2*x - I*(a^3 - a*b^2)*d*f*cosh(1) - I*(a^3 - a*b^2)*d*f*sinh(1) + 3*(2*a^2
*b*d*f^2*x + 2*a^2*b*d*f*cosh(1) + 2*a^2*b*d*f*sinh(1) - I*(a^3 - a*b^2)*d*
f^2*x - I*(a^3 - a*b^2)*d*f*cosh(1) - I*(a^3 - a*b^2)*d*f*sinh(1))*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 + 4*((2*a^2*b*d*f^2*x + 2*a^2*b*d*f*cosh(1) + 2*a^
2*b*d*f*sinh(1) - I*(a^3 - a*b^2)*d*f^2*x - I*(a^3 - a*b^2)*d*f*cosh(1) - I
*(a^3 - a*b^2)*d*f*sinh(1))*cosh(d*x + c)^3 + (...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2\tanh^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sech(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*tanh(c + d*x)**2*sech(c + d*x)/(a + b*sinh(c + d*x)),
 x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(c+d\,x)^2\,(e+f\,x)^2}{\cosh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(c + d*x)^2*(e + f*x)^2)/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
[Out] int((tanh(c + d*x)^2*(e + f*x)^2)/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.388 $\int \frac{(e+fx)\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=760

$$-\frac{a(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{b^2d}+\frac{2a^3(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2d}+\frac{a^3(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d}+\frac{a^2b(e+fx)\log\left(1+\right.}{\left(a^2+b^2\right.}$$

```
[Out] -a*(f*x+e)*arctan(exp(d*x+c))/b^2/d+2*a^3*(f*x+e)*arctan(exp(d*x+c))/(a^2+b
^2)^2/d+a^3*(f*x+e)*arctan(exp(d*x+c))/b^2/(a^2+b^2)/d-a^2*b*(f*x+e)*ln(1+e
xp(2*d*x+2*c))/(a^2+b^2)^2/d+a^2*b*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(
1/2)))/(a^2+b^2)^2/d+a^2*b*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(
a^2+b^2)^2/d+1/2*I*a^3*f*polylog(2,I*exp(d*x+c))/b^2/(a^2+b^2)/d^2+I*a^3*f*
polylog(2,I*exp(d*x+c))/(a^2+b^2)^2/d^2-1/2*I*a*f*polylog(2,I*exp(d*x+c))/b
^2/d^2+1/2*I*a*f*polylog(2,-I*exp(d*x+c))/b^2/d^2-I*a^3*f*polylog(2,-I*exp(
d*x+c))/(a^2+b^2)^2/d^2-1/2*I*a^3*f*polylog(2,-I*exp(d*x+c))/b^2/(a^2+b^2)/
d^2-1/2*a^2*b*f*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^2+a^2*b*f*polylog(
2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2+a^2*b*f*polylog(2,-b*e
xp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2-1/2*a*f*sech(d*x+c)/b^2/d^2+
1/2*a^3*f*sech(d*x+c)/b^2/(a^2+b^2)/d^2-1/2*(f*x+e)*sech(d*x+c)^2/b/d+1/2*a
^2*(f*x+e)*sech(d*x+c)^2/b/(a^2+b^2)/d+1/2*f*tanh(d*x+c)/b/d^2-1/2*a^2*f*ta
nh(d*x+c)/b/(a^2+b^2)/d^2-1/2*a*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/b^2/d+1/2*a
^3*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/b^2/(a^2+b^2)/d
```

Rubi [A]
time = 0.94, antiderivative size = 760, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {5702, 5559, 3852, 8, 4270, 4265, 2317, 2438, 5692, 5680, 2221, 6874, 3799}

$$\frac{a^2bf\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^2}+\frac{a^2bf\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^2}-\frac{a^2bf\text{Li}_2\left(-e^{2(c+dx)}\right)}{2d^2\left(a^2+b^2\right)^2}-\frac{a^2f\tanh(c+dx)}{2bd^2\left(a^2+b^2\right)}+\frac{a^2b(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{d\left(a^2+b^2\right)^2}+\frac{a^2b(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{d\left(a^2+b^2\right)^2}-\frac{a^2b(e+fx)\log\left(e^{2(c+dx)}+1\right)}{d\left(a^2+b^2\right)^2}+\frac{a^2(e+fx)\text{sech}^2(c+dx)}{2bd\left(a^2+b^2\right)}+\frac{a^3(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{b^2d\left(a^2+b^2\right)}+\frac{2a^3(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{d\left(a^2+b^2\right)^2}-\frac{ia^3f\text{Li}_2\left(-ie^{c+dx}\right)}{2b^2d^2\left(a^2+b^2\right)}-\frac{ia^3f\text{Li}_2\left(-ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)^2}+\frac{ia^3f\text{Li}_2\left(ie^{c+dx}\right)}{2b^2d^2\left(a^2+b^2\right)}+\frac{ia^3f\text{Li}_2\left(ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)^2}+\frac{a^3f\text{sech}(c+dx)}{2b^2d^2\left(a^2+b^2\right)}+\frac{a^3(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2b^2d\left(a^2+b^2\right)}-\frac{a(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{b^2d}+\frac{iaf\text{Li}_2\left(-ie^{c+dx}\right)}{2b^2d^2}-\frac{iaf\text{Li}_2\left(ie^{c+dx}\right)}{2b^2d^2}-\frac{af\text{sech}(c+dx)}{2b^2d^2}-\frac{a(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2b^2d}+\frac{f\tanh(c+dx)}{2bd^2}-\frac{(e+fx)\text{sech}^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a*(e + f*x)*ArcTan[E^(c + d*x)])/(b^2*d)) + (2*a^3*(e + f*x)*ArcTan[E^(c
 + d*x)])/((a^2 + b^2)^2*d) + (a^3*(e + f*x)*ArcTan[E^(c + d*x)])/(b^2*(a^2
 + b^2)*d) + (a^2*b*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
])/((a^2 + b^2)^2*d) + (a^2*b*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/((a^2 + b^2)^2*d) - (a^2*b*(e + f*x)*Log[1 + E^(2*(c + d*x))])
/((a^2 + b^2)^2*d) + ((I/2)*a*f*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*d^2) - (
I*a^3*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - ((I/2)*a^3*f*Po
lyLog[2, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) - ((I/2)*a*f*PolyLog[2, I
*E^(c + d*x)])/(b^2*d^2) + (I*a^3*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)
^2*d^2) + ((I/2)*a^3*f*PolyLog[2, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) + (
```

```
a^2*b*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^
2*d^2) + (a^2*b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a
^2 + b^2)^2*d^2) - (a^2*b*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*
d^2) - (a*f*Sech[c + d*x])/(2*b^2*d^2) + (a^3*f*Sech[c + d*x])/(2*b^2*(a^2
+ b^2)*d^2) - ((e + f*x)*Sech[c + d*x]^2)/(2*b*d) + (a^2*(e + f*x)*Sech[c +
 d*x]^2)/(2*b*(a^2 + b^2)*d) + (f*Tanh[c + d*x])/(2*b*d^2) - (a^2*f*Tanh[c
+ d*x])/(2*b*(a^2 + b^2)*d^2) - (a*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(
2*b^2*d) + (a^3*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*b^2*(a^2 + b^2)*d
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5702

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
```

```
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{sech}^2(c+dx)\tanh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+}}{b}$$

$$= -\frac{(e+fx)\text{sech}^2(c+dx)}{2bd} - \frac{a\int (e+fx)\text{sech}^3(c+dx)\,dx}{b^2} + \frac{a^2\int}{}$$

$$= -\frac{af\text{sech}(c+dx)}{2b^2d^2} - \frac{(e+fx)\text{sech}^2(c+dx)}{2bd} - \frac{a(e+fx)\text{sech}(c-}{2b}$$

$$= -\frac{a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} - \frac{af\text{sech}(c+dx)}{2b^2d^2} - \frac{(e+fx)\text{sech}^2(c}{2bd}$$

$$= -\frac{a^2b(e+fx)^2}{2\left(a^2+b^2\right)^2 f} - \frac{a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} - \frac{af\text{sech}(c+dx)}{2b^2d^2} -$$

$$= -\frac{a^2b(e+fx)^2}{2\left(a^2+b^2\right)^2 f} - \frac{a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{a^2b(e+fx)\log\Big(1}{(a^2+}$$

$$= -\frac{a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} + \frac{a^3(e-}{}$$

$$= -\frac{a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} + \frac{a^3(e-}{}$$

$$= -\frac{a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} + \frac{a^3(e-}{}$$

$$= -\frac{a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} + \frac{a^3(e-}{}$$

Mathematica [A]
time = 6.79, size = 588, normalized size = 0.77

$$\frac{2a^2b\left(-\frac{1}{2}f(c+dx)^2+f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+f(c+dx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)+de\log(a+b\sinh(c+dx))-cf\log(a+b\sinh(c+dx))+f\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)+f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)+a\left(2abde(c+dx)-2abcf(c+dx)+abf(c+dx)^2+2a^2de\text{ArcTan}\left(e^{c+dx}\right)-2b^2de\text{ArcTan}\left(e^{c+dx}\right)-2a^2cf\text{ArcTan}\left(e^{c+dx}\right)+2b^2cf\text{ArcTan}\left(e^{c+dx}\right)+ia^2f(c+dx)\log\left(1-ie^{c+dx}\right)-ib^2f(c+dx)\log\left(1-ie^{c+dx}\right)-ia^2f(c+dx)\log\left(1+ie^{c+dx}\right)+ib^2f(c+dx)\log\left(1+ie^{c+dx}\right)-2abde\log\left(1+e^{2(c+dx)}\right)+2abcf\log\left(1+e^{2(c+dx)}\right)-2abf(c+dx)\log\left(1+e^{2(c+dx)}\right)-i\left(a^2-b^2\right)f\text{PolyLog}\left(2,-ie^{c+dx}\right)+i\left(a^2-b^2\right)f\text{PolyLog}\left(2,ie^{c+dx}\right)-abf\text{PolyLog}\left(2,-e^{2(c+dx)}\right)\right)-\left(a^2+b^2\right)d(e+fx)\text{sech}^2(c+dx)(b+a\sinh(c+dx))+\left(a^2+b^2\right)f\text{sech}(c+dx)(-a+b\sinh(c+dx))}{2\left(a^2+b^2\right)^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (2*a^2*b*(-1/2*(f*(c + d*x)^2) + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLo
g[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))]]) + a*(2*a*b*d*e*(c + d*x) - 2*a*b*c*f*(c + d*x)
+ a*b*f*(c + d*x)^2 + 2*a^2*d*e*ArcTan[E^(c + d*x)] - 2*b^2*d*e*ArcTan[E^(c
 + d*x)] - 2*a^2*c*f*ArcTan[E^(c + d*x)] + 2*b^2*c*f*ArcTan[E^(c + d*x)] +
I*a^2*f*(c + d*x)*Log[1 - I*E^(c + d*x)] - I*b^2*f*(c + d*x)*Log[1 - I*E^(c
 + d*x)] - I*a^2*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + I*b^2*f*(c + d*x)*Log
[1 + I*E^(c + d*x)] - 2*a*b*d*e*Log[1 + E^(2*(c + d*x))] + 2*a*b*c*f*Log[1
+ E^(2*(c + d*x))] - 2*a*b*f*(c + d*x)*Log[1 + E^(2*(c + d*x))] - I*(a^2 -
b^2)*f*PolyLog[2, (-I)*E^(c + d*x)] + I*(a^2 - b^2)*f*PolyLog[2, I*E^(c + d
*x)] - a*b*f*PolyLog[2, -E^(2*(c + d*x))]) - (a^2 + b^2)*d*(e + f*x)*Sech[c
 + d*x]^2*(b + a*Sinh[c + d*x]) + (a^2 + b^2)*f*Sech[c + d*x]*(-a + b*Sinh[
c + d*x]))/(2*(a^2 + b^2)^2*d^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2067 vs. 2(701) = 1402.
time = 4.27, size = 2068, normalized size = 2.72

method	result	size
risch	Expression too large to display	2068

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVER
BOSE)
```

```
[Out] -I*a/d^2/(a^2+b^2)*b^2*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))+2*a^2/d/(a^2+b
^2)*b*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2
)))*x+2*a^2/d^2/(a^2+b^2)*b*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/
2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2*a^2/d^2/(a^2+b^2)*b*f*c/(2*a^2+2*b^2)*ln(b*
exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2*a^2/d^2/(a^2+b^2)*b*f*c/(2*a^2+2*b^2)*ln
(1+exp(2*d*x+2*c))+2*a^2/d^2/(a^2+b^2)*b*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(
a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-2*a^2/d/(a^2+b^2)*b*f/(2*a^2+2*b^2
)*ln(1+I*exp(d*x+c))*x-2*a^2/d^2/(a^2+b^2)*b*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x
+c))*c-2*a^2/d^2/(a^2+b^2)*b*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c-2*a^2/d/(
```

```
a^2+b^2)*b*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x+I*a/d^2/(a^2+b^2)*b^2*f/(2*
a^2+2*b^2)*dilog(1+I*exp(d*x+c))+2*a^2/d/(a^2+b^2)*b*f/(2*a^2+2*b^2)*ln((-b
*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d^2/(a^2+b^2)^(1/2
)*a*b*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2
/d^2/(a^2+b^2)*a*b^2*f*c/(2*a^2+2*b^2)*arctan(exp(d*x+c))-1/d/(a^2+b^2)^(1/
2)*e*a*b/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2*
a^2/d^2/(a^2+b^2)*b*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))+2*a^2/d/(a^2+b^2)
*b*e/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d^2/(a^2+b^2)*a^
3*f*c/(2*a^2+2*b^2)*arctan(exp(d*x+c))+I*a/d/(a^2+b^2)*b^2*f/(2*a^2+2*b^2)*
ln(1+I*exp(d*x+c))*x+I*a/d^2/(a^2+b^2)*b^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c
))*c-I/d/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x-I/d^2/(a^2+b^2)
*a^3*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c+I/d/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)
*ln(1-I*exp(d*x+c))*x+I/d^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c)
)*c+1/d/(a^2+b^2)^(3/2)*e*a*b^3/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2
*a)/(a^2+b^2)^(1/2))+1/d/(a^2+b^2)^(3/2)*e*a^3*b/(2*a^2+2*b^2)*arctanh(1/2*
(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d/(a^2+b^2)*a^3*e/(2*a^2+2*b^2)*arc
tan(exp(d*x+c))+I/d^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))-I
*a/d^2/(a^2+b^2)*b^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c+2*a^2/d^2/(a^2+b^
2)*b*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1
/2)))+2*a^2/d^2/(a^2+b^2)*b*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^
(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2*a^2/d^2/(a^2+b^2)*b*f/(2*a^2+2*b^2)*dilog(
1-I*exp(d*x+c))-2*a^2/d/(a^2+b^2)*b*e/(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))-2/
d/(a^2+b^2)*e*a*b^2/(2*a^2+2*b^2)*arctan(exp(d*x+c))-I/d^2/(a^2+b^2)*a^3*f/
(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))-1/d^2/(a^2+b^2)^(3/2)*a^3*b*f*c/(2*a^2+
2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d^2/(a^2+b^2)^(3
/2)*a*b^3*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2
))-I*a/d/(a^2+b^2)*b^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x-(a*d*f*x*exp(3*
d*x+3*c)+a*d*e*exp(3*d*x+3*c)+2*b*d*f*x*exp(2*d*x+2*c)-a*d*f*x*exp(d*x+c)+a
*f*exp(3*d*x+3*c)+2*b*d*e*exp(2*d*x+2*c)-a*d*e*exp(d*x+c)+b*f*exp(2*d*x+2*c
)+a*f*exp(d*x+c)+b*f)/d^2/(a^2+b^2)/(1+exp(2*d*x+2*c))^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -f*(((a*d*x*e^(3*c) + a*e^(3*c))*e^(3*d*x) + (2*b*d*x*e^(2*c) + b*e^(2*c))*
e^(2*d*x) - (a*d*x*e^c - a*e^c)*e^(d*x) + b)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*
e^(4*c) + b^2*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c)
)*e^(2*d*x)) + 2*integrate(-(a^3*b*x*e^(d*x + c) - a^2*b^2*x)/(a^4*b + 2*a^
2*b^3 + b^5 - (a^4*b*e^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) -
```

```
 2*(a^5*e^c + 2*a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) - 2*integrate(1/2*(2*
a^2*b*x + (a^3*e^c - a*b^2*e^c)*x*e^(d*x))/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^
(2*c) + 2*a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x)) + (a^2*b*log(-2*a*
e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b^4)*d) - a^2*b*
log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a^3 - a*b^2)*arcta
n(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) - (a*e^(-d*x - c) + 2*b*e^(-2*d
*x - 2*c) - a*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*e^(-2*d*x - 2*c
) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d))*e
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5682 vs. $2(690) = 1380$.
time = 0.52, size = 5682, normalized size = 7.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -1/2*(2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*cosh(1) + (a^3 + a*b^2)*d*si
nh(1) + (a^3 + a*b^2)*f)*cosh(d*x + c)^3 + 2*((a^3 + a*b^2)*d*f*x + (a^3 +
a*b^2)*d*cosh(1) + (a^3 + a*b^2)*d*sinh(1) + (a^3 + a*b^2)*f)*sinh(d*x + c)
^3 + 2*(2*(a^2*b + b^3)*d*f*x + 2*(a^2*b + b^3)*d*cosh(1) + 2*(a^2*b + b^3)
*d*sinh(1) + (a^2*b + b^3)*f)*cosh(d*x + c)^2 + 2*(2*(a^2*b + b^3)*d*f*x +
2*(a^2*b + b^3)*d*cosh(1) + 2*(a^2*b + b^3)*d*sinh(1) + (a^2*b + b^3)*f + 3
*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*cosh(1) + (a^3 + a*b^2)*d*sinh(1) +
 (a^3 + a*b^2)*f)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(a^2*b + b^3)*f - 2*((
a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*cosh(1) + (a^3 + a*b^2)*d*sinh(1) - (a
^3 + a*b^2)*f)*cosh(d*x + c) - 2*(a^2*b*f*cosh(d*x + c)^4 + 4*a^2*b*f*cosh(
d*x + c)*sinh(d*x + c)^3 + a^2*b*f*sinh(d*x + c)^4 + 2*a^2*b*f*cosh(d*x + c
)^2 + a^2*b*f + 2*(3*a^2*b*f*cosh(d*x + c)^2 + a^2*b*f)*sinh(d*x + c)^2 + 4
*(a^2*b*f*cosh(d*x + c)^3 + a^2*b*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*
cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2) - b)/b + 1) - 2*(a^2*b*f*cosh(d*x + c)^4 + 4*a^2*b*f*cosh(
d*x + c)*sinh(d*x + c)^3 + a^2*b*f*sinh(d*x + c)^4 + 2*a^2*b*f*cosh(d*x + c
)^2 + a^2*b*f + 2*(3*a^2*b*f*cosh(d*x + c)^2 + a^2*b*f)*sinh(d*x + c)^2 + 4
*(a^2*b*f*cosh(d*x + c)^3 + a^2*b*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*
cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2) - b)/b + 1) + ((2*a^2*b*f - I*(a^3 - a*b^2)*f)*cosh(d*x +
c)^4 + 4*(2*a^2*b*f - I*(a^3 - a*b^2)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2
*a^2*b*f - I*(a^3 - a*b^2)*f)*sinh(d*x + c)^4 + 2*a^2*b*f + 2*(2*a^2*b*f -
I*(a^3 - a*b^2)*f)*cosh(d*x + c)^2 + 2*(2*a^2*b*f + 3*(2*a^2*b*f - I*(a^3 -
 a*b^2)*f)*cosh(d*x + c)^2 - I*(a^3 - a*b^2)*f)*sinh(d*x + c)^2 - I*(a^3 -
a*b^2)*f + 4*((2*a^2*b*f - I*(a^3 - a*b^2)*f)*cosh(d*x + c)^3 + (2*a^2*b*f
- I*(a^3 - a*b^2)*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(I*cosh(d*x + c) +
```

```
I*sinh(d*x + c)) + ((2*a^2*b*f + I*(a^3 - a*b^2)*f)*cosh(d*x + c)^4 + 4*(2*
a^2*b*f + I*(a^3 - a*b^2)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2*b*f + I
*(a^3 - a*b^2)*f)*sinh(d*x + c)^4 + 2*a^2*b*f + 2*(2*a^2*b*f + I*(a^3 - a*b
^2)*f)*cosh(d*x + c)^2 + 2*(2*a^2*b*f + 3*(2*a^2*b*f + I*(a^3 - a*b^2)*f)*c
osh(d*x + c)^2 + I*(a^3 - a*b^2)*f)*sinh(d*x + c)^2 + I*(a^3 - a*b^2)*f + 4
*((2*a^2*b*f + I*(a^3 - a*b^2)*f)*cosh(d*x + c)^3 + (2*a^2*b*f + I*(a^3 - a
*b^2)*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(-I*cosh(d*x + c) - I*sinh(d*x
+ c)) + 2*(a^2*b*c*f - a^2*b*d*cosh(1) + (a^2*b*c*f - a^2*b*d*cosh(1) - a^2
*b*d*sinh(1))*cosh(d*x + c)^4 - a^2*b*d*sinh(1) + 4*(a^2*b*c*f - a^2*b*d*co
sh(1) - a^2*b*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b*c*f - a^2*b
*d*cosh(1) - a^2*b*d*sinh(1))*sinh(d*x + c)^4 + 2*(a^2*b*c*f - a^2*b*d*cosh
(1) - a^2*b*d*sinh(1))*cosh(d*x + c)^2 + 2*(a^2*b*c*f - a^2*b*d*cosh(1) - a
^2*b*d*sinh(1) + 3*(a^2*b*c*f - a^2*b*d*cosh(1) - a^2*b*d*sinh(1))*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 + 4*((a^2*b*c*f - a^2*b*d*cosh(1) - a^2*b*d*sinh(1
))*cosh(d*x + c)^3 + (a^2*b*c*f - a^2*b*d*cosh(1) - a^2*b*d*sinh(1))*cosh(d
*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqr
t((a^2 + b^2)/b^2) + 2*a) + 2*(a^2*b*c*f - a^2*b*d*cosh(1) + (a^2*b*c*f - a
^2*b*d*cosh(1) - a^2*b*d*sinh(1))*cosh(d*x + c)^4 - a^2*b*d*sinh(1) + 4*(a^
2*b*c*f - a^2*b*d*cosh(1) - a^2*b*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3
+ (a^2*b*c*f - a^2*b*d*cosh(1) - a^2*b*d*sinh(1))*sinh(d*x + c)^4 + 2*(a^2*
b*c*f - a^2*b*d*cosh(1) - a^2*b*d*sinh(1))*cosh(d*x + c)^2 + 2*(a^2*b*c*f -
 a^2*b*d*cosh(1) - a^2*b*d*sinh(1) + 3*(a^2*b*c*f - a^2*b*d*cosh(1) - a^2*b
*d*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^2*b*c*f - a^2*b*d*cosh
(1) - a^2*b*d*sinh(1))*cosh(d*x + c)^3 + (a^2*b*c*f - a^2*b*d*cosh(1) - a^2
*b*d*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sin
h(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(a^2*b*d*f*x + a^2*b*c*f
+ (a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^4 + 4*(a^2*b*d*f*x + a^2*b*c*f)*c
osh(d*x + c)*sinh(d*x + c)^3 + (a^2*b*d*f*x + a^2*b*c*f)*sinh(d*x + c)^4 +
2*(a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*f*x + a^2*b*c*f +
3*(a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^2*b*d*
f*x + a^2*b*c*f)*cosh(d*x + c)^3 + (a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c))
*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(a^2*b*d*f*x + a^2*b*c
*f + (a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^4 + 4*(a^2*b*d*f*x + a^2*b*c*f
)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b*d*f*x + a^2*b*c*f)*sinh(d*x + c)^4
 + 2*(a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*f*x + a^2*b*c*f
 + 3*(a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^2*b
*d*f*x + a^2*b*c*f)*cosh(d*x + c)^3 + (a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x +
c))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (2*a^2*b*c*f - 2*a^2*
b*d*cosh(1) + (2*a^2*b*c*f - 2*a^2*b*d*cosh(1) ...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\tanh^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*tanh(c + d*x)**2*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(c+d\,x)^2\,(e+f\,x)}{\cosh(c+d\,x)\,\left(a+b\sinh(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(c + d*x)^2*(e + f*x))/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((tanh(c + d*x)^2*(e + f*x))/(cosh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.389 $\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=121

$$\frac{a(a^2-b^2)\operatorname{ArcTan}(\sinh(c+dx))}{2(a^2+b^2)^2 d}-\frac{a^2b\log(\cosh(c+dx))}{(a^2+b^2)^2 d}+\frac{a^2b\log(a+b\sinh(c+dx))}{(a^2+b^2)^2 d}-\frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2(a^2+b^2)d}$$

```
[Out] 1/2*a*(a^2-b^2)*arctan(sinh(d*x+c))/(a^2+b^2)^2/d-a^2*b*ln(cosh(d*x+c))/(a^
2+b^2)^2/d+a^2*b*ln(a+b*sinh(d*x+c))/(a^2+b^2)^2/d-1/2*sech(d*x+c)^2*(b+a*s
inh(d*x+c))/(a^2+b^2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2916, 12, 1661, 815, 649, 209, 266}

$$\frac{a(a^2-b^2)\operatorname{ArcTan}(\sinh(c+dx))}{2d(a^2+b^2)^2}+\frac{a^2b\log(a+b\sinh(c+dx))}{d(a^2+b^2)^2}-\frac{a^2b\log(\cosh(c+dx))}{d(a^2+b^2)^2}-\frac{\operatorname{sech}^2(c+dx)(a\sinh(c+dx)+b)}{2d(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*(a^2 - b^2)*ArcTan[Sinh[c + d*x]])/(2*(a^2 + b^2)^2*d) - (a^2*b*Log[Cosh
[c + d*x]])/((a^2 + b^2)^2*d) + (a^2*b*Log[a + b*Sinh[c + d*x]])/((a^2 + b^
2)^2*d) - (Sech[c + d*x]^2*(b + a*Sinh[c + d*x]))/(2*(a^2 + b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b^3\text{Subst}\left(\int \frac{x^2}{b^2(a+x)(-b^2-x^2)^2}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \frac{x^2}{(a+x)(-b^2-x^2)^2}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= -\frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2(a^2+b^2)d} - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^2}{a^2+b^2}-\frac{ab^2x}{a^2+b^2}}{(a+x)(-b^2-x^2)}\,dx, x, b\sinh\right.}{2bd}$$

$$= -\frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2(a^2+b^2)d} - \frac{\text{Subst}\left(\int \left(-\frac{2a^2b^2}{(a^2+b^2)^2(a+x)} - \frac{ab^2(a^2-}{(a^2+b^2)}\right.\right.}{2bd}$$

$$= \frac{a^2b\log(a+b\sinh(c+dx))}{(a^2+b^2)^2d} - \frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2(a^2+b^2)d} + \frac{(ab)\text{S}}{}$$

$$= \frac{a^2b\log(a+b\sinh(c+dx))}{(a^2+b^2)^2d} - \frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2(a^2+b^2)d} - \frac{(a^2b)}{}$$

$$= \frac{a(a^2-b^2)\tan^{-1}(\sinh(c+dx))}{2(a^2+b^2)^2d} - \frac{a^2b\log(\cosh(c+dx))}{(a^2+b^2)^2d} + \frac{a^2b\log(a+b\sinh}{(a^2+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.28, size = 130, normalized size = 1.07

$$-\frac{a((a^2+b^2)\text{ArcTan}(\sinh(c+dx))+a((ia+b)\log(i-\sinh(c+dx))+(-ia+b)\log(i+\sinh(c+dx))-2b\log(a+b\sinh(c+dx))))+b(a^2+b^2)\operatorname{sech}^2(c+dx)+a(a^2+b^2)\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a^2+b^2)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/2*(a*((a^2 + b^2)*ArcTan[Sinh[c + d*x]] + a*((I*a + b)*Log[I - Sinh[c +
d*x]] + ((-I)*a + b)*Log[I + Sinh[c + d*x]] - 2*b*Log[a + b*Sinh[c + d*x]])
) + b*(a^2 + b^2)*Sech[c + d*x]^2 + a*(a^2 + b^2)*Sech[c + d*x]*Tanh[c + d*
x])/((a^2 + b^2)^2*d)
```

Maple [A]
time = 1.48, size = 209, normalized size = 1.73

method	result
derivativedivides	$\frac{\frac{\frac{2\left(\left(\frac{1}{2}a^3+\frac{1}{2}ab^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^2b+b^3\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{1}{2}a^3-\frac{1}{2}ab^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+a\left(-ab\ln\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\left(a\right.\right.}{a^4+2a^2b^2+b^4}}{d}$

default	$\frac{\frac{\frac{2\left(\left(\frac{1}{2}a^3+\frac{1}{2}ab^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^2b+b^3\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{1}{2}a^3-\frac{1}{2}ab^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+a\left(-ab\ln\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\right.}{a^4+2a^2b^2+b^4}}{d}$
risch	$\frac{2a^2bd^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{2a^2bdc}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{2a^2bx}{a^4+2a^2b^2+b^4}-\frac{2a^2bc}{d(a^4+2a^2b^2+b^4)}-\frac{e^{dx+c}\left(a\,e^{2dx+2c}+2b\,e^{dx+c}\right.}{d(a^2+b^2)(1+e^{2dx+2c})^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/(a^4+2*a^2*b^2+b^4)*(((1/2*a^3+1/2*a*b^2)*tanh(1/2*d*x+1/2*c)^3+(a^2
*b+b^3)*tanh(1/2*d*x+1/2*c)^2+(-1/2*a^3-1/2*a*b^2)*tanh(1/2*d*x+1/2*c))/(ta
nh(1/2*d*x+1/2*c)^2+1)^2+1/2*a*(-a*b*ln(tanh(1/2*d*x+1/2*c)^2+1)+(a^2-b^2)*
arctan(tanh(1/2*d*x+1/2*c))))+4*a^2*b/(4*a^4+8*a^2*b^2+4*b^4)*ln(a*tanh(1/2
*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)-a))
```

Maxima [A]
time = 0.49, size = 219, normalized size = 1.81

$$\frac{a^2b\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^4+2\,a^2b^2+b^4)d}-\frac{a^2b\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^4+2\,a^2b^2+b^4)d}-\frac{(a^3-ab^2)\arctan\left(e^{(-dx-c)}\right)}{(a^4+2\,a^2b^2+b^4)d}-\frac{ae^{(-dx-c)}+2\,be^{(-2\,dx-2\,c)}-ae^{(-3\,dx-3\,c)}}{(a^2+b^2+2\,(a^2+b^2)e^{(-2\,dx-2\,c)}+(a^2+b^2)e^{(-4\,dx-4\,c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] a^2*b*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b
^4)*d) - a^2*b*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a^3
 - a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) - (a*e^(-d*x - c
) + 2*b*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*
e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 917 vs. $2(117) = 234$.
time = 0.35, size = 917, normalized size = 7.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -((a^3 + a*b^2)*cosh(d*x + c)^3 + (a^3 + a*b^2)*sinh(d*x + c)^3 + 2*(a^2*b
+ b^3)*cosh(d*x + c)^2 + (2*a^2*b + 2*b^3 + 3*(a^3 + a*b^2)*cosh(d*x + c))*
sinh(d*x + c)^2 - ((a^3 - a*b^2)*cosh(d*x + c)^4 + 4*(a^3 - a*b^2)*cosh(d*x
 + c)*sinh(d*x + c)^3 + (a^3 - a*b^2)*sinh(d*x + c)^4 + a^3 - a*b^2 + 2*(a^
```

```
3 - a*b^2)*cosh(d*x + c)^2 + 2*(a^3 - a*b^2 + 3*(a^3 - a*b^2)*cosh(d*x + c)
^2)*sinh(d*x + c)^2 + 4*((a^3 - a*b^2)*cosh(d*x + c)^3 + (a^3 - a*b^2)*cosh
(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - (a^3 + a*
b^2)*cosh(d*x + c) - (a^2*b*cosh(d*x + c)^4 + 4*a^2*b*cosh(d*x + c)*sinh(d*
x + c)^3 + a^2*b*sinh(d*x + c)^4 + 2*a^2*b*cosh(d*x + c)^2 + a^2*b + 2*(3*a
^2*b*cosh(d*x + c)^2 + a^2*b)*sinh(d*x + c)^2 + 4*(a^2*b*cosh(d*x + c)^3 +
a^2*b*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x +
 c) - sinh(d*x + c))) + (a^2*b*cosh(d*x + c)^4 + 4*a^2*b*cosh(d*x + c)*sinh
(d*x + c)^3 + a^2*b*sinh(d*x + c)^4 + 2*a^2*b*cosh(d*x + c)^2 + a^2*b + 2*(
3*a^2*b*cosh(d*x + c)^2 + a^2*b)*sinh(d*x + c)^2 + 4*(a^2*b*cosh(d*x + c)^3
 + a^2*b*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) -
 sinh(d*x + c))) - (a^3 + a*b^2 - 3*(a^3 + a*b^2)*cosh(d*x + c)^2 - 4*(a^2*
b + b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x
+ c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 +
 2*a^2*b^2 + b^4)*d*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x
+ c)^2 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2
+ b^4)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d + 4*((a^4 + 2*a^2*b^2
 + b^4)*d*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c))*sinh(d
*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)**2*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(117) = 234.
time = 0.49, size = 281, normalized size = 2.32

$$\frac{\frac{4a^2b^2\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2a\right|\right)}{a^4b+2a^2b^3+b^5}-\frac{2a^2b\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^4+2a^2b^2+b^4}+\frac{\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)\left(a^3-ab^2\right)}{a^4+2a^2b^2+b^4}+\frac{2\left(a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-2a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)-2ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)-4b^3\right)}{\left(a^4+2a^2b^2+b^4\right)\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(4*a^2*b^2*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^4*b + 2*a^
2*b^3 + b^5) - 2*a^2*b*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^4 + 2*a^2
*b^2 + b^4) + (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(a^3
- a*b^2)/(a^4 + 2*a^2*b^2 + b^4) + 2*(a^2*b*(e^(d*x + c) - e^(-d*x - c))^2
- 2*a^3*(e^(d*x + c) - e^(-d*x - c)) - 2*a*b^2*(e^(d*x + c) - e^(-d*x - c))
 - 4*b^3)/((a^4 + 2*a^2*b^2 + b^4)*((e^(d*x + c) - e^(-d*x - c))^2 + 4)))/d
```

Mupad [B]
time = 2.02, size = 339, normalized size = 2.80

$$\frac{\frac{2\,b}{d\,(a^2+b^2)}+\frac{2\,a\,\mathrm{e}^{c+d\,x}}{d\,(a^2+b^2)}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1}-\frac{\frac{2\,(a^2\,b+b^3)}{d\,{(a^2+b^2)}^2}+\frac{\mathrm{e}^{c+d\,x}\,(a^3+a\,b^2)}{d\,{(a^2+b^2)}^2}}{\mathrm{e}^{2\,c+2\,d\,x}+1}-\frac{a\,\ln\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)}{2\,(-1\mathrm{i}\,d\,a^2+2\,d\,a\,b+1\mathrm{i}\,d\,b^2)}+\frac{a^2\,b\,\ln\left(2\,a^7\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-a^2\,b^5-14\,a^4\,b^3-a^6\,b+a^6\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+2\,a^3\,b^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+28\,a^5\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+a^2\,b^5\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+14\,a^4\,b^3\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\right)}{d\,a^4+2\,d\,a^2\,b^2+d\,b^4}-\frac{a\,\ln\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)\,1\mathrm{i}}{2\,(-d\,a^2+2\mathrm{i}\,d\,a\,b+d\,b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2/(cosh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] ((2*b)/(d*(a^2 + b^2)) + (2*a*exp(c + d*x))/(d*(a^2 + b^2)))/(2*exp(2*c + 2
*d*x) + exp(4*c + 4*d*x) + 1) - ((2*(a^2*b + b^3))/(d*(a^2 + b^2)^2) + (exp
(c + d*x)*(a*b^2 + a^3))/(d*(a^2 + b^2)^2))/(exp(2*c + 2*d*x) + 1) - (a*log
(exp(c + d*x) + 1i)*1i)/(2*(b^2*d - a^2*d + a*b*d*2i)) - (a*log(exp(c + d*x
)*1i + 1))/(2*(b^2*d*1i - a^2*d*1i + 2*a*b*d)) + (a^2*b*log(2*a^7*exp(d*x)*
exp(c) - a^2*b^5 - 14*a^4*b^3 - a^6*b + a^6*b*exp(2*c)*exp(2*d*x) + 2*a^3*b
^4*exp(d*x)*exp(c) + 28*a^5*b^2*exp(d*x)*exp(c) + a^2*b^5*exp(2*c)*exp(2*d*
x) + 14*a^4*b^3*exp(2*c)*exp(2*d*x)))/(a^4*d + b^4*d + 2*a^2*b^2*d)
```

3.390 $\int \frac{\text{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{\text{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(Sech[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][(Sech[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\text{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx = \int \frac{\text{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} \, dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(Sech[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\text{sech}(dx+c)\left(\tanh^2(dx+c)\right)}{(fx+e)\left(a+b\sinh(dx+c)\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] (b*f - (a*d*f*x*e^(3*c) - a*f*e^(3*c) + a*d*e^(3*c + 1))*e^(3*d*x) - (2*b*d
*f*x*e^(2*c) - b*f*e^(2*c) + 2*b*d*e^(2*c + 1))*e^(2*d*x) + (a*d*f*x*e^c +
a*d*e^(c + 1) + a*f*e^c)*e^(d*x))/((a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*(a^2
*d^2*f + b^2*d^2*f)*x*e + (a^2*d^2 + b^2*d^2)*e^2 + ((a^2*d^2*f^2*e^(4*c) +
 b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^2*d^2*f*e^(4*c) + b^2*d^2*f*e^(4*c))*x*e +
 (a^2*d^2*e^(4*c) + b^2*d^2*e^(4*c))*e^2)*e^(4*d*x) + 2*((a^2*d^2*f^2*e^(2*
c) + b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^2*d^2*f*e^(2*c) + b^2*d^2*f*e^(2*c))*x
*e + (a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^2)*e^(2*d*x)) + 2*integrate(1/2*
(2*a^2*b*d^2*f^2*x^2 + 4*a^2*b*d^2*f*x*e + 2*a^2*b*d^2*e^2 + 2*a^2*b*f^2 +
2*b^3*f^2 + (2*a^3*f^2*e^c + 2*a*b^2*f^2*e^c + (a^3*d^2*f^2*e^c - a*b^2*d^2
*f^2*e^c)*x^2 + 2*(a^3*d^2*f*e^c - a*b^2*d^2*f*e^c)*x*e + (a^3*d^2*e^c - a*
b^2*d^2*e^c)*e^2)*e^(d*x))/((a^4*d^2*f^3 + 2*a^2*b^2*d^2*f^3 + b^4*d^2*f^3)
*x^3 + 3*(a^4*d^2*f^2 + 2*a^2*b^2*d^2*f^2 + b^4*d^2*f^2)*x^2*e + 3*(a^4*d^2
*f + 2*a^2*b^2*d^2*f + b^4*d^2*f)*x*e^2 + (a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^
2)*e^3 + ((a^4*d^2*f^3*e^(2*c) + 2*a^2*b^2*d^2*f^3*e^(2*c) + b^4*d^2*f^3*e^
(2*c))*x^3 + 3*(a^4*d^2*f^2*e^(2*c) + 2*a^2*b^2*d^2*f^2*e^(2*c) + b^4*d^2*f
^2*e^(2*c))*x^2*e + 3*(a^4*d^2*f*e^(2*c) + 2*a^2*b^2*d^2*f*e^(2*c) + b^4*d^
2*f*e^(2*c))*x*e^2 + (a^4*d^2*e^(2*c) + 2*a^2*b^2*d^2*e^(2*c) + b^4*d^2*e^(
2*c))*e^3)*e^(2*d*x)), x) - 2*integrate(-(a^3*b*e^(d*x + c) - a^2*b^2)/((a^
4*b*f + 2*a^2*b^3*f + b^5*f)*x + (a^4*b + 2*a^2*b^3 + b^5)*e - ((a^4*b*f*e^
(2*c) + 2*a^2*b^3*f*e^(2*c) + b^5*f*e^(2*c))*x + (a^4*b*e^(2*c) + 2*a^2*b^3
*e^(2*c) + b^5*e^(2*c))*e)*e^(2*d*x) - 2*((a^5*f*e^c + 2*a^3*b^2*f*e^c + a*
b^4*f*e^c)*x + (a^5*e^c + 2*a^3*b^2*e^c + a*b^4*e^c)*e)*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(sech(d*x + c)*tanh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(c+dx)\operatorname{sech}(c+dx)}{(a+b\sinh(c+dx))\,(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)**2*sech(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)),
x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tanh(c+d\,x)^2}{\cosh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2/(cosh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(tanh(c + d*x)^2/(cosh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.391 $\int \frac{(e+fx)^3 \cosh(c+dx) \sinh^3(c+dx)}{a+b\sinh(c+dx)} \, dx$

Optimal. Leaf size=792

$$-\frac{3af^3x}{8b^2d^3}-\frac{a(e+fx)^3}{4b^2d}+\frac{a^3(e+fx)^4}{4b^4f}-\frac{6a^2f^3\cosh(c+dx)}{b^3d^4}+\frac{14f^3\cosh(c+dx)}{9bd^4}-\frac{3a^2f(e+fx)^2\cosh(c+dx)}{b^3d^2}$$

```
[Out] -3/8*a*f^3*x/b^2/d^3-6*a^2*f^3*cosh(d*x+c)/b^3/d^4-a^3*(f*x+e)^3*ln(1+b*exp
(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d-a^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+
b^2)^(1/2)))/b^4/d-6*a^3*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b
^4/d^4-6*a^3*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d^4-1/4*a
*(f*x+e)^3/b^2/d-2/27*f^3*cosh(d*x+c)^3/b/d^4+14/9*f^3*cosh(d*x+c)/b/d^4-3*
a^2*f*(f*x+e)^2*cosh(d*x+c)/b^3/d^2+6*a^2*f^2*(f*x+e)*sinh(d*x+c)/b^3/d^3+3
/8*a*f^3*cosh(d*x+c)*sinh(d*x+c)/b^2/d^4-3/4*a*f^2*(f*x+e)*sinh(d*x+c)^2/b^
2/d^3+1/4*a^3*(f*x+e)^4/b^4/f-3*a^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/b^4/d^2-3*a^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))/b^4/d^2+6*a^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))/b^4/d^3+6*a^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/b^4/d^3+1/3*(f*x+e)^3*sinh(d*x+c)^3/b/d-1/3*f*(f*x+e)^2*cosh(d*x+c)*s
inh(d*x+c)^2/b/d^2+3/4*a*f*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/b^2/d^2-1/2*a*
(f*x+e)^3*sinh(d*x+c)^2/b^2/d+2/9*f^2*(f*x+e)*sinh(d*x+c)^3/b/d^3+2/3*f*(f*
x+e)^2*cosh(d*x+c)/b/d^2-4/3*f^2*(f*x+e)*sinh(d*x+c)/b/d^3+a^2*(f*x+e)^3*si
nh(d*x+c)/b^3/d
```

Rubi [A]
time = 0.85, antiderivative size = 792, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 15, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.441$, Rules used = {5698, 5554, 3392, 3377, 2718, 2713, 32, 2715, 8, 5680, 2221, 2611, 6744, 2320, 6724}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-3*a*f^3*x)/(8*b^2*d^3) - (a*(e + f*x)^3)/(4*b^2*d) + (a^3*(e + f*x)^4)/(4
*b^4*f) - (6*a^2*f^3*Cosh[c + d*x])/(b^3*d^4) + (14*f^3*Cosh[c + d*x])/(9*b
*d^4) - (3*a^2*f*(e + f*x)^2*Cosh[c + d*x])/(b^3*d^2) + (2*f*(e + f*x)^2*Co
sh[c + d*x])/(3*b*d^2) - (2*f^3*Cosh[c + d*x]^3)/(27*b*d^4) - (a^3*(e + f*x
)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a^3*(e + f*x
)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^4*d) - (3*a^3*f*(e +
 f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^2) - (
3*a^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
```

```
b^4*d^2) + (6*a^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2]))])/(b^4*d^3) + (6*a^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2]))])/(b^4*d^3) - (6*a^3*f^3*PolyLog[4, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/(b^4*d^4) - (6*a^3*f^3*PolyLog[4, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/(b^4*d^4) + (6*a^2*f^2*(e + f*x)*Sinh[c + d*x])/
(b^3*d^3) - (4*f^2*(e + f*x)*Sinh[c + d*x])/(3*b*d^3) + (a^2*(e + f*x)^3*Si
nh[c + d*x])/(b^3*d) + (3*a*f^3*Cosh[c + d*x]*Sinh[c + d*x])/(8*b^2*d^4) +
(3*a*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^2*d^2) - (3*a*f^2*(e +
 f*x)*Sinh[c + d*x]^2)/(4*b^2*d^3) - (a*(e + f*x)^3*Sinh[c + d*x]^2)/(2*b^2
*d) - (f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^2)/(3*b*d^2) + (2*f^2*(e +
 f*x)*Sinh[c + d*x]^3)/(9*b*d^3) + ((e + f*x)^3*Sinh[c + d*x]^3)/(3*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}dx = \frac{\int (e+fx)^3\cosh(c+dx)\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\cosh(c+}{a+b\sinh}}{b}$$

$$= \frac{(e+fx)^3\sinh^3(c+dx)}{3bd} - \frac{a\int (e+fx)^3\cosh(c+dx)\sinh(c+}{b^2}$$

$$= -\frac{a(e+fx)^3\sinh^2(c+dx)}{2b^2d} - \frac{f(e+fx)^2\cosh(c+dx)\sinh^2(}{3bd^2}$$

$$= \frac{a^3(e+fx)^4}{4b^4f} + \frac{2f(e+fx)^2\cosh(c+dx)}{3bd^2} + \frac{a^2(e+fx)^3\sinh(}{b^3d}$$

$$= -\frac{a(e+fx)^3}{4b^2d} + \frac{a^3(e+fx)^4}{4b^4f} + \frac{2f^3\cosh(c+dx)}{9bd^4} - \frac{3a^2f(e+}{}$$

$$= -\frac{3af^3x}{8b^2d^3} - \frac{a(e+fx)^3}{4b^2d} + \frac{a^3(e+fx)^4}{4b^4f} + \frac{14f^3\cosh(c+dx)}{9bd^4} -$$

$$= -\frac{3af^3x}{8b^2d^3} - \frac{a(e+fx)^3}{4b^2d} + \frac{a^3(e+fx)^4}{4b^4f} - \frac{6a^2f^3\cosh(c+dx)}{b^3d^4} -$$

$$= -\frac{3af^3x}{8b^2d^3} - \frac{a(e+fx)^3}{4b^2d} + \frac{a^3(e+fx)^4}{4b^4f} - \frac{6a^2f^3\cosh(c+dx)}{b^3d^4} -$$

$$= -\frac{3af^3x}{8b^2d^3} - \frac{a(e+fx)^3}{4b^2d} + \frac{a^3(e+fx)^4}{4b^4f} - \frac{6a^2f^3\cosh(c+dx)}{b^3d^4} -$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3901 vs. $2(792) = 1584$.
time = 6.47, size = 3901, normalized size = 4.93

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (1296*a^3*c^2*d^2*e^2*E^(3*c)*f + 2592*a^3*c*d^3*e^2*E^(3*c)*f*x + 1296*a^3
*d^4*e^2*E^(3*c)*f*x^2 + 864*a^3*d^4*e*E^(3*c)*f^2*x^3 + 216*a^3*d^4*E^(3*c
```

)*f^3*x^4 - 2592*a^2*b*d*e*E^(2*c)*f^2*Cosh[d*x] + 648*b^3*d*e*E^(2*c)*f^2*
Cosh[d*x] + 2592*a^2*b*d*e*E^(4*c)*f^2*Cosh[d*x] - 648*b^3*d*e*E^(4*c)*f^2*
Cosh[d*x] - 2592*a^2*b*E^(2*c)*f^3*Cosh[d*x] + 648*b^3*E^(2*c)*f^3*Cosh[d*x
] - 2592*a^2*b*E^(4*c)*f^3*Cosh[d*x] + 648*b^3*E^(4*c)*f^3*Cosh[d*x] - 2592
*a^2*b*d^2*e*E^(2*c)*f^2*x*Cosh[d*x] + 648*b^3*d^2*e*E^(2*c)*f^2*x*Cosh[d*x
] - 2592*a^2*b*d^2*e*E^(4*c)*f^2*x*Cosh[d*x] + 648*b^3*d^2*e*E^(4*c)*f^2*x*
Cosh[d*x] - 2592*a^2*b*d*E^(2*c)*f^3*x*Cosh[d*x] + 648*b^3*d*E^(2*c)*f^3*x*
Cosh[d*x] + 2592*a^2*b*d*E^(4*c)*f^3*x*Cosh[d*x] - 648*b^3*d*E^(4*c)*f^3*x*
Cosh[d*x] - 1296*a^2*b*d^3*e*E^(2*c)*f^2*x^2*Cosh[d*x] + 324*b^3*d^3*e*E^(2
*c)*f^2*x^2*Cosh[d*x] + 1296*a^2*b*d^3*e*E^(4*c)*f^2*x^2*Cosh[d*x] - 324*b^
3*d^3*e*E^(4*c)*f^2*x^2*Cosh[d*x] - 1296*a^2*b*d^2*E^(2*c)*f^3*x^2*Cosh[d*x
] + 324*b^3*d^2*E^(2*c)*f^3*x^2*Cosh[d*x] - 1296*a^2*b*d^2*E^(4*c)*f^3*x^2*
Cosh[d*x] + 324*b^3*d^2*E^(4*c)*f^3*x^2*Cosh[d*x] - 432*a^2*b*d^3*E^(2*c)*f
^3*x^3*Cosh[d*x] + 108*b^3*d^3*E^(2*c)*f^3*x^3*Cosh[d*x] + 432*a^2*b*d^3*E^
(4*c)*f^3*x^3*Cosh[d*x] - 108*b^3*d^3*E^(4*c)*f^3*x^3*Cosh[d*x] - 162*a*b^2
*d*e*E^c*f^2*Cosh[2*d*x] - 162*a*b^2*d*e*E^(5*c)*f^2*Cosh[2*d*x] - 81*a*b^2
*E^c*f^3*Cosh[2*d*x] + 81*a*b^2*E^(5*c)*f^3*Cosh[2*d*x] - 324*a*b^2*d^2*e*E
^c*f^2*x*Cosh[2*d*x] + 324*a*b^2*d^2*e*E^(5*c)*f^2*x*Cosh[2*d*x] - 162*a*b^
2*d*E^c*f^3*x*Cosh[2*d*x] - 162*a*b^2*d*E^(5*c)*f^3*x*Cosh[2*d*x] - 324*a*b
^2*d^3*e*E^c*f^2*x^2*Cosh[2*d*x] - 324*a*b^2*d^3*e*E^(5*c)*f^2*x^2*Cosh[2*d
*x] - 162*a*b^2*d^2*E^c*f^3*x^2*Cosh[2*d*x] + 162*a*b^2*d^2*E^(5*c)*f^3*x^2
*Cosh[2*d*x] - 108*a*b^2*d^3*E^c*f^3*x^3*Cosh[2*d*x] - 108*a*b^2*d^3*E^(5*c
)*f^3*x^3*Cosh[2*d*x] - 24*b^3*d*e*f^2*Cosh[3*d*x] + 24*b^3*d*e*E^(6*c)*f^2
*Cosh[3*d*x] - 8*b^3*f^3*Cosh[3*d*x] - 8*b^3*E^(6*c)*f^3*Cosh[3*d*x] - 72*b
^3*d^2*e*f^2*x*Cosh[3*d*x] - 72*b^3*d^2*e*E^(6*c)*f^2*x*Cosh[3*d*x] - 24*b^
3*d*f^3*x*Cosh[3*d*x] + 24*b^3*d*E^(6*c)*f^3*x*Cosh[3*d*x] - 108*b^3*d^3*e*
f^2*x^2*Cosh[3*d*x] + 108*b^3*d^3*e*E^(6*c)*f^2*x^2*Cosh[3*d*x] - 36*b^3*d^
2*f^3*x^2*Cosh[3*d*x] - 36*b^3*d^2*E^(6*c)*f^3*x^2*Cosh[3*d*x] - 36*b^3*d^3
*f^3*x^3*Cosh[3*d*x] + 36*b^3*d^3*E^(6*c)*f^3*x^3*Cosh[3*d*x] - 2592*a^2*b*
d^2*e^2*E^(3*c)*f*Cosh[c + d*x] + 648*b^3*d^2*e^2*E^(3*c)*f*Cosh[c + d*x] -
 216*a*b^2*d^3*e^3*E^(3*c)*Cosh[2*(c + d*x)] - 648*a*b^2*d^3*e^2*E^(3*c)*f*
x*Cosh[2*(c + d*x)] - 72*b^3*d^2*e^2*E^(3*c)*f*Cosh[3*(c + d*x)] - 2592*a^3
*c*d^2*e^2*E^(3*c)*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2592*
a^3*d^3*e^2*E^(3*c)*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 25
92*a^3*c*d^2*e^2*E^(3*c)*f*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] -
 2592*a^3*d^3*e^2*E^(3*c)*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
] - 2592*a^3*d^3*e*E^(3*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[
(a^2 + b^2)*E^(2*c)])] - 864*a^3*d^3*E^(3*c)*f^3*x^3*Log[1 + (b*E^(2*c + d*
x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - 2592*a^3*d^3*e*E^(3*c)*f^2*x^2*L
og[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 864*a^3*d^3
*E^(3*c)*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c
)])] - 864*a^3*d^3*e^3*E^(3*c)*Log[a + b*Sinh[c + d*x]] + 2592*a^3*c*d^2*e^
2*E^(3*c)*f*Log[a + b*Sinh[c + d*x]] - 2592*a^3*d^2*e^2*E^(3*c)*f*PolyLog[2
, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2592*a^3*d^2*e^2*E^(3*c)*f*Poly
Log[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 5184*a^3*d^2*e*E^(3*c)*f

```
^2*x*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] -
 2592*a^3*d^2*E^(3*c)*f^3*x^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[
(a^2 + b^2)*E^(2*c)]))] - 5184*a^3*d^2*e*E^(3*c)*f^2*x*PolyLog[2, -((b*E^(2
*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2592*a^3*d^2*E^(3*c)*f^3
*x^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] +
 5184*a^3*d*e*E^(3*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2
 + b^2)*E^(2*c)]))] + 5184*a^3*d*E^(3*c)*f^3*x*PolyLog[3, -((b*E^(2*c + d*x
))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 5184*a^3*d*e*E^(3*c)*f^2*PolyLog
[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 5184*a^3*d*
E^(3*c)*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2
*c)]))] - 5184*a^3*E^(3*c)*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c - Sqrt
[(a^2 + b^2)*E^(2*c)]))] - 5184*a^3*E^(3*c)*f^3*PolyLog[4, -((b*E^(2*c + d*
x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2592*a^2*b*d*e*E^(2*c)*f^2*Sinh
[d*x] - 648*b^3*d*e*E^(2*c)*f^2*Sinh[d*x] + 2592*a^2*b*d*e*E^(4*c)*f^2*Sinh
[d*x] - 648*b^3*d*e*E^(4*c)*f^2*Sinh[d*x] + 2592*a^2*b*E^(2*c)*f^3*Sinh[d*x
] - 648*b^3*E^(2*c)*f^3*Sinh[d*x] - 2592*a^2*b*E^(4*c)*f^3*Sinh[d*x] + 648*
b^3*E^(4*c)*f^3*Sinh[d*x] + 2592*a^2*b*d^2*e*E^(2*c)*f^2*x*Sinh[d*x] - 648*
b^3*d^2*e*E^(2*c)*f^2*x*Sinh[d*x] - 2592*a^2*b*d^2*e*E^(4*c)*f^2*x*Sinh[d*x
] + 648*b^3*d^2*e*E^(4*c)*f^2*x*Sinh[d*x] + 2592*a^2*b*d*E^(2*c)*f^3*x*Sinh
[d*x] - 648*b^3*d*E^(2*c)*f^3*x*Sinh[d*x] + 259...
```

Maple [**F**]
time = 2.32, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \cosh(dx+c) \left(\sinh^3(dx+c)\right)}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -1/24*(24*(d*x + c)*a^3/(b^4*d) + 24*a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*
x - 2*c) - b)/(b^4*d) + (3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 - b^2)*e^(-2*d
*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-3*d*
x - 3*c) + 3*(4*a^2 - b^2)*e^(-d*x - c))/(b^3*d))*e^3 - 1/864*(216*a^3*d^4*
f^3*x^4*e^(3*c) + 864*a^3*d^4*f^2*x^3*e^(3*c + 1) + 1296*a^3*d^4*f*x^2*e^(3
```

```
*c + 2) - 4*(9*b^3*d^3*f^3*x^3*e^(6*c) - 2*b^3*f^3*e^(6*c) - 9*b^3*d^2*f*e^
(6*c + 2) + 6*b^3*d*f^2*e^(6*c + 1) - 9*(b^3*d^2*f^3*e^(6*c) - 3*b^3*d^3*f^
2*e^(6*c + 1))*x^2 + 3*(2*b^3*d*f^3*e^(6*c) + 9*b^3*d^3*f*e^(6*c + 2) - 6*b
^3*d^2*f^2*e^(6*c + 1))*x)*e^(3*d*x) + 27*(4*a*b^2*d^3*f^3*x^3*e^(5*c) - 3*
a*b^2*f^3*e^(5*c) - 6*a*b^2*d^2*f*e^(5*c + 2) + 6*a*b^2*d*f^2*e^(5*c + 1) -
 6*(a*b^2*d^2*f^3*e^(5*c) - 2*a*b^2*d^3*f^2*e^(5*c + 1))*x^2 + 6*(a*b^2*d*f
^3*e^(5*c) + 2*a*b^2*d^3*f*e^(5*c + 2) - 2*a*b^2*d^2*f^2*e^(5*c + 1))*x)*e^
(2*d*x) + 108*(24*a^2*b*f^3*e^(4*c) - 6*b^3*f^3*e^(4*c) - (4*a^2*b*d^3*f^3*
e^(4*c) - b^3*d^3*f^3*e^(4*c))*x^3 + 3*(4*a^2*b*d^2*f^3*e^(4*c) - b^3*d^2*f
^3*e^(4*c) - (4*a^2*b*d^3*f^2*e^(4*c) - b^3*d^3*f^2*e^(4*c))*e)*x^2 - 3*(8*
a^2*b*d*f^3*e^(4*c) - 2*b^3*d*f^3*e^(4*c) + (4*a^2*b*d^3*f*e^(4*c) - b^3*d^
3*f*e^(4*c))*e^2 - 2*(4*a^2*b*d^2*f^2*e^(4*c) - b^3*d^2*f^2*e^(4*c))*e)*x +
 3*(4*a^2*b*d^2*f*e^(4*c) - b^3*d^2*f*e^(4*c))*e^2 - 6*(4*a^2*b*d*f^2*e^(4*
c) - b^3*d*f^2*e^(4*c))*e)*e^(d*x) + 108*(24*a^2*b*f^3*e^(2*c) - 6*b^3*f^3*
e^(2*c) + (4*a^2*b*d^3*f^3*e^(2*c) - b^3*d^3*f^3*e^(2*c))*x^3 + 3*(4*a^2*b*
d^2*f^3*e^(2*c) - b^3*d^2*f^3*e^(2*c) + (4*a^2*b*d^3*f^2*e^(2*c) - b^3*d^3*
f^2*e^(2*c))*e)*x^2 + 3*(8*a^2*b*d*f^3*e^(2*c) - 2*b^3*d*f^3*e^(2*c) + (4*a
^2*b*d^3*f*e^(2*c) - b^3*d^3*f*e^(2*c))*e^2 + 2*(4*a^2*b*d^2*f^2*e^(2*c) -
b^3*d^2*f^2*e^(2*c))*e)*x + 3*(4*a^2*b*d^2*f*e^(2*c) - b^3*d^2*f*e^(2*c))*e
^2 + 6*(4*a^2*b*d*f^2*e^(2*c) - b^3*d*f^2*e^(2*c))*e)*e^(-d*x) + 27*(4*a*b^
2*d^3*f^3*x^3*e^c + 6*a*b^2*d^2*f*e^(c + 2) + 6*a*b^2*d*f^2*e^(c + 1) + 3*a
*b^2*f^3*e^c + 6*(2*a*b^2*d^3*f^2*e^(c + 1) + a*b^2*d^2*f^3*e^c)*x^2 + 6*(2
*a*b^2*d^3*f*e^(c + 2) + 2*a*b^2*d^2*f^2*e^(c + 1) + a*b^2*d*f^3*e^c)*x)*e^
(-2*d*x) + 4*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^2*f*e^2 + 6*b^3*d*f^2*e + 2*b^3*f
^3 + 9*(3*b^3*d^3*f^2*e + b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*f*e^2 + 6*b^3*d^2
*f^2*e + 2*b^3*d*f^3)*x)*e^(-3*d*x))*e^(-3*c)/(b^4*d^4) + integrate(-2*(a^3
*b*f^3*x^3 + 3*a^3*b*f^2*x^2*e + 3*a^3*b*f*x*e^2 - (a^4*f^3*x^3*e^c + 3*a^4
*f^2*x^2*e^(c + 1) + 3*a^4*f*x*e^(c + 2))*e^(d*x))/(b^5*e^(2*d*x + 2*c) + 2
*a*b^4*e^(d*x + c) - b^5), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 13122 vs. $2(757) = 1514$.
time = 0.54, size = 13122, normalized size = 16.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -1/864*(36*b^3*d^3*f^3*x^3 + 36*b^3*d^2*f^3*x^2 + 36*b^3*d^3*cosh(1)^3 + 36
*b^3*d^3*sinh(1)^3 + 24*b^3*d*f^3*x - 4*(9*b^3*d^3*f^3*x^3 - 9*b^3*d^2*f^3*
x^2 + 9*b^3*d^3*cosh(1)^3 + 9*b^3*d^3*sinh(1)^3 + 6*b^3*d*f^3*x - 2*b^3*f^3
 + 9*(3*b^3*d^3*f*x - b^3*d^2*f)*cosh(1)^2 + 9*(3*b^3*d^3*f*x + 3*b^3*d^3*c
osh(1) - b^3*d^2*f)*sinh(1)^2 + 3*(9*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 2*
```

b^3*d*f^2)*cosh(1) + 3*(9*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 9*b^3*d^3*cos
h(1)^2 + 2*b^3*d*f^2 + 6*(3*b^3*d^3*f*x - b^3*d^2*f)*cosh(1))*sinh(1))*cosh
(d*x + c)^6 - 4*(9*b^3*d^3*f^3*x^3 - 9*b^3*d^2*f^3*x^2 + 9*b^3*d^3*cosh(1)^
3 + 9*b^3*d^3*sinh(1)^3 + 6*b^3*d*f^3*x - 2*b^3*f^3 + 9*(3*b^3*d^3*f*x - b^
3*d^2*f)*cosh(1)^2 + 9*(3*b^3*d^3*f*x + 3*b^3*d^3*cosh(1) - b^3*d^2*f)*sinh
(1)^2 + 3*(9*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 2*b^3*d*f^2)*cosh(1) + 3*(
9*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 9*b^3*d^3*cosh(1)^2 + 2*b^3*d*f^2 + 6
*(3*b^3*d^3*f*x - b^3*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^6 + 8*b^3*f^3
+ 27*(4*a*b^2*d^3*f^3*x^3 - 6*a*b^2*d^2*f^3*x^2 + 4*a*b^2*d^3*cosh(1)^3 + 4
*a*b^2*d^3*sinh(1)^3 + 6*a*b^2*d*f^3*x - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*f*x -
a*b^2*d^2*f)*cosh(1)^2 + 6*(2*a*b^2*d^3*f*x + 2*a*b^2*d^3*cosh(1) - a*b^2*
d^2*f)*sinh(1)^2 + 6*(2*a*b^2*d^3*f^2*x^2 - 2*a*b^2*d^2*f^2*x + a*b^2*d*f^2
)*cosh(1) + 6*(2*a*b^2*d^3*f^2*x^2 - 2*a*b^2*d^2*f^2*x + 2*a*b^2*d^3*cosh(1
)^2 + a*b^2*d*f^2 + 2*(2*a*b^2*d^3*f*x - a*b^2*d^2*f)*cosh(1))*sinh(1))*cos
h(d*x + c)^5 + 3*(36*a*b^2*d^3*f^3*x^3 - 54*a*b^2*d^2*f^3*x^2 + 36*a*b^2*d^
3*cosh(1)^3 + 36*a*b^2*d^3*sinh(1)^3 + 54*a*b^2*d*f^3*x - 27*a*b^2*f^3 + 54
*(2*a*b^2*d^3*f*x - a*b^2*d^2*f)*cosh(1)^2 + 54*(2*a*b^2*d^3*f*x + 2*a*b^2*
d^3*cosh(1) - a*b^2*d^2*f)*sinh(1)^2 + 54*(2*a*b^2*d^3*f^2*x^2 - 2*a*b^2*d^
2*f^2*x + a*b^2*d*f^2)*cosh(1) - 8*(9*b^3*d^3*f^3*x^3 - 9*b^3*d^2*f^3*x^2 +
9*b^3*d^3*cosh(1)^3 + 9*b^3*d^3*sinh(1)^3 + 6*b^3*d*f^3*x - 2*b^3*f^3 + 9*
(3*b^3*d^3*f*x - b^3*d^2*f)*cosh(1)^2 + 9*(3*b^3*d^3*f*x + 3*b^3*d^3*cosh(1
) - b^3*d^2*f)*sinh(1)^2 + 3*(9*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 2*b^3*d
*f^2)*cosh(1) + 3*(9*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 9*b^3*d^3*cosh(1)^
2 + 2*b^3*d*f^2 + 6*(3*b^3*d^3*f*x - b^3*d^2*f)*cosh(1))*sinh(1))*cosh(d*x
+ c) + 54*(2*a*b^2*d^3*f^2*x^2 - 2*a*b^2*d^2*f^2*x + 2*a*b^2*d^3*cosh(1)^2
+ a*b^2*d*f^2 + 2*(2*a*b^2*d^3*f*x - a*b^2*d^2*f)*cosh(1))*sinh(1))*sinh(d*
x + c)^5 - 108*((4*a^2*b - b^3)*d^3*f^3*x^3 - 3*(4*a^2*b - b^3)*d^2*f^3*x^2
+ (4*a^2*b - b^3)*d^3*cosh(1)^3 + (4*a^2*b - b^3)*d^3*sinh(1)^3 + 6*(4*a^2
*b - b^3)*d*f^3*x - 6*(4*a^2*b - b^3)*f^3 + 3*((4*a^2*b - b^3)*d^3*f*x - (4
*a^2*b - b^3)*d^2*f)*cosh(1)^2 + 3*((4*a^2*b - b^3)*d^3*f*x + (4*a^2*b - b^
3)*d^3*cosh(1) - (4*a^2*b - b^3)*d^2*f)*sinh(1)^2 + 3*((4*a^2*b - b^3)*d^3*
f^2*x^2 - 2*(4*a^2*b - b^3)*d^2*f^2*x + 2*(4*a^2*b - b^3)*d*f^2)*cosh(1) +
3*((4*a^2*b - b^3)*d^3*f^2*x^2 - 2*(4*a^2*b - b^3)*d^2*f^2*x + (4*a^2*b - b
^3)*d^3*cosh(1)^2 + 2*(4*a^2*b - b^3)*d*f^2 + 2*((4*a^2*b - b^3)*d^3*f*x -
(4*a^2*b - b^3)*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^4 - 3*(36*(4*a^2*b -
b^3)*d^3*f^3*x^3 - 108*(4*a^2*b - b^3)*d^2*f^3*x^2 + 36*(4*a^2*b - b^3)*d^
3*cosh(1)^3 + 36*(4*a^2*b - b^3)*d^3*sinh(1)^3 + 216*(4*a^2*b - b^3)*d*f^3*
x - 216*(4*a^2*b - b^3)*f^3 + 108*((4*a^2*b - b^3)*d^3*f*x - (4*a^2*b - b^3
)*d^2*f)*cosh(1)^2 + 20*(9*b^3*d^3*f^3*x^3 - 9*b^3*d^2*f^3*x^2 + 9*b^3*d^3*
cosh(1)^3 + 9*b^3*d^3*sinh(1)^3 + 6*b^3*d*f^3*x - 2*b^3*f^3 + 9*(3*b^3*d^3*
f*x - b^3*d^2*f)*cosh(1)^2 + 9*(3*b^3*d^3*f*x + 3*b^3*d^3*cosh(1) - b^3*d^2
*f)*sinh(1)^2 + 3*(9*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 2*b^3*d*f^2)*cosh(
1) + 3*(9*b^3*d^3*f^2*x^2 - 6*b^3*d^2*f^2*x + 9*b^3*d^3*cosh(1)^2 + 2*b^3*d
f^2 + 6(3*b^3*d^3*f*x - b^3*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 10
8*((4*a^2*b - b^3)*d^3*f*x + (4*a^2*b - b^3)*d^3*cosh(1) - (4*a^2*b - b^3)*

```
d^2*f)*sinh(1)^2 + 108*((4*a^2*b - b^3)*d^3*f^2*x^2 - 2*(4*a^2*b - b^3)*d^2
*f^2*x + 2*(4*a^2*b - b^3)*d*f^2)*cosh(1) - 45*(4*a*b^2*d^3*f^3*x^3 - 6*a*b
^2*d^2*f^3*x^2 + 4*a*b^2*d^3*cosh(1)^3 + 4*a*b^2*d^3*sinh(1)^3 + 6*a*b^2*d*
f^3*x - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*f*x - a*b^2*d^2*f)*cosh(1)^2 + 6*(2*a*
b^2*d^3*f*x + 2*a*b^2*d^3*cosh(1) - a*b^2*d^2*f)*sinh(1)^2 + 6*(2*a*b^2*d^3
*f^2*x^2 - 2*a*b^2*d^2*f^2*x + a*b^2*d*f^2)*cosh(1) + 6*(2*a*b^2*d^3*f^2*x^
2 - 2*a*b^2*d^2*f^2*x + 2*a*b^2*d^3*cosh(1)^2 + a*b^2*d*f^2 + 2*(2*a*b^2*d^
3*f*x - a*b^2*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c) + 108*((4*a^2*b - b^3)
*d^3*f^2*x^2 - 2*(4*a^2*b - b^3)*d^2*f^2*x + (4*a^2*b - b^3)*d^3*cosh(1)^2
+ 2*(4*a^2*b - b^3)*d*f^2 + 2*((4*a^2*b - b^3)*d^3*f*x - (4*a^2*b - b^3)*d^
2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^4 - 216*(a^3*d^4*f^3*x^4 - 2*a^3*c^4*f
^3 + 4*(a^3*d^4*x + 2*a^3*c*d^3)*cosh(1)^3 + 4*(a^3*d^4*x + 2*a^3*c*d^3)*si
nh(1)^3 + 6*(a^3*d^4*f*x^2 - 2*a^3*c^2*d^2*f)*cosh(1)^2 + 6*(a^3*d^4*f*x^2
- 2*a^3*c^2*d^2*f + 2*(a^3*d^4*x + 2*a^3*c*d^3)*cosh(1))*sinh(1)^2 + 4*(a^3
*d^4*f^2*x^3 + 2*a^3*c^3*d*f^2)*cosh(1) + 4*(a^3*d^4*f^2*x^3 + 2*a^3*c^3*d*
f^2 + 3*(a^3*d^4*x + 2*a^3*c*d^3)*cosh(1)^2 + 3*(a^3*d^4*f*x^2 - 2*a^3*c^2*
d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^3 - 2*(1...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="giac")
[Out] integrate((f*x + e)^3*cosh(d*x + c)*sinh(d*x + c)^3/(b*sinh(d*x + c) + a),
x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^3\,{(e+f\,x)}^3}{a+b\,\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.392 $\int \frac{(e+fx)^2 \cosh(c+dx) \sinh^3(c+dx)}{a+b\sinh(c+dx)}\, dx$

Optimal. Leaf size=578

$$-\frac{aefx}{2b^2d}-\frac{af^2x^2}{4b^2d}+\frac{a^3(e+fx)^3}{3b^4f}-\frac{2a^2f(e+fx)\cosh(c+dx)}{b^3d^2}+\frac{4f(e+fx)\cosh(c+dx)}{9bd^2}-\frac{a^3(e+fx)^2\log\left(1-\right.}{b^4d}$$

```
[Out] -1/2*a*e*f*x/b^2/d-1/4*a*f^2*x^2/b^2/d+1/3*a^3*(f*x+e)^3/b^4/f-2*a^2*f*(f*x
+e)*cosh(d*x+c)/b^3/d^2+4/9*f*(f*x+e)*cosh(d*x+c)/b/d^2-a^3*(f*x+e)^2*ln(1+
b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d-a^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))/b^4/d-2*a^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))/b^4/d^2-2*a^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2
)))/b^4/d^2+2*a^3*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d^3+
2*a^3*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d^3+2*a^2*f^2*si
nh(d*x+c)/b^3/d^3-4/9*f^2*sinh(d*x+c)/b/d^3+a^2*(f*x+e)^2*sinh(d*x+c)/b^3/d
+1/2*a*f*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/b^2/d^2-1/4*a*f^2*sinh(d*x+c)^2/b^
2/d^3-1/2*a*(f*x+e)^2*sinh(d*x+c)^2/b^2/d-2/9*f*(f*x+e)*cosh(d*x+c)*sinh(d*
x+c)^2/b/d^2+2/27*f^2*sinh(d*x+c)^3/b/d^3+1/3*(f*x+e)^2*sinh(d*x+c)^3/b/d
```

Rubi [A]
time = 0.66, antiderivative size = 578, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5698, 5554, 3391, 3377, 2717, 5680, 2221, 2611, 2320, 6724}

$$\frac{a^3(e+fx)^3}{3b^4f}+\frac{2a^2f^2\sinh(c+dx)}{b^3d^3}-\frac{2a^2f(e+fx)\cosh(c+dx)}{b^3d^2}+\frac{a^2(e+fx)^2\sinh(c+dx)}{b^3d}+\frac{2a^3f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^3}+\frac{2a^3f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^3}-\frac{2a^3f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{2a^3f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{a^3(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^4d}-\frac{a^3(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^4d}-\frac{af^2\sinh^2(c+dx)}{4b^2d^3}+\frac{af(e+fx)\sinh(c+dx)\cosh(c+dx)}{2b^2d^2}-\frac{a(e+fx)^2\sinh^2(c+dx)}{2b^2d}-\frac{aefx}{2b^2d}-\frac{af^2x^2}{4b^2d}+\frac{2f^2\sinh^3(c+dx)}{27bd^3}-\frac{4f^2\sinh(c+dx)}{9bd^3}+\frac{4f(e+fx)\cosh(c+dx)}{9bd^2}-\frac{2f(e+fx)\sinh^2(c+dx)\cosh(c+dx)}{9bd^2}+\frac{(e+fx)^2\sinh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/2*(a*e*f*x)/(b^2*d) - (a*f^2*x^2)/(4*b^2*d) + (a^3*(e + f*x)^3)/(3*b^4*f
) - (2*a^2*f*(e + f*x)*Cosh[c + d*x])/(b^3*d^2) + (4*f*(e + f*x)*Cosh[c + d
*x])/(9*b*d^2) - (a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])])/(b^4*d) - (a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])])/(b^4*d) - (2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2]))])/(b^4*d^2) - (2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/(b^4*d^2) + (2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2]))])/(b^4*d^3) + (2*a^3*f^2*PolyLog[3, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^3) + (2*a^2*f^2*Sinh[c + d*x])/(b^3*d^3
) - (4*f^2*Sinh[c + d*x])/(9*b*d^3) + (a^2*(e + f*x)^2*Sinh[c + d*x])/(b^3*
d) + (a*f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^2*d^2) - (a*f^2*Sinh[
c + d*x]^2)/(4*b^2*d^3) - (a*(e + f*x)^2*Sinh[c + d*x]^2)/(2*b^2*d) - (2*f*
(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x]^2)/(9*b*d^2) + (2*f^2*Sinh[c + d*x]^3
)/(27*b*d^3) + ((e + f*x)^2*Sinh[c + d*x]^3)/(3*b*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
```

```
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh(c+dx)\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\cosh(c+dx}{a+b\sinh(c}}{b}$$

$$= \frac{(e+fx)^2\sinh^3(c+dx)}{3bd} - \frac{a\int (e+fx)^2\cosh(c+dx)\sinh(c+}{b^2}$$

$$= -\frac{a(e+fx)^2\sinh^2(c+dx)}{2b^2d} - \frac{2f(e+fx)\cosh(c+dx)\sinh^2(c}{9bd^2}$$

$$= \frac{a^3(e+fx)^3}{3b^4f} + \frac{4f(e+fx)\cosh(c+dx)}{9bd^2} + \frac{a^2(e+fx)^2\sinh(c-}{b^3d}$$

$$= -\frac{aefx}{2b^2d} - \frac{af^2x^2}{4b^2d} + \frac{a^3(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx)\cosh(c+dx)}{b^3d^2}$$

$$= -\frac{aefx}{2b^2d} - \frac{af^2x^2}{4b^2d} + \frac{a^3(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx)\cosh(c+dx)}{b^3d^2}$$

$$= -\frac{aefx}{2b^2d} - \frac{af^2x^2}{4b^2d} + \frac{a^3(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx)\cosh(c+dx)}{b^3d^2}$$

$$= -\frac{aefx}{2b^2d} - \frac{af^2x^2}{4b^2d} + \frac{a^3(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx)\cosh(c+dx)}{b^3d^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1945 vs. $2(578) = 1156$.
time = 3.30, size = 1945, normalized size = 3.37

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (432*a^3*c^2*d*e*E^(3*c)*f + 864*a^3*c*d^2*e*E^(3*c)*f*x + 432*a^3*d^3*e*E^
(3*c)*f*x^2 + 144*a^3*d^3*E^(3*c)*f^2*x^3 - 432*a^2*b*E^(2*c)*f^2*Cosh[d*x]
 + 108*b^3*E^(2*c)*f^2*Cosh[d*x] + 432*a^2*b*E^(4*c)*f^2*Cosh[d*x] - 108*b^
3*E^(4*c)*f^2*Cosh[d*x] - 432*a^2*b*d*E^(2*c)*f^2*x*Cosh[d*x] + 108*b^3*d*E
^(2*c)*f^2*x*Cosh[d*x] - 432*a^2*b*d*E^(4*c)*f^2*x*Cosh[d*x] + 108*b^3*d*E^
(4*c)*f^2*x*Cosh[d*x] - 216*a^2*b*d^2*E^(2*c)*f^2*x^2*Cosh[d*x] + 54*b^3*d^
```

2*E^(2*c)*f^2*x^2*Cosh[d*x] + 216*a^2*b*d^2*E^(4*c)*f^2*x^2*Cosh[d*x] - 54*b^3*d^2*E^(4*c)*f^2*x^2*Cosh[d*x] - 27*a*b^2*E^c*f^2*Cosh[2*d*x] - 27*a*b^2*E^(5*c)*f^2*Cosh[2*d*x] - 54*a*b^2*d*E^c*f^2*x*Cosh[2*d*x] + 54*a*b^2*d*E^(5*c)*f^2*x*Cosh[2*d*x] - 54*a*b^2*d^2*E^c*f^2*x^2*Cosh[2*d*x] - 54*a*b^2*d^2*E^(5*c)*f^2*x^2*Cosh[2*d*x] - 4*b^3*f^2*Cosh[3*d*x] + 4*b^3*E^(6*c)*f^2*Cosh[3*d*x] - 12*b^3*d*f^2*x*Cosh[3*d*x] - 12*b^3*d*E^(6*c)*f^2*x*Cosh[3*d*x] - 18*b^3*d^2*f^2*x^2*Cosh[3*d*x] + 18*b^3*d^2*E^(6*c)*f^2*x^2*Cosh[3*d*x] - 864*a^2*b*d*e*E^(3*c)*f*Cosh[c + d*x] + 216*b^3*d*e*E^(3*c)*f*Cosh[c + d*x] - 108*a*b^2*d^2*e^2*E^(3*c)*Cosh[2*(c + d*x)] - 216*a*b^2*d^2*e*E^(3*c)*f*x*Cosh[2*(c + d*x)] - 24*b^3*d*e*E^(3*c)*f*Cosh[3*(c + d*x)] - 864*a^3*c*d*e*E^(3*c)*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 864*a^3*d^2*e*E^(3*c)*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 864*a^3*c*d*e*E^(3*c)*f*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 864*a^3*d^2*e*E^(3*c)*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 432*a^3*d^2*E^(3*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - 432*a^3*d^2*E^(3*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 432*a^3*d^2*e^2*E^(3*c)*Log[a + b*Sinh[c + d*x]] + 864*a^3*c*d*e*E^(3*c)*f*Log[a + b*Sinh[c + d*x]] - 864*a^3*d*e*E^(3*c)*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 864*a^3*d*e*E^(3*c)*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 864*a^3*d*E^(3*c)*f^2*x*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 864*a^3*d*E^(3*c)*f^2*x*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 864*a^3*E^(3*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 864*a^3*E^(3*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 432*a^2*b*E^(2*c)*f^2*Sinh[d*x] - 108*b^3*E^(2*c)*f^2*Sinh[d*x] + 432*a^2*b*E^(4*c)*f^2*Sinh[d*x] - 108*b^3*E^(4*c)*f^2*Sinh[d*x] + 432*a^2*b*d*E^(2*c)*f^2*x*Sinh[d*x] - 108*b^3*d*E^(2*c)*f^2*x*Sinh[d*x] - 432*a^2*b*d*E^(4*c)*f^2*x*Sinh[d*x] + 108*b^3*d*E^(4*c)*f^2*x*Sinh[d*x] + 216*a^2*b*d^2*E^(2*c)*f^2*x^2*Sinh[d*x] - 54*b^3*d^2*E^(2*c)*f^2*x^2*Sinh[d*x] + 216*a^2*b*d^2*E^(4*c)*f^2*x^2*Sinh[d*x] - 54*b^3*d^2*E^(4*c)*f^2*x^2*Sinh[d*x] + 27*a*b^2*E^c*f^2*Sinh[2*d*x] - 27*a*b^2*E^(5*c)*f^2*Sinh[2*d*x] + 54*a*b^2*d*E^c*f^2*x*Sinh[2*d*x] + 54*a*b^2*d*E^(5*c)*f^2*x*Sinh[2*d*x] + 54*a*b^2*d^2*E^c*f^2*x^2*Sinh[2*d*x] - 54*a*b^2*d^2*E^(5*c)*f^2*x^2*Sinh[2*d*x] + 4*b^3*f^2*Sinh[3*d*x] + 4*b^3*E^(6*c)*f^2*Sinh[3*d*x] + 12*b^3*d*f^2*x*Sinh[3*d*x] - 12*b^3*d*E^(6*c)*f^2*x*Sinh[3*d*x] + 18*b^3*d^2*f^2*x^2*Sinh[3*d*x] + 18*b^3*d^2*E^(6*c)*f^2*x^2*Sinh[3*d*x] + 432*a^2*b*d^2*e^2*E^(3*c)*Sinh[c + d*x] - 108*b^3*d^2*e^2*E^(3*c)*Sinh[c + d*x] + 864*a^2*b*d^2*e*E^(3*c)*f*x*Sinh[c + d*x] - 216*b^3*d^2*e*E^(3*c)*f*x*Sinh[c + d*x] + 108*a*b^2*d*e*E^(3*c)*f*Sinh[2*(c + d*x)] + 36*b^3*d^2*e^2*E^(3*c)*Sinh[3*(c + d*x)] + 72*b^3*d^2*e*E^(3*c)*f*x*Sinh[3*(c + d*x)])/(432*b^4*d^3*E^(3*c))

Maple [F]

time = 2.30, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \cosh(dx+c)\left(\sinh^3(dx+c)\right)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
[Out] int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
[Out] -1/24*(24*(d*x + c)*a^3/(b^4*d) + 24*a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*
x - 2*c) - b)/(b^4*d) + (3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 - b^2)*e^(-2*d
*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-3*d*
x - 3*c) + 3*(4*a^2 - b^2)*e^(-d*x - c))/(b^3*d))*e^2 - 1/432*(144*a^3*d^3*
f^2*x^3*e^(3*c) + 432*a^3*d^3*f*x^2*e^(3*c + 1) - 2*(9*b^3*d^2*f^2*x^2*e^(6
*c) + 2*b^3*f^2*e^(6*c) - 6*b^3*d*f*e^(6*c + 1) - 6*(b^3*d*f^2*e^(6*c) - 3*
b^3*d^2*f*e^(6*c + 1))*x)*e^(3*d*x) + 27*(2*a*b^2*d^2*f^2*x^2*e^(5*c) + a*b
^2*f^2*e^(5*c) - 2*a*b^2*d*f*e^(5*c + 1) - 2*(a*b^2*d*f^2*e^(5*c) - 2*a*b^2
*d^2*f*e^(5*c + 1))*x)*e^(2*d*x) - 54*(8*a^2*b*f^2*e^(4*c) - 2*b^3*f^2*e^(4
*c) + (4*a^2*b*d^2*f^2*e^(4*c) - b^3*d^2*f^2*e^(4*c))*x^2 - 2*(4*a^2*b*d*f^
2*e^(4*c) - b^3*d*f^2*e^(4*c) - (4*a^2*b*d^2*f*e^(4*c) - b^3*d^2*f*e^(4*c))
*e)*x - 2*(4*a^2*b*d*f*e^(4*c) - b^3*d*f*e^(4*c))*e)*e^(d*x) + 54*(8*a^2*b*
f^2*e^(2*c) - 2*b^3*f^2*e^(2*c) + (4*a^2*b*d^2*f^2*e^(2*c) - b^3*d^2*f^2*e^
(2*c))*x^2 + 2*(4*a^2*b*d*f^2*e^(2*c) - b^3*d*f^2*e^(2*c) + (4*a^2*b*d^2*f*
e^(2*c) - b^3*d^2*f*e^(2*c))*e)*x + 2*(4*a^2*b*d*f*e^(2*c) - b^3*d*f*e^(2*c
))*e)*e^(-d*x) + 27*(2*a*b^2*d^2*f^2*x^2*e^c + 2*a*b^2*d*f*e^(c + 1) + a*b^
2*f^2*e^c + 2*(2*a*b^2*d^2*f*e^(c + 1) + a*b^2*d*f^2*e^c)*x)*e^(-2*d*x) + 2
*(9*b^3*d^2*f^2*x^2 + 6*b^3*d*f*e + 2*b^3*f^2 + 6*(3*b^3*d^2*f*e + b^3*d*f^
2)*x)*e^(-3*d*x))*e^(-3*c)/(b^4*d^3) + integrate(-2*(a^3*b*f^2*x^2 + 2*a^3*
b*f*x*e - (a^4*f^2*x^2*e^c + 2*a^4*f*x*e^(c + 1))*e^(d*x))/(b^5*e^(2*d*x +
2*c) + 2*a*b^4*e^(d*x + c) - b^5), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6483 vs. 2(549) = 1098.
time = 0.42, size = 6483, normalized size = 11.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -1/432*(18*b^3*d^2*f^2*x^2 + 12*b^3*d*f^2*x + 18*b^3*d^2*cosh(1)^2 - 2*(9*b
^3*d^2*f^2*x^2 - 6*b^3*d*f^2*x + 9*b^3*d^2*cosh(1)^2 + 9*b^3*d^2*sinh(1)^2
+ 2*b^3*f^2 + 6*(3*b^3*d^2*f*x - b^3*d*f)*cosh(1) + 6*(3*b^3*d^2*f*x + 3*b^
3*d^2*cosh(1) - b^3*d*f)*sinh(1))*cosh(d*x + c)^6 + 18*b^3*d^2*sinh(1)^2 -
2*(9*b^3*d^2*f^2*x^2 - 6*b^3*d*f^2*x + 9*b^3*d^2*cosh(1)^2 + 9*b^3*d^2*sinh
(1)^2 + 2*b^3*f^2 + 6*(3*b^3*d^2*f*x - b^3*d*f)*cosh(1) + 6*(3*b^3*d^2*f*x
+ 3*b^3*d^2*cosh(1) - b^3*d*f)*sinh(1))*sinh(d*x + c)^6 + 27*(2*a*b^2*d^2*f
^2*x^2 - 2*a*b^2*d*f^2*x + 2*a*b^2*d^2*cosh(1)^2 + 2*a*b^2*d^2*sinh(1)^2 +
a*b^2*f^2 + 2*(2*a*b^2*d^2*f*x - a*b^2*d*f)*cosh(1) + 2*(2*a*b^2*d^2*f*x +
2*a*b^2*d^2*cosh(1) - a*b^2*d*f)*sinh(1))*cosh(d*x + c)^5 + 3*(18*a*b^2*d^2
*f^2*x^2 - 18*a*b^2*d*f^2*x + 18*a*b^2*d^2*cosh(1)^2 + 18*a*b^2*d^2*sinh(1)
^2 + 9*a*b^2*f^2 + 18*(2*a*b^2*d^2*f*x - a*b^2*d*f)*cosh(1) - 4*(9*b^3*d^2*
f^2*x^2 - 6*b^3*d*f^2*x + 9*b^3*d^2*cosh(1)^2 + 9*b^3*d^2*sinh(1)^2 + 2*b^3
*f^2 + 6*(3*b^3*d^2*f*x - b^3*d*f)*cosh(1) + 6*(3*b^3*d^2*f*x + 3*b^3*d^2*c
osh(1) - b^3*d*f)*sinh(1))*cosh(d*x + c) + 18*(2*a*b^2*d^2*f*x + 2*a*b^2*d^
2*cosh(1) - a*b^2*d*f)*sinh(1))*sinh(d*x + c)^5 + 4*b^3*f^2 - 54*((4*a^2*b
- b^3)*d^2*f^2*x^2 - 2*(4*a^2*b - b^3)*d*f^2*x + (4*a^2*b - b^3)*d^2*cosh(1
)^2 + (4*a^2*b - b^3)*d^2*sinh(1)^2 + 2*(4*a^2*b - b^3)*f^2 + 2*((4*a^2*b -
 b^3)*d^2*f*x - (4*a^2*b - b^3)*d*f)*cosh(1) + 2*((4*a^2*b - b^3)*d^2*f*x +
 (4*a^2*b - b^3)*d^2*cosh(1) - (4*a^2*b - b^3)*d*f)*sinh(1))*cosh(d*x + c)^
4 - 3*(18*(4*a^2*b - b^3)*d^2*f^2*x^2 - 36*(4*a^2*b - b^3)*d*f^2*x + 18*(4*
a^2*b - b^3)*d^2*cosh(1)^2 + 18*(4*a^2*b - b^3)*d^2*sinh(1)^2 + 36*(4*a^2*b
 - b^3)*f^2 + 10*(9*b^3*d^2*f^2*x^2 - 6*b^3*d*f^2*x + 9*b^3*d^2*cosh(1)^2 +
 9*b^3*d^2*sinh(1)^2 + 2*b^3*f^2 + 6*(3*b^3*d^2*f*x - b^3*d*f)*cosh(1) + 6*
(3*b^3*d^2*f*x + 3*b^3*d^2*cosh(1) - b^3*d*f)*sinh(1))*cosh(d*x + c)^2 + 36
*((4*a^2*b - b^3)*d^2*f*x - (4*a^2*b - b^3)*d*f)*cosh(1) - 45*(2*a*b^2*d^2*
f^2*x^2 - 2*a*b^2*d*f^2*x + 2*a*b^2*d^2*cosh(1)^2 + 2*a*b^2*d^2*sinh(1)^2 +
 a*b^2*f^2 + 2*(2*a*b^2*d^2*f*x - a*b^2*d*f)*cosh(1) + 2*(2*a*b^2*d^2*f*x +
 2*a*b^2*d^2*cosh(1) - a*b^2*d*f)*sinh(1))*cosh(d*x + c) + 36*((4*a^2*b - b
^3)*d^2*f*x + (4*a^2*b - b^3)*d^2*cosh(1) - (4*a^2*b - b^3)*d*f)*sinh(1))*s
inh(d*x + c)^4 - 144*(a^3*d^3*f^2*x^3 + 2*a^3*c^3*f^2 + 3*(a^3*d^3*x + 2*a^
3*c*d^2)*cosh(1)^2 + 3*(a^3*d^3*x + 2*a^3*c*d^2)*sinh(1)^2 + 3*(a^3*d^3*f*x
^2 - 2*a^3*c^2*d*f)*cosh(1) + 3*(a^3*d^3*f*x^2 - 2*a^3*c^2*d*f + 2*(a^3*d^3
*x + 2*a^3*c*d^2)*cosh(1))*sinh(1))*cosh(d*x + c)^3 - 2*(72*a^3*d^3*f^2*x^3
 + 144*a^3*c^3*f^2 + 20*(9*b^3*d^2*f^2*x^2 - 6*b^3*d*f^2*x + 9*b^3*d^2*cosh
(1)^2 + 9*b^3*d^2*sinh(1)^2 + 2*b^3*f^2 + 6*(3*b^3*d^2*f*x - b^3*d*f)*cosh(
1) + 6*(3*b^3*d^2*f*x + 3*b^3*d^2*cosh(1) - b^3*d*f)*sinh(1))*cosh(d*x + c)
^3 + 216*(a^3*d^3*x + 2*a^3*c*d^2)*cosh(1)^2 - 135*(2*a*b^2*d^2*f^2*x^2 - 2
*a*b^2*d*f^2*x + 2*a*b^2*d^2*cosh(1)^2 + 2*a*b^2*d^2*sinh(1)^2 + a*b^2*f^2
+ 2*(2*a*b^2*d^2*f*x - a*b^2*d*f)*cosh(1) + 2*(2*a*b^2*d^2*f*x + 2*a*b^2*d^
```

```
2*cosh(1) - a*b^2*d*f)*sinh(1))*cosh(d*x + c)^2 + 216*(a^3*d^3*x + 2*a^3*c*
d^2)*sinh(1)^2 + 216*(a^3*d^3*f*x^2 - 2*a^3*c^2*d*f)*cosh(1) + 108*((4*a^2*
b - b^3)*d^2*f^2*x^2 - 2*(4*a^2*b - b^3)*d*f^2*x + (4*a^2*b - b^3)*d^2*cosh
(1)^2 + (4*a^2*b - b^3)*d^2*sinh(1)^2 + 2*(4*a^2*b - b^3)*f^2 + 2*((4*a^2*b
 - b^3)*d^2*f*x - (4*a^2*b - b^3)*d*f)*cosh(1) + 2*((4*a^2*b - b^3)*d^2*f*x
 + (4*a^2*b - b^3)*d^2*cosh(1) - (4*a^2*b - b^3)*d*f)*sinh(1))*cosh(d*x + c
) + 216*(a^3*d^3*f*x^2 - 2*a^3*c^2*d*f + 2*(a^3*d^3*x + 2*a^3*c*d^2)*cosh(1
))*sinh(1))*sinh(d*x + c)^3 + 54*((4*a^2*b - b^3)*d^2*f^2*x^2 + 2*(4*a^2*b
- b^3)*d*f^2*x + (4*a^2*b - b^3)*d^2*cosh(1)^2 + (4*a^2*b - b^3)*d^2*sinh(1
)^2 + 2*(4*a^2*b - b^3)*f^2 + 2*((4*a^2*b - b^3)*d^2*f*x + (4*a^2*b - b^3)*
d*f)*cosh(1) + 2*((4*a^2*b - b^3)*d^2*f*x + (4*a^2*b - b^3)*d^2*cosh(1) + (
4*a^2*b - b^3)*d*f)*sinh(1))*cosh(d*x + c)^2 + 6*(9*(4*a^2*b - b^3)*d^2*f^2
*x^2 + 18*(4*a^2*b - b^3)*d*f^2*x + 9*(4*a^2*b - b^3)*d^2*cosh(1)^2 - 5*(9*
b^3*d^2*f^2*x^2 - 6*b^3*d*f^2*x + 9*b^3*d^2*cosh(1)^2 + 9*b^3*d^2*sinh(1)^2
 + 2*b^3*f^2 + 6*(3*b^3*d^2*f*x - b^3*d*f)*cosh(1) + 6*(3*b^3*d^2*f*x + 3*b
^3*d^2*cosh(1) - b^3*d*f)*sinh(1))*cosh(d*x + c)^4 + 9*(4*a^2*b - b^3)*d^2*
sinh(1)^2 + 45*(2*a*b^2*d^2*f^2*x^2 - 2*a*b^2*d*f^2*x + 2*a*b^2*d^2*cosh(1)
^2 + 2*a*b^2*d^2*sinh(1)^2 + a*b^2*f^2 + 2*(2*a*b^2*d^2*f*x - a*b^2*d*f)*co
sh(1) + 2*(2*a*b^2*d^2*f*x + 2*a*b^2*d^2*cosh(1) - a*b^2*d*f)*sinh(1))*cosh
(d*x + c)^3 + 18*(4*a^2*b - b^3)*f^2 - 54*((4*a^2*b - b^3)*d^2*f^2*x^2 - 2*
(4*a^2*b - b^3)*d*f^2*x + (4*a^2*b - b^3)*d^2*cosh(1)^2 + (4*a^2*b - b^3)*d
^2*sinh(1)^2 + 2*(4*a^2*b - b^3)*f^2 + 2*((4*a^2*b - b^3)*d^2*f*x - (4*a^2*
b - b^3)*d*f)*cosh(1) + 2*((4*a^2*b - b^3)*d^2*f*x + (4*a^2*b - b^3)*d^2*co
sh(1) - (4*a^2*b - b^3)*d*f)*sinh(1))*cosh(d*x + c)^2 + 18*((4*a^2*b - b^3)
*d^2*f*x + (4*a^2*b - b^3)*d*f)*cosh(1) - 72*(a^3*d^3*f^2*x^3 + 2*a^3*c^3*f
^2 + 3*(a^3*d^3*x + 2*a^3*c*d^2)*cosh(1)^2 + 3*(a^3*d^3*x + 2*a^3*c*d^2)*si
nh(1)^2 + 3*(a^3*d^3*f*x^2 - 2*a^3*c^2*d*f)*cos...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)*sinh(d*x + c)^3/(b*sinh(d*x + c) + a),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^3\,(e+f\,x)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.393 $\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=348

$$-\frac{afx}{4b^2d}+\frac{a^3(e+fx)^2}{2b^4f}-\frac{a^2f\cosh(c+dx)}{b^3d^2}+\frac{f\cosh(c+dx)}{3bd^2}-\frac{f\cosh^3(c+dx)}{9bd^2}-\frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d}$$

```
[Out] -1/4*a*f*x/b^2/d+1/2*a^3*(f*x+e)^2/b^4/f-a^2*f*cosh(d*x+c)/b^3/d^2+1/3*f*co
sh(d*x+c)/b/d^2-1/9*f*cosh(d*x+c)^3/b/d^2-a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/b^4/d-a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/
b^4/d-a^3*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^4/d^2-a^3*f*poly
log(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^4/d^2+a^2*(f*x+e)*sinh(d*x+c)/b^
3/d+1/4*a*f*cosh(d*x+c)*sinh(d*x+c)/b^2/d^2-1/2*a*(f*x+e)*sinh(d*x+c)^2/b^2
/d+1/3*(f*x+e)*sinh(d*x+c)^3/b/d
```

Rubi [A]
time = 0.38, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {5698, 5554, 2713, 2715, 8, 3377, 2718, 5680, 2221, 2317, 2438}

$$\frac{a^3(e+fx)^2}{2b^4f}-\frac{a^2f\cosh(c+dx)}{b^3d^2}+\frac{a^2(e+fx)\sinh(c+dx)}{b^3d}-\frac{a^3f\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{a^3f\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{a^3(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^4d}-\frac{a^3(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^4d}+\frac{af\sinh(c+dx)\cosh(c+dx)}{4b^2d^2}-\frac{a(e+fx)\sinh^2(c+dx)}{2b^2d}-\frac{afx}{4b^2d}-\frac{f\cosh^3(c+dx)}{9bd^2}+\frac{f\cosh(c+dx)}{3bd^2}+\frac{(e+fx)\sinh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/4*(a*f*x)/(b^2*d) + (a^3*(e + f*x)^2)/(2*b^4*f) - (a^2*f*Cosh[c + d*x])/
(b^3*d^2) + (f*Cosh[c + d*x])/(3*b*d^2) - (f*Cosh[c + d*x]^3)/(9*b*d^2) - (
a^3*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a^
3*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^4*d) - (a^3*
f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^2) - (a^3*f*
PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^2) + (a^2*(e +
 f*x)*Sinh[c + d*x])/(b^3*d) + (a*f*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^2*d^2
) - (a*(e + f*x)*Sinh[c + d*x]^2)/(2*b^2*d) + ((e + f*x)*Sinh[c + d*x]^3)/(
3*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh(c+dx)\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\cosh(c+dx)\sin}{a+b\sinh(c+d}}{b}$$

$$= \frac{(e+fx)\sinh^3(c+dx)}{3bd} - \frac{a\int (e+fx)\cosh(c+dx)\sinh(c+dx}{b^2}$$

$$= -\frac{a(e+fx)\sinh^2(c+dx)}{2b^2d} + \frac{(e+fx)\sinh^3(c+dx)}{3bd} + \frac{a^2\int (e+}{}$$

$$= \frac{a^3(e+fx)^2}{2b^4f} + \frac{f\cosh(c+dx)}{3bd^2} - \frac{f\cosh^3(c+dx)}{9bd^2} + \frac{a^2(e+fx)}{b}$$

$$= -\frac{afx}{4b^2d} + \frac{a^3(e+fx)^2}{2b^4f} - \frac{a^2f\cosh(c+dx)}{b^3d^2} + \frac{f\cosh(c+dx)}{3bd^2} -$$

$$= -\frac{afx}{4b^2d} + \frac{a^3(e+fx)^2}{2b^4f} - \frac{a^2f\cosh(c+dx)}{b^3d^2} + \frac{f\cosh(c+dx)}{3bd^2} -$$

$$= -\frac{afx}{4b^2d} + \frac{a^3(e+fx)^2}{2b^4f} - \frac{a^2f\cosh(c+dx)}{b^3d^2} + \frac{f\cosh(c+dx)}{3bd^2} -$$

Mathematica [A]
time = 1.72, size = 460, normalized size = 1.32

$$-\frac{-36a^3c^2f - 72a^3cdfx - 36a^3d^2fx^2 + 72a^2bf\cosh(c+dx) - 18b^3f\cosh(c+dx) + 18ab^2de\cosh(2(c+dx)) + 18ab^2dfx\cosh(2(c+dx)) + 2b^3f\cosh(3(c+dx)) + 72a^3cf\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) + 72a^3dfx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) + 72a^3cf\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right) + 72a^3dfx\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right) + 72a^3de\log(a+b\sinh(c+dx)) - 72a^3cf\log(a+b\sinh(c+dx)) + 72a^3f\,\mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right) + 72a^3f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right) - 72a^2bde\sinh(c+dx) + 18b^3de\sinh(c+dx) - 72a^2bdfx\sinh(c+dx) + 18b^3dfx\sinh(c+dx) - 9ab^2f\sinh(2(c+dx)) - 6b^3de\sinh(3(c+dx)) - 6b^3dfx\sinh(3(c+dx))}{72b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] -1/72*(-36*a^3*c^2*f - 72*a^3*c*d*f*x - 36*a^3*d^2*f*x^2 + 72*a^2*b*f*Cosh[
c + d*x] - 18*b^3*f*Cosh[c + d*x] + 18*a*b^2*d*e*Cosh[2*(c + d*x)] + 18*a*b
^2*d*f*x*Cosh[2*(c + d*x)] + 2*b^3*f*Cosh[3*(c + d*x)] + 72*a^3*c*f*Log[1 +
 (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 72*a^3*d*f*x*Log[1 + (b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2])] + 72*a^3*c*f*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])] + 72*a^3*d*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] +
 72*a^3*d*e*Log[a + b*Sinh[c + d*x]] - 72*a^3*c*f*Log[a + b*Sinh[c + d*x]]
+ 72*a^3*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 72*a^3*f*Po
lyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 72*a^2*b*d*e*Sinh[c +
d*x] + 18*b^3*d*e*Sinh[c + d*x] - 72*a^2*b*d*f*x*Sinh[c + d*x] + 18*b^3*d*f
*x*Sinh[c + d*x] - 9*a*b^2*f*Sinh[2*(c + d*x)] - 6*b^3*d*e*Sinh[3*(c + d*x)
] - 6*b^3*d*f*x*Sinh[3*(c + d*x)])/(b^4*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(322) = 644$.
time = 3.08, size = 671, normalized size = 1.93

method	result
risch	$\frac{a^3 f x^2}{2b^4} - \frac{a^3 e x}{b^4} + \frac{(3dxf+3de-f)e^{3dx+3c}}{72d^2 b} - \frac{a(2dxf+2de-f)e^{2dx+2c}}{16b^2 d^2} + \frac{(4a^2 dfx - b^2 dfx + 4a^2 de - b^2 de - 4a^2 f + f b^2)e^{dx+c}}{8b^3 d^2} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/2*a^3*f*x^2/b^4-a^3*e*x/b^4+1/72*(3*d*f*x+3*d*e-f)/d^2/b*exp(3*d*x+3*c)-1
/16*a*(2*d*f*x+2*d*e-f)/b^2/d^2*exp(2*d*x+2*c)+1/8*(4*a^2*d*f*x-b^2*d*f*x+4
*a^2*d*e-b^2*d*e-4*a^2*f+b^2*f)/b^3/d^2*exp(d*x+c)-1/8*(4*a^2-b^2)*(d*f*x+d
*e+f)/b^3/d^2*exp(-d*x-c)-1/16*a*(2*d*f*x+2*d*e+f)/b^2/d^2*exp(-2*d*x-2*c)-
1/72*(3*d*f*x+3*d*e+f)/d^2/b*exp(-3*d*x-3*c)-2/d^2*a^3/b^4*f*c*ln(exp(d*x+c
))+1/d^2*a^3/b^4*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d*a^3/b^4*e*ln
(exp(d*x+c))-1/d*a^3/b^4*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d*a^3/b^
4*c*f*x+1/d^2*a^3/b^4*f*c^2-1/d*a^3/b^4*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d^2*a^3/b^4*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/
2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d*a^3/b^4*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2
)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*a^3/b^4*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2
)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d^2*a^3/b^4*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(
1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2*a^3/b^4*f*dilog((-b*exp(d*x+c)+(a^2+b^2)
^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/144*f*((72*a^3*d^2*x^2*e^(3*c) - 2*(3*b^3*d*x*e^(6*c) - b^3*e^(6*c))*e^(
3*d*x) + 9*(2*a*b^2*d*x*e^(5*c) - a*b^2*e^(5*c))*e^(2*d*x) + 18*(4*a^2*b*e^
(4*c) - b^3*e^(4*c) - (4*a^2*b*d*e^(4*c) - b^3*d*e^(4*c))*x)*e^(d*x) + 18*(
4*a^2*b*e^(2*c) - b^3*e^(2*c) + (4*a^2*b*d*e^(2*c) - b^3*d*e^(2*c))*x)*e^(-
d*x) + 9*(2*a*b^2*d*x*e^c + a*b^2*e^c)*e^(-2*d*x) + 2*(3*b^3*d*x + b^3)*e^(
-3*d*x))*e^(-3*c)/(b^4*d^2) - 9*integrate(32*(a^4*x*e^(d*x + c) - a^3*b*x)/
(b^5*e^(2*d*x + 2*c) + 2*a*b^4*e^(d*x + c) - b^5), x)) - 1/24*(24*(d*x + c)
*a^3/(b^4*d) + 24*a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^4*
d) + (3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 - b^2)*e^(-2*d*x - 2*c))*e^(3*d*x
 + 3*c)/(b^3*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-3*d*x - 3*c) + 3*(4*a^2
 - b^2)*e^(-d*x - c))/(b^3*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2579 vs. $2(326) = 652$.
time = 0.40, size = 2579, normalized size = 7.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] 1/144*(2*(3*b^3*d*f*x + 3*b^3*d*cosh(1) + 3*b^3*d*sinh(1) - b^3*f)*cosh(d*x
 + c)^6 + 2*(3*b^3*d*f*x + 3*b^3*d*cosh(1) + 3*b^3*d*sinh(1) - b^3*f)*sinh(
d*x + c)^6 - 6*b^3*d*f*x - 9*(2*a*b^2*d*f*x + 2*a*b^2*d*cosh(1) + 2*a*b^2*d
*sinh(1) - a*b^2*f)*cosh(d*x + c)^5 - 3*(6*a*b^2*d*f*x + 6*a*b^2*d*cosh(1)
+ 6*a*b^2*d*sinh(1) - 3*a*b^2*f - 4*(3*b^3*d*f*x + 3*b^3*d*cosh(1) + 3*b^3*
d*sinh(1) - b^3*f)*cosh(d*x + c))*sinh(d*x + c)^5 - 6*b^3*d*cosh(1) + 18*((
4*a^2*b - b^3)*d*f*x + (4*a^2*b - b^3)*d*cosh(1) + (4*a^2*b - b^3)*d*sinh(1
) - (4*a^2*b - b^3)*f)*cosh(d*x + c)^4 - 6*b^3*d*sinh(1) + 3*(6*(4*a^2*b -
b^3)*d*f*x + 6*(4*a^2*b - b^3)*d*cosh(1) + 10*(3*b^3*d*f*x + 3*b^3*d*cosh(1
) + 3*b^3*d*sinh(1) - b^3*f)*cosh(d*x + c)^2 + 6*(4*a^2*b - b^3)*d*sinh(1)
- 6*(4*a^2*b - b^3)*f - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*cosh(1) + 2*a*b^2*d*s
inh(1) - a*b^2*f)*cosh(d*x + c))*sinh(d*x + c)^4 - 2*b^3*f + 72*(a^3*d^2*f*
x^2 - 2*a^3*c^2*f + 2*(a^3*d^2*x + 2*a^3*c*d)*cosh(1) + 2*(a^3*d^2*x + 2*a^
3*c*d)*sinh(1))*cosh(d*x + c)^3 + 2*(36*a^3*d^2*f*x^2 - 72*a^3*c^2*f + 20*(
```

3*b^3*d*f*x + 3*b^3*d*cosh(1) + 3*b^3*d*sinh(1) - b^3*f)*cosh(d*x + c)^3 - 45*(2*a*b^2*d*f*x + 2*a*b^2*d*cosh(1) + 2*a*b^2*d*sinh(1) - a*b^2*f)*cosh(d*x + c)^2 + 72*(a^3*d^2*x + 2*a^3*c*d)*cosh(1) + 36*((4*a^2*b - b^3)*d*f*x + (4*a^2*b - b^3)*d*cosh(1) + (4*a^2*b - b^3)*d*sinh(1) - (4*a^2*b - b^3)*f)*cosh(d*x + c) + 72*(a^3*d^2*x + 2*a^3*c*d)*sinh(1))*sinh(d*x + c)^3 - 18*((4*a^2*b - b^3)*d*f*x + (4*a^2*b - b^3)*d*cosh(1) + (4*a^2*b - b^3)*d*sinh(1) + (4*a^2*b - b^3)*f)*cosh(d*x + c)^2 + 6*(5*(3*b^3*d*f*x + 3*b^3*d*cosh(1) + 3*b^3*d*sinh(1) - b^3*f)*cosh(d*x + c)^4 - 3*(4*a^2*b - b^3)*d*f*x - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*cosh(1) + 2*a*b^2*d*sinh(1) - a*b^2*f)*cosh(d*x + c)^3 - 3*(4*a^2*b - b^3)*d*cosh(1) + 18*((4*a^2*b - b^3)*d*f*x + (4*a^2*b - b^3)*d*cosh(1) + (4*a^2*b - b^3)*d*sinh(1) - (4*a^2*b - b^3)*f)*cosh(d*x + c)^2 - 3*(4*a^2*b - b^3)*d*sinh(1) - 3*(4*a^2*b - b^3)*f + 36*(a^3*d^2*f*x^2 - 2*a^3*c^2*f + 2*(a^3*d^2*x + 2*a^3*c*d)*cosh(1) + 2*(a^3*d^2*x + 2*a^3*c*d)*sinh(1))*cosh(d*x + c))*sinh(d*x + c)^2 - 9*(2*a*b^2*d*f*x + 2*a*b^2*d*cosh(1) + 2*a*b^2*d*sinh(1) + a*b^2*f)*cosh(d*x + c) - 144*(a^3*f*cosh(d*x + c)^3 + 3*a^3*f*cosh(d*x + c)^2*sinh(d*x + c) + 3*a^3*f*cosh(d*x + c)*sinh(d*x + c)^2 + a^3*f*sinh(d*x + c)^3)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 144*(a^3*f*cosh(d*x + c)^3 + 3*a^3*f*cosh(d*x + c)^2*sinh(d*x + c) + 3*a^3*f*cosh(d*x + c)*sinh(d*x + c)^2 + a^3*f*sinh(d*x + c)^3)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 144*((a^3*c*f - a^3*d*cosh(1) - a^3*d*sinh(1))*cosh(d*x + c)^3 + 3*(a^3*c*f - a^3*d*cosh(1) - a^3*d*sinh(1))*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3*c*f - a^3*d*cosh(1) - a^3*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3*c*f - a^3*d*cosh(1) - a^3*d*sinh(1))*sinh(d*x + c)^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 144*((a^3*c*f - a^3*d*cosh(1) - a^3*d*sinh(1))*cosh(d*x + c)^3 + 3*(a^3*c*f - a^3*d*cosh(1) - a^3*d*sinh(1))*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3*c*f - a^3*d*cosh(1) - a^3*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3*c*f - a^3*d*cosh(1) - a^3*d*sinh(1))*sinh(d*x + c)^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 144*((a^3*d*f*x + a^3*c*f)*cosh(d*x + c)^3 + 3*(a^3*d*f*x + a^3*c*f)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3*d*f*x + a^3*c*f)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3*d*f*x + a^3*c*f)*sinh(d*x + c)^3)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 144*((a^3*d*f*x + a^3*c*f)*cosh(d*x + c)^3 + 3*(a^3*d*f*x + a^3*c*f)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3*d*f*x + a^3*c*f)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3*d*f*x + a^3*c*f)*sinh(d*x + c)^3)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 3*(6*a*b^2*d*f*x - 4*(3*b^3*d*f*x + 3*b^3*d*cosh(1) + 3*b^3*d*sinh(1) - b^3*f)*cosh(d*x + c)^5 + 6*a*b^2*d*cosh(1) + 15*(2*a*b^2*d*f*x + 2*a*b^2*d*cosh(1) + 2*a*b^2*d*sinh(1) - a*b^2*f)*cosh(d*x + c)^4 + 6*a*b^2*d*sinh(1) + 3*a*b^2*f - 24*((4*a^2*b - b^3)*d*f*x + (4*a^2*b - b^3)*d*cosh(1) + (4*a^2*b - b^3)*d*sinh(1) - (4*a^2*b - b^3)*f)*cosh(d*x + c)^3 - 72*(a^3*d^2*f*x^2 - 2*a^3*c^2*f + 2*(a^3*d^2*x + 2*a^3*c*d)*cosh(1) + 2*(a^3*d^2*x + 2*a^3

```
*c*d)*sinh(1))*cosh(d*x + c)^2 + 12*((4*a^2*b - b^3)*d*f*x + (4*a^2*b - b^3
)*d*cosh(1) + (4*a^2*b - b^3)*d*sinh(1) + (4*a^2*b - b^3)*f)*cosh(d*x + c))
*sinh(d*x + c))/(b^4*d^2*cosh(d*x + c)^3 + 3*b^4*d^2*cosh(d*x + c)^2*sinh(d
*x + c) + 3*b^4*d^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^4*d^2*sinh(d*x + c)^3
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)*sinh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^3\,(e+f\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.394 $\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=76

$$-\frac{a^3\log(a+b\sinh(c+dx))}{b^4d}+\frac{a^2\sinh(c+dx)}{b^3d}-\frac{a\sinh^2(c+dx)}{2b^2d}+\frac{\sinh^3(c+dx)}{3bd}$$

```
[Out] -a^3*ln(a+b*sinh(d*x+c))/b^4/d+a^2*sinh(d*x+c)/b^3/d-1/2*a*sinh(d*x+c)^2/b^
2/d+1/3*sinh(d*x+c)^3/b/d
```

Rubi [A]
time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$-\frac{a^3\log(a+b\sinh(c+dx))}{b^4d}+\frac{a^2\sinh(c+dx)}{b^3d}-\frac{a\sinh^2(c+dx)}{2b^2d}+\frac{\sinh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a^3*Log[a + b*Sinh[c + d*x]])/(b^4*d)) + (a^2*Sinh[c + d*x])/(b^3*d) - (
a*Sinh[c + d*x]^2)/(2*b^2*d) + Sinh[c + d*x]^3/(3*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{b^3(a+x)}\,dx, x, b\sinh(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{a+x}\,dx, x, b\sinh(c+dx)\right)}{b^4d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2-ax+x^2-\frac{a^3}{a+x}\right)\,dx, x, b\sinh(c+dx)\right)}{b^4d}$$
$$= -\frac{a^3\log(a+b\sinh(c+dx))}{b^4d} + \frac{a^2\sinh(c+dx)}{b^3d} - \frac{a\sinh^2(c+dx)}{2b^2d} + \frac{\sinh^3}{}$$

Mathematica [A]
time = 0.15, size = 71, normalized size = 0.93

$$\frac{-3ab^2\cosh(2(c+dx)) - 12a^3\log(a+b\sinh(c+dx)) - 3b(-4a^2+b^2)\sinh(c+dx) + b^3\sinh(3(c+dx))}{12b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-3*a*b^2*Cosh[2*(c + d*x)] - 12*a^3*Log[a + b*Sinh[c + d*x]] - 3*b*(-4*a^2
 + b^2)*Sinh[c + d*x] + b^3*Sinh[3*(c + d*x)])/(12*b^4*d)
```

Maple [A]
time = 0.58, size = 65, normalized size = 0.86

method	result
derivativedivides	$\frac{\frac{\frac{\left(\sinh^3(dx+c)\right)b^2}{3}-\frac{ba\left(\sinh^2(dx+c)\right)}{2}+a^2\sinh(dx+c)}{b^3}-\frac{a^3\ln(a+b\sinh(dx+c))}{b^4}}{d}$
default	$\frac{\frac{\frac{\left(\sinh^3(dx+c)\right)b^2}{3}-\frac{ba\left(\sinh^2(dx+c)\right)}{2}+a^2\sinh(dx+c)}{b^3}-\frac{a^3\ln(a+b\sinh(dx+c))}{b^4}}{d}$
risch	$\frac{a^3x}{b^4}+\frac{e^{3dx+3c}}{24bd}-\frac{a\,e^{2dx+2c}}{8b^2d}+\frac{e^{dx+c}a^2}{2b^3d}-\frac{e^{dx+c}}{8bd}-\frac{e^{-dx-c}a^2}{2b^3d}+\frac{e^{-dx-c}}{8bd}-\frac{a\,e^{-2dx-2c}}{8b^2d}-\frac{e^{-3dx-3c}}{24bd}+\frac{2a^3c}{b^4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^3*(1/3*sinh(d*x+c)^3*b^2-1/2*b*a*sinh(d*x+c)^2+a^2*sinh(d*x+c))-a^
3/b^4*ln(a+b*sinh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(72) = 144$.

time = 0.27, size = 171, normalized size = 2.25

$$-\frac{(dx+c)a^3}{b^4d}-\frac{a^3\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{b^4d}-\frac{\left(3\,abe^{(-dx-c)}-b^2-3\,(4\,a^2-b^2)e^{(-2\,dx-2\,c)}\right)e^{(3\,dx+3\,c)}}{24\,b^3d}-\frac{3\,abe^{(-2\,dx-2\,c)}+b^2e^{(-3\,dx-3\,c)}+3\,(4\,a^2-b^2)e^{(-dx-c)}}{24\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -(d*x + c)*a^3/(b^4*d) - a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b
)/(b^4*d) - 1/24*(3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 - b^2)*e^(-2*d*x - 2*
c))*e^(3*d*x + 3*c)/(b^3*d) - 1/24*(3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-3*d*x
- 3*c) + 3*(4*a^2 - b^2)*e^(-d*x - c))/(b^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(72) = 144$.
time = 0.37, size = 602, normalized size = 7.92

$$\frac{b^3\cosh(dx+c)^6+b^3\sinh(dx+c)^6+24\,a^3dx\cosh(dx+c)^3-3\,ab^2\cosh(dx+c)^5+3\,(2\,b^3\cosh(dx+c)-ab^2)\sinh(dx+c)^5+3\,(4\,a^2b-b^3)\cosh(dx+c)^4+3\,(5\,b^3\cosh(dx+c)^2-5\,ab^2\cosh(dx+c)+4\,a^2b-b^3)\sinh(dx+c)^4-3\,ab^2\cosh(dx+c)+2\,(10\,b^3\cosh(dx+c)^3+12\,a^3dx-15\,ab^2\cosh(dx+c)^2+6\,(4\,a^2b-b^3)\cosh(dx+c))\sinh(dx+c)^3-b^3-3\,(4\,a^2b-b^3)\cosh(dx+c)^2+3\,(5\,b^3\cosh(dx+c)^4+24\,a^3dx\cosh(dx+c)-10\,ab^2\cosh(dx+c)^3-4\,a^2b+b^3+6\,(4\,a^2b-b^3)\cosh(dx+c)^2)\sinh(dx+c)^2-24\,(a^3\cosh(dx+c)^3+3\,a^3\cosh(dx+c)^2\sinh(dx+c)+3\,a^3\cosh(dx+c)\sinh(dx+c)^2+a^3\sinh(dx+c)^3)\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)+3\,(2\,b^3\cosh(dx+c)^5+24\,a^3dx\cosh(dx+c)^2-5\,ab^2\cosh(dx+c)^4+4\,(4\,a^2b-b^3)\cosh(dx+c)^3-ab^2-2\,(4\,a^2b-b^3)\cosh(dx+c))\sinh(dx+c)}{24\,(b^4d\cosh(dx+c)^3+3\,b^4d\cosh(dx+c)^2\sinh(dx+c)+3\,b^4d\cosh(dx+c)\sinh(dx+c)^2+b^4d\sinh(dx+c)^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/24*(b^3*cosh(d*x + c)^6 + b^3*sinh(d*x + c)^6 + 24*a^3*d*x*cosh(d*x + c)^
3 - 3*a*b^2*cosh(d*x + c)^5 + 3*(2*b^3*cosh(d*x + c) - a*b^2)*sinh(d*x + c)
^5 + 3*(4*a^2*b - b^3)*cosh(d*x + c)^4 + 3*(5*b^3*cosh(d*x + c)^2 - 5*a*b^2
*cosh(d*x + c) + 4*a^2*b - b^3)*sinh(d*x + c)^4 - 3*a*b^2*cosh(d*x + c) + 2
*(10*b^3*cosh(d*x + c)^3 + 12*a^3*d*x - 15*a*b^2*cosh(d*x + c)^2 + 6*(4*a^2
*b - b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - b^3 - 3*(4*a^2*b - b^3)*cosh(d*x
 + c)^2 + 3*(5*b^3*cosh(d*x + c)^4 + 24*a^3*d*x*cosh(d*x + c) - 10*a*b^2*co
sh(d*x + c)^3 - 4*a^2*b + b^3 + 6*(4*a^2*b - b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 - 24*(a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c)^2*sinh(d*x + c) + 3
*a^3*cosh(d*x + c)*sinh(d*x + c)^2 + a^3*sinh(d*x + c)^3)*log(2*(b*sinh(d*x
 + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 3*(2*b^3*cosh(d*x + c)^5 + 24
*a^3*d*x*cosh(d*x + c)^2 - 5*a*b^2*cosh(d*x + c)^4 + 4*(4*a^2*b - b^3)*cosh
(d*x + c)^3 - a*b^2 - 2*(4*a^2*b - b^3)*cosh(d*x + c))*sinh(d*x + c))/(b^4*
d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^4*d*cosh(d*
x + c)*sinh(d*x + c)^2 + b^4*d*sinh(d*x + c)^3)
```

Sympy [A]
time = 0.60, size = 105, normalized size = 1.38

$$\begin{cases} \frac{x\sinh^{3}(c)\cosh(c)}{a} & \text{for}\: b=0 \wedge d=0 \\ \frac{\sinh^{4}(c+dx)}{4ad} & \text{for}\: b=0 \\ \frac{x\sinh^{3}(c)\cosh(c)}{a+b\sinh(c)} & \text{for}\: d=0 \\ -\frac{a^{3}\log\left(\frac{a}{b}+\sinh(c+dx)\right)}{b^{4}d}+\frac{a^{2}\sinh(c+dx)}{b^{3}d}-\frac{a\cosh^{2}(c+dx)}{2b^{2}d}+\frac{\sinh^{3}(c+dx)}{3bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Piecewise((x*sinh(c)**3*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (sinh(c + d*x)**4/
(4*a*d), Eq(b, 0)), (x*sinh(c)**3*cosh(c)/(a + b*sinh(c)), Eq(d, 0)), (-a**
3*log(a/b + sinh(c + d*x))/(b**4*d) + a**2*sinh(c + d*x)/(b**3*d) - a*cosh(
c + d*x)**2/(2*b**2*d) + sinh(c + d*x)**3/(3*b*d), True))
```

Giac [A]
time = 0.45, size = 117, normalized size = 1.54

$$-\frac{\frac{24\,a^3\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{b^4}-\frac{b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3-3\,ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+12\,a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{b^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(24*a^3*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/b^4 - (b^2*(e^
(d*x + c) - e^(-d*x - c))^3 - 3*a*b*(e^(d*x + c) - e^(-d*x - c))^2 + 12*a^2
*(e^(d*x + c) - e^(-d*x - c)))/b^3)/d
```

Mupad [B]
time = 0.14, size = 63, normalized size = 0.83

$$-\frac{a^3\,\ln\left(a+b\,\mathrm{sinh}(c+d\,x)\right)-\frac{b^3\,{\mathrm{sinh}(c+d\,x)}^3}{3}+\frac{a\,b^2\,{\mathrm{sinh}(c+d\,x)}^2}{2}-a^2\,b\,\mathrm{sinh}(c+d\,x)}{b^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*sinh(c + d*x)^3)/(a + b*sinh(c + d*x)),x)
```

```
[Out] -(a^3*log(a + b*sinh(c + d*x)) - (b^3*sinh(c + d*x)^3)/3 + (a*b^2*sinh(c +
d*x)^2)/2 - a^2*b*sinh(c + d*x))/(b^4*d)
```

3.395 $\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cosh[c + d*x]*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][(Cosh[c + d*x]*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(Cosh[c + d*x]*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)\left(\sinh^3(dx+c)\right)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/8*e^(-3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b*f) - 1/4*a*e^
(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b^2*f) + 1/4*a*e^(2*c
- 2*d*e/f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b^2*f) - 1/8*e^(3*c - 3*d*e
/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/(b*f) - 1/8*(4*a^2 - b^2)*e^(-c + d
*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^3*f) - 1/8*(4*a^2*e^c - b^2*e^c)*
e^(-d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^3*f) - a^3*log(f*x + e)/(b^
4*f) + 1/16*integrate(-32*(a^4*e^(d*x + c) - a^3*b)/(b^5*f*x + b^5*e - (b^5
*f*x*e^(2*c) + b^5*e^(2*c + 1))*e^(2*d*x) - 2*(a*b^4*f*x*e^c + a*b^4*e^(c +
 1))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(cosh(d*x + c)*sinh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate(cosh(d*x + c)*sinh(d*x + c)^3/((f*x + e)*(b*sinh(d*x + c) + a)),
x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh(c+d\,x)\,\sinh(c+d\,x)^3}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*sinh(c + d*x)^3)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((cosh(c + d*x)*sinh(c + d*x)^3)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.396 $\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal. Leaf size=1038

$$\frac{3a^2ef^2x}{4b^3d^2}+\frac{3a^2f^3x^2}{8b^3d^2}+\frac{a^4(e+fx)^4}{4b^5f}+\frac{a^2(e+fx)^4}{8b^3f}-\frac{(e+fx)^4}{32bf}-\frac{6a^3f^2(e+fx)\cosh(c+dx)}{b^4d^3}-\frac{4af^2(e+fx)\cosh}{3b^2d^3}$$

```
[Out] 3/8*a^2*f^3*x^2/b^3/d^2-3/8*a^2*f^3*cosh(d*x+c)^2/b^3/d^4-1/3*a*(f*x+e)^3*c
osh(d*x+c)^3/b^2/d-3/128*f*(f*x+e)^2*cosh(4*d*x+4*c)/b/d^2+6*a^3*f^3*sinh(d
*x+c)/b^4/d^4+2/27*a*f^3*sinh(d*x+c)^3/b^2/d^4+3/256*f^2*(f*x+e)*sinh(4*d*x
+4*c)/b/d^3-3*a^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*
(a^2+b^2)^(1/2)/b^5/d^2+3*a^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d^2+6*a^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+
c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d^3-6*a^3*f^2*(f*x+e)*polylog(3
,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d^3+1/8*a^2*(f*x+e)
^4/b^3/f+3/4*a^2*f^2*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/b^3/d^3+1/3*a*f*(f*x+e
)^2*cosh(d*x+c)^2*sinh(d*x+c)/b^2/d^2-4/3*a*f^2*(f*x+e)*cosh(d*x+c)/b^2/d^3
+2/3*a*f*(f*x+e)^2*sinh(d*x+c)/b^2/d^2-1/32*(f*x+e)^4/b/f-a^3*(f*x+e)^3*cos
h(d*x+c)/b^4/d+1/4*a^4*(f*x+e)^4/b^5/f-3/1024*f^3*cosh(4*d*x+4*c)/b/d^4+1/3
2*(f*x+e)^3*sinh(4*d*x+4*c)/b/d+a^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2
)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d-6*a^3*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b
^2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d^4+6*a^3*f^3*polylog(4,-b*exp(d*x+c)/(a+(a
^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d^4-a^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d+3/4*a^2*e*f^2*x/b^3/d^2-6*a^3*f^2*(
f*x+e)*cosh(d*x+c)/b^4/d^3-3/4*a^2*f*(f*x+e)^2*cosh(d*x+c)^2/b^3/d^2-2/9*a*
f^2*(f*x+e)*cosh(d*x+c)^3/b^2/d^3+3*a^3*f*(f*x+e)^2*sinh(d*x+c)/b^4/d^2+1/2
*a^2*(f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/b^3/d+14/9*a*f^3*sinh(d*x+c)/b^2/d^4
```

Rubi [A]
time = 1.30, antiderivative size = 1038, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 18, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5698, 5556, 3377, 2718, 5555, 3392, 2717, 2713, 32, 3391, 5684, 3403, 2296, 2221, 2611, 6744, 2320, 6724}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (3*a^2*e*f^2*x)/(4*b^3*d^2) + (3*a^2*f^3*x^2)/(8*b^3*d^2) + (a^4*(e + f*x)^
4)/(4*b^5*f) + (a^2*(e + f*x)^4)/(8*b^3*f) - (e + f*x)^4/(32*b*f) - (6*a^3*
f^2*(e + f*x)*Cosh[c + d*x])/(b^4*d^3) - (4*a*f^2*(e + f*x)*Cosh[c + d*x])/
(3*b^2*d^3) - (a^3*(e + f*x)^3*Cosh[c + d*x])/(b^4*d) - (3*a^2*f^3*Cosh[c +
```

```
d*x]^2)/(8*b^3*d^4) - (3*a^2*f*(e + f*x)^2*Cosh[c + d*x]^2)/(4*b^3*d^2) -
(2*a*f^2*(e + f*x)*Cosh[c + d*x]^3)/(9*b^2*d^3) - (a*(e + f*x)^3*Cosh[c + d
*x]^3)/(3*b^2*d) - (3*f^3*Cosh[4*c + 4*d*x])/(1024*b*d^4) - (3*f*(e + f*x)^
2*Cosh[4*c + 4*d*x])/(128*b*d^2) - (a^3*Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 +
 (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^5*d) + (a^3*Sqrt[a^2 + b^2]*(e
+ f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^5*d) - (3*a^3*S
qrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/(b^5*d^2) + (3*a^3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^2) + (6*a^3*Sqrt[a^2 + b^2]*f^2*
(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^3) -
 (6*a^3*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/(b^5*d^3) - (6*a^3*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^4) + (6*a^3*Sqrt[a^2 + b^2]*f^3*P
olyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^4) + (6*a^3*f^3
*Sinh[c + d*x])/(b^4*d^4) + (14*a*f^3*Sinh[c + d*x])/(9*b^2*d^4) + (3*a^3*f
*(e + f*x)^2*Sinh[c + d*x])/(b^4*d^2) + (2*a*f*(e + f*x)^2*Sinh[c + d*x])/(
3*b^2*d^2) + (3*a^2*f^2*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^3*d^3)
+ (a^2*(e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^3*d) + (a*f*(e + f*x)^
2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b^2*d^2) + (2*a*f^3*Sinh[c + d*x]^3)/(2
7*b^2*d^4) + (3*f^2*(e + f*x)*Sinh[4*c + 4*d*x])/(256*b*d^3) + ((e + f*x)^3
*Sinh[4*c + 4*d*x])/(32*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
```

```
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5555

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\cosh^2}{a+b\,\mathrm{s}}}{}$$

$$= -\frac{a\int (e+fx)^3\cosh^2(c+dx)\sinh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^3\,\mathrm{c}}{a}}{}$$

$$= -\frac{(e+fx)^4}{32bf} - \frac{a(e+fx)^3\cosh^3(c+dx)}{3b^2d} + \frac{a^2\int (e+fx)^3\cos}{b^3}$$

$$= -\frac{(e+fx)^4}{32bf} - \frac{3a^2f(e+fx)^2\cosh^2(c+dx)}{4b^3d^2} - \frac{2af^2(e+fx)}{9b}$$

$$= \frac{a^4(e+fx)^4}{4b^5f} + \frac{a^2(e+fx)^4}{8b^3f} - \frac{(e+fx)^4}{32bf} - \frac{a^3(e+fx)^3\cosh}{b^4d}$$

$$= \frac{3a^2ef^2x}{4b^3d^2} + \frac{3a^2f^3x^2}{8b^3d^2} + \frac{a^4(e+fx)^4}{4b^5f} + \frac{a^2(e+fx)^4}{8b^3f} - \frac{(e+f}{32bf}$$

$$= \frac{3a^2ef^2x}{4b^3d^2} + \frac{3a^2f^3x^2}{8b^3d^2} + \frac{a^4(e+fx)^4}{4b^5f} + \frac{a^2(e+fx)^4}{8b^3f} - \frac{(e+f}{32bf}$$

$$= \frac{3a^2ef^2x}{4b^3d^2} + \frac{3a^2f^3x^2}{8b^3d^2} + \frac{a^4(e+fx)^4}{4b^5f} + \frac{a^2(e+fx)^4}{8b^3f} - \frac{(e+f}{32bf}$$

$$= \frac{3a^2ef^2x}{4b^3d^2} + \frac{3a^2f^3x^2}{8b^3d^2} + \frac{a^4(e+fx)^4}{4b^5f} + \frac{a^2(e+fx)^4}{8b^3f} - \frac{(e+f}{32bf}$$

$$= \frac{3a^2ef^2x}{4b^3d^2} + \frac{3a^2f^3x^2}{8b^3d^2} + \frac{a^4(e+fx)^4}{4b^5f} + \frac{a^2(e+fx)^4}{8b^3f} - \frac{(e+f}{32bf}$$

$$= \frac{3a^2ef^2x}{4b^3d^2} + \frac{3a^2f^3x^2}{8b^3d^2} + \frac{a^4(e+fx)^4}{4b^5f} + \frac{a^2(e+fx)^4}{8b^3f} - \frac{(e+f}{32bf}$$

Mathematica [C] Result contains complex when optimal does not.
time = 26.35, size = 6428, normalized size = 6.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x
]),x]
```

[Out] Result too large to show

Maple [F]
time = 1.91, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \left(\cosh^2 (dx+c)\right) \left(\sinh^3 (dx+c)\right)}{a+b\sinh (dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)

[Out] int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")

[Out] -1/192*(192*sqrt(a^2 + b^2)*a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b^5*d) + (8*a*b^2*e^(-d*x - c) - 24*a^2*b*e^(-2*d*x - 2*c) - 3*b^3 + 24*(4*a^3 + a*b^2)*e^(-3*d*x - 3*c))*e^(4*d*x + 4*c)/(b^4*d) - 24*(8*a^4 + 4*a^2*b^2 - b^4)*(d*x + c)/(b^5*d) + (24*a^2*b*e^(-2*d*x - 2*c) + 8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e^(-4*d*x - 4*c) + 24*(4*a^3 + a*b^2)*e^(-d*x - c))/(b^4*d))*e^3 + 1/55296*(1728*(8*a^4*d^4*f^3*e^(4*c) + 4*a^2*b^2*d^4*f^3*e^(4*c) - b^4*d^4*f^3*e^(4*c))*x^4 + 6912*(8*a^4*d^4*f^2*e^(4*c) + 4*a^2*b^2*d^4*f^2*e^(4*c) - b^4*d^4*f^2*e^(4*c))*x^3*e + 10368*(8*a^4*d^4*f*e^(4*c) + 4*a^2*b^2*d^4*f*e^(4*c) - b^4*d^4*f*e^(4*c))*x^2*e^2 + 27*(32*b^4*d^3*f^3*x^3*e^(8*c) - 3*b^4*f^3*e^(8*c) - 24*b^4*d^2*f*e^(8*c + 2) + 12*b^4*d*f^2*e^(8*c + 1) - 24*(b^4*d^2*f^3*e^(8*c) - 4*b^4*d^3*f^2*e^(8*c + 1))*x^2 + 12*(b^4*d*f^3*e^(8*c) + 8*b^4*d^3*f*e^(8*c + 2) - 4*b^4*d^2*f^2*e^(8*c + 1))*x)*e^(4*d*x) - 256*(9*a*b^3*d^3*f^3*x^3*e^(7*c) - 2*a*b^3*f^3*e^(7*c) - 9*a*b^3*d^2*f*e^(7*c + 2) + 6*a*b^3*d*f^2*e^(7*c + 1) - 9*(a*b^3*d^2*f^3*e^(7*c) - 3*a*b^3*d^3*f^2*e^(7*c + 1))*x^2 + 3*(2*a*b^3*d*f^3*e^(7*c) + 9*a*b^3*d^3*f*e^(7*c + 2) - 6*a*b^3*d^2*f^2*e^(7*c + 1))*x)*e^(3*d*x) + 1728*(4*a^2*b^2*d^3*f^3*x^3*e^(6*c) - 3*a^2*b^2*f^3*e^(6*c) - 6*a^2*b^2*d^2*f*e^(6*c + 2) + 6*a^2*b^2*d*f^2*e^(6*c + 1) - 6*(a^2*b^2*d^2*f^3*e^(6*c) - 2*a^2*b^2*d^3*f^2*e^(6*c + 1))*x^2 + 6*(a^2*b^2*d*f^3*e^(6*c) + 2*a^2*b^2*d^3*f*e^(6*c + 2) - 2*a^2*b^2*d^2*f^2*e^(6*c + 1))*x)*e^(2*d*x) + 6912*(24*a^3*b*f^3*e^(5*c) + 6*a*b^3*f^3*e^(5*c) - (4*a^3*b*d^3*f^3*e^(5*c) + a*b^3*d^3*f^3*e^(5*c))*x^3 + 3*(4*a^3*b*d^2*f^3*e^(5*c) + a*b^3*d^2*f^3*e^(5*c) - (4*a^3*b*d^3*f^2*e^(5*c) + a*b^3*d^3*f^2*e^(5*c))*e)*x^2 - 3*(8*a^3*b*d*f^3*e^(5*c) + 2*a*b^3*d*f^3*e^(5*c) + (4*a^3*b*d^3*f*e

```
^(5*c) + a*b^3*d^3*f*e^(5*c))*e^2 - 2*(4*a^3*b*d^2*f^2*e^(5*c) + a*b^3*d^2*
f^2*e^(5*c))*e)*x + 3*(4*a^3*b*d^2*f*e^(5*c) + a*b^3*d^2*f*e^(5*c))*e^2 - 6
*(4*a^3*b*d*f^2*e^(5*c) + a*b^3*d*f^2*e^(5*c))*e)*e^(d*x) - 6912*(24*a^3*b*
f^3*e^(3*c) + 6*a*b^3*f^3*e^(3*c) + (4*a^3*b*d^3*f^3*e^(3*c) + a*b^3*d^3*f^
3*e^(3*c))*x^3 + 3*(4*a^3*b*d^2*f^3*e^(3*c) + a*b^3*d^2*f^3*e^(3*c) + (4*a^
3*b*d^3*f^2*e^(3*c) + a*b^3*d^3*f^2*e^(3*c))*e)*x^2 + 3*(8*a^3*b*d*f^3*e^(3
*c) + 2*a*b^3*d*f^3*e^(3*c) + (4*a^3*b*d^3*f*e^(3*c) + a*b^3*d^3*f*e^(3*c))
*e^2 + 2*(4*a^3*b*d^2*f^2*e^(3*c) + a*b^3*d^2*f^2*e^(3*c))*e)*x + 3*(4*a^3*
b*d^2*f*e^(3*c) + a*b^3*d^2*f*e^(3*c))*e^2 + 6*(4*a^3*b*d*f^2*e^(3*c) + a*b
^3*d*f^2*e^(3*c))*e)*e^(-d*x) - 1728*(4*a^2*b^2*d^3*f^3*x^3*e^(2*c) + 3*a^2
*b^2*f^3*e^(2*c) + 6*a^2*b^2*d^2*f*e^(2*c + 2) + 6*a^2*b^2*d*f^2*e^(2*c + 1
) + 6*(a^2*b^2*d^2*f^3*e^(2*c) + 2*a^2*b^2*d^3*f^2*e^(2*c + 1))*x^2 + 6*(a^
2*b^2*d*f^3*e^(2*c) + 2*a^2*b^2*d^3*f*e^(2*c + 2) + 2*a^2*b^2*d^2*f^2*e^(2*
c + 1))*x)*e^(-2*d*x) - 256*(9*a*b^3*d^3*f^3*x^3*e^c + 9*a*b^3*d^2*f*e^(c +
 2) + 6*a*b^3*d*f^2*e^(c + 1) + 2*a*b^3*f^3*e^c + 9*(3*a*b^3*d^3*f^2*e^(c +
 1) + a*b^3*d^2*f^3*e^c)*x^2 + 3*(9*a*b^3*d^3*f*e^(c + 2) + 6*a*b^3*d^2*f^2
*e^(c + 1) + 2*a*b^3*d*f^3*e^c)*x)*e^(-3*d*x) - 27*(32*b^4*d^3*f^3*x^3 + 24
*b^4*d^2*f*e^2 + 12*b^4*d*f^2*e + 3*b^4*f^3 + 24*(4*b^4*d^3*f^2*e + b^4*d^2
*f^3)*x^2 + 12*(8*b^4*d^3*f*e^2 + 4*b^4*d^2*f^2*e + b^4*d*f^3)*x)*e^(-4*d*x
))*e^(-4*c)/(b^5*d^4) - integrate(2*((a^5*f^3*e^c + a^3*b^2*f^3*e^c)*x^3 +
3*(a^5*f^2*e^c + a^3*b^2*f^2*e^c)*x^2*e + 3*(a^5*f*e^c + a^3*b^2*f*e^c)*x*e
^2)*e^(d*x)/(b^6*e^(2*d*x + 2*c) + 2*a*b^5*e^(d*x + c) - b^6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 20130 vs. $2(980) = 1960$.
time = 0.64, size = 20130, normalized size = 19.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="fricas")
```

```
[Out] -1/55296*(864*b^4*d^3*f^3*x^3 + 648*b^4*d^2*f^3*x^2 + 864*b^4*d^3*cosh(1)^3
 + 864*b^4*d^3*sinh(1)^3 + 324*b^4*d*f^3*x - 27*(32*b^4*d^3*f^3*x^3 - 24*b^
4*d^2*f^3*x^2 + 32*b^4*d^3*cosh(1)^3 + 32*b^4*d^3*sinh(1)^3 + 12*b^4*d*f^3*
x - 3*b^4*f^3 + 24*(4*b^4*d^3*f*x - b^4*d^2*f)*cosh(1)^2 + 24*(4*b^4*d^3*f*
x + 4*b^4*d^3*cosh(1) - b^4*d^2*f)*sinh(1)^2 + 12*(8*b^4*d^3*f^2*x^2 - 4*b^
4*d^2*f^2*x + b^4*d*f^2)*cosh(1) + 12*(8*b^4*d^3*f^2*x^2 - 4*b^4*d^2*f^2*x
+ 8*b^4*d^3*cosh(1)^2 + b^4*d*f^2 + 4*(4*b^4*d^3*f*x - b^4*d^2*f)*cosh(1))*
sinh(1))*cosh(d*x + c)^8 - 27*(32*b^4*d^3*f^3*x^3 - 24*b^4*d^2*f^3*x^2 + 32
*b^4*d^3*cosh(1)^3 + 32*b^4*d^3*sinh(1)^3 + 12*b^4*d*f^3*x - 3*b^4*f^3 + 24
*(4*b^4*d^3*f*x - b^4*d^2*f)*cosh(1)^2 + 24*(4*b^4*d^3*f*x + 4*b^4*d^3*cosh
(1) - b^4*d^2*f)*sinh(1)^2 + 12*(8*b^4*d^3*f^2*x^2 - 4*b^4*d^2*f^2*x + b^4*
d*f^2)*cosh(1) + 12*(8*b^4*d^3*f^2*x^2 - 4*b^4*d^2*f^2*x + 8*b^4*d^3*cosh(1
```

)^2 + b^4*d*f^2 + 4*(4*b^4*d^3*f*x - b^4*d^2*f)*cosh(1))*sinh(1))*sinh(d*x
+ c)^8 + 256*(9*a*b^3*d^3*f^3*x^3 - 9*a*b^3*d^2*f^3*x^2 + 9*a*b^3*d^3*cosh(
1)^3 + 9*a*b^3*d^3*sinh(1)^3 + 6*a*b^3*d*f^3*x - 2*a*b^3*f^3 + 9*(3*a*b^3*d
^3*f*x - a*b^3*d^2*f)*cosh(1)^2 + 9*(3*a*b^3*d^3*f*x + 3*a*b^3*d^3*cosh(1)
- a*b^3*d^2*f)*sinh(1)^2 + 3*(9*a*b^3*d^3*f^2*x^2 - 6*a*b^3*d^2*f^2*x + 2*a
*b^3*d*f^2)*cosh(1) + 3*(9*a*b^3*d^3*f^2*x^2 - 6*a*b^3*d^2*f^2*x + 9*a*b^3*
d^3*cosh(1)^2 + 2*a*b^3*d*f^2 + 6*(3*a*b^3*d^3*f*x - a*b^3*d^2*f)*cosh(1))*
sinh(1))*cosh(d*x + c)^7 + 8*(288*a*b^3*d^3*f^3*x^3 - 288*a*b^3*d^2*f^3*x^2
 + 288*a*b^3*d^3*cosh(1)^3 + 288*a*b^3*d^3*sinh(1)^3 + 192*a*b^3*d*f^3*x -
64*a*b^3*f^3 + 288*(3*a*b^3*d^3*f*x - a*b^3*d^2*f)*cosh(1)^2 + 288*(3*a*b^3
*d^3*f*x + 3*a*b^3*d^3*cosh(1) - a*b^3*d^2*f)*sinh(1)^2 + 96*(9*a*b^3*d^3*f
^2*x^2 - 6*a*b^3*d^2*f^2*x + 2*a*b^3*d*f^2)*cosh(1) - 27*(32*b^4*d^3*f^3*x^
3 - 24*b^4*d^2*f^3*x^2 + 32*b^4*d^3*cosh(1)^3 + 32*b^4*d^3*sinh(1)^3 + 12*b
^4*d*f^3*x - 3*b^4*f^3 + 24*(4*b^4*d^3*f*x - b^4*d^2*f)*cosh(1)^2 + 24*(4*b
^4*d^3*f*x + 4*b^4*d^3*cosh(1) - b^4*d^2*f)*sinh(1)^2 + 12*(8*b^4*d^3*f^2*x
^2 - 4*b^4*d^2*f^2*x + b^4*d*f^2)*cosh(1) + 12*(8*b^4*d^3*f^2*x^2 - 4*b^4*d
^2*f^2*x + 8*b^4*d^3*cosh(1)^2 + b^4*d*f^2 + 4*(4*b^4*d^3*f*x - b^4*d^2*f)*
cosh(1))*sinh(1))*cosh(d*x + c) + 96*(9*a*b^3*d^3*f^2*x^2 - 6*a*b^3*d^2*f^2
*x + 9*a*b^3*d^3*cosh(1)^2 + 2*a*b^3*d*f^2 + 6*(3*a*b^3*d^3*f*x - a*b^3*d^2
*f)*cosh(1))*sinh(1))*sinh(d*x + c)^7 + 81*b^4*f^3 - 1728*(4*a^2*b^2*d^3*f^
3*x^3 - 6*a^2*b^2*d^2*f^3*x^2 + 4*a^2*b^2*d^3*cosh(1)^3 + 4*a^2*b^2*d^3*sin
h(1)^3 + 6*a^2*b^2*d*f^3*x - 3*a^2*b^2*f^3 + 6*(2*a^2*b^2*d^3*f*x - a^2*b^2
*d^2*f)*cosh(1)^2 + 6*(2*a^2*b^2*d^3*f*x + 2*a^2*b^2*d^3*cosh(1) - a^2*b^2*
d^2*f)*sinh(1)^2 + 6*(2*a^2*b^2*d^3*f^2*x^2 - 2*a^2*b^2*d^2*f^2*x + a^2*b^2
*d*f^2)*cosh(1) + 6*(2*a^2*b^2*d^3*f^2*x^2 - 2*a^2*b^2*d^2*f^2*x + 2*a^2*b^
2*d^3*cosh(1)^2 + a^2*b^2*d*f^2 + 2*(2*a^2*b^2*d^3*f*x - a^2*b^2*d^2*f)*cos
h(1))*sinh(1))*cosh(d*x + c)^6 - 4*(1728*a^2*b^2*d^3*f^3*x^3 - 2592*a^2*b^2
*d^2*f^3*x^2 + 1728*a^2*b^2*d^3*cosh(1)^3 + 1728*a^2*b^2*d^3*sinh(1)^3 + 25
92*a^2*b^2*d*f^3*x - 1296*a^2*b^2*f^3 + 2592*(2*a^2*b^2*d^3*f*x - a^2*b^2*d
^2*f)*cosh(1)^2 + 189*(32*b^4*d^3*f^3*x^3 - 24*b^4*d^2*f^3*x^2 + 32*b^4*d^3
*cosh(1)^3 + 32*b^4*d^3*sinh(1)^3 + 12*b^4*d*f^3*x - 3*b^4*f^3 + 24*(4*b^4*
d^3*f*x - b^4*d^2*f)*cosh(1)^2 + 24*(4*b^4*d^3*f*x + 4*b^4*d^3*cosh(1) - b^
4*d^2*f)*sinh(1)^2 + 12*(8*b^4*d^3*f^2*x^2 - 4*b^4*d^2*f^2*x + b^4*d*f^2)*c
osh(1) + 12*(8*b^4*d^3*f^2*x^2 - 4*b^4*d^2*f^2*x + 8*b^4*d^3*cosh(1)^2 + b^
4*d*f^2 + 4*(4*b^4*d^3*f*x - b^4*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 +
 2592*(2*a^2*b^2*d^3*f*x + 2*a^2*b^2*d^3*cosh(1) - a^2*b^2*d^2*f)*sinh(1)^2
 + 2592*(2*a^2*b^2*d^3*f^2*x^2 - 2*a^2*b^2*d^2*f^2*x + a^2*b^2*d*f^2)*cosh(
1) - 448*(9*a*b^3*d^3*f^3*x^3 - 9*a*b^3*d^2*f^3*x^2 + 9*a*b^3*d^3*cosh(1)^3
 + 9*a*b^3*d^3*sinh(1)^3 + 6*a*b^3*d*f^3*x - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*f
*x - a*b^3*d^2*f)*cosh(1)^2 + 9*(3*a*b^3*d^3*f*x + 3*a*b^3*d^3*cosh(1) - a*
b^3*d^2*f)*sinh(1)^2 + 3*(9*a*b^3*d^3*f^2*x^2 - 6*a*b^3*d^2*f^2*x + 2*a*b^3
*d*f^2)*cosh(1) + 3*(9*a*b^3*d^3*f^2*x^2 - 6*a*b^3*d^2*f^2*x + 9*a*b^3*d^3*
cosh(1)^2 + 2*a*b^3*d*f^2 + 6*(3*a*b^3*d^3*f*x - a*b^3*d^2*f)*cosh(1))*sinh
(1))*cosh(d*x + c) + 2592*(2*a^2*b^2*d^3*f^2*x^2 - 2*a^2*b^2*d^2*f^2*x + 2*
a^2*b^2*d^3*cosh(1)^2 + a^2*b^2*d*f^2 + 2*(2*a^2*b^2*d^3*f*x - a^2*b^2*d^2*

```
f)*cosh(1))*sinh(1))*sinh(d*x + c)^6 + 6912*((4*a^3*b + a*b^3)*d^3*f^3*x^3
- 3*(4*a^3*b + a*b^3)*d^2*f^3*x^2 + (4*a^3*b + a*b^3)*d^3*cosh(1)^3 + (4*a^
3*b + a*b^3)*d^3*sinh(1)^3 + 6*(4*a^3*b + a*b^3)*d*f^3*x - 6*(4*a^3*b + a*b
^3)*f^3 + 3*((4*a^3*b + a*b^3)*d^3*f*x - (4*a^3*b + a*b^3)*d^2*f)*cosh(1)^2
 + 3*((4*a^3*b + a*b^3)*d^3*f*x + (4*a^3*b + a*b^3)*d^3*cosh(1) - (4*a^3*b
+ a*b^3)*d^2*f)*sinh(1)^2 + 3*((4*a^3*b + a*b^3)*d^3*f^2*x^2 - 2*(4*a^3*b +
 a*b^3)*d^2*f^2*x + 2*(4*a^3*b + a*b^3)*d*f^2)*cosh(1) + 3*((4*a^3*b + a*b^
3)*d^3*f^2*x^2 - 2*(4*a^3*b + a*b^3)*d^2*f^2*x + (4*a^3*b + a*b^3)*d^3*cosh
(1)^2 + 2*(4*a^3*b + a*b^3)*d*f^2 + 2*((4*a^3*b + a*b^3)*d^3*f*x - (4*a^3*b
 + a*b^3)*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**2*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="giac")
[Out] integrate((f*x + e)^3*cosh(d*x + c)^2*sinh(d*x + c)^3/(b*sinh(d*x + c) + a)
, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^2\,\sinh(c+d\,x)^3\,{(e+f\,x)}^3}{a+b\,\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.397 $\int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$

Optimal. Leaf size=755

$$\frac{a^2f^2x}{4b^3d^2}+\frac{a^4(e+fx)^3}{3b^5f}+\frac{a^2(e+fx)^3}{6b^3f}-\frac{(e+fx)^3}{24bf}-\frac{2a^3f^2\cosh(c+dx)}{b^4d^3}-\frac{4af^2\cosh(c+dx)}{9b^2d^3}-\frac{a^3(e+fx)^2\cosh(c}{b^4d}$$

```
[Out] 1/4*a^2*f^2*x/b^3/d^2+1/3*a^4*(f*x+e)^3/b^5/f+1/6*a^2*(f*x+e)^3/b^3/f-1/24*
(f*x+e)^3/b/f-2*a^3*f^2*cosh(d*x+c)/b^4/d^3-4/9*a*f^2*cosh(d*x+c)/b^2/d^3-a
^3*(f*x+e)^2*cosh(d*x+c)/b^4/d-1/2*a^2*f*(f*x+e)*cosh(d*x+c)^2/b^3/d^2-2/27
*a*f^2*cosh(d*x+c)^3/b^2/d^3-1/3*a*(f*x+e)^2*cosh(d*x+c)^3/b^2/d-1/64*f*(f*
x+e)*cosh(4*d*x+4*c)/b/d^2+2*a^3*f*(f*x+e)*sinh(d*x+c)/b^4/d^2+4/9*a*f*(f*x
+e)*sinh(d*x+c)/b^2/d^2+1/4*a^2*f^2*cosh(d*x+c)*sinh(d*x+c)/b^3/d^3+1/2*a^2
*(f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/b^3/d+2/9*a*f*(f*x+e)*cosh(d*x+c)^2*sinh
(d*x+c)/b^2/d^2+1/256*f^2*sinh(4*d*x+4*c)/b/d^3+1/32*(f*x+e)^2*sinh(4*d*x+4
*c)/b/d-a^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2
)/b^5/d+a^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2
)/b^5/d-2*a^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b
^2)^(1/2)/b^5/d^2+2*a^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2
)))*(a^2+b^2)^(1/2)/b^5/d^2+2*a^3*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^
(1/2)))*(a^2+b^2)^(1/2)/b^5/d^3-2*a^3*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d^3
```

Rubi [A]
time = 1.05, antiderivative size = 755, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 18, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5698, 5556, 3377, 2717, 5555, 3391, 2718, 3392, 32, 2715, 8, 5684, 3403, 2296, 2221, 2611, 2320, 6724}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*f^2*x)/(4*b^3*d^2) + (a^4*(e + f*x)^3)/(3*b^5*f) + (a^2*(e + f*x)^3)/(
6*b^3*f) - (e + f*x)^3/(24*b*f) - (2*a^3*f^2*Cosh[c + d*x])/(b^4*d^3) - (4*
a*f^2*Cosh[c + d*x])/(9*b^2*d^3) - (a^3*(e + f*x)^2*Cosh[c + d*x])/(b^4*d)
- (a^2*f*(e + f*x)*Cosh[c + d*x]^2)/(2*b^3*d^2) - (2*a*f^2*Cosh[c + d*x]^3)
/(27*b^2*d^3) - (a*(e + f*x)^2*Cosh[c + d*x]^3)/(3*b^2*d) - (f*(e + f*x)*Co
sh[4*c + 4*d*x])/(64*b*d^2) - (a^3*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^5*d) + (a^3*Sqrt[a^2 + b^2]*(e + f*x
)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^5*d) - (2*a^3*Sqrt[a
^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])
```

```
/(b^5*d^2) + (2*a^3*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/(b^5*d^2) + (2*a^3*Sqrt[a^2 + b^2]*f^2*PolyLog[3
, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^3) - (2*a^3*Sqrt[a^2 +
b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^3) +
(2*a^3*f*(e + f*x)*Sinh[c + d*x])/(b^4*d^2) + (4*a*f*(e + f*x)*Sinh[c + d*x
])/(9*b^2*d^2) + (a^2*f^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^3*d^3) + (a^2*(
e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^3*d) + (2*a*f*(e + f*x)*Cosh[c
 + d*x]^2*Sinh[c + d*x])/(9*b^2*d^2) + (f^2*Sinh[4*c + 4*d*x])/(256*b*d^3)
+ ((e + f*x)^2*Sinh[4*c + 4*d*x])/(32*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5555

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\cosh^2(c}{a+b\sin}}{b}$$

$$= -\frac{a\int (e+fx)^2\cosh^2(c+dx)\sinh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^2\cos}{a+}}{}$$

$$= -\frac{(e+fx)^3}{24bf} - \frac{a(e+fx)^2\cosh^3(c+dx)}{3b^2d} + \frac{a^2\int (e+fx)^2\cosh}{b^3}$$

$$= -\frac{(e+fx)^3}{24bf} - \frac{a^2f(e+fx)\cosh^2(c+dx)}{2b^3d^2} - \frac{2af^2\cosh^3(c+d}{27b^2d^3}$$

$$= \frac{a^4(e+fx)^3}{3b^5f} + \frac{a^2(e+fx)^3}{6b^3f} - \frac{(e+fx)^3}{24bf} - \frac{a^3(e+fx)^2\cosh(c}{b^4d}$$

$$= \frac{a^2f^2x}{4b^3d^2} + \frac{a^4(e+fx)^3}{3b^5f} + \frac{a^2(e+fx)^3}{6b^3f} - \frac{(e+fx)^3}{24bf} - \frac{4af^2\cosh}{9b^2}$$

$$= \frac{a^2f^2x}{4b^3d^2} + \frac{a^4(e+fx)^3}{3b^5f} + \frac{a^2(e+fx)^3}{6b^3f} - \frac{(e+fx)^3}{24bf} - \frac{2a^3f^2\cos}{b^4}$$

$$= \frac{a^2f^2x}{4b^3d^2} + \frac{a^4(e+fx)^3}{3b^5f} + \frac{a^2(e+fx)^3}{6b^3f} - \frac{(e+fx)^3}{24bf} - \frac{2a^3f^2\cos}{b^4}$$

$$= \frac{a^2f^2x}{4b^3d^2} + \frac{a^4(e+fx)^3}{3b^5f} + \frac{a^2(e+fx)^3}{6b^3f} - \frac{(e+fx)^3}{24bf} - \frac{2a^3f^2\cos}{b^4}$$

$$= \frac{a^2f^2x}{4b^3d^2} + \frac{a^4(e+fx)^3}{3b^5f} + \frac{a^2(e+fx)^3}{6b^3f} - \frac{(e+fx)^3}{24bf} - \frac{2a^3f^2\cos}{b^4}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.83, size = 4023, normalized size = 5.33

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x
]),x]
```

```
[Out] -1/8*(e^2*(c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]
])/(Sqrt[-a^2 - b^2]*d)))/b - (e*f*(x^2 + ((2*I)*a*Pi*ArcTanh[(-b + a*Tanh[
```

(c + d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d^2) + (2*a*(2*((-I)*c + A
rcCos[((-I)*a)/b])*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sq
rt[-a^2 - b^2]] + ((-2*I)*c + Pi - (2*I)*d*x)*ArcTanh[((a - I*b)*Tan[((2*I)
*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] - (ArcCos[((-I)*a)/b] + (2*I)*Ar
cTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]])*Log[
((I*a + b)*(a + I*(b + Sqrt[-a^2 - b^2]))*(-I + Cot[((2*I)*c + Pi + (2*I)*d
x)/4]))/(b(I*a + b + I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]
))] - (ArcCos[((-I)*a)/b] - (2*I)*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2
*I)*d*x)/4])/Sqrt[-a^2 - b^2]])*Log[((I*a + b)*(I*a - b + Sqrt[-a^2 - b^2])
*(I + Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(a - I*b + Sqrt[-a^2 - b^2]*Co
t[((2*I)*c + Pi + (2*I)*d*x)/4]))] + (ArcCos[((-I)*a)/b] - (2*I)*ArcTanh[((
a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] - (2*I)*ArcTa
nh[((a - I*b)*Tan[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]])*Log[-((
(-1)^(3/4)*Sqrt[-a^2 - b^2]*E^(-1/2*c - (d*x)/2))/(Sqrt[2]*Sqrt[(-I)*b]*Sqr
t[a + b*Sinh[c + d*x]]))] + (ArcCos[((-I)*a)/b] + (2*I)*(ArcTanh[((a + I*b)
*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] + ArcTanh[((a - I*b)*
Tan[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]]))*Log[((-1)^(1/4)*Sqrt
[-a^2 - b^2]*E^((c + d*x)/2))/(Sqrt[2]*Sqrt[(-I)*b]*Sqrt[a + b*Sinh[c + d*x
]])] + I*(PolyLog[2, ((I*a + Sqrt[-a^2 - b^2])*(I*a + b - I*Sqrt[-a^2 - b^2
]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(I*a + b + I*Sqrt[-a^2 - b^2]*Cot[
((2*I)*c + Pi + (2*I)*d*x)/4]))] - PolyLog[2, ((a + I*Sqrt[-a^2 - b^2])*(-a
 + I*b + Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(I*a + b +
 I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))])))/(Sqrt[-a^2 - b^
2]*d^2)))/(8*b) - (f^2*(x^3 - (3*a*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2])] - d^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] +
2*d*x*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*x*PolyLog[2,
 -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*PolyLog[3, (b*E^(c + d*x))/(
-a + Sqrt[a^2 + b^2])] + 2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
]))]))/(Sqrt[a^2 + b^2]*d^3)))/(24*b) - (f^2*(2*(4*a^2 + b^2)*x^3 - (6*a*(4
*a^2 + 3*b^2)*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^2
*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, (b*E
^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*x*PolyLog[2, -((b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2]))] - 2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])
] + 2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^2 + b^
2]*d^3) - (24*a*b*Cosh[d*x]*((2 + d^2*x^2)*Cosh[c] - 2*d*x*Sinh[c]))/d^3 +
(3*b^2*Cosh[2*d*x]*(-2*d*x*Cosh[2*c] + (1 + 2*d^2*x^2)*Sinh[2*c]))/d^3 - (2
4*a*b*(-2*d*x*Cosh[c] + (2 + d^2*x^2)*Sinh[c])*Sinh[d*x])/d^3 + (3*b^2*((1
+ 2*d^2*x^2)*Cosh[2*c] - 2*d*x*Sinh[2*c])*Sinh[2*d*x])/d^3))/(96*b^3) - (e^
2*((4*a^2 + b^2)*(c + d*x) - (2*a*(4*a^2 + 3*b^2)*ArcTan[(b - a*Tanh[(c + d
*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 4*a*b*Cosh[c + d*x] + b^2*Sin
h[2*(c + d*x)]))/(16*b^3*d) - (e*f*((4*a^2 + b^2)*(-c + d*x)*(c + d*x) - 8*
a*b*d*x*Cosh[c + d*x] - b^2*Cosh[2*(c + d*x)] - (2*a*(4*a^2 + 3*b^2)*(2*c*A
rcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + (c + d*x)
Log[1 + (b(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] - (c +
d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])] + P

```
olyLog[2, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])] - Pol
yLog[2, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))]))/Sqr
t[a^2 + b^2] + 8*a*b*Sinh[c + d*x] + 2*b^2*d*x*Sinh[2*(c + d*x)]))/(16*b^3*
d^2) + (e^2*(6*(16*a^4 + 12*a^2*b^2 + b^4)*(c + d*x) - (12*a*(16*a^4 + 20*a
^2*b^2 + 5*b^4)*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a
^2 - b^2] - 48*a*b*(2*a^2 + b^2)*Cosh[c + d*x] - 8*a*b^3*Cosh[3*(c + d*x)]
+ 6*b^2*(4*a^2 + b^2)*Sinh[2*(c + d*x)] + 3*b^4*Sinh[4*(c + d*x)]))/(96*b^5
*d) + (e*f*(-576*a^4*c^2 - 432*a^2*b^2*c^2 - 36*b^4*c^2 + 576*a^4*d^2*x^2 +
 432*a^2*b^2*d^2*x^2 + 36*b^4*d^2*x^2 - 576*a*b*(2*a^2 + b^2)*d*x*Cosh[c +
d*x] - 36*(4*a^2*b^2 + b^4)*Cosh[2*(c + d*x)] - 96*a*b^3*d*x*Cosh[3*(c + d*
x)] - 9*b^4*Cosh[4*(c + d*x)] + (72*a*(16*a^4 + 20*a^2*b^2 + 5*b^4)*(2*Sqrt
[-(a^2 + b^2)^2]*c*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]] + a^2*(c +
d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + b^2*(c + d*x)*Log[1 +
 (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - a^2*(c + d*x)*Log[1 + (b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2])] - b^2*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2])] + (a^2 + b^2)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b
^2])] - (a^2 + b^2)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/
(Sqrt[-a^2 - b^2]*Sqrt[-(a^2 + b^2)^2]) + 1152*a^3*b*Sinh[c + d*x] + 576*a*
b^3*Sinh[c + d*x] + 288*a^2*b^2*d*x*Sinh[2*(c + d*x)] + 72*b^4*d*x*Sinh[2*(
c + d*x)] + 32*a*b^3*Sinh[3*(c + d*x)] + 36*b^4...
```

Maple [F]
time = 1.88, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \left(\cosh^2(dx+c)\right) \left(\sinh^3(dx+c)\right)}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="maxima")
```

```
[Out] -1/192*(192*sqrt(a^2 + b^2)*a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/
(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b^5*d) + (8*a*b^2*e^(-d*x - c) - 2
4*a^2*b*e^(-2*d*x - 2*c) - 3*b^3 + 24*(4*a^3 + a*b^2)*e^(-3*d*x - 3*c))*e^(
4*d*x + 4*c)/(b^4*d) - 24*(8*a^4 + 4*a^2*b^2 - b^4)*(d*x + c)/(b^5*d) + (24
*a^2*b*e^(-2*d*x - 2*c) + 8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e^(-4*d*x - 4*c)
```

```
 + 24*(4*a^3 + a*b^2)*e^(-d*x - c))/(b^4*d))*e^2 + 1/13824*(576*(8*a^4*d^3*
f^2*e^(4*c) + 4*a^2*b^2*d^3*f^2*e^(4*c) - b^4*d^3*f^2*e^(4*c))*x^3 + 1728*(
8*a^4*d^3*f*e^(4*c) + 4*a^2*b^2*d^3*f*e^(4*c) - b^4*d^3*f*e^(4*c))*x^2*e +
27*(8*b^4*d^2*f^2*x^2*e^(8*c) + b^4*f^2*e^(8*c) - 4*b^4*d*f*e^(8*c + 1) - 4
*(b^4*d*f^2*e^(8*c) - 4*b^4*d^2*f*e^(8*c + 1))*x)*e^(4*d*x) - 64*(9*a*b^3*d
^2*f^2*x^2*e^(7*c) + 2*a*b^3*f^2*e^(7*c) - 6*a*b^3*d*f*e^(7*c + 1) - 6*(a*b
^3*d*f^2*e^(7*c) - 3*a*b^3*d^2*f*e^(7*c + 1))*x)*e^(3*d*x) + 864*(2*a^2*b^2
*d^2*f^2*x^2*e^(6*c) + a^2*b^2*f^2*e^(6*c) - 2*a^2*b^2*d*f*e^(6*c + 1) - 2*
(a^2*b^2*d*f^2*e^(6*c) - 2*a^2*b^2*d^2*f*e^(6*c + 1))*x)*e^(2*d*x) - 1728*(
8*a^3*b*f^2*e^(5*c) + 2*a*b^3*f^2*e^(5*c) + (4*a^3*b*d^2*f^2*e^(5*c) + a*b^
3*d^2*f^2*e^(5*c))*x^2 - 2*(4*a^3*b*d*f^2*e^(5*c) + a*b^3*d*f^2*e^(5*c) - (
4*a^3*b*d^2*f*e^(5*c) + a*b^3*d^2*f*e^(5*c))*e)*x - 2*(4*a^3*b*d*f*e^(5*c)
+ a*b^3*d*f*e^(5*c))*e)*e^(d*x) - 1728*(8*a^3*b*f^2*e^(3*c) + 2*a*b^3*f^2*e
^(3*c) + (4*a^3*b*d^2*f^2*e^(3*c) + a*b^3*d^2*f^2*e^(3*c))*x^2 + 2*(4*a^3*b
*d*f^2*e^(3*c) + a*b^3*d*f^2*e^(3*c) + (4*a^3*b*d^2*f*e^(3*c) + a*b^3*d^2*f
*e^(3*c))*e)*x + 2*(4*a^3*b*d*f*e^(3*c) + a*b^3*d*f*e^(3*c))*e)*e^(-d*x) -
864*(2*a^2*b^2*d^2*f^2*x^2*e^(2*c) + a^2*b^2*f^2*e^(2*c) + 2*a^2*b^2*d*f*e^
(2*c + 1) + 2*(a^2*b^2*d*f^2*e^(2*c) + 2*a^2*b^2*d^2*f*e^(2*c + 1))*x)*e^(-
2*d*x) - 64*(9*a*b^3*d^2*f^2*x^2*e^c + 6*a*b^3*d*f*e^(c + 1) + 2*a*b^3*f^2*
e^c + 6*(3*a*b^3*d^2*f*e^(c + 1) + a*b^3*d*f^2*e^c)*x)*e^(-3*d*x) - 27*(8*b
^4*d^2*f^2*x^2 + 4*b^4*d*f*e + b^4*f^2 + 4*(4*b^4*d^2*f*e + b^4*d*f^2)*x)*e
^(-4*d*x))*e^(-4*c)/(b^5*d^3) - integrate(2*((a^5*f^2*e^c + a^3*b^2*f^2*e^c
)*x^2 + 2*(a^5*f*e^c + a^3*b^2*f*e^c)*x*e)*e^(d*x)/(b^6*e^(2*d*x + 2*c) + 2
*a*b^5*e^(d*x + c) - b^6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 9951 vs. $2(709) = 1418$.
time = 0.53, size = 9951, normalized size = 13.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="fricas")
```

```
[Out] -1/13824*(216*b^4*d^2*f^2*x^2 - 27*(8*b^4*d^2*f^2*x^2 - 4*b^4*d*f^2*x + 8*b
^4*d^2*cosh(1)^2 + 8*b^4*d^2*sinh(1)^2 + b^4*f^2 + 4*(4*b^4*d^2*f*x - b^4*d
*f)*cosh(1) + 4*(4*b^4*d^2*f*x + 4*b^4*d^2*cosh(1) - b^4*d*f)*sinh(1))*cosh
(d*x + c)^8 - 27*(8*b^4*d^2*f^2*x^2 - 4*b^4*d*f^2*x + 8*b^4*d^2*cosh(1)^2 +
 8*b^4*d^2*sinh(1)^2 + b^4*f^2 + 4*(4*b^4*d^2*f*x - b^4*d*f)*cosh(1) + 4*(4
*b^4*d^2*f*x + 4*b^4*d^2*cosh(1) - b^4*d*f)*sinh(1))*sinh(d*x + c)^8 + 108*
b^4*d*f^2*x + 216*b^4*d^2*cosh(1)^2 + 64*(9*a*b^3*d^2*f^2*x^2 - 6*a*b^3*d*f
^2*x + 9*a*b^3*d^2*cosh(1)^2 + 9*a*b^3*d^2*sinh(1)^2 + 2*a*b^3*f^2 + 6*(3*a
*b^3*d^2*f*x - a*b^3*d*f)*cosh(1) + 6*(3*a*b^3*d^2*f*x + 3*a*b^3*d^2*cosh(1
) - a*b^3*d*f)*sinh(1))*cosh(d*x + c)^7 + 216*b^4*d^2*sinh(1)^2 + 8*(72*a*b
```

^3*d^2*f^2*x^2 - 48*a*b^3*d*f^2*x + 72*a*b^3*d^2*cosh(1)^2 + 72*a*b^3*d^2*s
inh(1)^2 + 16*a*b^3*f^2 + 48*(3*a*b^3*d^2*f*x - a*b^3*d*f)*cosh(1) - 27*(8*
b^4*d^2*f^2*x^2 - 4*b^4*d*f^2*x + 8*b^4*d^2*cosh(1)^2 + 8*b^4*d^2*sinh(1)^2
 + b^4*f^2 + 4*(4*b^4*d^2*f*x - b^4*d*f)*cosh(1) + 4*(4*b^4*d^2*f*x + 4*b^4
*d^2*cosh(1) - b^4*d*f)*sinh(1))*cosh(d*x + c) + 48*(3*a*b^3*d^2*f*x + 3*a*
b^3*d^2*cosh(1) - a*b^3*d*f)*sinh(1))*sinh(d*x + c)^7 - 864*(2*a^2*b^2*d^2*
f^2*x^2 - 2*a^2*b^2*d*f^2*x + 2*a^2*b^2*d^2*cosh(1)^2 + 2*a^2*b^2*d^2*sinh(
1)^2 + a^2*b^2*f^2 + 2*(2*a^2*b^2*d^2*f*x - a^2*b^2*d*f)*cosh(1) + 2*(2*a^2
*b^2*d^2*f*x + 2*a^2*b^2*d^2*cosh(1) - a^2*b^2*d*f)*sinh(1))*cosh(d*x + c)^
6 - 4*(432*a^2*b^2*d^2*f^2*x^2 - 432*a^2*b^2*d*f^2*x + 432*a^2*b^2*d^2*cosh
(1)^2 + 432*a^2*b^2*d^2*sinh(1)^2 + 216*a^2*b^2*f^2 + 189*(8*b^4*d^2*f^2*x^
2 - 4*b^4*d*f^2*x + 8*b^4*d^2*cosh(1)^2 + 8*b^4*d^2*sinh(1)^2 + b^4*f^2 + 4
*(4*b^4*d^2*f*x - b^4*d*f)*cosh(1) + 4*(4*b^4*d^2*f*x + 4*b^4*d^2*cosh(1) -
 b^4*d*f)*sinh(1))*cosh(d*x + c)^2 + 432*(2*a^2*b^2*d^2*f*x - a^2*b^2*d*f)*
cosh(1) - 112*(9*a*b^3*d^2*f^2*x^2 - 6*a*b^3*d*f^2*x + 9*a*b^3*d^2*cosh(1)^
2 + 9*a*b^3*d^2*sinh(1)^2 + 2*a*b^3*f^2 + 6*(3*a*b^3*d^2*f*x - a*b^3*d*f)*c
osh(1) + 6*(3*a*b^3*d^2*f*x + 3*a*b^3*d^2*cosh(1) - a*b^3*d*f)*sinh(1))*cos
h(d*x + c) + 432*(2*a^2*b^2*d^2*f*x + 2*a^2*b^2*d^2*cosh(1) - a^2*b^2*d*f)*
sinh(1))*sinh(d*x + c)^6 + 27*b^4*f^2 + 1728*((4*a^3*b + a*b^3)*d^2*f^2*x^2
 - 2*(4*a^3*b + a*b^3)*d*f^2*x + (4*a^3*b + a*b^3)*d^2*cosh(1)^2 + (4*a^3*b
 + a*b^3)*d^2*sinh(1)^2 + 2*(4*a^3*b + a*b^3)*f^2 + 2*((4*a^3*b + a*b^3)*d^
2*f*x - (4*a^3*b + a*b^3)*d*f)*cosh(1) + 2*((4*a^3*b + a*b^3)*d^2*f*x + (4*
a^3*b + a*b^3)*d^2*cosh(1) - (4*a^3*b + a*b^3)*d*f)*sinh(1))*cosh(d*x + c)^
5 + 24*(72*(4*a^3*b + a*b^3)*d^2*f^2*x^2 - 144*(4*a^3*b + a*b^3)*d*f^2*x +
72*(4*a^3*b + a*b^3)*d^2*cosh(1)^2 + 72*(4*a^3*b + a*b^3)*d^2*sinh(1)^2 - 6
3*(8*b^4*d^2*f^2*x^2 - 4*b^4*d*f^2*x + 8*b^4*d^2*cosh(1)^2 + 8*b^4*d^2*sinh
(1)^2 + b^4*f^2 + 4*(4*b^4*d^2*f*x - b^4*d*f)*cosh(1) + 4*(4*b^4*d^2*f*x +
4*b^4*d^2*cosh(1) - b^4*d*f)*sinh(1))*cosh(d*x + c)^3 + 144*(4*a^3*b + a*b^
3)*f^2 + 56*(9*a*b^3*d^2*f^2*x^2 - 6*a*b^3*d*f^2*x + 9*a*b^3*d^2*cosh(1)^2
+ 9*a*b^3*d^2*sinh(1)^2 + 2*a*b^3*f^2 + 6*(3*a*b^3*d^2*f*x - a*b^3*d*f)*cos
h(1) + 6*(3*a*b^3*d^2*f*x + 3*a*b^3*d^2*cosh(1) - a*b^3*d*f)*sinh(1))*cosh(
d*x + c)^2 + 144*((4*a^3*b + a*b^3)*d^2*f*x - (4*a^3*b + a*b^3)*d*f)*cosh(1
) - 216*(2*a^2*b^2*d^2*f^2*x^2 - 2*a^2*b^2*d*f^2*x + 2*a^2*b^2*d^2*cosh(1)^
2 + 2*a^2*b^2*d^2*sinh(1)^2 + a^2*b^2*f^2 + 2*(2*a^2*b^2*d^2*f*x - a^2*b^2*
d*f)*cosh(1) + 2*(2*a^2*b^2*d^2*f*x + 2*a^2*b^2*d^2*cosh(1) - a^2*b^2*d*f)*
sinh(1))*cosh(d*x + c) + 144*((4*a^3*b + a*b^3)*d^2*f*x + (4*a^3*b + a*b^3)
*d^2*cosh(1) - (4*a^3*b + a*b^3)*d*f)*sinh(1))*sinh(d*x + c)^5 - 576*((8*a^
4 + 4*a^2*b^2 - b^4)*d^3*f^2*x^3 + 3*(8*a^4 + 4*a^2*b^2 - b^4)*d^3*f*x^2*co
sh(1) + 3*(8*a^4 + 4*a^2*b^2 - b^4)*d^3*x*cosh(1)^2 + 3*(8*a^4 + 4*a^2*b^2
- b^4)*d^3*x*sinh(1)^2 + 3*((8*a^4 + 4*a^2*b^2 - b^4)*d^3*f*x^2 + 2*(8*a^4
+ 4*a^2*b^2 - b^4)*d^3*x*cosh(1))*sinh(1))*cosh(d*x + c)^4 - 2*(288*(8*a^4
+ 4*a^2*b^2 - b^4)*d^3*f^2*x^3 + 864*(8*a^4 + 4*a^2*b^2 - b^4)*d^3*f*x^2*co
sh(1) + 864*(8*a^4 + 4*a^2*b^2 - b^4)*d^3*x*cosh(1)^2 + 864*(8*a^4 + 4*a^2*
b^2 - b^4)*d^3*x*sinh(1)^2 + 945*(8*b^4*d^2*f^2*x^2 - 4*b^4*d*f^2*x + 8*b^4
*d^2*cosh(1)^2 + 8*b^4*d^2*sinh(1)^2 + b^4*f^2 + 4*(4*b^4*d^2*f*x - b^4*d*f

```
)*cosh(1) + 4*(4*b^4*d^2*f*x + 4*b^4*d^2*cosh(1) - b^4*d*f)*sinh(1))*cosh(d
*x + c)^4 - 1120*(9*a*b^3*d^2*f^2*x^2 - 6*a*b^3*d*f^2*x + 9*a*b^3*d^2*cosh(
1)^2 + 9*a*b^3*d^2*sinh(1)^2 + 2*a*b^3*f^2 + 6*(3*a*b^3*d^2*f*x - a*b^3*d*f
)*cosh(1) + 6*(3*a*b^3*d^2*f*x + 3*a*b^3*d^2*cosh(1) - a*b^3*d*f)*sinh(1))*
cosh(d*x + c)^3 + 6480*(2*a^2*b^2*d^2*f^2*x^2 - 2*a^2*b^2*d*f^2*x + 2*a^2*b
^2*d^2*cosh(1)^2 + 2*a^2*b^2*d^2*sinh(1)^2 + a^2*b^2*f^2 + 2*(2*a^2*b^2*d^2
*f*x - a^2*b^2*d*f)*cosh(1) + 2*(2*a^2*b^2*d^2*f*x + 2*a^2*b^2*d^2*cosh(1)
- a^2*b^2*d*f)*sinh(1))*cosh(d*x + c)^2 - 4320*((4*a^3*b + a*b^3)*d^2*f^2*x
^2 - 2*(4*a^3*b + a*b^3)*d*f^2*x + (4*a^3*b + a*b^3)*d^2*cosh(1)^2 + (4*a^3
*b + a*b^3)*d^2*sinh(1)^2 + 2*(4*a^3*b + a*b^3)*f^2 + 2*((4*a^3*b + a*b^3)*
d^2*f*x - (4*a^3*b + a*b^3)*d*f)*cosh(1) + 2*((4*a^3*b + a*b^3)*d^2*f*x + (
4*a^3*b + a*b^3)*d^2*cosh(1) - (4*a^3*b + a*b^3...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**2*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)^2*sinh(d*x + c)^3/(b*sinh(d*x + c) + a)
, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{cosh}\left(c+d\,x\right)}^2\,{\mathrm{sinh}\left(c+d\,x\right)}^3\,{\left(e+f\,x\right)}^2}{a+b\,\mathrm{sinh}\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.398 $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=474

$$\frac{a^4ex}{b^5}+\frac{a^2ex}{2b^3}+\frac{a^4fx^2}{2b^5}+\frac{a^2fx^2}{4b^3}-\frac{(e+fx)^2}{16bf}-\frac{a^3(e+fx)\cosh(c+dx)}{b^4d}-\frac{a^2f\cosh^2(c+dx)}{4b^3d^2}-\frac{a(e+fx)\cosh^3(c+}{3b^2d}$$

```
[Out] a^4*e*x/b^5+1/2*a^2*e*x/b^3+1/2*a^4*f*x^2/b^5+1/4*a^2*f*x^2/b^3-1/16*(f*x+e
)^2/b/f-a^3*(f*x+e)*cosh(d*x+c)/b^4/d-1/4*a^2*f*cosh(d*x+c)^2/b^3/d^2-1/3*a
*(f*x+e)*cosh(d*x+c)^3/b^2/d-1/128*f*cosh(4*d*x+4*c)/b/d^2+a^3*f*sinh(d*x+c
)/b^4/d^2+1/3*a*f*sinh(d*x+c)/b^2/d^2+1/2*a^2*(f*x+e)*cosh(d*x+c)*sinh(d*x+
c)/b^3/d+1/9*a*f*sinh(d*x+c)^3/b^2/d^2+1/32*(f*x+e)*sinh(4*d*x+4*c)/b/d-a^3
*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d+a^3*(
f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d-a^3*f*p
olylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d^2+a^3*f*p
olylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/b^5/d^2
```

Rubi [A]
time = 0.62, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 14, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5698, 5556, 3377, 2718, 5555, 2713, 3391, 5684, 2717, 3403, 2296, 2221, 2317, 2438}

$$\frac{a^4ex}{b^5}+\frac{a^4fx^2}{2b^5}+\frac{a^3f\sinh(c+dx)}{b^4d^2}-\frac{a^3(e+fx)\cosh(c+dx)}{b^4d}-\frac{a^2f\cosh^2(c+dx)}{4b^3d^2}+\frac{a^2(e+fx)\sinh(c+dx)\cosh(c+dx)}{2b^3d}+\frac{a^2ex}{2b^3}+\frac{a^2fx^2}{4b^3}-\frac{a^3f\sqrt{a^2+b^2}\,\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^2}+\frac{a^3f\sqrt{a^2+b^2}\,\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^5d^2}-\frac{a^3\sqrt{a^2+b^2}(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^5d}+\frac{a^3\sqrt{a^2+b^2}(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^5d}+\frac{af\sinh^3(c+dx)}{9b^2d^2}+\frac{af\sinh(c+dx)}{3b^2d^2}-\frac{a(e+fx)\cosh^3(c+dx)}{3b^2d}-\frac{f\cosh(4c+4dx)}{128bd^2}+\frac{(e+fx)\sinh(4c+4dx)}{32bd}-\frac{(e+fx)^2}{16bf}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^4*e*x)/b^5 + (a^2*e*x)/(2*b^3) + (a^4*f*x^2)/(2*b^5) + (a^2*f*x^2)/(4*b^
3) - (e + f*x)^2/(16*b*f) - (a^3*(e + f*x)*Cosh[c + d*x])/(b^4*d) - (a^2*f*
Cosh[c + d*x]^2)/(4*b^3*d^2) - (a*(e + f*x)*Cosh[c + d*x]^3)/(3*b^2*d) - (f
*Cosh[4*c + 4*d*x])/(128*b*d^2) - (a^3*Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b
*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^5*d) + (a^3*Sqrt[a^2 + b^2]*(e + f
*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^5*d) - (a^3*Sqrt[a^2
 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^2) +
 (a^3*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/(b^5*d^2) + (a^3*f*Sinh[c + d*x])/(b^4*d^2) + (a*f*Sinh[c + d*x])/(3*b^2
*d^2) + (a^2*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^3*d) + (a*f*Sinh[c
 + d*x]^3)/(9*b^2*d^2) + ((e + f*x)*Sinh[4*c + 4*d*x])/(32*b*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5555

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh^2(c+dx)\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\cosh^2(c+}{a+b\sinh}}{b}$$

$$= -\frac{a\int (e+fx)\cosh^2(c+dx)\sinh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)\cosh}{a+b}}{}$$

$$= -\frac{(e+fx)^2}{16bf} - \frac{a(e+fx)\cosh^3(c+dx)}{3b^2d} + \frac{a^2\int (e+fx)\cosh^2}{b^3}$$

$$= -\frac{(e+fx)^2}{16bf} - \frac{a^2f\cosh^2(c+dx)}{4b^3d^2} - \frac{a(e+fx)\cosh^3(c+dx)}{3b^2d}$$

$$= \frac{a^4ex}{b^5} + \frac{a^2ex}{2b^3} + \frac{a^4fx^2}{2b^5} + \frac{a^2fx^2}{4b^3} - \frac{(e+fx)^2}{16bf} - \frac{a^3(e+fx)\mathrm{co}}{b^4d}$$

$$= \frac{a^4ex}{b^5} + \frac{a^2ex}{2b^3} + \frac{a^4fx^2}{2b^5} + \frac{a^2fx^2}{4b^3} - \frac{(e+fx)^2}{16bf} - \frac{a^3(e+fx)\mathrm{co}}{b^4d}$$

$$= \frac{a^4ex}{b^5} + \frac{a^2ex}{2b^3} + \frac{a^4fx^2}{2b^5} + \frac{a^2fx^2}{4b^3} - \frac{(e+fx)^2}{16bf} - \frac{a^3(e+fx)\mathrm{co}}{b^4d}$$

$$= \frac{a^4ex}{b^5} + \frac{a^2ex}{2b^3} + \frac{a^4fx^2}{2b^5} + \frac{a^2fx^2}{4b^3} - \frac{(e+fx)^2}{16bf} - \frac{a^3(e+fx)\mathrm{co}}{b^4d}$$

$$= \frac{a^4ex}{b^5} + \frac{a^2ex}{2b^3} + \frac{a^4fx^2}{2b^5} + \frac{a^2fx^2}{4b^3} - \frac{(e+fx)^2}{16bf} - \frac{a^3(e+fx)\mathrm{co}}{b^4d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.13, size = 2275, normalized size = 4.80

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (-144*b^4*e*(c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^
2]])/(Sqrt[-a^2 - b^2]*d)) - 72*b^4*f*(x^2 + ((2*I)*a*Pi*ArcTanh[(-b + a*Ta
nh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d^2) + (2*a*(2*((-I)*c
+ ArcCos[((-I)*a)/b])*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])
/Sqrt[-a^2 - b^2]] + ((-2*I)*c + Pi - (2*I)*d*x)*ArcTanh[((a - I*b)*Tan[((2
```

```
*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] - (ArcCos[((-I)*a)/b] + (2*I)
*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]])*L
og[((I*a + b)*(a + I*(b + Sqrt[-a^2 - b^2]))*(-I + Cot[((2*I)*c + Pi + (2*I
)*d*x)/4]))/(b*(I*a + b + I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)
/4]))] - (ArcCos[((-I)*a)/b] - (2*I)*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi +
 (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]])*Log[((I*a + b)*(I*a - b + Sqrt[-a^2 - b^
2])*(I + Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(a - I*b + Sqrt[-a^2 - b^2]
*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))] + (ArcCos[((-I)*a)/b] - (2*I)*ArcTanh
[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] - (2*I)*Ar
cTanh[((a - I*b)*Tan[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]])*Log[
-(((-1)^(3/4)*Sqrt[-a^2 - b^2]*E^(-1/2*c - (d*x)/2))/(Sqrt[2]*Sqrt[(-I)*b]*
Sqrt[a + b*Sinh[c + d*x]]))] + (ArcCos[((-I)*a)/b] + (2*I)*(ArcTanh[((a + I
*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] + ArcTanh[((a - I*
b)*Tan[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]]))*Log[((-1)^(1/4)*S
qrt[-a^2 - b^2]*E^((c + d*x)/2))/(Sqrt[2]*Sqrt[(-I)*b]*Sqrt[a + b*Sinh[c +
d*x]])] + I*(PolyLog[2, ((I*a + Sqrt[-a^2 - b^2])*(I*a + b - I*Sqrt[-a^2 -
b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(I*a + b + I*Sqrt[-a^2 - b^2]*C
ot[((2*I)*c + Pi + (2*I)*d*x)/4]))] - PolyLog[2, ((a + I*Sqrt[-a^2 - b^2])*
(-a + I*b + Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(I*a +
b + I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))])))/(Sqrt[-a^2 -
 b^2]*d^2)) - (72*b^2*e*((4*a^2 + b^2)*(c + d*x) - (2*a*(4*a^2 + 3*b^2)*Arc
Tan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 4*a*b*C
osh[c + d*x] + b^2*Sinh[2*(c + d*x)]))/d - (36*b^2*f*((4*a^2 + b^2)*(-c + d
*x)*(c + d*x) - 8*a*b*d*x*Cosh[c + d*x] - b^2*Cosh[2*(c + d*x)] - (2*a*(4*a
^2 + 3*b^2)*(2*c*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 +
 b^2]] + (c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^
2 + b^2])] - (c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqr
t[a^2 + b^2])] + PolyLog[2, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[
a^2 + b^2])] - PolyLog[2, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a
^2 + b^2]))]))/Sqrt[a^2 + b^2] + 8*a*b*Sinh[c + d*x] + 2*b^2*d*x*Sinh[2*(c
+ d*x)]))/d^2 + (12*e*(6*(16*a^4 + 12*a^2*b^2 + b^4)*(c + d*x) - (12*a*(16*
a^4 + 20*a^2*b^2 + 5*b^4)*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]
])/Sqrt[-a^2 - b^2] - 48*a*b*(2*a^2 + b^2)*Cosh[c + d*x] - 8*a*b^3*Cosh[3*(
c + d*x)] + 6*b^2*(4*a^2 + b^2)*Sinh[2*(c + d*x)] + 3*b^4*Sinh[4*(c + d*x)]
))/d + (f*(-576*a^4*c^2 - 432*a^2*b^2*c^2 - 36*b^4*c^2 + 576*a^4*d^2*x^2 +
432*a^2*b^2*d^2*x^2 + 36*b^4*d^2*x^2 - 576*a*b*(2*a^2 + b^2)*d*x*Cosh[c + d
*x] - 36*(4*a^2*b^2 + b^4)*Cosh[2*(c + d*x)] - 96*a*b^3*d*x*Cosh[3*(c + d*x
)] - 9*b^4*Cosh[4*(c + d*x)] + (72*a*(16*a^4 + 20*a^2*b^2 + 5*b^4)*(2*Sqrt[
-(a^2 + b^2)^2]*c*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]] + a^2*(c + d
*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + b^2*(c + d*x)*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - a^2*(c + d*x)*Log[1 + (b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2])] - b^2*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2])] + (a^2 + b^2)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^
2])] - (a^2 + b^2)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(
Sqrt[-a^2 - b^2]*Sqrt[-(a^2 + b^2)^2]) + 1152*a^3*b*Sinh[c + d*x] + 576*a*b
```

```
^3*Sinh[c + d*x] + 288*a^2*b^2*d*x*Sinh[2*(c + d*x)] + 72*b^4*d*x*Sinh[2*(c
 + d*x)] + 32*a*b^3*Sinh[3*(c + d*x)] + 36*b^4*d*x*Sinh[4*(c + d*x)]))/d^2)
/(1152*b^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1212 vs. $2(432) = 864$.
time = 1.62, size = 1213, normalized size = 2.56

method	result
risch	$-\frac{2a^5 fc \operatorname{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{d^2 b^5 \sqrt{a^2+b^2}} - \frac{2a^3 fc \operatorname{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{d^2 b^3 \sqrt{a^2+b^2}} + \frac{a^3 f \ln\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right) x}{d\, b^3 \sqrt{a^2+b^2}} + \frac{a^3 f \ln\left(\frac{b\,\mathrm{e}^{d}}{}\right.}{d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNV
ERBOSE)
```

```
[Out] -2/d^2*a^5/b^5*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^
2)^(1/2))-2/d^2*a^3/b^3*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a
)/(a^2+b^2)^(1/2))+1/d*a^3/b^3*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)
^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2*a^3/b^3*f/(a^2+b^2)^(1/2)*ln((b*exp(
d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d*a^5/b^5*f/(a^2+b^2)^(1
/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d^2*a^5/
b^5*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1
/2)))*c+1/d*a^5/b^5*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(
a+(a^2+b^2)^(1/2)))*x+1/d^2*a^5/b^5*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2
+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d*a^3/b^3*f/(a^2+b^2)^(1/2)*ln((-b*
exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d^2*a^3/b^3*f/(a^2+
b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/2
56*(4*d*f*x+4*d*e-f)/d^2/b*exp(4*d*x+4*c)-1/8*a*(4*a^2*d*f*x+b^2*d*f*x+4*a^
2*d*e+b^2*d*e-4*a^2*f-b^2*f)/b^4/d^2*exp(d*x+c)+1/4*a^2*f*x^2/b^3-1/16*f*x^
2/b-1/d^2*a^5/b^5*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)
/(-a+(a^2+b^2)^(1/2)))+1/d^2*a^5/b^5*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+
(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2*a^3/b^3*f/(a^2+b^2)^(1/2)*dil
og((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/d^2*a^3/b^3*f/
(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))
-1/256*(4*d*f*x+4*d*e+f)/d^2/b*exp(-4*d*x-4*c)+1/2*a^4*f*x^2/b^5-1/8*a*(4*a
^2+b^2)*(d*f*x+d*e+f)/b^4/d^2*exp(-d*x-c)+1/16*a^2*(2*d*f*x+2*d*e-f)/b^3/d^
2*exp(2*d*x+2*c)-1/16*a^2*(2*d*f*x+2*d*e+f)/b^3/d^2*exp(-2*d*x-2*c)+2/d*a^5
/b^5*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/
d*a^3/b^3*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2
))+1/2*a^2*e*x/b^3-1/72*a*(3*d*f*x+3*d*e-f)/b^2/d^2*exp(3*d*x+3*c)-1/8*e*x/
b+a^4*e*x/b^5-1/72*a*(3*d*f*x+3*d*e+f)/b^2/d^2*exp(-3*d*x-3*c)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -1/2304*(4608*(a^5*e^c + a^3*b^2*e^c)*integrate(x*e^(d*x)/(b^6*e^(2*d*x + 2
*c) + 2*a*b^5*e^(d*x + c) - b^6), x) - (144*(8*a^4*d^2*e^(4*c) + 4*a^2*b^2*
d^2*e^(4*c) - b^4*d^2*e^(4*c))*x^2 + 9*(4*b^4*d*x*e^(8*c) - b^4*e^(8*c))*e^
(4*d*x) - 32*(3*a*b^3*d*x*e^(7*c) - a*b^3*e^(7*c))*e^(3*d*x) + 144*(2*a^2*b
^2*d*x*e^(6*c) - a^2*b^2*e^(6*c))*e^(2*d*x) + 288*(4*a^3*b*e^(5*c) + a*b^3*
e^(5*c) - (4*a^3*b*d*e^(5*c) + a*b^3*d*e^(5*c))*x)*e^(d*x) - 288*(4*a^3*b*e
^(3*c) + a*b^3*e^(3*c) + (4*a^3*b*d*e^(3*c) + a*b^3*d*e^(3*c))*x)*e^(-d*x)
 - 144*(2*a^2*b^2*d*x*e^(2*c) + a^2*b^2*e^(2*c))*e^(-2*d*x) - 32*(3*a*b^3*d*
x*e^c + a*b^3*e^c)*e^(-3*d*x) - 9*(4*b^4*d*x + b^4)*e^(-4*d*x))*e^(-4*c)/(b
^5*d^2))*f - 1/192*(192*sqrt(a^2 + b^2)*a^3*log((b*e^(-d*x - c) - a - sqrt(
a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(b^5*d) + (8*a*b^2*e^(-
d*x - c) - 24*a^2*b*e^(-2*d*x - 2*c) - 3*b^3 + 24*(4*a^3 + a*b^2)*e^(-3*d*x
 - 3*c))*e^(4*d*x + 4*c)/(b^4*d) - 24*(8*a^4 + 4*a^2*b^2 - b^4)*(d*x + c)/(
b^5*d) + (24*a^2*b*e^(-2*d*x - 2*c) + 8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e^(-
4*d*x - 4*c) + 24*(4*a^3 + a*b^2)*e^(-d*x - c))/(b^4*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3950 vs. $2(439) = 878$.
time = 0.45, size = 3950, normalized size = 8.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/2304*(9*(4*b^4*d*f*x + 4*b^4*d*cosh(1) + 4*b^4*d*sinh(1) - b^4*f)*cosh(d*
x + c)^8 + 9*(4*b^4*d*f*x + 4*b^4*d*cosh(1) + 4*b^4*d*sinh(1) - b^4*f)*sinh
(d*x + c)^8 - 32*(3*a*b^3*d*f*x + 3*a*b^3*d*cosh(1) + 3*a*b^3*d*sinh(1) - a
*b^3*f)*cosh(d*x + c)^7 - 8*(12*a*b^3*d*f*x + 12*a*b^3*d*cosh(1) + 12*a*b^3
*d*sinh(1) - 4*a*b^3*f - 9*(4*b^4*d*f*x + 4*b^4*d*cosh(1) + 4*b^4*d*sinh(1)
 - b^4*f)*cosh(d*x + c))*sinh(d*x + c)^7 - 36*b^4*d*f*x + 144*(2*a^2*b^2*d*
f*x + 2*a^2*b^2*d*cosh(1) + 2*a^2*b^2*d*sinh(1) - a^2*b^2*f)*cosh(d*x + c)^
6 + 4*(72*a^2*b^2*d*f*x + 72*a^2*b^2*d*cosh(1) + 72*a^2*b^2*d*sinh(1) - 36*
a^2*b^2*f + 63*(4*b^4*d*f*x + 4*b^4*d*cosh(1) + 4*b^4*d*sinh(1) - b^4*f)*co
sh(d*x + c)^2 - 56*(3*a*b^3*d*f*x + 3*a*b^3*d*cosh(1) + 3*a*b^3*d*sinh(1) -
 a*b^3*f)*cosh(d*x + c))*sinh(d*x + c)^6 - 36*b^4*d*cosh(1) - 288*((4*a^3*b
```

+ a*b^3)*d*f*x + (4*a^3*b + a*b^3)*d*cosh(1) + (4*a^3*b + a*b^3)*d*sinh(1)
- (4*a^3*b + a*b^3)*f)*cosh(d*x + c)^5 - 36*b^4*d*sinh(1) - 24*(12*(4*a^3*
b + a*b^3)*d*f*x - 21*(4*b^4*d*f*x + 4*b^4*d*cosh(1) + 4*b^4*d*sinh(1) - b^
4*f)*cosh(d*x + c)^3 + 12*(4*a^3*b + a*b^3)*d*cosh(1) + 28*(3*a*b^3*d*f*x +
3*a*b^3*d*cosh(1) + 3*a*b^3*d*sinh(1) - a*b^3*f)*cosh(d*x + c)^2 + 12*(4*a
^3*b + a*b^3)*d*sinh(1) - 12*(4*a^3*b + a*b^3)*f - 36*(2*a^2*b^2*d*f*x + 2*
a^2*b^2*d*cosh(1) + 2*a^2*b^2*d*sinh(1) - a^2*b^2*f)*cosh(d*x + c))*sinh(d*
x + c)^5 - 9*b^4*f + 144*((8*a^4 + 4*a^2*b^2 - b^4)*d^2*f*x^2 + 2*(8*a^4 +
4*a^2*b^2 - b^4)*d^2*x*cosh(1) + 2*(8*a^4 + 4*a^2*b^2 - b^4)*d^2*x*sinh(1))
*cosh(d*x + c)^4 + 2*(72*(8*a^4 + 4*a^2*b^2 - b^4)*d^2*f*x^2 + 144*(8*a^4 +
4*a^2*b^2 - b^4)*d^2*x*cosh(1) + 315*(4*b^4*d*f*x + 4*b^4*d*cosh(1) + 4*b^
4*d*sinh(1) - b^4*f)*cosh(d*x + c)^4 + 144*(8*a^4 + 4*a^2*b^2 - b^4)*d^2*x*
sinh(1) - 560*(3*a*b^3*d*f*x + 3*a*b^3*d*cosh(1) + 3*a*b^3*d*sinh(1) - a*b^
3*f)*cosh(d*x + c)^3 + 1080*(2*a^2*b^2*d*f*x + 2*a^2*b^2*d*cosh(1) + 2*a^2*
b^2*d*sinh(1) - a^2*b^2*f)*cosh(d*x + c)^2 - 720*((4*a^3*b + a*b^3)*d*f*x +
(4*a^3*b + a*b^3)*d*cosh(1) + (4*a^3*b + a*b^3)*d*sinh(1) - (4*a^3*b + a*b
^3)*f)*cosh(d*x + c))*sinh(d*x + c)^4 - 288*((4*a^3*b + a*b^3)*d*f*x + (4*a
^3*b + a*b^3)*d*cosh(1) + (4*a^3*b + a*b^3)*d*sinh(1) + (4*a^3*b + a*b^3)*f
)*cosh(d*x + c)^3 + 8*(63*(4*b^4*d*f*x + 4*b^4*d*cosh(1) + 4*b^4*d*sinh(1)
- b^4*f)*cosh(d*x + c)^5 - 140*(3*a*b^3*d*f*x + 3*a*b^3*d*cosh(1) + 3*a*b^3
*d*sinh(1) - a*b^3*f)*cosh(d*x + c)^4 - 36*(4*a^3*b + a*b^3)*d*f*x + 360*(2
*a^2*b^2*d*f*x + 2*a^2*b^2*d*cosh(1) + 2*a^2*b^2*d*sinh(1) - a^2*b^2*f)*cos
h(d*x + c)^3 - 36*(4*a^3*b + a*b^3)*d*cosh(1) - 360*((4*a^3*b + a*b^3)*d*f*
x + (4*a^3*b + a*b^3)*d*cosh(1) + (4*a^3*b + a*b^3)*d*sinh(1) - (4*a^3*b +
a*b^3)*f)*cosh(d*x + c)^2 - 36*(4*a^3*b + a*b^3)*d*sinh(1) - 36*(4*a^3*b +
a*b^3)*f + 72*((8*a^4 + 4*a^2*b^2 - b^4)*d^2*f*x^2 + 2*(8*a^4 + 4*a^2*b^2 -
b^4)*d^2*x*cosh(1) + 2*(8*a^4 + 4*a^2*b^2 - b^4)*d^2*x*sinh(1))*cosh(d*x +
c))*sinh(d*x + c)^3 - 144*(2*a^2*b^2*d*f*x + 2*a^2*b^2*d*cosh(1) + 2*a^2*b
^2*d*sinh(1) + a^2*b^2*f)*cosh(d*x + c)^2 - 12*(24*a^2*b^2*d*f*x - 21*(4*b^
4*d*f*x + 4*b^4*d*cosh(1) + 4*b^4*d*sinh(1) - b^4*f)*cosh(d*x + c)^6 + 24*a
^2*b^2*d*cosh(1) + 56*(3*a*b^3*d*f*x + 3*a*b^3*d*cosh(1) + 3*a*b^3*d*sinh(1
) - a*b^3*f)*cosh(d*x + c)^5 + 24*a^2*b^2*d*sinh(1) + 12*a^2*b^2*f - 180*(2
*a^2*b^2*d*f*x + 2*a^2*b^2*d*cosh(1) + 2*a^2*b^2*d*sinh(1) - a^2*b^2*f)*cos
h(d*x + c)^4 + 240*((4*a^3*b + a*b^3)*d*f*x + (4*a^3*b + a*b^3)*d*cosh(1) +
(4*a^3*b + a*b^3)*d*sinh(1) - (4*a^3*b + a*b^3)*f)*cosh(d*x + c)^3 - 72*((
8*a^4 + 4*a^2*b^2 - b^4)*d^2*f*x^2 + 2*(8*a^4 + 4*a^2*b^2 - b^4)*d^2*x*cosh
(1) + 2*(8*a^4 + 4*a^2*b^2 - b^4)*d^2*x*sinh(1))*cosh(d*x + c)^2 + 72*((4*a
^3*b + a*b^3)*d*f*x + (4*a^3*b + a*b^3)*d*cosh(1) + (4*a^3*b + a*b^3)*d*sin
h(1) + (4*a^3*b + a*b^3)*f)*cosh(d*x + c))*sinh(d*x + c)^2 - 2304*(a^3*b*f*
cosh(d*x + c)^4 + 4*a^3*b*f*cosh(d*x + c)^3*sinh(d*x + c) + 6*a^3*b*f*cosh(
d*x + c)^2*sinh(d*x + c)^2 + 4*a^3*b*f*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*
b*f*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(
d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b
+ 1) + 2304*(a^3*b*f*cosh(d*x + c)^4 + 4*a^3*b*f*cosh(d*x + c)^3*sinh(d*x
+ c) + 6*a^3*b*f*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^3*b*f*cosh(d*x + c)*

```
sinh(d*x + c)^3 + a^3*b*f*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*dilog((a*c
osh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((
a^2 + b^2)/b^2) - b)/b + 1) - 2304*((a^3*b*c*f - a^3*b*d*cosh(1) - a^3*b*d*
sinh(1))*cosh(d*x + c)^4 + 4*(a^3*b*c*f - a^3*b*d*cosh(1) - a^3*b*d*sinh(1)
)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3*b*c*f - a^3*b*d*cosh(1) - a^3*b*d*
sinh(1))*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3*b*c*f - a^3*b*d*cosh(1) -
 a^3*b*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b*c*f - a^3*b*d*cosh
(1) - a^3*b*d*sinh(1))*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(
d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2304*((a^
3*b*c*f - a^3*b*d*cosh(1) - a^3*b*d*sinh(1))*cosh(d*x + c)^4 + 4*(a^3*b*c*f
 - a^3*b*d*cosh(1) - a^3*b*d*sinh(1))*cosh(d*x ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**2*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="giac")
[Out] integrate((f*x + e)*cosh(d*x + c)^2*sinh(d*x + c)^3/(b*sinh(d*x + c) + a),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^2\,\sinh(c+d\,x)^3\,(e+f\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.399 $\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=184

$$\frac{(8a^4+4a^2b^2-b^4)\,x}{8b^5}+\frac{2a^3\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^5d}-\frac{a(3a^2+b^2)\cosh(c+dx)}{3b^4d}+\frac{(4a^2+b^2)\cosh(}{}$$

```
[Out] 1/8*(8*a^4+4*a^2*b^2-b^4)*x/b^5-1/3*a*(3*a^2+b^2)*cosh(d*x+c)/b^4/d+1/8*(4*
a^2+b^2)*cosh(d*x+c)*sinh(d*x+c)/b^3/d-1/3*a*cosh(d*x+c)*sinh(d*x+c)^2/b^2/
d+1/4*cosh(d*x+c)*sinh(d*x+c)^3/b/d+2*a^3*arctanh((b-a*tanh(1/2*d*x+1/2*c))
/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/b^5/d
```

Rubi [A]
time = 0.55, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2968, 3129, 3128, 3102, 2814, 2739, 632, 210}

$$-\frac{a(3a^2+b^2)\cosh(c+dx)}{3b^4d}+\frac{(4a^2+b^2)\sinh(c+dx)\cosh(c+dx)}{8b^3d}+\frac{x(8a^4+4a^2b^2-b^4)}{8b^5}+\frac{2a^3\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^5d}-\frac{a\sinh^2(c+dx)\cosh(c+dx)}{3b^2d}+\frac{\sinh^3(c+dx)\cosh(c+dx)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((8*a^4 + 4*a^2*b^2 - b^4)*x)/(8*b^5) + (2*a^3*Sqrt[a^2 + b^2]*ArcTanh[(b -
 a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b^5*d) - (a*(3*a^2 + b^2)*Cosh[c +
 d*x])/(3*b^4*d) + ((4*a^2 + b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(8*b^3*d) -
(a*Cosh[c + d*x]*Sinh[c + d*x]^2)/(3*b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x]^
3)/(4*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3129

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x]
)^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n
 + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(
a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0
])))
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh^3(c+dx)\left(1+\sinh^2(c+dx)\right)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{\cosh(c+dx)\sinh^3(c+dx)}{4bd} + \frac{\int \frac{\sinh^2(c+dx)\left(-3a+b\sinh(c+dx)-4a\sinh^2(c+dx)\right)}{a+b\sinh(c+dx)}}{4b}$$

$$= -\frac{a\cosh(c+dx)\sinh^2(c+dx)}{3b^2d} + \frac{\cosh(c+dx)\sinh^3(c+dx)}{4bd} + \frac{\int \frac{\sinh(}{}}{}$$

$$= \frac{(4a^2+b^2)\cosh(c+dx)\sinh(c+dx)}{8b^3d} - \frac{a\cosh(c+dx)\sinh^2(c+dx)}{3b^2d} +$$

$$= -\frac{a(3a^2+b^2)\cosh(c+dx)}{3b^4d} + \frac{(4a^2+b^2)\cosh(c+dx)\sinh(c+dx)}{8b^3d} - $$

$$= \frac{(8a^4+4a^2b^2-b^4)\,x}{8b^5} - \frac{a(3a^2+b^2)\cosh(c+dx)}{3b^4d} + \frac{(4a^2+b^2)\cosh(c+}{8b^3}$$

$$= \frac{(8a^4+4a^2b^2-b^4)\,x}{8b^5} - \frac{a(3a^2+b^2)\cosh(c+dx)}{3b^4d} + \frac{(4a^2+b^2)\cosh(c+}{8b^3}$$

$$= \frac{(8a^4+4a^2b^2-b^4)\,x}{8b^5} - \frac{a(3a^2+b^2)\cosh(c+dx)}{3b^4d} + \frac{(4a^2+b^2)\cosh(c+}{8b^3}$$

$$= \frac{(8a^4+4a^2b^2-b^4)\,x}{8b^5} + \frac{2a^3\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^5d} - \frac{a(3}{}$$

Mathematica [A]
time = 1.46, size = 153, normalized size = 0.83

$$\frac{-24ab(4a^2+b^2)\cosh(c+dx)-8ab^3\cosh(3(c+dx))+3\left(4(8a^4+4a^2b^2-b^4)(c+dx)+64a^3\sqrt{-a^2-b^2}\,\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)+8a^2b^2\sinh(2(c+dx))+b^4\sinh(4(c+dx))\right)}{96b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-24*a*b*(4*a^2 + b^2)*Cosh[c + d*x] - 8*a*b^3*Cosh[3*(c + d*x)] + 3*(4*(8*
a^4 + 4*a^2*b^2 - b^4)*(c + d*x) + 64*a^3*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Ta
nh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] + 8*a^2*b^2*Sinh[2*(c + d*x)] + b^4*Sinh
[4*(c + d*x)]))/(96*b^5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(169) = 338$.
time = 1.18, size = 355, normalized size = 1.93

method	result
risch	$\frac{x a^4}{b^5}+\frac{x a^2}{2 b^3}-\frac{x}{8 b}+\frac{e^{4 d x+4 c}}{64 b d}-\frac{a e^{3 d x+3 c}}{24 b^2 d}+\frac{a^2 e^{2 d x+2 c}}{8 b^3 d}-\frac{a^3 e^{d x+c}}{2 b^4 d}-\frac{a e^{d x+c}}{8 b^2 d}-\frac{a^3 e^{-d x-c}}{2 b^4 d}-\frac{a e^{-d x-c}}{8 b^2 d}-$
derivativedivides	$\frac{1}{4 b\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{-3 b-2 a}{6 b^2\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{-4 a^2-4 a b-3 b^2}{8 b^3\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{\left(-8 a^4-4 a^2 b^2+b^4\right)\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}{8 b^5}-\frac{-8 a^3-4 a^2}{8 b^4\left(\tanh\right.}$
default	$\frac{1}{4 b\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{-3 b-2 a}{6 b^2\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{-4 a^2-4 a b-3 b^2}{8 b^3\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{\left(-8 a^4-4 a^2 b^2+b^4\right)\ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}{8 b^5}-\frac{-8 a^3-4 a^2}{8 b^4\left(\tanh\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/b/(tanh(1/2*d*x+1/2*c)-1)^4-1/6*(-3*b-2*a)/b^2/(tanh(1/2*d*x+1/2*c
)-1)^3-1/8*(-4*a^2-4*a*b-3*b^2)/b^3/(tanh(1/2*d*x+1/2*c)-1)^2+1/8/b^5*(-8*a
^4-4*a^2*b^2+b^4)*ln(tanh(1/2*d*x+1/2*c)-1)-1/8*(-8*a^3-4*a^2*b-4*a*b^2-b^3
)/b^4/(tanh(1/2*d*x+1/2*c)-1)-1/4/b/(tanh(1/2*d*x+1/2*c)+1)^4-1/6*(-3*b+2*a
)/b^2/(tanh(1/2*d*x+1/2*c)+1)^3-1/8*(4*a^2-4*a*b+3*b^2)/b^3/(tanh(1/2*d*x+1
/2*c)+1)^2+1/8*(8*a^4+4*a^2*b^2-b^4)/b^5*ln(tanh(1/2*d*x+1/2*c)+1)-1/8*(8*a
^3-4*a^2*b+4*a*b^2-b^3)/b^4/(tanh(1/2*d*x+1/2*c)+1)-2*a^3*(a^2+b^2)^(1/2)/b
^5*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [A]
time = 0.50, size = 257, normalized size = 1.40

$$-\frac{\sqrt{a^2+b^2}\,a^3\log\left(\frac{b e^{(-d x-c)}-a-\sqrt{a^2+b^2}}{b e^{(-d x-c)}-a+\sqrt{a^2+b^2}}\right)}{b^5 d}-\frac{\left(8\,a b^2 e^{(-d x-c)}-24\,a^2 b e^{(-2\,d x-2\,c)}-3\,b^3+24\,(4\,a^3+a b^2)e^{(-3\,d x-3\,c)}\right)e^{(4\,d x+4\,c)}}{192\,b^4 d}+\frac{(8\,a^4+4\,a^2 b^2-b^4)(d x+c)}{8\,b^5 d}-\frac{24\,a^2 b e^{(-2\,d x-2\,c)}+8\,a b^2 e^{(-3\,d x-3\,c)}+3\,b^3 e^{(-4\,d x-4\,c)}+24\,(4\,a^3+a b^2)e^{(-d x-c)}}{192\,b^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxim
a")
```

```
[Out] -sqrt(a^2 + b^2)*a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x
- c) - a + sqrt(a^2 + b^2)))/(b^5*d) - 1/192*(8*a*b^2*e^(-d*x - c) - 24*a^2
*b*e^(-2*d*x - 2*c) - 3*b^3 + 24*(4*a^3 + a*b^2)*e^(-3*d*x - 3*c))*e^(4*d*x
 + 4*c)/(b^4*d) + 1/8*(8*a^4 + 4*a^2*b^2 - b^4)*(d*x + c)/(b^5*d) - 1/192*(
24*a^2*b*e^(-2*d*x - 2*c) + 8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e^(-4*d*x - 4*
c) + 24*(4*a^3 + a*b^2)*e^(-d*x - c))/(b^4*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(171) = 342$.

time = 0.41, size = 1134, normalized size = 6.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="frica
s")
```

```
[Out] 1/192*(3*b^4*cosh(d*x + c)^8 + 3*b^4*sinh(d*x + c)^8 - 8*a*b^3*cosh(d*x + c
)^7 + 24*a^2*b^2*cosh(d*x + c)^6 + 8*(3*b^4*cosh(d*x + c) - a*b^3)*sinh(d*x
 + c)^7 + 24*(8*a^4 + 4*a^2*b^2 - b^4)*d*x*cosh(d*x + c)^4 + 4*(21*b^4*cosh
(d*x + c)^2 - 14*a*b^3*cosh(d*x + c) + 6*a^2*b^2)*sinh(d*x + c)^6 - 24*a^2*
b^2*cosh(d*x + c)^2 - 24*(4*a^3*b + a*b^3)*cosh(d*x + c)^5 + 24*(7*b^4*cosh
(d*x + c)^3 - 7*a*b^3*cosh(d*x + c)^2 + 6*a^2*b^2*cosh(d*x + c) - 4*a^3*b -
 a*b^3)*sinh(d*x + c)^5 - 8*a*b^3*cosh(d*x + c) + 2*(105*b^4*cosh(d*x + c)^
4 - 140*a*b^3*cosh(d*x + c)^3 + 180*a^2*b^2*cosh(d*x + c)^2 + 12*(8*a^4 + 4
*a^2*b^2 - b^4)*d*x - 60*(4*a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 -
 3*b^4 - 24*(4*a^3*b + a*b^3)*cosh(d*x + c)^3 + 8*(21*b^4*cosh(d*x + c)^5 -
 35*a*b^3*cosh(d*x + c)^4 + 60*a^2*b^2*cosh(d*x + c)^3 - 12*a^3*b - 3*a*b^3
 + 12*(8*a^4 + 4*a^2*b^2 - b^4)*d*x*cosh(d*x + c) - 30*(4*a^3*b + a*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c)^3 + 12*(7*b^4*cosh(d*x + c)^6 - 14*a*b^3*cosh(
d*x + c)^5 + 30*a^2*b^2*cosh(d*x + c)^4 + 12*(8*a^4 + 4*a^2*b^2 - b^4)*d*x*
cosh(d*x + c)^2 - 2*a^2*b^2 - 20*(4*a^3*b + a*b^3)*cosh(d*x + c)^3 - 6*(4*a
^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 192*(a^3*cosh(d*x + c)^4 + 4
*a^3*cosh(d*x + c)^3*sinh(d*x + c) + 6*a^3*cosh(d*x + c)^2*sinh(d*x + c)^2
+ 4*a^3*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*sinh(d*x + c)^4)*sqrt(a^2 + b^2
)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*
a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(
b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c
)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 8*(
3*b^4*cosh(d*x + c)^7 - 7*a*b^3*cosh(d*x + c)^6 + 18*a^2*b^2*cosh(d*x + c)^
5 + 12*(8*a^4 + 4*a^2*b^2 - b^4)*d*x*cosh(d*x + c)^3 - 6*a^2*b^2*cosh(d*x +
 c) - 15*(4*a^3*b + a*b^3)*cosh(d*x + c)^4 - a*b^3 - 9*(4*a^3*b + a*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c))/(b^5*d*cosh(d*x + c)^4 + 4*b^5*d*cosh(d*x + c
)^3*sinh(d*x + c) + 6*b^5*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^5*d*cosh(
d*x + c)*sinh(d*x + c)^3 + b^5*d*sinh(d*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 258, normalized size = 1.40

$$\frac{\frac{24\,(8\,a^4+4\,a^2b^2-b^4)(dx+c)}{b^5}+\frac{3\,b^3e^{(4\,dx+4\,c)}-8\,ab^2e^{(3\,dx+3\,c)}+24\,a^2be^{(2\,dx+2\,c)}-96\,a^3e^{(dx+c)}-24\,ab^2e^{(dx+c)}}{b^4}-\frac{(24\,a^2b^2e^{(2\,dx+2\,c)}+8\,ab^3e^{(dx+c)}+3\,b^4+24\,(4\,a^3b+ab^3)e^{(3\,dx+3\,c)})e^{(-4\,dx-4\,c)}}{b^5}-\frac{192\,(a^5+a^3b^2)\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^5}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac"
)

[Out] 1/192*(24*(8*a^4 + 4*a^2*b^2 - b^4)*(d*x + c)/b^5 + (3*b^3*e^(4*d*x + 4*c)
- 8*a*b^2*e^(3*d*x + 3*c) + 24*a^2*b*e^(2*d*x + 2*c) - 96*a^3*e^(d*x + c) -
 24*a*b^2*e^(d*x + c))/b^4 - (24*a^2*b^2*e^(2*d*x + 2*c) + 8*a*b^3*e^(d*x +
 c) + 3*b^4 + 24*(4*a^3*b + a*b^3)*e^(3*d*x + 3*c))*e^(-4*d*x - 4*c)/b^5 -
192*(a^5 + a^3*b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(
2*b*e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^5))/d
```

Mupad [B]
time = 0.73, size = 330, normalized size = 1.79

$$\frac{x\,(8\,a^4+4\,a^2\,b^2-b^4)}{8\,b^5}-\frac{\mathrm{e}^{-4\,c-4\,d\,x}}{64\,b\,d}+\frac{\mathrm{e}^{4\,c+4\,d\,x}}{64\,b\,d}-\frac{a\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,b^2\,d}-\frac{a\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,b^2\,d}-\frac{\mathrm{e}^{c+d\,x}\,(4\,a^3+a\,b^2)}{8\,b^4\,d}-\frac{a^2\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,b^3\,d}+\frac{a^2\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,b^3\,d}-\frac{\mathrm{e}^{-c-d\,x}\,(4\,a^3+a\,b^2)}{8\,b^4\,d}-\frac{a^3\,\ln\left(\frac{2\,a^3\,\mathrm{e}^{c+d\,x}\,(a^2+b^2)}{b^6}-\frac{2\,a^3\,\sqrt{a^2+b^2}\,(b-a\,\mathrm{e}^{c+d\,x})}{b^6}\right)\,\sqrt{a^2+b^2}}{b^5\,d}+\frac{a^3\,\ln\left(\frac{2\,a^3\,\sqrt{a^2+b^2}\,(b-a\,\mathrm{e}^{c+d\,x})}{b^6}+\frac{2\,a^3\,\mathrm{e}^{c+d\,x}\,(a^2+b^2)}{b^6}\right)\,\sqrt{a^2+b^2}}{b^5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*sinh(c + d*x)^3)/(a + b*sinh(c + d*x)),x)

[Out] (x*(8*a^4 - b^4 + 4*a^2*b^2))/(8*b^5) - exp(- 4*c - 4*d*x)/(64*b*d) + exp(4
*c + 4*d*x)/(64*b*d) - (a*exp(- 3*c - 3*d*x))/(24*b^2*d) - (a*exp(3*c + 3*d
*x))/(24*b^2*d) - (exp(c + d*x)*(a*b^2 + 4*a^3))/(8*b^4*d) - (a^2*exp(- 2*c
 - 2*d*x))/(8*b^3*d) + (a^2*exp(2*c + 2*d*x))/(8*b^3*d) - (exp(- c - d*x)*(
a*b^2 + 4*a^3))/(8*b^4*d) - (a^3*log((2*a^3*exp(c + d*x)*(a^2 + b^2))/b^6 -
 (2*a^3*(a^2 + b^2)^(1/2)*(b - a*exp(c + d*x)))/b^6)*(a^2 + b^2)^(1/2))/(b^
5*d) + (a^3*log((2*a^3*(a^2 + b^2)^(1/2)*(b - a*exp(c + d*x)))/b^6 + (2*a^3
*exp(c + d*x)*(a^2 + b^2))/b^6)*(a^2 + b^2)^(1/2))/(b^5*d)
```

3.400 $\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=39

$$\text{Int}\left(\frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cosh[c + d*x]^2*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Cosh[c + d*x]^2*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x]
)),x]
```

```
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\cosh^2(dx+c)\right)\left(\sinh^3(dx+c)\right)}{(fx+e)\,(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -2*(a^5*e^c + a^3*b^2*e^c)*integrate(-e^(d*x)/(b^6*f*x + b^6*e - (b^6*f*x*e
^(2*c) + b^6*e^(2*c + 1))*e^(2*d*x) - 2*(a*b^5*f*x*e^c + a*b^5*e^(c + 1))*e
^(d*x)), x) - 1/16*e^(-4*c + 4*d*e/f)*exp_integral_e(1, 4*(f*x + e)*d/f)/(b
*f) - 1/8*a*e^(-3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b^2*f) -
 1/4*a^2*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b^3*f) - 1/
4*a^2*e^(2*c - 2*d*e/f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b^3*f) + 1/8*a
*e^(3*c - 3*d*e/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/(b^2*f) - 1/16*e^(4*
c - 4*d*e/f)*exp_integral_e(1, -4*(f*x + e)*d/f)/(b*f) - 1/8*(4*a^3 + a*b^2
)*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^4*f) + 1/8*(4*a^3*e^c
+ a*b^2*e^c)*e^(-d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^4*f) + 1/8*(8*
a^4 + 4*a^2*b^2 - b^4)*log(f*x + e)/(b^5*f)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] integral(cosh(d*x + c)^2*sinh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(
d*x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*sinh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate(cosh(d*x + c)^2*sinh(d*x + c)^3/((f*x + e)*(b*sinh(d*x + c) + a))
, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh(c+d\,x)^2\,\sinh(c+d\,x)^3}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*sinh(c + d*x)^3)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((cosh(c + d*x)^2*sinh(c + d*x)^3)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.401 $\int \frac{(e+fx)^3 \cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1443

$$-\frac{3a^3f^3x}{8b^4d^3}+\frac{45af^3x}{256b^2d^3}-\frac{a^3(e+fx)^3}{4b^4d}+\frac{3a(e+fx)^3}{32b^2d}+\frac{a^3(a^2+b^2)(e+fx)^4}{4b^6f}-\frac{6a^4f^3\cosh(c+dx)}{b^5d^4}-\frac{40a^2f^3\cosh(c}{9b^3d^4}$$

```
[Out] -3*a^3*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b
^6/d^2-3*a^3*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/
2)))/b^6/d^2+6*a^3*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))/b^6/d^3+6*a^3*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(
a^2+b^2)^(1/2)))/b^6/d^3-3/8*a^3*f^3*x/b^4/d^3+1/4*a^3*(a^2+b^2)*(f*x+e)^4/
b^6/f-6*a^4*f^3*cosh(d*x+c)/b^5/d^4-2/27*a^2*f^3*cosh(d*x+c)^3/b^3/d^4-1/4*
a*(f*x+e)^3*cosh(d*x+c)^4/b^2/d-1/48*f*(f*x+e)^2*cosh(3*d*x+3*c)/b/d^2-3/40
0*f*(f*x+e)^2*cosh(5*d*x+5*c)/b/d^2-3*a^4*f*(f*x+e)^2*cosh(d*x+c)/b^5/d^2+4
5/256*a*f^3*x/b^2/d^3-40/9*a^2*f^3*cosh(d*x+c)/b^3/d^4+3/4*a^3*f*(f*x+e)^2*
cosh(d*x+c)*sinh(d*x+c)/b^4/d^2+2/9*a^2*f^2*(f*x+e)*cosh(d*x+c)^2*sinh(d*x+
c)/b^3/d^3+3/16*a*f*(f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/b^2/d^2+3/32*a*(f*x
+e)^3/b^2/d-1/8*(f*x+e)^3*sinh(d*x+c)/b/d+3/4*f^3*cosh(d*x+c)/b/d^4-9/32*a*
f^2*(f*x+e)*cosh(d*x+c)^2/b^2/d^3-1/3*a^2*f*(f*x+e)^2*cosh(d*x+c)^3/b^3/d^2
-3/32*a*f^2*(f*x+e)*cosh(d*x+c)^4/b^2/d^3+6*a^4*f^2*(f*x+e)*sinh(d*x+c)/b^5
/d^3+3/8*a^3*f^3*cosh(d*x+c)*sinh(d*x+c)/b^4/d^4+1/3*a^2*(f*x+e)^3*cosh(d*x
+c)^2*sinh(d*x+c)/b^3/d+3/128*a*f^3*cosh(d*x+c)^3*sinh(d*x+c)/b^2/d^4-3/4*a
^3*f^2*(f*x+e)*sinh(d*x+c)^2/b^4/d^3-2*a^2*f*(f*x+e)^2*cosh(d*x+c)/b^3/d^2+
40/9*a^2*f^2*(f*x+e)*sinh(d*x+c)/b^3/d^3+45/256*a*f^3*cosh(d*x+c)*sinh(d*x+
c)/b^2/d^4+a^4*(f*x+e)^3*sinh(d*x+c)/b^5/d-1/4*a^3*(f*x+e)^3/b^4/d-1/216*f^
3*cosh(3*d*x+3*c)/b/d^4-3/5000*f^3*cosh(5*d*x+5*c)/b/d^4+1/48*(f*x+e)^3*sin
h(3*d*x+3*c)/b/d+1/80*(f*x+e)^3*sinh(5*d*x+5*c)/b/d-a^3*(a^2+b^2)*(f*x+e)^3
*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^6/d-a^3*(a^2+b^2)*(f*x+e)^3*ln(1+
b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^6/d-6*a^3*(a^2+b^2)*f^3*polylog(4,-b*ex
p(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^6/d^4-6*a^3*(a^2+b^2)*f^3*polylog(4,-b*exp(
d*x+c)/(a+(a^2+b^2)^(1/2)))/b^6/d^4+9/32*a*f*(f*x+e)^2*cosh(d*x+c)*sinh(d*x
+c)/b^2/d^2+3/8*f*(f*x+e)^2*cosh(d*x+c)/b/d^2-3/4*f^2*(f*x+e)*sinh(d*x+c)/b
/d^3+2/3*a^2*(f*x+e)^3*sinh(d*x+c)/b^3/d-1/2*a^3*(f*x+e)^3*sinh(d*x+c)^2/b^
4/d+1/72*f^2*(f*x+e)*sinh(3*d*x+3*c)/b/d^3+3/1000*f^2*(f*x+e)*sinh(5*d*x+5*
c)/b/d^3
```

Rubi [A]
time = 1.52, antiderivative size = 1443, normalized size of antiderivative = 1.00, number of steps used = 55, number of rules used = 18, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5698, 5556, 3377, 2718, 5555, 3392, 32, 2715, 8, 3391, 5684, 5554, 5680, 2221, 2611, 6744, 2320, 6724}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-3*a^3*f^3*x)/(8*b^4*d^3) + (45*a*f^3*x)/(256*b^2*d^3) - (a^3*(e + f*x)^3)
/(4*b^4*d) + (3*a*(e + f*x)^3)/(32*b^2*d) + (a^3*(a^2 + b^2)*(e + f*x)^4)/(
4*b^6*f) - (6*a^4*f^3*Cosh[c + d*x])/(b^5*d^4) - (40*a^2*f^3*Cosh[c + d*x])
/(9*b^3*d^4) + (3*f^3*Cosh[c + d*x])/(4*b*d^4) - (3*a^4*f*(e + f*x)^2*Cosh[
c + d*x])/(b^5*d^2) - (2*a^2*f*(e + f*x)^2*Cosh[c + d*x])/(b^3*d^2) + (3*f*
(e + f*x)^2*Cosh[c + d*x])/(8*b*d^2) - (9*a*f^2*(e + f*x)*Cosh[c + d*x]^2)/
(32*b^2*d^3) - (2*a^2*f^3*Cosh[c + d*x]^3)/(27*b^3*d^4) - (a^2*f*(e + f*x)^
2*Cosh[c + d*x]^3)/(3*b^3*d^2) - (3*a*f^2*(e + f*x)*Cosh[c + d*x]^4)/(32*b^
2*d^3) - (a*(e + f*x)^3*Cosh[c + d*x]^4)/(4*b^2*d) - (f^3*Cosh[3*c + 3*d*x]
)/(216*b*d^4) - (f*(e + f*x)^2*Cosh[3*c + 3*d*x])/(48*b*d^2) - (3*f^3*Cosh[
5*c + 5*d*x])/(5000*b*d^4) - (3*f*(e + f*x)^2*Cosh[5*c + 5*d*x])/(400*b*d^2
) - (a^3*(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])])/(b^6*d) - (a^3*(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2])])/(b^6*d) - (3*a^3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^6*d^2) - (3*a^3*(a^2 + b^2)*f*(e
 + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^6*d^2) +
 (6*a^3*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/(b^6*d^3) + (6*a^3*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^6*d^3) - (6*a^3*(a^2 + b^2)*f^3*Pol
yLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^6*d^4) - (6*a^3*(a^2
+ b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^6*d^4)
+ (6*a^4*f^2*(e + f*x)*Sinh[c + d*x])/(b^5*d^3) + (40*a^2*f^2*(e + f*x)*Sin
h[c + d*x])/(9*b^3*d^3) - (3*f^2*(e + f*x)*Sinh[c + d*x])/(4*b*d^3) + (a^4*
(e + f*x)^3*Sinh[c + d*x])/(b^5*d) + (2*a^2*(e + f*x)^3*Sinh[c + d*x])/(3*b
^3*d) - ((e + f*x)^3*Sinh[c + d*x])/(8*b*d) + (3*a^3*f^3*Cosh[c + d*x]*Sinh
[c + d*x])/(8*b^4*d^4) + (45*a*f^3*Cosh[c + d*x]*Sinh[c + d*x])/(256*b^2*d^
4) + (3*a^3*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^4*d^2) + (9*a*f
*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(32*b^2*d^2) + (2*a^2*f^2*(e + f*
x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(9*b^3*d^3) + (a^2*(e + f*x)^3*Cosh[c + d
*x]^2*Sinh[c + d*x])/(3*b^3*d) + (3*a*f^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(1
28*b^2*d^4) + (3*a*f*(e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(16*b^2*d^2
) - (3*a^3*f^2*(e + f*x)*Sinh[c + d*x]^2)/(4*b^4*d^3) - (a^3*(e + f*x)^3*Si
nh[c + d*x]^2)/(2*b^4*d) + (f^2*(e + f*x)*Sinh[3*c + 3*d*x])/(72*b*d^3) + (
(e + f*x)^3*Sinh[3*c + 3*d*x])/(48*b*d) + (3*f^2*(e + f*x)*Sinh[5*c + 5*d*x
])/(1000*b*d^3) + ((e + f*x)^3*Sinh[5*c + 5*d*x])/(80*b*d)
```

<u>Rule 8</u>

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

<u>Rule 32</u>

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
```

```
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5555

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
```

```
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\cosh^3}{a+b\,\mathrm{s}}}{}$$

$$= -\frac{a\int (e+fx)^3\cosh^3(c+dx)\sinh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^3\,\mathrm{c}}{a}}{}$$

$$= -\frac{a(e+fx)^3\cosh^4(c+dx)}{4b^2d} + \frac{a^2\int (e+fx)^3\cosh^3(c+dx)\,d}{b^3}$$

$$= -\frac{a^2f(e+fx)^2\cosh^3(c+dx)}{3b^3d^2} - \frac{3af^2(e+fx)\cosh^4(c+dx)}{32b^2d^3}$$

$$= \frac{a^3(a^2+b^2)(e+fx)^4}{4b^6f} + \frac{3f(e+fx)^2\cosh(c+dx)}{8bd^2} - \frac{9af^2(e}{}$$

$$= \frac{3a(e+fx)^3}{32b^2d} + \frac{a^3(a^2+b^2)(e+fx)^4}{4b^6f} - \frac{3a^4f(e+fx)^2\cosh(}{b^5d^2}$$

$$= \frac{45af^3x}{256b^2d^3} - \frac{a^3(e+fx)^3}{4b^4d} + \frac{3a(e+fx)^3}{32b^2d} + \frac{a^3(a^2+b^2)(e+f}{4b^6f}$$

$$= -\frac{3a^3f^3x}{8b^4d^3} + \frac{45af^3x}{256b^2d^3} - \frac{a^3(e+fx)^3}{4b^4d} + \frac{3a(e+fx)^3}{32b^2d} + \frac{a^3(a^2}{}$$

$$= -\frac{3a^3f^3x}{8b^4d^3} + \frac{45af^3x}{256b^2d^3} - \frac{a^3(e+fx)^3}{4b^4d} + \frac{3a(e+fx)^3}{32b^2d} + \frac{a^3(a^2}{}$$

$$= -\frac{3a^3f^3x}{8b^4d^3} + \frac{45af^3x}{256b^2d^3} - \frac{a^3(e+fx)^3}{4b^4d} + \frac{3a(e+fx)^3}{32b^2d} + \frac{a^3(a^2}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 5157 vs. $2(1443) = 2886$.
time = 17.91, size = 5157, normalized size = 3.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x
]),x]
```

[Out] Result too large to show

Maple [F]
time = 2.50, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 (\cosh^3(dx+c)) (\sinh^3(dx+c))}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="maxima")
```

```
[Out] -1/960*((15*a*b^3*e^(-d*x - c) - 6*b^4 - 10*(4*a^2*b^2 + b^4)*e^(-2*d*x - 2
*c) + 60*(2*a^3*b + a*b^3)*e^(-3*d*x - 3*c) - 60*(8*a^4 + 6*a^2*b^2 - b^4)*
e^(-4*d*x - 4*c))*e^(5*d*x + 5*c)/(b^5*d) + 960*(a^5 + a^3*b^2)*(d*x + c)/(
b^6*d) + (15*a*b^3*e^(-4*d*x - 4*c) + 6*b^4*e^(-5*d*x - 5*c) + 60*(8*a^4 +
6*a^2*b^2 - b^4)*e^(-d*x - c) + 60*(2*a^3*b + a*b^3)*e^(-2*d*x - 2*c) + 10*
(4*a^2*b^2 + b^4)*e^(-3*d*x - 3*c))/(b^5*d) + 960*(a^5 + a^3*b^2)*log(-2*a*
e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^6*d))*e^3 - 1/34560000*(8640000*(
a^5*d^4*f^3*e^(5*c) + a^3*b^2*d^4*f^3*e^(5*c))*x^4 + 34560000*(a^5*d^4*f^2*
e^(5*c) + a^3*b^2*d^4*f^2*e^(5*c))*x^3*e + 51840000*(a^5*d^4*f*e^(5*c) + a^
3*b^2*d^4*f*e^(5*c))*x^2*e^2 - 1728*(125*b^5*d^3*f^3*x^3*e^(10*c) - 6*b^5*f
^3*e^(10*c) - 75*b^5*d^2*f*e^(10*c + 2) + 30*b^5*d*f^2*e^(10*c + 1) - 75*(b
^5*d^2*f^3*e^(10*c) - 5*b^5*d^3*f^2*e^(10*c + 1))*x^2 + 15*(2*b^5*d*f^3*e^(
10*c) + 25*b^5*d^3*f*e^(10*c + 2) - 10*b^5*d^2*f^2*e^(10*c + 1))*x)*e^(5*d*
x) + 16875*(32*a*b^4*d^3*f^3*x^3*e^(9*c) - 3*a*b^4*f^3*e^(9*c) - 24*a*b^4*d
^2*f*e^(9*c + 2) + 12*a*b^4*d*f^2*e^(9*c + 1) - 24*(a*b^4*d^2*f^3*e^(9*c) -
 4*a*b^4*d^3*f^2*e^(9*c + 1))*x^2 + 12*(a*b^4*d*f^3*e^(9*c) + 8*a*b^4*d^3*f
*e^(9*c + 2) - 4*a*b^4*d^2*f^2*e^(9*c + 1))*x)*e^(4*d*x) + 40000*(8*a^2*b^3
*f^3*e^(8*c) + 2*b^5*f^3*e^(8*c) - 9*(4*a^2*b^3*d^3*f^3*e^(8*c) + b^5*d^3*f
^3*e^(8*c))*x^3 + 9*(4*a^2*b^3*d^2*f^3*e^(8*c) + b^5*d^2*f^3*e^(8*c) - 3*(4
*a^2*b^3*d^3*f^2*e^(8*c) + b^5*d^3*f^2*e^(8*c))*e)*x^2 - 3*(8*a^2*b^3*d*f^3
*e^(8*c) + 2*b^5*d*f^3*e^(8*c) + 9*(4*a^2*b^3*d^3*f*e^(8*c) + b^5*d^3*f*e^(
8*c))*e^2 - 6*(4*a^2*b^3*d^2*f^2*e^(8*c) + b^5*d^2*f^2*e^(8*c))*e)*x + 9*(4
*a^2*b^3*d^2*f*e^(8*c) + b^5*d^2*f*e^(8*c))*e^2 - 6*(4*a^2*b^3*d*f^2*e^(8*c
) + b^5*d*f^2*e^(8*c))*e)*e^(3*d*x) - 540000*(6*a^3*b^2*f^3*e^(7*c) + 3*a*b
```

```
^4*f^3*e^(7*c) - 4*(2*a^3*b^2*d^3*f^3*e^(7*c) + a*b^4*d^3*f^3*e^(7*c))*x^3
+ 6*(2*a^3*b^2*d^2*f^3*e^(7*c) + a*b^4*d^2*f^3*e^(7*c) - 2*(2*a^3*b^2*d^3*f
^2*e^(7*c) + a*b^4*d^3*f^2*e^(7*c))*e)*x^2 - 6*(2*a^3*b^2*d*f^3*e^(7*c) + a
*b^4*d*f^3*e^(7*c) + 2*(2*a^3*b^2*d^3*f*e^(7*c) + a*b^4*d^3*f*e^(7*c))*e^2
- 2*(2*a^3*b^2*d^2*f^2*e^(7*c) + a*b^4*d^2*f^2*e^(7*c))*e)*x + 6*(2*a^3*b^2
*d^2*f*e^(7*c) + a*b^4*d^2*f*e^(7*c))*e^2 - 6*(2*a^3*b^2*d*f^2*e^(7*c) + a*
b^4*d*f^2*e^(7*c))*e)*e^(2*d*x) + 2160000*(48*a^4*b*f^3*e^(6*c) + 36*a^2*b^
3*f^3*e^(6*c) - 6*b^5*f^3*e^(6*c) - (8*a^4*b*d^3*f^3*e^(6*c) + 6*a^2*b^3*d^
3*f^3*e^(6*c) - b^5*d^3*f^3*e^(6*c))*x^3 + 3*(8*a^4*b*d^2*f^3*e^(6*c) + 6*a
^2*b^3*d^2*f^3*e^(6*c) - b^5*d^2*f^3*e^(6*c) - (8*a^4*b*d^3*f^2*e^(6*c) + 6
*a^2*b^3*d^3*f^2*e^(6*c) - b^5*d^3*f^2*e^(6*c))*e)*x^2 - 3*(16*a^4*b*d*f^3*
e^(6*c) + 12*a^2*b^3*d*f^3*e^(6*c) - 2*b^5*d*f^3*e^(6*c) + (8*a^4*b*d^3*f*e
^(6*c) + 6*a^2*b^3*d^3*f*e^(6*c) - b^5*d^3*f*e^(6*c))*e^2 - 2*(8*a^4*b*d^2*
f^2*e^(6*c) + 6*a^2*b^3*d^2*f^2*e^(6*c) - b^5*d^2*f^2*e^(6*c))*e)*x + 3*(8*
a^4*b*d^2*f*e^(6*c) + 6*a^2*b^3*d^2*f*e^(6*c) - b^5*d^2*f*e^(6*c))*e^2 - 6*
(8*a^4*b*d*f^2*e^(6*c) + 6*a^2*b^3*d*f^2*e^(6*c) - b^5*d*f^2*e^(6*c))*e)*e^
(d*x) + 2160000*(48*a^4*b*f^3*e^(4*c) + 36*a^2*b^3*f^3*e^(4*c) - 6*b^5*f^3*
e^(4*c) + (8*a^4*b*d^3*f^3*e^(4*c) + 6*a^2*b^3*d^3*f^3*e^(4*c) - b^5*d^3*f^
3*e^(4*c))*x^3 + 3*(8*a^4*b*d^2*f^3*e^(4*c) + 6*a^2*b^3*d^2*f^3*e^(4*c) - b
^5*d^2*f^3*e^(4*c) + (8*a^4*b*d^3*f^2*e^(4*c) + 6*a^2*b^3*d^3*f^2*e^(4*c) -
 b^5*d^3*f^2*e^(4*c))*e)*x^2 + 3*(16*a^4*b*d*f^3*e^(4*c) + 12*a^2*b^3*d*f^3
*e^(4*c) - 2*b^5*d*f^3*e^(4*c) + (8*a^4*b*d^3*f*e^(4*c) + 6*a^2*b^3*d^3*f*e
^(4*c) - b^5*d^3*f*e^(4*c))*e^2 + 2*(8*a^4*b*d^2*f^2*e^(4*c) + 6*a^2*b^3*d^
2*f^2*e^(4*c) - b^5*d^2*f^2*e^(4*c))*e)*x + 3*(8*a^4*b*d^2*f*e^(4*c) + 6*a^
2*b^3*d^2*f*e^(4*c) - b^5*d^2*f*e^(4*c))*e^2 + 6*(8*a^4*b*d*f^2*e^(4*c) + 6
*a^2*b^3*d*f^2*e^(4*c) - b^5*d*f^2*e^(4*c))*e)*e^(-d*x) + 540000*(6*a^3*b^2
*f^3*e^(3*c) + 3*a*b^4*f^3*e^(3*c) + 4*(2*a^3*b^2*d^3*f^3*e^(3*c) + a*b^4*d
^3*f^3*e^(3*c))*x^3 + 6*(2*a^3*b^2*d^2*f^3*e^(3*c) + a*b^4*d^2*f^3*e^(3*c)
+ 2*(2*a^3*b^2*d^3*f^2*e^(3*c) + a*b^4*d^3*f^2*e^(3*c))*e)*x^2 + 6*(2*a^3*b
^2*d*f^3*e^(3*c) + a*b^4*d*f^3*e^(3*c) + 2*(2*a^3*b^2*d^3*f*e^(3*c) + a*b^4
*d^3*f*e^(3*c))*e^2 + 2*(2*a^3*b^2*d^2*f^2*e^(3*c) + a*b^4*d^2*f^2*e^(3*c))
*e)*x + 6*(2*a^3*b^2*d^2*f*e^(3*c) + a*b^4*d^2*f*e^(3*c))*e^2 + 6*(2*a^3*b^
2*d*f^2*e^(3*c) + a*b^4*d*f^2*e^(3*c))*e)*e^(-2*d*x) + 40000*(8*a^2*b^3*f^3
*e^(2*c) + 2*b^5*f^3*e^(2*c) + 9*(4*a^2*b^3*d^3*f^3*e^(2*c) + b^5*d^3*f^3*e
^(2*c))*x^3 + 9*(4*a^2*b^3*d^2*f^3*e^(2*c) + b^5*d^2*f^3*e^(2*c) + 3*(4*a^2
*b^3*d^3*f^2*e^(2*c) + b^5*d^3*f^2*e^(2*c))*e)*x^2 + 3*(8*a^2*b^3*d*f^3*e^(
2*c) + 2*b^5*d*f^3*e^(2*c) + 9*(4*a^2*b^3*d^3*f*e^(2*c) + b^5*d^3*f*e^(2*c)
)*e^2 + 6*(4*a^2*b^3*d^2*f^2*e^(2*c) + b^5*d^2*f^2*e^(2*c))*e)*x + 9*(4*a^2
*b^3*d^2*f*e^(2*c) + b^5*d^2*f*e^(2*c))*e^2 + 6*(4*a^2*b^3*d*f^2*e^(2*c) +
b^5*d*f^2*e^(2*c))*e)*e^(-3*d*x) + 16875*(32*a*b^4*d^3*f^3*x^3*e^c + 24*a*b
^4*d^2*f*e^(c + 2) + 12*a*b^4*d*f^2*e^(c + 1) + 3*a*b^4*f^3*e^c + 24*(4*a*b
^4*d^3*f^2*e^(c + 1) + a*b^4*d^2*f^3*e^c)*x^2 + 12*(8*a*b^4*d^3*f*e^(c + 2)
 + 4*a*b^4*d^2*f^2*e^(c + 1) + a*b^4*d*f^3*e^c)...
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 36449 vs.

2(1382) = 2764.
time = 0.84, size = 36449, normalized size = 25.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="fricas")
```

```
[Out] -1/34560000*(216000*b^5*d^3*f^3*x^3 + 129600*b^5*d^2*f^3*x^2 + 216000*b^5*d
^3*cosh(1)^3 - 1728*(125*b^5*d^3*f^3*x^3 - 75*b^5*d^2*f^3*x^2 + 125*b^5*d^3
*cosh(1)^3 + 125*b^5*d^3*sinh(1)^3 + 30*b^5*d*f^3*x - 6*b^5*f^3 + 75*(5*b^5
*d^3*f*x - b^5*d^2*f)*cosh(1)^2 + 75*(5*b^5*d^3*f*x + 5*b^5*d^3*cosh(1) - b
^5*d^2*f)*sinh(1)^2 + 15*(25*b^5*d^3*f^2*x^2 - 10*b^5*d^2*f^2*x + 2*b^5*d*f
^2)*cosh(1) + 15*(25*b^5*d^3*f^2*x^2 - 10*b^5*d^2*f^2*x + 25*b^5*d^3*cosh(1
)^2 + 2*b^5*d*f^2 + 10*(5*b^5*d^3*f*x - b^5*d^2*f)*cosh(1))*sinh(1))*cosh(d
*x + c)^10 + 216000*b^5*d^3*sinh(1)^3 - 1728*(125*b^5*d^3*f^3*x^3 - 75*b^5*
d^2*f^3*x^2 + 125*b^5*d^3*cosh(1)^3 + 125*b^5*d^3*sinh(1)^3 + 30*b^5*d*f^3*
x - 6*b^5*f^3 + 75*(5*b^5*d^3*f*x - b^5*d^2*f)*cosh(1)^2 + 75*(5*b^5*d^3*f*
x + 5*b^5*d^3*cosh(1) - b^5*d^2*f)*sinh(1)^2 + 15*(25*b^5*d^3*f^2*x^2 - 10*
b^5*d^2*f^2*x + 2*b^5*d*f^2)*cosh(1) + 15*(25*b^5*d^3*f^2*x^2 - 10*b^5*d^2*
f^2*x + 25*b^5*d^3*cosh(1)^2 + 2*b^5*d*f^2 + 10*(5*b^5*d^3*f*x - b^5*d^2*f)
*cosh(1))*sinh(1))*sinh(d*x + c)^10 + 51840*b^5*d*f^3*x + 16875*(32*a*b^4*d
^3*f^3*x^3 - 24*a*b^4*d^2*f^3*x^2 + 32*a*b^4*d^3*cosh(1)^3 + 32*a*b^4*d^3*s
inh(1)^3 + 12*a*b^4*d*f^3*x - 3*a*b^4*f^3 + 24*(4*a*b^4*d^3*f*x - a*b^4*d^2
*f)*cosh(1)^2 + 24*(4*a*b^4*d^3*f*x + 4*a*b^4*d^3*cosh(1) - a*b^4*d^2*f)*si
nh(1)^2 + 12*(8*a*b^4*d^3*f^2*x^2 - 4*a*b^4*d^2*f^2*x + a*b^4*d*f^2)*cosh(1
) + 12*(8*a*b^4*d^3*f^2*x^2 - 4*a*b^4*d^2*f^2*x + 8*a*b^4*d^3*cosh(1)^2 + a
*b^4*d*f^2 + 4*(4*a*b^4*d^3*f*x - a*b^4*d^2*f)*cosh(1))*sinh(1))*cosh(d*x +
 c)^9 + 135*(4000*a*b^4*d^3*f^3*x^3 - 3000*a*b^4*d^2*f^3*x^2 + 4000*a*b^4*d
^3*cosh(1)^3 + 4000*a*b^4*d^3*sinh(1)^3 + 1500*a*b^4*d*f^3*x - 375*a*b^4*f^
3 + 3000*(4*a*b^4*d^3*f*x - a*b^4*d^2*f)*cosh(1)^2 + 3000*(4*a*b^4*d^3*f*x
+ 4*a*b^4*d^3*cosh(1) - a*b^4*d^2*f)*sinh(1)^2 + 1500*(8*a*b^4*d^3*f^2*x^2
- 4*a*b^4*d^2*f^2*x + a*b^4*d*f^2)*cosh(1) - 128*(125*b^5*d^3*f^3*x^3 - 75*
b^5*d^2*f^3*x^2 + 125*b^5*d^3*cosh(1)^3 + 125*b^5*d^3*sinh(1)^3 + 30*b^5*d*
f^3*x - 6*b^5*f^3 + 75*(5*b^5*d^3*f*x - b^5*d^2*f)*cosh(1)^2 + 75*(5*b^5*d^
3*f*x + 5*b^5*d^3*cosh(1) - b^5*d^2*f)*sinh(1)^2 + 15*(25*b^5*d^3*f^2*x^2 -
 10*b^5*d^2*f^2*x + 2*b^5*d*f^2)*cosh(1) + 15*(25*b^5*d^3*f^2*x^2 - 10*b^5*
d^2*f^2*x + 25*b^5*d^3*cosh(1)^2 + 2*b^5*d*f^2 + 10*(5*b^5*d^3*f*x - b^5*d^
2*f)*cosh(1))*sinh(1))*cosh(d*x + c) + 1500*(8*a*b^4*d^3*f^2*x^2 - 4*a*b^4*
d^2*f^2*x + 8*a*b^4*d^3*cosh(1)^2 + a*b^4*d*f^2 + 4*(4*a*b^4*d^3*f*x - a*b^
4*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^9 - 40000*(9*(4*a^2*b^3 + b^5)*d^3
*f^3*x^3 - 9*(4*a^2*b^3 + b^5)*d^2*f^3*x^2 + 9*(4*a^2*b^3 + b^5)*d^3*cosh(1
)^3 + 9*(4*a^2*b^3 + b^5)*d^3*sinh(1)^3 + 6*(4*a^2*b^3 + b^5)*d*f^3*x - 2*(
4*a^2*b^3 + b^5)*f^3 + 9*(3*(4*a^2*b^3 + b^5)*d^3*f*x - (4*a^2*b^3 + b^5)*d
```

```
^2*f)*cosh(1)^2 + 9*(3*(4*a^2*b^3 + b^5)*d^3*f*x + 3*(4*a^2*b^3 + b^5)*d^3*
cosh(1) - (4*a^2*b^3 + b^5)*d^2*f)*sinh(1)^2 + 3*(9*(4*a^2*b^3 + b^5)*d^3*f
^2*x^2 - 6*(4*a^2*b^3 + b^5)*d^2*f^2*x + 2*(4*a^2*b^3 + b^5)*d*f^2)*cosh(1)
 + 3*(9*(4*a^2*b^3 + b^5)*d^3*f^2*x^2 - 6*(4*a^2*b^3 + b^5)*d^2*f^2*x + 9*(
4*a^2*b^3 + b^5)*d^3*cosh(1)^2 + 2*(4*a^2*b^3 + b^5)*d*f^2 + 6*(3*(4*a^2*b^
3 + b^5)*d^3*f*x - (4*a^2*b^3 + b^5)*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)
^8 - 5*(72000*(4*a^2*b^3 + b^5)*d^3*f^3*x^3 - 72000*(4*a^2*b^3 + b^5)*d^2*f
^3*x^2 + 72000*(4*a^2*b^3 + b^5)*d^3*cosh(1)^3 + 72000*(4*a^2*b^3 + b^5)*d^
3*sinh(1)^3 + 48000*(4*a^2*b^3 + b^5)*d*f^3*x - 16000*(4*a^2*b^3 + b^5)*f^3
 + 72000*(3*(4*a^2*b^3 + b^5)*d^3*f*x - (4*a^2*b^3 + b^5)*d^2*f)*cosh(1)^2
+ 15552*(125*b^5*d^3*f^3*x^3 - 75*b^5*d^2*f^3*x^2 + 125*b^5*d^3*cosh(1)^3 +
 125*b^5*d^3*sinh(1)^3 + 30*b^5*d*f^3*x - 6*b^5*f^3 + 75*(5*b^5*d^3*f*x - b
^5*d^2*f)*cosh(1)^2 + 75*(5*b^5*d^3*f*x + 5*b^5*d^3*cosh(1) - b^5*d^2*f)*si
nh(1)^2 + 15*(25*b^5*d^3*f^2*x^2 - 10*b^5*d^2*f^2*x + 2*b^5*d*f^2)*cosh(1)
+ 15*(25*b^5*d^3*f^2*x^2 - 10*b^5*d^2*f^2*x + 25*b^5*d^3*cosh(1)^2 + 2*b^5*
d*f^2 + 10*(5*b^5*d^3*f*x - b^5*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 +
72000*(3*(4*a^2*b^3 + b^5)*d^3*f*x + 3*(4*a^2*b^3 + b^5)*d^3*cosh(1) - (4*a
^2*b^3 + b^5)*d^2*f)*sinh(1)^2 + 24000*(9*(4*a^2*b^3 + b^5)*d^3*f^2*x^2 - 6
*(4*a^2*b^3 + b^5)*d^2*f^2*x + 2*(4*a^2*b^3 + b^5)*d*f^2)*cosh(1) - 30375*(
32*a*b^4*d^3*f^3*x^3 - 24*a*b^4*d^2*f^3*x^2 + 32*a*b^4*d^3*cosh(1)^3 + 32*a
*b^4*d^3*sinh(1)^3 + 12*a*b^4*d*f^3*x - 3*a*b^4*f^3 + 24*(4*a*b^4*d^3*f*x -
 a*b^4*d^2*f)*cosh(1)^2 + 24*(4*a*b^4*d^3*f*x + 4*a*b^4*d^3*cosh(1) - a*b^4
*d^2*f)*sinh(1)^2 + 12*(8*a*b^4*d^3*f^2*x^2 - 4*a*b^4*d^2*f^2*x + a*b^4*d*f
^2)*cosh(1) + 12*(8*a*b^4*d^3*f^2*x^2 - 4*a*b^4*d^2*f^2*x + 8*a*b^4*d^3*cos
h(1)^2 + a*b^4*d*f^2 + 4*(4*a*b^4*d^3*f*x - a*b^4*d^2*f)*cosh(1))*sinh(1))*
cosh(d*x + c) + 24000*(9*(4*a^2*b^3 + b^5)*d^3*f^2*x^2 - 6*(4*a^2*b^3 + b^5
)*d^2*f^2*x + 9*(4*a^2*b^3 + b^5)*d^3*cosh(1)^2 + 2*(4*a^2*b^3 + b^5)*d*f^2
 + 6*(3*(4*a^2*b^3 + b^5)*d^3*f*x - (4*a^2*b^3 + b^5)*d^2*f)*cosh(1))*sinh(
1))*sinh(d*x + c)^8 + 10368*b^5*f^3 + 540000*(4*(2*a^3*b^2 + a*b^4)*d^3*f^3
*x^3 - 6*(2*a^3*b^2 + a*b^4)*d^2*f^3*x^2 + 4*(2*a^3*b^2 + a*b^4)*d^3*cosh(1
)^3 + 4*(2*a^3*b^2 + a*b^4)*d^3*sinh(1)^3 + 6*(...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**3*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)^3*sinh(d*x + c)^3/(b*sinh(d*x + c) + a)
, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^3\,\sinh(c+d\,x)^3\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.402 $\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1049

$$-\frac{a^3efx}{2b^4d}+\frac{3aefx}{16b^2d}-\frac{a^3f^2x^2}{4b^4d}+\frac{3af^2x^2}{32b^2d}+\frac{a^3(a^2+b^2)\,(e+fx)^3}{3b^6f}-\frac{2a^4f(e+fx)\cosh(c+dx)}{b^5d^2}-\frac{4a^2f(e+fx)\cosh}{3b^3d^2}$$

```
[Out] -2*a^3*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^6
/d^2-2*a^3*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))
/b^6/d^2+1/2*a^3*f*(f*x+e)*cosh(d*x+c)*sinh(d*x+c)/b^4/d^2+1/8*a*f*(f*x+e)*
cosh(d*x+c)^3*sinh(d*x+c)/b^2/d^2+1/4*f*(f*x+e)*cosh(d*x+c)/b/d^2-1/8*(f*x+
e)^2*sinh(d*x+c)/b/d-1/4*f^2*sinh(d*x+c)/b/d^3-1/2*a^3*e*f*x/b^4/d-2*a^4*f*
(f*x+e)*cosh(d*x+c)/b^5/d^2-2/9*a^2*f*(f*x+e)*cosh(d*x+c)^3/b^3/d^2+3/16*a*
e*f*x/b^2/d-4/3*a^2*f*(f*x+e)*cosh(d*x+c)/b^3/d^2+a^4*(f*x+e)^2*sinh(d*x+c)
/b^5/d+1/216*f^2*sinh(3*d*x+3*c)/b/d^3+1/48*(f*x+e)^2*sinh(3*d*x+3*c)/b/d+1
/1000*f^2*sinh(5*d*x+5*c)/b/d^3+1/80*(f*x+e)^2*sinh(5*d*x+5*c)/b/d-a^3*(a^2
+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^6/d-a^3*(a^2+b^2)*
(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^6/d+2*a^3*(a^2+b^2)*f^2*
polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^6/d^3+2*a^3*(a^2+b^2)*f^2*po
lylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^6/d^3+3/16*a*f*(f*x+e)*cosh(d*
x+c)*sinh(d*x+c)/b^2/d^2+1/3*a^2*(f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/b^3/d+
3/32*a*f^2*x^2/b^2/d+14/9*a^2*f^2*sinh(d*x+c)/b^3/d^3-1/4*a^3*f^2*x^2/b^4/d
+1/3*a^3*(a^2+b^2)*(f*x+e)^3/b^6/f-3/32*a*f^2*cosh(d*x+c)^2/b^2/d^3-1/32*a*
f^2*cosh(d*x+c)^4/b^2/d^3-1/4*a*(f*x+e)^2*cosh(d*x+c)^4/b^2/d-1/72*f*(f*x+e
)*cosh(3*d*x+3*c)/b/d^2-1/200*f*(f*x+e)*cosh(5*d*x+5*c)/b/d^2+2*a^4*f^2*sin
h(d*x+c)/b^5/d^3+2/3*a^2*(f*x+e)^2*sinh(d*x+c)/b^3/d-1/4*a^3*f^2*sinh(d*x+c
)^2/b^4/d^3-1/2*a^3*(f*x+e)^2*sinh(d*x+c)^2/b^4/d+2/27*a^2*f^2*sinh(d*x+c)^
3/b^3/d^3
```

Rubi [A]
time = 1.11, antiderivative size = 1049, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 15, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5698, 5556, 3377, 2717, 5555, 3391, 3392, 2713, 5684, 5554, 5680, 2221, 2611, 2320, 6724}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/2*(a^3*e*f*x)/(b^4*d) + (3*a*e*f*x)/(16*b^2*d) - (a^3*f^2*x^2)/(4*b^4*d)
 + (3*a*f^2*x^2)/(32*b^2*d) + (a^3*(a^2 + b^2)*(e + f*x)^3)/(3*b^6*f) - (2*
a^4*f*(e + f*x)*Cosh[c + d*x])/(b^5*d^2) - (4*a^2*f*(e + f*x)*Cosh[c + d*x]
```

```
)/(3*b^3*d^2) + (f*(e + f*x)*Cosh[c + d*x])/(4*b*d^2) - (3*a*f^2*Cosh[c + d
*x]^2)/(32*b^2*d^3) - (2*a^2*f*(e + f*x)*Cosh[c + d*x]^3)/(9*b^3*d^2) - (a*
f^2*Cosh[c + d*x]^4)/(32*b^2*d^3) - (a*(e + f*x)^2*Cosh[c + d*x]^4)/(4*b^2*
d) - (f*(e + f*x)*Cosh[3*c + 3*d*x])/(72*b*d^2) - (f*(e + f*x)*Cosh[5*c + 5
*d*x])/(200*b*d^2) - (a^3*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])])/(b^6*d) - (a^3*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^6*d) - (2*a^3*(a^2 + b^2)*f*(e + f*x)
*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^6*d^2) - (2*a^3*(
a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/(b^6*d^2) + (2*a^3*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2]))])/(b^6*d^3) + (2*a^3*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/(b^6*d^3) + (2*a^4*f^2*Sinh[c + d*x])/(b^5*d^
3) + (14*a^2*f^2*Sinh[c + d*x])/(9*b^3*d^3) - (f^2*Sinh[c + d*x])/(4*b*d^3)
 + (a^4*(e + f*x)^2*Sinh[c + d*x])/(b^5*d) + (2*a^2*(e + f*x)^2*Sinh[c + d*
x])/(3*b^3*d) - ((e + f*x)^2*Sinh[c + d*x])/(8*b*d) + (a^3*f*(e + f*x)*Cosh
[c + d*x]*Sinh[c + d*x])/(2*b^4*d^2) + (3*a*f*(e + f*x)*Cosh[c + d*x]*Sinh[
c + d*x])/(16*b^2*d^2) + (a^2*(e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3
*b^3*d) + (a*f*(e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x])/(8*b^2*d^2) - (a^3*
f^2*Sinh[c + d*x]^2)/(4*b^4*d^3) - (a^3*(e + f*x)^2*Sinh[c + d*x]^2)/(2*b^4
*d) + (2*a^2*f^2*Sinh[c + d*x]^3)/(27*b^3*d^3) + (f^2*Sinh[3*c + 3*d*x])/(2
16*b*d^3) + ((e + f*x)^2*Sinh[3*c + 3*d*x])/(48*b*d) + (f^2*Sinh[5*c + 5*d*
x])/(1000*b*d^3) + ((e + f*x)^2*Sinh[5*c + 5*d*x])/(80*b*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5555

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
```

```
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\cosh^3}{a+b\,\mathrm{s}}}{}$$

$$= -\frac{a\int (e+fx)^2\cosh^3(c+dx)\sinh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^2\,\mathrm{c}}{a}}{}$$

$$= -\frac{a(e+fx)^2\cosh^4(c+dx)}{4b^2d} + \frac{a^2\int (e+fx)^2\cosh^3(c+dx)\,d}{b^3}$$

$$= -\frac{2a^2f(e+fx)\cosh^3(c+dx)}{9b^3d^2} - \frac{af^2\cosh^4(c+dx)}{32b^2d^3} - \frac{a(e+}{}$$

$$= \frac{a^3(a^2+b^2)(e+fx)^3}{3b^6f} + \frac{f(e+fx)\cosh(c+dx)}{4bd^2} - \frac{3af^2\cosh}{32b}$$

$$= \frac{3aefx}{16b^2d} + \frac{3af^2x^2}{32b^2d} + \frac{a^3(a^2+b^2)(e+fx)^3}{3b^6f} - \frac{2a^4f(e+fx)\,\mathrm{co}}{b^5d^2}$$

$$= -\frac{a^3efx}{2b^4d} + \frac{3aefx}{16b^2d} - \frac{a^3f^2x^2}{4b^4d} + \frac{3af^2x^2}{32b^2d} + \frac{a^3(a^2+b^2)(e+f}{3b^6f}$$

$$= -\frac{a^3efx}{2b^4d} + \frac{3aefx}{16b^2d} - \frac{a^3f^2x^2}{4b^4d} + \frac{3af^2x^2}{32b^2d} + \frac{a^3(a^2+b^2)(e+f}{3b^6f}$$

$$= -\frac{a^3efx}{2b^4d} + \frac{3aefx}{16b^2d} - \frac{a^3f^2x^2}{4b^4d} + \frac{3af^2x^2}{32b^2d} + \frac{a^3(a^2+b^2)(e+f}{3b^6f}$$

Mathematica [A]
time = 7.54, size = 1170, normalized size = 1.12

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x
]),x]
```

```
[Out] ((-8*a^3*(a^2 + b^2)*e^2*x*Coth[c])/b^6 - (8*a^3*(a^2 + b^2)*e*f*x^2*Coth[c
])/b^6 - (8*a^3*(a^2 + b^2)*f^2*x^3*Coth[c])/(3*b^6) - (8*a^3*(a^2 + b^2)*(
2*d^3*E^(2*c)*x*(3*e^2 + 3*e*f*x + f^2*x^2) + 3*(1 - E^(2*c))*(d^2*e^2*Log[
2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*d^2*e*f*x*Log[1 + (b*E^(2*c
```

```
 + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + d^2*f^2*x^2*Log[1 + (b*E^(2
*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*d^2*e*f*x*Log[1 + (b*E^
(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + d^2*f^2*x^2*Log[1 + (b*
E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*d*f*(e + f*x)*PolyL
og[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2*d*f*(e
+ f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]
 - 2*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])
)] - 2*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)
]))])))/(3*b^6*d^3*(1 - E^(2*c))) + ((-8*a^4 - 6*a^2*b^2 + b^4)*(2*f^2 + 2*
d*f*(e + f*x) + d^2*(e + f*x)^2)*(Cosh[c + d*x] - Sinh[c + d*x]))/(2*b^5*d^
3) + ((8*a^4 + 6*a^2*b^2 - b^4)*(2*f^2 - 2*d*f*(e + f*x) + d^2*(e + f*x)^2)
*(Cosh[c + d*x] + Sinh[c + d*x]))/(2*b^5*d^3) + (a*(2*a^2 + b^2)*(f^2 + 2*d
*f*(e + f*x) + 2*d^2*(e + f*x)^2)*(-Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]))
/(4*b^4*d^3) - (a*(2*a^2 + b^2)*(f^2 - 2*d*f*(e + f*x) + 2*d^2*(e + f*x)^2)
*(Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]))/(4*b^4*d^3) + ((4*a^2 + b^2)*(2*f
^2 + 6*d*f*(e + f*x) + 9*d^2*(e + f*x)^2)*(-Cosh[3*(c + d*x)] + Sinh[3*(c +
 d*x)]))/(108*b^3*d^3) + ((4*a^2 + b^2)*(2*f^2 - 6*d*f*(e + f*x) + 9*d^2*(e
 + f*x)^2)*(Cosh[3*(c + d*x)] + Sinh[3*(c + d*x)]))/(108*b^3*d^3) + (a*(f^2
 + 4*d*f*(e + f*x) + 8*d^2*(e + f*x)^2)*(-Cosh[4*(c + d*x)] + Sinh[4*(c + d
*x)]))/(64*b^2*d^3) - (a*(f^2 - 4*d*f*(e + f*x) + 8*d^2*(e + f*x)^2)*(Cosh[
4*(c + d*x)] + Sinh[4*(c + d*x)]))/(64*b^2*d^3) + ((2*f^2 + 10*d*f*(e + f*x
) + 25*d^2*(e + f*x)^2)*(-Cosh[5*(c + d*x)] + Sinh[5*(c + d*x)]))/(500*b*d^
3) + ((2*f^2 - 10*d*f*(e + f*x) + 25*d^2*(e + f*x)^2)*(Cosh[5*(c + d*x)] +
Sinh[5*(c + d*x)]))/(500*b*d^3))/8
```

Maple [F]
time = 2.60, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^{2} \left(\cosh^{3}\left(dx+c\right)\right) \left(\sinh^{3}\left(dx+c\right)\right)}{a+b\sinh\left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
[Out] int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="maxima")
```

```
[Out] -1/960*((15*a*b^3*e^(-d*x - c) - 6*b^4 - 10*(4*a^2*b^2 + b^4)*e^(-2*d*x - 2
*c) + 60*(2*a^3*b + a*b^3)*e^(-3*d*x - 3*c) - 60*(8*a^4 + 6*a^2*b^2 - b^4)*
e^(-4*d*x - 4*c))*e^(5*d*x + 5*c)/(b^5*d) + 960*(a^5 + a^3*b^2)*(d*x + c)/(
b^6*d) + (15*a*b^3*e^(-4*d*x - 4*c) + 6*b^4*e^(-5*d*x - 5*c) + 60*(8*a^4 +
6*a^2*b^2 - b^4)*e^(-d*x - c) + 60*(2*a^3*b + a*b^3)*e^(-2*d*x - 2*c) + 10*
(4*a^2*b^2 + b^4)*e^(-3*d*x - 3*c))/(b^5*d) + 960*(a^5 + a^3*b^2)*log(-2*a*
e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^6*d))*e^2 - 1/1728000*(576000*(a^
5*d^3*f^2*e^(5*c) + a^3*b^2*d^3*f^2*e^(5*c))*x^3 + 1728000*(a^5*d^3*f*e^(5*
c) + a^3*b^2*d^3*f*e^(5*c))*x^2*e - 432*(25*b^5*d^2*f^2*x^2*e^(10*c) + 2*b^
5*f^2*e^(10*c) - 10*b^5*d*f*e^(10*c + 1) - 10*(b^5*d*f^2*e^(10*c) - 5*b^5*d
^2*f*e^(10*c + 1))*x)*e^(5*d*x) + 3375*(8*a*b^4*d^2*f^2*x^2*e^(9*c) + a*b^4
*f^2*e^(9*c) - 4*a*b^4*d*f*e^(9*c + 1) - 4*(a*b^4*d*f^2*e^(9*c) - 4*a*b^4*d
^2*f*e^(9*c + 1))*x)*e^(4*d*x) - 2000*(8*a^2*b^3*f^2*e^(8*c) + 2*b^5*f^2*e^
(8*c) + 9*(4*a^2*b^3*d^2*f^2*e^(8*c) + b^5*d^2*f^2*e^(8*c))*x^2 - 6*(4*a^2*
b^3*d*f^2*e^(8*c) + b^5*d*f^2*e^(8*c) - 3*(4*a^2*b^3*d^2*f*e^(8*c) + b^5*d^
2*f*e^(8*c))*e)*x - 6*(4*a^2*b^3*d*f*e^(8*c) + b^5*d*f*e^(8*c))*e)*e^(3*d*x
) + 54000*(2*a^3*b^2*f^2*e^(7*c) + a*b^4*f^2*e^(7*c) + 2*(2*a^3*b^2*d^2*f^2
*e^(7*c) + a*b^4*d^2*f^2*e^(7*c))*x^2 - 2*(2*a^3*b^2*d*f^2*e^(7*c) + a*b^4*
d*f^2*e^(7*c) - 2*(2*a^3*b^2*d^2*f*e^(7*c) + a*b^4*d^2*f*e^(7*c))*e)*x - 2*
(2*a^3*b^2*d*f*e^(7*c) + a*b^4*d*f*e^(7*c))*e)*e^(2*d*x) - 108000*(16*a^4*b
*f^2*e^(6*c) + 12*a^2*b^3*f^2*e^(6*c) - 2*b^5*f^2*e^(6*c) + (8*a^4*b*d^2*f^
2*e^(6*c) + 6*a^2*b^3*d^2*f^2*e^(6*c) - b^5*d^2*f^2*e^(6*c))*x^2 - 2*(8*a^4
*b*d*f^2*e^(6*c) + 6*a^2*b^3*d*f^2*e^(6*c) - b^5*d*f^2*e^(6*c) - (8*a^4*b*d
^2*f*e^(6*c) + 6*a^2*b^3*d^2*f*e^(6*c) - b^5*d^2*f*e^(6*c))*e)*x - 2*(8*a^4
*b*d*f*e^(6*c) + 6*a^2*b^3*d*f*e^(6*c) - b^5*d*f*e^(6*c))*e)*e^(d*x) + 1080
00*(16*a^4*b*f^2*e^(4*c) + 12*a^2*b^3*f^2*e^(4*c) - 2*b^5*f^2*e^(4*c) + (8*
a^4*b*d^2*f^2*e^(4*c) + 6*a^2*b^3*d^2*f^2*e^(4*c) - b^5*d^2*f^2*e^(4*c))*x^
2 + 2*(8*a^4*b*d*f^2*e^(4*c) + 6*a^2*b^3*d*f^2*e^(4*c) - b^5*d*f^2*e^(4*c)
+ (8*a^4*b*d^2*f*e^(4*c) + 6*a^2*b^3*d^2*f*e^(4*c) - b^5*d^2*f*e^(4*c))*e)*
x + 2*(8*a^4*b*d*f*e^(4*c) + 6*a^2*b^3*d*f*e^(4*c) - b^5*d*f*e^(4*c))*e)*e^
(-d*x) + 54000*(2*a^3*b^2*f^2*e^(3*c) + a*b^4*f^2*e^(3*c) + 2*(2*a^3*b^2*d^
2*f^2*e^(3*c) + a*b^4*d^2*f^2*e^(3*c))*x^2 + 2*(2*a^3*b^2*d*f^2*e^(3*c) + a
*b^4*d*f^2*e^(3*c) + 2*(2*a^3*b^2*d^2*f*e^(3*c) + a*b^4*d^2*f*e^(3*c))*e)*x
 + 2*(2*a^3*b^2*d*f*e^(3*c) + a*b^4*d*f*e^(3*c))*e)*e^(-2*d*x) + 2000*(8*a^
2*b^3*f^2*e^(2*c) + 2*b^5*f^2*e^(2*c) + 9*(4*a^2*b^3*d^2*f^2*e^(2*c) + b^5*
d^2*f^2*e^(2*c))*x^2 + 6*(4*a^2*b^3*d*f^2*e^(2*c) + b^5*d*f^2*e^(2*c) + 3*(
4*a^2*b^3*d^2*f*e^(2*c) + b^5*d^2*f*e^(2*c))*e)*x + 6*(4*a^2*b^3*d*f*e^(2*c
) + b^5*d*f*e^(2*c))*e)*e^(-3*d*x) + 3375*(8*a*b^4*d^2*f^2*x^2*e^c + 4*a*b^
4*d*f*e^(c + 1) + a*b^4*f^2*e^c + 4*(4*a*b^4*d^2*f*e^(c + 1) + a*b^4*d*f^2*
e^c)*x)*e^(-4*d*x) + 432*(25*b^5*d^2*f^2*x^2 + 10*b^5*d*f*e + 2*b^5*f^2 + 1
0*(5*b^5*d^2*f*e + b^5*d*f^2)*x)*e^(-5*d*x))*e^(-5*c)/(b^6*d^3) + integrate
(-2*((a^5*b*f^2 + a^3*b^3*f^2)*x^2 + 2*(a^5*b*f + a^3*b^3*f)*x*e - ((a^6*f^
2*e^c + a^4*b^2*f^2*e^c)*x^2 + 2*(a^6*f*e^c + a^4*b^2*f*e^c)*x*e)*e^(d*x))/
(b^7*e^(2*d*x + 2*c) + 2*a*b^6*e^(d*x + c) - b^7), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17953 vs. $2(997) = 1994$.
time = 0.63, size = 17953, normalized size = 17.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="fricas")
```

```
[Out] -1/1728000*(10800*b^5*d^2*f^2*x^2 - 432*(25*b^5*d^2*f^2*x^2 - 10*b^5*d*f^2*
x + 25*b^5*d^2*cosh(1)^2 + 25*b^5*d^2*sinh(1)^2 + 2*b^5*f^2 + 10*(5*b^5*d^2
*f*x - b^5*d*f)*cosh(1) + 10*(5*b^5*d^2*f*x + 5*b^5*d^2*cosh(1) - b^5*d*f)*
sinh(1))*cosh(d*x + c)^10 - 432*(25*b^5*d^2*f^2*x^2 - 10*b^5*d*f^2*x + 25*b
^5*d^2*cosh(1)^2 + 25*b^5*d^2*sinh(1)^2 + 2*b^5*f^2 + 10*(5*b^5*d^2*f*x - b
^5*d*f)*cosh(1) + 10*(5*b^5*d^2*f*x + 5*b^5*d^2*cosh(1) - b^5*d*f)*sinh(1))
*sinh(d*x + c)^10 + 3375*(8*a*b^4*d^2*f^2*x^2 - 4*a*b^4*d*f^2*x + 8*a*b^4*d
^2*cosh(1)^2 + 8*a*b^4*d^2*sinh(1)^2 + a*b^4*f^2 + 4*(4*a*b^4*d^2*f*x - a*b
^4*d*f)*cosh(1) + 4*(4*a*b^4*d^2*f*x + 4*a*b^4*d^2*cosh(1) - a*b^4*d*f)*sin
h(1))*cosh(d*x + c)^9 + 135*(200*a*b^4*d^2*f^2*x^2 - 100*a*b^4*d*f^2*x + 20
0*a*b^4*d^2*cosh(1)^2 + 200*a*b^4*d^2*sinh(1)^2 + 25*a*b^4*f^2 + 100*(4*a*b
^4*d^2*f*x - a*b^4*d*f)*cosh(1) - 32*(25*b^5*d^2*f^2*x^2 - 10*b^5*d*f^2*x +
 25*b^5*d^2*cosh(1)^2 + 25*b^5*d^2*sinh(1)^2 + 2*b^5*f^2 + 10*(5*b^5*d^2*f*
x - b^5*d*f)*cosh(1) + 10*(5*b^5*d^2*f*x + 5*b^5*d^2*cosh(1) - b^5*d*f)*sin
h(1))*cosh(d*x + c) + 100*(4*a*b^4*d^2*f*x + 4*a*b^4*d^2*cosh(1) - a*b^4*d*
f)*sinh(1))*sinh(d*x + c)^9 + 4320*b^5*d*f^2*x + 10800*b^5*d^2*cosh(1)^2 -
2000*(9*(4*a^2*b^3 + b^5)*d^2*f^2*x^2 - 6*(4*a^2*b^3 + b^5)*d*f^2*x + 9*(4*
a^2*b^3 + b^5)*d^2*cosh(1)^2 + 9*(4*a^2*b^3 + b^5)*d^2*sinh(1)^2 + 2*(4*a^2
*b^3 + b^5)*f^2 + 6*(3*(4*a^2*b^3 + b^5)*d^2*f*x - (4*a^2*b^3 + b^5)*d*f)*c
osh(1) + 6*(3*(4*a^2*b^3 + b^5)*d^2*f*x + 3*(4*a^2*b^3 + b^5)*d^2*cosh(1) -
 (4*a^2*b^3 + b^5)*d*f)*sinh(1))*cosh(d*x + c)^8 + 10800*b^5*d^2*sinh(1)^2
 - 5*(3600*(4*a^2*b^3 + b^5)*d^2*f^2*x^2 - 2400*(4*a^2*b^3 + b^5)*d*f^2*x +
3600*(4*a^2*b^3 + b^5)*d^2*cosh(1)^2 + 3600*(4*a^2*b^3 + b^5)*d^2*sinh(1)^2
 + 800*(4*a^2*b^3 + b^5)*f^2 + 3888*(25*b^5*d^2*f^2*x^2 - 10*b^5*d*f^2*x +
25*b^5*d^2*cosh(1)^2 + 25*b^5*d^2*sinh(1)^2 + 2*b^5*f^2 + 10*(5*b^5*d^2*f*x
 - b^5*d*f)*cosh(1) + 10*(5*b^5*d^2*f*x + 5*b^5*d^2*cosh(1) - b^5*d*f)*sinh
(1))*cosh(d*x + c)^2 + 2400*(3*(4*a^2*b^3 + b^5)*d^2*f*x - (4*a^2*b^3 + b^5
)*d*f)*cosh(1) - 6075*(8*a*b^4*d^2*f^2*x^2 - 4*a*b^4*d*f^2*x + 8*a*b^4*d^2*
cosh(1)^2 + 8*a*b^4*d^2*sinh(1)^2 + a*b^4*f^2 + 4*(4*a*b^4*d^2*f*x - a*b^4*
d*f)*cosh(1) + 4*(4*a*b^4*d^2*f*x + 4*a*b^4*d^2*cosh(1) - a*b^4*d*f)*sinh(1
))*cosh(d*x + c) + 2400*(3*(4*a^2*b^3 + b^5)*d^2*f*x + 3*(4*a^2*b^3 + b^5)*
d^2*cosh(1) - (4*a^2*b^3 + b^5)*d*f)*sinh(1))*sinh(d*x + c)^8 + 54000*(2*(2
*a^3*b^2 + a*b^4)*d^2*f^2*x^2 - 2*(2*a^3*b^2 + a*b^4)*d*f^2*x + 2*(2*a^3*b^
2 + a*b^4)*d^2*cosh(1)^2 + 2*(2*a^3*b^2 + a*b^4)*d^2*sinh(1)^2 + (2*a^3*b^2
 + a*b^4)*f^2 + 2*(2*(2*a^3*b^2 + a*b^4)*d^2*f*x - (2*a^3*b^2 + a*b^4)*d*f)
```

```
*cosh(1) + 2*(2*(2*a^3*b^2 + a*b^4)*d^2*f*x + 2*(2*a^3*b^2 + a*b^4)*d^2*cos
h(1) - (2*a^3*b^2 + a*b^4)*d*f)*sinh(1))*cosh(d*x + c)^7 + 20*(5400*(2*a^3*
b^2 + a*b^4)*d^2*f^2*x^2 - 5400*(2*a^3*b^2 + a*b^4)*d*f^2*x + 5400*(2*a^3*b
^2 + a*b^4)*d^2*cosh(1)^2 + 5400*(2*a^3*b^2 + a*b^4)*d^2*sinh(1)^2 - 2592*(
25*b^5*d^2*f^2*x^2 - 10*b^5*d*f^2*x + 25*b^5*d^2*cosh(1)^2 + 25*b^5*d^2*sin
h(1)^2 + 2*b^5*f^2 + 10*(5*b^5*d^2*f*x - b^5*d*f)*cosh(1) + 10*(5*b^5*d^2*f
*x + 5*b^5*d^2*cosh(1) - b^5*d*f)*sinh(1))*cosh(d*x + c)^3 + 2700*(2*a^3*b^
2 + a*b^4)*f^2 + 6075*(8*a*b^4*d^2*f^2*x^2 - 4*a*b^4*d*f^2*x + 8*a*b^4*d^2*
cosh(1)^2 + 8*a*b^4*d^2*sinh(1)^2 + a*b^4*f^2 + 4*(4*a*b^4*d^2*f*x - a*b^4*
d*f)*cosh(1) + 4*(4*a*b^4*d^2*f*x + 4*a*b^4*d^2*cosh(1) - a*b^4*d*f)*sinh(1
))*cosh(d*x + c)^2 + 5400*(2*(2*a^3*b^2 + a*b^4)*d^2*f*x - (2*a^3*b^2 + a*b
^4)*d*f)*cosh(1) - 800*(9*(4*a^2*b^3 + b^5)*d^2*f^2*x^2 - 6*(4*a^2*b^3 + b^
5)*d*f^2*x + 9*(4*a^2*b^3 + b^5)*d^2*cosh(1)^2 + 9*(4*a^2*b^3 + b^5)*d^2*si
nh(1)^2 + 2*(4*a^2*b^3 + b^5)*f^2 + 6*(3*(4*a^2*b^3 + b^5)*d^2*f*x - (4*a^2
*b^3 + b^5)*d*f)*cosh(1) + 6*(3*(4*a^2*b^3 + b^5)*d^2*f*x + 3*(4*a^2*b^3 +
b^5)*d^2*cosh(1) - (4*a^2*b^3 + b^5)*d*f)*sinh(1))*cosh(d*x + c) + 5400*(2*
(2*a^3*b^2 + a*b^4)*d^2*f*x + 2*(2*a^3*b^2 + a*b^4)*d^2*cosh(1) - (2*a^3*b^
2 + a*b^4)*d*f)*sinh(1))*sinh(d*x + c)^7 + 864*b^5*f^2 - 108000*((8*a^4*b +
 6*a^2*b^3 - b^5)*d^2*f^2*x^2 - 2*(8*a^4*b + 6*a^2*b^3 - b^5)*d*f^2*x + (8*
a^4*b + 6*a^2*b^3 - b^5)*d^2*cosh(1)^2 + (8*a^4*b + 6*a^2*b^3 - b^5)*d^2*si
nh(1)^2 + 2*(8*a^4*b + 6*a^2*b^3 - b^5)*f^2 + 2*((8*a^4*b + 6*a^2*b^3 - b^5
)*d^2*f*x - (8*a^4*b + 6*a^2*b^3 - b^5)*d*f)*cosh(1) + 2*((8*a^4*b + 6*a^2*
b^3 - b^5)*d^2*f*x + (8*a^4*b + 6*a^2*b^3 - b^5)*d^2*cosh(1) - (8*a^4*b + 6
*a^2*b^3 - b^5)*d*f)*sinh(1))*cosh(d*x + c)^6 - 20*(5400*(8*a^4*b + 6*a^2*b
^3 - b^5)*d^2*f^2*x^2 - 10800*(8*a^4*b + 6*a^2*b^3 - b^5)*d*f^2*x + 5400*(8
*a^4*b + 6*a^2*b^3 - b^5)*d^2*cosh(1)^2 + 4536*(25*b^5*d^2*f^2*x^2 - 10*b^5
*d*f^2*x + 25*b^5*d^2*cosh(1)^2 + 25*b^5*d^2*sinh(1)^2 + 2*b^5*f^2 + 10*(5*
b^5*d^2*f*x - b^5*d*f)*cosh(1) + 10*(5*b^5*d^2*f*x + 5*b^5*d^2*cosh(1) - b^
5*d*f)*sinh(1))*cosh(d*x + c)^4 + 5400*(8*a^4*b + 6*a^2*b^3 - b^5)*d^2*sinh
(1)^2 - 14175*(8*a*b^4*d^2*f^2*x^2 - 4*a*b^4*d*f^2*x + 8*a*b^4*d^2*cosh(1)^
2 + 8*a*b^4*d^2*sinh(1)^2 + a*b^4*f^2 + 4*(4*a*b^4*d^2*f*x - a*b^4*d*f)*cos
h(1) + 4*(4*a*b^4*d^2*f*x + 4*a*b^4*d^2*cosh(1)...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**3*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)^3*sinh(d*x + c)^3/(b*sinh(d*x + c) + a)
, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^3\,\sinh(c+d\,x)^3\,(e+f\,x)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.403 $\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=641

$$-\frac{a^3fx}{4b^4d}+\frac{3afx}{32b^2d}+\frac{a^3(a^2+b^2)(e+fx)^2}{2b^6f}-\frac{a^4f\cosh(c+dx)}{b^5d^2}-\frac{2a^2f\cosh(c+dx)}{3b^3d^2}+\frac{f\cosh(c+dx)}{8bd^2}-\frac{a^2f\cosh^3(}{9b^3d}$$

```
[Out] -1/4*a^3*f*x/b^4/d+3/32*a*f*x/b^2/d+1/2*a^3*(a^2+b^2)*(f*x+e)^2/b^6/f-a^4*f
*cosh(d*x+c)/b^5/d^2-2/3*a^2*f*cosh(d*x+c)/b^3/d^2+1/8*f*cosh(d*x+c)/b/d^2-
1/9*a^2*f*cosh(d*x+c)^3/b^3/d^2-1/4*a*(f*x+e)*cosh(d*x+c)^4/b^2/d-1/144*f*c
osh(3*d*x+3*c)/b/d^2-1/400*f*cosh(5*d*x+5*c)/b/d^2-a^3*(a^2+b^2)*(f*x+e)*ln
(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^6/d-a^3*(a^2+b^2)*(f*x+e)*ln(1+b*exp
(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^6/d-a^3*(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/
(a-(a^2+b^2)^(1/2)))/b^6/d^2-a^3*(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a+(a^
2+b^2)^(1/2)))/b^6/d^2+a^4*(f*x+e)*sinh(d*x+c)/b^5/d+2/3*a^2*(f*x+e)*sinh(d
*x+c)/b^3/d-1/8*(f*x+e)*sinh(d*x+c)/b/d+1/4*a^3*f*cosh(d*x+c)*sinh(d*x+c)/b
^4/d^2+3/32*a*f*cosh(d*x+c)*sinh(d*x+c)/b^2/d^2+1/3*a^2*(f*x+e)*cosh(d*x+c)
^2*sinh(d*x+c)/b^3/d+1/16*a*f*cosh(d*x+c)^3*sinh(d*x+c)/b^2/d^2-1/2*a^3*(f*
x+e)*sinh(d*x+c)^2/b^4/d+1/48*(f*x+e)*sinh(3*d*x+3*c)/b/d+1/80*(f*x+e)*sinh
(5*d*x+5*c)/b/d
```

Rubi [A]
time = 0.65, antiderivative size = 641, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5698, 5556, 3377, 2718, 5555, 2715, 8, 3391, 5684, 5554, 5680, 2221, 2317, 2438}

$$-\frac{a^4f\cosh(c+dx)}{b^5d^2}+\frac{a^4(e+fx)\sinh(c+dx)}{b^5d}+\frac{a^3f\sinh(c+dx)\cosh(c+dx)}{4b^4d^2}-\frac{a^3(e+fx)\sinh^2(c+dx)}{2b^4d}-\frac{a^3fx}{4b^4d}-\frac{a^2f\cosh^3(c+dx)}{9b^3d^2}-\frac{2a^2f\cosh(c+dx)}{3b^3d^2}+\frac{2a^2(e+fx)\sinh(c+dx)}{3b^3d}+\frac{a^2(e+fx)\sinh(c+dx)\cosh^2(c+dx)}{3b^3d}-\frac{a^3f(a^2+b^2)\,\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^6d^2}-\frac{a^3f(a^2+b^2)\,\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^6d^2}-\frac{a^3(a^2+b^2)(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^6d}-\frac{a^3(a^2+b^2)(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^6d}+\frac{a^3(a^2+b^2)(e+fx)^2}{2b^6f}+\frac{af\sinh(c+dx)\cosh^3(c+dx)}{16b^2d^2}+\frac{3af\sinh(c+dx)\cosh(c+dx)}{32b^2d^2}-\frac{a(e+fx)\cosh^4(c+dx)}{4b^2d}+\frac{3afx}{32b^2d}+\frac{f\cosh(c+dx)}{8bd^2}-\frac{f\cosh(3c+3dx)}{144bd^2}-\frac{f\cosh(5c+5dx)}{400bd^2}-\frac{(e+fx)\sinh(c+dx)}{8bd}+\frac{(e+fx)\sinh(3c+3dx)}{48bd}+\frac{(e+fx)\sinh(5c+5dx)}{80bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/4*(a^3*f*x)/(b^4*d) + (3*a*f*x)/(32*b^2*d) + (a^3*(a^2 + b^2)*(e + f*x)^
2)/(2*b^6*f) - (a^4*f*Cosh[c + d*x])/(b^5*d^2) - (2*a^2*f*Cosh[c + d*x])/(3
*b^3*d^2) + (f*Cosh[c + d*x])/(8*b*d^2) - (a^2*f*Cosh[c + d*x]^3)/(9*b^3*d^
2) - (a*(e + f*x)*Cosh[c + d*x]^4)/(4*b^2*d) - (f*Cosh[3*c + 3*d*x])/(144*b
*d^2) - (f*Cosh[5*c + 5*d*x])/(400*b*d^2) - (a^3*(a^2 + b^2)*(e + f*x)*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^6*d) - (a^3*(a^2 + b^2)*(e +
 f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^6*d) - (a^3*(a^2 +
 b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^6*d^2) - (
a^3*(a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^
6*d^2) + (a^4*(e + f*x)*Sinh[c + d*x])/(b^5*d) + (2*a^2*(e + f*x)*Sinh[c +
d*x])/(3*b^3*d) - ((e + f*x)*Sinh[c + d*x])/(8*b*d) + (a^3*f*Cosh[c + d*x]*
```

```
Sinh[c + d*x])/(4*b^4*d^2) + (3*a*f*Cosh[c + d*x]*Sinh[c + d*x])/(32*b^2*d^
2) + (a^2*(e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b^3*d) + (a*f*Cosh[c
+ d*x]^3*Sinh[c + d*x])/(16*b^2*d^2) - (a^3*(e + f*x)*Sinh[c + d*x]^2)/(2*b
^4*d) + ((e + f*x)*Sinh[3*c + 3*d*x])/(48*b*d) + ((e + f*x)*Sinh[5*c + 5*d*
x])/(80*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5555

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[(c + d*x)^m*(Cosh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5698

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*(Sinh[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh^3(c+dx)\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\cosh^3(c+dx\cdots}{a+b\sinh(c\cdots}}{b}$$

$$= -\frac{a\int (e+fx)\cosh^3(c+dx)\sinh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)\cosh^3(\cdots}{a+b\sin\cdots}}{b\cdots}$$

$$= -\frac{a(e+fx)\cosh^4(c+dx)}{4b^2d} + \frac{a^2\int (e+fx)\cosh^3(c+dx)\,dx}{b^3} - \cdots$$

$$= -\frac{a^2f\cosh^3(c+dx)}{9b^3d^2} - \frac{a(e+fx)\cosh^4(c+dx)}{4b^2d} - \frac{(e+fx)\sin\cdots}{8bd\cdots}$$

$$= \frac{a^3(a^2+b^2)(e+fx)^2}{2b^6f} + \frac{f\cosh(c+dx)}{8bd^2} - \frac{a^2f\cosh^3(c+dx)}{9b^3d^2} - \cdots$$

$$= \frac{3afx}{32b^2d} + \frac{a^3(a^2+b^2)(e+fx)^2}{2b^6f} - \frac{a^4f\cosh(c+dx)}{b^5d^2} - \frac{2a^2f\cosh\cdots}{3b^3\cdots}$$

$$= -\frac{a^3fx}{4b^4d} + \frac{3afx}{32b^2d} + \frac{a^3(a^2+b^2)(e+fx)^2}{2b^6f} - \frac{a^4f\cosh(c+dx)}{b^5d^2} - \cdots$$

$$= -\frac{a^3fx}{4b^4d} + \frac{3afx}{32b^2d} + \frac{a^3(a^2+b^2)(e+fx)^2}{2b^6f} - \frac{a^4f\cosh(c+dx)}{b^5d^2} - \cdots$$

Mathematica [A]
time = 2.85, size = 958, normalized size = 1.49

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] -1/28800*(-14400*a^5*c^2*f - 14400*a^3*b^2*c^2*f - 28800*a^5*c*d*f*x - 2880
0*a^3*b^2*c*d*f*x - 14400*a^5*d^2*f*x^2 - 14400*a^3*b^2*d^2*f*x^2 + 28800*a
^4*b*f*Cosh[c + d*x] + 21600*a^2*b^3*f*Cosh[c + d*x] - 3600*b^5*f*Cosh[c +
d*x] + 7200*a^3*b^2*d*e*Cosh[2*(c + d*x)] + 3600*a*b^4*d*e*Cosh[2*(c + d*x)
] + 7200*a^3*b^2*d*f*x*Cosh[2*(c + d*x)] + 3600*a*b^4*d*f*x*Cosh[2*(c + d*x
)] + 800*a^2*b^3*f*Cosh[3*(c + d*x)] + 200*b^5*f*Cosh[3*(c + d*x)] + 900*a*
b^4*d*e*Cosh[4*(c + d*x)] + 900*a*b^4*d*f*x*Cosh[4*(c + d*x)] + 72*b^5*f*Co
sh[5*(c + d*x)] + 28800*a^5*c*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])] + 28800*a^3*b^2*c*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 28
800*a^5*d*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 28800*a^3*b^
2*d*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 28800*a^5*c*f*Log[
1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 28800*a^3*b^2*c*f*Log[1 + (b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 28800*a^5*d*f*x*Log[1 + (b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2])] + 28800*a^3*b^2*d*f*x*Log[1 + (b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2])] + 28800*a^5*d*e*Log[a + b*Sinh[c + d*x]] + 28800*a^3*b^2
*d*e*Log[a + b*Sinh[c + d*x]] - 28800*a^5*c*f*Log[a + b*Sinh[c + d*x]] - 28
800*a^3*b^2*c*f*Log[a + b*Sinh[c + d*x]] + 28800*a^3*(a^2 + b^2)*f*PolyLog[
2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 28800*a^3*(a^2 + b^2)*f*PolyLo
g[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 28800*a^4*b*d*e*Sinh[c + d
*x] - 21600*a^2*b^3*d*e*Sinh[c + d*x] + 3600*b^5*d*e*Sinh[c + d*x] - 28800*
a^4*b*d*f*x*Sinh[c + d*x] - 21600*a^2*b^3*d*f*x*Sinh[c + d*x] + 3600*b^5*d*
f*x*Sinh[c + d*x] - 3600*a^3*b^2*f*Sinh[2*(c + d*x)] - 1800*a*b^4*f*Sinh[2*
(c + d*x)] - 2400*a^2*b^3*d*e*Sinh[3*(c + d*x)] - 600*b^5*d*e*Sinh[3*(c + d
*x)] - 2400*a^2*b^3*d*f*x*Sinh[3*(c + d*x)] - 600*b^5*d*f*x*Sinh[3*(c + d*x
)] - 225*a*b^4*f*Sinh[4*(c + d*x)] - 360*b^5*d*e*Sinh[5*(c + d*x)] - 360*b^
5*d*f*x*Sinh[5*(c + d*x)])/(b^6*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. $2(593) = 1186$.
time = 3.72, size = 1363, normalized size = 2.13

method	result	size
risch	Expression too large to display	1363

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNV
ERBOSE)
```

```
[Out] -1/800*(5*d*f*x+5*d*e+f)/d^2/b*exp(-5*d*x-5*c)+1/16*(8*a^4*d*f*x+6*a^2*b^2*
d*f*x-b^4*d*f*x+8*a^4*d*e+6*a^2*b^2*d*e-b^4*d*e-8*a^4*f-6*a^2*b^2*f+b^4*f)/
b^5/d^2*exp(d*x+c)+2/d*a^5/b^6*f*c*x+1/2*a^3*f*x^2/b^4-1/d*a^3/b^4*e*ln(b*e
xp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d*a^3/b^4*e*ln(exp(d*x+c))+2/d*a^3/b^4*c*
f*x-2/d^2*a^3/b^4*f*c*ln(exp(d*x+c))+1/d^2*a^3/b^4*f*c*ln(b*exp(2*d*x+2*c)+
```

```
2*a*exp(d*x+c)-b)-1/d*a^3/b^4*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a
^2+b^2)^(1/2)))*x+1/d^2*a^3/b^4*f*c^2+1/2*a^5/b^6*f*x^2-a^5/b^6*e*x-1/d^2*a
^3/b^4*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d*a
^3/b^4*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/256*a
*(4*d*f*x+4*d*e-f)/b^2/d^2*exp(4*d*x+4*c)-1/d^2*a^3/b^4*f*ln((b*exp(d*x+c)+
(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d^2*a^3/b^4*f*dilog((b*exp(d*x+
c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2*a^3/b^4*f*dilog((-b*exp(d*
x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-a^3*e*x/b^4+1/800*(5*d*f*x+5*
d*e-f)/d^2/b*exp(5*d*x+5*c)+1/288*(12*a^2*d*f*x+3*b^2*d*f*x+12*a^2*d*e+3*b^
2*d*e-4*a^2*f-b^2*f)/b^3/d^2*exp(3*d*x+3*c)-1/288*(4*a^2+b^2)*(3*d*f*x+3*d*
e+f)/b^3/d^2*exp(-3*d*x-3*c)-1/256*a*(4*d*f*x+4*d*e+f)/b^2/d^2*exp(-4*d*x-4
*c)-1/d^2*a^5/b^6*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2
)))*c-2/d^2*a^5/b^6*f*c*ln(exp(d*x+c))-1/32*a*(4*a^2*d*f*x+2*b^2*d*f*x+4*a^
2*d*e+2*b^2*d*e-2*a^2*f-b^2*f)/b^4/d^2*exp(2*d*x+2*c)-1/16*(8*a^4+6*a^2*b^2
-b^4)*(d*f*x+d*e+f)/b^5/d^2*exp(-d*x-c)+1/d^2*a^5/b^6*f*c^2-1/d*a^5/b^6*e*l
n(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d*a^5/b^6*e*ln(exp(d*x+c))-1/d^2*a^5
/b^6*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2*a^
5/b^6*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/d^2
*a^5/b^6*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d*a^5/b^6*f*ln((b*exp(
d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*a^5/b^6*f*ln((b*exp(
d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d*a^5/b^6*f*ln((-b*exp(d
*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/32*a*(2*a^2+b^2)*(2*d*f*
x+2*d*e+f)/b^4/d^2*exp(-2*d*x-2*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -1/57600*f*((28800*(a^5*d^2*e^(5*c) + a^3*b^2*d^2*e^(5*c))*x^2 - 72*(5*b^5*
d*x*e^(10*c) - b^5*e^(10*c))*e^(5*d*x) + 225*(4*a*b^4*d*x*e^(9*c) - a*b^4*e
^(9*c))*e^(4*d*x) + 200*(4*a^2*b^3*e^(8*c) + b^5*e^(8*c) - 3*(4*a^2*b^3*d*e
^(8*c) + b^5*d*e^(8*c))*x)*e^(3*d*x) - 1800*(2*a^3*b^2*e^(7*c) + a*b^4*e^(7
*c) - 2*(2*a^3*b^2*d*e^(7*c) + a*b^4*d*e^(7*c))*x)*e^(2*d*x) + 3600*(8*a^4*
b*e^(6*c) + 6*a^2*b^3*e^(6*c) - b^5*e^(6*c) - (8*a^4*b*d*e^(6*c) + 6*a^2*b^
3*d*e^(6*c) - b^5*d*e^(6*c))*x)*e^(d*x) + 3600*(8*a^4*b*e^(4*c) + 6*a^2*b^3
*e^(4*c) - b^5*e^(4*c) + (8*a^4*b*d*e^(4*c) + 6*a^2*b^3*d*e^(4*c) - b^5*d*e
^(4*c))*x)*e^(-d*x) + 1800*(2*a^3*b^2*e^(3*c) + a*b^4*e^(3*c) + 2*(2*a^3*b^
2*d*e^(3*c) + a*b^4*d*e^(3*c))*x)*e^(-2*d*x) + 200*(4*a^2*b^3*e^(2*c) + b^5
*e^(2*c) + 3*(4*a^2*b^3*d*e^(2*c) + b^5*d*e^(2*c))*x)*e^(-3*d*x) + 225*(4*a
*b^4*d*x*e^c + a*b^4*e^c)*e^(-4*d*x) + 72*(5*b^5*d*x + b^5)*e^(-5*d*x))*e^(
```

```
-5*c)/(b^6*d^2) - 900*integrate(128*((a^6*e^c + a^4*b^2*e^c)*x*e^(d*x) - (a
^5*b + a^3*b^3)*x)/(b^7*e^(2*d*x + 2*c) + 2*a*b^6*e^(d*x + c) - b^7), x)) -
 1/960*((15*a*b^3*e^(-d*x - c) - 6*b^4 - 10*(4*a^2*b^2 + b^4)*e^(-2*d*x - 2
*c) + 60*(2*a^3*b + a*b^3)*e^(-3*d*x - 3*c) - 60*(8*a^4 + 6*a^2*b^2 - b^4)*
e^(-4*d*x - 4*c))*e^(5*d*x + 5*c)/(b^5*d) + 960*(a^5 + a^3*b^2)*(d*x + c)/(
b^6*d) + (15*a*b^3*e^(-4*d*x - 4*c) + 6*b^4*e^(-5*d*x - 5*c) + 60*(8*a^4 +
6*a^2*b^2 - b^4)*e^(-d*x - c) + 60*(2*a^3*b + a*b^3)*e^(-2*d*x - 2*c) + 10*
(4*a^2*b^2 + b^4)*e^(-3*d*x - 3*c))/(b^5*d) + 960*(a^5 + a^3*b^2)*log(-2*a*
e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^6*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6966 vs. $2(602) = 1204$.
time = 0.44, size = 6966, normalized size = 10.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/57600*(72*(5*b^5*d*f*x + 5*b^5*d*cosh(1) + 5*b^5*d*sinh(1) - b^5*f)*cosh(
d*x + c)^10 + 72*(5*b^5*d*f*x + 5*b^5*d*cosh(1) + 5*b^5*d*sinh(1) - b^5*f)*
sinh(d*x + c)^10 - 225*(4*a*b^4*d*f*x + 4*a*b^4*d*cosh(1) + 4*a*b^4*d*sinh(
1) - a*b^4*f)*cosh(d*x + c)^9 - 45*(20*a*b^4*d*f*x + 20*a*b^4*d*cosh(1) + 2
0*a*b^4*d*sinh(1) - 5*a*b^4*f - 16*(5*b^5*d*f*x + 5*b^5*d*cosh(1) + 5*b^5*d
*sinh(1) - b^5*f)*cosh(d*x + c))*sinh(d*x + c)^9 + 200*(3*(4*a^2*b^3 + b^5)
*d*f*x + 3*(4*a^2*b^3 + b^5)*d*cosh(1) + 3*(4*a^2*b^3 + b^5)*d*sinh(1) - (4
*a^2*b^3 + b^5)*f)*cosh(d*x + c)^8 + 5*(120*(4*a^2*b^3 + b^5)*d*f*x + 120*(
4*a^2*b^3 + b^5)*d*cosh(1) + 648*(5*b^5*d*f*x + 5*b^5*d*cosh(1) + 5*b^5*d*s
inh(1) - b^5*f)*cosh(d*x + c)^2 + 120*(4*a^2*b^3 + b^5)*d*sinh(1) - 40*(4*a
^2*b^3 + b^5)*f - 405*(4*a*b^4*d*f*x + 4*a*b^4*d*cosh(1) + 4*a*b^4*d*sinh(1
) - a*b^4*f)*cosh(d*x + c))*sinh(d*x + c)^8 - 360*b^5*d*f*x - 1800*(2*(2*a^
3*b^2 + a*b^4)*d*f*x + 2*(2*a^3*b^2 + a*b^4)*d*cosh(1) + 2*(2*a^3*b^2 + a*b
^4)*d*sinh(1) - (2*a^3*b^2 + a*b^4)*f)*cosh(d*x + c)^7 - 20*(180*(2*a^3*b^2
 + a*b^4)*d*f*x - 432*(5*b^5*d*f*x + 5*b^5*d*cosh(1) + 5*b^5*d*sinh(1) - b^
5*f)*cosh(d*x + c)^3 + 180*(2*a^3*b^2 + a*b^4)*d*cosh(1) + 405*(4*a*b^4*d*f
*x + 4*a*b^4*d*cosh(1) + 4*a*b^4*d*sinh(1) - a*b^4*f)*cosh(d*x + c)^2 + 180
*(2*a^3*b^2 + a*b^4)*d*sinh(1) - 90*(2*a^3*b^2 + a*b^4)*f - 80*(3*(4*a^2*b^
3 + b^5)*d*f*x + 3*(4*a^2*b^3 + b^5)*d*cosh(1) + 3*(4*a^2*b^3 + b^5)*d*sinh
(1) - (4*a^2*b^3 + b^5)*f)*cosh(d*x + c))*sinh(d*x + c)^7 - 360*b^5*d*cosh(
1) + 3600*((8*a^4*b + 6*a^2*b^3 - b^5)*d*f*x + (8*a^4*b + 6*a^2*b^3 - b^5)*
d*cosh(1) + (8*a^4*b + 6*a^2*b^3 - b^5)*d*sinh(1) - (8*a^4*b + 6*a^2*b^3 -
b^5)*f)*cosh(d*x + c)^6 - 360*b^5*d*sinh(1) + 20*(756*(5*b^5*d*f*x + 5*b^5*
d*cosh(1) + 5*b^5*d*sinh(1) - b^5*f)*cosh(d*x + c)^4 + 180*(8*a^4*b + 6*a^2
*b^3 - b^5)*d*f*x - 945*(4*a*b^4*d*f*x + 4*a*b^4*d*cosh(1) + 4*a*b^4*d*sinh
```

```
(1) - a*b^4*f)*cosh(d*x + c)^3 + 180*(8*a^4*b + 6*a^2*b^3 - b^5)*d*cosh(1)
+ 280*(3*(4*a^2*b^3 + b^5)*d*f*x + 3*(4*a^2*b^3 + b^5)*d*cosh(1) + 3*(4*a^2
*b^3 + b^5)*d*sinh(1) - (4*a^2*b^3 + b^5)*f)*cosh(d*x + c)^2 + 180*(8*a^4*b
 + 6*a^2*b^3 - b^5)*d*sinh(1) - 180*(8*a^4*b + 6*a^2*b^3 - b^5)*f - 630*(2*
(2*a^3*b^2 + a*b^4)*d*f*x + 2*(2*a^3*b^2 + a*b^4)*d*cosh(1) + 2*(2*a^3*b^2
+ a*b^4)*d*sinh(1) - (2*a^3*b^2 + a*b^4)*f)*cosh(d*x + c))*sinh(d*x + c)^6
- 72*b^5*f + 28800*((a^5 + a^3*b^2)*d^2*f*x^2 - 2*(a^5 + a^3*b^2)*c^2*f + 2
*((a^5 + a^3*b^2)*d^2*x + 2*(a^5 + a^3*b^2)*c*d)*cosh(1) + 2*((a^5 + a^3*b^
2)*d^2*x + 2*(a^5 + a^3*b^2)*c*d)*sinh(1))*cosh(d*x + c)^5 + 2*(14400*(a^5
+ a^3*b^2)*d^2*f*x^2 + 9072*(5*b^5*d*f*x + 5*b^5*d*cosh(1) + 5*b^5*d*sinh(1
) - b^5*f)*cosh(d*x + c)^5 - 14175*(4*a*b^4*d*f*x + 4*a*b^4*d*cosh(1) + 4*a
*b^4*d*sinh(1) - a*b^4*f)*cosh(d*x + c)^4 - 28800*(a^5 + a^3*b^2)*c^2*f + 5
600*(3*(4*a^2*b^3 + b^5)*d*f*x + 3*(4*a^2*b^3 + b^5)*d*cosh(1) + 3*(4*a^2*b
^3 + b^5)*d*sinh(1) - (4*a^2*b^3 + b^5)*f)*cosh(d*x + c)^3 - 18900*(2*(2*a^
3*b^2 + a*b^4)*d*f*x + 2*(2*a^3*b^2 + a*b^4)*d*cosh(1) + 2*(2*a^3*b^2 + a*b
^4)*d*sinh(1) - (2*a^3*b^2 + a*b^4)*f)*cosh(d*x + c)^2 + 28800*((a^5 + a^3*
b^2)*d^2*x + 2*(a^5 + a^3*b^2)*c*d)*cosh(1) + 10800*((8*a^4*b + 6*a^2*b^3 -
 b^5)*d*f*x + (8*a^4*b + 6*a^2*b^3 - b^5)*d*cosh(1) + (8*a^4*b + 6*a^2*b^3
- b^5)*d*sinh(1) - (8*a^4*b + 6*a^2*b^3 - b^5)*f)*cosh(d*x + c) + 28800*((a
^5 + a^3*b^2)*d^2*x + 2*(a^5 + a^3*b^2)*c*d)*sinh(1))*sinh(d*x + c)^5 - 360
0*((8*a^4*b + 6*a^2*b^3 - b^5)*d*f*x + (8*a^4*b + 6*a^2*b^3 - b^5)*d*cosh(1
) + (8*a^4*b + 6*a^2*b^3 - b^5)*d*sinh(1) + (8*a^4*b + 6*a^2*b^3 - b^5)*f)*
cosh(d*x + c)^4 + 10*(1512*(5*b^5*d*f*x + 5*b^5*d*cosh(1) + 5*b^5*d*sinh(1)
 - b^5*f)*cosh(d*x + c)^6 - 2835*(4*a*b^4*d*f*x + 4*a*b^4*d*cosh(1) + 4*a*b
^4*d*sinh(1) - a*b^4*f)*cosh(d*x + c)^5 + 1400*(3*(4*a^2*b^3 + b^5)*d*f*x +
 3*(4*a^2*b^3 + b^5)*d*cosh(1) + 3*(4*a^2*b^3 + b^5)*d*sinh(1) - (4*a^2*b^3
 + b^5)*f)*cosh(d*x + c)^4 - 360*(8*a^4*b + 6*a^2*b^3 - b^5)*d*f*x - 6300*(
2*(2*a^3*b^2 + a*b^4)*d*f*x + 2*(2*a^3*b^2 + a*b^4)*d*cosh(1) + 2*(2*a^3*b^
2 + a*b^4)*d*sinh(1) - (2*a^3*b^2 + a*b^4)*f)*cosh(d*x + c)^3 - 360*(8*a^4*
b + 6*a^2*b^3 - b^5)*d*cosh(1) + 5400*((8*a^4*b + 6*a^2*b^3 - b^5)*d*f*x +
(8*a^4*b + 6*a^2*b^3 - b^5)*d*cosh(1) + (8*a^4*b + 6*a^2*b^3 - b^5)*d*sinh(
1) - (8*a^4*b + 6*a^2*b^3 - b^5)*f)*cosh(d*x + c)^2 - 360*(8*a^4*b + 6*a^2*
b^3 - b^5)*d*sinh(1) - 360*(8*a^4*b + 6*a^2*b^3 - b^5)*f + 14400*((a^5 + a^
3*b^2)*d^2*f*x^2 - 2*(a^5 + a^3*b^2)*c^2*f + 2*((a^5 + a^3*b^2)*d^2*x + 2*(
a^5 + a^3*b^2)*c*d)*cosh(1) + 2*((a^5 + a^3*b^2)*d^2*x + 2*(a^5 + a^3*b^2)*
c*d)*sinh(1))*cosh(d*x + c))*sinh(d*x + c)^4 - 1800*(2*(2*a^3*b^2 + a*b^4)*
d*f*x + 2*(2*a^3*b^2 + a*b^4)*d*cosh(1) + 2*(2*a^3*b^2 + a*b^4)*d*sinh(1) +
 (2*a^3*b^2 + a*b^4)*f)*cosh(d*x + c)^3 + 20*(432*(5*b^5*d*f*x + 5*b^5*d*co
sh(1) + 5*b^5*d*sinh(1) - b^5*f)*cosh(d*x + c)^7 - 945*(4*a*b^4*d*f*x + 4*a
*b^4*d*cosh(1) + 4*a*b^4*d*sinh(1) - a*b^4*f)*cosh(d*x + c)^6 + 560*(3*(4*a
^2*b^3 + b^5)*d*f*x + 3*(4*a^2*b^3 + b^5)*d*cosh(1) + 3*(4*a^2*b^3 + b^5)*d
*sinh(1) - (4*a^2*b^3 + b^5)*f)*cosh(d*x + c)^5...
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**3*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)^3*sinh(d*x + c)^3/(b*sinh(d*x + c) + a),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^3\,\sinh(c+d\,x)^3\,(e+f\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.404 $\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=141

$$-\frac{a^3(a^2+b^2)\log(a+b\sinh(c+dx))}{b^6d}+\frac{a^2(a^2+b^2)\sinh(c+dx)}{b^5d}-\frac{a(a^2+b^2)\sinh^2(c+dx)}{2b^4d}+\frac{(a^2+b^2)\sinh^3(c+dx)}{3b^3d}$$

```
[Out] -a^3*(a^2+b^2)*ln(a+b*sinh(d*x+c))/b^6/d+a^2*(a^2+b^2)*sinh(d*x+c)/b^5/d-1/
2*a*(a^2+b^2)*sinh(d*x+c)^2/b^4/d+1/3*(a^2+b^2)*sinh(d*x+c)^3/b^3/d-1/4*a*s
inh(d*x+c)^4/b^2/d+1/5*sinh(d*x+c)^5/b/d
```

Rubi [A]
time = 0.15, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 908}

$$\frac{a^2(a^2+b^2)\sinh(c+dx)}{b^5d}-\frac{a(a^2+b^2)\sinh^2(c+dx)}{2b^4d}+\frac{(a^2+b^2)\sinh^3(c+dx)}{3b^3d}-\frac{a^3(a^2+b^2)\log(a+b\sinh(c+dx))}{b^6d}-\frac{a\sinh^4(c+dx)}{4b^2d}+\frac{\sinh^5(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a^3*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/(b^6*d)) + (a^2*(a^2 + b^2)*Si
nh[c + d*x])/(b^5*d) - (a*(a^2 + b^2)*Sinh[c + d*x]^2)/(2*b^4*d) + ((a^2 +
b^2)*Sinh[c + d*x]^3)/(3*b^3*d) - (a*Sinh[c + d*x]^4)/(4*b^2*d) + Sinh[c +
d*x]^5/(5*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^3(-b^2-x^2)}{b^3(a+x)}\,dx, x, b\sinh(c+dx)\right)}{b^3 d}$$
$$= -\frac{\text{Subst}\left(\int \frac{x^3(-b^2-x^2)}{a+x}\,dx, x, b\sinh(c+dx)\right)}{b^6 d}$$
$$= -\frac{\text{Subst}\left(\int \left(-a^2(a^2+b^2)+a(a^2+b^2)\,x-(a^2+b^2)\,x^2+ax^3-x^4+\right.\right.}{b^6 d}$$
$$= -\frac{a^3(a^2+b^2)\log(a+b\sinh(c+dx))}{b^6 d}+\frac{a^2(a^2+b^2)\sinh(c+dx)}{b^5 d}-\frac{a(a}{}$$

Mathematica [A]
time = 0.30, size = 123, normalized size = 0.87

$$-\frac{\frac{60a^3(a^2+b^2)\log(a+b\sinh(c+dx))}{b^6}-\frac{60a^2(a^2+b^2)\sinh(c+dx)}{b^5}+\frac{30a(a^2+b^2)\sinh^2(c+dx)}{b^4}-\frac{20(a^2+b^2)\sinh^3(c+dx)}{b^3}+\frac{15a\sinh^4(c+dx)}{b^2}-\frac{12\sinh^5(c+dx)}{b}}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/60*((60*a^3*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/b^6 - (60*a^2*(a^2 + b
^2)*Sinh[c + d*x])/b^5 + (30*a*(a^2 + b^2)*Sinh[c + d*x]^2)/b^4 - (20*(a^2
+ b^2)*Sinh[c + d*x]^3)/b^3 + (15*a*Sinh[c + d*x]^4)/b^2 - (12*Sinh[c + d*x
]^5)/b)/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(133) = 266.
time = 1.16, size = 421, normalized size = 2.99

method	result
derivativedivides	$-\frac{1}{5b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}-\frac{2b+a}{4b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{4a^2+6ab+7b^2}{12b^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{4a^3+4a^2b+5a\,b^2+3b^3}{8b^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{a\left(8a^3+4a^2b+8a\,b^2+3\right.}{8b^5\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right.}$
default	$-\frac{1}{5b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}-\frac{2b+a}{4b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{4a^2+6ab+7b^2}{12b^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{4a^3+4a^2b+5a\,b^2+3b^3}{8b^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{a\left(8a^3+4a^2b+8a\,b^2+3\right.}{8b^5\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right.}$
risch	$\frac{e^{3dx+3c}}{96bd}-\frac{a\,e^{-2dx-2c}}{16b^2d}-\frac{a\,e^{4dx+4c}}{64b^2d}+\frac{a^5x}{b^6}-\frac{e^{-5dx-5c}}{160bd}+\frac{e^{5dx+5c}}{160bd}-\frac{a\,e^{2dx+2c}}{16b^2d}-\frac{e^{-3dx-3c}}{96bd}-\frac{e^{dx+c}}{16bd}+\frac{a}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5/b/(tanh(1/2*d*x+1/2*c)-1)^5-1/4*(2*b+a)/b^2/(tanh(1/2*d*x+1/2*c)-
1)^4-1/12*(4*a^2+6*a*b+7*b^2)/b^3/(tanh(1/2*d*x+1/2*c)-1)^3-1/8*(4*a^3+4*a^
2*b+5*a*b^2+3*b^3)/b^4/(tanh(1/2*d*x+1/2*c)-1)^2-1/8*a*(8*a^3+4*a^2*b+8*a*b
^2+3*b^3)/b^5/(tanh(1/2*d*x+1/2*c)-1)+a^3*(a^2+b^2)/b^6*ln(tanh(1/2*d*x+1/2
*c)-1)-1/5/b/(tanh(1/2*d*x+1/2*c)+1)^5-1/4*(-2*b+a)/b^2/(tanh(1/2*d*x+1/2*c
)+1)^4-1/12*(4*a^2-6*a*b+7*b^2)/b^3/(tanh(1/2*d*x+1/2*c)+1)^3-1/8*(4*a^3-4*
a^2*b+5*a*b^2-3*b^3)/b^4/(tanh(1/2*d*x+1/2*c)+1)^2-1/8*a*(8*a^3-4*a^2*b+8*a
*b^2-3*b^3)/b^5/(tanh(1/2*d*x+1/2*c)+1)+a^3*(a^2+b^2)/b^6*ln(tanh(1/2*d*x+1
/2*c)+1)-2*a^3/b^6*(1/2*a^2+1/2*b^2)*ln(a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/
2*d*x+1/2*c)-a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(133) = 266$.
time = 0.28, size = 300, normalized size = 2.13

$$-\frac{\left(15\,ab^3e^{(-dx-c)} - 6\,b^4 - 10\,(4\,a^2b^2 + b^4)e^{(-2\,dx-2\,c)} + 60\,(2\,a^3b + ab^3)e^{(-3\,dx-3\,c)} - 60\,(8\,a^4 + 6\,a^2b^2 - b^4)e^{(-4\,dx-4\,c)}\right)e^{(5\,dx+5\,c)}}{960\,b^5d} - \frac{(a^5 + a^3b^2)(dx + c)}{b^6d} - \frac{15\,ab^3e^{(-4\,dx-4\,c)} + 6\,b^4e^{(-5\,dx-5\,c)} + 60\,(8\,a^4 + 6\,a^2b^2 - b^4)e^{(-dx-c)} + 60\,(2\,a^3b + ab^3)e^{(-2\,dx-2\,c)} + 10\,(4\,a^2b^2 + b^4)e^{(-3\,dx-3\,c)}}{960\,b^5d} - \frac{(a^5 + a^3b^2)\log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{b^6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxim
a")
```

```
[Out] -1/960*(15*a*b^3*e^(-d*x - c) - 6*b^4 - 10*(4*a^2*b^2 + b^4)*e^(-2*d*x - 2*
c) + 60*(2*a^3*b + a*b^3)*e^(-3*d*x - 3*c) - 60*(8*a^4 + 6*a^2*b^2 - b^4)*e
^(-4*d*x - 4*c))*e^(5*d*x + 5*c)/(b^5*d) - (a^5 + a^3*b^2)*(d*x + c)/(b^6*d
) - 1/960*(15*a*b^3*e^(-4*d*x - 4*c) + 6*b^4*e^(-5*d*x - 5*c) + 60*(8*a^4 +
 6*a^2*b^2 - b^4)*e^(-d*x - c) + 60*(2*a^3*b + a*b^3)*e^(-2*d*x - 2*c) + 10
*(4*a^2*b^2 + b^4)*e^(-3*d*x - 3*c))/(b^5*d) - (a^5 + a^3*b^2)*log(-2*a*e^(
-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^6*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1660 vs. $2(133) = 266$.
time = 0.43, size = 1660, normalized size = 11.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="frica
s")
```

```
[Out] 1/960*(6*b^5*cosh(d*x + c)^10 + 6*b^5*sinh(d*x + c)^10 - 15*a*b^4*cosh(d*x
+ c)^9 + 15*(4*b^5*cosh(d*x + c) - a*b^4)*sinh(d*x + c)^9 + 10*(4*a^2*b^3 +
 b^5)*cosh(d*x + c)^8 + 5*(54*b^5*cosh(d*x + c)^2 - 27*a*b^4*cosh(d*x + c)
+ 8*a^2*b^3 + 2*b^5)*sinh(d*x + c)^8 + 960*(a^5 + a^3*b^2)*d*x*cosh(d*x + c
)^5 - 60*(2*a^3*b^2 + a*b^4)*cosh(d*x + c)^7 + 20*(36*b^5*cosh(d*x + c)^3 -
 27*a*b^4*cosh(d*x + c)^2 - 6*a^3*b^2 - 3*a*b^4 + 4*(4*a^2*b^3 + b^5)*cosh(
d*x + c))*sinh(d*x + c)^7 + 60*(8*a^4*b + 6*a^2*b^3 - b^5)*cosh(d*x + c)^6
```

```
+ 20*(63*b^5*cosh(d*x + c)^4 - 63*a*b^4*cosh(d*x + c)^3 + 24*a^4*b + 18*a^2
*b^3 - 3*b^5 + 14*(4*a^2*b^3 + b^5)*cosh(d*x + c)^2 - 21*(2*a^3*b^2 + a*b^4
)*cosh(d*x + c))*sinh(d*x + c)^6 - 15*a*b^4*cosh(d*x + c) + 2*(756*b^5*cosh
(d*x + c)^5 - 945*a*b^4*cosh(d*x + c)^4 + 280*(4*a^2*b^3 + b^5)*cosh(d*x +
c)^3 + 480*(a^5 + a^3*b^2)*d*x - 630*(2*a^3*b^2 + a*b^4)*cosh(d*x + c)^2 +
180*(8*a^4*b + 6*a^2*b^3 - b^5)*cosh(d*x + c))*sinh(d*x + c)^5 - 6*b^5 - 60
*(8*a^4*b + 6*a^2*b^3 - b^5)*cosh(d*x + c)^4 + 10*(126*b^5*cosh(d*x + c)^6
- 189*a*b^4*cosh(d*x + c)^5 - 48*a^4*b - 36*a^2*b^3 + 6*b^5 + 70*(4*a^2*b^3
 + b^5)*cosh(d*x + c)^4 + 480*(a^5 + a^3*b^2)*d*x*cosh(d*x + c) - 210*(2*a^
3*b^2 + a*b^4)*cosh(d*x + c)^3 + 90*(8*a^4*b + 6*a^2*b^3 - b^5)*cosh(d*x +
c)^2)*sinh(d*x + c)^4 - 60*(2*a^3*b^2 + a*b^4)*cosh(d*x + c)^3 + 20*(36*b^5
*cosh(d*x + c)^7 - 63*a*b^4*cosh(d*x + c)^6 + 28*(4*a^2*b^3 + b^5)*cosh(d*x
 + c)^5 - 6*a^3*b^2 - 3*a*b^4 + 480*(a^5 + a^3*b^2)*d*x*cosh(d*x + c)^2 - 1
05*(2*a^3*b^2 + a*b^4)*cosh(d*x + c)^4 + 60*(8*a^4*b + 6*a^2*b^3 - b^5)*cos
h(d*x + c)^3 - 12*(8*a^4*b + 6*a^2*b^3 - b^5)*cosh(d*x + c))*sinh(d*x + c)^
3 - 10*(4*a^2*b^3 + b^5)*cosh(d*x + c)^2 + 10*(27*b^5*cosh(d*x + c)^8 - 54*
a*b^4*cosh(d*x + c)^7 + 28*(4*a^2*b^3 + b^5)*cosh(d*x + c)^6 + 960*(a^5 + a
^3*b^2)*d*x*cosh(d*x + c)^3 - 126*(2*a^3*b^2 + a*b^4)*cosh(d*x + c)^5 - 4*a
^2*b^3 - b^5 + 90*(8*a^4*b + 6*a^2*b^3 - b^5)*cosh(d*x + c)^4 - 36*(8*a^4*b
 + 6*a^2*b^3 - b^5)*cosh(d*x + c)^2 - 18*(2*a^3*b^2 + a*b^4)*cosh(d*x + c))
*sinh(d*x + c)^2 - 960*((a^5 + a^3*b^2)*cosh(d*x + c)^5 + 5*(a^5 + a^3*b^2)
*cosh(d*x + c)^4*sinh(d*x + c) + 10*(a^5 + a^3*b^2)*cosh(d*x + c)^3*sinh(d*
x + c)^2 + 10*(a^5 + a^3*b^2)*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*(a^5 + a^
3*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 + (a^5 + a^3*b^2)*sinh(d*x + c)^5)*log
(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 5*(12*b^5*cosh(
d*x + c)^9 - 27*a*b^4*cosh(d*x + c)^8 + 16*(4*a^2*b^3 + b^5)*cosh(d*x + c)^
7 + 960*(a^5 + a^3*b^2)*d*x*cosh(d*x + c)^4 - 84*(2*a^3*b^2 + a*b^4)*cosh(d
*x + c)^6 + 72*(8*a^4*b + 6*a^2*b^3 - b^5)*cosh(d*x + c)^5 - 3*a*b^4 - 48*(
8*a^4*b + 6*a^2*b^3 - b^5)*cosh(d*x + c)^3 - 36*(2*a^3*b^2 + a*b^4)*cosh(d*
x + c)^2 - 4*(4*a^2*b^3 + b^5)*cosh(d*x + c))*sinh(d*x + c))/(b^6*d*cosh(d*
x + c)^5 + 5*b^6*d*cosh(d*x + c)^4*sinh(d*x + c) + 10*b^6*d*cosh(d*x + c)^3
*sinh(d*x + c)^2 + 10*b^6*d*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*b^6*d*cosh(
d*x + c)*sinh(d*x + c)^4 + b^6*d*sinh(d*x + c)^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]

time = 0.45, size = 258, normalized size = 1.83

$$\frac{\frac{6\,b^4\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5-15\,ab^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^4+40\,a^2b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+40\,b^4\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3-120\,a^3b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-120\,ab^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+480\,a^4\left(e^{(dx+c)}-e^{(-dx-c)}\right)+480\,a^2b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{b^5}-\frac{960\,(a^5+a^3b^2)\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{b^6}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac"
)
```

```
[Out] 1/960*((6*b^4*(e^(d*x + c) - e^(-d*x - c))^5 - 15*a*b^3*(e^(d*x + c) - e^(-
d*x - c))^4 + 40*a^2*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 40*b^4*(e^(d*x +
c) - e^(-d*x - c))^3 - 120*a^3*b*(e^(d*x + c) - e^(-d*x - c))^2 - 120*a*b^3
*(e^(d*x + c) - e^(-d*x - c))^2 + 480*a^4*(e^(d*x + c) - e^(-d*x - c)) + 48
0*a^2*b^2*(e^(d*x + c) - e^(-d*x - c)))/b^5 - 960*(a^5 + a^3*b^2)*log(abs(b
*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/b^6)/d
```

Mupad [B]
time = 0.75, size = 307, normalized size = 2.18

$$\frac{e^{5\,c+5\,d\,x}}{160\,b\,d}-\frac{e^{-5\,c-5\,d\,x}}{160\,b\,d}-\frac{a\,e^{-4\,c-4\,d\,x}}{64\,b^2\,d}-\frac{a\,e^{4\,c+4\,d\,x}}{64\,b^2\,d}-\frac{\ln\left(2\,a\,e^{d\,x}\,e^{c}-b+b\,e^{2\,c}\,e^{2\,d\,x}\right)\,\left(a^5+a^3\,b^2\right)}{b^6\,d}-\frac{e^{-c-d\,x}\,\left(8\,a^4+6\,a^2\,b^2-b^4\right)}{16\,b^5\,d}+\frac{a^3\,x\,\left(a^2+b^2\right)}{b^6}-\frac{e^{-2\,c-2\,d\,x}\,\left(2\,a^3+a\,b^2\right)}{16\,b^4\,d}-\frac{e^{2\,c+2\,d\,x}\,\left(2\,a^3+a\,b^2\right)}{16\,b^4\,d}+\frac{e^{c+d\,x}\,\left(8\,a^4+6\,a^2\,b^2-b^4\right)}{16\,b^5\,d}-\frac{e^{-3\,c-3\,d\,x}\,\left(4\,a^2+b^2\right)}{96\,b^3\,d}+\frac{e^{3\,c+3\,d\,x}\,\left(4\,a^2+b^2\right)}{96\,b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^3*sinh(c + d*x)^3)/(a + b*sinh(c + d*x)),x)
```

```
[Out] exp(5*c + 5*d*x)/(160*b*d) - exp(- 5*c - 5*d*x)/(160*b*d) - (a*exp(- 4*c -
4*d*x))/(64*b^2*d) - (a*exp(4*c + 4*d*x))/(64*b^2*d) - (log(2*a*exp(d*x)*ex
p(c) - b + b*exp(2*c)*exp(2*d*x))*(a^5 + a^3*b^2))/(b^6*d) - (exp(- c - d*x
)*(8*a^4 - b^4 + 6*a^2*b^2))/(16*b^5*d) + (a^3*x*(a^2 + b^2))/b^6 - (exp(-
2*c - 2*d*x)*(a*b^2 + 2*a^3))/(16*b^4*d) - (exp(2*c + 2*d*x)*(a*b^2 + 2*a^3
))/(16*b^4*d) + (exp(c + d*x)*(8*a^4 - b^4 + 6*a^2*b^2))/(16*b^5*d) - (exp(
- 3*c - 3*d*x)*(4*a^2 + b^2))/(96*b^3*d) + (exp(3*c + 3*d*x)*(4*a^2 + b^2))
/(96*b^3*d)
```

3.405 $\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=39

$$\operatorname{Int}\left(\frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

[Out] Unintegrable(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

[In] Int[(Cosh[c + d*x]^3*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]

[Out] Defer[Int][(Cosh[c + d*x]^3*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])), x]

Rubi steps

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

[In] Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]

[Out] $Aborted

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\cosh^3(dx+c)\right)\left(\sinh^3(dx+c)\right)}{(fx+e)\,(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -1/32*e^(-5*c + 5*d*e/f)*exp_integral_e(1, 5*(f*x + e)*d/f)/(b*f) - 1/16*a*
e^(-4*c + 4*d*e/f)*exp_integral_e(1, 4*(f*x + e)*d/f)/(b^2*f) + 1/16*a*e^(4
*c - 4*d*e/f)*exp_integral_e(1, -4*(f*x + e)*d/f)/(b^2*f) - 1/32*e^(5*c - 5
*d*e/f)*exp_integral_e(1, -5*(f*x + e)*d/f)/(b*f) - 1/32*(4*a^2 + b^2)*e^(-
3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b^3*f) - 1/32*(4*a^2*e^(
3*c) + b^2*e^(3*c))*e^(-3*d*e/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/(b^3*f
) - 1/8*(2*a^3 + a*b^2)*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/
f)/(b^4*f) + 1/8*(2*a^3*e^(2*c) + a*b^2*e^(2*c))*e^(-2*d*e/f)*exp_integral_
e(1, -2*(f*x + e)*d/f)/(b^4*f) - 1/16*(8*a^4 + 6*a^2*b^2 - b^4)*e^(-c + d*e
/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^5*f) - 1/16*(8*a^4*e^c + 6*a^2*b^2*
e^c - b^4*e^c)*e^(-d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^5*f) - (a^5
+ a^3*b^2)*log(f*x + e)/(b^6*f) + 1/64*integrate(128*(a^5*b + a^3*b^3 - (a^
6*e^c + a^4*b^2*e^c)*e^(d*x))/(b^7*f*x + b^7*e - (b^7*f*x*e^(2*c) + b^7*e^(
2*c + 1))*e^(2*d*x) - 2*(a*b^6*f*x*e^c + a*b^6*e^(c + 1))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] integral(cosh(d*x + c)^3*sinh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(
d*x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*sinh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate(cosh(d*x + c)^3*sinh(d*x + c)^3/((f*x + e)*(b*sinh(d*x + c) + a))
, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh(c+d\,x)^3\,\sinh(c+d\,x)^3}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^3*sinh(c + d*x)^3)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((cosh(c + d*x)^3*sinh(c + d*x)^3)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.406 $\int \frac{(e+fx)^3 \sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1519

$$\frac{a(e+fx)^4}{4b^2f}+\frac{2a^2(e+fx)^3\text{ArcTan}\left(e^{c+dx}\right)}{b^3d}-\frac{2(e+fx)^3\text{ArcTan}\left(e^{c+dx}\right)}{bd}-\frac{2a^4(e+fx)^3\text{ArcTan}\left(e^{c+dx}\right)}{b^3\left(a^2+b^2\right)d}-\frac{6f^3\cos}{}$$

```
[Out] -3*a^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^
2)/d^2-3*a^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(
a^2+b^2)/d^2+6*a^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))
/b^2/(a^2+b^2)/d^3+6*a^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))/b^2/(a^2+b^2)/d^3+1/4*a*(f*x+e)^4/b^2/f-3*I*f*(f*x+e)^2*polylog(2,I*
exp(d*x+c))/b/d^2-6*I*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/b/d^3-6*I*a^2*f^
3*polylog(4,-I*exp(d*x+c))/b^3/d^4+3*I*a^4*f*(f*x+e)^2*polylog(2,-I*exp(d*x
+c))/b^3/(a^2+b^2)/d^2+6*I*a^4*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/b^3/(a^2
+b^2)/d^3+3*I*f*(f*x+e)^2*polylog(2,-I*exp(d*x+c))/b/d^2+3/2*a^3*f*(f*x+e)^
2*polylog(2,-exp(2*d*x+2*c))/b^2/(a^2+b^2)/d^2+6*I*f^2*(f*x+e)*polylog(3,I*
exp(d*x+c))/b/d^3-3/2*a^3*f^2*(f*x+e)*polylog(3,-exp(2*d*x+2*c))/b^2/(a^2+b
^2)/d^3+6*I*a^2*f^3*polylog(4,I*exp(d*x+c))/b^3/d^4-3*I*a^2*f*(f*x+e)^2*pol
ylog(2,-I*exp(d*x+c))/b^3/d^2-6*I*a^2*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/b
^3/d^3-6*I*a^4*f^3*polylog(4,I*exp(d*x+c))/b^3/(a^2+b^2)/d^4+(f*x+e)^3*sinh
(d*x+c)/b/d-2*(f*x+e)^3*arctan(exp(d*x+c))/b/d+6*I*a^4*f^3*polylog(4,-I*exp
(d*x+c))/b^3/(a^2+b^2)/d^4-3*I*a^4*f*(f*x+e)^2*polylog(2,I*exp(d*x+c))/b^3/
(a^2+b^2)/d^2-6*I*a^4*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/b^3/(a^2+b^2)/d^
3-6*f^3*cosh(d*x+c)/b/d^4+6*I*f^3*polylog(4,-I*exp(d*x+c))/b/d^4-a*(f*x+e)^
3*ln(1+exp(2*d*x+2*c))/b^2/d-6*a^3*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)
^(1/2)))/b^2/(a^2+b^2)/d^4+a^3*(f*x+e)^3*ln(1+exp(2*d*x+2*c))/b^2/(a^2+b^2)
/d-a^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d-a^3
*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d-6*a^3*f^3
*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d^4-2*a^4*(f*x+
e)^3*arctan(exp(d*x+c))/b^3/(a^2+b^2)/d-3/2*a*f*(f*x+e)^2*polylog(2,-exp(2*
d*x+2*c))/b^2/d^2+3/2*a*f^2*(f*x+e)*polylog(3,-exp(2*d*x+2*c))/b^2/d^3+3/4*
a^3*f^3*polylog(4,-exp(2*d*x+2*c))/b^2/(a^2+b^2)/d^4-3/4*a*f^3*polylog(4,-e
xp(2*d*x+2*c))/b^2/d^4-6*I*f^3*polylog(4,I*exp(d*x+c))/b/d^4+2*a^2*(f*x+e)^
3*arctan(exp(d*x+c))/b^3/d-3*f*(f*x+e)^2*cosh(d*x+c)/b/d^2+6*f^2*(f*x+e)*si
nh(d*x+c)/b/d^3+3*I*a^2*f*(f*x+e)^2*polylog(2,I*exp(d*x+c))/b^3/d^2+6*I*a^2
*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/b^3/d^3
```

Rubi [A]
time = 1.71, antiderivative size = 1519, normalized size of antiderivative = 1.00, number of steps used = 61, number of rules used = 15, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.441$, Rules used = {5700, 5557, 3377, 2718, 4265, 2611, 6744, 2320, 6724, 3799, 2221, 5686, 5692,

5680, 6874}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*(e + f*x)^4)/(4*b^2*f) + (2*a^2*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b^3*d)
 - (2*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b*d) - (2*a^4*(e + f*x)^3*ArcTan[E^
(c + d*x)])/(b^3*(a^2 + b^2)*d) - (6*f^3*Cosh[c + d*x])/(b*d^4) - (3*f*(e +
 f*x)^2*Cosh[c + d*x])/(b*d^2) - (a^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d) - (a^3*(e + f*x)^3*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d) - (a*(e + f*x)^3*Log
[1 + E^(2*(c + d*x))])/(b^2*d) + (a^3*(e + f*x)^3*Log[1 + E^(2*(c + d*x))])
/(b^2*(a^2 + b^2)*d) - ((3*I)*a^2*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)
])/(b^3*d^2) + ((3*I)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) +
 ((3*I)*a^4*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^
2) + ((3*I)*a^2*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b^3*d^2) - ((3*I)
*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b*d^2) - ((3*I)*a^4*f*(e + f*x)^
2*PolyLog[2, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) - (3*a^3*f*(e + f*x)^2*P
olyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^2)
- (3*a^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/(b^2*(a^2 + b^2)*d^2) - (3*a*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/
(2*b^2*d^2) + (3*a^3*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*b^2*(a^
2 + b^2)*d^2) + ((6*I)*a^2*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(b^3
*d^3) - ((6*I)*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(b*d^3) - ((6*I)
*a^4*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^3) - ((
6*I)*a^2*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(b^3*d^3) + ((6*I)*f^2*(e
 + f*x)*PolyLog[3, I*E^(c + d*x)])/(b*d^3) + ((6*I)*a^4*f^2*(e + f*x)*PolyL
og[3, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^3) + (6*a^3*f^2*(e + f*x)*PolyLog[
3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^3) + (6*a^
3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*
(a^2 + b^2)*d^3) + (3*a*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*
d^3) - (3*a^3*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*(a^2 + b^2
)*d^3) - ((6*I)*a^2*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(b^3*d^4) + ((6*I)*f^
3*PolyLog[4, (-I)*E^(c + d*x)])/(b*d^4) + ((6*I)*a^4*f^3*PolyLog[4, (-I)*E^
(c + d*x)])/(b^3*(a^2 + b^2)*d^4) + ((6*I)*a^2*f^3*PolyLog[4, I*E^(c + d*x)
])/(b^3*d^4) - ((6*I)*f^3*PolyLog[4, I*E^(c + d*x)])/(b*d^4) - ((6*I)*a^4*f
^3*PolyLog[4, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^4) - (6*a^3*f^3*PolyLog[4,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^4) - (6*a^3*
f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*
d^4) - (3*a*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*b^2*d^4) + (3*a^3*f^3*Poly
Log[4, -E^(2*(c + d*x))])/(4*b^2*(a^2 + b^2)*d^4) + (6*f^2*(e + f*x)*Sinh[c
 + d*x])/(b*d^3) + ((e + f*x)^3*Sinh[c + d*x])/(b*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
```

```
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5557

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5686

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*Sech[c +
d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5700

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - Di
st[a/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*(Tanh[c + d*x]^n/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^3\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\sinh(c+dx)\tanh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\sinh(c+d}{a+b\sinh(}}{b}$$

$$= -\frac{a\int (e+fx)^3\tanh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b^2} +$$

$$= \frac{a(e+fx)^4}{4b^2f} - \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} + \frac{(e+fx)^3\sinh(c+}{bd}$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^3\tan^{-1}}{bd}$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{a^3(e+fx)^4}{4b^2(a^2+b^2)f} + \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^3d} -$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{a^3(e+fx)^4}{4b^2(a^2+b^2)f} + \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^3d} -$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^3\tan^{-1}}{bd}$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^3\tan^{-1}}{bd}$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^3\tan^{-1}}{bd}$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^3\tan^{-1}}{bd}$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^3\tan^{-1}}{bd}$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^3\tan^{-1}}{bd}$$

<u>**Mathematica**</u> [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3060 vs. $2(1519) = 3038$.

time = 18.17, size = 3060, normalized size = 2.01

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] -1/4*(8*b*d^3*e^3*(1 + E^(2*c))*ArcTan[E^(c + d*x)] - 4*a*d^3*e^3*E^(2*c)*(
2*d*x - Log[1 + E^(2*(c + d*x))]) + 4*a*d^3*e^3*Log[1 + E^(2*(c + d*x))] +
(12*I)*b*d^2*e^2*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E
^(c + d*x)]) - PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) -
6*a*d^2*e^2*E^(2*c)*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2,
-E^(2*(c + d*x))]) + 6*a*d^2*e^2*f*(2*d*x*Log[1 + E^(2*(c + d*x))] + PolyLo
g[2, -E^(2*(c + d*x))]) + (12*I)*b*d*e*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I
*E^(c + d*x)] - d^2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c
 + d*x)] + 2*d*x*PolyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)]
 - 2*PolyLog[3, I*E^(c + d*x)]) + 6*a*d*e*f^2*(2*d^2*x^2*Log[1 + E^(2*(c +
d*x))] + 2*d*x*PolyLog[2, -E^(2*(c + d*x))] - PolyLog[3, -E^(2*(c + d*x))])
 - 2*a*d*e*E^(2*c)*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))]) - 6*
d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))]) + (4*I)*
b*(1 + E^(2*c))*f^3*(d^3*x^3*Log[1 - I*E^(c + d*x)] - d^3*x^3*Log[1 + I*E^(
c + d*x)] - 3*d^2*x^2*PolyLog[2, (-I)*E^(c + d*x)] + 3*d^2*x^2*PolyLog[2, I
*E^(c + d*x)] + 6*d*x*PolyLog[3, (-I)*E^(c + d*x)] - 6*d*x*PolyLog[3, I*E^(
c + d*x)] - 6*PolyLog[4, (-I)*E^(c + d*x)] + 6*PolyLog[4, I*E^(c + d*x)]) -
 a*E^(2*c)*f^3*(2*d^4*x^4 - 4*d^3*x^3*Log[1 + E^(2*(c + d*x))] - 6*d^2*x^2*
PolyLog[2, -E^(2*(c + d*x))] + 6*d*x*PolyLog[3, -E^(2*(c + d*x))] - 3*PolyL
og[4, -E^(2*(c + d*x))]) + a*f^3*(4*d^3*x^3*Log[1 + E^(2*(c + d*x))] + 6*d^
2*x^2*PolyLog[2, -E^(2*(c + d*x))] - 6*d*x*PolyLog[3, -E^(2*(c + d*x))] + 3
*PolyLog[4, -E^(2*(c + d*x))]))/((a^2 + b^2)*d^4*(1 + E^(2*c))) + (a^3*((4*
e^3*E^(2*c)*x)/(-1 + E^(2*c)) + (6*e^2*E^(2*c)*f*x^2)/(-1 + E^(2*c)) + (4*e
*E^(2*c)*f^2*x^3)/(-1 + E^(2*c)) + (E^(2*c)*f^3*x^4)/(-1 + E^(2*c)) + (4*a*
Sqrt[-(a^2 + b^2)^2]*e^3*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^
2 + b^2)^(3/2)*d) + (4*a*Sqrt[-(a^2 + b^2)^2]*e^3*ArcTanh[(a + b*E^(c + d*x
))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) - (2*e^3*Log[2*a*E^(c + d*x) +
b*(-1 + E^(2*(c + d*x)))])/d - (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c
- Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(
a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (2*f^3*x^3*Log[1 + (b*E^(2*c + d*x
))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e^2*f*x*Log[1 + (b*E^(2*c +
 d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*f^2*x^2*Log[1 + (b*E^
(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (2*f^3*x^3*Log[1 + (
b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*f*(e + f*x)^2
*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2
- (6*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)
*E^(2*c)]))])/d^2 + (12*e*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[
```

```
(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*
E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*e*f^2*PolyLog[3, -((b*E^(2*c
+ d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*f^3*x*PolyLog[3, -
((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (12*f^3*Pol
yLog[4, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^4 - (1
2*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])
/d^4))/(2*b^2*(a^2 + b^2)) - (3*x^2*(a^3*e^2*f + a*b^2*e^2*f + 2*a^3*e^2*f*
Cosh[2*c] - 2*a*b^2*e^2*f*Cosh[2*c] + a^3*e^2*f*Cosh[4*c] + a*b^2*e^2*f*Cos
h[4*c] + 2*a^3*e^2*f*Sinh[2*c] - 2*a*b^2*e^2*f*Sinh[2*c] + a^3*e^2*f*Sinh[4
*c] + a*b^2*e^2*f*Sinh[4*c]))/(2*b^2*(a^2 + b^2)*(-1 + Cosh[2*c] + Sinh[2*c
])*(1 + Cosh[2*c] + Sinh[2*c])) - (x^3*(a^3*e*f^2 + a*b^2*e*f^2 + 2*a^3*e*f
^2*Cosh[2*c] - 2*a*b^2*e*f^2*Cosh[2*c] + a^3*e*f^2*Cosh[4*c] + a*b^2*e*f^2*
Cosh[4*c] + 2*a^3*e*f^2*Sinh[2*c] - 2*a*b^2*e*f^2*Sinh[2*c] + a^3*e*f^2*Sin
h[4*c] + a*b^2*e*f^2*Sinh[4*c]))/(b^2*(a^2 + b^2)*(-1 + Cosh[2*c] + Sinh[2*
c])*(1 + Cosh[2*c] + Sinh[2*c])) - (x^4*(a^3*f^3 + a*b^2*f^3 + 2*a^3*f^3*Co
sh[2*c] - 2*a*b^2*f^3*Cosh[2*c] + a^3*f^3*Cosh[4*c] + a*b^2*f^3*Cosh[4*c] +
 2*a^3*f^3*Sinh[2*c] - 2*a*b^2*f^3*Sinh[2*c] + a^3*f^3*Sinh[4*c] + a*b^2*f^
3*Sinh[4*c]))/(4*b^2*(a^2 + b^2)*(-1 + Cosh[2*c] + Sinh[2*c])*(1 + Cosh[2*c
] + Sinh[2*c])) + x*(-((a*e^3)/((a^2 + b^2)*(-1 + Cosh[2*c] + Sinh[2*c])*(1
 + Cosh[2*c] + Sinh[2*c]))) - (a^3*e^3)/(b^2*(a^2 + b^2)*(-1 + Cosh[2*c] +
Sinh[2*c])*(1 + Cosh[2*c] + Sinh[2*c])) + (2*a*e^3*Cosh[2*c] + 2*a*e^3*Sinh
[2*c])/((a^2 + b^2)*(-1 + Cosh[2*c] + Sinh[2*c])*(1 + Cosh[2*c] + Sinh[2*c]
)) + ((-2*a^3*e^3*Cosh[2*c])/b^2 - (2*a^3*e^3*Sinh[2*c])/b^2)/((a^2 + b^2)*
(-1 + Cosh[2*c] + Sinh[2*c])*(1 + Cosh[2*c] + Sinh[2*c])) + (-(a*e^3*Cosh[4
*c]) - a*e^3*Sinh[4*c])/((a^2 + b^2)*(-1 + Cosh[2*c] + Sinh[2*c])*(1 + Cosh
[2*c] + Sinh[2*c])) + (-((a^3*e^3*Cosh[4*c])/b^2) - (a^3*e^3*Sinh[4*c])/b^2
)/((a^2 + b^2)*(-1 + Cosh[2*c] + Sinh[2*c])*(1 + Cosh[2*c] + Sinh[2*c]))) +
 (-1/2*(f^3*x^3*Cosh[c])/(b*d) + (f^3*x^3*Sinh[c])/(2*b*d) + (d^3*e^3 + 3*d
^2*e^2*f + 6*d*e*f^2 + 6*f^3)*(-1/2*Cosh[c]/(b*d^4) + Sinh[c]/(2*b*d^4)) +
(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*((-3*x*Cosh[c])...
```

Maple [F]
time = 5.05, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \left(\sinh^2(dx+c)\right) \tanh(dx+c)}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)`

[Out] `int((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)`

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -1/2*(2*a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b^2 + b^4
)*d) - 4*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + 2*a*log(e^(-2*d*x - 2*c)
+ 1)/((a^2 + b^2)*d) + 2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x
- c)/(b*d))*e^3 - 1/4*(a*d^4*f^3*x^4*e^c + 4*a*d^4*f^2*x^3*e^(c + 1) + 6*a*
d^4*f*x^2*e^(c + 2) - 2*(b*d^3*f^3*x^3*e^(2*c) - 6*b*f^3*e^(2*c) - 3*b*d^2*
f*e^(2*c + 2) + 6*b*d*f^2*e^(2*c + 1) - 3*(b*d^2*f^3*e^(2*c) - b*d^3*f^2*e^
(2*c + 1))*x^2 + 3*(2*b*d*f^3*e^(2*c) + b*d^3*f*e^(2*c + 2) - 2*b*d^2*f^2*e
^(2*c + 1))*x)*e^(d*x) + 2*(b*d^3*f^3*x^3 + 3*b*d^2*f*e^2 + 6*b*d*f^2*e + 6
*b*f^3 + 3*(b*d^3*f^2*e + b*d^2*f^3)*x^2 + 3*(b*d^3*f*e^2 + 2*b*d^2*f^2*e +
 2*b*d*f^3)*x)*e^(-d*x))*e^(-c)/(b^2*d^4) + integrate(2*(a^3*b*f^3*x^3 + 3*
a^3*b*f^2*x^2*e + 3*a^3*b*f*x*e^2 - (a^4*f^3*x^3*e^c + 3*a^4*f^2*x^2*e^(c +
 1) + 3*a^4*f*x*e^(c + 2))*e^(d*x))/(a^2*b^3 + b^5 - (a^2*b^3*e^(2*c) + b^5
*e^(2*c))*e^(2*d*x) - 2*(a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) - integrate(
-2*(a*f^3*x^3 + 3*a*f^2*x^2*e + 3*a*f*x*e^2 - (b*f^3*x^3*e^c + 3*b*f^2*x^2*
e^(c + 1) + 3*b*f*x*e^(c + 2))*e^(d*x))/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(
2*c))*e^(2*d*x)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8301 vs. $2(1409) = 2818$.
time = 0.57, size = 8301, normalized size = 5.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -1/4*(2*(a^2*b + b^3)*d^3*f^3*x^3 + 6*(a^2*b + b^3)*d^2*f^3*x^2 + 2*(a^2*b
+ b^3)*d^3*cosh(1)^3 + 2*(a^2*b + b^3)*d^3*sinh(1)^3 + 12*(a^2*b + b^3)*d*f
^3*x + 12*(a^2*b + b^3)*f^3 + 6*((a^2*b + b^3)*d^3*f*x + (a^2*b + b^3)*d^2*
f)*cosh(1)^2 - 2*((a^2*b + b^3)*d^3*f^3*x^3 - 3*(a^2*b + b^3)*d^2*f^3*x^2 +
 (a^2*b + b^3)*d^3*cosh(1)^3 + (a^2*b + b^3)*d^3*sinh(1)^3 + 6*(a^2*b + b^3
)*d*f^3*x - 6*(a^2*b + b^3)*f^3 + 3*((a^2*b + b^3)*d^3*f*x - (a^2*b + b^3)*
d^2*f)*cosh(1)^2 + 3*((a^2*b + b^3)*d^3*f*x + (a^2*b + b^3)*d^3*cosh(1) - (
a^2*b + b^3)*d^2*f)*sinh(1)^2 + 3*((a^2*b + b^3)*d^3*f^2*x^2 - 2*(a^2*b + b
^3)*d^2*f^2*x + 2*(a^2*b + b^3)*d*f^2)*cosh(1) + 3*((a^2*b + b^3)*d^3*f^2*x
^2 - 2*(a^2*b + b^3)*d^2*f^2*x + (a^2*b + b^3)*d^3*cosh(1)^2 + 2*(a^2*b + b
^3)*d*f^2 + 2*((a^2*b + b^3)*d^3*f*x - (a^2*b + b^3)*d^2*f)*cosh(1))*sinh(1
))*cosh(d*x + c)^2 + 6*((a^2*b + b^3)*d^3*f*x + (a^2*b + b^3)*d^3*cosh(1) +
 (a^2*b + b^3)*d^2*f)*sinh(1)^2 - 2*((a^2*b + b^3)*d^3*f^3*x^3 - 3*(a^2*b +
 b^3)*d^2*f^3*x^2 + (a^2*b + b^3)*d^3*cosh(1)^3 + (a^2*b + b^3)*d^3*sinh(1)
```

^3 + 6*(a^2*b + b^3)*d*f^3*x - 6*(a^2*b + b^3)*f^3 + 3*((a^2*b + b^3)*d^3*f
*x - (a^2*b + b^3)*d^2*f)*cosh(1)^2 + 3*((a^2*b + b^3)*d^3*f*x + (a^2*b + b
^3)*d^3*cosh(1) - (a^2*b + b^3)*d^2*f)*sinh(1)^2 + 3*((a^2*b + b^3)*d^3*f^2
x^2 - 2(a^2*b + b^3)*d^2*f^2*x + 2*(a^2*b + b^3)*d*f^2)*cosh(1) + 3*((a^2
*b + b^3)*d^3*f^2*x^2 - 2*(a^2*b + b^3)*d^2*f^2*x + (a^2*b + b^3)*d^3*cosh(
1)^2 + 2*(a^2*b + b^3)*d*f^2 + 2*((a^2*b + b^3)*d^3*f*x - (a^2*b + b^3)*d^2
*f)*cosh(1))*sinh(1))*sinh(d*x + c)^2 + 6*((a^2*b + b^3)*d^3*f^2*x^2 + 2*(a
^2*b + b^3)*d^2*f^2*x + 2*(a^2*b + b^3)*d*f^2)*cosh(1) - ((a^3 + a*b^2)*d^4
*f^3*x^4 - 2*(a^3 + a*b^2)*c^4*f^3 + 4*((a^3 + a*b^2)*d^4*x + 2*(a^3 + a*b^
2)*c*d^3)*cosh(1)^3 + 4*((a^3 + a*b^2)*d^4*x + 2*(a^3 + a*b^2)*c*d^3)*sinh(
1)^3 + 6*((a^3 + a*b^2)*d^4*f*x^2 - 2*(a^3 + a*b^2)*c^2*d^2*f)*cosh(1)^2 +
6*((a^3 + a*b^2)*d^4*f*x^2 - 2*(a^3 + a*b^2)*c^2*d^2*f + 2*((a^3 + a*b^2)*d
^4*x + 2*(a^3 + a*b^2)*c*d^3)*cosh(1))*sinh(1)^2 + 4*((a^3 + a*b^2)*d^4*f^2
x^3 + 2(a^3 + a*b^2)*c^3*d*f^2)*cosh(1) + 4*((a^3 + a*b^2)*d^4*f^2*x^3 +
2*(a^3 + a*b^2)*c^3*d*f^2 + 3*((a^3 + a*b^2)*d^4*x + 2*(a^3 + a*b^2)*c*d^3)
cosh(1)^2 + 3((a^3 + a*b^2)*d^4*f*x^2 - 2*(a^3 + a*b^2)*c^2*d^2*f)*cosh(1
))*sinh(1))*cosh(d*x + c) + 12*((a^3*d^2*f^3*x^2 + 2*a^3*d^2*f^2*x*cosh(1)
+ a^3*d^2*f*cosh(1)^2 + a^3*d^2*f*sinh(1)^2 + 2*(a^3*d^2*f^2*x + a^3*d^2*f*
cosh(1))*sinh(1))*cosh(d*x + c) + (a^3*d^2*f^3*x^2 + 2*a^3*d^2*f^2*x*cosh(1
) + a^3*d^2*f*cosh(1)^2 + a^3*d^2*f*sinh(1)^2 + 2*(a^3*d^2*f^2*x + a^3*d^2*
f*cosh(1))*sinh(1))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c)
 + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) +
12*((a^3*d^2*f^3*x^2 + 2*a^3*d^2*f^2*x*cosh(1) + a^3*d^2*f*cosh(1)^2 + a^3*
d^2*f*sinh(1)^2 + 2*(a^3*d^2*f^2*x + a^3*d^2*f*cosh(1))*sinh(1))*cosh(d*x +
 c) + (a^3*d^2*f^3*x^2 + 2*a^3*d^2*f^2*x*cosh(1) + a^3*d^2*f*cosh(1)^2 + a^
3*d^2*f*sinh(1)^2 + 2*(a^3*d^2*f^2*x + a^3*d^2*f*cosh(1))*sinh(1))*sinh(d*x
 + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 12*((a*b^2*d^2*f^3*x^2 + I*b
^3*d^2*f^3*x^2 + 2*a*b^2*d^2*f^2*x*cosh(1) + 2*I*b^3*d^2*f^2*x*cosh(1) + a*
b^2*d^2*f*cosh(1)^2 + I*b^3*d^2*f*cosh(1)^2 + a*b^2*d^2*f*sinh(1)^2 + I*b^3
*d^2*f*sinh(1)^2 + 2*(a*b^2*d^2*f^2*x + a*b^2*d^2*f*cosh(1))*sinh(1) + 2*I*
(b^3*d^2*f^2*x + b^3*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c) + (a*b^2*d^2*f^3
*x^2 + I*b^3*d^2*f^3*x^2 + 2*a*b^2*d^2*f^2*x*cosh(1) + 2*I*b^3*d^2*f^2*x*co
sh(1) + a*b^2*d^2*f*cosh(1)^2 + I*b^3*d^2*f*cosh(1)^2 + a*b^2*d^2*f*sinh(1)
^2 + I*b^3*d^2*f*sinh(1)^2 + 2*(a*b^2*d^2*f^2*x + a*b^2*d^2*f*cosh(1))*sinh
(1) + 2*I*(b^3*d^2*f^2*x + b^3*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c))*dilog
(I*cosh(d*x + c) + I*sinh(d*x + c)) + 12*((a*b^2*d^2*f^3*x^2 - I*b^3*d^2*f^
3*x^2 + 2*a*b^2*d^2*f^2*x*cosh(1) - 2*I*b^3*d^2*f^2*x*cosh(1) + a*b^2*d^2*f
*cosh(1)^2 - I*b^3*d^2*f*cosh(1)^2 + a*b^2*d^2*f*sinh(1)^2 - I*b^3*d^2*f*si
nh(1)^2 + 2*(a*b^2*d^2*f^2*x + a*b^2*d^2*f*cosh(1))*sinh(1) - 2*I*(b^3*d^2*
f^2*x + b^3*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c) + (a*b^2*d^2*f^3*x^2 - I*
b^3*d^2*f^3*x^2 + 2*a*b^2*d^2*f^2*x*cosh(1) - 2*I*b^3*d^2*f^2*x*cosh(1) + a
*b^2*d^2*f*cosh(1)^2 - I*b^3*d^2*f*cosh(1)^2 + a*b^2*d^2*f*sinh(1)^2 - I*b^
3*d^2*f*sinh(1)^2 + 2*(a*b^2*d^2*f^2*x + a*b^2*d^2*f*cosh(1))*sinh(1) - 2*I
*(b^3*d^2*f^2*x + b^3*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c))*dilog(-I*cosh(

```
d*x + c) - I*sinh(d*x + c)) - 4*((a^3*c^3*f^3 - 3*a^3*c^2*d*f^2*cosh(1) + 3
*a^3*c*d^2*f*cosh(1)^2 - a^3*d^3*cosh(1)^3 - a^3*d^3*sinh(1)^3 + 3*(a^3*c*d
^2*f - a^3*d^3*cosh(1))*sinh(1)^2 - 3*(a^3*c^2*d*f^2 - 2*a^3*c*d^2*f*cosh(1
) + a^3*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c) + (a^3*c^3*f^3 - 3*a^3*c^2*d*
f^2*cosh(1) + 3*a^3*c*d^2*f*cosh(1)^2 - a^3*d^3*cosh(1)^3 - a^3*d^3*sinh(1)
^3 + 3*(a^3*c*d^2*f - a^3*d^3*cosh(1))*sinh(1)^2 - 3*(a^3*c^2*d*f^2 - 2*a^3
*c*d^2*f*cosh(1) + a^3*d^3*cosh(1)^2)*sinh(1))*sinh(d*x + c))*log(2*b*cosh(
d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3 \sinh^2(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)**3*sinh(c + d*x)**2*tanh(c + d*x)/(a + b*sinh(c + d*x)),
 x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^2 \tanh(c+d\,x)\,\left(e+f\,x\right)^3}{a+b\sinh(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)^2*tanh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
[Out] int((sinh(c + d*x)^2*tanh(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.407 $\int \frac{(e+fx)^2\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1067

$$\frac{a(e+fx)^3}{3b^2f}+\frac{2a^2(e+fx)^2\mathrm{ArcTan}\left(e^{c+dx}\right)}{b^3d}-\frac{2(e+fx)^2\mathrm{ArcTan}\left(e^{c+dx}\right)}{bd}-\frac{2a^4(e+fx)^2\mathrm{ArcTan}\left(e^{c+dx}\right)}{b^3\left(a^2+b^2\right)d}-\frac{2f(e$$

```
[Out] -2*a^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)
/d^2-2*a^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+
b^2)/d^2+1/3*a*(f*x+e)^3/b^2/f-2*a^4*(f*x+e)^2*arctan(exp(d*x+c))/b^3/(a^2+
b^2)/d-a*f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/b^2/d^2+2*I*f^2*polylog(3,I*e
xp(d*x+c))/b/d^3-1/2*a^3*f^2*polylog(3,-exp(2*d*x+2*c))/b^2/(a^2+b^2)/d^3-2
*I*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b/d^2-2*I*a^2*f^2*polylog(3,I*exp(d*x+
c))/b^3/d^3+2*I*a^4*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/b^3/(a^2+b^2)/d^2-2*
f*(f*x+e)*cosh(d*x+c)/b/d^2+(f*x+e)^2*sinh(d*x+c)/b/d-2*(f*x+e)^2*arctan(ex
p(d*x+c))/b/d+2*I*a^2*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b^3/d^2+2*I*a^4*f^2
*polylog(3,I*exp(d*x+c))/b^3/(a^2+b^2)/d^3-2*I*a^4*f*(f*x+e)*polylog(2,I*ex
p(d*x+c))/b^3/(a^2+b^2)/d^2+2*f^2*sinh(d*x+c)/b/d^3+2*a^2*(f*x+e)^2*arctan(
exp(d*x+c))/b^3/d+1/2*a*f^2*polylog(3,-exp(2*d*x+2*c))/b^2/d^3-2*I*f^2*poly
log(3,-I*exp(d*x+c))/b/d^3+2*I*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/b/d^2+a^3
*f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/b^2/(a^2+b^2)/d^2+2*I*a^2*f^2*polylog
(3,-I*exp(d*x+c))/b^3/d^3-2*I*a^2*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/b^3/d^
2-2*I*a^4*f^2*polylog(3,-I*exp(d*x+c))/b^3/(a^2+b^2)/d^3-a*(f*x+e)^2*ln(1+e
xp(2*d*x+2*c))/b^2/d+a^3*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/b^2/(a^2+b^2)/d-a^3
*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d-a^3*(f*x+
e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d+2*a^3*f^2*polyl
og(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d^3+2*a^3*f^2*polylog
(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d^3
```

Rubi [A]
time = 1.24, antiderivative size = 1067, normalized size of antiderivative = 1.00, number of steps used = 50, number of rules used = 14, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5700, 5557, 3377, 2717, 4265, 2611, 2320, 6724, 3799, 2221, 5686, 5692, 5680, 6874}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*(e + f*x)^3)/(3*b^2*f) + (2*a^2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^3*d)
 - (2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b*d) - (2*a^4*(e + f*x)^2*ArcTan[E^
(c + d*x)])/(b^3*(a^2 + b^2)*d) - (2*f*(e + f*x)*Cosh[c + d*x])/(b*d^2) - (
```

```
a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*(a^2 +
 b^2)*d) - (a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])
/(b^2*(a^2 + b^2)*d) - (a*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b^2*d) + (
a^3*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b^2*(a^2 + b^2)*d) - ((2*I)*a^2*
f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^3*d^2) + ((2*I)*f*(e + f*x)*Po
lyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + ((2*I)*a^4*f*(e + f*x)*PolyLog[2, (-I
)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) + ((2*I)*a^2*f*(e + f*x)*PolyLog[2, I
*E^(c + d*x)])/(b^3*d^2) - ((2*I)*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b
*d^2) - ((2*I)*a^4*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*
d^2) - (2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
))])/(b^2*(a^2 + b^2)*d^2) - (2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^2) - (a*f*(e + f*x)*PolyLog[2
, -E^(2*(c + d*x))])/(b^2*d^2) + (a^3*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x
))])/(b^2*(a^2 + b^2)*d^2) + ((2*I)*a^2*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(
b^3*d^3) - ((2*I)*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b*d^3) - ((2*I)*a^4*f^
2*PolyLog[3, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^3) - ((2*I)*a^2*f^2*Poly
Log[3, I*E^(c + d*x)])/(b^3*d^3) + ((2*I)*f^2*PolyLog[3, I*E^(c + d*x)])/(b
*d^3) + ((2*I)*a^4*f^2*PolyLog[3, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^3) + (
2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*(a^2 +
 b^2)*d^3) + (2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/(b^2*(a^2 + b^2)*d^3) + (a*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*d^3)
 - (a^3*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*(a^2 + b^2)*d^3) + (2*f^2*
Sinh[c + d*x])/(b*d^3) + ((e + f*x)^2*Sinh[c + d*x])/(b*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5557

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5686

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*Sech[c +
d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5700

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - Di
st[a/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*(Tanh[c + d*x]^n/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2 \sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2 \sinh(c+dx)\tanh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\sinh(c+d}{a+b\sinh(}}{b}$$

$$= -\frac{a\int (e+fx)^2\tanh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b^2} +$$

$$= \frac{a(e+fx)^3}{3b^2f} - \frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} + \frac{(e+fx)^2\sinh(c+}{bd}$$

$$= \frac{a(e+fx)^3}{3b^2f} + \frac{2a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^2\tan^{-1}}{bd}$$

$$= \frac{a(e+fx)^3}{3b^2f} + \frac{a^3(e+fx)^3}{3b^2\left(a^2+b^2\right)f} + \frac{2a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} -$$

$$= \frac{a(e+fx)^3}{3b^2f} + \frac{a^3(e+fx)^3}{3b^2\left(a^2+b^2\right)f} + \frac{2a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} -$$

$$= \frac{a(e+fx)^3}{3b^2f} + \frac{2a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^2\tan^{-1}}{bd}$$

$$= \frac{a(e+fx)^3}{3b^2f} + \frac{2a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^2\tan^{-1}}{bd}$$

$$= \frac{a(e+fx)^3}{3b^2f} + \frac{2a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^2\tan^{-1}}{bd}$$

$$= \frac{a(e+fx)^3}{3b^2f} + \frac{2a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^2\tan^{-1}}{bd}$$

$$= \frac{a(e+fx)^3}{3b^2f} + \frac{2a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^2\tan^{-1}}{bd}$$

Mathematica [A]
time = 7.83, size = 1948, normalized size = 1.83

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (6*a^3*d^3*e^2*E^c*x + 6*a*b^2*d^3*e^2*E^c*x + 6*a^3*d^3*e*E^c*f*x^2 + 6*a*
b^2*d^3*e*E^c*f*x^2 + 2*a^3*d^3*E^c*f^2*x^3 + 2*a*b^2*d^3*E^c*f^2*x^3 - 12*
b^3*d^2*e^2*E^c*ArcTan[E^(c + d*x)] - 3*a^2*b*d^2*e^2*Cosh[d*x] - 3*b^3*d^2
*e^2*Cosh[d*x] + 3*a^2*b*d^2*e^2*E^(2*c)*Cosh[d*x] + 3*b^3*d^2*e^2*E^(2*c)*
Cosh[d*x] - 6*a^2*b*d*e*f*Cosh[d*x] - 6*b^3*d*e*f*Cosh[d*x] - 6*a^2*b*d*e*E
^(2*c)*f*Cosh[d*x] - 6*b^3*d*e*E^(2*c)*f*Cosh[d*x] - 6*a^2*b*f^2*Cosh[d*x]
 - 6*b^3*f^2*Cosh[d*x] + 6*a^2*b*E^(2*c)*f^2*Cosh[d*x] + 6*b^3*E^(2*c)*f^2*C
osh[d*x] - 6*a^2*b*d^2*e*f*x*Cosh[d*x] - 6*b^3*d^2*e*f*x*Cosh[d*x] + 6*a^2*
b*d^2*e*E^(2*c)*f*x*Cosh[d*x] + 6*b^3*d^2*e*E^(2*c)*f*x*Cosh[d*x] - 6*a^2*b
*d*f^2*x*Cosh[d*x] - 6*b^3*d*f^2*x*Cosh[d*x] - 6*a^2*b*d*E^(2*c)*f^2*x*Cosh
[d*x] - 6*b^3*d*E^(2*c)*f^2*x*Cosh[d*x] - 3*a^2*b*d^2*f^2*x^2*Cosh[d*x] - 3
*b^3*d^2*f^2*x^2*Cosh[d*x] + 3*a^2*b*d^2*E^(2*c)*f^2*x^2*Cosh[d*x] + 3*b^3*
d^2*E^(2*c)*f^2*x^2*Cosh[d*x] - (12*I)*b^3*d^2*e*E^c*f*x*Log[1 - I*E^(c + d
*x)] - (6*I)*b^3*d^2*E^c*f^2*x^2*Log[1 - I*E^(c + d*x)] + (12*I)*b^3*d^2*e*
E^c*f*x*Log[1 + I*E^(c + d*x)] + (6*I)*b^3*d^2*E^c*f^2*x^2*Log[1 + I*E^(c +
 d*x)] - 6*a*b^2*d^2*e^2*E^c*Log[1 + E^(2*(c + d*x))] - 12*a*b^2*d^2*e*E^c*
f*x*Log[1 + E^(2*(c + d*x))] - 6*a*b^2*d^2*E^c*f^2*x^2*Log[1 + E^(2*(c + d*
x))] - 6*a^3*d^2*e^2*E^c*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] -
12*a^3*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^
(2*c)])] - 6*a^3*d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a
^2 + b^2)*E^(2*c)])] - 12*a^3*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^
c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 6*a^3*d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c +
 d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + (12*I)*b^3*d*E^c*f*(e + f*x)*
PolyLog[2, (-I)*E^(c + d*x)] - (12*I)*b^3*d*E^c*f*(e + f*x)*PolyLog[2, I*E^
(c + d*x)] - 6*a*b^2*d*e*E^c*f*PolyLog[2, -E^(2*(c + d*x))] - 6*a*b^2*d*E^c
*f^2*x*PolyLog[2, -E^(2*(c + d*x))] - 12*a^3*d*e*E^c*f*PolyLog[2, -((b*E^(2
*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 12*a^3*d*E^c*f^2*x*PolyL
og[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 12*a^3*d*
e*E^c*f*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))
] - 12*a^3*d*E^c*f^2*x*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 +
b^2)*E^(2*c)]))] - (12*I)*b^3*E^c*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (12*I)
*b^3*E^c*f^2*PolyLog[3, I*E^(c + d*x)] + 3*a*b^2*E^c*f^2*PolyLog[3, -E^(2*(
c + d*x))] + 12*a^3*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a
^2 + b^2)*E^(2*c)]))] + 12*a^3*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^
c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 3*a^2*b*d^2*e^2*Sinh[d*x] + 3*b^3*d^2*e^
2*Sinh[d*x] + 3*a^2*b*d^2*e^2*E^(2*c)*Sinh[d*x] + 3*b^3*d^2*e^2*E^(2*c)*Sin
h[d*x] + 6*a^2*b*d*e*f*Sinh[d*x] + 6*b^3*d*e*f*Sinh[d*x] - 6*a^2*b*d*e*E^(2
*c)*f*Sinh[d*x] - 6*b^3*d*e*E^(2*c)*f*Sinh[d*x] + 6*a^2*b*f^2*Sinh[d*x] + 6
*b^3*f^2*Sinh[d*x] + 6*a^2*b*E^(2*c)*f^2*Sinh[d*x] + 6*b^3*E^(2*c)*f^2*Sinh
[d*x] + 6*a^2*b*d^2*e*f*x*Sinh[d*x] + 6*b^3*d^2*e*f*x*Sinh[d*x] + 6*a^2*b*d
^2*e*E^(2*c)*f*x*Sinh[d*x] + 6*b^3*d^2*e*E^(2*c)*f*x*Sinh[d*x] + 6*a^2*b*d*
f^2*x*Sinh[d*x] + 6*b^3*d*f^2*x*Sinh[d*x] - 6*a^2*b*d*E^(2*c)*f^2*x*Sinh[d*
```

```
x] - 6*b^3*d*E^(2*c)*f^2*x*Sinh[d*x] + 3*a^2*b*d^2*f^2*x^2*Sinh[d*x] + 3*b^
3*d^2*f^2*x^2*Sinh[d*x] + 3*a^2*b*d^2*E^(2*c)*f^2*x^2*Sinh[d*x] + 3*b^3*d^2
*E^(2*c)*f^2*x^2*Sinh[d*x])/(6*b^2*(a^2 + b^2)*d^3*E^c)
```

Maple [F]
time = 4.86, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2\left(\sinh^2(dx+c)\right)\tanh(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -1/2*(2*a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b^2 + b^4
)*d) - 4*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + 2*a*log(e^(-2*d*x - 2*c)
+ 1)/((a^2 + b^2)*d) + 2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x
- c)/(b*d))*e^2 - 1/6*(2*a*d^3*f^2*x^3*e^c + 6*a*d^3*f*x^2*e^(c + 1) - 3*(b
*d^2*f^2*x^2*e^(2*c) + 2*b*f^2*e^(2*c) - 2*b*d*f*e^(2*c + 1) - 2*(b*d*f^2*e
^(2*c) - b*d^2*f*e^(2*c + 1))*x)*e^(d*x) + 3*(b*d^2*f^2*x^2 + 2*b*d*f*e + 2
*b*f^2 + 2*(b*d^2*f*e + b*d*f^2)*x)*e^(-d*x))*e^(-c)/(b^2*d^3) + integrate(
2*(a^3*b*f^2*x^2 + 2*a^3*b*f*x*e - (a^4*f^2*x^2*e^c + 2*a^4*f*x*e^(c + 1))*
e^(d*x))/(a^2*b^3 + b^5 - (a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^
3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) - integrate(-2*(a*f^2*x^2 + 2*a*f*x*e -
 (b*f^2*x^2*e^c + 2*b*f*x*e^(c + 1))*e^(d*x))/(a^2 + b^2 + (a^2*e^(2*c) + b
^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4087 vs. 2(990) = 1980.
time = 0.44, size = 4087, normalized size = 3.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -1/6*(3*(a^2*b + b^3)*d^2*f^2*x^2 + 6*(a^2*b + b^3)*d*f^2*x + 3*(a^2*b + b^
3)*d^2*cosh(1)^2 + 3*(a^2*b + b^3)*d^2*sinh(1)^2 + 6*(a^2*b + b^3)*f^2 - 3*
((a^2*b + b^3)*d^2*f^2*x^2 - 2*(a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d^2*co
sh(1)^2 + (a^2*b + b^3)*d^2*sinh(1)^2 + 2*(a^2*b + b^3)*f^2 + 2*((a^2*b + b
^3)*d^2*f*x - (a^2*b + b^3)*d*f)*cosh(1) + 2*((a^2*b + b^3)*d^2*f*x + (a^2*
b + b^3)*d^2*cosh(1) - (a^2*b + b^3)*d*f)*sinh(1))*cosh(d*x + c)^2 - 3*((a^
2*b + b^3)*d^2*f^2*x^2 - 2*(a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d^2*cosh(1
)^2 + (a^2*b + b^3)*d^2*sinh(1)^2 + 2*(a^2*b + b^3)*f^2 + 2*((a^2*b + b^3)*
d^2*f*x - (a^2*b + b^3)*d*f)*cosh(1) + 2*((a^2*b + b^3)*d^2*f*x + (a^2*b +
b^3)*d^2*cosh(1) - (a^2*b + b^3)*d*f)*sinh(1))*sinh(d*x + c)^2 + 6*((a^2*b
+ b^3)*d^2*f*x + (a^2*b + b^3)*d*f)*cosh(1) - 2*((a^3 + a*b^2)*d^3*f^2*x^3
+ 2*(a^3 + a*b^2)*c^3*f^2 + 3*((a^3 + a*b^2)*d^3*x + 2*(a^3 + a*b^2)*c*d^2)
*cosh(1)^2 + 3*((a^3 + a*b^2)*d^3*x + 2*(a^3 + a*b^2)*c*d^2)*sinh(1)^2 + 3*
((a^3 + a*b^2)*d^3*f*x^2 - 2*(a^3 + a*b^2)*c^2*d*f)*cosh(1) + 3*((a^3 + a*b
^2)*d^3*f*x^2 - 2*(a^3 + a*b^2)*c^2*d*f + 2*((a^3 + a*b^2)*d^3*x + 2*(a^3 +
 a*b^2)*c*d^2)*cosh(1))*sinh(1))*cosh(d*x + c) + 12*((a^3*d*f^2*x + a^3*d*f
*cosh(1) + a^3*d*f*sinh(1))*cosh(d*x + c) + (a^3*d*f^2*x + a^3*d*f*cosh(1)
+ a^3*d*f*sinh(1))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c)
+ (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 1
2*((a^3*d*f^2*x + a^3*d*f*cosh(1) + a^3*d*f*sinh(1))*cosh(d*x + c) + (a^3*d
*f^2*x + a^3*d*f*cosh(1) + a^3*d*f*sinh(1))*sinh(d*x + c))*dilog((a*cosh(d*
x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b + 1) + 12*((a*b^2*d*f^2*x + I*b^3*d*f^2*x + a*b^2*d*f*cosh
(1) + I*b^3*d*f*cosh(1) + a*b^2*d*f*sinh(1) + I*b^3*d*f*sinh(1))*cosh(d*x +
 c) + (a*b^2*d*f^2*x + I*b^3*d*f^2*x + a*b^2*d*f*cosh(1) + I*b^3*d*f*cosh(1
) + a*b^2*d*f*sinh(1) + I*b^3*d*f*sinh(1))*sinh(d*x + c))*dilog(I*cosh(d*x
+ c) + I*sinh(d*x + c)) + 12*((a*b^2*d*f^2*x - I*b^3*d*f^2*x + a*b^2*d*f*co
sh(1) - I*b^3*d*f*cosh(1) + a*b^2*d*f*sinh(1) - I*b^3*d*f*sinh(1))*cosh(d*x
 + c) + (a*b^2*d*f^2*x - I*b^3*d*f^2*x + a*b^2*d*f*cosh(1) - I*b^3*d*f*cosh
(1) + a*b^2*d*f*sinh(1) - I*b^3*d*f*sinh(1))*sinh(d*x + c))*dilog(-I*cosh(d
*x + c) - I*sinh(d*x + c)) + 6*((a^3*c^2*f^2 - 2*a^3*c*d*f*cosh(1) + a^3*d^
2*cosh(1)^2 + a^3*d^2*sinh(1)^2 - 2*(a^3*c*d*f - a^3*d^2*cosh(1))*sinh(1))*
cosh(d*x + c) + (a^3*c^2*f^2 - 2*a^3*c*d*f*cosh(1) + a^3*d^2*cosh(1)^2 + a^
3*d^2*sinh(1)^2 - 2*(a^3*c*d*f - a^3*d^2*cosh(1))*sinh(1))*sinh(d*x + c))*l
og(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
 + 6*((a^3*c^2*f^2 - 2*a^3*c*d*f*cosh(1) + a^3*d^2*cosh(1)^2 + a^3*d^2*sinh
(1)^2 - 2*(a^3*c*d*f - a^3*d^2*cosh(1))*sinh(1))*cosh(d*x + c) + (a^3*c^2*f
^2 - 2*a^3*c*d*f*cosh(1) + a^3*d^2*cosh(1)^2 + a^3*d^2*sinh(1)^2 - 2*(a^3*c
*d*f - a^3*d^2*cosh(1))*sinh(1))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b
*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 6*((a^3*d^2*f^2*x^2 - a
^3*c^2*f^2 + 2*(a^3*d^2*f*x + a^3*c*d*f)*cosh(1) + 2*(a^3*d^2*f*x + a^3*c*d
*f)*sinh(1))*cosh(d*x + c) + (a^3*d^2*f^2*x^2 - a^3*c^2*f^2 + 2*(a^3*d^2*f*
x + a^3*c*d*f)*cosh(1) + 2*(a^3*d^2*f*x + a^3*c*d*f)*sinh(1))*sinh(d*x + c)
)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x +
 c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 6*((a^3*d^2*f^2*x^2 - a^3*c^2*f^2 + 2*
```

```
(a^3*d^2*f*x + a^3*c*d*f)*cosh(1) + 2*(a^3*d^2*f*x + a^3*c*d*f)*sinh(1))*co
sh(d*x + c) + (a^3*d^2*f^2*x^2 - a^3*c^2*f^2 + 2*(a^3*d^2*f*x + a^3*c*d*f)*
cosh(1) + 2*(a^3*d^2*f*x + a^3*c*d*f)*sinh(1))*sinh(d*x + c))*log(-(a*cosh(
d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2) - b)/b) + 6*((a*b^2*c^2*f^2 + I*b^3*c^2*f^2 - 2*a*b^2*c*d*f*cos
h(1) - 2*I*b^3*c*d*f*cosh(1) + a*b^2*d^2*cosh(1)^2 + I*b^3*d^2*cosh(1)^2 +
a*b^2*d^2*sinh(1)^2 + I*b^3*d^2*sinh(1)^2 - 2*(a*b^2*c*d*f - a*b^2*d^2*cosh
(1))*sinh(1) - 2*I*(b^3*c*d*f - b^3*d^2*cosh(1))*sinh(1))*cosh(d*x + c) + (
a*b^2*c^2*f^2 + I*b^3*c^2*f^2 - 2*a*b^2*c*d*f*cosh(1) - 2*I*b^3*c*d*f*cosh(
1) + a*b^2*d^2*cosh(1)^2 + I*b^3*d^2*cosh(1)^2 + a*b^2*d^2*sinh(1)^2 + I*b^
3*d^2*sinh(1)^2 - 2*(a*b^2*c*d*f - a*b^2*d^2*cosh(1))*sinh(1) - 2*I*(b^3*c*
d*f - b^3*d^2*cosh(1))*sinh(1))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x
 + c) + I) + 6*((a*b^2*c^2*f^2 - I*b^3*c^2*f^2 - 2*a*b^2*c*d*f*cosh(1) + 2*
I*b^3*c*d*f*cosh(1) + a*b^2*d^2*cosh(1)^2 - I*b^3*d^2*cosh(1)^2 + a*b^2*d^2
*sinh(1)^2 - I*b^3*d^2*sinh(1)^2 - 2*(a*b^2*c*d*f - a*b^2*d^2*cosh(1))*sinh
(1) + 2*I*(b^3*c*d*f - b^3*d^2*cosh(1))*sinh(1))*cosh(d*x + c) + (a*b^2*c^2
*f^2 - I*b^3*c^2*f^2 - 2*a*b^2*c*d*f*cosh(1) + 2*I*b^3*c*d*f*cosh(1) + a*b^
2*d^2*cosh(1)^2 - I*b^3*d^2*cosh(1)^2 + a*b^2*d^2*sinh(1)^2 - I*b^3*d^2*sin
h(1)^2 - 2*(a*b^2*c*d*f - a*b^2*d^2*cosh(1))*sinh(1) + 2*I*(b^3*c*d*f - b^3
*d^2*cosh(1))*sinh(1))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - I
) + 6*((a*b^2*d^2*f^2*x^2 - I*b^3*d^2*f^2*x^2 - a*b^2*c^2*f^2 + I*b^3*c^2*f
^2 + 2*(a*b^2*d^2*f*x + a*b^2*c*d*f)*cosh(1) - ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2 \sinh^2(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*sinh(c + d*x)**2*tanh(c + d*x)/(a + b*sinh(c + d*x)),
 x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^2\,\tanh(c+d\,x)\,\left(e+f\,x\right)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)^2*tanh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
[Out] int((sinh(c + d*x)^2*tanh(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.408 $\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=631

$$\frac{a(e+fx)^2}{2b^2f}+\frac{2a^2(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{b^3d}-\frac{2(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{bd}-\frac{2a^4(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{b^3\left(a^2+b^2\right)d}-\frac{f\cosh(c}{bd}$$

```
[Out] 1/2*a*(f*x+e)^2/b^2/f+2*a^2*(f*x+e)*arctan(exp(d*x+c))/b^3/d-2*(f*x+e)*arct
an(exp(d*x+c))/b/d-2*a^4*(f*x+e)*arctan(exp(d*x+c))/b^3/(a^2+b^2)/d-f*cosh(
d*x+c)/b/d^2-a*(f*x+e)*ln(1+exp(2*d*x+2*c))/b^2/d+a^3*(f*x+e)*ln(1+exp(2*d*
x+2*c))/b^2/(a^2+b^2)/d-a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/
b^2/(a^2+b^2)/d-a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2
+b^2)/d+I*a^2*f*polylog(2,I*exp(d*x+c))/b^3/d^2-I*a^4*f*polylog(2,I*exp(d*x
+c))/b^3/(a^2+b^2)/d^2+I*a^4*f*polylog(2,-I*exp(d*x+c))/b^3/(a^2+b^2)/d^2-I
*f*polylog(2,I*exp(d*x+c))/b/d^2+I*f*polylog(2,-I*exp(d*x+c))/b/d^2-I*a^2*f
*polylog(2,-I*exp(d*x+c))/b^3/d^2-1/2*a*f*polylog(2,-exp(2*d*x+2*c))/b^2/d^
2+1/2*a^3*f*polylog(2,-exp(2*d*x+2*c))/b^2/(a^2+b^2)/d^2-a^3*f*polylog(2,-b
*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d^2-a^3*f*polylog(2,-b*exp(d
*x+c)/(a+(a^2+b^2)^(1/2)))/b^2/(a^2+b^2)/d^2+(f*x+e)*sinh(d*x+c)/b/d
```

Rubi [A]
time = 0.70, antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {5700, 5557, 3377, 2718, 4265, 2317, 2438, 3799, 2221, 5686, 5692, 5680, 6874}

$$\frac{2a^2(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{b^3d}-\frac{ia^2f\text{Li}_2\left(-ie^{c+dx}\right)}{b^3d^2}+\frac{ia^2f\text{Li}_2\left(ie^{c+dx}\right)}{b^3d^2}-\frac{2a^4(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{b^3d\left(a^2+b^2\right)}+\frac{ia^4f\text{Li}_2\left(-ie^{c+dx}\right)}{b^3d^2\left(a^2+b^2\right)}-\frac{ia^4f\text{Li}_2\left(ie^{c+dx}\right)}{b^3d^2\left(a^2+b^2\right)}-\frac{a^3f\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2\left(a^2+b^2\right)}-\frac{a^3f\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2d^2\left(a^2+b^2\right)}+\frac{a^3f\text{Li}_2\left(-e^{2(c+dx)}\right)}{2b^2d^2\left(a^2+b^2\right)}-\frac{a^3(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^2d\left(a^2+b^2\right)}-\frac{a^3(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^2d\left(a^2+b^2\right)}+\frac{a^3(e+fx)\log\left(e^{2(c+dx)}+1\right)}{b^2d\left(a^2+b^2\right)}-\frac{af\text{Li}_2\left(-e^{2(c+dx)}\right)}{2b^2d^2}-\frac{a(e+fx)\log\left(e^{2(c+dx)}+1\right)}{b^2d}+\frac{a(e+fx)^2}{2b^2f}-\frac{2(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{bd}+\frac{if\text{Li}_2\left(-ie^{c+dx}\right)}{bd^2}-\frac{if\text{Li}_2\left(ie^{c+dx}\right)}{bd^2}-\frac{f\cosh(c+dx)}{bd^2}+\frac{(e+fx)\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*(e + f*x)^2)/(2*b^2*f) + (2*a^2*(e + f*x)*ArcTan[E^(c + d*x)])/(b^3*d) -
 (2*(e + f*x)*ArcTan[E^(c + d*x)])/(b*d) - (2*a^4*(e + f*x)*ArcTan[E^(c + d
*x)])/(b^3*(a^2 + b^2)*d) - (f*Cosh[c + d*x])/(b*d^2) - (a^3*(e + f*x)*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d) - (a^3*(e +
 f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d) -
 (a*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(b^2*d) + (a^3*(e + f*x)*Log[1 + E^
(2*(c + d*x))])/(b^2*(a^2 + b^2)*d) - (I*a^2*f*PolyLog[2, (-I)*E^(c + d*x)]
)/(b^3*d^2) + (I*f*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + (I*a^4*f*PolyLog
[2, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) + (I*a^2*f*PolyLog[2, I*E^(c +
 d*x)])/(b^3*d^2) - (I*f*PolyLog[2, I*E^(c + d*x)])/(b*d^2) - (I*a^4*f*Poly
Log[2, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) - (a^3*f*PolyLog[2, -((b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^2) - (a^3*f*PolyLog[2,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^2) - (a*f*Pol
```

```
yLog[2, -E^(2*(c + d*x))])/(2*b^2*d^2) + (a^3*f*PolyLog[2, -E^(2*(c + d*x))
])/(2*b^2*(a^2 + b^2)*d^2) + ((e + f*x)*Sinh[c + d*x])/(b*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5557

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5686

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*Sech[c +
d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5700

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - Di
st[a/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*(Tanh[c + d*x]^n/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\sinh(c+dx)\tanh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\sinh(c+dx)\tan}{a+b\sinh(c+dx}}{b}$$

$$= -\frac{a\int (e+fx)\tanh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b^2} + \int(e$$

$$= \frac{a(e+fx)^2}{2b^2f} - \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} + \frac{(e+fx)\sinh(c+dx)}{bd}$$

$$= \frac{a(e+fx)^2}{2b^2f} + \frac{2a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)\tan^{-1}\left(e^{c}\right.}{bd}$$

$$= \frac{a(e+fx)^2}{2b^2f} + \frac{a^3(e+fx)^2}{2b^2(a^2+b^2)f} + \frac{2a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - 2($$

$$= \frac{a(e+fx)^2}{2b^2f} + \frac{a^3(e+fx)^2}{2b^2(a^2+b^2)f} + \frac{2a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - 2($$

$$= \frac{a(e+fx)^2}{2b^2f} + \frac{2a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)\tan^{-1}\left(e^{c}\right.}{bd}$$

$$= \frac{a(e+fx)^2}{2b^2f} + \frac{2a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)\tan^{-1}\left(e^{c}\right.}{bd}$$

$$= \frac{a(e+fx)^2}{2b^2f} + \frac{2a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)\tan^{-1}\left(e^{c}\right.}{bd}$$

$$= \frac{a(e+fx)^2}{2b^2f} + \frac{2a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)\tan^{-1}\left(e^{c}\right.}{bd}$$

Mathematica [A]
time = 3.26, size = 481, normalized size = 0.76

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] -((((f*Cosh[c + d*x])/b + (a^3*(-1/2*(f*(c + d*x)^2) + f*(c + d*x)*Log[1 + (
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2])] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh
[c + d*x]] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyL
og[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(b^2*(a^2 + b^2)) + (-(a*
d*e*(c + d*x)) + a*c*f*(c + d*x) + (a*f*(c + d*x)^2)/2 + 2*b*d*e*ArcTan[Cos
h[c + d*x] + Sinh[c + d*x]] - 2*b*c*f*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]]
 + 2*b*f*(c + d*x)*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]] + a*f*(c + d*x)*Lo
g[2*Cosh[c + d*x]*(Cosh[c + d*x] - Sinh[c + d*x])] + a*d*e*Log[1 + Cosh[2*(
c + d*x)] + Sinh[2*(c + d*x)]] - a*c*f*Log[1 + Cosh[2*(c + d*x)] + Sinh[2*(
c + d*x)]] - I*b*f*PolyLog[2, (-I)*(Cosh[c + d*x] + Sinh[c + d*x])] + I*b*f
*PolyLog[2, I*(Cosh[c + d*x] + Sinh[c + d*x])] - (a*f*PolyLog[2, -Cosh[2*(c
 + d*x)] + Sinh[2*(c + d*x)]])/2)/(a^2 + b^2) - (d*(e + f*x)*Sinh[c + d*x])
/b)/d^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4065 vs. $2(592) = 1184$.
time = 5.85, size = 4066, normalized size = 6.44

method	result	size
risch	Expression too large to display	4066

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVER
BOSE)
```

```
[Out] -2/d^2*a^2*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1
/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d/b^2*e*a^3/(a^2+b^2)*ln(b*exp(2*d*x+2*c)+2*a
*exp(d*x+c)-b)-2/d^2/b^2*a*f*c*ln(exp(d*x+c))-1/d^2/b^2*a^3*f/(a^2+b^2)*dil
og((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2/b^2*a^3*f/
(a^2+b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/2/d
*a*f/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-2
/d^2*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))*a-4/d*b*e/(2*a^2+2*b^2)*arctan(e
xp(d*x+c))+2/d/b^2*e*a*ln(exp(d*x+c))-1/2/d^2*a*f/(a^2+b^2)*dilog((-b*exp(d
*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/2/d^2*a*f/(a^2+b^2)*dilog(
(b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/d*e/(2*a^2+2*b^2)*a
*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d*e/(2*a^2+2*b^2)*a*ln(1+exp(2*d*x
+2*c))-1/d^2*a^2*f/(a^2+b^2)^(3/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/
(-a+(a^2+b^2)^(1/2)))+1/d^2*a^2*f/(a^2+b^2)^(3/2)*dilog((b*exp(d*x+c)+(a^2+
b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2/b^2*a^4*f/(a^2+b^2)^(3/2)*dilog((-
b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/d^2/b^2*a^4*f/(a^2+
b^2)^(3/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+4/d^
2*b*f*c/(2*a^2+2*b^2)*arctan(exp(d*x+c))+2/d/b^2*e*a^4/(a^2+b^2)^(3/2)*arct
anh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d*f/(2*a^2+2*b^2)*ln(1-I*ex
p(d*x+c))*a*x-2/d^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*c-1/d*a^2*f/(a^2+b
```

```
^2)^(3/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d^
2*a^2*f/(a^2+b^2)^(3/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^
(1/2)))*c+2/d^2*a^2*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a
^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/2/d^2*a*f*c/(a^2+b^2)*ln(b*exp(2*d
*x+2*c)+2*a*exp(d*x+c)-b)+2/d^2/b^2*a^4*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*dil
og((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2/d^2/b^2*a^4*f/
(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^
2+b^2)^(1/2)))+1/d^2/b^2*a^3*f*c/(a^2+b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+
c)-b)-2/d^2/b^2*a^4*f*c/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a
^2+b^2)^(1/2))-1/d/b^2*a^3*f/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)
/(-a+(a^2+b^2)^(1/2)))*x-1/d^2/b^2*a^3*f/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b
^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d/b^2*a^3*f/(a^2+b^2)*ln((b*exp(d*x+
c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2/b^2*a^3*f/(a^2+b^2)*ln((
b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d/b^2*a^4*f/(a^2+b
^2)^(3/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2/
b^2*a^4*f/(a^2+b^2)^(3/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^
(1/2)))*c-1/d/b^2*a^4*f/(a^2+b^2)^(3/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a
)/(-a+(a^2+b^2)^(1/2)))*x-1/d^2/b^2*a^4*f/(a^2+b^2)^(3/2)*ln((-b*exp(d*x+c)
+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/d/b^2*a^4*e/(2*a^2+2*b^2)/(a^
2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d/b^2*a^2*
e/(2*a^2+2*b^2)*(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^
(1/2))-2*I/d*b*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x-2*I/d^2*b*f/(2*a^2+2*b^
2)*ln(1-I*exp(d*x+c))*c+2*I/d*b*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x+2*I/d^
2*b*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c+1/2*a*f*x^2/b^2-2/d/b^2*a^4*f/(2*a
^2+2*b^2)/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^
(1/2)))*x+2/d/b^2*a^4*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^
2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-2/d^2*a^2*f/(2*a^2+2*b^2)/(a^2+b^2)
^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-2/d*a^2*f
/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+
b^2)^(1/2)))*x+2/d^2*a^2*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+
(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+2/d*a^2*f/(2*a^2+2*b^2)/(a^2+b^2
)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d^2/
b^2*f*a*c^2+2/d*e*a^2/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2
+b^2)^(1/2))-2/d*e/(2*a^2+2*b^2)*(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c
)+2*a)/(a^2+b^2)^(1/2))-1/2/d*e*a/(a^2+b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x
+c)-b)+1/d^2*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a
^2+b^2)^(1/2)))*a+1/d^2*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)
+a)/(a+(a^2+b^2)^(1/2)))*a-2/d^2*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))*a-1/
2/d^2*a*f/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))
)*c+2/d^2*a*f*c/(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))-2/d^2*a^2*f*c/(a^2+b^2)^
(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d^2*f*c/(2*a^2+2*
b^2)*(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d*
a^2*f/(a^2+b^2)^(3/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2
)))*x+1/d^2*a^2*f/(a^2+b^2)^(3/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a
^2+b^2)^(1/2)))*c-1/2/d*a*f/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/
```

```
(-a+(a^2+b^2)^(1/2)))*x-1/2/d^2*a*f/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(
1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+
b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a*x+1/d^2*f...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/4*f*(2*(a*d^2*x^2*e^c - (b*d*x*e^(2*c) - b*e^(2*c))*e^(d*x) + (b*d*x + b
)*e^(-d*x))*e^(-c)/(b^2*d^2) - integrate(-8*(a^4*x*e^(d*x + c) - a^3*b*x)/(
a^2*b^3 + b^5 - (a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^3*b^2*e^c
+ a*b^4*e^c)*e^(d*x)), x) + integrate(8*(b*x*e^(d*x + c) - a*x)/(a^2 + b^2
+ (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)) - 1/2*(2*a^3*log(-2*a*e^(-d*x
 - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b^2 + b^4)*d) - 4*b*arctan(e^(-d*x -
c))/((a^2 + b^2)*d) + 2*a*log(e^(-2*d*x - 2*c) + 1)/((a^2 + b^2)*d) + 2*(d*
x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d))*e
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1646 vs. 2(579) = 1158.
time = 0.44, size = 1646, normalized size = 2.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -1/2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*cosh(1) - ((a^2*b + b^3)*d*f*x
+ (a^2*b + b^3)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1) - (a^2*b + b^3)*f)*cosh
(d*x + c)^2 + (a^2*b + b^3)*d*sinh(1) - ((a^2*b + b^3)*d*f*x + (a^2*b + b^3
)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1) - (a^2*b + b^3)*f)*sinh(d*x + c)^2 +
(a^2*b + b^3)*f - ((a^3 + a*b^2)*d^2*f*x^2 - 2*(a^3 + a*b^2)*c^2*f + 2*((a^
3 + a*b^2)*d^2*x + 2*(a^3 + a*b^2)*c*d)*cosh(1) + 2*((a^3 + a*b^2)*d^2*x +
2*(a^3 + a*b^2)*c*d)*sinh(1))*cosh(d*x + c) + 2*(a^3*f*cosh(d*x + c) + a^3*
f*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a^3*f*cosh(d*x
+ c) + a^3*f*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*((a*b
^2*f + I*b^3*f)*cosh(d*x + c) + (a*b^2*f + I*b^3*f)*sinh(d*x + c))*dilog(I*
cosh(d*x + c) + I*sinh(d*x + c)) + 2*((a*b^2*f - I*b^3*f)*cosh(d*x + c) + (
a*b^2*f - I*b^3*f)*sinh(d*x + c))*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c))
```

```
 - 2*((a^3*c*f - a^3*d*cosh(1) - a^3*d*sinh(1))*cosh(d*x + c) + (a^3*c*f -
a^3*d*cosh(1) - a^3*d*sinh(1))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*s
inh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*((a^3*c*f - a^3*d*cosh(
1) - a^3*d*sinh(1))*cosh(d*x + c) + (a^3*c*f - a^3*d*cosh(1) - a^3*d*sinh(1
))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2
 + b^2)/b^2) + 2*a) + 2*((a^3*d*f*x + a^3*c*f)*cosh(d*x + c) + (a^3*d*f*x +
 a^3*c*f)*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*((a^3*d*f*x +
 a^3*c*f)*cosh(d*x + c) + (a^3*d*f*x + a^3*c*f)*sinh(d*x + c))*log(-(a*cosh
(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2) - b)/b) - 2*((a*b^2*c*f + I*b^3*c*f - a*b^2*d*cosh(1) - I*b^3*
d*cosh(1) - a*b^2*d*sinh(1) - I*b^3*d*sinh(1))*cosh(d*x + c) + (a*b^2*c*f +
 I*b^3*c*f - a*b^2*d*cosh(1) - I*b^3*d*cosh(1) - a*b^2*d*sinh(1) - I*b^3*d*
sinh(1))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + I) - 2*((a*b^2*
c*f - I*b^3*c*f - a*b^2*d*cosh(1) + I*b^3*d*cosh(1) - a*b^2*d*sinh(1) + I*b
^3*d*sinh(1))*cosh(d*x + c) + (a*b^2*c*f - I*b^3*c*f - a*b^2*d*cosh(1) + I*
b^3*d*cosh(1) - a*b^2*d*sinh(1) + I*b^3*d*sinh(1))*sinh(d*x + c))*log(cosh(
d*x + c) + sinh(d*x + c) - I) + 2*((a*b^2*d*f*x - I*b^3*d*f*x + a*b^2*c*f -
 I*b^3*c*f)*cosh(d*x + c) + (a*b^2*d*f*x - I*b^3*d*f*x + a*b^2*c*f - I*b^3*
c*f)*sinh(d*x + c))*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) + 2*((a*b^2*
d*f*x + I*b^3*d*f*x + a*b^2*c*f + I*b^3*c*f)*cosh(d*x + c) + (a*b^2*d*f*x +
 I*b^3*d*f*x + a*b^2*c*f + I*b^3*c*f)*sinh(d*x + c))*log(-I*cosh(d*x + c) -
 I*sinh(d*x + c) + 1) - ((a^3 + a*b^2)*d^2*f*x^2 - 2*(a^3 + a*b^2)*c^2*f +
2*((a^3 + a*b^2)*d^2*x + 2*(a^3 + a*b^2)*c*d)*cosh(1) + 2*((a^2*b + b^3)*d*
f*x + (a^2*b + b^3)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1) - (a^2*b + b^3)*f)*
cosh(d*x + c) + 2*((a^3 + a*b^2)*d^2*x + 2*(a^3 + a*b^2)*c*d)*sinh(1))*sinh
(d*x + c))/((a^2*b^2 + b^4)*d^2*cosh(d*x + c) + (a^2*b^2 + b^4)*d^2*sinh(d*
x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*sinh(c + d*x)**2*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^2\,\tanh(c+d\,x)\,\,(e+f\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)^2*tanh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((sinh(c + d*x)^2*tanh(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.409 $\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=89

$$-\frac{b\text{ArcTan}(\sinh(c+dx))}{(a^2+b^2)\,d}-\frac{a\log(\cosh(c+dx))}{(a^2+b^2)\,d}-\frac{a^3\log(a+b\sinh(c+dx))}{b^2\,(a^2+b^2)\,d}+\frac{\sinh(c+dx)}{bd}$$

```
[Out] -b*arctan(sinh(d*x+c))/(a^2+b^2)/d-a*ln(cosh(d*x+c))/(a^2+b^2)/d-a^3*ln(a+b
*sinh(d*x+c))/b^2/(a^2+b^2)/d+sinh(d*x+c)/b/d
```

Rubi [A]
time = 0.14, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2916, 12, 1643, 649, 209, 266}

$$-\frac{b\text{ArcTan}(\sinh(c+dx))}{d\,(a^2+b^2)}-\frac{a\log(\cosh(c+dx))}{d\,(a^2+b^2)}-\frac{a^3\log(a+b\sinh(c+dx))}{b^2d\,(a^2+b^2)}+\frac{\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((b*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d)) - (a*Log[Cosh[c + d*x]])/((a^2
 + b^2)*d) - (a^3*Log[a + b*Sinh[c + d*x]])/(b^2*(a^2 + b^2)*d) + Sinh[c +
d*x]/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{b\text{Subst}\left(\int \frac{x^3}{b^3(a+x)(-b^2-x^2)}\,dx, x, b\sinh(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{x^3}{(a+x)(-b^2-x^2)}\,dx, x, b\sinh(c+dx)\right)}{b^2d}\\
&= -\frac{\text{Subst}\left(\int \left(-1+\frac{a^3}{(a^2+b^2)(a+x)}+\frac{b^4+ab^2x}{(a^2+b^2)(b^2+x^2)}\right)\,dx, x, b\sinh(c+dx)\right)}{b^2d}\\
&= -\frac{a^3\log(a+b\sinh(c+dx))}{b^2(a^2+b^2)\,d}+\frac{\sinh(c+dx)}{bd}-\frac{\text{Subst}\left(\int \frac{b^4+ab^2x}{b^2+x^2}\,dx, x, b\right.}{b^2(a^2+b^2)}\\
&= -\frac{a^3\log(a+b\sinh(c+dx))}{b^2(a^2+b^2)\,d}+\frac{\sinh(c+dx)}{bd}-\frac{a\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\,\text{s}\right.}{(a^2+b^2)\,d}\\
&= -\frac{b\tan^{-1}(\sinh(c+dx))}{(a^2+b^2)\,d}-\frac{a\log(\cosh(c+dx))}{(a^2+b^2)\,d}-\frac{a^3\log(a+b\sinh(c+}{b^2(a^2+b^2)\,d}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.16, size = 91, normalized size = 1.02

$$-\frac{\frac{\log(i-\sinh(c+dx))}{a+ib}+\frac{\log(i+\sinh(c+dx))}{a-ib}+\frac{2a^3\log(a+b\sinh(c+dx))}{b^2(a^2+b^2)}-\frac{2\sinh(c+dx)}{b}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/2*(Log[I - Sinh[c + d*x]]/(a + I*b) + Log[I + Sinh[c + d*x]]/(a - I*b) +
 (2*a^3*Log[a + b*Sinh[c + d*x]])/(b^2*(a^2 + b^2)) - (2*Sinh[c + d*x])/b)/
d
```

Maple [A]
time = 1.69, size = 169, normalized size = 1.90

method	result
derivativedivides	$\frac{-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{-8a\ln\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-16b\,\text{ar}}{8a^2+8b^2}}{d}$
default	$\frac{-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{-8a\ln\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-16b\,\text{ar}}{8a^2+8b^2}}{d}$
risch	$-\frac{ax}{b^2}+\frac{e^{dx+c}}{2bd}-\frac{e^{-dx-c}}{2bd}+\frac{2a\,d^2x}{a^2d^2+b^2d^2}+\frac{2adc}{a^2d^2+b^2d^2}+\frac{2a^3x}{b^2(a^2+b^2)}+\frac{2a^3c}{b^2d(a^2+b^2)}+\frac{i\ln(e^{dx+c}-i)b}{(a^2+b^2)d}-\frac{\ln(e^{dx}}{(a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b/(tanh(1/2*d*x+1/2*c)+1)+a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/b/(tanh
(1/2*d*x+1/2*c)-1)+a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)+16/(8*a^2+8*b^2)*(-1/2*a
*ln(tanh(1/2*d*x+1/2*c)^2+1)-b*arctan(tanh(1/2*d*x+1/2*c)))-a^3/b^2/(a^2+b^
2)*ln(a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)-a))
```

Maxima [A]
time = 0.49, size = 147, normalized size = 1.65

$$-\frac{a^3\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^2b^2+b^4)d}+\frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d}-\frac{a\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d}-\frac{(dx+c)a}{b^2d}+\frac{e^{(dx+c)}}{2\,bd}-\frac{e^{(-dx-c)}}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b^2 + b^4)*d) +
2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a^
2 + b^2)*d) - (d*x + c)*a/(b^2*d) + 1/2*e^(d*x + c)/(b*d) - 1/2*e^(-d*x - c
)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(89) = 178.
time = 0.38, size = 288, normalized size = 3.24

$$\frac{2\,(a^3+ab^2)dx\cosh(dx+c)-a^2b-b^3+(a^2b+b^3)\cosh(dx+c)^2+(a^2b+b^3)\sinh(dx+c)^2-4\,(b^3\cosh(dx+c)+b^3\sinh(dx+c))\arctan(\cosh(dx+c)+\sinh(dx+c))-2\,(a^3\cosh(dx+c)+a^3\sinh(dx+c))\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)-2\,(ab^2\cosh(dx+c)+ab^2\sinh(dx+c))\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)+2\,((a^3+ab^2)dx+(a^2b+b^3)\cosh(dx+c))\sinh(dx+c)}{2\,((a^2b^2+b^4)d\cosh(dx+c)+(a^2b^2+b^4)d\sinh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*(a^3 + a*b^2)*d*x*cosh(d*x + c) - a^2*b - b^3 + (a^2*b + b^3)*cosh(d
*x + c)^2 + (a^2*b + b^3)*sinh(d*x + c)^2 - 4*(b^3*cosh(d*x + c) + b^3*sinh
(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*(a^3*cosh(d*x + c) + a
^3*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c
))) - 2*(a*b^2*cosh(d*x + c) + a*b^2*sinh(d*x + c))*log(2*cosh(d*x + c)/(co
sh(d*x + c) - sinh(d*x + c))) + 2*((a^3 + a*b^2)*d*x + (a^2*b + b^3)*cosh(d
*x + c))*sinh(d*x + c))/((a^2*b^2 + b^4)*d*cosh(d*x + c) + (a^2*b^2 + b^4)*
d*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sinh(c + d*x)**2*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A]
time = 0.46, size = 145, normalized size = 1.63

$$-\frac{\frac{2\,a^3\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^2b^2+b^4}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)b}{a^2+b^2}+\frac{a\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^2+b^2}-\frac{e^{(dx+c)}-e^{(-dx-c)}}{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^3*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^2*b^2 + b^4)
+ (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*b/(a^2 + b^2) + a
*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2 + b^2) - (e^(d*x + c) - e^(-d
*x - c))/b)/d
```

Mupad [B]
time = 1.64, size = 249, normalized size = 2.80

$$\frac{\mathrm{e}^{c+d\,x}}{2\,b\,d}-\frac{\ln\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)}{a\,d-b\,d\,1\mathrm{i}}-\frac{a^3\,\ln\left(2\,a^4\,b^3-b^7-a^2\,b^5-a^6\,b+2\,a^7\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+b^7\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+a^6\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+2\,a^3\,b^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-4\,a^5\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+a^2\,b^5\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-2\,a^4\,b^3\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+2\,a\,b^6\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right)}{d\,a^2\,b^2+d\,b^4}-\frac{\mathrm{e}^{-c-d\,x}}{2\,b\,d}+\frac{a\,x}{b^2}-\frac{\ln\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)\,1\mathrm{i}}{-b\,d+a\,d\,1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)^2*tanh(c + d*x))/(a + b*sinh(c + d*x)),x)
```

```
[Out] exp(c + d*x)/(2*b*d) - (log(exp(c + d*x)*1i + 1)*1i)/(a*d*1i - b*d) - log(e
xp(c + d*x) + 1i)/(a*d - b*d*1i) - (a^3*log(2*a^4*b^3 - b^7 - a^2*b^5 - a^6
*b + 2*a^7*exp(d*x)*exp(c) + b^7*exp(2*c)*exp(2*d*x) + a^6*b*exp(2*c)*exp(2
*d*x) + 2*a^3*b^4*exp(d*x)*exp(c) - 4*a^5*b^2*exp(d*x)*exp(c) + a^2*b^5*exp
(2*c)*exp(2*d*x) - 2*a^4*b^3*exp(2*c)*exp(2*d*x) + 2*a*b^6*exp(d*x)*exp(c))
)/(b^4*d + a^2*b^2*d) - exp(- c - d*x)/(2*b*d) + (a*x)/b^2
```

3.410 $\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sinh[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Sinh[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$$\$\text{Aborted}$$

Verification is not applicable to the result.

```
[In] Integrate[(Sinh[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sinh^2(dx+c)\right)\tanh(dx+c)}{(fx+e)\,(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/2*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b*f) - 1/2*e^(c - d*e
/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b*f) - a*log(f*x + e)/(b^2*f) + 1/4*
integrate(-8*(a^4*e^(d*x + c) - a^3*b)/((a^2*b^3*f + b^5*f)*x + (a^2*b^3 +
b^5)*e - ((a^2*b^3*f*e^(2*c) + b^5*f*e^(2*c))*x + (a^2*b^3*e^(2*c) + b^5*e^
(2*c))*e)*e^(2*d*x) - 2*((a^3*b^2*f*e^c + a*b^4*f*e^c)*x + (a^3*b^2*e^c + a
*b^4*e^c)*e)*e^(d*x)), x) - 1/4*integrate(8*(b*e^(d*x + c) - a)/((a^2*f + b
^2*f)*x + (a^2 + b^2)*e + ((a^2*f*e^(2*c) + b^2*f*e^(2*c))*x + (a^2*e^(2*c)
 + b^2*e^(2*c))*e)*e^(2*d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(sinh(d*x + c)^2*tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sinh(c + d*x)**2*tanh(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)),
x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sinh(c+d\,x)^2\,\tanh(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)^2*tanh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((sinh(c + d*x)^2*tanh(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.411 $\int \frac{(e+fx)^3\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1294

$$\frac{a^2(e+fx)^3}{b^3d}-\frac{(e+fx)^3}{bd}-\frac{a^4(e+fx)^3}{b^3\left(a^2+b^2\right)d}+\frac{(e+fx)^4}{4bf}-\frac{6af(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{b^2d^2}+\frac{6a^3f(e+fx)^2\text{ArcTan}\big(}{b^2\left(a^2+b^2\right)d^2}$$

```
[Out] -3*a^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)
^(3/2)/d^2+3*a^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b
/(a^2+b^2)^(3/2)/d^2+6*a^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)
^(1/2)))/b/(a^2+b^2)^(3/2)/d^3-6*a^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a
+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d^3+3*a^4*f*(f*x+e)^2*ln(1+exp(2*d*x+2
*c))/b^3/(a^2+b^2)/d^2+6*I*a*f^3*polylog(3,I*exp(d*x+c))/b^2/d^4-6*I*a*f^2*
(f*x+e)*polylog(2,I*exp(d*x+c))/b^2/d^3-6*I*a^3*f^3*polylog(3,I*exp(d*x+c))
/b^2/(a^2+b^2)/d^4-6*a*f*(f*x+e)^2*arctan(exp(d*x+c))/b^2/d^2-3*a^2*f^2*(f*
x+e)*polylog(2,-exp(2*d*x+2*c))/b^3/d^3-3/2*a^4*f^3*polylog(3,-exp(2*d*x+2*
c))/b^3/(a^2+b^2)/d^4-a^3*(f*x+e)^3*sech(d*x+c)/b^2/(a^2+b^2)/d-a^4*(f*x+e)
^3*tanh(d*x+c)/b^3/(a^2+b^2)/d-6*I*a*f^3*polylog(3,-I*exp(d*x+c))/b^2/d^4+6
*I*a^3*f^2*(f*x+e)*polylog(2,I*exp(d*x+c))/b^2/(a^2+b^2)/d^3-3/2*f^3*polylo
g(3,-exp(2*d*x+2*c))/b/d^4-(f*x+e)^3*tanh(d*x+c)/b/d+3*f^2*(f*x+e)*polylog(
2,-exp(2*d*x+2*c))/b/d^3+6*I*a*f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/b^2/d^3
+6*I*a^3*f^3*polylog(3,-I*exp(d*x+c))/b^2/(a^2+b^2)/d^4-6*I*a^3*f^2*(f*x+e)
*polylog(2,-I*exp(d*x+c))/b^2/(a^2+b^2)/d^3-(f*x+e)^3/b/d+1/4*(f*x+e)^4/b/f
+3*f*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/b/d^2-a^4*(f*x+e)^3/b^3/(a^2+b^2)/d+6*a
^3*f*(f*x+e)^2*arctan(exp(d*x+c))/b^2/(a^2+b^2)/d^2+3*a^4*f^2*(f*x+e)*polyl
og(2,-exp(2*d*x+2*c))/b^3/(a^2+b^2)/d^3-3*a^2*f*(f*x+e)^2*ln(1+exp(2*d*x+2*
c))/b^3/d^2-a^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2
)^(3/2)/d+a^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^
(3/2)/d-6*a^3*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)^
(3/2)/d^4+6*a^3*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2
)^(3/2)/d^4+a^2*(f*x+e)^3/b^3/d+3/2*a^2*f^3*polylog(3,-exp(2*d*x+2*c))/b^3/
d^4+a*(f*x+e)^3*sech(d*x+c)/b^2/d+a^2*(f*x+e)^3*tanh(d*x+c)/b^3/d
```

Rubi [A]
time = 1.89, antiderivative size = 1294, normalized size of antiderivative = 1.00, number of steps used = 53, number of rules used = 18, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {5700, 3801, 3799, 2221, 2611, 2320, 6724, 32, 5686, 5559, 4265, 5702, 4269, 5692, 3403, 2296, 6744, 6874}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*(e + f*x)^3)/(b^3*d) - (e + f*x)^3/(b*d) - (a^4*(e + f*x)^3)/(b^3*(a^2
 + b^2)*d) + (e + f*x)^4/(4*b*f) - (6*a*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/
(b^2*d^2) + (6*a^3*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2)
 - (a^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2
 + b^2)^(3/2)*d) + (a^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])])/(b*(a^2 + b^2)^(3/2)*d) - (3*a^2*f*(e + f*x)^2*Log[1 + E^(2*(c + d
*x))])/(b^3*d^2) + (3*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b*d^2) + (3*
a^4*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b^3*(a^2 + b^2)*d^2) + ((6*I)*
a*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*d^3) - ((6*I)*a^3*f^2*(e
 + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) - ((6*I)*a*f^2*
(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b^2*d^3) + ((6*I)*a^3*f^2*(e + f*x)*P
olyLog[2, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) - (3*a^3*f*(e + f*x)^2*Poly
Log[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^2)
 + (3*a^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/(b*(a^2 + b^2)^(3/2)*d^2) - (3*a^2*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d
*x))])/(b^3*d^3) + (3*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(b*d^3) +
 (3*a^4*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(b^3*(a^2 + b^2)*d^3) -
 ((6*I)*a*f^3*PolyLog[3, (-I)*E^(c + d*x)])/(b^2*d^4) + ((6*I)*a^3*f^3*Poly
Log[3, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^4) + ((6*I)*a*f^3*PolyLog[3, I
*E^(c + d*x)])/(b^2*d^4) - ((6*I)*a^3*f^3*PolyLog[3, I*E^(c + d*x)])/(b^2*(
a^2 + b^2)*d^4) + (6*a^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) - (6*a^3*f^2*(e + f*x)*PolyLog[
3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) + (
3*a^2*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*b^3*d^4) - (3*f^3*PolyLog[3, -E^
(2*(c + d*x))])/(2*b*d^4) - (3*a^4*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*b^3
*(a^2 + b^2)*d^4) - (6*a^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/(b*(a^2 + b^2)^(3/2)*d^4) + (6*a^3*f^3*PolyLog[4, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^4) + (a*(e + f*x)^3*Sech
[c + d*x])/(b^2*d) - (a^3*(e + f*x)^3*Sech[c + d*x])/(b^2*(a^2 + b^2)*d) +
(a^2*(e + f*x)^3*Tanh[c + d*x])/(b^3*d) - ((e + f*x)^3*Tanh[c + d*x])/(b*d)
 - (a^4*(e + f*x)^3*Tanh[c + d*x])/(b^3*(a^2 + b^2)*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5686

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*Sech[c +
d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5700

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - Di
st[a/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*(Tanh[c + d*x]^n/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5702

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^3 \sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\tanh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)^3\tanh(c+dx)}{bd} - \frac{a\int (e+fx)^3\text{sech}(c+dx)\tanh(c+$$

$$= -\frac{(e+fx)^3}{bd} + \frac{(e+fx)^4}{4bf} + \frac{a(e+fx)^3\text{sech}(c+dx)}{b^2d} - (e+fx$$

$$= -\frac{(e+fx)^3}{bd} + \frac{(e+fx)^4}{4bf} - \frac{6af(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d^2} + 3f($$

$$= \frac{a^2(e+fx)^3}{b^3d} - \frac{(e+fx)^3}{bd} + \frac{(e+fx)^4}{4bf} - \frac{6af(e+fx)^2\tan^{-1}(}{b^2d^2}$$

$$= \frac{a^2(e+fx)^3}{b^3d} - \frac{(e+fx)^3}{bd} + \frac{(e+fx)^4}{4bf} - \frac{6af(e+fx)^2\tan^{-1}(}{b^2d^2}$$

$$= \frac{a^2(e+fx)^3}{b^3d} - \frac{(e+fx)^3}{bd} + \frac{(e+fx)^4}{4bf} - \frac{6af(e+fx)^2\tan^{-1}(}{b^2d^2}$$

$$= \frac{a^2(e+fx)^3}{b^3d} - \frac{(e+fx)^3}{bd} - \frac{a^4(e+fx)^3}{b^3(a^2+b^2)d} + \frac{(e+fx)^4}{4bf} - 6af($$

$$= \frac{a^2(e+fx)^3}{b^3d} - \frac{(e+fx)^3}{bd} - \frac{a^4(e+fx)^3}{b^3(a^2+b^2)d} + \frac{(e+fx)^4}{4bf} - 6af($$

$$= \frac{a^2(e+fx)^3}{b^3d} - \frac{(e+fx)^3}{bd} - \frac{a^4(e+fx)^3}{b^3(a^2+b^2)d} + \frac{(e+fx)^4}{4bf} - 6af($$

$$= \frac{a^2(e+fx)^3}{b^3d} - \frac{(e+fx)^3}{bd} - \frac{a^4(e+fx)^3}{b^3(a^2+b^2)d} + \frac{(e+fx)^4}{4bf} - 6af($$

$$= \frac{a^2(e+fx)^3}{b^3d} - \frac{(e+fx)^3}{bd} - \frac{a^4(e+fx)^3}{b^3(a^2+b^2)d} + \frac{(e+fx)^4}{4bf} - 6af($$

Mathematica [A]

time = 11.73, size = 1632, normalized size = 1.26

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(4*b) - (f*(12*a*d^2*e^2*(1
 + E^(2*c))*ArcTan[E^(c + d*x)] + 6*b*d^2*e^2*E^(2*c)*(2*d*x - Log[1 + E^(2
*(c + d*x))]) - 6*b*d^2*e^2*Log[1 + E^(2*(c + d*x))] + (12*I)*a*d*e*(1 + E^
(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - PolyLog[2
, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) + 6*b*d*e*E^(2*c)*f*(2*d*x
*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))]) - 6*b*d*e
*f*(2*d*x*Log[1 + E^(2*(c + d*x))] + PolyLog[2, -E^(2*(c + d*x))]) + (6*I)*
a*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Log[1 + I*E^(
c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2, I*E^(c +
d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d*x)]) - 3*b
*f^2*(2*d^2*x^2*Log[1 + E^(2*(c + d*x))] + 2*d*x*PolyLog[2, -E^(2*(c + d*x)
)] - PolyLog[3, -E^(2*(c + d*x))]) + b*E^(2*c)*f^2*(2*d^2*x^2*(2*d*x - 3*Lo
g[1 + E^(2*(c + d*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3,
 -E^(2*(c + d*x))])))/(2*(a^2 + b^2)*d^4*(1 + E^(2*c))) + (a^3*(2*d^3*e^3*S
qrt[(a^2 + b^2)*E^(2*c)]*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]] + 3*S
qrt[-a^2 - b^2]*d^3*e^2*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^
2 + b^2)*E^(2*c)])] + 3*Sqrt[-a^2 - b^2]*d^3*e*E^c*f^2*x^2*Log[1 + (b*E^(2*
c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + Sqrt[-a^2 - b^2]*d^3*E^c*f
^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*S
qrt[-a^2 - b^2]*d^3*e^2*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^
2 + b^2)*E^(2*c)])] - 3*Sqrt[-a^2 - b^2]*d^3*e*E^c*f^2*x^2*Log[1 + (b*E^(2*
c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - Sqrt[-a^2 - b^2]*d^3*E^c*f
^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 3*S
qrt[-a^2 - b^2]*d^2*E^c*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c
 - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 3*Sqrt[-a^2 - b^2]*d^2*E^c*f*(e + f*x)^2*
PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*Sq
rt[-a^2 - b^2]*d*e*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^
2 + b^2)*E^(2*c)]))] - 6*Sqrt[-a^2 - b^2]*d*E^c*f^3*x*PolyLog[3, -((b*E^(2*
c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[-a^2 - b^2]*d*e*E^
c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]
+ 6*Sqrt[-a^2 - b^2]*d*E^c*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sq
rt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[-a^2 - b^2]*E^c*f^3*PolyLog[4, -((b*E^(
2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[-a^2 - b^2]*E^c*
f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/
(b*(-a^2 - b^2)^(3/2)*d^4*Sqrt[(a^2 + b^2)*E^(2*c)]) + ((e + f*x)^3*Sech[c
+ d*x]*(a - b*Sech[c]*Sinh[d*x]))/((a^2 + b^2)*d)
```

Maple [F]
time = 2.29, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \sinh(dx+c)\left(\tanh^2(dx+c)\right)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -6*a*f^3*integrate(x^2*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x
 + 2*c) + a^2*d + b^2*d), x) - 6*b*f^3*integrate(x^2/(a^2*d*e^(2*d*x + 2*c)
 + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 12*b*f^2*e*integrate(x/(a^2*
d*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 3*b*f*(2*(
d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 + b^2)*d^2))*e^
2 - 12*a*f^2*integrate(x*e^(d*x + c + 1)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*e^(
2*d*x + 2*c) + a^2*d + b^2*d), x) - (a^3*log((b*e^(-d*x - c) - a - sqrt(a^2
 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2*b + b^3)*sqrt(a^2 +
b^2)*d) - 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c)
)*d) - (d*x + c)/(b*d))*e^3 - 6*a*f*arctan(e^(d*x + c))*e^2/((a^2 + b^2)*d^
2) + 1/4*((a^2*d*f^3 + b^2*d*f^3)*x^4 + 24*b^2*f*x*e^2 + 4*(2*b^2*f^3 + (a^
2*d*f^2 + b^2*d*f^2)*e)*x^3 + 6*(4*b^2*f^2*e + (a^2*d*f + b^2*d*f)*e^2)*x^2
 + ((a^2*d*f^3*e^(2*c) + b^2*d*f^3*e^(2*c))*x^4 + 4*(a^2*d*f^2*e^(2*c) + b^
2*d*f^2*e^(2*c))*x^3*e + 6*(a^2*d*f*e^(2*c) + b^2*d*f*e^(2*c))*x^2*e^2)*e^(
2*d*x) + 8*(a*b*f^3*x^3*e^c + 3*a*b*f^2*x^2*e^(c + 1) + 3*a*b*f*x*e^(c + 2)
)*e^(d*x))/(a^2*b*d + b^3*d + (a^2*b*d*e^(2*c) + b^3*d*e^(2*c))*e^(2*d*x))
- integrate(-2*(a^3*f^3*x^3*e^c + 3*a^3*f^2*x^2*e^(c + 1) + 3*a^3*f*x*e^(c
+ 2))*e^(d*x)/(a^2*b^2 + b^4 - (a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x) -
2*(a^3*b*e^c + a*b^3*e^c)*e^(d*x)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 12510 vs. 2(1222) = 2444.
time = 0.64, size = 12510, normalized size = 9.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/4*((a^4 + 2*a^2*b^2 + b^4)*d^4*f^3*x^4 - 8*(a^2*b^2 + b^4)*c^3*f^3 + 4*((
a^4 + 2*a^2*b^2 + b^4)*d^4*x + 2*(a^2*b^2 + b^4)*d^3)*cosh(1)^3 + 4*((a^4 +
 2*a^2*b^2 + b^4)*d^4*x + 2*(a^2*b^2 + b^4)*d^3)*sinh(1)^3 + 6*((a^4 + 2*a^
2*b^2 + b^4)*d^4*f*x^2 - 4*(a^2*b^2 + b^4)*c*d^2*f)*cosh(1)^2 + ((a^4 + 2*a
^2*b^2 + b^4)*d^4*f^3*x^4 - 8*(a^2*b^2 + b^4)*d^3*f^3*x^3 + 4*(a^4 + 2*a^2*
b^2 + b^4)*d^4*x*cosh(1)^3 + 4*(a^4 + 2*a^2*b^2 + b^4)*d^4*x*sinh(1)^3 - 8*
(a^2*b^2 + b^4)*c^3*f^3 + 6*((a^4 + 2*a^2*b^2 + b^4)*d^4*f*x^2 - 4*(a^2*b^2
 + b^4)*d^3*f*x - 4*(a^2*b^2 + b^4)*c*d^2*f)*cosh(1)^2 + 6*((a^4 + 2*a^2*b^
2 + b^4)*d^4*f*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^4*x*cosh(1) - 4*(a^2*b^2 +
 b^4)*d^3*f*x - 4*(a^2*b^2 + b^4)*c*d^2*f)*sinh(1)^2 + 4*((a^4 + 2*a^2*b^2
+ b^4)*d^4*f^2*x^3 - 6*(a^2*b^2 + b^4)*d^3*f^2*x^2 + 6*(a^2*b^2 + b^4)*c^2*
d*f^2)*cosh(1) + 4*((a^4 + 2*a^2*b^2 + b^4)*d^4*f^2*x^3 - 6*(a^2*b^2 + b^4)
*d^3*f^2*x^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*d^4*x*cosh(1)^2 + 6*(a^2*b^2 + b^4
)*c^2*d*f^2 + 3*((a^4 + 2*a^2*b^2 + b^4)*d^4*f*x^2 - 4*(a^2*b^2 + b^4)*d^3*
f*x - 4*(a^2*b^2 + b^4)*c*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 6*((a^
4 + 2*a^2*b^2 + b^4)*d^4*f*x^2 - 4*(a^2*b^2 + b^4)*c*d^2*f + 2*((a^4 + 2*a^
2*b^2 + b^4)*d^4*x + 2*(a^2*b^2 + b^4)*d^3)*cosh(1))*sinh(1)^2 + ((a^4 + 2*
a^2*b^2 + b^4)*d^4*f^3*x^4 - 8*(a^2*b^2 + b^4)*d^3*f^3*x^3 + 4*(a^4 + 2*a^2
*b^2 + b^4)*d^4*x*cosh(1)^3 + 4*(a^4 + 2*a^2*b^2 + b^4)*d^4*x*sinh(1)^3 - 8
*(a^2*b^2 + b^4)*c^3*f^3 + 6*((a^4 + 2*a^2*b^2 + b^4)*d^4*f*x^2 - 4*(a^2*b^
2 + b^4)*d^3*f*x - 4*(a^2*b^2 + b^4)*c*d^2*f)*cosh(1)^2 + 6*((a^4 + 2*a^2*b
^2 + b^4)*d^4*f*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^4*x*cosh(1) - 4*(a^2*b^2
+ b^4)*d^3*f*x - 4*(a^2*b^2 + b^4)*c*d^2*f)*sinh(1)^2 + 4*((a^4 + 2*a^2*b^2
 + b^4)*d^4*f^2*x^3 - 6*(a^2*b^2 + b^4)*d^3*f^2*x^2 + 6*(a^2*b^2 + b^4)*c^2
*d*f^2)*cosh(1) + 4*((a^4 + 2*a^2*b^2 + b^4)*d^4*f^2*x^3 - 6*(a^2*b^2 + b^4
)*d^3*f^2*x^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*d^4*x*cosh(1)^2 + 6*(a^2*b^2 + b^
4)*c^2*d*f^2 + 3*((a^4 + 2*a^2*b^2 + b^4)*d^4*f*x^2 - 4*(a^2*b^2 + b^4)*d^3
*f*x - 4*(a^2*b^2 + b^4)*c*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 12*(a
^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*f^2*x*cosh(1) + a^3*b*d^2*f*cosh(1)^2 + a^3*
b*d^2*f*sinh(1)^2 + (a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*f^2*x*cosh(1) + a^3*b*
d^2*f*cosh(1)^2 + a^3*b*d^2*f*sinh(1)^2 + 2*(a^3*b*d^2*f^2*x + a^3*b*d^2*f*
cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*f^2*
x*cosh(1) + a^3*b*d^2*f*cosh(1)^2 + a^3*b*d^2*f*sinh(1)^2 + 2*(a^3*b*d^2*f^
2*x + a^3*b*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d^
2*f^3*x^2 + 2*a^3*b*d^2*f^2*x*cosh(1) + a^3*b*d^2*f*cosh(1)^2 + a^3*b*d^2*f
*sinh(1)^2 + 2*(a^3*b*d^2*f^2*x + a^3*b*d^2*f*cosh(1))*sinh(1))*sinh(d*x +
c)^2 + 2*(a^3*b*d^2*f^2*x + a^3*b*d^2*f*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/
b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 12*(a^3*b*d^2*f^3*x^2 + 2*a^3*
b*d^2*f^2*x*cosh(1) + a^3*b*d^2*f*cosh(1)^2 + a^3*b*d^2*f*sinh(1)^2 + (a^3*
```

```
b*d^2*f^3*x^2 + 2*a^3*b*d^2*f^2*x*cosh(1) + a^3*b*d^2*f*cosh(1)^2 + a^3*b*d
^2*f*sinh(1)^2 + 2*(a^3*b*d^2*f^2*x + a^3*b*d^2*f*cosh(1))*sinh(1))*cosh(d*
x + c)^2 + 2*(a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*f^2*x*cosh(1) + a^3*b*d^2*f*c
osh(1)^2 + a^3*b*d^2*f*sinh(1)^2 + 2*(a^3*b*d^2*f^2*x + a^3*b*d^2*f*cosh(1)
)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*f
^2*x*cosh(1) + a^3*b*d^2*f*cosh(1)^2 + a^3*b*d^2*f*sinh(1)^2 + 2*(a^3*b*d^2
*f^2*x + a^3*b*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c)^2 + 2*(a^3*b*d^2*f^2*x
 + a^3*b*d^2*f*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x +
c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2) - b)/b + 1) - 4*(a^3*b*c^3*f^3 - 3*a^3*b*c^2*d*f^2*cosh(1) + 3*a^3*b*
c*d^2*f*cosh(1)^2 - a^3*b*d^3*cosh(1)^3 - a^3*b*d^3*sinh(1)^3 + (a^3*b*c^3*
f^3 - 3*a^3*b*c^2*d*f^2*cosh(1) + 3*a^3*b*c*d^2*f*cosh(1)^2 - a^3*b*d^3*cos
h(1)^3 - a^3*b*d^3*sinh(1)^3 + 3*(a^3*b*c*d^2*f - a^3*b*d^3*cosh(1))*sinh(1
)^2 - 3*(a^3*b*c^2*d*f^2 - 2*a^3*b*c*d^2*f*cosh(1) + a^3*b*d^3*cosh(1)^2)*s
inh(1))*cosh(d*x + c)^2 + 3*(a^3*b*c*d^2*f - a^3*b*d^3*cosh(1))*sinh(1)^2 +
 2*(a^3*b*c^3*f^3 - 3*a^3*b*c^2*d*f^2*cosh(1) + 3*a^3*b*c*d^2*f*cosh(1)^2 -
 a^3*b*d^3*cosh(1)^3 - a^3*b*d^3*sinh(1)^3 + 3*(a^3*b*c*d^2*f - a^3*b*d^3*c
osh(1))*sinh(1)^2 - 3*(a^3*b*c^2*d*f^2 - 2*a^3*b*c*d^2*f*cosh(1) + a^3*b*d^
3*cosh(1)^2)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*c^3*f^3 - 3*a^3*
b*c^2*d*f^2*cosh(1) + 3*a^3*b*c*d^2*f*cosh(1)^2 - a^3*b*d^3*cosh(1)^3 - a^3
*b*d^3*sinh(1)^3 + 3*(a^3*b*c*d^2*f - a^3*b*d^3*cosh(1))*sinh(1)^2 - 3*(a^3
*b*c^2*d*f^2 - 2*a^3*b*c*d^2*f*cosh(1) + a^3*b*d^3*cosh(1)^2)*sinh(1))*sinh
(d*x + c)^2 - 3*(a^3*b*c^2*d*f^2 - 2*a^3*b*c*d^2*f*cosh(1) + a^3*b*d^3*cosh
(1)^2)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x
+ c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 4*(a^3*b*c^3*f^3 - 3*a^3*b*c^2*d*
f^2*cosh(1) + 3*a^3*b*c*d^2*f*cosh(1)^2 - a^3*b*d^3*cosh(1)^3 - a^3*b*d^3*s
inh(1)^3 + (a^3*b*c^3*f^3 - 3*a^3*b*c^2*d*f^2*c...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3 \sinh(c+dx) \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*sinh(c + d*x)*tanh(c + d*x)**2/(a + b*sinh(c + d*x)),
 x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)^2\,{(e+f\,x)}^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*tanh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((sinh(c + d*x)*tanh(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.412 $\int \frac{(e+fx)^2 \sinh(c+dx) \tanh^2(c+dx)}{a+b\sinh(c+dx)}\, dx$

Optimal. Leaf size=904

$$\frac{a^2(e+fx)^2}{b^3d} - \frac{(e+fx)^2}{bd} - \frac{a^4(e+fx)^2}{b^3(a^2+b^2)d} + \frac{(e+fx)^3}{3bf} - \frac{4af(e+fx)\text{ArcTan}(e^{c+dx})}{b^2d^2} + \frac{4a^3f(e+fx)\text{ArcTan}(e^{c+c}}{b^2(a^2+b^2)d^2}$$

```
[Out] 2*a^4*f*(f*x+e)*ln(1+exp(2*d*x+2*c))/b^3/(a^2+b^2)/d^2+4*a^3*f*(f*x+e)*arct
an(exp(d*x+c))/b^2/(a^2+b^2)/d^2+2*I*a*f^2*polylog(2,-I*exp(d*x+c))/b^2/d^3
-2*I*a^3*f^2*polylog(2,-I*exp(d*x+c))/b^2/(a^2+b^2)/d^3+a^2*(f*x+e)^2/b^3/d
+f^2*polylog(2,-exp(2*d*x+2*c))/b/d^3-(f*x+e)^2*tanh(d*x+c)/b/d+2*I*a^3*f^2
*polylog(2,I*exp(d*x+c))/b^2/(a^2+b^2)/d^3-(f*x+e)^2/b/d+1/3*(f*x+e)^3/b/f+
2*f*(f*x+e)*ln(1+exp(2*d*x+2*c))/b/d^2-2*a^2*f*(f*x+e)*ln(1+exp(2*d*x+2*c))
/b^3/d^2-a^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(
3/2)/d+a^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/
2)/d+2*a^3*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(3/
2)/d^3-2*a^3*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/b/(a^2+b^2)^(
3/2)/d^3-2*a^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/b/(a^
2+b^2)^(3/2)/d^2+2*a^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)
))/b/(a^2+b^2)^(3/2)/d^2-4*a*f*(f*x+e)*arctan(exp(d*x+c))/b^2/d^2+a^4*f^2*p
olylog(2,-exp(2*d*x+2*c))/b^3/(a^2+b^2)/d^3-a^3*(f*x+e)^2*sech(d*x+c)/b^2/(
a^2+b^2)/d-a^4*(f*x+e)^2*tanh(d*x+c)/b^3/(a^2+b^2)/d-2*I*a*f^2*polylog(2,I*
exp(d*x+c))/b^2/d^3-a^4*(f*x+e)^2/b^3/(a^2+b^2)/d-a^2*f^2*polylog(2,-exp(2*
d*x+2*c))/b^3/d^3+a*(f*x+e)^2*sech(d*x+c)/b^2/d+a^2*(f*x+e)^2*tanh(d*x+c)/b
^3/d
```

Rubi [A]
time = 1.45, antiderivative size = 904, normalized size of antiderivative = 1.00, number of steps used = 44, number of rules used = 19, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.559$, Rules used = {5700, 3801, 3799, 2221, 2317, 2438, 32, 5686, 5559, 4265, 5702, 4269, 5692, 3403, 2296, 2611, 2320, 6724, 6874}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*(e + f*x)^2)/(b^3*d) - (e + f*x)^2/(b*d) - (a^4*(e + f*x)^2)/(b^3*(a^2
 + b^2)*d) + (e + f*x)^3/(3*b*f) - (4*a*f*(e + f*x)*ArcTan[E^(c + d*x)])/(b
^2*d^2) + (4*a^3*f*(e + f*x)*ArcTan[E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) - (
a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b
^2)^(3/2)*d) + (a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
])])/(b*(a^2 + b^2)^(3/2)*d) - (2*a^2*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])
```

```
/(b^3*d^2) + (2*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(b*d^2) + (2*a^4*f*(e
 + f*x)*Log[1 + E^(2*(c + d*x))])/(b^3*(a^2 + b^2)*d^2) + ((2*I)*a*f^2*Poly
Log[2, (-I)*E^(c + d*x)])/(b^2*d^3) - ((2*I)*a^3*f^2*PolyLog[2, (-I)*E^(c +
 d*x)])/(b^2*(a^2 + b^2)*d^3) - ((2*I)*a*f^2*PolyLog[2, I*E^(c + d*x)])/(b^
2*d^3) + ((2*I)*a^3*f^2*PolyLog[2, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) -
(2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b
*(a^2 + b^2)^(3/2)*d^2) + (2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^2) - (a^2*f^2*PolyLog[2, -E^
(2*(c + d*x))])/(b^3*d^3) + (f^2*PolyLog[2, -E^(2*(c + d*x))])/(b*d^3) + (a
^4*f^2*PolyLog[2, -E^(2*(c + d*x))])/(b^3*(a^2 + b^2)*d^3) + (2*a^3*f^2*Pol
yLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3
) - (2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^
2 + b^2)^(3/2)*d^3) + (a*(e + f*x)^2*Sech[c + d*x])/(b^2*d) - (a^3*(e + f*x
)^2*Sech[c + d*x])/(b^2*(a^2 + b^2)*d) + (a^2*(e + f*x)^2*Tanh[c + d*x])/(b
^3*d) - ((e + f*x)^2*Tanh[c + d*x])/(b*d) - (a^4*(e + f*x)^2*Tanh[c + d*x])
/(b^3*(a^2 + b^2)*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5686

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*Sech[c +
d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5700

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - Di
st[a/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*(Tanh[c + d*x]^n/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5702

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1)/
```

```
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\tanh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)^2\tanh(c+dx)}{bd} - \frac{a\int (e+fx)^2\text{sech}(c+dx)\tanh(c}{b^2}$$

$$= -\frac{(e+fx)^2}{bd} + \frac{(e+fx)^3}{3bf} + \frac{a(e+fx)^2\text{sech}(c+dx)}{b^2d} - \frac{(e+f}{}$$

$$= -\frac{(e+fx)^2}{bd} + \frac{(e+fx)^3}{3bf} - \frac{4af(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d^2} + \frac{2f(}{}$$

$$= \frac{a^2(e+fx)^2}{b^3d} - \frac{(e+fx)^2}{bd} + \frac{(e+fx)^3}{3bf} - \frac{4af(e+fx)\tan^{-1}(}{b^2d^2}$$

$$= \frac{a^2(e+fx)^2}{b^3d} - \frac{(e+fx)^2}{bd} + \frac{(e+fx)^3}{3bf} - \frac{4af(e+fx)\tan^{-1}(}{b^2d^2}$$

$$= \frac{a^2(e+fx)^2}{b^3d} - \frac{(e+fx)^2}{bd} + \frac{(e+fx)^3}{3bf} - \frac{4af(e+fx)\tan^{-1}(}{b^2d^2}$$

$$= \frac{a^2(e+fx)^2}{b^3d} - \frac{(e+fx)^2}{bd} - \frac{a^4(e+fx)^2}{b^3(a^2+b^2)d} + \frac{(e+fx)^3}{3bf} - \frac{4af}{}$$

$$= \frac{a^2(e+fx)^2}{b^3d} - \frac{(e+fx)^2}{bd} - \frac{a^4(e+fx)^2}{b^3(a^2+b^2)d} + \frac{(e+fx)^3}{3bf} - \frac{4af}{}$$

$$= \frac{a^2(e+fx)^2}{b^3d} - \frac{(e+fx)^2}{bd} - \frac{a^4(e+fx)^2}{b^3(a^2+b^2)d} + \frac{(e+fx)^3}{3bf} - \frac{4af}{}$$

$$= \frac{a^2(e+fx)^2}{b^3d} - \frac{(e+fx)^2}{bd} - \frac{a^4(e+fx)^2}{b^3(a^2+b^2)d} + \frac{(e+fx)^3}{3bf} - \frac{4af}{}$$

Mathematica [A]
time = 9.48, size = 1211, normalized size = 1.34

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2))/(3*b) - (a^3*((2*d^2*e^2*ArcTan[(a + b*E^(c
 + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + (2*d^2*e*E^c*f*x*Log[1 + (b*
E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*
c)] + (d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*
E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c
+ d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] - (
d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)
])])/Sqrt[(a^2 + b^2)*E^(2*c)] + (2*d*E^c*f*(e + f*x)*PolyLog[2, -((b*E^(2*
c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)]
- (2*d*E^c*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 +
b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*E^c*f^2*PolyLog[3, -((b*E^
(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c
)] + (2*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^
(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)]))/(b*(a^2 + b^2)*d^3) + (2*b*e*f*Sech[
c]*(Cosh[c]*Log[Cosh[c]*Cosh[d*x] + Sinh[c]*Sinh[d*x]] - d*x*Sinh[c]))/((a^
2 + b^2)*d^2*(Cosh[c]^2 - Sinh[c]^2)) - (4*a*e*f*ArcTan[(Sinh[c] + Cosh[c]*
Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]])/((a^2 + b^2)*d^2*Sqrt[Cosh[c]^
2 - Sinh[c]^2]) + (b*f^2*Csch[c]*((d^2*x^2)/E^ArcTanh[Coth[c]] - (I*Coth[c]
*(-(d*x*(-Pi + (2*I)*ArcTanh[Coth[c]])) - Pi*Log[1 + E^(2*d*x)] - 2*(I*d*x
+ I*ArcTanh[Coth[c]])*Log[1 - E^((2*I)*(I*d*x + I*ArcTanh[Coth[c]]))] + Pi*
Log[Cosh[d*x]] + (2*I)*ArcTanh[Coth[c]]*Log[I*Sinh[d*x + ArcTanh[Coth[c]]]]
 + I*PolyLog[2, E^((2*I)*(I*d*x + I*ArcTanh[Coth[c]]))]))/Sqrt[1 - Coth[c]^
2])*Sech[c])/((a^2 + b^2)*d^3*Sqrt[Csch[c]^2*(-Cosh[c]^2 + Sinh[c]^2)]) - (
2*a*f^2*(((-I)*Csch[c]*(I*(d*x + ArcTanh[Coth[c]])*(Log[1 - E^(-(d*x) - Arc
Tanh[Coth[c]])] - Log[1 + E^(-(d*x) - ArcTanh[Coth[c]])]) + I*(PolyLog[2, -
E^(-(d*x) - ArcTanh[Coth[c]])] - PolyLog[2, E^(-(d*x) - ArcTanh[Coth[c]])])
))/Sqrt[1 - Coth[c]^2] - (2*ArcTan[(Sinh[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[C
osh[c]^2 - Sinh[c]^2]]*ArcTanh[Coth[c]])/Sqrt[Cosh[c]^2 - Sinh[c]^2]))/((a^
2 + b^2)*d^3) + (Sech[c]*Sech[c + d*x]*(a*e^2*Cosh[c] + 2*a*e*f*x*Cosh[c] +
 a*f^2*x^2*Cosh[c] - b*e^2*Sinh[d*x] - 2*b*e*f*x*Sinh[d*x] - b*f^2*x^2*Sinh
[d*x]))/((a^2 + b^2)*d)
```

Maple [F]
time = 2.11, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \sinh(dx+c) \left(\tanh^2(dx+c)\right)}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -2*b*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 + b^
2)*d^2))*e - 4*a*f^2*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*d
*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 4*b*f^2*integrate(x/(a^2*d*e^(2*d*x
 + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - (a^3*log((b*e^(-d*x
- c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2*b
 + b^3)*sqrt(a^2 + b^2)*d) - 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^2 + b^
2)*e^(-2*d*x - 2*c))*d) - (d*x + c)/(b*d))*e^2 - 4*a*f*arctan(e^(d*x + c))*
e/((a^2 + b^2)*d^2) + 1/3*(12*b^2*f*x*e + (a^2*d*f^2 + b^2*d*f^2)*x^3 + 3*(
2*b^2*f^2 + (a^2*d*f + b^2*d*f)*e)*x^2 + ((a^2*d*f^2*e^(2*c) + b^2*d*f^2*e^
(2*c))*x^3 + 3*(a^2*d*f*e^(2*c) + b^2*d*f*e^(2*c))*x^2*e)*e^(2*d*x) + 6*(a*
b*f^2*x^2*e^c + 2*a*b*f*x*e^(c + 1))*e^(d*x))/(a^2*b*d + b^3*d + (a^2*b*d*e
^(2*c) + b^3*d*e^(2*c))*e^(2*d*x)) - integrate(-2*(a^3*f^2*x^2*e^c + 2*a^3*
f*x*e^(c + 1))*e^(d*x)/(a^2*b^2 + b^4 - (a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(
2*d*x) - 2*(a^3*b*e^c + a*b^3*e^c)*e^(d*x)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5647 vs. 2(859) = 1718.
time = 0.52, size = 5647, normalized size = 6.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/3*((a^4 + 2*a^2*b^2 + b^4)*d^3*f^2*x^3 + 6*(a^2*b^2 + b^4)*c^2*f^2 + 3*((
a^4 + 2*a^2*b^2 + b^4)*d^3*x + 2*(a^2*b^2 + b^4)*d^2)*cosh(1)^2 + ((a^4 + 2
*a^2*b^2 + b^4)*d^3*f^2*x^3 - 6*(a^2*b^2 + b^4)*d^2*f^2*x^2 + 3*(a^4 + 2*a^
2*b^2 + b^4)*d^3*x*cosh(1)^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*d^3*x*sinh(1)^2 +
6*(a^2*b^2 + b^4)*c^2*f^2 + 3*((a^4 + 2*a^2*b^2 + b^4)*d^3*f*x^2 - 4*(a^2*b
^2 + b^4)*d^2*f*x - 4*(a^2*b^2 + b^4)*c*d*f)*cosh(1) + 3*((a^4 + 2*a^2*b^2
+ b^4)*d^3*f*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*x*cosh(1) - 4*(a^2*b^2 + b
^4)*d^2*f*x - 4*(a^2*b^2 + b^4)*c*d*f)*sinh(1))*cosh(d*x + c)^2 + 3*((a^4 +
 2*a^2*b^2 + b^4)*d^3*x + 2*(a^2*b^2 + b^4)*d^2)*sinh(1)^2 + ((a^4 + 2*a^2*
b^2 + b^4)*d^3*f^2*x^3 - 6*(a^2*b^2 + b^4)*d^2*f^2*x^2 + 3*(a^4 + 2*a^2*b^2
 + b^4)*d^3*x*cosh(1)^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*d^3*x*sinh(1)^2 + 6*(a^
```

2*b^2 + b^4)*c^2*f^2 + 3*((a^4 + 2*a^2*b^2 + b^4)*d^3*f*x^2 - 4*(a^2*b^2 + b^4)*d^2*f*x - 4*(a^2*b^2 + b^4)*c*d*f)*cosh(1) + 3*((a^4 + 2*a^2*b^2 + b^4)*d^3*f*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*x*cosh(1) - 4*(a^2*b^2 + b^4)*d^2*f*x - 4*(a^2*b^2 + b^4)*c*d*f)*sinh(1))*sinh(d*x + c)^2 - 6*(a^3*b*d*f^2*x + a^3*b*d*f*cosh(1) + a^3*b*d*f*sinh(1) + (a^3*b*d*f^2*x + a^3*b*d*f*cosh(1) + a^3*b*d*f*sinh(1))*cosh(d*x + c)^2 + 2*(a^3*b*d*f^2*x + a^3*b*d*f*cosh(1) + a^3*b*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d*f^2*x + a^3*b*d*f*cosh(1) + a^3*b*d*f*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 6*(a^3*b*d*f^2*x + a^3*b*d*f*cosh(1) + a^3*b*d*f*sinh(1) + (a^3*b*d*f^2*x + a^3*b*d*f*cosh(1) + a^3*b*d*f*sinh(1))*cosh(d*x + c)^2 + 2*(a^3*b*d*f^2*x + a^3*b*d*f*cosh(1) + a^3*b*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d*f^2*x + a^3*b*d*f*cosh(1) + a^3*b*d*f*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 3*(a^3*b*c^2*f^2 - 2*a^3*b*c*d*f*cosh(1) + a^3*b*d^2*cosh(1)^2 + a^3*b*d^2*sinh(1)^2 + (a^3*b*c^2*f^2 - 2*a^3*b*c*d*f*cosh(1) + a^3*b*d^2*cosh(1)^2 + a^3*b*d^2*sinh(1)^2 - 2*(a^3*b*c*d*f - a^3*b*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(a^3*b*c^2*f^2 - 2*a^3*b*c*d*f*cosh(1) + a^3*b*d^2*cosh(1)^2 + a^3*b*d^2*sinh(1)^2 - 2*(a^3*b*c*d*f - a^3*b*d^2*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*c^2*f^2 - 2*a^3*b*c*d*f*cosh(1) + a^3*b*d^2*cosh(1)^2 + a^3*b*d^2*sinh(1)^2 - 2*(a^3*b*c*d*f - a^3*b*d^2*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 2*(a^3*b*c*d*f - a^3*b*d^2*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(a^3*b*c^2*f^2 - 2*a^3*b*c*d*f*cosh(1) + a^3*b*d^2*cosh(1)^2 + a^3*b*d^2*sinh(1)^2 + (a^3*b*c^2*f^2 - 2*a^3*b*c*d*f*cosh(1) + a^3*b*d^2*cosh(1)^2 + a^3*b*d^2*sinh(1)^2 - 2*(a^3*b*c*d*f - a^3*b*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(a^3*b*c^2*f^2 - 2*a^3*b*c*d*f*cosh(1) + a^3*b*d^2*cosh(1)^2 + a^3*b*d^2*sinh(1)^2 - 2*(a^3*b*c*d*f - a^3*b*d^2*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*c^2*f^2 - 2*a^3*b*c*d*f*cosh(1) + a^3*b*d^2*cosh(1)^2 + a^3*b*d^2*sinh(1)^2 - 2*(a^3*b*c*d*f - a^3*b*d^2*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 2*(a^3*b*c*d*f - a^3*b*d^2*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(a^3*b*d^2*f^2*x^2 - a^3*b*c^2*f^2 + (a^3*b*d^2*f^2*x^2 - a^3*b*c^2*f^2 + 2*(a^3*b*d^2*f*x + a^3*b*c*d*f)*cosh(1) + 2*(a^3*b*d^2*f*x + a^3*b*c*d*f)*sinh(1))*cosh(d*x + c)^2 + 2*(a^3*b*d^2*f^2*x^2 - a^3*b*c^2*f^2 + 2*(a^3*b*d^2*f*x + a^3*b*c*d*f)*cosh(1) + 2*(a^3*b*d^2*f*x + a^3*b*c*d*f)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d^2*f^2*x^2 - a^3*b*c^2*f^2 + 2*(a^3*b*d^2*f*x + a^3*b*c*d*f)*cosh(1) + 2*(a^3*b*d^2*f*x + a^3*b*c*d*f)*sinh(1))*sinh(d*x + c)^2 + 2*(a^3*b*d^2*f*x + a^3*b*c*d*f)*cosh(1) + 2*(a^3*b*d^2*f*x + a^3*b*c*d*f)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 3*(a^3*b*d^2*f^2*x^2 - a^3*b*c^2*f^2 + (a^3*b*d^2*f^2*x^2 - a^3*b*c^2*f^2 + 2*(a^3*b*d^2*f*x + a^3*b*c*d*f)*cosh(1) + 2*(a^3*b*d^2*f*x + a^3*b*c*d*f)*sinh(1))*cosh

```
(d*x + c)^2 + 2*(a^3*b*d^2*f^2*x^2 - a^3*b*c^2*f^2 + 2*(a^3*b*d^2*f*x + a^3
*b*c*d*f)*cosh(1) + 2*(a^3*b*d^2*f*x + a^3*b*c*d*f)*sinh(1))*cosh(d*x + c)*
sinh(d*x + c) + (a^3*b*d^2*f^2*x^2 - a^3*b*c^2*f^2 + 2*(a^3*b*d^2*f*x + a^3
*b*c*d*f)*cosh(1) + 2*(a^3*b*d^2*f*x + a^3*b*c*d*f)*sinh(1))*sinh(d*x + c)^
2 + 2*(a^3*b*d^2*f*x + a^3*b*c*d*f)*cosh(1) + 2*(a^3*b*d^2*f*x + a^3*b*c*d*
f)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) -
 (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 6*(a^3
*b*f^2*cosh(d*x + c)^2 + 2*a^3*b*f^2*cosh(d*x + c)*sinh(d*x + c) + a^3*b*f^
2*sinh(d*x + c)^2 + a^3*b*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x
 + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2))/b) - 6*(a^3*b*f^2*cosh(d*x + c)^2 + 2...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2 \sinh{(c+dx)} \tanh^2{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*sinh(c + d*x)*tanh(c + d*x)**2/(a + b*sinh(c + d*x)),
 x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)^2\,{(e+f\,x)}^2}{a+b\sinh{(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*tanh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((sinh(c + d*x)*tanh(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.413 $\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=454

$$\frac{ex}{b}+\frac{fx^2}{2b}-\frac{af\text{ArcTan}(\sinh(c+dx))}{b^2d^2}+\frac{a^3f\text{ArcTan}(\sinh(c+dx))}{b^2\left(a^2+b^2\right)d^2}-\frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d}+\frac{a^3(e\cdots}{}$$

```
[Out] e*x/b+1/2*f*x^2/b-a*f*arctan(sinh(d*x+c))/b^2/d^2+a^3*f*arctan(sinh(d*x+c))
/b^2/(a^2+b^2)/d^2-a^2*f*ln(cosh(d*x+c))/b^3/d^2+f*ln(cosh(d*x+c))/b/d^2+a^
4*f*ln(cosh(d*x+c))/b^3/(a^2+b^2)/d^2-a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2
+b^2)^(1/2)))/b/(a^2+b^2)^(3/2)/d+a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2
)^(1/2)))/b/(a^2+b^2)^(3/2)/d-a^3*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1
/2)))/b/(a^2+b^2)^(3/2)/d^2+a^3*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2
)))/b/(a^2+b^2)^(3/2)/d^2+a*(f*x+e)*sech(d*x+c)/b^2/d-a^3*(f*x+e)*sech(d*x+
c)/b^2/(a^2+b^2)/d+a^2*(f*x+e)*tanh(d*x+c)/b^3/d-(f*x+e)*tanh(d*x+c)/b/d-a^
4*(f*x+e)*tanh(d*x+c)/b^3/(a^2+b^2)/d
```

Rubi [A]
time = 0.71, antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 15, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {5700, 3801, 3556, 5686, 5559, 3855, 5702, 4269, 5692, 3403, 2296, 2221, 2317, 2438, 6874}

$$-\frac{a^2 f\log(\cosh(c+dx))}{b^3d^2}+\frac{a^2(e+fx)\tanh(c+dx)}{b^3d}+\frac{a^4 f\log(\cosh(c+dx))}{b^3d^2\left(a^2+b^2\right)}-\frac{a^4(e+fx)\tanh(c+dx)}{b^3d\left(a^2+b^2\right)}+\frac{a^3f\text{ArcTan}(\sinh(c+dx))}{b^2d^2\left(a^2+b^2\right)}-\frac{a^3f\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2\left(a^2+b^2\right)^{3/2}}+\frac{a^3f\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2\left(a^2+b^2\right)^{3/2}}-\frac{a^3(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd\left(a^2+b^2\right)^{3/2}}+\frac{a^3(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd\left(a^2+b^2\right)^{3/2}}-\frac{a^3(e+fx)\text{sech}(c+dx)}{b^2d\left(a^2+b^2\right)}-\frac{af\text{ArcTan}(\sinh(c+dx))}{b^2d^2}+\frac{a(e+fx)\text{sech}(c+dx)}{b^2d}+\frac{f\log(\cosh(c+dx))}{bd^2}-\frac{(e+fx)\tanh(c+dx)}{bd}+\frac{ex}{b}+\frac{fx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e*x)/b + (f*x^2)/(2*b) - (a*f*ArcTan[Sinh[c + d*x]])/(b^2*d^2) + (a^3*f*Ar
cTan[Sinh[c + d*x]])/(b^2*(a^2 + b^2)*d^2) - (a^3*(e + f*x)*Log[1 + (b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)^(3/2)*d) + (a^3*(e + f*x)*L
og[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)^(3/2)*d) - (a
^2*f*Log[Cosh[c + d*x]])/(b^3*d^2) + (f*Log[Cosh[c + d*x]])/(b*d^2) + (a^4*
f*Log[Cosh[c + d*x]])/(b^3*(a^2 + b^2)*d^2) - (a^3*f*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^2) + (a^3*f*PolyLog[
2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^2) + (
a*(e + f*x)*Sech[c + d*x])/(b^2*d) - (a^3*(e + f*x)*Sech[c + d*x])/(b^2*(a^
2 + b^2)*d) + (a^2*(e + f*x)*Tanh[c + d*x])/(b^3*d) - ((e + f*x)*Tanh[c + d
*x])/(b*d) - (a^4*(e + f*x)*Tanh[c + d*x])/(b^3*(a^2 + b^2)*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5686

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*Sech[c +
d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5700

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - Di
st[a/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*(Tanh[c + d*x]^n/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5702

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\tanh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)\tanh(c+dx)}{bd} - \frac{a\int (e+fx)\text{sech}(c+dx)\tanh(c+}{b^2}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} + \frac{f\log(\cosh(c+dx))}{bd^2} + \frac{a(e+fx)\text{sech}(c+dx)}{b^2d} -$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{af\tan^{-1}(\sinh(c+dx))}{b^2d^2} + \frac{f\log(\cosh(c+dx))}{bd^2} +$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{af\tan^{-1}(\sinh(c+dx))}{b^2d^2} - \frac{a^2f\log(\cosh(c+dx))}{b^3d^2}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{af\tan^{-1}(\sinh(c+dx))}{b^2d^2} - \frac{a^2f\log(\cosh(c+dx))}{b^3d^2}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{af\tan^{-1}(\sinh(c+dx))}{b^2d^2} - \frac{a^3(e+fx)\log\left(1+\frac{}{a-}\right.}{b\left(a^2+b^2\right)^{3/}}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{af\tan^{-1}(\sinh(c+dx))}{b^2d^2} + \frac{a^3f\tan^{-1}(\sinh(c+dx}{b^2\left(a^2+b^2\right)d^2}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{af\tan^{-1}(\sinh(c+dx))}{b^2d^2} + \frac{a^3f\tan^{-1}(\sinh(c+dx}{b^2\left(a^2+b^2\right)d^2}$$

Mathematica [A]
time = 2.81, size = 317, normalized size = 0.70

$$\frac{-\frac{(c+dx)(cf-d(2e+fx))}{b}-\frac{4af\,\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a^2+b^2}+\frac{2bf\log(\cosh(c+dx))}{a^2+b^2}+\frac{2a^3\left(2de\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-2cf\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+f(c+dx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)-f\,\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)+f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)}{b(a^2+b^2)^{3/2}}+\frac{2d(e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))}{a^2+b^2}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (-(((c + d*x)*(c*f - d*(2*e + f*x)))/b) - (4*a*f*ArcTan[Tanh[(c + d*x)/2]])
/(a^2 + b^2) + (2*b*f*Log[Cosh[c + d*x]])/(a^2 + b^2) + (2*a^3*(2*d*e*ArcTa
nh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*c*f*ArcTanh[(a + b*E^(c + d*x))
/Sqrt[a^2 + b^2]] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - f*PolyLo
g[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))]))/(b*(a^2 + b^2)^(3/2)) + (2*d*(e + f*x)*Sech[c
+ d*x]*(a - b*Sinh[c + d*x]))/(a^2 + b^2))/(2*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1896 vs. $2(432) = 864$.
time = 6.38, size = 1897, normalized size = 4.18

method	result	size
risch	Expression too large to display	1897

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVER
BOSE)
```

```
[Out] -2/d^2/b*a^3*f*c/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+
2*a)/(a^2+b^2)^(1/2))-4/d^2*b^2/(a^2+b^2)*f/(2*a^2+2*b^2)*a*arctan(exp(d*x+
c))-2/d*b/(a^2+b^2)^(3/2)*a^3*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(
1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+2/d*b/(a^2+b^2)^(3/2)*a^3*f/(2*a^2+2*b^2)*l
n((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-2/d/b/(a^2+b^2)^(
3/2)*a^5*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)
^(1/2)))*x-2/d^2/b/(a^2+b^2)^(3/2)*a^5*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a
^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+2*(f*x+e)*(a*exp(d*x+c)+b)/d/(a^2+
b^2)/(1+exp(2*d*x+2*c))+2/d^2/b/(a^2+b^2)^(3/2)*a^5*f/(2*a^2+2*b^2)*ln((b*e
xp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-2/d^2/b/(a^2+b^2)^(3/2)
*a^5*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/
d^2*b/(a^2+b^2)^(3/2)*a^3*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))*c+1/2*f*x^2/b+2/d/b*e*a^3/(2*a^2+2*b^2)/(a^2+b^2)
^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/2/d^2*b^3/(a^2+b
^2)^2*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d^2*b/(a^2+b^2)*f*ln(exp(d*
x+c))-2/d^2/b/(a^2+b^2)^(3/2)*a^5*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2
+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+2/d/b/(a^2+b^2)^(3/2)*a^5*e/(2*a^2+2*b
^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d^2*b^3/(a^2+b^2)^(
3/2)*a*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/
d^2*b/(a^2+b^2)^(1/2)*a*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a
^2+b^2)^(1/2))+2/d^2*b/(a^2+b^2)^(3/2)*a^3*f/(2*a^2+2*b^2)*dilog((b*exp(d*x
+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2/d^2*b/(a^2+b^2)^(3/2)*a^3*f/(
2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+
```

```
2/d^2*b/(a^2+b^2)^(3/2)*a^3*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a
)/(a^2+b^2)^(1/2))+2/d^2/b/(a^2+b^2)^(3/2)*a^5*f/(2*a^2+2*b^2)*dilog((b*exp
(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/d/(a^2+b^2)^(3/2)*e*a^3*b
/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d^2*b^3/
(a^2+b^2)*f/(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))+1/d^2*b/(a^2+b^2)^2*f*ln(b*e
xp(2*d*x+2*c)+2*a*exp(d*x+c)-b)*a^2+2/d^2*b/(a^2+b^2)^(3/2)*a^3*f/(2*a^2+2*
b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/d/b/(a^2+
b^2)^(3/2)*a^5*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+
b^2)^(1/2)))*x+2/d^2*b/(a^2+b^2)*a^2*f/(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))-2
/d^2*b/(a^2+b^2)^(5/2)*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*
a^3-2/d^2*b^3/(a^2+b^2)^(5/2)*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^
(1/2))*a-2/d^2*b/(a^2+b^2)*a^2*f/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(
d*x+c)-b)+e*x/b-2/d^2/(a^2+b^2)^(3/2)*a^3*b*f*c/(2*a^2+2*b^2)*arctanh(1/2*(
2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-4/d^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*ar
ctan(exp(d*x+c))-1/d^2*b^3/(a^2+b^2)*f/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*
a*exp(d*x+c)-b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/2*(4*a^3*integrate(-x*e^(d*x + c)/(a^2*b^2 + b^4 - (a^2*b^2*e^(2*c) + b^
4*e^(2*c))*e^(2*d*x) - 2*(a^3*b*e^c + a*b^3*e^c)*e^(d*x)), x) - ((a^2*d*e^(
2*c) + b^2*d*e^(2*c))*x^2*e^(2*d*x) + 4*a*b*x*e^(d*x + c) + 4*b^2*x + (a^2*
d + b^2*d)*x^2)/(a^2*b*d + b^3*d + (a^2*b*d*e^(2*c) + b^3*d*e^(2*c))*e^(2*d
*x)) + 4*b*x/((a^2 + b^2)*d) + 4*a*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) -
2*b*log(e^(2*d*x + 2*c) + 1)/((a^2 + b^2)*d^2))*f - (a^3*log((b*e^(-d*x - c
) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^2*b +
b^3)*sqrt(a^2 + b^2)*d) - 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^2 + b^2)*
e^(-2*d*x - 2*c))*d) - (d*x + c)/(b*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1794 vs. $2(438) = 876$.
time = 0.47, size = 1794, normalized size = 3.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] 1/2*((a^4 + 2*a^2*b^2 + b^4)*d^2*f*x^2 + ((a^4 + 2*a^2*b^2 + b^4)*d^2*f*x^2
 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*x*cosh(1) + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*
x*sinh(1) - 4*(a^2*b^2 + b^4)*d*f*x)*cosh(d*x + c)^2 + ((a^4 + 2*a^2*b^2 +
b^4)*d^2*f*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*x*cosh(1) + 2*(a^4 + 2*a^2*b
^2 + b^4)*d^2*x*sinh(1) - 4*(a^2*b^2 + b^4)*d*f*x)*sinh(d*x + c)^2 - 2*(a^3
*b*f*cosh(d*x + c)^2 + 2*a^3*b*f*cosh(d*x + c)*sinh(d*x + c) + a^3*b*f*sinh
(d*x + c)^2 + a^3*b*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sin
h(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)
/b + 1) + 2*(a^3*b*f*cosh(d*x + c)^2 + 2*a^3*b*f*cosh(d*x + c)*sinh(d*x + c
) + a^3*b*f*sinh(d*x + c)^2 + a^3*b*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(
d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2) - b)/b + 1) - 2*(a^3*b*c*f - a^3*b*d*cosh(1) - a^3*b*d*sinh(1)
+ (a^3*b*c*f - a^3*b*d*cosh(1) - a^3*b*d*sinh(1))*cosh(d*x + c)^2 + 2*(a^3*
b*c*f - a^3*b*d*cosh(1) - a^3*b*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a
^3*b*c*f - a^3*b*d*cosh(1) - a^3*b*d*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 +
b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/
b^2) + 2*a) + 2*(a^3*b*c*f - a^3*b*d*cosh(1) - a^3*b*d*sinh(1) + (a^3*b*c*f
 - a^3*b*d*cosh(1) - a^3*b*d*sinh(1))*cosh(d*x + c)^2 + 2*(a^3*b*c*f - a^3*
b*d*cosh(1) - a^3*b*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*c*f - a
^3*b*d*cosh(1) - a^3*b*d*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*lo
g(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
- 2*(a^3*b*d*f*x + a^3*b*c*f + (a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x + c)^2 +
2*(a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d*f*x + a^
3*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*s
inh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
b)/b) + 2*(a^3*b*d*f*x + a^3*b*c*f + (a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x + c
)^2 + 2*(a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d*f*
x + a^3*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c)
 + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2) - b)/b) - 4*((a^3*b + a*b^3)*f*cosh(d*x + c)^2 + 2*(a^3*b + a*b^3)*f*co
sh(d*x + c)*sinh(d*x + c) + (a^3*b + a*b^3)*f*sinh(d*x + c)^2 + (a^3*b + a*
b^3)*f)*arctan(cosh(d*x + c) + sinh(d*x + c)) + 2*((a^4 + 2*a^2*b^2 + b^4)*
d^2*x + 2*(a^2*b^2 + b^4)*d)*cosh(1) + 4*((a^3*b + a*b^3)*d*f*x + (a^3*b +
a*b^3)*d*cosh(1) + (a^3*b + a*b^3)*d*sinh(1))*cosh(d*x + c) + 2*((a^2*b^2 +
 b^4)*f*cosh(d*x + c)^2 + 2*(a^2*b^2 + b^4)*f*cosh(d*x + c)*sinh(d*x + c) +
 (a^2*b^2 + b^4)*f*sinh(d*x + c)^2 + (a^2*b^2 + b^4)*f)*log(2*cosh(d*x + c)
/(cosh(d*x + c) - sinh(d*x + c))) + 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*x + 2*(a
^2*b^2 + b^4)*d)*sinh(1) + 2*(2*(a^3*b + a*b^3)*d*f*x + 2*(a^3*b + a*b^3)*d
*cosh(1) + 2*(a^3*b + a*b^3)*d*sinh(1) + ((a^4 + 2*a^2*b^2 + b^4)*d^2*f*x^2
 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*x*cosh(1) + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*
x*sinh(1) - 4*(a^2*b^2 + b^4)*d*f*x)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b
+ 2*a^2*b^3 + b^5)*d^2*cosh(d*x + c)^2 + 2*(a^4*b + 2*a^2*b^3 + b^5)*d^2*co
sh(d*x + c)*sinh(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*d^2*sinh(d*x + c)^2 +
 (a^4*b + 2*a^2*b^3 + b^5)*d^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\sinh{(c+dx)}\tanh^{2}{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*sinh(c + d*x)*tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)^2\,(e+f\,x)}{a+b\sinh{(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*tanh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((sinh(c + d*x)*tanh(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.414 $\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=121

$$\frac{a^2x}{b\,(a^2+b^2)}+\frac{bx}{a^2+b^2}+\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}\,d}+\frac{a\mathrm{sech}(c+dx)}{(a^2+b^2)\,d}-\frac{b\tanh(c+dx)}{(a^2+b^2)\,d}$$

```
[Out] a^2*x/b/(a^2+b^2)+b*x/(a^2+b^2)+2*a^3*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^
2+b^2)^(1/2))/b/(a^2+b^2)^(3/2)/d+a*sech(d*x+c)/(a^2+b^2)/d-b*tanh(d*x+c)/(
a^2+b^2)/d
```

Rubi [A]
time = 0.18, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2981, 2686, 8, 3554, 2814, 2739, 632, 210}

$$-\frac{b\tanh(c+dx)}{d\,(a^2+b^2)}+\frac{a\mathrm{sech}(c+dx)}{d\,(a^2+b^2)}+\frac{a^2x}{b\,(a^2+b^2)}+\frac{bx}{a^2+b^2}+\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd\,(a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*x)/(b*(a^2 + b^2)) + (b*x)/(a^2 + b^2) + (2*a^3*ArcTanh[(b - a*Tanh[(c
 + d*x)/2])/Sqrt[a^2 + b^2]])/(b*(a^2 + b^2)^(3/2)*d) + (a*Sech[c + d*x])/(
(a^2 + b^2)*d) - (b*Tanh[c + d*x])/((a^2 + b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\int \operatorname{sech}(c+dx)\tanh(c+dx)\,dx}{a^2+b^2} + \frac{a^2\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx}{a^2+b^2} + \frac{b\int \tanh^2(c}{a^2+}$$

$$= \frac{a^2x}{b(a^2+b^2)} - \frac{b\tanh(c+dx)}{(a^2+b^2)\,d} - \frac{a^3\int \frac{1}{a+b\sinh(c+dx)}\,dx}{b(a^2+b^2)} + \frac{b\int 1\,dx}{a^2+b^2} + \frac{a\text{Subst}}{}$$

$$= \frac{a^2x}{b(a^2+b^2)} + \frac{bx}{a^2+b^2} + \frac{a\text{sech}(c+dx)}{(a^2+b^2)\,d} - \frac{b\tanh(c+dx)}{(a^2+b^2)\,d} + \frac{(2ia^3)\,\text{Subst}(}{}$$

$$= \frac{a^2x}{b(a^2+b^2)} + \frac{bx}{a^2+b^2} + \frac{a\text{sech}(c+dx)}{(a^2+b^2)\,d} - \frac{b\tanh(c+dx)}{(a^2+b^2)\,d} - \frac{(4ia^3)\,\text{Subst}(}{}$$

$$= \frac{a^2x}{b(a^2+b^2)} + \frac{bx}{a^2+b^2} + \frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}\,d} + \frac{a\text{sech}(c+dx)}{(a^2+b^2)\,d}$$

Mathematica [A]
time = 0.38, size = 96, normalized size = 0.79

$$\frac{\frac{c+dx}{b} + \frac{2a^3\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{b(-a^2-b^2)^{3/2}} + \frac{\text{sech}(c+dx)(a-b\sinh(c+dx))}{a^2+b^2}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
[Out] ((c + d*x)/b + (2*a^3*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(
b*(-a^2 - b^2)^(3/2)) + (Sech[c + d*x]*(a - b*Sinh[c + d*x]))/(a^2 + b^2))/
d
```

Maple [A]
time = 1.44, size = 125, normalized size = 1.03

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b} - \frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b} - \frac{2a^3\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{\frac{3}{2}}} - \frac{2\left(b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^2+b^2\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b} - \frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b} - \frac{2a^3\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{\frac{3}{2}}} - \frac{2\left(b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^2+b^2\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d}$

risch	$\frac{x}{b}+\frac{2a\,\mathrm{e}^{dx+c}+2b}{d(a^2+b^2)(1+\mathrm{e}^{2dx+2c})}+\frac{a^3\ln\left(\mathrm{e}^{dx+c}+\frac{a(a^2+b^2)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b(a^2+b^2)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}db}-\frac{a^3\ln\left(\mathrm{e}^{dx+c}+\frac{a(a^2+b^2)^{\frac{3}{2}}-a^4-2a^2b^2-}{b(a^2+b^2)^{\frac{3}{2}}}\right.}{(a^2+b^2)^{\frac{3}{2}}db}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b*ln(tanh(1/2*d*x+1/2*c)+1)-1/b*ln(tanh(1/2*d*x+1/2*c)-1)-2/b*a^3/(a
^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-2/
(a^2+b^2)*(b*tanh(1/2*d*x+1/2*c)-a)/(tanh(1/2*d*x+1/2*c)^2+1))
```

Maxima [A]
time = 0.46, size = 141, normalized size = 1.17

$$-\frac{a^3\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{(a^2b+b^3)\sqrt{a^2+b^2}\,d}+\frac{2\left(ae^{(-dx-c)}-b\right)}{(a^2+b^2+(a^2+b^2)e^{(-2\,dx-2\,c)})d}+\frac{dx+c}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -a^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(
a^2 + b^2)))/((a^2*b + b^3)*sqrt(a^2 + b^2)*d) + 2*(a*e^(-d*x - c) - b)/((a
^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*d) + (d*x + c)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 459 vs. 2(118) = 236.
time = 0.44, size = 459, normalized size = 3.79

$$\frac{(a^4+2\,a^2b^2+b^4)dx\cosh(dx+c)^2+(a^4+2\,a^2b^2+b^4)dx\sinh(dx+c)^2+2\,a^2b^2+2\,b^4+(a^4+2\,a^2b^2+b^4)dx+(a^3\cosh(dx+c)^2+2\,a^3\cosh(dx+c)\sinh(dx+c)+a^3\sinh(dx+c)^2+a^3)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2+b^2+2\,(b^2\cosh(dx+c)+ab)\sinh(dx+c)+2\sqrt{a^2+b^2}(b\cosh(dx+c)+b\sinh(dx+c)+a)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)-b}\right)+2\,(a^3b+ab^3)\cosh(dx+c)+2\,(a^3b+ab^3+(a^4+2\,a^2b^2+b^4)dx\cosh(dx+c))\sinh(dx+c)}{(a^4b+2\,a^2b^3+b^5)d\cosh(dx+c)^2+2\,(a^4b+2\,a^2b^3+b^5)d\cosh(dx+c)\sinh(dx+c)+(a^4b+2\,a^2b^3+b^5)d\sinh(dx+c)^2+(a^4b+2\,a^2b^3+b^5)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] ((a^4 + 2*a^2*b^2 + b^4)*d*x*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d*x*
sinh(d*x + c)^2 + 2*a^2*b^2 + 2*b^4 + (a^4 + 2*a^2*b^2 + b^4)*d*x + (a^3*co
sh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*x + c)^2 + a
^3)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*
cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2
*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^
2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*
x + c) - b)) + 2*(a^3*b + a*b^3)*cosh(d*x + c) + 2*(a^3*b + a*b^3 + (a^4 +
```

```
2*a^2*b^2 + b^4)*d*x*cosh(d*x + c))*sinh(d*x + c))/((a^4*b + 2*a^2*b^3 + b^
5)*d*cosh(d*x + c)^2 + 2*(a^4*b + 2*a^2*b^3 + b^5)*d*cosh(d*x + c)*sinh(d*x
 + c) + (a^4*b + 2*a^2*b^3 + b^5)*d*sinh(d*x + c)^2 + (a^4*b + 2*a^2*b^3 +
b^5)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh (c+dx) \tanh ^2(c+dx)}{a+b \sinh (c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
[Out] Integral(sinh(c + d*x)*tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [A]
time = 0.48, size = 128, normalized size = 1.06

$$-\frac{\frac{a^3 \log \left(\frac{\left|2\, be^{(dx+c)}+2\, a-2\, \sqrt{a^2+b^2}\right|}{\left|2\, be^{(dx+c)}+2\, a+2\, \sqrt{a^2+b^2}\right|}\right)}{(a^2 b+b^3)\sqrt{a^2+b^2}} - \frac{dx+c}{b} - \frac{2\,(ae^{(dx+c)}+b)}{(a^2+b^2)(e^{(2\,dx+2\,c)}+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] -(a^3*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c
) + 2*a + 2*sqrt(a^2 + b^2)))/((a^2*b + b^3)*sqrt(a^2 + b^2)) - (d*x + c)/b
 - 2*(a*e^(d*x + c) + b)/((a^2 + b^2)*(e^(2*d*x + 2*c) + 1)))/d
```

Mupad [B]
time = 2.00, size = 468, normalized size = 3.87

$$\frac{\frac{2b}{d(a^2+b^2)}+\frac{2a\,e^{c+dx}}{d(a^2+b^2)}}{e^{2c+2dx}+1}+\frac{x}{b}+\frac{2\,\mathrm{atan}\left(\left(e^{dx}\,e^{c}\left(\frac{2a^3}{b^3 d\,(a^2 b+b^3)\sqrt{a^6}\,(a^2+b^2)}+\frac{2\left(a\,b^3 d\sqrt{a^6}+a^3 b\,d\sqrt{a^6}\right)}{a^2 b^2 (a^2 b+b^3)\sqrt{-b^2 d^2 (a^2+b^2)^3}\,\sqrt{-a^6 b^2 d^2-3a^4 b^4 d^2-3a^2 b^6 d^2-b^8 d^2}}\right)-\frac{2\left(b^4 d\sqrt{a^6}+a^2 b^2 d\sqrt{a^6}\right)}{a^2 b^2 (a^2 b+b^3)\sqrt{-b^2 d^2 (a^2+b^2)^3}\,\sqrt{-a^6 b^2 d^2-3a^4 b^4 d^2-3a^2 b^6 d^2-b^8 d^2}}\right)\left(\frac{b^4\sqrt{-a^6 b^2 d^2-3a^4 b^4 d^2-3a^2 b^6 d^2-b^8 d^2}}{2}+\frac{a^2 b^2\sqrt{-a^6 b^2 d^2-3a^4 b^4 d^2-3a^2 b^6 d^2-b^8 d^2}}{2}\right)\right)\sqrt{a^6}}{\sqrt{-a^6 b^2 d^2-3a^4 b^4 d^2-3a^2 b^6 d^2-b^8 d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*tanh(c + d*x)^2)/(a + b*sinh(c + d*x)),x)
[Out] ((2*b)/(d*(a^2 + b^2)) + (2*a*exp(c + d*x))/(d*(a^2 + b^2)))/(exp(2*c + 2*d
*x) + 1) + x/b + (2*atan((exp(d*x)*exp(c)*((2*a^3)/(b^3*d*(a^2*b + b^3)*(a^
6)^(1/2)*(a^2 + b^2)) + (2*(a*b^3*d*(a^6)^(1/2) + a^3*b*d*(a^6)^(1/2)))/(a^
2*b^2*(a^2*b + b^3)*(-b^2*d^2*(a^2 + b^2)^3)^(1/2)*(- b^8*d^2 - 3*a^2*b^6*d
^2 - 3*a^4*b^4*d^2 - a^6*b^2*d^2)^(1/2))) - (2*(b^4*d*(a^6)^(1/2) + a^2*b^2
*d*(a^6)^(1/2)))/(a^2*b^2*(a^2*b + b^3)*(-b^2*d^2*(a^2 + b^2)^3)^(1/2)*(- b
^8*d^2 - 3*a^2*b^6*d^2 - 3*a^4*b^4*d^2 - a^6*b^2*d^2)^(1/2)))*((b^4*(- b^8*
d^2 - 3*a^2*b^6*d^2 - 3*a^4*b^4*d^2 - a^6*b^2*d^2)^(1/2))/2 + (a^2*b^2*(- b
^8*d^2 - 3*a^2*b^6*d^2 - 3*a^4*b^4*d^2 - a^6*b^2*d^2)^(1/2))/2))*(a^6)^(1/2
))/(- b^8*d^2 - 3*a^2*b^6*d^2 - 3*a^4*b^4*d^2 - a^6*b^2*d^2)^(1/2)
```

3.415 $\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sinh[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Sinh[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$$\$\text{Aborted}$$

Verification is not applicable to the result.

```
[In] Integrate[(Sinh[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(dx+c)\left(\tanh^2(dx+c)\right)}{(fx+e)\,(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -2*a^3*integrate(-e^(d*x + c)/((a^2*b^2*f + b^4*f)*x + (a^2*b^2 + b^4)*e -
((a^2*b^2*f*e^(2*c) + b^4*f*e^(2*c))*x + (a^2*b^2*e^(2*c) + b^4*e^(2*c))*e)
*e^(2*d*x) - 2*((a^3*b*f*e^c + a*b^3*f*e^c)*x + (a^3*b*e^c + a*b^3*e^c)*e)*
e^(d*x)), x) + 2*(a*e^(d*x + c) + b)/((a^2*d*f + b^2*d*f)*x + (a^2*d + b^2*
d)*e + ((a^2*d*f*e^(2*c) + b^2*d*f*e^(2*c))*x + (a^2*d*e^(2*c) + b^2*d*e^(2
*c))*e)*e^(2*d*x)) + log(f*x + e)/(b*f) + 1/2*integrate(4*(a*f*e^(d*x + c)
+ b*f)/((a^2*d*f^2 + b^2*d*f^2)*x^2 + 2*(a^2*d*f + b^2*d*f)*x*e + (a^2*d +
b^2*d)*e^2 + ((a^2*d*f^2*e^(2*c) + b^2*d*f^2*e^(2*c))*x^2 + 2*(a^2*d*f*e^(2
*c) + b^2*d*f*e^(2*c))*x*e + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^2)*e^(2*d*x)
), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(sinh(d*x + c)*tanh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sinh(c + d*x)*tanh(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)),
x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sinh(c+d\,x)\,\tanh(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*tanh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((sinh(c + d*x)*tanh(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.416 $\int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1479

$$\frac{a^2(e+fx)^2\mathrm{ArcTan}\left(e^{c+dx}\right)}{b^3d}+\frac{(e+fx)^2\mathrm{ArcTan}\left(e^{c+dx}\right)}{bd}-\frac{2a^4(e+fx)^2\mathrm{ArcTan}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)^2d}-\frac{a^4(e+fx)^2\mathrm{ArcTan}\left(e^{c}\right.}{b^3\left(a^2+b^2\right)d}$$

```
[Out] I*a^2*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b^3/d^2+I*a^4*f^2*polylog(3,I*exp(d
*x+c))/b^3/(a^2+b^2)/d^3-a^4*f*(f*x+e)*sech(d*x+c)/b^3/(a^2+b^2)/d^2-1/2*a^
3*(f*x+e)^2*sech(d*x+c)^2/b^2/(a^2+b^2)/d-a*f*(f*x+e)*tanh(d*x+c)/b^2/d^2+1
/2*a^2*(f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/b^3/d-I*a^2*f^2*polylog(3,I*exp(d*
x+c))/b^3/d^3-a^4*(f*x+e)^2*arctan(exp(d*x+c))/b^3/(a^2+b^2)/d-a^3*f^2*ln(c
osh(d*x+c))/b^2/(a^2+b^2)/d^3-2*a^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a
^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2-2*a^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2+2*I*a^4*f*(f*x+e)*polylog(2,-I*exp(d*x+c)
)/b/(a^2+b^2)^2/d^2+I*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b/d^2+(f*x+e)^2*arc
tan(exp(d*x+c))/b/d+f^2*arctan(sinh(d*x+c))/b/d^3+I*a^4*f*(f*x+e)*polylog(2
,-I*exp(d*x+c))/b^3/(a^2+b^2)/d^2+2*I*a^4*f^2*polylog(3,I*exp(d*x+c))/b/(a^
2+b^2)^2/d^3-2*I*a^4*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b/(a^2+b^2)^2/d^2-I*
a^4*f*(f*x+e)*polylog(2,I*exp(d*x+c))/b^3/(a^2+b^2)/d^2+a^2*(f*x+e)^2*arcta
n(exp(d*x+c))/b^3/d+a^3*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/(a^2+b^2)^2/d+a*f^2*
ln(cosh(d*x+c))/b^2/d^3-a^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2))
)/(a^2+b^2)^2/d-a^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b
^2)^2/d+2*a^3*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/
d^3+2*a^3*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^3+
I*f^2*polylog(3,-I*exp(d*x+c))/b/d^3-f*(f*x+e)*sech(d*x+c)/b/d^2-1/2*(f*x+e
)^2*sech(d*x+c)*tanh(d*x+c)/b/d-I*f^2*polylog(3,I*exp(d*x+c))/b/d^3-I*f*(f*
x+e)*polylog(2,-I*exp(d*x+c))/b/d^2-2*a^4*(f*x+e)^2*arctan(exp(d*x+c))/b/(a
^2+b^2)^2/d+a^4*f^2*arctan(sinh(d*x+c))/b^3/(a^2+b^2)/d^3+a^3*f*(f*x+e)*pol
ylog(2,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^2+I*a^2*f^2*polylog(3,-I*exp(d*x+c))/
b^3/d^3+a^2*f*(f*x+e)*sech(d*x+c)/b^3/d^2+a^3*f*(f*x+e)*tanh(d*x+c)/b^2/(a^
2+b^2)/d^2-1/2*a^4*(f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/b^3/(a^2+b^2)/d-I*a^2*
f*(f*x+e)*polylog(2,-I*exp(d*x+c))/b^3/d^2-2*I*a^4*f^2*polylog(3,-I*exp(d*x
+c))/b/(a^2+b^2)^2/d^3-I*a^4*f^2*polylog(3,-I*exp(d*x+c))/b^3/(a^2+b^2)/d^3
-a^2*f^2*arctan(sinh(d*x+c))/b^3/d^3-1/2*a^3*f^2*polylog(3,-exp(2*d*x+2*c))
/(a^2+b^2)^2/d^3+1/2*a*(f*x+e)^2*sech(d*x+c)^2/b^2/d
```

Rubi [A]
time = 2.03, antiderivative size = 1479, normalized size of antiderivative = 1.00, number of steps used = 71, number of rules used = 17, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {5686, 5563, 4265, 2611, 2320, 6724, 4271, 3855, 5702, 5559, 4269, 3556, 5692, 5680, 2221, 6874, 3799}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Tanh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^3*d) + ((e + f*x)^2*ArcTan[E^(c +
d*x)])/(b*d) - (2*a^4*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b*(a^2 + b^2)^2*d)
- (a^4*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^3*(a^2 + b^2)*d) - (a^2*f^2*ArcT
an[Sinh[c + d*x]])/(b^3*d^3) + (f^2*ArcTan[Sinh[c + d*x]])/(b*d^3) + (a^4*f
^2*ArcTan[Sinh[c + d*x]])/(b^3*(a^2 + b^2)*d^3) - (a^3*(e + f*x)^2*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) - (a^3*(e + f*x)^
2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) + (a^3*
(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)^2*d) + (a*f^2*Log[Cosh[c
 + d*x]])/(b^2*d^3) - (a^3*f^2*Log[Cosh[c + d*x]])/(b^2*(a^2 + b^2)*d^3) -
(I*a^2*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^3*d^2) - (I*f*(e + f*x)
*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + ((2*I)*a^4*f*(e + f*x)*PolyLog[2,
(-I)*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^2) + (I*a^4*f*(e + f*x)*PolyLog[2, (-
I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) + (I*a^2*f*(e + f*x)*PolyLog[2, I*E^
(c + d*x)])/(b^3*d^2) + (I*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b*d^2) -
 ((2*I)*a^4*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^2) -
(I*a^4*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) - (2*a^
3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 +
 b^2)^2*d^2) - (2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))])/((a^2 + b^2)^2*d^2) + (a^3*f*(e + f*x)*PolyLog[2, -E^(2*(c + d
*x))])/((a^2 + b^2)^2*d^2) + (I*a^2*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b^3*
d^3) + (I*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b*d^3) - ((2*I)*a^4*f^2*PolyLo
g[3, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^3) - (I*a^4*f^2*PolyLog[3, (-I)*
E^(c + d*x)])/(b^3*(a^2 + b^2)*d^3) - (I*a^2*f^2*PolyLog[3, I*E^(c + d*x)])
/(b^3*d^3) - (I*f^2*PolyLog[3, I*E^(c + d*x)])/(b*d^3) + ((2*I)*a^4*f^2*Pol
yLog[3, I*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^3) + (I*a^4*f^2*PolyLog[3, I*E^(
c + d*x)])/(b^3*(a^2 + b^2)*d^3) + (2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) + (2*a^3*f^2*PolyLog[3, -((b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) - (a^3*f^2*PolyLog
[3, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^3) + (a^2*f*(e + f*x)*Sech[c + d*
x])/(b^3*d^2) - (f*(e + f*x)*Sech[c + d*x])/(b*d^2) - (a^4*f*(e + f*x)*Sech
[c + d*x])/(b^3*(a^2 + b^2)*d^2) + (a*(e + f*x)^2*Sech[c + d*x]^2)/(2*b^2*d
) - (a^3*(e + f*x)^2*Sech[c + d*x]^2)/(2*b^2*(a^2 + b^2)*d) - (a*f*(e + f*x
)*Tanh[c + d*x])/(b^2*d^2) + (a^3*f*(e + f*x)*Tanh[c + d*x])/(b^2*(a^2 + b^
2)*d^2) + (a^2*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*b^3*d) - ((e + f
*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*b*d) - (a^4*(e + f*x)^2*Sech[c + d*x]
*Tanh[c + d*x])/(2*b^3*(a^2 + b^2)*d)
```

<u>Rule 2221</u>

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5563

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5686

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*Sech[c +
d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5702

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\mathrm{sech}(c+dx)\tanh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\mathrm{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}}{b}$$

$$= -\frac{a\int (e+fx)^2\mathrm{sech}^2(c+dx)\tanh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^2\mathrm{sech}^2(c+dx)\tanh(}{a+b\sinh(c+dx)}}{b^2}$$

$$= \frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{f(e+fx)\mathrm{sech}(c+dx)}{bd^2} + \frac{a(e+fx)^2\mathrm{sech}^2(c+}{2b^2d}$$

$$= \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} + \frac{f^2\tan^{-1}(\sinh(c+dx))}{bd^3} - \frac{2if(e+fx)\mathrm{Li}_2\left(-ie^{c}\right.}{bd^2}$$

$$= \frac{a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{a^2f^2\tan^{-1}(\sinh(c}{b^3d^3}$$

$$= \frac{a^3(e+fx)^3}{3\left(a^2+b^2\right)^2f} + \frac{a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{a}{}$$

$$= \frac{a^3(e+fx)^3}{3\left(a^2+b^2\right)^2f} + \frac{a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{a}{}$$

$$= \frac{a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(e+fx)^2\tan^{-}}{b\left(a^2+b^2\right)^2}$$

$$= \frac{a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(e+fx)^2\tan^{-}}{b\left(a^2+b^2\right)^2}$$

$$= \frac{a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(e+fx)^2\tan^{-}}{b\left(a^2+b^2\right)^2}$$

$$= \frac{a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(e+fx)^2\tan^{-}}{b\left(a^2+b^2\right)^2}$$

$$= \frac{a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(e+fx)^2\tan^{-}}{b\left(a^2+b^2\right)^2}$$

Mathematica [A]

time = 20.91, size = 2713, normalized size = 1.83

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Tanh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-12*a^3*d^3*e^2*E^(2*c)*x - 12*a^3*d*E^(2*c)*f^2*x - 12*a*b^2*d*E^(2*c)*f^
2*x - 12*a^3*d^3*e*E^(2*c)*f*x^2 - 4*a^3*d^3*E^(2*c)*f^2*x^3 + 18*a^2*b*d^2
*e^2*ArcTan[E^(c + d*x)] + 6*b^3*d^2*e^2*ArcTan[E^(c + d*x)] + 18*a^2*b*d^2
*e^2*E^(2*c)*ArcTan[E^(c + d*x)] + 6*b^3*d^2*e^2*E^(2*c)*ArcTan[E^(c + d*x)
] + 12*a^2*b*f^2*ArcTan[E^(c + d*x)] + 12*b^3*f^2*ArcTan[E^(c + d*x)] + 12*
a^2*b*E^(2*c)*f^2*ArcTan[E^(c + d*x)] + 12*b^3*E^(2*c)*f^2*ArcTan[E^(c + d*
x)] + (18*I)*a^2*b*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + (6*I)*b^3*d^2*e*f*x*L
og[1 - I*E^(c + d*x)] + (18*I)*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x
)] + (6*I)*b^3*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (9*I)*a^2*b*d^2*f
^2*x^2*Log[1 - I*E^(c + d*x)] + (3*I)*b^3*d^2*f^2*x^2*Log[1 - I*E^(c + d*x)
] + (9*I)*a^2*b*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (3*I)*b^3*d^2*
E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] - (18*I)*a^2*b*d^2*e*f*x*Log[1 + I*E
^(c + d*x)] - (6*I)*b^3*d^2*e*f*x*Log[1 + I*E^(c + d*x)] - (18*I)*a^2*b*d^2
*e*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (6*I)*b^3*d^2*e*E^(2*c)*f*x*Log[1 +
 I*E^(c + d*x)] - (9*I)*a^2*b*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - (3*I)*b^
3*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - (9*I)*a^2*b*d^2*E^(2*c)*f^2*x^2*Log[
1 + I*E^(c + d*x)] - (3*I)*b^3*d^2*E^(2*c)*f^2*x^2*Log[1 + I*E^(c + d*x)] +
 6*a^3*d^2*e^2*Log[1 + E^(2*(c + d*x))] + 6*a^3*d^2*e^2*E^(2*c)*Log[1 + E^(
2*(c + d*x))] + 6*a^3*f^2*Log[1 + E^(2*(c + d*x))] + 6*a*b^2*f^2*Log[1 + E^
(2*(c + d*x))] + 6*a^3*E^(2*c)*f^2*Log[1 + E^(2*(c + d*x))] + 6*a*b^2*E^(2*
c)*f^2*Log[1 + E^(2*(c + d*x))] + 12*a^3*d^2*e*f*x*Log[1 + E^(2*(c + d*x))]
 + 12*a^3*d^2*e*E^(2*c)*f*x*Log[1 + E^(2*(c + d*x))] + 6*a^3*d^2*f^2*x^2*Lo
g[1 + E^(2*(c + d*x))] + 6*a^3*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(2*(c + d*x))]
 - (6*I)*b*(3*a^2 + b^2)*d*(1 + E^(2*c))*f*(e + f*x)*PolyLog[2, (-I)*E^(c +
 d*x)] + (6*I)*b*(3*a^2 + b^2)*d*(1 + E^(2*c))*f*(e + f*x)*PolyLog[2, I*E^(
c + d*x)] + 6*a^3*d*e*f*PolyLog[2, -E^(2*(c + d*x))] + 6*a^3*d*e*E^(2*c)*f*
PolyLog[2, -E^(2*(c + d*x))] + 6*a^3*d*f^2*x*PolyLog[2, -E^(2*(c + d*x))] +
 6*a^3*d*E^(2*c)*f^2*x*PolyLog[2, -E^(2*(c + d*x))] + (18*I)*a^2*b*f^2*Poly
Log[3, (-I)*E^(c + d*x)] + (6*I)*b^3*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (18
*I)*a^2*b*E^(2*c)*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (6*I)*b^3*E^(2*c)*f^2*
PolyLog[3, (-I)*E^(c + d*x)] - (18*I)*a^2*b*f^2*PolyLog[3, I*E^(c + d*x)] -
 (6*I)*b^3*f^2*PolyLog[3, I*E^(c + d*x)] - (18*I)*a^2*b*E^(2*c)*f^2*PolyLog
[3, I*E^(c + d*x)] - (6*I)*b^3*E^(2*c)*f^2*PolyLog[3, I*E^(c + d*x)] - 3*a^
3*f^2*PolyLog[3, -E^(2*(c + d*x))] - 3*a^3*E^(2*c)*f^2*PolyLog[3, -E^(2*(c
+ d*x))])/(6*(a^2 + b^2)^2*d^3*(1 + E^(2*c))) + (a^3*((2*E^(2*c)*x*(3*e^2 +
 3*e*f*x + f^2*x^2))/(-1 + E^(2*c)) - (3*(d^2*e^2*Log[2*a*E^(c + d*x) + b*(
-1 + E^(2*(c + d*x)))] + 2*d^2*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqr
t[(a^2 + b^2)*E^(2*c)])] + d^2*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - S
```

```
qrt[(a^2 + b^2)*E^(2*c)])] + 2*d^2*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c +
 Sqrt[(a^2 + b^2)*E^(2*c)])] + d^2*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c
 + Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d
*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2*d*f*(e + f*x)*PolyLog[2, -((
b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*f^2*PolyLog[3, -
((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*f^2*PolyLog[3,
 -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/d^3))/(3*(a^2
+ b^2)^2) + (Csch[c]*Sech[c]*Sech[c + d*x]^2*(-6*a^3*e*f - 6*a*b^2*e*f - 12
*a^3*d^2*e^2*x - 6*a^3*f^2*x - 6*a*b^2*f^2*x - 12*a^3*d^2*e*f*x^2 - 4*a^3*d
^2*f^2*x^3 + 6*a^3*e*f*Cosh[2*c] + 6*a*b^2*e*f*Cosh[2*c] + 6*a^3*f^2*x*Cosh
[2*c] + 6*a*b^2*f^2*x*Cosh[2*c] + 6*a^3*e*f*Cosh[2*d*x] + 6*a*b^2*e*f*Cosh[
2*d*x] + 6*a^3*f^2*x*Cosh[2*d*x] + 6*a*b^2*f^2*x*Cosh[2*d*x] + 3*a^2*b*d*e^
2*Cosh[c - d*x] + 3*b^3*d*e^2*Cosh[c - d*x] + 6*a^2*b*d*e*f*x*Cosh[c - d*x]
 + 6*b^3*d*e*f*x*Cosh[c - d*x] + 3*a^2*b*d*f^2*x^2*Cosh[c - d*x] + 3*b^3*d*
f^2*x^2*Cosh[c - d*x] - 3*a^2*b*d*e^2*Cosh[3*c + d*x] - 3*b^3*d*e^2*Cosh[3*
c + d*x] - 6*a^2*b*d*e*f*x*Cosh[3*c + d*x] - 6*b^3*d*e*f*x*Cosh[3*c + d*x]
- 3*a^2*b*d*f^2*x^2*Cosh[3*c + d*x] - 3*b^3*d*f^2*x^2*Cosh[3*c + d*x] - 6*a
^3*e*f*Cosh[2*c + 2*d*x] - 6*a*b^2*e*f*Cosh[2*c + 2*d*x] - 12*a^3*d^2*e^2*x
*Cosh[2*c + 2*d*x] - 6*a^3*f^2*x*Cosh[2*c + 2*d*x] - 6*a*b^2*f^2*x*Cosh[2*c
 + 2*d*x] - 12*a^3*d^2*e*f*x^2*Cosh[2*c + 2*d*x] - 4*a^3*d^2*f^2*x^3*Cosh[2
*c + 2*d*x] + 6*a^3*d*e^2*Sinh[2*c] + 6*a*b^2*d*e^2*Sinh[2*c] + 12*a^3*d*e*
f*x*Sinh[2*c] + 12*a*b^2*d*e*f*x*Sinh[2*c] + 6*a^3*d*f^2*x^2*Sinh[2*c] + 6*
a*b^2*d*f^2*x^2*Sinh[2*c] - 6*a^2*b*e*f*Sinh[c - d*x] - 6*b^3*e*f*Sinh[c -
d*x] - 6*a^2*b*f^2*x*Sinh[c - d*x] - 6*b^3*f^2*x*Sinh[c - d*x] - 6*a^2*b*e*
f*Sinh[3*c + d*x] - 6*b^3*e*f*Sinh[3*c + d*x] - 6*a^2*b*f^2*x*Sinh[3*c + d*
x] - 6*b^3*f^2*x*Sinh[3*c + d*x]))/(24*(a^2 + b^2)^2*d^2)
```

Maple [**F**]
time = 2.73, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \left(\tanh^3(dx+c)\right)}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 3*a^2*b*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*
b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2
 + b^4*d^2), x) + b^3*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x +
 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 +
 2*a^2*b^2*d^2 + b^4*d^2), x) - 2*a^3*d^2*f^2*integrate(x^2/(a^4*d^2*e^(2*d
*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d
^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 4*a^3*d^2*f*e*integrate(x/(a^4*d^2*e^(2
*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4
*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 6*a^2*b*d^2*f*integrate(x*e^(d*x + c
+ 1)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(
2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*b^3*d^2*f*integra
te(x*e^(d*x + c + 1)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*
c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - a^3
*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/
((a^4 + 2*a^2*b^2 + b^4)*d^3)) - a*b^2*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 +
 b^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) + 2*a^
2*b*f^2*arctan(e^(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3) + 2*b^3*f^2*arcta
n(e^(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3) - (a^3*log(-2*a*e^(-d*x - c) +
 b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b^4)*d) - a^3*log(e^(-2*d*x -
2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) + (3*a^2*b + b^3)*arctan(e^(-d*x - c)
)/((a^4 + 2*a^2*b^2 + b^4)*d) + (b*e^(-d*x - c) - 2*a*e^(-2*d*x - 2*c) - b*
e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*e^(-2*d*x - 2*c) + (a^2 + b^2
)*e^(-4*d*x - 4*c))*d))*e^2 + (2*a*f^2*x + 2*a*f*e - (b*d*f^2*x^2*e^(3*c) +
 2*b*f*e^(3*c + 1) + 2*(b*f^2*e^(3*c) + b*d*f*e^(3*c + 1))*x)*e^(3*d*x) + 2
*(a*d*f^2*x^2*e^(2*c) + a*f*e^(2*c + 1) + (a*f^2*e^(2*c) + 2*a*d*f*e^(2*c +
 1))*x)*e^(2*d*x) + (b*d*f^2*x^2*e^c - 2*b*f*e^(c + 1) + 2*(b*d*f*e^(c + 1)
 - b*f^2*e^c)*x)*e^(d*x))/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^(4*c) + b^2*d^2*e
^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*x)) + inte
grate(2*(a^3*b*f^2*x^2 + 2*a^3*b*f*x*e - (a^4*f^2*x^2*e^c + 2*a^4*f*x*e^(c
+ 1))*e^(d*x))/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b*e^(2*c) + 2*a^2*b^3*e^(2*c
) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c + 2*a^3*b^2*e^c + a*b^4*e^c)*e^(d*x
)), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 15580 vs. $2(1382) = 2764$.
time = 0.71, size = 15580, normalized size = 10.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*cosh(d*x + c)^4 + 4*(
(a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*sinh(d*x + c)^4 + 4*(a^3 + a*b
^2)*c*f^2 - 4*(a^3 + a*b^2)*d*f*cosh(1) + 2*((a^2*b + b^3)*d^2*f^2*x^2 + 2*
```

(a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d^2*cosh(1)^2 + (a^2*b + b^3)*d^2*sin
h(1)^2 + 2*((a^2*b + b^3)*d^2*f*x + (a^2*b + b^3)*d*f)*cosh(1) + 2*((a^2*b
+ b^3)*d^2*f*x + (a^2*b + b^3)*d^2*cosh(1) + (a^2*b + b^3)*d*f)*sinh(1))*co
sh(d*x + c)^3 - 4*(a^3 + a*b^2)*d*f*sinh(1) + 2*((a^2*b + b^3)*d^2*f^2*x^2
+ 2*(a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d^2*cosh(1)^2 + (a^2*b + b^3)*d^2
sinh(1)^2 + 2((a^2*b + b^3)*d^2*f*x + (a^2*b + b^3)*d*f)*cosh(1) + 8*((a^
3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*cosh(d*x + c) + 2*((a^2*b + b^3)*
d^2*f*x + (a^2*b + b^3)*d^2*cosh(1) + (a^2*b + b^3)*d*f)*sinh(1))*sinh(d*x
+ c)^3 - 4*((a^3 + a*b^2)*d^2*f^2*x^2 - (a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^
2)*d^2*cosh(1)^2 + (a^3 + a*b^2)*d^2*sinh(1)^2 - 2*(a^3 + a*b^2)*c*f^2 + (2
*(a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*d*f)*cosh(1) + (2*(a^3 + a*b^2)*d^2*
f*x + 2*(a^3 + a*b^2)*d^2*cosh(1) + (a^3 + a*b^2)*d*f)*sinh(1))*cosh(d*x +
c)^2 - 2*(2*(a^3 + a*b^2)*d^2*f^2*x^2 - 2*(a^3 + a*b^2)*d*f^2*x + 2*(a^3 +
a*b^2)*d^2*cosh(1)^2 + 2*(a^3 + a*b^2)*d^2*sinh(1)^2 - 4*(a^3 + a*b^2)*c*f^
2 - 12*((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*cosh(d*x + c)^2 + 2*(2
*(a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*d*f)*cosh(1) - 3*((a^2*b + b^3)*d^2*
f^2*x^2 + 2*(a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d^2*cosh(1)^2 + (a^2*b +
b^3)*d^2*sinh(1)^2 + 2*((a^2*b + b^3)*d^2*f*x + (a^2*b + b^3)*d*f)*cosh(1)
+ 2*((a^2*b + b^3)*d^2*f*x + (a^2*b + b^3)*d^2*cosh(1) + (a^2*b + b^3)*d*f)
*sinh(1))*cosh(d*x + c) + 2*(2*(a^3 + a*b^2)*d^2*f*x + 2*(a^3 + a*b^2)*d^2*
cosh(1) + (a^3 + a*b^2)*d*f)*sinh(1))*sinh(d*x + c)^2 - 2*((a^2*b + b^3)*d^
2*f^2*x^2 - 2*(a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d^2*cosh(1)^2 + (a^2*b
+ b^3)*d^2*sinh(1)^2 + 2*((a^2*b + b^3)*d^2*f*x - (a^2*b + b^3)*d*f)*cosh(1
) + 2*((a^2*b + b^3)*d^2*f*x + (a^2*b + b^3)*d^2*cosh(1) - (a^2*b + b^3)*d*
f)*sinh(1))*cosh(d*x + c) + 4*(a^3*d*f^2*x + a^3*d*f*cosh(1) + a^3*d*f*sinh
(1) + (a^3*d*f^2*x + a^3*d*f*cosh(1) + a^3*d*f*sinh(1))*cosh(d*x + c)^4 + 4
*(a^3*d*f^2*x + a^3*d*f*cosh(1) + a^3*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x +
c)^3 + (a^3*d*f^2*x + a^3*d*f*cosh(1) + a^3*d*f*sinh(1))*sinh(d*x + c)^4 +
2*(a^3*d*f^2*x + a^3*d*f*cosh(1) + a^3*d*f*sinh(1))*cosh(d*x + c)^2 + 2*(a
^3*d*f^2*x + a^3*d*f*cosh(1) + a^3*d*f*sinh(1) + 3*(a^3*d*f^2*x + a^3*d*f*c
osh(1) + a^3*d*f*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3*d*f^2*
x + a^3*d*f*cosh(1) + a^3*d*f*sinh(1))*cosh(d*x + c)^3 + (a^3*d*f^2*x + a^3
*d*f*cosh(1) + a^3*d*f*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh
(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2) - b)/b + 1) + 4*(a^3*d*f^2*x + a^3*d*f*cosh(1) + a^3*d*f*sinh(
1) + (a^3*d*f^2*x + a^3*d*f*cosh(1) + a^3*d*f*sinh(1))*cosh(d*x + c)^4 + 4*
(a^3*d*f^2*x + a^3*d*f*cosh(1) + a^3*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x +
c)^3 + (a^3*d*f^2*x + a^3*d*f*cosh(1) + a^3*d*f*sinh(1))*sinh(d*x + c)^4 +
2*(a^3*d*f^2*x + a^3*d*f*cosh(1) + a^3*d*f*sinh(1))*cosh(d*x + c)^2 + 2*(a^
3*d*f^2*x + a^3*d*f*cosh(1) + a^3*d*f*sinh(1) + 3*(a^3*d*f^2*x + a^3*d*f*co
sh(1) + a^3*d*f*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3*d*f^2*x
+ a^3*d*f*cosh(1) + a^3*d*f*sinh(1))*cosh(d*x + c)^3 + (a^3*d*f^2*x + a^3*
d*f*cosh(1) + a^3*d*f*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(
d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2) - b)/b + 1) - 2*(2*a^3*d*f^2*x + 2*a^3*d*f*cosh(1) + 2*a^3*d*f*

```
sinh(1) + I*(3*a^2*b + b^3)*d*f^2*x + (2*a^3*d*f^2*x + 2*a^3*d*f*cosh(1) +
2*a^3*d*f*sinh(1) + I*(3*a^2*b + b^3)*d*f^2*x + I*(3*a^2*b + b^3)*d*f*cosh(
1) + I*(3*a^2*b + b^3)*d*f*sinh(1))*cosh(d*x + c)^4 + 4*(2*a^3*d*f^2*x + 2*
a^3*d*f*cosh(1) + 2*a^3*d*f*sinh(1) + I*(3*a^2*b + b^3)*d*f^2*x + I*(3*a^2*
b + b^3)*d*f*cosh(1) + I*(3*a^2*b + b^3)*d*f*sinh(1))*cosh(d*x + c)*sinh(d*
x + c)^3 + (2*a^3*d*f^2*x + 2*a^3*d*f*cosh(1) + 2*a^3*d*f*sinh(1) + I*(3*a^
2*b + b^3)*d*f^2*x + I*(3*a^2*b + b^3)*d*f*cosh(1) + I*(3*a^2*b + b^3)*d*f*
sinh(1))*sinh(d*x + c)^4 + I*(3*a^2*b + b^3)*d*f*cosh(1) + I*(3*a^2*b + b^3
)*d*f*sinh(1) + 2*(2*a^3*d*f^2*x + 2*a^3*d*f*cosh(1) + 2*a^3*d*f*sinh(1) +
I*(3*a^2*b + b^3)*d*f^2*x + I*(3*a^2*b + b^3)*d*f*cosh(1) + I*(3*a^2*b + b^
3)*d*f*sinh(1))*cosh(d*x + c)^2 + 2*(2*a^3*d*f^2*x + 2*a^3*d*f*cosh(1) + 2*
a^3*d*f*sinh(1) + I*(3*a^2*b + b^3)*d*f^2*x + I*(3*a^2*b + b^3)*d*f*cosh(1)
 + I*(3*a^2*b + b^3)*d*f*sinh(1) + 3*(2*a^3*d*f^2*x + 2*a^3*d*f*cosh(1) + 2
*a^3*d*f*sinh(1) + I*(3*a^2*b + b^3)*d*f^2*x + I*(3*a^2*b + b^3)*d*f*cosh(1
) + I*(3*a^2*b + b^3)*d*f*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((2
*a^3*d*f^2*x + 2*a^3*d*f*cosh(1) + 2*a^3*d*f*sinh(1) + I*(3*a^2*b + b^3)*d*
f^2*x + I*(3*a^2*b + b^3)*d*f*cosh(1) + I*(3*a^2*b + b^3)*d*f*sinh(1))*cosh
(d*x + c)^3 + (2*a^3*d*f^2*x + 2*a^3*d*f*cosh(1) + 2*a^3*d*f*sinh(1) + I*(3
*a^2*b + b^3)*d*f^2*x + I*(3*a^2*b + b^3)*d*f*c...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2 \tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*tanh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*tanh(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(c+d\,x)^3\,(e+f\,x)^2}{a+b\sinh(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
[Out] int((tanh(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.417 $\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=894

$$\frac{a^2(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{b^3d}+\frac{(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{bd}-\frac{2a^4(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)^2d}-\frac{a^4(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{b^3\left(a^2+b^2\right)d}$$

```
[Out] -2*a^4*(f*x+e)*arctan(exp(d*x+c))/b/(a^2+b^2)^2/d-1/2*a^4*f*sech(d*x+c)/b^3
/(a^2+b^2)/d^2-1/2*a^3*(f*x+e)*sech(d*x+c)^2/b^2/(a^2+b^2)/d+1/2*a^3*f*tanh
(d*x+c)/b^2/(a^2+b^2)/d^2+1/2*a^2*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/b^3/d-1/2
*I*a^2*f*polylog(2,-I*exp(d*x+c))/b^3/d^2-a^4*(f*x+e)*arctan(exp(d*x+c))/b^
3/(a^2+b^2)/d+1/2*a^3*f*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)^2/d^2+1/2*a^2*
f*sech(d*x+c)/b^3/d^2+1/2*a*(f*x+e)*sech(d*x+c)^2/b^2/d-1/2*a*f*tanh(d*x+c)
/b^2/d^2+(f*x+e)*arctan(exp(d*x+c))/b/d+1/2*I*a^4*f*polylog(2,-I*exp(d*x+c)
)/b^3/(a^2+b^2)/d^2+a^2*(f*x+e)*arctan(exp(d*x+c))/b^3/d+a^3*(f*x+e)*ln(1+e
xp(2*d*x+2*c))/(a^2+b^2)^2/d-a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/
2)))/(a^2+b^2)^2/d-a^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+
b^2)^2/d-a^3*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2
-a^3*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/(a^2+b^2)^2/d^2-1/2*f*s
ech(d*x+c)/b/d^2-1/2*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/b/d-1/2*I*f*polylog(2,
-I*exp(d*x+c))/b/d^2+1/2*I*f*polylog(2,I*exp(d*x+c))/b/d^2+I*a^4*f*polylog(
2,-I*exp(d*x+c))/b/(a^2+b^2)^2/d^2+1/2*I*a^2*f*polylog(2,I*exp(d*x+c))/b^3/
d^2-1/2*a^4*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/b^3/(a^2+b^2)/d-I*a^4*f*polylog
(2,I*exp(d*x+c))/b/(a^2+b^2)^2/d^2-1/2*I*a^4*f*polylog(2,I*exp(d*x+c))/b^3/
(a^2+b^2)/d^2
```

Rubi [A]
time = 1.11, antiderivative size = 894, normalized size of antiderivative = 1.00, number of steps used = 55, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {5686, 5563, 4265, 2317, 2438, 4270, 5702, 5559, 3852, 8, 5692, 5680, 2221, 6874, 3799}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Tanh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*(e + f*x)*ArcTan[E^(c + d*x)])/(b^3*d) + ((e + f*x)*ArcTan[E^(c + d*x)
])/(b*d) - (2*a^4*(e + f*x)*ArcTan[E^(c + d*x)])/(b*(a^2 + b^2)^2*d) - (a^4
*(e + f*x)*ArcTan[E^(c + d*x)])/(b^3*(a^2 + b^2)*d) - (a^3*(e + f*x)*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) - (a^3*(e + f*x
)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) + (a^3*
(e + f*x)*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)^2*d) - ((I/2)*a^2*f*PolyLo
```

```
g[2, (-I)*E^(c + d*x)])/(b^3*d^2) - ((I/2)*f*PolyLog[2, (-I)*E^(c + d*x)])/
(b*d^2) + (I*a^4*f*PolyLog[2, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^2) + ((
I/2)*a^4*f*PolyLog[2, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) + ((I/2)*a^2
*f*PolyLog[2, I*E^(c + d*x)])/(b^3*d^2) + ((I/2)*f*PolyLog[2, I*E^(c + d*x)
])/(b*d^2) - (I*a^4*f*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^2) - ((
I/2)*a^4*f*PolyLog[2, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) - (a^3*f*PolyLo
g[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) - (a^3*
f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2)
 + (a^3*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^2) + (a^2*f*Sech
[c + d*x])/(2*b^3*d^2) - (f*Sech[c + d*x])/(2*b*d^2) - (a^4*f*Sech[c + d*x]
)/(2*b^3*(a^2 + b^2)*d^2) + (a*(e + f*x)*Sech[c + d*x]^2)/(2*b^2*d) - (a^3*
(e + f*x)*Sech[c + d*x]^2)/(2*b^2*(a^2 + b^2)*d) - (a*f*Tanh[c + d*x])/(2*b
^2*d^2) + (a^3*f*Tanh[c + d*x])/(2*b^2*(a^2 + b^2)*d^2) + (a^2*(e + f*x)*Se
ch[c + d*x]*Tanh[c + d*x])/(2*b^3*d) - ((e + f*x)*Sech[c + d*x]*Tanh[c + d*
x])/(2*b*d) - (a^4*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*b^3*(a^2 + b^2
)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5563

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5686

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[(e + f*x)^m*Sech[c +
d*x]*(Tanh[c + d*x]^(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5702

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*(Tanh[c + d*x]^(n - 1)/
(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{sech}(c+dx)\tanh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{a\int (e+fx)\text{sech}^2(c+dx)\tanh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}}{b^2}$$

$$= \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{f\text{sech}(c+dx)}{2bd^2} + \frac{a(e+fx)\text{sech}^2(c+dx)}{2b^2d} - \frac{(e+}{}$$

$$= \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} + \frac{a^2f\text{sech}(c+dx)}{2b^3d^2} - \frac{f\text{sech}(c+dx)}{2bd^2} + \frac{a(e+fx)\text{sech}}{2b^2d}$$

$$= \frac{a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{ifLi_2\left(-ie^{c+dx}\right)}{bd^2} + \frac{ifL}{}$$

$$= \frac{a^3(e+fx)^2}{2\left(a^2+b^2\right)^2f} + \frac{a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{ifLi_2}{}$$

$$= \frac{a^3(e+fx)^2}{2\left(a^2+b^2\right)^2f} + \frac{a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{a^3(e+}{}$$

$$= \frac{a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(e+fx)\tan^{-1}\left(e^{c+}\right.}{b\left(a^2+b^2\right)^2d}$$

$$= \frac{a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(e+fx)\tan^{-1}\left(e^{c+}\right.}{b\left(a^2+b^2\right)^2d}$$

$$= \frac{a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(e+fx)\tan^{-1}\left(e^{c+}\right.}{b\left(a^2+b^2\right)^2d}$$

$$= \frac{a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(e+fx)\tan^{-1}\left(e^{c+}\right.}{b\left(a^2+b^2\right)^2d}$$

Mathematica [A]
time = 5.47, size = 588, normalized size = 0.66

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Tanh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*a^3*d*e*(c + d*x) + 2*a^3*c*f*(c + d*x) + 6*a^2*b*d*e*ArcTan[E^(c + d*x
)] + 2*b^3*d*e*ArcTan[E^(c + d*x)] - 6*a^2*b*c*f*ArcTan[E^(c + d*x)] - 2*b^
3*c*f*ArcTan[E^(c + d*x)] + (3*I)*a^2*b*f*(c + d*x)*Log[1 - I*E^(c + d*x)]
+ I*b^3*f*(c + d*x)*Log[1 - I*E^(c + d*x)] - (3*I)*a^2*b*f*(c + d*x)*Log[1
+ I*E^(c + d*x)] - I*b^3*f*(c + d*x)*Log[1 + I*E^(c + d*x)] - 2*a^3*f*(c +
d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*a^3*f*(c + d*x)*Log
[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*a^3*d*e*Log[1 + E^(2*(c + d
*x))] - 2*a^3*c*f*Log[1 + E^(2*(c + d*x))] + 2*a^3*f*(c + d*x)*Log[1 + E^(2
*(c + d*x))] - 2*a^3*d*e*Log[a + b*Sinh[c + d*x]] + 2*a^3*c*f*Log[a + b*Sin
h[c + d*x]] - I*b*(3*a^2 + b^2)*f*PolyLog[2, (-I)*E^(c + d*x)] + I*b*(3*a^2
 + b^2)*f*PolyLog[2, I*E^(c + d*x)] - 2*a^3*f*PolyLog[2, (b*E^(c + d*x))/(-
a + Sqrt[a^2 + b^2])] - 2*a^3*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2]))] + a^3*f*PolyLog[2, -E^(2*(c + d*x))] - (a^2 + b^2)*f*Sech[c + d*x
]*(b + a*Sinh[c + d*x]) + (a^2 + b^2)*d*(e + f*x)*Sech[c + d*x]^2*(a - b*Si
nh[c + d*x]))/(2*(a^2 + b^2)^2*d^2)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2283 vs. $2(820) = 1640$.
time = 5.72, size = 2284, normalized size = 2.55

method	result	size
risch	Expression too large to display	2284

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*b^3/d/(a^2+b^2)*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x-I*b^3/d^2/(a^2+b^2)
*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c+I*b^3/d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*l
n(1-I*exp(d*x+c))*c+I*b^3/d/(a^2+b^2)*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x-
6*b/d^2/(a^2+b^2)*a^2*f*c/(2*a^2+2*b^2)*arctan(exp(d*x+c))+b^4/d^2/(a^2+b^2
)^(3/2)*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
-3*b^2/d/(a^2+b^2)^(3/2)*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(
a^2+b^2)^(1/2))*a^2-2/d^2/(a^2+b^2)*a^3*f*c/(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*
c))+2/d/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x+2/d^2/(a^2+b^2)*
a^3*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c-b^4/d/(a^2+b^2)^(3/2)*e/(2*a^2+2*b
^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2*b^3/d^2/(a^2+b^2)*f
*c/(2*a^2+2*b^2)*arctan(exp(d*x+c))-2/d/(a^2+b^2)*a^3*e/(2*a^2+2*b^2)*ln(b*
exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+3*I/d^2/(a^2+b^2)*a^2*f/(2*a^2+2*b^2)*ln(1
-I*exp(d*x+c))*b*c+6*b/d/(a^2+b^2)*e*a^2/(2*a^2+2*b^2)*arctan(exp(d*x+c))+I
*b^3/d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))-2/d^2/(a^2+b^2)*a^
3*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/
2)))-2/d^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2
)+a)/(a+(a^2+b^2)^(1/2)))+2/d/(a^2+b^2)*a^3*e/(2*a^2+2*b^2)*ln(1+exp(2*d*x+
2*c))-2/d^2/(a^2+b^2)^(1/2)*a^2*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+
```

```
c)+2*a)/(a^2+b^2)^(1/2))+2/d^2/(a^2+b^2)^(3/2)*a^4*f*c/(2*a^2+2*b^2)*arctan
h(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-3*I*b/d/(a^2+b^2)*a^2*f/(2*a^2+
2*b^2)*ln(1+I*exp(d*x+c))*x-3*I*b/d^2/(a^2+b^2)*a^2*f/(2*a^2+2*b^2)*ln(1+I*
exp(d*x+c))*c+2/d^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))+2/d
^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))+2/d^2/(a^2+b^2)*a^3*
f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c-2/d/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*ln((
-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-2/d^2/(a^2+b^2)*a^
3*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))
)*c-2/d/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(
a+(a^2+b^2)^(1/2)))*x-2/d^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+
(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/d/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)
*ln(1-I*exp(d*x+c))*x+2/d^2/(a^2+b^2)*a^3*f*c/(2*a^2+2*b^2)*ln(b*exp(2*d*x+
2*c)+2*a*exp(d*x+c)-b)+3*b^2/d^2/(a^2+b^2)^(3/2)*f*c/(2*a^2+2*b^2)*arctanh(
1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2-1/d^2*f*c*b^2/(2*a^2+2*b^2)/(
a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-3*I*b/d^2/
(a^2+b^2)*a^2*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))+3*I/d^2/(a^2+b^2)*a^2*f
/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))*b+2/d/(a^2+b^2)^(1/2)*a^2*e/(2*a^2+2*b
^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d/(a^2+b^2)^(3/2)*a
^4*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-I*b^3/
d^2/(a^2+b^2)*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))+2*b^3/d/(a^2+b^2)*e/(2*
a^2+2*b^2)*arctan(exp(d*x+c))+1/d*e*b^2/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arcta
nh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+3*I/d/(a^2+b^2)*a^2*f/(2*a^2+2
*b^2)*ln(1-I*exp(d*x+c))*b*x+(-b*d*f*x*exp(3*d*x+3*c)+2*a*d*f*x*exp(2*d*x+2
*c)-b*d*e*exp(3*d*x+3*c)+2*a*d*e*exp(2*d*x+2*c)+b*d*f*x*exp(d*x+c)-b*f*exp(
3*d*x+3*c)+a*f*exp(2*d*x+2*c)+b*d*e*exp(d*x+c)-f*b*exp(d*x+c)+f*a)/d^2/(a^2
+b^2)/(1+exp(2*d*x+2*c))^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -f*(((b*d*x*e^(3*c) + b*e^(3*c))*e^(3*d*x) - (2*a*d*x*e^(2*c) + a*e^(2*c))*
e^(2*d*x) - (b*d*x*e^c - b*e^c)*e^(d*x) - a)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*
e^(4*c) + b^2*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c)
)*e^(2*d*x)) - integrate(-2*(a^4*x*e^(d*x + c) - a^3*b*x)/(a^4*b + 2*a^2*b^
3 + b^5 - (a^4*b*e^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(
a^5*e^c + 2*a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) - integrate(-(2*a^3*x - (
3*a^2*b*e^c + b^3*e^c)*x*e^(d*x))/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^(2*c) + 2
*a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x)) - (a^3*log(-2*a*e^(-d*x - c
) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b^4)*d) - a^3*log(e^(-2*d*x
 - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) + (3*a^2*b + b^3)*arctan(e^(-d*x -
```

```
c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (b*e^(-d*x - c) - 2*a*e^(-2*d*x - 2*c) -
 b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*e^(-2*d*x - 2*c) + (a^2 +
b^2)*e^(-4*d*x - 4*c))*d))*e
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5468 vs. $2(805) = 1610$.
time = 0.53, size = 5468, normalized size = 6.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*cosh(1) + (a^2*b + b^3)*d*si
nh(1) + (a^2*b + b^3)*f)*cosh(d*x + c)^3 + 2*((a^2*b + b^3)*d*f*x + (a^2*b
+ b^3)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1) + (a^2*b + b^3)*f)*sinh(d*x + c)
^3 - 2*(2*(a^3 + a*b^2)*d*f*x + 2*(a^3 + a*b^2)*d*cosh(1) + 2*(a^3 + a*b^2)
*d*sinh(1) + (a^3 + a*b^2)*f)*cosh(d*x + c)^2 - 2*(2*(a^3 + a*b^2)*d*f*x +
2*(a^3 + a*b^2)*d*cosh(1) + 2*(a^3 + a*b^2)*d*sinh(1) + (a^3 + a*b^2)*f - 3
*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1) +
 (a^2*b + b^3)*f)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(a^3 + a*b^2)*f - 2*((
a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1) - (a
^2*b + b^3)*f)*cosh(d*x + c) + 2*(a^3*f*cosh(d*x + c)^4 + 4*a^3*f*cosh(d*x
+ c)*sinh(d*x + c)^3 + a^3*f*sinh(d*x + c)^4 + 2*a^3*f*cosh(d*x + c)^2 + a^
3*f + 2*(3*a^3*f*cosh(d*x + c)^2 + a^3*f)*sinh(d*x + c)^2 + 4*(a^3*f*cosh(d
*x + c)^3 + a^3*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*
sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
 b)/b + 1) + 2*(a^3*f*cosh(d*x + c)^4 + 4*a^3*f*cosh(d*x + c)*sinh(d*x + c)
^3 + a^3*f*sinh(d*x + c)^4 + 2*a^3*f*cosh(d*x + c)^2 + a^3*f + 2*(3*a^3*f*c
osh(d*x + c)^2 + a^3*f)*sinh(d*x + c)^2 + 4*(a^3*f*cosh(d*x + c)^3 + a^3*f*
cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - ((2*a
^3*f + I*(3*a^2*b + b^3)*f)*cosh(d*x + c)^4 + 4*(2*a^3*f + I*(3*a^2*b + b^3
)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^3*f + I*(3*a^2*b + b^3)*f)*sinh(d
*x + c)^4 + 2*a^3*f + 2*(2*a^3*f + I*(3*a^2*b + b^3)*f)*cosh(d*x + c)^2 + 2
*(2*a^3*f + 3*(2*a^3*f + I*(3*a^2*b + b^3)*f)*cosh(d*x + c)^2 + I*(3*a^2*b
+ b^3)*f)*sinh(d*x + c)^2 + I*(3*a^2*b + b^3)*f + 4*((2*a^3*f + I*(3*a^2*b
+ b^3)*f)*cosh(d*x + c)^3 + (2*a^3*f + I*(3*a^2*b + b^3)*f)*cosh(d*x + c))*
sinh(d*x + c))*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) - ((2*a^3*f - I*(3*
a^2*b + b^3)*f)*cosh(d*x + c)^4 + 4*(2*a^3*f - I*(3*a^2*b + b^3)*f)*cosh(d*
x + c)*sinh(d*x + c)^3 + (2*a^3*f - I*(3*a^2*b + b^3)*f)*sinh(d*x + c)^4 +
2*a^3*f + 2*(2*a^3*f - I*(3*a^2*b + b^3)*f)*cosh(d*x + c)^2 + 2*(2*a^3*f +
3*(2*a^3*f - I*(3*a^2*b + b^3)*f)*cosh(d*x + c)^2 - I*(3*a^2*b + b^3)*f)*si
nh(d*x + c)^2 - I*(3*a^2*b + b^3)*f + 4*((2*a^3*f - I*(3*a^2*b + b^3)*f)*co
sh(d*x + c)^3 + (2*a^3*f - I*(3*a^2*b + b^3)*f)*cosh(d*x + c))*sinh(d*x + c
```

```
))*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) - 2*(a^3*c*f - a^3*d*cosh(1) +
 (a^3*c*f - a^3*d*cosh(1) - a^3*d*sinh(1))*cosh(d*x + c)^4 - a^3*d*sinh(1)
+ 4*(a^3*c*f - a^3*d*cosh(1) - a^3*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3
 + (a^3*c*f - a^3*d*cosh(1) - a^3*d*sinh(1))*sinh(d*x + c)^4 + 2*(a^3*c*f -
 a^3*d*cosh(1) - a^3*d*sinh(1))*cosh(d*x + c)^2 + 2*(a^3*c*f - a^3*d*cosh(1
) - a^3*d*sinh(1) + 3*(a^3*c*f - a^3*d*cosh(1) - a^3*d*sinh(1))*cosh(d*x +
c)^2)*sinh(d*x + c)^2 + 4*((a^3*c*f - a^3*d*cosh(1) - a^3*d*sinh(1))*cosh(d
*x + c)^3 + (a^3*c*f - a^3*d*cosh(1) - a^3*d*sinh(1))*cosh(d*x + c))*sinh(d
*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b
^2) + 2*a) - 2*(a^3*c*f - a^3*d*cosh(1) + (a^3*c*f - a^3*d*cosh(1) - a^3*d*
sinh(1))*cosh(d*x + c)^4 - a^3*d*sinh(1) + 4*(a^3*c*f - a^3*d*cosh(1) - a^3
*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*c*f - a^3*d*cosh(1) - a^3*
d*sinh(1))*sinh(d*x + c)^4 + 2*(a^3*c*f - a^3*d*cosh(1) - a^3*d*sinh(1))*co
sh(d*x + c)^2 + 2*(a^3*c*f - a^3*d*cosh(1) - a^3*d*sinh(1) + 3*(a^3*c*f - a
^3*d*cosh(1) - a^3*d*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3*c*
f - a^3*d*cosh(1) - a^3*d*sinh(1))*cosh(d*x + c)^3 + (a^3*c*f - a^3*d*cosh(
1) - a^3*d*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2
*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(a^3*d*f*x + a^3*c*
f + (a^3*d*f*x + a^3*c*f)*cosh(d*x + c)^4 + 4*(a^3*d*f*x + a^3*c*f)*cosh(d*
x + c)*sinh(d*x + c)^3 + (a^3*d*f*x + a^3*c*f)*sinh(d*x + c)^4 + 2*(a^3*d*f
*x + a^3*c*f)*cosh(d*x + c)^2 + 2*(a^3*d*f*x + a^3*c*f + 3*(a^3*d*f*x + a^3
*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3*d*f*x + a^3*c*f)*cosh(d*x
+ c)^3 + (a^3*d*f*x + a^3*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d
*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2) - b)/b) + 2*(a^3*d*f*x + a^3*c*f + (a^3*d*f*x + a^3*c*f)*cosh(d*
x + c)^4 + 4*(a^3*d*f*x + a^3*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*d*f
*x + a^3*c*f)*sinh(d*x + c)^4 + 2*(a^3*d*f*x + a^3*c*f)*cosh(d*x + c)^2 + 2
*(a^3*d*f*x + a^3*c*f + 3*(a^3*d*f*x + a^3*c*f)*cosh(d*x + c)^2)*sinh(d*x +
 c)^2 + 4*((a^3*d*f*x + a^3*c*f)*cosh(d*x + c)^3 + (a^3*d*f*x + a^3*c*f)*co
sh(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*co
sh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (2*a^3*c*f -
 2*a^3*d*cosh(1) + (2*a^3*c*f - 2*a^3*d*cosh(1) - 2*a^3*d*sinh(1) + I*(3*a^
2*b + b^3)*c*f - I*(3*a^2*b + b^3)*d*cosh(1) - I*(3*a^2*b + b^3)*d*sinh(1))
*cosh(d*x + c)^4 - 2*a^3*d*sinh(1) + 4*(2*a^3*c*f - 2*a^3*d*cosh(1) - 2*a^3
*d*sinh(1) + I*(3*a^2*b + b^3)*c*f - I*(3*a^2*b...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)*tanh(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(c+d\,x)^3\,(e+f\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tanh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((tanh(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.418 $\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=120

$$\frac{b(3a^2+b^2)\operatorname{ArcTan}(\sinh(c+dx))}{2(a^2+b^2)^2 d}+\frac{a^3\log(\cosh(c+dx))}{(a^2+b^2)^2 d}-\frac{a^3\log(a+b\sinh(c+dx))}{(a^2+b^2)^2 d}+\frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2)d}$$

```
[Out] 1/2*b*(3*a^2+b^2)*arctan(sinh(d*x+c))/(a^2+b^2)^2/d+a^3*ln(cosh(d*x+c))/(a^
2+b^2)^2/d-a^3*ln(a+b*sinh(d*x+c))/(a^2+b^2)^2/d+1/2*sech(d*x+c)^2*(a-b*sin
h(d*x+c))/(a^2+b^2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2800, 1661, 815, 649, 209, 266}

$$\frac{b(3a^2+b^2)\operatorname{ArcTan}(\sinh(c+dx))}{2d(a^2+b^2)^2}+\frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))}{2d(a^2+b^2)}-\frac{a^3\log(a+b\sinh(c+dx))}{d(a^2+b^2)^2}+\frac{a^3\log(\cosh(c+dx))}{d(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*(3*a^2 + b^2)*ArcTan[Sinh[c + d*x]])/(2*(a^2 + b^2)^2*d) + (a^3*Log[Cosh
[c + d*x]])/((a^2 + b^2)^2*d) - (a^3*Log[a + b*Sinh[c + d*x]])/((a^2 + b^2)
^2*d) + (Sech[c + d*x]^2*(a - b*Sinh[c + d*x]))/(2*(a^2 + b^2)*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)(-b^2-x^2)^2}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= \frac{\text{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2)\,d} - \frac{\text{Subst}\left(\int \frac{\frac{ab^4}{a^2+b^2}+\frac{b^2(2a^2+b^2)x}{a^2+b^2}}{(a+x)(-b^2-x^2)}\,dx, x, b\sinh(c+\right.}{2b^2d}$$

$$= \frac{\text{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2)\,d} - \frac{\text{Subst}\left(\int \left(\frac{2a^3b^2}{(a^2+b^2)^2(a+x)} - \frac{b^2(3a^2b^2+b^4+2a^3x)}{(a^2+b^2)^2(b^2+x^2)}\right)\right.}{2b^2d}$$

$$= -\frac{a^3\log(a+b\sinh(c+dx))}{(a^2+b^2)^2\,d} + \frac{\text{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2)\,d} + \frac{\text{Subst}\left(\int \frac{3a^2}{}\right.}{}$$

$$= -\frac{a^3\log(a+b\sinh(c+dx))}{(a^2+b^2)^2\,d} + \frac{\text{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2)\,d} + \frac{a^3\text{Subst}\left(\int \right.}{}$$

$$= \frac{b(3a^2+b^2)\tan^{-1}(\sinh(c+dx))}{2(a^2+b^2)^2\,d} + \frac{a^3\log(\cosh(c+dx))}{(a^2+b^2)^2\,d} - \frac{a^3\log(a+b\sinh(c+}{(a^2+b^2)^2\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.31, size = 152, normalized size = 1.27

$$-\frac{b(a^2+b^2)\text{ArcTan}(\sinh(c+dx)) - (a^3-i(2a^2b+b^3))\log(i-\sinh(c+dx)) - (a^3+i(2a^2b+b^3))\log(i+\sinh(c+dx)) + 2a^3\log(a+b\sinh(c+dx)) - a(a^2+b^2)\text{sech}^2(c+dx) + b(a^2+b^2)\text{sech}(c+dx)\tanh(c+dx)}{2(a^2+b^2)^2\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/2*(b*(a^2 + b^2)*ArcTan[Sinh[c + d*x]] - (a^3 - I*(2*a^2*b + b^3))*Log[I
 - Sinh[c + d*x]] - (a^3 + I*(2*a^2*b + b^3))*Log[I + Sinh[c + d*x]] + 2*a^
3*Log[a + b*Sinh[c + d*x]] - a*(a^2 + b^2)*Sech[c + d*x]^2 + b*(a^2 + b^2)*
Sech[c + d*x]*Tanh[c + d*x])/((a^2 + b^2)^2*d)
```

Maple [A]
time = 1.47, size = 210, normalized size = 1.75

method	result
derivativedivides	$\frac{\frac{\frac{2\left(\left(\frac{1}{2}a^2b+\frac{1}{2}b^3\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^3-ab^2\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+a^3\ln\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\left(3a^2b\right.}{a^4+2a^2b^2+b^4}}{d}$
default	$\frac{\frac{\frac{2\left(\left(\frac{1}{2}a^2b+\frac{1}{2}b^3\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^3-ab^2\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+a^3\ln\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\left(3a^2b\right.}{a^4+2a^2b^2+b^4}}{d}$
risch	$-\frac{2a^3d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{2a^3dc}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{2a^3x}{a^4+2a^2b^2+b^4}+\frac{2a^3c}{d(a^4+2a^2b^2+b^4)}+\frac{e^{dx+c}\left(-b\,e^{2dx+2c}+2a\,e^{dx}\right.}{d(a^2+b^2)(1+e^{2dx+2c})}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/(a^4+2*a^2*b^2+b^4)*(((1/2*a^2*b+1/2*b^3)*tanh(1/2*d*x+1/2*c)^3+(-a^
3-a*b^2)*tanh(1/2*d*x+1/2*c)^2+(-1/2*a^2*b-1/2*b^3)*tanh(1/2*d*x+1/2*c))/(t
anh(1/2*d*x+1/2*c)^2+1)^2+1/2*a^3*ln(tanh(1/2*d*x+1/2*c)^2+1)+1/2*(3*a^2*b+
b^3)*arctan(tanh(1/2*d*x+1/2*c)))-8*a^3/(8*a^4+16*a^2*b^2+8*b^4)*ln(a*tanh(
1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)-a))
```

Maxima [A]
time = 0.49, size = 217, normalized size = 1.81

$$-\frac{a^3\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^4+2\,a^2b^2+b^4)d}+\frac{a^3\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^4+2\,a^2b^2+b^4)d}-\frac{(3\,a^2b+b^3)\arctan\left(e^{(-dx-c)}\right)}{(a^4+2\,a^2b^2+b^4)d}-\frac{be^{(-dx-c)}-2\,ae^{(-2\,dx-2\,c)}-be^{(-3\,dx-3\,c)}}{\left(a^2+b^2+2\,(a^2+b^2)e^{(-2\,dx-2\,c)}+(a^2+b^2)e^{(-4\,dx-4\,c)}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b^
4)*d) + a^3*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (3*a^2*
b + b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) - (b*e^(-d*x - c)
 - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*e
^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(117) = 234$.
time = 0.50, size = 896, normalized size = 7.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((a^2*b + b^3)*cosh(d*x + c)^3 + (a^2*b + b^3)*sinh(d*x + c)^3 - 2*(a^3 +
a*b^2)*cosh(d*x + c)^2 - (2*a^3 + 2*a*b^2 - 3*(a^2*b + b^3)*cosh(d*x + c))*
sinh(d*x + c)^2 - ((3*a^2*b + b^3)*cosh(d*x + c)^4 + 4*(3*a^2*b + b^3)*cosh
(d*x + c)*sinh(d*x + c)^3 + (3*a^2*b + b^3)*sinh(d*x + c)^4 + 3*a^2*b + b^3
 + 2*(3*a^2*b + b^3)*cosh(d*x + c)^2 + 2*(3*a^2*b + b^3 + 3*(3*a^2*b + b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((3*a^2*b + b^3)*cosh(d*x + c)^3 + (3
*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x
 + c)) - (a^2*b + b^3)*cosh(d*x + c) + (a^3*cosh(d*x + c)^4 + 4*a^3*cosh(d*
x + c)*sinh(d*x + c)^3 + a^3*sinh(d*x + c)^4 + 2*a^3*cosh(d*x + c)^2 + a^3
+ 2*(3*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^2 + 4*(a^3*cosh(d*x + c)^3
+ a^3*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x +
 c) - sinh(d*x + c))) - (a^3*cosh(d*x + c)^4 + 4*a^3*cosh(d*x + c)*sinh(d*x
 + c)^3 + a^3*sinh(d*x + c)^4 + 2*a^3*cosh(d*x + c)^2 + a^3 + 2*(3*a^3*cosh
(d*x + c)^2 + a^3)*sinh(d*x + c)^2 + 4*(a^3*cosh(d*x + c)^3 + a^3*cosh(d*x
+ c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) -
 (a^2*b + b^3 - 3*(a^2*b + b^3)*cosh(d*x + c)^2 + 4*(a^3 + a*b^2)*cosh(d*x
+ c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^4 + 4*(a^4 +
2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*
d*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4
 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d)*sinh(d*x
 + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d + 4*((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x
 + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(c + dx)}{a + b \sinh(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(117) = 234$.

time = 0.48, size = 279, normalized size = 2.32

$$-\frac{\frac{4\,a^3b\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^4b+2\,a^2b^3+b^5}-\frac{2\,a^3\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^4+2\,a^2b^2+b^4}-\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(3\,a^2b+b^3\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\left(a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+2\,a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)-4\,ab^2\right)}{\left(a^4+2\,a^2b^2+b^4\right)\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(4*a^3*b*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^4*b + 2*a^2
*b^3 + b^5) - 2*a^3*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^4 + 2*a^2*b^
2 + b^4) - (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(3*a^2*b
 + b^3)/(a^4 + 2*a^2*b^2 + b^4) + 2*(a^3*(e^(d*x + c) - e^(-d*x - c))^2 + 2
*a^2*b*(e^(d*x + c) - e^(-d*x - c)) + 2*b^3*(e^(d*x + c) - e^(-d*x - c)) -
4*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*((e^(d*x + c) - e^(-d*x - c))^2 + 4)))/d
```

Mupad [B]

time = 2.37, size = 381, normalized size = 3.18

$$\frac{\frac{2\left(a^3+a\,b^2\right)}{d\left(a^2+b^2\right)^2}-\frac{\mathrm{e}^{c+d\,x}\left(a^2\,b+b^3\right)}{d\left(a^2+b^2\right)^2}}{\mathrm{e}^{2\,c+2\,d\,x}+1}-\frac{\frac{2\,a}{d\left(a^2+b^2\right)}-\frac{2\,b\,\mathrm{e}^{c+d\,x}}{d\left(a^2+b^2\right)}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1}+\frac{\ln\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)\,\left(2\,a+b\,1\mathrm{i}\right)}{2\,\left(d\,a^2+2\mathrm{i}\,d\,a\,b-d\,b^2\right)}+\frac{\ln\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)\,\left(b+a\,2\mathrm{i}\right)}{2\,\left(1\mathrm{i}\,d\,a^2+2\,d\,a\,b-1\mathrm{i}\,d\,b^2\right)}-\frac{a^3\,\ln\left(32\,a^7\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-b^7-6\,a^2\,b^5-9\,a^4\,b^3-16\,a^6\,b+b^7\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+16\,a^6\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+12\,a^3\,b^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+18\,a^5\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+6\,a^2\,b^5\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+9\,a^4\,b^3\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+2\,a\,b^6\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\right)}{d\,a^4+2\,d\,a^2\,b^2+d\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3/(a + b*sinh(c + d*x)),x)
```

```
[Out] ((2*(a*b^2 + a^3))/(d*(a^2 + b^2)^2) - (exp(c + d*x)*(a^2*b + b^3))/(d*(a^2
 + b^2)^2))/(exp(2*c + 2*d*x) + 1) - ((2*a)/(d*(a^2 + b^2)) - (2*b*exp(c +
d*x))/(d*(a^2 + b^2)))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + (log(e
xp(c + d*x)*1i + 1)*(2*a + b*1i))/(2*(a^2*d - b^2*d + a*b*d*2i)) + (log(exp
(c + d*x) + 1i)*(a*2i + b))/(2*(a^2*d*1i - b^2*d*1i + 2*a*b*d)) - (a^3*log(
32*a^7*exp(d*x)*exp(c) - b^7 - 6*a^2*b^5 - 9*a^4*b^3 - 16*a^6*b + b^7*exp(2
*c)*exp(2*d*x) + 16*a^6*b*exp(2*c)*exp(2*d*x) + 12*a^3*b^4*exp(d*x)*exp(c)
+ 18*a^5*b^2*exp(d*x)*exp(c) + 6*a^2*b^5*exp(2*c)*exp(2*d*x) + 9*a^4*b^3*ex
p(2*c)*exp(2*d*x) + 2*a*b^6*exp(d*x)*exp(c)))/(a^4*d + b^4*d + 2*a^2*b^2*d)
```

3.419 $\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

[Out] Unintegrable(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

[In] Int[Tanh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]

[Out] Defer[Int][Tanh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]

Rubi steps

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

[In] Integrate[Tanh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]

[Out] $Aborted

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(dx+c)}{(fx+e)\,(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
[Out] int(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
[Out] -(a*f + (b*d*f*x*e^(3*c) - b*f*e^(3*c) + b*d*e^(3*c + 1))*e^(3*d*x) - (2*a*
d*f*x*e^(2*c) - a*f*e^(2*c) + 2*a*d*e^(2*c + 1))*e^(2*d*x) - (b*d*f*x*e^c +
 b*d*e^(c + 1) + b*f*e^c)*e^(d*x))/((a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*(a^
2*d^2*f + b^2*d^2*f)*x*e + (a^2*d^2 + b^2*d^2)*e^2 + ((a^2*d^2*f^2*e^(4*c)
+ b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^2*d^2*f*e^(4*c) + b^2*d^2*f*e^(4*c))*x*e
+ (a^2*d^2*e^(4*c) + b^2*d^2*e^(4*c))*e^2)*e^(4*d*x) + 2*((a^2*d^2*f^2*e^(2
*c) + b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^2*d^2*f*e^(2*c) + b^2*d^2*f*e^(2*c))*
x*e + (a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^2)*e^(2*d*x)) + integrate(-(2*a
^3*d^2*f^2*x^2 + 4*a^3*d^2*f*x*e + 2*a^3*d^2*e^2 + 2*a^3*f^2 + 2*a*b^2*f^2
- (2*a^2*b*f^2*e^c + 2*b^3*f^2*e^c + (3*a^2*b*d^2*f^2*e^c + b^3*d^2*f^2*e^c
)*x^2 + 2*(3*a^2*b*d^2*f*e^c + b^3*d^2*f*e^c)*x*e + (3*a^2*b*d^2*e^c + b^3*
d^2*e^c)*e^2)*e^(d*x))/((a^4*d^2*f^3 + 2*a^2*b^2*d^2*f^3 + b^4*d^2*f^3)*x^3
 + 3*(a^4*d^2*f^2 + 2*a^2*b^2*d^2*f^2 + b^4*d^2*f^2)*x^2*e + 3*(a^4*d^2*f +
 2*a^2*b^2*d^2*f + b^4*d^2*f)*x*e^2 + (a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2)*e
^3 + ((a^4*d^2*f^3*e^(2*c) + 2*a^2*b^2*d^2*f^3*e^(2*c) + b^4*d^2*f^3*e^(2*c
))*x^3 + 3*(a^4*d^2*f^2*e^(2*c) + 2*a^2*b^2*d^2*f^2*e^(2*c) + b^4*d^2*f^2*e
^(2*c))*x^2*e + 3*(a^4*d^2*f*e^(2*c) + 2*a^2*b^2*d^2*f*e^(2*c) + b^4*d^2*f*
e^(2*c))*x*e^2 + (a^4*d^2*e^(2*c) + 2*a^2*b^2*d^2*e^(2*c) + b^4*d^2*e^(2*c)
)*e^3)*e^(2*d*x)), x) + integrate(-2*(a^4*e^(d*x + c) - a^3*b)/((a^4*b*f +
2*a^2*b^3*f + b^5*f)*x + (a^4*b + 2*a^2*b^3 + b^5)*e - ((a^4*b*f*e^(2*c) +
2*a^2*b^3*f*e^(2*c) + b^5*f*e^(2*c))*x + (a^4*b*e^(2*c) + 2*a^2*b^3*e^(2*c)
 + b^5*e^(2*c))*e)*e^(2*d*x) - 2*((a^5*f*e^c + 2*a^3*b^2*f*e^c + a*b^4*f*e^
c)*x + (a^5*e^c + 2*a^3*b^2*e^c + a*b^4*e^c)*e)*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
[Out] integral(tanh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)**3/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\tanh(c+d\,x)}^3}{(e+f\,x)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(tanh(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.420 $\int \frac{(e+fx)^3 \coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=451

$$-\frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad}-\frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad}+\frac{(e+fx)^3\log\left(1-e^{2(c+dx)}\right)}{ad}-\frac{3f(e\cdots}{\cdots}$$

```
[Out] (f*x+e)^3*ln(1-exp(2*d*x+2*c))/a/d-(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)
^(1/2)))/a/d-(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d+3/2*f*(f*
x+e)^2*polylog(2,exp(2*d*x+2*c))/a/d^2-3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c
)/(a-(a^2+b^2)^(1/2)))/a/d^2-3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+
b^2)^(1/2)))/a/d^2-3/2*f^2*(f*x+e)*polylog(3,exp(2*d*x+2*c))/a/d^3+6*f^2*(f
*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d^3+6*f^2*(f*x+e)*poly
log(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d^3+3/4*f^3*polylog(4,exp(2*d*x+
2*c))/a/d^4-6*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d^4-6*f^3*
polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d^4
```

Rubi [A]
time = 0.54, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5688, 3797, 2221, 2611, 6744, 2320, 6724, 5680}

$$-\frac{6f^3\text{Li}_4\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^4}-\frac{6f^3\text{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^4}+\frac{6f^2(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3}+\frac{6f^2(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^3}-\frac{3f(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2}-\frac{3f(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^2}-\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{ad}-\frac{(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{ad}+\frac{3f^3\text{Li}_4\left(e^{2(c+dx)}\right)}{4ad^4}-\frac{3f^2(e+fx)\text{Li}_3\left(e^{2(c+dx)}\right)}{2ad^3}+\frac{3f(e+fx)^2\text{Li}_2\left(e^{2(c+dx)}\right)}{2ad^2}+\frac{(e+fx)^3\log\left(1-e^{2(c+dx)}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*d)) - ((e
 + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*d) + ((e + f*x
)^3*Log[1 - E^(2*(c + d*x))])/(a*d) - (3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*d^2) - (3*f*(e + f*x)^2*PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*d^2) + (3*f*(e + f*x)^2*PolyLog[
2, E^(2*(c + d*x))])/(2*a*d^2) + (6*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2]))])/(a*d^3) + (6*f^2*(e + f*x)*PolyLog[3, -((b*E^(
c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*d^3) - (3*f^2*(e + f*x)*PolyLog[3, E^
(2*(c + d*x))])/(2*a*d^3) - (6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(a*d^4) - (6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/(a*d^4) + (3*f^3*PolyLog[4, E^(2*(c + d*x))])/(4*a*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5688

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x]^
(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\coth(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{2\int \frac{e^{2(c+dx)}(e+fx)^3}{1-e^{2(c+dx)}}\,dx}{a} - \frac{b\int \frac{e^{c+dx}(e+fx)^3}{a-\sqrt{a^2+b^2}+be^{c+dx}}\,dx}{a} - \frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+\sqrt{a^2+b^2}+be^{c+dx}}}{a}$$

$$= -\frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} +$$

$$= -\frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} +$$

$$= -\frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} +$$

$$= -\frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} +$$

$$= -\frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} +$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1002 vs. $2(451) = 902$.
time = 5.22, size = 1002, normalized size = 2.22

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/4*(-4*d^3*e^3*Log[1 - E^(2*(c + d*x))] - 12*d^3*e^2*f*x*Log[1 - E^(2*(c
+ d*x))] - 12*d^3*e*f^2*x^2*Log[1 - E^(2*(c + d*x))] - 4*d^3*f^3*x^3*Log[1
- E^(2*(c + d*x))] + 4*d^3*e^3*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))]
 + 12*d^3*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*
c)])] + 12*d^3*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2
)*E^(2*c)])] + 4*d^3*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 +
 b^2)*E^(2*c)])] + 12*d^3*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(
a^2 + b^2)*E^(2*c)])] + 12*d^3*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c +
 Sqrt[(a^2 + b^2)*E^(2*c)])] + 4*d^3*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E
^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 6*d^2*f*(e + f*x)^2*PolyLog[2, E^(2*(c +
 d*x))] + 12*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt
[(a^2 + b^2)*E^(2*c)]))] + 12*d^2*e^2*f*PolyLog[2, -((b*E^(2*c + d*x))/(a*E
^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 24*d^2*e*f^2*x*PolyLog[2, -((b*E^(2*c +
 d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 12*d^2*f^3*x^2*PolyLog[2, -(
(b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*d*e*f^2*PolyLog
[3, E^(2*(c + d*x))] + 6*d*f^3*x*PolyLog[3, E^(2*(c + d*x))] - 24*d*e*f^2*P
olyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 24*d*
f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))]
- 24*d*e*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*
c)]))] - 24*d*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2
)*E^(2*c)]))] - 3*f^3*PolyLog[4, E^(2*(c + d*x))] + 24*f^3*PolyLog[4, -((b*
E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 24*f^3*PolyLog[4, -(
(b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/(a*d^4)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \coth(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*d) - log(e^(-d*x - c)
+ 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d))*e^3 + 3*(d*x*log(e^(d*x + c) + 1)
```

```
 + dilog(-e^(d*x + c)))*f*e^2/(a*d^2) + 3*(d*x*log(-e^(d*x + c) + 1) + dilo
g(e^(d*x + c)))*f*e^2/(a*d^2) + 3*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dil
og(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2*e/(a*d^3) + 3*(d^2*x^2*l
og(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c))
)*f^2*e/(a*d^3) + (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x +
 c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*f^3/(a*
d^4) + (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*
x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*f^3/(a*d^4) - 1/2*(d
^4*f^3*x^4 + 4*d^4*f^2*x^3*e + 6*d^4*f*x^2*e^2)/(a*d^4) + integrate(-2*(b*f
^3*x^3 + 3*b*f^2*x^2*e + 3*b*f*x*e^2 - (a*f^3*x^3*e^c + 3*a*f^2*x^2*e^(c +
1) + 3*a*f*x*e^(c + 2))*e^(d*x))/(a*b*e^(2*d*x + 2*c) + 2*a^2*e^(d*x + c) -
 a*b), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1816 vs. $2(427) = 854$.
time = 0.42, size = 1816, normalized size = 4.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(6*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*f^3*polylog(4, (a*cosh(d*x +
 c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2
)/b^2))/b) - 6*f^3*polylog(4, cosh(d*x + c) + sinh(d*x + c)) - 6*f^3*polylo
g(4, -cosh(d*x + c) - sinh(d*x + c)) + 3*(d^2*f^3*x^2 + 2*d^2*f^2*x*cosh(1)
 + d^2*f*cosh(1)^2 + d^2*f*sinh(1)^2 + 2*(d^2*f^2*x + d^2*f*cosh(1))*sinh(1
))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 3*(d^2*f^3*x^2 + 2*d^2*f^2*x*cos
h(1) + d^2*f*cosh(1)^2 + d^2*f*sinh(1)^2 + 2*(d^2*f^2*x + d^2*f*cosh(1))*si
nh(1))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(d^2*f^3*x^2 + 2*d^2*f^2*x
*cosh(1) + d^2*f*cosh(1)^2 + d^2*f*sinh(1)^2 + 2*(d^2*f^2*x + d^2*f*cosh(1)
)*sinh(1))*dilog(cosh(d*x + c) + sinh(d*x + c)) - 3*(d^2*f^3*x^2 + 2*d^2*f^
2*x*cosh(1) + d^2*f*cosh(1)^2 + d^2*f*sinh(1)^2 + 2*(d^2*f^2*x + d^2*f*cosh
(1))*sinh(1))*dilog(-cosh(d*x + c) - sinh(d*x + c)) - (c^3*f^3 - 3*c^2*d*f^
2*cosh(1) + 3*c*d^2*f*cosh(1)^2 - d^3*cosh(1)^3 - d^3*sinh(1)^3 + 3*(c*d^2*
f - d^3*cosh(1))*sinh(1)^2 - 3*(c^2*d*f^2 - 2*c*d^2*f*cosh(1) + d^3*cosh(1)
^2)*sinh(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^
2)/b^2) + 2*a) - (c^3*f^3 - 3*c^2*d*f^2*cosh(1) + 3*c*d^2*f*cosh(1)^2 - d^3
*cosh(1)^3 - d^3*sinh(1)^3 + 3*(c*d^2*f - d^3*cosh(1))*sinh(1)^2 - 3*(c^2*d
*f^2 - 2*c*d^2*f*cosh(1) + d^3*cosh(1)^2)*sinh(1))*log(2*b*cosh(d*x + c) +
2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (d^3*f^3*x^3 + c^3*f
^3 + 3*(d^3*f*x + c*d^2*f)*cosh(1)^2 + 3*(d^3*f*x + c*d^2*f)*sinh(1)^2 + 3*
```

```
(d^3*f^2*x^2 - c^2*d*f^2)*cosh(1) + 3*(d^3*f^2*x^2 - c^2*d*f^2 + 2*(d^3*f*x
 + c*d^2*f)*cosh(1))*sinh(1))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (d^3*f^3*x
^3 + c^3*f^3 + 3*(d^3*f*x + c*d^2*f)*cosh(1)^2 + 3*(d^3*f*x + c*d^2*f)*sinh
(1)^2 + 3*(d^3*f^2*x^2 - c^2*d*f^2)*cosh(1) + 3*(d^3*f^2*x^2 - c^2*d*f^2 +
2*(d^3*f*x + c*d^2*f)*cosh(1))*sinh(1))*log(-(a*cosh(d*x + c) + a*sinh(d*x
+ c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) -
(d^3*f^3*x^3 + 3*d^3*f^2*x^2*cosh(1) + 3*d^3*f*x*cosh(1)^2 + d^3*cosh(1)^3
+ d^3*sinh(1)^3 + 3*(d^3*f*x + d^3*cosh(1))*sinh(1)^2 + 3*(d^3*f^2*x^2 + 2*
d^3*f*x*cosh(1) + d^3*cosh(1)^2)*sinh(1))*log(cosh(d*x + c) + sinh(d*x + c)
 + 1) + (c^3*f^3 - 3*c^2*d*f^2*cosh(1) + 3*c*d^2*f*cosh(1)^2 - d^3*cosh(1)^
3 - d^3*sinh(1)^3 + 3*(c*d^2*f - d^3*cosh(1))*sinh(1)^2 - 3*(c^2*d*f^2 - 2*
c*d^2*f*cosh(1) + d^3*cosh(1)^2)*sinh(1))*log(cosh(d*x + c) + sinh(d*x + c)
 - 1) - (d^3*f^3*x^3 + c^3*f^3 + 3*(d^3*f*x + c*d^2*f)*cosh(1)^2 + 3*(d^3*f
*x + c*d^2*f)*sinh(1)^2 + 3*(d^3*f^2*x^2 - c^2*d*f^2)*cosh(1) + 3*(d^3*f^2*
x^2 - c^2*d*f^2 + 2*(d^3*f*x + c*d^2*f)*cosh(1))*sinh(1))*log(-cosh(d*x + c
) - sinh(d*x + c) + 1) - 6*(d*f^3*x + d*f^2*cosh(1) + d*f^2*sinh(1))*polylo
g(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2))/b) - 6*(d*f^3*x + d*f^2*cosh(1) + d*f^2*sinh(1))*
polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*(d*f^3*x + d*f^2*cosh(1) + d*f^2*sin
h(1))*polylog(3, cosh(d*x + c) + sinh(d*x + c)) + 6*(d*f^3*x + d*f^2*cosh(1
) + d*f^2*sinh(1))*polylog(3, -cosh(d*x + c) - sinh(d*x + c)))/(a*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3 \coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)**3*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)^3*coth(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(c+d\,x)\,\left(e+f\,x\right)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
[Out] int((coth(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.421 $\int \frac{(e+fx)^2 \coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=325

$$-\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad}-\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad}+\frac{(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{ad}-\frac{2f(e$$

```
[Out] (f*x+e)^2*ln(1-exp(2*d*x+2*c))/a/d-(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)
^(1/2)))/a/d-(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d+f*(f*x+e)
*polylog(2,exp(2*d*x+2*c))/a/d^2-2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^
2+b^2)^(1/2)))/a/d^2-2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)
))/a/d^2-1/2*f^2*polylog(3,exp(2*d*x+2*c))/a/d^3+2*f^2*polylog(3,-b*exp(d*x
+c)/(a-(a^2+b^2)^(1/2)))/a/d^3+2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(
1/2)))/a/d^3
```

Rubi [A]

time = 0.51, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5688, 3797, 2221, 2611, 2320, 6724, 5680}

$$\frac{2f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3}+\frac{2f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^3}-\frac{2f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2}-\frac{2f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^2}-\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{ad}-\frac{(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{ad}-\frac{f^2\text{Li}_3\left(e^{2(c+dx)}\right)}{2ad^3}+\frac{f(e+fx)\text{Li}_2\left(e^{2(c+dx)}\right)}{ad^2}+\frac{(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*d)) - ((e
 + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*d) + ((e + f*x
)^2*Log[1 - E^(2*(c + d*x))])/(a*d) - (2*f*(e + f*x)*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/(a*d^2) - (2*f*(e + f*x)*PolyLog[2, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*d^2) + (f*(e + f*x)*PolyLog[2, E^(2*
(c + d*x))])/(a*d^2) + (2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2]))])/(a*d^3) + (2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
))])/(a*d^3) - (f^2*PolyLog[3, E^(2*(c + d*x))])/(2*a*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5688

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x]^
(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\coth(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{2\int \frac{e^{2(c+dx)}(e+fx)^2}{1-e^{2(c+dx)}}\,dx}{a} - \frac{b\int \frac{e^{c+dx}(e+fx)^2}{a-\sqrt{a^2+b^2}+be^{c+dx}}\,dx}{a} - \frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+\sqrt{a^2+b^2}+be^{c+d}}}{a}$$

$$= -\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} +$$

$$= -\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} +$$

$$= -\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} +$$

$$= -\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} +$$

Mathematica [A]
time = 4.25, size = 492, normalized size = 1.51

$$\frac{2d^2(e+fx)^2\log\left(1-e^{2(c+dx)}\right)+2df(e+fx)\text{PolyLog}\left(2,e^{2(c+dx)}\right)-f^2\text{PolyLog}\left(3,e^{2(c+dx)}\right)-2\left(d^2e^2\log\left(2ae^{c+dx}+b\left(-1+e^{2(c+dx)}\right)\right)+2d^2efx\log\left(1+\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)\,e^{2c}}}\right)+d^2f^2x^2\log\left(1+\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)\,e^{2c}}}\right)+2d^2efx\log\left(1+\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)\,e^{2c}}}\right)+d^2f^2x^2\log\left(1+\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)\,e^{2c}}}\right)+2df(e+fx)\text{PolyLog}\left(2,-\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)\,e^{2c}}}\right)+2df(e+fx)\text{PolyLog}\left(2,-\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)\,e^{2c}}}\right)-2f^2\text{PolyLog}\left(3,-\frac{be^{2c+dx}}{ae^c-\sqrt{(a^2+b^2)\,e^{2c}}}\right)-2f^2\text{PolyLog}\left(3,-\frac{be^{2c+dx}}{ae^c+\sqrt{(a^2+b^2)\,e^{2c}}}\right)\right)}{2ad^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]

[Out] (2*d^2*(e + f*x)^2*Log[1 - E^(2*(c + d*x))] + 2*d*f*(e + f*x)*PolyLog[2, E^
(2*(c + d*x))] - f^2*PolyLog[3, E^(2*(c + d*x))] - 2*(d^2*e^2*Log[2*a*E^(c
+ d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*d^2*e*f*x*Log[1 + (b*E^(2*c + d*x))/
(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + d^2*f^2*x^2*Log[1 + (b*E^(2*c + d*x)
)/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*d^2*e*f*x*Log[1 + (b*E^(2*c + d*
x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + d^2*f^2*x^2*Log[1 + (b*E^(2*c +
d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*d*f*(e + f*x)*PolyLog[2, -((
b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2*d*f*(e + f*x)*Po
lyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*f^2*
PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*f^
2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(2
*a*d^3)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \coth(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*d) - log(e^(-d*x - c)
+ 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d))*e^2 + 2*(d*x*log(e^(d*x + c) + 1)
 + dilog(-e^(d*x + c)))*f*e/(a*d^2) + 2*(d*x*log(-e^(d*x + c) + 1) + dilog(
e^(d*x + c)))*f*e/(a*d^2) + (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^
(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2/(a*d^3) + (d^2*x^2*log(-e^(d*x
 + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*f^2/(a*d
^3) - 2/3*(d^3*f^2*x^3 + 3*d^3*f*x^2*e)/(a*d^3) + integrate(-2*(b*f^2*x^2 +
 2*b*f*x*e - (a*f^2*x^2*e^c + 2*a*f*x*e^(c + 1))*e^(d*x))/(a*b*e^(2*d*x + 2
*c) + 2*a^2*e^(d*x + c) - a*b), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 990 vs. 2(308) = 616.
time = 0.40, size = 990, normalized size = 3.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*f^2*polylog(3, (a*cosh(d*x +
c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2))/b) - 2*f^2*polylog(3, cosh(d*x + c) + sinh(d*x + c)) - 2*f^2*polylog
(3, -cosh(d*x + c) - sinh(d*x + c)) - 2*(d*f^2*x + d*f*cosh(1) + d*f*sinh(1
))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(d*f^2*x + d*f*cosh(1) + d*f*s
```

```
inh(1))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sin
h(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(d*f^2*x + d*f*cosh(1) +
d*f*sinh(1))*dilog(cosh(d*x + c) + sinh(d*x + c)) + 2*(d*f^2*x + d*f*cosh(1
) + d*f*sinh(1))*dilog(-cosh(d*x + c) - sinh(d*x + c)) - (c^2*f^2 - 2*c*d*f
*cosh(1) + d^2*cosh(1)^2 + d^2*sinh(1)^2 - 2*(c*d*f - d^2*cosh(1))*sinh(1))
*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*
a) - (c^2*f^2 - 2*c*d*f*cosh(1) + d^2*cosh(1)^2 + d^2*sinh(1)^2 - 2*(c*d*f
- d^2*cosh(1))*sinh(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqr
t((a^2 + b^2)/b^2) + 2*a) - (d^2*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)*co
sh(1) + 2*(d^2*f*x + c*d*f)*sinh(1))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c
) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (d^
2*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)*cosh(1) + 2*(d^2*f*x + c*d*f)*sin
h(1))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (d^2*f^2*x^2 + 2*d^2*f*x*cosh(1) +
 d^2*cosh(1)^2 + d^2*sinh(1)^2 + 2*(d^2*f*x + d^2*cosh(1))*sinh(1))*log(cos
h(d*x + c) + sinh(d*x + c) + 1) + (c^2*f^2 - 2*c*d*f*cosh(1) + d^2*cosh(1)^
2 + d^2*sinh(1)^2 - 2*(c*d*f - d^2*cosh(1))*sinh(1))*log(cosh(d*x + c) + si
nh(d*x + c) - 1) + (d^2*f^2*x^2 - c^2*f^2 + 2*(d^2*f*x + c*d*f)*cosh(1) + 2
*(d^2*f*x + c*d*f)*sinh(1))*log(-cosh(d*x + c) - sinh(d*x + c) + 1))/(a*d^3
)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2 \coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*coth(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(c+d\,x)\,{\left(e+f\,x\right)}^2}{a+b\,\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
[Out] int((coth(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.422 $\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=205

$$-\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad}-\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad}+\frac{(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad}-\frac{f\text{PolyL}}{}$$

```
[Out] (f*x+e)*ln(1-exp(2*d*x+2*c))/a/d-(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/
2)))/a/d-(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d+1/2*f*polylog(2
,exp(2*d*x+2*c))/a/d^2-f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/d^2
-f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/d^2
```

Rubi [A]

time = 0.27, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5688, 3797, 2221, 2317, 2438, 5680}

$$-\frac{f\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2}-\frac{f\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^2}-\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{ad}-\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{ad}+\frac{f\text{Li}_2\left(e^{2(c+dx)}\right)}{2ad^2}+\frac{(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*d)) - ((e +
 f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*d) + ((e + f*x)*Lo
g[1 - E^(2*(c + d*x))])/(a*d) - (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(a*d^2) - (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
]))])/(a*d^2) + (f*PolyLog[2, E^(2*(c + d*x))])/(2*a*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5688

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x]^
(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\coth(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{2\int \frac{e^{2(c+dx)}(e+fx)}{1-e^{2(c+dx)}}\,dx}{a} - \frac{b\int \frac{e^{c+dx}(e+fx)}{a-\sqrt{a^2+b^2}+be^{c+dx}}\,dx}{a} - \frac{b\int \frac{e^{c+dx}(e+fx)}{a+\sqrt{a^2+b^2}+be^{c+dx}}\,\epsilon}{a}$$

$$= -\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e}{}$$

$$= -\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e}{}$$

$$= -\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e}{}$$

Mathematica [A]
time = 0.69, size = 236, normalized size = 1.15

$$-\frac{-f(c+dx)^2 - f(c+dx)\log\left(1-e^{-2(c+dx)}\right) + f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) + f(c+dx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right) - de\log(\sinh(c+dx)) + cf\log(\sinh(c+dx)) + de\log(a+b\sinh(c+dx)) - cf\log(a+b\sinh(c+dx)) + \frac{1}{2}f\,\mathrm{PolyLog}\left(2,e^{-2(c+dx)}\right) + f\,\mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right) + f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((-(f*(c + d*x)^2) - f*(c + d*x)*Log[1 - E^(-2*(c + d*x))] + f*(c + d*x)*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2])] - d*e*Log[Sinh[c + d*x]] + c*f*Log[Sinh[c +
 d*x]] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + (f*P
olyLog[2, E^(-2*(c + d*x))])/2 + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^
2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*d^2
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(191) = 382$.
time = 3.05, size = 451, normalized size = 2.20

method	result
risch	$\frac{e\ln(\mathrm{e}^{dx+c}-1)}{da} - \frac{e\ln(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-b)}{da} + \frac{e\ln(\mathrm{e}^{dx+c}+1)}{da} - \frac{f\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)x}{da} - \frac{f\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a}}{-a+\sqrt{a}}\right)}{d^2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*e/a*ln(exp(d*x+c)-1)-1/d*e/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/d*
e/a*ln(exp(d*x+c)+1)-1/d*f/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+
b^2)^(1/2)))*x-1/d^2*f/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)
^(1/2)))*c-1/d*f/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))
*x-1/d^2*f/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d
*f/a*ln(exp(d*x+c)+1)*x+1/d^2*f/a*dilog(exp(d*x+c)+1)-1/d^2*f*dilog(exp(d*x
+c))/a-1/d^2*f/a*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2
)))-1/d^2*f/a*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1
/d^2*f*c/a*ln(exp(d*x+c)-1)+1/d^2*f*c/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-
b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*d) - log(e^(-d*x - c)
+ 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d))*e + f*integrate(2*x*(e^(d*x + c)
+ e^(-d*x - c))/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(e^(d*x + c) - e^(-
d*x - c))), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(191) = 382$.
time = 0.38, size = 500, normalized size = 2.44

$$-\frac{f\mathrm{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}+1\right)+f\mathrm{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}+1\right)-f\mathrm{Li}_2(\cosh(dx+c)+\sinh(dx+c))-f\mathrm{Li}_2(-\cosh(dx+c)-\sinh(dx+c))-(cf-d\cosh(1)-d\sinh(1))\log\left(2b\cosh(dx+c)+2b\sinh(dx+c)+2b\sqrt{\frac{a^2+b^2}{b^2}}+2a\right)-(cf-d\cosh(1)-d\sinh(1))\log\left(2b\cosh(dx+c)+2b\sinh(dx+c)-2b\sqrt{\frac{a^2+b^2}{b^2}}+2a\right)+(dfx+cf)\log\left(-\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}\right)+(dfx+cf)\log\left(-\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}\right)-(dfx+d\cosh(1)+d\sinh(1))\log(\cosh(dx+c)+\sinh(dx+c)+1)+(cf-d\cosh(1)-d\sinh(1))\log(\cosh(dx+c)+\sinh(dx+c)-1)-(dfx+cf)\log(-\cosh(dx+c)-\sinh(dx+c)+1)}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + f*dilog((a*cosh(d*x + c) + a*si
nh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b
)/b + 1) - f*dilog(cosh(d*x + c) + sinh(d*x + c)) - f*dilog(-cosh(d*x + c)
- sinh(d*x + c)) - (c*f - d*cosh(1) - d*sinh(1))*log(2*b*cosh(d*x + c) + 2*
b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (c*f - d*cosh(1) - d*s
inh(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^
```

```
2) + 2*a) + (d*f*x + c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (d*f*x + c*f)*
log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b) - (d*f*x + d*cosh(1) + d*sinh(1))*log(cosh
(d*x + c) + sinh(d*x + c) + 1) + (c*f - d*cosh(1) - d*sinh(1))*log(cosh(d*x
 + c) + sinh(d*x + c) - 1) - (d*f*x + c*f)*log(-cosh(d*x + c) - sinh(d*x +
c) + 1))/(a*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] integrate((f*x + e)*coth(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(c+d\,x)\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
[Out] int((coth(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.423 $\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{\log(\sinh(c+dx))}{ad}-\frac{\log(a+b\sinh(c+dx))}{ad}$$

```
[Out] ln(sinh(d*x+c))/a/d-ln(a+b*sinh(d*x+c))/a/d
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2800, 36, 29, 31}

$$\frac{\log(\sinh(c+dx))}{ad}-\frac{\log(a+b\sinh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Log[Sinh[c + d*x]]/(a*d) - Log[a + b*Sinh[c + d*x]]/(a*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\sinh(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh(c+dx)\right)}{ad}$$
$$= \frac{\log(\sinh(c+dx))}{ad} - \frac{\log(a+b\sinh(c+dx))}{ad}$$

Mathematica [A]
time = 0.03, size = 28, normalized size = 0.82

$$\frac{\log(\sinh(c+dx)) - \log(a+b\sinh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/(a + b*Sinh[c + d*x]),x]
[Out] (Log[Sinh[c + d*x]] - Log[a + b*Sinh[c + d*x]])/(a*d)
```

Maple [A]
time = 0.74, size = 33, normalized size = 0.97

method	result	size
derivativedivides	$\frac{-\frac{\ln(a+b\sinh(dx+c))}{a}+\frac{\ln(\sinh(dx+c))}{a}}{d}$	33
default	$\frac{-\frac{\ln(a+b\sinh(dx+c))}{a}+\frac{\ln(\sinh(dx+c))}{a}}{d}$	33
risch	$\frac{\ln(e^{2dx+2c}-1)}{ad}-\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{ad}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/a*ln(a+b*sinh(d*x+c))+1/a*ln(sinh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(34) = 68$.
time = 0.28, size = 75, normalized size = 2.21

$$-\frac{\log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{ad} + \frac{\log\left(e^{(-dx-c)} + 1\right)}{ad} + \frac{\log\left(e^{(-dx-c)} - 1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*d) + log(e^(-d*x - c) +
 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d)
```

Fricas [A]
time = 0.40, size = 67, normalized size = 1.97

$$-\frac{\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)-\log\left(\frac{2\,\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) - log(2*sinh
(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A]
time = 0.48, size = 61, normalized size = 1.79

$$-\frac{\frac{\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a}-\frac{\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -(log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/a - log(abs(e^(d*x + c) -
e^(-d*x - c)))/a)/d
```

Mupad [B]
time = 0.41, size = 254, normalized size = 7.47

$$\frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-a^2\,d^2}+b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-a^2\,d^2}-2\,a\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{-a^2\,d^2}-b\,\mathrm{e}^{3\,c}\,\mathrm{e}^{3\,d\,x}\,\sqrt{-a^2\,d^2}}{a^2\,d}\right)}{\sqrt{-a^2\,d^2}}-\frac{2\,\mathrm{atan}\left(\left(4\,a^4\,b\,d\,\sqrt{-a^2\,d^2}+4\,a^2\,b^3\,d\,\sqrt{-a^2\,d^2}\right)\,\left(\frac{1}{8\,a\,b\,d^2\,{\left(a^2+b^2\right)}^2}-\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(\frac{1}{16\,b^2\,d^2\,{\left(a^2+b^2\right)}^2}-\frac{{\left(a^2+2\,b^2\right)}^2}{16\,a^4\,b^2\,d^2\,{\left(a^2+b^2\right)}^2}\right)+\frac{a^2+2\,b^2}{8\,a^3\,b\,d^2\,{\left(a^2+b^2\right)}^2}\right)\right)}{\sqrt{-a^2\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)/(a + b*sinh(c + d*x)),x)
```

```
[Out] (2*atan((a*(-a^2*d^2)^(1/2) + b*exp(d*x)*exp(c)*(-a^2*d^2)^(1/2) - 2*a*exp(
2*c)*exp(2*d*x)*(-a^2*d^2)^(1/2) - b*exp(3*c)*exp(3*d*x)*(-a^2*d^2)^(1/2))/
(a^2*d)))/(-a^2*d^2)^(1/2) - (2*atan((4*a^4*b*d*(-a^2*d^2)^(1/2) + 4*a^2*b^
3*d*(-a^2*d^2)^(1/2))*(1/(8*a*b*d^2*(a^2 + b^2)^2) - exp(d*x)*exp(c)*(1/(16
*b^2*d^2*(a^2 + b^2)^2) - (a^2 + 2*b^2)^2/(16*a^4*b^2*d^2*(a^2 + b^2)^2)) +
 (a^2 + 2*b^2)/(8*a^3*b*d^2*(a^2 + b^2)^2))))/(-a^2*d^2)^(1/2)
```

3.424 $\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Coth[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][Coth[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 21.32, size = 0, normalized size = 0.00

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Coth[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Integrate[Coth[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(coth(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(coth(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(coth(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\coth(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))),x)
[Out] int(coth(c + d*x)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.425 $\int \frac{(e+fx)^3 \cosh(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal. Leaf size=638

$$\frac{(e+fx)^4}{4bf}-\frac{2(e+fx)^3 \tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{\sqrt{a^2+b^2}\,(e+fx)^3 \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd}+\frac{\sqrt{a^2+b^2}\,(e+fx)^3 \text{l}}{a}$$

```
[Out] 1/4*(f*x+e)^4/b/f-2*(f*x+e)^3*arctanh(exp(d*x+c))/a/d-3*f*(f*x+e)^2*polylog
(2,-exp(d*x+c))/a/d^2+3*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a/d^2+6*f^2*(f*x+
e)*polylog(3,-exp(d*x+c))/a/d^3-6*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a/d^3-6
*f^3*polylog(4,-exp(d*x+c))/a/d^4+6*f^3*polylog(4,exp(d*x+c))/a/d^4-(f*x+e)
^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a/b/d+(f*x+e)^3*l
n(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a/b/d-3*f*(f*x+e)^2*p
olylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a/b/d^2+3*f*(f*
x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a/b/d^2
+6*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)
/a/b/d^3-6*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^
2)^(1/2)/a/b/d^3-6*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^
2)^(1/2)/a/b/d^4+6*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^
2)^(1/2)/a/b/d^4
```

Rubi [A]
time = 0.91, antiderivative size = 638, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 14, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5704, 5558, 3377, 2717, 4267, 2611, 6744, 2320, 6724, 5684, 32, 3403, 2296, 2221}

$$-\frac{6f^3\sqrt{a^2+b^2}\,\text{Li}_4\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^4}+\frac{6f^3\sqrt{a^2+b^2}\,\text{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd^4}+\frac{6f^2\sqrt{a^2+b^2}\,(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^3}-\frac{6f^2\sqrt{a^2+b^2}\,(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd^3}-\frac{3f\sqrt{a^2+b^2}\,(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^2}+\frac{3f\sqrt{a^2+b^2}\,(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd^2}-\frac{\sqrt{a^2+b^2}\,(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{abd}+\frac{\sqrt{a^2+b^2}\,(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{abd}-\frac{6f^3\text{Li}_4\left(-e^{c+dx}\right)}{ad^4}+\frac{6f^3\text{Li}_4\left(e^{c+dx}\right)}{ad^4}+\frac{6f^2(e+fx)\text{Li}_3\left(-e^{c+dx}\right)}{ad^3}-\frac{6f^2(e+fx)\text{Li}_3\left(e^{c+dx}\right)}{ad^3}-\frac{3f(e+fx)^2\text{Li}_2\left(-e^{c+dx}\right)}{ad^2}+\frac{3f(e+fx)^2\text{Li}_2\left(e^{c+dx}\right)}{ad^2}-\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{(e+fx)^4}{4bf}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e + f*x)^4/(4*b*f) - (2*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) - (Sqrt[a^
2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*b*d
) + (Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(a*b*d) - (3*f*(e + f*x)^2*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (3*f*(
e + f*x)^2*PolyLog[2, E^(c + d*x)])/(a*d^2) - (3*Sqrt[a^2 + b^2]*f*(e + f*x
)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b*d^2) + (3*Sq
rt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2]))])/(a*b*d^2) + (6*f^2*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) - (6*
f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a*d^3) + (6*Sqrt[a^2 + b^2]*f^2*(e
+ f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b*d^3) - (6
*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
```

```
b^2]))])/(a*b*d^3) - (6*f^3*PolyLog[4, -E^(c + d*x)])/(a*d^4) + (6*f^3*Pol
yLog[4, E^(c + d*x)])/(a*d^4) - (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b*d^4) + (6*Sqrt[a^2 + b^2]*f^3*PolyLo
g[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*b*d^4)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5704

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\cosh(c+dx)\coth(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cosh^2(c+d}{a+b\sinh(c+dx)}}{a}$$

$$= \frac{\int (e+fx)^3\operatorname{csch}(c+dx)\,dx}{a} + \frac{\int (e+fx)^3\,dx}{b} - \frac{(a^2+b^2)\int \frac{}{a+l}}{ab}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{(2(a^2+b^2))\int \frac{e^c}{-b+2a}}{ab}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{3f(e+fx)^2\mathrm{Li}_2\left(-e\right.}{ad^2}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}\,(e+fx)^3}{}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}\,(e+fx)^3}{}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}\,(e+fx)^3}{}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}\,(e+fx)^3}{}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}\,(e+fx)^3}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1374 vs. $2(638) = 1276$.
time = 4.02, size = 1374, normalized size = 2.15

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(4*b) + (-2*d^3*e^3*ArcTanh
[E^(c + d*x)] + 3*d^3*e^2*f*x*Log[1 - E^(c + d*x)] + 3*d^3*e*f^2*x^2*Log[1
```

```
- E^(c + d*x)] + d^3*f^3*x^3*Log[1 - E^(c + d*x)] - 3*d^3*e^2*f*x*Log[1 + E
^(c + d*x)] - 3*d^3*e*f^2*x^2*Log[1 + E^(c + d*x)] - d^3*f^3*x^3*Log[1 + E^
(c + d*x)] - 3*d^2*f*(e + f*x)^2*PolyLog[2, -E^(c + d*x)] + 3*d^2*f*(e + f*
x)^2*PolyLog[2, E^(c + d*x)] + 6*d*e*f^2*PolyLog[3, -E^(c + d*x)] + 6*d*f^3
*x*PolyLog[3, -E^(c + d*x)] - 6*d*e*f^2*PolyLog[3, E^(c + d*x)] - 6*d*f^3*x
*PolyLog[3, E^(c + d*x)] - 6*f^3*PolyLog[4, -E^(c + d*x)] + 6*f^3*PolyLog[4
, E^(c + d*x)])/(a*d^4) + (Sqrt[-a^2 - b^2]*(2*d^3*e^3*Sqrt[(a^2 + b^2)*E^(
2*c)]*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]] + 3*Sqrt[-a^2 - b^2]*d^3
*e^2*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])]
 + 3*Sqrt[-a^2 - b^2]*d^3*e*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c -
Sqrt[(a^2 + b^2)*E^(2*c)])] + Sqrt[-a^2 - b^2]*d^3*E^c*f^3*x^3*Log[1 + (b*E
^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*Sqrt[-a^2 - b^2]*d^3
*e^2*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])]
 - 3*Sqrt[-a^2 - b^2]*d^3*e*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c +
Sqrt[(a^2 + b^2)*E^(2*c)])] - Sqrt[-a^2 - b^2]*d^3*E^c*f^3*x^3*Log[1 + (b*E
^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 3*Sqrt[-a^2 - b^2]*d^2
*E^c*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)
*E^(2*c)]))] - 3*Sqrt[-a^2 - b^2]*d^2*E^c*f*(e + f*x)^2*PolyLog[2, -((b*E^(
2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[-a^2 - b^2]*d*e*
E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))
] - 6*Sqrt[-a^2 - b^2]*d*E^c*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c -
Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[-a^2 - b^2]*d*e*E^c*f^2*PolyLog[3, -(
(b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[-a^2 - b^2
]*d*E^c*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2
*c)]))] + 6*Sqrt[-a^2 - b^2]*E^c*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c
- Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[-a^2 - b^2]*E^c*f^3*PolyLog[4, -((b
*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(a*b*d^4*Sqrt[(a^2
+ b^2)*E^(2*c)])
```

Maple [F]
time = 3.24, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \cosh (dx+c) \coth (dx+c)}{a+b\sinh (dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] ((d*x + c)/(b*d) - log(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d
) - sqrt(a^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x -
 c) - a + sqrt(a^2 + b^2)))/(a*b*d))*e^3 + 1/4*(f^3*x^4 + 4*f^2*x^3*e + 6*f
*x^2*e^2)/b - 3*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*f*e^2/(a*d
^2) + 3*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))*f*e^2/(a*d^2) - 3*
(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e
^(d*x + c)))*f^2*e/(a*d^3) + 3*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog
(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*f^2*e/(a*d^3) - (d^3*x^3*log(e^(
d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x +
c)) + 6*polylog(4, -e^(d*x + c)))*f^3/(a*d^4) + (d^3*x^3*log(-e^(d*x + c) +
 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*poly
log(4, e^(d*x + c)))*f^3/(a*d^4) - integrate(2*((a^2*f^3*e^c + b^2*f^3*e^c)
*x^3 + 3*(a^2*f^2*e^c + b^2*f^2*e^c)*x^2*e + 3*(a^2*f*e^c + b^2*f*e^c)*x*e^
2)*e^(d*x)/(a*b^2*e^(2*d*x + 2*c) + 2*a^2*b*e^(d*x + c) - a*b^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2193 vs. $2(597) = 1194$.
time = 0.46, size = 2193, normalized size = 3.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] 1/4*(a*d^4*f^3*x^4 + 4*a*d^4*f^2*x^3*cosh(1) + 6*a*d^4*f*x^2*cosh(1)^2 + 4*
a*d^4*x*cosh(1)^3 + 4*a*d^4*x*sinh(1)^3 - 24*b*f^3*sqrt((a^2 + b^2)/b^2)*po
lylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2))/b) + 24*b*f^3*sqrt((a^2 + b^2)/b^2)*polylog(4
, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2))/b) + 24*b*f^3*polylog(4, cosh(d*x + c) + sinh(d*x +
c)) - 24*b*f^3*polylog(4, -cosh(d*x + c) - sinh(d*x + c)) + 6*(a*d^4*f*x^2
+ 2*a*d^4*x*cosh(1))*sinh(1)^2 - 12*(b*d^2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1)
+ b*d^2*f*cosh(1)^2 + b*d^2*f*sinh(1)^2 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))
*sinh(1))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) +
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 12*
(b*d^2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^2*f*cosh(1)^2 + b*d^2*f*sinh(1
)^2 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*dilo
g((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 4*(b*c^3*f^3 - 3*b*c^2*d*f^2*cosh(1) +
3*b*c*d^2*f*cosh(1)^2 - b*d^3*cosh(1)^3 - b*d^3*sinh(1)^3 + 3*(b*c*d^2*f -
b*d^3*cosh(1))*sinh(1)^2 - 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(1) + b*d^3*cos
h(1)^2)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x
```

```
 + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 4*(b*c^3*f^3 - 3*b*c^2*d*f^2*cos
h(1) + 3*b*c*d^2*f*cosh(1)^2 - b*d^3*cosh(1)^3 - b*d^3*sinh(1)^3 + 3*(b*c*d
^2*f - b*d^3*cosh(1))*sinh(1)^2 - 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(1) + b*
d^3*cosh(1)^2)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*s
inh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 4*(b*d^3*f^3*x^3 + b*c^3*
f^3 + 3*(b*d^3*f*x + b*c*d^2*f)*cosh(1)^2 + 3*(b*d^3*f*x + b*c*d^2*f)*sinh(
1)^2 + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2)*cosh(1) + 3*(b*d^3*f^2*x^2 - b*c^2*d
*f^2 + 2*(b*d^3*f*x + b*c*d^2*f)*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*lo
g(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2) - b)/b) + 4*(b*d^3*f^3*x^3 + b*c^3*f^3 + 3*(b*d^3*f*
x + b*c*d^2*f)*cosh(1)^2 + 3*(b*d^3*f*x + b*c*d^2*f)*sinh(1)^2 + 3*(b*d^3*f
^2*x^2 - b*c^2*d*f^2)*cosh(1) + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2 + 2*(b*d^3*f
*x + b*c*d^2*f)*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x +
c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2) - b)/b) + 24*(b*d*f^3*x + b*d*f^2*cosh(1) + b*d*f^2*sinh(1))*sqrt((a^
2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
 c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 24*(b*d*f^3*x + b*d*f^2*
cosh(1) + b*d*f^2*sinh(1))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c
) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2))/b) + 12*(b*d^2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^2*f*cosh(1)^2 +
b*d^2*f*sinh(1)^2 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh(1))*dilog(cosh(d
*x + c) + sinh(d*x + c)) - 12*(b*d^2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^
2*f*cosh(1)^2 + b*d^2*f*sinh(1)^2 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh(
1))*dilog(-cosh(d*x + c) - sinh(d*x + c)) - 4*(b*d^3*f^3*x^3 + 3*b*d^3*f^2*
x^2*cosh(1) + 3*b*d^3*f*x*cosh(1)^2 + b*d^3*cosh(1)^3 + b*d^3*sinh(1)^3 + 3
*(b*d^3*f*x + b*d^3*cosh(1))*sinh(1)^2 + 3*(b*d^3*f^2*x^2 + 2*b*d^3*f*x*cos
h(1) + b*d^3*cosh(1)^2)*sinh(1))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 4
*(b*c^3*f^3 - 3*b*c^2*d*f^2*cosh(1) + 3*b*c*d^2*f*cosh(1)^2 - b*d^3*cosh(1)
^3 - b*d^3*sinh(1)^3 + 3*(b*c*d^2*f - b*d^3*cosh(1))*sinh(1)^2 - 3*(b*c^2*d
*f^2 - 2*b*c*d^2*f*cosh(1) + b*d^3*cosh(1)^2)*sinh(1))*log(cosh(d*x + c) +
sinh(d*x + c) - 1) + 4*(b*d^3*f^3*x^3 + b*c^3*f^3 + 3*(b*d^3*f*x + b*c*d^2*
f)*cosh(1)^2 + 3*(b*d^3*f*x + b*c*d^2*f)*sinh(1)^2 + 3*(b*d^3*f^2*x^2 - b*c
^2*d*f^2)*cosh(1) + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2 + 2*(b*d^3*f*x + b*c*d^2
*f)*cosh(1))*sinh(1))*log(-cosh(d*x + c) - sinh(d*x + c) + 1) - 24*(b*d*f^3
*x + b*d*f^2*cosh(1) + b*d*f^2*sinh(1))*polylog(3, cosh(d*x + c) + sinh(d*x
 + c)) + 24*(b*d*f^3*x + b*d*f^2*cosh(1) + b*d*f^2*sinh(1))*polylog(3, -cos
h(d*x + c) - sinh(d*x + c)) + 4*(a*d^4*f^2*x^3 + 3*a*d^4*f*x^2*cosh(1) + 3*
a*d^4*x*cosh(1)^2)*sinh(1))/(a*b*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3 \cosh(c+dx) \coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*cosh(c + d*x)*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)\,\left(e+f\,x\right)^3}{a+b\,\mathrm{sinh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*coth(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((cosh(c + d*x)*coth(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.426 $\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=462

$$\frac{(e+fx)^3}{3bf}-\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{\sqrt{a^2+b^2}\,(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd}+\frac{\sqrt{a^2+b^2}\,(e+fx)^2\log}{ab}$$

```
[Out] 1/3*(f*x+e)^3/b/f-2*(f*x+e)^2*arctanh(exp(d*x+c))/a/d-2*f*(f*x+e)*polylog(2
,-exp(d*x+c))/a/d^2+2*f*(f*x+e)*polylog(2,exp(d*x+c))/a/d^2+2*f^2*polylog(3
,-exp(d*x+c))/a/d^3-2*f^2*polylog(3,exp(d*x+c))/a/d^3-(f*x+e)^2*ln(1+b*exp(
d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a/b/d+(f*x+e)^2*ln(1+b*exp(d*x+
c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a/b/d-2*f*(f*x+e)*polylog(2,-b*exp(
d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a/b/d^2+2*f*(f*x+e)*polylog(2,-
b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a/b/d^2+2*f^2*polylog(3,-
b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a/b/d^3-2*f^2*polylog(3,-
b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a/b/d^3
```

Rubi [A]
time = 0.77, antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {5704, 5558, 3377, 2718, 4267, 2611, 2320, 6724, 5684, 32, 3403, 2296, 2221}

$$\frac{2f^2\sqrt{a^2+b^2}\,\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^3}-\frac{2f^2\sqrt{a^2+b^2}\,\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd^3}-\frac{2f\sqrt{a^2+b^2}\,(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^2}+\frac{2f\sqrt{a^2+b^2}\,(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd^2}-\frac{\sqrt{a^2+b^2}\,(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{abd}+\frac{\sqrt{a^2+b^2}\,(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{abd}+\frac{2f^2\text{Li}_3\left(-e^{c+dx}\right)}{ad^3}-\frac{2f^2\text{Li}_3\left(e^{c+dx}\right)}{ad^3}-\frac{2f(e+fx)\text{Li}_2\left(-e^{c+dx}\right)}{ad^2}+\frac{2f(e+fx)\text{Li}_2\left(e^{c+dx}\right)}{ad^2}-\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{(e+fx)^3}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e + f*x)^3/(3*b*f) - (2*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (Sqrt[a^
2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*b*d
) + (Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(a*b*d) - (2*f*(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (2*f*(e
+ f*x)*PolyLog[2, E^(c + d*x)])/(a*d^2) - (2*Sqrt[a^2 + b^2]*f*(e + f*x)*Po
lyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b*d^2) + (2*Sqrt[a^2
 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
a*b*d^2) + (2*f^2*PolyLog[3, -E^(c + d*x)])/(a*d^3) - (2*f^2*PolyLog[3, E^(
c + d*x)])/(a*d^3) + (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2]))])/(a*b*d^3) - (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*b*d^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
```

```
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5704

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh(c+dx)\coth(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cosh^2(c+d}{a+b\sinh(c+dx)}}{a}$$

$$= \frac{\int (e+fx)^2\operatorname{csch}(c+dx)\,dx}{a} + \frac{\int (e+fx)^2\,dx}{b} - \frac{(a^2+b^2)\int}{ab}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{(2(a^2+b^2))\int}{ab}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2f(e+fx)\mathrm{Li}_2\left(-e^{c}\right.}{ad^2}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}\,(e+fx)^2}{}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}\,(e+fx)^2}{}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}\,(e+fx)^2}{}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}\,(e+fx)^2}{}$$

Mathematica [A]
time = 3.16, size = 815, normalized size = 1.76

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2))/(3*b) + (-2*d^2*e^2*ArcTanh[E^(c + d*x)] +
2*d^2*e*f*x*Log[1 - E^(c + d*x)] + d^2*f^2*x^2*Log[1 - E^(c + d*x)] - 2*d^2
*e*f*x*Log[1 + E^(c + d*x)] - d^2*f^2*x^2*Log[1 + E^(c + d*x)] - 2*d*f*(e +
 f*x)*PolyLog[2, -E^(c + d*x)] + 2*d*f*(e + f*x)*PolyLog[2, E^(c + d*x)] +
2*f^2*PolyLog[3, -E^(c + d*x)] - 2*f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) - (
(a^2 + b^2)*((2*d^2*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[
-a^2 - b^2] + (2*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2
```

```
 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] + (d^2*E^c*f^2*x^2*Log[1 + (b
*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2
*c)] - (2*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)
*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] - (d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c
 + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] +
(2*d*E^c*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^
2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*d*E^c*f*(e + f*x)*PolyLog[2,
 -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2
)*E^(2*c)] - (2*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 +
 b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] + (2*E^c*f^2*PolyLog[3, -((b*E
^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*
c)]))/(a*b*d^3)
```

Maple [**F**]
time = 3.16, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \cosh(dx+c) \coth(dx+c)}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] ((d*x + c)/(b*d) - log(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d
) - sqrt(a^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x -
 c) - a + sqrt(a^2 + b^2)))/(a*b*d))*e^2 + 1/3*(f^2*x^3 + 3*f*x^2*e)/b - 2*
(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*f*e/(a*d^2) + 2*(d*x*log(-
e^(d*x + c) + 1) + dilog(e^(d*x + c)))*f*e/(a*d^2) - (d^2*x^2*log(e^(d*x +
c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2/(a*d^
3) + (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(
3, e^(d*x + c)))*f^2/(a*d^3) - integrate(2*((a^2*f^2*e^c + b^2*f^2*e^c)*x^2
 + 2*(a^2*f*e^c + b^2*f*e^c)*x*e)*e^(d*x)/(a*b^2*e^(2*d*x + 2*c) + 2*a^2*b*
e^(d*x + c) - a*b^2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1227 vs. $2(429) = 858$.

time = 0.43, size = 1227, normalized size = 2.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] 1/3*(a*d^3*f^2*x^3 + 3*a*d^3*f*x^2*cosh(1) + 3*a*d^3*x*cosh(1)^2 + 3*a*d^3*
x*sinh(1)^2 + 6*b*f^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a
*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))
/b) - 6*b*f^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*
x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*
b*f^2*polylog(3, cosh(d*x + c) + sinh(d*x + c)) + 6*b*f^2*polylog(3, -cosh(
d*x + c) - sinh(d*x + c)) - 6*(b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1))*s
qrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 6*(b*d*f^2*x +
 b*d*f*cosh(1) + b*d*f*sinh(1))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c
) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2) - b)/b + 1) + 3*(b*c^2*f^2 - 2*b*c*d*f*cosh(1) + b*d^2*cosh(1)^2 + b*d
^2*sinh(1)^2 - 2*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*l
og(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
 - 3*(b*c^2*f^2 - 2*b*c*d*f*cosh(1) + b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 - 2
*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x
+ c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(b*d^2*f^2*
x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*cosh(1) + 2*(b*d^2*f*x + b*c*d*f)
*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 3*(b*d^2
*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*cosh(1) + 2*(b*d^2*f*x + b*c
*d*f)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c
) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 6*(
b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1))*dilog(cosh(d*x + c) + sinh(d*x +
 c)) - 6*(b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1))*dilog(-cosh(d*x + c) -
 sinh(d*x + c)) - 3*(b*d^2*f^2*x^2 + 2*b*d^2*f*x*cosh(1) + b*d^2*cosh(1)^2
+ b*d^2*sinh(1)^2 + 2*(b*d^2*f*x + b*d^2*cosh(1))*sinh(1))*log(cosh(d*x + c
) + sinh(d*x + c) + 1) + 3*(b*c^2*f^2 - 2*b*c*d*f*cosh(1) + b*d^2*cosh(1)^2
 + b*d^2*sinh(1)^2 - 2*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*log(cosh(d*x + c)
 + sinh(d*x + c) - 1) + 3*(b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d
*f)*cosh(1) + 2*(b*d^2*f*x + b*c*d*f)*sinh(1))*log(-cosh(d*x + c) - sinh(d*
x + c) + 1) + 3*(a*d^3*f*x^2 + 2*a*d^3*x*cosh(1))*sinh(1))/(a*b*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2 \cosh(c+dx) \coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*cosh(c + d*x)*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)\,{(e+f\,x)}^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*coth(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((cosh(c + d*x)*coth(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.427 $\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=286

$$\frac{ex}{b}+\frac{fx^2}{2b}-\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{\sqrt{a^2+b^2}\,(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd}+\frac{\sqrt{a^2+b^2}\,(e+fx)\log\left(\right.}{abd}$$

```
[Out] e*x/b+1/2*f*x^2/b-2*(f*x+e)*arctanh(exp(d*x+c))/a/d-f*polylog(2,-exp(d*x+c)
)/a/d^2+f*polylog(2,exp(d*x+c))/a/d^2-(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2
)^(1/2)))*(a^2+b^2)^(1/2)/a/b/d+(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2
)))*(a^2+b^2)^(1/2)/a/b/d-f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a
^2+b^2)^(1/2)/a/b/d^2+f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b
^2)^(1/2)/a/b/d^2
```

Rubi [A]
time = 0.43, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {5704, 5558, 3377, 2717, 4267, 2317, 2438, 5684, 3403, 2296, 2221}

$$-\frac{f\sqrt{a^2+b^2}\,\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^2}+\frac{f\sqrt{a^2+b^2}\,\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{abd^2}-\frac{\sqrt{a^2+b^2}\,(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{abd}+\frac{\sqrt{a^2+b^2}\,(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{abd}-\frac{f\text{Li}_2\left(-e^{c+dx}\right)}{ad^2}+\frac{f\text{Li}_2\left(e^{c+dx}\right)}{ad^2}-\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{ex}{b}+\frac{fx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e*x)/b + (f*x^2)/(2*b) - (2*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d) - (Sqrt[
a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*b*d
) + (Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)])/(a*b*d) - (f*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (f*PolyLog[2, E^(c + d
*x)])/(a*d^2) - (Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(a*b*d^2) + (Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(a*b*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
```

```
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5704

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh(c+dx)\coth(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,d}{a}$$

$$= \frac{\int (e+fx)\text{csch}(c+dx)\,dx}{a} + \frac{\int (e+fx)\,dx}{b} - \frac{(a^2+b^2)\int \frac{e+}{a+b\sin}}{ab}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\left(2(a^2+b^2)\right)\int \frac{e^{c+}}{-b+2ae^{c}}}{ab}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\left(2\sqrt{a^2+b^2}\right)\int \frac{}{2a-2}}{a}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}\,(e+fx)\log}{ab}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}\,(e+fx)\log}{ab}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}\,(e+fx)\log}{ab}$$

Mathematica [A]

time = 1.31, size = 339, normalized size = 1.19

$$\frac{-a(c+dx)(cf-d(2e+fx))+2bde\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-2bcf\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+2bf\left((c+dx)\left(\log\left(1-e^{-c-dx}\right)-\log\left(1+e^{-c-dx}\right)\right)+\operatorname{PolyLog}(2,-e^{-c-dx})-\operatorname{PolyLog}(2,e^{-c-dx})\right)+2\sqrt{a^2+b^2}\left(2de\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-2cf\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+f(c+dx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)-f\operatorname{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)+f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)}{2abd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-(a*(c + d*x)*(c*f - d*(2*e + f*x))) + 2*b*d*e*Log[Tanh[(c + d*x)/2]] - 2*
b*c*f*Log[Tanh[(c + d*x)/2]] + 2*b*f*((c + d*x)*(Log[1 - E^(-c - d*x)] - Lo
g[1 + E^(-c - d*x)]) + PolyLog[2, -E^(-c - d*x)] - PolyLog[2, E^(-c - d*x)]
) + 2*Sqrt[a^2 + b^2]*(2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] -
 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - f*(c + d*x)*Log[1 + (
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2])] - f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2
])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(2*a*b*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(261) = 522$.
time = 13.16, size = 970, normalized size = 3.39

method	result
risch	$\frac{f\,x^2}{2b}+\frac{ex}{b}-\frac{2afc\,\mathrm{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{d^2b\sqrt{a^2+b^2}}-\frac{f\,\mathrm{dilog}(\mathrm{e}^{dx+c}+1)}{d^2a}-\frac{f\,\mathrm{dilog}(\mathrm{e}^{dx+c})}{d^2a}+\frac{2ae\,\mathrm{arctanh}\left(\frac{2b\,\mathrm{e}^{dx+c}+2a}{2\sqrt{a^2+b^2}}\right)}{db\sqrt{a^2+b^2}}-fc$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/2*f*x^2/b+e*x/b-2/d^2/b*a*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)
+2*a)/(a^2+b^2)^(1/2))-1/d^2*f/a*dilog(exp(d*x+c)+1)-1/d^2*f*dilog(exp(d*x+
c))/a+2/d/b*e*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^
(1/2))-1/d^2*f*c/a*ln(exp(d*x+c)-1)-1/d*a/b*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*
x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d^2*a/b*f/(a^2+b^2)^(1/2)
*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d*a/b*f/(a^
2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d
^2*a/b*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(
1/2)))*c-2/d^2*b*f*c/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^
2+b^2)^(1/2))-1/d*f/a*ln(exp(d*x+c)+1)*x-1/d^2*b*f/a/(a^2+b^2)^(1/2)*dilog(
(-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/d^2*b*f/a/(a^2+b^
2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d*b*
f/a/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2
)))*x-1/d^2*b*f/a/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+
(a^2+b^2)^(1/2)))*c+1/d*b*f/a/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1
/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2*b*f/a/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+
```

```
(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d^2/b*a*f/(a^2+b^2)^(1/2)*dilog
((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/d^2/b*a*f/(a^2+b
^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/d*b
*e/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d*
e/a*ln(exp(d*x+c)+1)+1/d*e/a*ln(exp(d*x+c)-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] -1/2*(4*(a^2*e^c + b^2*e^c)*integrate(x*e^(d*x)/(a*b^2*e^(2*d*x + 2*c) + 2*
a^2*b*e^(d*x + c) - a*b^2), x) - x^2/b - 2*integrate(x/(a*e^(d*x + c) + a),
 x) - 2*integrate(x/(a*e^(d*x + c) - a), x))*f + ((d*x + c)/(b*d) - log(e^(
-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d) - sqrt(a^2 + b^2)*log((b
*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2))
)/(a*b*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 635 vs. 2(261) = 522.
time = 0.40, size = 635, normalized size = 2.22

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] 1/2*(a*d^2*f*x^2 + 2*a*d^2*x*cosh(1) + 2*a*d^2*x*sinh(1) - 2*b*f*sqrt((a^2
+ b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*b*f*sqrt((a^2 + b^2)/
b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*b*f*dilog(cosh(d*x + c) + si
nh(d*x + c)) - 2*b*f*dilog(-cosh(d*x + c) - sinh(d*x + c)) - 2*(b*c*f - b*d
*cosh(1) - b*d*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*s
inh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(b*c*f - b*d*cosh(1) -
b*d*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c
) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(b*d*f*x + b*c*f)*sqrt((a^2 + b^2)
/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(b*d*f*x + b*c*f)*sqrt((a^2 + b^
2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(b*d*f*x + b*d*cosh(1) + b*d*s
```

```
inh(1))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 2*(b*c*f - b*d*cosh(1) - b
*d*sinh(1))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(b*d*f*x + b*c*f)*lo
g(-cosh(d*x + c) - sinh(d*x + c) + 1))/(a*b*d^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)*cosh(c + d*x)*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*coth(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)*coth(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.428 $\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=71

$$\frac{x}{b}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{abd}$$

```
[Out] x/b-arctanh(cosh(d*x+c))/a/d+2*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^
(1/2))*(a^2+b^2)^(1/2)/a/b/d
```

Rubi [A]
time = 0.16, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2968, 3137, 2739, 632, 210, 3855}

$$\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{abd}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] x/b - ArcTanh[Cosh[c + d*x]]/(a*d) + (2*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh
[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[C*(x
/(b*d)), x] + (Dist[(A*b^2 + a^2*C)/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f
*x]), x], x] - Dist[(c^2*C + A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f
*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx &= \int \frac{\operatorname{csch}(c+dx)\left(1+\sinh^2(c+dx)\right)}{a+b\sinh(c+dx)}\,dx\\
&= \frac{x}{b}+\frac{\int \operatorname{csch}(c+dx)\,dx}{a}-\frac{(a^2+b^2)\int \frac{1}{a+b\sinh(c+dx)}\,dx}{ab}\\
&= \frac{x}{b}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{(2i(a^2+b^2))\operatorname{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx,x,\tan\left(\right.\right.}{abd}\\
&= \frac{x}{b}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}-\frac{(4i(a^2+b^2))\operatorname{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx,x,-2\right.}{abd}\\
&= \frac{x}{b}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{abd}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 80, normalized size = 1.13

$$
\frac{a(c+dx)+2\sqrt{-a^2-b^2}\,\operatorname{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)+b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{abd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*(c + d*x) + 2*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^
2 - b^2]] + b*Log[Tanh[(c + d*x)/2]])/(a*b*d)
```

Maple [A]
time = 2.54, size = 109, normalized size = 1.54

method	result
derivativedivides	$\frac{-\frac{(2a^2+2b^2)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{ab\sqrt{a^2+b^2}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}}{d}$
default	$\frac{-\frac{(2a^2+2b^2)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{ab\sqrt{a^2+b^2}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}}{d}$
risch	$\frac{x}{b}+\frac{\ln(e^{dx+c}-1)}{da}-\frac{\ln(e^{dx+c}+1)}{da}+\frac{\sqrt{a^2+b^2}\,\ln\left(e^{dx+c}+\frac{a+\sqrt{a^2+b^2}}{b}\right)}{dba}-\frac{\sqrt{a^2+b^2}\,\ln\left(e^{dx+c}-\right.}{dba}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-(2*a^2+2*b^2)/a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c
)-2*b)/(a^2+b^2)^(1/2))+1/a*ln(tanh(1/2*d*x+1/2*c))+1/b*ln(tanh(1/2*d*x+1/2
*c)+1)-1/b*ln(tanh(1/2*d*x+1/2*c)-1))
```

Maxima [A]
time = 0.49, size = 126, normalized size = 1.77

$$\frac{dx+c}{bd}-\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}+\frac{\log\left(e^{(-dx-c)}-1\right)}{ad}-\frac{\sqrt{a^2+b^2}\,\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{abd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(b*d) - log(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d)
 - sqrt(a^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x -
c) - a + sqrt(a^2 + b^2)))/(a*b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(68) = 136$.
time = 0.41, size = 209, normalized size = 2.94

$$\frac{adx-b\log\left(\cosh(dx+c)+\sinh(dx+c)+1\right)+b\log\left(\cosh(dx+c)+\sinh(dx+c)-1\right)+\sqrt{a^2+b^2}\,\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2ab\cosh(dx+c)+2a^2+b^2+2\left(b^2\cosh(dx+c)+ab\right)\sinh(dx+c)+2\sqrt{a^2+b^2}\left(b\cosh(dx+c)+b\sinh(dx+c)+a\right)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2a\cosh(dx+c)+2\left(b\cosh(dx+c)+a\right)\sinh(dx+c)-b}\right)}{abd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (a*d*x - b*log(cosh(d*x + c) + sinh(d*x + c) + 1) + b*log(cosh(d*x + c) + s
inh(d*x + c) - 1) + sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x
 + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*s
inh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(
b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x +
 c) + a)*sinh(d*x + c) - b)))/(a*b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(cosh(c + d*x)*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A]
time = 0.47, size = 113, normalized size = 1.59

$$\frac{\frac{dx+c}{b} - \frac{\log\left(e^{(dx+c)}+1\right)}{a} + \frac{\log\left(\left|e^{(dx+c)}-1\right|\right)}{a} - \frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)/b - log(e^(d*x + c) + 1)/a + log(abs(e^(d*x + c) - 1))/a - sqrt(
a^2 + b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*
x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(a*b))/d
```

Mupad [B]
time = 0.47, size = 384, normalized size = 5.41

$$\frac{x}{b}+\frac{\ln\left(32\,a\,b^2+32\,a^3-32\,a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-32\,a\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\right)}{a\,d}-\frac{\ln\left(32\,a\,b^2+32\,a^3+32\,a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+32\,a\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\right)}{a\,d}-\frac{\ln\left(128\,a^5\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-64\,a^2\,b^3-64\,a^4\,b-32\,a\,b^3\,\sqrt{a^2+b^2}-64\,a^3\,b\,\sqrt{a^2+b^2}+160\,a^3\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+128\,a^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{a^2+b^2}+32\,a\,b^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+96\,a^2\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{a\,b\,d}+\frac{\ln\left(128\,a^5\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-64\,a^2\,b^3-64\,a^4\,b+32\,a\,b^3\,\sqrt{a^2+b^2}+64\,a^3\,b\,\sqrt{a^2+b^2}+160\,a^3\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-128\,a^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{a^2+b^2}+32\,a\,b^4\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-96\,a^2\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{a\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*coth(c + d*x))/(a + b*sinh(c + d*x)),x)
```

```
[Out] x/b + log(32*a*b^2 + 32*a^3 - 32*a^3*exp(d*x)*exp(c) - 32*a*b^2*exp(d*x)*ex
p(c))/(a*d) - log(32*a*b^2 + 32*a^3 + 32*a^3*exp(d*x)*exp(c) + 32*a*b^2*exp
```

```
(d*x)*exp(c))/(a*d) - (log(128*a^5*exp(d*x)*exp(c) - 64*a^2*b^3 - 64*a^4*b
- 32*a*b^3*(a^2 + b^2)^(1/2) - 64*a^3*b*(a^2 + b^2)^(1/2) + 160*a^3*b^2*exp
(d*x)*exp(c) + 128*a^4*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) + 32*a*b^4*exp(d*x
)*exp(c) + 96*a^2*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))
/(a*b*d) + (log(128*a^5*exp(d*x)*exp(c) - 64*a^2*b^3 - 64*a^4*b + 32*a*b^3*
(a^2 + b^2)^(1/2) + 64*a^3*b*(a^2 + b^2)^(1/2) + 160*a^3*b^2*exp(d*x)*exp(c
) - 128*a^4*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) + 32*a*b^4*exp(d*x)*exp(c) -
96*a^2*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a*b*d)
```

3.429 $\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=35

$$\text{Int}\left(\frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cosh[c + d*x]*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Cosh[c + d*x]*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

Rubi steps

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 42.45, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cosh[c + d*x]*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x
]
```

```
[Out] Integrate[(Cosh[c + d*x]*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)\coth(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] -2*(a^2*e^c + b^2*e^c)*integrate(-e^(d*x)/(a*b^2*f*x + a*b^2*e - (a*b^2*f*x
*e^(2*c) + a*b^2*e^(2*c + 1))*e^(2*d*x) - 2*(a^2*b*f*x*e^c + a^2*b*e^(c + 1
))*e^(d*x)), x) + log(f*x + e)/(b*f) + integrate(1/(a*f*x + a*e + (a*f*x*e^
c + a*e^(c + 1))*e^(d*x)), x) + integrate(-1/(a*f*x + a*e - (a*f*x*e^c + a*
e^(c + 1))*e^(d*x)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] integral(cosh(d*x + c)*coth(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
+ c)), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(cosh(c + d*x)*coth(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*coth(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((cosh(c + d*x)*coth(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.430 $\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=656

$$-\frac{(e+fx)^4}{4af}+\frac{(a^2+b^2)(e+fx)^4}{4ab^2f}-\frac{6f^3\cosh(c+dx)}{bd^4}-\frac{3f(e+fx)^2\cosh(c+dx)}{bd^2}-\frac{(a^2+b^2)(e+fx)^3\log\left(1\right.}{ab^2d}$$

```
[Out] -1/4*(f*x+e)^4/a/f+1/4*(a^2+b^2)*(f*x+e)^4/a/b^2/f-6*f^3*cosh(d*x+c)/b/d^4-
3*f*(f*x+e)^2*cosh(d*x+c)/b/d^2+(f*x+e)^3*ln(1-exp(2*d*x+2*c))/a/d-(a^2+b^2
)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/b^2/d-(a^2+b^2)*(f*x+e
)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/b^2/d+3/2*f*(f*x+e)^2*polylog(
2,exp(2*d*x+2*c))/a/d^2-3*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-
(a^2+b^2)^(1/2)))/a/b^2/d^2-3*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)
/(a+(a^2+b^2)^(1/2)))/a/b^2/d^2-3/2*f^2*(f*x+e)*polylog(3,exp(2*d*x+2*c))/a
/d^3+6*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a
/b^2/d^3+6*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)
))/a/b^2/d^3+3/4*f^3*polylog(4,exp(2*d*x+2*c))/a/d^4-6*(a^2+b^2)*f^3*polylo
g(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/b^2/d^4-6*(a^2+b^2)*f^3*polylog(4,
-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/b^2/d^4+6*f^2*(f*x+e)*sinh(d*x+c)/b/d^
3+(f*x+e)^3*sinh(d*x+c)/b/d
```

Rubi [A]

time = 0.91, antiderivative size = 656, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 17, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5704, 5558, 5554, 3392, 32, 2715, 8, 3797, 2221, 2611, 6744, 2320, 6724, 5684, 3377, 2718, 5680}

$$-\frac{6f^3(a^2+b^2)\text{Li}_4\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^4}-\frac{6f^3(a^2+b^2)\text{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d^4}+\frac{6f^2(a^2+b^2)(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^3}+\frac{6f^2(a^2+b^2)(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d^3}-\frac{3f(a^2+b^2)(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^2}-\frac{3f(a^2+b^2)(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d^2}-\frac{(a^2+b^2)(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{ab^2d}-\frac{(a^2+b^2)(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{ab^2d}+\frac{(a^2+b^2)(e+fx)^4}{4ab^2f}+\frac{3f^3\text{Li}_4\left(e^{2(c+dx)}\right)}{4ad^4}-\frac{3f^2(e+fx)\text{Li}_3\left(e^{2(c+dx)}\right)}{2ad^3}+\frac{3f(e+fx)^2\text{Li}_2\left(e^{2(c+dx)}\right)}{2ad^2}+\frac{(e+fx)^3\log\left(1-e^{2(c+dx)}\right)}{ad}-\frac{(e+fx)^4}{4af}-\frac{6f^3\cosh(c+dx)}{bd^4}+\frac{6f^2(e+fx)\sinh(c+dx)}{bd^3}-\frac{3f(e+fx)^2\cosh(c+dx)}{bd^2}+\frac{(e+fx)^3\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/4*(e + f*x)^4/(a*f) + ((a^2 + b^2)*(e + f*x)^4)/(4*a*b^2*f) - (6*f^3*Cos
h[c + d*x])/(b*d^4) - (3*f*(e + f*x)^2*Cosh[c + d*x])/(b*d^2) - ((a^2 + b^2
)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*b^2*d) - (
(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*
b^2*d) + ((e + f*x)^3*Log[1 - E^(2*(c + d*x))])/(a*d) - (3*(a^2 + b^2)*f*(e
 + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b^2*d^2)
 - (3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2]))])/(a*b^2*d^2) + (3*f*(e + f*x)^2*PolyLog[2, E^(2*(c + d*x))])/(2*a
*d^2) + (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2]))])/(a*b^2*d^3) + (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*b^2*d^3) - (3*f^2*(e + f*x)*PolyLo
```

```
g[3, E^(2*(c + d*x))])/(2*a*d^3) - (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b^2*d^4) - (6*(a^2 + b^2)*f^3*PolyLog[4
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*b^2*d^4) + (3*f^3*PolyLog[4
, E^(2*(c + d*x))])/(4*a*d^4) + (6*f^2*(e + f*x)*Sinh[c + d*x])/(b*d^3) + (
(e + f*x)^3*Sinh[c + d*x])/(b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
```

```
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5704

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\cosh^2(c+dx)\coth(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cosh^3(c\cdot}{a+b\sinh(c+d\cdot}}{a}$$

$$= \frac{\int (e+fx)^3\coth(c+dx)\,dx}{a} + \frac{\int (e+fx)^3\cosh(c+dx)\,dx}{b} -$$

$$= -\frac{(e+fx)^4}{4af} + \frac{(a^2+b^2)(e+fx)^4}{4ab^2f} + \frac{(e+fx)^3\sinh(c+dx)}{bd}$$

$$= -\frac{(e+fx)^4}{4af} + \frac{(a^2+b^2)(e+fx)^4}{4ab^2f} - \frac{3f(e+fx)^2\cosh(c+dx}{bd^2}$$

$$= -\frac{(e+fx)^4}{4af} + \frac{(a^2+b^2)(e+fx)^4}{4ab^2f} - \frac{3f(e+fx)^2\cosh(c+dx}{bd^2}$$

$$= -\frac{(e+fx)^4}{4af} + \frac{(a^2+b^2)(e+fx)^4}{4ab^2f} - \frac{6f^3\cosh(c+dx)}{bd^4} - 3f($$

$$= -\frac{(e+fx)^4}{4af} + \frac{(a^2+b^2)(e+fx)^4}{4ab^2f} - \frac{6f^3\cosh(c+dx)}{bd^4} - 3f($$

$$= -\frac{(e+fx)^4}{4af} + \frac{(a^2+b^2)(e+fx)^4}{4ab^2f} - \frac{6f^3\cosh(c+dx)}{bd^4} - 3f($$

Mathematica [C] Result contains complex when optimal does not.
time = 27.74, size = 14209, normalized size = 21.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 5.77, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3\left(\cosh^2(dx+c)\right)\coth(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -1/2*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) - 2*lo
g(e^(-d*x - c) + 1)/(a*d) - 2*log(e^(-d*x - c) - 1)/(a*d) + 2*(a^2 + b^2)*l
og(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*b^2*d))*e^3 + 3*(d*x*log(
e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*f*e^2/(a*d^2) + 3*(d*x*log(-e^(d*x
+ c) + 1) + dilog(e^(d*x + c)))*f*e^2/(a*d^2) + 3*(d^2*x^2*log(e^(d*x + c)
+ 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2*e/(a*d^3
) + 3*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog
(3, e^(d*x + c)))*f^2*e/(a*d^3) + (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2
*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*
x + c)))*f^3/(a*d^4) + (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(
d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*f^3/
(a*d^4) - 1/2*(d^4*f^3*x^4 + 4*d^4*f^2*x^3*e + 6*d^4*f*x^2*e^2)/(a*d^4) - 1
/4*(a*d^4*f^3*x^4*e^c + 4*a*d^4*f^2*x^3*e^(c + 1) + 6*a*d^4*f*x^2*e^(c + 2)
 - 2*(b*d^3*f^3*x^3*e^(2*c) - 6*b*f^3*e^(2*c) - 3*b*d^2*f*e^(2*c + 2) + 6*b
*d*f^2*e^(2*c + 1) - 3*(b*d^2*f^3*e^(2*c) - b*d^3*f^2*e^(2*c + 1))*x^2 + 3*
(2*b*d*f^3*e^(2*c) + b*d^3*f*e^(2*c + 2) - 2*b*d^2*f^2*e^(2*c + 1))*x)*e^(d
*x) + 2*(b*d^3*f^3*x^3 + 3*b*d^2*f*e^2 + 6*b*d*f^2*e + 6*b*f^3 + 3*(b*d^3*f
^2*e + b*d^2*f^3)*x^2 + 3*(b*d^3*f*e^2 + 2*b*d^2*f^2*e + 2*b*d*f^3)*x)*e^(-
d*x))*e^(-c)/(b^2*d^4) + integrate(-2*((a^2*b*f^3 + b^3*f^3)*x^3 + 3*(a^2*b
*f^2 + b^3*f^2)*x^2*e + 3*(a^2*b*f + b^3*f)*x*e^2 - ((a^3*f^3*e^c + a*b^2*f
^3*e^c)*x^3 + 3*(a^3*f^2*e^c + a*b^2*f^2*e^c)*x^2*e + 3*(a^3*f*e^c + a*b^2*
f*e^c)*x*e^2)*e^(d*x))/(a*b^3*e^(2*d*x + 2*c) + 2*a^2*b^2*e^(d*x + c) - a*b
^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5885 vs. $2(633) = 1266$.
time = 0.52, size = 5885, normalized size = 8.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -1/4*(2*a*b*d^3*f^3*x^3 + 6*a*b*d^2*f^3*x^2 + 2*a*b*d^3*cosh(1)^3 + 2*a*b*d
^3*sinh(1)^3 + 12*a*b*d*f^3*x + 12*a*b*f^3 + 6*(a*b*d^3*f*x + a*b*d^2*f)*co
sh(1)^2 - 2*(a*b*d^3*f^3*x^3 - 3*a*b*d^2*f^3*x^2 + a*b*d^3*cosh(1)^3 + a*b*
d^3*sinh(1)^3 + 6*a*b*d*f^3*x - 6*a*b*f^3 + 3*(a*b*d^3*f*x - a*b*d^2*f)*cos
h(1)^2 + 3*(a*b*d^3*f*x + a*b*d^3*cosh(1) - a*b*d^2*f)*sinh(1)^2 + 3*(a*b*d
^3*f^2*x^2 - 2*a*b*d^2*f^2*x + 2*a*b*d*f^2)*cosh(1) + 3*(a*b*d^3*f^2*x^2 -
2*a*b*d^2*f^2*x + a*b*d^3*cosh(1)^2 + 2*a*b*d*f^2 + 2*(a*b*d^3*f*x - a*b*d^
2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 6*(a*b*d^3*f*x + a*b*d^3*cosh(1) +
 a*b*d^2*f)*sinh(1)^2 - 2*(a*b*d^3*f^3*x^3 - 3*a*b*d^2*f^3*x^2 + a*b*d^3*co
sh(1)^3 + a*b*d^3*sinh(1)^3 + 6*a*b*d*f^3*x - 6*a*b*f^3 + 3*(a*b*d^3*f*x -
a*b*d^2*f)*cosh(1)^2 + 3*(a*b*d^3*f*x + a*b*d^3*cosh(1) - a*b*d^2*f)*sinh(1
)^2 + 3*(a*b*d^3*f^2*x^2 - 2*a*b*d^2*f^2*x + 2*a*b*d*f^2)*cosh(1) + 3*(a*b*
d^3*f^2*x^2 - 2*a*b*d^2*f^2*x + a*b*d^3*cosh(1)^2 + 2*a*b*d*f^2 + 2*(a*b*d^
3*f*x - a*b*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^2 + 6*(a*b*d^3*f^2*x^2 +
 2*a*b*d^2*f^2*x + 2*a*b*d*f^2)*cosh(1) - (a^2*d^4*f^3*x^4 - 2*a^2*c^4*f^3
+ 4*(a^2*d^4*x + 2*a^2*c*d^3)*cosh(1)^3 + 4*(a^2*d^4*x + 2*a^2*c*d^3)*sinh(
1)^3 + 6*(a^2*d^4*f*x^2 - 2*a^2*c^2*d^2*f)*cosh(1)^2 + 6*(a^2*d^4*f*x^2 - 2
*a^2*c^2*d^2*f + 2*(a^2*d^4*x + 2*a^2*c*d^3)*cosh(1))*sinh(1)^2 + 4*(a^2*d^
4*f^2*x^3 + 2*a^2*c^3*d*f^2)*cosh(1) + 4*(a^2*d^4*f^2*x^3 + 2*a^2*c^3*d*f^2
 + 3*(a^2*d^4*x + 2*a^2*c*d^3)*cosh(1)^2 + 3*(a^2*d^4*f*x^2 - 2*a^2*c^2*d^2
*f)*cosh(1))*sinh(1))*cosh(d*x + c) + 12*(((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2
 + b^2)*d^2*f^2*x*cosh(1) + (a^2 + b^2)*d^2*f*cosh(1)^2 + (a^2 + b^2)*d^2*f
*sinh(1)^2 + 2*((a^2 + b^2)*d^2*f^2*x + (a^2 + b^2)*d^2*f*cosh(1))*sinh(1))
*cosh(d*x + c) + ((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*f^2*x*cosh(1)
 + (a^2 + b^2)*d^2*f*cosh(1)^2 + (a^2 + b^2)*d^2*f*sinh(1)^2 + 2*((a^2 + b^
2)*d^2*f^2*x + (a^2 + b^2)*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c))*dilog((a*
cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2) - b)/b + 1) + 12*(((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)
*d^2*f^2*x*cosh(1) + (a^2 + b^2)*d^2*f*cosh(1)^2 + (a^2 + b^2)*d^2*f*sinh(1
)^2 + 2*((a^2 + b^2)*d^2*f^2*x + (a^2 + b^2)*d^2*f*cosh(1))*sinh(1))*cosh(d
*x + c) + ((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*f^2*x*cosh(1) + (a^2
 + b^2)*d^2*f*cosh(1)^2 + (a^2 + b^2)*d^2*f*sinh(1)^2 + 2*((a^2 + b^2)*d^2*
f^2*x + (a^2 + b^2)*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c))*dilog((a*cosh(d*
x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b + 1) - 12*((b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^
2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh
(1))*sinh(1))*cosh(d*x + c) + (b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) +
b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*co
sh(1))*sinh(1))*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)) - 12*((
b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f
*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c) +
 (b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2
```

```
*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c)
)*dilog(-cosh(d*x + c) - sinh(d*x + c)) - 4*(((a^2 + b^2)*c^3*f^3 - 3*(a^2
+ b^2)*c^2*d*f^2*cosh(1) + 3*(a^2 + b^2)*c*d^2*f*cosh(1)^2 - (a^2 + b^2)*d^
3*cosh(1)^3 - (a^2 + b^2)*d^3*sinh(1)^3 + 3*((a^2 + b^2)*c*d^2*f - (a^2 + b
^2)*d^3*cosh(1))*sinh(1)^2 - 3*((a^2 + b^2)*c^2*d*f^2 - 2*(a^2 + b^2)*c*d^2
*f*cosh(1) + (a^2 + b^2)*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c) + ((a^2 + b^
2)*c^3*f^3 - 3*(a^2 + b^2)*c^2*d*f^2*cosh(1) + 3*(a^2 + b^2)*c*d^2*f*cosh(1
)^2 - (a^2 + b^2)*d^3*cosh(1)^3 - (a^2 + b^2)*d^3*sinh(1)^3 + 3*((a^2 + b^2
)*c*d^2*f - (a^2 + b^2)*d^3*cosh(1))*sinh(1)^2 - 3*((a^2 + b^2)*c^2*d*f^2 -
 2*(a^2 + b^2)*c*d^2*f*cosh(1) + (a^2 + b^2)*d^3*cosh(1)^2)*sinh(1))*sinh(d
*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b
^2) + 2*a) - 4*(((a^2 + b^2)*c^3*f^3 - 3*(a^2 + b^2)*c^2*d*f^2*cosh(1) + 3*
(a^2 + b^2)*c*d^2*f*cosh(1)^2 - (a^2 + b^2)*d^3*cosh(1)^3 - (a^2 + b^2)*d^3
*sinh(1)^3 + 3*((a^2 + b^2)*c*d^2*f - (a^2 + b^2)*d^3*cosh(1))*sinh(1)^2 -
3*((a^2 + b^2)*c^2*d*f^2 - 2*(a^2 + b^2)*c*d^2*f*cosh(1) + (a^2 + b^2)*d^3*
cosh(1)^2)*sinh(1))*cosh(d*x + c) + ((a^2 + b^2)*c^3*f^3 - 3*(a^2 + b^2)*c^
2*d*f^2*cosh(1) + 3*(a^2 + b^2)*c*d^2*f*cosh(1)^2 - (a^2 + b^2)*d^3*cosh(1)
^3 - (a^2 + b^2)*d^3*sinh(1)^3 + 3*((a^2 + b^2)*c*d^2*f - (a^2 + b^2)*d^3*c
osh(1))*sinh(1)^2 - 3*((a^2 + b^2)*c^2*d*f^2 - 2*(a^2 + b^2)*c*d^2*f*cosh(1
) + (a^2 + b^2)*d^3*cosh(1)^2)*sinh(1))*sinh(d*x + c))*log(2*b*cosh(d*x + c
) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 4*(((a^2 + b^2)*
d^3*f^3*x^3 + (a^2 + b^2)*c^3*f^3 + 3*((a^2 + b^2)*d^3*f*x + (a^2 + b^2)*c*
d^2*f)*cosh(1)^2 + 3*((a^2 + b^2)*d^3*f*x + (a^2 + b^2)*c*d^2*f)*sinh(1)^2
+ 3*((a^2 + b^2)*d^3*f^2*x^2 - (a^2 + b^2)*c^2*d*f^2)*cosh(1) + 3*((a^2 + b
^2)*d^3*f^2*x^2 - (a^2 + b^2)*c^2*d*f^2 + 2*((a...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3 \cosh^2(c+dx) \coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*cosh(c + d*x)**2*coth(c + d*x)/(a + b*sinh(c + d*x)),
 x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 624.79Not invertible
 Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^2\coth(c+d\,x)\,\left(e+f\,x\right)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*coth(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((cosh(c + d*x)^2*coth(c + d*x)*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.431 $\int \frac{(e+fx)^2 \cosh^2(c+dx) \coth(c+dx)}{a+b\sinh(c+dx)}\, dx$

Optimal. Leaf size=486

$$-\frac{(e+fx)^3}{3af}+\frac{(a^2+b^2)(e+fx)^3}{3ab^2f}-\frac{2f(e+fx)\cosh(c+dx)}{bd^2}-\frac{(a^2+b^2)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d}-\cdots$$

```
[Out] -1/3*(f*x+e)^3/a/f+1/3*(a^2+b^2)*(f*x+e)^3/a/b^2/f-2*f*(f*x+e)*cosh(d*x+c)/
b/d^2+(f*x+e)^2*ln(1-exp(2*d*x+2*c))/a/d-(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x
+c)/(a-(a^2+b^2)^(1/2)))/a/b^2/d-(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(
a^2+b^2)^(1/2)))/a/b^2/d+f*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d^2-2*(a^2+b
^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/b^2/d^2-2*(a^2
+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/b^2/d^2-1/2*
f^2*polylog(3,exp(2*d*x+2*c))/a/d^3+2*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)
/(a-(a^2+b^2)^(1/2)))/a/b^2/d^3+2*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)/(a+
(a^2+b^2)^(1/2)))/a/b^2/d^3+2*f^2*sinh(d*x+c)/b/d^3+(f*x+e)^2*sinh(d*x+c)/b
/d
```

Rubi [A]
time = 0.72, antiderivative size = 486, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$, Rules used = {5704, 5558, 5554, 3391, 3797, 2221, 2611, 2320, 6724, 5684, 3377, 2717, 5680}

$$\frac{2f^2(a^2+b^2)\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^3}+\frac{2f^2(a^2+b^2)\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d^3}-\frac{2f(a^2+b^2)(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^2}-\frac{2f(a^2+b^2)(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d^2}-\frac{(a^2+b^2)(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{ab^2d}-\frac{(a^2+b^2)(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{ab^2d}+\frac{(a^2+b^2)(e+fx)^3}{3ab^2f}-\frac{f^2\text{Li}_3\left(e^{2(c+dx)}\right)}{2ad^3}+\frac{f(e+fx)\text{Li}_2\left(e^{2(c+dx)}\right)}{ad^2}+\frac{(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{ad}-\frac{(e+fx)^3}{3af}+\frac{2f^2\sinh(c+dx)}{bd^3}-\frac{2f(e+fx)\cosh(c+dx)}{bd^2}+\frac{(e+fx)^2\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/3*(e + f*x)^3/(a*f) + ((a^2 + b^2)*(e + f*x)^3)/(3*a*b^2*f) - (2*f*(e +
f*x)*Cosh[c + d*x])/(b*d^2) - ((a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2])])/(a*b^2*d) - ((a^2 + b^2)*(e + f*x)^2*Log[1 + (b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*b^2*d) + ((e + f*x)^2*Log[1 - E^(2
*(c + d*x))])/(a*d) - (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2]))])/(a*b^2*d^2) - (2*(a^2 + b^2)*f*(e + f*x)*PolyLo
g[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*b^2*d^2) + (f*(e + f*x)*
PolyLog[2, E^(2*(c + d*x))])/(a*d^2) + (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b^2*d^3) + (2*(a^2 + b^2)*f^2*PolyL
og[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*b^2*d^3) - (f^2*PolyLog
[3, E^(2*(c + d*x))])/(2*a*d^3) + (2*f^2*Sinh[c + d*x])/(b*d^3) + ((e + f*x
)^2*Sinh[c + d*x])/(b*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5704

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh^2(c+dx)\coth(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cosh^3(c}{a+b\sinh(c+d}}{a}$$

$$= \frac{\int (e+fx)^2\coth(c+dx)\,dx}{a} + \frac{\int (e+fx)^2\cosh(c+dx)\,dx}{b} -$$

$$= -\frac{(e+fx)^3}{3af} + \frac{(a^2+b^2)(e+fx)^3}{3ab^2f} + \frac{(e+fx)^2\sinh(c+dx)}{bd}$$

$$= -\frac{(e+fx)^3}{3af} + \frac{(a^2+b^2)(e+fx)^3}{3ab^2f} - \frac{2f(e+fx)\cosh(c+dx}{bd^2}$$

$$= -\frac{(e+fx)^3}{3af} + \frac{(a^2+b^2)(e+fx)^3}{3ab^2f} - \frac{2f(e+fx)\cosh(c+dx}{bd^2}$$

$$= -\frac{(e+fx)^3}{3af} + \frac{(a^2+b^2)(e+fx)^3}{3ab^2f} - \frac{2f(e+fx)\cosh(c+dx}{bd^2}$$

$$= -\frac{(e+fx)^3}{3af} + \frac{(a^2+b^2)(e+fx)^3}{3ab^2f} - \frac{2f(e+fx)\cosh(c+dx}{bd^2}$$

Mathematica [A]
time = 5.18, size = 700, normalized size = 1.44

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] ((-2*a*x*(3*e^2 + 3*e*f*x + f^2*x^2)*Coth[c])/b^2 + ((-4*E^(2*c)*x*(3*e^2 +
 3*e*f*x + f^2*x^2))/(-1 + E^(2*c)) + (6*(e + f*x)^2*Log[1 - E^(2*(c + d*x)
)])/d + (6*f*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/d^2 - (3*f^2*PolyLog[3,
 E^(2*(c + d*x))])/d^3)/a + (2*(a^2 + b^2)*((2*E^(2*c)*x*(3*e^2 + 3*e*f*x +
 f^2*x^2))/(-1 + E^(2*c)) - (3*(d^2*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*
(c + d*x)))] + 2*d^2*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b
^2)*E^(2*c)])] + d^2*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 +
 b^2)*E^(2*c)])] + 2*d^2*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2
```

```
 + b^2)*E^(2*c)])] + d^2*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a
^2 + b^2)*E^(2*c)])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^
c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(2*c +
 d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*f^2*PolyLog[3, -((b*E^(2*c
 + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*f^2*PolyLog[3, -((b*E^(2
*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/d^3))/(a*b^2) + (6*Cosh[
d*x]*(-2*d*f*(e + f*x)*Cosh[c] + (2*f^2 + d^2*(e + f*x)^2)*Sinh[c]))/(b*d^3
) + (6*((2*f^2 + d^2*(e + f*x)^2)*Cosh[c] - 2*d*f*(e + f*x)*Sinh[c])*Sinh[d
*x])/(b*d^3))/6
```

Maple [F]
time = 5.74, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \left(\cosh^2(dx+c)\right) \coth(dx+c)}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)

[Out] int((f*x+e)^2*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")

[Out] -1/2*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) - 2*lo
g(e^(-d*x - c) + 1)/(a*d) - 2*log(e^(-d*x - c) - 1)/(a*d) + 2*(a^2 + b^2)*l
og(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*b^2*d))*e^2 + 2*(d*x*log(
e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*f*e/(a*d^2) + 2*(d*x*log(-e^(d*x +
c) + 1) + dilog(e^(d*x + c)))*f*e/(a*d^2) + (d^2*x^2*log(e^(d*x + c) + 1) +
 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2/(a*d^3) + (d^2
*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x
 + c)))*f^2/(a*d^3) - 2/3*(d^3*f^2*x^3 + 3*d^3*f*x^2*e)/(a*d^3) - 1/6*(2*a*
d^3*f^2*x^3*e^c + 6*a*d^3*f*x^2*e^(c + 1) - 3*(b*d^2*f^2*x^2*e^(2*c) + 2*b*
f^2*e^(2*c) - 2*b*d*f*e^(2*c + 1) - 2*(b*d*f^2*e^(2*c) - b*d^2*f*e^(2*c + 1
))*x)*e^(d*x) + 3*(b*d^2*f^2*x^2 + 2*b*d*f*e + 2*b*f^2 + 2*(b*d^2*f*e + b*d
*f^2)*x)*e^(-d*x))*e^(-c)/(b^2*d^3) + integrate(-2*((a^2*b*f^2 + b^3*f^2)*x
^2 + 2*(a^2*b*f + b^3*f)*x*e - ((a^3*f^2*e^c + a*b^2*f^2*e^c)*x^2 + 2*(a^3*
f*e^c + a*b^2*f*e^c)*x*e)*e^(d*x))/(a*b^3*e^(2*d*x + 2*c) + 2*a^2*b^2*e^(d*
x + c) - a*b^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2987 vs. $2(469) = 938$.
time = 0.48, size = 2987, normalized size = 6.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -1/6*(3*a*b*d^2*f^2*x^2 + 6*a*b*d*f^2*x + 3*a*b*d^2*cosh(1)^2 + 3*a*b*d^2*s
inh(1)^2 + 6*a*b*f^2 - 3*(a*b*d^2*f^2*x^2 - 2*a*b*d*f^2*x + a*b*d^2*cosh(1)
^2 + a*b*d^2*sinh(1)^2 + 2*a*b*f^2 + 2*(a*b*d^2*f*x - a*b*d*f)*cosh(1) + 2*
(a*b*d^2*f*x + a*b*d^2*cosh(1) - a*b*d*f)*sinh(1))*cosh(d*x + c)^2 - 3*(a*b
*d^2*f^2*x^2 - 2*a*b*d*f^2*x + a*b*d^2*cosh(1)^2 + a*b*d^2*sinh(1)^2 + 2*a*
b*f^2 + 2*(a*b*d^2*f*x - a*b*d*f)*cosh(1) + 2*(a*b*d^2*f*x + a*b*d^2*cosh(1
) - a*b*d*f)*sinh(1))*sinh(d*x + c)^2 + 6*(a*b*d^2*f*x + a*b*d*f)*cosh(1) -
 2*(a^2*d^3*f^2*x^3 + 2*a^2*c^3*f^2 + 3*(a^2*d^3*x + 2*a^2*c*d^2)*cosh(1)^2
 + 3*(a^2*d^3*x + 2*a^2*c*d^2)*sinh(1)^2 + 3*(a^2*d^3*f*x^2 - 2*a^2*c^2*d*f
)*cosh(1) + 3*(a^2*d^3*f*x^2 - 2*a^2*c^2*d*f + 2*(a^2*d^3*x + 2*a^2*c*d^2)*
cosh(1))*sinh(1))*cosh(d*x + c) + 12*(((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*
f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*cosh(d*x + c) + ((a^2 + b^2)*d*f^2*x +
 (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*sinh(d*x + c))*dilog((a
*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b + 1) + 12*(((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*
cosh(1) + (a^2 + b^2)*d*f*sinh(1))*cosh(d*x + c) + ((a^2 + b^2)*d*f^2*x + (
a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*sinh(d*x + c))*dilog((a*c
osh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((
a^2 + b^2)/b^2) - b)/b + 1) - 12*((b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*
sinh(1))*cosh(d*x + c) + (b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*
sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)) - 12*((b^2*d*f^2*x + b^
2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh(d*x + c) + (b^2*d*f^2*x + b^2*d*f*cos
h(1) + b^2*d*f*sinh(1))*sinh(d*x + c))*dilog(-cosh(d*x + c) - sinh(d*x + c)
) + 6*(((a^2 + b^2)*c^2*f^2 - 2*(a^2 + b^2)*c*d*f*cosh(1) + (a^2 + b^2)*d^2
*cosh(1)^2 + (a^2 + b^2)*d^2*sinh(1)^2 - 2*((a^2 + b^2)*c*d*f - (a^2 + b^2)
*d^2*cosh(1))*sinh(1))*cosh(d*x + c) + ((a^2 + b^2)*c^2*f^2 - 2*(a^2 + b^2)
*c*d*f*cosh(1) + (a^2 + b^2)*d^2*cosh(1)^2 + (a^2 + b^2)*d^2*sinh(1)^2 - 2*
((a^2 + b^2)*c*d*f - (a^2 + b^2)*d^2*cosh(1))*sinh(1))*sinh(d*x + c))*log(2
*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 6
*(((a^2 + b^2)*c^2*f^2 - 2*(a^2 + b^2)*c*d*f*cosh(1) + (a^2 + b^2)*d^2*cosh
(1)^2 + (a^2 + b^2)*d^2*sinh(1)^2 - 2*((a^2 + b^2)*c*d*f - (a^2 + b^2)*d^2*
cosh(1))*sinh(1))*cosh(d*x + c) + ((a^2 + b^2)*c^2*f^2 - 2*(a^2 + b^2)*c*d*
f*cosh(1) + (a^2 + b^2)*d^2*cosh(1)^2 + (a^2 + b^2)*d^2*sinh(1)^2 - 2*((a^2
 + b^2)*c*d*f - (a^2 + b^2)*d^2*cosh(1))*sinh(1))*sinh(d*x + c))*log(2*b*co
sh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 6*(((a
```

```
^2 + b^2)*d^2*f^2*x^2 - (a^2 + b^2)*c^2*f^2 + 2*((a^2 + b^2)*d^2*f*x + (a^2
 + b^2)*c*d*f)*cosh(1) + 2*((a^2 + b^2)*d^2*f*x + (a^2 + b^2)*c*d*f)*sinh(1
))*cosh(d*x + c) + ((a^2 + b^2)*d^2*f^2*x^2 - (a^2 + b^2)*c^2*f^2 + 2*((a^2
 + b^2)*d^2*f*x + (a^2 + b^2)*c*d*f)*cosh(1) + 2*((a^2 + b^2)*d^2*f*x + (a^
2 + b^2)*c*d*f)*sinh(1))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x
+ c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) +
6*(((a^2 + b^2)*d^2*f^2*x^2 - (a^2 + b^2)*c^2*f^2 + 2*((a^2 + b^2)*d^2*f*x
+ (a^2 + b^2)*c*d*f)*cosh(1) + 2*((a^2 + b^2)*d^2*f*x + (a^2 + b^2)*c*d*f)*
sinh(1))*cosh(d*x + c) + ((a^2 + b^2)*d^2*f^2*x^2 - (a^2 + b^2)*c^2*f^2 + 2
*((a^2 + b^2)*d^2*f*x + (a^2 + b^2)*c*d*f)*cosh(1) + 2*((a^2 + b^2)*d^2*f*x
 + (a^2 + b^2)*c*d*f)*sinh(1))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sin
h(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)
/b) - 6*((b^2*d^2*f^2*x^2 + 2*b^2*d^2*f*x*cosh(1) + b^2*d^2*cosh(1)^2 + b^2
*d^2*sinh(1)^2 + 2*(b^2*d^2*f*x + b^2*d^2*cosh(1))*sinh(1))*cosh(d*x + c) +
 (b^2*d^2*f^2*x^2 + 2*b^2*d^2*f*x*cosh(1) + b^2*d^2*cosh(1)^2 + b^2*d^2*sin
h(1)^2 + 2*(b^2*d^2*f*x + b^2*d^2*cosh(1))*sinh(1))*sinh(d*x + c))*log(cosh
(d*x + c) + sinh(d*x + c) + 1) - 6*((b^2*c^2*f^2 - 2*b^2*c*d*f*cosh(1) + b^
2*d^2*cosh(1)^2 + b^2*d^2*sinh(1)^2 - 2*(b^2*c*d*f - b^2*d^2*cosh(1))*sinh(
1))*cosh(d*x + c) + (b^2*c^2*f^2 - 2*b^2*c*d*f*cosh(1) + b^2*d^2*cosh(1)^2
+ b^2*d^2*sinh(1)^2 - 2*(b^2*c*d*f - b^2*d^2*cosh(1))*sinh(1))*sinh(d*x + c
))*log(cosh(d*x + c) + sinh(d*x + c) - 1) - 6*((b^2*d^2*f^2*x^2 - b^2*c^2*f
^2 + 2*(b^2*d^2*f*x + b^2*c*d*f)*cosh(1) + 2*(b^2*d^2*f*x + b^2*c*d*f)*sinh
(1))*cosh(d*x + c) + (b^2*d^2*f^2*x^2 - b^2*c^2*f^2 + 2*(b^2*d^2*f*x + b^2*
c*d*f)*cosh(1) + 2*(b^2*d^2*f*x + b^2*c*d*f)*sinh(1))*sinh(d*x + c))*log(-c
osh(d*x + c) - sinh(d*x + c) + 1) - 12*((a^2 + b^2)*f^2*cosh(d*x + c) + (a^
2 + b^2)*f^2*sinh(d*x + c))*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) +
 (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 12*((a^2 +
 b^2)*f^2*cosh(d*x + c) + (a^2 + b^2)*f^2*sinh(d*x + c))*polylog(3, (a*cosh
(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2))/b) + 12*(b^2*f^2*cosh(d*x + c) + b^2*f^2*sinh(d*x + c))*polyl
og(3, cosh(d*x + c) + sinh(d*x + c)) + 12*(b^2*f^2*cosh(d*x + c) + b^2*f^2*
sinh(d*x + c))*polylog(3, -cosh(d*x + c) - sinh...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2\cosh^2{(c+dx)}\coth{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*cosh(c + d*x)**2*coth(c + d*x)/(a + b*sinh(c + d*x)),
 x)
```

Giac [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^2\,\coth(c+d\,x)\,\left(e+f\,x\right)^2}{a+b\,\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*coth(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((cosh(c + d*x)^2*coth(c + d*x)*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.432 $\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=322

$$-\frac{(e+fx)^2}{2af}+\frac{(a^2+b^2)(e+fx)^2}{2ab^2f}-\frac{f\cosh(c+dx)}{bd^2}-\frac{(a^2+b^2)(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d}-\frac{(a^2+b^2)(e\cdot}{}$$

```
[Out] -1/2*(f*x+e)^2/a/f+1/2*(a^2+b^2)*(f*x+e)^2/a/b^2/f-f*cosh(d*x+c)/b/d^2+(f*x
+e)*ln(1-exp(2*d*x+2*c))/a/d-(a^2+b^2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))/a/b^2/d-(a^2+b^2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))
/a/b^2/d+1/2*f*polylog(2,exp(2*d*x+2*c))/a/d^2-(a^2+b^2)*f*polylog(2,-b*exp
(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/b^2/d^2-(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/
(a+(a^2+b^2)^(1/2)))/a/b^2/d^2+(f*x+e)*sinh(d*x+c)/b/d
```

Rubi [A]
time = 0.42, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {5704, 5558, 5554, 2715, 8, 3797, 2221, 2317, 2438, 5684, 3377, 2718, 5680}

$$-\frac{f(a^2+b^2)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^2}-\frac{f(a^2+b^2)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ab^2d^2}-\frac{(a^2+b^2)(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{ab^2d}-\frac{(a^2+b^2)(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{ab^2d}+\frac{(a^2+b^2)(e+fx)^2}{2ab^2f}+\frac{f\text{Li}_2\left(e^{2(c+dx)}\right)}{2ad^2}+\frac{(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad}-\frac{(e+fx)^2}{2af}-\frac{f\cosh(c+dx)}{bd^2}+\frac{(e+fx)\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/2*(e + f*x)^2/(a*f) + ((a^2 + b^2)*(e + f*x)^2)/(2*a*b^2*f) - (f*Cosh[c
+ d*x])/(b*d^2) - ((a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2])])/(a*b^2*d) - ((a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2])])/(a*b^2*d) + ((e + f*x)*Log[1 - E^(2*(c + d*x))])/(a*d
) - ((a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a
*b^2*d^2) - ((a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
]))])/(a*b^2*d^2) + (f*PolyLog[2, E^(2*(c + d*x))])/(2*a*d^2) + ((e + f*x)*
Sinh[c + d*x])/(b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
```

```
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5704

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh^2(c+dx)\coth(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cosh^3(c+dx}{a+b\sinh(c+dx)}}{a}$$

$$= \frac{\int (e+fx)\coth(c+dx)\,dx}{a} + \frac{\int (e+fx)\cosh(c+dx)\,dx}{b} - \frac{(a}{}$$

$$= -\frac{(e+fx)^2}{2af} + \frac{(a^2+b^2)(e+fx)^2}{2ab^2f} + \frac{(e+fx)\sinh(c+dx)}{bd} -$$

$$= -\frac{(e+fx)^2}{2af} + \frac{(a^2+b^2)(e+fx)^2}{2ab^2f} - \frac{f\cosh(c+dx)}{bd^2} - \frac{(a^2+b}{}$$

$$= -\frac{(e+fx)^2}{2af} + \frac{(a^2+b^2)(e+fx)^2}{2ab^2f} - \frac{f\cosh(c+dx)}{bd^2} - \frac{(a^2+b}{}$$

$$= -\frac{(e+fx)^2}{2af} + \frac{(a^2+b^2)(e+fx)^2}{2ab^2f} - \frac{f\cosh(c+dx)}{bd^2} - \frac{(a^2+b}{}$$

Mathematica [A]
time = 1.20, size = 296, normalized size = 0.92

$$\frac{-abf\cosh(c+dx)+b^2de\log(\sinh(c+dx))-b^2cf\log(\sinh(c+dx))+\frac{1}{2}b^2f\left((c+dx)\left(c+dx+2\log\left(1-e^{-2(c+dx)}\right)\right)-\text{PolyLog}\left(2,e^{-2(c+dx)}\right)\right)+(a^2+b^2)\left(\frac{1}{2}f(c+dx)^2-f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)-f(c+dx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)-de\log(a+b\sinh(c+dx))+cf\log(a+b\sinh(c+dx))-f\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)-f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)+abd(e+fx)\sinh(c+dx)}{ab^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (-(a*b*f*Cosh[c + d*x]) + b^2*d*e*Log[Sinh[c + d*x]] - b^2*c*f*Log[Sinh[c +
 d*x]] + (b^2*f*((c + d*x)*(c + d*x + 2*Log[1 - E^(-2*(c + d*x))]) - PolyLo
g[2, E^(-2*(c + d*x))]))/2 + (a^2 + b^2)*((f*(c + d*x)^2)/2 - f*(c + d*x)*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2])] - d*e*Log[a + b*Sinh[c + d*x]] + c*f*Log[a
+ b*Sinh[c + d*x]] - f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] -
 f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) + a*b*d*(e + f*x)*
Sinh[c + d*x])/(a*b^2*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(304) = 608$.
time = 10.71, size = 932, normalized size = 2.89

method	result
risch	$-\frac{af\,\mathrm{dilog}\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right)}{d^2b^2}-\frac{2afc\ln(\mathrm{e}^{dx+c})}{d^2b^2}+\frac{2ea\ln(\mathrm{e}^{dx+c})}{d\,b^2}-\frac{af\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)c}{d^2b^2}-\frac{af\ln\left(\frac{b\,\mathrm{e}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVER
BOSE)
```

```
[Out] -2/d^2/b^2*a*f*c*ln(exp(d*x+c))+2/d/b^2*e*a*ln(exp(d*x+c))-1/d^2/b^2*a*f*ln
((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d/b^2*a*f*ln((
b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2/b^2*a*f*ln((b*
exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d^2/b^2*a*f*c*ln(b*e
xp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d*a/b^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/
2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/2*a*f*x^2/b^2-1/d/b^2*a*e*ln(b*exp(2*d*x+2*
c)+2*a*exp(d*x+c)-b)-1/d^2/b^2*a*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(
a+(a^2+b^2)^(1/2)))-1/d^2/b^2*a*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(
-a+(a^2+b^2)^(1/2)))+1/d^2/b^2*f*a*c^2+1/2*(d*f*x+d*e-f)/d^2/b*exp(d*x+c)+2
/d/b^2*f*a*c*x+1/d^2*f/a*dilog(exp(d*x+c)+1)-1/d^2*f*dilog(exp(d*x+c))/a-a*
e*x/b^2-1/d^2*f/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*
c-1/d*f/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d^
2*f/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d*f/a*
ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/2*(d*f*x+d*e+f
)/d^2/b*exp(-d*x-c)+1/d*e/a*ln(exp(d*x+c)-1)-1/d*e/a*ln(b*exp(2*d*x+2*c)+2*
a*exp(d*x+c)-b)+1/d*e/a*ln(exp(d*x+c)+1)+1/d*f/a*ln(exp(d*x+c)+1)*x-1/d^2*f
*c/a*ln(exp(d*x+c)-1)+1/d^2*f*c/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d
^2*f/a*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2*f/
a*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/4*f*(2*(a*d^2*x^2*e^c - (b*d*x*e^(2*c) - b*e^(2*c))*e^(d*x) + (b*d*x + b
)*e^(-d*x))*e^(-c)/(b^2*d^2) - integrate(8*((a^3*e^c + a*b^2*e^c)*x*e^(d*x)
 - (a^2*b + b^3)*x)/(a*b^3*e^(2*d*x + 2*c) + 2*a^2*b^2*e^(d*x + c) - a*b^3)
, x) + 4*integrate(x/(a*e^(d*x + c) + a), x) - 4*integrate(x/(a*e^(d*x + c)
 - a), x)) - 1/2*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/
(b*d) - 2*log(e^(-d*x - c) + 1)/(a*d) - 2*log(e^(-d*x - c) - 1)/(a*d) + 2*(
a^2 + b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*b^2*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1266 vs. $2(307) = 614$.
time = 0.44, size = 1266, normalized size = 3.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -1/2*(a*b*d*f*x + a*b*d*cosh(1) + a*b*d*sinh(1) + a*b*f - (a*b*d*f*x + a*b*
d*cosh(1) + a*b*d*sinh(1) - a*b*f)*cosh(d*x + c)^2 - (a*b*d*f*x + a*b*d*cos
h(1) + a*b*d*sinh(1) - a*b*f)*sinh(d*x + c)^2 - (a^2*d^2*f*x^2 - 2*a^2*c^2*
f + 2*(a^2*d^2*x + 2*a^2*c*d)*cosh(1) + 2*(a^2*d^2*x + 2*a^2*c*d)*sinh(1))*
cosh(d*x + c) + 2*((a^2 + b^2)*f*cosh(d*x + c) + (a^2 + b^2)*f*sinh(d*x + c
))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*((a^2 + b^2)*f*cosh(d*x + c) +
 (a^2 + b^2)*f*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b^
2*f*cosh(d*x + c) + b^2*f*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c
)) - 2*(b^2*f*cosh(d*x + c) + b^2*f*sinh(d*x + c))*dilog(-cosh(d*x + c) - s
inh(d*x + c)) - 2*(((a^2 + b^2)*c*f - (a^2 + b^2)*d*cosh(1) - (a^2 + b^2)*d
*sinh(1))*cosh(d*x + c) + ((a^2 + b^2)*c*f - (a^2 + b^2)*d*cosh(1) - (a^2 +
 b^2)*d*sinh(1))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) +
 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(((a^2 + b^2)*c*f - (a^2 + b^2)*d*cos
h(1) - (a^2 + b^2)*d*sinh(1))*cosh(d*x + c) + ((a^2 + b^2)*c*f - (a^2 + b^2
)*d*cosh(1) - (a^2 + b^2)*d*sinh(1))*sinh(d*x + c))*log(2*b*cosh(d*x + c) +
 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(((a^2 + b^2)*d*f
*x + (a^2 + b^2)*c*f)*cosh(d*x + c) + ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)
*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(((a^2 + b^2)*d*f*x +
(a^2 + b^2)*c*f)*cosh(d*x + c) + ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*sinh
(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*s
inh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*((b^2*d*f*x + b^2*d*cosh(1)
 + b^2*d*sinh(1))*cosh(d*x + c) + (b^2*d*f*x + b^2*d*cosh(1) + b^2*d*sinh(1
))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*((b^2*c*f - b^
2*d*cosh(1) - b^2*d*sinh(1))*cosh(d*x + c) + (b^2*c*f - b^2*d*cosh(1) - b^2
*d*sinh(1))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) - 2*((b^2
*d*f*x + b^2*c*f)*cosh(d*x + c) + (b^2*d*f*x + b^2*c*f)*sinh(d*x + c))*log(
-cosh(d*x + c) - sinh(d*x + c) + 1) - (a^2*d^2*f*x^2 - 2*a^2*c^2*f + 2*(a^2
*d^2*x + 2*a^2*c*d)*cosh(1) + 2*(a*b*d*f*x + a*b*d*cosh(1) + a*b*d*sinh(1)
- a*b*f)*cosh(d*x + c) + 2*(a^2*d^2*x + 2*a^2*c*d)*sinh(1))*sinh(d*x + c))/
(a*b^2*d^2*cosh(d*x + c) + a*b^2*d^2*sinh(d*x + c))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*cosh(c + d*x)**2*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cosh(c+d\,x)}^2\,\coth(c+d\,x)\,\left(e+f\,x\right)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*coth(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((cosh(c + d*x)^2*coth(c + d*x)*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.433 $\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=57

$$\frac{\log(\sinh(c+dx))}{ad}-\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{ab^2d}+\frac{\sinh(c+dx)}{bd}$$

```
[Out] ln(sinh(d*x+c))/a/d-(a^2+b^2)*ln(a+b*sinh(d*x+c))/a/b^2/d+sinh(d*x+c)/b/d
```

Rubi [A]
time = 0.09, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2916, 12, 908}

$$-\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{ab^2d}+\frac{\log(\sinh(c+dx))}{ad}+\frac{\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Log[Sinh[c + d*x]]/(a*d) - ((a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/(a*b^2*d)
 + Sinh[c + d*x]/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{b(-b^2-x^2)}{x(a+x)}\,dx, x, b\sinh(c+dx)\right)}{b^3d}$$
$$= -\frac{\text{Subst}\left(\int \frac{-b^2-x^2}{x(a+x)}\,dx, x, b\sinh(c+dx)\right)}{b^2d}$$
$$= -\frac{\text{Subst}\left(\int \left(-1-\frac{b^2}{ax}+\frac{a^2+b^2}{a(a+x)}\right)\,dx, x, b\sinh(c+dx)\right)}{b^2d}$$
$$= \frac{\log(\sinh(c+dx))}{ad} - \frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{ab^2d} + \frac{\sinh(c+dx)}{bd}$$

Mathematica [A]
time = 0.07, size = 48, normalized size = 0.84

$$\frac{\frac{\log(\sinh(c+dx))}{a} - \left(\frac{1}{a}+\frac{a}{b^2}\right)\log(a+b\sinh(c+dx)) + \frac{\sinh(c+dx)}{b}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (Log[Sinh[c + d*x]]/a - (a^(-1) + a/b^2)*Log[a + b*Sinh[c + d*x]] + Sinh[c
+ d*x]/b)/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(57) = 114$.
time = 5.35, size = 138, normalized size = 2.42

method	result
risch	$\frac{ax}{b^2} + \frac{e^{dx+c}}{2bd} - \frac{e^{-dx-c}}{2bd} + \frac{2ac}{b^2d} + \frac{\ln(e^{2dx+2c}-1)}{ad} - \frac{a\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{b^2d} - \frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{ad}$
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a} - \frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)} + \frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2} + \frac{(-a^2-b^2)\ln\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{b^2a} - \frac{}{b(\tan}}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a} - \frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)} + \frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2} + \frac{(-a^2-b^2)\ln\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{b^2a} - \frac{}{b(\tan}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*ln(tanh(1/2*d*x+1/2*c))-1/b/(tanh(1/2*d*x+1/2*c)+1)+a/b^2*ln(tanh(
1/2*d*x+1/2*c)+1)+1/b^2/a*(-a^2-b^2)*ln(a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/
```

```
2*d*x+1/2*c)-a)-1/b/(tanh(1/2*d*x+1/2*c)-1)+a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(57) = 114$.
time = 0.27, size = 130, normalized size = 2.28

$$-\frac{(dx+c)a}{b^2d}+\frac{e^{(dx+c)}}{2\,bd}-\frac{e^{(-dx-c)}}{2\,bd}+\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}+\frac{\log\left(e^{(-dx-c)}-1\right)}{ad}-\frac{(a^2+b^2)\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{ab^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -(d*x + c)*a/(b^2*d) + 1/2*e^(d*x + c)/(b*d) - 1/2*e^(-d*x - c)/(b*d) + log
(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d) - (a^2 + b^2)*log(-2
*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*b^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(57) = 114$.
time = 0.44, size = 203, normalized size = 3.56

$$\frac{2\,a^2dx\cosh\left(dx+c\right)+ab\cosh\left(dx+c\right)^2+ab\sinh\left(dx+c\right)^2-ab-2\,\left((a^2+b^2)\cosh\left(dx+c\right)+(a^2+b^2)\sinh\left(dx+c\right)\right)\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)+2\,\left(b^2\cosh\left(dx+c\right)+b^2\sinh\left(dx+c\right)\right)\log\left(\frac{2\,\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)+2\,\left(a^2dx+ab\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{2\,\left(ab^2d\cosh\left(dx+c\right)+ab^2d\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*a^2*d*x*cosh(d*x + c) + a*b*cosh(d*x + c)^2 + a*b*sinh(d*x + c)^2 -
a*b - 2*((a^2 + b^2)*cosh(d*x + c) + (a^2 + b^2)*sinh(d*x + c))*log(2*(b*si
nh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 2*(b^2*cosh(d*x + c) +
b^2*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 2
*(a^2*d*x + a*b*cosh(d*x + c))*sinh(d*x + c))/(a*b^2*d*cosh(d*x + c) + a*b^
2*d*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2\left(c+dx\right)\coth\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(cosh(c + d*x)**2*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A]
time = 0.47, size = 94, normalized size = 1.65

$$\frac{\frac{e^{(dx+c)}-e^{(-dx-c)}}{b}+\frac{2\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a}-\frac{2\left(a^2+b^2\right)\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2a\right|\right)}{ab^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((e^(d*x + c) - e^(-d*x - c))/b + 2*log(abs(e^(d*x + c) - e^(-d*x - c))
)/a - 2*(a^2 + b^2)*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a*b^2))
/d
```

Mupad [B]
time = 0.49, size = 360, normalized size = 6.32

$$\frac{e^{c+dx}}{2bd}-\frac{e^{-c-dx}}{2bd}-\frac{\ln\left(8a^5e^{dx}e^c-16b^5-16a^2b^3-4a^4b+16b^5e^{2c}e^{2dx}+4a^4be^{2c}e^{2dx}+32a^3b^2e^{dx}e^c+16a^2b^3e^{2c}e^{2dx}+32ab^4e^{dx}e^c\right)}{ad}+\frac{\ln\left(4a^6+16b^6+32a^2b^4+20a^4b^2-4a^6e^{2c}e^{2dx}-16b^6e^{2c}e^{2dx}-32a^2b^4e^{2c}e^{2dx}-20a^4b^2e^{2c}e^{2dx}\right)}{ad}+\frac{ax}{b^2}-\frac{a\ln\left(8a^5e^{dx}e^c-16b^5-16a^2b^3-4a^4b+16b^5e^{2c}e^{2dx}+4a^4be^{2c}e^{2dx}+32a^3b^2e^{dx}e^c+16a^2b^3e^{2c}e^{2dx}+32ab^4e^{dx}e^c\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*coth(c + d*x))/(a + b*sinh(c + d*x)),x)
```

```
[Out] exp(c + d*x)/(2*b*d) - exp(- c - d*x)/(2*b*d) - log(8*a^5*exp(d*x)*exp(c) -
 16*b^5 - 16*a^2*b^3 - 4*a^4*b + 16*b^5*exp(2*c)*exp(2*d*x) + 4*a^4*b*exp(2
*c)*exp(2*d*x) + 32*a^3*b^2*exp(d*x)*exp(c) + 16*a^2*b^3*exp(2*c)*exp(2*d*x
) + 32*a*b^4*exp(d*x)*exp(c))/(a*d) + log(4*a^6 + 16*b^6 + 32*a^2*b^4 + 20*
a^4*b^2 - 4*a^6*exp(2*c)*exp(2*d*x) - 16*b^6*exp(2*c)*exp(2*d*x) - 32*a^2*b
^4*exp(2*c)*exp(2*d*x) - 20*a^4*b^2*exp(2*c)*exp(2*d*x))/(a*d) + (a*x)/b^2
- (a*log(8*a^5*exp(d*x)*exp(c) - 16*b^5 - 16*a^2*b^3 - 4*a^4*b + 16*b^5*exp
(2*c)*exp(2*d*x) + 4*a^4*b*exp(2*c)*exp(2*d*x) + 32*a^3*b^2*exp(d*x)*exp(c)
 + 16*a^2*b^3*exp(2*c)*exp(2*d*x) + 32*a*b^4*exp(d*x)*exp(c)))/(b^2*d)
```

3.434 $\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cosh[c + d*x]^2*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Cosh[c + d*x]^2*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$$\text{\$Aborted}$$

Verification is not applicable to the result.

```
[In] Integrate[(Cosh[c + d*x]^2*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\cosh^2\left(dx+c\right)\right)\coth\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/2*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b*f) - 1/2*e^(c - d*e
/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b*f) - a*log(f*x + e)/(b^2*f) + 1/4*
integrate(8*(a^2*b + b^3 - (a^3*e^c + a*b^2*e^c)*e^(d*x))/(a*b^3*f*x + a*b^
3*e - (a*b^3*f*x*e^(2*c) + a*b^3*e^(2*c + 1))*e^(2*d*x) - 2*(a^2*b^2*f*x*e^
c + a^2*b^2*e^(c + 1))*e^(d*x)), x) - integrate(1/(a*f*x + a*e + (a*f*x*e^c
 + a*e^(c + 1))*e^(d*x)), x) + integrate(-1/(a*f*x + a*e - (a*f*x*e^c + a*e
^(c + 1))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(cosh(d*x + c)^2*coth(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(cosh(c + d*x)**2*coth(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)),
x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh(c+d\,x)^2\,\coth(c+d\,x)}{(e+f\,x)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)^2*coth(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((cosh(c + d*x)^2*coth(c + d*x))/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.435 $\int \frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1049

$$-\frac{2b(e+fx)^3\operatorname{ArcTan}\left(e^{c+dx}\right)}{(a^2+b^2)\,d}-\frac{2(e+fx)^3\tanh^{-1}\left(e^{2c+2dx}\right)}{ad}-\frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)\,d}-\frac{b^2(e+fx}{}$$

```
[Out] 3/4*f^3*polylog(4,exp(2*d*x+2*c))/a/d^4+b^2*(f*x+e)^3*ln(1+exp(2*d*x+2*c))/
a/(a^2+b^2)/d-b^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b
^2)/d-b^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d-6*
b^2*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^4+3/2*f*
(f*x+e)^2*polylog(2,exp(2*d*x+2*c))/a/d^2-3/2*f^2*(f*x+e)*polylog(3,exp(2*d
*x+2*c))/a/d^3-2*(f*x+e)^3*arctanh(exp(2*d*x+2*c))/a/d-3/4*f^3*polylog(4,-e
xp(2*d*x+2*c))/a/d^4+3*I*b*f*(f*x+e)^2*polylog(2,-I*exp(d*x+c))/(a^2+b^2)/d
^2+6*I*b*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/(a^2+b^2)/d^3+3/2*b^2*f*(f*x+e
)^2*polylog(2,-exp(2*d*x+2*c))/a/(a^2+b^2)/d^2-3/2*b^2*f^2*(f*x+e)*polylog(
3,-exp(2*d*x+2*c))/a/(a^2+b^2)/d^3+6*I*b*f^3*polylog(4,-I*exp(d*x+c))/(a^2+
b^2)/d^4-3*I*b*f*(f*x+e)^2*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^2-6*I*b*f^2*
(f*x+e)*polylog(3,-I*exp(d*x+c))/(a^2+b^2)/d^3-3*b^2*f*(f*x+e)^2*polylog(2,
-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^2-3*b^2*f*(f*x+e)^2*polylo
g(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^2+6*b^2*f^2*(f*x+e)*po
lylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^3+6*b^2*f^2*(f*x+e
)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^3-6*b^2*f^3*po
lylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^4-2*b*(f*x+e)^3*ar
ctan(exp(d*x+c))/(a^2+b^2)/d-3/2*f*(f*x+e)^2*polylog(2,-exp(2*d*x+2*c))/a/d
^2+3/2*f^2*(f*x+e)*polylog(3,-exp(2*d*x+2*c))/a/d^3+3/4*b^2*f^3*polylog(4,-
exp(2*d*x+2*c))/a/(a^2+b^2)/d^4-6*I*b*f^3*polylog(4,I*exp(d*x+c))/(a^2+b^2)
/d^4
```

Rubi [A]
time = 1.11, antiderivative size = 1049, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {5708, 5569, 4267, 2611, 6744, 2320, 6724, 5692, 5680, 2221, 6874, 4265, 3799}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*b*(e + f*x)^3*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) - (2*(e + f*x)^3*Arc
Tanh[E^(2*c + 2*d*x)])/(a*d) - (b^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) - (b^2*(e + f*x)^3*Log[1 + (b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) + (b^2*(e + f*x)^3*Log[1 +
```

```
E^(2*(c + d*x))])/(a*(a^2 + b^2)*d) + ((3*I)*b*f*(e + f*x)^2*PolyLog[2, (-
I)*E^(c + d*x)])/((a^2 + b^2)*d^2) - ((3*I)*b*f*(e + f*x)^2*PolyLog[2, I*E^
(c + d*x)])/((a^2 + b^2)*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^2) - (3*b^2*f*(e + f*x)^2*
PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^2) +
 (3*b^2*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*a*(a^2 + b^2)*d^2) -
 (3*f*(e + f*x)^2*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a*d^2) + (3*f*(e + f*x)^
2*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d^2) - ((6*I)*b*f^2*(e + f*x)*PolyLog[3
, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) + ((6*I)*b*f^2*(e + f*x)*PolyLog[3,
I*E^(c + d*x)])/((a^2 + b^2)*d^3) + (6*b^2*f^2*(e + f*x)*PolyLog[3, -((b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^3) + (6*b^2*f^2*(e + f
*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^
3) - (3*b^2*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*a*(a^2 + b^2)*d^
3) + (3*f^2*(e + f*x)*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a*d^3) - (3*f^2*(e +
 f*x)*PolyLog[3, E^(2*c + 2*d*x)])/(2*a*d^3) + ((6*I)*b*f^3*PolyLog[4, (-I)
*E^(c + d*x)])/((a^2 + b^2)*d^4) - ((6*I)*b*f^3*PolyLog[4, I*E^(c + d*x)])/
((a^2 + b^2)*d^4) - (6*b^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/(a*(a^2 + b^2)*d^4) - (6*b^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^4) + (3*b^2*f^3*PolyLog[4, -E^(2*(c
+ d*x))])/(4*a*(a^2 + b^2)*d^4) - (3*f^3*PolyLog[4, -E^(2*c + 2*d*x)])/(4*a
*d^4) + (3*f^3*PolyLog[4, E^(2*c + 2*d*x)])/(4*a*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5708

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= \frac{2\int (e+fx)^3\text{csch}(2c+2dx)\,dx}{a} - \frac{b\int (e+fx)^3\text{sech}(c+dx)(a-}{a(a^2+b^2)}$$

$$= \frac{b^2(e+fx)^4}{4a(a^2+b^2)f} - \frac{2(e+fx)^3\tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b\int (a(e+fx)^3\text{se}}{}$$

$$= \frac{b^2(e+fx)^4}{4a(a^2+b^2)f} - \frac{2(e+fx)^3\tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)^3\log\Big(}{a(a^2}$$

$$= -\frac{2b(e+fx)^3\tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)^3\tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(}{}$$

$$= -\frac{2b(e+fx)^3\tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)^3\tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(}{}$$

$$= -\frac{2b(e+fx)^3\tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)^3\tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(}{}$$

$$= -\frac{2b(e+fx)^3\tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)^3\tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(}{}$$

$$= -\frac{2b(e+fx)^3\tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)^3\tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(}{}$$

$$= -\frac{2b(e+fx)^3\tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)^3\tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 9140 vs. 2(1049) = 2098.
time = 23.28, size = 9140, normalized size = 8.71

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] Result too large to show
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c) \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -(b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^3 + a*b^2)*d) - 2
*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(e^(-2*d*x - 2*c) + 1)/((a^2
 + b^2)*d) - log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d))*e^3
 + 3*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*f*e^2/(a*d^2) + 3*(d*
x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))*f*e^2/(a*d^2) + 3*(d^2*x^2*lo
g(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c))
)*f^2*e/(a*d^3) + 3*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c
)) - 2*polylog(3, e^(d*x + c)))*f^2*e/(a*d^3) + (d^3*x^3*log(e^(d*x + c) +
1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*pol
ylog(4, -e^(d*x + c)))*f^3/(a*d^4) + (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2
*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d
*x + c)))*f^3/(a*d^4) - 1/2*(d^4*f^3*x^4 + 4*d^4*f^2*x^3*e + 6*d^4*f*x^2*e^
2)/(a*d^4) + integrate(2*(b^3*f^3*x^3 + 3*b^3*f^2*x^2*e + 3*b^3*f*x*e^2 - (
a*b^2*f^3*x^3*e^c + 3*a*b^2*f^2*x^2*e^(c + 1) + 3*a*b^2*f*x*e^(c + 2))*e^(d
*x))/(a^3*b + a*b^3 - (a^3*b*e^(2*c) + a*b^3*e^(2*c))*e^(2*d*x) - 2*(a^4*e^
c + a^2*b^2*e^c)*e^(d*x)), x) - integrate(-2*(a*f^3*x^3 + 3*a*f^2*x^2*e + 3
*a*f*x*e^2 - (b*f^3*x^3*e^c + 3*b*f^2*x^2*e^(c + 1) + 3*b*f*x*e^(c + 2))*e^
(d*x))/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4128 vs. 2(981) = 1962.
time = 0.49, size = 4128, normalized size = 3.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -(6*b^2*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*b^2*f^3*polylog(4, (a*co
sh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2))/b) - 6*(a^2 + b^2)*f^3*polylog(4, cosh(d*x + c) + sinh(d*x
+ c)) - 6*(a^2 + b^2)*f^3*polylog(4, -cosh(d*x + c) - sinh(d*x + c)) + 3*(b
^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*
sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*dilog((a*cosh(d*
x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b + 1) + 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*
d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1
))*sinh(1))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*((a^2 + b^2)*d^2*f^3*
x^2 + 2*(a^2 + b^2)*d^2*f^2*x*cosh(1) + (a^2 + b^2)*d^2*f*cosh(1)^2 + (a^2
+ b^2)*d^2*f*sinh(1)^2 + 2*((a^2 + b^2)*d^2*f^2*x + (a^2 + b^2)*d^2*f*cosh(
1))*sinh(1))*dilog(cosh(d*x + c) + sinh(d*x + c)) + 3*(a^2*d^2*f^3*x^2 + I*
a*b*d^2*f^3*x^2 + 2*a^2*d^2*f^2*x*cosh(1) + 2*I*a*b*d^2*f^2*x*cosh(1) + a^2
*d^2*f*cosh(1)^2 + I*a*b*d^2*f*cosh(1)^2 + a^2*d^2*f*sinh(1)^2 + I*a*b*d^2*
f*sinh(1)^2 + 2*(a^2*d^2*f^2*x + a^2*d^2*f*cosh(1))*sinh(1) + 2*I*(a*b*d^2*
f^2*x + a*b*d^2*f*cosh(1))*sinh(1))*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)
) + 3*(a^2*d^2*f^3*x^2 - I*a*b*d^2*f^3*x^2 + 2*a^2*d^2*f^2*x*cosh(1) - 2*I*
a*b*d^2*f^2*x*cosh(1) + a^2*d^2*f*cosh(1)^2 - I*a*b*d^2*f*cosh(1)^2 + a^2*d
^2*f*sinh(1)^2 - I*a*b*d^2*f*sinh(1)^2 + 2*(a^2*d^2*f^2*x + a^2*d^2*f*cosh(
1))*sinh(1) - 2*I*(a*b*d^2*f^2*x + a*b*d^2*f*cosh(1))*sinh(1))*dilog(-I*cos
h(d*x + c) - I*sinh(d*x + c)) - 3*((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*
d^2*f^2*x*cosh(1) + (a^2 + b^2)*d^2*f*cosh(1)^2 + (a^2 + b^2)*d^2*f*sinh(1)
^2 + 2*((a^2 + b^2)*d^2*f^2*x + (a^2 + b^2)*d^2*f*cosh(1))*sinh(1))*dilog(-
cosh(d*x + c) - sinh(d*x + c)) - (b^2*c^3*f^3 - 3*b^2*c^2*d*f^2*cosh(1) + 3
*b^2*c*d^2*f*cosh(1)^2 - b^2*d^3*cosh(1)^3 - b^2*d^3*sinh(1)^3 + 3*(b^2*c*d
^2*f - b^2*d^3*cosh(1))*sinh(1)^2 - 3*(b^2*c^2*d*f^2 - 2*b^2*c*d^2*f*cosh(1
) + b^2*d^3*cosh(1)^2)*sinh(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) +
 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^2*c^3*f^3 - 3*b^2*c^2*d*f^2*cosh(1)
+ 3*b^2*c*d^2*f*cosh(1)^2 - b^2*d^3*cosh(1)^3 - b^2*d^3*sinh(1)^3 + 3*(b^2*
c*d^2*f - b^2*d^3*cosh(1))*sinh(1)^2 - 3*(b^2*c^2*d*f^2 - 2*b^2*c*d^2*f*cos
h(1) + b^2*d^3*cosh(1)^2)*sinh(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c
) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^2*d^3*f^3*x^3 + b^2*c^3*f^3 + 3*(
b^2*d^3*f*x + b^2*c*d^2*f)*cosh(1)^2 + 3*(b^2*d^3*f*x + b^2*c*d^2*f)*sinh(1
)^2 + 3*(b^2*d^3*f^2*x^2 - b^2*c^2*d*f^2)*cosh(1) + 3*(b^2*d^3*f^2*x^2 - b^
2*c^2*d*f^2 + 2*(b^2*d^3*f*x + b^2*c*d^2*f)*cosh(1))*sinh(1))*log(-(a*cosh(
d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2) - b)/b) + (b^2*d^3*f^3*x^3 + b^2*c^3*f^3 + 3*(b^2*d^3*f*x + b^2
```

```
*c*d^2*f)*cosh(1)^2 + 3*(b^2*d^3*f*x + b^2*c*d^2*f)*sinh(1)^2 + 3*(b^2*d^3*
f^2*x^2 - b^2*c^2*d*f^2)*cosh(1) + 3*(b^2*d^3*f^2*x^2 - b^2*c^2*d*f^2 + 2*(
b^2*d^3*f*x + b^2*c*d^2*f)*cosh(1))*sinh(1))*log(-(a*cosh(d*x + c) + a*sinh
(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/
b) - ((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*f^2*x^2*cosh(1) + 3*(a^2
+ b^2)*d^3*f*x*cosh(1)^2 + (a^2 + b^2)*d^3*cosh(1)^3 + (a^2 + b^2)*d^3*sinh
(1)^3 + 3*((a^2 + b^2)*d^3*f*x + (a^2 + b^2)*d^3*cosh(1))*sinh(1)^2 + 3*((a
^2 + b^2)*d^3*f^2*x^2 + 2*(a^2 + b^2)*d^3*f*x*cosh(1) + (a^2 + b^2)*d^3*cos
h(1)^2)*sinh(1))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a^2*c^3*f^3 + I*
a*b*c^3*f^3 - 3*a^2*c^2*d*f^2*cosh(1) - 3*I*a*b*c^2*d*f^2*cosh(1) + 3*a^2*c
*d^2*f*cosh(1)^2 + 3*I*a*b*c*d^2*f*cosh(1)^2 - a^2*d^3*cosh(1)^3 - I*a*b*d^
3*cosh(1)^3 - a^2*d^3*sinh(1)^3 - I*a*b*d^3*sinh(1)^3 + 3*(a^2*c*d^2*f - a^
2*d^3*cosh(1))*sinh(1)^2 + 3*I*(a*b*c*d^2*f - a*b*d^3*cosh(1))*sinh(1)^2 -
3*(a^2*c^2*d*f^2 - 2*a^2*c*d^2*f*cosh(1) + a^2*d^3*cosh(1)^2)*sinh(1) - 3*I
*(a*b*c^2*d*f^2 - 2*a*b*c*d^2*f*cosh(1) + a*b*d^3*cosh(1)^2)*sinh(1))*log(c
osh(d*x + c) + sinh(d*x + c) + I) - (a^2*c^3*f^3 - I*a*b*c^3*f^3 - 3*a^2*c^
2*d*f^2*cosh(1) + 3*I*a*b*c^2*d*f^2*cosh(1) + 3*a^2*c*d^2*f*cosh(1)^2 - 3*I
*a*b*c*d^2*f*cosh(1)^2 - a^2*d^3*cosh(1)^3 + I*a*b*d^3*cosh(1)^3 - a^2*d^3*
sinh(1)^3 + I*a*b*d^3*sinh(1)^3 + 3*(a^2*c*d^2*f - a^2*d^3*cosh(1))*sinh(1)
^2 - 3*I*(a*b*c*d^2*f - a*b*d^3*cosh(1))*sinh(1)^2 - 3*(a^2*c^2*d*f^2 - 2*a
^2*c*d^2*f*cosh(1) + a^2*d^3*cosh(1)^2)*sinh(1) + 3*I*(a*b*c^2*d*f^2 - 2*a*
b*c*d^2*f*cosh(1) + a*b*d^3*cosh(1)^2)*sinh(1))*log(cosh(d*x + c) + sinh(d*
x + c) - I) + ((a^2 + b^2)*c^3*f^3 - 3*(a^2 + b^2)*c^2*d*f^2*cosh(1) + 3*(a
^2 + b^2)*c*d^2*f*cosh(1)^2 - (a^2 + b^2)*d^3*cosh(1)^3 - (a^2 + b^2)*d^3*s
inh(1)^3 + 3*((a^2 + b^2)*c*d^2*f - (a^2 + b^2)*d^3*cosh(1))*sinh(1)^2 - 3*
((a^2 + b^2)*c^2*d*f^2 - 2*(a^2 + b^2)*c*d^2*f*cosh(1) + (a^2 + b^2)*d^3*co
sh(1)^2)*sinh(1))*log(cosh(d*x + c) + sinh(d*x ...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6439 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate((f*x + e)^3*csch(d*x + c)*sech(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^3}{\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
[Out] int((e + f*x)^3/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.436 $\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=734

$$-\frac{2b(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d}-\frac{2(e+fx)^2\tanh^{-1}\left(e^{2c+2dx}\right)}{ad}-\frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)d}-\frac{b^2(e+f$$

```
[Out] -2*b*(f*x+e)^2*arctan(exp(d*x+c))/(a^2+b^2)/d-2*(f*x+e)^2*arctanh(exp(2*d*x
+2*c))/a/d+b^2*(f*x+e)^2*ln(1+exp(2*d*x+2*c))/a/(a^2+b^2)/d-b^2*(f*x+e)^2*l
n(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d-b^2*(f*x+e)^2*ln(1+b*ex
p(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d-2*I*b*f*(f*x+e)*polylog(2,I*exp
(d*x+c))/(a^2+b^2)/d^2+2*I*b*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/(a^2+b^2)/d
^2+b^2*f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/a/(a^2+b^2)/d^2-f*(f*x+e)*polyl
og(2,-exp(2*d*x+2*c))/a/d^2+f*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d^2-2*b^2
*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^2-2*b
^2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^2+2
*I*b*f^2*polylog(3,I*exp(d*x+c))/(a^2+b^2)/d^3-2*I*b*f^2*polylog(3,-I*exp(d
*x+c))/(a^2+b^2)/d^3-1/2*b^2*f^2*polylog(3,-exp(2*d*x+2*c))/a/(a^2+b^2)/d^3
+1/2*f^2*polylog(3,-exp(2*d*x+2*c))/a/d^3-1/2*f^2*polylog(3,exp(2*d*x+2*c))
/a/d^3+2*b^2*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d
^3+2*b^2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d^3
```

Rubi [A]
time = 0.84, antiderivative size = 734, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5708, 5569, 4267, 2611, 2320, 6724, 5692, 5680, 2221, 6874, 4265, 3799}

$$-\frac{2b(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{d\left(a^2+b^2\right)}+\frac{2b^2f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3\left(a^2+b^2\right)}+\frac{2b^2f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^3\left(a^2+b^2\right)}-\frac{b^2f^2\text{Li}_3\left(-e^{2(c+dx)}\right)}{2ad^3\left(a^2+b^2\right)}-\frac{2ibf^2\text{Li}_3\left(-ie^{c+dx}\right)}{d^3\left(a^2+b^2\right)}+\frac{2ibf^2\text{Li}_3\left(ie^{c+dx}\right)}{d^3\left(a^2+b^2\right)}-\frac{2b^2f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\left(a^2+b^2\right)}-\frac{2b^2f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^2\left(a^2+b^2\right)}+\frac{b^2f(e+fx)\text{Li}_2\left(-e^{2(c+dx)}\right)}{ad^2\left(a^2+b^2\right)}+\frac{2ibf(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)}-\frac{2ibf(e+fx)\text{Li}_2\left(ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)}-\frac{b^2(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{ad\left(a^2+b^2\right)}-\frac{b^2(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{ad\left(a^2+b^2\right)}+\frac{b^2(e+fx)^2\log\left(e^{2(c+dx)}+1\right)}{ad\left(a^2+b^2\right)}+\frac{f^2\text{Li}_3\left(-e^{2c+2dx}\right)}{2ad^3}-\frac{f^2\text{Li}_3\left(e^{2c+2dx}\right)}{2ad^3}-\frac{f(e+fx)\text{Li}_2\left(-e^{2c+2dx}\right)}{ad^2}+\frac{f(e+fx)\text{Li}_2\left(e^{2c+2dx}\right)}{ad^2}-\frac{2(e+fx)^2\tanh^{-1}\left(e^{2c+2dx}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*b*(e + f*x)^2*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) - (2*(e + f*x)^2*Arc
Tanh[E^(2*c + 2*d*x)])/(a*d) - (b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) - (b^2*(e + f*x)^2*Log[1 + (b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) + (b^2*(e + f*x)^2*Log[1 +
 E^(2*(c + d*x))])/(a*(a^2 + b^2)*d) + ((2*I)*b*f*(e + f*x)*PolyLog[2, (-I)
*E^(c + d*x)])/((a^2 + b^2)*d^2) - ((2*I)*b*f*(e + f*x)*PolyLog[2, I*E^(c +
 d*x)])/((a^2 + b^2)*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^2) - (2*b^2*f*(e + f*x)*PolyLog[
2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^2) + (b^2*f*
(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(a*(a^2 + b^2)*d^2) - (f*(e + f*x)*
PolyLog[2, -E^(2*c + 2*d*x)])/(a*d^2) + (f*(e + f*x)*PolyLog[2, E^(2*c + 2*
```

```
d*x)])/(a*d^2) - ((2*I)*b*f^2*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^
3) + ((2*I)*b*f^2*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)*d^3) + (2*b^2*f^2
*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^3)
+ (2*b^2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2
+ b^2)*d^3) - (b^2*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*a*(a^2 + b^2)*d^3)
+ (f^2*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a*d^3) - (f^2*PolyLog[3, E^(2*c + 2
*d*x)])/(2*a*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5708

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= \frac{2\int (e+fx)^2\text{csch}(2c+2dx)\,dx}{a} - \frac{b\int (e+fx)^2\text{sech}(c+dx)(a-}{a\,(a^2+b^2)}$$

$$= \frac{b^2(e+fx)^3}{3a\,(a^2+b^2)\,f} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b\int (a(e+fx)^2\text{se}}{}$$

$$= \frac{b^2(e+fx)^3}{3a\,(a^2+b^2)\,f} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(e+fx)^2\log\Big(}{a\,(a^2}$$

$$= -\frac{2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(}{}$$

$$= -\frac{2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(}{}$$

$$= -\frac{2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(}{}$$

$$= -\frac{2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(}{}$$

$$= -\frac{2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)\,d} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2950 vs. 2(734) = 1468.
time = 22.47, size = 2950, normalized size = 4.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x
]

[Out] 2*((a*(-(d^3*E^c*x*(3*e^2 + 3*e*f*x + f^2*x^2)) + 3*d^2*(1 + E^c)*(e + f*x)
^2*Log[1 + E^(c + d*x)] + 6*d*(1 + E^c)*f*(e + f*x)*PolyLog[2, -E^(c + d*x)
] - 6*(1 + E^c)*f^2*PolyLog[3, -E^(c + d*x)]))/(6*(a^2 + b^2)*d^3*(1 + E^c)
) + (d^2*((-I)*d*E^c*x*((-3*I)*b*e*f*x + a*(3*e^2 + 3*e*f*x + f^2*x^2)) + 3
*(1 + I*E^c)*((-2*I)*b*e*f*x + a*(e + f*x)^2)*Log[1 + I*E^(c + d*x)]) + 6*d
*(1 + I*E^c)*f*((-I)*b*e + a*(e + f*x))*PolyLog[2, (-I)*E^(c + d*x)] - (6*I
)*a*(-I + E^c)*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(6*(a - I*b)*((-I)*a + b)*
d^3*(-I + E^c)) - ((I/2)*b*((-2*I)*d^2*e^2*ArcTan[E^(c + d*x)] + d^2*f^2*x^
2*Log[1 - I*E^(c + d*x)] - d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*f^2*x*P
olyLog[2, (-I)*E^(c + d*x)] + 2*d*f^2*x*PolyLog[2, I*E^(c + d*x)] + 2*f^2*P
olyLog[3, (-I)*E^(c + d*x)] - 2*f^2*PolyLog[3, I*E^(c + d*x)]))/((a^2 + b^2
)*d^3) - ((-I)*b*d^3*e*E^(2*c)*f*x^2 + 2*a*d^2*e^2*ArcTan[1 - (1 + I)*E^(c
+ d*x)] + (2*I)*a*d^2*e^2*E^(2*c)*ArcTan[1 - (1 + I)*E^(c + d*x)] + (2*I)*a
*d^2*e*f*x*Log[1 - E^(c + d*x)] - 2*a*d^2*e*E^(2*c)*f*x*Log[1 - E^(c + d*x)
] + I*a*d^2*f^2*x^2*Log[1 - E^(c + d*x)] - a*d^2*E^(2*c)*f^2*x^2*Log[1 - E^
(c + d*x)] - (2*I)*a*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + 2*b*d^2*e*f*x*Log[1
 - I*E^(c + d*x)] + 2*a*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (2*I)*b*
d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] - I*a*d^2*f^2*x^2*Log[1 - I*E^(c +
 d*x)] + a*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + 2*d*(-I + E^(2*c))*
f*(I*b*e + a*(e + f*x))*PolyLog[2, I*E^(c + d*x)] - 2*a*d*(-I + E^(2*c))*f*
(e + f*x)*PolyLog[2, E^(c + d*x)] + (2*I)*a*f^2*PolyLog[3, I*E^(c + d*x)] -
 2*a*E^(2*c)*f^2*PolyLog[3, I*E^(c + d*x)] - (2*I)*a*f^2*PolyLog[3, E^(c +
d*x)] + 2*a*E^(2*c)*f^2*PolyLog[3, E^(c + d*x)])/(2*(a^2 + b^2)*d^3*(-I + E
^(2*c))) - (b^2*(4*d^3*E^(2*c)*x*(3*e^2 + 3*e*f*x + f^2*x^2) - 6*d^2*(-1 +
E^(2*c))*(e + f*x)^2*Log[1 - E^(2*(c + d*x))] - 6*d*(-1 + E^(2*c))*f*(e + f
*x)*PolyLog[2, E^(2*(c + d*x))] + 3*(-1 + E^(2*c))*f^2*PolyLog[3, E^(2*(c +
 d*x))]))/(12*a*(a^2 + b^2)*d^3*(-1 + E^(2*c))) + (b^2*((2*E^(2*c)*x*(3*e^2
 + 3*e*f*x + f^2*x^2))/(-1 + E^(2*c)) - (3*(d^2*e^2*Log[2*a*E^(c + d*x) + b
*(-1 + E^(2*(c + d*x)))] + 2*d^2*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - S
qrt[(a^2 + b^2)*E^(2*c)])] + d^2*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c -
 Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*d^2*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c
 + Sqrt[(a^2 + b^2)*E^(2*c)])] + d^2*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E
^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(2*c +
 d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2*d*f*(e + f*x)*PolyLog[2, -
((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*f^2*PolyLog[3,
 -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*f^2*PolyLog[
3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/d^3))/(6*a*(
a^2 + b^2)) - (b^2*x*(3*e^2 + 3*e*f*x + f^2*x^2)*Csch[c/2]*Sech[c/2]*Sech[c
])/(24*a*(a^2 + b^2)) + (x*Csch[c/2]*Sech[c/2]*(a^2*e^2 + b^2*e^2 - a^2*e^2
*Cosh[c] - I*a^2*e^2*Sinh[c]))/(8*a*(a^2 + b^2)*(Cosh[c/2] - I*Sinh[c/2])*(
```

```
Cosh[c/2] + I*Sinh[c/2])) + (b^2*e*f*x^2*Cosh[2*c])/(a*(a^2 + b^2)*(-1 + Co
sh[2*c] + Sinh[2*c])*(1 + Cosh[2*c] + Sinh[2*c])) + (b^2*f^2*x^3*Cosh[2*c])
/(3*a*(a^2 + b^2)*(-1 + Cosh[2*c] + Sinh[2*c])*(1 + Cosh[2*c] + Sinh[2*c]))
 + (b^2*e*f*x^2*Sinh[2*c])/(a*(a^2 + b^2)*(-1 + Cosh[2*c] + Sinh[2*c])*(1 +
 Cosh[2*c] + Sinh[2*c])) + (b^2*f^2*x^3*Sinh[2*c])/(3*a*(a^2 + b^2)*(-1 + C
osh[2*c] + Sinh[2*c])*(1 + Cosh[2*c] + Sinh[2*c])) - ((1/2 - I/2)*a*e*f*x^2
*Cosh[c])/((a^2 + b^2)*(-1 - (1 + I)*Cosh[c] - (2*I)*Cosh[2*c] + (1 - I)*Co
sh[3*c] + Cosh[4*c] - (1 + I)*Sinh[c] - (2*I)*Sinh[2*c] + (1 - I)*Sinh[3*c]
 + Sinh[4*c])) + (b*e*f*x^2*Cosh[c])/(2*(a^2 + b^2)*(-1 - (1 + I)*Cosh[c] -
 (2*I)*Cosh[2*c] + (1 - I)*Cosh[3*c] + Cosh[4*c] - (1 + I)*Sinh[c] - (2*I)*
Sinh[2*c] + (1 - I)*Sinh[3*c] + Sinh[4*c])) - ((1/6 - I/6)*a*f^2*x^3*Cosh[c
])/((a^2 + b^2)*(-1 - (1 + I)*Cosh[c] - (2*I)*Cosh[2*c] + (1 - I)*Cosh[3*c]
 + Cosh[4*c] - (1 + I)*Sinh[c] - (2*I)*Sinh[2*c] + (1 - I)*Sinh[3*c] + Sinh
[4*c])) - ((1/2 + I/2)*a*e*f*x^2*Cosh[3*c])/((a^2 + b^2)*(-1 - (1 + I)*Cosh
[c] - (2*I)*Cosh[2*c] + (1 - I)*Cosh[3*c] + Cosh[4*c] - (1 + I)*Sinh[c] - (
2*I)*Sinh[2*c] + (1 - I)*Sinh[3*c] + Sinh[4*c])) - (b*e*f*x^2*Cosh[3*c])/(2
*(a^2 + b^2)*(-1 - (1 + I)*Cosh[c] - (2*I)*Cosh[2*c] + (1 - I)*Cosh[3*c] +
Cosh[4*c] - (1 + I)*Sinh[c] - (2*I)*Sinh[2*c] + (1 - I)*Sinh[3*c] + Sinh[4*
c])) - ((1/6 + I/6)*a*f^2*x^3*Cosh[3*c])/((a^2 + b^2)*(-1 - (1 + I)*Cosh[c]
 - (2*I)*Cosh[2*c] + (1 - I)*Cosh[3*c] + Cosh[4*c] - (1 + I)*Sinh[c] - (2*I
)*Sinh[2*c] + (1 - I)*Sinh[3*c] + Sinh[4*c])) - ((1/2 - I/2)*a*e*f*x^2*Sinh
[c])/((a^2 + b^2)*(-1 - (1 + I)*Cosh[c] - (2*I)*Cosh[2*c] + (1 - I)*Cosh[3*
c] + Cosh[4*c] - (1 + I)*Sinh[c] - (2*I)*Sinh[2*c] + (1 - I)*Sinh[3*c] + Si
nh[4*c])) + (b*e*f*x^2*Sinh[c])/(2*(a^2 + b^2)*(-1 - (1 + I)*Cosh[c] - (2*I
)*Cosh[2*c] + (1 - I)*Cosh[3*c] + Cosh[4*c] - (1 + I)*Sinh[c] - (2*I)*Sinh[
2*c] + (1 - I)*Sinh[3*c] + Sinh[4*c])) - ((1/6 - I/6)*a*f^2*x^3*Sinh[c])/((
a^2 + b^2)*(-1 - (1 + I)*Cosh[c] - (2*I)*Cosh[2...
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c) \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -(b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^3 + a*b^2)*d) - 2
*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(e^(-2*d*x - 2*c) + 1)/((a^2
 + b^2)*d) - log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d))*e^2
 + 2*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*f*e/(a*d^2) + 2*(d*x*
log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))*f*e/(a*d^2) + (d^2*x^2*log(e^(d
*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2/
(a*d^3) + (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*pol
ylog(3, e^(d*x + c)))*f^2/(a*d^3) - 2/3*(d^3*f^2*x^3 + 3*d^3*f*x^2*e)/(a*d^
3) + integrate(2*(b^3*f^2*x^2 + 2*b^3*f*x*e - (a*b^2*f^2*x^2*e^c + 2*a*b^2*
f*x*e^(c + 1))*e^(d*x))/(a^3*b + a*b^3 - (a^3*b*e^(2*c) + a*b^3*e^(2*c))*e^
(2*d*x) - 2*(a^4*e^c + a^2*b^2*e^c)*e^(d*x)), x) - integrate(-2*(a*f^2*x^2
+ 2*a*f*x*e - (b*f^2*x^2*e^c + 2*b*f*x*e^(c + 1))*e^(d*x))/(a^2 + b^2 + (a^
2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2074 vs. $2(689) = 1378$.
time = 0.47, size = 2074, normalized size = 2.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] (2*b^2*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*b^2*f^2*polylog(3, (a*cos
h(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2))/b) - 2*(a^2 + b^2)*f^2*polylog(3, cosh(d*x + c) + sinh(d*x +
 c)) - 2*(a^2 + b^2)*f^2*polylog(3, -cosh(d*x + c) - sinh(d*x + c)) - 2*(b^
2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*dilog((a*cosh(d*x + c) + a*s
inh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
b)/b + 1) - 2*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*dilog((a*co
sh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b + 1) + 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(
1) + (a^2 + b^2)*d*f*sinh(1))*dilog(cosh(d*x + c) + sinh(d*x + c)) - 2*(a^2
*d*f^2*x + I*a*b*d*f^2*x + a^2*d*f*cosh(1) + I*a*b*d*f*cosh(1) + a^2*d*f*si
nh(1) + I*a*b*d*f*sinh(1))*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) - 2*(a^
2*d*f^2*x - I*a*b*d*f^2*x + a^2*d*f*cosh(1) - I*a*b*d*f*cosh(1) + a^2*d*f*s
inh(1) - I*a*b*d*f*sinh(1))*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + 2*(
(a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*di
log(-cosh(d*x + c) - sinh(d*x + c)) - (b^2*c^2*f^2 - 2*b^2*c*d*f*cosh(1) +
b^2*d^2*cosh(1)^2 + b^2*d^2*sinh(1)^2 - 2*(b^2*c*d*f - b^2*d^2*cosh(1))*sin
h(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2)
```

```
 + 2*a) - (b^2*c^2*f^2 - 2*b^2*c*d*f*cosh(1) + b^2*d^2*cosh(1)^2 + b^2*d^2*
sinh(1)^2 - 2*(b^2*c*d*f - b^2*d^2*cosh(1))*sinh(1))*log(2*b*cosh(d*x + c)
+ 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^2*d^2*f^2*x^2 -
 b^2*c^2*f^2 + 2*(b^2*d^2*f*x + b^2*c*d*f)*cosh(1) + 2*(b^2*d^2*f*x + b^2*c
*d*f)*sinh(1))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^2*d^2*f^2*x^2 - b^2*c^
2*f^2 + 2*(b^2*d^2*f*x + b^2*c*d*f)*cosh(1) + 2*(b^2*d^2*f*x + b^2*c*d*f)*s
inh(1))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^
2 + b^2)*d^2*f*x*cosh(1) + (a^2 + b^2)*d^2*cosh(1)^2 + (a^2 + b^2)*d^2*sinh
(1)^2 + 2*((a^2 + b^2)*d^2*f*x + (a^2 + b^2)*d^2*cosh(1))*sinh(1))*log(cosh
(d*x + c) + sinh(d*x + c) + 1) - (a^2*c^2*f^2 + I*a*b*c^2*f^2 - 2*a^2*c*d*f
*cosh(1) - 2*I*a*b*c*d*f*cosh(1) + a^2*d^2*cosh(1)^2 + I*a*b*d^2*cosh(1)^2
+ a^2*d^2*sinh(1)^2 + I*a*b*d^2*sinh(1)^2 - 2*(a^2*c*d*f - a^2*d^2*cosh(1))
*sinh(1) - 2*I*(a*b*c*d*f - a*b*d^2*cosh(1))*sinh(1))*log(cosh(d*x + c) + s
inh(d*x + c) + I) - (a^2*c^2*f^2 - I*a*b*c^2*f^2 - 2*a^2*c*d*f*cosh(1) + 2*
I*a*b*c*d*f*cosh(1) + a^2*d^2*cosh(1)^2 - I*a*b*d^2*cosh(1)^2 + a^2*d^2*sin
h(1)^2 - I*a*b*d^2*sinh(1)^2 - 2*(a^2*c*d*f - a^2*d^2*cosh(1))*sinh(1) + 2*
I*(a*b*c*d*f - a*b*d^2*cosh(1))*sinh(1))*log(cosh(d*x + c) + sinh(d*x + c)
- I) + ((a^2 + b^2)*c^2*f^2 - 2*(a^2 + b^2)*c*d*f*cosh(1) + (a^2 + b^2)*d^2
*cosh(1)^2 + (a^2 + b^2)*d^2*sinh(1)^2 - 2*((a^2 + b^2)*c*d*f - (a^2 + b^2)
*d^2*cosh(1))*sinh(1))*log(cosh(d*x + c) + sinh(d*x + c) - 1) - (a^2*d^2*f^
2*x^2 - I*a*b*d^2*f^2*x^2 - a^2*c^2*f^2 + I*a*b*c^2*f^2 + 2*(a^2*d^2*f*x +
a^2*c*d*f)*cosh(1) - 2*I*(a*b*d^2*f*x + a*b*c*d*f)*cosh(1) + 2*(a^2*d^2*f*x
 + a^2*c*d*f)*sinh(1) - 2*I*(a*b*d^2*f*x + a*b*c*d*f)*sinh(1))*log(I*cosh(d
*x + c) + I*sinh(d*x + c) + 1) - (a^2*d^2*f^2*x^2 + I*a*b*d^2*f^2*x^2 - a^2
*c^2*f^2 - I*a*b*c^2*f^2 + 2*(a^2*d^2*f*x + a^2*c*d*f)*cosh(1) + 2*I*(a*b*d
^2*f*x + a*b*c*d*f)*cosh(1) + 2*(a^2*d^2*f*x + a^2*c*d*f)*sinh(1) + 2*I*(a*
b*d^2*f*x + a*b*c*d*f)*sinh(1))*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1)
 + ((a^2 + b^2)*d^2*f^2*x^2 - (a^2 + b^2)*c^2*f^2 + 2*((a^2 + b^2)*d^2*f*x
+ (a^2 + b^2)*c*d*f)*cosh(1) + 2*((a^2 + b^2)*d^2*f*x + (a^2 + b^2)*c*d*f)*
sinh(1))*log(-cosh(d*x + c) - sinh(d*x + c) + 1) + 2*(a^2*f^2 + I*a*b*f^2)*
polylog(3, I*cosh(d*x + c) + I*sinh(d*x + c)) + 2*(a^2*f^2 - I*a*b*f^2)*pol
ylog(3, -I*cosh(d*x + c) - I*sinh(d*x + c)))/((a^3 + a*b^2)*d^3)
```

Sympy [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3436 deep
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate((f*x + e)^2*csch(d*x + c)*sech(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^2}{\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((e + f*x)^2/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.437 $\int \frac{(e+fx)\mathbf{csch}(c+dx)\mathbf{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=439

$$-\frac{2b(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d}-\frac{2(e+fx)\tanh^{-1}\left(e^{2c+2dx}\right)}{ad}-\frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)d}-\frac{b^2(e+fx)\text{lo}}{a}$$

```
[Out] -2*b*(f*x+e)*arctan(exp(d*x+c))/(a^2+b^2)/d-2*(f*x+e)*arctanh(exp(2*d*x+2*c
))/a/d+b^2*(f*x+e)*ln(1+exp(2*d*x+2*c))/a/(a^2+b^2)/d-b^2*(f*x+e)*ln(1+b*ex
p(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d-b^2*(f*x+e)*ln(1+b*exp(d*x+c)/(
a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)/d+I*b*f*polylog(2,-I*exp(d*x+c))/(a^2+b^2)/
d^2-I*b*f*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^2+1/2*b^2*f*polylog(2,-exp(2*
d*x+2*c))/a/(a^2+b^2)/d^2-1/2*f*polylog(2,-exp(2*d*x+2*c))/a/d^2+1/2*f*poly
log(2,exp(2*d*x+2*c))/a/d^2-b^2*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2
)))/a/(a^2+b^2)/d^2-b^2*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a
^2+b^2)/d^2
```

Rubi [A]
time = 0.48, antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {5708, 5569, 4267, 2317, 2438, 5692, 5680, 2221, 6874, 4265, 3799}

$$-\frac{2b(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{d\left(a^2+b^2\right)}-\frac{b^2f\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\left(a^2+b^2\right)}-\frac{b^2f\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^2\left(a^2+b^2\right)}+\frac{b^2f\text{Li}_2\left(-e^{2(c+dx)}\right)}{2ad^2\left(a^2+b^2\right)}+\frac{ibf\text{Li}_2\left(-ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)}-\frac{ibf\text{Li}_2\left(ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)}-\frac{b^2(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{ad\left(a^2+b^2\right)}-\frac{b^2(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{ad\left(a^2+b^2\right)}+\frac{b^2(e+fx)\log\left(e^{2(c+dx)}+1\right)}{ad\left(a^2+b^2\right)}-\frac{f\text{Li}_2\left(-e^{2c+2dx}\right)}{2ad^2}+\frac{f\text{Li}_2\left(e^{2c+2dx}\right)}{2ad^2}-\frac{2(e+fx)\tanh^{-1}\left(e^{2c+2dx}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*b*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) - (2*(e + f*x)*ArcTanh
[E^(2*c + 2*d*x)])/(a*d) - (b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2])])/(a*(a^2 + b^2)*d) - (b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) + (b^2*(e + f*x)*Log[1 + E^(2*(c +
 d*x))])/(a*(a^2 + b^2)*d) + (I*b*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b
^2)*d^2) - (I*b*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) - (b^2*f*Pol
yLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^2) - (b
^2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d
^2) + (b^2*f*PolyLog[2, -E^(2*(c + d*x))])/(2*a*(a^2 + b^2)*d^2) - (f*PolyL
og[2, -E^(2*c + 2*d*x)])/(2*a*d^2) + (f*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d
^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5708

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{csch}(c+dx)\text{sech}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= \frac{2\int (e+fx)\text{csch}(2c+2dx)\,dx}{a} - \frac{b\int (e+fx)\text{sech}(c+dx)(a-b}{a\left(a^2+b^2\right)}$$

$$= \frac{b^2(e+fx)^2}{2a\left(a^2+b^2\right)f} - \frac{2(e+fx)\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b\int (a(e+fx)\text{sec}}{}$$

$$= \frac{b^2(e+fx)^2}{2a\left(a^2+b^2\right)f} - \frac{2(e+fx)\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(e+fx)\log\left(1\right.}{a\left(a^2\right.}$$

$$= -\frac{2b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} - \frac{2(e+fx)\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(e}{}$$

$$= -\frac{2b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} - \frac{2(e+fx)\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(e}{}$$

$$= -\frac{2b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} - \frac{2(e+fx)\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(e}{}$$

$$= -\frac{2b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} - \frac{2(e+fx)\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(e}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1541 vs. $2(439) = 878$.
time = 1.72, size = 1541, normalized size = 3.51

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] 2*(((-1/4*I)*(a^2 - b^2)*(d*e - c*f)*(c + d*x))/(a*(a^2 + b^2)*d^2) - ((I/8
)*(a^2 - b^2)*f*(c + d*x)^2)/(a*(a^2 + b^2)*d^2) - (e*ArcTanh[1 - (2*I)*Tan
h[(c + d*x)/2]])/((a - I*b)*d) + (I*b*e*ArcTanh[1 - (2*I)*Tanh[(c + d*x)/2]
])/(a*(a - I*b)*d) + (c*f*ArcTanh[1 - (2*I)*Tanh[(c + d*x)/2]])/((a - I*b)*
d^2) - (I*b*c*f*ArcTanh[1 - (2*I)*Tanh[(c + d*x)/2]])/(a*(a - I*b)*d^2) + (
```

```
e*Log[Cosh[(c + d*x)/2]])/(2*a*d) - (c*f*Log[Cosh[(c + d*x)/2]])/(2*a*d^2)
- (e*((-1/2*I)*(c + d*x) + Log[Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]]))/(
2*(a + I*b)*d) + (c*f*((-1/2*I)*(c + d*x) + Log[Cosh[(c + d*x)/2] + I*Sinh[
(c + d*x)/2]]))/(2*(a + I*b)*d^2) - ((I/4)*b*e*((-I)*(c + d*x) + 2*ArcTanh[
1 - (2*I)*Tanh[(c + d*x)/2]] + Log[-1 + Cosh[c + d*x] + I*Sinh[c + d*x]]))/
(a*(a - I*b)*d) + ((I/4)*b*c*f*((-I)*(c + d*x) + 2*ArcTanh[1 - (2*I)*Tanh[(
c + d*x)/2]] + Log[-1 + Cosh[c + d*x] + I*Sinh[c + d*x]]))/(a*(a - I*b)*d^2
) + (I*f*((-1/8*I)*(c + d*x)^2 - (I/2)*(c + d*x)*Log[1 + E^(-c - d*x)] + (I
/2)*PolyLog[2, -E^(-c - d*x)]))/(a*d^2) - ((I/2)*b*f*((-1/2*I)*(c + d*x)^2
+ (I/4)*(3*Pi*(c + d*x) + (1 - I)*(c + d*x)^2 + 2*(Pi - (2*I)*(c + d*x))*Lo
g[1 + I*E^(-c - d*x)] - 4*Pi*Log[1 + E^(c + d*x)] - 2*Pi*Log[-Cos[(Pi + (2*
I)*(c + d*x))/4]] + 4*Pi*Log[Cosh[(c + d*x)/2]] + (4*I)*PolyLog[2, (-I)*E^(
-c - d*x)])))/(a*(a - I*b)*d^2) + ((I/2)*f*((c + d*x)^2/4 + (-3*Pi*(c + d*x
) - (1 - I)*(c + d*x)^2 - 2*(Pi - (2*I)*(c + d*x))*Log[1 + I*E^(-c - d*x)]
+ 4*Pi*Log[1 + E^(c + d*x)] + 2*Pi*Log[-Cos[(Pi + (2*I)*(c + d*x))/4]] - 4*
Pi*Log[Cosh[(c + d*x)/2]] - (4*I)*PolyLog[2, (-I)*E^(-c - d*x)])/4 - (I/2)*
(-1/2*(c + d*x)^2 + 2*(c + d*x)*Log[1 - E^(c + d*x)] + 2*PolyLog[2, E^(c +
d*x)])))/((a - I*b)*d^2) + (b*f*((c + d*x)^2/4 + (-3*Pi*(c + d*x) - (1 - I)
*(c + d*x)^2 - 2*(Pi - (2*I)*(c + d*x))*Log[1 + I*E^(-c - d*x)] + 4*Pi*Log[
1 + E^(c + d*x)] + 2*Pi*Log[-Cos[(Pi + (2*I)*(c + d*x))/4]] - 4*Pi*Log[Cosh
[(c + d*x)/2]] - (4*I)*PolyLog[2, (-I)*E^(-c - d*x)])/4 - (I/2)*(-1/2*(c +
d*x)^2 + 2*(c + d*x)*Log[1 - E^(c + d*x)] + 2*PolyLog[2, E^(c + d*x)])))/(2
*a*(a - I*b)*d^2) - (b^2*(-1/2*(f*(c + d*x)^2) + f*(c + d*x)*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2])] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c +
d*x]] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2,
 -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(2*a*(a^2 + b^2)*d^2) + (I*f*(
(E^((I/4)*Pi)*(c + d*x)^2)/4 - ((Pi*(c + d*x))/4 - Pi*Log[1 + E^(c + d*x)]
- 2*(Pi/4 + (I/2)*(c + d*x))*Log[1 - E^((2*I)*(Pi/4 + (I/2)*(c + d*x)))] +
Pi*Log[Cosh[(c + d*x)/2]] + (Pi*Log[Sin[Pi/4 + (I/2)*(c + d*x)]])/2 + I*Pol
yLog[2, E^((2*I)*(Pi/4 + (I/2)*(c + d*x)))])/Sqrt[2]))/(Sqrt[2]*(a + I*b)*d
^2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1064 vs. $2(411) = 822$.
time = 3.16, size = 1065, normalized size = 2.43

method	result
risch	$\frac{4if\ln\left(1+i\mathrm{e}^{dx+c}\right)bx}{d\left(4a^2+4b^2\right)}+\frac{4if\ln\left(1+i\mathrm{e}^{dx+c}\right)bc}{d^2\left(4a^2+4b^2\right)}-\frac{4if\ln\left(1-i\mathrm{e}^{dx+c}\right)bx}{d\left(4a^2+4b^2\right)}-\frac{4f\,\mathrm{dilog}\left(1+i\mathrm{e}^{dx+c}\right)a}{d^2\left(4a^2+4b^2\right)}-\frac{4f\,\mathrm{dilog}\left(1-i\mathrm{e}^{dx+c}\right)a}{d^2\left(4a^2+4b^2\right)}-\frac{8eb\,\mathrm{arcta}}{d(4a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBO
SE)
```

```
[Out] -4/d^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*a*c-4/d^2*f/(4*a^2+4*b^2)*dilog(1
+I*exp(d*x+c))*a-4/d^2*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))*a-8/d*e/(4*a^2
+4*b^2)*b*arctan(exp(d*x+c))-4/d*e/(4*a^2+4*b^2)*a*ln(1+exp(2*d*x+2*c))-1/d
*f*b^2/a/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)
))*x-1/d^2*f*b^2/a/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+
b^2)^(1/2)))*c-1/d*f*b^2/a/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a
+(a^2+b^2)^(1/2)))*x-1/d^2*f*b^2/a/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/
2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d^2*f/a*dilog(exp(d*x+c)+1)-1/d^2*f*dilog(ex
p(d*x+c))/a-4/d*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*a*x-1/d^2*f*b^2/a/(a^2+b
^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+4/d^2*f*c/(
4*a^2+4*b^2)*a*ln(1+exp(2*d*x+2*c))-1/d^2*f*b^2/a/(a^2+b^2)*dilog((-b*exp(d
*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d*e*b^2/a/(a^2+b^2)*ln(b*e
xp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-4/d*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*a*x-
4/d^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*a*c+1/d^2*f*c*b^2/a/(a^2+b^2)*ln(b
*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+4*I/d*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*
b*x+4*I/d^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*b*c-4*I/d*f/(4*a^2+4*b^2)*ln
(1-I*exp(d*x+c))*b*x-4*I/d^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*b*c+8/d^2*f
*c/(4*a^2+4*b^2)*b*arctan(exp(d*x+c))+4*I/d^2*f/(4*a^2+4*b^2)*dilog(1+I*exp
(d*x+c))*b-4*I/d^2*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))*b+1/d*e/a*ln(exp(d
*x+c)-1)+1/d*e/a*ln(exp(d*x+c)+1)+1/d*f/a*ln(exp(d*x+c)+1)*x-1/d^2*f*c/a*ln
(exp(d*x+c)-1)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] -(b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^3 + a*b^2)*d) - 2
*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(e^(-2*d*x - 2*c) + 1)/((a^2
 + b^2)*d) - log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d))*e +
 4*f*integrate(2*x/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(e^(d*x + c) + e
^(-d*x - c))*(e^(d*x + c) - e^(-d*x - c))), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 888 vs. $2(405) = 810$.
time = 0.42, size = 888, normalized size = 2.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] -(b^2*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sin
h(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + b^2*f*dilog((a*cosh(d*x + c
) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2) - b)/b + 1) - (a^2 + b^2)*f*dilog(cosh(d*x + c) + sinh(d*x + c)) - (a^
2 + b^2)*f*dilog(-cosh(d*x + c) - sinh(d*x + c)) + (a^2*f + I*a*b*f)*dilog(
I*cosh(d*x + c) + I*sinh(d*x + c)) + (a^2*f - I*a*b*f)*dilog(-I*cosh(d*x +
c) - I*sinh(d*x + c)) - (b^2*c*f - b^2*d*cosh(1) - b^2*d*sinh(1))*log(2*b*c
osh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^2*
c*f - b^2*d*cosh(1) - b^2*d*sinh(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
 c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^2*d*f*x + b^2*c*f)*log(-(a*cosh
(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2) - b)/b) + (b^2*d*f*x + b^2*c*f)*log(-(a*cosh(d*x + c) + a*sinh
(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/
b) - ((a^2 + b^2)*d*f*x + (a^2 + b^2)*d*cosh(1) + (a^2 + b^2)*d*sinh(1))*lo
g(cosh(d*x + c) + sinh(d*x + c) + 1) - (a^2*c*f + I*a*b*c*f - a^2*d*cosh(1)
 - I*a*b*d*cosh(1) - a^2*d*sinh(1) - I*a*b*d*sinh(1))*log(cosh(d*x + c) + s
inh(d*x + c) + I) - (a^2*c*f - I*a*b*c*f - a^2*d*cosh(1) + I*a*b*d*cosh(1)
- a^2*d*sinh(1) + I*a*b*d*sinh(1))*log(cosh(d*x + c) + sinh(d*x + c) - I) +
 ((a^2 + b^2)*c*f - (a^2 + b^2)*d*cosh(1) - (a^2 + b^2)*d*sinh(1))*log(cosh
(d*x + c) + sinh(d*x + c) - 1) + (a^2*d*f*x - I*a*b*d*f*x + a^2*c*f - I*a*b
*c*f)*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) + (a^2*d*f*x + I*a*b*d*f*x
 + a^2*c*f + I*a*b*c*f)*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1) - ((a^2
 + b^2)*d*f*x + (a^2 + b^2)*c*f)*log(-cosh(d*x + c) - sinh(d*x + c) + 1))/(
(a^3 + a*b^2)*d^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] integrate((f*x + e)*csch(d*x + c)*sech(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
[Out] int((e + f*x)/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.438 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=90

$$-\frac{b\text{ArcTan}(\sinh(c+dx))}{(a^2+b^2)\,d}-\frac{a\log(\cosh(c+dx))}{(a^2+b^2)\,d}+\frac{\log(\sinh(c+dx))}{ad}-\frac{b^2\log(a+b\sinh(c+dx))}{a\,(a^2+b^2)\,d}$$

```
[Out] -b*arctan(sinh(d*x+c))/(a^2+b^2)/d-a*ln(cosh(d*x+c))/(a^2+b^2)/d+ln(sinh(d*
x+c))/a/d-b^2*ln(a+b*sinh(d*x+c))/a/(a^2+b^2)/d
```

Rubi [A]
time = 0.13, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2916, 12, 908, 649, 209, 266}

$$-\frac{b\text{ArcTan}(\sinh(c+dx))}{d\,(a^2+b^2)}-\frac{b^2\log(a+b\sinh(c+dx))}{ad\,(a^2+b^2)}-\frac{a\log(\cosh(c+dx))}{d\,(a^2+b^2)}+\frac{\log(\sinh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((b*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d)) - (a*Log[Cosh[c + d*x]])/((a^2
 + b^2)*d) + Log[Sinh[c + d*x]]/(a*d) - (b^2*Log[a + b*Sinh[c + d*x]])/(a*(
a^2 + b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{b\text{Subst}\left(\int \frac{b}{x(a+x)(-b^2-x^2)}\,dx, x, b\sinh(c+dx)\right)}{d}\\
&= -\frac{b^2\text{Subst}\left(\int \frac{1}{x(a+x)(-b^2-x^2)}\,dx, x, b\sinh(c+dx)\right)}{d}\\
&= -\frac{b^2\text{Subst}\left(\int \left(-\frac{1}{ab^2x}+\frac{1}{a(a^2+b^2)(a+x)}+\frac{b^2+ax}{b^2(a^2+b^2)(b^2+x^2)}\right) dx, x, b\sinh(c+dx\\
&= \frac{\log(\sinh(c+dx))}{ad}-\frac{b^2\log(a+b\sinh(c+dx))}{a\left(a^2+b^2\right)d}-\frac{\text{Subst}\left(\int \frac{b^2+ax}{b^2+x^2}\,dx, x, b\,\text{s}}{\left(a^2+b^2\right)c}\\
&= \frac{\log(\sinh(c+dx))}{ad}-\frac{b^2\log(a+b\sinh(c+dx))}{a\left(a^2+b^2\right)d}-\frac{a\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b}{\left(a^2+b^2\right)c}\\
&= -\frac{b\tan^{-1}(\sinh(c+dx))}{\left(a^2+b^2\right)d}-\frac{a\log(\cosh(c+dx))}{\left(a^2+b^2\right)d}+\frac{\log(\sinh(c+dx))}{ad}-\frac{b^2}{}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.13, size = 92, normalized size = 1.02

$$-\frac{\frac{\log(i-\sinh(c+dx))}{a+ib}-\frac{2\log(\sinh(c+dx))}{a}+\frac{\log(i+\sinh(c+dx))}{a-ib}+\frac{2b^2\log(a+b\sinh(c+dx))}{a(a^2+b^2)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/2*(Log[I - Sinh[c + d*x]]/(a + I*b) - (2*Log[Sinh[c + d*x]])/a + Log[I +
 Sinh[c + d*x]]/(a - I*b) + (2*b^2*Log[a + b*Sinh[c + d*x]])/(a*(a^2 + b^2)
))/d
```

Maple [A]
time = 1.34, size = 108, normalized size = 1.20

method	result
derivativedivides	$\frac{\frac{-a\ln\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-2b\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2+b^2}-\frac{b^2\ln\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^2+b^2\right)a}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
default	$\frac{\frac{-a\ln\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-2b\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2+b^2}-\frac{b^2\ln\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^2+b^2\right)a}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
risch	$\frac{2a\,d^2x}{a^2d^2+b^2d^2}+\frac{2adc}{a^2d^2+b^2d^2}-\frac{2x}{a}-\frac{2c}{ad}+\frac{2b^2x}{a(a^2+b^2)}+\frac{2b^2c}{ad(a^2+b^2)}+\frac{i\ln(\mathrm{e}^{dx+c}-i)b}{(a^2+b^2)d}-\frac{\ln(\mathrm{e}^{dx+c}-i)a}{(a^2+b^2)d}-\frac{i\ln(\mathrm{e}^{dx}}{(a^2+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)*(-a*ln(tanh(1/2*d*x+1/2*c)^2+1)-2*b*arctan(tanh(1/2*d*x+1/
2*c)))-b^2/(a^2+b^2)/a*ln(a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)-a
)+1/a*ln(tanh(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.47, size = 138, normalized size = 1.53

$$-\frac{b^2\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^3+ab^2)d}+\frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d}-\frac{a\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}+\frac{\log\left(e^{(-dx-c)}-1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^3 + a*b^2)*d) + 2*
b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a^2
+ b^2)*d) + log(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d)
```

Fricas [A]
time = 0.42, size = 134, normalized size = 1.49

$$-\frac{2\,ab\arctan\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)+b^2\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)+a^2\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)-(a^2+b^2)\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{(a^3+ab^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*a*b*arctan(cosh(d*x + c) + sinh(d*x + c)) + b^2*log(2*(b*sinh(d*x + c)
+ a)/(cosh(d*x + c) - sinh(d*x + c))) + a^2*log(2*cosh(d*x + c)/(cosh(d*x +
```

```
 c) - sinh(d*x + c))) - (a^2 + b^2)*log(2*sinh(d*x + c)/(cosh(d*x + c) - si
nh(d*x + c))))/((a^3 + a*b^2)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(csch(c + d*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A]
time = 0.47, size = 147, normalized size = 1.63

$$-\frac{\frac{2\,b^3\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^3b+ab^3}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)b}{a^2+b^2}+\frac{a\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^2+b^2}-\frac{2\,\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^3*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^3*b + a*b^3)
+ (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*b/(a^2 + b^2) + a
*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2 + b^2) - 2*log(abs(e^(d*x + c
) - e^(-d*x - c)))/a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(1/(cosh(c + d*x)*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.439 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=35

$$\operatorname{Int}\left(\frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csch[c + d*x]*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Csch[c + d*x]*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 19.71, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csch[c + d*x]*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x
]
```

```
[Out] Integrate[(Csch[c + d*x]*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)}{(fx+e)\left(a+b\sinh\left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] integrate(csch(d*x + c)*sech(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] integral(csch(d*x + c)*sech(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
+ c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c + dx)\operatorname{sech}(c + dx)}{(a + b\sinh(c + dx))(e + fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(csch(c + d*x)*sech(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] integrate(csch(d*x + c)*sech(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(1/(cosh(c + d*x)*sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.440 $\int \frac{(e+fx)^3\mathbf{csch}(c+dx)\mathbf{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1164

$$-\frac{b(e+fx)^3}{(a^2+b^2)\,d}-\frac{6f(e+fx)^2\mathrm{ArcTan}\left(e^{c+dx}\right)}{ad^2}+\frac{6b^2f(e+fx)^2\mathrm{ArcTan}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d^2}-\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{b^3(\cdots}{\cdots}$$

```
[Out] 6*I*b^2*f^2*(f*x+e)*polylog(2,I*exp(d*x+c))/a/(a^2+b^2)/d^3-3*f*(f*x+e)^2*p
olylog(2,-exp(d*x+c))/a/d^2+3*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a/d^2+6*f^2
*(f*x+e)*polylog(3,-exp(d*x+c))/a/d^3-6*f*(f*x+e)^2*arctan(exp(d*x+c))/a/d^
2-6*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a/d^3-b*(f*x+e)^3/(a^2+b^2)/d+(f*x+e)
^3*sech(d*x+c)/a/d+6*I*b^2*f^3*polylog(3,-I*exp(d*x+c))/a/(a^2+b^2)/d^4-6*I
*b^2*f^2*(f*x+e)*polylog(2,-I*exp(d*x+c))/a/(a^2+b^2)/d^3-2*(f*x+e)^3*arcta
nh(exp(d*x+c))/a/d-6*f^3*polylog(4,-exp(d*x+c))/a/d^4+6*f^3*polylog(4,exp(d
*x+c))/a/d^4+3*b*f^2*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)/d^3+6*I*f
^3*polylog(3,I*exp(d*x+c))/a/d^4-b^2*(f*x+e)^3*sech(d*x+c)/a/(a^2+b^2)/d-6*
I*f^2*(f*x+e)*polylog(2,I*exp(d*x+c))/a/d^3-3*b^3*f*(f*x+e)^2*polylog(2,-b*
exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d^2+3*b^3*f*(f*x+e)^2*pol
ylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d^2+6*b^3*f^2*(
f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d^3-6
*b^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(
3/2)/d^3-3/2*b*f^3*polylog(3,-exp(2*d*x+2*c))/(a^2+b^2)/d^4-b*(f*x+e)^3*tan
h(d*x+c)/(a^2+b^2)/d-6*I*f^3*polylog(3,-I*exp(d*x+c))/a/d^4+6*b^2*f*(f*x+e)
^2*arctan(exp(d*x+c))/a/(a^2+b^2)/d^2+6*I*f^2*(f*x+e)*polylog(2,-I*exp(d*x+
c))/a/d^3-6*I*b^2*f^3*polylog(3,I*exp(d*x+c))/a/(a^2+b^2)/d^4+3*b*f*(f*x+e)
^2*ln(1+exp(2*d*x+2*c))/(a^2+b^2)/d^2-b^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a
^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d+b^3*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2
+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d-6*b^3*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2
+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d^4+6*b^3*f^3*polylog(4,-b*exp(d*x+c)/(a+(a
^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d^4
```

Rubi [A]
time = 1.76, antiderivative size = 1164, normalized size of antiderivative = 1.00, number of steps used = 53, number of rules used = 22, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {5708, 2702, 327, 213, 5570, 6873, 12, 6874, 6408, 4267, 2611, 6744, 2320, 6724, 4265, 5692, 3403, 2296, 2221, 4269, 3799, 5559}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((b*(e + f*x)^3)/((a^2 + b^2)*d)) - (6*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/
(a*d^2) + (6*b^2*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/(a*(a^2 + b^2)*d^2) - (
```

```
2*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) - (b^3*(e + f*x)^3*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^(3/2)*d) + (b^3*(e + f*x)^
3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^(3/2)*d) +
 (3*b*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)*d^2) - (3*f*(e +
 f*x)^2*PolyLog[2, -E^(c + d*x)])/(a*d^2) + ((6*I)*f^2*(e + f*x)*PolyLog[2,
 (-I)*E^(c + d*x)])/(a*d^3) - ((6*I)*b^2*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c
 + d*x)])/(a*(a^2 + b^2)*d^3) - ((6*I)*f^2*(e + f*x)*PolyLog[2, I*E^(c + d*
x)])/(a*d^3) + ((6*I)*b^2*f^2*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(a*(a^2
+ b^2)*d^3) + (3*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)])/(a*d^2) - (3*b^3*f*
(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 +
 b^2)^(3/2)*d^2) + (3*b^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^2) + (3*b*f^2*(e + f*x)*PolyLog[2
, -E^(2*(c + d*x))])/((a^2 + b^2)*d^3) + (6*f^2*(e + f*x)*PolyLog[3, -E^(c
+ d*x)])/(a*d^3) - ((6*I)*f^3*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^4) + ((6*I
)*b^2*f^3*PolyLog[3, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^4) + ((6*I)*f^3*Po
lyLog[3, I*E^(c + d*x)])/(a*d^4) - ((6*I)*b^2*f^3*PolyLog[3, I*E^(c + d*x)]
)/(a*(a^2 + b^2)*d^4) - (6*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a*d^3) +
 (6*b^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])
/(a*(a^2 + b^2)^(3/2)*d^3) - (6*b^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^3) - (3*b*f^3*PolyLog[3
, -E^(2*(c + d*x))])/(2*(a^2 + b^2)*d^4) - (6*f^3*PolyLog[4, -E^(c + d*x)])
/(a*d^4) + (6*f^3*PolyLog[4, E^(c + d*x)])/(a*d^4) - (6*b^3*f^3*PolyLog[4,
-((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^4) + (6*b
^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)
^(3/2)*d^4) + ((e + f*x)^3*Sech[c + d*x])/(a*d) - (b^2*(e + f*x)^3*Sech[c +
 d*x])/(a*(a^2 + b^2)*d) - (b*(e + f*x)^3*Tanh[c + d*x])/((a^2 + b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5708

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\text{csch}(c+dx)\text{sech}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\text{sech}^2(c+\ldots}{a+b\sinh(c+dx)}}{a}$$

$$= -\frac{(e+fx)^3\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(e+fx)^3\text{sech}(c+dx)}{ad} - $$

$$= -\frac{(e+fx)^3\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(e+fx)^3\text{sech}(c+dx)}{ad} - $$

$$= -\frac{(e+fx)^3\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(e+fx)^3\text{sech}(c+dx)}{ad} - $$

$$= -\frac{(e+fx)^3\tanh^{-1}(\cosh(c+dx))}{ad} - \frac{b^3(e+fx)^3\log\left(1+\frac{}{a-\sqrt{}}\right.}{a\left(a^2+b^2\right)^{3/2}d}$$

$$= -\frac{b(e+fx)^3}{(a^2+b^2)\,d} + \frac{6b^2f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d^2} - \frac{(e+fx)^3\tanh}{}$$

$$= -\frac{b(e+fx)^3}{(a^2+b^2)\,d} - \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{6b^2f(e+fx)^2\tan}{a\left(a^2+b^2\right.}$$

$$= -\frac{b(e+fx)^3}{(a^2+b^2)\,d} - \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{6b^2f(e+fx)^2\tan}{a\left(a^2+b^2\right.}$$

$$= -\frac{b(e+fx)^3}{(a^2+b^2)\,d} - \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{6b^2f(e+fx)^2\tan}{a\left(a^2+b^2\right.}$$

$$= -\frac{b(e+fx)^3}{(a^2+b^2)\,d} - \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{6b^2f(e+fx)^2\tan}{a\left(a^2+b^2\right.}$$

$$= -\frac{b(e+fx)^3}{(a^2+b^2)\,d} - \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{6b^2f(e+fx)^2\tan}{a\left(a^2+b^2\right.}$$

$$= -\frac{b(e+fx)^3}{(a^2+b^2)\,d} - \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{6b^2f(e+fx)^2\tan}{a\left(a^2+b^2\right.}$$

$$= -\frac{b(e+fx)^3}{(a^2+b^2)\,d} - \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{6b^2f(e+fx)^2\tan}{a\left(a^2+b^2\right.}$$

Mathematica [A]
time = 16.32, size = 1991, normalized size = 1.71

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] 4*((f*Csch[c + d*x]*((-4*b*d^3*E^(2*c)*x*(3*e^2 + 3*e*f*x + f^2*x^2))/(1 +
E^(2*c)) + 3*(-4*a*d^2*e^2*ArcTan[E^(c + d*x)] - (4*I)*a*d^2*e*f*x*Log[1 -
I*E^(c + d*x)] - (2*I)*a*d^2*f^2*x^2*Log[1 - I*E^(c + d*x)] + (4*I)*a*d^2*e
*f*x*Log[1 + I*E^(c + d*x)] + (2*I)*a*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] +
2*b*d^2*e^2*Log[1 + E^(2*(c + d*x))] + 4*b*d^2*e*f*x*Log[1 + E^(2*(c + d*x)
)] + 2*b*d^2*f^2*x^2*Log[1 + E^(2*(c + d*x))] + (4*I)*a*d*f*(e + f*x)*PolyL
og[2, (-I)*E^(c + d*x)] - (4*I)*a*d*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)] +
 2*b*d*e*f*PolyLog[2, -E^(2*(c + d*x))] + 2*b*d*f^2*x*PolyLog[2, -E^(2*(c +
 d*x))] - (4*I)*a*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (4*I)*a*f^2*PolyLog[3,
 I*E^(c + d*x)] - b*f^2*PolyLog[3, -E^(2*(c + d*x))]))*(a + b*Sinh[c + d*x]
))/(8*(a^2 + b^2)*d^4*(b + a*Csch[c + d*x])) + (Csch[c + d*x]*(d^3*e^3*Log[
1 - E^(c + d*x)] + 3*d^3*e^2*f*x*Log[1 - E^(c + d*x)] + 3*d^3*e*f^2*x^2*Log
[1 - E^(c + d*x)] + d^3*f^3*x^3*Log[1 - E^(c + d*x)] - d^3*e^3*Log[1 + E^(c
 + d*x)] - 3*d^3*e^2*f*x*Log[1 + E^(c + d*x)] - 3*d^3*e*f^2*x^2*Log[1 + E^(
c + d*x)] - d^3*f^3*x^3*Log[1 + E^(c + d*x)] - 3*d^2*f*(e + f*x)^2*PolyLog[
2, -E^(c + d*x)] + 3*d^2*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)] + 6*d*e*f^2*
PolyLog[3, -E^(c + d*x)] + 6*d*f^3*x*PolyLog[3, -E^(c + d*x)] - 6*d*e*f^2*P
olyLog[3, E^(c + d*x)] - 6*d*f^3*x*PolyLog[3, E^(c + d*x)] - 6*f^3*PolyLog[
4, -E^(c + d*x)] + 6*f^3*PolyLog[4, E^(c + d*x)])*(a + b*Sinh[c + d*x]))/(4
*a*d^4*(b + a*Csch[c + d*x])) + (b^3*Csch[c + d*x]*(2*d^3*e^3*Sqrt[(a^2 + b
^2)*E^(2*c)]*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]] + 3*Sqrt[-a^2 - b
^2]*d^3*e^2*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(
2*c)])] + 3*Sqrt[-a^2 - b^2]*d^3*e*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a
*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + Sqrt[-a^2 - b^2]*d^3*E^c*f^3*x^3*Log[1
 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - 3*Sqrt[-a^2 - b
^2]*d^3*e^2*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(
2*c)])] - 3*Sqrt[-a^2 - b^2]*d^3*e*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a
*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - Sqrt[-a^2 - b^2]*d^3*E^c*f^3*x^3*Log[1
 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 3*Sqrt[-a^2 - b
^2]*d^2*E^c*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2
 + b^2)*E^(2*c)]))] - 3*Sqrt[-a^2 - b^2]*d^2*E^c*f*(e + f*x)^2*PolyLog[2, -
((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[-a^2 - b^
2]*d*e*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(
2*c)]))] - 6*Sqrt[-a^2 - b^2]*d*E^c*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a
*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[-a^2 - b^2]*d*e*E^c*f^2*PolyLo
g[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[-a^
```

```
2 - b^2]*d*E^c*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^
2)*E^(2*c)]))] + 6*Sqrt[-a^2 - b^2]*E^c*f^3*PolyLog[4, -((b*E^(2*c + d*x))/
(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[-a^2 - b^2]*E^c*f^3*PolyLog[
4, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])*(a + b*Sinh[c
 + d*x]))/(4*a*(-a^2 - b^2)^(3/2)*d^4*Sqrt[(a^2 + b^2)*E^(2*c)]*(b + a*Csch
[c + d*x])) + (Csch[c + d*x]*Sech[c]*Sech[c + d*x]*(a*e^3*Cosh[c] + 3*a*e^2
*f*x*Cosh[c] + 3*a*e*f^2*x^2*Cosh[c] + a*f^3*x^3*Cosh[c] - b*e^3*Sinh[d*x]
- 3*b*e^2*f*x*Sinh[d*x] - 3*b*e*f^2*x^2*Sinh[d*x] - b*f^3*x^3*Sinh[d*x])*(a
 + b*Sinh[c + d*x]))/(4*(a^2 + b^2)*d*(b + a*Csch[c + d*x])))
```

Maple [F]
time = 2.72, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c) \operatorname{sech}(dx+c)^2}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -6*a*f^3*integrate(x^2*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x
+ 2*c) + a^2*d + b^2*d), x) - 6*b*f^3*integrate(x^2/(a^2*d*e^(2*d*x + 2*c)
+ b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 12*b*f^2*e*integrate(x/(a^2*
d*e^(2*d*x + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 3*b*f*(2*(
d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 + b^2)*d^2))*e^
2 - 12*a*f^2*integrate(x*e^(d*x + c + 1)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*e^(
2*d*x + 2*c) + a^2*d + b^2*d), x) - (b^3*log((b*e^(-d*x - c) - a - sqrt(a^2
 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^3 + a*b^2)*sqrt(a^2 +
b^2)*d) - 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c)
)*d) + log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d))*e^3 - 6*a
*f*arctan(e^(d*x + c))*e^2/((a^2 + b^2)*d^2) + 2*(b*f^3*x^3 + 3*b*f^2*x^2*e
 + 3*b*f*x*e^2 + (a*f^3*x^3*e^c + 3*a*f^2*x^2*e^(c + 1) + 3*a*f*x*e^(c + 2)
)*e^(d*x))/(a^2*d + b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) - 3*
(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*f*e^2/(a*d^2) + 3*(d*x*log
(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))*f*e^2/(a*d^2) - 3*(d^2*x^2*log(e^(
d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2
```

```
*e/(a*d^3) + 3*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) -
2*polylog(3, e^(d*x + c)))*f^2*e/(a*d^3) - (d^3*x^3*log(e^(d*x + c) + 1) +
3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(
4, -e^(d*x + c)))*f^3/(a*d^4) + (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*
dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x +
c)))*f^3/(a*d^4) - integrate(-2*(b^3*f^3*x^3*e^c + 3*b^3*f^2*x^2*e^(c + 1)
+ 3*b^3*f*x*e^(c + 2))*e^(d*x)/(a^3*b + a*b^3 - (a^3*b*e^(2*c) + a*b^3*e^(2
*c))*e^(2*d*x) - 2*(a^4*e^c + a^2*b^2*e^c)*e^(d*x)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 16781 vs. $2(1087) = 2174$.
time = 0.72, size = 16781, normalized size = 14.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -(2*(a^3*b + a*b^3)*c^3*f^3 - 6*(a^3*b + a*b^3)*c^2*d*f^2*cosh(1) + 6*(a^3*
b + a*b^3)*c*d^2*f*cosh(1)^2 - 2*(a^3*b + a*b^3)*d^3*cosh(1)^3 - 2*(a^3*b +
 a*b^3)*d^3*sinh(1)^3 + 2*((a^3*b + a*b^3)*d^3*f^3*x^3 + (a^3*b + a*b^3)*c^
3*f^3 + 3*((a^3*b + a*b^3)*d^3*f*x + (a^3*b + a*b^3)*c*d^2*f)*cosh(1)^2 + 3
*((a^3*b + a*b^3)*d^3*f*x + (a^3*b + a*b^3)*c*d^2*f)*sinh(1)^2 + 3*((a^3*b
+ a*b^3)*d^3*f^2*x^2 - (a^3*b + a*b^3)*c^2*d*f^2)*cosh(1) + 3*((a^3*b + a*b
^3)*d^3*f^2*x^2 - (a^3*b + a*b^3)*c^2*d*f^2 + 2*((a^3*b + a*b^3)*d^3*f*x +
(a^3*b + a*b^3)*c*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 6*((a^3*b + a*
b^3)*c*d^2*f - (a^3*b + a*b^3)*d^3*cosh(1))*sinh(1)^2 + 2*((a^3*b + a*b^3)*
d^3*f^3*x^3 + (a^3*b + a*b^3)*c^3*f^3 + 3*((a^3*b + a*b^3)*d^3*f*x + (a^3*b
 + a*b^3)*c*d^2*f)*cosh(1)^2 + 3*((a^3*b + a*b^3)*d^3*f*x + (a^3*b + a*b^3)
*c*d^2*f)*sinh(1)^2 + 3*((a^3*b + a*b^3)*d^3*f^2*x^2 - (a^3*b + a*b^3)*c^2*
d*f^2)*cosh(1) + 3*((a^3*b + a*b^3)*d^3*f^2*x^2 - (a^3*b + a*b^3)*c^2*d*f^2
 + 2*((a^3*b + a*b^3)*d^3*f*x + (a^3*b + a*b^3)*c*d^2*f)*cosh(1))*sinh(1))*
sinh(d*x + c)^2 + 3*(b^4*d^2*f^3*x^2 + 2*b^4*d^2*f^2*x*cosh(1) + b^4*d^2*f*
cosh(1)^2 + b^4*d^2*f*sinh(1)^2 + (b^4*d^2*f^3*x^2 + 2*b^4*d^2*f^2*x*cosh(1
) + b^4*d^2*f*cosh(1)^2 + b^4*d^2*f*sinh(1)^2 + 2*(b^4*d^2*f^2*x + b^4*d^2*
f*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(b^4*d^2*f^3*x^2 + 2*b^4*d^2*f^2*x*
cosh(1) + b^4*d^2*f*cosh(1)^2 + b^4*d^2*f*sinh(1)^2 + 2*(b^4*d^2*f^2*x + b^
4*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (b^4*d^2*f^3*x^2 +
2*b^4*d^2*f^2*x*cosh(1) + b^4*d^2*f*cosh(1)^2 + b^4*d^2*f*sinh(1)^2 + 2*(b^
4*d^2*f^2*x + b^4*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c)^2 + 2*(b^4*d^2*f^2*
x + b^4*d^2*f*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c
) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2) - b)/b + 1) - 3*(b^4*d^2*f^3*x^2 + 2*b^4*d^2*f^2*x*cosh(1) + b^4*d^2*f
*cosh(1)^2 + b^4*d^2*f*sinh(1)^2 + (b^4*d^2*f^3*x^2 + 2*b^4*d^2*f^2*x*cosh(
```

```
1) + b^4*d^2*f*cosh(1)^2 + b^4*d^2*f*sinh(1)^2 + 2*(b^4*d^2*f^2*x + b^4*d^2
*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(b^4*d^2*f^3*x^2 + 2*b^4*d^2*f^2*x
*cosh(1) + b^4*d^2*f*cosh(1)^2 + b^4*d^2*f*sinh(1)^2 + 2*(b^4*d^2*f^2*x + b
^4*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (b^4*d^2*f^3*x^2 +
 2*b^4*d^2*f^2*x*cosh(1) + b^4*d^2*f*cosh(1)^2 + b^4*d^2*f*sinh(1)^2 + 2*(b
^4*d^2*f^2*x + b^4*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c)^2 + 2*(b^4*d^2*f^2
*x + b^4*d^2*f*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x +
c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2) - b)/b + 1) + (b^4*c^3*f^3 - 3*b^4*c^2*d*f^2*cosh(1) + 3*b^4*c*d^2*f*
cosh(1)^2 - b^4*d^3*cosh(1)^3 - b^4*d^3*sinh(1)^3 + (b^4*c^3*f^3 - 3*b^4*c^
2*d*f^2*cosh(1) + 3*b^4*c*d^2*f*cosh(1)^2 - b^4*d^3*cosh(1)^3 - b^4*d^3*sin
h(1)^3 + 3*(b^4*c*d^2*f - b^4*d^3*cosh(1))*sinh(1)^2 - 3*(b^4*c^2*d*f^2 - 2
*b^4*c*d^2*f*cosh(1) + b^4*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c)^2 + 3*(b^4
*c*d^2*f - b^4*d^3*cosh(1))*sinh(1)^2 + 2*(b^4*c^3*f^3 - 3*b^4*c^2*d*f^2*co
sh(1) + 3*b^4*c*d^2*f*cosh(1)^2 - b^4*d^3*cosh(1)^3 - b^4*d^3*sinh(1)^3 + 3
*(b^4*c*d^2*f - b^4*d^3*cosh(1))*sinh(1)^2 - 3*(b^4*c^2*d*f^2 - 2*b^4*c*d^2
*f*cosh(1) + b^4*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (b^4
*c^3*f^3 - 3*b^4*c^2*d*f^2*cosh(1) + 3*b^4*c*d^2*f*cosh(1)^2 - b^4*d^3*cosh
(1)^3 - b^4*d^3*sinh(1)^3 + 3*(b^4*c*d^2*f - b^4*d^3*cosh(1))*sinh(1)^2 - 3
*(b^4*c^2*d*f^2 - 2*b^4*c*d^2*f*cosh(1) + b^4*d^3*cosh(1)^2)*sinh(1))*sinh(
d*x + c)^2 - 3*(b^4*c^2*d*f^2 - 2*b^4*c*d^2*f*cosh(1) + b^4*d^3*cosh(1)^2)*
sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) +
2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^4*c^3*f^3 - 3*b^4*c^2*d*f^2*cosh(1) +
 3*b^4*c*d^2*f*cosh(1)^2 - b^4*d^3*cosh(1)^3 - b^4*d^3*sinh(1)^3 + (b^4*c^3
*f^3 - 3*b^4*c^2*d*f^2*cosh(1) + 3*b^4*c*d^2*f*cosh(1)^2 - b^4*d^3*cosh(1)^
3 - b^4*d^3*sinh(1)^3 + 3*(b^4*c*d^2*f - b^4*d^3*cosh(1))*sinh(1)^2 - 3*(b^
4*c^2*d*f^2 - 2*b^4*c*d^2*f*cosh(1) + b^4*d^3*cosh(1)^2)*sinh(1))*cosh(d*x
+ c)^2 + 3*(b^4*c*d^2*f - b^4*d^3*cosh(1))*sinh(1)^2 + 2*(b^4*c^3*f^3 - 3*b
^4*c^2*d*f^2*cosh(1) + 3*b^4*c*d^2*f*cosh(1)^2 - b^4*d^3*cosh(1)^3 - b^4*d^
3*sinh(1)^3 + 3*(b^4*c*d^2*f - b^4*d^3*cosh(1))*sinh(1)^2 - 3*(b^4*c^2*d*f^
2 - 2*b^4*c*d^2*f*cosh(1) + b^4*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c)*sinh(
d*x + c) + (b^4*c^3*f^3 - 3*b^4*c^2*d*f^2*cosh(1) + 3*b^4*c*d^2*f*cosh(1)^2
 - b^4*d^3*cosh(1)^3 - b^4*d^3*sinh(1)^3 + 3*(b^4*c*d^2*f - b^4*d^3*cosh(1)
)*sinh(1)^2 - 3*(b^4*c^2*d*f^2 - 2*b^4*c*d^2*f*cosh(1) + b^4*d^3*cosh(1)^2)
*sinh(1))*sinh(d*x + c)^2 - 3*(b^4*c^2*d*f^2 - 2*b^4*c*d^2*f*cosh(1) + b^4*
d^3*cosh(1)^2)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*s
inh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^4*d^3*f^3*x^3 + b^4*c^
3*f^3 + 3*(b^4*d^3*f*x + b^4*c*d^2*f)*cosh(1)^2 + (b^4*d^3*f^3*x^3 + b^4*c^
3*f^3 + 3*(b^4*d^3*f*x + b^4*c*d^2*f)*cosh(1)^2 + 3*(b^4*d^3*f*x + b^4*c*d^
2*f)*sinh(1)^2 + 3*(b^4*d^3*f^2*x^2 - b^4*c^2*d*f^2)*cosh(1) + 3*(b^4*d^3*f
^2*x^2 - b^4*c^2*d*f^2 + 2*(b^4*d^3*f*x + b^4*c...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^3}{\cosh\left(c+d\,x\right)^2\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(cosh(c + d*x)^2*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((e + f*x)^3/(cosh(c + d*x)^2*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.441 $\int \frac{(e+fx)^2\mathbf{csch}(c+dx)\mathbf{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=795

$$-\frac{b(e+fx)^2}{(a^2+b^2)\,d}-\frac{4f(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{ad^2}+\frac{4b^2f(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{a\,(a^2+b^2)\,d^2}-\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{b^3(e\ \cdots}{}$$

[Out] -b*(f*x+e)^2/(a^2+b^2)/d-4*f*(f*x+e)*arctan(exp(d*x+c))/a/d^2+4*b^2*f*(f*x+e)*arctan(exp(d*x+c))/a/(a^2+b^2)/d^2-2*(f*x+e)^2*arctanh(exp(d*x+c))/a/d+2*b*f*(f*x+e)*ln(1+exp(2*d*x+2*c))/(a^2+b^2)/d^2-b^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d+b^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d-2*f*(f*x+e)*polylog(2,-exp(d*x+c))/a/d^2-2*I*b^2*f^2*polylog(2,-I*exp(d*x+c))/a/(a^2+b^2)/d^3-2*I*f^2*polylog(2,I*exp(d*x+c))/a/d^3+2*I*b^2*f^2*polylog(2,I*exp(d*x+c))/a/(a^2+b^2)/d^3+2*I*f^2*polylog(2,-I*exp(d*x+c))/a/d^3+2*f*(f*x+e)*polylog(2,exp(d*x+c))/a/d^2+b*f^2*polylog(2,-exp(2*d*x+2*c))/(a^2+b^2)/d^3-2*b^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d^2+2*b^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d^2+2*f^2*polylog(3,-exp(d*x+c))/a/d^3-2*f^2*polylog(3,exp(d*x+c))/a/d^3+2*b^3*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d^3-2*b^3*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d^3+(f*x+e)^2*sech(d*x+c)/a/d-b^2*(f*x+e)^2*sech(d*x+c)/a/(a^2+b^2)/d-b*(f*x+e)^2*tanh(d*x+c)/(a^2+b^2)/d

Rubi [A]
time = 1.28, antiderivative size = 795, normalized size of antiderivative = 1.00, number of steps used = 44, number of rules used = 23, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.676$, Rules used = {5708, 2702, 327, 213, 5570, 6873, 12, 6874, 6408, 4267, 2611, 2320, 6724, 4265, 2317, 2438, 5692, 3403, 2296, 2221, 4269, 3799, 5559}

Antiderivative was successfully verified.

[In] Int[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]

[Out] -((b*(e + f*x)^2)/((a^2 + b^2)*d)) - (4*f*(e + f*x)*ArcTan[E^(c + d*x)])/(a*d^2) + (4*b^2*f*(e + f*x)*ArcTan[E^(c + d*x)])/(a*(a^2 + b^2)*d^2) - (2*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^(3/2)*d) + (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^(3/2)*d) + (2*b*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)*d^2) - (2*f*(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) + ((2*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^3) - ((2*I)*b^2*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^3

```
) - ((2*I)*f^2*PolyLog[2, I*E^(c + d*x)])/(a*d^3) + ((2*I)*b^2*f^2*PolyLog[
2, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^3) + (2*f*(e + f*x)*PolyLog[2, E^(c + d
*x)])/(a*d^2) - (2*b^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^2) + (2*b^3*f*(e + f*x)*PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^2) + (b*f^2*
PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^2)*d^3) + (2*f^2*PolyLog[3, -E^(c +
 d*x)])/(a*d^3) - (2*f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) + (2*b^3*f^2*Poly
Log[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^3)
 - (2*b^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2
 + b^2)^(3/2)*d^3) + ((e + f*x)^2*Sech[c + d*x])/(a*d) - (b^2*(e + f*x)^2*S
ech[c + d*x])/(a*(a^2 + b^2)*d) - (b*(e + f*x)^2*Tanh[c + d*x])/((a^2 + b^2
)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
```

```
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
```

```
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5708

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}}{a}$$

$$= -\frac{(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(e+fx)^2\text{sech}(c+dx)}{ad} - \frac{b}{}$$

$$= -\frac{(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(e+fx)^2\text{sech}(c+dx)}{ad} - \frac{b}{}$$

$$= -\frac{(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(e+fx)^2\text{sech}(c+dx)}{ad} - \frac{(}{}$$

$$= -\frac{(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{ad} - \frac{b^3(e+fx)^2\log\left(1+\frac{be}{a-\sqrt{a}}\right.}{a\left(a^2+b^2\right)^{3/2}d}$$

$$= -\frac{b(e+fx)^2}{(a^2+b^2)\,d} + \frac{4b^2f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d^2} - \frac{(e+fx)^2\tanh^{-1}(}{ad}$$

$$= -\frac{b(e+fx)^2}{(a^2+b^2)\,d} - \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{4b^2f(e+fx)\tan^{-1}}{a\left(a^2+b^2\right)d}$$

$$= -\frac{b(e+fx)^2}{(a^2+b^2)\,d} - \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{4b^2f(e+fx)\tan^{-1}}{a\left(a^2+b^2\right)d}$$

$$= -\frac{b(e+fx)^2}{(a^2+b^2)\,d} - \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{4b^2f(e+fx)\tan^{-1}}{a\left(a^2+b^2\right)d}$$

$$= -\frac{b(e+fx)^2}{(a^2+b^2)\,d} - \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{4b^2f(e+fx)\tan^{-1}}{a\left(a^2+b^2\right)d}$$

$$= -\frac{b(e+fx)^2}{(a^2+b^2)\,d} - \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{4b^2f(e+fx)\tan^{-1}}{a\left(a^2+b^2\right)d}$$

$$= -\frac{b(e+fx)^2}{(a^2+b^2)\,d} - \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{4b^2f(e+fx)\tan^{-1}}{a\left(a^2+b^2\right)d}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1592 vs. 2(795) = 1590.
time = 11.56, size = 1592, normalized size = 2.00

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] 4*((Csch[c + d*x]*(d^2*e^2*Log[1 - E^(c + d*x)] + 2*d^2*e*f*x*Log[1 - E^(c
+ d*x)] + d^2*f^2*x^2*Log[1 - E^(c + d*x)] - d^2*e^2*Log[1 + E^(c + d*x)] -
 2*d^2*e*f*x*Log[1 + E^(c + d*x)] - d^2*f^2*x^2*Log[1 + E^(c + d*x)] - 2*d*
f*(e + f*x)*PolyLog[2, -E^(c + d*x)] + 2*d*f*(e + f*x)*PolyLog[2, E^(c + d*
x)] + 2*f^2*PolyLog[3, -E^(c + d*x)] - 2*f^2*PolyLog[3, E^(c + d*x)])*(a +
b*Sinh[c + d*x]))/(4*a*d^3*(b + a*Csch[c + d*x])) - (b^3*Csch[c + d*x]*((2*
d^2*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + (2
*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]
)])/Sqrt[(a^2 + b^2)*E^(2*c)] + (d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/
(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*d^2*e*
E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqr
t[(a^2 + b^2)*E^(2*c)] - (d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c
+ Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] + (2*d*E^c*f*(e +
f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/
Sqrt[(a^2 + b^2)*E^(2*c)] - (2*d*E^c*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d
*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*
E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))
])/Sqrt[(a^2 + b^2)*E^(2*c)] + (2*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a
*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)])*(a + b*Sinh
[c + d*x]))/(4*a*(a^2 + b^2)*d^3*(b + a*Csch[c + d*x])) + (b*e*f*Csch[c + d
*x]*Sech[c]*(Cosh[c]*Log[Cosh[c]*Cosh[d*x] + Sinh[c]*Sinh[d*x]] - d*x*Sinh[
c])*(a + b*Sinh[c + d*x]))/(2*(a^2 + b^2)*d^2*(b + a*Csch[c + d*x])*(Cosh[c
]^2 - Sinh[c]^2)) - (a*e*f*ArcTan[(Sinh[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Co
sh[c]^2 - Sinh[c]^2]]*Csch[c + d*x]*(a + b*Sinh[c + d*x]))/((a^2 + b^2)*d^2
*(b + a*Csch[c + d*x])*Sqrt[Cosh[c]^2 - Sinh[c]^2]) + (b*f^2*Csch[c]*Csch[c
 + d*x]*((d^2*x^2)/E^ArcTanh[Coth[c]] - (I*Coth[c]*(-(d*x*(-Pi + (2*I)*ArcT
anh[Coth[c]])) - Pi*Log[1 + E^(2*d*x)] - 2*(I*d*x + I*ArcTanh[Coth[c]])*Log
[1 - E^((2*I)*(I*d*x + I*ArcTanh[Coth[c]]))] + Pi*Log[Cosh[d*x]] + (2*I)*Ar
cTanh[Coth[c]]*Log[I*Sinh[d*x + ArcTanh[Coth[c]]]] + I*PolyLog[2, E^((2*I)*
(I*d*x + I*ArcTanh[Coth[c]]))]))/Sqrt[1 - Coth[c]^2])*Sech[c]*(a + b*Sinh[c
 + d*x]))/(4*(a^2 + b^2)*d^3*(b + a*Csch[c + d*x])*Sqrt[Csch[c]^2*(-Cosh[c]
^2 + Sinh[c]^2)]) - (a*f^2*Csch[c + d*x]*(((-I)*Csch[c]*(I*(d*x + ArcTanh[C
oth[c]])*(Log[1 - E^(-(d*x) - ArcTanh[Coth[c]])] - Log[1 + E^(-(d*x) - ArcT
anh[Coth[c]])]) + I*(PolyLog[2, -E^(-(d*x) - ArcTanh[Coth[c]])] - PolyLog[2
, E^(-(d*x) - ArcTanh[Coth[c]])])))/Sqrt[1 - Coth[c]^2] - (2*ArcTan[(Sinh[c
```

```
] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]]*ArcTanh[Coth[c]])/S
qrt[Cosh[c]^2 - Sinh[c]^2])*(a + b*Sinh[c + d*x]))/(2*(a^2 + b^2)*d^3*(b +
a*Csch[c + d*x])) + (Csch[c + d*x]*Sech[c]*Sech[c + d*x]*(a*e^2*Cosh[c] + 2
*a*e*f*x*Cosh[c] + a*f^2*x^2*Cosh[c] - b*e^2*Sinh[d*x] - 2*b*e*f*x*Sinh[d*x
] - b*f^2*x^2*Sinh[d*x])*(a + b*Sinh[c + d*x]))/(4*(a^2 + b^2)*d*(b + a*Csc
h[c + d*x])))
```

Maple [F]
time = 2.62, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)\operatorname{sech}(dx+c)^2}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -2*b*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 + b^
2)*d^2))*e - 4*a*f^2*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*d
*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 4*b*f^2*integrate(x/(a^2*d*e^(2*d*x
 + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - (b^3*log((b*e^(-d*x
- c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^3 +
 a*b^2)*sqrt(a^2 + b^2)*d) - 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^2 + b^
2)*e^(-2*d*x - 2*c))*d) + log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) -
1)/(a*d))*e^2 - 4*a*f*arctan(e^(d*x + c))*e/((a^2 + b^2)*d^2) + 2*(b*f^2*x^
2 + 2*b*f*x*e + (a*f^2*x^2*e^c + 2*a*f*x*e^(c + 1))*e^(d*x))/(a^2*d + b^2*d
 + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x)) - 2*(d*x*log(e^(d*x + c) + 1)
 + dilog(-e^(d*x + c)))*f*e/(a*d^2) + 2*(d*x*log(-e^(d*x + c) + 1) + dilog(
e^(d*x + c)))*f*e/(a*d^2) - (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^
(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2/(a*d^3) + (d^2*x^2*log(-e^(d*x
 + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*f^2/(a*d
^3) - integrate(-2*(b^3*f^2*x^2*e^c + 2*b^3*f*x*e^(c + 1))*e^(d*x)/(a^3*b +
 a*b^3 - (a^3*b*e^(2*c) + a*b^3*e^(2*c))*e^(2*d*x) - 2*(a^4*e^c + a^2*b^2*e
^c)*e^(d*x)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7596 vs. $2(743) = 1486$.
time = 0.51, size = 7596, normalized size = 9.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] (2*(a^3*b + a*b^3)*c^2*f^2 - 4*(a^3*b + a*b^3)*c*d*f*cosh(1) + 2*(a^3*b + a
*b^3)*d^2*cosh(1)^2 + 2*(a^3*b + a*b^3)*d^2*sinh(1)^2 - 2*((a^3*b + a*b^3)*
d^2*f^2*x^2 - (a^3*b + a*b^3)*c^2*f^2 + 2*((a^3*b + a*b^3)*d^2*f*x + (a^3*b
 + a*b^3)*c*d*f)*cosh(1) + 2*((a^3*b + a*b^3)*d^2*f*x + (a^3*b + a*b^3)*c*d
*f)*sinh(1))*cosh(d*x + c)^2 - 2*((a^3*b + a*b^3)*d^2*f^2*x^2 - (a^3*b + a*
b^3)*c^2*f^2 + 2*((a^3*b + a*b^3)*d^2*f*x + (a^3*b + a*b^3)*c*d*f)*cosh(1)
+ 2*((a^3*b + a*b^3)*d^2*f*x + (a^3*b + a*b^3)*c*d*f)*sinh(1))*sinh(d*x + c
)^2 - 2*(b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1) + (b^4*d*f^2*x + b
^4*d*f*cosh(1) + b^4*d*f*sinh(1))*cosh(d*x + c)^2 + 2*(b^4*d*f^2*x + b^4*d*
f*cosh(1) + b^4*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (b^4*d*f^2*x + b
^4*d*f*cosh(1) + b^4*d*f*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*di
log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b^4*d*f^2*x + b^4*d*f*cosh(1) + b^
4*d*f*sinh(1) + (b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1))*cosh(d*x
+ c)^2 + 2*(b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1))*cosh(d*x + c)*
sinh(d*x + c) + (b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1))*sinh(d*x
+ c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b^4*
c^2*f^2 - 2*b^4*c*d*f*cosh(1) + b^4*d^2*cosh(1)^2 + b^4*d^2*sinh(1)^2 + (b^
4*c^2*f^2 - 2*b^4*c*d*f*cosh(1) + b^4*d^2*cosh(1)^2 + b^4*d^2*sinh(1)^2 - 2
*(b^4*c*d*f - b^4*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(b^4*c^2*f^2 -
2*b^4*c*d*f*cosh(1) + b^4*d^2*cosh(1)^2 + b^4*d^2*sinh(1)^2 - 2*(b^4*c*d*f
- b^4*d^2*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (b^4*c^2*f^2 - 2*
b^4*c*d*f*cosh(1) + b^4*d^2*cosh(1)^2 + b^4*d^2*sinh(1)^2 - 2*(b^4*c*d*f -
b^4*d^2*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 2*(b^4*c*d*f - b^4*d^2*cosh(1))
*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) +
 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^4*c^2*f^2 - 2*b^4*c*d*f*cosh(1) + b^
4*d^2*cosh(1)^2 + b^4*d^2*sinh(1)^2 + (b^4*c^2*f^2 - 2*b^4*c*d*f*cosh(1) +
b^4*d^2*cosh(1)^2 + b^4*d^2*sinh(1)^2 - 2*(b^4*c*d*f - b^4*d^2*cosh(1))*sin
h(1))*cosh(d*x + c)^2 + 2*(b^4*c^2*f^2 - 2*b^4*c*d*f*cosh(1) + b^4*d^2*cosh
(1)^2 + b^4*d^2*sinh(1)^2 - 2*(b^4*c*d*f - b^4*d^2*cosh(1))*sinh(1))*cosh(d
*x + c)*sinh(d*x + c) + (b^4*c^2*f^2 - 2*b^4*c*d*f*cosh(1) + b^4*d^2*cosh(1
)^2 + b^4*d^2*sinh(1)^2 - 2*(b^4*c*d*f - b^4*d^2*cosh(1))*sinh(1))*sinh(d*x
 + c)^2 - 2*(b^4*c*d*f - b^4*d^2*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*lo
g(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
```

```
- (b^4*d^2*f^2*x^2 - b^4*c^2*f^2 + (b^4*d^2*f^2*x^2 - b^4*c^2*f^2 + 2*(b^4*
d^2*f*x + b^4*c*d*f)*cosh(1) + 2*(b^4*d^2*f*x + b^4*c*d*f)*sinh(1))*cosh(d*
x + c)^2 + 2*(b^4*d^2*f^2*x^2 - b^4*c^2*f^2 + 2*(b^4*d^2*f*x + b^4*c*d*f)*c
osh(1) + 2*(b^4*d^2*f*x + b^4*c*d*f)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) +
 (b^4*d^2*f^2*x^2 - b^4*c^2*f^2 + 2*(b^4*d^2*f*x + b^4*c*d*f)*cosh(1) + 2*(
b^4*d^2*f*x + b^4*c*d*f)*sinh(1))*sinh(d*x + c)^2 + 2*(b^4*d^2*f*x + b^4*c*
d*f)*cosh(1) + 2*(b^4*d^2*f*x + b^4*c*d*f)*sinh(1))*sqrt((a^2 + b^2)/b^2)*l
og(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2) - b)/b) + (b^4*d^2*f^2*x^2 - b^4*c^2*f^2 + (b^4*d^2
*f^2*x^2 - b^4*c^2*f^2 + 2*(b^4*d^2*f*x + b^4*c*d*f)*cosh(1) + 2*(b^4*d^2*f
*x + b^4*c*d*f)*sinh(1))*cosh(d*x + c)^2 + 2*(b^4*d^2*f^2*x^2 - b^4*c^2*f^2
 + 2*(b^4*d^2*f*x + b^4*c*d*f)*cosh(1) + 2*(b^4*d^2*f*x + b^4*c*d*f)*sinh(1
))*cosh(d*x + c)*sinh(d*x + c) + (b^4*d^2*f^2*x^2 - b^4*c^2*f^2 + 2*(b^4*d^
2*f*x + b^4*c*d*f)*cosh(1) + 2*(b^4*d^2*f*x + b^4*c*d*f)*sinh(1))*sinh(d*x
+ c)^2 + 2*(b^4*d^2*f*x + b^4*c*d*f)*cosh(1) + 2*(b^4*d^2*f*x + b^4*c*d*f)*
sinh(1))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(b^4*f^
2*cosh(d*x + c)^2 + 2*b^4*f^2*cosh(d*x + c)*sinh(d*x + c) + b^4*f^2*sinh(d*
x + c)^2 + b^4*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*s
inh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b
) - 2*(b^4*f^2*cosh(d*x + c)^2 + 2*b^4*f^2*cosh(d*x + c)*sinh(d*x + c) + b^
4*f^2*sinh(d*x + c)^2 + b^4*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d
*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2))/b) + 2*((a^4 + a^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*f*x
*cosh(1) + (a^4 + a^2*b^2)*d^2*cosh(1)^2 + (a^4 + a^2*b^2)*d^2*sinh(1)^2 +
2*((a^4 + a^2*b^2)*d^2*f*x + (a^4 + a^2*b^2)*d^2*cosh(1))*sinh(1))*cosh(d*x
 + c) + 2*((a^4 + 2*a^2*b^2 + b^4)*d*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d*f*co
sh(1) + (a^4 + 2*a^2*b^2 + b^4)*d*f*sinh(1) + ((a^4 + 2*a^2*b^2 + b^4)*d*f^
2*x + (a^4 + 2*a^2*b^2 + b^4)*d*f*cosh(1) + (a^4 + 2*a^2*b^2 + b^4)*d*f*sin
h(1))*cosh(d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d*f^2*x + (a^4 + 2*a^2*b
^2 + b^4)*d*f*cosh(1) + (a^4 + 2*a^2*b^2 + b^4)*d*f*sinh(1))*cosh(d*x + c)*
sinh(d*x + c) + ((a^4 + 2*a^2*b^2 + b^4)*d*f^2*...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6439 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^2}{\cosh\left(c+d\,x\right)^2\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(cosh(c + d*x)^2*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((e + f*x)^2/(cosh(c + d*x)^2*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.442 $\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=442

$$-\frac{f\text{ArcTan}(\sinh(c+dx))}{ad^2}+\frac{b^2f\text{ArcTan}(\sinh(c+dx))}{a\left(a^2+b^2\right)d^2}-\frac{2fx\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{fx\tanh^{-1}(\cosh(c+dx))}{ad}-\frac{(e+$$

```
[Out] -f*arctan(sinh(d*x+c))/a/d^2+b^2*f*arctan(sinh(d*x+c))/a/(a^2+b^2)/d^2-2*f*
x*arctanh(exp(d*x+c))/a/d+f*x*arctanh(cosh(d*x+c))/a/d-(f*x+e)*arctanh(cosh
(d*x+c))/a/d+b*f*ln(cosh(d*x+c))/(a^2+b^2)/d^2-b^3*(f*x+e)*ln(1+b*exp(d*x+c
)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d+b^3*(f*x+e)*ln(1+b*exp(d*x+c)/(a
+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^(3/2)/d-f*polylog(2,-exp(d*x+c))/a/d^2+f*pol
ylog(2,exp(d*x+c))/a/d^2-b^3*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))
/a/(a^2+b^2)^(3/2)/d^2+b^3*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a
/(a^2+b^2)^(3/2)/d^2+(f*x+e)*sech(d*x+c)/a/d-b^2*(f*x+e)*sech(d*x+c)/a/(a^2
+b^2)/d-b*(f*x+e)*tanh(d*x+c)/(a^2+b^2)/d
```

Rubi [A]
time = 0.64, antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 19, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.594$, Rules used = {5708, 2702, 327, 213, 5570, 6406, 12, 4267, 2317, 2438, 3855, 5692, 3403, 2296, 2221, 6874, 4269, 3556, 5559}

$$\frac{b^2f\text{ArcTan}(\sinh(c+dx))}{ad^2\left(a^2+b^2\right)}+\frac{bf\log(\cosh(c+dx))}{d^2\left(a^2+b^2\right)}-\frac{b(e+fx)\tanh(c+dx)}{d\left(a^2+b^2\right)}-\frac{b^2(e+fx)\operatorname{sech}(c+dx)}{ad\left(a^2+b^2\right)}-\frac{b^3f\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\left(a^2+b^2\right)^{3/2}}+\frac{b^3f\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^2\left(a^2+b^2\right)^{3/2}}-\frac{b^3(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{ad\left(a^2+b^2\right)^{3/2}}+\frac{b^3(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{ad\left(a^2+b^2\right)^{3/2}}-\frac{f\text{ArcTan}(\sinh(c+dx))}{ad^2}-\frac{f\text{Li}_2\left(-e^{c+dx}\right)}{ad^2}+\frac{f\text{Li}_2\left(e^{c+dx}\right)}{ad^2}+\frac{(e+fx)\operatorname{sech}(c+dx)}{ad}-\frac{(e+fx)\tanh^{-1}(\cosh(c+dx))}{ad}-\frac{2fx\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{fx\tanh^{-1}(\cosh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((f*ArcTan[Sinh[c + d*x]])/(a*d^2)) + (b^2*f*ArcTan[Sinh[c + d*x]])/(a*(a^
2 + b^2)*d^2) - (2*f*x*ArcTanh[E^(c + d*x)])/(a*d) + (f*x*ArcTanh[Cosh[c +
d*x]])/(a*d) - ((e + f*x)*ArcTanh[Cosh[c + d*x]])/(a*d) - (b^3*(e + f*x)*Lo
g[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^(3/2)*d) + (b^
3*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^
(3/2)*d) + (b*f*Log[Cosh[c + d*x]])/((a^2 + b^2)*d^2) - (f*PolyLog[2, -E^(c
 + d*x)])/(a*d^2) + (f*PolyLog[2, E^(c + d*x)])/(a*d^2) - (b^3*f*PolyLog[2,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^2) + (b^
3*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3
/2)*d^2) + ((e + f*x)*Sech[c + d*x])/(a*d) - (b^2*(e + f*x)*Sech[c + d*x])/
(a*(a^2 + b^2)*d) - (b*(e + f*x)*Tanh[c + d*x])/((a^2 + b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
```

```
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
```

```
p]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5708

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6406

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(1 - u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(e+fx)\text{sech}(c+dx)}{ad} - \frac{b\int (}{}$$

$$= -\frac{(e+fx)\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(e+fx)\text{sech}(c+dx)}{ad} - \frac{b\int (}{}$$

$$= -\frac{f\tan^{-1}(\sinh(c+dx))}{ad^2} + \frac{fx\tanh^{-1}(\cosh(c+dx))}{ad} - \frac{(e+fx)}{}$$

$$= -\frac{f\tan^{-1}(\sinh(c+dx))}{ad^2} + \frac{fx\tanh^{-1}(\cosh(c+dx))}{ad} - \frac{(e+fx)}{}$$

$$= -\frac{f\tan^{-1}(\sinh(c+dx))}{ad^2} + \frac{b^2f\tan^{-1}(\sinh(c+dx))}{a(a^2+b^2)d^2} - \frac{2fx\tanh^{-1}}{a}$$

$$= -\frac{f\tan^{-1}(\sinh(c+dx))}{ad^2} + \frac{b^2f\tan^{-1}(\sinh(c+dx))}{a(a^2+b^2)d^2} - \frac{2fx\tanh^{-1}}{a}$$

$$= -\frac{f\tan^{-1}(\sinh(c+dx))}{ad^2} + \frac{b^2f\tan^{-1}(\sinh(c+dx))}{a(a^2+b^2)d^2} - \frac{2fx\tanh^{-1}}{a}$$

Mathematica [A]

time = 4.37, size = 459, normalized size = 1.04

$$\frac{\text{csch}(c+dx)(a+b\sinh(c+dx))\left(-\frac{2af\,\text{ArcTan}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a^2+b^2}+\frac{bf\log(\cosh(c+dx))}{a^2+b^2}+\frac{de\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a}-\frac{cf\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a}+\frac{f\left((c+dx)\left(\log\left(1-e^{-c-dx}\right)-\log\left(1+e^{-c-dx}\right)\right)+\text{PolyLog}\left(2,-e^{-c-dx}\right)-\text{PolyLog}\left(2,e^{-c-dx}\right)\right)}{a}+\frac{b^3\left(2de\tanh^{-1}\left(\frac{a+b\cosh(c+dx)+b\sinh(c+dx)}{\sqrt{a^2+b^2}}\right)-2cf\tanh^{-1}\left(\frac{a+b\cosh(c+dx)+b\sinh(c+dx)}{\sqrt{a^2+b^2}}\right)-f(c+dx)\log\left(1+\frac{b(\cosh(c+dx)+\sinh(c+dx))}{a-\sqrt{a^2+b^2}}\right)+f(c+dx)\log\left(1+\frac{b(\cosh(c+dx)+\sinh(c+dx))}{a+\sqrt{a^2+b^2}}\right)-f\,\text{PolyLog}\left(2,\frac{b(\cosh(c+dx)+\sinh(c+dx))}{-a+\sqrt{a^2+b^2}}\right)+f\,\text{PolyLog}\left(2,-\frac{b(\cosh(c+dx)+\sinh(c+dx))}{a+\sqrt{a^2+b^2}}\right)\right)}{a(a^2+b^2)^{3/2}}+\frac{d(e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))}{a^2+b^2}\right)}{d^2(b+a\,\text{csch}(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (Csch[c + d*x]*(a + b*Sinh[c + d*x])*((-2*a*f*ArcTan[Tanh[(c + d*x)/2]])/(a
^2 + b^2) + (b*f*Log[Cosh[c + d*x]])/(a^2 + b^2) + (d*e*Log[Tanh[(c + d*x)/
2]])/a - (c*f*Log[Tanh[(c + d*x)/2]])/a + (f*((c + d*x)*(Log[1 - E^(-c - d*
x)] - Log[1 + E^(-c - d*x)]) + PolyLog[2, -E^(-c - d*x)] - PolyLog[2, E^(-c
 - d*x)]))/a + (b^3*(2*d*e*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/
```

```
Sqrt[a^2 + b^2]] - 2*c*f*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sq
rt[a^2 + b^2]] - f*(c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a
 - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x
]))/(a + Sqrt[a^2 + b^2])] - f*PolyLog[2, (b*(Cosh[c + d*x] + Sinh[c + d*x]
))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*(Cosh[c + d*x] + Sinh[c + d*
x]))/(a + Sqrt[a^2 + b^2]))]))/(a*(a^2 + b^2)^(3/2)) + (d*(e + f*x)*Sech[c
+ d*x]*(a - b*Sinh[c + d*x]))/(a^2 + b^2)))/(d^2*(b + a*Csch[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1814 vs. $2(419) = 838$.
time = 5.83, size = 1815, normalized size = 4.11

method	result	size
risch	Expression too large to display	1815

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVER
BOSE)
```

```
[Out] 4/d^2/(a^2+b^2)*a^2*b*f/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))-1/d/(a^2+b^2)*b^
2*f/a*ln(exp(d*x+c)+1)*x+1/d/(a^2+b^2)^(3/2)*b^3*e/a*arctanh(1/2*(2*b*exp(d
*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d/(a^2+b^2)^(5/2)*a^3*b*e*arctanh(1/2*(2*b*ex
p(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d^2/(a^2+b^2)^(3/2)*f*b^3/a*arctanh(1/2*(2
*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d^2/(a^2+b^2)^(5/2)*f*b^5/a*arctanh(1
/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d^2/(a^2+b^2)^(3/2)*a*f*b*arctan
h(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d^2/(a^2+b^2)^(5/2)*f*b^5/a*d
ilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/d/(a^2+b^2)^(5
/2)*b^5*e/a*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d^2/(a^2+b^
2)*b^2*f*c/a*ln(exp(d*x+c)-1)-1/d^2/(a^2+b^2)^(5/2)*f*b^5/a*dilog((-b*exp(d
*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d/(a^2+b^2)^(5/2)*a*b^3*f*
ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d^2/(a^2+b^2
)^(5/2)*a*b^3*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-
1/d/(a^2+b^2)^(5/2)*f*b^5/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b
^2)^(1/2)))*x-1/d^2/(a^2+b^2)^(5/2)*a*b^3*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/
2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d^2/(a^2+b^2)^(3/2)*b^3*f*c/a*arctanh(1/2*(
2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d^2/(a^2+b^2)^(5/2)*b^5*f*c/a*arctan
h(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d^2/(a^2+b^2)^(5/2)*f*a^3*b*c
*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d/(a^2+b^2)^(5/2)*a*b^
3*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d/(a^2+b^2
)^(5/2)*f*b^5/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+
2*(f*x+e)*(a*exp(d*x+c)+b)/d/(a^2+b^2)/(1+exp(2*d*x+2*c))+1/d^2/(a^2+b^2)^(
5/2)*f*b^5/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d
^2/(a^2+b^2)^(5/2)*f*b^5/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^
2)^(1/2)))*c-2/d^2*b/(a^2+b^2)*f*ln(exp(d*x+c))-8/d^2/(a^2+b^2)*a^3*f/(4*a^
2+4*b^2)*arctan(exp(d*x+c))-1/d^2/(a^2+b^2)*b^2*f/a*dilog(exp(d*x+c)+1)-1/d
```

```
^2/(a^2+b^2)*b^2*f*dilog(exp(d*x+c))/a+1/d*b*e/(a^2+b^2)^(3/2)*arctanh(1/2*
(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a-1/d^2*b/(a^2+b^2)^(5/2)*f*arctanh(1
/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^3-2/d^2*b^3/(a^2+b^2)^(5/2)*f*ar
ctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a-1/d^2*b*f*c/(a^2+b^2)^(3/
2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a-1/d^2/(a^2+b^2)*a*f*
c*ln(exp(d*x+c)-1)+4/d^2/(a^2+b^2)*f*b^3/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))
-1/d/(a^2+b^2)*b^2*e/a*ln(exp(d*x+c)+1)+1/d/(a^2+b^2)*b^2*e/a*ln(exp(d*x+c)
-1)-1/d/(a^2+b^2)*ln(exp(d*x+c)+1)*a*f*x-8/d^2/(a^2+b^2)*a*b^2*f/(4*a^2+4*b
^2)*arctan(exp(d*x+c))+1/d^2/(a^2+b^2)^(5/2)*a*b^3*f*dilog((b*exp(d*x+c)+(a
^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2/(a^2+b^2)^(5/2)*a*b^3*f*dilog((
-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d/(a^2+b^2)*a*e*ln
(exp(d*x+c)+1)+1/d/(a^2+b^2)*a*e*ln(exp(d*x+c)-1)-1/d^2/(a^2+b^2)*a*f*dilog
(exp(d*x+c)+1)-1/d^2/(a^2+b^2)*a*f*dilog(exp(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -(8*b^3*integrate(-1/4*x*e^(d*x + c)/(a^3*b + a*b^3 - (a^3*b*e^(2*c) + a*b^
3*e^(2*c))*e^(2*d*x) - 2*(a^4*e^c + a^2*b^2*e^c)*e^(d*x)), x) - 2*(a*x*e^(d
*x + c) + b*x)/(a^2*d + b^2*d + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^(2*d*x))
+ 2*b*x/((a^2 + b^2)*d) + 2*a*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) - b*log
(e^(2*d*x + 2*c) + 1)/((a^2 + b^2)*d^2) - 8*integrate(1/8*x/(a*e^(d*x + c)
+ a), x) - 8*integrate(1/8*x/(a*e^(d*x + c) - a), x))*f - (b^3*log((b*e^(-d
*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^
3 + a*b^2)*sqrt(a^2 + b^2)*d) - 2*(a*e^(-d*x - c) - b)/((a^2 + b^2 + (a^2 +
 b^2)*e^(-2*d*x - 2*c))*d) + log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c)
 - 1)/(a*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2461 vs. $2(421) = 842$.
time = 0.44, size = 2461, normalized size = 5.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -(2*(a^3*b + a*b^3)*d*f*x*cosh(d*x + c)^2 + 2*(a^3*b + a*b^3)*d*f*x*sinh(d*
x + c)^2 - 2*(a^3*b + a*b^3)*d*cosh(1) - 2*(a^3*b + a*b^3)*d*sinh(1) + (b^4
```

*f*cosh(d*x + c)^2 + 2*b^4*f*cosh(d*x + c)*sinh(d*x + c) + b^4*f*sinh(d*x + c)^2 + b^4*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^4*f*cosh(d*x + c)^2 + 2*b^4*f*cosh(d*x + c)*sinh(d*x + c) + b^4*f*sinh(d*x + c)^2 + b^4*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1) + (b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*cosh(d*x + c)^2 + 2*(b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1) + (b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*cosh(d*x + c)^2 + 2*(b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^4*d*f*x + b^4*c*f + (b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^2 + 2*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)*sinh(d*x + c) + (b^4*d*f*x + b^4*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^4*d*f*x + b^4*c*f + (b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^2 + 2*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)*sinh(d*x + c) + (b^4*d*f*x + b^4*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*((a^4 + a^2*b^2)*f*cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*f*cosh(d*x + c)*sinh(d*x + c) + (a^4 + a^2*b^2)*f*sinh(d*x + c)^2 + (a^4 + a^2*b^2)*f)*arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*((a^4 + a^2*b^2)*d*f*x + (a^4 + a^2*b^2)*d*cosh(1) + (a^4 + a^2*b^2)*d*sinh(1))*cosh(d*x + c) - ((a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*f*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) + ((a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*f*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*f)*dilog(-cosh(d*x + c) - sinh(d*x + c)) - ((a^3*b + a*b^3)*f*cosh(d*x + c)^2 + 2*(a^3*b + a*b^3)*f*cosh(d*x + c)*sinh(d*x + c) + (a^3*b + a*b^3)*f*sinh(d*x + c)^2 + (a^3*b + a*b^3)*f)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + ((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*d*cosh(1) + ((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*d*cosh(1) + (a^4 + 2*a^2*b^2 + b^4)*d*sinh(1))*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d*sinh(1) + 2*((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*d*cosh(1) + (a^4 + 2*a^2*b^2 + b^4)*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + ((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*d*cosh(1) + (a^4 + 2*a^2*b^2 + b^4)*d*sinh(1))*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a^4 + 2*a^2*b^2 + b^4)*c*f - (a^4 + 2*a^2*b^2 + b^4)*d*cosh(1) + ((a^4 + 2*a^2*b^2 + b^4)*c*f - (a^4 + 2*a^2*b^2 + b^4)*d*cosh(1) - (a^4 + 2*a^2*b^2 + b^4)*d*sinh(1))*cosh(d*x + c)^2 - (a^4 + 2*a^2*b^2 + b^4)*d*sinh(1) + 2*((a^4 + 2*a

```
^2*b^2 + b^4)*c*f - (a^4 + 2*a^2*b^2 + b^4)*d*cosh(1) - (a^4 + 2*a^2*b^2 +
b^4)*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + ((a^4 + 2*a^2*b^2 + b^4)*c*f
- (a^4 + 2*a^2*b^2 + b^4)*d*cosh(1) - (a^4 + 2*a^2*b^2 + b^4)*d*sinh(1))*si
nh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) - 1) - ((a^4 + 2*a^2*b^2 +
 b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*c*f + ((a^4 + 2*a^2*b^2 + b^4)*d*f*x
+ (a^4 + 2*a^2*b^2 + b^4)*c*f)*cosh(d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^4)
*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*c*f)*cosh(d*x + c)*sinh(d*x + c) + ((a^4 +
 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*c*f)*sinh(d*x + c)^2)*log
(-cosh(d*x + c) - sinh(d*x + c) + 1) + 2*(2*(a^3*b + a*b^3)*d*f*x*cosh(d*x
+ c) - (a^4 + a^2*b^2)*d*f*x - (a^4 + a^2*b^2)*d*cosh(1) - (a^4 + a^2*b^2)*
d*sinh(1))*sinh(d*x + c))/((a^5 + 2*a^3*b^2 + a*b^4)*d^2*cosh(d*x + c)^2 +
2*(a^5 + 2*a^3*b^2 + a*b^4)*d^2*cosh(d*x + c)*sinh(d*x + c) + (a^5 + 2*a^3*
b^2 + a*b^4)*d^2*sinh(d*x + c)^2 + (a^5 + 2*a^3*b^2 + a*b^4)*d^2)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3436 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{{\cosh\left(c+d\,x\right)}^2\,\sinh\left(c+d\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(cosh(c + d*x)^2*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((e + f*x)/(cosh(c + d*x)^2*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.443 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=113

$$-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{2b^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{3/2}d}+\frac{\operatorname{sech}(c+dx)}{ad}-\frac{b\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a\left(a^2+b^2\right)d}$$

```
[Out] -arctanh(cosh(d*x+c))/a/d+2*b^3*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)
^(1/2))/a/(a^2+b^2)^(3/2)/d+sech(d*x+c)/a/d-b*sech(d*x+c)*(b+a*sinh(d*x+c))
/a/(a^2+b^2)/d
```

Rubi [A]
time = 0.19, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2977, 2702, 327, 213, 2775, 12, 2739, 632, 210}

$$-\frac{b\operatorname{sech}(c+dx)(a\sinh(c+dx)+b)}{ad\left(a^2+b^2\right)}+\frac{2b^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{ad\left(a^2+b^2\right)^{3/2}}+\frac{\operatorname{sech}(c+dx)}{ad}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(ArcTanh[Cosh[c + d*x]]/(a*d)) + (2*b^3*ArcTanh[(b - a*Tanh[(c + d*x)/2])/
Sqrt[a^2 + b^2]])/(a*(a^2 + b^2)^(3/2)*d) + Sech[c + d*x]/(a*d) - (b*Sech[c
 + d*x]*(b + a*Sinh[c + d*x]))/(a*(a^2 + b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = i\int \left(-\frac{i\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a}+\frac{ib\operatorname{sech}^2(c+dx)}{a(a+b\sinh(c+dx))}\right)dx$$

$$= \frac{\int \operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)\,dx}{a}-\frac{b\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{b\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a(a^2+b^2)d}-\frac{b\int \frac{b^2}{a+b\sinh(c+dx)}\,dx}{a(a^2+b^2)}+\frac{\text{Subst}\left(\int -\right.}{}$$

$$= \frac{\operatorname{sech}(c+dx)}{ad}-\frac{b\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a(a^2+b^2)d}-\frac{b^3\int \frac{1}{a+b\sinh(c+dx)}\,dx}{a(a^2+b^2)}$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{\operatorname{sech}(c+dx)}{ad}-\frac{b\operatorname{sech}(c+dx)(b+a\sinh(c+}{a(a^2+b^2)d}$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{\operatorname{sech}(c+dx)}{ad}-\frac{b\operatorname{sech}(c+dx)(b+a\sinh(c+}{a(a^2+b^2)d}$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{2b^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{3/2}d}+\frac{\operatorname{sech}(c+dx}{ad}$$

Mathematica [A]
time = 0.21, size = 171, normalized size = 1.51

$$-\frac{-2b^3\operatorname{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)+a^2\sqrt{-a^2-b^2}\,\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+b^2\sqrt{-a^2-b^2}\,\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+a^2\sqrt{-a^2-b^2}\operatorname{sech}(c+dx)-ab\sqrt{-a^2-b^2}\tanh(c+dx)}{a(-a^2-b^2)^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((-2*b^3*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] + a^2*Sqrt[-a^
2 - b^2]*Log[Tanh[(c + d*x)/2]] + b^2*Sqrt[-a^2 - b^2]*Log[Tanh[(c + d*x)/2
]] + a^2*Sqrt[-a^2 - b^2]*Sech[c + d*x] - a*b*Sqrt[-a^2 - b^2]*Tanh[c + d*x
])/(a*(-a^2 - b^2)^(3/2)*d))
```

Maple [A]
time = 1.64, size = 106, normalized size = 0.94

method	result
derivativedivides	$\frac{-\frac{2\left(b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^2+b^2\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b^3\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$

default	$$\frac{-\frac{2\left(b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^2+b^2\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b^3\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$$
risch	$$\frac{2a\,e^{dx+c}+2b}{d(a^2+b^2)(1+e^{2dx+2c})}-\frac{\ln(e^{dx+c}+1)}{da}+\frac{\ln(e^{dx+c}-1)}{da}+\frac{b^3\ln\left(e^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}\,da}-\frac{b^3\ln\left(e^{dx+\dots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/(a^2+b^2)*(b*tanh(1/2*d*x+1/2*c)-a)/(tanh(1/2*d*x+1/2*c)^2+1)+1/a*l
n(tanh(1/2*d*x+1/2*c))-2/a*b^3/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*d*
x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [A]
time = 0.48, size = 168, normalized size = 1.49

$$-\frac{b^3\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{(a^3+ab^2)\sqrt{a^2+b^2}\,d}+\frac{2\left(ae^{(-dx-c)}-b\right)}{(a^2+b^2+(a^2+b^2)e^{(-2\,dx-2\,c)})d}-\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}+\frac{\log\left(e^{(-dx-c)}-1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -b^3*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(
a^2 + b^2)))/((a^3 + a*b^2)*sqrt(a^2 + b^2)*d) + 2*(a*e^(-d*x - c) - b)/((a
^2 + b^2 + (a^2 + b^2)*e^(-2*d*x - 2*c))*d) - log(e^(-d*x - c) + 1)/(a*d) +
 log(e^(-d*x - c) - 1)/(a*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(110) = 220$.
time = 0.50, size = 581, normalized size = 5.14

$$\frac{2a^3b+2ab^3+\left(b^3\cosh(dx+c)^2+2b^3\cosh(dx+c)\sinh(dx+c)+b^3\sinh(dx+c)^2+b^3\right)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2ab\cosh(dx+c)+2a^2+b^2+2\left(b^2\cosh(dx+c)+ab\right)\sinh(dx+c)+2\sqrt{a^2+b^2}\left(b\cosh(dx+c)+b\sinh(dx+c)+a\right)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2a\cosh(dx+c)+2\left(b\cosh(dx+c)+a\right)\sinh(dx+c)-b}\right)+2\left(a^4+a^2b^2\right)\cosh(dx+c)-\left(a^4+2a^2b^2+b^4+\left(a^4+2a^2b^2+b^4\right)\cosh(dx+c)^2+2\left(a^4+2a^2b^2+b^4\right)\cosh(dx+c)\sinh(dx+c)+\left(a^4+2a^2b^2+b^4\right)\sinh(dx+c)^2\right)\log\left(\cosh(dx+c)+\sinh(dx+c)+1\right)+\left(a^4+2a^2b^2+b^4+\left(a^4+2a^2b^2+b^4\right)\cosh(dx+c)^2+2\left(a^4+2a^2b^2+b^4\right)\cosh(dx+c)\sinh(dx+c)+\left(a^4+2a^2b^2+b^4\right)\sinh(dx+c)^2\right)\log\left(\cosh(dx+c)+\sinh(dx+c)-1\right)+2\left(a^4+a^2b^2\right)\sinh(dx+c)}{\left(a^5+2a^3b^2+ab^4\right)d\cosh(dx+c)^2+2\left(a^5+2a^3b^2+ab^4\right)d\cosh(dx+c)\sinh(dx+c)+\left(a^5+2a^3b^2+ab^4\right)d\sinh(dx+c)^2+\left(a^5+2a^3b^2+ab^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] (2*a^3*b + 2*a*b^3 + (b^3*cosh(d*x + c)^2 + 2*b^3*cosh(d*x + c)*sinh(d*x +
c) + b^3*sinh(d*x + c)^2 + b^3)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 +
b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x +
 c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x
```

```
+ c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(
b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*(a^4 + a^2*b^2)*cosh(d*x + c)
- (a^4 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(a^4
 + 2*a^2*b^2 + b^4)*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*s
inh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + (a^4 + 2*a^2*b^2 +
 b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*
cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*sinh(d*x + c)^2)*log(
cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(a^4 + a^2*b^2)*sinh(d*x + c))/((a^5
 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c)^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*d*cos
h(d*x + c)*sinh(d*x + c) + (a^5 + 2*a^3*b^2 + a*b^4)*d*sinh(d*x + c)^2 + (a
^5 + 2*a^3*b^2 + a*b^4)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(csch(c + d*x)*sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [A]
time = 0.43, size = 146, normalized size = 1.29

$$-\frac{\frac{b^3\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^3+ab^2)\sqrt{a^2+b^2}}+\frac{\log\left(e^{(dx+c)}+1\right)}{a}-\frac{\log\left(\left|e^{(dx+c)}-1\right|\right)}{a}-\frac{2\,(ae^{(dx+c)}+b)}{(a^2+b^2)\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b^3*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c
) + 2*a + 2*sqrt(a^2 + b^2)))/((a^3 + a*b^2)*sqrt(a^2 + b^2)) + log(e^(d*x
+ c) + 1)/a - log(abs(e^(d*x + c) - 1))/a - 2*(a*e^(d*x + c) + b)/((a^2 + b
^2)*(e^(2*d*x + 2*c) + 1)))/d
```

Mupad [B]
time = 4.82, size = 668, normalized size = 5.91

$$\frac{\frac{2b}{d\,(a^2+b^2)}+\frac{2\,a\,e^{c+d\,x}}{d\,(a^2+b^2)}}{e^{2\,c+2\,d\,x}+1}+\frac{\ln\left(e^{c+d\,x}-1\right)}{a\,d}-\frac{\ln\left(e^{c+d\,x}+1\right)}{a\,d}-\frac{b^3\,\ln\left(\frac{32\,\left(-4\,e^{c+d\,x}\,a^3+2\,a^2\,b-5\,e^{c+d\,x}\,a\,b^2+2\,b^3\right)}{b^2\,(a^2+b^2)^2}-\frac{128\,a^{10}\,e^{c+d\,x}-64\,a^9\,b-96\,a\,b^9+64\,b^7\,\sqrt{(a^2+b^2)^3}-384\,a^3\,b^7-512\,a^5\,b^5-288\,a^7\,b^3+288\,a^2\,b^8\,e^{c+d\,x}+960\,a^4\,b^6\,e^{c+d\,x}+1152\,a^6\,b^4\,e^{c+d\,x}+608\,a^8\,b^2\,e^{c+d\,x}-64\,a\,b^6\,e^{c+d\,x}\,\sqrt{(a^2+b^2)^3}+32\,a^3\,b^4\,e^{c+d\,x}\,\sqrt{(a^2+b^2)^3}}{b^2\,\left((a^2+b^2)^3\right)^{3/2}\,(a^2+b^2)}\right)\,\sqrt{(a^2+b^2)^3}}{d\,a^7+3\,d\,a^5\,b^2+3\,d\,a^3\,b^4+d\,a\,b^6}+\frac{b^3\,\ln\left(\frac{32\,\left(-4\,e^{c+d\,x}\,a^3+2\,a^2\,b-5\,e^{c+d\,x}\,a\,b^2+2\,b^3\right)}{b^2\,(a^2+b^2)^2}-\frac{96\,a\,b^9+64\,a^9\,b-128\,a^{10}\,e^{c+d\,x}+64\,b^7\,\sqrt{(a^2+b^2)^3}+384\,a^3\,b^7+512\,a^5\,b^5+288\,a^7\,b^3-288\,a^2\,b^8\,e^{c+d\,x}-960\,a^4\,b^6\,e^{c+d\,x}-1152\,a^6\,b^4\,e^{c+d\,x}-608\,a^8\,b^2\,e^{c+d\,x}-64\,a\,b^6\,e^{c+d\,x}\,\sqrt{(a^2+b^2)^3}+32\,a^3\,b^4\,e^{c+d\,x}\,\sqrt{(a^2+b^2)^3}}{b^2\,\left((a^2+b^2)^3\right)^{3/2}\,(a^2+b^2)}\right)\,\sqrt{(a^2+b^2)^3}}{d\,a^7+3\,d\,a^5\,b^2+3\,d\,a^3\,b^4+d\,a\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] ((2*b)/(d*(a^2 + b^2)) + (2*a*exp(c + d*x))/(d*(a^2 + b^2)))/(exp(2*c + 2*d
*x) + 1) + log(exp(c + d*x) - 1)/(a*d) - log(exp(c + d*x) + 1)/(a*d) - (b^3
*log((32*(2*a^2*b - 4*a^3*exp(c + d*x) + 2*b^3 - 5*a*b^2*exp(c + d*x)))/(b^
2*(a^2 + b^2)^2) - (128*a^10*exp(c + d*x) - 64*a^9*b - 96*a*b^9 + 64*b^7*((
a^2 + b^2)^3)^(1/2) - 384*a^3*b^7 - 512*a^5*b^5 - 288*a^7*b^3 + 288*a^2*b^8
*exp(c + d*x) + 960*a^4*b^6*exp(c + d*x) + 1152*a^6*b^4*exp(c + d*x) + 608*
a^8*b^2*exp(c + d*x) - 64*a*b^6*exp(c + d*x)*((a^2 + b^2)^3)^(1/2) + 32*a^3
*b^4*exp(c + d*x)*((a^2 + b^2)^3)^(1/2))/(b^2*((a^2 + b^2)^3)^(3/2)*(a^2 +
b^2)))*((a^2 + b^2)^3)^(1/2))/(a^7*d + 3*a^3*b^4*d + 3*a^5*b^2*d + a*b^6*d)
 + (b^3*log((32*(2*a^2*b - 4*a^3*exp(c + d*x) + 2*b^3 - 5*a*b^2*exp(c + d*x
)))/(b^2*(a^2 + b^2)^2) - (96*a*b^9 + 64*a^9*b - 128*a^10*exp(c + d*x) + 64
*b^7*((a^2 + b^2)^3)^(1/2) + 384*a^3*b^7 + 512*a^5*b^5 + 288*a^7*b^3 - 288*
a^2*b^8*exp(c + d*x) - 960*a^4*b^6*exp(c + d*x) - 1152*a^6*b^4*exp(c + d*x)
 - 608*a^8*b^2*exp(c + d*x) - 64*a*b^6*exp(c + d*x)*((a^2 + b^2)^3)^(1/2) +
 32*a^3*b^4*exp(c + d*x)*((a^2 + b^2)^3)^(1/2))/(b^2*((a^2 + b^2)^3)^(3/2)*
(a^2 + b^2)))*((a^2 + b^2)^3)^(1/2))/(a^7*d + 3*a^3*b^4*d + 3*a^5*b^2*d + a
*b^6*d)
```

3.444 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csch[c + d*x]*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Csch[c + d*x]*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 172.01, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csch[c + d*x]*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] Integrate[(Csch[c + d*x]*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]))
, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)^2}{(fx+e)\,(a+b\sinh{(dx+c)})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -8*b^3*integrate(-1/4*e^(d*x + c)/((a^3*b*f + a*b^3*f)*x + (a^3*b + a*b^3)*
e - ((a^3*b*f*e^(2*c) + a*b^3*f*e^(2*c))*x + (a^3*b*e^(2*c) + a*b^3*e^(2*c)
)*e)*e^(2*d*x) - 2*((a^4*f*e^c + a^2*b^2*f*e^c)*x + (a^4*e^c + a^2*b^2*e^c)
*e)*e^(d*x)), x) + 2*(a*e^(d*x + c) + b)/((a^2*d*f + b^2*d*f)*x + (a^2*d +
b^2*d)*e + ((a^2*d*f*e^(2*c) + b^2*d*f*e^(2*c))*x + (a^2*d*e^(2*c) + b^2*d*
e^(2*c))*e)*e^(2*d*x)) + 8*integrate(1/4*(a*f*e^(d*x + c) + b*f)/((a^2*d*f^
2 + b^2*d*f^2)*x^2 + 2*(a^2*d*f + b^2*d*f)*x*e + (a^2*d + b^2*d)*e^2 + ((a^
2*d*f^2*e^(2*c) + b^2*d*f^2*e^(2*c))*x^2 + 2*(a^2*d*f*e^(2*c) + b^2*d*f*e^(
2*c))*x*e + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^2)*e^(2*d*x)), x) + 8*integra
te(1/8/(a*f*x + a*e + (a*f*x*e^c + a*e^(c + 1))*e^(d*x)), x) + 8*integrate(
-1/8/(a*f*x + a*e - (a*f*x*e^c + a*e^(c + 1))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(csch(d*x + c)*sech(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cosh\left(c+d\,x\right)^2\,\sinh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(1/(cosh(c + d*x)^2*sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.445 $\int \frac{(e+fx)^2\mathbf{csch}(c+dx)\mathbf{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1185

$$\frac{efx}{ad}+\frac{f^2x^2}{2ad}-\frac{2b^3(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d}-\frac{b(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{\left(a^2+b^2\right) d}+\frac{bf^2\text{ArcTan}(\sinh(c+dx))}{\left(a^2+b^2\right) d^3}-\frac{2(e+fx)}{}$$

```
[Out] -1/2*f^2*polylog(3,exp(2*d*x+2*c))/a/d^3+1/2*f^2*x^2/a/d+I*b*f^2*polylog(3,
I*exp(d*x+c))/(a^2+b^2)/d^3-1/2*b^4*f^2*polylog(3,-exp(2*d*x+2*c))/a/(a^2+b
^2)^2/d^3-b*f*(f*x+e)*sech(d*x+c)/(a^2+b^2)/d^2-1/2*b^2*(f*x+e)^2*sech(d*x+
c)^2/a/(a^2+b^2)/d-1/2*b*(f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a^2+b^2)/d-2*I*
b^3*f^2*polylog(3,-I*exp(d*x+c))/(a^2+b^2)^2/d^3-I*b*f^2*polylog(3,-I*exp(d
*x+c))/(a^2+b^2)/d^3+f^2*ln(cosh(d*x+c))/a/d^3-2*(f*x+e)^2*arctanh(exp(2*d*
x+2*c))/a/d+1/2*f^2*polylog(3,-exp(2*d*x+2*c))/a/d^3-1/2*(f*x+e)^2*tanh(d*x
+c)^2/a/d+2*I*b^3*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/(a^2+b^2)^2/d^2+b^2*f*
(f*x+e)*tanh(d*x+c)/a/(a^2+b^2)/d^2-2*I*b^3*f*(f*x+e)*polylog(2,I*exp(d*x+c
))/(a^2+b^2)^2/d^2-I*b*f*(f*x+e)*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^2+e*f*
x/a/d-f*(f*x+e)*tanh(d*x+c)/a/d^2-2*b^4*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(
a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^2/d^2-2*b^4*f*(f*x+e)*polylog(2,-b*exp(d*x+
c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^2/d^2+I*b*f*(f*x+e)*polylog(2,-I*exp(d*
x+c))/(a^2+b^2)/d^2+b^4*f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/a/(a^2+b^2)^2/
d^2+2*I*b^3*f^2*polylog(3,I*exp(d*x+c))/(a^2+b^2)^2/d^3+b^4*(f*x+e)^2*ln(1+
exp(2*d*x+2*c))/a/(a^2+b^2)^2/d-b^2*f^2*ln(cosh(d*x+c))/a/(a^2+b^2)/d^3-b^4
*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^2/d-b^4*(f*x+
e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^2/d+2*b^4*f^2*polyl
og(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^2/d^3+2*b^4*f^2*polylog
(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^2/d^3-b*(f*x+e)^2*arctan(
exp(d*x+c))/(a^2+b^2)/d-f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/a/d^2+f*(f*x+e
)*polylog(2,exp(2*d*x+2*c))/a/d^2-2*b^3*(f*x+e)^2*arctan(exp(d*x+c))/(a^2+b
^2)^2/d+b*f^2*arctan(sinh(d*x+c))/(a^2+b^2)/d^3
```

Rubi [A]
time = 1.73, antiderivative size = 1185, normalized size of antiderivative = 1.00, number of steps used = 57, number of rules used = 23, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.676$, Rules used = {5708, 2700, 14, 5570, 6873, 12, 6874, 2631, 4267, 2611, 2320, 6724, 3801, 3556, 5692, 5680, 2221, 4265, 3799, 4271, 3855, 5559, 4269}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e*f*x)/(a*d) + (f^2*x^2)/(2*a*d) - (2*b^3*(e + f*x)^2*ArcTan[E^(c + d*x)])
/((a^2 + b^2)^2*d) - (b*(e + f*x)^2*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) +
```

(b*f^2*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d^3) - (2*(e + f*x)^2*ArcTanh[E^(2*c + 2*d*x)])/(a*d) - (b^4*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^2*d) - (b^4*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^2*d) + (b^4*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(a*(a^2 + b^2)^2*d) + (f^2*Log[Cosh[c + d*x]])/(a*d^3) - (b^2*f^2*Log[Cosh[c + d*x]])/(a*(a^2 + b^2)*d^3) + ((2*I)*b^3*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) + (I*b*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) - ((2*I)*b^3*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - (I*b*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) - (2*b^4*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^2*d^2) - (2*b^4*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^2*d^2) + (b^4*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(a*(a^2 + b^2)^2*d^2) - (f*(e + f*x)*PolyLog[2, -E^(2*c + 2*d*x)])/(a*d^2) + (f*(e + f*x)*PolyLog[2, E^(2*c + 2*d*x)])/(a*d^2) - ((2*I)*b^3*f^2*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^3) - (I*b*f^2*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) + ((2*I)*b^3*f^2*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)^2*d^3) + (I*b*f^2*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)*d^3) + (2*b^4*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^2*d^3) + (2*b^4*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^2*d^3) - (b^4*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*a*(a^2 + b^2)^2*d^3) + (f^2*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a*d^3) - (f^2*PolyLog[3, E^(2*c + 2*d*x)])/(2*a*d^3) - (b*f*(e + f*x)*Sech[c + d*x])/((a^2 + b^2)*d^2) - (b^2*(e + f*x)^2*Sech[c + d*x]^2)/(2*a*(a^2 + b^2)*d) - (f*(e + f*x)*Tanh[c + d*x])/(a*d^2) + (b^2*f*(e + f*x)*Tanh[c + d*x])/(a*(a^2 + b^2)*d^2) - (b*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*(a^2 + b^2)*d) - ((e + f*x)^2*Tanh[c + d*x]^2)/(2*a*d)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_) + (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2631

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a +
b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
```

```
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5708

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{csch}(c+dx)\text{sech}^3(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}}{a}$$

$$= \frac{(e+fx)^2\log(\tanh(c+dx))}{ad} - \frac{(e+fx)^2\tanh^2(c+dx)}{2ad} - \frac{b\int (e$$

$$= \frac{(e+fx)^2\log(\tanh(c+dx))}{ad} - \frac{(e+fx)^2\tanh^2(c+dx)}{2ad} - \frac{b^3\int ($$

$$= \frac{b^4(e+fx)^3}{3a\left(a^2+b^2\right)^2 f} + \frac{(e+fx)^2\log(\tanh(c+dx))}{ad} - \frac{(e+fx)^2\tanh}{2ad}$$

$$= \frac{b^4(e+fx)^3}{3a\left(a^2+b^2\right)^2 f} - \frac{b^4(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^2 d} - \frac{b^4(e+}{}$$

$$= -\frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right) d} + \frac{bf^2}{}$$

$$= -\frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right) d} + \frac{bf^2}{}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{b(e+fx)^2\tan^{-1}\left(\right.}{\left(a^2+b^2\right) d}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{b(e+fx)^2\tan^{-1}\left(\right.}{\left(a^2+b^2\right) d}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{b(e+fx)^2\tan^{-1}\left(\right.}{\left(a^2+b^2\right) d}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{b(e+fx)^2\tan^{-1}\left(\right.}{\left(a^2+b^2\right) d}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{b(e+fx)^2\tan^{-1}\left(\right.}{\left(a^2+b^2\right) d}$$

<u>**Mathematica**</u> **[B]** Both result and optimal contain complex but leaf count is larger than

twice the leaf count of optimal. 3310 vs. 2(1185) = 2370.
time = 24.80, size = 3310, normalized size = 2.79

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] -1/6*(-12*a^3*d^3*e^2*E^(2*c)*x - 24*a*b^2*d^3*e^2*E^(2*c)*x + 12*a^3*d*E^(
2*c)*f^2*x + 12*a*b^2*d*E^(2*c)*f^2*x - 12*a^3*d^3*e*E^(2*c)*f*x^2 - 24*a*b
^2*d^3*e*E^(2*c)*f*x^2 - 4*a^3*d^3*E^(2*c)*f^2*x^3 - 8*a*b^2*d^3*E^(2*c)*f^
2*x^3 + 6*a^2*b*d^2*e^2*ArcTan[E^(c + d*x)] + 18*b^3*d^2*e^2*ArcTan[E^(c +
d*x)] + 6*a^2*b*d^2*e^2*E^(2*c)*ArcTan[E^(c + d*x)] + 18*b^3*d^2*e^2*E^(2*c
)*ArcTan[E^(c + d*x)] - 12*a^2*b*f^2*ArcTan[E^(c + d*x)] - 12*b^3*f^2*ArcTa
n[E^(c + d*x)] - 12*a^2*b*E^(2*c)*f^2*ArcTan[E^(c + d*x)] - 12*b^3*E^(2*c)*
f^2*ArcTan[E^(c + d*x)] + (6*I)*a^2*b*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + (1
8*I)*b^3*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + (6*I)*a^2*b*d^2*e*E^(2*c)*f*x*L
og[1 - I*E^(c + d*x)] + (18*I)*b^3*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)]
 + (3*I)*a^2*b*d^2*f^2*x^2*Log[1 - I*E^(c + d*x)] + (9*I)*b^3*d^2*f^2*x^2*L
og[1 - I*E^(c + d*x)] + (3*I)*a^2*b*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*
x)] + (9*I)*b^3*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] - (6*I)*a^2*b*d^
2*e*f*x*Log[1 + I*E^(c + d*x)] - (18*I)*b^3*d^2*e*f*x*Log[1 + I*E^(c + d*x)
] - (6*I)*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (18*I)*b^3*d^2*e
*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (3*I)*a^2*b*d^2*f^2*x^2*Log[1 + I*E^(
c + d*x)] - (9*I)*b^3*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - (3*I)*a^2*b*d^2*
E^(2*c)*f^2*x^2*Log[1 + I*E^(c + d*x)] - (9*I)*b^3*d^2*E^(2*c)*f^2*x^2*Log[
1 + I*E^(c + d*x)] + 6*a^3*d^2*e^2*Log[1 + E^(2*(c + d*x))] + 12*a*b^2*d^2*
e^2*Log[1 + E^(2*(c + d*x))] + 6*a^3*d^2*e^2*E^(2*c)*Log[1 + E^(2*(c + d*x)
)] + 12*a*b^2*d^2*e^2*E^(2*c)*Log[1 + E^(2*(c + d*x))] - 6*a^3*f^2*Log[1 +
E^(2*(c + d*x))] - 6*a*b^2*f^2*Log[1 + E^(2*(c + d*x))] - 6*a^3*E^(2*c)*f^2
*Log[1 + E^(2*(c + d*x))] - 6*a*b^2*E^(2*c)*f^2*Log[1 + E^(2*(c + d*x))] +
12*a^3*d^2*e*f*x*Log[1 + E^(2*(c + d*x))] + 24*a*b^2*d^2*e*f*x*Log[1 + E^(2
*(c + d*x))] + 12*a^3*d^2*e*E^(2*c)*f*x*Log[1 + E^(2*(c + d*x))] + 24*a*b^2
*d^2*e*E^(2*c)*f*x*Log[1 + E^(2*(c + d*x))] + 6*a^3*d^2*f^2*x^2*Log[1 + E^(
2*(c + d*x))] + 12*a*b^2*d^2*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 6*a^3*d^2*E
^(2*c)*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 12*a*b^2*d^2*E^(2*c)*f^2*x^2*Log[
1 + E^(2*(c + d*x))] - (6*I)*b*(a^2 + 3*b^2)*d*(1 + E^(2*c))*f*(e + f*x)*Po
lyLog[2, (-I)*E^(c + d*x)] + (6*I)*b*(a^2 + 3*b^2)*d*(1 + E^(2*c))*f*(e + f
*x)*PolyLog[2, I*E^(c + d*x)] + 6*a^3*d*e*f*PolyLog[2, -E^(2*(c + d*x))] +
12*a*b^2*d*e*f*PolyLog[2, -E^(2*(c + d*x))] + 6*a^3*d*e*E^(2*c)*f*PolyLog[2
, -E^(2*(c + d*x))] + 12*a*b^2*d*e*E^(2*c)*f*PolyLog[2, -E^(2*(c + d*x))] +
 6*a^3*d*f^2*x*PolyLog[2, -E^(2*(c + d*x))] + 12*a*b^2*d*f^2*x*PolyLog[2, -
E^(2*(c + d*x))] + 6*a^3*d*E^(2*c)*f^2*x*PolyLog[2, -E^(2*(c + d*x))] + 12*
a*b^2*d*E^(2*c)*f^2*x*PolyLog[2, -E^(2*(c + d*x))] + (6*I)*a^2*b*f^2*PolyLo
```

```
g[3, (-I)*E^(c + d*x)] + (18*I)*b^3*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (6*I
)*a^2*b*E^(2*c)*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (18*I)*b^3*E^(2*c)*f^2*P
olyLog[3, (-I)*E^(c + d*x)] - (6*I)*a^2*b*f^2*PolyLog[3, I*E^(c + d*x)] - (
18*I)*b^3*f^2*PolyLog[3, I*E^(c + d*x)] - (6*I)*a^2*b*E^(2*c)*f^2*PolyLog[3
, I*E^(c + d*x)] - (18*I)*b^3*E^(2*c)*f^2*PolyLog[3, I*E^(c + d*x)] - 3*a^3
*f^2*PolyLog[3, -E^(2*(c + d*x))] - 6*a*b^2*f^2*PolyLog[3, -E^(2*(c + d*x))
] - 3*a^3*E^(2*c)*f^2*PolyLog[3, -E^(2*(c + d*x))] - 6*a*b^2*E^(2*c)*f^2*Po
lyLog[3, -E^(2*(c + d*x))])/((a^2 + b^2)^2*d^3*(1 + E^(2*c))) + ((-4*E^(2*c
)*x*(3*e^2 + 3*e*f*x + f^2*x^2))/(-1 + E^(2*c)) + (6*(e + f*x)^2*Log[1 - E^
(2*(c + d*x))])/d + (6*f*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/d^2 - (3*f^
2*PolyLog[3, E^(2*(c + d*x))])/d^3)/(6*a) + (b^4*((2*E^(2*c)*x*(3*e^2 + 3*e
*f*x + f^2*x^2))/(-1 + E^(2*c)) - (3*(d^2*e^2*Log[2*a*E^(c + d*x) + b*(-1 +
 E^(2*(c + d*x)))] + 2*d^2*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a
^2 + b^2)*E^(2*c)])] + d^2*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[
(a^2 + b^2)*E^(2*c)])] + 2*d^2*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqr
t[(a^2 + b^2)*E^(2*c)])] + d^2*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + S
qrt[(a^2 + b^2)*E^(2*c)])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))
/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^
(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*f^2*PolyLog[3, -((b*
E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*f^2*PolyLog[3, -((
b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/d^3))/(3*a*(a^2 +
b^2)^2) + (Csch[c]*Sech[c]*Sech[c + d*x]^2*(-6*a^3*e*f - 6*a*b^2*e*f + 12*a
^3*d^2*e^2*x + 24*a*b^2*d^2*e^2*x - 6*a^3*f^2*x - 6*a*b^2*f^2*x + 12*a^3*d^
2*e*f*x^2 + 24*a*b^2*d^2*e*f*x^2 + 4*a^3*d^2*f^2*x^3 + 8*a*b^2*d^2*f^2*x^3
+ 6*a^3*e*f*Cosh[2*c] + 6*a*b^2*e*f*Cosh[2*c] + 6*a^3*f^2*x*Cosh[2*c] + 6*a
*b^2*f^2*x*Cosh[2*c] + 6*a^3*e*f*Cosh[2*d*x] + 6*a*b^2*e*f*Cosh[2*d*x] + 6*
a^3*f^2*x*Cosh[2*d*x] + 6*a*b^2*f^2*x*Cosh[2*d*x] + 3*a^2*b*d*e^2*Cosh[c -
d*x] + 3*b^3*d*e^2*Cosh[c - d*x] + 6*a^2*b*d*e*f*x*Cosh[c - d*x] + 6*b^3*d*
e*f*x*Cosh[c - d*x] + 3*a^2*b*d*f^2*x^2*Cosh[c - d*x] + 3*b^3*d*f^2*x^2*Cos
h[c - d*x] - 3*a^2*b*d*e^2*Cosh[3*c + d*x] - 3*b^3*d*e^2*Cosh[3*c + d*x] -
6*a^2*b*d*e*f*x*Cosh[3*c + d*x] - 6*b^3*d*e*f*x...
```

Maple [F]
time = 3.02, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c) \operatorname{sech}(dx+c)^3}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
[Out] int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -a^2*b*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b
^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2
+ b^4*d^2), x) - 3*b^3*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x
+ 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2
+ 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*a^3*d^2*f^2*integrate(x^2/(a^4*d^2*e^(2*
d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*
d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 4*a*b^2*d^2*f^2*integrate(x^2/(a^4*d^2
*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c)
+ a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 4*a^3*d^2*f*e*integrate(x/(a^4*d
^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c
) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 8*a*b^2*d^2*f*e*integrate(x/(a
^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x +
 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 2*a^2*b*d^2*f*integrate(x*
e^(d*x + c + 1)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) +
b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 6*b^3*d^
2*f*integrate(x*e^(d*x + c + 1)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^
(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2
), x) - a^3*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x +
 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) - a*b^2*f^2*(2*(d*x + c)/((a^4 +
2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d
^3)) + 2*a^2*b*f^2*arctan(e^(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3) + 2*b^
3*f^2*arctan(e^(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3) - (b^4*log(-2*a*e^(
-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + 2*a^3*b^2 + a*b^4)*d) - (a^2*b
+ 3*b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (a^3 + 2*a*b^2)
*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) + (b*e^(-d*x - c) -
2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*e^(-
2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d) - log(e^(-d*x - c) + 1)/(a*
d) - log(e^(-d*x - c) - 1)/(a*d))*e^2 + (2*a*f^2*x + 2*a*f*e - (b*d*f^2*x^2
*e^(3*c) + 2*b*f*e^(3*c + 1) + 2*(b*f^2*e^(3*c) + b*d*f*e^(3*c + 1))*x)*e^(
3*d*x) + 2*(a*d*f^2*x^2*e^(2*c) + a*f*e^(2*c + 1) + (a*f^2*e^(2*c) + 2*a*d*
f*e^(2*c + 1))*x)*e^(2*d*x) + (b*d*f^2*x^2*e^c - 2*b*f*e^(c + 1) + 2*(b*d*f
*e^(c + 1) - b*f^2*e^c)*x)*e^(d*x))/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^(4*c) +
 b^2*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*
x)) + 2*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*f*e/(a*d^2) + 2*(d
*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))*f*e/(a*d^2) + (d^2*x^2*log(e
^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f
^2/(a*d^3) + (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*
polylog(3, e^(d*x + c)))*f^2/(a*d^3) - 2/3*(d^3*f^2*x^3 + 3*d^3*f*x^2*e)/(a
*d^3) + integrate(2*(b^5*f^2*x^2 + 2*b^5*f*x*e - (a*b^4*f^2*x^2*e^c + 2*a*b
^4*f*x*e^(c + 1))*e^(d*x))/(a^5*b + 2*a^3*b^3 + a*b^5 - (a^5*b*e^(2*c) + 2*
```

```
a^3*b^3*e^(2*c) + a*b^5*e^(2*c))*e^(2*d*x) - 2*(a^6*e^c + 2*a^4*b^2*e^c + a
^2*b^4*e^c)*e^(d*x)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 25021 vs. 2(1119) = 2238.
time = 0.81, size = 25021, normalized size = 21.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -1/2*(4*((a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*c*f^2)*cosh(d*x + c)^4 +
 4*((a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*c*f^2)*sinh(d*x + c)^4 + 4*(a
^4 + a^2*b^2)*c*f^2 - 4*(a^4 + a^2*b^2)*d*f*cosh(1) + 2*((a^3*b + a*b^3)*d^
2*f^2*x^2 + 2*(a^3*b + a*b^3)*d*f^2*x + (a^3*b + a*b^3)*d^2*cosh(1)^2 + (a^
3*b + a*b^3)*d^2*sinh(1)^2 + 2*((a^3*b + a*b^3)*d^2*f*x + (a^3*b + a*b^3)*d
*f)*cosh(1) + 2*((a^3*b + a*b^3)*d^2*f*x + (a^3*b + a*b^3)*d^2*cosh(1) + (a
^3*b + a*b^3)*d*f)*sinh(1))*cosh(d*x + c)^3 - 4*(a^4 + a^2*b^2)*d*f*sinh(1)
 + 2*((a^3*b + a*b^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d*f^2*x + (a^3*b + a*
b^3)*d^2*cosh(1)^2 + (a^3*b + a*b^3)*d^2*sinh(1)^2 + 2*((a^3*b + a*b^3)*d^2
*f*x + (a^3*b + a*b^3)*d*f)*cosh(1) + 8*((a^4 + a^2*b^2)*d*f^2*x + (a^4 + a
^2*b^2)*c*f^2)*cosh(d*x + c) + 2*((a^3*b + a*b^3)*d^2*f*x + (a^3*b + a*b^3)
*d^2*cosh(1) + (a^3*b + a*b^3)*d*f)*sinh(1))*sinh(d*x + c)^3 - 4*((a^4 + a^
2*b^2)*d^2*f^2*x^2 - (a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*d^2*cosh(1)^
2 + (a^4 + a^2*b^2)*d^2*sinh(1)^2 - 2*(a^4 + a^2*b^2)*c*f^2 + (2*(a^4 + a^2
*b^2)*d^2*f*x + (a^4 + a^2*b^2)*d*f)*cosh(1) + (2*(a^4 + a^2*b^2)*d^2*f*x +
 2*(a^4 + a^2*b^2)*d^2*cosh(1) + (a^4 + a^2*b^2)*d*f)*sinh(1))*cosh(d*x + c
)^2 - 2*(2*(a^4 + a^2*b^2)*d^2*f^2*x^2 - 2*(a^4 + a^2*b^2)*d*f^2*x + 2*(a^4
 + a^2*b^2)*d^2*cosh(1)^2 + 2*(a^4 + a^2*b^2)*d^2*sinh(1)^2 - 4*(a^4 + a^2*
b^2)*c*f^2 - 12*((a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*c*f^2)*cosh(d*x
+ c)^2 + 2*(2*(a^4 + a^2*b^2)*d^2*f*x + (a^4 + a^2*b^2)*d*f)*cosh(1) - 3*((
a^3*b + a*b^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d*f^2*x + (a^3*b + a*b^3)*d^
2*cosh(1)^2 + (a^3*b + a*b^3)*d^2*sinh(1)^2 + 2*((a^3*b + a*b^3)*d^2*f*x +
(a^3*b + a*b^3)*d*f)*cosh(1) + 2*((a^3*b + a*b^3)*d^2*f*x + (a^3*b + a*b^3)
*d^2*cosh(1) + (a^3*b + a*b^3)*d*f)*sinh(1))*cosh(d*x + c) + 2*(2*(a^4 + a^
2*b^2)*d^2*f*x + 2*(a^4 + a^2*b^2)*d^2*cosh(1) + (a^4 + a^2*b^2)*d*f)*sinh(
1))*sinh(d*x + c)^2 - 2*((a^3*b + a*b^3)*d^2*f^2*x^2 - 2*(a^3*b + a*b^3)*d*
f^2*x + (a^3*b + a*b^3)*d^2*cosh(1)^2 + (a^3*b + a*b^3)*d^2*sinh(1)^2 + 2*(
(a^3*b + a*b^3)*d^2*f*x - (a^3*b + a*b^3)*d*f)*cosh(1) + 2*((a^3*b + a*b^3)
*d^2*f*x + (a^3*b + a*b^3)*d^2*cosh(1) - (a^3*b + a*b^3)*d*f)*sinh(1))*cosh
(d*x + c) + 4*(b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1) + (b^4*d*f^2
*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1))*cosh(d*x + c)^4 + 4*(b^4*d*f^2*x +
b^4*d*f*cosh(1) + b^4*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*d*f
```

```
^2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1))*sinh(d*x + c)^4 + 2*(b^4*d*f^2*x
+ b^4*d*f*cosh(1) + b^4*d*f*sinh(1))*cosh(d*x + c)^2 + 2*(b^4*d*f^2*x + b^4
*d*f*cosh(1) + b^4*d*f*sinh(1) + 3*(b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f
*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*d*f^2*x + b^4*d*f*cosh
(1) + b^4*d*f*sinh(1))*cosh(d*x + c)^3 + (b^4*d*f^2*x + b^4*d*f*cosh(1) + b
^4*d*f*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*si
nh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b
)/b + 1) + 4*(b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1) + (b^4*d*f^2*
x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1))*cosh(d*x + c)^4 + 4*(b^4*d*f^2*x + b
^4*d*f*cosh(1) + b^4*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*d*f^
2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1))*sinh(d*x + c)^4 + 2*(b^4*d*f^2*x +
 b^4*d*f*cosh(1) + b^4*d*f*sinh(1))*cosh(d*x + c)^2 + 2*(b^4*d*f^2*x + b^4*
d*f*cosh(1) + b^4*d*f*sinh(1) + 3*(b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f*
sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*d*f^2*x + b^4*d*f*cosh(
1) + b^4*d*f*sinh(1))*cosh(d*x + c)^3 + (b^4*d*f^2*x + b^4*d*f*cosh(1) + b^
4*d*f*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sin
h(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)
/b + 1) - 4*((a^4 + 2*a^2*b^2 + b^4)*d*f^2*x + ((a^4 + 2*a^2*b^2 + b^4)*d*f
^2*x + (a^4 + 2*a^2*b^2 + b^4)*d*f*cosh(1) + (a^4 + 2*a^2*b^2 + b^4)*d*f*si
nh(1))*cosh(d*x + c)^4 + 4*((a^4 + 2*a^2*b^2 + b^4)*d*f^2*x + (a^4 + 2*a^2*
b^2 + b^4)*d*f*cosh(1) + (a^4 + 2*a^2*b^2 + b^4)*d*f*sinh(1))*cosh(d*x + c)
*sinh(d*x + c)^3 + ((a^4 + 2*a^2*b^2 + b^4)*d*f^2*x + (a^4 + 2*a^2*b^2 + b^
4)*d*f*cosh(1) + (a^4 + 2*a^2*b^2 + b^4)*d*f*sinh(1))*sinh(d*x + c)^4 + (a^
4 + 2*a^2*b^2 + b^4)*d*f*cosh(1) + (a^4 + 2*a^2*b^2 + b^4)*d*f*sinh(1) + 2*
((a^4 + 2*a^2*b^2 + b^4)*d*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d*f*cosh(1) + (a
^4 + 2*a^2*b^2 + b^4)*d*f*sinh(1))*cosh(d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 +
b^4)*d*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d*f*cosh(1) + (a^4 + 2*a^2*b^2 + b^4
)*d*f*sinh(1) + 3*((a^4 + 2*a^2*b^2 + b^4)*d*f^2*x + (a^4 + 2*a^2*b^2 + b^4
)*d*f*cosh(1) + (a^4 + 2*a^2*b^2 + b^4)*d*f*sinh(1))*cosh(d*x + c)^2)*sinh(
d*x + c)^2 + 4*(((a^4 + 2*a^2*b^2 + b^4)*d*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*
d*f*cosh(1) + (a^4 + 2*a^2*b^2 + b^4)*d*f*sinh(1))*cosh(d*x + c)^3 + ((a^4
+ 2*a^2*b^2 + b^4)*d*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d*f*cosh(1) + (a^4 + 2
*a^2*b^2 + b^4)*d*f*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*dilog(cosh(d*x +
 c) + sinh(d*x + c)) + 2*(2*(a^4 + 2*a^2*b^2)*d*f^2*x + I*(a^3*b + 3*a*b^3)
*d*f^2*x + (2*(a^4 + 2*a^2*b^2)*d*f^2*x + I*(a^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^2}{\cosh\left(c+d\,x\right)^3\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(cosh(c + d*x)^3*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((e + f*x)^2/(cosh(c + d*x)^3*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.446 $\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=746

$$\frac{fx}{2ad}-\frac{2b^3(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d}-\frac{b(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d}-\frac{2fx\tanh^{-1}\left(e^{2c+2dx}\right)}{ad}-\frac{b^4(e+fx)\log\left(1+\frac{\ldots}{a-\ldots}\right.}{a\left(a^2+b^2\right)^2\ldots}$$

[Out] 1/2*f*x/a/d-2*b^3*(f*x+e)*arctan(exp(d*x+c))/(a^2+b^2)^2/d-b*(f*x+e)*arctan(exp(d*x+c))/(a^2+b^2)/d-2*f*x*arctanh(exp(2*d*x+2*c))/a/d+b^4*(f*x+e)*ln(1+exp(2*d*x+2*c))/a/(a^2+b^2)^2/d-b^4*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^2/d-b^4*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^2/d-f*x*ln(tanh(d*x+c))/a/d+(f*x+e)*ln(tanh(d*x+c))/a/d-1/2*I*b*f*polylog(2,I*exp(d*x+c))/(a^2+b^2)/d^2+1/2*I*b*f*polylog(2,-I*exp(d*x+c))/(a^2+b^2)/d^2-I*b^3*f*polylog(2,I*exp(d*x+c))/(a^2+b^2)^2/d^2+I*b^3*f*polylog(2,-I*exp(d*x+c))/(a^2+b^2)^2/d^2+1/2*b^4*f*polylog(2,-exp(2*d*x+2*c))/a/(a^2+b^2)^2/d^2-1/2*f*polylog(2,-exp(2*d*x+2*c))/a/d^2+1/2*f*polylog(2,exp(2*d*x+2*c))/a/d^2-b^4*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a/(a^2+b^2)^2/d^2-b^4*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a/(a^2+b^2)^2/d^2-1/2*b*f*sech(d*x+c)/(a^2+b^2)/d^2-1/2*b^2*(f*x+e)*sech(d*x+c)^2/a/(a^2+b^2)/d-1/2*f*tanh(d*x+c)/a/d^2+1/2*b^2*f*tanh(d*x+c)/a/(a^2+b^2)/d^2-1/2*b*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a^2+b^2)/d-1/2*(f*x+e)*tanh(d*x+c)^2/a/d

Rubi [A]
time = 0.88, antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 20, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5708, 2700, 14, 5570, 2628, 12, 4267, 2317, 2438, 3554, 8, 5692, 5680, 2221, 6874, 4265, 3799, 4270, 5559, 3852}

$$-\frac{b(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{d\left(a^2+b^2\right)}-\frac{2b^3(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{d\left(a^2+b^2\right)^2}+\frac{ibf\text{Li}_2\left(-ie^{c+dx}\right)}{2d^2\left(a^2+b^2\right)}-\frac{ibf\text{Li}_2\left(ie^{c+dx}\right)}{2d^2\left(a^2+b^2\right)}+\frac{b^2f\tanh(c+dx)}{2ad^2\left(a^2+b^2\right)}-\frac{bf\text{sech}(c+dx)}{2d^2\left(a^2+b^2\right)}-\frac{b^2(e+fx)\text{sech}^2(c+dx)}{2ad\left(a^2+b^2\right)}-\frac{b(e+fx)\tanh(c+dx)\text{sech}(c+dx)}{2d\left(a^2+b^2\right)}-\frac{b^4f\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\left(a^2+b^2\right)^2}-\frac{b^4f\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad^2\left(a^2+b^2\right)^2}+\frac{b^4f\text{Li}_2\left(-e^{2(c+dx)}\right)}{2ad^2\left(a^2+b^2\right)^2}-\frac{b^4(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{ad\left(a^2+b^2\right)^2}-\frac{b^4(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{ad\left(a^2+b^2\right)^2}+\frac{b^4(e+fx)\log\left(e^{2(c+dx)}+1\right)}{ad\left(a^2+b^2\right)^2}+\frac{ib^3f\text{Li}_2\left(-ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)^2}-\frac{ib^3f\text{Li}_2\left(ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)^2}-\frac{f\text{Li}_2\left(-e^{2c+2dx}\right)}{2ad^2}+\frac{f\text{Li}_2\left(e^{2c+2dx}\right)}{2ad^2}-\frac{f\tanh(c+dx)}{2ad^2}-\frac{(e+fx)\tanh^2(c+dx)}{2ad}+\frac{(e+fx)\log(\tanh(c+dx))}{ad}-\frac{2fx\tanh^{-1}\left(e^{2c+2dx}\right)}{ad}-\frac{fx\log(\tanh(c+dx))}{ad}+\frac{fx}{2ad}$$

Antiderivative was successfully verified.

[In] Int[((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]

[Out] (f*x)/(2*a*d) - (2*b^3*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)^2*d) - (b*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) - (2*f*x*ArcTanh[E^(2*c + 2*d*x)])/(a*d) - (b^4*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^2*d) - (b^4*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^2*d) + (b^4*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a*(a^2 + b^2)^2*d) - (f*x*Log[Tanh[c + d*x]])/(a*d) + ((e + f*x)*Log[Tanh[c + d*x]])/(a*d) + (I*b^3*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) + ((I/2)*b*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) - (I*b^3*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - ((I/2)*b*f*PolyLog[2, I

```
*E^(c + d*x)])/((a^2 + b^2)*d^2) - (b^4*f*PolyLog[2, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^2*d^2) - (b^4*f*PolyLog[2, -((b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^2*d^2) + (b^4*f*PolyLog[2, -E
^(2*(c + d*x))])/(2*a*(a^2 + b^2)^2*d^2) - (f*PolyLog[2, -E^(2*c + 2*d*x)])
/(2*a*d^2) + (f*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d^2) - (b*f*Sech[c + d*x]
)/(2*(a^2 + b^2)*d^2) - (b^2*(e + f*x)*Sech[c + d*x]^2)/(2*a*(a^2 + b^2)*d)
 - (f*Tanh[c + d*x])/(2*a*d^2) + (b^2*f*Tanh[c + d*x])/(2*a*(a^2 + b^2)*d^2
) - (b*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*(a^2 + b^2)*d) - ((e + f*x
)*Tanh[c + d*x]^2)/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
```

```
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5708

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}}{a}$$

$$= \frac{(e+fx)\log(\tanh(c+dx))}{ad} - \frac{(e+fx)\tanh^2(c+dx)}{2ad} - \frac{b\int (e}{}$$

$$= \frac{(e+fx)\log(\tanh(c+dx))}{ad} - \frac{(e+fx)\tanh^2(c+dx)}{2ad} - \frac{b^3\int (e}{}$$

$$= \frac{b^4(e+fx)^2}{2a\left(a^2+b^2\right)^2 f} - \frac{fx\log(\tanh(c+dx))}{ad} + \frac{(e+fx)\log(\tanh(c}{ad}$$

$$= \frac{fx}{2ad} + \frac{b^4(e+fx)^2}{2a\left(a^2+b^2\right)^2 f} - \frac{b^4(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^2 d} -$$

$$= \frac{fx}{2ad} - \frac{2b^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right) d} -$$

$$= \frac{fx}{2ad} - \frac{2b^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right) d} -$$

$$= \frac{fx}{2ad} - \frac{2b^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right) d} -$$

$$= \frac{fx}{2ad} - \frac{2b^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right) d} -$$

Mathematica [A]
time = 9.06, size = 886, normalized size = 1.19

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (e*Log[Sinh[c + d*x]])/(a*d) - (c*f*Log[Sinh[c + d*x]])/(a*d^2) - (I*f*(I*(
c + d*x)*Log[1 - E^(-2*(c + d*x))] - (I/2)*(-(c + d*x)^2 + PolyLog[2, E^(-2
*(c + d*x))])))/(a*d^2) - (b^4*(-1/2*(f*(c + d*x)^2) + f*(c + d*x)*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2])] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sin
h[c + d*x]] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*Poly
Log[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a*(a^2 + b^2)^2*d^2) -
(-2*a^3*d*e*(c + d*x) - 4*a*b^2*d*e*(c + d*x) + 2*a^3*c*f*(c + d*x) + 4*a*b
^2*c*f*(c + d*x) - a^3*f*(c + d*x)^2 - 2*a*b^2*f*(c + d*x)^2 + 2*a^2*b*d*e*
ArcTan[E^(c + d*x)] + 6*b^3*d*e*ArcTan[E^(c + d*x)] - 2*a^2*b*c*f*ArcTan[E^
(c + d*x)] - 6*b^3*c*f*ArcTan[E^(c + d*x)] + I*a^2*b*f*(c + d*x)*Log[1 - I*
E^(c + d*x)] + (3*I)*b^3*f*(c + d*x)*Log[1 - I*E^(c + d*x)] - I*a^2*b*f*(c
+ d*x)*Log[1 + I*E^(c + d*x)] - (3*I)*b^3*f*(c + d*x)*Log[1 + I*E^(c + d*x)
] + 2*a^3*d*e*Log[1 + E^(2*(c + d*x))] + 4*a*b^2*d*e*Log[1 + E^(2*(c + d*x)
)] - 2*a^3*c*f*Log[1 + E^(2*(c + d*x))] - 4*a*b^2*c*f*Log[1 + E^(2*(c + d*x
))] + 2*a^3*f*(c + d*x)*Log[1 + E^(2*(c + d*x))] + 4*a*b^2*f*(c + d*x)*Log[
1 + E^(2*(c + d*x))] - I*b*(a^2 + 3*b^2)*f*PolyLog[2, (-I)*E^(c + d*x)] + I
*b*(a^2 + 3*b^2)*f*PolyLog[2, I*E^(c + d*x)] + a^3*f*PolyLog[2, -E^(2*(c +
d*x))] + 2*a*b^2*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^2) + (S
ech[c + d*x]*(-(b*f) - a*f*Sinh[c + d*x]))/(2*(a^2 + b^2)*d^2) + (Sech[c +
d*x]^2*(a*d*e - a*c*f + a*f*(c + d*x) - b*d*e*Sinh[c + d*x] + b*c*f*Sinh[c
+ d*x] - b*f*(c + d*x)*Sinh[c + d*x]))/(2*(a^2 + b^2)*d^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2579 vs. 2(693) = 1386.
time = 8.08, size = 2580, normalized size = 3.46

method	result	size
risch	Expression too large to display	2580

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/d/(a^2+b^2)*b^2*f/a*ln(exp(d*x+c)+1)*x+4/d^2/(a^2+b^2)*a^2*f*c/(4*a^2+4*b
^2)*b*arctan(exp(d*x+c))-8/d/(a^2+b^2)*b^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c
))*a*x-8/d^2/(a^2+b^2)*b^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*a*c-8/d/(a^2+
b^2)*b^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*a*x-8/d^2/(a^2+b^2)*b^2*f/(4*a^
2+4*b^2)*ln(1+I*exp(d*x+c))*a*c-1/2/d^2/(a^2+b^2)^(5/2)*a^2*b^2*f*c*arctanh
(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+8/d^2/(a^2+b^2)*b^2*f*c/(4*a^2+4
*b^2)*a*ln(1+exp(2*d*x+2*c))+2*I/d^2/(a^2+b^2)*a^2*f/(4*a^2+4*b^2)*dilog(1+
I*exp(d*x+c))*b-2*I/d^2/(a^2+b^2)*a^2*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))
*b+6*I/d/(a^2+b^2)*b^3*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x+6*I/d^2/(a^2+b^
```

```
2)*b^3*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*c-6*I/d/(a^2+b^2)*b^3*f/(4*a^2+4*
b^2)*ln(1-I*exp(d*x+c))*x-6*I/d^2/(a^2+b^2)*b^3*f/(4*a^2+4*b^2)*ln(1-I*exp(
d*x+c))*c-1/d^2/(a^2+b^2)*b^2*f*c/a*ln(exp(d*x+c)-1)+12/d^2/(a^2+b^2)*b^3*f
*c/(4*a^2+4*b^2)*arctan(exp(d*x+c))+1/2/d^2/(a^2+b^2)^(3/2)*b^2*f*c*arctanh
(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d^2/(a^2+b^2)^2*b^4*f*c/a*ln(b
*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-4/d/(a^2+b^2)*a^2*e/(4*a^2+4*b^2)*b*arcta
n(exp(d*x+c))-1/d/(a^2+b^2)^2*b^4*f/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/
(-a+(a^2+b^2)^(1/2)))*x-1/d^2/(a^2+b^2)^2*b^4*f/a*ln((-b*exp(d*x+c)+(a^2+b^
2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d/(a^2+b^2)^2*b^4*f/a*ln((b*exp(d*x+c
)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/2/d/(a^2+b^2)^(5/2)*e*a^2*b^2
*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-8/d/(a^2+b^2)*b^2*e/(4*a
^2+4*b^2)*a*ln(1+exp(2*d*x+2*c))-6*I/d^2/(a^2+b^2)*b^3*f/(4*a^2+4*b^2)*dilo
g(1-I*exp(d*x+c))+6*I/d^2/(a^2+b^2)*b^3*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c
))-4/d^2/(a^2+b^2)*a^3*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))-1/2/d/(a^2+b^2
)^(3/2)*b^2*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-4/d/(a^2+b^
2)*a^3*e/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))-1/d^2/(a^2+b^2)^2*b^4*f/a*dilog
((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2/(a^2+b^2)^2*
b^4*f/a*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d/(a^
2+b^2)^2*b^4*e/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/2/d/(a^2+b^2)^(5/2
)*b^4*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-12/d/(a^2+b^2)*b^
3*e/(4*a^2+4*b^2)*arctan(exp(d*x+c))-4/d^2/(a^2+b^2)*a^3*f/(4*a^2+4*b^2)*di
log(1+I*exp(d*x+c))+1/d^2/(a^2+b^2)*b^2*f/a*dilog(exp(d*x+c)+1)-1/d^2/(a^2+
b^2)*b^2*f*dilog(exp(d*x+c))/a+2*I/d/(a^2+b^2)*a^2*f/(4*a^2+4*b^2)*ln(1+I*e
xp(d*x+c))*b*x+2*I/d^2/(a^2+b^2)*a^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*b*c
-2*I/d/(a^2+b^2)*a^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*b*x-2*I/d^2/(a^2+b^
2)*a^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*b*c-1/d^2/(a^2+b^2)^2*b^4*f/a*ln(
(b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-4/d/(a^2+b^2)*a^3*f
/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x-4/d^2/(a^2+b^2)*a^3*f/(4*a^2+4*b^2)*ln(
1+I*exp(d*x+c))*c-4/d/(a^2+b^2)*a^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x-4/
d^2/(a^2+b^2)*a^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*c-8/d^2/(a^2+b^2)*b^2*
f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))*a-8/d^2/(a^2+b^2)*b^2*f/(4*a^2+4*b^2)
*dilog(1-I*exp(d*x+c))*a+4/d^2/(a^2+b^2)*a^3*f*c/(4*a^2+4*b^2)*ln(1+exp(2*d
*x+2*c))-1/2/d^2/(a^2+b^2)^(5/2)*b^4*f*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(
a^2+b^2)^(1/2))-1/d^2/(a^2+b^2)*a*f*c*ln(exp(d*x+c)-1)+1/d/(a^2+b^2)*b^2*e/
a*ln(exp(d*x+c)+1)+1/d/(a^2+b^2)*b^2*e/a*ln(exp(d*x+c)-1)+1/d/(a^2+b^2)*ln(
exp(d*x+c)+1)*a*f*x+(-b*d*f*x*exp(3*d*x+3*c)+2*a*d*f*x*exp(2*d*x+2*c)-b*d*e
*exp(3*d*x+3*c)+2*a*d*e*exp(2*d*x+2*c)+b*d*f*x*exp(d*x+c)-b*f*exp(3*d*x+3*c
)+a*f*exp(2*d*x+2*c)+b*d*e*exp(d*x+c)-f*b*exp(d*x+c)+f*a)/d^2/(a^2+b^2)/(1+
exp(2*d*x+2*c))^2+1/d/(a^2+b^2)*a*e*ln(exp(d*x+c)+1)+1/d/(a^2+b^2)*a*e*ln(e
xp(d*x+c)-1)+1/d^2/(a^2+b^2)*a*f*dilog(exp(d*x+c)+1)-1/d^2/(a^2+b^2)*a*f*di
log(exp(d*x+c))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -f*(((b*d*x*e^(3*c) + b*e^(3*c))*e^(3*d*x) - (2*a*d*x*e^(2*c) + a*e^(2*c))*
e^(2*d*x) - (b*d*x*e^c - b*e^c)*e^(d*x) - a)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*
e^(4*c) + b^2*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c)
)*e^(2*d*x)) - 16*integrate(-1/8*(a*b^4*x*e^(d*x + c) - b^5*x)/(a^5*b + 2*a
^3*b^3 + a*b^5 - (a^5*b*e^(2*c) + 2*a^3*b^3*e^(2*c) + a*b^5*e^(2*c))*e^(2*d
*x) - 2*(a^6*e^c + 2*a^4*b^2*e^c + a^2*b^4*e^c)*e^(d*x)), x) + 16*integrate
(1/16*((a^2*b*e^c + 3*b^3*e^c)*x*e^(d*x) - 2*(a^3 + 2*a*b^2)*x)/(a^4 + 2*a^
2*b^2 + b^4 + (a^4*e^(2*c) + 2*a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x
) + 16*integrate(1/16*x/(a*e^(d*x + c) + a), x) - 16*integrate(1/16*x/(a*e^
(d*x + c) - a), x)) - (b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/
((a^5 + 2*a^3*b^2 + a*b^4)*d) - (a^2*b + 3*b^3)*arctan(e^(-d*x - c))/((a^4
+ 2*a^2*b^2 + b^4)*d) + (a^3 + 2*a*b^2)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2
*a^2*b^2 + b^4)*d) + (b*e^(-d*x - c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x -
 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x
 - 4*c))*d) - log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d))*e
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8973 vs. $2(685) = 1370$.
time = 0.62, size = 8973, normalized size = 12.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -1/2*(2*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*cosh(1) + (a^3*b + a*b^3
)*d*sinh(1) + (a^3*b + a*b^3)*f)*cosh(d*x + c)^3 + 2*((a^3*b + a*b^3)*d*f*x
 + (a^3*b + a*b^3)*d*cosh(1) + (a^3*b + a*b^3)*d*sinh(1) + (a^3*b + a*b^3)*
f)*sinh(d*x + c)^3 - 2*(2*(a^4 + a^2*b^2)*d*f*x + 2*(a^4 + a^2*b^2)*d*cosh(
1) + 2*(a^4 + a^2*b^2)*d*sinh(1) + (a^4 + a^2*b^2)*f)*cosh(d*x + c)^2 - 2*(
2*(a^4 + a^2*b^2)*d*f*x + 2*(a^4 + a^2*b^2)*d*cosh(1) + 2*(a^4 + a^2*b^2)*d
*sinh(1) + (a^4 + a^2*b^2)*f - 3*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d
*cosh(1) + (a^3*b + a*b^3)*d*sinh(1) + (a^3*b + a*b^3)*f)*cosh(d*x + c))*si
nh(d*x + c)^2 - 2*(a^4 + a^2*b^2)*f - 2*((a^3*b + a*b^3)*d*f*x + (a^3*b + a
*b^3)*d*cosh(1) + (a^3*b + a*b^3)*d*sinh(1) - (a^3*b + a*b^3)*f)*cosh(d*x +
 c) + 2*(b^4*f*cosh(d*x + c)^4 + 4*b^4*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^
4*f*sinh(d*x + c)^4 + 2*b^4*f*cosh(d*x + c)^2 + b^4*f + 2*(3*b^4*f*cosh(d*x
 + c)^2 + b^4*f)*sinh(d*x + c)^2 + 4*(b^4*f*cosh(d*x + c)^3 + b^4*f*cosh(d*
x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b^4*f*cos
```

h(d*x + c)^4 + 4*b^4*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^4*f*sinh(d*x + c)^
4 + 2*b^4*f*cosh(d*x + c)^2 + b^4*f + 2*(3*b^4*f*cosh(d*x + c)^2 + b^4*f)*s
inh(d*x + c)^2 + 4*(b^4*f*cosh(d*x + c)^3 + b^4*f*cosh(d*x + c))*sinh(d*x +
 c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*((a^4 + 2*a^2*b^2 + b^4)*f*c
osh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c)*sinh(d*x + c)^3
 + (a^4 + 2*a^2*b^2 + b^4)*f*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*f*c
osh(d*x + c)^2 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c)^2 + (a^4 + 2*
a^2*b^2 + b^4)*f)*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*f + 4*((a^4 + 2
*a^2*b^2 + b^4)*f*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c)
)*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)) + ((2*(a^4 + 2*a^2*b^
2)*f + I*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c)^4 + 4*(2*(a^4 + 2*a^2*b^2)*f +
I*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*(a^4 + 2*a^2*b^2)
f + I(a^3*b + 3*a*b^3)*f)*sinh(d*x + c)^4 + 2*(2*(a^4 + 2*a^2*b^2)*f + I*
(a^3*b + 3*a*b^3)*f)*cosh(d*x + c)^2 + 2*(3*(2*(a^4 + 2*a^2*b^2)*f + I*(a^3
*b + 3*a*b^3)*f)*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2)*f + I*(a^3*b + 3*a*b
^3)*f)*sinh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2)*f + I*(a^3*b + 3*a*b^3)*f + 4*
((2*(a^4 + 2*a^2*b^2)*f + I*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c)^3 + (2*(a^4
 + 2*a^2*b^2)*f + I*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c))*sinh(d*x + c))*dilog
(I*cosh(d*x + c) + I*sinh(d*x + c)) + ((2*(a^4 + 2*a^2*b^2)*f - I*(a^3*b +
3*a*b^3)*f)*cosh(d*x + c)^4 + 4*(2*(a^4 + 2*a^2*b^2)*f - I*(a^3*b + 3*a*b^3
)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*(a^4 + 2*a^2*b^2)*f - I*(a^3*b + 3*
a*b^3)*f)*sinh(d*x + c)^4 + 2*(2*(a^4 + 2*a^2*b^2)*f - I*(a^3*b + 3*a*b^3)*
f)*cosh(d*x + c)^2 + 2*(3*(2*(a^4 + 2*a^2*b^2)*f - I*(a^3*b + 3*a*b^3)*f)*c
osh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2)*f - I*(a^3*b + 3*a*b^3)*f)*sinh(d*x +
c)^2 + 2*(a^4 + 2*a^2*b^2)*f - I*(a^3*b + 3*a*b^3)*f + 4*((2*(a^4 + 2*a^2*b
^2)*f - I*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c)^3 + (2*(a^4 + 2*a^2*b^2)*f - I
*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(-I*cosh(d*x + c)
 - I*sinh(d*x + c)) - 2*((a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c)^4 + 4*(a^4
 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4
)*f*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c)^2 + 2*(3*(a
^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*f)*sinh(d
*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*f + 4*((a^4 + 2*a^2*b^2 + b^4)*f*cosh(d
*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c))*sinh(d*x + c))*dilog(-
cosh(d*x + c) - sinh(d*x + c)) - 2*(b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1)
 + (b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*cosh(d*x + c)^4 + 4*(b^4*c*f -
 b^4*d*cosh(1) - b^4*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*c*f -
b^4*d*cosh(1) - b^4*d*sinh(1))*sinh(d*x + c)^4 + 2*(b^4*c*f - b^4*d*cosh(1)
 - b^4*d*sinh(1))*cosh(d*x + c)^2 + 2*(b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh
(1) + 3*(b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 + 4*((b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*cosh(d*x + c)^3 + (b
^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*log(2
*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2
*(b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1) + (b^4*c*f - b^4*d*cosh(1) - b^4*
d*sinh(1))*cosh(d*x + c)^4 + 4*(b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*co

```
sh(d*x + c)*sinh(d*x + c)^3 + (b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*sin
h(d*x + c)^4 + 2*(b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*cosh(d*x + c)^2
+ 2*(b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1) + 3*(b^4*c*f - b^4*d*cosh(1) -
 b^4*d*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*c*f - b^4*d*cosh
(1) - b^4*d*sinh(1))*cosh(d*x + c)^3 + (b^4*c*f - b^4*d*cosh(1) - b^4*d*sin
h(1))*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(b^4*...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6439 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{{\cosh\left(c+d\,x\right)}^3\,\sinh\left(c+d\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(cosh(c + d*x)^3*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((e + f*x)/(cosh(c + d*x)^3*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.447 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=160

$$-\frac{b^3\text{ArcTan}(\sinh(c+dx))}{(a^2+b^2)^2 d}-\frac{b\text{ArcTan}(\sinh(c+dx))}{2(a^2+b^2)d}-\frac{a(a^2+2b^2)\log(\cosh(c+dx))}{(a^2+b^2)^2 d}+\frac{\log(\sinh(c+dx))}{ad}-\frac{b^4}{}$$

```
[Out] -b^3*arctan(sinh(d*x+c))/(a^2+b^2)^2/d-1/2*b*arctan(sinh(d*x+c))/(a^2+b^2)/
d-a*(a^2+2*b^2)*ln(cosh(d*x+c))/(a^2+b^2)^2/d+ln(sinh(d*x+c))/a/d-b^4*ln(a+
b*sinh(d*x+c))/a/(a^2+b^2)^2/d+1/2*sech(d*x+c)^2*(a-b*sinh(d*x+c))/(a^2+b^2
)/d
```

Rubi [A]
time = 0.19, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2916, 12, 908, 653, 209, 649, 266}

$$-\frac{b\text{ArcTan}(\sinh(c+dx))}{2d(a^2+b^2)}-\frac{b^3\text{ArcTan}(\sinh(c+dx))}{d(a^2+b^2)^2}-\frac{a(a^2+2b^2)\log(\cosh(c+dx))}{d(a^2+b^2)^2}+\frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))}{2d(a^2+b^2)}-\frac{b^4\log(a+b\sinh(c+dx))}{ad(a^2+b^2)^2}+\frac{\log(\sinh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((b^3*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)^2*d)) - (b*ArcTan[Sinh[c + d*x]]
)/(2*(a^2 + b^2)*d) - (a*(a^2 + 2*b^2)*Log[Cosh[c + d*x]])/((a^2 + b^2)^2*d
) + Log[Sinh[c + d*x]]/(a*d) - (b^4*Log[a + b*Sinh[c + d*x]])/(a*(a^2 + b^2
)^2*d) + (Sech[c + d*x]^2*(a - b*Sinh[c + d*x]))/(2*(a^2 + b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b^3\text{Subst}\left(\int \frac{b}{x(a+x)(-b^2-x^2)^2}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= \frac{b^4\text{Subst}\left(\int \frac{1}{x(a+x)(-b^2-x^2)^2}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= \frac{b^4\text{Subst}\left(\int \left(\frac{1}{ab^4x} - \frac{1}{a(a^2+b^2)^2(a+x)} + \frac{-b^2-ax}{b^2(a^2+b^2)(b^2+x^2)^2} + \frac{-b^4-a(a^2+2b^2)x}{b^4(a^2+b^2)^2(b^2+x^2)}\right) dx}{d}$$

$$= \frac{\log(\sinh(c+dx))}{ad} - \frac{b^4\log(a+b\sinh(c+dx))}{a(a^2+b^2)^2 d} + \frac{\text{Subst}\left(\int \frac{-b^4-a(a^2+2b^2)x}{b^2+x^2}\right.}{(a^2+}$$

$$= \frac{\log(\sinh(c+dx))}{ad} - \frac{b^4\log(a+b\sinh(c+dx))}{a(a^2+b^2)^2 d} + \frac{\operatorname{sech}^2(c+dx)(a-b\sin}{2(a^2+b^2)}$$

$$= -\frac{b^3\tan^{-1}(\sinh(c+dx))}{(a^2+b^2)^2 d} - \frac{b\tan^{-1}(\sinh(c+dx))}{2(a^2+b^2)d} - \frac{a(a^2+2b^2)\log(\cos}{(a^2+b^2)^2}$$

Mathematica [A]
time = 0.59, size = 196, normalized size = 1.22

$$-\frac{ab(a^2+b^2)\text{ArcTan}(\sinh(c+dx)) - 2(a^2+b^2)^2\log(\sinh(c+dx)) + a\left(a^3+2ab^2+(-b^2)^{3/2}\right)\log\left(\sqrt{-b^2}-b\sinh(c+dx)\right) + 2b^4\log(a+b\sinh(c+dx)) + a\left(a^3+2ab^2-(-b^2)^{3/2}\right)\log\left(\sqrt{-b^2}+b\sinh(c+dx)\right) - a^2(a^2+b^2)\operatorname{sech}^2(c+dx) + ab(a^2+b^2)\operatorname{sech}(c+dx)\tanh(c+dx)}{2a(a^2+b^2)^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]

[Out] -1/2*(a*b*(a^2 + b^2)*ArcTan[Sinh[c + d*x]] - 2*(a^2 + b^2)^2*Log[Sinh[c +
d*x]] + a*(a^3 + 2*a*b^2 + (-b^2)^(3/2))*Log[Sqrt[-b^2] - b*Sinh[c + d*x]]
+ 2*b^4*Log[a + b*Sinh[c + d*x]] + a*(a^3 + 2*a*b^2 - (-b^2)^(3/2))*Log[Sqr
t[-b^2] + b*Sinh[c + d*x]] - a^2*(a^2 + b^2)*Sech[c + d*x]^2 + a*b*(a^2 + b
^2)*Sech[c + d*x]*Tanh[c + d*x])/(a*(a^2 + b^2)^2*d)
```

Maple [A]
time = 1.76, size = 230, normalized size = 1.44

method	result
derivativedivides	$$\frac{-\frac{b^4\ln\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a\left(a^4+2a^2b^2+b^4\right)} - 2\left(\frac{\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^3+ab^2\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^2b+\frac{1}{2}b\right.}{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}\right.}{d}$$
default	$$\frac{-\frac{b^4\ln\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a\left(a^4+2a^2b^2+b^4\right)} - 2\left(\frac{\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^3+ab^2\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^2b+\frac{1}{2}b\right.}{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}\right.}{d}$$

risch	$\frac{2a^3d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{2a^3dc}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{4a\,b^2d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{4a\,b^2dc}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{2x}{a}-\frac{2c}{ad}+\frac{}{a(a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b^4/a/(a^4+2*a^2*b^2+b^4)*ln(a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x
+1/2*c)-a)-2/(a^4+2*a^2*b^2+b^4)*(((-1/2*a^2*b-1/2*b^3)*tanh(1/2*d*x+1/2*c)
^3+(a^3+a*b^2)*tanh(1/2*d*x+1/2*c)^2+(1/2*a^2*b+1/2*b^3)*tanh(1/2*d*x+1/2*c
))/(tanh(1/2*d*x+1/2*c)^2+1)^2+1/4*(2*a^3+4*a*b^2)*ln(tanh(1/2*d*x+1/2*c)^2
+1)+1/2*(a^2*b+3*b^3)*arctan(tanh(1/2*d*x+1/2*c)))+1/a*ln(tanh(1/2*d*x+1/2*
c)))
```

Maxima [**A**]
time = 0.49, size = 265, normalized size = 1.66

$$-\frac{b^4\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^5+2\,a^3b^2+ab^4)d}+\frac{(a^2b+3\,b^3)\arctan\left(e^{(-dx-c)}\right)}{(a^4+2\,a^2b^2+b^4)d}-\frac{(a^3+2\,ab^2)\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^4+2\,a^2b^2+b^4)d}-\frac{be^{(-dx-c)}-2\,ae^{(-2\,dx-2\,c)}-be^{(-3\,dx-3\,c)}}{(a^2+b^2+2\,(a^2+b^2)e^{(-2\,dx-2\,c)}+(a^2+b^2)e^{(-4\,dx-4\,c)})d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}+\frac{\log\left(e^{(-dx-c)}-1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + 2*a^3*b^2 + a*
b^4)*d) + (a^2*b + 3*b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d)
- (a^3 + 2*a*b^2)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (
b*e^(-d*x - c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2
*(a^2 + b^2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d) + log(e^(-
d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1279 vs. $2(157) = 314$.
time = 0.56, size = 1279, normalized size = 7.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -((a^3*b + a*b^3)*cosh(d*x + c)^3 + (a^3*b + a*b^3)*sinh(d*x + c)^3 - 2*(a^
4 + a^2*b^2)*cosh(d*x + c)^2 - (2*a^4 + 2*a^2*b^2 - 3*(a^3*b + a*b^3)*cosh(
d*x + c))*sinh(d*x + c)^2 + ((a^3*b + 3*a*b^3)*cosh(d*x + c)^4 + 4*(a^3*b +
 3*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b + 3*a*b^3)*sinh(d*x + c)^4
 + a^3*b + 3*a*b^3 + 2*(a^3*b + 3*a*b^3)*cosh(d*x + c)^2 + 2*(a^3*b + 3*a*b
^3 + 3*(a^3*b + 3*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3*b + 3*a
```

```
*b^3)*cosh(d*x + c)^3 + (a^3*b + 3*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*arc
tan(cosh(d*x + c) + sinh(d*x + c)) - (a^3*b + a*b^3)*cosh(d*x + c) + (b^4*c
osh(d*x + c)^4 + 4*b^4*cosh(d*x + c)*sinh(d*x + c)^3 + b^4*sinh(d*x + c)^4
+ 2*b^4*cosh(d*x + c)^2 + b^4 + 2*(3*b^4*cosh(d*x + c)^2 + b^4)*sinh(d*x +
c)^2 + 4*(b^4*cosh(d*x + c)^3 + b^4*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*
sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + ((a^4 + 2*a^2*b^2)*co
sh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 +
2*a^2*b^2)*sinh(d*x + c)^4 + a^4 + 2*a^2*b^2 + 2*(a^4 + 2*a^2*b^2)*cosh(d*x
 + c)^2 + 2*(a^4 + 2*a^2*b^2 + 3*(a^4 + 2*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*
x + c)^2 + 4*((a^4 + 2*a^2*b^2)*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2)*cosh(d*
x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)))
 - ((a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*cos
h(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*sinh(d*x + c)^4 + a^4
+ 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(a^4 + 2*
a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 4*((a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*cosh
(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c
))) - (a^3*b + a*b^3 - 3*(a^3*b + a*b^3)*cosh(d*x + c)^2 + 4*(a^4 + a^2*b^2
)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c)^
4 + 4*(a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^5 + 2*
a^3*b^2 + a*b^4)*d*sinh(d*x + c)^4 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x
 + c)^2 + 2*(3*(a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c)^2 + (a^5 + 2*a^3*b
^2 + a*b^4)*d)*sinh(d*x + c)^2 + (a^5 + 2*a^3*b^2 + a*b^4)*d + 4*((a^5 + 2*
a^3*b^2 + a*b^4)*d*cosh(d*x + c)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x +
 c))*sinh(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3436 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(157) = 314.
time = 0.46, size = 343, normalized size = 2.14

$$-\frac{\frac{4 b^5 \log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2 a\right|\right)}{a^5 b+2 a^3 b^3+a b^5}+\frac{\left(\pi+2 \arctan\left(\frac{1}{2}\left(e^{(2 dx+2 c)}-1\right)e^{(-dx-c)}\right)\right)\left(a^2 b+3 b^3\right)}{a^4+2 a^2 b^2+b^4}+\frac{2\left(a^3+2 a b^2\right)\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^4+2 a^2 b^2+b^4}-\frac{4 \log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a}-\frac{2\left(a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+2 a b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-2 a^2 b\left(e^{(dx+c)}-e^{(-dx-c)}\right)-2 b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)+8 a^3+12 a b^2\right)}{\left(a^4+2 a^2 b^2+b^4\right)\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(4*b^5*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^5*b + 2*a^3*b
^3 + a*b^5) + (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(a^2*
b + 3*b^3)/(a^4 + 2*a^2*b^2 + b^4) + 2*(a^3 + 2*a*b^2)*log((e^(d*x + c) - e
^(-d*x - c))^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) - 4*log(abs(e^(d*x + c) - e^(-d
*x - c)))/a - 2*(a^3*(e^(d*x + c) - e^(-d*x - c))^2 + 2*a*b^2*(e^(d*x + c)
- e^(-d*x - c))^2 - 2*a^2*b*(e^(d*x + c) - e^(-d*x - c)) - 2*b^3*(e^(d*x +
c) - e^(-d*x - c)) + 8*a^3 + 12*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*((e^(d*x +
c) - e^(-d*x - c))^2 + 4)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^3\,\sinh\left(c+d\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(1/(cosh(c + d*x)^3*sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.448 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=37

$$\operatorname{Int}\left(\frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csch[c + d*x]*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][(Csch[c + d*x]*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 123.54, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csch[c + d*x]*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
[Out] Integrate[(Csch[c + d*x]*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x]))
, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)^3}{(fx+e)\,(a+b\sinh{(dx+c)})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -(a*f + (b*d*f*x*e^(3*c) - b*f*e^(3*c) + b*d*e^(3*c + 1))*e^(3*d*x) - (2*a*
d*f*x*e^(2*c) - a*f*e^(2*c) + 2*a*d*e^(2*c + 1))*e^(2*d*x) - (b*d*f*x*e^c +
 b*d*e^(c + 1) + b*f*e^c)*e^(d*x))/((a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*(a^
2*d^2*f + b^2*d^2*f)*x*e + (a^2*d^2 + b^2*d^2)*e^2 + ((a^2*d^2*f^2*e^(4*c)
+ b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^2*d^2*f*e^(4*c) + b^2*d^2*f*e^(4*c))*x*e
+ (a^2*d^2*e^(4*c) + b^2*d^2*e^(4*c))*e^2)*e^(4*d*x) + 2*((a^2*d^2*f^2*e^(2
*c) + b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^2*d^2*f*e^(2*c) + b^2*d^2*f*e^(2*c))*
x*e + (a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^2)*e^(2*d*x)) - 16*integrate(1/
16*(2*a^3*f^2 + 2*a*b^2*f^2 - 2*(a^3*d^2*f^2 + 2*a*b^2*d^2*f^2)*x^2 - 4*(a^
3*d^2*f + 2*a*b^2*d^2*f)*x*e - 2*(a^3*d^2 + 2*a*b^2*d^2)*e^2 - (2*a^2*b*f^2
*e^c + 2*b^3*f^2*e^c - (a^2*b*d^2*f^2*e^c + 3*b^3*d^2*f^2*e^c)*x^2 - 2*(a^2
*b*d^2*f*e^c + 3*b^3*d^2*f*e^c)*x*e - (a^2*b*d^2*e^c + 3*b^3*d^2*e^c)*e^2)*
e^(d*x))/((a^4*d^2*f^3 + 2*a^2*b^2*d^2*f^3 + b^4*d^2*f^3)*x^3 + 3*(a^4*d^2*
f^2 + 2*a^2*b^2*d^2*f^2 + b^4*d^2*f^2)*x^2*e + 3*(a^4*d^2*f + 2*a^2*b^2*d^2
*f + b^4*d^2*f)*x*e^2 + (a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2)*e^3 + ((a^4*d^2
*f^3*e^(2*c) + 2*a^2*b^2*d^2*f^3*e^(2*c) + b^4*d^2*f^3*e^(2*c))*x^3 + 3*(a^
4*d^2*f^2*e^(2*c) + 2*a^2*b^2*d^2*f^2*e^(2*c) + b^4*d^2*f^2*e^(2*c))*x^2*e
+ 3*(a^4*d^2*f*e^(2*c) + 2*a^2*b^2*d^2*f*e^(2*c) + b^4*d^2*f*e^(2*c))*x*e^2
 + (a^4*d^2*e^(2*c) + 2*a^2*b^2*d^2*e^(2*c) + b^4*d^2*e^(2*c))*e^3)*e^(2*d*
x)), x) + 16*integrate(-1/8*(a*b^4*e^(d*x + c) - b^5)/((a^5*b*f + 2*a^3*b^3
*f + a*b^5*f)*x + (a^5*b + 2*a^3*b^3 + a*b^5)*e - ((a^5*b*f*e^(2*c) + 2*a^3
*b^3*f*e^(2*c) + a*b^5*f*e^(2*c))*x + (a^5*b*e^(2*c) + 2*a^3*b^3*e^(2*c) +
a*b^5*e^(2*c))*e)*e^(2*d*x) - 2*((a^6*f*e^c + 2*a^4*b^2*f*e^c + a^2*b^4*f*e
^c)*x + (a^6*e^c + 2*a^4*b^2*e^c + a^2*b^4*e^c)*e)*e^(d*x)), x) - 16*integr
ate(1/16/(a*f*x + a*e + (a*f*x*e^c + a*e^(c + 1))*e^(d*x)), x) + 16*integra
te(-1/16/(a*f*x + a*e - (a*f*x*e^c + a*e^(c + 1))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(csch(d*x + c)*sech(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3437 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cosh\left(c+d\,x\right)^3\,\sinh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(1/(cosh(c + d*x)^3*sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.449 $\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=601

$$-\frac{6f(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad^2}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{ad}+\frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d}+\frac{b(e+fx)^3\log\left(\right.}{}$$

```
[Out] -6*f*(f*x+e)^2*arctanh(exp(d*x+c))/a/d^2-(f*x+e)^3*csch(d*x+c)/a/d-b*(f*x+e
)^3*ln(1-exp(2*d*x+2*c))/a^2/d+b*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(
1/2)))/a^2/d+b*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d-6*f^2
*(f*x+e)*polylog(2,-exp(d*x+c))/a/d^3+6*f^2*(f*x+e)*polylog(2,exp(d*x+c))/a
/d^3-3/2*b*f*(f*x+e)^2*polylog(2,exp(2*d*x+2*c))/a^2/d^2+3*b*f*(f*x+e)^2*po
lylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^2+3*b*f*(f*x+e)^2*polylog(
2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^2+6*f^3*polylog(3,-exp(d*x+c))/a
/d^4-6*f^3*polylog(3,exp(d*x+c))/a/d^4+3/2*b*f^2*(f*x+e)*polylog(3,exp(2*d*
x+2*c))/a^2/d^3-6*b*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2))
)/a^2/d^3-6*b*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/
d^3-3/4*b*f^3*polylog(4,exp(2*d*x+2*c))/a^2/d^4+6*b*f^3*polylog(4,-b*exp(d*
x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^4+6*b*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))/a^2/d^4
```

Rubi [A]

time = 0.70, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {5706, 5560, 4267, 2611, 2320, 6724, 5688, 3797, 2221, 6744, 5680}

$$\frac{6bf^3\text{Li}_4\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^4}+\frac{6bf^3\text{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^4}-\frac{6bf^2(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3}-\frac{6bf^2(e+fx)\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^3}+\frac{3bf(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2}+\frac{3bf(e+fx)^2\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^2}+\frac{b(e+fx)^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^2d}+\frac{b(e+fx)^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{a^2d}-\frac{3bf^3\text{Li}_4\left(e^{2(c+dx)}\right)}{4a^2d^4}+\frac{3bf^2(e+fx)\text{Li}_3\left(e^{2(c+dx)}\right)}{2a^2d^3}-\frac{3bf(e+fx)^2\text{Li}_2\left(e^{2(c+dx)}\right)}{2a^2d^2}-\frac{b(e+fx)^3\log\left(1-e^{2(c+dx)}\right)}{a^2d}+\frac{6f^3\text{Li}_3\left(-e^{c+dx}\right)}{ad^4}-\frac{6f^3\text{Li}_3\left(e^{c+dx}\right)}{ad^4}-\frac{6f^2(e+fx)\text{Li}_2\left(-e^{c+dx}\right)}{ad^3}+\frac{6f^2(e+fx)\text{Li}_2\left(e^{c+dx}\right)}{ad^3}-\frac{6f(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad^2}-\frac{(e+fx)^3\operatorname{csch}(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-6*f*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d^2) - ((e + f*x)^3*Csch[c + d*x
])/(a*d) + (b*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(
a^2*d) + (b*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^
2*d) - (b*(e + f*x)^3*Log[1 - E^(2*(c + d*x))])/(a^2*d) - (6*f^2*(e + f*x)*
PolyLog[2, -E^(c + d*x)])/(a*d^3) + (6*f^2*(e + f*x)*PolyLog[2, E^(c + d*x)
])/(a*d^3) + (3*b*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2]))])/(a^2*d^2) + (3*b*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2]))])/(a^2*d^2) - (3*b*f*(e + f*x)^2*PolyLog[2, E^(2*(c + d*
x))])/(2*a^2*d^2) + (6*f^3*PolyLog[3, -E^(c + d*x)])/(a*d^4) - (6*f^3*PolyL
og[3, E^(c + d*x)])/(a*d^4) - (6*b*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^3) + (3*b*f^2*(e + f*x)*PolyLo
```

```
g[3, E^(2*(c + d*x))])/(2*a^2*d^3) + (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/(a^2*d^4) + (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/(a^2*d^4) - (3*b*f^3*PolyLog[4, E^(2*(c + d*x))])/
(4*a^2*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5688

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x]^
(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5706

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3 \coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3 \coth(c+dx)\text{csch}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3 \coth(c+dx)}{a+b\sinh(c+dx)}}{a}$$

$$= -\frac{(e+fx)^3\text{csch}(c+dx)}{ad} - \frac{b\int (e+fx)^3 \coth(c+dx)\,dx}{a^2} + \frac{b^2}{}$$

$$= -\frac{6f(e+fx)^2 \tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^3\text{csch}(c+dx)}{ad} + \frac{(2b)}{}$$

$$= -\frac{6f(e+fx)^2 \tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^3\text{csch}(c+dx)}{ad} + \frac{b(e}{}$$

$$= -\frac{6f(e+fx)^2 \tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^3\text{csch}(c+dx)}{ad} + \frac{b(e}{}$$

$$= -\frac{6f(e+fx)^2 \tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^3\text{csch}(c+dx)}{ad} + \frac{b(e}{}$$

$$= -\frac{6f(e+fx)^2 \tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^3\text{csch}(c+dx)}{ad} + \frac{b(e}{}$$

$$= -\frac{6f(e+fx)^2 \tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^3\text{csch}(c+dx)}{ad} + \frac{b(e}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.22, size = 7478, normalized size = 12.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] Result too large to show
```

Maple [F]
time = 2.44, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \coth(dx+c)\,\text{csch}(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] (2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) + b*log(-2*a*e^(-d*x - c) + b*
e^(-2*d*x - 2*c) - b)/(a^2*d) - b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(
-d*x - c) - 1)/(a^2*d))*e^3 - 2*(f^3*x^3*e^c + 3*f^2*x^2*e^(c + 1) + 3*f*x*
e^(c + 2))*e^(d*x)/(a*d*e^(2*d*x + 2*c) - a*d) - 3*f*e^2*log(e^(d*x + c) +
1)/(a*d^2) + 3*f*e^2*log(e^(d*x + c) - 1)/(a*d^2) - (d^3*x^3*log(e^(d*x + c
) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6
*polylog(4, -e^(d*x + c)))*b*f^3/(a^2*d^4) - (d^3*x^3*log(-e^(d*x + c) + 1)
 + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polylog
(4, e^(d*x + c)))*b*f^3/(a^2*d^4) - 3*(b*d*f*e^2 + 2*a*f^2*e)*(d*x*log(e^(d
*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) - 3*(b*d*f*e^2 - 2*a*f^2*e)*(
d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^2*d^3) - 3*(b*d*f^2*e +
a*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylo
g(3, -e^(d*x + c)))/(a^2*d^4) - 3*(b*d*f^2*e - a*f^3)*(d^2*x^2*log(-e^(d*x
+ c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))/(a^2*d^4)
 + 1/4*(b*d^4*f^3*x^4 + 4*(b*d*f^2*e + a*f^3)*d^3*x^3 + 6*(b*d^2*f*e^2 + 2*
a*d*f^2*e)*d^2*x^2)/(a^2*d^4) + 1/4*(b*d^4*f^3*x^4 + 4*(b*d*f^2*e - a*f^3)*
d^3*x^3 + 6*(b*d^2*f*e^2 - 2*a*d*f^2*e)*d^2*x^2)/(a^2*d^4) - integrate(-2*(
b^2*f^3*x^3 + 3*b^2*f^2*x^2*e + 3*b^2*f*x*e^2 - (a*b*f^3*x^3*e^c + 3*a*b*f^
2*x^2*e^(c + 1) + 3*a*b*f*x*e^(c + 2))*e^(d*x))/(a^2*b*e^(2*d*x + 2*c) + 2*
a^3*e^(d*x + c) - a^2*b), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7283 vs. $2(574) = 1148$.
time = 0.48, size = 7283, normalized size = 12.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -(2*(a*d^3*f^3*x^3 + 3*a*d^3*f^2*x^2*cosh(1) + 3*a*d^3*f*x*cosh(1)^2 + a*d^
3*cosh(1)^3 + a*d^3*sinh(1)^3 + 3*(a*d^3*f*x + a*d^3*cosh(1))*sinh(1)^2 + 3
*(a*d^3*f^2*x^2 + 2*a*d^3*f*x*cosh(1) + a*d^3*cosh(1)^2)*sinh(1))*cosh(d*x
+ c) + 3*(b*d^2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^2*f*cosh(1)^2 + b*d^2
*f*sinh(1)^2 - (b*d^2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^2*f*cosh(1)^2 +
 b*d^2*f*sinh(1)^2 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh(1))*cosh(d*x +
c)^2 - 2*(b*d^2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^2*f*cosh(1)^2 + b*d^2
*f*sinh(1)^2 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)*sin
h(d*x + c) - (b*d^2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^2*f*cosh(1)^2 + b
*d^2*f*sinh(1)^2 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c)
^2 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh(1))*dilog((a*cosh(d*x + c) + a*
sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
 b)/b + 1) + 3*(b*d^2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^2*f*cosh(1)^2 +
 b*d^2*f*sinh(1)^2 - (b*d^2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^2*f*cosh(
1)^2 + b*d^2*f*sinh(1)^2 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh(1))*cosh(
d*x + c)^2 - 2*(b*d^2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^2*f*cosh(1)^2 +
 b*d^2*f*sinh(1)^2 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh(1))*cosh(d*x +
c)*sinh(d*x + c) - (b*d^2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^2*f*cosh(1)
^2 + b*d^2*f*sinh(1)^2 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh(1))*sinh(d*
x + c)^2 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh(1))*dilog((a*cosh(d*x + c
) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2) - b)/b + 1) - 3*(b*d^2*f^3*x^2 - 2*a*d*f^3*x + b*d^2*f*cosh(1)^2 + b*d
^2*f*sinh(1)^2 - (b*d^2*f^3*x^2 - 2*a*d*f^3*x + b*d^2*f*cosh(1)^2 + b*d^2*f
*sinh(1)^2 + 2*(b*d^2*f^2*x - a*d*f^2)*cosh(1) + 2*(b*d^2*f^2*x + b*d^2*f*c
osh(1) - a*d*f^2)*sinh(1))*cosh(d*x + c)^2 - 2*(b*d^2*f^3*x^2 - 2*a*d*f^3*x
 + b*d^2*f*cosh(1)^2 + b*d^2*f*sinh(1)^2 + 2*(b*d^2*f^2*x - a*d*f^2)*cosh(1
) + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1) - a*d*f^2)*sinh(1))*cosh(d*x + c)*sinh
(d*x + c) - (b*d^2*f^3*x^2 - 2*a*d*f^3*x + b*d^2*f*cosh(1)^2 + b*d^2*f*sinh
(1)^2 + 2*(b*d^2*f^2*x - a*d*f^2)*cosh(1) + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1
) - a*d*f^2)*sinh(1))*sinh(d*x + c)^2 + 2*(b*d^2*f^2*x - a*d*f^2)*cosh(1) +
 2*(b*d^2*f^2*x + b*d^2*f*cosh(1) - a*d*f^2)*sinh(1))*dilog(cosh(d*x + c) +
 sinh(d*x + c)) - 3*(b*d^2*f^3*x^2 + 2*a*d*f^3*x + b*d^2*f*cosh(1)^2 + b*d^
2*f*sinh(1)^2 - (b*d^2*f^3*x^2 + 2*a*d*f^3*x + b*d^2*f*cosh(1)^2 + b*d^2*f*
sinh(1)^2 + 2*(b*d^2*f^2*x + a*d*f^2)*cosh(1) + 2*(b*d^2*f^2*x + b*d^2*f*co
sh(1) + a*d*f^2)*sinh(1))*cosh(d*x + c)^2 - 2*(b*d^2*f^3*x^2 + 2*a*d*f^3*x
+ b*d^2*f*cosh(1)^2 + b*d^2*f*sinh(1)^2 + 2*(b*d^2*f^2*x + a*d*f^2)*cosh(1)
 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1) + a*d*f^2)*sinh(1))*cosh(d*x + c)*sinh(
d*x + c) - (b*d^2*f^3*x^2 + 2*a*d*f^3*x + b*d^2*f*cosh(1)^2 + b*d^2*f*sinh(
1)^2 + 2*(b*d^2*f^2*x + a*d*f^2)*cosh(1) + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1)
 + a*d*f^2)*sinh(1))*sinh(d*x + c)^2 + 2*(b*d^2*f^2*x + a*d*f^2)*cosh(1) +
2*(b*d^2*f^2*x + b*d^2*f*cosh(1) + a*d*f^2)*sinh(1))*dilog(-cosh(d*x + c) -
 sinh(d*x + c)) - (b*c^3*f^3 - 3*b*c^2*d*f^2*cosh(1) + 3*b*c*d^2*f*cosh(1)^
2 - b*d^3*cosh(1)^3 - b*d^3*sinh(1)^3 - (b*c^3*f^3 - 3*b*c^2*d*f^2*cosh(1)
+ 3*b*c*d^2*f*cosh(1)^2 - b*d^3*cosh(1)^3 - b*d^3*sinh(1)^3 + 3*(b*c*d^2*f
- b*d^3*cosh(1))*sinh(1)^2 - 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(1) + b*d^3*c
```

```
osh(1)^2)*sinh(1))*cosh(d*x + c)^2 + 3*(b*c*d^2*f - b*d^3*cosh(1))*sinh(1)^
2 - 2*(b*c^3*f^3 - 3*b*c^2*d*f^2*cosh(1) + 3*b*c*d^2*f*cosh(1)^2 - b*d^3*co
sh(1)^3 - b*d^3*sinh(1)^3 + 3*(b*c*d^2*f - b*d^3*cosh(1))*sinh(1)^2 - 3*(b*
c^2*d*f^2 - 2*b*c*d^2*f*cosh(1) + b*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c)*s
inh(d*x + c) - (b*c^3*f^3 - 3*b*c^2*d*f^2*cosh(1) + 3*b*c*d^2*f*cosh(1)^2 -
 b*d^3*cosh(1)^3 - b*d^3*sinh(1)^3 + 3*(b*c*d^2*f - b*d^3*cosh(1))*sinh(1)^
2 - 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(1) + b*d^3*cosh(1)^2)*sinh(1))*sinh(d
*x + c)^2 - 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(1) + b*d^3*cosh(1)^2)*sinh(1)
)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2
*a) - (b*c^3*f^3 - 3*b*c^2*d*f^2*cosh(1) + 3*b*c*d^2*f*cosh(1)^2 - b*d^3*co
sh(1)^3 - b*d^3*sinh(1)^3 - (b*c^3*f^3 - 3*b*c^2*d*f^2*cosh(1) + 3*b*c*d^2*
f*cosh(1)^2 - b*d^3*cosh(1)^3 - b*d^3*sinh(1)^3 + 3*(b*c*d^2*f - b*d^3*cosh
(1))*sinh(1)^2 - 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(1) + b*d^3*cosh(1)^2)*si
nh(1))*cosh(d*x + c)^2 + 3*(b*c*d^2*f - b*d^3*cosh(1))*sinh(1)^2 - 2*(b*c^3
*f^3 - 3*b*c^2*d*f^2*cosh(1) + 3*b*c*d^2*f*cosh(1)^2 - b*d^3*cosh(1)^3 - b*
d^3*sinh(1)^3 + 3*(b*c*d^2*f - b*d^3*cosh(1))*sinh(1)^2 - 3*(b*c^2*d*f^2 -
2*b*c*d^2*f*cosh(1) + b*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c)*sinh(d*x + c)
 - (b*c^3*f^3 - 3*b*c^2*d*f^2*cosh(1) + 3*b*c*d^2*f*cosh(1)^2 - b*d^3*cosh(
1)^3 - b*d^3*sinh(1)^3 + 3*(b*c*d^2*f - b*d^3*cosh(1))*sinh(1)^2 - 3*(b*c^2
*d*f^2 - 2*b*c*d^2*f*cosh(1) + b*d^3*cosh(1)^2)*sinh(1))*sinh(d*x + c)^2 -
3*(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(1) + b*d^3*cosh(1)^2)*sinh(1))*log(2*b*co
sh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(c+d\,x)\,{\left(e+f\,x\right)}^3}{\mathrm{sinh}\left(c+d\,x\right)\,\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(c + d*x)*(e + f*x)^3)/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
[Out] int((coth(c + d*x)*(e + f*x)^3)/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.450 $\int \frac{(e+fx)^2 \coth(c+dx)\mathbf{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=419

$$-\frac{4f(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^2}-\frac{(e+fx)^2\text{csch}(c+dx)}{ad}+\frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d}+\frac{b(e+fx)^2\log\left(1\right.}{a}$$

```
[Out] -4*f*(f*x+e)*arctanh(exp(d*x+c))/a/d^2-(f*x+e)^2*csch(d*x+c)/a/d-b*(f*x+e)^
2*ln(1-exp(2*d*x+2*c))/a^2/d+b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/
2)))/a^2/d+b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d-2*f^2*p
olylog(2,-exp(d*x+c))/a/d^3+2*f^2*polylog(2,exp(d*x+c))/a/d^3-b*f*(f*x+e)*p
olylog(2,exp(2*d*x+2*c))/a^2/d^2+2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(
a^2+b^2)^(1/2)))/a^2/d^2+2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)
^(1/2)))/a^2/d^2+1/2*b*f^2*polylog(3,exp(2*d*x+2*c))/a^2/d^3-2*b*f^2*polylo
g(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^3-2*b*f^2*polylog(3,-b*exp(d*x
+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^3
```

Rubi [A]
time = 0.56, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5706, 5560, 4267, 2317, 2438, 5688, 3797, 2221, 2611, 2320, 6724, 5680}

$$-\frac{2bf^2\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3}-\frac{2bf^2\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^3}+\frac{2bf(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2}+\frac{2bf(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^2}+\frac{b(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^2d}+\frac{b(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{a^2d}+\frac{bf^2\text{Li}_3\left(e^{2(c+dx)}\right)}{2a^2d^3}-\frac{bf(e+fx)\text{Li}_2\left(e^{2(c+dx)}\right)}{a^2d^2}-\frac{b(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{a^2d}-\frac{2f^2\text{Li}_2\left(-e^{c+dx}\right)}{ad^3}+\frac{2f^2\text{Li}_2\left(e^{c+dx}\right)}{ad^3}-\frac{4f(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^2}-\frac{(e+fx)^2\text{csch}(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-4*f*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d^2) - ((e + f*x)^2*Csch[c + d*x])
/(a*d) + (b*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^
2*d) + (b*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*
d) - (b*(e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a^2*d) - (2*f^2*PolyLog[2, -
E^(c + d*x)])/(a*d^3) + (2*f^2*PolyLog[2, E^(c + d*x)])/(a*d^3) + (2*b*f*(e
 + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^2) + (
2*b*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*
d^2) - (b*f*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a^2*d^2) - (2*b*f^2*Pol
yLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^3) - (2*b*f^2*Pol
yLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^3) + (b*f^2*PolyL
og[3, E^(2*(c + d*x))])/(2*a^2*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5688

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x]^
(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5706

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2 \coth(c+dx)\mathrm{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2 \coth(c+dx)\mathrm{csch}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}}{a}$$

$$= -\frac{(e+fx)^2\mathrm{csch}(c+dx)}{ad} - \frac{b\int (e+fx)^2\coth(c+dx)\,dx}{a^2} + \frac{b^2}{}$$

$$= -\frac{4f(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^2\mathrm{csch}(c+dx)}{ad} + \frac{(2b)}{}$$

$$= -\frac{4f(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^2\mathrm{csch}(c+dx)}{ad} + \frac{b(e+}{}$$

$$= -\frac{4f(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^2\mathrm{csch}(c+dx)}{ad} + \frac{b(e+}{}$$

$$= -\frac{4f(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^2\mathrm{csch}(c+dx)}{ad} + \frac{b(e+}{}$$

$$= -\frac{4f(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^2\mathrm{csch}(c+dx)}{ad} + \frac{b(e+}{}$$

Mathematica [A]
time = 9.42, size = 783, normalized size = 1.87

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (-8*a*d*e*f*ArcTanh[E^(c + d*x)] - 2*a*d^2*e^2*Csch[c + d*x] - 4*a*d^2*e*f*
x*Csch[c + d*x] - 2*a*d^2*f^2*x^2*Csch[c + d*x] + 4*a*d*f^2*x*Log[1 - E^(c
+ d*x)] - 4*a*d*f^2*x*Log[1 + E^(c + d*x)] - 2*b*d^2*e^2*Log[1 - E^(2*(c +
d*x))] - 4*b*d^2*e*f*x*Log[1 - E^(2*(c + d*x))] - 2*b*d^2*f^2*x^2*Log[1 - E
^(2*(c + d*x))] + 2*b*d^2*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x))
)] + 4*b*d^2*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)])] + 2*b*d^2*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2
)*E^(2*c)])] + 4*b*d^2*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 +
 b^2)*E^(2*c)])] + 2*b*d^2*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[
(a^2 + b^2)*E^(2*c)])] - 4*a*f^2*PolyLog[2, -E^(c + d*x)] + 4*a*f^2*PolyLog
```

```
[2, E^(c + d*x)] - 2*b*d*e*f*PolyLog[2, E^(2*(c + d*x))] - 2*b*d*f^2*x*Poly
Log[2, E^(2*(c + d*x))] + 4*b*d*e*f*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c -
 Sqrt[(a^2 + b^2)*E^(2*c)]))] + 4*b*d*f^2*x*PolyLog[2, -((b*E^(2*c + d*x))/
(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 4*b*d*e*f*PolyLog[2, -((b*E^(2*c +
d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 4*b*d*f^2*x*PolyLog[2, -((b*E
^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + b*f^2*PolyLog[3, E^(2
*(c + d*x))] - 4*b*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 +
b^2)*E^(2*c)]))] - 4*b*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^
2 + b^2)*E^(2*c)]))])/(2*a^2*d^3)
```

Maple [F]
time = 2.12, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \coth(dx+c)\operatorname{csch}(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] (2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) + b*log(-2*a*e^(-d*x - c) + b*
e^(-2*d*x - 2*c) - b)/(a^2*d) - b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(
-d*x - c) - 1)/(a^2*d))*e^2 - 2*(f^2*x^2*e^c + 2*f*x*e^(c + 1))*e^(d*x)/(a*
d*e^(2*d*x + 2*c) - a*d) - 2*f*e*log(e^(d*x + c) + 1)/(a*d^2) + 2*f*e*log(e
^(d*x + c) - 1)/(a*d^2) - (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d
*x + c)) - 2*polylog(3, -e^(d*x + c)))*b*f^2/(a^2*d^3) - (d^2*x^2*log(-e^(d
*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*b*f^2/
(a^2*d^3) - 2*(b*d*f*e + a*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x +
 c)))/(a^2*d^3) - 2*(b*d*f*e - a*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^
(d*x + c)))/(a^2*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*d*f*e + a*f^2)*d^2*x^2)/(
a^2*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*d*f*e - a*f^2)*d^2*x^2)/(a^2*d^3) - in
tegrate(-2*(b^2*f^2*x^2 + 2*b^2*f*x*e - (a*b*f^2*x^2*e^c + 2*a*b*f*x*e^(c +
 1))*e^(d*x))/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(d*x + c) - a^2*b), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3408 vs. 2(399) = 798.

time = 0.46, size = 3408, normalized size = 8.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -(2*(a*d^2*f^2*x^2 + 2*a*d^2*f*x*cosh(1) + a*d^2*cosh(1)^2 + a*d^2*sinh(1)^
2 + 2*(a*d^2*f*x + a*d^2*cosh(1))*sinh(1))*cosh(d*x + c) + 2*(b*d*f^2*x + b
*d*f*cosh(1) + b*d*f*sinh(1) - (b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1))*
cosh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1))*cosh(d*x +
c)*sinh(d*x + c) - (b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1))*sinh(d*x + c
)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b*d*f^2*x + b*d*f*cosh(1) +
 b*d*f*sinh(1) - (b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1))*cosh(d*x + c)^
2 - 2*(b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x +
c) - (b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1))*sinh(d*x + c)^2)*dilog((a*
cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2) - b)/b + 1) - 2*(b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1)
 - a*f^2 - (b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1) - a*f^2)*cosh(d*x + c
)^2 - 2*(b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1) - a*f^2)*cosh(d*x + c)*s
inh(d*x + c) - (b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1) - a*f^2)*sinh(d*x
 + c)^2)*dilog(cosh(d*x + c) + sinh(d*x + c)) - 2*(b*d*f^2*x + b*d*f*cosh(1
) + b*d*f*sinh(1) + a*f^2 - (b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1) + a*
f^2)*cosh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1) + a*f^2
)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1)
+ a*f^2)*sinh(d*x + c)^2)*dilog(-cosh(d*x + c) - sinh(d*x + c)) + (b*c^2*f^
2 - 2*b*c*d*f*cosh(1) + b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 - (b*c^2*f^2 - 2*
b*c*d*f*cosh(1) + b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 - 2*(b*c*d*f - b*d^2*co
sh(1))*sinh(1))*cosh(d*x + c)^2 - 2*(b*c^2*f^2 - 2*b*c*d*f*cosh(1) + b*d^2*
cosh(1)^2 + b*d^2*sinh(1)^2 - 2*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*cosh(d*x
 + c)*sinh(d*x + c) - (b*c^2*f^2 - 2*b*c*d*f*cosh(1) + b*d^2*cosh(1)^2 + b*
d^2*sinh(1)^2 - 2*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 2*(b
*c*d*f - b*d^2*cosh(1))*sinh(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c)
+ 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b*c^2*f^2 - 2*b*c*d*f*cosh(1) + b*d^2
*cosh(1)^2 + b*d^2*sinh(1)^2 - (b*c^2*f^2 - 2*b*c*d*f*cosh(1) + b*d^2*cosh(
1)^2 + b*d^2*sinh(1)^2 - 2*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*cosh(d*x + c)
^2 - 2*(b*c^2*f^2 - 2*b*c*d*f*cosh(1) + b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 -
 2*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - (b*c^2*
f^2 - 2*b*c*d*f*cosh(1) + b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 - 2*(b*c*d*f -
b*d^2*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 2*(b*c*d*f - b*d^2*cosh(1))*sinh(
1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) +
 2*a) + (b*d^2*f^2*x^2 - b*c^2*f^2 - (b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*
f*x + b*c*d*f)*cosh(1) + 2*(b*d^2*f*x + b*c*d*f)*sinh(1))*cosh(d*x + c)^2 -
```

```
 2*(b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*cosh(1) + 2*(b*d^2*
f*x + b*c*d*f)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*f^2*x^2 - b*c^
2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*cosh(1) + 2*(b*d^2*f*x + b*c*d*f)*sinh(1))*
sinh(d*x + c)^2 + 2*(b*d^2*f*x + b*c*d*f)*cosh(1) + 2*(b*d^2*f*x + b*c*d*f)
*sinh(1))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b*d^2*f^2*x^2 - b*c^2*f^2 - (
b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*cosh(1) + 2*(b*d^2*f*x
+ b*c*d*f)*sinh(1))*cosh(d*x + c)^2 - 2*(b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d
^2*f*x + b*c*d*f)*cosh(1) + 2*(b*d^2*f*x + b*c*d*f)*sinh(1))*cosh(d*x + c)*
sinh(d*x + c) - (b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*cosh(1
) + 2*(b*d^2*f*x + b*c*d*f)*sinh(1))*sinh(d*x + c)^2 + 2*(b*d^2*f*x + b*c*d
*f)*cosh(1) + 2*(b*d^2*f*x + b*c*d*f)*sinh(1))*log(-(a*cosh(d*x + c) + a*si
nh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b
)/b) - (b*d^2*f^2*x^2 + 2*a*d*f^2*x + b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 - (
b*d^2*f^2*x^2 + 2*a*d*f^2*x + b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 + 2*(b*d^2*
f*x + a*d*f)*cosh(1) + 2*(b*d^2*f*x + b*d^2*cosh(1) + a*d*f)*sinh(1))*cosh(
d*x + c)^2 - 2*(b*d^2*f^2*x^2 + 2*a*d*f^2*x + b*d^2*cosh(1)^2 + b*d^2*sinh(
1)^2 + 2*(b*d^2*f*x + a*d*f)*cosh(1) + 2*(b*d^2*f*x + b*d^2*cosh(1) + a*d*f
)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*f^2*x^2 + 2*a*d*f^2*x + b*d
^2*cosh(1)^2 + b*d^2*sinh(1)^2 + 2*(b*d^2*f*x + a*d*f)*cosh(1) + 2*(b*d^2*f
*x + b*d^2*cosh(1) + a*d*f)*sinh(1))*sinh(d*x + c)^2 + 2*(b*d^2*f*x + a*d*f
)*cosh(1) + 2*(b*d^2*f*x + b*d^2*cosh(1) + a*d*f)*sinh(1))*log(cosh(d*x + c
) + sinh(d*x + c) + 1) - (b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 - 2*(b*c + a)*d
*f*cosh(1) + (b*c^2 + 2*a*c)*f^2 - (b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 - 2*(
b*c + a)*d*f*cosh(1) + (b*c^2 + 2*a*c)*f^2 + 2*(b*d^2*cosh(1) - (b*c + a)*d
*f)*sinh(1))*cosh(d*x + c)^2 - 2*(b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 - 2*(b*
c + a)*d*f*cosh(1) + (b*c^2 + 2*a*c)*f^2 + 2*(b*d^2*cosh(1) - (b*c + a)*d*f
)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2
 - 2*(b*c + a)*d*f*cosh(1) + (b*c^2 + 2*a*c)*f^2 + 2*(b*d^2*cosh(1) - (b*c
+ a)*d*f)*sinh(1))*sinh(d*x + c)^2 + 2*(b*d^2*c...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2 \coth(c+dx) \operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*coth(c + d*x)*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(c+d\,x)\,\left(e+f\,x\right)^2}{\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(c + d*x)*(e + f*x)^2)/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((coth(c + d*x)*(e + f*x)^2)/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.451 $\int \frac{(e+fx)\coth(c+dx)\mathbf{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=243

$$-\frac{f\tanh^{-1}(\cosh(c+dx))}{ad^2}-\frac{(e+fx)\text{csch}(c+dx)}{ad}+\frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d}+\frac{b(e+fx)\log\left(1+\frac{}{a+}\right.}{a^2d}$$

```
[Out] -f*arctanh(cosh(d*x+c))/a/d^2-(f*x+e)*csch(d*x+c)/a/d-b*(f*x+e)*ln(1-exp(2*
d*x+2*c))/a^2/d+b*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d+b*(f
*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d-1/2*b*f*polylog(2,exp(2*
d*x+2*c))/a^2/d^2+b*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/d^2+
b*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/d^2
```

Rubi [A]

time = 0.33, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5706, 5560, 3855, 5688, 3797, 2221, 2317, 2438, 5680}

$$\frac{bf\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2}+\frac{bf\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^2}+\frac{b(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^2d}+\frac{b(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{a^2d}-\frac{bf\text{Li}_2\left(e^{2(c+dx)}\right)}{2a^2d^2}-\frac{b(e+fx)\log\left(1-e^{2(c+dx)}\right)}{a^2d}-\frac{f\tanh^{-1}(\cosh(c+dx))}{ad^2}-\frac{(e+fx)\text{csch}(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((f*ArcTanh[Cosh[c + d*x]])/(a*d^2)) - ((e + f*x)*Csch[c + d*x])/(a*d) + (
b*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*d) + (b*(e
 + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*d) - (b*(e + f
*x)*Log[1 - E^(2*(c + d*x))])/(a^2*d) + (b*f*PolyLog[2, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/(a^2*d^2) + (b*f*PolyLog[2, -((b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2]))])/(a^2*d^2) - (b*f*PolyLog[2, E^(2*(c + d*x))])/(2*a^2*d^
2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5688

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x]^
(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5706

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c + d*
```

```
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\coth(c+dx)\text{csch}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)\text{csch}(c+dx)}{ad} - \frac{b\int (e+fx)\coth(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{(e+}{a}}{}$$

$$= -\frac{f\tanh^{-1}(\cosh(c+dx))}{ad^2} - \frac{(e+fx)\text{csch}(c+dx)}{ad} + \frac{(2b)\int \frac{e^{2(c+d}}{1-e}}{a^2}$$

$$= -\frac{f\tanh^{-1}(\cosh(c+dx))}{ad^2} - \frac{(e+fx)\text{csch}(c+dx)}{ad} + \frac{b(e+fx)\,\text{lo}}{}$$

$$= -\frac{f\tanh^{-1}(\cosh(c+dx))}{ad^2} - \frac{(e+fx)\text{csch}(c+dx)}{ad} + \frac{b(e+fx)\,\text{lo}}{}$$

$$= -\frac{f\tanh^{-1}(\cosh(c+dx))}{ad^2} - \frac{(e+fx)\text{csch}(c+dx)}{ad} + \frac{b(e+fx)\,\text{lo}}{}$$

Mathematica [A]
time = 1.34, size = 416, normalized size = 1.71

$$\frac{-2bc^2f-4bcdfx-2bd^2fx^2-ade\coth\left(\frac{1}{2}(c+dx)\right)-adfx\coth\left(\frac{1}{2}(c+dx)\right)-2bcf\log\left(1-e^{-2(c+dx)}\right)-2bdfx\log\left(1-e^{-2(c+dx)}\right)+2bcf\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+2bdfx\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+2bcf\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)+2bdfx\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)-2bde\log(\sinh(c+dx))+2bcf\log(\sinh(c+dx))+2bde\log(a+b\sinh(c+dx))-2bcf\log(a+b\sinh(c+dx))+2af\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+bf\text{PolyLog}\left(2,e^{-2(c+dx)}\right)+2bf\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)+2bf\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)+ade\tanh\left(\frac{1}{2}(c+dx)\right)+adfx\tanh\left(\frac{1}{2}(c+dx)\right)}{2a^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]

[Out] (-2*b*c^2*f - 4*b*c*d*f*x - 2*b*d^2*f*x^2 - a*d*e*Coth[(c + d*x)/2] - a*d*f
**x*Coth[(c + d*x)/2] - 2*b*c*f*Log[1 - E^(-2*(c + d*x))] - 2*b*d*f*x*Log[1
- E^(-2*(c + d*x))] + 2*b*c*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
] + 2*b*d*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*b*c*f*Log[
1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*b*d*f*x*Log[1 + (b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2])] - 2*b*d*e*Log[Sinh[c + d*x]] + 2*b*c*f*Log[Sinh[
c + d*x]] + 2*b*d*e*Log[a + b*Sinh[c + d*x]] - 2*b*c*f*Log[a + b*Sinh[c + d
*x]] + 2*a*f*Log[Tanh[(c + d*x)/2]] + b*f*PolyLog[2, E^(-2*(c + d*x))] + 2*
b*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*b*f*PolyLog[2, -
```

((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + a*d*e*Tanh[(c + d*x)/2] + a*d*f*
x*Tanh[(c + d*x)/2])/(2*a^2*d^2)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(229) = 458$.
time = 3.19, size = 528, normalized size = 2.17

method	result
risch	$-\frac{2(fx+e)\,\mathrm{e}^{dx+c}}{da\left(\mathrm{e}^{2dx+2c}-1\right)}-\frac{bf\,\mathrm{dilog}\left(\mathrm{e}^{dx+c}+1\right)}{a^2d^2}+\frac{bf\,\mathrm{dilog}\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)}{d^2a^2}+\frac{bf\,\mathrm{dilog}\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right)}{d^2a^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBO
SE)
```

```
[Out] -2/d*(f*x+e)/a*exp(d*x+c)/(exp(2*d*x+2*c)-1)-1/d^2/a^2*b*f*dilog(exp(d*x+c)
+1)+1/d^2/a^2*b*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/
2)))+1/d^2/a^2*b*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2
)))+1/d^2/a^2*b*f*dilog(exp(d*x+c))-1/a^2/d*b*e*ln(exp(d*x+c)+1)+1/d/a^2*b*
e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/a^2/d*b*e*ln(exp(d*x+c)-1)-1/a^2/
d*b*f*ln(exp(d*x+c)+1)*x+1/d/a^2*b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(
-a+(a^2+b^2)^(1/2)))*x+1/d^2/a^2*b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(
-a+(a^2+b^2)^(1/2)))*c+1/d/a^2*b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(
a^2+b^2)^(1/2)))*x+1/d^2/a^2*b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^
2+b^2)^(1/2)))*c-1/d^2/a^2*b*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/a^
2/d^2*b*f*c*ln(exp(d*x+c)-1)-1/a/d^2*f*ln(exp(d*x+c)+1)+1/a/d^2*f*ln(exp(d*
x+c)-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] (2*b*d*integrate(1/2*x/(a^2*d*e^(d*x + c) + a^2*d), x) - 2*b*d*integrate(1/
2*x/(a^2*d*e^(d*x + c) - a^2*d), x) + a*((d*x + c)/(a^2*d^2) - log(e^(d*x +
 c) + 1)/(a^2*d^2)) - a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) - 1)/(a^2*d^
2)) - 2*x*e^(d*x + c)/(a*d*e^(2*d*x + 2*c) - a*d) - 2*integrate((a*b*x*e^(d
*x + c) - b^2*x)/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(d*x + c) - a^2*b), x))*f
 + (2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) + b*log(-2*a*e^(-d*x - c) +
 b*e^(-2*d*x - 2*c) - b)/(a^2*d) - b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(
e^(-d*x - c) - 1)/(a^2*d))*e
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1339 vs. $2(230) = 460$.

time = 0.40, size = 1339, normalized size = 5.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] -(2*(a*d*f*x + a*d*cosh(1) + a*d*sinh(1))*cosh(d*x + c) - (b*f*cosh(d*x + c
)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c) + b*f*sinh(d*x + c)^2 - b*f)*dilog(
(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sq
rt((a^2 + b^2)/b^2) - b)/b + 1) - (b*f*cosh(d*x + c)^2 + 2*b*f*cosh(d*x + c
)*sinh(d*x + c) + b*f*sinh(d*x + c)^2 - b*f)*dilog((a*cosh(d*x + c) + a*sin
h(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)
/b + 1) + (b*f*cosh(d*x + c)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c) + b*f*si
nh(d*x + c)^2 - b*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) + (b*f*cosh(d*x +
 c)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c) + b*f*sinh(d*x + c)^2 - b*f)*dilo
g(-cosh(d*x + c) - sinh(d*x + c)) - (b*c*f - b*d*cosh(1) - (b*c*f - b*d*cos
h(1) - b*d*sinh(1))*cosh(d*x + c)^2 - b*d*sinh(1) - 2*(b*c*f - b*d*cosh(1)
- b*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - (b*c*f - b*d*cosh(1) - b*d*sin
h(1))*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt
((a^2 + b^2)/b^2) + 2*a) - (b*c*f - b*d*cosh(1) - (b*c*f - b*d*cosh(1) - b*
d*sinh(1))*cosh(d*x + c)^2 - b*d*sinh(1) - 2*(b*c*f - b*d*cosh(1) - b*d*sin
h(1))*cosh(d*x + c)*sinh(d*x + c) - (b*c*f - b*d*cosh(1) - b*d*sinh(1))*sin
h(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b
^2)/b^2) + 2*a) + (b*d*f*x + b*c*f - (b*d*f*x + b*c*f)*cosh(d*x + c)^2 - 2*
(b*d*f*x + b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f*x + b*c*f)*sinh(d*x
+ c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b*d*f*x + b*c*f - (b*d*f*x + b*
c*f)*cosh(d*x + c)^2 - 2*(b*d*f*x + b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b
*d*f*x + b*c*f)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) -
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b*d*f*
x + b*d*cosh(1) - (b*d*f*x + b*d*cosh(1) + b*d*sinh(1) + a*f)*cosh(d*x + c)
^2 + b*d*sinh(1) - 2*(b*d*f*x + b*d*cosh(1) + b*d*sinh(1) + a*f)*cosh(d*x +
 c)*sinh(d*x + c) - (b*d*f*x + b*d*cosh(1) + b*d*sinh(1) + a*f)*sinh(d*x +
c)^2 + a*f)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (b*d*cosh(1) - (b*d*co
sh(1) + b*d*sinh(1) - (b*c + a)*f)*cosh(d*x + c)^2 + b*d*sinh(1) - 2*(b*d*c
osh(1) + b*d*sinh(1) - (b*c + a)*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*cosh
(1) + b*d*sinh(1) - (b*c + a)*f)*sinh(d*x + c)^2 - (b*c + a)*f)*log(cosh(d*
x + c) + sinh(d*x + c) - 1) - (b*d*f*x + b*c*f - (b*d*f*x + b*c*f)*cosh(d*x
 + c)^2 - 2*(b*d*f*x + b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f*x + b*c*
f)*sinh(d*x + c)^2)*log(-cosh(d*x + c) - sinh(d*x + c) + 1) + 2*(a*d*f*x +
```

```
a*d*cosh(1) + a*d*sinh(1))*sinh(d*x + c))/(a^2*d^2*cosh(d*x + c)^2 + 2*a^2*
d^2*cosh(d*x + c)*sinh(d*x + c) + a^2*d^2*sinh(d*x + c)^2 - a^2*d^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)*coth(c + d*x)*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(c+d\,x)\,\left(e+f\,x\right)}{\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(c + d*x)*(e + f*x))/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
[Out] int((coth(c + d*x)*(e + f*x))/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.452 $\int \frac{\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=50

$$-\frac{\text{csch}(c+dx)}{ad}-\frac{b\log(\sinh(c+dx))}{a^2d}+\frac{b\log(a+b\sinh(c+dx))}{a^2d}$$

```
[Out] -csch(d*x+c)/a/d-b*ln(sinh(d*x+c))/a^2/d+b*ln(a+b*sinh(d*x+c))/a^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.120$, Rules used = {2912, 12, 46}

$$-\frac{b\log(\sinh(c+dx))}{a^2d}+\frac{b\log(a+b\sinh(c+dx))}{a^2d}-\frac{\text{csch}(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(Csch[c + d*x]/(a*d)) - (b*Log[Sinh[c + d*x]])/(a^2*d) + (b*Log[a + b*Sinh
[c + d*x]])/(a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^2}{x^2(a+x)}\,dx, x, b\sinh(c+dx)\right)}{bd}$$
$$= \frac{b\text{Subst}\left(\int \frac{1}{x^2(a+x)}\,dx, x, b\sinh(c+dx)\right)}{d}$$
$$= \frac{b\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{1}{a^2x}+\frac{1}{a^2(a+x)}\right)\,dx, x, b\sinh(c+dx)\right)}{d}$$
$$= -\frac{\text{csch}(c+dx)}{ad}-\frac{b\log(\sinh(c+dx))}{a^2d}+\frac{b\log(a+b\sinh(c+dx))}{a^2d}$$

Mathematica [A]
time = 0.04, size = 50, normalized size = 1.00

$$-\frac{\text{csch}(c+dx)}{ad}-\frac{b\log(\sinh(c+dx))}{a^2d}+\frac{b\log(a+b\sinh(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(Csch[c + d*x]/(a*d)) - (b*Log[Sinh[c + d*x]])/(a^2*d) + (b*Log[a + b*Sinh
[c + d*x]])/(a^2*d)
```

Maple [A]
time = 0.76, size = 34, normalized size = 0.68

method	result	size
derivativedivides	$-\frac{\frac{\text{csch}(dx+c)}{a}-\frac{b\ln(a\,\text{csch}(dx+c)+b)}{a^2}}{d}$	34
default	$-\frac{\frac{\text{csch}(dx+c)}{a}-\frac{b\ln(a\,\text{csch}(dx+c)+b)}{a^2}}{d}$	34
risch	$-\frac{2\,e^{dx+c}}{da\left(e^{2dx+2c}-1\right)}-\frac{b\ln\left(e^{2dx+2c}-1\right)}{a^2d}+\frac{b\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{a^2d}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(1/a*csch(d*x+c)-1/a^2*b*ln(a*csch(d*x+c)+b))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(50) = 100.
time = 0.27, size = 110, normalized size = 2.20

$$\frac{2\,e^{(-dx-c)}}{(ae^{(-2\,dx-2\,c)}-a)d}+\frac{b\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{a^2d}-\frac{b\log\left(e^{(-dx-c)}+1\right)}{a^2d}-\frac{b\log\left(e^{(-dx-c)}-1\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) + b*log(-2*a*e^(-d*x - c) + b*e
^(-2*d*x - 2*c) - b)/(a^2*d) - b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-
d*x - c) - 1)/(a^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(50) = 100$.
time = 0.42, size = 211, normalized size = 4.22

$$-\frac{2\,a\cosh\left(dx+c\right)-\left(b\cosh\left(dx+c\right)^{2}+2\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b\sinh\left(dx+c\right)^{2}-b\right)\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)+\left(b\cosh\left(dx+c\right)^{2}+2\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b\sinh\left(dx+c\right)^{2}-b\right)\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)+2\,a\sinh\left(dx+c\right)}{a^{2}d\cosh\left(dx+c\right)^{2}+2\,a^{2}d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a^{2}d\sinh\left(dx+c\right)^{2}-a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*a*cosh(d*x + c) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c)
+ b*sinh(d*x + c)^2 - b)*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(
d*x + c))) + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(
d*x + c)^2 - b)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 2*a*
sinh(d*x + c))/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c)*sinh(d*x + c)
 + a^2*d*sinh(d*x + c)^2 - a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth\left(c+dx\right)\operatorname{csch}\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(50) = 100$.
time = 0.43, size = 110, normalized size = 2.20

$$\frac{\frac{b\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^{2}}-\frac{b\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a^{2}}+\frac{b\left(e^{(dx+c)}-e^{(-dx-c)}\right)-2\,a}{a^{2}\left(e^{(dx+c)}-e^{(-dx-c)}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (b*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/a^2 - b*log(abs(e^(d*x +
c) - e^(-d*x - c)))/a^2 + (b*(e^(d*x + c) - e^(-d*x - c)) - 2*a)/(a^2*(e^(d
*x + c) - e^(-d*x - c))))/d
```

Mupad [B]
time = 0.87, size = 409, normalized size = 8.18

$$\frac{\left(2\,\mathrm{atan}\left(\left(4\,a^3\,b\,d\,{\left(b^2\right)}^{5/2}\,\sqrt{-a^4\,d^2}+4\,a^5\,b\,d\,{\left(b^2\right)}^{3/2}\,\sqrt{-a^4\,d^2}\right)\,\left(\frac{1}{8\,a^3\,b^4\,d^2\,{\left(a^2+b^2\right)}^2}-{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}\,\left(\frac{1}{16\,a^2\,b^5\,d^2\,{\left(a^2+b^2\right)}^2}-\frac{{\left(a^2+2\,b^2\right)}^2}{16\,a^6\,b^5\,d^2\,{\left(a^2+b^2\right)}^2}\right)+\frac{a^2+2\,b^2}{8\,a^5\,b^4\,d^2\,{\left(a^2+b^2\right)}^2}\right)\right)+2\,\mathrm{atan}\left(-\frac{4\,a^3\,b^5\,\sqrt{-a^4\,d^2}+4\,a\,b^7\,\sqrt{-a^4\,d^2}-4\,b^8\,{\mathrm{e}}^{3\,c}\,{\mathrm{e}}^{3\,d\,x}\,\sqrt{-a^4\,d^2}+4\,b^8\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}\,\sqrt{-a^4\,d^2}-8\,a\,b^7\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}\,\sqrt{-a^4\,d^2}+4\,a^2\,b^6\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}\,\sqrt{-a^4\,d^2}-8\,a^3\,b^5\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}\,\sqrt{-a^4\,d^2}-4\,a^2\,b^6\,{\mathrm{e}}^{3\,c}\,{\mathrm{e}}^{3\,d\,x}\,\sqrt{-a^4\,d^2}}{b^4\,\left(4\,a^3\,d\,{\left(b^2\right)}^{3/2}+4\,a^5\,d\,\sqrt{b^2}\right)}\right)\right)\,\sqrt{b^2}}{\sqrt{-a^4\,d^2}}-\frac{1}{a\,d\,\mathrm{sinh}\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] ((2*atan((4*a^3*b*d*(b^2)^(5/2)*(-a^4*d^2)^(1/2) + 4*a^5*b*d*(b^2)^(3/2)*(-
a^4*d^2)^(1/2))*(1/(8*a^3*b^4*d^2*(a^2 + b^2)^2) - exp(d*x)*exp(c)*(1/(16*a
^2*b^5*d^2*(a^2 + b^2)^2) - (a^2 + 2*b^2)^2/(16*a^6*b^5*d^2*(a^2 + b^2)^2))
 + (a^2 + 2*b^2)/(8*a^5*b^4*d^2*(a^2 + b^2)^2))) + 2*atan(-(4*a^3*b^5*(-a^4
*d^2)^(1/2) + 4*a*b^7*(-a^4*d^2)^(1/2) - 4*b^8*exp(3*c)*exp(3*d*x)*(-a^4*d^
2)^(1/2) + 4*b^8*exp(d*x)*exp(c)*(-a^4*d^2)^(1/2) - 8*a*b^7*exp(2*c)*exp(2*
d*x)*(-a^4*d^2)^(1/2) + 4*a^2*b^6*exp(d*x)*exp(c)*(-a^4*d^2)^(1/2) - 8*a^3*
b^5*exp(2*c)*exp(2*d*x)*(-a^4*d^2)^(1/2) - 4*a^2*b^6*exp(3*c)*exp(3*d*x)*(-
a^4*d^2)^(1/2))/(b^4*(4*a^3*d*(b^2)^(3/2) + 4*a^5*d*(b^2)^(1/2)))))*(b^2)^(
1/2))/(-a^4*d^2)^(1/2) - 1/(a*d*sinh(c + d*x))
```

3.453 $\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=35

$$\operatorname{Int}\left(\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Coth[c + d*x]*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][(Coth[c + d*x]*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

Rubi steps

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 140.66, size = 0, normalized size = 0.00

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Coth[c + d*x]*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x
]
[Out] Integrate[(Coth[c + d*x]*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(dx+c)\operatorname{csch}(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] 2*e^(d*x + c)/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e^(2*c + 1))*e^(2*d
*x)) - 2*integrate(-1/2*(b*d*f*x + b*d*e + a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*f*
x*e + a^2*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*f*x*e^(c + 1) + a^2*d*e^(c +
 2))*e^(d*x)), x) + 2*integrate(1/2*(b*d*f*x + b*d*e - a*f)/(a^2*d*f^2*x^2
+ 2*a^2*d*f*x*e + a^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*d*f*x*e^(c + 1) +
a^2*d*e^(c + 2))*e^(d*x)), x) - 2*integrate(-(a*b*e^(d*x + c) - b^2)/(a^2*b
*f*x + a^2*b*e - (a^2*b*f*x*e^(2*c) + a^2*b*e^(2*c + 1))*e^(2*d*x) - 2*(a^3
*f*x*e^c + a^3*e^(c + 1))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] integral(coth(d*x + c)*csch(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
+ c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(c + dx)\operatorname{csch}(c + dx)}{(a + b\sinh(c + dx))(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)*csch(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\coth(c+d\,x)}{\sinh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)/(sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(coth(c + d*x)/(sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.454 $\int \frac{(e+fx)^3 \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=721

$$-\frac{(e+fx)^3}{ad}+\frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d}-\frac{(e+fx)^3\coth(c+dx)}{ad}+\frac{\sqrt{a^2+b^2}\,(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+}}\right.}{a^2d}$$

```
[Out] -(f*x+e)^3/a/d+2*b*(f*x+e)^3*arctanh(exp(d*x+c))/a^2/d-(f*x+e)^3*coth(d*x+c
)/a/d+3*f*(f*x+e)^2*ln(1-exp(2*d*x+2*c))/a/d^2+3*b*f*(f*x+e)^2*polylog(2,-e
xp(d*x+c))/a^2/d^2-3*b*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a^2/d^2+3*f^2*(f*x
+e)*polylog(2,exp(2*d*x+2*c))/a/d^3-6*b*f^2*(f*x+e)*polylog(3,-exp(d*x+c))/
a^2/d^3+6*b*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a^2/d^3-3/2*f^3*polylog(3,exp
(2*d*x+2*c))/a/d^4+6*b*f^3*polylog(4,-exp(d*x+c))/a^2/d^4-6*b*f^3*polylog(4
,exp(d*x+c))/a^2/d^4+(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+
b^2)^(1/2)/a^2/d-(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)
^(1/2)/a^2/d+3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^
2+b^2)^(1/2)/a^2/d^2-3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/
2)))*(a^2+b^2)^(1/2)/a^2/d^2-6*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+
b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^2/d^3+6*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/
(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^2/d^3+6*f^3*polylog(4,-b*exp(d*x+c)/
(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^2/d^4-6*f^3*polylog(4,-b*exp(d*x+c)/
(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^2/d^4
```

Rubi [A]
time = 1.17, antiderivative size = 721, normalized size of antiderivative = 1.00, number of steps used = 41, number of rules used = 17, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {5688, 3801, 3797, 2221, 2611, 2320, 6724, 32, 5704, 5558, 3377, 2717, 4267, 6744, 5684, 3403, 2296}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((e + f*x)^3/(a*d)) + (2*b*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a^2*d) - ((e
 + f*x)^3*Coth[c + d*x])/(a*d) + (Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*d) - (Sqrt[a^2 + b^2]*(e + f*x)^3*L
og[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*d) + (3*f*(e + f*x)^2*L
og[1 - E^(2*(c + d*x))])/(a*d^2) + (3*b*f*(e + f*x)^2*PolyLog[2, -E^(c + d*
x)])/(a^2*d^2) - (3*b*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)])/(a^2*d^2) + (3
*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/(a^2*d^2) - (3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^2) + (3*f^2*(e + f*x)*PolyLog[2, E
^(2*(c + d*x))])/(a*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a^
```

```
2*d^3) + (6*b*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a^2*d^3) - (6*Sqrt[a^
2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]
)/(a^2*d^3) + (6*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2]))])/(a^2*d^3) - (3*f^3*PolyLog[3, E^(2*(c + d*x))])/(
2*a*d^4) + (6*b*f^3*PolyLog[4, -E^(c + d*x)])/(a^2*d^4) - (6*b*f^3*PolyLog[
4, E^(c + d*x)])/(a^2*d^4) + (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^4) - (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^4)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5688

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x]^
(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5704

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\coth^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{\int (e+fx)^3\,dx}{a} - \frac{b\int (e+fx)^3\cosh(c+dx)\,\mathrm{co}}{a^2}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} - \frac{(e+fx)^3\coth(c+dx)}{ad} - \frac{\int (e+fx)^3\,dx}{a} - \frac{b\int}{}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{3f}{}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{3f}{}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{\sqrt{a}}{}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{\sqrt{a}}{}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{\sqrt{a}}{}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{\sqrt{a}}{}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{\sqrt{a}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.06, size = 4761, normalized size = 6.60

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (3*e^2*f*(((-I)*Pi*ArcTanh[(-b + a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/Sqr
t[a^2 + b^2] - (2*((-I)*c + Pi/2 - I*d*x)*ArcTanh[((a - I*b)*Cot[((-I)*c +
Pi/2 - I*d*x)/2])/Sqrt[-a^2 - b^2]] - 2*((-I)*c + ArcCos[((-I)*a)/b])*ArcTa
```

nh[((-a - I*b)*Tan[((-I)*c + Pi/2 - I*d*x)/2])/Sqrt[-a^2 - b^2]] + (ArcCos[
((-I)*a)/b] - (2*I)*(ArcTanh[((a - I*b)*Cot[((-I)*c + Pi/2 - I*d*x)/2])/Sqr
t[-a^2 - b^2]] - ArcTanh[((-a - I*b)*Tan[((-I)*c + Pi/2 - I*d*x)/2])/Sqrt[-
a^2 - b^2]]))*Log[Sqrt[-a^2 - b^2]/(Sqrt[2]*Sqrt[(-I)*b]*E^((I/2)*((-I)*c +
 Pi/2 - I*d*x))*Sqrt[a + b*Sinh[c + d*x]])] + (ArcCos[((-I)*a)/b] + (2*I)*(
ArcTanh[((a - I*b)*Cot[((-I)*c + Pi/2 - I*d*x)/2])/Sqrt[-a^2 - b^2]] - ArcT
anh[((-a - I*b)*Tan[((-I)*c + Pi/2 - I*d*x)/2])/Sqrt[-a^2 - b^2]]))*Log[(Sq
rt[-a^2 - b^2]*E^((I/2)*((-I)*c + Pi/2 - I*d*x)))/(Sqrt[2]*Sqrt[(-I)*b]*Sqr
t[a + b*Sinh[c + d*x]])] - (ArcCos[((-I)*a)/b] + (2*I)*ArcTanh[((-a - I*b)*
Tan[((-I)*c + Pi/2 - I*d*x)/2])/Sqrt[-a^2 - b^2]])*Log[1 - (I*(a - I*Sqrt[-
a^2 - b^2])*(a - I*b - Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x)/2]))/(b
*(a - I*b + Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x)/2]))] + (-ArcCos[(
(-I)*a)/b] + (2*I)*ArcTanh[((-a - I*b)*Tan[((-I)*c + Pi/2 - I*d*x)/2])/Sqrt
[-a^2 - b^2]])*Log[1 - (I*(a + I*Sqrt[-a^2 - b^2])*(a - I*b - Sqrt[-a^2 - b
^2]*Tan[((-I)*c + Pi/2 - I*d*x)/2]))/(b*(a - I*b + Sqrt[-a^2 - b^2]*Tan[((-
I)*c + Pi/2 - I*d*x)/2]))] + I*(PolyLog[2, (I*(a - I*Sqrt[-a^2 - b^2])*(a -
 I*b - Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x)/2]))/(b*(a - I*b + Sqrt
[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x)/2]))] - PolyLog[2, (I*(a + I*Sqrt[
-a^2 - b^2])*(a - I*b - Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x)/2]))/(
b*(a - I*b + Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x)/2]))]))/Sqrt[-a^2
 - b^2]))/d^2 + (3*b^2*e^2*f*(((-I)*Pi*ArcTanh[(-b + a*Tanh[(c + d*x)/2])/S
qrt[a^2 + b^2]])/Sqrt[a^2 + b^2] - (2*((-I)*c + Pi/2 - I*d*x)*ArcTanh[((a -
 I*b)*Cot[((-I)*c + Pi/2 - I*d*x)/2])/Sqrt[-a^2 - b^2]] - 2*((-I)*c + ArcCo
s[((-I)*a)/b])*ArcTanh[((-a - I*b)*Tan[((-I)*c + Pi/2 - I*d*x)/2])/Sqrt[-a^
2 - b^2]] + (ArcCos[((-I)*a)/b] - (2*I)*(ArcTanh[((a - I*b)*Cot[((-I)*c + P
i/2 - I*d*x)/2])/Sqrt[-a^2 - b^2]] - ArcTanh[((-a - I*b)*Tan[((-I)*c + Pi/2
 - I*d*x)/2])/Sqrt[-a^2 - b^2]]))*Log[Sqrt[-a^2 - b^2]/(Sqrt[2]*Sqrt[(-I)*b
]*E^((I/2)*((-I)*c + Pi/2 - I*d*x))*Sqrt[a + b*Sinh[c + d*x]])] + (ArcCos[(
(-I)*a)/b] + (2*I)*(ArcTanh[((a - I*b)*Cot[((-I)*c + Pi/2 - I*d*x)/2])/Sqrt
[-a^2 - b^2]] - ArcTanh[((-a - I*b)*Tan[((-I)*c + Pi/2 - I*d*x)/2])/Sqrt[-a
^2 - b^2]]))*Log[(Sqrt[-a^2 - b^2]*E^((I/2)*((-I)*c + Pi/2 - I*d*x)))/(Sqrt
[2]*Sqrt[(-I)*b]*Sqrt[a + b*Sinh[c + d*x]])] - (ArcCos[((-I)*a)/b] + (2*I)*
ArcTanh[((-a - I*b)*Tan[((-I)*c + Pi/2 - I*d*x)/2])/Sqrt[-a^2 - b^2]])*Log[
1 - (I*(a - I*Sqrt[-a^2 - b^2])*(a - I*b - Sqrt[-a^2 - b^2]*Tan[((-I)*c + P
i/2 - I*d*x)/2]))/(b*(a - I*b + Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x
)/2]))] + (-ArcCos[((-I)*a)/b] + (2*I)*ArcTanh[((-a - I*b)*Tan[((-I)*c + Pi
/2 - I*d*x)/2])/Sqrt[-a^2 - b^2]])*Log[1 - (I*(a + I*Sqrt[-a^2 - b^2])*(a -
 I*b - Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x)/2]))/(b*(a - I*b + Sqrt
[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x)/2]))] + I*(PolyLog[2, (I*(a - I*Sq
rt[-a^2 - b^2])*(a - I*b - Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x)/2])
)/(b*(a - I*b + Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x)/2]))] - PolyLo
g[2, (I*(a + I*Sqrt[-a^2 - b^2])*(a - I*b - Sqrt[-a^2 - b^2]*Tan[((-I)*c +
Pi/2 - I*d*x)/2]))/(b*(a - I*b + Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*
x)/2]))]))/Sqrt[-a^2 - b^2]))/(a^2*d^2) + (3*e*f^2*(d^2*x^2*Log[1 + (b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2])] - d^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt

```
[a^2 + b^2])] + 2*d*x*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] -
2*d*x*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*PolyLog[3, (
b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*PolyLog[3, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))]))/(Sqrt[a^2 + b^2]*d^3) + (3*b^2*e*f^2*(d^2*x^2*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^2*x^2*Log[1 + (b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 +
b^2])] - 2*d*x*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*Pol
yLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*PolyLog[3, -((b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^2*Sqrt[a^2 + b^2]*d^3) - (12*a*d^3*e^2*E
^(2*c)*f*x + 12*a*d^3*e*E^(2*c)*f^2*x^2 + 4*a*d^3*E^(2*c)*f^3*x^3 + 4*b*d^3
*e^3*ArcTanh[E^(c + d*x)] - 4*b*d^3*e^3*E^(2*c)*ArcTanh[E^(c + d*x)] - 6*b*
d^3*e^2*f*x*Log[1 - E^(c + d*x)] + 6*b*d^3*e^2*E^(2*c)*f*x*Log[1 - E^(c + d
*x)] - 6*b*d^3*e*f^2*x^2*Log[1 - E^(c + d*x)] + 6*b*d^3*e*E^(2*c)*f^2*x^2*L
og[1 - E^(c + d*x)] - 2*b*d^3*f^3*x^3*Log[1 - E^(c + d*x)] + 2*b*d^3*E^(2*c
)*f^3*x^3*Log[1 - E^(c + d*x)] + 6*b*d^3*e^2*f*x*Log[1 + E^(c + d*x)] - 6*b
*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(c + d*x)] + 6*b*d^3*e*f^2*x^2*Log[1 + E^(c
+ d*x)] - 6*b*d^3*e*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] + 2*b*d^3*f^3*x^3*
Log[1 + E^(c + d*x)] - 2*b*d^3*E^(2*c)*f^3*x^3*Log[1 + E^(c + d*x)] + 6*a*d
^2*e^2*f*Log[1 - E^(2*(c + d*x))] - 6*a*d^2*e^2...
```

Maple [F]
time = 2.36, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \left(\coth^2(dx+c)\right)}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + sqrt(a
^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a +
sqrt(a^2 + b^2)))/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d))*e^3 - 6*f*x*e^2
/(a*d) - 2*(f^3*x^3 + 3*f^2*x^2*e + 3*f*x*e^2)/(a*d*e^(2*d*x + 2*c) - a*d)
+ 3*f*e^2*log(e^(d*x + c) + 1)/(a*d^2) + 3*f*e^2*log(e^(d*x + c) - 1)/(a*d^
2) + (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*
polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*b*f^3/(a^2*d^4) - (d
```

```
^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog
(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*b*f^3/(a^2*d^4) + 3*(b*d*f*e^
2 + 2*a*f^2*e)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) -
 3*(b*d*f*e^2 - 2*a*f^2*e)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))
/(a^2*d^3) + 3*(b*d*f^2*e + a*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*di
log(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^2*d^4) - 3*(b*d*f^2*e -
a*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylo
g(3, e^(d*x + c)))/(a^2*d^4) - 1/4*(b*d^4*f^3*x^4 + 4*(b*d*f^2*e + a*f^3)*d
^3*x^3 + 6*(b*d^2*f*e^2 + 2*a*d*f^2*e)*d^2*x^2)/(a^2*d^4) + 1/4*(b*d^4*f^3*
x^4 + 4*(b*d*f^2*e - a*f^3)*d^3*x^3 + 6*(b*d^2*f*e^2 - 2*a*d*f^2*e)*d^2*x^2
)/(a^2*d^4) + integrate(2*((a^2*f^3*e^c + b^2*f^3*e^c)*x^3 + 3*(a^2*f^2*e^c
 + b^2*f^2*e^c)*x^2*e + 3*(a^2*f*e^c + b^2*f*e^c)*x*e^2)*e^(d*x)/(a^2*b*e^(
2*d*x + 2*c) + 2*a^3*e^(d*x + c) - a^2*b), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7711 vs. $2(682) = 1364$.
time = 0.52, size = 7711, normalized size = 10.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*a*c^3*f^3 - 6*a*c^2*d*f^2*cosh(1) + 6*a*c*d^2*f*cosh(1)^2 - 2*a*d^3*cosh
(1)^3 - 2*a*d^3*sinh(1)^3 - 2*(a*d^3*f^3*x^3 + a*c^3*f^3 + 3*(a*d^3*f*x + a
*c*d^2*f)*cosh(1)^2 + 3*(a*d^3*f*x + a*c*d^2*f)*sinh(1)^2 + 3*(a*d^3*f^2*x^
2 - a*c^2*d*f^2)*cosh(1) + 3*(a*d^3*f^2*x^2 - a*c^2*d*f^2 + 2*(a*d^3*f*x +
a*c*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 6*(a*c*d^2*f - a*d^3*cosh(1)
)*sinh(1)^2 - 4*(a*d^3*f^3*x^3 + a*c^3*f^3 + 3*(a*d^3*f*x + a*c*d^2*f)*cosh
(1)^2 + 3*(a*d^3*f*x + a*c*d^2*f)*sinh(1)^2 + 3*(a*d^3*f^2*x^2 - a*c^2*d*f^
2)*cosh(1) + 3*(a*d^3*f^2*x^2 - a*c^2*d*f^2 + 2*(a*d^3*f*x + a*c*d^2*f)*cos
h(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - 2*(a*d^3*f^3*x^3 + a*c^3*f^3 +
 3*(a*d^3*f*x + a*c*d^2*f)*cosh(1)^2 + 3*(a*d^3*f*x + a*c*d^2*f)*sinh(1)^2
+ 3*(a*d^3*f^2*x^2 - a*c^2*d*f^2)*cosh(1) + 3*(a*d^3*f^2*x^2 - a*c^2*d*f^2
+ 2*(a*d^3*f*x + a*c*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 3*(b*d^2*f^
3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^2*f*cosh(1)^2 + b*d^2*f*sinh(1)^2 - (b*
d^2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^2*f*cosh(1)^2 + b*d^2*f*sinh(1)^2
 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 - 2*(b*d^2*f^
3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^2*f*cosh(1)^2 + b*d^2*f*sinh(1)^2 + 2*(
b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - (b*d^
2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^2*f*cosh(1)^2 + b*d^2*f*sinh(1)^2 +
 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c)^2 + 2*(b*d^2*f^2*
x + b*d^2*f*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c)
+ a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2) - b)/b + 1) + 3*(b*d^2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^2*f*cosh(1)
```

```
^2 + b*d^2*f*sinh(1)^2 - (b*d^2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^2*f*c
osh(1)^2 + b*d^2*f*sinh(1)^2 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh(1))*c
osh(d*x + c)^2 - 2*(b*d^2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^2*f*cosh(1)
^2 + b*d^2*f*sinh(1)^2 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh(1))*cosh(d*
x + c)*sinh(d*x + c) - (b*d^2*f^3*x^2 + 2*b*d^2*f^2*x*cosh(1) + b*d^2*f*cos
h(1)^2 + b*d^2*f*sinh(1)^2 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh(1))*sin
h(d*x + c)^2 + 2*(b*d^2*f^2*x + b*d^2*f*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/
b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b*c^3*f^3 - 3*b*c^2*d*f^2*cos
h(1) + 3*b*c*d^2*f*cosh(1)^2 - b*d^3*cosh(1)^3 - b*d^3*sinh(1)^3 - (b*c^3*f
^3 - 3*b*c^2*d*f^2*cosh(1) + 3*b*c*d^2*f*cosh(1)^2 - b*d^3*cosh(1)^3 - b*d^
3*sinh(1)^3 + 3*(b*c*d^2*f - b*d^3*cosh(1))*sinh(1)^2 - 3*(b*c^2*d*f^2 - 2*
b*c*d^2*f*cosh(1) + b*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c)^2 + 3*(b*c*d^2*
f - b*d^3*cosh(1))*sinh(1)^2 - 2*(b*c^3*f^3 - 3*b*c^2*d*f^2*cosh(1) + 3*b*c
*d^2*f*cosh(1)^2 - b*d^3*cosh(1)^3 - b*d^3*sinh(1)^3 + 3*(b*c*d^2*f - b*d^3
*cosh(1))*sinh(1)^2 - 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(1) + b*d^3*cosh(1)^
2)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - (b*c^3*f^3 - 3*b*c^2*d*f^2*cosh(1
) + 3*b*c*d^2*f*cosh(1)^2 - b*d^3*cosh(1)^3 - b*d^3*sinh(1)^3 + 3*(b*c*d^2*
f - b*d^3*cosh(1))*sinh(1)^2 - 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(1) + b*d^3
*cosh(1)^2)*sinh(1))*sinh(d*x + c)^2 - 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(1)
 + b*d^3*cosh(1)^2)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) +
2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b*c^3*f^3 - 3*b*c^2
*d*f^2*cosh(1) + 3*b*c*d^2*f*cosh(1)^2 - b*d^3*cosh(1)^3 - b*d^3*sinh(1)^3
- (b*c^3*f^3 - 3*b*c^2*d*f^2*cosh(1) + 3*b*c*d^2*f*cosh(1)^2 - b*d^3*cosh(1
)^3 - b*d^3*sinh(1)^3 + 3*(b*c*d^2*f - b*d^3*cosh(1))*sinh(1)^2 - 3*(b*c^2*
d*f^2 - 2*b*c*d^2*f*cosh(1) + b*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c)^2 + 3
*(b*c*d^2*f - b*d^3*cosh(1))*sinh(1)^2 - 2*(b*c^3*f^3 - 3*b*c^2*d*f^2*cosh(
1) + 3*b*c*d^2*f*cosh(1)^2 - b*d^3*cosh(1)^3 - b*d^3*sinh(1)^3 + 3*(b*c*d^2
*f - b*d^3*cosh(1))*sinh(1)^2 - 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(1) + b*d^
3*cosh(1)^2)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - (b*c^3*f^3 - 3*b*c^2*d*
f^2*cosh(1) + 3*b*c*d^2*f*cosh(1)^2 - b*d^3*cosh(1)^3 - b*d^3*sinh(1)^3 + 3
*(b*c*d^2*f - b*d^3*cosh(1))*sinh(1)^2 - 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(
1) + b*d^3*cosh(1)^2)*sinh(1))*sinh(d*x + c)^2 - 3*(b*c^2*d*f^2 - 2*b*c*d^2
*f*cosh(1) + b*d^3*cosh(1)^2)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d
*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b*d^3*f^3
*x^3 + b*c^3*f^3 + 3*(b*d^3*f*x + b*c*d^2*f)*cosh(1)^2 - (b*d^3*f^3*x^3 + b
*c^3*f^3 + 3*(b*d^3*f*x + b*c*d^2*f)*cosh(1)^2 + 3*(b*d^3*f*x + b*c*d^2*f)*
sinh(1)^2 + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2)*cosh(1) + 3*(b*d^3*f^2*x^2 - b*
c^2*d*f^2 + 2*(b*d^3*f*x + b*c*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 3
*(b*d^3*f*x + b*c*d^2*f)*sinh(1)^2 - 2*(b*d^3*f^3*x^3 + b*c^3*f^3 + 3*(b*d^
3*f*x + b*c*d^2*f)*cosh(1)^2 + 3*(b*d^3*f*x + b*c*d^2*f)*sinh(1)^2 + 3*(b*d
^3*f^2*x^2 - b*c^2*d*f^2)*cosh(1) + 3*(b*d^3*f^2*x^2 - b*c^2*d*f^2 + 2*(b*d
^3*f*x + b*c*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - (b*d^3*
f^3*x^3 + b*c^3*f^3 + 3*(b*d^3*f*x + b*c*d^2*f)*cosh(1)^2 + 3*(b*d^3*f*x +
b*c*d^2*f)*sinh(1)^2 + 3*(b*d^3*f^2*x^2 - b*c^2...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*coth(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{coth}\left(c+d\,x\right)}^2\,{\left(e+f\,x\right)}^3}{a+b\,\mathrm{sinh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((coth(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.455 $\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=517

$$-\frac{(e+fx)^2}{ad}+\frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d}-\frac{(e+fx)^2\coth(c+dx)}{ad}+\frac{\sqrt{a^2+b^2}\,(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+}}\right.}{a^2d}$$

```
[Out] -(f*x+e)^2/a/d+2*b*(f*x+e)^2*arctanh(exp(d*x+c))/a^2/d-(f*x+e)^2*coth(d*x+c
)/a/d+2*f*(f*x+e)*ln(1-exp(2*d*x+2*c))/a/d^2+2*b*f*(f*x+e)*polylog(2,-exp(d
*x+c))/a^2/d^2-2*b*f*(f*x+e)*polylog(2,exp(d*x+c))/a^2/d^2+f^2*polylog(2,ex
p(2*d*x+2*c))/a/d^3-2*b*f^2*polylog(3,-exp(d*x+c))/a^2/d^3+2*b*f^2*polylog(
3,exp(d*x+c))/a^2/d^3+(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2
+b^2)^(1/2)/a^2/d-(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2
)^(1/2)/a^2/d+2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2
+b^2)^(1/2)/a^2/d^2-2*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2))
)*(a^2+b^2)^(1/2)/a^2/d^2-2*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2))
)*(a^2+b^2)^(1/2)/a^2/d^3+2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2))
)*(a^2+b^2)^(1/2)/a^2/d^3
```

Rubi [A]

time = 0.92, antiderivative size = 517, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 18, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {5688, 3801, 3797, 2221, 2317, 2438, 32, 5704, 5558, 3377, 2718, 4267, 2611, 2320, 6724, 5684, 3403, 2296}

$$-\frac{2f^2\sqrt{a^2+b^2}\,\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3}+\frac{2f^2\sqrt{a^2+b^2}\,\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^3}+\frac{2f\sqrt{a^2+b^2}\,(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2}-\frac{2f\sqrt{a^2+b^2}\,(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^2}+\frac{\sqrt{a^2+b^2}\,(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^2d}-\frac{\sqrt{a^2+b^2}\,(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{a^2d}-\frac{2bf^2\text{Li}_3\left(-e^{c+dx}\right)}{a^2d^3}+\frac{2bf^2\text{Li}_3\left(e^{c+dx}\right)}{a^2d^3}+\frac{2bf(e+fx)\text{Li}_2\left(-e^{c+dx}\right)}{a^2d^2}-\frac{2bf(e+fx)\text{Li}_2\left(e^{c+dx}\right)}{a^2d^2}+\frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d}+\frac{f^2\text{Li}_2\left(e^{2(c+dx)}\right)}{ad^3}+\frac{2f(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad^2}-\frac{(e+fx)^2\coth(c+dx)}{ad}-\frac{(e+fx)^2}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((e + f*x)^2/(a*d)) + (2*b*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^2*d) - ((e
 + f*x)^2*Coth[c + d*x])/(a*d) + (Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*d) - (Sqrt[a^2 + b^2]*(e + f*x)^2*L
og[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*d) + (2*f*(e + f*x)*Log
[1 - E^(2*(c + d*x))])/(a*d^2) + (2*b*f*(e + f*x)*PolyLog[2, -E^(c + d*x)])
/(a^2*d^2) - (2*b*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^2*d^2) + (2*Sqrt[
a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]
)/(a^2*d^2) - (2*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(a^2*d^2) + (f^2*PolyLog[2, E^(2*(c + d*x))])/(a*d^
3) - (2*b*f^2*PolyLog[3, -E^(c + d*x)])/(a^2*d^3) + (2*b*f^2*PolyLog[3, E^(
c + d*x)])/(a^2*d^3) - (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/(a^2*d^3) + (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5688

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x]^
(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5704

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\coth^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{\int (e+fx)^2\,dx}{a} - \frac{b\int (e+fx)^2\cosh(c+dx)\,\text{co}}{a^2}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} - \frac{(e+fx)^2\coth(c+dx)}{ad} - \frac{\int (e+fx)^2\,dx}{a} - b\int$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + 2f$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + 2f$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \sqrt{a}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \sqrt{a}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \sqrt{a}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \sqrt{a}$$

Mathematica [A]
time = 8.28, size = 972, normalized size = 1.88

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*((-4*a*d^2*e*E^(2*c)*f*x)/(-1 + E^(2*c)) - (2*a*d^2*E^(2*c)*f^2*x^2)/(-1
 + E^(2*c)) + 2*b*d^2*e^2*ArcTanh[E^(c + d*x)] - 2*b*d^2*e*f*x*Log[1 - E^(c
 + d*x)] - b*d^2*f^2*x^2*Log[1 - E^(c + d*x)] + 2*b*d^2*e*f*x*Log[1 + E^(c
+ d*x)] + b*d^2*f^2*x^2*Log[1 + E^(c + d*x)] + 2*a*d*e*f*Log[1 - E^(2*(c +
d*x))] + 2*a*d*f^2*x*Log[1 - E^(2*(c + d*x))] + 2*b*d*f*(e + f*x)*PolyLog[2
, -E^(c + d*x)] - 2*b*d*f*(e + f*x)*PolyLog[2, E^(c + d*x)] + a*f^2*PolyLog
```

```
[2, E^(2*(c + d*x))] - 2*b*f^2*PolyLog[3, -E^(c + d*x)] + 2*b*f^2*PolyLog[3
, E^(c + d*x)]) + 2*(a^2 + b^2)*((2*d^2*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt
[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + (2*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x)
)/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] + (d^2*E^
c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/S
qrt[(a^2 + b^2)*E^(2*c)] - (2*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^
c + Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] - (d^2*E^c*f^2*x
^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^
2 + b^2)*E^(2*c)] + (2*d*E^c*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*
E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*d*E^c*f*
(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]
))])/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/
(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] + (2*E^c*f
^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sq
rt[(a^2 + b^2)*E^(2*c)]) + a*d^2*(e + f*x)^2*Csch[c/2]*Csch[(c + d*x)/2]*Si
nh[(d*x)/2] - a*d^2*(e + f*x)^2*Sech[c/2]*Sech[(c + d*x)/2]*Sinh[(d*x)/2])/
(2*a^2*d^3)
```

Maple [F]
time = 2.12, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \left(\coth^2(dx+c)\right)}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
[Out] int((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
[Out] (b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + sqrt(a
^2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a +
sqrt(a^2 + b^2)))/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d))*e^2 - 4*f*x*e/(
a*d) - 2*(f^2*x^2 + 2*f*x*e)/(a*d*e^(2*d*x + 2*c) - a*d) + 2*f*e*log(e^(d*x
 + c) + 1)/(a*d^2) + 2*f*e*log(e^(d*x + c) - 1)/(a*d^2) + (d^2*x^2*log(e^(d
*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*b*f^
2/(a^2*d^3) - (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2
*polylog(3, e^(d*x + c)))*b*f^2/(a^2*d^3) + 2*(b*d*f*e + a*f^2)*(d*x*log(e^
(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) - 2*(b*d*f*e - a*f^2)*(d*x*
```

```
log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^2*d^3) - 1/3*(b*d^3*f^2*x^3
+ 3*(b*d*f*e + a*f^2)*d^2*x^2)/(a^2*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*d*f*e
- a*f^2)*d^2*x^2)/(a^2*d^3) + integrate(2*((a^2*f^2*e^c + b^2*f^2*e^c)*x^2
+ 2*(a^2*f*e^c + b^2*f*e^c)*x*e)*e^(d*x)/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(
d*x + c) - a^2*b), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3633 vs. $2(487) = 974$.
time = 0.46, size = 3633, normalized size = 7.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")

[Out] -(2*a*c^2*f^2 - 4*a*c*d*f*cosh(1) + 2*a*d^2*cosh(1)^2 + 2*a*d^2*sinh(1)^2 +
 2*(a*d^2*f^2*x^2 - a*c^2*f^2 + 2*(a*d^2*f*x + a*c*d*f)*cosh(1) + 2*(a*d^2*
f*x + a*c*d*f)*sinh(1))*cosh(d*x + c)^2 + 4*(a*d^2*f^2*x^2 - a*c^2*f^2 + 2*
(a*d^2*f*x + a*c*d*f)*cosh(1) + 2*(a*d^2*f*x + a*c*d*f)*sinh(1))*cosh(d*x +
 c)*sinh(d*x + c) + 2*(a*d^2*f^2*x^2 - a*c^2*f^2 + 2*(a*d^2*f*x + a*c*d*f)*
cosh(1) + 2*(a*d^2*f*x + a*c*d*f)*sinh(1))*sinh(d*x + c)^2 + 2*(b*d*f^2*x +
 b*d*f*cosh(1) + b*d*f*sinh(1) - (b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1)
)*cosh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1))*cosh(d*x
+ c)*sinh(d*x + c) - (b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1))*sinh(d*x +
 c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b*d
*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1) - (b*d*f^2*x + b*d*f*cosh(1) + b*d*f
*sinh(1))*cosh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1))*c
osh(d*x + c)*sinh(d*x + c) - (b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1))*si
nh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x +
c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1)
- (b*c^2*f^2 - 2*b*c*d*f*cosh(1) + b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 - (b*c
^2*f^2 - 2*b*c*d*f*cosh(1) + b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 - 2*(b*c*d*f
 - b*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^2 - 2*(b*c^2*f^2 - 2*b*c*d*f*cosh(
1) + b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 - 2*(b*c*d*f - b*d^2*cosh(1))*sinh(1
))*cosh(d*x + c)*sinh(d*x + c) - (b*c^2*f^2 - 2*b*c*d*f*cosh(1) + b*d^2*cos
h(1)^2 + b*d^2*sinh(1)^2 - 2*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*sinh(d*x +
c)^2 - 2*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*c
osh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b*c^
2*f^2 - 2*b*c*d*f*cosh(1) + b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 - (b*c^2*f^2
- 2*b*c*d*f*cosh(1) + b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 - 2*(b*c*d*f - b*d^
2*cosh(1))*sinh(1))*cosh(d*x + c)^2 - 2*(b*c^2*f^2 - 2*b*c*d*f*cosh(1) + b*
d^2*cosh(1)^2 + b*d^2*sinh(1)^2 - 2*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*cosh
(d*x + c)*sinh(d*x + c) - (b*c^2*f^2 - 2*b*c*d*f*cosh(1) + b*d^2*cosh(1)^2
+ b*d^2*sinh(1)^2 - 2*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*sinh(d*x + c)^2 -
```

```
2*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x
 + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b*d^2*f^2*x
^2 - b*c^2*f^2 - (b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*cosh(
1) + 2*(b*d^2*f*x + b*c*d*f)*sinh(1))*cosh(d*x + c)^2 - 2*(b*d^2*f^2*x^2 -
b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*cosh(1) + 2*(b*d^2*f*x + b*c*d*f)*sinh(
1))*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x
 + b*c*d*f)*cosh(1) + 2*(b*d^2*f*x + b*c*d*f)*sinh(1))*sinh(d*x + c)^2 + 2*
(b*d^2*f*x + b*c*d*f)*cosh(1) + 2*(b*d^2*f*x + b*c*d*f)*sinh(1))*sqrt((a^2
+ b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b*d^2*f^2*x^2 - b*c^2*f^2 -
 (b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*cosh(1) + 2*(b*d^2*f*
x + b*c*d*f)*sinh(1))*cosh(d*x + c)^2 - 2*(b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b
*d^2*f*x + b*c*d*f)*cosh(1) + 2*(b*d^2*f*x + b*c*d*f)*sinh(1))*cosh(d*x + c
)*sinh(d*x + c) - (b*d^2*f^2*x^2 - b*c^2*f^2 + 2*(b*d^2*f*x + b*c*d*f)*cosh
(1) + 2*(b*d^2*f*x + b*c*d*f)*sinh(1))*sinh(d*x + c)^2 + 2*(b*d^2*f*x + b*c
*d*f)*cosh(1) + 2*(b*d^2*f*x + b*c*d*f)*sinh(1))*sqrt((a^2 + b^2)/b^2)*log(
-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 + b^2)/b^2) - b)/b) + 2*(b*f^2*cosh(d*x + c)^2 + 2*b*f^2*cosh(d*x
+ c)*sinh(d*x + c) + b*f^2*sinh(d*x + c)^2 - b*f^2)*sqrt((a^2 + b^2)/b^2)*p
olylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*(b*f^2*cosh(d*x + c)^2 + 2*b*f^2*cosh
(d*x + c)*sinh(d*x + c) + b*f^2*sinh(d*x + c)^2 - b*f^2)*sqrt((a^2 + b^2)/b
^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*(b*d*f^2*x + b*d*f*cosh(1) + b*d
*f*sinh(1) - a*f^2 - (b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1) - a*f^2)*co
sh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1) - a*f^2)*cosh(
d*x + c)*sinh(d*x + c) - (b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(1) - a*f^2
)*sinh(d*x + c)^2)*dilog(cosh(d*x + c) + sinh(d*x + c)) + 2*(b*d*f^2*x + b*
d*f*cosh(1) + b*d*f*sinh(1) + a*f^2 - (b*d*f^2*x + b*d*f*cosh(1) + b*d*f*si
nh(1) + a*f^2)*cosh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*f*cosh(1) + b*d*f*sinh(
1) + a*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f^2*x + b*d*f*cosh(1) + b*d*
f*sinh(1) + a*f^2)*sinh(d*x + c)^2)*dilog(-cosh(d*x + c) - sinh(d*x + c)) +
 (b*d^2*f^2*x^2 + 2*a*d*f^2*x + b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 - (b*d^2*
f^2*x^2 + 2*a*d*f^2*x + b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 + 2*(b*d^2*f*x +
a*d*f)*cosh(1) + 2*(b*d^2*f*x + b*d^2*cosh(1) + a*d*f)*sinh(1))*cosh(d*x +
c)^2 - 2*(b*d^2*f^2*x^2 + 2*a*d*f^2*x + b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 +
 2*(b*d^2*f*x + a*d*f)*cosh(1) + 2*(b*d^2*f*x +...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)**2*coth(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(c+d\,x)^2\,{(e+f\,x)}^2}{a+b\,\mathrm{sinh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
[Out] int((coth(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.456 $\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=294

$$\frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d}-\frac{(e+fx)\coth(c+dx)}{ad}+\frac{\sqrt{a^2+b^2}\,(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d}-\frac{\sqrt{a^2+b^2}\,(}{}$$

```
[Out] 2*b*(f*x+e)*arctanh(exp(d*x+c))/a^2/d-(f*x+e)*coth(d*x+c)/a/d+f*ln(sinh(d*x
+c))/a/d^2+b*f*polylog(2,-exp(d*x+c))/a^2/d^2-b*f*polylog(2,exp(d*x+c))/a^2
/d^2+(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^2/d-(
f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^2/d+f*polyl
og(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^2/d^2-f*polylog(2
,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^2/d^2
```

Rubi [A]
time = 0.49, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {5688, 3801, 3556, 5704, 5558, 3377, 2717, 4267, 2317, 2438, 5684, 3403, 2296, 2221}

$$\frac{f\sqrt{a^2+b^2}\,\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2}-\frac{f\sqrt{a^2+b^2}\,\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^2}+\frac{\sqrt{a^2+b^2}\,(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^2d}-\frac{\sqrt{a^2+b^2}\,(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{a^2d}+\frac{bf\text{Li}_2\left(-e^{c+dx}\right)}{a^2d^2}-\frac{bf\text{Li}_2\left(e^{c+dx}\right)}{a^2d^2}+\frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d}+\frac{f\log(\sinh(c+dx))}{ad^2}-\frac{(e+fx)\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*b*(e + f*x)*ArcTanh[E^(c + d*x)])/(a^2*d) - ((e + f*x)*Coth[c + d*x])/(a
*d) + (Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])])/(a^2*d) - (Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2])])/(a^2*d) + (f*Log[Sinh[c + d*x]])/(a*d^2) + (b*f*PolyLog[2,
-E^(c + d*x)])/(a^2*d^2) - (b*f*PolyLog[2, E^(c + d*x)])/(a^2*d^2) + (Sqrt[
a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^2
) - (Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/(a^2*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5688

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x]^
(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5704

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\coth^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)\coth(c+dx)}{ad} + \frac{\int (e+fx)\,dx}{a} - \frac{b\int (e+fx)\cosh(c+dx)\coth(c}{a^2}$$

$$= \frac{ex}{a} + \frac{fx^2}{2a} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{f\log(\sinh(c+dx))}{ad^2} - \frac{\int (e+fx)\,dx}{a}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{f\log(\sinh(c+dx))}{ad^2}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{f\log(\sinh(c+dx))}{ad^2}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}\,(e+fx)\,\mathrm{lo}}{a}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}\,(e+fx)\,\mathrm{lo}}{a}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}\,(e+fx)\,\mathrm{lo}}{a}$$

Mathematica [A]
time = 2.61, size = 364, normalized size = 1.24

$$\frac{-ad(e+fx)\coth\left(\frac{1}{2}(c+dx)\right) + 2af\log(\sinh(c+dx)) - 2bde\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + 2bcf\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + 2bf\left(-\left((c+dx)\left(\log\left(1-e^{-c-dx}\right) - \log\left(1+e^{-c-dx}\right)\right)\right) - \mathrm{PolyLog}(2,-e^{-c-dx}) + \mathrm{PolyLog}(2,e^{-c-dx})\right) + 2\sqrt{a^2+b^2}\left(-2de\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) + 2cf\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) + f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) - f(c+dx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right) + f\mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right) - f\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right) - ad(e+fx)\tanh\left(\frac{1}{2}(c+dx)\right)}{2a^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-(a*d*(e + f*x)*Coth[(c + d*x)/2]) + 2*a*f*Log[Sinh[c + d*x]] - 2*b*d*e*Lo
g[Tanh[(c + d*x)/2]] + 2*b*c*f*Log[Tanh[(c + d*x)/2]] + 2*b*f*(-((c + d*x)*
(Log[1 - E^(-c - d*x)] - Log[1 + E^(-c - d*x)])) - PolyLog[2, -E^(-c - d*x)
] + PolyLog[2, E^(-c - d*x)]) + 2*Sqrt[a^2 + b^2]*(-2*d*e*ArcTanh[(a + b*E^
(c + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 +
b^2]] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c +
 d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(c
 + d*x))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2]))]) - a*d*(e + f*x)*Tanh[(c + d*x)/2])/(2*a^2*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1016 vs. $2(271) = 542$.

time = 5.40, size = 1017, normalized size = 3.46

method	result
risch	$-\frac{2f\ln\left(e^{dx+c}\right)}{a\,d^2}+\frac{f\ln\left(e^{dx+c}+1\right)}{a\,d^2}+\frac{f\ln\left(e^{dx+c}-1\right)}{a\,d^2}+\frac{bf\,\mathrm{dilog}\left(e^{dx+c}+1\right)}{a^2d^2}-\frac{f\ln\left(\frac{b\,e^{dx+c}+\sqrt{a^2+b^2}+a}{a+\sqrt{a^2+b^2}}\right)x}{d\sqrt{a^2+b^2}}+\frac{f\ln\left(\frac{-b\,e^{dx}}{-}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a/d^2*f*ln(exp(d*x+c))+1/a/d^2*f*ln(exp(d*x+c)+1)+1/a/d^2*f*ln(exp(d*x+c
)-1)+1/d^2/a^2*b*f*dilog(exp(d*x+c)+1)+1/d^2/a^2*b*f*dilog(exp(d*x+c))+2/d^
2*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d
*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))
*x+1/d*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)
^(1/2)))*x-1/d^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(
a^2+b^2)^(1/2)))*c+1/d^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2
)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/a^2/d^2*b*f*c*ln(exp(d*x+c)-1)+1/a^2/d*b*f*l
n(exp(d*x+c)+1)*x-2/d*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a
^2+b^2)^(1/2))-1/d^2*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+
a)/(a+(a^2+b^2)^(1/2)))+1/d^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b
^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/a^2/d*b^2*f/(a^2+b^2)^(1/2)*ln((-b*exp
(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/a^2/d^2*b^2*f/(a^2+b^2
)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/a^2/
d*b^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1
/2)))*x-1/a^2/d^2*b^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)
/(a+(a^2+b^2)^(1/2)))*c+1/a^2/d*b*e*ln(exp(d*x+c)+1)-1/a^2/d*b*e*ln(exp(d*x
+c)-1)-2/d*(f*x+e)/a/(exp(2*d*x+2*c)-1)+1/d^2/a^2*b^2*f/(a^2+b^2)^(1/2)*dil
og((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2/a^2*b^2*f/
(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))
-2/d/a^2*b^2*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(
1/2))+2/d^2/a^2*b^2*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a
^2+b^2)^(1/2))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(b*d*integrate(x/(a^2*d*e^(d*x + c) + a^2*d), x) + b*d*integrate(x/(a^2*d*
e^(d*x + c) - a^2*d), x) + a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) + 1)/(a
^2*d^2)) + a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) - 1)/(a^2*d^2)) - 2*(a^
2*e^c + b^2*e^c)*integrate(x*e^(d*x)/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(d*x
+ c) - a^2*b), x) + 2*x/(a*d*e^(2*d*x + 2*c) - a*d))*f + (b*log(e^(-d*x - c
) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + sqrt(a^2 + b^2)*log((b*e
^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/
(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1447 vs. $2(271) = 542$.
time = 0.41, size = 1447, normalized size = 4.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*a*c*f - 2*a*d*cosh(1) - 2*(a*d*f*x + a*c*f)*cosh(d*x + c)^2 - 2*a*d*sinh
(1) - 4*(a*d*f*x + a*c*f)*cosh(d*x + c)*sinh(d*x + c) - 2*(a*d*f*x + a*c*f)
*sinh(d*x + c)^2 + (b*f*cosh(d*x + c)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c)
 + b*f*sinh(d*x + c)^2 - b*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c)
+ a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2) - b)/b + 1) - (b*f*cosh(d*x + c)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c) +
 b*f*sinh(d*x + c)^2 - b*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) +
a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
 - b)/b + 1) - (b*c*f - b*d*cosh(1) - (b*c*f - b*d*cosh(1) - b*d*sinh(1))*c
osh(d*x + c)^2 - b*d*sinh(1) - 2*(b*c*f - b*d*cosh(1) - b*d*sinh(1))*cosh(d
*x + c)*sinh(d*x + c) - (b*c*f - b*d*cosh(1) - b*d*sinh(1))*sinh(d*x + c)^2
)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqr
t((a^2 + b^2)/b^2) + 2*a) + (b*c*f - b*d*cosh(1) - (b*c*f - b*d*cosh(1) - b
*d*sinh(1))*cosh(d*x + c)^2 - b*d*sinh(1) - 2*(b*c*f - b*d*cosh(1) - b*d*si
nh(1))*cosh(d*x + c)*sinh(d*x + c) - (b*c*f - b*d*cosh(1) - b*d*sinh(1))*si
nh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
 c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b*d*f*x + b*c*f - (b*d*f*x + b*c*
f)*cosh(d*x + c)^2 - 2*(b*d*f*x + b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d
*f*x + b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c)
+ a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2) - b)/b) + (b*d*f*x + b*c*f - (b*d*f*x + b*c*f)*cosh(d*x + c)^2 - 2*(b*d*
f*x + b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f*x + b*c*f)*sinh(d*x + c)^
2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b*f*cosh(d*x +
 c)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c) + b*f*sinh(d*x + c)^2 - b*f)*dilo
g(cosh(d*x + c) + sinh(d*x + c)) + (b*f*cosh(d*x + c)^2 + 2*b*f*cosh(d*x +
c)*sinh(d*x + c) + b*f*sinh(d*x + c)^2 - b*f)*dilog(-cosh(d*x + c) - sinh(d
```

```
*x + c)) - (b*d*f*x + b*d*cosh(1) - (b*d*f*x + b*d*cosh(1) + b*d*sinh(1) +
a*f)*cosh(d*x + c)^2 + b*d*sinh(1) - 2*(b*d*f*x + b*d*cosh(1) + b*d*sinh(1)
 + a*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f*x + b*d*cosh(1) + b*d*sinh(1)
+ a*f)*sinh(d*x + c)^2 + a*f)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + (b*d
*cosh(1) - (b*d*cosh(1) + b*d*sinh(1) - (b*c + a)*f)*cosh(d*x + c)^2 + b*d*
sinh(1) - 2*(b*d*cosh(1) + b*d*sinh(1) - (b*c + a)*f)*cosh(d*x + c)*sinh(d*
x + c) - (b*d*cosh(1) + b*d*sinh(1) - (b*c + a)*f)*sinh(d*x + c)^2 - (b*c +
 a)*f)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + (b*d*f*x + b*c*f - (b*d*f*x
 + b*c*f)*cosh(d*x + c)^2 - 2*(b*d*f*x + b*c*f)*cosh(d*x + c)*sinh(d*x + c)
 - (b*d*f*x + b*c*f)*sinh(d*x + c)^2)*log(-cosh(d*x + c) - sinh(d*x + c) +
1))/(a^2*d^2*cosh(d*x + c)^2 + 2*a^2*d^2*cosh(d*x + c)*sinh(d*x + c) + a^2*
d^2*sinh(d*x + c)^2 - a^2*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*coth(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(c+d\,x)^2\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((coth(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.457 $\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d}-\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d}-\frac{\coth(c+dx)}{ad}$$

```
[Out] b*arctanh(cosh(d*x+c))/a^2/d-coth(d*x+c)/a/d-2*arctanh((b-a*tanh(1/2*d*x+1/
2*c))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/a^2/d
```

Rubi [A]
time = 0.19, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2802, 3135, 3080, 3855, 2739, 632, 210}

$$-\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d}+\frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d}-\frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Cosh[c + d*x]])/(a^2*d) - (2*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh
[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2*d) - Coth[c + d*x]/(a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\operatorname{csch}^2(c+dx)\left(1+\sinh^2(c+dx)\right)}{a+b\sinh(c+dx)}\,dx$$
$$= -\frac{\coth(c+dx)}{ad} + \frac{i\int \frac{\operatorname{csch}(c+dx)(ib-ia\sinh(c+dx))}{a+b\sinh(c+dx)}\,dx}{a}$$
$$= -\frac{\coth(c+dx)}{ad} - \frac{b\int \operatorname{csch}(c+dx)\,dx}{a^2} + \frac{(a^2+b^2)\int \frac{1}{a+b\sinh(c+dx)}\,dx}{a^2}$$
$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{\coth(c+dx)}{ad} - \frac{(2i(a^2+b^2))\,\text{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx, x$$
$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{\coth(c+dx)}{ad} + \frac{(4i(a^2+b^2))\,\text{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx\right.}{a^2d}$$
$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d} - \frac{\coth(c+dx}{ad}$$

Mathematica [A]
time = 0.37, size = 98, normalized size = 1.27

$$-\frac{4\sqrt{-a^2-b^2}\,\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)+a\coth\left(\frac{1}{2}(c+dx)\right)+2b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+a\tanh\left(\frac{1}{2}(c+dx)\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/2*(4*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]]
 + a*Coth[(c + d*x)/2] + 2*b*Log[Tanh[(c + d*x)/2]] + a*Tanh[(c + d*x)/2])/
(a^2*d)
```

Maple [A]
time = 1.42, size = 105, normalized size = 1.36

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{\left(-4a^2-4b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2a^2\sqrt{a^2+b^2}}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{\left(-4a^2-4b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2a^2\sqrt{a^2+b^2}}}{d}$

risch	$-\frac{2}{da\left(\mathrm{e}^{2dx+2c}-1\right)}+\frac{b\ln\left(\mathrm{e}^{dx+c}+1\right)}{a^{2}d}-\frac{b\ln\left(\mathrm{e}^{dx+c}-1\right)}{a^{2}d}+\frac{\sqrt{a^{2}+b^{2}}\,\ln\left(\mathrm{e}^{dx+c}-\frac{-a+\sqrt{a^{2}+b^{2}}}{b}\right)}{d\,a^{2}}-\frac{\sqrt{a^{2}+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a*tanh(1/2*d*x+1/2*c)-1/2/a/tanh(1/2*d*x+1/2*c)-1/a^2*b*ln(tanh(1
/2*d*x+1/2*c))-1/2/a^2*(-4*a^2-4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh
(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [A]
time = 0.48, size = 134, normalized size = 1.74

$$\frac{b\log\left(e^{(-dx-c)}+1\right)}{a^{2}d}-\frac{b\log\left(e^{(-dx-c)}-1\right)}{a^{2}d}+\frac{\sqrt{a^{2}+b^{2}}\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^{2}+b^{2}}}{be^{(-dx-c)}-a+\sqrt{a^{2}+b^{2}}}\right)}{a^{2}d}+\frac{2}{\left(ae^{(-2\,dx-2\,c)}-a\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + sqrt(a^
2 + b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + s
qrt(a^2 + b^2)))/(a^2*d) + 2/((a*e^(-2*d*x - 2*c) - a)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(74) = 148$.
time = 0.37, size = 360, normalized size = 4.68

$$\frac{\sqrt{a^{2}+b^{2}}\left(\cosh\left(dx+c\right)^{2}+2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^{2}-1\right)\log\left(\frac{b^{2}\cosh(dx+c)^{2}+b^{2}\sinh(dx+c)^{2}+2\,ab\cosh(dx+c)+2\,a^{2}+b^{2}+2\,(b^{2}\cosh(dx+c)+ab)\sinh(dx+c)-2\sqrt{a^{2}+b^{2}}\,(b\cosh(dx+c)+b\sinh(dx+c)+a)}{b\cosh(dx+c)^{2}+b\sinh(dx+c)^{2}+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)-b}\right)+\left(b\cosh\left(dx+c\right)^{2}+2\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b\sinh\left(dx+c\right)^{2}-b\right)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+1\right)-\left(b\cosh\left(dx+c\right)^{2}+2\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b\sinh\left(dx+c\right)^{2}-b\right)\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1\right)-2\,a}{a^{2}d\cosh\left(dx+c\right)^{2}+2\,a^{2}d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a^{2}d\sinh\left(dx+c\right)^{2}-a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*
x + c)^2 - 1)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d
*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(
a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*
sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c)
 - b)) + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
+ c)^2 - b)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (b*cosh(d*x + c)^2 + 2
*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*log(cosh(d*x + c) +
 sinh(d*x + c) - 1) - 2*a)/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c)*s
inh(d*x + c) + a^2*d*sinh(d*x + c)^2 - a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [A]
time = 0.47, size = 120, normalized size = 1.56

$$\frac{\frac{b\log\left(e^{(dx+c)}+1\right)}{a^2} - \frac{b\log\left(\left|e^{(dx+c)}-1\right|\right)}{a^2} + \frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{a^2} - \frac{2}{a\left(e^{(2\,dx+2\,c)}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (b*log(e^(d*x + c) + 1)/a^2 - b*log(abs(e^(d*x + c) - 1))/a^2 + sqrt(a^2 +
b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c)
 + 2*a + 2*sqrt(a^2 + b^2)))/a^2 - 2/(a*(e^(2*d*x + 2*c) - 1)))/d
```

Mupad [B]
time = 0.64, size = 380, normalized size = 4.94

$$\frac{2}{a\,d-a\,d\,e^{2\,c+2\,d\,x}} - \frac{b\,\ln\left(32\,a^2+32\,b^2-32\,a^2\,e^{d\,x}\,e^c-32\,b^2\,e^{d\,x}\,e^c\right)}{a^2\,d} + \frac{b\,\ln\left(32\,a^2+32\,b^2+32\,a^2\,e^{d\,x}\,e^c+32\,b^2\,e^{d\,x}\,e^c\right)}{a^2\,d} + \frac{\ln\left(128\,a^4\,e^{d\,x}\,e^c-64\,a\,b^3-64\,a^3\,b-32\,b^3\,\sqrt{a^2+b^2}+32\,b^4\,e^{d\,x}\,e^c-64\,a^2\,b\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,e^{d\,x}\,e^c+128\,a^3\,e^{d\,x}\,e^c\,\sqrt{a^2+b^2}+96\,a\,b^2\,e^{d\,x}\,e^c\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{a^2\,d} - \frac{\ln\left(32\,b^3\,\sqrt{a^2+b^2}-64\,a\,b^3-64\,a^3\,b+128\,a^4\,e^{d\,x}\,e^c+32\,b^4\,e^{d\,x}\,e^c+64\,a^2\,b\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,e^{d\,x}\,e^c-128\,a^3\,e^{d\,x}\,e^c\,\sqrt{a^2+b^2}-96\,a\,b^2\,e^{d\,x}\,e^c\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2/(a + b*sinh(c + d*x)),x)
```

```
[Out] 2/(a*d - a*d*exp(2*c + 2*d*x)) - (b*log(32*a^2 + 32*b^2 - 32*a^2*exp(d*x)*e
xp(c) - 32*b^2*exp(d*x)*exp(c)))/(a^2*d) + (b*log(32*a^2 + 32*b^2 + 32*a^2*
exp(d*x)*exp(c) + 32*b^2*exp(d*x)*exp(c)))/(a^2*d) + (log(128*a^4*exp(d*x)*
exp(c) - 64*a*b^3 - 64*a^3*b - 32*b^3*(a^2 + b^2)^(1/2) + 32*b^4*exp(d*x)*e
xp(c) - 64*a^2*b*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(d*x)*exp(c) + 128*a^3*
exp(d*x)*exp(c)*(a^2 + b^2)^(1/2) + 96*a*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1
/2))*(a^2 + b^2)^(1/2))/(a^2*d) - (log(32*b^3*(a^2 + b^2)^(1/2) - 64*a*b^3
- 64*a^3*b + 128*a^4*exp(d*x)*exp(c) + 32*b^4*exp(d*x)*exp(c) + 64*a^2*b*(a
^2 + b^2)^(1/2) + 160*a^2*b^2*exp(d*x)*exp(c) - 128*a^3*exp(d*x)*exp(c)*(a^
2 + b^2)^(1/2) - 96*a*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1
/2))/(a^2*d)
```

3.458 $\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is not applicable to the result.

```
[In] Int[Coth[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][Coth[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Mathematica [A]
time = 142.65, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Coth[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Integrate[Coth[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(dx+c)}{(fx+e)(a+b\sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*(a^2*e^c + b^2*e^c)*integrate(-e^(d*x)/(a^2*b*f*x + a^2*b*e - (a^2*b*f*x*
e^(2*c) + a^2*b*e^(2*c + 1))*e^(2*d*x) - 2*(a^3*f*x*e^c + a^3*e^(c + 1))*e^
(d*x)), x) + 2/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e^(2*c + 1))*e^(2*
d*x)) - integrate(-(b*d*f*x + b*d*e + a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*f*x*e +
 a^2*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*f*x*e^(c + 1) + a^2*d*e^(c + 2))*
e^(d*x)), x) - integrate((b*d*f*x + b*d*e - a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*f
*x*e + a^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*d*f*x*e^(c + 1) + a^2*d*e^(c
+ 2))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(coth(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\coth(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(coth(c + d*x)^2/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.459 $\int \frac{(e+fx)^3 \cosh(c+dx) \coth^2(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal. Leaf size=718

$$\frac{b(e+fx)^4}{4a^2f} - \frac{(a^2+b^2)(e+fx)^4}{4a^2bf} - \frac{6f(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^3\operatorname{csch}(c+dx)}{ad} + \frac{(a^2+b^2)(e+fx)^3}{}$$

```
[Out] 1/4*b*(f*x+e)^4/a^2/f-1/4*(a^2+b^2)*(f*x+e)^4/a^2/b/f-6*f*(f*x+e)^2*arctanh
(exp(d*x+c))/a/d^2-(f*x+e)^3*csch(d*x+c)/a/d-b*(f*x+e)^3*ln(1-exp(2*d*x+2*c
))/a^2/d+(a^2+b^2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/b/d
+(a^2+b^2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/b/d-6*f^2*(
f*x+e)*polylog(2,-exp(d*x+c))/a/d^3+6*f^2*(f*x+e)*polylog(2,exp(d*x+c))/a/d
^3-3/2*b*f*(f*x+e)^2*polylog(2,exp(2*d*x+2*c))/a^2/d^2+3*(a^2+b^2)*f*(f*x+e
)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/b/d^2+3*(a^2+b^2)*f*(f
*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/b/d^2+6*f^3*polylo
g(3,-exp(d*x+c))/a/d^4-6*f^3*polylog(3,exp(d*x+c))/a/d^4+3/2*b*f^2*(f*x+e)*
polylog(3,exp(2*d*x+2*c))/a^2/d^3-6*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*exp(
d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/b/d^3-6*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*
exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/b/d^3-3/4*b*f^3*polylog(4,exp(2*d*x+2*c
))/a^2/d^4+6*(a^2+b^2)*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2
/b/d^4+6*(a^2+b^2)*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/b/d
^4
```

Rubi [A]

time = 1.16, antiderivative size = 718, normalized size of antiderivative = 1.00, number of steps used = 48, number of rules used = 19, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.559$, Rules used = {5704, 5558, 3377, 2718, 5560, 4267, 2611, 2320, 6724, 5554, 3392, 32, 2715, 8, 3797, 2221, 6744, 5684, 5680}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*(e + f*x)^4)/(4*a^2*f) - ((a^2 + b^2)*(e + f*x)^4)/(4*a^2*b*f) - (6*f*(e
 + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d^2) - ((e + f*x)^3*Csch[c + d*x])/(a*d)
 + ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])
/(a^2*b*d) + ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2])])/(a^2*b*d) - (b*(e + f*x)^3*Log[1 - E^(2*(c + d*x))])/(a^2*d) - (
6*f^2*(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^3) + (6*f^2*(e + f*x)*PolyLo
g[2, E^(c + d*x)])/(a*d^3) + (3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*b*d^2) + (3*(a^2 + b^2)*f*(e + f*
x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^2) - (3
```

```
*b*f*(e + f*x)^2*PolyLog[2, E^(2*(c + d*x))])/(2*a^2*d^2) + (6*f^3*PolyLog[
3, -E^(c + d*x)])/(a*d^4) - (6*f^3*PolyLog[3, E^(c + d*x)])/(a*d^4) - (6*(a
^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))
])/(a^2*b*d^3) - (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^3) + (3*b*f^2*(e + f*x)*PolyLog[3, E^(2*(
c + d*x))])/(2*a^2*d^3) + (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/(a^2*b*d^4) + (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^4) - (3*b*f^3*PolyLog[4, E^(2*
(c + d*x))])/(4*a^2*d^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
```

```
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5704

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3 \cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3 \cosh(c+dx)\coth^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cosh^2(c\cdots}{a+b\sinh\cdots}}{a\cdots}$$

$$= \frac{\int (e+fx)^3 \cosh(c+dx)\,dx}{a} + \frac{\int (e+fx)^3 \coth(c+dx)\operatorname{csch}(\cdots}{a}$$

$$= -\frac{(e+fx)^3\operatorname{csch}(c+dx)}{ad} + \frac{(e+fx)^3\sinh(c+dx)}{ad} - \frac{\int (e+f\cdots}{\cdots}$$

$$= \frac{b(e+fx)^4}{4a^2f} - \frac{(a^2+b^2)(e+fx)^4}{4a^2bf} - \frac{6f(e+fx)^2\tanh^{-1}(e^{c+\cdots}}{ad^2}$$

$$= \frac{b(e+fx)^4}{4a^2f} - \frac{(a^2+b^2)(e+fx)^4}{4a^2bf} - \frac{6f(e+fx)^2\tanh^{-1}(e^{c+\cdots}}{ad^2}$$

$$= \frac{b(e+fx)^4}{4a^2f} - \frac{(a^2+b^2)(e+fx)^4}{4a^2bf} - \frac{6f(e+fx)^2\tanh^{-1}(e^{c+\cdots}}{ad^2}$$

$$= \frac{b(e+fx)^4}{4a^2f} - \frac{(a^2+b^2)(e+fx)^4}{4a^2bf} - \frac{6f(e+fx)^2\tanh^{-1}(e^{c+\cdots}}{ad^2}$$

$$= \frac{b(e+fx)^4}{4a^2f} - \frac{(a^2+b^2)(e+fx)^4}{4a^2bf} - \frac{6f(e+fx)^2\tanh^{-1}(e^{c+\cdots}}{ad^2}$$

$$= \frac{b(e+fx)^4}{4a^2f} - \frac{(a^2+b^2)(e+fx)^4}{4a^2bf} - \frac{6f(e+fx)^2\tanh^{-1}(e^{c+\cdots}}{ad^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.20, size = 13888, normalized size = 19.34

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 3.01, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \cosh(dx+c) \left(\coth^2(dx+c)\right)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] ((d*x + c)/(b*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-
d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + (a^2 + b^2)*log(-
2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^2*b*d))*e^3 - 1/4*(a*d*f^3*x^
4 + 4*a*d*f^2*x^3*e + 6*a*d*f*x^2*e^2 - (a*d*f^3*x^4*e^(2*c) + 4*a*d*f^2*x^
3*e^(2*c + 1) + 6*a*d*f*x^2*e^(2*c + 2))*e^(2*d*x) + 8*(b*f^3*x^3*e^c + 3*b
*f^2*x^2*e^(c + 1) + 3*b*f*x*e^(c + 2))*e^(d*x))/(a*b*d*e^(2*d*x + 2*c) - a
*b*d) - 3*f*e^2*log(e^(d*x + c) + 1)/(a*d^2) + 3*f*e^2*log(e^(d*x + c) - 1)
/(a*d^2) - (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) -
6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*b*f^3/(a^2*d^4
) - (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*p
olylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*b*f^3/(a^2*d^4) - 3*(b*
d*f*e^2 + 2*a*f^2*e)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*
d^3) - 3*(b*d*f*e^2 - 2*a*f^2*e)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x
+ c)))/(a^2*d^3) - 3*(b*d*f^2*e + a*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2*
d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^2*d^4) - 3*(b*d*f^
2*e - a*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*
polylog(3, e^(d*x + c)))/(a^2*d^4) + 1/4*(b*d^4*f^3*x^4 + 4*(b*d*f^2*e + a*
f^3)*d^3*x^3 + 6*(b*d^2*f*e^2 + 2*a*d*f^2*e)*d^2*x^2)/(a^2*d^4) + 1/4*(b*d^
4*f^3*x^4 + 4*(b*d*f^2*e - a*f^3)*d^3*x^3 + 6*(b*d^2*f*e^2 - 2*a*d*f^2*e)*d
```

```
^2*x^2)/(a^2*d^4) - integrate(-2*((a^2*b*f^3 + b^3*f^3)*x^3 + 3*(a^2*b*f^2
+ b^3*f^2)*x^2*e + 3*(a^2*b*f + b^3*f)*x*e^2 - ((a^3*f^3*e^c + a*b^2*f^3*e^
c)*x^3 + 3*(a^3*f^2*e^c + a*b^2*f^2*e^c)*x^2*e + 3*(a^3*f*e^c + a*b^2*f*e^c
)*x*e^2)*e^(d*x))/(a^2*b^2*e^(2*d*x + 2*c) + 2*a^3*b*e^(d*x + c) - a^2*b^2)
, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 10474 vs. $2(689) = 1378$.
time = 0.56, size = 10474, normalized size = 14.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/4*(a^2*d^4*f^3*x^4 - 2*a^2*c^4*f^3 + 4*(a^2*d^4*x + 2*a^2*c*d^3)*cosh(1)^
3 + 4*(a^2*d^4*x + 2*a^2*c*d^3)*sinh(1)^3 + 6*(a^2*d^4*f*x^2 - 2*a^2*c^2*d^
2*f)*cosh(1)^2 - (a^2*d^4*f^3*x^4 - 2*a^2*c^4*f^3 + 4*(a^2*d^4*x + 2*a^2*c*
d^3)*cosh(1)^3 + 4*(a^2*d^4*x + 2*a^2*c*d^3)*sinh(1)^3 + 6*(a^2*d^4*f*x^2 -
 2*a^2*c^2*d^2*f)*cosh(1)^2 + 6*(a^2*d^4*f*x^2 - 2*a^2*c^2*d^2*f + 2*(a^2*d
^4*x + 2*a^2*c*d^3)*cosh(1))*sinh(1)^2 + 4*(a^2*d^4*f^2*x^3 + 2*a^2*c^3*d*f
^2)*cosh(1) + 4*(a^2*d^4*f^2*x^3 + 2*a^2*c^3*d*f^2 + 3*(a^2*d^4*x + 2*a^2*c
*d^3)*cosh(1)^2 + 3*(a^2*d^4*f*x^2 - 2*a^2*c^2*d^2*f)*cosh(1))*sinh(1))*cos
h(d*x + c)^2 + 6*(a^2*d^4*f*x^2 - 2*a^2*c^2*d^2*f + 2*(a^2*d^4*x + 2*a^2*c*
d^3)*cosh(1))*sinh(1)^2 - (a^2*d^4*f^3*x^4 - 2*a^2*c^4*f^3 + 4*(a^2*d^4*x +
 2*a^2*c*d^3)*cosh(1)^3 + 4*(a^2*d^4*x + 2*a^2*c*d^3)*sinh(1)^3 + 6*(a^2*d^
4*f*x^2 - 2*a^2*c^2*d^2*f)*cosh(1)^2 + 6*(a^2*d^4*f*x^2 - 2*a^2*c^2*d^2*f +
 2*(a^2*d^4*x + 2*a^2*c*d^3)*cosh(1))*sinh(1)^2 + 4*(a^2*d^4*f^2*x^3 + 2*a^
2*c^3*d*f^2)*cosh(1) + 4*(a^2*d^4*f^2*x^3 + 2*a^2*c^3*d*f^2 + 3*(a^2*d^4*x
+ 2*a^2*c*d^3)*cosh(1)^2 + 3*(a^2*d^4*f*x^2 - 2*a^2*c^2*d^2*f)*cosh(1))*sin
h(1))*sinh(d*x + c)^2 + 4*(a^2*d^4*f^2*x^3 + 2*a^2*c^3*d*f^2)*cosh(1) - 8*(
a*b*d^3*f^3*x^3 + 3*a*b*d^3*f^2*x^2*cosh(1) + 3*a*b*d^3*f*x*cosh(1)^2 + a*b
*d^3*cosh(1)^3 + a*b*d^3*sinh(1)^3 + 3*(a*b*d^3*f*x + a*b*d^3*cosh(1))*sinh
(1)^2 + 3*(a*b*d^3*f^2*x^2 + 2*a*b*d^3*f*x*cosh(1) + a*b*d^3*cosh(1)^2)*sin
h(1))*cosh(d*x + c) - 12*((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*f^2*x
*cosh(1) + (a^2 + b^2)*d^2*f*cosh(1)^2 + (a^2 + b^2)*d^2*f*sinh(1)^2 - ((a^
2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*f^2*x*cosh(1) + (a^2 + b^2)*d^2*f*
cosh(1)^2 + (a^2 + b^2)*d^2*f*sinh(1)^2 + 2*((a^2 + b^2)*d^2*f^2*x + (a^2 +
 b^2)*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^2*f^3*x^2
+ 2*(a^2 + b^2)*d^2*f^2*x*cosh(1) + (a^2 + b^2)*d^2*f*cosh(1)^2 + (a^2 + b^
2)*d^2*f*sinh(1)^2 + 2*((a^2 + b^2)*d^2*f^2*x + (a^2 + b^2)*d^2*f*cosh(1))*
sinh(1))*cosh(d*x + c)*sinh(d*x + c) - ((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 +
b^2)*d^2*f^2*x*cosh(1) + (a^2 + b^2)*d^2*f*cosh(1)^2 + (a^2 + b^2)*d^2*f*si
nh(1)^2 + 2*((a^2 + b^2)*d^2*f^2*x + (a^2 + b^2)*d^2*f*cosh(1))*sinh(1))*si
```

```
nh(d*x + c)^2 + 2*((a^2 + b^2)*d^2*f^2*x + (a^2 + b^2)*d^2*f*cosh(1))*sinh(
1))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 12*((a^2 + b^2)*d^2*f^3*x^2 + 2
*(a^2 + b^2)*d^2*f^2*x*cosh(1) + (a^2 + b^2)*d^2*f*cosh(1)^2 + (a^2 + b^2)*
d^2*f*sinh(1)^2 - ((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*f^2*x*cosh(1
) + (a^2 + b^2)*d^2*f*cosh(1)^2 + (a^2 + b^2)*d^2*f*sinh(1)^2 + 2*((a^2 + b
^2)*d^2*f^2*x + (a^2 + b^2)*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 - 2*((a
^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*f^2*x*cosh(1) + (a^2 + b^2)*d^2*f
*cosh(1)^2 + (a^2 + b^2)*d^2*f*sinh(1)^2 + 2*((a^2 + b^2)*d^2*f^2*x + (a^2
+ b^2)*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - ((a^2 + b^2)*d
^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*f^2*x*cosh(1) + (a^2 + b^2)*d^2*f*cosh(1)^2
+ (a^2 + b^2)*d^2*f*sinh(1)^2 + 2*((a^2 + b^2)*d^2*f^2*x + (a^2 + b^2)*d^2*
f*cosh(1))*sinh(1))*sinh(d*x + c)^2 + 2*((a^2 + b^2)*d^2*f^2*x + (a^2 + b^2
)*d^2*f*cosh(1))*sinh(1))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 12*(b^2*d
^2*f^3*x^2 - 2*a*b*d*f^3*x + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 - (b
^2*d^2*f^3*x^2 - 2*a*b*d*f^3*x + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2
+ 2*(b^2*d^2*f^2*x - a*b*d*f^2)*cosh(1) + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh
(1) - a*b*d*f^2)*sinh(1))*cosh(d*x + c)^2 - 2*(b^2*d^2*f^3*x^2 - 2*a*b*d*f^
3*x + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x - a*b*d*
f^2)*cosh(1) + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1) - a*b*d*f^2)*sinh(1))*c
osh(d*x + c)*sinh(d*x + c) - (b^2*d^2*f^3*x^2 - 2*a*b*d*f^3*x + b^2*d^2*f*c
osh(1)^2 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x - a*b*d*f^2)*cosh(1) + 2*
(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1) - a*b*d*f^2)*sinh(1))*sinh(d*x + c)^2 +
2*(b^2*d^2*f^2*x - a*b*d*f^2)*cosh(1) + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1
) - a*b*d*f^2)*sinh(1))*dilog(cosh(d*x + c) + sinh(d*x + c)) + 12*(b^2*d^2*
f^3*x^2 + 2*a*b*d*f^3*x + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 - (b^2*
d^2*f^3*x^2 + 2*a*b*d*f^3*x + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + 2
*(b^2*d^2*f^2*x + a*b*d*f^2)*cosh(1) + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1)
 + a*b*d*f^2)*sinh(1))*cosh(d*x + c)^2 - 2*(b^2*d^2*f^3*x^2 + 2*a*b*d*f^3*x
 + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + a*b*d*f^2
)*cosh(1) + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1) + a*b*d*f^2)*sinh(1))*cosh
(d*x + c)*sinh(d*x + c) - (b^2*d^2*f^3*x^2 + 2*a*b*d*f^3*x + b^2*d^2*f*cosh
(1)^2 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + a*b*d*f^2)*cosh(1) + 2*(b^
2*d^2*f^2*x + b^2*d^2*f*cosh(1) + a*b*d*f^2)*sinh(1))*sinh(d*x + c)^2 + 2*(
b^2*d^2*f^2*x + a*b*d*f^2)*cosh(1) + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1) +
 a*b*d*f^2)*sinh(1))*dilog(-cosh(d*x + c) - sinh(d*x + c)) + 4*((a^2 + b^2)
*c^3*f^3 - 3*(a^2 + b^2)*c^2*d*f^2*cosh(1) + 3*...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3 \cosh (c+dx) \coth^2 (c+dx)}{a+b \sinh (c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*cosh(c + d*x)*coth(c + d*x)**2/(a + b*sinh(c + d*x)),
 x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)^2\,(e+f\,x)^3}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*coth(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((cosh(c + d*x)*coth(c + d*x)^2*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.460 $\int \frac{(e+fx)^2 \cosh(c+dx) \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=518

$$\frac{b(e+fx)^3}{3a^2f}-\frac{(a^2+b^2)(e+fx)^3}{3a^2bf}-\frac{4f(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^2}-\frac{(e+fx)^2\text{csch}(c+dx)}{ad}+\frac{(a^2+b^2)(e+fx)^2\log}{a^2}$$

```
[Out] 1/3*b*(f*x+e)^3/a^2/f-1/3*(a^2+b^2)*(f*x+e)^3/a^2/b/f-4*f*(f*x+e)*arctanh(e
xp(d*x+c))/a/d^2-(f*x+e)^2*csch(d*x+c)/a/d-b*(f*x+e)^2*ln(1-exp(2*d*x+2*c))
/a^2/d+(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/b/d+(
a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/b/d-2*f^2*pol
ylog(2,-exp(d*x+c))/a/d^3+2*f^2*polylog(2,exp(d*x+c))/a/d^3-b*f*(f*x+e)*pol
ylog(2,exp(2*d*x+2*c))/a^2/d^2+2*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c
)/(a-(a^2+b^2)^(1/2)))/a^2/b/d^2+2*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x
+c)/(a+(a^2+b^2)^(1/2)))/a^2/b/d^2+1/2*b*f^2*polylog(3,exp(2*d*x+2*c))/a^2/
d^3-2*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/b/d^3-
2*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/b/d^3
```

Rubi [A]
time = 0.89, antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 17, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5704, 5558, 3377, 2717, 5560, 4267, 2317, 2438, 5554, 3391, 3797, 2221, 2611, 2320, 6724, 5684, 5680}

$$-\frac{2f^2(a^2+b^2)\,\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^3}-\frac{2f^2(a^2+b^2)\,\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2bd^3}+\frac{2f(a^2+b^2)(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^2}+\frac{2f(a^2+b^2)(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2bd^2}+\frac{(a^2+b^2)(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^2bd}+\frac{(a^2+b^2)(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{a^2bd}-\frac{(a^2+b^2)(e+fx)^3}{3a^2bf}+\frac{bf^2\text{Li}_3\left(e^{2(c+dx)}\right)}{2a^2d^3}-\frac{bf(e+fx)\text{Li}_2\left(e^{2(c+dx)}\right)}{a^2d^2}-\frac{b(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{a^2d}+\frac{b(e+fx)^3}{3a^2f}-\frac{2f^2\text{Li}_2\left(-e^{c+dx}\right)}{ad^3}+\frac{2f^2\text{Li}_2\left(e^{c+dx}\right)}{ad^3}-\frac{4f(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^2}-\frac{(e+fx)^2\text{csch}(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*(e + f*x)^3)/(3*a^2*f) - ((a^2 + b^2)*(e + f*x)^3)/(3*a^2*b*f) - (4*f*(e
 + f*x)*ArcTanh[E^(c + d*x)])/(a*d^2) - ((e + f*x)^2*Csch[c + d*x])/(a*d) +
 ((a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(
a^2*b*d) + ((a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])])/(a^2*b*d) - (b*(e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a^2*d) - (2*
f^2*PolyLog[2, -E^(c + d*x)])/(a*d^3) + (2*f^2*PolyLog[2, E^(c + d*x)])/(a*
d^3) + (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/(a^2*b*d^2) + (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^2) - (b*f*(e + f*x)*PolyLog[2, E^
(2*(c + d*x))])/(a^2*d^2) - (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/(a^2*b*d^3) - (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^3) + (b*f^2*PolyLog[3, E^(2*
(c + d*x))])/(2*a^2*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
```

```
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5704

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh(c+dx)\coth^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cosh^2(c+}{a+b\sinh(}}{a}$$

$$= \frac{\int (e+fx)^2\cosh(c+dx)\,dx}{a} + \frac{\int (e+fx)^2\coth(c+dx)\operatorname{csch}(c}{a}$$

$$= -\frac{(e+fx)^2\operatorname{csch}(c+dx)}{ad} + \frac{(e+fx)^2\sinh(c+dx)}{ad} - \frac{\int (e+fx}{}$$

$$= \frac{b(e+fx)^3}{3a^2f} - \frac{(a^2+b^2)(e+fx)^3}{3a^2bf} - \frac{4f(e+fx)\tanh^{-1}(e^{c+dx})}{ad^2}$$

$$= \frac{b(e+fx)^3}{3a^2f} - \frac{(a^2+b^2)(e+fx)^3}{3a^2bf} - \frac{4f(e+fx)\tanh^{-1}(e^{c+dx})}{ad^2}$$

$$= \frac{b(e+fx)^3}{3a^2f} - \frac{(a^2+b^2)(e+fx)^3}{3a^2bf} - \frac{4f(e+fx)\tanh^{-1}(e^{c+dx})}{ad^2}$$

$$= \frac{b(e+fx)^3}{3a^2f} - \frac{(a^2+b^2)(e+fx)^3}{3a^2bf} - \frac{4f(e+fx)\tanh^{-1}(e^{c+dx})}{ad^2}$$

$$= \frac{b(e+fx)^3}{3a^2f} - \frac{(a^2+b^2)(e+fx)^3}{3a^2bf} - \frac{4f(e+fx)\tanh^{-1}(e^{c+dx})}{ad^2}$$

Mathematica [A]
time = 10.13, size = 978, normalized size = 1.89

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (-12*b*e^2*x + (12*b*e^2*E^(2*c)*x)/(-1 + E^(2*c)) + (12*b*e*f*x^2)/(-1 + E
^(2*c)) + (4*b*f^2*x^3)/(-1 + E^(2*c)) - (24*a*e*f*ArcTanh[E^(c + d*x)])/d^
2 + (6*b*e^2*(2*d*x - Log[1 - E^(2*(c + d*x))]))/d + (12*a*f^2*(d*x*(Log[1
- E^(c + d*x)] - Log[1 + E^(c + d*x)]) - PolyLog[2, -E^(c + d*x)] + PolyLog
[2, E^(c + d*x)]))/d^3 + (6*b*e*f*(2*d*x*(d*x - Log[1 - E^(2*(c + d*x))]) -
 PolyLog[2, E^(2*(c + d*x))]))/d^2 + (b*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 - E
^(2*(c + d*x))]) - 6*d*x*PolyLog[2, E^(2*(c + d*x))] + 3*PolyLog[3, E^(2*(c
```

```
 + d*x))]))/d^3)/(6*a^2) + ((a^2 + b^2)*((-2*E^(2*c)*x*(3*e^2 + 3*e*f*x + f
^2*x^2))/(-1 + E^(2*c)) + (3*(d^2*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c
 + d*x)))] + 2*d^2*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2
)*E^(2*c)])] + d^2*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b
^2)*E^(2*c)])] + 2*d^2*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 +
 b^2)*E^(2*c)])] + d^2*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2
 + b^2)*E^(2*c)])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c
- Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d
*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*f^2*PolyLog[3, -((b*E^(2*c +
 d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*f^2*PolyLog[3, -((b*E^(2*c
 + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/d^3))/(3*a^2*b) + ((-3*b*e
^2 - 6*b*e*f*x - 3*b*f^2*x^2 + 3*a*d*e^2*x*Cosh[c] + 3*a*d*e*f*x^2*Cosh[c]
+ a*d*f^2*x^3*Cosh[c])*Csch[c/2]*Sech[c/2])/(6*a*b*d) + (Csch[c/2]*Csch[c/2
 + (d*x)/2]*(e^2*Sinh[(d*x)/2] + 2*e*f*x*Sinh[(d*x)/2] + f^2*x^2*Sinh[(d*x)
/2]))/(2*a*d) + (Sech[c/2]*Sech[c/2 + (d*x)/2]*(e^2*Sinh[(d*x)/2] + 2*e*f*x
*Sinh[(d*x)/2] + f^2*x^2*Sinh[(d*x)/2]))/(2*a*d)
```

Maple [F]
time = 1.58, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^{2} \cosh\left(dx+c\right) \left(\coth^{2}\left(dx+c\right)\right)}{a+b\sinh\left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] ((d*x + c)/(b*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-
d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + (a^2 + b^2)*log(-
2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^2*b*d))*e^2 - 1/3*(a*d*f^2*x^
3 + 3*a*d*f*x^2*e - (a*d*f^2*x^3*e^(2*c) + 3*a*d*f*x^2*e^(2*c + 1))*e^(2*d*
x) + 6*(b*f^2*x^2*e^c + 2*b*f*x*e^(c + 1))*e^(d*x))/(a*b*d*e^(2*d*x + 2*c)
- a*b*d) - 2*f*e*log(e^(d*x + c) + 1)/(a*d^2) + 2*f*e*log(e^(d*x + c) - 1)/
(a*d^2) - (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*pol
ylog(3, -e^(d*x + c)))*b*f^2/(a^2*d^3) - (d^2*x^2*log(-e^(d*x + c) + 1) + 2
*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*b*f^2/(a^2*d^3) - 2*(b
```

```
*d*f*e + a*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3)
- 2*(b*d*f*e - a*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^2
*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*d*f*e + a*f^2)*d^2*x^2)/(a^2*d^3) + 1/3*(
b*d^3*f^2*x^3 + 3*(b*d*f*e - a*f^2)*d^2*x^2)/(a^2*d^3) - integrate(-2*((a^2
*b*f^2 + b^3*f^2)*x^2 + 2*(a^2*b*f + b^3*f)*x*e - ((a^3*f^2*e^c + a*b^2*f^2
*e^c)*x^2 + 2*(a^3*f*e^c + a*b^2*f*e^c)*x*e)*e^(d*x))/(a^2*b^2*e^(2*d*x + 2
*c) + 2*a^3*b*e^(d*x + c) - a^2*b^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5069 vs. $2(496) = 992$.
time = 0.45, size = 5069, normalized size = 9.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/3*(a^2*d^3*f^2*x^3 + 2*a^2*c^3*f^2 + 3*(a^2*d^3*x + 2*a^2*c*d^2)*cosh(1)^
2 - (a^2*d^3*f^2*x^3 + 2*a^2*c^3*f^2 + 3*(a^2*d^3*x + 2*a^2*c*d^2)*cosh(1)^
2 + 3*(a^2*d^3*x + 2*a^2*c*d^2)*sinh(1)^2 + 3*(a^2*d^3*f*x^2 - 2*a^2*c^2*d*
f)*cosh(1) + 3*(a^2*d^3*f*x^2 - 2*a^2*c^2*d*f + 2*(a^2*d^3*x + 2*a^2*c*d^2)
*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 3*(a^2*d^3*x + 2*a^2*c*d^2)*sinh(1)^2
- (a^2*d^3*f^2*x^3 + 2*a^2*c^3*f^2 + 3*(a^2*d^3*x + 2*a^2*c*d^2)*cosh(1)^2
+ 3*(a^2*d^3*x + 2*a^2*c*d^2)*sinh(1)^2 + 3*(a^2*d^3*f*x^2 - 2*a^2*c^2*d*f)
*cosh(1) + 3*(a^2*d^3*f*x^2 - 2*a^2*c^2*d*f + 2*(a^2*d^3*x + 2*a^2*c*d^2)*c
osh(1))*sinh(1))*sinh(d*x + c)^2 + 3*(a^2*d^3*f*x^2 - 2*a^2*c^2*d*f)*cosh(1
) - 6*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*f*x*cosh(1) + a*b*d^2*cosh(1)^2 + a*b*d^
2*sinh(1)^2 + 2*(a*b*d^2*f*x + a*b*d^2*cosh(1))*sinh(1))*cosh(d*x + c) - 6*
((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1) -
((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*c
osh(d*x + c)^2 - 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(1) + (a^2 +
b^2)*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - ((a^2 + b^2)*d*f^2*x + (a^2
 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*sinh(d*x + c)^2)*dilog((a*co
sh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b + 1) - 6*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(
1) + (a^2 + b^2)*d*f*sinh(1) - ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(
1) + (a^2 + b^2)*d*f*sinh(1))*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*f^2*x + (a
^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c
) - ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1
))*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 6*(b^2*d*f^2*x
+ b^2*d*f*cosh(1) + b^2*d*f*sinh(1) - a*b*f^2 - (b^2*d*f^2*x + b^2*d*f*cosh
(1) + b^2*d*f*sinh(1) - a*b*f^2)*cosh(d*x + c)^2 - 2*(b^2*d*f^2*x + b^2*d*f
*cosh(1) + b^2*d*f*sinh(1) - a*b*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b^2*d*
```

```
f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1) - a*b*f^2)*sinh(d*x + c)^2)*dilog
(cosh(d*x + c) + sinh(d*x + c)) + 6*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*
f*sinh(1) + a*b*f^2 - (b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1) + a*
b*f^2)*cosh(d*x + c)^2 - 2*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1)
 + a*b*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b^2*d*f^2*x + b^2*d*f*cosh(1) +
b^2*d*f*sinh(1) + a*b*f^2)*sinh(d*x + c)^2)*dilog(-cosh(d*x + c) - sinh(d*x
 + c)) - 3*((a^2 + b^2)*c^2*f^2 - 2*(a^2 + b^2)*c*d*f*cosh(1) + (a^2 + b^2)
*d^2*cosh(1)^2 + (a^2 + b^2)*d^2*sinh(1)^2 - ((a^2 + b^2)*c^2*f^2 - 2*(a^2
+ b^2)*c*d*f*cosh(1) + (a^2 + b^2)*d^2*cosh(1)^2 + (a^2 + b^2)*d^2*sinh(1)^
2 - 2*((a^2 + b^2)*c*d*f - (a^2 + b^2)*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^
2 - 2*((a^2 + b^2)*c^2*f^2 - 2*(a^2 + b^2)*c*d*f*cosh(1) + (a^2 + b^2)*d^2*
cosh(1)^2 + (a^2 + b^2)*d^2*sinh(1)^2 - 2*((a^2 + b^2)*c*d*f - (a^2 + b^2)*
d^2*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - ((a^2 + b^2)*c^2*f^2 -
2*(a^2 + b^2)*c*d*f*cosh(1) + (a^2 + b^2)*d^2*cosh(1)^2 + (a^2 + b^2)*d^2*s
inh(1)^2 - 2*((a^2 + b^2)*c*d*f - (a^2 + b^2)*d^2*cosh(1))*sinh(1))*sinh(d*
x + c)^2 - 2*((a^2 + b^2)*c*d*f - (a^2 + b^2)*d^2*cosh(1))*sinh(1))*log(2*b
*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(
(a^2 + b^2)*c^2*f^2 - 2*(a^2 + b^2)*c*d*f*cosh(1) + (a^2 + b^2)*d^2*cosh(1)
^2 + (a^2 + b^2)*d^2*sinh(1)^2 - ((a^2 + b^2)*c^2*f^2 - 2*(a^2 + b^2)*c*d*f
*cosh(1) + (a^2 + b^2)*d^2*cosh(1)^2 + (a^2 + b^2)*d^2*sinh(1)^2 - 2*((a^2
+ b^2)*c*d*f - (a^2 + b^2)*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^2 - 2*((a^2
+ b^2)*c^2*f^2 - 2*(a^2 + b^2)*c*d*f*cosh(1) + (a^2 + b^2)*d^2*cosh(1)^2 +
(a^2 + b^2)*d^2*sinh(1)^2 - 2*((a^2 + b^2)*c*d*f - (a^2 + b^2)*d^2*cosh(1))
*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - ((a^2 + b^2)*c^2*f^2 - 2*(a^2 + b^2
)*c*d*f*cosh(1) + (a^2 + b^2)*d^2*cosh(1)^2 + (a^2 + b^2)*d^2*sinh(1)^2 - 2
*((a^2 + b^2)*c*d*f - (a^2 + b^2)*d^2*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 2
*((a^2 + b^2)*c*d*f - (a^2 + b^2)*d^2*cosh(1))*sinh(1))*log(2*b*cosh(d*x +
c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*((a^2 + b^2)*
d^2*f^2*x^2 - (a^2 + b^2)*c^2*f^2 - ((a^2 + b^2)*d^2*f^2*x^2 - (a^2 + b^2)*
c^2*f^2 + 2*((a^2 + b^2)*d^2*f*x + (a^2 + b^2)*c*d*f)*cosh(1) + 2*((a^2 + b
^2)*d^2*f*x + (a^2 + b^2)*c*d*f)*sinh(1))*cosh(d*x + c)^2 - 2*((a^2 + b^2)*
d^2*f^2*x^2 - (a^2 + b^2)*c^2*f^2 + 2*((a^2 + b^2)*d^2*f*x + (a^2 + b^2)*c*
d*f)*cosh(1) + 2*((a^2 + b^2)*d^2*f*x + (a^2 + b^2)*c*d*f)*sinh(1))*cosh(d*
x + c)*sinh(d*x + c) - ((a^2 + b^2)*d^2*f^2*x^2 - (a^2 + b^2)*c^2*f^2 + 2*(
(a^2 + b^2)*d^2*f*x + (a^2 + b^2)*c*d*f)*cosh(1) + 2*((a^2 + b^2)*d^2*f*x +
 (a^2 + b^2)*c*d*f)*sinh(1))*sinh(d*x + c)^2 + 2*((a^2 + b^2)*d^2*f*x + (a^
2 + b^2)*c*d*f)*cosh(1) + 2*((a^2 + b^2)*d^2*f*x + (a^2 + b^2)*c*d*f)*sinh(
1))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 3*((a^2 + b^2)*d^2*f^2*x^2 - (a^2 +
b^2)*c^2*f^2 - ((a^2 + b^2)*d^2*f^2*x^2 - (a^2 ...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^2\cosh{(c+dx)}\coth^2{(c+dx)}}{a+b\sinh{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*cosh(c + d*x)*coth(c + d*x)**2/(a + b*sinh(c + d*x)),
 x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)^2\,{(e+f\,x)}^2}{a+b\,\mathrm{sinh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*coth(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((cosh(c + d*x)*coth(c + d*x)^2*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.461 $\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=324

$$\frac{b(e+fx)^2}{2a^2f}-\frac{(a^2+b^2)(e+fx)^2}{2a^2bf}-\frac{f\tanh^{-1}(\cosh(c+dx))}{ad^2}-\frac{(e+fx)\operatorname{csch}(c+dx)}{ad}+\frac{(a^2+b^2)(e+fx)\log\Big(}{a^2bd}$$

```
[Out] 1/2*b*(f*x+e)^2/a^2/f-1/2*(a^2+b^2)*(f*x+e)^2/a^2/b/f-f*arctanh(cosh(d*x+c)
)/a/d^2-(f*x+e)*csch(d*x+c)/a/d-b*(f*x+e)*ln(1-exp(2*d*x+2*c))/a^2/d+(a^2+b
^2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/b/d+(a^2+b^2)*(f*x+e
)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/b/d-1/2*b*f*polylog(2,exp(2*d*
x+2*c))/a^2/d^2+(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^
2/b/d^2+(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/b/d^2
```

Rubi [A]
time = 0.52, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 15, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {5704, 5558, 3377, 2718, 5560, 3855, 5554, 2715, 8, 3797, 2221, 2317, 2438, 5684, 5680}

$$\frac{f(a^2+b^2)\operatorname{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^2}+\frac{f(a^2+b^2)\operatorname{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2bd^2}+\frac{(a^2+b^2)(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^2bd}+\frac{(a^2+b^2)(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{a^2bd}-\frac{(a^2+b^2)(e+fx)^2}{2a^2bf}-\frac{bf\operatorname{Li}_2\left(e^{2(c+dx)}\right)}{2a^2d^2}-\frac{b(e+fx)\log\left(1-e^{2(c+dx)}\right)}{a^2d}+\frac{b(e+fx)^2}{2a^2f}-\frac{f\tanh^{-1}(\cosh(c+dx))}{ad^2}-\frac{(e+fx)\operatorname{csch}(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*(e + f*x)^2)/(2*a^2*f) - ((a^2 + b^2)*(e + f*x)^2)/(2*a^2*b*f) - (f*ArcT
anh[Cosh[c + d*x]])/(a*d^2) - ((e + f*x)*Csch[c + d*x])/(a*d) + ((a^2 + b^2
)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*b*d) + ((a
^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*b*
d) - (b*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a^2*d) + ((a^2 + b^2)*f*PolyLo
g[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*b*d^2) + ((a^2 + b^2)*
f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^2) - (b*f*
PolyLog[2, E^(2*(c + d*x))])/(2*a^2*d^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5704

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh(c+dx)\coth^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cosh^2(c+dx)\,c}{a+b\sinh(c+d}}{a}$$

$$= \frac{\int (e+fx)\cosh(c+dx)\,dx}{a} + \frac{\int (e+fx)\coth(c+dx)\operatorname{csch}(c+d}{a}$$

$$= -\frac{(e+fx)\operatorname{csch}(c+dx)}{ad} + \frac{(e+fx)\sinh(c+dx)}{ad} - \frac{\int (e+fx)\,c}{}$$

$$= \frac{b(e+fx)^2}{2a^2f} - \frac{(a^2+b^2)(e+fx)^2}{2a^2bf} - \frac{f\tanh^{-1}(\cosh(c+dx))}{ad^2} -$$

$$= \frac{b(e+fx)^2}{2a^2f} - \frac{(a^2+b^2)(e+fx)^2}{2a^2bf} - \frac{f\tanh^{-1}(\cosh(c+dx))}{ad^2} -$$

$$= \frac{b(e+fx)^2}{2a^2f} - \frac{(a^2+b^2)(e+fx)^2}{2a^2bf} - \frac{f\tanh^{-1}(\cosh(c+dx))}{ad^2} -$$

$$= \frac{b(e+fx)^2}{2a^2f} - \frac{(a^2+b^2)(e+fx)^2}{2a^2bf} - \frac{f\tanh^{-1}(\cosh(c+dx))}{ad^2} -$$

Mathematica [A]
time = 1.77, size = 313, normalized size = 0.97

$$\frac{-ad(e+fx)\coth\left(\frac{1}{2}(c+dx)\right) - 2bde\log(\sinh(c+dx)) + 2bcf\log(\sinh(c+dx)) + 2af\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + bf\left(-\left((c+dx)\left(c+dx+2\log\left(1-e^{-2(c+dx)}\right)\right)\right) + \text{PolyLog}\left(2,e^{-2(c+dx)}\right)\right) + \frac{2(a^2+b^2)\left(-\frac{1}{2}f(c+dx)^2 + f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) + f(c+dx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right) + de\log(a+b\sinh(c+dx)) - cf\log(a+b\sinh(c+dx)) + f\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right) + f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)}{b} + ad(e+fx)\tanh\left(\frac{1}{2}(c+dx)\right)}{2a^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (-(a*d*(e + f*x)*Coth[(c + d*x)/2]) - 2*b*d*e*Log[Sinh[c + d*x]] + 2*b*c*f*
Log[Sinh[c + d*x]] + 2*a*f*Log[Tanh[(c + d*x)/2]] + b*f*(-((c + d*x)*(c + d
*x + 2*Log[1 - E^(-2*(c + d*x))])) + PolyLog[2, E^(-2*(c + d*x))]) + (2*(a^
2 + b^2)*(-1/2*(f*(c + d*x)^2) + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLo
g[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))]))/b + a*d*(e + f*x)*Tanh[(c + d*x)/2])/(2*a^2*d^
2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(306) = 612$.
time = 13.30, size = 938, normalized size = 2.90

method	result
risch	$-\frac{f\ln(\mathrm{e}^{dx+c}+1)}{a\,d^2}+\frac{f\ln(\mathrm{e}^{dx+c}-1)}{a\,d^2}+\frac{bf\,\mathrm{dilog}\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}-a}{-a+\sqrt{a^2+b^2}}\right)}{d^2a^2}-\frac{bf\,\mathrm{dilog}(\mathrm{e}^{dx+c}+1)}{a^2d^2}+\frac{bf\ln\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^2-}}{a+\sqrt{a^2+}}\right.}{d\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/a/d^2*f*ln(exp(d*x+c)+1)+1/a/d^2*f*ln(exp(d*x+c)-1)+b/d/a^2*f*ln((b*exp(
d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/a^2/d^2*b*f*c*ln(exp(d*x
+c)-1)-1/a^2/d*b*f*ln(exp(d*x+c)+1)*x-1/2*f*x^2/b-1/a^2/d*b*e*ln(exp(d*x+c)
+1)-1/a^2/d*b*e*ln(exp(d*x+c)-1)-1/d^2/b*f*c^2-2/d/b*e*ln(exp(d*x+c))+1/b/d
*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/b/d^2*f*dilog((-b*exp(d*x+c)+(a^
2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/b/d^2*f*dilog((b*exp(d*x+c)+(a^2+b^
2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+e*x/b+1/d/a^2*b*f*ln((-b*exp(d*x+c)+(a^2+b
^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d^2/a^2*b*f*ln((-b*exp(d*x+c)+(a^2+b
^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d^2/a^2*b*f*ln((b*exp(d*x+c)+(a^2+b^
2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d^2/a^2*b*f*c*ln(b*exp(2*d*x+2*c)+2*a*
exp(d*x+c)-b)-2/d*(f*x+e)/a*exp(d*x+c)/(exp(2*d*x+2*c)-1)+2/d^2/b*f*c*ln(ex
p(d*x+c))-2/d/b*c*f*x+b/d^2/a^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(
-a+(a^2+b^2)^(1/2)))+b/d^2/a^2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+
(a^2+b^2)^(1/2)))-b/d^2/a^2*f*dilog(exp(d*x+c)+1)+1/b/d*ln((-b*exp(d*x+c)+(
a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*f*x+1/b/d*ln((b*exp(d*x+c)+(a^2+b^2
)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*f*x+1/b/d^2*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/
2)-a)/(-a+(a^2+b^2)^(1/2)))*c*f+1/b/d^2*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)
/(a+(a^2+b^2)^(1/2)))*c*f-1/b/d^2*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)
+b/d^2/a^2*f*dilog(exp(d*x+c))+b/d/a^2*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)
-b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] 1/2*(2*b*d*integrate(x/(a^2*d*e^(d*x + c) + a^2*d), x) - 2*b*d*integrate(x/
(a^2*d*e^(d*x + c) - a^2*d), x) + 2*a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c
```

```
) + 1)/(a^2*d^2)) - 2*a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) - 1)/(a^2*d^
2)) + (a*d*x^2*e^(2*d*x + 2*c) - a*d*x^2 - 4*b*x*e^(d*x + c))/(a*b*d*e^(2*d
*x + 2*c) - a*b*d) - integrate(4*((a^3*e^c + a*b^2*e^c)*x*e^(d*x) - (a^2*b
+ b^3)*x)/(a^2*b^2*e^(2*d*x + 2*c) + 2*a^3*b*e^(d*x + c) - a^2*b^2), x))*f
+ ((d*x + c)/(b*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^
(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + (a^2 + b^2)*log
(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^2*b*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2006 vs. $2(309) = 618$.
time = 0.44, size = 2006, normalized size = 6.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] 1/2*(a^2*d^2*f*x^2 - 2*a^2*c^2*f - (a^2*d^2*f*x^2 - 2*a^2*c^2*f + 2*(a^2*d^
2*x + 2*a^2*c*d)*cosh(1) + 2*(a^2*d^2*x + 2*a^2*c*d)*sinh(1))*cosh(d*x + c)
^2 - (a^2*d^2*f*x^2 - 2*a^2*c^2*f + 2*(a^2*d^2*x + 2*a^2*c*d)*cosh(1) + 2*(
a^2*d^2*x + 2*a^2*c*d)*sinh(1))*sinh(d*x + c)^2 + 2*(a^2*d^2*x + 2*a^2*c*d)
*cosh(1) - 4*(a*b*d*f*x + a*b*d*cosh(1) + a*b*d*sinh(1))*cosh(d*x + c) + 2*
((a^2 + b^2)*f*cosh(d*x + c)^2 + 2*(a^2 + b^2)*f*cosh(d*x + c)*sinh(d*x + c
) + (a^2 + b^2)*f*sinh(d*x + c)^2 - (a^2 + b^2)*f)*dilog((a*cosh(d*x + c) +
 a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2
) - b)/b + 1) + 2*((a^2 + b^2)*f*cosh(d*x + c)^2 + 2*(a^2 + b^2)*f*cosh(d*x
 + c)*sinh(d*x + c) + (a^2 + b^2)*f*sinh(d*x + c)^2 - (a^2 + b^2)*f)*dilog(
(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sq
rt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b^2*f*cosh(d*x + c)^2 + 2*b^2*f*cosh(d
*x + c)*sinh(d*x + c) + b^2*f*sinh(d*x + c)^2 - b^2*f)*dilog(cosh(d*x + c)
+ sinh(d*x + c)) - 2*(b^2*f*cosh(d*x + c)^2 + 2*b^2*f*cosh(d*x + c)*sinh(d*
x + c) + b^2*f*sinh(d*x + c)^2 - b^2*f)*dilog(-cosh(d*x + c) - sinh(d*x + c
)) + 2*((a^2 + b^2)*c*f - (a^2 + b^2)*d*cosh(1) - ((a^2 + b^2)*c*f - (a^2 +
 b^2)*d*cosh(1) - (a^2 + b^2)*d*sinh(1))*cosh(d*x + c)^2 - (a^2 + b^2)*d*si
nh(1) - 2*((a^2 + b^2)*c*f - (a^2 + b^2)*d*cosh(1) - (a^2 + b^2)*d*sinh(1))
*cosh(d*x + c)*sinh(d*x + c) - ((a^2 + b^2)*c*f - (a^2 + b^2)*d*cosh(1) - (
a^2 + b^2)*d*sinh(1))*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x
 + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*((a^2 + b^2)*c*f - (a^2 + b^2)
*d*cosh(1) - ((a^2 + b^2)*c*f - (a^2 + b^2)*d*cosh(1) - (a^2 + b^2)*d*sinh(
1))*cosh(d*x + c)^2 - (a^2 + b^2)*d*sinh(1) - 2*((a^2 + b^2)*c*f - (a^2 + b
^2)*d*cosh(1) - (a^2 + b^2)*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - ((a^2
+ b^2)*c*f - (a^2 + b^2)*d*cosh(1) - (a^2 + b^2)*d*sinh(1))*sinh(d*x + c)^2
)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2
*a) - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f - ((a^2 + b^2)*d*f*x + (a^2 +
```

```
b^2)*c*f)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*
x + c)*sinh(d*x + c) - ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*sinh(d*x + c)^
2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*
f - ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*
d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c) - ((a^2 + b^2)*d*f*x +
 (a^2 + b^2)*c*f)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
- (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(b^
2*d*f*x + b^2*d*cosh(1) + b^2*d*sinh(1) + a*b*f - (b^2*d*f*x + b^2*d*cosh(1
) + b^2*d*sinh(1) + a*b*f)*cosh(d*x + c)^2 - 2*(b^2*d*f*x + b^2*d*cosh(1) +
 b^2*d*sinh(1) + a*b*f)*cosh(d*x + c)*sinh(d*x + c) - (b^2*d*f*x + b^2*d*co
sh(1) + b^2*d*sinh(1) + a*b*f)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*
x + c) + 1) + 2*(b^2*d*cosh(1) + b^2*d*sinh(1) - (b^2*d*cosh(1) + b^2*d*sin
h(1) - (b^2*c + a*b)*f)*cosh(d*x + c)^2 - 2*(b^2*d*cosh(1) + b^2*d*sinh(1)
- (b^2*c + a*b)*f)*cosh(d*x + c)*sinh(d*x + c) - (b^2*d*cosh(1) + b^2*d*sin
h(1) - (b^2*c + a*b)*f)*sinh(d*x + c)^2 - (b^2*c + a*b)*f)*log(cosh(d*x + c
) + sinh(d*x + c) - 1) + 2*(b^2*d*f*x + b^2*c*f - (b^2*d*f*x + b^2*c*f)*cos
h(d*x + c)^2 - 2*(b^2*d*f*x + b^2*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b^2*d
*f*x + b^2*c*f)*sinh(d*x + c)^2)*log(-cosh(d*x + c) - sinh(d*x + c) + 1) +
2*(a^2*d^2*x + 2*a^2*c*d)*sinh(1) - 2*(2*a*b*d*f*x + 2*a*b*d*cosh(1) + 2*a*
b*d*sinh(1) + (a^2*d^2*f*x^2 - 2*a^2*c^2*f + 2*(a^2*d^2*x + 2*a^2*c*d)*cosh
(1) + 2*(a^2*d^2*x + 2*a^2*c*d)*sinh(1))*cosh(d*x + c))*sinh(d*x + c))/(a^2
*b*d^2*cosh(d*x + c)^2 + 2*a^2*b*d^2*cosh(d*x + c)*sinh(d*x + c) + a^2*b*d^
2*sinh(d*x + c)^2 - a^2*b*d^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*cosh(c + d*x)*coth(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)^2\,(e+f\,x)}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*coth(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)),x)
[Out] int((cosh(c + d*x)*coth(c + d*x)^2*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.462 $\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=59

$$-\frac{\operatorname{csch}(c+dx)}{ad}-\frac{b\log(\sinh(c+dx))}{a^2d}+\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{a^2bd}$$

```
[Out] -csch(d*x+c)/a/d-b*ln(sinh(d*x+c))/a^2/d+(a^2+b^2)*ln(a+b*sinh(d*x+c))/a^2/
b/d
```

Rubi [A]
time = 0.08, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {2916, 12, 908}

$$\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{a^2bd}-\frac{b\log(\sinh(c+dx))}{a^2d}-\frac{\operatorname{csch}(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(Csch[c + d*x]/(a*d)) - (b*Log[Sinh[c + d*x]])/(a^2*d) + ((a^2 + b^2)*Log[
a + b*Sinh[c + d*x]])/(a^2*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{b^2(-b^2-x^2)}{x^2(a+x)}\,dx,x,b\sinh(c+dx)\right)}{b^3d}$$
$$= -\frac{\text{Subst}\left(\int \frac{-b^2-x^2}{x^2(a+x)}\,dx,x,b\sinh(c+dx)\right)}{bd}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{b^2}{ax^2}+\frac{b^2}{a^2x}+\frac{-a^2-b^2}{a^2(a+x)}\right)dx,x,b\sinh(c+dx)\right)}{bd}$$
$$= -\frac{\text{csch}(c+dx)}{ad}-\frac{b\log(\sinh(c+dx))}{a^2d}+\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{a^2bd}$$

Mathematica [A]
time = 0.08, size = 52, normalized size = 0.88

$$\frac{-ab\text{csch}(c+dx)-b^2\log(\sinh(c+dx))+(a^2+b^2)\log(a+b\sinh(c+dx))}{a^2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-(a*b*Csch[c + d*x]) - b^2*Log[Sinh[c + d*x]] + (a^2 + b^2)*Log[a + b*Sinh
[c + d*x]])/(a^2*b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(59) = 118.
time = 2.41, size = 135, normalized size = 2.29

method	result
risch	$-\frac{x}{b}-\frac{2c}{bd}-\frac{2\,e^{dx+c}}{da(e^{2dx+2c}-1)}-\frac{b\ln(e^{2dx+2c}-1)}{a^2d}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{bd}+\frac{b\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{a^2d}$
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}+\frac{(2a^2+2b^2)\ln\left(a\left(\tanh^2\left(\frac{dx}{2}\right.\right.\right.}{2a}}{d}$
default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}+\frac{(2a^2+2b^2)\ln\left(a\left(\tanh^2\left(\frac{dx}{2}\right.\right.\right.}{2a}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/a*tanh(1/2*d*x+1/2*c)-1/2/a/tanh(1/2*d*x+1/2*c)-1/a^2*b*ln(tanh(1/
2*d*x+1/2*c))-1/b*ln(tanh(1/2*d*x+1/2*c)+1)-1/b*ln(tanh(1/2*d*x+1/2*c)-1)+1
```

```
/2/a^2/b*(2*a^2+2*b^2)*ln(a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)-a
))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(59) = 118$.
time = 0.27, size = 131, normalized size = 2.22

$$\frac{dx+c}{bd}+\frac{2\,e^{(-dx-c)}}{(ae^{(-2\,dx-2\,c)}-a)d}-\frac{b\log\left(e^{(-dx-c)}+1\right)}{a^2d}-\frac{b\log\left(e^{(-dx-c)}-1\right)}{a^2d}+\frac{(a^2+b^2)\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{a^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] (d*x + c)/(b*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-d
*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + (a^2 + b^2)*log(-2
*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^2*b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(59) = 118$.
time = 0.39, size = 299, normalized size = 5.07

$$-\frac{a^2dx\cosh\left(dx+c\right)^2+a^2dx\sinh\left(dx+c\right)^2-a^2dx+2\,ab\cosh\left(dx+c\right)-\left((a^2+b^2)\cosh\left(dx+c\right)^2+2\,(a^2+b^2)\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(a^2+b^2)\sinh\left(dx+c\right)^2-a^2-b^2\right)\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)+\left(b^2\cosh\left(dx+c\right)^2+2\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b^2\sinh\left(dx+c\right)^2-b^2\right)\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)+2\,(a^2dx\cosh\left(dx+c\right)+ab)\sinh\left(dx+c\right)}{a^2bd\cosh\left(dx+c\right)^2+2\,a^2bd\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a^2bd\sinh\left(dx+c\right)^2-a^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -(a^2*d*x*cosh(d*x + c)^2 + a^2*d*x*sinh(d*x + c)^2 - a^2*d*x + 2*a*b*cosh(
d*x + c) - ((a^2 + b^2)*cosh(d*x + c)^2 + 2*(a^2 + b^2)*cosh(d*x + c)*sinh(
d*x + c) + (a^2 + b^2)*sinh(d*x + c)^2 - a^2 - b^2)*log(2*(b*sinh(d*x + c)
+ a)/(cosh(d*x + c) - sinh(d*x + c))) + (b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d
*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - b^2)*log(2*sinh(d*x + c)/(cos
h(d*x + c) - sinh(d*x + c))) + 2*(a^2*d*x*cosh(d*x + c) + a*b)*sinh(d*x + c
))/(a^2*b*d*cosh(d*x + c)^2 + 2*a^2*b*d*cosh(d*x + c)*sinh(d*x + c) + a^2*b
*d*sinh(d*x + c)^2 - a^2*b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh\left(c+dx\right)\coth^2\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(cosh(c + d*x)*coth(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(59) = 118$.

time = 0.45, size = 121, normalized size = 2.05

$$-\frac{\frac{b\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a^2}-\frac{(a^2+b^2)\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^2b}-\frac{b\left(e^{(dx+c)}-e^{(-dx-c)}\right)-2\,a}{a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b*log(abs(e^(d*x + c) - e^(-d*x - c)))/a^2 - (a^2 + b^2)*log(abs(b*(e^(d*
x + c) - e^(-d*x - c)) + 2*a))/(a^2*b) - (b*(e^(d*x + c) - e^(-d*x - c)) -
2*a)/(a^2*(e^(d*x + c) - e^(-d*x - c))))/d
```

Mupad [B]

time = 0.47, size = 356, normalized size = 6.03

$$\frac{2\,e^{c+d\,x}}{a\,d-a\,d\,e^{2\,c+2\,d\,x}}-\frac{x}{b}+\frac{\ln\left(8\,a^5\,e^{d\,x}\,e^c-16\,b^5-16\,a^2\,b^3-4\,a^4\,b+16\,b^5\,e^{2\,c}\,e^{2\,d\,x}+4\,a^4\,b\,e^{2\,c}\,e^{2\,d\,x}+32\,a^3\,b^2\,e^{d\,x}\,e^c+16\,a^2\,b^3\,e^{2\,c}\,e^{2\,d\,x}+32\,a\,b^4\,e^{d\,x}\,e^c\right)}{b\,d}+\frac{b\,\ln\left(8\,a^5\,e^{d\,x}\,e^c-16\,b^5-16\,a^2\,b^3-4\,a^4\,b+16\,b^5\,e^{2\,c}\,e^{2\,d\,x}+4\,a^4\,b\,e^{2\,c}\,e^{2\,d\,x}+32\,a^3\,b^2\,e^{d\,x}\,e^c+16\,a^2\,b^3\,e^{2\,c}\,e^{2\,d\,x}+32\,a\,b^4\,e^{d\,x}\,e^c\right)}{a^2\,d}-\frac{b\,\ln\left(4\,a^6+16\,b^6+32\,a^2\,b^4+20\,a^4\,b^2-4\,a^6\,e^{2\,c}\,e^{2\,d\,x}-16\,b^6\,e^{2\,c}\,e^{2\,d\,x}-32\,a^2\,b^4\,e^{2\,c}\,e^{2\,d\,x}-20\,a^4\,b^2\,e^{2\,c}\,e^{2\,d\,x}\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*coth(c + d*x)^2)/(a + b*sinh(c + d*x)),x)
```

```
[Out] (2*exp(c + d*x))/(a*d - a*d*exp(2*c + 2*d*x)) - x/b + log(8*a^5*exp(d*x)*ex
p(c) - 16*b^5 - 16*a^2*b^3 - 4*a^4*b + 16*b^5*exp(2*c)*exp(2*d*x) + 4*a^4*b
*exp(2*c)*exp(2*d*x) + 32*a^3*b^2*exp(d*x)*exp(c) + 16*a^2*b^3*exp(2*c)*exp
(2*d*x) + 32*a*b^4*exp(d*x)*exp(c))/(b*d) + (b*log(8*a^5*exp(d*x)*exp(c) -
16*b^5 - 16*a^2*b^3 - 4*a^4*b + 16*b^5*exp(2*c)*exp(2*d*x) + 4*a^4*b*exp(2*
c)*exp(2*d*x) + 32*a^3*b^2*exp(d*x)*exp(c) + 16*a^2*b^3*exp(2*c)*exp(2*d*x)
 + 32*a*b^4*exp(d*x)*exp(c)))/(a^2*d) - (b*log(4*a^6 + 16*b^6 + 32*a^2*b^4
+ 20*a^4*b^2 - 4*a^6*exp(2*c)*exp(2*d*x) - 16*b^6*exp(2*c)*exp(2*d*x) - 32*
a^2*b^4*exp(2*c)*exp(2*d*x) - 20*a^4*b^2*exp(2*c)*exp(2*d*x)))/(a^2*d)
```

3.463 $\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cosh[c + d*x]*Coth[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Cosh[c + d*x]*Coth[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] Integrate[(Cosh[c + d*x]*Coth[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)\left(\coth^2(dx+c)\right)}{(fx+e)\,(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] 2*e^(d*x + c)/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e^(2*c + 1))*e^(2*d
*x)) + log(f*x + e)/(b*f) - 1/2*integrate(-2*(b*d*f*x + b*d*e + a*f)/(a^2*d
*f^2*x^2 + 2*a^2*d*f*x*e + a^2*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*f*x*e^(
c + 1) + a^2*d*e^(c + 2))*e^(d*x)), x) + 1/2*integrate(2*(b*d*f*x + b*d*e -
 a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*f*x*e + a^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a
^2*d*f*x*e^(c + 1) + a^2*d*e^(c + 2))*e^(d*x)), x) - 1/2*integrate(4*(a^2*b
 + b^3 - (a^3*e^c + a*b^2*e^c)*e^(d*x))/(a^2*b^2*f*x + a^2*b^2*e - (a^2*b^2
*f*x*e^(2*c) + a^2*b^2*e^(2*c + 1))*e^(2*d*x) - 2*(a^3*b*f*x*e^c + a^3*b*e^
(c + 1))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(cosh(d*x + c)*coth(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(cosh(c + d*x)*coth(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)),
x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cosh(c+d\,x)\,\coth(c+d\,x)^2}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(c + d*x)*coth(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))),x)
[Out] int((cosh(c + d*x)*coth(c + d*x)^2)/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.464 $\int \frac{(e+fx)^3\mathbf{csch}^2(c+dx)\mathbf{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1428

$$-\frac{2(e+fx)^3\text{ArcTan}\left(e^{c+dx}\right)}{ad}+\frac{2b^2(e+fx)^3\text{ArcTan}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d}-\frac{6f(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad^2}+\frac{2b(e+fx)^3\tanh^{-1}}{a^2d}$$

```
[Out] -6*f^2*(f*x+e)*polylog(2,-exp(d*x+c))/a/d^3+6*f^2*(f*x+e)*polylog(2,exp(d*x
+c))/a/d^3-3/4*b*f^3*polylog(4,exp(2*d*x+2*c))/a^2/d^4+3*I*b^2*f*(f*x+e)^2*
polylog(2,I*exp(d*x+c))/a/(a^2+b^2)/d^2+6*I*b^2*f^2*(f*x+e)*polylog(3,-I*ex
p(d*x+c))/a/(a^2+b^2)/d^3-2*(f*x+e)^3*arctan(exp(d*x+c))/a/d+6*f^3*polylog(
3,-exp(d*x+c))/a/d^4-6*f^3*polylog(3,exp(d*x+c))/a/d^4-(f*x+e)^3*csch(d*x+c
)/a/d-b^3*(f*x+e)^3*ln(1+exp(2*d*x+2*c))/a^2/(a^2+b^2)/d+b^3*(f*x+e)^3*ln(1
+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d+b^3*(f*x+e)^3*ln(1+b*exp
(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d+6*b^3*f^3*polylog(4,-b*exp(d*x
+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^4+6*b^3*f^3*polylog(4,-b*exp(d*x+c
)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^4+2*b^2*(f*x+e)^3*arctan(exp(d*x+c))
/a/(a^2+b^2)/d+3/2*b*f*(f*x+e)^2*polylog(2,-exp(2*d*x+2*c))/a^2/d^2-3/2*b*f
^2*(f*x+e)*polylog(3,-exp(2*d*x+2*c))/a^2/d^3+6*I*f^3*polylog(4,-I*exp(d*x+
c))/a/d^4-3/4*b^3*f^3*polylog(4,-exp(2*d*x+2*c))/a^2/(a^2+b^2)/d^4-3*I*f*(f
*x+e)^2*polylog(2,I*exp(d*x+c))/a/d^2-6*I*f^2*(f*x+e)*polylog(3,-I*exp(d*x+
c))/a/d^3-3/2*b*f*(f*x+e)^2*polylog(2,exp(2*d*x+2*c))/a^2/d^2+3/2*b*f^2*(f*
x+e)*polylog(3,exp(2*d*x+2*c))/a^2/d^3+3*I*f*(f*x+e)^2*polylog(2,-I*exp(d*x
+c))/a/d^2-3/2*b^3*f*(f*x+e)^2*polylog(2,-exp(2*d*x+2*c))/a^2/(a^2+b^2)/d^2
+6*I*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/a/d^3+3/2*b^3*f^2*(f*x+e)*polylog(
3,-exp(2*d*x+2*c))/a^2/(a^2+b^2)/d^3-6*I*b^2*f^3*polylog(4,-I*exp(d*x+c))/a
/(a^2+b^2)/d^4+6*I*b^2*f^3*polylog(4,I*exp(d*x+c))/a/(a^2+b^2)/d^4-3*I*b^2*
f*(f*x+e)^2*polylog(2,-I*exp(d*x+c))/a/(a^2+b^2)/d^2-6*I*b^2*f^2*(f*x+e)*po
lylog(3,I*exp(d*x+c))/a/(a^2+b^2)/d^3+2*b*(f*x+e)^3*arctanh(exp(2*d*x+2*c))
/a^2/d+3/4*b*f^3*polylog(4,-exp(2*d*x+2*c))/a^2/d^4-6*I*f^3*polylog(4,I*exp
(d*x+c))/a/d^4+3*b^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)
))/a^2/(a^2+b^2)/d^2+3*b^3*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)
^(1/2)))/a^2/(a^2+b^2)/d^2-6*b^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^
2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^3-6*b^3*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/
(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^3-6*f*(f*x+e)^2*arctanh(exp(d*x+c))/a/
d^2
```

Rubi [A]
time = 1.68, antiderivative size = 1428, normalized size of antiderivative = 1.00, number of steps used = 64, number of rules used = 20, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {5708, 2701, 327, 213, 5570, 6873, 12, 6874, 5313, 4265, 2611, 6744, 2320, 6724, 4267, 5569, 5692, 5680, 2221, 3799}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*(e + f*x)^3*ArcTan[E^(c + d*x)])/(a*d) + (2*b^2*(e + f*x)^3*ArcTan[E^(c
 + d*x)])/(a*(a^2 + b^2)*d) - (6*f*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d^2
) + (2*b*(e + f*x)^3*ArcTanh[E^(2*c + 2*d*x)])/(a^2*d) - ((e + f*x)^3*Csch[
c + d*x])/(a*d) + (b^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2])])/(a^2*(a^2 + b^2)*d) + (b^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d) - (b^3*(e + f*x)^3*Log[1 + E^(2*(c
+ d*x))])/(a^2*(a^2 + b^2)*d) - (6*f^2*(e + f*x)*PolyLog[2, -E^(c + d*x)])/
(a*d^3) + ((3*I)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) - ((3*
I)*b^2*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) - ((
3*I)*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + ((3*I)*b^2*f*(e + f
*x)^2*PolyLog[2, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) + (6*f^2*(e + f*x)*Pol
yLog[2, E^(c + d*x)])/(a*d^3) + (3*b^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^2) + (3*b^3*f*(e + f*x)^
2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^
2) - (3*b^3*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*a^2*(a^2 + b^2)*
d^2) + (3*b*f*(e + f*x)^2*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a^2*d^2) - (3*b*
f*(e + f*x)^2*PolyLog[2, E^(2*c + 2*d*x)])/(2*a^2*d^2) + (6*f^3*PolyLog[3,
-E^(c + d*x)])/(a*d^4) - ((6*I)*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])
/(a*d^3) + ((6*I)*b^2*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(a*(a^2 +
 b^2)*d^3) + ((6*I)*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(a*d^3) - ((6*
I)*b^2*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^3) - (6*f^
3*PolyLog[3, E^(c + d*x)])/(a*d^4) - (6*b^3*f^2*(e + f*x)*PolyLog[3, -((b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^3) - (6*b^3*f^2*(e
+ f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^
2)*d^3) + (3*b^3*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*a^2*(a^2 +
b^2)*d^3) - (3*b*f^2*(e + f*x)*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a^2*d^3) +
(3*b*f^2*(e + f*x)*PolyLog[3, E^(2*c + 2*d*x)])/(2*a^2*d^3) + ((6*I)*f^3*Po
lyLog[4, (-I)*E^(c + d*x)])/(a*d^4) - ((6*I)*b^2*f^3*PolyLog[4, (-I)*E^(c +
 d*x)])/(a*(a^2 + b^2)*d^4) - ((6*I)*f^3*PolyLog[4, I*E^(c + d*x)])/(a*d^4)
 + ((6*I)*b^2*f^3*PolyLog[4, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^4) + (6*b^3*f
^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d
^4) + (6*b^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2
*(a^2 + b^2)*d^4) - (3*b^3*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*a^2*(a^2 +
b^2)*d^4) + (3*b*f^3*PolyLog[4, -E^(2*c + 2*d*x)])/(4*a^2*d^4) - (3*b*f^3*P
olyLog[4, E^(2*c + 2*d*x)])/(4*a^2*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
```

```
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
```

```
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5313

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcTan[u])/(d*(m + 1))), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 + u^2)), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
```

```
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5708

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^3\mathrm{csch}^2(c+dx)\mathrm{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\mathrm{csch}^2(c+dx)\mathrm{sech}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\mathrm{csch}(c+dx}{a+b\sinh(c}}{a}$$

$$= -\frac{(e+fx)^3\tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)^3\mathrm{csch}(c+dx)}{ad} - \frac{b}{}$$

$$= -\frac{(e+fx)^3\tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)^3\mathrm{csch}(c+dx)}{ad} - \frac{(}{}$$

$$= -\frac{b^3(e+fx)^4}{4a^2(a^2+b^2)f} - \frac{(e+fx)^3\tan^{-1}(\sinh(c+dx))}{ad} + \frac{2b(e+f}{}$$

$$= -\frac{b^3(e+fx)^4}{4a^2(a^2+b^2)f} - \frac{(e+fx)^3\tan^{-1}(\sinh(c+dx))}{ad} + \frac{2b(e+f}{}$$

$$= \frac{2b^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a(a^2+b^2)d} - \frac{(e+fx)^3\tan^{-1}(\sinh(c+dx))}{ad} +$$

$$= \frac{2b^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a(a^2+b^2)d} - \frac{6f(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{2}{}$$

$$= \frac{2b^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a(a^2+b^2)d} - \frac{6f(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{2}{}$$

$$= -\frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a(a^2+b^2)d} - \frac{6f}{}$$

$$= -\frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a(a^2+b^2)d} - \frac{6f}{}$$

$$= -\frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a(a^2+b^2)d} - \frac{6f}{}$$

$$= -\frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a(a^2+b^2)d} - \frac{6f}{}$$

$$= -\frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a(a^2+b^2)d} - \frac{6f}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 8578 vs. 2(1428) = 2856.
time = 10.27, size = 8578, normalized size = 6.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 2.51, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c)^2 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] (b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + a^2*b^2)*d) +
2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^
2 + b^2)*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-d*x -
 c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d))*e^3 - 2*(f^3*x^3*e^c +
3*f^2*x^2*e^(c + 1) + 3*f*x*e^(c + 2))*e^(d*x)/(a*d*e^(2*d*x + 2*c) - a*d)
- 3*f*e^2*log(e^(d*x + c) + 1)/(a*d^2) + 3*f*e^2*log(e^(d*x + c) - 1)/(a*d^
2) - (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*
polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*b*f^3/(a^2*d^4) - (d
^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog
(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*b*f^3/(a^2*d^4) - 3*(b*d*f*e^
2 + 2*a*f^2*e)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) -
 3*(b*d*f*e^2 - 2*a*f^2*e)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))
/(a^2*d^3) - 3*(b*d*f^2*e + a*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*di
```

```
log(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^2*d^4) - 3*(b*d*f^2*e -
a*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylo
g(3, e^(d*x + c)))/(a^2*d^4) + 1/4*(b*d^4*f^3*x^4 + 4*(b*d*f^2*e + a*f^3)*d
^3*x^3 + 6*(b*d^2*f*e^2 + 2*a*d*f^2*e)*d^2*x^2)/(a^2*d^4) + 1/4*(b*d^4*f^3*
x^4 + 4*(b*d*f^2*e - a*f^3)*d^3*x^3 + 6*(b*d^2*f*e^2 - 2*a*d*f^2*e)*d^2*x^2
)/(a^2*d^4) - integrate(2*(b^4*f^3*x^3 + 3*b^4*f^2*x^2*e + 3*b^4*f*x*e^2 -
(a*b^3*f^3*x^3*e^c + 3*a*b^3*f^2*x^2*e^(c + 1) + 3*a*b^3*f*x*e^(c + 2))*e^(
d*x))/(a^4*b + a^2*b^3 - (a^4*b*e^(2*c) + a^2*b^3*e^(2*c))*e^(2*d*x) - 2*(a
^5*e^c + a^3*b^2*e^c)*e^(d*x)), x) - integrate(2*(b*f^3*x^3 + 3*b*f^2*x^2*e
 + 3*b*f*x*e^2 + (a*f^3*x^3*e^c + 3*a*f^2*x^2*e^(c + 1) + 3*a*f*x*e^(c + 2)
)*e^(d*x))/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 17737 vs. $2(1329) = 2658$.
time = 0.72, size = 17737, normalized size = 12.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -(2*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*f^2*x^2*cosh(1) + 3*(a
^3 + a*b^2)*d^3*f*x*cosh(1)^2 + (a^3 + a*b^2)*d^3*cosh(1)^3 + (a^3 + a*b^2)
*d^3*sinh(1)^3 + 3*((a^3 + a*b^2)*d^3*f*x + (a^3 + a*b^2)*d^3*cosh(1))*sinh
(1)^2 + 3*((a^3 + a*b^2)*d^3*f^2*x^2 + 2*(a^3 + a*b^2)*d^3*f*x*cosh(1) + (a
^3 + a*b^2)*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c) + 3*(b^3*d^2*f^3*x^2 + 2*
b^3*d^2*f^2*x*cosh(1) + b^3*d^2*f*cosh(1)^2 + b^3*d^2*f*sinh(1)^2 - (b^3*d^
2*f^3*x^2 + 2*b^3*d^2*f^2*x*cosh(1) + b^3*d^2*f*cosh(1)^2 + b^3*d^2*f*sinh(
1)^2 + 2*(b^3*d^2*f^2*x + b^3*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 - 2*(
b^3*d^2*f^3*x^2 + 2*b^3*d^2*f^2*x*cosh(1) + b^3*d^2*f*cosh(1)^2 + b^3*d^2*f
*sinh(1)^2 + 2*(b^3*d^2*f^2*x + b^3*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)*s
inh(d*x + c) - (b^3*d^2*f^3*x^2 + 2*b^3*d^2*f^2*x*cosh(1) + b^3*d^2*f*cosh(
1)^2 + b^3*d^2*f*sinh(1)^2 + 2*(b^3*d^2*f^2*x + b^3*d^2*f*cosh(1))*sinh(1))
*sinh(d*x + c)^2 + 2*(b^3*d^2*f^2*x + b^3*d^2*f*cosh(1))*sinh(1))*dilog((a*
cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2) - b)/b + 1) + 3*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*f^2*x*cosh(1)
 + b^3*d^2*f*cosh(1)^2 + b^3*d^2*f*sinh(1)^2 - (b^3*d^2*f^3*x^2 + 2*b^3*d^2
*f^2*x*cosh(1) + b^3*d^2*f*cosh(1)^2 + b^3*d^2*f*sinh(1)^2 + 2*(b^3*d^2*f^2
*x + b^3*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 - 2*(b^3*d^2*f^3*x^2 + 2*b
^3*d^2*f^2*x*cosh(1) + b^3*d^2*f*cosh(1)^2 + b^3*d^2*f*sinh(1)^2 + 2*(b^3*d
^2*f^2*x + b^3*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - (b^3*d
^2*f^3*x^2 + 2*b^3*d^2*f^2*x*cosh(1) + b^3*d^2*f*cosh(1)^2 + b^3*d^2*f*sinh
(1)^2 + 2*(b^3*d^2*f^2*x + b^3*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c)^2 + 2*
(b^3*d^2*f^2*x + b^3*d^2*f*cosh(1))*sinh(1))*dilog((a*cosh(d*x + c) + a*sin
```

h(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)
/b + 1) - 3*((a^2*b + b^3)*d^2*f^3*x^2 - 2*(a^3 + a*b^2)*d*f^3*x + (a^2*b +
b^3)*d^2*f*cosh(1)^2 + (a^2*b + b^3)*d^2*f*sinh(1)^2 - ((a^2*b + b^3)*d^2*
f^3*x^2 - 2*(a^3 + a*b^2)*d*f^3*x + (a^2*b + b^3)*d^2*f*cosh(1)^2 + (a^2*b
+ b^3)*d^2*f*sinh(1)^2 + 2*((a^2*b + b^3)*d^2*f^2*x - (a^3 + a*b^2)*d*f^2)*
cosh(1) + 2*((a^2*b + b^3)*d^2*f^2*x + (a^2*b + b^3)*d^2*f*cosh(1) - (a^3 +
a*b^2)*d*f^2)*sinh(1))*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d^2*f^3*x^2 - 2*
(a^3 + a*b^2)*d*f^3*x + (a^2*b + b^3)*d^2*f*cosh(1)^2 + (a^2*b + b^3)*d^2*f
sinh(1)^2 + 2((a^2*b + b^3)*d^2*f^2*x - (a^3 + a*b^2)*d*f^2)*cosh(1) + 2*
((a^2*b + b^3)*d^2*f^2*x + (a^2*b + b^3)*d^2*f*cosh(1) - (a^3 + a*b^2)*d*f^
2)*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d^2*f^3*x^2 - 2*(a
^3 + a*b^2)*d*f^3*x + (a^2*b + b^3)*d^2*f*cosh(1)^2 + (a^2*b + b^3)*d^2*f*s
inh(1)^2 + 2*((a^2*b + b^3)*d^2*f^2*x - (a^3 + a*b^2)*d*f^2)*cosh(1) + 2*((
a^2*b + b^3)*d^2*f^2*x + (a^2*b + b^3)*d^2*f*cosh(1) - (a^3 + a*b^2)*d*f^2)
*sinh(1))*sinh(d*x + c)^2 + 2*((a^2*b + b^3)*d^2*f^2*x - (a^3 + a*b^2)*d*f^
2)*cosh(1) + 2*((a^2*b + b^3)*d^2*f^2*x + (a^2*b + b^3)*d^2*f*cosh(1) - (a^
3 + a*b^2)*d*f^2)*sinh(1))*dilog(cosh(d*x + c) + sinh(d*x + c)) + 3*(-I*a^3
*d^2*f^3*x^2 + a^2*b*d^2*f^3*x^2 - 2*I*a^3*d^2*f^2*x*cosh(1) + 2*a^2*b*d^2*
f^2*x*cosh(1) - I*a^3*d^2*f*cosh(1)^2 + a^2*b*d^2*f*cosh(1)^2 - I*a^3*d^2*f
*sinh(1)^2 + a^2*b*d^2*f*sinh(1)^2 + (I*a^3*d^2*f^3*x^2 - a^2*b*d^2*f^3*x^2
+ 2*I*a^3*d^2*f^2*x*cosh(1) - 2*a^2*b*d^2*f^2*x*cosh(1) + I*a^3*d^2*f*cosh
(1)^2 - a^2*b*d^2*f*cosh(1)^2 + I*a^3*d^2*f*sinh(1)^2 - a^2*b*d^2*f*sinh(1)
^2 + 2*I*(a^3*d^2*f^2*x + a^3*d^2*f*cosh(1))*sinh(1) - 2*(a^2*b*d^2*f^2*x +
a^2*b*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(I*a^3*d^2*f^3*x^2 - a^2
*b*d^2*f^3*x^2 + 2*I*a^3*d^2*f^2*x*cosh(1) - 2*a^2*b*d^2*f^2*x*cosh(1) + I*
a^3*d^2*f*cosh(1)^2 - a^2*b*d^2*f*cosh(1)^2 + I*a^3*d^2*f*sinh(1)^2 - a^2*b
*d^2*f*sinh(1)^2 + 2*I*(a^3*d^2*f^2*x + a^3*d^2*f*cosh(1))*sinh(1) - 2*(a^2
*b*d^2*f^2*x + a^2*b*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) +
(I*a^3*d^2*f^3*x^2 - a^2*b*d^2*f^3*x^2 + 2*I*a^3*d^2*f^2*x*cosh(1) - 2*a^2*
b*d^2*f^2*x*cosh(1) + I*a^3*d^2*f*cosh(1)^2 - a^2*b*d^2*f*cosh(1)^2 + I*a^3
*d^2*f*sinh(1)^2 - a^2*b*d^2*f*sinh(1)^2 + 2*I*(a^3*d^2*f^2*x + a^3*d^2*f*c
osh(1))*sinh(1) - 2*(a^2*b*d^2*f^2*x + a^2*b*d^2*f*cosh(1))*sinh(1))*sinh(d
*x + c)^2 - 2*I*(a^3*d^2*f^2*x + a^3*d^2*f*cosh(1))*sinh(1) + 2*(a^2*b*d^2*
f^2*x + a^2*b*d^2*f*cosh(1))*sinh(1))*dilog(I*cosh(d*x + c) + I*sinh(d*x +
c)) + 3*(I*a^3*d^2*f^3*x^2 + a^2*b*d^2*f^3*x^2 + 2*I*a^3*d^2*f^2*x*cosh(1)
+ 2*a^2*b*d^2*f^2*x*cosh(1) + I*a^3*d^2*f*cosh(1)^2 + a^2*b*d^2*f*cosh(1)^2
+ I*a^3*d^2*f*sinh(1)^2 + a^2*b*d^2*f*sinh(1)^2 + (-I*a^3*d^2*f^3*x^2 - a^
2*b*d^2*f^3*x^2 - 2*I*a^3*d^2*f^2*x*cosh(1) - 2*a^2*b*d^2*f^2*x*cosh(1) - I
*a^3*d^2*f*cosh(1)^2 - a^2*b*d^2*f*cosh(1)^2 - I*a^3*d^2*f*sinh(1)^2 - a^2*
b*d^2*f*sinh(1)^2 - 2*I*(a^3*d^2*f^2*x + a^3*d^2*f*cosh(1))*sinh(1) - 2*(a^
2*b*d^2*f^2*x + a^2*b*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(-I*a^3*d
^2*f^3*x^2 - a^2*b*d^2*f^3*x^2 - 2*I*a^3*d^2*f^2*x*cosh(1) - 2*a^2*b*d^2*f^
2*x*cosh(1) - I*a^3*d^2*f*cosh(1)^2 - a^2*b*d^2*f*cosh(1)^2 - I*a^3*d^2*f*s
inh(1)^2 - a^2*b*d^2*f*sinh(1)^2 - 2*I*(a^3*d^2...

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)**2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^3}{\cosh\left(c+d\,x\right)\,{\sinh\left(c+d\,x\right)}^2\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(cosh(c + d*x)*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((e + f*x)^3/(cosh(c + d*x)*sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.465 $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=982

$$-\frac{2(e+fx)^2\operatorname{ArcTan}\left(e^{c+dx}\right)}{ad}+\frac{2b^2(e+fx)^2\operatorname{ArcTan}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d}-\frac{4f(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^2}+\frac{2b(e+fx)^2\tanh^{-1}}{a^2d}$$

```
[Out] -4*f*(f*x+e)*arctanh(exp(d*x+c))/a/d^2+1/2*b*f^2*polylog(3,exp(2*d*x+2*c))/
a^2/d^3+2*I*b^2*f*(f*x+e)*polylog(2,I*exp(d*x+c))/a/(a^2+b^2)/d^2+b*f*(f*x+
e)*polylog(2,-exp(2*d*x+2*c))/a^2/d^2-b*f*(f*x+e)*polylog(2,exp(2*d*x+2*c))
/a^2/d^2+2*I*f^2*polylog(3,I*exp(d*x+c))/a/d^3+1/2*b^3*f^2*polylog(3,-exp(2
*d*x+2*c))/a^2/(a^2+b^2)/d^3-2*I*f*(f*x+e)*polylog(2,I*exp(d*x+c))/a/d^2-2*
(f*x+e)^2*arctan(exp(d*x+c))/a/d-2*f^2*polylog(2,-exp(d*x+c))/a/d^3+2*f^2*p
olylog(2,exp(d*x+c))/a/d^3-(f*x+e)^2*csch(d*x+c)/a/d-b^3*(f*x+e)^2*ln(1+exp
(2*d*x+2*c))/a^2/(a^2+b^2)/d+b^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(
1/2)))/a^2/(a^2+b^2)/d+b^3*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))
/a^2/(a^2+b^2)/d-2*b^3*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2
/(a^2+b^2)/d^3-2*b^3*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(
a^2+b^2)/d^3+2*b^2*(f*x+e)^2*arctan(exp(d*x+c))/a/(a^2+b^2)/d-1/2*b*f^2*pol
ylog(3,-exp(2*d*x+2*c))/a^2/d^3-2*I*f^2*polylog(3,-I*exp(d*x+c))/a/d^3+2*I*
f*(f*x+e)*polylog(2,-I*exp(d*x+c))/a/d^2-b^3*f*(f*x+e)*polylog(2,-exp(2*d*x
+2*c))/a^2/(a^2+b^2)/d^2-2*I*b^2*f^2*polylog(3,I*exp(d*x+c))/a/(a^2+b^2)/d^
3+2*I*b^2*f^2*polylog(3,-I*exp(d*x+c))/a/(a^2+b^2)/d^3-2*I*b^2*f*(f*x+e)*po
lylog(2,-I*exp(d*x+c))/a/(a^2+b^2)/d^2+2*b*(f*x+e)^2*arctanh(exp(2*d*x+2*c)
)/a^2/d+2*b^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a
^2+b^2)/d^2+2*b^3*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^
2/(a^2+b^2)/d^2
```

Rubi [A]
time = 1.26, antiderivative size = 982, normalized size of antiderivative = 1.00, number of steps used = 53, number of rules used = 21, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.618$, Rules used = {5708, 2701, 327, 213, 5570, 6873, 12, 6874, 5313, 4265, 2611, 2320, 6724, 4267, 2317, 2438, 5569, 5692, 5680, 2221, 3799}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(a*d) + (2*b^2*(e + f*x)^2*ArcTan[E^(c
 + d*x)])/(a*(a^2 + b^2)*d) - (4*f*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d^2)
+ (2*b*(e + f*x)^2*ArcTanh[E^(2*c + 2*d*x)])/(a^2*d) - ((e + f*x)^2*Csch[c
+ d*x])/(a*d) + (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
```

```
2])])/(a^2*(a^2 + b^2)*d) + (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d) - (b^3*(e + f*x)^2*Log[1 + E^(2*(c +
d*x))])/(a^2*(a^2 + b^2)*d) - (2*f^2*PolyLog[2, -E^(c + d*x)])/(a*d^3) + ((
2*I)*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) - ((2*I)*b^2*f*(e +
f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) - ((2*I)*f*(e + f*x)
*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + ((2*I)*b^2*f*(e + f*x)*PolyLog[2, I*E
^(c + d*x)])/(a*(a^2 + b^2)*d^2) + (2*f^2*PolyLog[2, E^(c + d*x)])/(a*d^3)
+ (2*b^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/
(a^2*(a^2 + b^2)*d^2) + (2*b^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^2) - (b^3*f*(e + f*x)*PolyLog[2, -
E^(2*(c + d*x))])/(a^2*(a^2 + b^2)*d^2) + (b*f*(e + f*x)*PolyLog[2, -E^(2*c
 + 2*d*x)])/(a^2*d^2) - (b*f*(e + f*x)*PolyLog[2, E^(2*c + 2*d*x)])/(a^2*d^
2) - ((2*I)*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^3) + ((2*I)*b^2*f^2*Poly
Log[3, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^3) + ((2*I)*f^2*PolyLog[3, I*E^(
c + d*x)])/(a*d^3) - ((2*I)*b^2*f^2*PolyLog[3, I*E^(c + d*x)])/(a*(a^2 + b^
2)*d^3) - (2*b^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/
(a^2*(a^2 + b^2)*d^3) - (2*b^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2]))])/(a^2*(a^2 + b^2)*d^3) + (b^3*f^2*PolyLog[3, -E^(2*(c + d*x))]
)/(2*a^2*(a^2 + b^2)*d^3) - (b*f^2*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a^2*d^3
) + (b*f^2*PolyLog[3, E^(2*c + 2*d*x)])/(2*a^2*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
```

```
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5313

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcTan[u])/(d*(m + 1))), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 + u^2)), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5708

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\text{csch}(c+dx}{a+b\sinh(c}}{a}$$

$$= -\frac{(e+fx)^2\tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)^2\text{csch}(c+dx)}{ad} - \frac{b}{}$$

$$= -\frac{(e+fx)^2\tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)^2\text{csch}(c+dx)}{ad} - ($$

$$= -\frac{b^3(e+fx)^3}{3a^2(a^2+b^2)f} - \frac{(e+fx)^2\tan^{-1}(\sinh(c+dx))}{ad} + \frac{2b(e+f}{}$$

$$= -\frac{b^3(e+fx)^3}{3a^2(a^2+b^2)f} - \frac{(e+fx)^2\tan^{-1}(\sinh(c+dx))}{ad} + \frac{2b(e+f}{}$$

$$= \frac{2b^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a(a^2+b^2)d} - \frac{(e+fx)^2\tan^{-1}(\sinh(c+dx))}{ad} +$$

$$= \frac{2b^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a(a^2+b^2)d} - \frac{4f(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{2b}{}$$

$$= \frac{2b^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a(a^2+b^2)d} - \frac{4f(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{2b}{}$$

$$= -\frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a(a^2+b^2)d} - \frac{4f}{}$$

$$= -\frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a(a^2+b^2)d} - \frac{4f}{}$$

$$= -\frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a(a^2+b^2)d} - \frac{4f}{}$$

$$= -\frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a(a^2+b^2)d} - \frac{4f}{}$$

Mathematica [A]

time = 9.86, size = 1467, normalized size = 1.49

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] -1/6*(12*b*d^3*e^2*E^(2*c)*x + 12*b*d^3*e*E^(2*c)*f*x^2 + 4*b*d^3*E^(2*c)*f
^2*x^3 + 12*a*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] - 6*b*d^2*e^2*(1 +
E^(2*c))*Log[1 + E^(2*(c + d*x))] + (12*I)*a*d*e*(1 + E^(2*c))*f*(d*x*(Log[
1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - PolyLog[2, (-I)*E^(c + d*x)]
 + PolyLog[2, I*E^(c + d*x)]) - 6*b*d*e*(1 + E^(2*c))*f*(2*d*x*Log[1 + E^(2
*(c + d*x))] + PolyLog[2, -E^(2*(c + d*x))]) + (6*I)*a*(1 + E^(2*c))*f^2*(d
^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*Poly
Log[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (
-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d*x)]) - 3*b*(1 + E^(2*c))*f^2*(2*
d^2*x^2*Log[1 + E^(2*(c + d*x))] + 2*d*x*PolyLog[2, -E^(2*(c + d*x))] - Pol
yLog[3, -E^(2*(c + d*x))]))/((a^2 + b^2)*d^3*(1 + E^(2*c))) + (-12*b*e^2*x
+ (12*b*e^2*E^(2*c)*x)/(-1 + E^(2*c)) + (12*b*e*f*x^2)/(-1 + E^(2*c)) + (4*
b*f^2*x^3)/(-1 + E^(2*c)) - (24*a*e*f*ArcTanh[E^(c + d*x)])/d^2 + (6*b*e^2*
(2*d*x - Log[1 - E^(2*(c + d*x))]))/d + (12*a*f^2*(d*x*(Log[1 - E^(c + d*x)
] - Log[1 + E^(c + d*x)]) - PolyLog[2, -E^(c + d*x)] + PolyLog[2, E^(c + d*
x)]))/d^3 + (6*b*e*f*(2*d*x*(d*x - Log[1 - E^(2*(c + d*x))]) - PolyLog[2, E
^(2*(c + d*x))]))/d^2 + (b*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 - E^(2*(c + d*x)
)]) - 6*d*x*PolyLog[2, E^(2*(c + d*x))] + 3*PolyLog[3, E^(2*(c + d*x))]))/d
^3)/(6*a^2) + (b^3*((-2*E^(2*c)*x*(3*e^2 + 3*e*f*x + f^2*x^2))/(-1 + E^(2*c
)) + (3*(d^2*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*d^2*e*
f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + d^2*f^
2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*d^
2*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + d^
2*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] +
2*d*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^
(2*c)]))] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a
^2 + b^2)*E^(2*c)]))] - 2*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[
(a^2 + b^2)*E^(2*c)]))] - 2*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqr
t[(a^2 + b^2)*E^(2*c)]))]))/d^3))/(3*a^2*(a^2 + b^2)) + ((-3*a*b*d*e^2*x -
3*a*b*d*e*f*x^2 - a*b*d*f^2*x^3 - 3*a^2*e^2*Cosh[c] - 3*b^2*e^2*Cosh[c] - 6
*a^2*e*f*x*Cosh[c] - 6*b^2*e*f*x*Cosh[c] - 3*a^2*f^2*x^2*Cosh[c] - 3*b^2*f^
2*x^2*Cosh[c])*Csch[c/2]*Sech[c/2]*Sech[c])/(6*a*(a^2 + b^2)*d) + (Csch[c/2
]*Csch[c/2 + (d*x)/2]*(e^2*Sinh[(d*x)/2] + 2*e*f*x*Sinh[(d*x)/2] + f^2*x^2*
Sinh[(d*x)/2]))/(2*a*d) + (Sech[c/2]*Sech[c/2 + (d*x)/2]*(e^2*Sinh[(d*x)/2]
 + 2*e*f*x*Sinh[(d*x)/2] + f^2*x^2*Sinh[(d*x)/2]))/(2*a*d)
```

Maple [F]

time = 2.24, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^2 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] (b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + a^2*b^2)*d) +
2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^
2 + b^2)*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-d*x -
 c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d))*e^2 - 2*(f^2*x^2*e^c +
2*f*x*e^(c + 1))*e^(d*x)/(a*d*e^(2*d*x + 2*c) - a*d) - 2*f*e*log(e^(d*x + c
) + 1)/(a*d^2) + 2*f*e*log(e^(d*x + c) - 1)/(a*d^2) - (d^2*x^2*log(e^(d*x +
 c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*b*f^2/(a
^2*d^3) - (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*pol
ylog(3, e^(d*x + c)))*b*f^2/(a^2*d^3) - 2*(b*d*f*e + a*f^2)*(d*x*log(e^(d*x
 + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) - 2*(b*d*f*e - a*f^2)*(d*x*log(
-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^2*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*
(b*d*f*e + a*f^2)*d^2*x^2)/(a^2*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*d*f*e - a*
f^2)*d^2*x^2)/(a^2*d^3) - integrate(2*(b^4*f^2*x^2 + 2*b^4*f*x*e - (a*b^3*f
^2*x^2*e^c + 2*a*b^3*f*x*e^(c + 1))*e^(d*x))/(a^4*b + a^2*b^3 - (a^4*b*e^(2
*c) + a^2*b^3*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c + a^3*b^2*e^c)*e^(d*x)), x) -
 integrate(2*(b*f^2*x^2 + 2*b*f*x*e + (a*f^2*x^2*e^c + 2*a*f*x*e^(c + 1))*e
^(d*x))/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8198 vs. 2(914) = 1828.
time = 0.57, size = 8198, normalized size = 8.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -(2*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*f*x*cosh(1) + (a^3 + a
*b^2)*d^2*cosh(1)^2 + (a^3 + a*b^2)*d^2*sinh(1)^2 + 2*((a^3 + a*b^2)*d^2*f*
x + (a^3 + a*b^2)*d^2*cosh(1))*sinh(1))*cosh(d*x + c) + 2*(b^3*d*f^2*x + b^
3*d*f*cosh(1) + b^3*d*f*sinh(1) - (b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*
sinh(1))*cosh(d*x + c)^2 - 2*(b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh(
1))*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3*d*f*
sinh(1))*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b^3*d*
f^2*x + b^3*d*f*cosh(1) + b^3*d*f*sinh(1) - (b^3*d*f^2*x + b^3*d*f*cosh(1)
+ b^3*d*f*sinh(1))*cosh(d*x + c)^2 - 2*(b^3*d*f^2*x + b^3*d*f*cosh(1) + b^3
*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*f^2*x + b^3*d*f*cosh(1)
+ b^3*d*f*sinh(1))*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c
) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) -
 2*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*f*cosh(1) + (a^2*b + b^3)*d*f*s
inh(1) - (a^3 + a*b^2)*f^2 - ((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*f*cos
h(1) + (a^2*b + b^3)*d*f*sinh(1) - (a^3 + a*b^2)*f^2)*cosh(d*x + c)^2 - 2*(
(a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*f*cosh(1) + (a^2*b + b^3)*d*f*sinh(
1) - (a^3 + a*b^2)*f^2)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d*f^2*
x + (a^2*b + b^3)*d*f*cosh(1) + (a^2*b + b^3)*d*f*sinh(1) - (a^3 + a*b^2)*f
^2)*sinh(d*x + c)^2)*dilog(cosh(d*x + c) + sinh(d*x + c)) + 2*(-I*a^3*d*f^2
*x + a^2*b*d*f^2*x - I*a^3*d*f*cosh(1) + a^2*b*d*f*cosh(1) - I*a^3*d*f*sinh
(1) + a^2*b*d*f*sinh(1) + (I*a^3*d*f^2*x - a^2*b*d*f^2*x + I*a^3*d*f*cosh(1
) - a^2*b*d*f*cosh(1) + I*a^3*d*f*sinh(1) - a^2*b*d*f*sinh(1))*cosh(d*x + c
)^2 + 2*(I*a^3*d*f^2*x - a^2*b*d*f^2*x + I*a^3*d*f*cosh(1) - a^2*b*d*f*cosh
(1) + I*a^3*d*f*sinh(1) - a^2*b*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c) +
(I*a^3*d*f^2*x - a^2*b*d*f^2*x + I*a^3*d*f*cosh(1) - a^2*b*d*f*cosh(1) + I*
a^3*d*f*sinh(1) - a^2*b*d*f*sinh(1))*sinh(d*x + c)^2)*dilog(I*cosh(d*x + c)
 + I*sinh(d*x + c)) + 2*(I*a^3*d*f^2*x + a^2*b*d*f^2*x + I*a^3*d*f*cosh(1)
+ a^2*b*d*f*cosh(1) + I*a^3*d*f*sinh(1) + a^2*b*d*f*sinh(1) + (-I*a^3*d*f^2
*x - a^2*b*d*f^2*x - I*a^3*d*f*cosh(1) - a^2*b*d*f*cosh(1) - I*a^3*d*f*sinh
(1) - a^2*b*d*f*sinh(1))*cosh(d*x + c)^2 + 2*(-I*a^3*d*f^2*x - a^2*b*d*f^2*
x - I*a^3*d*f*cosh(1) - a^2*b*d*f*cosh(1) - I*a^3*d*f*sinh(1) - a^2*b*d*f*s
inh(1))*cosh(d*x + c)*sinh(d*x + c) + (-I*a^3*d*f^2*x - a^2*b*d*f^2*x - I*a
^3*d*f*cosh(1) - a^2*b*d*f*cosh(1) - I*a^3*d*f*sinh(1) - a^2*b*d*f*sinh(1))
*sinh(d*x + c)^2)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) - 2*((a^2*b + b
^3)*d*f^2*x + (a^2*b + b^3)*d*f*cosh(1) + (a^2*b + b^3)*d*f*sinh(1) + (a^3
+ a*b^2)*f^2 - ((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*f*cosh(1) + (a^2*b
+ b^3)*d*f*sinh(1) + (a^3 + a*b^2)*f^2)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*
d*f^2*x + (a^2*b + b^3)*d*f*cosh(1) + (a^2*b + b^3)*d*f*sinh(1) + (a^3 + a*
b^2)*f^2)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d*f^2*x + (a^2*b + b
^3)*d*f*cosh(1) + (a^2*b + b^3)*d*f*sinh(1) + (a^3 + a*b^2)*f^2)*sinh(d*x +
 c)^2)*dilog(-cosh(d*x + c) - sinh(d*x + c)) + (b^3*c^2*f^2 - 2*b^3*c*d*f*c
osh(1) + b^3*d^2*cosh(1)^2 + b^3*d^2*sinh(1)^2 - (b^3*c^2*f^2 - 2*b^3*c*d*f
*cosh(1) + b^3*d^2*cosh(1)^2 + b^3*d^2*sinh(1)^2 - 2*(b^3*c*d*f - b^3*d^2*c
osh(1))*sinh(1))*cosh(d*x + c)^2 - 2*(b^3*c^2*f^2 - 2*b^3*c*d*f*cosh(1) + b
```

```
^3*d^2*cosh(1)^2 + b^3*d^2*sinh(1)^2 - 2*(b^3*c*d*f - b^3*d^2*cosh(1))*sinh
(1))*cosh(d*x + c)*sinh(d*x + c) - (b^3*c^2*f^2 - 2*b^3*c*d*f*cosh(1) + b^3
*d^2*cosh(1)^2 + b^3*d^2*sinh(1)^2 - 2*(b^3*c*d*f - b^3*d^2*cosh(1))*sinh(1
))*sinh(d*x + c)^2 - 2*(b^3*c*d*f - b^3*d^2*cosh(1))*sinh(1))*log(2*b*cosh(
d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^3*c^2*
f^2 - 2*b^3*c*d*f*cosh(1) + b^3*d^2*cosh(1)^2 + b^3*d^2*sinh(1)^2 - (b^3*c^
2*f^2 - 2*b^3*c*d*f*cosh(1) + b^3*d^2*cosh(1)^2 + b^3*d^2*sinh(1)^2 - 2*(b^
3*c*d*f - b^3*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^2 - 2*(b^3*c^2*f^2 - 2*b^
3*c*d*f*cosh(1) + b^3*d^2*cosh(1)^2 + b^3*d^2*sinh(1)^2 - 2*(b^3*c*d*f - b^
3*d^2*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - (b^3*c^2*f^2 - 2*b^3*
c*d*f*cosh(1) + b^3*d^2*cosh(1)^2 + b^3*d^2*sinh(1)^2 - 2*(b^3*c*d*f - b^3*
d^2*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 2*(b^3*c*d*f - b^3*d^2*cosh(1))*sin
h(1))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2)
 + 2*a) + (b^3*d^2*f^2*x^2 - b^3*c^2*f^2 - (b^3*d^2*f^2*x^2 - b^3*c^2*f^2 +
 2*(b^3*d^2*f*x + b^3*c*d*f)*cosh(1) + 2*(b^3*d^2*f*x + b^3*c*d*f)*sinh(1))
*cosh(d*x + c)^2 - 2*(b^3*d^2*f^2*x^2 - b^3*c^2*f^2 + 2*(b^3*d^2*f*x + b^3*
c*d*f)*cosh(1) + 2*(b^3*d^2*f*x + b^3*c*d*f)*sinh(1))*cosh(d*x + c)*sinh(d*
x + c) - (b^3*d^2*f^2*x^2 - b^3*c^2*f^2 + 2*(b^3*d^2*f*x + b^3*c*d*f)*cosh(
1) + 2*(b^3*d^2*f*x + b^3*c*d*f)*sinh(1))*sinh(d*x + c)^2 + 2*(b^3*d^2*f*x
+ b^3*c*d*f)*cosh(1) + 2*(b^3*d^2*f*x + b^3*c*d*f)*sinh(1))*log(-(a*cosh(d*
x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b) + (b^3*d^2*f^2*x^2 - b^3*c^2*...
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)**2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6439 deep
```

Giac [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
[Out] Timed out
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^2}{\cosh\left(c+d\,x\right)\,{\sinh\left(c+d\,x\right)}^2\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(cosh(c + d*x)*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
[Out] int((e + f*x)^2/(cosh(c + d*x)*sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.466 $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=591

$$-\frac{2fx\text{ArcTan}\left(e^{c+dx}\right)}{ad}+\frac{2b^2(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d}+\frac{fx\text{ArcTan}(\sinh(c+dx))}{ad}-\frac{(e+fx)\text{ArcTan}(\sinh(c+d}{ad}$$

```
[Out] -2*f*x*arctan(exp(d*x+c))/a/d+2*b^2*(f*x+e)*arctan(exp(d*x+c))/a/(a^2+b^2)/
d+f*x*arctan(sinh(d*x+c))/a/d-(f*x+e)*arctan(sinh(d*x+c))/a/d+2*b*(f*x+e)*a
rctanh(exp(2*d*x+2*c))/a^2/d-f*arctanh(cosh(d*x+c))/a/d^2-(f*x+e)*csch(d*x+
c)/a/d-b^3*(f*x+e)*ln(1+exp(2*d*x+2*c))/a^2/(a^2+b^2)/d+b^3*(f*x+e)*ln(1+b*
exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d+b^3*(f*x+e)*ln(1+b*exp(d*x+
c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d-I*b^2*f*polylog(2,-I*exp(d*x+c))/a/
(a^2+b^2)/d^2-I*f*polylog(2,I*exp(d*x+c))/a/d^2+I*b^2*f*polylog(2,I*exp(d*x
+c))/a/(a^2+b^2)/d^2+I*f*polylog(2,-I*exp(d*x+c))/a/d^2-1/2*b^3*f*polylog(2
,-exp(2*d*x+2*c))/a^2/(a^2+b^2)/d^2+1/2*b*f*polylog(2,-exp(2*d*x+2*c))/a^2/
d^2-1/2*b*f*polylog(2,exp(2*d*x+2*c))/a^2/d^2+b^3*f*polylog(2,-b*exp(d*x+c)
/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^2+b^3*f*polylog(2,-b*exp(d*x+c)/(a+(a
^2+b^2)^(1/2)))/a^2/(a^2+b^2)/d^2
```

Rubi [A]
time = 0.62, antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 18, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5708, 2701, 327, 213, 5570, 5311, 12, 4265, 2317, 2438, 3855, 5569, 4267, 5692, 5680, 2221, 6874, 3799}

$$\frac{2b^2(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{ad\left(a^2+b^2\right)}-\frac{ib^2f\text{Li}_2\left(-ie^{c+dx}\right)}{ad^2\left(a^2+b^2\right)}+\frac{ib^2f\text{Li}_2\left(ie^{c+dx}\right)}{ad^2\left(a^2+b^2\right)}+\frac{b^3f\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2\left(a^2+b^2\right)}+\frac{b^3f\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^2\left(a^2+b^2\right)}-\frac{b^3f\text{Li}_2\left(-e^{2(c+dx)}\right)}{2a^2d^2\left(a^2+b^2\right)}+\frac{b^3(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^2d\left(a^2+b^2\right)}+\frac{b^3(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{a^2d\left(a^2+b^2\right)}-\frac{b^3(e+fx)\log\left(e^{2(c+dx)}+1\right)}{a^2d\left(a^2+b^2\right)}+\frac{bf\text{Li}_2\left(-e^{2c+2dx}\right)}{2a^2d^2}-\frac{bf\text{Li}_2\left(e^{2c+2dx}\right)}{2a^2d^2}+\frac{2b(e+fx)\tanh^{-1}\left(e^{2c+2dx}\right)}{a^2d}-\frac{(e+fx)\text{ArcTan}(\sinh(c+dx))}{ad}-\frac{2fx\text{ArcTan}\left(e^{c+dx}\right)}{ad}+\frac{fx\text{ArcTan}(\sinh(c+dx))}{ad}+\frac{if\text{Li}_2\left(-ie^{c+dx}\right)}{ad^2}-\frac{if\text{Li}_2\left(ie^{c+dx}\right)}{ad^2}-\frac{f\tanh^{-1}(\cosh(c+dx))}{ad^2}-\frac{(e+fx)\text{csch}(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*f*x*ArcTan[E^(c + d*x)])/(a*d) + (2*b^2*(e + f*x)*ArcTan[E^(c + d*x)])/
(a*(a^2 + b^2)*d) + (f*x*ArcTan[Sinh[c + d*x]])/(a*d) - ((e + f*x)*ArcTan[S
inh[c + d*x]])/(a*d) + (2*b*(e + f*x)*ArcTanh[E^(2*c + 2*d*x)])/(a^2*d) - (
f*ArcTanh[Cosh[c + d*x]])/(a*d^2) - ((e + f*x)*Csch[c + d*x])/(a*d) + (b^3*
(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*
d) + (b^3*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*(a
^2 + b^2)*d) - (b^3*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a^2*(a^2 + b^2)*d)
 + (I*f*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) - (I*b^2*f*PolyLog[2, (-I)*E^
(c + d*x)])/(a*(a^2 + b^2)*d^2) - (I*f*PolyLog[2, I*E^(c + d*x)])/(a*d^2) +
 (I*b^2*f*PolyLog[2, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) + (b^3*f*PolyLog[2
, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^2) + (b^3*f
*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^2
```

```
) - (b^3*f*PolyLog[2, -E^(2*(c + d*x))])/(2*a^2*(a^2 + b^2)*d^2) + (b*f*Pol
yLog[2, -E^(2*c + 2*d*x)])/(2*a^2*d^2) - (b*f*PolyLog[2, E^(2*c + 2*d*x)])/
(2*a^2*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
```

```
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5311

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
```

```
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5708

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\text{csch}(c+dx)\text{se}}{a+b\sinh(c+d}}{a}$$

$$= -\frac{(e+fx)\tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)\text{csch}(c+dx)}{ad} - \frac{b\int(}{}$$

$$= -\frac{(e+fx)\tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)\text{csch}(c+dx)}{ad} - \frac{(2b)}{}$$

$$= -\frac{b^3(e+fx)^2}{2a^2(a^2+b^2)f} + \frac{fx\tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)\tan^{-1}(\text{s}}{ad}$$

$$= -\frac{b^3(e+fx)^2}{2a^2(a^2+b^2)f} + \frac{fx\tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)\tan^{-1}(\text{s}}{ad}$$

$$= -\frac{2fx\tan^{-1}(e^{c+dx})}{ad} + \frac{2b^2(e+fx)\tan^{-1}(e^{c+dx})}{a(a^2+b^2)d} + \frac{fx\tan^{-1}(\text{si}}{a}$$

$$= -\frac{2fx\tan^{-1}(e^{c+dx})}{ad} + \frac{2b^2(e+fx)\tan^{-1}(e^{c+dx})}{a(a^2+b^2)d} + \frac{fx\tan^{-1}(\text{si}}{a}$$

$$= -\frac{2fx\tan^{-1}(e^{c+dx})}{ad} + \frac{2b^2(e+fx)\tan^{-1}(e^{c+dx})}{a(a^2+b^2)d} + \frac{fx\tan^{-1}(\text{si}}{a}$$

$$= -\frac{2fx\tan^{-1}(e^{c+dx})}{ad} + \frac{2b^2(e+fx)\tan^{-1}(e^{c+dx})}{a(a^2+b^2)d} + \frac{fx\tan^{-1}(\text{si}}{a}$$

Mathematica [A]
time = 5.38, size = 591, normalized size = 1.00

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (-((d*(e + f*x)*Coth[(c + d*x)/2])/a) - (2*b*d*e*Log[Sinh[c + d*x]])/a^2 +
(2*b*c*f*Log[Sinh[c + d*x]])/a^2 + (2*f*Log[Tanh[(c + d*x)/2]])/a + (b*f*(-
((c + d*x)*(c + d*x + 2*Log[1 - E^(-2*(c + d*x))])) + PolyLog[2, E^(-2*(c +
```

```
d*x))]))/a^2 + (2*b^3*(-1/2*(f*(c + d*x)^2) + f*(c + d*x)*Log[1 + (b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2])] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*
x]] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^2*(a^2 + b^2)) + (2*(-(b*d*e*
(c + d*x)) + b*c*f*(c + d*x) + (b*f*(c + d*x)^2)/2 - 2*a*d*e*ArcTan[Cosh[c
+ d*x] + Sinh[c + d*x]] + 2*a*c*f*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]] - 2
*a*f*(c + d*x)*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]] + b*f*(c + d*x)*Log[2*
Cosh[c + d*x]*(Cosh[c + d*x] - Sinh[c + d*x])] + b*d*e*Log[1 + Cosh[2*(c +
d*x)] + Sinh[2*(c + d*x)]] - b*c*f*Log[1 + Cosh[2*(c + d*x)] + Sinh[2*(c +
d*x)]] + I*a*f*PolyLog[2, (-I)*(Cosh[c + d*x] + Sinh[c + d*x])] - I*a*f*Pol
yLog[2, I*(Cosh[c + d*x] + Sinh[c + d*x])] - (b*f*PolyLog[2, -Cosh[2*(c + d
*x)] + Sinh[2*(c + d*x)]])/2))/(a^2 + b^2) + (d*(e + f*x)*Tanh[(c + d*x)/2]
)/a)/(2*d^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1528 vs. 2(556) = 1112.
time = 5.84, size = 1529, normalized size = 2.59

method	result	size
risch	Expression too large to display	1529

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/a/d^2*f*ln(exp(d*x+c)+1)+1/a/d^2*f*ln(exp(d*x+c)-1)-1/d^2*b*f*c/a/(a^2+b
^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/a^2/d^2*b*f*c
*ln(exp(d*x+c)-1)-1/a^2/d*b*f*ln(exp(d*x+c)+1)*x-1/d*a*b*e/(a^2+b^2)^(3/2)*
arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d^2/a^2*b^3*f/(a^2+b^2)
*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/d^2/a^2*b^3*
f/(a^2+b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+8
/d^2*a*f*c/(4*a^2+4*b^2)*arctan(exp(d*x+c))-1/d/a*b^3*e/(a^2+b^2)^(3/2)*arc
tanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d^2*a*b*f/(a^2+b^2)^(3/2)*
arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d/a^2*b^3*e/(a^2+b^2)*l
n(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/a^2/d*b*e*ln(exp(d*x+c)+1)-1/a^2/d*b
*e*ln(exp(d*x+c)-1)+1/d^2/a^2*b^3*f/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1
/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d^2/a^2*b^3*f*c/(a^2+b^2)*ln(b*exp(2*d*x+2*
c)+2*a*exp(d*x+c)-b)+1/d^2/a^2*b^3*f/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^
(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d^2/a*b^3*f*c/(a^2+b^2)^(3/2)*arctanh(1/
2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-4*I/d*a*f/(4*a^2+4*b^2)*ln(1-I*exp(
d*x+c))*x+4*I/d*a*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x+4*I/d^2*a*f/(4*a^2+4
*b^2)*ln(1+I*exp(d*x+c))*c-4*I/d^2*a*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*c+1
/d*b*e/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+
1/d^2/a*f*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2
```

```
))-1/d^2/a*b^3*f/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)
^(1/2))+4/d*b*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x+4/d*b*f/(4*a^2+4*b^2)*ln
(1+I*exp(d*x+c))*x-4/d^2*b*f*c/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))+4/d^2*b*f
/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*c+4/d^2*b*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+
c))*c+4*I/d^2*a*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))-4*I/d^2*a*f/(4*a^2+4*
b^2)*dilog(1-I*exp(d*x+c))+1/d/a^2*b^3*f/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b
^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d/a^2*b^3*f/(a^2+b^2)*ln((b*exp(d*x+
c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2*a*b*f*c/(a^2+b^2)^(3/2)*
arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d*(f*x+e)/a*exp(d*x+c)/
(exp(2*d*x+2*c)-1)+4/d*b*e/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))+4/d^2*b*f/(4*
a^2+4*b^2)*dilog(1+I*exp(d*x+c))+4/d^2*b*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+
c))-8/d*a*e/(4*a^2+4*b^2)*arctan(exp(d*x+c))-b/d^2/a^2*f*dilog(exp(d*x+c)+1
)+b/d^2/a^2*f*dilog(exp(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] (8*b*d*integrate(1/8*x/(a^2*d*e^(d*x + c) + a^2*d), x) - 8*b*d*integrate(1/
8*x/(a^2*d*e^(d*x + c) - a^2*d), x) + a*((d*x + c)/(a^2*d^2) - log(e^(d*x +
 c) + 1)/(a^2*d^2)) - a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) - 1)/(a^2*d^
2)) - 2*x*e^(d*x + c)/(a*d*e^(2*d*x + 2*c) - a*d) - 8*integrate(-1/4*(a*b^3
*x*e^(d*x + c) - b^4*x)/(a^4*b + a^2*b^3 - (a^4*b*e^(2*c) + a^2*b^3*e^(2*c)
)*e^(2*d*x) - 2*(a^5*e^c + a^3*b^2*e^c)*e^(d*x)), x) - 8*integrate(1/4*(a*x
*e^(d*x + c) + b*x)/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
)*f + (b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + a^2*b^2)
*d) + 2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1
)/((a^2 + b^2)*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(
-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d))*e
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2955 vs. $2(546) = 1092$.
time = 0.52, size = 2955, normalized size = 5.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -(2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*cosh(1) + (a^3 + a*b^2)*d*sinh(1
))*cosh(d*x + c) - (b^3*f*cosh(d*x + c)^2 + 2*b^3*f*cosh(d*x + c)*sinh(d*x
+ c) + b^3*f*sinh(d*x + c)^2 - b^3*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x +
 c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1)
 - (b^3*f*cosh(d*x + c)^2 + 2*b^3*f*cosh(d*x + c)*sinh(d*x + c) + b^3*f*sin
h(d*x + c)^2 - b^3*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*
x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + ((a^2*b + b^3
)*f*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*f*cosh(d*x + c)*sinh(d*x + c) + (a^2*
b + b^3)*f*sinh(d*x + c)^2 - (a^2*b + b^3)*f)*dilog(cosh(d*x + c) + sinh(d*
x + c)) - (I*a^3*f - a^2*b*f + (-I*a^3*f + a^2*b*f)*cosh(d*x + c)^2 - 2*(I*
a^3*f - a^2*b*f)*cosh(d*x + c)*sinh(d*x + c) + (-I*a^3*f + a^2*b*f)*sinh(d*
x + c)^2)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) - (-I*a^3*f - a^2*b*f +
(I*a^3*f + a^2*b*f)*cosh(d*x + c)^2 - 2*(-I*a^3*f - a^2*b*f)*cosh(d*x + c)*
sinh(d*x + c) + (I*a^3*f + a^2*b*f)*sinh(d*x + c)^2)*dilog(-I*cosh(d*x + c)
 - I*sinh(d*x + c)) + ((a^2*b + b^3)*f*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*f*
cosh(d*x + c)*sinh(d*x + c) + (a^2*b + b^3)*f*sinh(d*x + c)^2 - (a^2*b + b^
3)*f)*dilog(-cosh(d*x + c) - sinh(d*x + c)) - (b^3*c*f - b^3*d*cosh(1) - b^
3*d*sinh(1) - (b^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1))*cosh(d*x + c)^2 - 2
*(b^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) - (b
^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1))*sinh(d*x + c)^2)*log(2*b*cosh(d*x +
 c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^3*c*f - b^3
*d*cosh(1) - b^3*d*sinh(1) - (b^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1))*cosh
(d*x + c)^2 - 2*(b^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1))*cosh(d*x + c)*sin
h(d*x + c) - (b^3*c*f - b^3*d*cosh(1) - b^3*d*sinh(1))*sinh(d*x + c)^2)*log
(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) +
 (b^3*d*f*x + b^3*c*f - (b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^2 - 2*(b^3*d*f*
x + b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*f*x + b^3*c*f)*sinh(d*x +
 c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b^3*d*f*x + b^3*c*f - (b^3*d*f*x
 + b^3*c*f)*cosh(d*x + c)^2 - 2*(b^3*d*f*x + b^3*c*f)*cosh(d*x + c)*sinh(d*
x + c) - (b^3*d*f*x + b^3*c*f)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*s
inh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
b)/b) - ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*cosh(1) - ((a^2*b + b^3)*d*f
*x + (a^2*b + b^3)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1) + (a^3 + a*b^2)*f)*c
osh(d*x + c)^2 + (a^2*b + b^3)*d*sinh(1) - 2*((a^2*b + b^3)*d*f*x + (a^2*b
+ b^3)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1) + (a^3 + a*b^2)*f)*cosh(d*x + c)
*sinh(d*x + c) - ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*cosh(1) + (a^2*b +
b^3)*d*sinh(1) + (a^3 + a*b^2)*f)*sinh(d*x + c)^2 + (a^3 + a*b^2)*f)*log(co
sh(d*x + c) + sinh(d*x + c) + 1) - (-I*a^3*c*f + a^2*b*c*f + I*a^3*d*cosh(1
) - a^2*b*d*cosh(1) + I*a^3*d*sinh(1) - a^2*b*d*sinh(1) + (I*a^3*c*f - a^2*
b*c*f - I*a^3*d*cosh(1) + a^2*b*d*cosh(1) - I*a^3*d*sinh(1) + a^2*b*d*sinh(
1))*cosh(d*x + c)^2 - 2*(-I*a^3*c*f + a^2*b*c*f + I*a^3*d*cosh(1) - a^2*b*d
*cosh(1) + I*a^3*d*sinh(1) - a^2*b*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) +
 (I*a^3*c*f - a^2*b*c*f - I*a^3*d*cosh(1) + a^2*b*d*cosh(1) - I*a^3*d*sinh(
1) + a^2*b*d*sinh(1))*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) +
```

```
I) - (I*a^3*c*f + a^2*b*c*f - I*a^3*d*cosh(1) - a^2*b*d*cosh(1) - I*a^3*d*s
inh(1) - a^2*b*d*sinh(1) + (-I*a^3*c*f - a^2*b*c*f + I*a^3*d*cosh(1) + a^2*
b*d*cosh(1) + I*a^3*d*sinh(1) + a^2*b*d*sinh(1))*cosh(d*x + c)^2 - 2*(I*a^3
*c*f + a^2*b*c*f - I*a^3*d*cosh(1) - a^2*b*d*cosh(1) - I*a^3*d*sinh(1) - a^
2*b*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c) + (-I*a^3*c*f - a^2*b*c*f + I*a^
3*d*cosh(1) + a^2*b*d*cosh(1) + I*a^3*d*sinh(1) + a^2*b*d*sinh(1))*sinh(d*x
 + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) - I) - ((a^2*b + b^3)*d*cosh(1)
- ((a^2*b + b^3)*d*cosh(1) + (a^2*b + b^3)*d*sinh(1) - (a^3 + a*b^2 + (a^2*
b + b^3)*c)*f)*cosh(d*x + c)^2 + (a^2*b + b^3)*d*sinh(1) - 2*((a^2*b + b^3)
*d*cosh(1) + (a^2*b + b^3)*d*sinh(1) - (a^3 + a*b^2 + (a^2*b + b^3)*c)*f)*c
osh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d*cosh(1) + (a^2*b + b^3)*d*sin
h(1) - (a^3 + a*b^2 + (a^2*b + b^3)*c)*f)*sinh(d*x + c)^2 - (a^3 + a*b^2 +
(a^2*b + b^3)*c)*f)*log(cosh(d*x + c) + sinh(d*x + c) - 1) - (-I*a^3*d*f*x
- a^2*b*d*f*x - I*a^3*c*f - a^2*b*c*f + (I*a^3*d*f*x + a^2*b*d*f*x + I*a^3*
c*f + a^2*b*c*f)*cosh(d*x + c)^2 - 2*(-I*a^3*d*f*x - a^2*b*d*f*x - I*a^3*c*
f - a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (I*a^3*d*f*x + a^2*b*d*f*x + I
*a^3*c*f + a^2*b*c*f)*sinh(d*x + c)^2)*log(I*cosh(d*x + c) + I*sinh(d*x + c
) + 1) - (I*a^3*d*f*x - a^2*b*d*f*x + I*a^3*c*f - a^2*b*c*f + (-I*a^3*d*f*x
 + a^2*b*d*f*x - I*a^3*c*f + a^2*b*c*f)*cosh(d*x + c)^2 - 2*(I*a^3*d*f*x -
a^2*b*d*f*x + I*a^3*c*f - a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (-I*a^3*
d*f*x + a^2*b*d*f*x - I*a^3*c*f + a^2*b*c*f)*sinh(d*x + c)^2)*log(-I*cosh(d
*x + c) - I*sinh(d*x + c) + 1) - ((a^2*b + b^3)...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3436 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\cosh\left(c+d\,x\right)\,{\mathrm{sinh}\left(c+d\,x\right)}^2\,\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(cosh(c + d*x)*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
[Out] int((e + f*x)/(cosh(c + d*x)*sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.467 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=104

$$-\frac{a\text{ArcTan}(\sinh(c+dx))}{(a^2+b^2)\,d}-\frac{\operatorname{csch}(c+dx)}{ad}+\frac{b\log(\cosh(c+dx))}{(a^2+b^2)\,d}-\frac{b\log(\sinh(c+dx))}{a^2d}+\frac{b^3\log(a+b\sinh(c+d$$

```
[Out] -a*arctan(sinh(d*x+c))/(a^2+b^2)/d-csch(d*x+c)/a/d+b*ln(cosh(d*x+c))/(a^2+b
^2)/d-b*ln(sinh(d*x+c))/a^2/d+b^3*ln(a+b*sinh(d*x+c))/a^2/(a^2+b^2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2916, 12, 908, 649, 209, 266}

$$-\frac{a\text{ArcTan}(\sinh(c+dx))}{d(a^2+b^2)}+\frac{b\log(\cosh(c+dx))}{d(a^2+b^2)}+\frac{b^3\log(a+b\sinh(c+dx))}{a^2d(a^2+b^2)}-\frac{b\log(\sinh(c+dx))}{a^2d}-\frac{\operatorname{csch}(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d)) - Csch[c + d*x]/(a*d) + (b*Log
[Cosh[c + d*x]])/((a^2 + b^2)*d) - (b*Log[Sinh[c + d*x]])/(a^2*d) + (b^3*Lo
g[a + b*Sinh[c + d*x]])/(a^2*(a^2 + b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{b\text{Subst}\left(\int \frac{b^2}{x^2(a+x)(-b^2-x^2)}\,dx, x, b\sinh(c+dx)\right)}{d}\\
&= -\frac{b^3\text{Subst}\left(\int \frac{1}{x^2(a+x)(-b^2-x^2)}\,dx, x, b\sinh(c+dx)\right)}{d}\\
&= -\frac{b^3\text{Subst}\left(\int \left(-\frac{1}{ab^2x^2}+\frac{1}{a^2b^2x}-\frac{1}{a^2(a^2+b^2)(a+x)}+\frac{a-x}{b^2(a^2+b^2)(b^2+x^2)}\right)\,dx, x, b\sin\right.}{d}\\
&= -\frac{\operatorname{csch}(c+dx)}{ad}-\frac{b\log(\sinh(c+dx))}{a^2d}+\frac{b^3\log(a+b\sinh(c+dx))}{a^2(a^2+b^2)d}-\frac{b\text{Sub}}{}\\
&= -\frac{\operatorname{csch}(c+dx)}{ad}-\frac{b\log(\sinh(c+dx))}{a^2d}+\frac{b^3\log(a+b\sinh(c+dx))}{a^2(a^2+b^2)d}+\frac{b\text{Sub}}{}\\
&= -\frac{a\tan^{-1}(\sinh(c+dx))}{(a^2+b^2)d}-\frac{\operatorname{csch}(c+dx)}{ad}+\frac{b\log(\cosh(c+dx))}{(a^2+b^2)d}-\frac{b\log(\sin}{}
\end{aligned}$$

Mathematica [A]
time = 0.49, size = 160, normalized size = 1.54

$$-\frac{b^3\left(\frac{\operatorname{csch}(c+dx)}{ab^3}+\frac{\log(\sinh(c+dx))}{a^2b^2}-\frac{\left(b^2+a\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\sinh(c+dx)\right)}{2b^4(a^2+b^2)}-\frac{\log(a+b\sinh(c+dx))}{a^2(a^2+b^2)}-\frac{\left(1+\frac{a}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\sinh(c+dx)\right)}{2b^2(a^2+b^2)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((b^3*(Csch[c + d*x]/(a*b^3) + Log[Sinh[c + d*x]]/(a^2*b^2) - ((b^2 + a*Sq
rt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[c + d*x]])/(2*b^4*(a^2 + b^2)) - Log[a +
b*Sinh[c + d*x]]/(a^2*(a^2 + b^2)) - ((1 + a/Sqrt[-b^2])*Log[Sqrt[-b^2] + b
*Sinh[c + d*x]])/(2*b^2*(a^2 + b^2))))/d)
```

Maple [A]
time = 1.54, size = 140, normalized size = 1.35

method	result
derivativedivides	$\frac{\frac{2b\ln\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-4a\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2+2b^2}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+ b^3\ln\left(a\left(\tanh^2\left(\frac{dx}{2}\right.\right.\right.}{d}$ …
default	$\frac{\frac{2b\ln\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-4a\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2+2b^2}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+ b^3\ln\left(a\left(\tanh^2\left(\frac{dx}{2}\right.\right.\right.}{d}$ …
risch	$-\frac{2b\,d^2x}{a^2d^2+b^2d^2}-\frac{2bdc}{a^2d^2+b^2d^2}-\frac{2b^3x}{a^2(a^2+b^2)}-\frac{2b^3c}{a^2d(a^2+b^2)}+\frac{2bx}{a^2}+\frac{2bc}{a^2d}-\frac{2\,\mathrm{e}^{dx+c}}{da\left(\mathrm{e}^{2dx+2c}-1\right)}+\frac{i\ln\left(\mathrm{e}^{dx+c}-i\right)a}{(a^2+b^2)d}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/(a^2+b^2)*(2*b*ln(tanh(1/2*d*x+1/2*c)^2+1)-4*a*arctan(tanh(1/2*d*x
+1/2*c)))+1/2/a*tanh(1/2*d*x+1/2*c)-1/2/a/tanh(1/2*d*x+1/2*c)-1/a^2*b*ln(ta
nh(1/2*d*x+1/2*c))+b^3/a^2/(a^2+b^2)*ln(a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/
2*d*x+1/2*c)-a))
```

Maxima [A]
time = 0.48, size = 173, normalized size = 1.66

$$\frac{b^3\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^4+a^2b^2)d}+\frac{2\,a\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d}+\frac{b\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d}+\frac{2\,e^{(-dx-c)}}{(ae^{(-2\,dx-2\,c)}-a)d}-\frac{b\log\left(e^{(-dx-c)}+1\right)}{a^2d}-\frac{b\log\left(e^{(-dx-c)}-1\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + a^2*b^2)*d) + 2
*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^2
 + b^2)*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-d*x -
c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(104) = 208.
time = 0.43, size = 441, normalized size = 4.24

$$-\frac{2\left(a^3\cosh(dx+c)^2+2\,a^3\cosh(dx+c)\sinh(dx+c)+a^3\sinh(dx+c)^2-a^3\right)\arctan\left(\cosh(dx+c)+\sinh(dx+c)\right)+2\left(a^3+ab^2\right)\cosh(dx+c)-\left(b^3\cosh(dx+c)^2+2\,b^3\cosh(dx+c)\sinh(dx+c)+b^3\sinh(dx+c)^2-b^3\right)\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)-\left(a^2b\cosh(dx+c)^2+2\,a^2b\cosh(dx+c)\sinh(dx+c)+a^2b\sinh(dx+c)^2-a^2b\right)\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)-\left(a^2b+b^3-(a^2b+b^3)\cosh(dx+c)^2-2\,(a^2b+b^3)\cosh(dx+c)\sinh(dx+c)-(a^2b+b^3)\sinh(dx+c)^2\right)\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)+2\left(a^3+ab^2\right)\sinh(dx+c)}{(a^4+a^2b^2)d\cosh(dx+c)^2+2\,(a^4+a^2b^2)d\cosh(dx+c)\sinh(dx+c)+(a^4+a^2b^2)d\sinh(dx+c)^2-(a^4+a^2b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -(2*(a^3*cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*x
 + c)^2 - a^3)*arctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(a^3 + a*b^2)*cosh
(d*x + c) - (b^3*cosh(d*x + c)^2 + 2*b^3*cosh(d*x + c)*sinh(d*x + c) + b^3*
sinh(d*x + c)^2 - b^3)*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*
x + c))) - (a^2*b*cosh(d*x + c)^2 + 2*a^2*b*cosh(d*x + c)*sinh(d*x + c) + a
^2*b*sinh(d*x + c)^2 - a^2*b)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x
 + c))) - (a^2*b + b^3 - (a^2*b + b^3)*cosh(d*x + c)^2 - 2*(a^2*b + b^3)*co
sh(d*x + c)*sinh(d*x + c) - (a^2*b + b^3)*sinh(d*x + c)^2)*log(2*sinh(d*x +
 c)/(cosh(d*x + c) - sinh(d*x + c))) + 2*(a^3 + a*b^2)*sinh(d*x + c))/((a^4
 + a^2*b^2)*d*cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x
+ c) + (a^4 + a^2*b^2)*d*sinh(d*x + c)^2 - (a^4 + a^2*b^2)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(csch(c + d*x)**2*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A]
time = 0.46, size = 200, normalized size = 1.92

$$\frac{\frac{2\,b^4\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^4b+a^2b^3}-\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)a}{a^2+b^2}+\frac{b\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^2+b^2}-\frac{2\,b\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a^2}+\frac{2\,\left(b\left(e^{(dx+c)}-e^{(-dx-c)}\right)-2\,a\right)}{a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^4*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^4*b + a^2*b^3)
 - (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*a/(a^2 + b^2) +
b*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2 + b^2) - 2*b*log(abs(e^(d*x
+ c) - e^(-d*x - c)))/a^2 + 2*(b*(e^(d*x + c) - e^(-d*x - c)) - 2*a)/(a^2*(
e^(d*x + c) - e^(-d*x - c))))/d
```

Mupad [B]
time = 2.87, size = 142, normalized size = 1.37

$$\frac{\ln\left({\mathrm{e}}^{c+d\,x}+1{}\mathrm{i}\right)}{b\,d+a\,d\,1{}\mathrm{i}}+\frac{b^3\,\ln\left(2\,a\,{\mathrm{e}}^{c+d\,x}-b+b\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}{d\,a^4+d\,a^2\,b^2}-\frac{2\,{\mathrm{e}}^{c+d\,x}}{a\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}-\frac{b\,\ln\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}{a^2\,d}+\frac{\ln\left(1+{\mathrm{e}}^{c+d\,x}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{a\,d+b\,d\,1{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

```
[Out] log(exp(c + d*x) + 1i)/(a*d*1i + b*d) + (log(exp(c + d*x)*1i + 1)*1i)/(a*d
+ b*d*1i) + (b^3*log(2*a*exp(c + d*x) - b + b*exp(2*c + 2*d*x)))/(a^4*d + a
^2*b^2*d) - (2*exp(c + d*x))/(a*d*(exp(2*c + 2*d*x) - 1)) - (b*log(exp(2*c
+ 2*d*x) - 1))/(a^2*d)
```

3.468 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=37

$$\operatorname{Int}\left(\frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csch[c + d*x]^2*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Csch[c + d*x]^2*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 56.52, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csch[c + d*x]^2*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] Integrate[(Csch[c + d*x]^2*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(dx+c)^2 \operatorname{sech}(dx+c)}{(fx+e)\,(a+b\sinh\,(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] 2*e^(d*x + c)/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e^(2*c + 1))*e^(2*d
*x)) - 8*integrate(-1/8*(b*d*f*x + b*d*e + a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*f*
x*e + a^2*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*f*x*e^(c + 1) + a^2*d*e^(c +
 2))*e^(d*x)), x) + 8*integrate(1/8*(b*d*f*x + b*d*e - a*f)/(a^2*d*f^2*x^2
+ 2*a^2*d*f*x*e + a^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*d*f*x*e^(c + 1) +
a^2*d*e^(c + 2))*e^(d*x)), x) - 8*integrate(-1/4*(a*b^3*e^(d*x + c) - b^4)/
((a^4*b*f + a^2*b^3*f)*x + (a^4*b + a^2*b^3)*e - ((a^4*b*f*e^(2*c) + a^2*b^
3*f*e^(2*c))*x + (a^4*b*e^(2*c) + a^2*b^3*e^(2*c))*e)*e^(2*d*x) - 2*((a^5*f
*e^c + a^3*b^2*f*e^c)*x + (a^5*e^c + a^3*b^2*e^c)*e)*e^(d*x)), x) - 8*integ
rate(1/4*(a*e^(d*x + c) + b)/((a^2*f + b^2*f)*x + (a^2 + b^2)*e + ((a^2*f*e
^(2*c) + b^2*f*e^(2*c))*x + (a^2*e^(2*c) + b^2*e^(2*c))*e)*e^(2*d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(csch(d*x + c)^2*sech(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cosh\left(c+d\,x\right)\,{\sinh\left(c+d\,x\right)}^2\,\left(e+f\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(1/(cosh(c + d*x)*sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.469 $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=914

$$-\frac{2(e+fx)^2}{ad}+\frac{b^2(e+fx)^2}{a\left(a^2+b^2\right)d}+\frac{4bf(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{a^2d^2}-\frac{4b^3f(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d^2}+\frac{2b(e+fx)^2\tanh^{-}}{a^2d}$$

```
[Out] b^4*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d-
b^4*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d-
2*b^4*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/
d^3+2*b^4*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3
/2)/d^3+2*f*(f*x+e)*ln(1-exp(4*d*x+4*c))/a/d^2+4*b*f*(f*x+e)*arctan(exp(d*x
+c))/a^2/d^2+2*b*(f*x+e)^2*arctanh(exp(d*x+c))/a^2/d-2*b*f^2*polylog(3,-exp
(d*x+c))/a^2/d^3+2*b*f^2*polylog(3,exp(d*x+c))/a^2/d^3+2*b*f*(f*x+e)*polylo
g(2,-exp(d*x+c))/a^2/d^2-2*(f*x+e)^2*coth(2*d*x+2*c)/a/d+1/2*f^2*polylog(2,
exp(4*d*x+4*c))/a/d^3-2*(f*x+e)^2/a/d-2*b*f*(f*x+e)*polylog(2,exp(d*x+c))/a
^2/d^2+b^2*(f*x+e)^2/a/(a^2+b^2)/d-b*(f*x+e)^2*sech(d*x+c)/a^2/d-4*b^3*f*(f
*x+e)*arctan(exp(d*x+c))/a^2/(a^2+b^2)/d^2+2*I*b*f^2*polylog(2,I*exp(d*x+c)
)/a^2/d^3-2*I*b^3*f^2*polylog(2,I*exp(d*x+c))/a^2/(a^2+b^2)/d^3+2*I*b^3*f^2
*polylog(2,-I*exp(d*x+c))/a^2/(a^2+b^2)/d^3-b^2*f^2*polylog(2,-exp(2*d*x+2*
c))/a/(a^2+b^2)/d^3+b^3*(f*x+e)^2*sech(d*x+c)/a^2/(a^2+b^2)/d+b^2*(f*x+e)^2
*tanh(d*x+c)/a/(a^2+b^2)/d-2*I*b*f^2*polylog(2,-I*exp(d*x+c))/a^2/d^3-2*b^2
*f*(f*x+e)*ln(1+exp(2*d*x+2*c))/a/(a^2+b^2)/d^2+2*b^4*f*(f*x+e)*polylog(2,-
b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^2-2*b^4*f*(f*x+e)*p
olylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^2
```

Rubi [A]

time = 1.61, antiderivative size = 914, normalized size of antiderivative = 1.00, number of steps used = 51, number of rules used = 25, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.694$, Rules used = {5708, 5569, 4269, 3797, 2221, 2317, 2438, 2702, 327, 213, 5570, 6873, 12, 6874, 6408, 4267, 2611, 2320, 6724, 4265, 5692, 3403, 2296, 3799, 5559}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*(e + f*x)^2)/(a*d) + (b^2*(e + f*x)^2)/(a*(a^2 + b^2)*d) + (4*b*f*(e +
f*x)*ArcTan[E^(c + d*x)])/(a^2*d^2) - (4*b^3*f*(e + f*x)*ArcTan[E^(c + d*x)
])/(a^2*(a^2 + b^2)*d^2) + (2*b*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^2*d) -
 (2*(e + f*x)^2*Coth[2*c + 2*d*x])/(a*d) + (b^4*(e + f*x)^2*Log[1 + (b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d) - (b^4*(e + f*x)
^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d
```

```
) - (2*b^2*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a*(a^2 + b^2)*d^2) + (2*f
*(e + f*x)*Log[1 - E^(4*(c + d*x))])/(a*d^2) + (2*b*f*(e + f*x)*PolyLog[2,
-E^(c + d*x)])/(a^2*d^2) - ((2*I)*b*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*
d^3) + ((2*I)*b^3*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^3) +
 ((2*I)*b*f^2*PolyLog[2, I*E^(c + d*x)])/(a^2*d^3) - ((2*I)*b^3*f^2*PolyLog
[2, I*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^3) - (2*b*f*(e + f*x)*PolyLog[2, E^(
c + d*x)])/(a^2*d^2) + (2*b^4*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^2) - (2*b^4*f*(e + f*x)*PolyL
og[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^2
) - (b^2*f^2*PolyLog[2, -E^(2*(c + d*x))])/(a*(a^2 + b^2)*d^3) + (f^2*PolyL
og[2, E^(4*(c + d*x))])/(2*a*d^3) - (2*b*f^2*PolyLog[3, -E^(c + d*x)])/(a^2
*d^3) + (2*b*f^2*PolyLog[3, E^(c + d*x)])/(a^2*d^3) - (2*b^4*f^2*PolyLog[3,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^3) + (
2*b^4*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*(a^2 +
 b^2)^(3/2)*d^3) - (b*(e + f*x)^2*Sech[c + d*x])/(a^2*d) + (b^3*(e + f*x)^2
*Sech[c + d*x])/(a^2*(a^2 + b^2)*d) + (b^2*(e + f*x)^2*Tanh[c + d*x])/(a*(a
^2 + b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5708

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\text{csch}(c+\cdots}{a+b\sinh\cdots}}{a\cdots}$$

$$= \frac{4\int (e+fx)^2\text{csch}^2(2c+2dx)\,dx}{a} - \frac{b\int (e+fx)^2\text{csch}(c+dx)\text{s}\cdots}{a^2}$$

$$= \frac{b(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{2(e+fx)^2\coth(2c+2dx\cdots}{ad}$$

$$= -\frac{2(e+fx)^2}{ad} + \frac{b(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{2(e+fx\cdots}{\cdots}$$

$$= -\frac{2(e+fx)^2}{ad} + \frac{b(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{2(e+fx\cdots}{\cdots}$$

$$= -\frac{2(e+fx)^2}{ad} + \frac{b(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{2(e+fx\cdots}{\cdots}$$

$$= -\frac{2(e+fx)^2}{ad} + \frac{b^2(e+fx)^2}{a\left(a^2+b^2\right)d} - \frac{4b^3f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d^2} + \cdots$$

$$= -\frac{2(e+fx)^2}{ad} + \frac{b^2(e+fx)^2}{a\left(a^2+b^2\right)d} + \frac{4bf(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \cdots$$

$$= -\frac{2(e+fx)^2}{ad} + \frac{b^2(e+fx)^2}{a\left(a^2+b^2\right)d} + \frac{4bf(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \cdots$$

$$= -\frac{2(e+fx)^2}{ad} + \frac{b^2(e+fx)^2}{a\left(a^2+b^2\right)d} + \frac{4bf(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \cdots$$

$$= -\frac{2(e+fx)^2}{ad} + \frac{b^2(e+fx)^2}{a\left(a^2+b^2\right)d} + \frac{4bf(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \cdots$$

$$= -\frac{2(e+fx)^2}{ad} + \frac{b^2(e+fx)^2}{a\left(a^2+b^2\right)d} + \frac{4bf(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \cdots$$

$$= -\frac{2(e+fx)^2}{ad} + \frac{b^2(e+fx)^2}{a\left(a^2+b^2\right)d} + \frac{4bf(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \cdots$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3086 vs. 2(914) = 1828.
time = 25.87, size = 3086, normalized size = 3.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x
]),x]
```

```
[Out] 4*(-1/4*(a*f*(d*(d*E^c*x*(2*e + f*x) - 2*e*(-I + E^c)*Log[I - E^(c + d*x)]
- 2*(-I + E^c)*f*x*Log[1 + I*E^(c + d*x)]) - 2*(-I + E^c)*f*PolyLog[2, (-I)
*E^(c + d*x)]))/((a^2 + b^2)*d^3*(-I + E^c)) + ((1 + I)*a*d*f*((-2 + 2*I)*d
*e*E^(2*c)*x - (1 - I)*d*E^(2*c)*f*x^2 - 2*e*(-I + E^(2*c))*ArcTan[E^(c + d
*x)] + (2 + 2*I)*e*(1 + I*E^(2*c))*ArcTan[1 - (1 + I)*E^(c + d*x)] - (2*I)*
e*Log[1 - E^(c + d*x)] + 2*e*E^(2*c)*Log[1 - E^(c + d*x)] - (1 + I)*f*x*Log
[1 - E^(c + d*x)] + (1 - I)*E^(2*c)*f*x*Log[1 - E^(c + d*x)] - (1 + I)*f*x*
Log[1 - I*E^(c + d*x)] + (1 - I)*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] - e*Log
[1 + E^(2*(c + d*x))] - I*e*E^(2*c)*Log[1 + E^(2*(c + d*x))]) + 2*a*(-I + E
^(2*c))*f^2*PolyLog[2, I*E^(c + d*x)] + 2*a*(-I + E^(2*c))*f^2*PolyLog[2, E
^(c + d*x)])/(4*(a^2 + b^2)*d^3*(-I + E^(2*c))) - (b*(4*a*b*d^2*e*E^(2*c)*f
*x + 2*a*b*d^2*E^(2*c)*f^2*x^2 + 2*a^2*d^2*e^2*ArcTanh[E^(c + d*x)] + 2*b^2
*d^2*e^2*ArcTanh[E^(c + d*x)] - 2*a^2*d^2*e^2*E^(2*c)*ArcTanh[E^(c + d*x)]
- 2*b^2*d^2*e^2*E^(2*c)*ArcTanh[E^(c + d*x)] - 2*a^2*d^2*e*f*x*Log[1 - E^(c
 + d*x)] - 2*b^2*d^2*e*f*x*Log[1 - E^(c + d*x)] + 2*a^2*d^2*e*E^(2*c)*f*x*L
og[1 - E^(c + d*x)] + 2*b^2*d^2*e*E^(2*c)*f*x*Log[1 - E^(c + d*x)] - a^2*d^
2*f^2*x^2*Log[1 - E^(c + d*x)] - b^2*d^2*f^2*x^2*Log[1 - E^(c + d*x)] + a^2
*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + b^2*d^2*E^(2*c)*f^2*x^2*Log[1 -
 E^(c + d*x)] + 2*a^2*d^2*e*f*x*Log[1 + E^(c + d*x)] + 2*b^2*d^2*e*f*x*Log[
1 + E^(c + d*x)] - 2*a^2*d^2*e*E^(2*c)*f*x*Log[1 + E^(c + d*x)] - 2*b^2*d^2
*e*E^(2*c)*f*x*Log[1 + E^(c + d*x)] + a^2*d^2*f^2*x^2*Log[1 + E^(c + d*x)]
+ b^2*d^2*f^2*x^2*Log[1 + E^(c + d*x)] - a^2*d^2*E^(2*c)*f^2*x^2*Log[1 + E^
(c + d*x)] - b^2*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] + 2*a*b*d*e*f*Log
[1 - E^(2*(c + d*x))] - 2*a*b*d*e*E^(2*c)*f*Log[1 - E^(2*(c + d*x))] + 2*a*
b*d*f^2*x*Log[1 - E^(2*(c + d*x))] - 2*a*b*d*E^(2*c)*f^2*x*Log[1 - E^(2*(c
+ d*x))] - 2*(a^2 + b^2)*d*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -E^(c + d*
x)] + 2*(a^2 + b^2)*d*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, E^(c + d*x)] +
a*b*f^2*PolyLog[2, E^(2*(c + d*x))] - a*b*E^(2*c)*f^2*PolyLog[2, E^(2*(c +
d*x))] - 2*a^2*f^2*PolyLog[3, -E^(c + d*x)] - 2*b^2*f^2*PolyLog[3, -E^(c +
d*x)] + 2*a^2*E^(2*c)*f^2*PolyLog[3, -E^(c + d*x)] + 2*b^2*E^(2*c)*f^2*Poly
Log[3, -E^(c + d*x)] + 2*a^2*f^2*PolyLog[3, E^(c + d*x)] + 2*b^2*f^2*PolyLo
g[3, E^(c + d*x)] - 2*a^2*E^(2*c)*f^2*PolyLog[3, E^(c + d*x)] - 2*b^2*E^(2*
c)*f^2*PolyLog[3, E^(c + d*x)]))/(4*a^2*(a^2 + b^2)*d^3*(-1 + E^(2*c))) + (
b^4*((2*d^2*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b
^2] + (2*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*
```

```
E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] + (d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c
+ d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] - (
2*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)
])])/Sqrt[(a^2 + b^2)*E^(2*c)] - (d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))
/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] + (2*d*E^c
*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*
c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*d*E^c*f*(e + f*x)*PolyLog[2, -((b*E^
(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c
)] - (2*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^
(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] + (2*E^c*f^2*PolyLog[3, -((b*E^(2*c +
d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)]))/(4
*a^2*(a^2 + b^2)*d^3) + (a*e*f*Sech[c/2]*(Cosh[c/2]*Log[Cosh[c/2]*Cosh[(d*x
)/2] + Sinh[c/2]*Sinh[(d*x)/2]] - (d*x*Sinh[c/2])/2))/(2*(a^2 + b^2)*d^2*(C
osh[c/2]^2 - Sinh[c/2]^2)) + (a*f^2*Csch[c/2]*((d^2*x^2)/(4*E^ArcTanh[Coth[
c/2]]) - (I*Coth[c/2]*(-1/2*(d*x*(-Pi + (2*I)*ArcTanh[Coth[c/2]])) - Pi*Log
[1 + E^(d*x)] - 2*((I/2)*d*x + I*ArcTanh[Coth[c/2]])*Log[1 - E^((2*I)*((I/2
)*d*x + I*ArcTanh[Coth[c/2]]))] + Pi*Log[Cosh[(d*x)/2]] + (2*I)*ArcTanh[Cot
h[c/2]]*Log[I*Sinh[(d*x)/2 + ArcTanh[Coth[c/2]]]] + I*PolyLog[2, E^((2*I)*(
(I/2)*d*x + I*ArcTanh[Coth[c/2]]))]))/Sqrt[1 - Coth[c/2]^2])*Sech[c/2])/(2*
(a^2 + b^2)*d^3*Sqrt[Csch[c/2]^2*(-Cosh[c/2]^2 + Sinh[c/2]^2)]) - (e*f*x*Cs
ch[c/2]*Sech[c/2]*(a^2*Cosh[c] - b^2*Cosh[c] + a^2*Cosh[2*c] - I*a^2*Sinh[c
] - I*b^2*Sinh[c]))/(8*a*(a^2 + b^2)*d*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2]
 + I*Sinh[c/2])*(Cosh[c] + I*Sinh[c])) - (f^2*x^2*Csch[c/2]*Sech[c/2]*(a^2*
Cosh[c] - b^2*Cosh[c] + a^2*Cosh[2*c] - I*a^2*Sinh[c] - I*b^2*Sinh[c]))/(16
*a*(a^2 + b^2)*d*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[
c] + I*Sinh[c])) + (b*e*f*ArcTan[(Sinh[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cos
h[c]^2 - Sinh[c]^2]])/((a^2 + b^2)*d^2*Sqrt[Cosh[c]^2 - Sinh[c]^2]) + (b*f^
2*(((-I)*Csch[c]*(I*(d*x + ArcTanh[Coth[c]])*(Log[1 - E^(-(d*x) - ArcTanh[C
oth[c]])] - Log[1 + E^(-(d*x) - ArcTanh[Coth[c]])]) + I*(PolyLog[2, -E^(-(d
*x) - ArcTanh[Coth[c]])] - PolyLog[2, E^(-(d*x) - ArcTanh[Coth[c]])])))/Sqr
t[1 - Coth[c]^2] - (2*ArcTan[(Sinh[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]
^2 - Sinh[c]^2]]*ArcTanh[Coth[c]])/Sqrt[Cosh[c]...
```

Maple [F]
time = 2.78, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^2 \operatorname{sech}(dx+c)^2}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="maxima")
```

```
[Out] -2*a*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 + b^
2)*d^2))*e + 4*b*f^2*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*d
*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 4*a*f^2*integrate(x/(a^2*d*e^(2*d*x
 + 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + (b^4*log((b*e^(-d*x
- c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^4 +
 a^2*b^2)*sqrt(a^2 + b^2)*d) - 2*(a*b*e^(-d*x - c) + b^2*e^(-2*d*x - 2*c) -
 a*b*e^(-3*d*x - 3*c) + 2*a^2 + b^2)/((a^3 + a*b^2 - (a^3 + a*b^2)*e^(-4*d*
x - 4*c))*d) + b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a
^2*d))*e^2 - 4*f*x*e/(a*d) + 4*b*f*arctan(e^(d*x + c))*e/((a^2 + b^2)*d^2)
+ 2*((2*a^2*f^2 + b^2*f^2)*x^2 + 2*(2*a^2*f + b^2*f)*x*e + (a*b*f^2*x^2*e^(
3*c) + 2*a*b*f*x*e^(3*c + 1))*e^(3*d*x) + (b^2*f^2*x^2*e^(2*c) + 2*b^2*f*x*
e^(2*c + 1))*e^(2*d*x) - (a*b*f^2*x^2*e^c + 2*a*b*f*x*e^(c + 1))*e^(d*x))/(
a^3*d + a*b^2*d - (a^3*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x)) + 2*f*e*log(
e^(d*x + c) + 1)/(a*d^2) + 2*f*e*log(e^(d*x + c) - 1)/(a*d^2) + (d^2*x^2*lo
g(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c))
)*b*f^2/(a^2*d^3) - (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c
)) - 2*polylog(3, e^(d*x + c)))*b*f^2/(a^2*d^3) + 2*(b*d*f*e + a*f^2)*(d*x*
log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) - 2*(b*d*f*e - a*f^2)
*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^2*d^3) - 1/3*(b*d^3*f^
2*x^3 + 3*(b*d*f*e + a*f^2)*d^2*x^2)/(a^2*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*
d*f*e - a*f^2)*d^2*x^2)/(a^2*d^3) + integrate(-2*(b^4*f^2*x^2*e^c + 2*b^4*f
*x*e^(c + 1))*e^(d*x)/(a^4*b + a^2*b^3 - (a^4*b*e^(2*c) + a^2*b^3*e^(2*c))*
e^(2*d*x) - 2*(a^5*e^c + a^3*b^2*e^c)*e^(d*x)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 14354 vs. 2(862) = 1724.
time = 0.69, size = 14354, normalized size = 15.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="fricas")
```

```
[Out] -(2*(2*a^5 + 3*a^3*b^2 + a*b^4)*c^2*f^2 - 4*(2*a^5 + 3*a^3*b^2 + a*b^4)*c*d
*f*cosh(1) + 2*(2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*cosh(1)^2 + 2*((2*a^5 + 3*a^
3*b^2 + a*b^4)*d^2*f^2*x^2 - (2*a^5 + 3*a^3*b^2 + a*b^4)*c^2*f^2 + 2*((2*a^
```

5 + 3*a^3*b^2 + a*b^4)*d^2*f*x + (2*a^5 + 3*a^3*b^2 + a*b^4)*c*d*f)*cosh(1) + 2*((2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*f*x + (2*a^5 + 3*a^3*b^2 + a*b^4)*c*d *f)*sinh(1))*cosh(d*x + c)^4 + 2*(2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*sinh(1)^2 + 2*((2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*f^2*x^2 - (2*a^5 + 3*a^3*b^2 + a*b^4)* c^2*f^2 + 2*((2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*f*x + (2*a^5 + 3*a^3*b^2 + a*b ^4)*c*d*f)*cosh(1) + 2*((2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*f*x + (2*a^5 + 3*a^ 3*b^2 + a*b^4)*c*d*f)*sinh(1))*sinh(d*x + c)^4 + 2*((a^4*b + a^2*b^3)*d^2*f ^2*x^2 + 2*(a^4*b + a^2*b^3)*d^2*f*x*cosh(1) + (a^4*b + a^2*b^3)*d^2*cosh(1)^2 + (a^4*b + a^2*b^3)*d^2*sinh(1)^2 + 2*((a^4*b + a^2*b^3)*d^2*f*x + (a^4 *b + a^2*b^3)*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^3 + 2*((a^4*b + a^2*b^3)* d^2*f^2*x^2 + 2*(a^4*b + a^2*b^3)*d^2*f*x*cosh(1) + (a^4*b + a^2*b^3)*d^2*c osh(1)^2 + (a^4*b + a^2*b^3)*d^2*sinh(1)^2 + 4*((2*a^5 + 3*a^3*b^2 + a*b^4) *d^2*f^2*x^2 - (2*a^5 + 3*a^3*b^2 + a*b^4)*c^2*f^2 + 2*((2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*f*x + (2*a^5 + 3*a^3*b^2 + a*b^4)*c*d*f)*cosh(1) + 2*((2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*f*x + (2*a^5 + 3*a^3*b^2 + a*b^4)*c*d*f)*sinh(1))*c osh(d*x + c) + 2*((a^4*b + a^2*b^3)*d^2*f*x + (a^4*b + a^2*b^3)*d^2*cosh(1))*sinh(1))*sinh(d*x + c)^3 + 2*((a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(a^3*b^2 + a*b^4)*d^2*f*x*cosh(1) + (a^3*b^2 + a*b^4)*d^2*cosh(1)^2 + (a^3*b^2 + a*b ^4)*d^2*sinh(1)^2 + 2*((a^3*b^2 + a*b^4)*d^2*f*x + (a^3*b^2 + a*b^4)*d^2*co sh(1))*sinh(1))*cosh(d*x + c)^2 + 2*((a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(a^3 *b^2 + a*b^4)*d^2*f*x*cosh(1) + (a^3*b^2 + a*b^4)*d^2*cosh(1)^2 + (a^3*b^2 + a*b^4)*d^2*sinh(1)^2 + 6*((2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*f^2*x^2 - (2*a^ 5 + 3*a^3*b^2 + a*b^4)*c^2*f^2 + 2*((2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*f*x + (2*a^5 + 3*a^3*b^2 + a*b^4)*c*d*f)*cosh(1) + 2*((2*a^5 + 3*a^3*b^2 + a*b^4)* d^2*f*x + (2*a^5 + 3*a^3*b^2 + a*b^4)*c*d*f)*sinh(1))*cosh(d*x + c)^2 + 3*((a^4*b + a^2*b^3)*d^2*f^2*x^2 + 2*(a^4*b + a^2*b^3)*d^2*f*x*cosh(1) + (a^4* b + a^2*b^3)*d^2*cosh(1)^2 + (a^4*b + a^2*b^3)*d^2*sinh(1)^2 + 2*((a^4*b + a^2*b^3)*d^2*f*x + (a^4*b + a^2*b^3)*d^2*cosh(1))*sinh(1))*cosh(d*x + c) + 2*((a^3*b^2 + a*b^4)*d^2*f*x + (a^3*b^2 + a*b^4)*d^2*cosh(1))*sinh(1))*sinh (d*x + c)^2 + 2*(b^5*d*f^2*x + b^5*d*f*cosh(1) + b^5*d*f*sinh(1) - (b^5*d*f ^2*x + b^5*d*f*cosh(1) + b^5*d*f*sinh(1))*cosh(d*x + c)^4 - 4*(b^5*d*f^2*x + b^5*d*f*cosh(1) + b^5*d*f*sinh(1))*cosh(d*x + c)^3*sinh(d*x + c) - 6*(b^5 *d*f^2*x + b^5*d*f*cosh(1) + b^5*d*f*sinh(1))*cosh(d*x + c)^2*sinh(d*x + c) ^2 - 4*(b^5*d*f^2*x + b^5*d*f*cosh(1) + b^5*d*f*sinh(1))*cosh(d*x + c)*sinh (d*x + c)^3 - (b^5*d*f^2*x + b^5*d*f*cosh(1) + b^5*d*f*sinh(1))*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*c osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b^5* d*f^2*x + b^5*d*f*cosh(1) + b^5*d*f*sinh(1) - (b^5*d*f^2*x + b^5*d*f*cosh(1) + b^5*d*f*sinh(1))*cosh(d*x + c)^4 - 4*(b^5*d*f^2*x + b^5*d*f*cosh(1) + b ^5*d*f*sinh(1))*cosh(d*x + c)^3*sinh(d*x + c) - 6*(b^5*d*f^2*x + b^5*d*f*co sh(1) + b^5*d*f*sinh(1))*cosh(d*x + c)^2*sinh(d*x + c)^2 - 4*(b^5*d*f^2*x + b^5*d*f*cosh(1) + b^5*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 - (b^5*d* f^2*x + b^5*d*f*cosh(1) + b^5*d*f*sinh(1))*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh (d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^5*c^2*f^2 - 2*b^5*c*d*f*c

```
osh(1) + b^5*d^2*cosh(1)^2 + b^5*d^2*sinh(1)^2 - (b^5*c^2*f^2 - 2*b^5*c*d*f
*cosh(1) + b^5*d^2*cosh(1)^2 + b^5*d^2*sinh(1)^2 - 2*(b^5*c*d*f - b^5*d^2*c
osh(1))*sinh(1))*cosh(d*x + c)^4 - 4*(b^5*c^2*f^2 - 2*b^5*c*d*f*cosh(1) + b
^5*d^2*cosh(1)^2 + b^5*d^2*sinh(1)^2 - 2*(b^5*c*d*f - b^5*d^2*cosh(1))*sinh
(1))*cosh(d*x + c)^3*sinh(d*x + c) - 6*(b^5*c^2*f^2 - 2*b^5*c*d*f*cosh(1) +
 b^5*d^2*cosh(1)^2 + b^5*d^2*sinh(1)^2 - 2*(b^5*c*d*f - b^5*d^2*cosh(1))*si
nh(1))*cosh(d*x + c)^2*sinh(d*x + c)^2 - 4*(b^5*c^2*f^2 - 2*b^5*c*d*f*cosh(
1) + b^5*d^2*cosh(1)^2 + b^5*d^2*sinh(1)^2 - 2*(b^5*c*d*f - b^5*d^2*cosh(1)
)*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 - (b^5*c^2*f^2 - 2*b^5*c*d*f*cosh(
1) + b^5*d^2*cosh(1)^2 + b^5*d^2*sinh(1)^2 - 2*(b^5*c*d*f - b^5*d^2*cosh(1)
)*sinh(1))*sinh(d*x + c)^4 - 2*(b^5*c*d*f - b^5*d^2*cosh(1))*sinh(1))*sqrt(
(a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2
+ b^2)/b^2) + 2*a) + (b^5*c^2*f^2 - 2*b^5*c*d*f*cosh(1) + b^5*d^2*cosh(1)^2
 + b^5*d^2*sinh(1)^2 - (b^5*c^2*f^2 - 2*b^5*c*d*f*cosh(1) + b^5*d^2*cosh(1)
^2 + b^5*d^2*sinh(1)^2 - 2*(b^5*c*d*f - b^5*d^2*cosh(1))*sinh(1))*cosh(d*x
+ c)^4 - 4*(b^5*c^2*f^2 - 2*b^5*c*d*f*cosh(1) + b^5*d^2*cosh(1)^2 + b^5*d^2
*sinh(1)^2 - 2*(b^5*c*d*f - b^5*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^3*sinh(
d*x + c) - 6*(b^5*c^2*f^2 - 2*b^5*c*d*f*cosh(1) + b^5*d^2*cosh(1)^2 + b^5*d
^2*sinh(1)^2 - 2*(b^5*c*d*f - b^5*d^2*cosh(1))*...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)**2*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^2}{\cosh\left(c+d\,x\right)^2\sinh\left(c+d\,x\right)^2\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(cosh(c + d*x)^2*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
[Out] int((e + f*x)^2/(cosh(c + d*x)^2*sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.470 $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=499

$$\frac{bf\text{ArcTan}(\sinh(c+dx))}{a^2d^2}-\frac{b^3f\text{ArcTan}(\sinh(c+dx))}{a^2\left(a^2+b^2\right)d^2}+\frac{2bfx\tanh^{-1}\left(e^{c+dx}\right)}{a^2d}-\frac{bfx\tanh^{-1}(\cosh(c+dx))}{a^2d}+\frac{b(e$$

```
[Out] b*f*arctan(sinh(d*x+c))/a^2/d^2-b^3*f*arctan(sinh(d*x+c))/a^2/(a^2+b^2)/d^2
+2*b*f*x*arctanh(exp(d*x+c))/a^2/d-b*f*x*arctanh(cosh(d*x+c))/a^2/d+b*(f*x+
e)*arctanh(cosh(d*x+c))/a^2/d-2*(f*x+e)*coth(2*d*x+2*c)/a/d-b^2*f*ln(cosh(d
*x+c))/a/(a^2+b^2)/d^2+f*ln(sinh(2*d*x+2*c))/a/d^2+b^4*(f*x+e)*ln(1+b*exp(d
*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d-b^4*(f*x+e)*ln(1+b*exp(d*x
+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d+b*f*polylog(2,-exp(d*x+c))/a
^2/d^2-b*f*polylog(2,exp(d*x+c))/a^2/d^2+b^4*f*polylog(2,-b*exp(d*x+c)/(a-(
a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^2-b^4*f*polylog(2,-b*exp(d*x+c)/(a+(
a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^(3/2)/d^2-b*(f*x+e)*sech(d*x+c)/a^2/d+b^3*(f
*x+e)*sech(d*x+c)/a^2/(a^2+b^2)/d+b^2*(f*x+e)*tanh(d*x+c)/a/(a^2+b^2)/d
```

Rubi [A]
time = 0.80, antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 20, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {5708, 5569, 4269, 3556, 2702, 327, 213, 5570, 6406, 12, 4267, 2317, 2438, 3855, 5692, 3403, 2296, 2221, 6874, 5559}

$$-\frac{b^3f\text{ArcTan}(\sinh(c+dx))}{a^2d^2\left(a^2+b^2\right)}+\frac{bf\text{ArcTan}(\sinh(c+dx))}{a^2d^2}-\frac{b^2f\log(\cosh(c+dx))}{ad^2\left(a^2+b^2\right)}+\frac{b^2(e+fx)\tanh(c+dx)}{ad\left(a^2+b^2\right)}+\frac{b^4f\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2\left(a^2+b^2\right)^{3/2}}-\frac{b^4f\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^2d^2\left(a^2+b^2\right)^{3/2}}+\frac{b^4(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^2d\left(a^2+b^2\right)^{3/2}}-\frac{b^4(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{a^2d\left(a^2+b^2\right)^{3/2}}+\frac{b^3(e+fx)\text{sech}(c+dx)}{a^2d\left(a^2+b^2\right)}+\frac{bf\text{Li}_2\left(-e^{c+dx}\right)}{a^2d^2}-\frac{bf\text{Li}_2\left(e^{c+dx}\right)}{a^2d^2}-\frac{b(e+fx)\text{sech}(c+dx)}{a^2d}+\frac{b(e+fx)\tanh^{-1}(\cosh(c+dx))}{a^2d}+\frac{2bfx\tanh^{-1}\left(e^{c+dx}\right)}{a^2d}-\frac{bfx\tanh^{-1}(\cosh(c+dx))}{a^2d}+\frac{f\log(\sinh(2c+2dx))}{ad^2}-\frac{2(e+fx)\coth(2c+2dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*f*ArcTan[Sinh[c + d*x]])/(a^2*d^2) - (b^3*f*ArcTan[Sinh[c + d*x]])/(a^2*
(a^2 + b^2)*d^2) + (2*b*f*x*ArcTanh[E^(c + d*x)])/(a^2*d) - (b*f*x*ArcTanh[
Cosh[c + d*x]])/(a^2*d) + (b*(e + f*x)*ArcTanh[Cosh[c + d*x]])/(a^2*d) - (2
*(e + f*x)*Coth[2*c + 2*d*x])/(a*d) + (b^4*(e + f*x)*Log[1 + (b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d) - (b^4*(e + f*x)*Log[1
+ (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d) - (b^2*
f*Log[Cosh[c + d*x]])/(a*(a^2 + b^2)*d^2) + (f*Log[Sinh[2*c + 2*d*x]])/(a*d
^2) + (b*f*PolyLog[2, -E^(c + d*x)])/(a^2*d^2) - (b*f*PolyLog[2, E^(c + d*x
)])/(a^2*d^2) + (b^4*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]
)/(a^2*(a^2 + b^2)^(3/2)*d^2) - (b^4*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^2) - (b*(e + f*x)*Sech[c + d*x])
/(a^2*d) + (b^3*(e + f*x)*Sech[c + d*x])/(a^2*(a^2 + b^2)*d) + (b^2*(e + f*
x)*Tanh[c + d*x])/(a*(a^2 + b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
```

```
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5708

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6406

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(1 - u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\text{csch}(c+dx)\text{se}}{a+b\sinh(c+}}{a}$$

$$= \frac{4\int (e+fx)\text{csch}^2(2c+2dx)\,dx}{a} - \frac{b\int (e+fx)\text{csch}(c+dx)\text{sech}^2}{a^2}$$

$$= \frac{b(e+fx)\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{2(e+fx)\coth(2c+2dx)}{ad} -$$

$$= \frac{b(e+fx)\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{2(e+fx)\coth(2c+2dx)}{ad} +$$

$$= \frac{bf\tan^{-1}(\sinh(c+dx))}{a^2d^2} - \frac{bfx\tanh^{-1}(\cosh(c+dx))}{a^2d} + \frac{b(e+f}{}$$

$$= \frac{bf\tan^{-1}(\sinh(c+dx))}{a^2d^2} - \frac{bfx\tanh^{-1}(\cosh(c+dx))}{a^2d} + \frac{b(e+f}{}$$

$$= \frac{bf\tan^{-1}(\sinh(c+dx))}{a^2d^2} - \frac{b^3f\tan^{-1}(\sinh(c+dx))}{a^2(a^2+b^2)d^2} + \frac{2bfx\tanh}{a^2}$$

$$= \frac{bf\tan^{-1}(\sinh(c+dx))}{a^2d^2} - \frac{b^3f\tan^{-1}(\sinh(c+dx))}{a^2(a^2+b^2)d^2} + \frac{2bfx\tanh}{a^2}$$

$$= \frac{bf\tan^{-1}(\sinh(c+dx))}{a^2d^2} - \frac{b^3f\tan^{-1}(\sinh(c+dx))}{a^2(a^2+b^2)d^2} + \frac{2bfx\tanh}{a^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.97, size = 1862, normalized size = 3.73

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] 4*(-1/8*(f*(c + d*x))/((a + I*b)*d^2) + ((I/8)*((2 - I)*a^3*d*f + (3*I)*a^2
*b*d*f - I*a*b^2*d*f + I*b^3*d*f + a^2*b*c*d*f + I*a*b^2*c*d*f)*(c + d*x))/
```

```
(a*(a + I*b)*(a^2 + b^2)*d^3) - ((I/16)*b*f*(c + d*x)^2)/((a^2 + b^2)*d^2)
+ ((I/4)*f*ArcTan[(a*Cosh[(c + d*x)/2] - b*Cosh[(c + d*x)/2] + a*Sinh[(c +
d*x)/2] + b*Sinh[(c + d*x)/2])/(a*Cosh[(c + d*x)/2] + b*Cosh[(c + d*x)/2] -
 a*Sinh[(c + d*x)/2] + b*Sinh[(c + d*x)/2])])/((a + I*b)*d^2) - (a*f*ArcTan
h[1 - (2*I)*Tanh[(c + d*x)/2]])/(2*(a^2 + b^2)*d^2) - (b^2*f*ArcTanh[1 - (2
*I)*Tanh[(c + d*x)/2]])/(2*a*(a^2 + b^2)*d^2) - (b*c*f*ArcTanh[1 - (2*I)*Ta
nh[(c + d*x)/2]])/(2*(a^2 + b^2)*d^2) + ((-(d*e*Cosh[(c + d*x)/2]) + c*f*Co
sh[(c + d*x)/2] - f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(8*a*d^
2) + (a*f*Log[Cosh[(c + d*x)/2]])/(4*(a^2 + b^2)*d^2) + (b^2*f*Log[Cosh[(c
+ d*x)/2]])/(4*a*(a^2 + b^2)*d^2) - (b*c*f*Log[Cosh[(c + d*x)/2]])/(4*(a^2
+ b^2)*d^2) + (f*Log[Cosh[c + d*x]])/(8*(a + I*b)*d^2) + (a*f*((-I)*(c + d*
x) + 2*ArcTanh[1 - (2*I)*Tanh[(c + d*x)/2]] + Log[-1 + Cosh[c + d*x] + I*Si
nh[c + d*x]]))/(4*(a^2 + b^2)*d^2) + ((I/8)*b*f*((-I)*(c + d*x) + 2*ArcTanh
[1 - (2*I)*Tanh[(c + d*x)/2]] + Log[-1 + Cosh[c + d*x] + I*Sinh[c + d*x]]))
/((a^2 + b^2)*d^2) + (b^2*f*((-I)*(c + d*x) + 2*ArcTanh[1 - (2*I)*Tanh[(c +
 d*x)/2]] + Log[-1 + Cosh[c + d*x] + I*Sinh[c + d*x]]))/(8*a*(a^2 + b^2)*d^
2) + (b*c*f*((-I)*(c + d*x) + 2*ArcTanh[1 - (2*I)*Tanh[(c + d*x)/2]] + Log[
-1 + Cosh[c + d*x] + I*Sinh[c + d*x]]))/(8*(a^2 + b^2)*d^2) - (b*e*Log[Tanh
[(c + d*x)/2]])/(4*(a^2 + b^2)*d) - (b^3*e*Log[Tanh[(c + d*x)/2]])/(4*a^2*(
a^2 + b^2)*d) + (b^3*c*f*Log[Tanh[(c + d*x)/2]])/(4*a^2*(a^2 + b^2)*d^2) +
((I/2)*b*f*((-1/8*I)*(c + d*x)^2 - (I/2)*(c + d*x)*Log[1 + E^(-c - d*x)] +
(I/2)*PolyLog[2, -E^(-c - d*x)]))/((a^2 + b^2)*d^2) - (b*f*((-1/2*I)*(c + d
*x)^2 + (I/4)*(3*Pi*(c + d*x) + (1 - I)*(c + d*x)^2 + 2*(Pi - (2*I)*(c + d*
x))*Log[1 + I*E^(-c - d*x)] - 4*Pi*Log[1 + E^(c + d*x)] - 2*Pi*Log[-Cos[(Pi
 + (2*I)*(c + d*x))/4]] + 4*Pi*Log[Cosh[(c + d*x)/2]] + (4*I)*PolyLog[2, (-
I)*E^(-c - d*x)])))/(4*(a^2 + b^2)*d^2) + ((I/4)*b^3*f*(I*(c + d*x)*(Log[1
- E^(-c - d*x)] - Log[1 + E^(-c - d*x)]) + I*(PolyLog[2, -E^(-c - d*x)] - P
olyLog[2, E^(-c - d*x)])))/(a^2*(a^2 + b^2)*d^2) - ((I/4)*b*f*((c + d*x)^2/
4 + (-3*Pi*(c + d*x) - (1 - I)*(c + d*x)^2 - 2*(Pi - (2*I)*(c + d*x))*Log[1
 + I*E^(-c - d*x)] + 4*Pi*Log[1 + E^(c + d*x)] + 2*Pi*Log[-Cos[(Pi + (2*I)*
(c + d*x))/4]] - 4*Pi*Log[Cosh[(c + d*x)/2]] - (4*I)*PolyLog[2, (-I)*E^(-c
- d*x)])/4 - (I/2)*(-1/2*(c + d*x)^2 + 2*(c + d*x)*Log[1 - E^(c + d*x)] + 2
*PolyLog[2, E^(c + d*x)])))/((a^2 + b^2)*d^2) + (b^4*(-2*d*e*ArcTanh[(a + b
*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a + b*C
osh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*(
Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1
+ (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2,
 (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2,
 -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))]))/(4*a^2*(a^
2 + b^2)^(3/2)*d^2) + (Sech[(c + d*x)/2]*(-(d*e*Sinh[(c + d*x)/2]) + c*f*Si
nh[(c + d*x)/2] - f*(c + d*x)*Sinh[(c + d*x)/2]))/(8*a*d^2) + (Sech[c + d*x
]*(-(b*d*e) + b*c*f - b*f*(c + d*x) - a*d*e*Sinh[c + d*x] + a*c*f*Sinh[c +
d*x] - a*f*(c + d*x)*Sinh[c + d*x]))/(4*(a^2 + b^2)*d^2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1770 vs.

2(476) = 952.
time = 5.94, size = 1771, normalized size = 3.55

method	result	size
risch	Expression too large to display	1771

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNV
ERBOSE)
```

```
[Out] -1/d^2/(a^2+b^2)^(5/2)*a^2*b^2*f*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^
2)^(1/2))+1/d^2/(a^2+b^2)^(3/2)*b^2*f*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a
^2+b^2)^(1/2))+1/d/(a^2+b^2)^(5/2)*e*a^2*b^2*arctanh(1/2*(2*b*exp(d*x+c)+2*
a)/(a^2+b^2)^(1/2))-2/(a^2+b^2)/d^2/a*f*b^2*ln(exp(d*x+c))+1/(a^2+b^2)/d^2/
a*b^2*f*ln(exp(d*x+c)+1)+1/(a^2+b^2)/d^2/a*b^2*f*ln(exp(d*x+c)-1)+1/(a^2+b^
2)/d/a^2*b^3*e*ln(exp(d*x+c)+1)-1/(a^2+b^2)/d/a^2*b^3*e*ln(exp(d*x+c)-1)+1/
(a^2+b^2)/d^2/a^2*f*b^3*dilog(exp(d*x+c)+1)+1/(a^2+b^2)/d^2/a^2*f*b^3*dilog
(exp(d*x+c))+4/(a^2+b^2)/d^2*a^3*f/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))+1/(a^
2+b^2)/d*ln(exp(d*x+c)+1)*b*f*x+8/(a^2+b^2)/d^2*f*b^3/(4*a^2+4*b^2)*arctan(
exp(d*x+c))+1/(a^2+b^2)/d^2*b*f*c*ln(exp(d*x+c)-1)+1/(a^2+b^2)^(5/2)/d^2*b^
4*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/(a^2+b^
2)^(5/2)/d^2*b^4*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2
)))-1/(a^2+b^2)^(5/2)/d/a^2*b^6*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2
)^(1/2))-1/(a^2+b^2)^(3/2)/d/a^2*b^4*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^
2+b^2)^(1/2))+1/(a^2+b^2)/d/a^2*f*b^3*ln(exp(d*x+c)+1)*x+4/(a^2+b^2)/d^2*a*
b^2*f/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))-1/(a^2+b^2)^(5/2)/d^2/a^2*f*b^6*di
log((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/(a^2+b^2)/d^2/a
^2*b^3*f*c*ln(exp(d*x+c)-1)+8/(a^2+b^2)/d^2*a^2*b*f/(4*a^2+4*b^2)*arctan(ex
p(d*x+c))+1/(a^2+b^2)^(5/2)/d^2/a^2*f*b^6*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1
/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/(a^2+b^2)^(5/2)/d*b^4*f*ln((-b*exp(d*x+c)+(a
^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/(a^2+b^2)^(5/2)/d*b^4*f*ln((b*ex
p(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/(a^2+b^2)^(5/2)/d^2*b^
4*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/(a^2+b^2
)^(5/2)/d^2*b^4*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*
c-1/d/(a^2+b^2)^(3/2)*b^2*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2
))-2*(f*x+e)*(a*b*exp(3*d*x+3*c)+b^2*exp(2*d*x+2*c)-a*b*exp(d*x+c)+2*a^2+b^
2)/d/a/(exp(2*d*x+2*c)-1)/(a^2+b^2)/(1+exp(2*d*x+2*c))+1/(a^2+b^2)^(5/2)/d^
2/a^2*f*b^6*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/
(a^2+b^2)^(5/2)/d/a^2*f*b^6*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2
)^(1/2)))*x+1/(a^2+b^2)^(5/2)/d/a^2*f*b^6*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))*x-1/(a^2+b^2)^(5/2)/d^2/a^2*f*b^6*ln((b*exp(d*x+c
)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/(a^2+b^2)^(3/2)/d^2/a^2*b^4*f
*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)^(5/2)/d^2/
a^2*b^6*f*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)/d
```

```
*b*e*ln(exp(d*x+c)+1)-1/(a^2+b^2)/d*b*e*ln(exp(d*x+c)-1)+1/(a^2+b^2)/d^2*b*
f*dilog(exp(d*x+c)+1)+1/(a^2+b^2)/d^2*b*f*dilog(exp(d*x+c))-4/(a^2+b^2)/d^2
*a*f*ln(exp(d*x+c))+1/(a^2+b^2)/d^2*a*f*ln(exp(d*x+c)+1)+1/(a^2+b^2)/d^2*a*
f*ln(exp(d*x+c)-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] (16*b^4*integrate(-1/8*x*e^(d*x + c)/(a^4*b + a^2*b^3 - (a^4*b*e^(2*c) + a^
2*b^3*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c + a^3*b^2*e^c)*e^(d*x)), x) - 16*b*d*
integrate(1/16*x/(a^2*d*e^(d*x + c) + a^2*d), x) - 16*b*d*integrate(1/16*x/
(a^2*d*e^(d*x + c) - a^2*d), x) - a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c)
+ 1)/(a^2*d^2)) - a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) - 1)/(a^2*d^2))
+ 2*(a*b*x*e^(3*d*x + 3*c) + b^2*x*e^(2*d*x + 2*c) - a*b*x*e^(d*x + c) + (2
*a^2 + b^2)*x)/(a^3*d + a*b^2*d - (a^3*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*
x)) - 2*a*x/((a^2 + b^2)*d) + 2*b*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) + a
*log(e^(2*d*x + 2*c) + 1)/((a^2 + b^2)*d^2))*f + (b^4*log((b*e^(-d*x - c) -
 a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^4 + a^2*b
^2)*sqrt(a^2 + b^2)*d) - 2*(a*b*e^(-d*x - c) + b^2*e^(-2*d*x - 2*c) - a*b*e
^(-3*d*x - 3*c) + 2*a^2 + b^2)/((a^3 + a*b^2 - (a^3 + a*b^2)*e^(-4*d*x - 4*
c))*d) + b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d))
*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4647 vs. $2(479) = 958$.
time = 0.45, size = 4647, normalized size = 9.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -(2*((2*a^5 + 3*a^3*b^2 + a*b^4)*d*f*x + (a^5 + 2*a^3*b^2 + a*b^4)*c*f)*cos
h(d*x + c)^4 + 2*((2*a^5 + 3*a^3*b^2 + a*b^4)*d*f*x + (a^5 + 2*a^3*b^2 + a*
b^4)*c*f)*sinh(d*x + c)^4 + 2*((a^4*b + a^2*b^3)*d*f*x + (a^4*b + a^2*b^3)*
d*cosh(1) + (a^4*b + a^2*b^3)*d*sinh(1))*cosh(d*x + c)^3 + 2*((a^4*b + a^2*
b^3)*d*f*x + (a^4*b + a^2*b^3)*d*cosh(1) + (a^4*b + a^2*b^3)*d*sinh(1) + 4*
((2*a^5 + 3*a^3*b^2 + a*b^4)*d*f*x + (a^5 + 2*a^3*b^2 + a*b^4)*c*f)*cosh(d*
x + c))*sinh(d*x + c)^3 - 2*(a^5 + 2*a^3*b^2 + a*b^4)*c*f + 2*(2*a^5 + 3*a^
```

3*b^2 + a*b^4)*d*cosh(1) + 2*((a^3*b^2 + a*b^4)*d*f*x + (a^3*b^2 + a*b^4)*d
*cosh(1) + (a^3*b^2 + a*b^4)*d*sinh(1))*cosh(d*x + c)^2 + 2*(2*a^5 + 3*a^3*
b^2 + a*b^4)*d*sinh(1) + 2*((a^3*b^2 + a*b^4)*d*f*x + (a^3*b^2 + a*b^4)*d*c
osh(1) + 6*((2*a^5 + 3*a^3*b^2 + a*b^4)*d*f*x + (a^5 + 2*a^3*b^2 + a*b^4)*c
*f)*cosh(d*x + c)^2 + (a^3*b^2 + a*b^4)*d*sinh(1) + 3*((a^4*b + a^2*b^3)*d*
f*x + (a^4*b + a^2*b^3)*d*cosh(1) + (a^4*b + a^2*b^3)*d*sinh(1))*cosh(d*x +
 c))*sinh(d*x + c)^2 - (b^5*f*cosh(d*x + c)^4 + 4*b^5*f*cosh(d*x + c)^3*sin
h(d*x + c) + 6*b^5*f*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^5*f*cosh(d*x + c
)*sinh(d*x + c)^3 + b^5*f*sinh(d*x + c)^4 - b^5*f)*sqrt((a^2 + b^2)/b^2)*di
log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b^5*f*cosh(d*x + c)^4 + 4*b^5*f*cosh
(d*x + c)^3*sinh(d*x + c) + 6*b^5*f*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^5
*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^5*f*sinh(d*x + c)^4 - b^5*f)*sqrt((a^2
 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b^5*c*f - b^5*d*cosh(
1) - b^5*d*sinh(1) - (b^5*c*f - b^5*d*cosh(1) - b^5*d*sinh(1))*cosh(d*x + c
)^4 - 4*(b^5*c*f - b^5*d*cosh(1) - b^5*d*sinh(1))*cosh(d*x + c)^3*sinh(d*x
+ c) - 6*(b^5*c*f - b^5*d*cosh(1) - b^5*d*sinh(1))*cosh(d*x + c)^2*sinh(d*x
 + c)^2 - 4*(b^5*c*f - b^5*d*cosh(1) - b^5*d*sinh(1))*cosh(d*x + c)*sinh(d*
x + c)^3 - (b^5*c*f - b^5*d*cosh(1) - b^5*d*sinh(1))*sinh(d*x + c)^4)*sqrt(
(a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2
+ b^2)/b^2) + 2*a) - (b^5*c*f - b^5*d*cosh(1) - b^5*d*sinh(1) - (b^5*c*f -
b^5*d*cosh(1) - b^5*d*sinh(1))*cosh(d*x + c)^4 - 4*(b^5*c*f - b^5*d*cosh(1)
 - b^5*d*sinh(1))*cosh(d*x + c)^3*sinh(d*x + c) - 6*(b^5*c*f - b^5*d*cosh(1
) - b^5*d*sinh(1))*cosh(d*x + c)^2*sinh(d*x + c)^2 - 4*(b^5*c*f - b^5*d*cos
h(1) - b^5*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 - (b^5*c*f - b^5*d*cosh
(1) - b^5*d*sinh(1))*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*
x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^5*d*f*x
+ b^5*c*f - (b^5*d*f*x + b^5*c*f)*cosh(d*x + c)^4 - 4*(b^5*d*f*x + b^5*c*f)
*cosh(d*x + c)^3*sinh(d*x + c) - 6*(b^5*d*f*x + b^5*c*f)*cosh(d*x + c)^2*si
nh(d*x + c)^2 - 4*(b^5*d*f*x + b^5*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 - (b^
5*d*f*x + b^5*c*f)*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x
+ c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2) - b)/b) - (b^5*d*f*x + b^5*c*f - (b^5*d*f*x + b^5*c*f)*cosh(d*x + c
)^4 - 4*(b^5*d*f*x + b^5*c*f)*cosh(d*x + c)^3*sinh(d*x + c) - 6*(b^5*d*f*x
+ b^5*c*f)*cosh(d*x + c)^2*sinh(d*x + c)^2 - 4*(b^5*d*f*x + b^5*c*f)*cosh(d
*x + c)*sinh(d*x + c)^3 - (b^5*d*f*x + b^5*c*f)*sinh(d*x + c)^4)*sqrt((a^2
+ b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*((a^4*b + a^2*b^3)*f*cosh(
d*x + c)^4 + 4*(a^4*b + a^2*b^3)*f*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^4*b
 + a^2*b^3)*f*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^4*b + a^2*b^3)*f*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^4*b + a^2*b^3)*f*sinh(d*x + c)^4 - (a^4*b + a
^2*b^3)*f)*arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*((a^4*b + a^2*b^3)*d*f
*x + (a^4*b + a^2*b^3)*d*cosh(1) + (a^4*b + a^2*b^3)*d*sinh(1))*cosh(d*x +
c) + ((a^4*b + 2*a^2*b^3 + b^5)*f*cosh(d*x + c)^4 + 4*(a^4*b + 2*a^2*b^3 +

```
b^5)*f*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^4*b + 2*a^2*b^3 + b^5)*f*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*(a^4*b + 2*a^2*b^3 + b^5)*f*cosh(d*x + c)*sin
h(d*x + c)^3 + (a^4*b + 2*a^2*b^3 + b^5)*f*sinh(d*x + c)^4 - (a^4*b + 2*a^2
*b^3 + b^5)*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) - ((a^4*b + 2*a^2*b^3 +
 b^5)*f*cosh(d*x + c)^4 + 4*(a^4*b + 2*a^2*b^3 + b^5)*f*cosh(d*x + c)^3*sin
h(d*x + c) + 6*(a^4*b + 2*a^2*b^3 + b^5)*f*cosh(d*x + c)^2*sinh(d*x + c)^2
+ 4*(a^4*b + 2*a^2*b^3 + b^5)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + 2*
a^2*b^3 + b^5)*f*sinh(d*x + c)^4 - (a^4*b + 2*a^2*b^3 + b^5)*f)*dilog(-cosh
(d*x + c) - sinh(d*x + c)) - ((a^5 + a^3*b^2)*f*cosh(d*x + c)^4 + 4*(a^5 +
a^3*b^2)*f*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^5 + a^3*b^2)*f*cosh(d*x + c
)^2*sinh(d*x + c)^2 + 4*(a^5 + a^3*b^2)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (
a^5 + a^3*b^2)*f*sinh(d*x + c)^4 - (a^5 + a^3*b^2)*f)*log(2*cosh(d*x + c)/(
cosh(d*x + c) - sinh(d*x + c))) - (((a^4*b + 2*a^2*b^3 + b^5)*d*f*x + (a^4*
b + 2*a^2*b^3 + b^5)*d*cosh(1) + (a^4*b + 2*a^2*b^3 + b^5)*d*sinh(1) + (a^5
 + 2*a^3*b^2 + a*b^4)*f)*cosh(d*x + c)^4 + 4*((...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**2*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6439 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{{\mathrm{cosh}\left(c+d\,x\right)}^2\,{\mathrm{sinh}\left(c+d\,x\right)}^2\,\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(cosh(c + d*x)^2*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((e + f*x)/(cosh(c + d*x)^2*sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.471 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=144

$$\frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d}-\frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d}-\frac{\coth(c+dx)}{ad}-\frac{b\operatorname{sech}(c+dx)}{a^2d}+\frac{b^2\operatorname{sech}(c+dx)(b+c}{a^2\left(a^2+b\right.}$$

```
[Out] b*arctanh(cosh(d*x+c))/a^2/d-2*b^4*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b
^2)^(1/2))/a^2/(a^2+b^2)^(3/2)/d-coth(d*x+c)/a/d-b*sech(d*x+c)/a^2/d+b^2*se
ch(d*x+c)*(b+a*sinh(d*x+c))/a^2/(a^2+b^2)/d-tanh(d*x+c)/a/d
```

Rubi [A]
time = 0.23, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {2977, 2702, 327, 213, 2700, 14, 2775, 12, 2739, 632, 210}

$$\frac{b^2\operatorname{sech}(c+dx)(a\sinh(c+dx)+b)}{a^2d\left(a^2+b^2\right)}-\frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d\left(a^2+b^2\right)^{3/2}}-\frac{b\operatorname{sech}(c+dx)}{a^2d}+\frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d}-\frac{\tanh(c+dx)}{ad}-\frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Cosh[c + d*x]])/(a^2*d) - (2*b^4*ArcTanh[(b - a*Tanh[(c + d*x)/2
])/Sqrt[a^2 + b^2]])/(a^2*(a^2 + b^2)^(3/2)*d) - Coth[c + d*x]/(a*d) - (b*S
ech[c + d*x])/(a^2*d) + (b^2*Sech[c + d*x]*(b + a*Sinh[c + d*x]))/(a^2*(a^2
 + b^2)*d) - Tanh[c + d*x]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
```

```
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\int \left(\frac{b\text{csch}(c+dx)\text{sech}^2(c+dx)}{a^2} - \frac{\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a} - \frac{b}{a^2(a \cdots}\right.$$

$$= \frac{\int \text{csch}^2(c+dx)\text{sech}^2(c+dx)\,dx}{a} - \frac{b\int \text{csch}(c+dx)\text{sech}^2(c+dx)\,dx}{a^2} + \cdots$$

$$= \frac{b^2\text{sech}(c+dx)(b+a\sinh(c+dx))}{a^2(a^2+b^2)d} + \frac{b^2\int \frac{b^2}{a+b\sinh(c+dx)}\,dx}{a^2(a^2+b^2)} + \frac{i\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{b\text{sech}(c+dx)}{a^2d} + \frac{b^2\text{sech}(c+dx)(b+a\sinh(c+dx))}{a^2(a^2+b^2)d} + \frac{b^4\int \frac{1}{a+b\sinh(c+dx)}\cdots}{a^2(a^2+b^2)\cdots}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{\coth(c+dx)}{ad} - \frac{b\text{sech}(c+dx)}{a^2d} + \frac{b^2\text{sech}(c+d\cdots}{a^2\cdots}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{\coth(c+dx)}{ad} - \frac{b\text{sech}(c+dx)}{a^2d} + \frac{b^2\text{sech}(c+d\cdots}{a^2\cdots}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d} - \frac{\coth(c+dx\cdots}{ad}$$

Mathematica [A]
time = 2.02, size = 135, normalized size = 0.94

$$-\frac{\frac{4b^4\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{a^2(-a^2-b^2)^{3/2}} + \frac{\coth\left(\frac{1}{2}(c+dx)\right)}{a} + \frac{2b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a^2} + \frac{2\text{sech}(c+dx)(b+a\sinh(c+dx))}{a^2+b^2} + \frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

[Out] -1/2*((4*b^4*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(a^2*(-a^2 - b^2)^(3/2)) + Coth[(c + d*x)/2]/a + (2*b*Log[Tanh[(c + d*x)/2]])/a^2 + (2*Sech[c + d*x]*(b + a*Sinh[c + d*x]))/(a^2 + b^2) + Tanh[(c + d*x)/2]/a)/d

Maple [A]
time = 1.53, size = 139, normalized size = 0.97

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{-2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{\left(a^2+b^2\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2b^4\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{-2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{\left(a^2+b^2\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2b^4\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}}{d}$
risch	$-\frac{2\left(ab\,e^{3dx+3c}+b^2e^{2dx+2c}-ab\,e^{dx+c}+2a^2+b^2\right)}{ad\left(e^{2dx+2c}-1\right)\left(a^2+b^2\right)\left(1+e^{2dx+2c}\right)}+\frac{b^4\ln\left(e^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}d\,a^2}-\frac{b^4\ln\left(e^{dx+c}+\frac{a\left(a^2\right.}{}\right.}{\left(a^2+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/2/a*tanh(1/2*d*x+1/2*c)+2/(a^2+b^2)*(-a*tanh(1/2*d*x+1/2*c)-b)/(tanh(1/2*d*x+1/2*c)^2+1)+2/a^2*b^4/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-1/2/a/tanh(1/2*d*x+1/2*c)-1/a^2*b*ln(tanh(1/2*d*x+1/2*c)))

Maxima [A]
time = 0.49, size = 208, normalized size = 1.44

$$\frac{b^4\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{(a^4+a^2b^2)\sqrt{a^2+b^2}\,d}-\frac{2\left(abe^{(-dx-c)}+b^2e^{(-2\,dx-2\,c)}-abe^{(-3\,dx-3\,c)}+2\,a^2+b^2\right)}{(a^3+ab^2-(a^3+ab^2)e^{(-4\,dx-4\,c)})d}+\frac{b\log\left(e^{(-dx-c)}+1\right)}{a^2d}-\frac{b\log\left(e^{(-dx-c)}-1\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")

[Out] b^4*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)*d) - 2*(a*b*e^(-d*x - c) + b^2*e^(-2*d*x - 2*c) - a*b*e^(-3*d*x - 3*c) + 2*a^2 + b^2)/((a^3 + a*b^2 - (a^3 + a*b^2)*e^(-4*d*x - 4*c))*d) + b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1040 vs. $2(141) = 282$.

time = 0.50, size = 1040, normalized size = 7.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="frica
s")
```

```
[Out] -(4*a^5 + 6*a^3*b^2 + 2*a*b^4 + 2*(a^4*b + a^2*b^3)*cosh(d*x + c)^3 + 2*(a^
4*b + a^2*b^3)*sinh(d*x + c)^3 + 2*(a^3*b^2 + a*b^4)*cosh(d*x + c)^2 + 2*(a
^3*b^2 + a*b^4 + 3*(a^4*b + a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - (b^4*
cosh(d*x + c)^4 + 4*b^4*cosh(d*x + c)^3*sinh(d*x + c) + 6*b^4*cosh(d*x + c)
^2*sinh(d*x + c)^2 + 4*b^4*cosh(d*x + c)*sinh(d*x + c)^3 + b^4*sinh(d*x + c
)^4 - b^4)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 +
 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x +
 c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*
x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*
sinh(d*x + c) - b)) - 2*(a^4*b + a^2*b^3)*cosh(d*x + c) + (a^4*b + 2*a^2*b^
3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^4 - 4*(a^4*b + 2*a^2*b^3
+ b^5)*cosh(d*x + c)^3*sinh(d*x + c) - 6*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x
 + c)^2*sinh(d*x + c)^2 - 4*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)*sinh(d*
x + c)^3 - (a^4*b + 2*a^2*b^3 + b^5)*sinh(d*x + c)^4)*log(cosh(d*x + c) + s
inh(d*x + c) + 1) - (a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)*co
sh(d*x + c)^4 - 4*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^3*sinh(d*x + c) -
 6*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^2*sinh(d*x + c)^2 - 4*(a^4*b + 2
*a^2*b^3 + b^5)*cosh(d*x + c)*sinh(d*x + c)^3 - (a^4*b + 2*a^2*b^3 + b^5)*s
inh(d*x + c)^4)*log(cosh(d*x + c) + sinh(d*x + c) - 1) - 2*(a^4*b + a^2*b^3
 - 3*(a^4*b + a^2*b^3)*cosh(d*x + c)^2 - 2*(a^3*b^2 + a*b^4)*cosh(d*x + c))
*sinh(d*x + c))/((a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^4 + 4*(a^6 + 2
*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^6 + 2*a^4*b^2 +
a^2*b^4)*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^6 + 2*a^4*b^2 + a^2*b^4)*
d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d*sinh(d*x +
c)^4 - (a^6 + 2*a^4*b^2 + a^2*b^4)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3436 deep
```

Giac [A]
time = 0.48, size = 185, normalized size = 1.28

$$\frac{\frac{b^4\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+a^2b^2)\sqrt{a^2+b^2}}+\frac{b\log\left(e^{(dx+c)}+1\right)}{a^2}-\frac{b\log\left(\left|e^{(dx+c)}-1\right|\right)}{a^2}-\frac{2\left(abe^{(3\,dx+3\,c)}+b^2e^{(2\,dx+2\,c)}-abe^{(dx+c)}+2\,a^2+b^2\right)}{(a^3+ab^2)\left(e^{(4\,dx+4\,c)}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac"
)
```

```
[Out] (b^4*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c)
 + 2*a + 2*sqrt(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)) + b*log(e^(d
*x + c) + 1)/a^2 - b*log(abs(e^(d*x + c) - 1))/a^2 - 2*(a*b*e^(3*d*x + 3*c)
 + b^2*e^(2*d*x + 2*c) - a*b*e^(d*x + c) + 2*a^2 + b^2)/((a^3 + a*b^2)*(e^(
4*d*x + 4*c) - 1)))/d
```

Mupad [B]
time = 5.23, size = 768, normalized size = 5.33

$$\frac{b^4\ln\left(\frac{64\,b^8\,\sqrt{{\left(a^2+b^2\right)}^3}-96\,a\,b^{10}-384\,a^3\,b^8-512\,a^5\,b^6-288\,a^7\,b^4-64\,a^9\,b^2+288\,a^2\,b^9\,{\mathrm{e}}^{c+d\,x}+960\,a^4\,b^7\,{\mathrm{e}}^{c+d\,x}+1152\,a^6\,b^5\,{\mathrm{e}}^{c+d\,x}+608\,a^8\,b^3\,{\mathrm{e}}^{c+d\,x}+128\,a^{10}\,b\,{\mathrm{e}}^{c+d\,x}-64\,a\,b^7\,{\mathrm{e}}^{c+d\,x}\,\sqrt{{\left(a^2+b^2\right)}^3}+32\,a^3\,b^5\,{\mathrm{e}}^{c+d\,x}\,\sqrt{{\left(a^2+b^2\right)}^3}}{a^3\,{\left({\left(a^2+b^2\right)}^3\right)}^{3/2}\,\left(a^2+b^2\right)}-\frac{32\,b\,\left(-4\,{\mathrm{e}}^{c+d\,x}\,a^3+2\,a^2\,b-5\,{\mathrm{e}}^{c+d\,x}\,a\,b^2+2\,b^3\right)}{a^3\,{\left(a^2+b^2\right)}^2}\right)\,\sqrt{{\left(a^2+b^2\right)}^3}}{d\,a^8+3\,d\,a^6\,b^2+3\,d\,a^4\,b^4+d\,a^2\,b^6}-\frac{\frac{2\,b^4\,{\mathrm{e}}^{3\,c+3\,d\,x}}{d\,\left(a^2\,b^3+b^5\right)}-\frac{2\,b^4\,{\mathrm{e}}^{c+d\,x}}{d\,\left(a^2\,b^3+b^5\right)}+\frac{2\,b^3\,\left(2\,a^2+b^2\right)}{a\,d\,\left(a^2\,b^3+b^5\right)}+\frac{2\,b^5\,{\mathrm{e}}^{2\,c+2\,d\,x}}{a\,d\,\left(a^2\,b^3+b^5\right)}}{{\mathrm{e}}^{4\,c+4\,d\,x}-1}-\frac{b^4\ln\left(\frac{96\,a\,b^{10}+64\,b^8\,\sqrt{{\left(a^2+b^2\right)}^3}+384\,a^3\,b^8+512\,a^5\,b^6+288\,a^7\,b^4+64\,a^9\,b^2-288\,a^2\,b^9\,{\mathrm{e}}^{c+d\,x}-960\,a^4\,b^7\,{\mathrm{e}}^{c+d\,x}-1152\,a^6\,b^5\,{\mathrm{e}}^{c+d\,x}-608\,a^8\,b^3\,{\mathrm{e}}^{c+d\,x}-128\,a^{10}\,b\,{\mathrm{e}}^{c+d\,x}-64\,a\,b^7\,{\mathrm{e}}^{c+d\,x}\,\sqrt{{\left(a^2+b^2\right)}^3}+32\,a^3\,b^5\,{\mathrm{e}}^{c+d\,x}\,\sqrt{{\left(a^2+b^2\right)}^3}}{a^3\,{\left({\left(a^2+b^2\right)}^3\right)}^{3/2}\,\left(a^2+b^2\right)}-\frac{32\,b\,\left(-4\,{\mathrm{e}}^{c+d\,x}\,a^3+2\,a^2\,b-5\,{\mathrm{e}}^{c+d\,x}\,a\,b^2+2\,b^3\right)}{a^3\,{\left(a^2+b^2\right)}^2}\right)\,\sqrt{{\left(a^2+b^2\right)}^3}}{d\,a^8+3\,d\,a^6\,b^2+3\,d\,a^4\,b^4+d\,a^2\,b^6}-\frac{b\,\ln\left({\mathrm{e}}^{c+d\,x}-1\right)}{a^2\,d}+\frac{b\,\ln\left({\mathrm{e}}^{c+d\,x}+1\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

```
[Out] (b^4*log((64*b^8*((a^2 + b^2)^3)^(1/2) - 96*a*b^10 - 384*a^3*b^8 - 512*a^5*
b^6 - 288*a^7*b^4 - 64*a^9*b^2 + 288*a^2*b^9*exp(c + d*x) + 960*a^4*b^7*exp
(c + d*x) + 1152*a^6*b^5*exp(c + d*x) + 608*a^8*b^3*exp(c + d*x) + 128*a^10
*b*exp(c + d*x) - 64*a*b^7*exp(c + d*x)*((a^2 + b^2)^3)^(1/2) + 32*a^3*b^5*
exp(c + d*x)*((a^2 + b^2)^3)^(1/2))/(a^3*((a^2 + b^2)^3)^(3/2)*(a^2 + b^2))
 - (32*b*(2*a^2*b - 4*a^3*exp(c + d*x) + 2*b^3 - 5*a*b^2*exp(c + d*x)))/(a^
3*(a^2 + b^2)^2))*((a^2 + b^2)^3)^(1/2))/(a^8*d + a^2*b^6*d + 3*a^4*b^4*d +
 3*a^6*b^2*d) - ((2*b^4*exp(3*c + 3*d*x))/(d*(b^5 + a^2*b^3)) - (2*b^4*exp(
c + d*x))/(d*(b^5 + a^2*b^3)) + (2*b^3*(2*a^2 + b^2))/(a*d*(b^5 + a^2*b^3))
 + (2*b^5*exp(2*c + 2*d*x))/(a*d*(b^5 + a^2*b^3)))/(exp(4*c + 4*d*x) - 1) -
 (b^4*log((96*a*b^10 + 64*b^8*((a^2 + b^2)^3)^(1/2) + 384*a^3*b^8 + 512*a^5
*b^6 + 288*a^7*b^4 + 64*a^9*b^2 - 288*a^2*b^9*exp(c + d*x) - 960*a^4*b^7*ex
p(c + d*x) - 1152*a^6*b^5*exp(c + d*x) - 608*a^8*b^3*exp(c + d*x) - 128*a^1
0*b*exp(c + d*x) - 64*a*b^7*exp(c + d*x)*((a^2 + b^2)^3)^(1/2) + 32*a^3*b^5
*exp(c + d*x)*((a^2 + b^2)^3)^(1/2))/(a^3*((a^2 + b^2)^3)^(3/2)*(a^2 + b^2)
) - (32*b*(2*a^2*b - 4*a^3*exp(c + d*x) + 2*b^3 - 5*a*b^2*exp(c + d*x)))/(a
^3*(a^2 + b^2)^2))*((a^2 + b^2)^3)^(1/2))/(a^8*d + a^2*b^6*d + 3*a^4*b^4*d
+ 3*a^6*b^2*d) - (b*log(exp(c + d*x) - 1))/(a^2*d) + (b*log(exp(c + d*x) +
1))/(a^2*d)
```

3.472 $\int \frac{\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=39

$$\text{Int}\left(\frac{\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csch[c + d*x]^2*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Csch[c + d*x]^2*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi steps

$$\int \frac{\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 140.36, size = 0, normalized size = 0.00

$$\int \frac{\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csch[c + d*x]^2*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]
)),x]
```

```
[Out] Integrate[(Csch[c + d*x]^2*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]
)), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(dx+c)^2 \operatorname{sech}(dx+c)^2}{(fx+e)\,(a+b\sinh\,(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] 16*b^4*integrate(-1/8*e^(d*x + c)/((a^4*b*f + a^2*b^3*f)*x + (a^4*b + a^2*b
^3)*e - ((a^4*b*f*e^(2*c) + a^2*b^3*f*e^(2*c))*x + (a^4*b*e^(2*c) + a^2*b^3
*e^(2*c))*e)*e^(2*d*x) - 2*((a^5*f*e^c + a^3*b^2*f*e^c)*x + (a^5*e^c + a^3*
b^2*e^c)*e)*e^(d*x)), x) + 2*(a*b*e^(3*d*x + 3*c) + b^2*e^(2*d*x + 2*c) - a
*b*e^(d*x + c) + 2*a^2 + b^2)/((a^3*d*f + a*b^2*d*f)*x + (a^3*d + a*b^2*d)*
e - ((a^3*d*f*e^(4*c) + a*b^2*d*f*e^(4*c))*x + (a^3*d*e^(4*c) + a*b^2*d*e^(
4*c))*e)*e^(4*d*x)) - 16*integrate(-1/16*(b*d*f*x + b*d*e + a*f)/(a^2*d*f^2
*x^2 + 2*a^2*d*f*x*e + a^2*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*f*x*e^(c +
1) + a^2*d*e^(c + 2))*e^(d*x)), x) - 16*integrate(1/16*(b*d*f*x + b*d*e - a
*f)/(a^2*d*f^2*x^2 + 2*a^2*d*f*x*e + a^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2
*d*f*x*e^(c + 1) + a^2*d*e^(c + 2))*e^(d*x)), x) - 16*integrate(1/8*(b*f*e^
(d*x + c) - a*f)/((a^2*d*f^2 + b^2*d*f^2)*x^2 + 2*(a^2*d*f + b^2*d*f)*x*e +
 (a^2*d + b^2*d)*e^2 + ((a^2*d*f^2*e^(2*c) + b^2*d*f^2*e^(2*c))*x^2 + 2*(a^
2*d*f*e^(2*c) + b^2*d*f*e^(2*c))*x*e + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^2)
*e^(2*d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] integral(csch(d*x + c)^2*sech(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(
d*x + c)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*sech(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3437 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cosh\left(c+d\,x\right)^2\,\sinh\left(c+d\,x\right)^2\,\left(e+f\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(1/(cosh(c + d*x)^2*sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.473 $\int \frac{(e+fx)\mathbf{csch}^2(c+dx)\mathbf{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=978

$$-\frac{bfx}{2a^2d}-\frac{3fx\text{ArcTan}\left(e^{c+dx}\right)}{ad}+\frac{2b^4(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)^2d}+\frac{b^2(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d}+\frac{3fx\text{ArcTan}(\sinh(c-}{2ad}$$

[Out] -1/2*b*f*polylog(2,exp(2*d*x+2*c))/a^2/d^2+b^5*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^2/d^2+b^5*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^2/d^2-b^5*(f*x+e)*ln(1+exp(2*d*x+2*c))/a^2/(a^2+b^2)^2/d+b^5*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^2/d+b^5*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^2/(a^2+b^2)^2/d+b*f*x*ln(tanh(d*x+c))/a^2/d-b*(f*x+e)*ln(tanh(d*x+c))/a^2/d+b^2*(f*x+e)*arctan(exp(d*x+c))/a/(a^2+b^2)/d+1/2*b^2*f*sech(d*x+c)/a/(a^2+b^2)/d^2+1/2*b^3*(f*x+e)*sech(d*x+c)^2/a^2/(a^2+b^2)/d-1/2*b^3*f*tanh(d*x+c)/a^2/(a^2+b^2)/d^2-3/2*(f*x+e)*arctan(sinh(d*x+c))/a/d-f*arctanh(cosh(d*x+c))/a/d^2-3/2*(f*x+e)*csch(d*x+c)/a/d-1/2*f*sech(d*x+c)/a/d^2+I*b^4*f*polylog(2,I*exp(d*x+c))/a/(a^2+b^2)^2/d^2+1/2*b^2*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/a/(a^2+b^2)/d-I*b^4*f*polylog(2,-I*exp(d*x+c))/a/(a^2+b^2)^2/d^2-1/2*I*b^2*f*polylog(2,-I*exp(d*x+c))/a/(a^2+b^2)/d^2-3*f*x*arctan(exp(d*x+c))/a/d+3/2*f*x*arctan(sinh(d*x+c))/a/d+1/2*b*f*polylog(2,-exp(2*d*x+2*c))/a^2/d^2-1/2*b*f*x/a^2/d+1/2*(f*x+e)*csch(d*x+c)*sech(d*x+c)^2/a/d+1/2*b*f*tanh(d*x+c)/a^2/d^2+1/2*b*(f*x+e)*tanh(d*x+c)^2/a^2/d-3/2*I*f*polylog(2,I*exp(d*x+c))/a/d^2+1/2*I*b^2*f*polylog(2,I*exp(d*x+c))/a/(a^2+b^2)/d^2+2*b^4*(f*x+e)*arctan(exp(d*x+c))/a/(a^2+b^2)^2/d+2*b*f*x*arctanh(exp(2*d*x+2*c))/a^2/d+3/2*I*f*polylog(2,-I*exp(d*x+c))/a/d^2-1/2*b^5*f*polylog(2,-exp(2*d*x+2*c))/a^2/(a^2+b^2)^2/d^2

Rubi [A]
time = 1.15, antiderivative size = 978, normalized size of antiderivative = 1.00, number of steps used = 57, number of rules used = 27, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.794$, Rules used = {5708, 2701, 294, 327, 213, 5570, 5311, 12, 4265, 2317, 2438, 3855, 2702, 2700, 14, 2628, 4267, 3554, 8, 5692, 5680, 2221, 6874, 3799, 4270, 5559, 3852}

Antiderivative was successfully verified.

[In] Int[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]

[Out] -1/2*(b*f*x)/(a^2*d) - (3*f*x*ArcTan[E^(c + d*x)])/(a*d) + (2*b^4*(e + f*x)*ArcTan[E^(c + d*x)])/(a*(a^2 + b^2)^2*d) + (b^2*(e + f*x)*ArcTan[E^(c + d*x)])/(a*(a^2 + b^2)*d) + (3*f*x*ArcTan[Sinh[c + d*x]])/(2*a*d) - (3*(e + f*x)*ArcTan[Sinh[c + d*x]])/(2*a*d) + (2*b*f*x*ArcTanh[E^(2*c + 2*d*x)])/(a^2*d) - (f*ArcTanh[Cosh[c + d*x]])/(a*d^2) - (3*(e + f*x)*Csch[c + d*x])/(2*a

```
*d) + (b^5*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*(
a^2 + b^2)^2*d) + (b^5*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(a^2*(a^2 + b^2)^2*d) - (b^5*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a^2
*(a^2 + b^2)^2*d) + (b*f*x*Log[Tanh[c + d*x]])/(a^2*d) - (b*(e + f*x)*Log[T
anh[c + d*x]])/(a^2*d) + (((3*I)/2)*f*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2)
 - (I*b^4*f*PolyLog[2, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)^2*d^2) - ((I/2)*b^
2*f*PolyLog[2, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) - (((3*I)/2)*f*PolyLo
g[2, I*E^(c + d*x)])/(a*d^2) + (I*b^4*f*PolyLog[2, I*E^(c + d*x)])/(a*(a^2
+ b^2)^2*d^2) + ((I/2)*b^2*f*PolyLog[2, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^2)
 + (b^5*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*(a^2 +
 b^2)^2*d^2) + (b^5*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])
/(a^2*(a^2 + b^2)^2*d^2) - (b^5*f*PolyLog[2, -E^(2*(c + d*x))])/(2*a^2*(a^2
 + b^2)^2*d^2) + (b*f*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a^2*d^2) - (b*f*Poly
Log[2, E^(2*c + 2*d*x)])/(2*a^2*d^2) - (f*Sech[c + d*x])/(2*a*d^2) + (b^2*f
*Sech[c + d*x])/(2*a*(a^2 + b^2)*d^2) + (b^3*(e + f*x)*Sech[c + d*x]^2)/(2*
a^2*(a^2 + b^2)*d) + ((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^2)/(2*a*d) + (b
*f*Tanh[c + d*x])/(2*a^2*d^2) - (b^3*f*Tanh[c + d*x])/(2*a^2*(a^2 + b^2)*d^
2) + (b^2*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*a*(a^2 + b^2)*d) + (b*(
e + f*x)*Tanh[c + d*x]^2)/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5311

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5708

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{csch}^2(c+dx)\text{sech}^3(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c}}{a}$$

$$= -\frac{3(e+fx)\tan^{-1}(\sinh(c+dx))}{2ad} - \frac{3(e+fx)\text{csch}(c+dx)}{2ad} +$$

$$= -\frac{3(e+fx)\tan^{-1}(\sinh(c+dx))}{2ad} - \frac{3(e+fx)\text{csch}(c+dx)}{2ad} -$$

$$= \frac{3fx\tan^{-1}(\sinh(c+dx))}{2ad} - \frac{3(e+fx)\tan^{-1}(\sinh(c+dx))}{2ad} -$$

$$= -\frac{b^5(e+fx)^2}{2a^2\left(a^2+b^2\right)^2 f} + \frac{3fx\tan^{-1}(\sinh(c+dx))}{2ad} - \frac{3(e+fx)\tan}{}$$

$$= -\frac{bfx}{2a^2d} - \frac{b^5(e+fx)^2}{2a^2\left(a^2+b^2\right)^2 f} - \frac{3fx\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{3fx\tan^{-1}(\sinh}{2a}$$

$$= -\frac{bfx}{2a^2d} - \frac{3fx\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^4(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)^2 d} + \frac{b^2}{}$$

$$= -\frac{bfx}{2a^2d} - \frac{3fx\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^4(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)^2 d} + \frac{b^2}{}$$

$$= -\frac{bfx}{2a^2d} - \frac{3fx\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^4(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)^2 d} + \frac{b^2}{}$$

$$= -\frac{bfx}{2a^2d} - \frac{3fx\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^4(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)^2 d} + \frac{b^2}{}$$

Mathematica [A]
time = 9.22, size = 1337, normalized size = 1.37

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] 8*(((-(d*e*Cosh[(c + d*x)/2]) + c*f*Cosh[(c + d*x)/2] - f*(c + d*x)*Cosh[(c
 + d*x)/2])*Csch[(c + d*x)/2]*Csch[c + d*x]*(a + b*Sinh[c + d*x]))/(16*a*d^
2*(b + a*Csch[c + d*x])) - (b*e*Csch[c + d*x]*Log[Sinh[c + d*x]]*(a + b*Sin
h[c + d*x]))/(8*a^2*d*(b + a*Csch[c + d*x])) + (b*c*f*Csch[c + d*x]*Log[Sin
h[c + d*x]]*(a + b*Sinh[c + d*x]))/(8*a^2*d^2*(b + a*Csch[c + d*x])) + (f*C
sch[c + d*x]*Log[Tanh[(c + d*x)/2]]*(a + b*Sinh[c + d*x]))/(8*a*d^2*(b + a*
Csch[c + d*x])) + ((I/8)*b*f*Csch[c + d*x]*(I*(c + d*x)*Log[1 - E^(-2*(c +
d*x))] - (I/2)*(-(c + d*x)^2 + PolyLog[2, E^(-2*(c + d*x))]))*(a + b*Sinh[c
 + d*x]))/(a^2*d^2*(b + a*Csch[c + d*x])) + (b^5*Csch[c + d*x]*(-1/2*(f*(c
+ d*x)^2) + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*
(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d*e*Log[a + b*Si
nh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLog[2, (b*E^(c + d*x))/
(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2]))])*(a + b*Sinh[c + d*x]))/(8*a^2*(a^2 + b^2)^2*d^2*(b + a*Csch[c + d*x]
)) + (Csch[c + d*x]*(-2*a^2*b*d*e*(c + d*x) - 4*b^3*d*e*(c + d*x) + 2*a^2*b
*c*f*(c + d*x) + 4*b^3*c*f*(c + d*x) - a^2*b*f*(c + d*x)^2 - 2*b^3*f*(c + d
*x)^2 - 6*a^3*d*e*ArcTan[E^(c + d*x)] - 10*a*b^2*d*e*ArcTan[E^(c + d*x)] +
6*a^3*c*f*ArcTan[E^(c + d*x)] + 10*a*b^2*c*f*ArcTan[E^(c + d*x)] - (3*I)*a^
3*f*(c + d*x)*Log[1 - I*E^(c + d*x)] - (5*I)*a*b^2*f*(c + d*x)*Log[1 - I*E^
(c + d*x)] + (3*I)*a^3*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + (5*I)*a*b^2*f*(
c + d*x)*Log[1 + I*E^(c + d*x)] + 2*a^2*b*d*e*Log[1 + E^(2*(c + d*x))] + 4*
b^3*d*e*Log[1 + E^(2*(c + d*x))] - 2*a^2*b*c*f*Log[1 + E^(2*(c + d*x))] - 4
*b^3*c*f*Log[1 + E^(2*(c + d*x))] + 2*a^2*b*f*(c + d*x)*Log[1 + E^(2*(c + d
*x))] + 4*b^3*f*(c + d*x)*Log[1 + E^(2*(c + d*x))] + I*a*(3*a^2 + 5*b^2)*f*
PolyLog[2, (-I)*E^(c + d*x)] - I*a*(3*a^2 + 5*b^2)*f*PolyLog[2, I*E^(c + d*
x)] + a^2*b*f*PolyLog[2, -E^(2*(c + d*x))] + 2*b^3*f*PolyLog[2, -E^(2*(c +
d*x))])*(a + b*Sinh[c + d*x]))/(16*(a^2 + b^2)^2*d^2*(b + a*Csch[c + d*x]))
 + (Csch[c + d*x]*Sech[(c + d*x)/2]*(d*e*Sinh[(c + d*x)/2] - c*f*Sinh[(c +
d*x)/2] + f*(c + d*x)*Sinh[(c + d*x)/2])*(a + b*Sinh[c + d*x]))/(16*a*d^2*(
b + a*Csch[c + d*x])) + (Csch[c + d*x]*Sech[c + d*x]*(a + b*Sinh[c + d*x])*
(-(a*f) + b*f*Sinh[c + d*x]))/(16*(a^2 + b^2)*d^2*(b + a*Csch[c + d*x])) +
(Csch[c + d*x]*Sech[c + d*x]^2*(a + b*Sinh[c + d*x])*(-(b*d*e) + b*c*f - b*
f*(c + d*x) - a*d*e*Sinh[c + d*x] + a*c*f*Sinh[c + d*x] - a*f*(c + d*x)*Sin
h[c + d*x]))/(16*(a^2 + b^2)*d^2*(b + a*Csch[c + d*x])))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3279 vs. $2(904) = 1808$.
time = 5.92, size = 3280, normalized size = 3.35

method	result	size
risch	Expression too large to display	3280

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNV
ERBOSE)
```

```
[Out] 3/2/d*a*b*e/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2
))+2/d/a*b^3*e/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(
1/2))+1/d^2*a*b*f/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2
)^(1/2))+10*I/(a^2+b^2)/d*a*b^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x+10*I/(
a^2+b^2)/d^2*a*b^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*c-10*I/(a^2+b^2)/d^2*
a*b^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*c-10*I/(a^2+b^2)/d*a*b^2*f/(4*a^2+
4*b^2)*ln(1-I*exp(d*x+c))*x-2/d^2/a*b^3*f*c/(a^2+b^2)^(3/2)*arctanh(1/2*(2*
b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+4/(a^2+b^2)/d*a^2*e/(4*a^2+4*b^2)*b*ln(1
+exp(2*d*x+2*c))-6*I/(a^2+b^2)/d^2*a^3*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c)
)+6*I/(a^2+b^2)/d^2*a^3*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))-1/(a^2+b^2)/d
^2/a*b^2*f*ln(exp(d*x+c)+1)+1/(a^2+b^2)/d^2/a*b^2*f*ln(exp(d*x+c)-1)-1/(a^2
+b^2)/d/a^2*b^3*e*ln(exp(d*x+c)+1)-1/(a^2+b^2)/d/a^2*b^3*e*ln(exp(d*x+c)-1)
-1/(a^2+b^2)/d^2/a^2*f*b^3*dilog(exp(d*x+c)+1)+1/(a^2+b^2)/d^2/a^2*f*b^3*di
log(exp(d*x+c))-1/(a^2+b^2)/d*ln(exp(d*x+c)+1)*b*f*x+1/(a^2+b^2)/d^2*b*f*c*
ln(exp(d*x+c)-1)-1/(a^2+b^2)/d/a^2*f*b^3*ln(exp(d*x+c)+1)*x+1/(a^2+b^2)/d^2
/a^2*b^3*f*c*ln(exp(d*x+c)-1)+1/d^2/a*b^3*f/(a^2+b^2)^(3/2)*arctanh(1/2*(2*
b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+4/(a^2+b^2)/d*a^2*f/(4*a^2+4*b^2)*ln(1+I
*exp(d*x+c))*b*x+4/(a^2+b^2)/d*a^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*b*x+4
/(a^2+b^2)/d^2*a^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*b*c+4/(a^2+b^2)/d^2*a
^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*b*c-4/(a^2+b^2)/d^2*a^2*f*c/(4*a^2+4*
b^2)*b*ln(1+exp(2*d*x+2*c))+7/2/(a^2+b^2)^(5/2)/d^2*a*f*b^3*c*arctanh(1/2*(
2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)^(5/2)/d^2/a*f*b^5*c*arctan
h(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+20/(a^2+b^2)/d^2*a*b^2*f*c/(4*a
^2+4*b^2)*arctan(exp(d*x+c))+3/2/(a^2+b^2)^(5/2)/d^2*a^3*f*c*b*arctanh(1/2*
(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+6*I/(a^2+b^2)/d*a^3*f/(4*a^2+4*b^2)*l
n(1+I*exp(d*x+c))*x-6*I/(a^2+b^2)/d*a^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*
x+6*I/(a^2+b^2)/d^2*a^3*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*c-6*I/(a^2+b^2)/
d^2*a^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*c+10*I/(a^2+b^2)/d^2*a*b^2*f/(4*
a^2+4*b^2)*dilog(1+I*exp(d*x+c))-10*I/(a^2+b^2)/d^2*a*b^2*f/(4*a^2+4*b^2)*d
ilog(1-I*exp(d*x+c))-3/2/d^2*a*b*f*c/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d
*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)/d*b*e*ln(exp(d*x+c)+1)-1/(a^2+b^2)/
d*b*e*ln(exp(d*x+c)-1)-1/(a^2+b^2)/d^2*b*f*dilog(exp(d*x+c)+1)+1/(a^2+b^2)/
d^2*b*f*dilog(exp(d*x+c))-12/(a^2+b^2)/d*a^3*e/(4*a^2+4*b^2)*arctan(exp(d*x
+c))+1/(a^2+b^2)^2/d^2/a^2*f*b^5*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-
a+(a^2+b^2)^(1/2)))+1/(a^2+b^2)^2/d^2/a^2*f*b^5*dilog((b*exp(d*x+c)+(a^2+b^
2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/(a^2+b^2)^2/d/a^2*b^5*e*ln(b*exp(2*d*x+2
*c)+2*a*exp(d*x+c)-b)+8/(a^2+b^2)/d*b^3*e/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c)
)+8/(a^2+b^2)/d^2*f*b^3/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))+8/(a^2+b^2)/d^2
*f*b^3/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))-1/(a^2+b^2)/d^2*a*f*ln(exp(d*x+c
)+1)+1/(a^2+b^2)/d^2*a*f*ln(exp(d*x+c)-1)+1/(a^2+b^2)^2/d/a^2*f*b^5*ln((b*e
xp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-7/2/(a^2+b^2)^(5/2)/d*a
*b^3*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-20/(a^2+b^2)/d*a*b
^2*e/(4*a^2+4*b^2)*arctan(exp(d*x+c))-3/2/(a^2+b^2)^(5/2)/d*a^3*e*b*arctanh
(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)^2/d^2/a^2*f*b^5*c*ln
(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/(a^2+b^2)^2/d^2/a^2*f*b^5*ln((-b*exp(
```

```
d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/(a^2+b^2)^(5/2)/d/a*b^5
*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+12/(a^2+b^2)/d^2*a^3*f
*c/(4*a^2+4*b^2)*arctan(exp(d*x+c))+4/(a^2+b^2)/d^2*a^2*f/(4*a^2+4*b^2)*dil
og(1+I*exp(d*x+c))*b+4/(a^2+b^2)/d^2*a^2*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+
c))*b-1/(a^2+b^2)^(5/2)/d^2*a^3*f*b*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b
^2)^(1/2))-2/(a^2+b^2)^(5/2)/d^2*a*f*b^3*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(
a^2+b^2)^(1/2))-1/(a^2+b^2)^(5/2)/d^2/a*f*b^5*arctanh(1/2*(2*b*exp(d*x+c)+2
*a)/(a^2+b^2)^(1/2))+8/(a^2+b^2)/d^2*f*b^3/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))
*c+8/(a^2+b^2)/d^2*f*b^3/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*c+8/(a^2+b^2)/d*f
*b^3/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x+8/(a^2+b^2)/d*f*b^3/(4*a^2+4*b^2)*l
n(1-I*exp(d*x+c))*x-8/(a^2+b^2)/d^2*f*b^3*c/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*
c))+1/(a^2+b^2)^2/d^2/a^2*f*b^5*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2
+b^2)^(1/2)))*c+1/(a^2+b^2)^2/d/a^2*f*b^5*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))*x-(3*a^2*d*f*x*exp(5*d*x+5*c)+2*b^2*d*f*x*exp(5*d
*x+5*c)+3*a^2*d*e*exp(5*d*x+5*c)+2*a*b*d*f*x*exp(4*d*x+4*c)+2*b^2*d*e*exp(5
*d*x+5*c)+2*a^2*d*f*x*exp(3*d*x+3*c)+a^2*f*exp(5*d*x+5*c)+2*a*b*d*e*exp(4*d
*x+4*c)+4*b^2*d*f*x*exp(3*d*x+3*c)+2*a^2*d*e*exp(3*d*x+3*c)-2*a*b*d*f*x*exp
(2*d*x+2*c)+a*b*f*exp(4*d*x+4*c)+4*b^2*d*e*exp(3*d*x+3*c)+3*a^2*d*f*x*exp(d
*x+c)-2*a*b*d*e*exp(2*d*x+2*c)+2*b^2*d*f*x*exp(d*x+c)+3*a^2*d*e*exp(d*x+c)+
2*b^2*d*e*exp(d*x+c)-a^2*f*exp(d*x+c)-a*b*f)/d^2/(a^2+b^2)/(1+exp(2*d*x+2*c
))^2/a/(exp(2*d*x+2*c)-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] (32*b*d*integrate(1/32*x/(a^2*d*e^(d*x + c) + a^2*d), x) - 32*b*d*integrate
(1/32*x/(a^2*d*e^(d*x + c) - a^2*d), x) + a*((d*x + c)/(a^2*d^2) - log(e^(d
*x + c) + 1)/(a^2*d^2)) - a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) - 1)/(a^
2*d^2)) - (2*a*b*d*x*e^(2*d*x + 2*c) - 2*(a^2*d*e^(3*c) + 2*b^2*d*e^(3*c))*
x*e^(3*d*x) + a*b - (a^2*e^(5*c) + (3*a^2*d*e^(5*c) + 2*b^2*d*e^(5*c))*x)*e
^(5*d*x) - (2*a*b*d*x*e^(4*c) + a*b*e^(4*c))*e^(4*d*x) + (a^2*e^c - (3*a^2*
d*e^c + 2*b^2*d*e^c)*x)*e^(d*x))/(a^3*d^2 + a*b^2*d^2 - (a^3*d^2*e^(6*c) +
a*b^2*d^2*e^(6*c))*e^(6*d*x) - (a^3*d^2*e^(4*c) + a*b^2*d^2*e^(4*c))*e^(4*d
*x) + (a^3*d^2*e^(2*c) + a*b^2*d^2*e^(2*c))*e^(2*d*x)) - 32*integrate(-1/16
*(a*b^5*x*e^(d*x + c) - b^6*x)/(a^6*b + 2*a^4*b^3 + a^2*b^5 - (a^6*b*e^(2*c
) + 2*a^4*b^3*e^(2*c) + a^2*b^5*e^(2*c))*e^(2*d*x) - 2*(a^7*e^c + 2*a^5*b^2
*e^c + a^3*b^4*e^c)*e^(d*x)), x) - 32*integrate(1/32*((3*a^3*e^c + 5*a*b^2*
e^c)*x*e^(d*x) + 2*(a^2*b + 2*b^3)*x)/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^(2*c)
 + 2*a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x))*f + (b^5*log(-2*a*e^(-d
```

```
*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^6 + 2*a^4*b^2 + a^2*b^4)*d) + (3*a^3
+ 5*a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (a^2*b + 2*b^
3)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (2*a*b*e^(-2*d*x
 - 2*c) - 2*a*b*e^(-4*d*x - 4*c) + (3*a^2 + 2*b^2)*e^(-d*x - c) + 2*(a^2 +
2*b^2)*e^(-3*d*x - 3*c) + (3*a^2 + 2*b^2)*e^(-5*d*x - 5*c))/((a^3 + a*b^2 +
 (a^3 + a*b^2)*e^(-2*d*x - 2*c) - (a^3 + a*b^2)*e^(-4*d*x - 4*c) - (a^3 + a
*b^2)*e^(-6*d*x - 6*c))*d) - b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*
x - c) - 1)/(a^2*d))*e
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 17940 vs. $2(893) = 1786$.
time = 0.69, size = 17940, normalized size = 18.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -1/2*(2*((3*a^5 + 5*a^3*b^2 + 2*a*b^4)*d*f*x + (3*a^5 + 5*a^3*b^2 + 2*a*b^4
)*d*cosh(1) + (3*a^5 + 5*a^3*b^2 + 2*a*b^4)*d*sinh(1) + (a^5 + a^3*b^2)*f)*
cosh(d*x + c)^5 + 2*((3*a^5 + 5*a^3*b^2 + 2*a*b^4)*d*f*x + (3*a^5 + 5*a^3*b
^2 + 2*a*b^4)*d*cosh(1) + (3*a^5 + 5*a^3*b^2 + 2*a*b^4)*d*sinh(1) + (a^5 +
a^3*b^2)*f)*sinh(d*x + c)^5 + 2*(2*(a^4*b + a^2*b^3)*d*f*x + 2*(a^4*b + a^2
*b^3)*d*cosh(1) + 2*(a^4*b + a^2*b^3)*d*sinh(1) + (a^4*b + a^2*b^3)*f)*cosh
(d*x + c)^4 + 2*(2*(a^4*b + a^2*b^3)*d*f*x + 2*(a^4*b + a^2*b^3)*d*cosh(1)
+ 2*(a^4*b + a^2*b^3)*d*sinh(1) + (a^4*b + a^2*b^3)*f + 5*((3*a^5 + 5*a^3*b
^2 + 2*a*b^4)*d*f*x + (3*a^5 + 5*a^3*b^2 + 2*a*b^4)*d*cosh(1) + (3*a^5 + 5*
a^3*b^2 + 2*a*b^4)*d*sinh(1) + (a^5 + a^3*b^2)*f)*cosh(d*x + c))*sinh(d*x +
 c)^4 + 4*((a^5 + 3*a^3*b^2 + 2*a*b^4)*d*f*x + (a^5 + 3*a^3*b^2 + 2*a*b^4)*
d*cosh(1) + (a^5 + 3*a^3*b^2 + 2*a*b^4)*d*sinh(1))*cosh(d*x + c)^3 + 4*((a^
5 + 3*a^3*b^2 + 2*a*b^4)*d*f*x + (a^5 + 3*a^3*b^2 + 2*a*b^4)*d*cosh(1) + 5*
((3*a^5 + 5*a^3*b^2 + 2*a*b^4)*d*f*x + (3*a^5 + 5*a^3*b^2 + 2*a*b^4)*d*cosh
(1) + (3*a^5 + 5*a^3*b^2 + 2*a*b^4)*d*sinh(1) + (a^5 + a^3*b^2)*f)*cosh(d*x
 + c)^2 + (a^5 + 3*a^3*b^2 + 2*a*b^4)*d*sinh(1) + 2*(2*(a^4*b + a^2*b^3)*d*
f*x + 2*(a^4*b + a^2*b^3)*d*cosh(1) + 2*(a^4*b + a^2*b^3)*d*sinh(1) + (a^4*
b + a^2*b^3)*f)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*((a^4*b + a^2*b^3)*d*f*x
 + (a^4*b + a^2*b^3)*d*cosh(1) + (a^4*b + a^2*b^3)*d*sinh(1))*cosh(d*x + c)
^2 - 4*((a^4*b + a^2*b^3)*d*f*x - 5*((3*a^5 + 5*a^3*b^2 + 2*a*b^4)*d*f*x +
(3*a^5 + 5*a^3*b^2 + 2*a*b^4)*d*cosh(1) + (3*a^5 + 5*a^3*b^2 + 2*a*b^4)*d*s
inh(1) + (a^5 + a^3*b^2)*f)*cosh(d*x + c)^3 + (a^4*b + a^2*b^3)*d*cosh(1) -
 3*(2*(a^4*b + a^2*b^3)*d*f*x + 2*(a^4*b + a^2*b^3)*d*cosh(1) + 2*(a^4*b +
a^2*b^3)*d*sinh(1) + (a^4*b + a^2*b^3)*f)*cosh(d*x + c)^2 + (a^4*b + a^2*b^
3)*d*sinh(1) - 3*((a^5 + 3*a^3*b^2 + 2*a*b^4)*d*f*x + (a^5 + 3*a^3*b^2 + 2*
a*b^4)*d*cosh(1) + (a^5 + 3*a^3*b^2 + 2*a*b^4)*d*sinh(1))*cosh(d*x + c))*si
```

```
nh(d*x + c)^2 - 2*(a^4*b + a^2*b^3)*f + 2*((3*a^5 + 5*a^3*b^2 + 2*a*b^4)*d*
f*x + (3*a^5 + 5*a^3*b^2 + 2*a*b^4)*d*cosh(1) + (3*a^5 + 5*a^3*b^2 + 2*a*b^
4)*d*sinh(1) - (a^5 + a^3*b^2)*f)*cosh(d*x + c) - 2*(b^5*f*cosh(d*x + c)^6
+ 6*b^5*f*cosh(d*x + c)*sinh(d*x + c)^5 + b^5*f*sinh(d*x + c)^6 + b^5*f*cos
h(d*x + c)^4 - b^5*f*cosh(d*x + c)^2 - b^5*f + (15*b^5*f*cosh(d*x + c)^2 +
b^5*f)*sinh(d*x + c)^4 + 4*(5*b^5*f*cosh(d*x + c)^3 + b^5*f*cosh(d*x + c))*
sinh(d*x + c)^3 + (15*b^5*f*cosh(d*x + c)^4 + 6*b^5*f*cosh(d*x + c)^2 - b^5
*f)*sinh(d*x + c)^2 + 2*(3*b^5*f*cosh(d*x + c)^5 + 2*b^5*f*cosh(d*x + c)^3
- b^5*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x +
 c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1)
 - 2*(b^5*f*cosh(d*x + c)^6 + 6*b^5*f*cosh(d*x + c)*sinh(d*x + c)^5 + b^5*f
*sinh(d*x + c)^6 + b^5*f*cosh(d*x + c)^4 - b^5*f*cosh(d*x + c)^2 - b^5*f +
(15*b^5*f*cosh(d*x + c)^2 + b^5*f)*sinh(d*x + c)^4 + 4*(5*b^5*f*cosh(d*x +
c)^3 + b^5*f*cosh(d*x + c))*sinh(d*x + c)^3 + (15*b^5*f*cosh(d*x + c)^4 + 6
*b^5*f*cosh(d*x + c)^2 - b^5*f)*sinh(d*x + c)^2 + 2*(3*b^5*f*cosh(d*x + c)^
5 + 2*b^5*f*cosh(d*x + c)^3 - b^5*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*
cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2) - b)/b + 1) + 2*((a^4*b + 2*a^2*b^3 + b^5)*f*cosh(d*x + c)
^6 + 6*(a^4*b + 2*a^2*b^3 + b^5)*f*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4*b +
 2*a^2*b^3 + b^5)*f*sinh(d*x + c)^6 + (a^4*b + 2*a^2*b^3 + b^5)*f*cosh(d*x
+ c)^4 + (15*(a^4*b + 2*a^2*b^3 + b^5)*f*cosh(d*x + c)^2 + (a^4*b + 2*a^2*b
^3 + b^5)*f)*sinh(d*x + c)^4 - (a^4*b + 2*a^2*b^3 + b^5)*f*cosh(d*x + c)^2
+ 4*(5*(a^4*b + 2*a^2*b^3 + b^5)*f*cosh(d*x + c)^3 + (a^4*b + 2*a^2*b^3 + b
^5)*f*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^4*b + 2*a^2*b^3 + b^5)*f*cosh
(d*x + c)^4 + 6*(a^4*b + 2*a^2*b^3 + b^5)*f*cosh(d*x + c)^2 - (a^4*b + 2*a^
2*b^3 + b^5)*f)*sinh(d*x + c)^2 - (a^4*b + 2*a^2*b^3 + b^5)*f + 2*(3*(a^4*b
 + 2*a^2*b^3 + b^5)*f*cosh(d*x + c)^5 + 2*(a^4*b + 2*a^2*b^3 + b^5)*f*cosh(
d*x + c)^3 - (a^4*b + 2*a^2*b^3 + b^5)*f*cosh(d*x + c))*sinh(d*x + c))*dilo
g(cosh(d*x + c) + sinh(d*x + c)) - ((-I*(3*a^5 + 5*a^3*b^2)*f + 2*(a^4*b +
2*a^2*b^3)*f)*cosh(d*x + c)^6 - 6*(I*(3*a^5 + 5*a^3*b^2)*f - 2*(a^4*b + 2*a
^2*b^3)*f)*cosh(d*x + c)*sinh(d*x + c)^5 + (-I*(3*a^5 + 5*a^3*b^2)*f + 2*(a
^4*b + 2*a^2*b^3)*f)*sinh(d*x + c)^6 + (-I*(3*a^5 + 5*a^3*b^2)*f + 2*(a^4*b
 + 2*a^2*b^3)*f)*cosh(d*x + c)^4 - (15*(I*(3*a^5 + 5*a^3*b^2)*f - 2*(a^4*b
+ 2*a^2*b^3)*f)*cosh(d*x + c)^2 + I*(3*a^5 + 5*a^3*b^2)*f - 2*(a^4*b + 2*a^
2*b^3)*f)*sinh(d*x + c)^4 - 4*(5*(I*(3*a^5 + 5*a^3*b^2)*f - 2*(a^4*b + 2*a^
2*b^3)*f)*cosh(d*x + c)^3 + (I*(3*a^5 + 5*a^3*b^2)*f - 2*(a^4*b + 2*a^2*b^3
)*f)*cosh(d*x + c))*sinh(d*x + c)^3 + (I*(3*a^5 + 5*a^3*b^2)*f - 2*(a^4*b +
 2*a^2*b^3)*f)*cosh(d*x + c)^2 - (15*(I*(3*a^5 + 5*a^3*b^2)*f - 2*(a^4*b +
2*a^2*b^3)*f)*cosh(d*x + c)^4 + 6*(I*(3*a^5 + 5*a^3*b^2)*f - 2*(a^4*b + 2*a
^2*b^3)*f)*cosh(d*x + c)^2 - I*(3*a^5 + 5*a^3*b^2)*f + 2*(a^4*b + 2*a^2*b^3
)*f)*sinh(d*x + c)^2 + I*(3*a^5 + 5*a^3*b^2)*f ...
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**2*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\cosh\left(c+d\,x\right)^3\,\sinh\left(c+d\,x\right)^2\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(cosh(c + d*x)^3*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((e + f*x)/(cosh(c + d*x)^3*sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.474 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=180

$$-\frac{a\operatorname{ArcTan}(\sinh(c+dx))}{2(a^2+b^2)d}-\frac{a(a^2+2b^2)\operatorname{ArcTan}(\sinh(c+dx))}{(a^2+b^2)^2d}-\frac{\operatorname{csch}(c+dx)}{ad}+\frac{b(a^2+2b^2)\log(\cosh(c+dx))}{(a^2+b^2)^2d}$$

```
[Out] -1/2*a*arctan(sinh(d*x+c))/(a^2+b^2)/d-a*(a^2+2*b^2)*arctan(sinh(d*x+c))/(a
^2+b^2)^2/d-csch(d*x+c)/a/d+b*(a^2+2*b^2)*ln(cosh(d*x+c))/(a^2+b^2)^2/d-b*l
n(sinh(d*x+c))/a^2/d+b^5*ln(a+b*sinh(d*x+c))/a^2/(a^2+b^2)^2/d-1/2*sech(d*x
+c)^2*(b+a*sinh(d*x+c))/(a^2+b^2)/d
```

Rubi [A]
time = 0.18, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2916, 12, 908, 653, 209, 649, 266}

$$-\frac{a(a^2+2b^2)\operatorname{ArcTan}(\sinh(c+dx))}{d(a^2+b^2)^2}-\frac{a\operatorname{ArcTan}(\sinh(c+dx))}{2d(a^2+b^2)}+\frac{b(a^2+2b^2)\log(\cosh(c+dx))}{d(a^2+b^2)^2}-\frac{\operatorname{sech}^2(c+dx)(a\sinh(c+dx)+b)}{2d(a^2+b^2)}+\frac{b^5\log(a+b\sinh(c+dx))}{a^2d(a^2+b^2)^2}-\frac{b\log(\sinh(c+dx))}{a^2d}-\frac{\operatorname{csch}(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[c + d*x]^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/2*(a*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d) - (a*(a^2 + 2*b^2)*ArcTan[Si
nh[c + d*x]])/((a^2 + b^2)^2*d) - Csch[c + d*x]/(a*d) + (b*(a^2 + 2*b^2)*Lo
g[Cosh[c + d*x]])/((a^2 + b^2)^2*d) - (b*Log[Sinh[c + d*x]])/(a^2*d) + (b^5
*Log[a + b*Sinh[c + d*x]])/(a^2*(a^2 + b^2)^2*d) - (Sech[c + d*x]^2*(b + a*
Sinh[c + d*x]))/(2*(a^2 + b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b^3\text{Subst}\left(\int \frac{b^2}{x^2(a+x)(-b^2-x^2)^2}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= \frac{b^5\text{Subst}\left(\int \frac{1}{x^2(a+x)(-b^2-x^2)^2}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= \frac{b^5\text{Subst}\left(\int \left(\frac{1}{ab^4x^2} - \frac{1}{a^2b^4x} + \frac{1}{a^2(a^2+b^2)^2(a+x)} + \frac{-a+x}{b^2(a^2+b^2)(b^2+x^2)^2} - \frac{(a^2+2b^2)(a}{b^4(a^2+b^2)^2(b}\right.\right.}{d}$$

$$= -\frac{\operatorname{csch}(c+dx)}{ad} - \frac{b\log(\sinh(c+dx))}{a^2d} + \frac{b^5\log(a+b\sinh(c+dx))}{a^2(a^2+b^2)^2d} + \frac{b^3\text{S}}{}$$

$$= -\frac{\operatorname{csch}(c+dx)}{ad} - \frac{b\log(\sinh(c+dx))}{a^2d} + \frac{b^5\log(a+b\sinh(c+dx))}{a^2(a^2+b^2)^2d} - \frac{\text{sec}}{}$$

$$= -\frac{a\tan^{-1}(\sinh(c+dx))}{2(a^2+b^2)d} - \frac{a(a^2+2b^2)\tan^{-1}(\sinh(c+dx))}{(a^2+b^2)^2d} - \frac{\operatorname{csch}(c+d}{ad}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.82, size = 227, normalized size = 1.26

$$-\frac{\operatorname{csch}(c+dx)(a+b\sinh(c+dx))\left(\frac{a\text{ArcTan}(\sinh(c+dx))}{a^2+b^2} + \frac{2\operatorname{csch}(c+dx)}{a} - \frac{(ia+b)(a^2+2b^2)\log(i-\sinh(c+dx))}{(a^2+b^2)^2} + \frac{2b\log(\sinh(c+dx))}{a^2} + \frac{(ia-b)(a^2+2b^2)\log(i+\sinh(c+dx))}{(a^2+b^2)^2} - \frac{2b^5\log(a+b\sinh(c+dx))}{a^2(a^2+b^2)^2} + \frac{b\operatorname{sech}^2(c+dx)}{a^2+b^2} + \frac{a\operatorname{sech}(c+dx)\tanh(c+dx)}{a^2+b^2}\right)}{2d(b+a\operatorname{csch}(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[c + d*x]^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/2*(Csch[c + d*x]*(a + b*Sinh[c + d*x])*((a*ArcTan[Sinh[c + d*x]])/(a^2 +
 b^2) + (2*Csch[c + d*x])/a - ((I*a + b)*(a^2 + 2*b^2)*Log[I - Sinh[c + d*x
]])/(a^2 + b^2)^2 + (2*b*Log[Sinh[c + d*x]])/a^2 + ((I*a - b)*(a^2 + 2*b^2)
*Log[I + Sinh[c + d*x]])/(a^2 + b^2)^2 - (2*b^5*Log[a + b*Sinh[c + d*x]])/(
a^2*(a^2 + b^2)^2) + (b*Sech[c + d*x]^2)/(a^2 + b^2) + (a*Sech[c + d*x]*Tan
h[c + d*x])/(a^2 + b^2)))/(d*(b + a*Csch[c + d*x]))
```

Maple [A]
time = 1.68, size = 249, normalized size = 1.38

method	result
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a} - \frac{2\left(\frac{\left(-\frac{1}{2}a^3-\frac{1}{2}a\,b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^2b-b^3\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^3+\frac{1}{2}a\,b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2} + \frac{\left(-2a^2b-4b^3\right)\ln}{}\right.}{\left(a^2+b^2\right)^2}}{}$

default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{2\left(\frac{\left(-\frac{1}{2}a^3-\frac{1}{2}a\,b^2\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^2b-b^3\right)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^3+\frac{1}{2}a\,b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{\left(-2a^2b-4b^3\right.}{}\right.}{\left(a^2+b^2\right)^2}}{}$
risch	$-\frac{2a^2b\,d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{2a^2bdc}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{4b^3d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{4b^3dc}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{2b^5x}{a^2(a^4+2a^2b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/a*tanh(1/2*d*x+1/2*c)-2/(a^2+b^2)^2*(((-1/2*a^3-1/2*a*b^2)*tanh(1/
2*d*x+1/2*c)^3+(-a^2*b-b^3)*tanh(1/2*d*x+1/2*c)^2+(1/2*a^3+1/2*a*b^2)*tanh(
1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2+1)^2+1/4*(-2*a^2*b-4*b^3)*ln(tanh(1/
2*d*x+1/2*c)^2+1)+1/2*(3*a^3+5*a*b^2)*arctan(tanh(1/2*d*x+1/2*c)))-1/2/a/ta
nh(1/2*d*x+1/2*c)-1/a^2*b*ln(tanh(1/2*d*x+1/2*c))+b^5/(a^2+b^2)^2/a^2*ln(a*
tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)-a))
```

Maxima [A]
time = 0.51, size = 350, normalized size = 1.94

$$\frac{b^5\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^6+2\,a^4b^2+a^2b^4)d}+\frac{(3\,a^3+5\,ab^2)\arctan\left(e^{(-dx-c)}\right)}{(a^4+2\,a^2b^2+b^4)d}+\frac{(a^2b+2\,b^3)\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^4+2\,a^2b^2+b^4)d}-\frac{2\,abe^{(-2\,dx-2\,c)}-2\,abe^{(-4\,dx-4\,c)}+(3\,a^2+2\,b^2)e^{(-dx-c)}+2\,(a^2+2\,b^2)e^{(-3\,dx-3\,c)}+(3\,a^2+2\,b^2)e^{(-5\,dx-5\,c)}}{(a^3+ab^2+(a^3+ab^2)e^{(-2\,dx-2\,c)}-(a^3+ab^2)e^{(-4\,dx-4\,c)}-(a^3+ab^2)e^{(-6\,dx-6\,c)})d}-\frac{b\log\left(e^{(-dx-c)}+1\right)}{a^2d}-\frac{b\log\left(e^{(-dx-c)}-1\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxim
a")
```

```
[Out] b^5*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^6 + 2*a^4*b^2 + a^2
*b^4)*d) + (3*a^3 + 5*a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*
d) + (a^2*b + 2*b^3)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d)
- (2*a*b*e^(-2*d*x - 2*c) - 2*a*b*e^(-4*d*x - 4*c) + (3*a^2 + 2*b^2)*e^(-d*
x - c) + 2*(a^2 + 2*b^2)*e^(-3*d*x - 3*c) + (3*a^2 + 2*b^2)*e^(-5*d*x - 5*c
))/((a^3 + a*b^2 + (a^3 + a*b^2)*e^(-2*d*x - 2*c) - (a^3 + a*b^2)*e^(-4*d*x
 - 4*c) - (a^3 + a*b^2)*e^(-6*d*x - 6*c))*d) - b*log(e^(-d*x - c) + 1)/(a^2
*d) - b*log(e^(-d*x - c) - 1)/(a^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2568 vs. $2(176) = 352$.
time = 0.67, size = 2568, normalized size = 14.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="frica
s")
```

[Out] -((3*a^5 + 5*a^3*b^2 + 2*a*b^4)*cosh(d*x + c)^5 + (3*a^5 + 5*a^3*b^2 + 2*a*b^4)*sinh(d*x + c)^5 + 2*(a^4*b + a^2*b^3)*cosh(d*x + c)^4 + (2*a^4*b + 2*a^2*b^3 + 5*(3*a^5 + 5*a^3*b^2 + 2*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + 2*(a^5 + 3*a^3*b^2 + 2*a*b^4)*cosh(d*x + c)^3 + 2*(a^5 + 3*a^3*b^2 + 2*a*b^4 + 5*(3*a^5 + 5*a^3*b^2 + 2*a*b^4)*cosh(d*x + c)^2 + 4*(a^4*b + a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(a^4*b + a^2*b^3)*cosh(d*x + c)^2 - 2*(a^4*b + a^2*b^3 - 5*(3*a^5 + 5*a^3*b^2 + 2*a*b^4)*cosh(d*x + c)^3 - 6*(a^4*b + a^2*b^3)*cosh(d*x + c)^2 - 3*(a^5 + 3*a^3*b^2 + 2*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + ((3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^6 + 6*(3*a^5 + 5*a^3*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^5 + 5*a^3*b^2)*sinh(d*x + c)^6 - 3*a^5 - 5*a^3*b^2 + (3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^4 + (3*a^5 + 5*a^3*b^2 + 15*(3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^3 + (3*a^5 + 5*a^3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^2 - (3*a^5 + 5*a^3*b^2 - 15*(3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^4 - 6*(3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^5 + 2*(3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^3 - (3*a^5 + 5*a^3*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + (3*a^5 + 5*a^3*b^2 + 2*a*b^4)*cosh(d*x + c) - (b^5*cosh(d*x + c)^6 + 6*b^5*cosh(d*x + c)*sinh(d*x + c)^5 + b^5*sinh(d*x + c)^6 + b^5*cosh(d*x + c)^4 - b^5*cosh(d*x + c)^2 - b^5 + (15*b^5*cosh(d*x + c)^2 + b^5)*sinh(d*x + c)^4 + 4*(5*b^5*cosh(d*x + c)^3 + b^5*cosh(d*x + c))*sinh(d*x + c)^3 + (15*b^5*cosh(d*x + c)^4 + 6*b^5*cosh(d*x + c)^2 - b^5)*sinh(d*x + c)^2 + 2*(3*b^5*cosh(d*x + c)^5 + 2*b^5*cosh(d*x + c)^3 - b^5*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) - ((a^4*b + 2*a^2*b^3)*cosh(d*x + c)^6 + 6*(a^4*b + 2*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4*b + 2*a^2*b^3)*sinh(d*x + c)^6 - a^4*b - 2*a^2*b^3 + (a^4*b + 2*a^2*b^3)*cosh(d*x + c)^4 + (a^4*b + 2*a^2*b^3 + 15*(a^4*b + 2*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(a^4*b + 2*a^2*b^3)*cosh(d*x + c)^3 + (a^4*b + 2*a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - (a^4*b + 2*a^2*b^3)*cosh(d*x + c)^2 - (a^4*b + 2*a^2*b^3 - 15*(a^4*b + 2*a^2*b^3)*cosh(d*x + c)^4 - 6*(a^4*b + 2*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(a^4*b + 2*a^2*b^3)*cosh(d*x + c)^5 + 2*(a^4*b + 2*a^2*b^3)*cosh(d*x + c)^3 - (a^4*b + 2*a^2*b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + ((a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^6 + 6*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4*b + 2*a^2*b^3 + b^5)*sinh(d*x + c)^6 - a^4*b - 2*a^2*b^3 - b^5 + (a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^4 + (a^4*b + 2*a^2*b^3 + b^5 + 15*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^3 + (a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - (a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^2 - (a^4*b + 2*a^2*b^3 + b^5 - 15*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^4 - 6*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^5 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^3 - (a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + (3*a^5 + 5*a^3*b^2 + 2*a*b^4 + 5*(3*a^

```
5 + 5*a^3*b^2 + 2*a*b^4)*cosh(d*x + c)^4 + 8*(a^4*b + a^2*b^3)*cosh(d*x + c
)^3 + 6*(a^5 + 3*a^3*b^2 + 2*a*b^4)*cosh(d*x + c)^2 - 4*(a^4*b + a^2*b^3)*c
osh(d*x + c))*sinh(d*x + c))/((a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^6
 + 6*(a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^6 + 2
*a^4*b^2 + a^2*b^4)*d*sinh(d*x + c)^6 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(
d*x + c)^4 + (15*(a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^2 + (a^6 + 2*a
^4*b^2 + a^2*b^4)*d)*sinh(d*x + c)^4 - (a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d
*x + c)^2 + 4*(5*(a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^3 + (a^6 + 2*a
^4*b^2 + a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^6 + 2*a^4*b^2 +
 a^2*b^4)*d*cosh(d*x + c)^4 + 6*(a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)
^2 - (a^6 + 2*a^4*b^2 + a^2*b^4)*d)*sinh(d*x + c)^2 - (a^6 + 2*a^4*b^2 + a^
2*b^4)*d + 2*(3*(a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^5 + 2*(a^6 + 2*
a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^3 - (a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d
*x + c))*sinh(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6439 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. 2(176) = 352.
time = 0.49, size = 458, normalized size = 2.54

$$\frac{\frac{12\,b^6\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^6b+2\,a^4b^3+a^2b^5}-\frac{3\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(3\,a^3+5\,ab^2\right)}{a^4+2\,a^2b^2+b^4}+\frac{6\left(a^2b+2\,b^3\right)\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^4+2\,a^2b^2+b^4}-\frac{12\,b\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a^2}+\frac{4\left(b^5\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3-9\,a^5\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-15\,a^3b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-6\,ab^4\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-6\,a^4b\left(e^{(dx+c)}-e^{(-dx-c)}\right)-6\,a^2b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)+4\,b^5\left(e^{(dx+c)}-e^{(-dx-c)}\right)-24\,a^5-48\,a^3b^2-24\,ab^4\right)}{\left(a^6+2\,a^4b^2+a^2b^4\right)\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+4\,e^{(dx+c)}-4\,e^{(-dx-c)}\right)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac"
)
```

```
[Out] 1/12*(12*b^6*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^6*b + 2*a^4*
b^3 + a^2*b^5) - 3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*
(3*a^3 + 5*a*b^2)/(a^4 + 2*a^2*b^2 + b^4) + 6*(a^2*b + 2*b^3)*log((e^(d*x +
 c) - e^(-d*x - c))^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) - 12*b*log(abs(e^(d*x +
c) - e^(-d*x - c)))/a^2 + 4*(b^5*(e^(d*x + c) - e^(-d*x - c))^3 - 9*a^5*(e^
(d*x + c) - e^(-d*x - c))^2 - 15*a^3*b^2*(e^(d*x + c) - e^(-d*x - c))^2 - 6
*a*b^4*(e^(d*x + c) - e^(-d*x - c))^2 - 6*a^4*b*(e^(d*x + c) - e^(-d*x - c)
) - 6*a^2*b^3*(e^(d*x + c) - e^(-d*x - c)) + 4*b^5*(e^(d*x + c) - e^(-d*x -
 c)) - 24*a^5 - 48*a^3*b^2 - 24*a*b^4)/((a^6 + 2*a^4*b^2 + a^2*b^4)*((e^(d*
x + c) - e^(-d*x - c))^3 + 4*e^(d*x + c) - 4*e^(-d*x - c))))/d
```

Mupad [B]

time = 7.50, size = 398, normalized size = 2.21

$$\frac{2\,b}{d\,{\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}^2\,\left(a^2+b^2\right)}-\frac{b\,\ln\left({\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}-1\right)}{a^2\,d}+\frac{2\,b\,\ln\left(1+{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,1{}\mathrm{i}\right)}{d\,{\left(-b+a\,1{}\mathrm{i}\right)}^2}-\frac{2\,b^3}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)\,{\left(a^2+b^2\right)}^2}-\frac{2\,{\mathrm{e}}^{c+d\,x}}{a\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}+\frac{2\,b\,\ln\left({\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c+1{}\mathrm{i}\right)}{d\,{\left(b+a\,1{}\mathrm{i}\right)}^2}-\frac{2\,a^2\,b}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)\,{\left(a^2+b^2\right)}^2}-\frac{a^3\,{\mathrm{e}}^{c+d\,x}}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)\,{\left(a^2+b^2\right)}^2}+\frac{b^5\,\ln\left(2\,a\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c-b+b\,{\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}\right)}{a^2\,d\,{\left(a^2+b^2\right)}^2}+\frac{2\,a\,{\mathrm{e}}^{c+d\,x}}{d\,{\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}^2\,\left(a^2+b^2\right)}-\frac{a\,b^2\,{\mathrm{e}}^{c+d\,x}}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)\,{\left(a^2+b^2\right)}^2}-\frac{a\,\ln\left(1+{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,1{}\mathrm{i}\right)\,3{}\mathrm{i}}{2\,d\,{\left(-b+a\,1{}\mathrm{i}\right)}^2}+\frac{a\,\ln\left({\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c+1{}\mathrm{i}\right)\,3{}\mathrm{i}}{2\,d\,{\left(b+a\,1{}\mathrm{i}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

```
[Out] (2*b)/(d*(exp(2*c + 2*d*x) + 1)^2*(a^2 + b^2)) - (b*log(exp(2*c)*exp(2*d*x)
 - 1))/(a^2*d) - (a*log(exp(d*x)*exp(c)*1i + 1)*3i)/(2*d*(a*1i - b)^2) + (2
*b*log(exp(d*x)*exp(c)*1i + 1))/(d*(a*1i - b)^2) - (2*b^3)/(d*(exp(2*c + 2*
d*x) + 1)*(a^2 + b^2)^2) - (2*exp(c + d*x))/(a*d*(exp(2*c + 2*d*x) - 1)) +
(a*log(exp(d*x)*exp(c) + 1i)*3i)/(2*d*(a*1i + b)^2) + (2*b*log(exp(d*x)*exp
(c) + 1i))/(d*(a*1i + b)^2) - (2*a^2*b)/(d*(exp(2*c + 2*d*x) + 1)*(a^2 + b^
2)^2) - (a^3*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)*(a^2 + b^2)^2) + (b^5*
log(2*a*exp(d*x)*exp(c) - b + b*exp(2*c)*exp(2*d*x)))/(a^2*d*(a^2 + b^2)^2)
 + (2*a*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)^2*(a^2 + b^2)) - (a*b^2*exp
(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)*(a^2 + b^2)^2)
```

3.475 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=39

$$\operatorname{Int}\left(\frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csch[c + d*x]^2*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][(Csch[c + d*x]^2*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(Csch[c + d*x]^2*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x]
)),x]
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(dx+c)^2\operatorname{sech}(dx+c)^3}{(fx+e)\,(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)

[Out] int(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")

[Out] (a*b*f - (a^2*f*e^(5*c) - (3*a^2*d*f*e^(5*c) + 2*b^2*d*f*e^(5*c))*x - (3*a^
2*d*e^(5*c) + 2*b^2*d*e^(5*c))*e)*e^(5*d*x) + (2*a*b*d*f*x*e^(4*c) - a*b*f*
e^(4*c) + 2*a*b*d*e^(4*c + 1))*e^(4*d*x) + 2*((a^2*d*f*e^(3*c) + 2*b^2*d*f*
e^(3*c))*x + (a^2*d*e^(3*c) + 2*b^2*d*e^(3*c))*e)*e^(3*d*x) - 2*(a*b*d*f*x*
e^(2*c) + a*b*d*e^(2*c + 1))*e^(2*d*x) + (a^2*f*e^c + (3*a^2*d*f*e^c + 2*b^
2*d*f*e^c)*x + (3*a^2*d*e^c + 2*b^2*d*e^c)*e)*e^(d*x))/((a^3*d^2*f^2 + a*b^
2*d^2*f^2)*x^2 + 2*(a^3*d^2*f + a*b^2*d^2*f)*x*e + (a^3*d^2 + a*b^2*d^2)*e^
2 - ((a^3*d^2*f^2*e^(6*c) + a*b^2*d^2*f^2*e^(6*c))*x^2 + 2*(a^3*d^2*f*e^(6*
c) + a*b^2*d^2*f*e^(6*c))*x*e + (a^3*d^2*e^(6*c) + a*b^2*d^2*e^(6*c))*e^2)*
e^(6*d*x) - ((a^3*d^2*f^2*e^(4*c) + a*b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^3*d^2
*f*e^(4*c) + a*b^2*d^2*f*e^(4*c))*x*e + (a^3*d^2*e^(4*c) + a*b^2*d^2*e^(4*c
))*e^2)*e^(4*d*x) + ((a^3*d^2*f^2*e^(2*c) + a*b^2*d^2*f^2*e^(2*c))*x^2 + 2*
(a^3*d^2*f*e^(2*c) + a*b^2*d^2*f*e^(2*c))*x*e + (a^3*d^2*e^(2*c) + a*b^2*d^
2*e^(2*c))*e^2)*e^(2*d*x)) - 32*integrate(-1/32*(b*d*f*x + b*d*e + a*f)/(a^
2*d*f^2*x^2 + 2*a^2*d*f*x*e + a^2*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*f*x*
e^(c + 1) + a^2*d*e^(c + 2))*e^(d*x)), x) + 32*integrate(1/32*(b*d*f*x + b*
d*e - a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*f*x*e + a^2*d*e^2 + (a^2*d*f^2*x^2*e^c
+ 2*a^2*d*f*x*e^(c + 1) + a^2*d*e^(c + 2))*e^(d*x)), x) - 32*integrate(-1/3
2*(2*a^2*b*f^2 + 2*b^3*f^2 - 2*(a^2*b*d^2*f^2 + 2*b^3*d^2*f^2)*x^2 - 4*(a^2
*b*d^2*f + 2*b^3*d^2*f)*x*e - 2*(a^2*b*d^2 + 2*b^3*d^2)*e^2 + (2*a^3*f^2*e^
c + 2*a*b^2*f^2*e^c - (3*a^3*d^2*f^2*e^c + 5*a*b^2*d^2*f^2*e^c)*x^2 - 2*(3*
a^3*d^2*f*e^c + 5*a*b^2*d^2*f*e^c)*x*e - (3*a^3*d^2*e^c + 5*a*b^2*d^2*e^c)*
e^2)*e^(d*x))/((a^4*d^2*f^3 + 2*a^2*b^2*d^2*f^3 + b^4*d^2*f^3)*x^3 + 3*(a^4
*d^2*f^2 + 2*a^2*b^2*d^2*f^2 + b^4*d^2*f^2)*x^2*e + 3*(a^4*d^2*f + 2*a^2*b^
2*d^2*f + b^4*d^2*f)*x*e^2 + (a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2)*e^3 + ((a^
4*d^2*f^3*e^(2*c) + 2*a^2*b^2*d^2*f^3*e^(2*c) + b^4*d^2*f^3*e^(2*c))*x^3 +
3*(a^4*d^2*f^2*e^(2*c) + 2*a^2*b^2*d^2*f^2*e^(2*c) + b^4*d^2*f^2*e^(2*c))*x
^2*e + 3*(a^4*d^2*f*e^(2*c) + 2*a^2*b^2*d^2*f*e^(2*c) + b^4*d^2*f*e^(2*c))*
x*e^2 + (a^4*d^2*e^(2*c) + 2*a^2*b^2*d^2*e^(2*c) + b^4*d^2*e^(2*c))*e^3)*e^
(2*d*x)), x) - 32*integrate(-1/16*(a*b^5*e^(d*x + c) - b^6)/((a^6*b*f + 2*a
^4*b^3*f + a^2*b^5*f)*x + (a^6*b + 2*a^4*b^3 + a^2*b^5)*e - ((a^6*b*f*e^(2*

```
c) + 2*a^4*b^3*f*e^(2*c) + a^2*b^5*f*e^(2*c))*x + (a^6*b*e^(2*c) + 2*a^4*b^
3*e^(2*c) + a^2*b^5*e^(2*c))*e)*e^(2*d*x) - 2*((a^7*f*e^c + 2*a^5*b^2*f*e^c
 + a^3*b^4*f*e^c)*x + (a^7*e^c + 2*a^5*b^2*e^c + a^3*b^4*e^c)*e)*e^(d*x)),
x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] integral(csch(d*x + c)^2*sech(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(
d*x + c)), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*sech(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6440 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cosh\left(c+d\,x\right)^3\,\sinh\left(c+d\,x\right)^2\,\left(e+f\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(1/(cosh(c + d*x)^3*sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.476 $\int \frac{(e+fx)^3 \coth(c+dx)\mathrm{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=752

$$-\frac{3f(e+fx)^2}{2ad^2}+\frac{6bf(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2}-\frac{3f(e+fx)^2\coth(c+dx)}{2ad^2}+\frac{b(e+fx)^3\mathrm{csch}(c+dx)}{a^2d}-\frac{(e+fx)}{}$$

[Out] -3/2*f*(f*x+e)^2/a/d^2+6*b*f*(f*x+e)^2*arctanh(exp(d*x+c))/a^2/d^2-3/2*f*(f*x+e)^2*coth(d*x+c)/a/d^2+b*(f*x+e)^3*csch(d*x+c)/a^2/d-1/2*(f*x+e)^3*csch(d*x+c)^2/a/d+3*f^2*(f*x+e)*ln(1-exp(2*d*x+2*c))/a/d^3+b^2*(f*x+e)^3*ln(1-exp(2*d*x+2*c))/a^3/d-b^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d-b^2*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d+6*b*f^2*(f*x+e)*polylog(2,-exp(d*x+c))/a^2/d^3-6*b*f^2*(f*x+e)*polylog(2,exp(d*x+c))/a^2/d^3+3/2*f^3*polylog(2,exp(2*d*x+2*c))/a/d^4+3/2*b^2*f*(f*x+e)^2*polylog(2,exp(2*d*x+2*c))/a^3/d^2-3*b^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^2-3*b^2*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^2-6*b*f^3*polylog(3,-exp(d*x+c))/a^2/d^4+6*b*f^3*polylog(3,exp(d*x+c))/a^2/d^4-3/2*b^2*f^2*(f*x+e)*polylog(3,exp(2*d*x+2*c))/a^3/d^3+6*b^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^3+6*b^2*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^3+3/4*b^2*f^3*polylog(4,exp(2*d*x+2*c))/a^3/d^4-6*b^2*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^4-6*b^2*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^4

Rubi [A]
time = 0.92, antiderivative size = 752, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 14, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5706, 5560, 4269, 3797, 2221, 2317, 2438, 4267, 2611, 2320, 6724, 5688, 6744, 5680}

Antiderivative was successfully verified.

[In] Int[((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]

[Out] (-3*f*(e + f*x)^2)/(2*a*d^2) + (6*b*f*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^2*d^2) - (3*f*(e + f*x)^2*Coth[c + d*x])/(2*a*d^2) + (b*(e + f*x)^3*Csch[c + d*x])/(a^2*d) - ((e + f*x)^3*Csch[c + d*x]^2)/(2*a*d) - (b^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*d) - (b^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*d) + (3*f^2*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a*d^3) + (b^2*(e + f*x)^3*Log[1 - E^(2*(c + d*x))])/(a^3*d) + (6*b*f^2*(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a^2*d^3) - (6*b*f^2*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^2*d^3) - (3*b^2*f*(e + f*x)^2*Po

```
lyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2) - (3*b^2*f*(e
 + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) +
 (3*f^3*PolyLog[2, E^(2*(c + d*x))])/(2*a*d^4) + (3*b^2*f*(e + f*x)^2*PolyL
og[2, E^(2*(c + d*x))])/(2*a^3*d^2) - (6*b*f^3*PolyLog[3, -E^(c + d*x)])/(a
^2*d^4) + (6*b*f^3*PolyLog[3, E^(c + d*x)])/(a^2*d^4) + (6*b^2*f^2*(e + f*x
)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^3) + (6*b^2*
f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^
3) - (3*b^2*f^2*(e + f*x)*PolyLog[3, E^(2*(c + d*x))])/(2*a^3*d^3) - (6*b^2
*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^4) - (6*b
^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^4) + (3
*b^2*f^3*PolyLog[4, E^(2*(c + d*x))])/(4*a^3*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5688

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x]^
(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5706

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\coth(c+dx)\text{csch}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+}}{a}$$

$$= -\frac{(e+fx)^3\text{csch}^2(c+dx)}{2ad} - \frac{b\int (e+fx)^3\coth(c+dx)\text{csch}(c+}{a^2}$$

$$= -\frac{3f(e+fx)^2\coth(c+dx)}{2ad^2} + \frac{b(e+fx)^3\text{csch}(c+dx)}{a^2d} - \frac{(e+}{}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{6bf(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{3f(e+fx)^2\,c}{2ad}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{6bf(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{3f(e+fx)^2\,c}{2ad}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{6bf(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{3f(e+fx)^2\,c}{2ad}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{6bf(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{3f(e+fx)^2\,c}{2ad}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{6bf(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{3f(e+fx)^2\,c}{2ad}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{6bf(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{3f(e+fx)^2\,c}{2ad}$$

Mathematica [C] Result contains complex when optimal does not.
time = 48.62, size = 7955, normalized size = 10.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 2.74, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3\coth(dx+c)\,\text{csch}(dx+c)^2}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -(2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2*e^(-
2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + b^2*log(-2*a*e^(-d*x - c) +
 b*e^(-2*d*x - 2*c) - b)/(a^3*d) - b^2*log(e^(-d*x - c) + 1)/(a^3*d) - b^2*
log(e^(-d*x - c) - 1)/(a^3*d))*e^3 + (3*a*f^3*x^2 + 6*a*f^2*x*e + 3*a*f*e^2
 + 2*(b*d*f^3*x^3*e^(3*c) + 3*b*d*f^2*x^2*e^(3*c + 1) + 3*b*d*f*x*e^(3*c +
2))*e^(3*d*x) - (2*a*d*f^3*x^3*e^(2*c) + 3*(a*f^3*e^(2*c) + 2*a*d*f^2*e^(2*
c + 1))*x^2 + 3*a*f*e^(2*c + 2) + 6*(a*d*f*e^(2*c + 2) + a*f^2*e^(2*c + 1))
*x)*e^(2*d*x) - 2*(b*d*f^3*x^3*e^c + 3*b*d*f^2*x^2*e^(c + 1) + 3*b*d*f*x*e^
(c + 2))*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^
2*d^2) + (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*
d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*b^2*f^3/(a^3*d^4
) + (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*p
olylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*b^2*f^3/(a^3*d^4) - 3*(
b*d*f*e^2 + a*f^2*e)*x/(a^2*d^2) + 3*(b*d*f*e^2 - a*f^2*e)*x/(a^2*d^2) + 3*
(b*d*f*e^2 + a*f^2*e)*log(e^(d*x + c) + 1)/(a^2*d^3) - 3*(b*d*f*e^2 - a*f^2
*e)*log(e^(d*x + c) - 1)/(a^2*d^3) + 3*(b^2*d*f^2*e + a*b*f^3)*(d^2*x^2*log
(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))
/(a^3*d^4) + 3*(b^2*d*f^2*e - a*b*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d
*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))/(a^3*d^4) + 3*(b^2*d^2*f
*e^2 + 2*a*b*d*f^2*e + a^2*f^3)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x +
 c)))/(a^3*d^4) + 3*(b^2*d^2*f*e^2 - 2*a*b*d*f^2*e + a^2*f^3)*(d*x*log(-e^(
d*x + c) + 1) + dilog(e^(d*x + c)))/(a^3*d^4) - 1/4*(b^2*d^4*f^3*x^4 + 4*(b
^2*d*f^2*e + a*b*f^3)*d^3*x^3 + 6*(b^2*d^2*f*e^2 + 2*a*b*d*f^2*e + a^2*f^3)
*d^2*x^2)/(a^3*d^4) - 1/4*(b^2*d^4*f^3*x^4 + 4*(b^2*d*f^2*e - a*b*f^3)*d^3*
x^3 + 6*(b^2*d^2*f*e^2 - 2*a*b*d*f^2*e + a^2*f^3)*d^2*x^2)/(a^3*d^4) + inte
grate(-2*(b^3*f^3*x^3 + 3*b^3*f^2*x^2*e + 3*b^3*f*x*e^2 - (a*b^2*f^3*x^3*e^
c + 3*a*b^2*f^2*x^2*e^(c + 1) + 3*a*b^2*f*x*e^(c + 2))*e^(d*x))/(a^3*b*e^(2
*d*x + 2*c) + 2*a^4*e^(d*x + c) - a^3*b), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 20211 vs. $2(720) = 1440$.

time = 0.56, size = 20211, normalized size = 26.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] (3*a^2*c^2*f^3 - 6*a^2*c*d*f^2*cosh(1) + 3*a^2*d^2*f*cosh(1)^2 + 3*a^2*d^2*
f*sinh(1)^2 - 3*(a^2*d^2*f^3*x^2 - a^2*c^2*f^3 + 2*(a^2*d^2*f^2*x + a^2*c*d
*f^2)*cosh(1) + 2*(a^2*d^2*f^2*x + a^2*c*d*f^2)*sinh(1))*cosh(d*x + c)^4 -
3*(a^2*d^2*f^3*x^2 - a^2*c^2*f^3 + 2*(a^2*d^2*f^2*x + a^2*c*d*f^2)*cosh(1)
+ 2*(a^2*d^2*f^2*x + a^2*c*d*f^2)*sinh(1))*sinh(d*x + c)^4 + 2*(a*b*d^3*f^3
*x^3 + 3*a*b*d^3*f^2*x^2*cosh(1) + 3*a*b*d^3*f*x*cosh(1)^2 + a*b*d^3*cosh(1
)^3 + a*b*d^3*sinh(1)^3 + 3*(a*b*d^3*f*x + a*b*d^3*cosh(1))*sinh(1)^2 + 3*(
a*b*d^3*f^2*x^2 + 2*a*b*d^3*f*x*cosh(1) + a*b*d^3*cosh(1)^2)*sinh(1))*cosh(
d*x + c)^3 + 2*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*f^2*x^2*cosh(1) + 3*a*b*d^3*f*x
*cosh(1)^2 + a*b*d^3*cosh(1)^3 + a*b*d^3*sinh(1)^3 + 3*(a*b*d^3*f*x + a*b*d
^3*cosh(1))*sinh(1)^2 - 6*(a^2*d^2*f^3*x^2 - a^2*c^2*f^3 + 2*(a^2*d^2*f^2*x
 + a^2*c*d*f^2)*cosh(1) + 2*(a^2*d^2*f^2*x + a^2*c*d*f^2)*sinh(1))*cosh(d*x
 + c) + 3*(a*b*d^3*f^2*x^2 + 2*a*b*d^3*f*x*cosh(1) + a*b*d^3*cosh(1)^2)*sin
h(1))*sinh(d*x + c)^3 - (2*a^2*d^3*f^3*x^3 - 3*a^2*d^2*f^3*x^2 + 2*a^2*d^3*
cosh(1)^3 + 2*a^2*d^3*sinh(1)^3 + 6*a^2*c^2*f^3 + 3*(2*a^2*d^3*f*x + a^2*d^
2*f)*cosh(1)^2 + 3*(2*a^2*d^3*f*x + 2*a^2*d^3*cosh(1) + a^2*d^2*f)*sinh(1)^
2 + 6*(a^2*d^3*f^2*x^2 - a^2*d^2*f^2*x - 2*a^2*c*d*f^2)*cosh(1) + 6*(a^2*d^
3*f^2*x^2 - a^2*d^2*f^2*x + a^2*d^3*cosh(1)^2 - 2*a^2*c*d*f^2 + (2*a^2*d^3*
f*x + a^2*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 - (2*a^2*d^3*f^3*x^3 - 3
*a^2*d^2*f^3*x^2 + 2*a^2*d^3*cosh(1)^3 + 2*a^2*d^3*sinh(1)^3 + 6*a^2*c^2*f^
3 + 3*(2*a^2*d^3*f*x + a^2*d^2*f)*cosh(1)^2 + 18*(a^2*d^2*f^3*x^2 - a^2*c^2
*f^3 + 2*(a^2*d^2*f^2*x + a^2*c*d*f^2)*cosh(1) + 2*(a^2*d^2*f^2*x + a^2*c*d
*f^2)*sinh(1))*cosh(d*x + c)^2 + 3*(2*a^2*d^3*f*x + 2*a^2*d^3*cosh(1) + a^2
*d^2*f)*sinh(1)^2 + 6*(a^2*d^3*f^2*x^2 - a^2*d^2*f^2*x - 2*a^2*c*d*f^2)*cos
h(1) - 6*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*f^2*x^2*cosh(1) + 3*a*b*d^3*f*x*cosh(
1)^2 + a*b*d^3*cosh(1)^3 + a*b*d^3*sinh(1)^3 + 3*(a*b*d^3*f*x + a*b*d^3*cos
h(1))*sinh(1)^2 + 3*(a*b*d^3*f^2*x^2 + 2*a*b*d^3*f*x*cosh(1) + a*b*d^3*cosh
(1)^2)*sinh(1))*cosh(d*x + c) + 6*(a^2*d^3*f^2*x^2 - a^2*d^2*f^2*x + a^2*d^
3*cosh(1)^2 - 2*a^2*c*d*f^2 + (2*a^2*d^3*f*x + a^2*d^2*f)*cosh(1))*sinh(1))
*sinh(d*x + c)^2 - 2*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*f^2*x^2*cosh(1) + 3*a*b*d
^3*f*x*cosh(1)^2 + a*b*d^3*cosh(1)^3 + a*b*d^3*sinh(1)^3 + 3*(a*b*d^3*f*x +
 a*b*d^3*cosh(1))*sinh(1)^2 + 3*(a*b*d^3*f^2*x^2 + 2*a*b*d^3*f*x*cosh(1) +
a*b*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c) - 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*
f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + (b^2*d^2*f^3*x^
2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + 2
*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^4 + 4*(b^2*d^2*
f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)
```

```
^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x
+ c)^3 + (b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 +
 b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*sinh(
d*x + c)^4 - 2*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(
1)^2 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))
*cosh(d*x + c)^2 - 2*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f
*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 - 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cos
h(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d
^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(
1))*sinh(1))*sinh(d*x + c)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1
) + 4*((b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b
^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*cosh(d*
x + c)^3 - (b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2
 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*cos
h(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*co
sh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(b^2*d
^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh
(1)^2 + (b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 +
b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*cosh(d
*x + c)^4 + 4*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1
)^2 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*
cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1)
+ b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*
cosh(1))*sinh(1))*sinh(d*x + c)^4 - 2*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*co
sh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*
d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 - 2*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^
2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 - 3*(b^2*d^2*f^3*x^
2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + 2
*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(b^2*d^2*
f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*sinh(d*x + ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*coth(d*x+c)*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(c+d\,x)\,\left(e+f\,x\right)^3}{{\mathrm{sinh}\left(c+d\,x\right)}^2\,\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(c + d*x)*(e + f*x)^3)/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((coth(c + d*x)*(e + f*x)^3)/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.477 $\int \frac{(e+fx)^2 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=502

$$\frac{4bf(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2}-\frac{f(e+fx)\coth(c+dx)}{ad^2}+\frac{b(e+fx)^2\operatorname{csch}(c+dx)}{a^2d}-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2ad}-\frac{b}{}$$

```
[Out] 4*b*f*(f*x+e)*arctanh(exp(d*x+c))/a^2/d^2-f*(f*x+e)*coth(d*x+c)/a/d^2+b*(f*
x+e)^2*csch(d*x+c)/a^2/d-1/2*(f*x+e)^2*csch(d*x+c)^2/a/d+b^2*(f*x+e)^2*ln(1
-exp(2*d*x+2*c))/a^3/d+f^2*ln(sinh(d*x+c))/a/d^3-b^2*(f*x+e)^2*ln(1+b*exp(d
*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d-b^2*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/a^3/d+2*b*f^2*polylog(2,-exp(d*x+c))/a^2/d^3-2*b*f^2*polylog(2,e
xp(d*x+c))/a^2/d^3+b^2*f*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a^3/d^2-2*b^2*f*
(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^2-2*b^2*f*(f*x+e
)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^2-1/2*b^2*f^2*polylog(
3,exp(2*d*x+2*c))/a^3/d^3+2*b^2*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1
/2)))/a^3/d^3+2*b^2*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^
3
```

Rubi [A]
time = 0.69, antiderivative size = 502, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 14, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5706, 5560, 4269, 3556, 4267, 2317, 2438, 5688, 3797, 2221, 2611, 2320, 6724, 5680}

$$-\frac{b^2f^2\text{Li}_3\left(e^{2(c+dx)}\right)}{2a^3d^3}+\frac{b^2f(e+fx)\text{Li}_2\left(e^{2(c+dx)}\right)}{a^3d^2}+\frac{b^2(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{a^3d}+\frac{2bf^2\text{Li}_2\left(-e^{c+dx}\right)}{a^2d^3}-\frac{2bf^2\text{Li}_2\left(e^{c+dx}\right)}{a^2d^3}+\frac{4bf(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2}+\frac{b(e+fx)^2\operatorname{csch}(c+dx)}{a^2d}+\frac{2b^2f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^3}+\frac{2b^2f^2\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^3}-\frac{2b^2f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}-\frac{2b^2f(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2}-\frac{b^2(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^3d}-\frac{b^2(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{a^3d}+\frac{f^2\log(\sinh(c+dx))}{ad^3}-\frac{f(e+fx)\coth(c+dx)}{ad^2}-\frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (4*b*f*(e + f*x)*ArcTanh[E^(c + d*x)])/(a^2*d^2) - (f*(e + f*x)*Coth[c + d*
x])/(a*d^2) + (b*(e + f*x)^2*Csch[c + d*x])/(a^2*d) - ((e + f*x)^2*Csch[c +
 d*x]^2)/(2*a*d) - (b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/(a^3*d) - (b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])])/(a^3*d) + (b^2*(e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a^3*d) + (f^
2*Log[Sinh[c + d*x]])/(a*d^3) + (2*b*f^2*PolyLog[2, -E^(c + d*x)])/(a^2*d^3
) - (2*b*f^2*PolyLog[2, E^(c + d*x)])/(a^2*d^3) - (2*b^2*f*(e + f*x)*PolyLo
g[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2) - (2*b^2*f*(e + f
*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) + (b^2*
f*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a^3*d^2) + (2*b^2*f^2*PolyLog[3,
-((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^3) + (2*b^2*f^2*PolyLog[3
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^3) - (b^2*f^2*PolyLog[3
, E^(2*(c + d*x))])/(2*a^3*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
```

```
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5688

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x]^
(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5706

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
```

```
IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\coth(c+dx)\text{csch}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+}}{a}$$

$$= -\frac{(e+fx)^2\text{csch}^2(c+dx)}{2ad} - \frac{b\int (e+fx)^2\coth(c+dx)\text{csch}(c+}{a^2}$$

$$= -\frac{f(e+fx)\coth(c+dx)}{ad^2} + \frac{b(e+fx)^2\text{csch}(c+dx)}{a^2d} - \frac{(e+fx}{}$$

$$= \frac{4bf(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{f(e+fx)\coth(c+dx)}{ad^2} + \frac{b(e+}{}$$

$$= \frac{4bf(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{f(e+fx)\coth(c+dx)}{ad^2} + \frac{b(e+}{}$$

$$= \frac{4bf(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{f(e+fx)\coth(c+dx)}{ad^2} + \frac{b(e+}{}$$

$$= \frac{4bf(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{f(e+fx)\coth(c+dx)}{ad^2} + \frac{b(e+}{}$$

$$= \frac{4bf(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{f(e+fx)\coth(c+dx)}{ad^2} + \frac{b(e+}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1161 vs. $2(502) = 1004$.
time = 22.09, size = 1161, normalized size = 2.31

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (b*(e + f*x)^2*Csch[c])/(a^2*d) + ((-e^2 - 2*e*f*x - f^2*x^2)*Csch[c/2 + (d
*x)/2]^2)/(8*a*d) - (12*d*E^(2*c)*(b^2*d^2*e^2 + a^2*f^2)*x - 12*d*(-1 + E^
(2*c))*(b^2*d^2*e^2 + a^2*f^2)*x + 12*b^2*d^3*e*f*x^2 + 4*b^2*d^3*f^2*x^3 -
 24*a*b*d*e*(-1 + E^(2*c))*f*ArcTanh[E^(c + d*x)] + 6*b^2*d^2*e^2*(-1 + E^(
2*c))*(2*d*x - Log[1 - E^(2*(c + d*x))]) + 6*a^2*(-1 + E^(2*c))*f^2*(2*d*x
- Log[1 - E^(2*(c + d*x))]) + 12*a*b*(-1 + E^(2*c))*f^2*(d*x*(Log[1 - E^(c
+ d*x)] - Log[1 + E^(c + d*x)]) - PolyLog[2, -E^(c + d*x)] + PolyLog[2, E^(
c + d*x)]) + 6*b^2*d*e*(-1 + E^(2*c))*f*(2*d*x*(d*x - Log[1 - E^(2*(c + d*x
))]) - PolyLog[2, E^(2*(c + d*x))]) + b^2*(-1 + E^(2*c))*f^2*(2*d^2*x^2*(2*
d*x - 3*Log[1 - E^(2*(c + d*x))]) - 6*d*x*PolyLog[2, E^(2*(c + d*x))] + 3*P
olyLog[3, E^(2*(c + d*x))]))/(6*a^3*d^3*(-1 + E^(2*c))) + (b^2*((2*E^(2*c)*
x*(3*e^2 + 3*e*f*x + f^2*x^2))/(-1 + E^(2*c)) - (3*(d^2*e^2*Log[2*a*E^(c +
d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*d^2*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a
*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + d^2*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/
(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*d^2*e*f*x*Log[1 + (b*E^(2*c + d*x)
)/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + d^2*f^2*x^2*Log[1 + (b*E^(2*c + d*
x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*
E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2*d*f*(e + f*x)*Poly
Log[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*f^2*Po
lyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 2*f^2*
PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/d^3)
)/(3*a^3) + ((e^2 + 2*e*f*x + f^2*x^2)*Sech[c/2 + (d*x)/2]^2)/(8*a*d) + (Se
ch[c/2]*Sech[c/2 + (d*x)/2]*(-(b*d*e^2*Sinh[(d*x)/2]) - a*e*f*Sinh[(d*x)/2]
 - 2*b*d*e*f*x*Sinh[(d*x)/2] - a*f^2*x*Sinh[(d*x)/2] - b*d*f^2*x^2*Sinh[(d*
x)/2]))/(2*a^2*d^2) + (Csch[c/2]*Csch[c/2 + (d*x)/2]*(-(b*d*e^2*Sinh[(d*x)/
2]) + a*e*f*Sinh[(d*x)/2] - 2*b*d*e*f*x*Sinh[(d*x)/2] + a*f^2*x*Sinh[(d*x)/
2] - b*d*f^2*x^2*Sinh[(d*x)/2]))/(2*a^2*d^2)
```

Maple [F]
time = 2.47, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \coth(dx+c) \operatorname{csch}(dx+c)^2}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -(2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2*e^(-
2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + b^2*log(-2*a*e^(-d*x - c) +
 b*e^(-2*d*x - 2*c) - b)/(a^3*d) - b^2*log(e^(-d*x - c) + 1)/(a^3*d) - b^2*
log(e^(-d*x - c) - 1)/(a^3*d))*e^2 + 2*(a*f^2*x + a*f*e + (b*d*f^2*x^2*e^(3
*c) + 2*b*d*f*x*e^(3*c + 1))*e^(3*d*x) - (a*d*f^2*x^2*e^(2*c) + a*f*e^(2*c
+ 1) + (a*f^2*e^(2*c) + 2*a*d*f*e^(2*c + 1))*x)*e^(2*d*x) - (b*d*f^2*x^2*e^
c + 2*b*d*f*x*e^(c + 1))*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2
*d*x + 2*c) + a^2*d^2) + (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*
x + c)) - 2*polylog(3, -e^(d*x + c)))*b^2*f^2/(a^3*d^3) + (d^2*x^2*log(-e^(
d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*b^2*f
^2/(a^3*d^3) - (2*b*d*f*e + a*f^2)*x/(a^2*d^2) + (2*b*d*f*e - a*f^2)*x/(a^2
*d^2) + (2*b*d*f*e + a*f^2)*log(e^(d*x + c) + 1)/(a^2*d^3) - (2*b*d*f*e - a
*f^2)*log(e^(d*x + c) - 1)/(a^2*d^3) + 2*(b^2*d*f*e + a*b*f^2)*(d*x*log(e^(
d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^3*d^3) + 2*(b^2*d*f*e - a*b*f^2)*(d
*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^3*d^3) - 1/3*(b^2*d^3*f^2
*x^3 + 3*(b^2*d*f*e + a*b*f^2)*d^2*x^2)/(a^3*d^3) - 1/3*(b^2*d^3*f^2*x^3 +
3*(b^2*d*f*e - a*b*f^2)*d^2*x^2)/(a^3*d^3) + integrate(-2*(b^3*f^2*x^2 + 2*
b^3*f*x*e - (a*b^2*f^2*x^2*e^c + 2*a*b^2*f*x*e^(c + 1))*e^(d*x))/(a^3*b*e^(
2*d*x + 2*c) + 2*a^4*e^(d*x + c) - a^3*b), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 9084 vs. $2(482) = 964$.
time = 0.43, size = 9084, normalized size = 18.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -(2*a^2*c*f^2 - 2*a^2*d*f*cosh(1) + 2*(a^2*d*f^2*x + a^2*c*f^2)*cosh(d*x +
c)^4 - 2*a^2*d*f*sinh(1) + 2*(a^2*d*f^2*x + a^2*c*f^2)*sinh(d*x + c)^4 - 2*
(a*b*d^2*f^2*x^2 + 2*a*b*d^2*f*x*cosh(1) + a*b*d^2*cosh(1)^2 + a*b*d^2*sinh
(1)^2 + 2*(a*b*d^2*f*x + a*b*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^3 - 2*(a*b
*d^2*f^2*x^2 + 2*a*b*d^2*f*x*cosh(1) + a*b*d^2*cosh(1)^2 + a*b*d^2*sinh(1)^
2 - 4*(a^2*d*f^2*x + a^2*c*f^2)*cosh(d*x + c) + 2*(a*b*d^2*f*x + a*b*d^2*co
sh(1))*sinh(1))*sinh(d*x + c)^3 + 2*(a^2*d^2*f^2*x^2 - a^2*d*f^2*x + a^2*d^
2*cosh(1)^2 + a^2*d^2*sinh(1)^2 - 2*a^2*c*f^2 + (2*a^2*d^2*f*x + a^2*d*f)*c
osh(1) + (2*a^2*d^2*f*x + 2*a^2*d^2*cosh(1) + a^2*d*f)*sinh(1))*cosh(d*x +
c)^2 + 2*(a^2*d^2*f^2*x^2 - a^2*d*f^2*x + a^2*d^2*cosh(1)^2 + a^2*d^2*sinh(
1)^2 - 2*a^2*c*f^2 + 6*(a^2*d*f^2*x + a^2*c*f^2)*cosh(d*x + c)^2 + (2*a^2*d
```

^2*f*x + a^2*d*f)*cosh(1) - 3*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*f*x*cosh(1) + a*b*d^2*cosh(1)^2 + a*b*d^2*sinh(1)^2 + 2*(a*b*d^2*f*x + a*b*d^2*cosh(1))*sinh(1))*cosh(d*x + c) + (2*a^2*d^2*f*x + 2*a^2*d^2*cosh(1) + a^2*d*f)*sinh(1))*sinh(d*x + c)^2 + 2*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*f*x*cosh(1) + a*b*d^2*cosh(1)^2 + a*b*d^2*sinh(1)^2 + 2*(a*b*d^2*f*x + a*b*d^2*cosh(1))*sinh(1))*cosh(d*x + c) + 2*(b^2*d*f^2*x + b^2*d*f*cosh(1) + (b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh(d*x + c)^4 + b^2*d*f*sinh(1) + 4*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*sinh(d*x + c)^4 - 2*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh(d*x + c)^2 - 2*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1) - 3*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh(d*x + c)^3 - (b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b^2*d*f^2*x + b^2*d*f*cosh(1) + (b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh(d*x + c)^4 + b^2*d*f*sinh(1) + 4*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*sinh(d*x + c)^4 - 2*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh(d*x + c)^2 - 2*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1) - 3*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh(d*x + c)^3 - (b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b^2*d*f^2*x + b^2*d*f*cosh(1) + (b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1) - a*b*f^2)*cosh(d*x + c)^4 + b^2*d*f*sinh(1) + 4*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1) - a*b*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1) - a*b*f^2)*sinh(d*x + c)^4 - a*b*f^2 - 2*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1) - a*b*f^2)*cosh(d*x + c)^2 - 2*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1) - a*b*f^2 - 3*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1) - a*b*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1) - a*b*f^2)*cosh(d*x + c)^3 - (b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1) - a*b*f^2)*cosh(d*x + c))*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)) - 2*(b^2*d*f^2*x + b^2*d*f*cosh(1) + (b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1) + a*b*f^2)*cosh(d*x + c)^4 + b^2*d*f*sinh(1) + 4*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1) + a*b*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1) + a*b*f^2)*sinh(d*x + c)^4 + a*b*f^2 - 2*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1) + a*b*f^2)*cosh(d*x + c)^2 - 2*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1) + a*b*f^2 - 3*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1) + a*b*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1) + a*b*f^2)*cosh(d*x + c)^3 - (b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1) + a*b*f^2)*cosh(d*x + c))*sinh(d*x + c))*d

```
ilog(-cosh(d*x + c) - sinh(d*x + c)) + (b^2*c^2*f^2 - 2*b^2*c*d*f*cosh(1) +
 b^2*d^2*cosh(1)^2 + b^2*d^2*sinh(1)^2 + (b^2*c^2*f^2 - 2*b^2*c*d*f*cosh(1)
 + b^2*d^2*cosh(1)^2 + b^2*d^2*sinh(1)^2 - 2*(b^2*c*d*f - b^2*d^2*cosh(1))*
sinh(1))*cosh(d*x + c)^4 + 4*(b^2*c^2*f^2 - 2*b^2*c*d*f*cosh(1) + b^2*d^2*c
osh(1)^2 + b^2*d^2*sinh(1)^2 - 2*(b^2*c*d*f - b^2*d^2*cosh(1))*sinh(1))*cos
h(d*x + c)*sinh(d*x + c)^3 + (b^2*c^2*f^2 - 2*b^2*c*d*f*cosh(1) + b^2*d^2*c
osh(1)^2 + b^2*d^2*sinh(1)^2 - 2*(b^2*c*d*f - b^2*d^2*cosh(1))*sinh(1))*sin
h(d*x + c)^4 - 2*(b^2*c^2*f^2 - 2*b^2*c*d*f*cosh(1) + b^2*d^2*cosh(1)^2 + b
^2*d^2*sinh(1)^2 - 2*(b^2*c*d*f - b^2*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^2
 - 2*(b^2*c^2*f^2 - 2*b^2*c*d*f*cosh(1) + b^2*d^2*cosh(1)^2 + b^2*d^2*sinh(
1)^2 - 3*(b^2*c^2*f^2 - 2*b^2*c*d*f*cosh(1) + b...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*coth(d*x+c)*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(c+d\,x)\,\left(e+f\,x\right)^2}{{\mathrm{sinh}\left(c+d\,x\right)}^2\,\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(c + d*x)*(e + f*x)^2)/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((coth(c + d*x)*(e + f*x)^2)/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.478 $\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=298

$$\frac{bf\tanh^{-1}(\cosh(c+dx))}{a^2d^2}-\frac{f\coth(c+dx)}{2ad^2}+\frac{b(e+fx)\operatorname{csch}(c+dx)}{a^2d}-\frac{(e+fx)\operatorname{csch}^2(c+dx)}{2ad}-\frac{b^2(e+fx)\log}{}$$

```
[Out] b*f*arctanh(cosh(d*x+c))/a^2/d^2-1/2*f*coth(d*x+c)/a/d^2+b*(f*x+e)*csch(d*x
+c)/a^2/d-1/2*(f*x+e)*csch(d*x+c)^2/a/d+b^2*(f*x+e)*ln(1-exp(2*d*x+2*c))/a^
3/d-b^2*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d-b^2*(f*x+e)*ln
(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d+1/2*b^2*f*polylog(2,exp(2*d*x+2*
c))/a^3/d^2-b^2*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^2-b^2*
f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^2
```

Rubi [A]
time = 0.39, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {5706, 5560, 3852, 8, 3855, 5688, 3797, 2221, 2317, 2438, 5680}

$$\frac{b^2f\text{Li}_2\left(e^{2(c+dx)}\right)}{2a^3d^2}+\frac{b^2(e+fx)\log\left(1-e^{2(c+dx)}\right)}{a^3d}+\frac{bf\tanh^{-1}(\cosh(c+dx))}{a^2d^2}+\frac{b(e+fx)\operatorname{csch}(c+dx)}{a^2d}-\frac{b^2f\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}-\frac{b^2f\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2}-\frac{b^2(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^3d}-\frac{b^2(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{a^3d}-\frac{f\coth(c+dx)}{2ad^2}-\frac{(e+fx)\operatorname{csch}^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*f*ArcTanh[Cosh[c + d*x]])/(a^2*d^2) - (f*Coth[c + d*x])/(2*a*d^2) + (b*(
e + f*x)*Csch[c + d*x])/(a^2*d) - ((e + f*x)*Csch[c + d*x]^2)/(2*a*d) - (b^
2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*d) - (b^2*
(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*d) + (b^2*(e
 + f*x)*Log[1 - E^(2*(c + d*x))])/(a^3*d) - (b^2*f*PolyLog[2, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2) - (b^2*f*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) + (b^2*f*PolyLog[2, E^(2*(c + d*x))])
/(2*a^3*d^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5688

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x]^
(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5706

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\coth(c+dx)\operatorname{csch}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\coth(c+dx)\mathrm{C}\ldots}{a+b\sinh(c+\ldots}}{a}$$

$$= -\frac{(e+fx)\operatorname{csch}^2(c+dx)}{2ad} - \frac{b\int (e+fx)\coth(c+dx)\operatorname{csch}(c+d\ldots}{a^2}$$

$$= \frac{b(e+fx)\operatorname{csch}(c+dx)}{a^2d} - \frac{(e+fx)\operatorname{csch}^2(c+dx)}{2ad} + \frac{b^2\int (e+fx\ldots}{}$$

$$= \frac{bf\tanh^{-1}(\cosh(c+dx))}{a^2d^2} - \frac{f\coth(c+dx)}{2ad^2} + \frac{b(e+fx)\operatorname{csch}(c\ldots}{a^2d}$$

$$= \frac{bf\tanh^{-1}(\cosh(c+dx))}{a^2d^2} - \frac{f\coth(c+dx)}{2ad^2} + \frac{b(e+fx)\operatorname{csch}(c\ldots}{a^2d}$$

$$= \frac{bf\tanh^{-1}(\cosh(c+dx))}{a^2d^2} - \frac{f\coth(c+dx)}{2ad^2} + \frac{b(e+fx)\operatorname{csch}(c\ldots}{a^2d}$$

$$= \frac{bf\tanh^{-1}(\cosh(c+dx))}{a^2d^2} - \frac{f\coth(c+dx)}{2ad^2} + \frac{b(e+fx)\operatorname{csch}(c\ldots}{a^2d}$$

Mathematica [A]
time = 5.28, size = 376, normalized size = 1.26

$$\frac{2a(-af+2bd(e+fx))\coth\left(\frac{1}{2}(c+dx)\right)-a^2d(e+fx)\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+8b^2de\log(\sinh(c+dx))-8b^2cf\log(\sinh(c+dx))-8abf\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+4b^2f\left((c+dx)\left(c+dx+2\log\left(1-e^{-2(c+dx)}\right)\right)-\operatorname{PolyLog}\left(2,e^{-2(c+dx)}\right)\right)-8b^2\left(-\frac{1}{2}f(c+dx)^2+f(c+dx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)+f(c+dx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)+de\log(a+b\sinh(c+dx))-cf\log(a+b\sinh(c+dx))+f\operatorname{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2+b^2}}\right)+f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)\right)+a^2d(e+fx)\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)-2a(af+2bd(e+fx))\tanh\left(\frac{1}{2}(c+dx)\right)}{8a^3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (2*a*(-(a*f) + 2*b*d*(e + f*x))*Coth[(c + d*x)/2] - a^2*d*(e + f*x)*Csch[(c
 + d*x)/2]^2 + 8*b^2*d*e*Log[Sinh[c + d*x]] - 8*b^2*c*f*Log[Sinh[c + d*x]]
- 8*a*b*f*Log[Tanh[(c + d*x)/2]] + 4*b^2*f*((c + d*x)*(c + d*x + 2*Log[1 -
E^(-2*(c + d*x))]) - PolyLog[2, E^(-2*(c + d*x))]) - 8*b^2*(-1/2*(f*(c + d*
x)^2) + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c +
 d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d*e*Log[a + b*Sinh[c
 + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLog[2, (b*E^(c + d*x))/(-a
+ Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
]) + a^2*d*(e + f*x)*Sech[(c + d*x)/2]^2 - 2*a*(a*f + 2*b*d*(e + f*x))*Tanh
[(c + d*x)/2])/(8*a^3*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(280) = 560$.
time = 5.12, size = 649, normalized size = 2.18

method	result
risch	$-\frac{-2bdfx\,\mathrm{e}^{3dx+3c}+2adfx\,\mathrm{e}^{2dx+2c}-2bde\,\mathrm{e}^{3dx+3c}+2ade\,\mathrm{e}^{2dx+2c}+2bdfx\,\mathrm{e}^{dx+c}+af\,\mathrm{e}^{2dx+2c}+2bde\,\mathrm{e}^{dx+c}-fa}{d^2a^2\left(\mathrm{e}^{2dx+2c}-1\right)^2}+\frac{b^2f\,\mathrm{dilog}\left(\mathrm{e}^{dx+c}+1\right)}{d^2a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVER
BOSE)
```

```
[Out] -(-2*b*d*f*x*exp(3*d*x+3*c)+2*a*d*f*x*exp(2*d*x+2*c)-2*b*d*e*exp(3*d*x+3*c)
+2*a*d*e*exp(2*d*x+2*c)+2*b*d*f*x*exp(d*x+c)+a*f*exp(2*d*x+2*c)+2*b*d*e*exp
(d*x+c)-f*a)/d^2/a^2/(exp(2*d*x+2*c)-1)^2+1/d^2/a^3*b^2*f*dilog(exp(d*x+c)+
1)-1/d^2/a^3*b^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1
/2)))-1/d^2/a^3*b^2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(
1/2)))-1/d^2/a^3*b^2*f*dilog(exp(d*x+c))+1/d/a^3*b^2*e*ln(exp(d*x+c)+1)-1/d
/a^3*b^2*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/d/a^3*b^2*e*ln(exp(d*x+c
)-1)+1/d/a^3*b^2*f*ln(exp(d*x+c)+1)*x-1/d/a^3*b^2*f*ln((-b*exp(d*x+c)+(a^2+
b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d^2/a^3*b^2*f*ln((-b*exp(d*x+c)+(a^
2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d/a^3*b^2*f*ln((b*exp(d*x+c)+(a^2
+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2/a^3*b^2*f*ln((b*exp(d*x+c)+(a^2
+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d^2/a^3*b^2*f*c*ln(b*exp(2*d*x+2*c)
+2*a*exp(d*x+c)-b)-1/d^2/a^3*b^2*f*c*ln(exp(d*x+c)-1)+1/d^2/a^2*b*f*ln(exp(
d*x+c)+1)-1/d^2/a^2*b*f*ln(exp(d*x+c)-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -(4*b^2*d*integrate(1/4*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 4*b^2*d*integra
te(1/4*x/(a^3*d*e^(d*x + c) - a^3*d), x) + a*b*((d*x + c)/(a^3*d^2) - log(e
^(d*x + c) + 1)/(a^3*d^2)) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) - 1
)/(a^3*d^2)) - (2*b*d*x*e^(3*d*x + 3*c) - 2*b*d*x*e^(d*x + c) - (2*a*d*x*e^
(2*c) + a*e^(2*c))*e^(2*d*x) + a)/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2
*d*x + 2*c) + a^2*d^2) - 4*integrate(1/2*(a*b^2*x*e^(d*x + c) - b^3*x)/(a^3
*b*e^(2*d*x + 2*c) + 2*a^4*e^(d*x + c) - a^3*b), x))*f - (2*(b*e^(-d*x - c)
 - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2*e^(-2*d*x - 2*c) - a^2*
e^(-4*d*x - 4*c) - a^2)*d) + b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c)
 - b)/(a^3*d) - b^2*log(e^(-d*x - c) + 1)/(a^3*d) - b^2*log(e^(-d*x - c) -
1)/(a^3*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3272 vs. $2(282) = 564$.
time = 0.38, size = 3272, normalized size = 10.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] (2*(a*b*d*f*x + a*b*d*cosh(1) + a*b*d*sinh(1))*cosh(d*x + c)^3 + 2*(a*b*d*f
*x + a*b*d*cosh(1) + a*b*d*sinh(1))*sinh(d*x + c)^3 + a^2*f - (2*a^2*d*f*x
+ 2*a^2*d*cosh(1) + 2*a^2*d*sinh(1) + a^2*f)*cosh(d*x + c)^2 - (2*a^2*d*f*x
 + 2*a^2*d*cosh(1) + 2*a^2*d*sinh(1) + a^2*f - 6*(a*b*d*f*x + a*b*d*cosh(1)
 + a*b*d*sinh(1))*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(a*b*d*f*x + a*b*d*cos
h(1) + a*b*d*sinh(1))*cosh(d*x + c) - (b^2*f*cosh(d*x + c)^4 + 4*b^2*f*cosh
(d*x + c)*sinh(d*x + c)^3 + b^2*f*sinh(d*x + c)^4 - 2*b^2*f*cosh(d*x + c)^2
 + b^2*f + 2*(3*b^2*f*cosh(d*x + c)^2 - b^2*f)*sinh(d*x + c)^2 + 4*(b^2*f*c
osh(d*x + c)^3 - b^2*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c)
 + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2) - b)/b + 1) - (b^2*f*cosh(d*x + c)^4 + 4*b^2*f*cosh(d*x + c)*sinh(d*x +
 c)^3 + b^2*f*sinh(d*x + c)^4 - 2*b^2*f*cosh(d*x + c)^2 + b^2*f + 2*(3*b^2*
f*cosh(d*x + c)^2 - b^2*f)*sinh(d*x + c)^2 + 4*(b^2*f*cosh(d*x + c)^3 - b^2
*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) -
 (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b
^2*f*cosh(d*x + c)^4 + 4*b^2*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*f*sinh(d
*x + c)^4 - 2*b^2*f*cosh(d*x + c)^2 + b^2*f + 2*(3*b^2*f*cosh(d*x + c)^2 -
b^2*f)*sinh(d*x + c)^2 + 4*(b^2*f*cosh(d*x + c)^3 - b^2*f*cosh(d*x + c))*si
nh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)) + (b^2*f*cosh(d*x + c)^4
```

```
+ 4*b^2*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*f*sinh(d*x + c)^4 - 2*b^2*f*c
osh(d*x + c)^2 + b^2*f + 2*(3*b^2*f*cosh(d*x + c)^2 - b^2*f)*sinh(d*x + c)^
2 + 4*(b^2*f*cosh(d*x + c)^3 - b^2*f*cosh(d*x + c))*sinh(d*x + c))*dilog(-c
osh(d*x + c) - sinh(d*x + c)) + ((b^2*c*f - b^2*d*cosh(1) - b^2*d*sinh(1))*
cosh(d*x + c)^4 + 4*(b^2*c*f - b^2*d*cosh(1) - b^2*d*sinh(1))*cosh(d*x + c)
*sinh(d*x + c)^3 + (b^2*c*f - b^2*d*cosh(1) - b^2*d*sinh(1))*sinh(d*x + c)^
4 + b^2*c*f - b^2*d*cosh(1) - b^2*d*sinh(1) - 2*(b^2*c*f - b^2*d*cosh(1) -
b^2*d*sinh(1))*cosh(d*x + c)^2 - 2*(b^2*c*f - b^2*d*cosh(1) - b^2*d*sinh(1)
 - 3*(b^2*c*f - b^2*d*cosh(1) - b^2*d*sinh(1))*cosh(d*x + c)^2)*sinh(d*x +
c)^2 + 4*((b^2*c*f - b^2*d*cosh(1) - b^2*d*sinh(1))*cosh(d*x + c)^3 - (b^2*
c*f - b^2*d*cosh(1) - b^2*d*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*log(2*b*
cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + ((b^
2*c*f - b^2*d*cosh(1) - b^2*d*sinh(1))*cosh(d*x + c)^4 + 4*(b^2*c*f - b^2*d
*cosh(1) - b^2*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*c*f - b^2*d*
cosh(1) - b^2*d*sinh(1))*sinh(d*x + c)^4 + b^2*c*f - b^2*d*cosh(1) - b^2*d*
sinh(1) - 2*(b^2*c*f - b^2*d*cosh(1) - b^2*d*sinh(1))*cosh(d*x + c)^2 - 2*(
b^2*c*f - b^2*d*cosh(1) - b^2*d*sinh(1) - 3*(b^2*c*f - b^2*d*cosh(1) - b^2*
d*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*c*f - b^2*d*cosh(1) -
 b^2*d*sinh(1))*cosh(d*x + c)^3 - (b^2*c*f - b^2*d*cosh(1) - b^2*d*sinh(1))
*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) -
2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^2*d*f*x + (b^2*d*f*x + b^2*c*f)*cosh(
d*x + c)^4 + 4*(b^2*d*f*x + b^2*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d
*f*x + b^2*c*f)*sinh(d*x + c)^4 + b^2*c*f - 2*(b^2*d*f*x + b^2*c*f)*cosh(d*
x + c)^2 - 2*(b^2*d*f*x + b^2*c*f - 3*(b^2*d*f*x + b^2*c*f)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 4*((b^2*d*f*x + b^2*c*f)*cosh(d*x + c)^3 - (b^2*d*f*x +
 b^2*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x
+ c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) -
(b^2*d*f*x + (b^2*d*f*x + b^2*c*f)*cosh(d*x + c)^4 + 4*(b^2*d*f*x + b^2*c*f
)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*f*x + b^2*c*f)*sinh(d*x + c)^4 + b
^2*c*f - 2*(b^2*d*f*x + b^2*c*f)*cosh(d*x + c)^2 - 2*(b^2*d*f*x + b^2*c*f -
 3*(b^2*d*f*x + b^2*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d*f*x +
 b^2*c*f)*cosh(d*x + c)^3 - (b^2*d*f*x + b^2*c*f)*cosh(d*x + c))*sinh(d*x +
 c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b^2*d*f*x + (b^2*d*f*x + b^2*d*cos
h(1) + b^2*d*sinh(1) + a*b*f)*cosh(d*x + c)^4 + 4*(b^2*d*f*x + b^2*d*cosh(1
) + b^2*d*sinh(1) + a*b*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*f*x + b^2
*d*cosh(1) + b^2*d*sinh(1) + a*b*f)*sinh(d*x + c)^4 + b^2*d*cosh(1) + b^2*d
*sinh(1) + a*b*f - 2*(b^2*d*f*x + b^2*d*cosh(1) + b^2*d*sinh(1) + a*b*f)*co
sh(d*x + c)^2 - 2*(b^2*d*f*x + b^2*d*cosh(1) + b^2*d*sinh(1) + a*b*f - 3*(b
^2*d*f*x + b^2*d*cosh(1) + b^2*d*sinh(1) + a*b*f)*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 + 4*((b^2*d*f*x + b^2*d*cosh(1) + b^2*d*sinh(1) + a*b*f)*cosh(d*x +
 c)^3 - (b^2*d*f*x + b^2*d*cosh(1) + b^2*d*sinh(1) + a*b*f)*cosh(d*x + c))*
sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((b^2*d*cosh(1) + b
^2*d*sinh(1) - (b^2*c + a*b)*f)*cosh(d*x + c)^4 + 4*(b^2*d*cosh(1) + b^2*d*
sinh(1) - (b^2*c + a*b)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*cosh(1) +
```

```
 b^2*d*sinh(1) - (b^2*c + a*b)*f)*sinh(d*x + c)^4 + b^2*d*cosh(1) + b^2*d*s
inh(1) - 2*(b^2*d*cosh(1) + b^2*d*sinh(1) - (b^2*c + a*b)*f)*cosh(d*x + c)^
2 - 2*(b^2*d*cosh(1) + b^2*d*sinh(1) - 3*(b^2*d...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)*coth(c + d*x)*csch(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(c+d\,x)\,\left(e+f\,x\right)}{{\sinh\left(c+d\,x\right)}^2\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(c + d*x)*(e + f*x))/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)
[Out] int((coth(c + d*x)*(e + f*x))/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))), x)
```

3.479 $\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{b\operatorname{csch}(c+dx)}{a^2d}-\frac{\operatorname{csch}^2(c+dx)}{2ad}+\frac{b^2\log(\sinh(c+dx))}{a^3d}-\frac{b^2\log(a+b\sinh(c+dx))}{a^3d}$$

```
[Out] b*csch(d*x+c)/a^2/d-1/2*csch(d*x+c)^2/a/d+b^2*ln(sinh(d*x+c))/a^3/d-b^2*ln(
a+b*sinh(d*x+c))/a^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2912, 12, 46}

$$\frac{b^2\log(\sinh(c+dx))}{a^3d}-\frac{b^2\log(a+b\sinh(c+dx))}{a^3d}+\frac{b\operatorname{csch}(c+dx)}{a^2d}-\frac{\operatorname{csch}^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*Csch[c + d*x])/(a^2*d) - Csch[c + d*x]^2/(2*a*d) + (b^2*Log[Sinh[c + d*x
]])/(a^3*d) - (b^2*Log[a + b*Sinh[c + d*x]])/(a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^3}{x^3(a+x)}\,dx, x, b\sinh(c+dx)\right)}{bd}$$
$$= \frac{b^2\text{Subst}\left(\int \frac{1}{x^3(a+x)}\,dx, x, b\sinh(c+dx)\right)}{d}$$
$$= \frac{b^2\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{1}{a^2x^2}+\frac{1}{a^3x}-\frac{1}{a^3(a+x)}\right)dx, x, b\sinh(c+dx)\right)}{d}$$
$$= \frac{b\text{csch}(c+dx)}{a^2d}-\frac{\text{csch}^2(c+dx)}{2ad}+\frac{b^2\log(\sinh(c+dx))}{a^3d}-\frac{b^2\log(a+b\sin}{a^3d}$$

Mathematica [A]
time = 0.09, size = 60, normalized size = 0.83

$$\frac{2ab\text{csch}(c+dx)-a^2\text{csch}^2(c+dx)+2b^2(\log(\sinh(c+dx))-\log(a+b\sinh(c+dx)))}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a*b*Csch[c + d*x] - a^2*Csch[c + d*x]^2 + 2*b^2*(Log[Sinh[c + d*x]] - Lo
g[a + b*Sinh[c + d*x]]))/(2*a^3*d)
```

Maple [A]
time = 0.93, size = 65, normalized size = 0.90

method	result	size
derivativedivides	$\frac{-\frac{1}{2a\sinh(dx+c)^2}+\frac{b^2\ln(\sinh(dx+c))}{a^3}+\frac{b}{a^2\sinh(dx+c)}-\frac{b^2\ln(a+b\sinh(dx+c))}{a^3}}{d}$	65
default	$\frac{-\frac{1}{2a\sinh(dx+c)^2}+\frac{b^2\ln(\sinh(dx+c))}{a^3}+\frac{b}{a^2\sinh(dx+c)}-\frac{b^2\ln(a+b\sinh(dx+c))}{a^3}}{d}$	65
risch	$-\frac{2\,e^{dx+c}(-b\,e^{2dx+2c}+a\,e^{dx+c}+b)}{a^2d(e^{2dx+2c}-1)^2}-\frac{b^2\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{a^3d}+\frac{b^2\ln(e^{2dx+2c}-1)}{a^3d}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a/sinh(d*x+c)^2+1/a^3*b^2*ln(sinh(d*x+c))+1/a^2*b/sinh(d*x+c)-1/a
^3*b^2*ln(a+b*sinh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(70) = 140$.

time = 0.28, size = 161, normalized size = 2.24

$$-\frac{2\left(be^{(-dx-c)} - ae^{(-2\,dx-2\,c)} - be^{(-3\,dx-3\,c)}\right)}{\left(2\,a^2e^{(-2\,dx-2\,c)} - a^2e^{(-4\,dx-4\,c)} - a^2\right)d} - \frac{b^2\log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{a^3d} + \frac{b^2\log\left(e^{(-dx-c)} + 1\right)}{a^3d} + \frac{b^2\log\left(e^{(-dx-c)} - 1\right)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2*e^(-2
*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - b^2*log(-2*a*e^(-d*x - c) +
b*e^(-2*d*x - 2*c) - b)/(a^3*d) + b^2*log(e^(-d*x - c) + 1)/(a^3*d) + b^2*l
og(e^(-d*x - c) - 1)/(a^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 545 vs. $2(70) = 140$.
time = 0.34, size = 545, normalized size = 7.57

$$\frac{2\,ab\cosh\left(dx+c\right)^3 + 2\,ab\sinh\left(dx+c\right)^3 - 2\,a^2\cosh\left(dx+c\right)^2 - 2\,ab\cosh\left(dx+c\right) + 2\left(3\,ab\cosh\left(dx+c\right) - a^2\right)\sinh\left(dx+c\right)^2 - \left(b^2\cosh\left(dx+c\right)^4 + 4\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3 + b^2\sinh\left(dx+c\right)^4 - 2\,b^2\cosh\left(dx+c\right)^2 + 2\left(3\,b^2\cosh\left(dx+c\right)^2 - b^2\right)\sinh\left(dx+c\right)^2 + b^2 + 4\left(b^2\cosh\left(dx+c\right)^3 - b^2\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)\right)\log\left(\frac{2\left(b\sinh(dx+c)+a\right)}{\cosh(dx+c)-\sinh(dx+c)}\right) + \left(b^2\cosh\left(dx+c\right)^4 + 4\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3 + b^2\sinh\left(dx+c\right)^4 - 2\,b^2\cosh\left(dx+c\right)^2 + 2\left(3\,b^2\cosh\left(dx+c\right)^2 - b^2\right)\sinh\left(dx+c\right)^2 + b^2 + 4\left(b^2\cosh\left(dx+c\right)^3 - b^2\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)\right)\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right) + 2\left(3\,ab\cosh\left(dx+c\right)^2 - 2\,a^2\cosh\left(dx+c\right) - ab\right)\sinh\left(dx+c\right)}{a^3d\cosh\left(dx+c\right)^4 + 4\,a^3d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3 + a^3d\sinh\left(dx+c\right)^4 - 2\,a^3d\cosh\left(dx+c\right)^2 + a^3d + 2\left(3\,a^3d\cosh\left(dx+c\right)^2 - a^3d\right)\sinh\left(dx+c\right)^2 + 4\left(a^3d\cosh\left(dx+c\right)^3 - a^3d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] (2*a*b*cosh(d*x + c)^3 + 2*a*b*sinh(d*x + c)^3 - 2*a^2*cosh(d*x + c)^2 - 2*
a*b*cosh(d*x + c) + 2*(3*a*b*cosh(d*x + c) - a^2)*sinh(d*x + c)^2 - (b^2*co
sh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 -
 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^2 +
b^2 + 4*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*s
inh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + (b^2*cosh(d*x + c)^4 +
 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 - 2*b^2*cosh(d*x
 + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*co
sh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh
(d*x + c) - sinh(d*x + c))) + 2*(3*a*b*cosh(d*x + c)^2 - 2*a^2*cosh(d*x + c
) - a*b)*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)*sinh
(d*x + c)^3 + a^3*d*sinh(d*x + c)^4 - 2*a^3*d*cosh(d*x + c)^2 + a^3*d + 2*(
3*a^3*d*cosh(d*x + c)^2 - a^3*d)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^3
 - a^3*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(c+dx\right)\operatorname{csch}^2\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)

[Out] Integral(coth(c + d*x)*csch(c + d*x)**2/(a + b*sinh(c + d*x)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(70) = 140.
time = 0.45, size = 145, normalized size = 2.01

$$-\frac{\frac{2\,b^2\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^3}-\frac{2\,b^2\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a^3}+\frac{3\,b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-4\,ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)+4\,a^2}{a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")

[Out] -1/2*(2*b^2*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/a^3 - 2*b^2*log(abs(e^(d*x + c) - e^(-d*x - c)))/a^3 + (3*b^2*(e^(d*x + c) - e^(-d*x - c))^2 - 4*a*b*(e^(d*x + c) - e^(-d*x - c)) + 4*a^2)/(a^3*(e^(d*x + c) - e^(-d*x - c))^2))/d

Mupad [B]
time = 1.00, size = 470, normalized size = 6.53

$$-\frac{\frac{2}{a\,d}-\frac{2\,b\,e^{c+d\,x}}{a^2\,d}}{e^{2\,c+2\,d\,x}-1}-\frac{\left(2\,\mathrm{atan}\left(-\frac{4\,a^3\,b^5\,\sqrt{-a^6\,d^2}+4\,a\,b^7\,\sqrt{-a^6\,d^2}-4\,b^8\,e^{3\,c}\,e^{3\,d\,x}\,\sqrt{-a^6\,d^2}+4\,b^8\,e^{d\,x}\,e^{c}\,\sqrt{-a^6\,d^2}-8\,a\,b^7\,e^{2\,c}\,e^{2\,d\,x}\,\sqrt{-a^6\,d^2}+4\,a^2\,b^6\,e^{d\,x}\,e^{c}\,\sqrt{-a^6\,d^2}-8\,a^3\,b^5\,e^{2\,c}\,e^{2\,d\,x}\,\sqrt{-a^6\,d^2}-4\,a^2\,b^6\,e^{3\,c}\,e^{3\,d\,x}\,\sqrt{-a^6\,d^2}}{4\,a^4\,b\,d\,{\left(b^4\right)}^{3/2}+4\,a^6\,b^3\,d\,\sqrt{b^4}}\right)+2\,\mathrm{atan}\left(\left(4\,a^4\,b^5\,d\,\sqrt{b^4}\,\sqrt{-a^6\,d^2}+4\,a^6\,b^3\,d\,\sqrt{b^4}\,\sqrt{-a^6\,d^2}\right)\,\left(\frac{1}{8\,a^5\,b^5\,d^2\,{\left(a^2+b^2\right)}^2}-e^{d\,x}\,e^{c}\,\left(\frac{1}{16\,a^4\,b^6\,d^2\,{\left(a^2+b^2\right)}^2}-\frac{{\left(a^2+2\,b^2\right)}^2}{16\,a^8\,b^6\,d^2\,{\left(a^2+b^2\right)}^2}\right)+\frac{a^2+2\,b^2}{8\,a^7\,b^5\,d^2\,{\left(a^2+b^2\right)}^2}\right)\right)\right)\,\sqrt{b^4}}{\sqrt{-a^6\,d^2}}-\frac{2}{a\,d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)/(sinh(c + d*x)^2*(a + b*sinh(c + d*x))),x)

[Out] - (2/(a*d) - (2*b*exp(c + d*x))/(a^2*d))/(exp(2*c + 2*d*x) - 1) - ((2*atan(-(4*a^3*b^5*(-a^6*d^2)^(1/2) + 4*a*b^7*(-a^6*d^2)^(1/2) - 4*b^8*exp(3*c)*exp(3*d*x)*(-a^6*d^2)^(1/2) + 4*b^8*exp(d*x)*exp(c)*(-a^6*d^2)^(1/2) - 8*a*b^7*exp(2*c)*exp(2*d*x)*(-a^6*d^2)^(1/2) + 4*a^2*b^6*exp(d*x)*exp(c)*(-a^6*d^2)^(1/2) - 8*a^3*b^5*exp(2*c)*exp(2*d*x)*(-a^6*d^2)^(1/2) - 4*a^2*b^6*exp(3*c)*exp(3*d*x)*(-a^6*d^2)^(1/2))/(4*a^4*b*d*(b^4)^(3/2) + 4*a^6*b^3*d*(b^4)^(1/2))) + 2*atan((4*a^4*b^5*d*(b^4)^(1/2)*(-a^6*d^2)^(1/2) + 4*a^6*b^3*d*(b^4)^(1/2)*(-a^6*d^2)^(1/2))*(1/(8*a^5*b^5*d^2*(a^2 + b^2)^2) - exp(d*x)*exp(c)*(1/(16*a^4*b^6*d^2*(a^2 + b^2)^2) - (a^2 + 2*b^2)^2/(16*a^8*b^6*d^2*(a^2 + b^2)^2)) + (a^2 + 2*b^2)/(8*a^7*b^5*d^2*(a^2 + b^2)^2))))*(b^4)^(1/2))/(-a^6*d^2)^(1/2) - 2/(a*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))

3.480 $\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{\coth(c+dx)\text{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Coth[c + d*x]*Csch[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Coth[c + d*x]*Csch[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(Coth[c + d*x]*Csch[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(dx+c)\,\text{csch}(dx+c)^2}{(fx+e)\,(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -(a*f - 2*(b*d*f*x*e^(3*c) + b*d*e^(3*c + 1))*e^(3*d*x) + (2*a*d*f*x*e^(2*c
) - a*f*e^(2*c) + 2*a*d*e^(2*c + 1))*e^(2*d*x) + 2*(b*d*f*x*e^c + b*d*e^(c
+ 1))*e^(d*x))/(a^2*d^2*f^2*x^2 + 2*a^2*d^2*f*x*e + a^2*d^2*e^2 + (a^2*d^2*
f^2*x^2*e^(4*c) + 2*a^2*d^2*f*x*e^(4*c + 1) + a^2*d^2*e^(4*c + 2))*e^(4*d*x
) - 2*(a^2*d^2*f^2*x^2*e^(2*c) + 2*a^2*d^2*f*x*e^(2*c + 1) + a^2*d^2*e^(2*c
 + 2))*e^(2*d*x)) + 4*integrate(-1/4*(b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + a*b*d
*f*e + a^2*f^2 + (2*b^2*d^2*f*e + a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^
2*f^2*x^2*e + 3*a^3*d^2*f*x*e^2 + a^3*d^2*e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^
3*d^2*f^2*x^2*e^(c + 1) + 3*a^3*d^2*f*x*e^(c + 2) + a^3*d^2*e^(c + 3))*e^(d
*x)), x) - 4*integrate(1/4*(b^2*d^2*f^2*x^2 + b^2*d^2*e^2 - a*b*d*f*e + a^2
*f^2 + (2*b^2*d^2*f*e - a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*f^2*x^2*
e + 3*a^3*d^2*f*x*e^2 + a^3*d^2*e^3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*f^2*
x^2*e^(c + 1) + 3*a^3*d^2*f*x*e^(c + 2) + a^3*d^2*e^(c + 3))*e^(d*x)), x) +
 4*integrate(-1/2*(a*b^2*e^(d*x + c) - b^3)/(a^3*b*f*x + a^3*b*e - (a^3*b*f
*x*e^(2*c) + a^3*b*e^(2*c + 1))*e^(2*d*x) - 2*(a^4*f*x*e^c + a^4*e^(c + 1))
*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(coth(d*x + c)*csch(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth{\left(c + dx \right)} \operatorname{csch}^{2}{\left(c + dx \right)}}{\left(a + b \sinh{\left(c + dx \right)}\right) \left(e + fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)*csch(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)),
x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\coth(c+d\,x)}{\sinh\left(c+d\,x\right)^2\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)/(sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(coth(c + d*x)/(sinh(c + d*x)^2*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.481 $\int \frac{(e+fx)^3 \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1038

$$\frac{b(e+fx)^3}{a^2d}-\frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3}-\frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{2b^2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^3d}+\frac{b(e+f}{}$$

```
[Out] -b*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^3/d+b
*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^3/d-6*b
*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^3/d^4+6
*b*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^3/d^4
-3*f^3*polylog(2,-exp(d*x+c))/a/d^4+3*f^3*polylog(2,exp(d*x+c))/a/d^4-3/2*f
*(f*x+e)^2*polylog(2,-exp(d*x+c))/a/d^2+3/2*f*(f*x+e)^2*polylog(2,exp(d*x+c
))/a/d^2+3*f^2*(f*x+e)*polylog(3,-exp(d*x+c))/a/d^3-2*b^2*(f*x+e)^3*arctanh
(exp(d*x+c))/a^3/d+3/2*b*f^3*polylog(3,exp(2*d*x+2*c))/a^2/d^4-6*b^2*f^3*po
lylog(4,-exp(d*x+c))/a^3/d^4+6*b^2*f^3*polylog(4,exp(d*x+c))/a^3/d^4-6*f^2*
(f*x+e)*arctanh(exp(d*x+c))/a/d^3-3/2*f*(f*x+e)^2*csch(d*x+c)/a/d^2-1/2*(f*
x+e)^3*coth(d*x+c)*csch(d*x+c)/a/d-3*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a/d^
3+b*(f*x+e)^3/a^2/d+b*(f*x+e)^3*coth(d*x+c)/a^2/d-3*b^2*f*(f*x+e)^2*polylog
(2,-exp(d*x+c))/a^3/d^2+3*b^2*f*(f*x+e)^2*polylog(2,exp(d*x+c))/a^3/d^2-3*b
*f^2*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a^2/d^3+6*b^2*f^2*(f*x+e)*polylog(3,
-exp(d*x+c))/a^3/d^3-6*b^2*f^2*(f*x+e)*polylog(3,exp(d*x+c))/a^3/d^3-(f*x+e
)^3*arctanh(exp(d*x+c))/a/d-3*f^3*polylog(4,-exp(d*x+c))/a/d^4+3*f^3*polylo
g(4,exp(d*x+c))/a/d^4-3*b*f*(f*x+e)^2*ln(1-exp(2*d*x+2*c))/a^2/d^2-3*b*f*(f
*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^3/d^
2+3*b*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a^2+b^2)^(1
/2)/a^3/d^2+6*b*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*(a
^2+b^2)^(1/2)/a^3/d^3-6*b*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^
(1/2)))*(a^2+b^2)^(1/2)/a^3/d^3
```

Rubi [A]
time = 1.52, antiderivative size = 1038, normalized size of antiderivative = 1.00, number of steps used = 67, number of rules used = 22, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {5706, 5565, 4267, 2611, 6744, 2320, 6724, 4271, 2317, 2438, 5688, 3801, 3797, 2221, 32, 5704, 5558, 3377, 2717, 5684, 3403, 2296}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*(e + f*x)^3)/(a^2*d) - (6*f^2*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d^3) -
((e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) - (2*b^2*(e + f*x)^3*ArcTanh[E^(c
```

```
+ d*x)])/(a^3*d) + (b*(e + f*x)^3*Coth[c + d*x])/(a^2*d) - (3*f*(e + f*x)^2
*Csch[c + d*x])/(2*a*d^2) - ((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x])/(2*a*
d) - (b*Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/(a^3*d) + (b*Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])])/(a^3*d) - (3*b*f*(e + f*x)^2*Log[1 - E^(2*(c + d*x))
])/(a^2*d^2) - (3*f^3*PolyLog[2, -E^(c + d*x)])/(a*d^4) - (3*f*(e + f*x)^2*
PolyLog[2, -E^(c + d*x)])/(2*a*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -E^(c
 + d*x)])/(a^3*d^2) + (3*f^3*PolyLog[2, E^(c + d*x)])/(a*d^4) + (3*f*(e + f
*x)^2*PolyLog[2, E^(c + d*x)])/(2*a*d^2) + (3*b^2*f*(e + f*x)^2*PolyLog[2,
E^(c + d*x)])/(a^3*d^2) - (3*b*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2) + (3*b*Sqrt[a^2 + b^2]*f*
(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2)
 - (3*b*f^2*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a^2*d^3) + (3*f^2*(e +
f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) + (6*b^2*f^2*(e + f*x)*PolyLog[3, -E
^(c + d*x)])/(a^3*d^3) - (3*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a*d^3)
- (6*b^2*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a^3*d^3) + (6*b*Sqrt[a^2 +
 b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(
a^3*d^3) - (6*b*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/(a^3*d^3) + (3*b*f^3*PolyLog[3, E^(2*(c + d*x))])/
(2*a^2*d^4) - (3*f^3*PolyLog[4, -E^(c + d*x)])/(a*d^4) - (6*b^2*f^3*PolyLog
[4, -E^(c + d*x)])/(a^3*d^4) + (3*f^3*PolyLog[4, E^(c + d*x)])/(a*d^4) + (6
*b^2*f^3*PolyLog[4, E^(c + d*x)])/(a^3*d^4) - (6*b*Sqrt[a^2 + b^2]*f^3*Poly
Log[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^4) + (6*b*Sqrt[a^2
 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^4)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5565

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5688

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x]^
(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5704

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5706

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
```

```
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\coth^2(c+dx)\text{csch}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\coth^2(c+dx}{a+b\sinh(c+dx)}}{a}$$

$$= \frac{\int (e+fx)^3\text{csch}(c+dx)\,dx}{a} + \frac{\int (e+fx)^3\text{csch}^3(c+dx)\,dx}{a} - \frac{b}{}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{b(e+fx)^3\coth(c+dx)}{a^2d} - \frac{3f(e}{}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{b(e+fx)^4}{4a^2f} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} - \frac{(e+}{}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} - \frac{(e+fx)^3\tanh^{-1}(}{ad}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} - \frac{(e+fx)^3\tanh^{-1}(}{ad}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} - \frac{(e+fx)^3\tanh^{-1}(}{ad}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} - \frac{(e+fx)^3\tanh^{-1}(}{ad}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} - \frac{(e+fx)^3\tanh^{-1}(}{ad}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} - \frac{(e+fx)^3\tanh^{-1}(}{ad}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} - \frac{(e+fx)^3\tanh^{-1}(}{ad}$$

Mathematica [C] Result contains complex when optimal does not.
time = 39.99, size = 5829, normalized size = 5.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 3.34, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \left(\coth^2\left(dx+c\right)\right) \operatorname{csch}(dx+c)}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] 1/2*(2*(a*e^(-d*x - c) + 2*b*e^(-2*d*x - 2*c) + a*e^(-3*d*x - 3*c) - 2*b)/(
(2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - (a^2 + 2*b^2)*lo
g(e^(-d*x - c) + 1)/(a^3*d) + (a^2 + 2*b^2)*log(e^(-d*x - c) - 1)/(a^3*d) -
 2*(a^2*b + b^3)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c)
 - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3*d))*e^3 - (2*b*d*f^3*x^3 + 6*
b*d*f^2*x^2*e + 6*b*d*f*x*e^2 + (a*d*f^3*x^3*e^(3*c) + 3*(a*f^3*e^(3*c) + a
*d*f^2*e^(3*c + 1))*x^2 + 3*a*f*e^(3*c + 2) + 3*(a*d*f*e^(3*c + 2) + 2*a*f^
2*e^(3*c + 1))*x)*e^(3*d*x) - 2*(b*d*f^3*x^3*e^(2*c) + 3*b*d*f^2*x^2*e^(2*c
 + 1) + 3*b*d*f*x*e^(2*c + 2))*e^(2*d*x) + (a*d*f^3*x^3*e^c + 3*(a*d*f^2*e^
(c + 1) - a*f^3*e^c)*x^2 - 3*a*f*e^(c + 2) + 3*(a*d*f*e^(c + 2) - 2*a*f^2*e
^(c + 1))*x)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c)
+ a^2*d^2) + 3*(b*d*f*e^2 + a*f^2*e)*x/(a^2*d^2) + 3*(b*d*f*e^2 - a*f^2*e)*
x/(a^2*d^2) - 3*(b*d*f*e^2 + a*f^2*e)*log(e^(d*x + c) + 1)/(a^2*d^3) - 3*(b
*d*f*e^2 - a*f^2*e)*log(e^(d*x + c) - 1)/(a^2*d^3) - 1/2*(d^3*x^3*log(e^(d*
x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)
) + 6*polylog(4, -e^(d*x + c)))*(a^2*f^3 + 2*b^2*f^3)/(a^3*d^4) + 1/2*(d^3*
x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3,
 e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*(a^2*f^3 + 2*b^2*f^3)/(a^3*d^4)
- 3/2*(2*a*b*f^3 + (a^2*d*f^2 + 2*b^2*d*f^2)*e)*(d^2*x^2*log(e^(d*x + c) +
```

```
1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^3*d^4) - 3/
2*(2*a*b*f^3 - (a^2*d*f^2 + 2*b^2*d*f^2)*e)*(d^2*x^2*log(-e^(d*x + c) + 1)
+ 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))/(a^3*d^4) - 3/2*(4*
a*b*d*f^2*e + 2*a^2*f^3 + (a^2*d^2*f + 2*b^2*d^2*f)*e^2)*(d*x*log(e^(d*x +
c) + 1) + dilog(-e^(d*x + c)))/(a^3*d^4) - 3/2*(4*a*b*d*f^2*e - 2*a^2*f^3 -
 (a^2*d^2*f + 2*b^2*d^2*f)*e^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x +
 c)))/(a^3*d^4) + 1/8*((a^2*f^3 + 2*b^2*f^3)*d^4*x^4 + 4*(2*a*b*f^3 + (a^2*
d*f^2 + 2*b^2*d*f^2)*e)*d^3*x^3 + 6*(4*a*b*d*f^2*e + 2*a^2*f^3 + (a^2*d^2*f
 + 2*b^2*d^2*f)*e^2)*d^2*x^2)/(a^3*d^4) - 1/8*((a^2*f^3 + 2*b^2*f^3)*d^4*x^
4 - 4*(2*a*b*f^3 - (a^2*d*f^2 + 2*b^2*d*f^2)*e)*d^3*x^3 - 6*(4*a*b*d*f^2*e
- 2*a^2*f^3 - (a^2*d^2*f + 2*b^2*d^2*f)*e^2)*d^2*x^2)/(a^3*d^4) - integrate
(2*((a^2*b*f^3*e^c + b^3*f^3*e^c)*x^3 + 3*(a^2*b*f^2*e^c + b^3*f^2*e^c)*x^2
*e + 3*(a^2*b*f*e^c + b^3*f*e^c)*x*e^2)*e^(d*x)/(a^3*b*e^(2*d*x + 2*c) + 2*
a^4*e^(d*x + c) - a^3*b), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 24038 vs. $2(985) = 1970$.
time = 0.62, size = 24038, normalized size = 23.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/2*(4*a*b*c^3*f^3 - 12*a*b*c^2*d*f^2*cosh(1) + 12*a*b*c*d^2*f*cosh(1)^2 -
4*a*b*d^3*cosh(1)^3 - 4*a*b*d^3*sinh(1)^3 + 4*(a*b*d^3*f^3*x^3 + a*b*c^3*f^
3 + 3*(a*b*d^3*f*x + a*b*c*d^2*f)*cosh(1)^2 + 3*(a*b*d^3*f*x + a*b*c*d^2*f)
*sinh(1)^2 + 3*(a*b*d^3*f^2*x^2 - a*b*c^2*d*f^2)*cosh(1) + 3*(a*b*d^3*f^2*x
^2 - a*b*c^2*d*f^2 + 2*(a*b*d^3*f*x + a*b*c*d^2*f)*cosh(1))*sinh(1))*cosh(d
*x + c)^4 + 4*(a*b*d^3*f^3*x^3 + a*b*c^3*f^3 + 3*(a*b*d^3*f*x + a*b*c*d^2*f
)*cosh(1)^2 + 3*(a*b*d^3*f*x + a*b*c*d^2*f)*sinh(1)^2 + 3*(a*b*d^3*f^2*x^2
- a*b*c^2*d*f^2)*cosh(1) + 3*(a*b*d^3*f^2*x^2 - a*b*c^2*d*f^2 + 2*(a*b*d^3*
f*x + a*b*c*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^4 - 2*(a^2*d^3*f^3*x^3 +
 3*a^2*d^2*f^3*x^2 + a^2*d^3*cosh(1)^3 + a^2*d^3*sinh(1)^3 + 3*(a^2*d^3*f*x
 + a^2*d^2*f)*cosh(1)^2 + 3*(a^2*d^3*f*x + a^2*d^3*cosh(1) + a^2*d^2*f)*sin
h(1)^2 + 3*(a^2*d^3*f^2*x^2 + 2*a^2*d^2*f^2*x)*cosh(1) + 3*(a^2*d^3*f^2*x^2
 + 2*a^2*d^2*f^2*x + a^2*d^3*cosh(1)^2 + 2*(a^2*d^3*f*x + a^2*d^2*f)*cosh(1
))*sinh(1))*cosh(d*x + c)^3 - 2*(a^2*d^3*f^3*x^3 + 3*a^2*d^2*f^3*x^2 + a^2*
d^3*cosh(1)^3 + a^2*d^3*sinh(1)^3 + 3*(a^2*d^3*f*x + a^2*d^2*f)*cosh(1)^2 +
 3*(a^2*d^3*f*x + a^2*d^3*cosh(1) + a^2*d^2*f)*sinh(1)^2 + 3*(a^2*d^3*f^2*x
^2 + 2*a^2*d^2*f^2*x)*cosh(1) - 8*(a*b*d^3*f^3*x^3 + a*b*c^3*f^3 + 3*(a*b*d
^3*f*x + a*b*c*d^2*f)*cosh(1)^2 + 3*(a*b*d^3*f*x + a*b*c*d^2*f)*sinh(1)^2 +
 3*(a*b*d^3*f^2*x^2 - a*b*c^2*d*f^2)*cosh(1) + 3*(a*b*d^3*f^2*x^2 - a*b*c^2
*d*f^2 + 2*(a*b*d^3*f*x + a*b*c*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c) + 3*
```

(a^2*d^3*f^2*x^2 + 2*a^2*d^2*f^2*x + a^2*d^3*cosh(1)^2 + 2*(a^2*d^3*f*x + a^2*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^3 - 4*(a*b*d^3*f^3*x^3 + 2*a*b*c^3*f^3 - a*b*d^3*cosh(1)^3 - a*b*d^3*sinh(1)^3 + 3*(a*b*d^3*f*x + 2*a*b*c*d^2*f)*cosh(1)^2 + 3*(a*b*d^3*f*x + 2*a*b*c*d^2*f - a*b*d^3*cosh(1))*sinh(1)^2 + 3*(a*b*d^3*f^2*x^2 - 2*a*b*c^2*d*f^2)*cosh(1) + 3*(a*b*d^3*f^2*x^2 - 2*a*b*c^2*d*f^2 - a*b*d^3*cosh(1)^2 + 2*(a*b*d^3*f*x + 2*a*b*c*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 12*(a*b*c*d^2*f - a*b*d^3*cosh(1))*sinh(1)^2 - 2*(2*a*b*d^3*f^3*x^3 + 4*a*b*c^3*f^3 - 2*a*b*d^3*cosh(1)^3 - 2*a*b*d^3*sinh(1)^3 + 6*(a*b*d^3*f*x + 2*a*b*c*d^2*f)*cosh(1)^2 - 12*(a*b*d^3*f^3*x^3 + a*b*c^3*f^3 + 3*(a*b*d^3*f*x + a*b*c*d^2*f)*cosh(1)^2 + 3*(a*b*d^3*f*x + a*b*c*d^2*f)*sinh(1)^2 + 3*(a*b*d^3*f^2*x^2 - a*b*c^2*d*f^2)*cosh(1) + 3*(a*b*d^3*f^2*x^2 - a*b*c^2*d*f^2 + 2*(a*b*d^3*f*x + a*b*c*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 6*(a*b*d^3*f*x + 2*a*b*c*d^2*f - a*b*d^3*cosh(1))*sinh(1)^2 + 6*(a*b*d^3*f^2*x^2 - 2*a*b*c^2*d*f^2)*cosh(1) + 3*(a^2*d^3*f^3*x^3 + 3*a^2*d^2*f^3*x^2 + a^2*d^3*cosh(1)^3 + a^2*d^3*sinh(1)^3 + 3*(a^2*d^3*f*x + a^2*d^2*f)*cosh(1)^2 + 3*(a^2*d^3*f*x + a^2*d^3*cosh(1) + a^2*d^2*f)*sinh(1)^2 + 3*(a^2*d^3*f^2*x^2 + 2*a^2*d^2*f^2*x)*cosh(1) + 3*(a^2*d^3*f^2*x^2 + 2*a^2*d^2*f^2*x + a^2*d^3*cosh(1)^2 + 2*(a^2*d^3*f*x + a^2*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c) + 6*(a*b*d^3*f^2*x^2 - 2*a*b*c^2*d*f^2 - a*b*d^3*cosh(1)^2 + 2*(a*b*d^3*f*x + 2*a*b*c*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 6*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + (b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^4 + 4*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c)^4 - 2*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 - 2*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 - 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1) + 4*((b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^3 - (b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 6*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + (b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^4 + 4*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*f^2*x*cosh(1) + b^2*d^2*f*cosh(1)^2 + b^2*d^2*f

```
*sinh(1)^2 + 2*(b^2*d^2*f^2*x + b^2*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)*s
inh(d*x + c)^3 + (b^2*d^2*f^3*x^2 + 2*b^2*d^2*f...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*coth(d*x+c)**2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(c+d\,x)^2\,{(e+f\,x)}^3}{\sinh\left(c+d\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(c + d*x)^2*(e + f*x)^3)/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
[Out] int((coth(c + d*x)^2*(e + f*x)^3)/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.482 $\int \frac{(e+fx)^2 \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=714

$$\frac{b(e+fx)^2}{a^2d}-\frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^3d}-\frac{f^2\tanh^{-1}(\cosh(c+dx))}{ad^3}+\frac{b(e+fx)^2}{}$$

```
[Out] b*(f*x+e)^2/a^2/d-(f*x+e)^2*arctanh(exp(d*x+c))/a/d-2*b^2*(f*x+e)^2*arctanh
(exp(d*x+c))/a^3/d-f^2*arctanh(cosh(d*x+c))/a/d^3+b*(f*x+e)^2*coth(d*x+c)/a
^2/d-f*(f*x+e)*csch(d*x+c)/a/d^2-1/2*(f*x+e)^2*coth(d*x+c)*csch(d*x+c)/a/d-
2*b*f*(f*x+e)*ln(1-exp(2*d*x+2*c))/a^2/d^2-f*(f*x+e)*polylog(2,-exp(d*x+c))
/a/d^2-2*b^2*f*(f*x+e)*polylog(2,-exp(d*x+c))/a^3/d^2+f*(f*x+e)*polylog(2,e
xp(d*x+c))/a/d^2+2*b^2*f*(f*x+e)*polylog(2,exp(d*x+c))/a^3/d^2-b*f^2*polylo
g(2,exp(2*d*x+2*c))/a^2/d^3+f^2*polylog(3,-exp(d*x+c))/a/d^3+2*b^2*f^2*poly
log(3,-exp(d*x+c))/a^3/d^3-f^2*polylog(3,exp(d*x+c))/a/d^3-2*b^2*f^2*polylo
g(3,exp(d*x+c))/a^3/d^3-b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))*
(a^2+b^2)^(1/2)/a^3/d+b*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(a
^2+b^2)^(1/2)/a^3/d-2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2
)))*(a^2+b^2)^(1/2)/a^3/d^2+2*b*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b
^2)^(1/2)))*(a^2+b^2)^(1/2)/a^3/d^2+2*b*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2
+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^3/d^3-2*b*f^2*polylog(3,-b*exp(d*x+c)/(a+(a
^2+b^2)^(1/2)))*(a^2+b^2)^(1/2)/a^3/d^3
```

Rubi [A]

time = 1.20, antiderivative size = 714, normalized size of antiderivative = 1.00, number of steps used = 52, number of rules used = 22, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {5706, 5565, 4267, 2611, 2320, 6724, 4271, 3855, 5688, 3801, 3797, 2221, 2317, 2438, 32, 5704, 5558, 3377, 2718, 5684, 3403, 2296}

$$\frac{2b^2f^2\text{Li}_3\left(-e^{c+dx}\right)}{a^3d^3}-\frac{2b^2f^2\text{Li}_3\left(e^{c+dx}\right)}{a^3d^3}-\frac{2b^2f(e+fx)\text{Li}_2\left(-e^{c+dx}\right)}{a^3d^2}+\frac{2b^2f(e+fx)\text{Li}_2\left(e^{c+dx}\right)}{a^3d^2}-\frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^3d}-\frac{bf^2\text{Li}_2\left(e^{2(c+dx)}\right)}{a^2d^3}-\frac{2bf(e+fx)\log\left(1-e^{2(c+dx)}\right)}{a^2d^2}+\frac{b(e+fx)^2\coth(c+dx)}{a^2d}+\frac{b(e+fx)^2}{a^2d}+\frac{2bf^2\sqrt{a^2+b^2}\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^3}-\frac{2bf^2\sqrt{a^2+b^2}\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^3}-\frac{2bf\sqrt{a^2+b^2}(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}+\frac{2bf\sqrt{a^2+b^2}(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2}-\frac{b\sqrt{a^2+b^2}(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^3d}+\frac{b\sqrt{a^2+b^2}(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{a^3d}+\frac{f^2\text{Li}_3\left(-e^{c+dx}\right)}{ad^3}-\frac{f^2\text{Li}_3\left(e^{c+dx}\right)}{ad^3}-\frac{f^2\tanh^{-1}(\cosh(c+dx))}{ad^3}-\frac{f(e+fx)\text{Li}_2\left(-e^{c+dx}\right)}{ad^2}+\frac{f(e+fx)\text{Li}_2\left(e^{c+dx}\right)}{ad^2}-\frac{f(e+fx)\text{csch}(c+dx)}{ad^2}-\frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*(e + f*x)^2)/(a^2*d) - ((e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (2*b^2
*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^3*d) - (f^2*ArcTanh[Cosh[c + d*x]])/(
a*d^3) + (b*(e + f*x)^2*Coth[c + d*x])/(a^2*d) - (f*(e + f*x)*Csch[c + d*x]
)/(a*d^2) - ((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x])/(2*a*d) - (b*Sqrt[a^2
 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*d)
 + (b*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])])/(a^3*d) - (2*b*f*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a^2*d^2) - (f*
(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2,
 -E^(c + d*x)])/(a^3*d^2) + (f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^2) +
```

```
(2*b^2*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^3*d^2) - (2*b*Sqrt[a^2 + b^
2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d
^2) + (2*b*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/(a^3*d^2) - (b*f^2*PolyLog[2, E^(2*(c + d*x))])/(a^2*d^3)
 + (f^2*PolyLog[3, -E^(c + d*x)])/(a*d^3) + (2*b^2*f^2*PolyLog[3, -E^(c + d
*x)])/(a^3*d^3) - (f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) - (2*b^2*f^2*PolyLo
g[3, E^(c + d*x)])/(a^3*d^3) + (2*b*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^3) - (2*b*Sqrt[a^2 + b^2]*f^2*Pol
yLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5565

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5688

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x]^
(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5704

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5706

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2 \coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2 \coth^2(c+dx)\text{csch}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2 \coth^2(c+dx}{a+b\sinh(c+dx)}}{a}$$

$$= \frac{\int (e+fx)^2\text{csch}(c+dx)\,dx}{a} + \frac{\int (e+fx)^2\text{csch}^3(c+dx)\,dx}{a} - b$$

$$= -\frac{2(e+fx)^2 \tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{b(e+fx)^2 \coth(c+dx)}{a^2 d} - f(e$$

$$= \frac{b(e+fx)^2}{a^2 d} - \frac{b(e+fx)^3}{3a^2 f} - \frac{(e+fx)^2 \tanh^{-1}\left(e^{c+dx}\right)}{ad} - f^2 \tanh$$

$$= \frac{b(e+fx)^2}{a^2 d} - \frac{(e+fx)^2 \tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}}{a^3 d}$$

$$= \frac{b(e+fx)^2}{a^2 d} - \frac{(e+fx)^2 \tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}}{a^3 d}$$

$$= \frac{b(e+fx)^2}{a^2 d} - \frac{(e+fx)^2 \tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}}{a^3 d}$$

$$= \frac{b(e+fx)^2}{a^2 d} - \frac{(e+fx)^2 \tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}}{a^3 d}$$

$$= \frac{b(e+fx)^2}{a^2 d} - \frac{(e+fx)^2 \tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}}{a^3 d}$$

$$= \frac{b(e+fx)^2}{a^2 d} - \frac{(e+fx)^2 \tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}}{a^3 d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1803 vs. $2(714) = 1428$.
time = 21.67, size = 1803, normalized size = 2.53

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (8*a*b*d^2*e*E^(2*c)*f*x + 4*a*b*d^2*E^(2*c)*f^2*x^2 + 2*a^2*d^2*e^2*ArcTan
h[E^(c + d*x)] + 4*b^2*d^2*e^2*ArcTanh[E^(c + d*x)] - 2*a^2*d^2*e^2*E^(2*c)
```

```
*ArcTanh[E^(c + d*x)] - 4*b^2*d^2*e^2*E^(2*c)*ArcTanh[E^(c + d*x)] + 4*a^2*
f^2*ArcTanh[E^(c + d*x)] - 4*a^2*E^(2*c)*f^2*ArcTanh[E^(c + d*x)] - 2*a^2*d
^2*e*f*x*Log[1 - E^(c + d*x)] - 4*b^2*d^2*e*f*x*Log[1 - E^(c + d*x)] + 2*a^
2*d^2*e*E^(2*c)*f*x*Log[1 - E^(c + d*x)] + 4*b^2*d^2*e*E^(2*c)*f*x*Log[1 -
E^(c + d*x)] - a^2*d^2*f^2*x^2*Log[1 - E^(c + d*x)] - 2*b^2*d^2*f^2*x^2*Log
[1 - E^(c + d*x)] + a^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + 2*b^2*d^
2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + 2*a^2*d^2*e*f*x*Log[1 + E^(c + d*x
)] + 4*b^2*d^2*e*f*x*Log[1 + E^(c + d*x)] - 2*a^2*d^2*e*E^(2*c)*f*x*Log[1 +
 E^(c + d*x)] - 4*b^2*d^2*e*E^(2*c)*f*x*Log[1 + E^(c + d*x)] + a^2*d^2*f^2*
x^2*Log[1 + E^(c + d*x)] + 2*b^2*d^2*f^2*x^2*Log[1 + E^(c + d*x)] - a^2*d^2
*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] - 2*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 + E
^(c + d*x)] + 4*a*b*d*e*f*Log[1 - E^(2*(c + d*x))] - 4*a*b*d*e*E^(2*c)*f*Lo
g[1 - E^(2*(c + d*x))] + 4*a*b*d*f^2*x*Log[1 - E^(2*(c + d*x))] - 4*a*b*d*E
^(2*c)*f^2*x*Log[1 - E^(2*(c + d*x))] - 2*(a^2 + 2*b^2)*d*(-1 + E^(2*c))*f*
(e + f*x)*PolyLog[2, -E^(c + d*x)] + 2*(a^2 + 2*b^2)*d*(-1 + E^(2*c))*f*(e
+ f*x)*PolyLog[2, E^(c + d*x)] + 2*a*b*f^2*PolyLog[2, E^(2*(c + d*x))] - 2*
a*b*E^(2*c)*f^2*PolyLog[2, E^(2*(c + d*x))] - 2*a^2*f^2*PolyLog[3, -E^(c +
d*x)] - 4*b^2*f^2*PolyLog[3, -E^(c + d*x)] + 2*a^2*E^(2*c)*f^2*PolyLog[3, -
E^(c + d*x)] + 4*b^2*E^(2*c)*f^2*PolyLog[3, -E^(c + d*x)] + 2*a^2*f^2*PolyL
og[3, E^(c + d*x)] + 4*b^2*f^2*PolyLog[3, E^(c + d*x)] - 2*a^2*E^(2*c)*f^2*
PolyLog[3, E^(c + d*x)] - 4*b^2*E^(2*c)*f^2*PolyLog[3, E^(c + d*x)])/(2*a^3
*d^3*(-1 + E^(2*c))) - (b*(a^2 + b^2)*((2*d^2*e^2*ArcTan[(a + b*E^(c + d*x)
)/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + (2*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c
+ d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] + (
d^2*E^c*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)
])])/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*d^2*e*E^c*f*x*Log[1 + (b*E^(2*c + d*x))
/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/Sqrt[(a^2 + b^2)*E^(2*c)] - (d^2*E^c
*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/Sq
rt[(a^2 + b^2)*E^(2*c)] + (2*d*E^c*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x
))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*d*
E^c*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^
(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] - (2*E^c*f^2*PolyLog[3, -((b*E^(2*c +
d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/Sqrt[(a^2 + b^2)*E^(2*c)] + (2
*E^c*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])
)])/Sqrt[(a^2 + b^2)*E^(2*c)]))/(a^3*d^3) + (Csch[c]*Csch[c + d*x]^2*(2*b*d
*e^2*Cosh[c] + 4*b*d*e*f*x*Cosh[c] + 2*b*d*f^2*x^2*Cosh[c] + 2*a*e*f*Cosh[d
*x] + 2*a*f^2*x*Cosh[d*x] - 2*a*e*f*Cosh[2*c + d*x] - 2*a*f^2*x*Cosh[2*c +
d*x] - 2*b*d*e^2*Cosh[c + 2*d*x] - 4*b*d*e*f*x*Cosh[c + 2*d*x] - 2*b*d*f^2*
x^2*Cosh[c + 2*d*x] + a*d*e^2*Sinh[d*x] + 2*a*d*e*f*x*Sinh[d*x] + a*d*f^2*x
^2*Sinh[d*x] - a*d*e^2*Sinh[2*c + d*x] - 2*a*d*e*f*x*Sinh[2*c + d*x] - a*d*
f^2*x^2*Sinh[2*c + d*x]))/(4*a^2*d^2)
```

Maple [F]

time = 3.35, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \left(\coth^2 (dx+c)\right) \operatorname{csch}(dx+c)}{a+b\sinh (dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] 1/2*(2*(a*e^(-d*x - c) + 2*b*e^(-2*d*x - 2*c) + a*e^(-3*d*x - 3*c) - 2*b)/(
(2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - (a^2 + 2*b^2)*lo
g(e^(-d*x - c) + 1)/(a^3*d) + (a^2 + 2*b^2)*log(e^(-d*x - c) - 1)/(a^3*d) -
 2*(a^2*b + b^3)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c)
 - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3*d))*e^2 - (2*b*d*f^2*x^2 + 4*
b*d*f*x*e + (a*d*f^2*x^2*e^(3*c) + 2*a*f*e^(3*c + 1) + 2*(a*f^2*e^(3*c) + a
*d*f*e^(3*c + 1))*x)*e^(3*d*x) - 2*(b*d*f^2*x^2*e^(2*c) + 2*b*d*f*x*e^(2*c
+ 1))*e^(2*d*x) + (a*d*f^2*x^2*e^c - 2*a*f*e^(c + 1) + 2*(a*d*f*e^(c + 1) -
 a*f^2*e^c)*x)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c
) + a^2*d^2) + (2*b*d*f*e + a*f^2)*x/(a^2*d^2) + (2*b*d*f*e - a*f^2)*x/(a^2
*d^2) - (2*b*d*f*e + a*f^2)*log(e^(d*x + c) + 1)/(a^2*d^3) - (2*b*d*f*e - a
*f^2)*log(e^(d*x + c) - 1)/(a^2*d^3) - 1/2*(d^2*x^2*log(e^(d*x + c) + 1) +
2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*(a^2*f^2 + 2*b^2*f^
2)/(a^3*d^3) + 1/2*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)
) - 2*polylog(3, e^(d*x + c)))*(a^2*f^2 + 2*b^2*f^2)/(a^3*d^3) - (2*a*b*f^2
 + (a^2*d*f + 2*b^2*d*f)*e)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c))
)/(a^3*d^3) - (2*a*b*f^2 - (a^2*d*f + 2*b^2*d*f)*e)*(d*x*log(-e^(d*x + c) +
 1) + dilog(e^(d*x + c)))/(a^3*d^3) + 1/6*((a^2*f^2 + 2*b^2*f^2)*d^3*x^3 +
3*(2*a*b*f^2 + (a^2*d*f + 2*b^2*d*f)*e)*d^2*x^2)/(a^3*d^3) - 1/6*((a^2*f^2
+ 2*b^2*f^2)*d^3*x^3 - 3*(2*a*b*f^2 - (a^2*d*f + 2*b^2*d*f)*e)*d^2*x^2)/(a^
3*d^3) - integrate(2*((a^2*b*f^2*e^c + b^3*f^2*e^c)*x^2 + 2*(a^2*b*f*e^c +
b^3*f*e^c)*x*e)*e^(d*x)/(a^3*b*e^(2*d*x + 2*c) + 2*a^4*e^(d*x + c) - a^3*b)
, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11068 vs. 2(680) = 1360.
time = 0.45, size = 11068, normalized size = 15.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")

[Out] -1/2*(4*a*b*c^2*f^2 - 8*a*b*c*d*f*cosh(1) + 4*a*b*d^2*cosh(1)^2 + 4*a*b*d^2
*sinh(1)^2 - 4*(a*b*d^2*f^2*x^2 - a*b*c^2*f^2 + 2*(a*b*d^2*f*x + a*b*c*d*f)
*cosh(1) + 2*(a*b*d^2*f*x + a*b*c*d*f)*sinh(1))*cosh(d*x + c)^4 - 4*(a*b*d^
2*f^2*x^2 - a*b*c^2*f^2 + 2*(a*b*d^2*f*x + a*b*c*d*f)*cosh(1) + 2*(a*b*d^2*
f*x + a*b*c*d*f)*sinh(1))*sinh(d*x + c)^4 + 2*(a^2*d^2*f^2*x^2 + 2*a^2*d*f^
2*x + a^2*d^2*cosh(1)^2 + a^2*d^2*sinh(1)^2 + 2*(a^2*d^2*f*x + a^2*d*f)*cos
h(1) + 2*(a^2*d^2*f*x + a^2*d^2*cosh(1) + a^2*d*f)*sinh(1))*cosh(d*x + c)^3
 + 2*(a^2*d^2*f^2*x^2 + 2*a^2*d*f^2*x + a^2*d^2*cosh(1)^2 + a^2*d^2*sinh(1)
^2 + 2*(a^2*d^2*f*x + a^2*d*f)*cosh(1) - 8*(a*b*d^2*f^2*x^2 - a*b*c^2*f^2 +
 2*(a*b*d^2*f*x + a*b*c*d*f)*cosh(1) + 2*(a*b*d^2*f*x + a*b*c*d*f)*sinh(1))
*cosh(d*x + c) + 2*(a^2*d^2*f*x + a^2*d^2*cosh(1) + a^2*d*f)*sinh(1))*sinh(
d*x + c)^3 + 4*(a*b*d^2*f^2*x^2 - 2*a*b*c^2*f^2 - a*b*d^2*cosh(1)^2 - a*b*d
^2*sinh(1)^2 + 2*(a*b*d^2*f*x + 2*a*b*c*d*f)*cosh(1) + 2*(a*b*d^2*f*x + 2*a
*b*c*d*f - a*b*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(2*a*b*d^2*f^2*x^2
 - 4*a*b*c^2*f^2 - 2*a*b*d^2*cosh(1)^2 - 2*a*b*d^2*sinh(1)^2 - 12*(a*b*d^2*
f^2*x^2 - a*b*c^2*f^2 + 2*(a*b*d^2*f*x + a*b*c*d*f)*cosh(1) + 2*(a*b*d^2*f*
x + a*b*c*d*f)*sinh(1))*cosh(d*x + c)^2 + 4*(a*b*d^2*f*x + 2*a*b*c*d*f)*cos
h(1) + 3*(a^2*d^2*f^2*x^2 + 2*a^2*d*f^2*x + a^2*d^2*cosh(1)^2 + a^2*d^2*sin
h(1)^2 + 2*(a^2*d^2*f*x + a^2*d*f)*cosh(1) + 2*(a^2*d^2*f*x + a^2*d^2*cosh(
1) + a^2*d*f)*sinh(1))*cosh(d*x + c) + 4*(a*b*d^2*f*x + 2*a*b*c*d*f - a*b*d
^2*cosh(1))*sinh(1))*sinh(d*x + c)^2 + 4*(b^2*d*f^2*x + b^2*d*f*cosh(1) + (
b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh(d*x + c)^4 + b^2*d*f*
sinh(1) + 4*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh(d*x + c)
*sinh(d*x + c)^3 + (b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*sinh(d
*x + c)^4 - 2*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh(d*x +
c)^2 - 2*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1) - 3*(b^2*d*f^2*x
+ b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(
(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh(d*x + c)^3 - (b^2*d*
f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*sq
rt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 4*(b^2*d*f^2*x
+ b^2*d*f*cosh(1) + (b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh(
d*x + c)^4 + b^2*d*f*sinh(1) + 4*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*s
inh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*f^2*x + b^2*d*f*cosh(1) + b^
2*d*f*sinh(1))*sinh(d*x + c)^4 - 2*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f
*sinh(1))*cosh(d*x + c)^2 - 2*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh
(1) - 3*(b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh(d*x + c)^2)*
sinh(d*x + c)^2 + 4*((b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh
(d*x + c)^3 - (b^2*d*f^2*x + b^2*d*f*cosh(1) + b^2*d*f*sinh(1))*cosh(d*x +
c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*
```

```
x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b +
 1) - 2*(b^2*c^2*f^2 - 2*b^2*c*d*f*cosh(1) + b^2*d^2*cosh(1)^2 + b^2*d^2*si
nh(1)^2 + (b^2*c^2*f^2 - 2*b^2*c*d*f*cosh(1) + b^2*d^2*cosh(1)^2 + b^2*d^2*
sinh(1)^2 - 2*(b^2*c*d*f - b^2*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^4 + 4*(b
^2*c^2*f^2 - 2*b^2*c*d*f*cosh(1) + b^2*d^2*cosh(1)^2 + b^2*d^2*sinh(1)^2 -
2*(b^2*c*d*f - b^2*d^2*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (b
^2*c^2*f^2 - 2*b^2*c*d*f*cosh(1) + b^2*d^2*cosh(1)^2 + b^2*d^2*sinh(1)^2 -
2*(b^2*c*d*f - b^2*d^2*cosh(1))*sinh(1))*sinh(d*x + c)^4 - 2*(b^2*c^2*f^2 -
 2*b^2*c*d*f*cosh(1) + b^2*d^2*cosh(1)^2 + b^2*d^2*sinh(1)^2 - 2*(b^2*c*d*f
 - b^2*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^2 - 2*(b^2*c^2*f^2 - 2*b^2*c*d*f
*cosh(1) + b^2*d^2*cosh(1)^2 + b^2*d^2*sinh(1)^2 - 3*(b^2*c^2*f^2 - 2*b^2*c
*d*f*cosh(1) + b^2*d^2*cosh(1)^2 + b^2*d^2*sinh(1)^2 - 2*(b^2*c*d*f - b^2*d
^2*cosh(1))*sinh(1))*cosh(d*x + c)^2 - 2*(b^2*c*d*f - b^2*d^2*cosh(1))*sinh
(1))*sinh(d*x + c)^2 - 2*(b^2*c*d*f - b^2*d^2*cosh(1))*sinh(1) + 4*((b^2*c^
2*f^2 - 2*b^2*c*d*f*cosh(1) + b^2*d^2*cosh(1)^2 + b^2*d^2*sinh(1)^2 - 2*(b^
2*c*d*f - b^2*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^3 - (b^2*c^2*f^2 - 2*b^2*
c*d*f*cosh(1) + b^2*d^2*cosh(1)^2 + b^2*d^2*sinh(1)^2 - 2*(b^2*c*d*f - b^2*
d^2*cosh(1))*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*l
og(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
 + 2*(b^2*c^2*f^2 - 2*b^2*c*d*f*cosh(1) + b^2*d^2*cosh(1)^2 + b^2*d^2*sinh(
1)^2 + (b^2*c^2*f^2 - 2*b^2*c*d*f*cosh(1) + b^2*d^2*cosh(1)^2 + b^2*d^2*sin
h(1)^2 - 2*(b^2*c*d*f - b^2*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^4 + 4*(b^2*
c^2*f^2 - 2*b^2*c*d*f*cosh(1) + b^2*d^2*cosh(1)^2 + b^2*d^2*sinh(1)^2 - 2*(
b^2*c*d*f - b^2*d^2*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*
c^2*f^2 - 2*b^2*c*d*f*cosh(1) + b^2*d^2*cosh(1)^2 + b^2*d^2*sinh(1)^2 - 2*(
b^2*c*d*f - b^2*d^2*cosh(1))*sinh(1))*sinh(d*x + c)^4 - 2*(b^2*c^2*f^2 - 2*
b^2*c*d*f*cosh(1) + b^2*d^2*cosh(1)^2 + b^2*d^2...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*coth(d*x+c)**2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(c+d\,x)^2\,{(e+f\,x)}^2}{\sinh\left(c+d\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(c + d*x)^2*(e + f*x)^2)/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
[Out] int((coth(c + d*x)^2*(e + f*x)^2)/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.483 $\int \frac{(e+fx)\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=413

$$-\frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d}+\frac{b(e+fx)\coth(c+dx)}{a^2d}-\frac{f\operatorname{csch}(c+dx)}{2ad^2}-\frac{(e+fx)\,\mathrm{c}}{}$$

```
[Out] -(f*x+e)*arctanh(exp(d*x+c))/a/d-2*b^2*(f*x+e)*arctanh(exp(d*x+c))/a^3/d+b*
(f*x+e)*coth(d*x+c)/a^2/d-1/2*f*csch(d*x+c)/a/d^2-1/2*(f*x+e)*coth(d*x+c)*c
sch(d*x+c)/a/d-b*f*ln(sinh(d*x+c))/a^2/d^2-1/2*f*polylog(2,-exp(d*x+c))/a/d
^2-b^2*f*polylog(2,-exp(d*x+c))/a^3/d^2+1/2*f*polylog(2,exp(d*x+c))/a/d^2+b
^2*f*polylog(2,exp(d*x+c))/a^3/d^2-b*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)
^(1/2)))*(a^2+b^2)^(1/2)/a^3/d+b*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/
2)))*(a^2+b^2)^(1/2)/a^3/d-b*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))
*(a^2+b^2)^(1/2)/a^3/d^2+b*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))*(
a^2+b^2)^(1/2)/a^3/d^2
```

Rubi [A]
time = 0.65, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 17, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.531$, Rules used = {5706, 5565, 4267, 2317, 2438, 4270, 5688, 3801, 3556, 5704, 5558, 3377, 2717, 5684, 3403, 2296, 2221}

$$-\frac{b^2 f\,\mathrm{Li}_2\left(-e^{c+dx}\right)}{a^3d^2}+\frac{b^2 f\,\mathrm{Li}_2\left(e^{c+dx}\right)}{a^3d^2}-\frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d}-\frac{bf\log(\sinh(c+dx))}{a^2d^2}+\frac{b(e+fx)\coth(c+dx)}{a^2d}-\frac{bf\sqrt{a^2+b^2}\,\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}+\frac{bf\sqrt{a^2+b^2}\,\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2}-\frac{b\sqrt{a^2+b^2}(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^3d}+\frac{b\sqrt{a^2+b^2}(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{a^3d}-\frac{f\,\mathrm{Li}_2\left(-e^{c+dx}\right)}{2ad^2}+\frac{f\,\mathrm{Li}_2\left(e^{c+dx}\right)}{2ad^2}-\frac{f\operatorname{csch}(c+dx)}{2ad^2}-\frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(((e + f*x)*ArcTanh[E^(c + d*x)])/(a*d)) - (2*b^2*(e + f*x)*ArcTanh[E^(c +
 d*x)])/(a^3*d) + (b*(e + f*x)*Coth[c + d*x])/(a^2*d) - (f*Csch[c + d*x])/(
2*a*d^2) - ((e + f*x)*Coth[c + d*x]*Csch[c + d*x])/(2*a*d) - (b*Sqrt[a^2 +
b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*d) + (b
*Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/
(a^3*d) - (b*f*Log[Sinh[c + d*x]])/(a^2*d^2) - (f*PolyLog[2, -E^(c + d*x)])
/(2*a*d^2) - (b^2*f*PolyLog[2, -E^(c + d*x)])/(a^3*d^2) + (f*PolyLog[2, E^(
c + d*x)])/(2*a*d^2) + (b^2*f*PolyLog[2, E^(c + d*x)])/(a^3*d^2) - (b*Sqrt[
a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2
) + (b*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)])/(a^3*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5565

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5688

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x]^
(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5704

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5706

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Csch[c + d*x]^(p - 1)*(Coth[c + d*x]^n/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\coth^2(c+dx)\operatorname{csch}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,d}{a}$$

$$= \frac{\int (e+fx)\operatorname{csch}(c+dx)\,dx}{a} + \frac{\int (e+fx)\operatorname{csch}^3(c+dx)\,dx}{a} - \frac{b\int (e}{}$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{b(e+fx)\coth(c+dx)}{a^2d} - \frac{f\operatorname{csch}(c}{2a}$$

$$= -\frac{bex}{a^2} - \frac{bfx^2}{2a^2} - \frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{b(e+fx)\coth(c+}{a^2d}$$

$$= -\frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} + \frac{b(e+}{}$$

$$= -\frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} + \frac{b(e+}{}$$

$$= -\frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} + \frac{b(e+}{}$$

$$= -\frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} + \frac{b(e+}{}$$

$$= -\frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} + \frac{b(e+}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.36, size = 734, normalized size = 1.78

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] ((2*b*d*e*Cosh[(c + d*x)/2] - a*f*Cosh[(c + d*x)/2] - 2*b*c*f*Cosh[(c + d*x
)/2] + 2*b*f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(4*a^2*d^2) +
((-(d*e) + c*f - f*(c + d*x))*Csch[(c + d*x)/2]^2)/(8*a*d^2) - (b*f*Log[Sin
h[c + d*x]])/(a^2*d^2) + (e*Log[Tanh[(c + d*x)/2]])/(2*a*d) + (b^2*e*Log[Ta
nh[(c + d*x)/2]])/(a^3*d) - (c*f*Log[Tanh[(c + d*x)/2]])/(2*a*d^2) - (b^2*c
```

```
*f*Log[Tanh[(c + d*x)/2]])/(a^3*d^2) - ((I/2)*f*(I*(c + d*x)*(Log[1 - E^(-c
 - d*x)] - Log[1 + E^(-c - d*x)]) + I*(PolyLog[2, -E^(-c - d*x)] - PolyLog[
2, E^(-c - d*x)])))/(a*d^2) - (I*b^2*f*(I*(c + d*x)*(Log[1 - E^(-c - d*x)]
- Log[1 + E^(-c - d*x)]) + I*(PolyLog[2, -E^(-c - d*x)] - PolyLog[2, E^(-c
- d*x)])))/(a^3*d^2) + (b*Sqrt[a^2 + b^2]*(2*d*e*ArcTanh[(a + b*E^(c + d*x)
)/Sqrt[a^2 + b^2]] - 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - f
*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log
[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - f*PolyLog[2, (b*E^(c + d*x))/
(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2]))]))/(a^3*d^2) + ((-(d*e) + c*f - f*(c + d*x))*Sech[(c + d*x)/2]^2)/(8*a
*d^2) + (Sech[(c + d*x)/2]*(2*b*d*e*Sinh[(c + d*x)/2] + a*f*Sinh[(c + d*x)/
2] - 2*b*c*f*Sinh[(c + d*x)/2] + 2*b*f*(c + d*x)*Sinh[(c + d*x)/2]))/(4*a^2
*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1283 vs. $2(379) = 758$.

time = 11.73, size = 1284, normalized size = 3.11

method	result	size
risch	Expression too large to display	1284

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVER
BOSE)
```

```
[Out] -2/d^2*b*f*c/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(
1/2))-1/d*b*f/a/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a
^2+b^2)^(1/2)))*x-1/d^2*b*f/a/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(
1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d^2*b*f/a/(a^2+b^2)^(1/2)*dilog((-b*exp(d
*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2/a^3*b^3*f/(a^2+b^2)^(1
/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d/a^3*b^
3*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))
)*x-1/d/a^3*b^3*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+
(a^2+b^2)^(1/2)))*x-1/d/a^3*b^2*e*ln(exp(d*x+c)+1)+1/d/a^3*b^2*e*ln(exp(d*x
+c)-1)-1/d^2/a^2*b*f*ln(exp(d*x+c)+1)-1/d^2/a^2*b*f*ln(exp(d*x+c)-1)-2/d^2/
a^3*b^3*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2
))+1/d^2/a^3*b^3*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(
a^2+b^2)^(1/2)))*c+1/d^2*b*f/a/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2
)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/d*b*e/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*
exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/2/d^2*f/a*dilog(exp(d*x+c)+1)-1/2/d^2*f*
dilog(exp(d*x+c))/a+2/d/a^3*b^3*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+
c)+2*a)/(a^2+b^2)^(1/2))+1/d^2/a^3*b^3*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c
)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2/a^3*b^3*f/(a^2+b^2)^(1/2)*d
ilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d/a^3*b^2*f*
ln(exp(d*x+c)+1)*x-1/d^2/a^3*b^2*f*c*ln(exp(d*x+c)-1)+1/d*b*f/a/(a^2+b^2)^(
```

```
1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2*b*f/a
/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c
+1/2/d*e/a*ln(exp(d*x+c)-1)-1/2/d*e/a*ln(exp(d*x+c)+1)-1/2/d*f/a*ln(exp(d*x
+c)+1)*x-1/2/d^2*f*c/a*ln(exp(d*x+c)-1)-1/d^2/a^3*b^2*f*dilog(exp(d*x+c))+2
/d^2/a^2*b*f*ln(exp(d*x+c))-1/d^2/a^3*b^2*f*dilog(exp(d*x+c)+1)-(a*d*f*x*ex
p(3*d*x+3*c)+a*d*e*exp(3*d*x+3*c)-2*b*d*f*x*exp(2*d*x+2*c)+a*d*f*x*exp(d*x+
c)+a*f*exp(3*d*x+3*c)-2*b*d*e*exp(2*d*x+2*c)+a*d*e*exp(d*x+c)+2*b*d*f*x-a*f
*exp(d*x+c)+2*b*e*d)/d^2/a^2/(exp(2*d*x+2*c)-1)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] (2*a^2*d*integrate(1/4*x/(a^3*d*e^(d*x + c) + a^3*d), x) + 4*b^2*d*integrat
e(1/4*x/(a^3*d*e^(d*x + c) + a^3*d), x) + 2*a^2*d*integrate(1/4*x/(a^3*d*e^
(d*x + c) - a^3*d), x) + 4*b^2*d*integrate(1/4*x/(a^3*d*e^(d*x + c) - a^3*d
), x) + a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) + 1)/(a^3*d^2)) + a*b*((
d*x + c)/(a^3*d^2) - log(e^(d*x + c) - 1)/(a^3*d^2)) - 2*(a^2*b*e^c + b^3*e
^c)*integrate(x*e^(d*x)/(a^3*b*e^(2*d*x + 2*c) + 2*a^4*e^(d*x + c) - a^3*b)
, x) + (2*b*d*x*e^(2*d*x + 2*c) - 2*b*d*x - (a*d*x*e^(3*c) + a*e^(3*c))*e^(
3*d*x) - (a*d*x*e^c - a*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*
e^(2*d*x + 2*c) + a^2*d^2))*f + 1/2*(2*(a*e^(-d*x - c) + 2*b*e^(-2*d*x - 2*
c) + a*e^(-3*d*x - 3*c) - 2*b)/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4
*c) - a^2)*d) - (a^2 + 2*b^2)*log(e^(-d*x - c) + 1)/(a^3*d) + (a^2 + 2*b^2)
*log(e^(-d*x - c) - 1)/(a^3*d) - 2*(a^2*b + b^3)*log((b*e^(-d*x - c) - a -
sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a
^3*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4079 vs. 2(379) = 758.
time = 0.40, size = 4079, normalized size = 9.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] 1/2*(4*(a*b*d*f*x + a*b*c*f)*cosh(d*x + c)^4 + 4*(a*b*d*f*x + a*b*c*f)*sinh
(d*x + c)^4 + 4*a*b*c*f - 4*a*b*d*cosh(1) - 2*(a^2*d*f*x + a^2*d*cosh(1) +
a^2*d*sinh(1) + a^2*f)*cosh(d*x + c)^3 - 4*a*b*d*sinh(1) - 2*(a^2*d*f*x + a
```

^2*d*cosh(1) + a^2*d*sinh(1) + a^2*f - 8*(a*b*d*f*x + a*b*c*f)*cosh(d*x + c
))*sinh(d*x + c)^3 - 4*(a*b*d*f*x + 2*a*b*c*f - a*b*d*cosh(1) - a*b*d*sinh(
1))*cosh(d*x + c)^2 - 2*(2*a*b*d*f*x + 4*a*b*c*f - 2*a*b*d*cosh(1) - 2*a*b*
d*sinh(1) - 12*(a*b*d*f*x + a*b*c*f)*cosh(d*x + c)^2 + 3*(a^2*d*f*x + a^2*d
*cosh(1) + a^2*d*sinh(1) + a^2*f)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(b^2*f
*cosh(d*x + c)^4 + 4*b^2*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*f*sinh(d*x +
 c)^4 - 2*b^2*f*cosh(d*x + c)^2 + b^2*f + 2*(3*b^2*f*cosh(d*x + c)^2 - b^2*
f)*sinh(d*x + c)^2 + 4*(b^2*f*cosh(d*x + c)^3 - b^2*f*cosh(d*x + c))*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b
^2*f*cosh(d*x + c)^4 + 4*b^2*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*f*sinh(d
*x + c)^4 - 2*b^2*f*cosh(d*x + c)^2 + b^2*f + 2*(3*b^2*f*cosh(d*x + c)^2 -
b^2*f)*sinh(d*x + c)^2 + 4*(b^2*f*cosh(d*x + c)^3 - b^2*f*cosh(d*x + c))*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c)
 - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) -
2*((b^2*c*f - b^2*d*cosh(1) - b^2*d*sinh(1))*cosh(d*x + c)^4 + 4*(b^2*c*f -
 b^2*d*cosh(1) - b^2*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*c*f -
b^2*d*cosh(1) - b^2*d*sinh(1))*sinh(d*x + c)^4 + b^2*c*f - b^2*d*cosh(1) -
b^2*d*sinh(1) - 2*(b^2*c*f - b^2*d*cosh(1) - b^2*d*sinh(1))*cosh(d*x + c)^2
 - 2*(b^2*c*f - b^2*d*cosh(1) - b^2*d*sinh(1) - 3*(b^2*c*f - b^2*d*cosh(1)
- b^2*d*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*c*f - b^2*d*cos
h(1) - b^2*d*sinh(1))*cosh(d*x + c)^3 - (b^2*c*f - b^2*d*cosh(1) - b^2*d*si
nh(1))*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x
 + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*((b^2*c*f
- b^2*d*cosh(1) - b^2*d*sinh(1))*cosh(d*x + c)^4 + 4*(b^2*c*f - b^2*d*cosh(
1) - b^2*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*c*f - b^2*d*cosh(1
) - b^2*d*sinh(1))*sinh(d*x + c)^4 + b^2*c*f - b^2*d*cosh(1) - b^2*d*sinh(1
) - 2*(b^2*c*f - b^2*d*cosh(1) - b^2*d*sinh(1))*cosh(d*x + c)^2 - 2*(b^2*c*
f - b^2*d*cosh(1) - b^2*d*sinh(1) - 3*(b^2*c*f - b^2*d*cosh(1) - b^2*d*sinh
(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*c*f - b^2*d*cosh(1) - b^2*d
*sinh(1))*cosh(d*x + c)^3 - (b^2*c*f - b^2*d*cosh(1) - b^2*d*sinh(1))*cosh(
d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*
sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(b^2*d*f*x + (b^2*d*f*
x + b^2*c*f)*cosh(d*x + c)^4 + 4*(b^2*d*f*x + b^2*c*f)*cosh(d*x + c)*sinh(d
*x + c)^3 + (b^2*d*f*x + b^2*c*f)*sinh(d*x + c)^4 + b^2*c*f - 2*(b^2*d*f*x
+ b^2*c*f)*cosh(d*x + c)^2 - 2*(b^2*d*f*x + b^2*c*f - 3*(b^2*d*f*x + b^2*c*
f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d*f*x + b^2*c*f)*cosh(d*x + c
)^3 - (b^2*d*f*x + b^2*c*f)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(b^2*d*f*x + (b^2*d*f*x + b^2*c*f
)*cosh(d*x + c)^4 + 4*(b^2*d*f*x + b^2*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (b^2*d*f*x + b^2*c*f)*sinh(d*x + c)^4 + b^2*c*f - 2*(b^2*d*f*x + b^2*c*f)*
cosh(d*x + c)^2 - 2*(b^2*d*f*x + b^2*c*f - 3*(b^2*d*f*x + b^2*c*f)*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d*f*x + b^2*c*f)*cosh(d*x + c)^3 - (b^2*
d*f*x + b^2*c*f)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(

```
a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqr
t((a^2 + b^2)/b^2) - b)/b) - 2*(a^2*d*f*x + a^2*d*cosh(1) + a^2*d*sinh(1) -
 a^2*f)*cosh(d*x + c) + ((a^2 + 2*b^2)*f*cosh(d*x + c)^4 + 4*(a^2 + 2*b^2)*
f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*b^2)*f*sinh(d*x + c)^4 - 2*(a^2
+ 2*b^2)*f*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*b^2)*f*cosh(d*x + c)^2 - (a^2 +
2*b^2)*f)*sinh(d*x + c)^2 + (a^2 + 2*b^2)*f + 4*((a^2 + 2*b^2)*f*cosh(d*x +
 c)^3 - (a^2 + 2*b^2)*f*cosh(d*x + c))*sinh(d*x + c))*dilog(cosh(d*x + c) +
 sinh(d*x + c)) - ((a^2 + 2*b^2)*f*cosh(d*x + c)^4 + 4*(a^2 + 2*b^2)*f*cosh
(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*b^2)*f*sinh(d*x + c)^4 - 2*(a^2 + 2*b^
2)*f*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*b^2)*f*cosh(d*x + c)^2 - (a^2 + 2*b^2)
*f)*sinh(d*x + c)^2 + (a^2 + 2*b^2)*f + 4*((a^2 + 2*b^2)*f*cosh(d*x + c)^3
- (a^2 + 2*b^2)*f*cosh(d*x + c))*sinh(d*x + c))*dilog(-cosh(d*x + c) - sinh
(d*x + c)) - (((a^2 + 2*b^2)*d*f*x + 2*a*b*f + (a^2 + 2*b^2)*d*cosh(1) + (a
^2 + 2*b^2)*d*sinh(1))*cosh(d*x + c)^4 + 4*((a^2 + 2*b^2)*d*f*x + 2*a*b*f +
 (a^2 + 2*b^2)*d*cosh(1) + (a^2 + 2*b^2)*d*sinh(1))*cosh(d*x + c)*sinh(d*x
+ c)^3 + ((a^2 + 2*b^2)*d*f*x + 2*a*b*f + (a^2 + 2*b^2)*d*cosh(1) + (a^2 +
2*b^2)*d*sinh(1))*sinh(d*x + c)^4 + (a^2 + 2*b^2)*d*f*x + 2*a*b*f + (a^2 +
2*b^2)*d*cosh(1) - 2*((a^2 + 2*b^2)*d*f*x + 2*a*b*f + (a^2 + 2*b^2)*d*cosh(
1) + (a^2 + 2*b^2)*d*sinh(1))*cosh(d*x + c)^2 +...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e + fx)\coth^2(c + dx)\operatorname{csch}(c + dx)}{a + b\sinh(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)**2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*coth(c + d*x)**2*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(c + d\,x)^2\,(e + f\,x)}{\sinh(c + d\,x)\,\left(a + b\sinh(c + d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(c + d*x)^2*(e + f*x))/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
[Out] int((coth(c + d*x)^2*(e + f*x))/(sinh(c + d*x)*(a + b*sinh(c + d*x))), x)
```

3.484 $\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=111

$$-\frac{(a^2+2b^2)\tanh^{-1}(\cosh(c+dx))}{2a^3d}+\frac{2b\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3d}+\frac{b\coth(c+dx)}{a^2d}-\frac{\coth(c+dx)\text{cs}}{2ad}$$

```
[Out] -1/2*(a^2+2*b^2)*arctanh(cosh(d*x+c))/a^3/d+b*coth(d*x+c)/a^2/d-1/2*coth(d*
x+c)*csch(d*x+c)/a/d+2*b*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))
*(a^2+b^2)^(1/2)/a^3/d
```

Rubi [A]
time = 0.37, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2968, 3135, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{b\coth(c+dx)}{a^2d}+\frac{2b\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3d}-\frac{(a^2+2b^2)\tanh^{-1}(\cosh(c+dx))}{2a^3d}-\frac{\coth(c+dx)\text{csch}(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -1/2*((a^2 + 2*b^2)*ArcTanh[Cosh[c + d*x]])/(a^3*d) + (2*b*Sqrt[a^2 + b^2]*
ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^3*d) + (b*Coth[c + d
*x])/(a^2*d) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\operatorname{csch}^3(c+dx)\left(1+\sinh^2(c+dx)\right)}{a+b\sinh(c+dx)}\,dx$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} + \frac{i\int \frac{\operatorname{csch}^2(c+dx)\left(2ib-ia\sinh(c+dx)+ib\sinh^2(c+dx)\right)}{a+b\sinh(c+dx)}\,d}{2a}$$

$$= \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{\int \frac{\operatorname{csch}(c+dx)\left(-a^2-2b^2+ab\sinh(\right.}{a+b\sinh(c+dx)}}{2a^2}$$

$$= \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{\left(b\left(a^2+b^2\right)\right)\int \frac{1}{a+b\sinh(c+dx)}}{a^3}$$

$$= -\frac{\left(a^2+2b^2\right)\tanh^{-1}(\cosh(c+dx))}{2a^3d} + \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(}{2ad}$$

$$= -\frac{\left(a^2+2b^2\right)\tanh^{-1}(\cosh(c+dx))}{2a^3d} + \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(}{2ad}$$

$$= -\frac{\left(a^2+2b^2\right)\tanh^{-1}(\cosh(c+dx))}{2a^3d} + \frac{2b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx\right.}{\sqrt{a^2+b^2}}\right.}{a^3d}$$

Mathematica [A]
time = 1.02, size = 145, normalized size = 1.31

$$\frac{16b\sqrt{-a^2-b^2}\,\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)+4ab\coth\left(\frac{1}{2}(c+dx)\right)-a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+4\left(a^2+2b^2\right)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)+4ab\tanh\left(\frac{1}{2}(c+dx)\right)}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]

[Out] (16*b*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] +
 4*a*b*Coth[(c + d*x)/2] - a^2*Csch[(c + d*x)/2]^2 + 4*(a^2 + 2*b^2)*Log[Ta
nh[(c + d*x)/2]] - a^2*Sech[(c + d*x)/2]^2 + 4*a*b*Tanh[(c + d*x)/2])/(8*a^
3*d)
```

Maple [A]
time = 2.62, size = 140, normalized size = 1.26

method	result
derivativedivides	$\frac{\frac{a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{2b\sqrt{a^2+b^2}\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^3}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(2a^2+4b^2\right)\ln\left(\text{ta}\right.}{4a^3}$ over d
default	$\frac{\frac{a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{2b\sqrt{a^2+b^2}\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^3}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(2a^2+4b^2\right)\ln\left(\text{ta}\right.}{4a^3}$ over d
risch	$-\frac{a\,e^{3dx+3c}-2b\,e^{2dx+2c}+a\,e^{dx+c}+2b}{a^2d\left(e^{2dx+2c}-1\right)^2}+\frac{\sqrt{a^2+b^2}\,b\ln\left(e^{dx+c}+\frac{a+\sqrt{a^2+b^2}}{b}\right)}{d\,a^3}-\frac{\sqrt{a^2+b^2}\,b\ln\left(e^{dx+c}\right.}{d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a^2*(1/2*a*tanh(1/2*d*x+1/2*c)^2+2*b*tanh(1/2*d*x+1/2*c))-2*b*(a^2
+b^2)^(1/2)/a^3*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-
1/8/a/tanh(1/2*d*x+1/2*c)^2+1/4/a^3*(2*a^2+4*b^2)*ln(tanh(1/2*d*x+1/2*c))+1
/2*b/a^2/tanh(1/2*d*x+1/2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(104) = 208$.
time = 0.48, size = 217, normalized size = 1.95

$$\frac{ae^{(-dx-c)}+2\,be^{(-2\,dx-2\,c)}+ae^{(-3\,dx-3\,c)}-2\,b}{\left(2\,a^2e^{(-2\,dx-2\,c)}-a^2e^{(-4\,dx-4\,c)}-a^2\right)d}-\frac{\left(a^2+2\,b^2\right)\log\left(e^{(-dx-c)}+1\right)}{2\,a^3d}+\frac{\left(a^2+2\,b^2\right)\log\left(e^{(-dx-c)}-1\right)}{2\,a^3d}-\frac{\left(a^2b+b^3\right)\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] (a*e^(-d*x - c) + 2*b*e^(-2*d*x - 2*c) + a*e^(-3*d*x - 3*c) - 2*b)/((2*a^2*
e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - 1/2*(a^2 + 2*b^2)*log(e
^(-d*x - c) + 1)/(a^3*d) + 1/2*(a^2 + 2*b^2)*log(e^(-d*x - c) - 1)/(a^3*d)
- (a^2*b + b^3)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c)
- a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 892 vs. $2(104) = 208$.
time = 0.38, size = 892, normalized size = 8.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/2*(2*a^2*cosh(d*x + c)^3 + 2*a^2*sinh(d*x + c)^3 - 4*a*b*cosh(d*x + c)^2
 + 2*a^2*cosh(d*x + c) + 2*(3*a^2*cosh(d*x + c) - 2*a*b)*sinh(d*x + c)^2 -
2*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^
4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 4*(
b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(a^2 + b^2)*log
((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 +
 b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cos
h(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 +
 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 4*a*b +
((a^2 + 2*b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*b^2)*cosh(d*x + c)*sinh(d*x + c
)^3 + (a^2 + 2*b^2)*sinh(d*x + c)^4 - 2*(a^2 + 2*b^2)*cosh(d*x + c)^2 + 2*(
3*(a^2 + 2*b^2)*cosh(d*x + c)^2 - a^2 - 2*b^2)*sinh(d*x + c)^2 + a^2 + 2*b^
2 + 4*((a^2 + 2*b^2)*cosh(d*x + c)^3 - (a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*
x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - ((a^2 + 2*b^2)*cosh(d*x +
c)^4 + 4*(a^2 + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*b^2)*sinh(d
*x + c)^4 - 2*(a^2 + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*b^2)*cosh(d*x +
 c)^2 - a^2 - 2*b^2)*sinh(d*x + c)^2 + a^2 + 2*b^2 + 4*((a^2 + 2*b^2)*cosh(
d*x + c)^3 - (a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c)
+ sinh(d*x + c) - 1) + 2*(3*a^2*cosh(d*x + c)^2 - 4*a*b*cosh(d*x + c) + a^2
)*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)*sinh(d*x +
c)^3 + a^3*d*sinh(d*x + c)^4 - 2*a^3*d*cosh(d*x + c)^2 + a^3*d + 2*(3*a^3*d
*cosh(d*x + c)^2 - a^3*d)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^3 - a^3*
d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)**2*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A]
time = 0.45, size = 182, normalized size = 1.64

$$-\frac{\frac{(a^2+2b^2)\log\left(e^{(dx+c)}+1\right)}{a^3}-\frac{(a^2+2b^2)\log\left(\left|e^{(dx+c)}-1\right|\right)}{a^3}+\frac{2\,(a^2b+b^3)\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a^3}+\frac{2\,\left(ae^{(3\,dx+3\,c)}-2\,be^{(2\,dx+2\,c)}+ae^{(dx+c)}+2\,b\right)}{a^2\left(e^{(2\,dx+2\,c)}-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*((a^2 + 2*b^2)*log(e^(d*x + c) + 1)/a^3 - (a^2 + 2*b^2)*log(abs(e^(d*x
 + c) - 1))/a^3 + 2*(a^2*b + b^3)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^
2 + b^2))/abs(2*b*e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*
a^3) + 2*(a*e^(3*d*x + 3*c) - 2*b*e^(2*d*x + 2*c) + a*e^(d*x + c) + 2*b)/(a
^2*(e^(2*d*x + 2*c) - 1)^2))/d
```

Mupad [B]
time = 0.69, size = 628, normalized size = 5.66

$$\frac{e^{c+dx}}{a\,d-a\,d\,e^{2c+2dx}}-\frac{2\,e^{c+dx}}{a\,d-2\,a\,d\,e^{2c+2dx}+a\,d\,e^{4c+4dx}}-\frac{2\,b}{a^2\,d-a^2\,d\,e^{2c+2dx}}+\frac{\ln\left(4\,a^4+8\,b^4+12\,a^2\,b^2-4\,a^4\,e^{dx}\,e^c-8\,b^4\,e^{dx}\,e^c-12\,a^2\,b^2\,e^{dx}\,e^c\right)}{2\,a\,d}-\frac{\ln\left(4\,a^4+8\,b^4+12\,a^2\,b^2+4\,a^4\,e^{dx}\,e^c+8\,b^4\,e^{dx}\,e^c+12\,a^2\,b^2\,e^{dx}\,e^c\right)}{2\,a\,d}+\frac{b^2\,\ln\left(4\,a^4+8\,b^4+12\,a^2\,b^2-4\,a^4\,e^{dx}\,e^c-8\,b^4\,e^{dx}\,e^c-12\,a^2\,b^2\,e^{dx}\,e^c\right)}{a^3\,d}-\frac{b^2\,\ln\left(4\,a^4+8\,b^4+12\,a^2\,b^2+4\,a^4\,e^{dx}\,e^c+8\,b^4\,e^{dx}\,e^c+12\,a^2\,b^2\,e^{dx}\,e^c\right)}{a^3\,d}-\frac{b\,\ln\left(32\,a^4\,e^{dx}\,e^c-16\,a\,b^3-16\,a^3\,b-8\,b^3\,\sqrt{a^2+b^2}+8\,b^4\,e^{dx}\,e^c-16\,a^2\,b\,\sqrt{a^2+b^2}+40\,a^2\,b^2\,e^{dx}\,e^c+32\,a^3\,e^{dx}\,e^c\,\sqrt{a^2+b^2}+24\,a\,b^2\,e^{dx}\,e^c\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{a^3\,d}+\frac{b\,\ln\left(8\,b^3\,\sqrt{a^2+b^2}-16\,a\,b^3-16\,a^3\,b+32\,a^4\,e^{dx}\,e^c+8\,b^4\,e^{dx}\,e^c+16\,a^2\,b\,\sqrt{a^2+b^2}+40\,a^2\,b^2\,e^{dx}\,e^c-32\,a^3\,e^{dx}\,e^c\,\sqrt{a^2+b^2}-24\,a\,b^2\,e^{dx}\,e^c\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2/(sinh(c + d*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] exp(c + d*x)/(a*d - a*d*exp(2*c + 2*d*x)) - (2*exp(c + d*x))/(a*d - 2*a*d*e
xp(2*c + 2*d*x) + a*d*exp(4*c + 4*d*x)) - (2*b)/(a^2*d - a^2*d*exp(2*c + 2*
d*x)) + log(4*a^4 + 8*b^4 + 12*a^2*b^2 - 4*a^4*exp(d*x)*exp(c) - 8*b^4*exp(
d*x)*exp(c) - 12*a^2*b^2*exp(d*x)*exp(c))/(2*a*d) - log(4*a^4 + 8*b^4 + 12*
a^2*b^2 + 4*a^4*exp(d*x)*exp(c) + 8*b^4*exp(d*x)*exp(c) + 12*a^2*b^2*exp(d*
x)*exp(c))/(2*a*d) + (b^2*log(4*a^4 + 8*b^4 + 12*a^2*b^2 - 4*a^4*exp(d*x)*e
xp(c) - 8*b^4*exp(d*x)*exp(c) - 12*a^2*b^2*exp(d*x)*exp(c)))/(a^3*d) - (b^2
*log(4*a^4 + 8*b^4 + 12*a^2*b^2 + 4*a^4*exp(d*x)*exp(c) + 8*b^4*exp(d*x)*ex
p(c) + 12*a^2*b^2*exp(d*x)*exp(c)))/(a^3*d) - (b*log(32*a^4*exp(d*x)*exp(c)
 - 16*a*b^3 - 16*a^3*b - 8*b^3*(a^2 + b^2)^(1/2) + 8*b^4*exp(d*x)*exp(c) -
16*a^2*b*(a^2 + b^2)^(1/2) + 40*a^2*b^2*exp(d*x)*exp(c) + 32*a^3*exp(d*x)*e
xp(c)*(a^2 + b^2)^(1/2) + 24*a*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2
+ b^2)^(1/2))/(a^3*d) + (b*log(8*b^3*(a^2 + b^2)^(1/2) - 16*a*b^3 - 16*a^3*
b + 32*a^4*exp(d*x)*exp(c) + 8*b^4*exp(d*x)*exp(c) + 16*a^2*b*(a^2 + b^2)^(
1/2) + 40*a^2*b^2*exp(d*x)*exp(c) - 32*a^3*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2
) - 24*a*b^2*exp(d*x)*exp(c)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^3*d)
```

3.485 $\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=37

$$\operatorname{Int}\left(\frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Coth[c + d*x]^2*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Coth[c + d*x]^2*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] Integrate[(Coth[c + d*x]^2*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\coth^2\left(dx+c\right)\right)\operatorname{csch}(dx+c)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -2*(a^2*b*e^c + b^3*e^c)*integrate(-e^(d*x)/(a^3*b*f*x + a^3*b*e - (a^3*b*f
*x*e^(2*c) + a^3*b*e^(2*c + 1))*e^(2*d*x) - 2*(a^4*f*x*e^c + a^4*e^(c + 1))
*e^(d*x)), x) - (2*b*d*f*x + 2*b*d*e + (a*d*f*x*e^(3*c) - a*f*e^(3*c) + a*d
*e^(3*c + 1))*e^(3*d*x) - 2*(b*d*f*x*e^(2*c) + b*d*e^(2*c + 1))*e^(2*d*x) +
 (a*d*f*x*e^c + a*d*e^(c + 1) + a*f*e^c)*e^(d*x))/(a^2*d^2*f^2*x^2 + 2*a^2*
d^2*f*x*e + a^2*d^2*e^2 + (a^2*d^2*f^2*x^2*e^(4*c) + 2*a^2*d^2*f*x*e^(4*c +
 1) + a^2*d^2*e^(4*c + 2))*e^(4*d*x) - 2*(a^2*d^2*f^2*x^2*e^(2*c) + 2*a^2*d
^2*f*x*e^(2*c + 1) + a^2*d^2*e^(2*c + 2))*e^(2*d*x)) + 2*integrate(-1/4*(2*
a*b*d*f*e + 2*a^2*f^2 + (a^2*d^2*f^2 + 2*b^2*d^2*f^2)*x^2 + 2*(a*b*d*f^2 +
(a^2*d^2*f + 2*b^2*d^2*f)*e)*x + (a^2*d^2 + 2*b^2*d^2)*e^2)/(a^3*d^2*f^3*x^
3 + 3*a^3*d^2*f^2*x^2*e + 3*a^3*d^2*f*x*e^2 + a^3*d^2*e^3 - (a^3*d^2*f^3*x^
3*e^c + 3*a^3*d^2*f^2*x^2*e^(c + 1) + 3*a^3*d^2*f*x*e^(c + 2) + a^3*d^2*e^(
c + 3))*e^(d*x)), x) + 2*integrate(-1/4*(2*a*b*d*f*e - 2*a^2*f^2 - (a^2*d^2
*f^2 + 2*b^2*d^2*f^2)*x^2 + 2*(a*b*d*f^2 - (a^2*d^2*f + 2*b^2*d^2*f)*e)*x -
 (a^2*d^2 + 2*b^2*d^2)*e^2)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*f^2*x^2*e + 3*a^3*
d^2*f*x*e^2 + a^3*d^2*e^3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*f^2*x^2*e^(c +
 1) + 3*a^3*d^2*f*x*e^(c + 2) + a^3*d^2*e^(c + 3))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(coth(d*x + c)^2*csch(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)**2*csch(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)),
x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\coth(c+d\,x)^2}{\sinh\left(c+d\,x\right)\,\left(e+f\,x\right)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2/(sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(coth(c + d*x)^2/(sinh(c + d*x)*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.486 $\int \frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=972

$$-\frac{3f(e+fx)^2}{2ad^2}+\frac{(e+fx)^3}{2ad}-\frac{(e+fx)^4}{4af}-\frac{b^2(e+fx)^4}{4a^3f}+\frac{(a^2+b^2)(e+fx)^4}{4a^3f}+\frac{6bf(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2}-3$$

```
[Out] 3*f^2*(f*x+e)*ln(1-exp(2*d*x+2*c))/a/d^3+b^2*(f*x+e)^3*ln(1-exp(2*d*x+2*c))
/a^3/d+3/4*f^3*polylog(4,exp(2*d*x+2*c))/a/d^4-1/4*b^2*(f*x+e)^4/a^3/f+1/4*
(a^2+b^2)*(f*x+e)^4/a^3/f-1/2*(f*x+e)^3*coth(d*x+c)^2/a/d+3/2*f*(f*x+e)^2*p
olylog(2,exp(2*d*x+2*c))/a/d^2-3/2*f^2*(f*x+e)*polylog(3,exp(2*d*x+2*c))/a/
d^3+6*b*f*(f*x+e)^2*arctanh(exp(d*x+c))/a^2/d^2+6*b*f^2*(f*x+e)*polylog(2,-
exp(d*x+c))/a^2/d^3-6*b*f^2*(f*x+e)*polylog(2,exp(d*x+c))/a^2/d^3+3/2*b^2*f
*(f*x+e)^2*polylog(2,exp(2*d*x+2*c))/a^3/d^2-3/2*b^2*f^2*(f*x+e)*polylog(3,
exp(2*d*x+2*c))/a^3/d^3-3/2*f*(f*x+e)^2/a/d^2+3/2*f^3*polylog(2,exp(2*d*x+2
*c))/a/d^4+1/2*(f*x+e)^3/a/d-1/4*(f*x+e)^4/a/f-6*(a^2+b^2)*f^3*polylog(4,-b
*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^4-6*(a^2+b^2)*f^3*polylog(4,-b*exp(d
*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/d^4-(a^2+b^2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a
-(a^2+b^2)^(1/2)))/a^3/d-(a^2+b^2)*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)
^(1/2)))/a^3/d-3*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)
^(1/2)))/a^3/d^2-3*(a^2+b^2)*f*(f*x+e)^2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/a^3/d^2+6*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a-(a^2+
b^2)^(1/2)))/a^3/d^3+6*(a^2+b^2)*f^2*(f*x+e)*polylog(3,-b*exp(d*x+c)/(a+(a^
2+b^2)^(1/2)))/a^3/d^3+(f*x+e)^3*ln(1-exp(2*d*x+2*c))/a/d-3/2*f*(f*x+e)^2*c
oth(d*x+c)/a/d^2-6*b*f^3*polylog(3,-exp(d*x+c))/a^2/d^4+6*b*f^3*polylog(3,e
xp(d*x+c))/a^2/d^4+3/4*b^2*f^3*polylog(4,exp(2*d*x+2*c))/a^3/d^4+b*(f*x+e)^
3*csch(d*x+c)/a^2/d
```

Rubi [A]
time = 1.57, antiderivative size = 972, normalized size of antiderivative = 1.00, number of steps used = 62, number of rules used = 23, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.821$, Rules used = {5688, 3801, 3797, 2221, 2317, 2438, 32, 2611, 6744, 2320, 6724, 5704, 5558, 3377, 2718, 5560, 4267, 5554, 3392, 2715, 8, 5684, 5680}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-3*f*(e + f*x)^2)/(2*a*d^2) + (e + f*x)^3/(2*a*d) - (e + f*x)^4/(4*a*f) -
(b^2*(e + f*x)^4)/(4*a^3*f) + ((a^2 + b^2)*(e + f*x)^4)/(4*a^3*f) + (6*b*f*
(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^2*d^2) - (3*f*(e + f*x)^2*Coth[c + d*x
])/(2*a*d^2) - ((e + f*x)^3*Coth[c + d*x]^2)/(2*a*d) + (b*(e + f*x)^3*Csch[
```

```
c + d*x])/(a^2*d) - ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])])/(a^3*d) - ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2])])/(a^3*d) + (3*f^2*(e + f*x)*Log[1 - E^(2*(c + d*x)
)])/(a*d^3) + ((e + f*x)^3*Log[1 - E^(2*(c + d*x))])/(a*d) + (b^2*(e + f*x)
^3*Log[1 - E^(2*(c + d*x))])/(a^3*d) + (6*b*f^2*(e + f*x)*PolyLog[2, -E^(c
+ d*x)])/(a^2*d^3) - (6*b*f^2*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^2*d^3)
- (3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/(a^3*d^2) - (3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) + (3*f^3*PolyLog[2, E^(2*(c + d*x))
])/(2*a*d^4) + (3*f*(e + f*x)^2*PolyLog[2, E^(2*(c + d*x))])/(2*a*d^2) + (3
*b^2*f*(e + f*x)^2*PolyLog[2, E^(2*(c + d*x))])/(2*a^3*d^2) - (6*b*f^3*Poly
Log[3, -E^(c + d*x)])/(a^2*d^4) + (6*b*f^3*PolyLog[3, E^(c + d*x)])/(a^2*d^
4) + (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/(a^3*d^3) + (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^3) - (3*f^2*(e + f*x)*PolyLog[3, E^
(2*(c + d*x))])/(2*a*d^3) - (3*b^2*f^2*(e + f*x)*PolyLog[3, E^(2*(c + d*x))
])/(2*a^3*d^3) - (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2]))])/(a^3*d^4) - (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/(a^3*d^4) + (3*f^3*PolyLog[4, E^(2*(c + d*x))])/(4
*a*d^4) + (3*b^2*f^3*PolyLog[4, E^(2*(c + d*x))])/(4*a^3*d^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
```

```
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5688

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x]^
(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5704

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\coth^3(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^3\coth^2(c+dx)}{2ad} + \frac{\int (e+fx)^3\coth(c+dx)\,dx}{a} - \frac{b\int (e+fx)^3\cos$$

$$= -\frac{(e+fx)^4}{4af} - \frac{3f(e+fx)^2\coth(c+dx)}{2ad^2} - \frac{(e+fx)^3\coth^2(c+dx)}{2ad} - \frac{2\int e$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{(e+fx)^4}{4af} - \frac{3f(e+fx)^2\coth(c+dx)}{2ad^2} - (e$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{(e+fx)^4}{4af} - \frac{b^2(e+fx)^4}{4a^3f} + \frac{(a^2+b^2)(e+fx}{4a^3f}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{(e+fx)^4}{4af} - \frac{b^2(e+fx)^4}{4a^3f} + \frac{(a^2+b^2)(e+fx}{4a^3f}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{(e+fx)^4}{4af} - \frac{b^2(e+fx)^4}{4a^3f} + \frac{(a^2+b^2)(e+fx}{4a^3f}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{(e+fx)^4}{4af} - \frac{b^2(e+fx)^4}{4a^3f} + \frac{(a^2+b^2)(e+fx}{4a^3f}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{(e+fx)^4}{4af} - \frac{b^2(e+fx)^4}{4a^3f} + \frac{(a^2+b^2)(e+fx}{4a^3f}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{(e+fx)^4}{4af} - \frac{b^2(e+fx)^4}{4a^3f} + \frac{(a^2+b^2)(e+fx}{4a^3f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 55.32, size = 14876, normalized size = 15.30

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
[Out] Result too large to show
```

Maple [F]
time = 3.12, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 (\coth^3(dx+c))}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2*e^(-
2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + (a^2 + b^2)*log(-2*a*e^(-d*
x - c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) - (a^2 + b^2)*log(e^(-d*x - c) + 1
)/(a^3*d) - (a^2 + b^2)*log(e^(-d*x - c) - 1)/(a^3*d))*e^3 + (3*a*f^3*x^2 +
 6*a*f^2*x*e + 3*a*f*e^2 + 2*(b*d*f^3*x^3*e^(3*c) + 3*b*d*f^2*x^2*e^(3*c +
1) + 3*b*d*f*x*e^(3*c + 2))*e^(3*d*x) - (2*a*d*f^3*x^3*e^(2*c) + 3*(a*f^3*e
^(2*c) + 2*a*d*f^2*e^(2*c + 1))*x^2 + 3*a*f*e^(2*c + 2) + 6*(a*d*f*e^(2*c +
 2) + a*f^2*e^(2*c + 1))*x)*e^(2*d*x) - 2*(b*d*f^3*x^3*e^c + 3*b*d*f^2*x^2*
e^(c + 1) + 3*b*d*f*x*e^(c + 2))*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*
d^2*e^(2*d*x + 2*c) + a^2*d^2) - 3*(b*d*f*e^2 + a*f^2*e)*x/(a^2*d^2) + 3*(b
*d*f*e^2 - a*f^2*e)*x/(a^2*d^2) + 3*(b*d*f*e^2 + a*f^2*e)*log(e^(d*x + c) +
 1)/(a^2*d^3) - 3*(b*d*f*e^2 - a*f^2*e)*log(e^(d*x + c) - 1)/(a^2*d^3) + (d
^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog
(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*(a^2*f^3 + b^2*f^3)/(a^3*d^
4) + (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*
polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*(a^2*f^3 + b^2*f^3)/(a
^3*d^4) + 3*(a*b*f^3 + (a^2*d*f^2 + b^2*d*f^2)*e)*(d^2*x^2*log(e^(d*x + c)
+ 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^3*d^4) -
3*(a*b*f^3 - (a^2*d*f^2 + b^2*d*f^2)*e)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*
d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))/(a^3*d^4) + 3*(2*a*b*d*
f^2*e + a^2*f^3 + (a^2*d^2*f + b^2*d^2*f)*e^2)*(d*x*log(e^(d*x + c) + 1) +
dilog(-e^(d*x + c)))/(a^3*d^4) - 3*(2*a*b*d*f^2*e - a^2*f^3 - (a^2*d^2*f +
b^2*d^2*f)*e^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^3*d^4)
- 1/4*((a^2*f^3 + b^2*f^3)*d^4*x^4 + 4*(a*b*f^3 + (a^2*d*f^2 + b^2*d*f^2)*e
)*d^3*x^3 + 6*(2*a*b*d*f^2*e + a^2*f^3 + (a^2*d^2*f + b^2*d^2*f)*e^2)*d^2*x
^2)/(a^3*d^4) - 1/4*((a^2*f^3 + b^2*f^3)*d^4*x^4 - 4*(a*b*f^3 - (a^2*d*f^2
```

```
+ b^2*d*f^2)*e)*d^3*x^3 - 6*(2*a*b*d*f^2*e - a^2*f^3 - (a^2*d^2*f + b^2*d^2
*f)*e^2)*d^2*x^2)/(a^3*d^4) + integrate(-2*((a^2*b*f^3 + b^3*f^3)*x^3 + 3*(
a^2*b*f^2 + b^3*f^2)*x^2*e + 3*(a^2*b*f + b^3*f)*x*e^2 - ((a^3*f^3*e^c + a*
b^2*f^3*e^c)*x^3 + 3*(a^3*f^2*e^c + a*b^2*f^2*e^c)*x^2*e + 3*(a^3*f*e^c + a
*b^2*f*e^c)*x*e^2)*e^(d*x))/(a^3*b*e^(2*d*x + 2*c) + 2*a^4*e^(d*x + c) - a^
3*b), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 24387 vs. $2(932) = 1864$.
time = 0.66, size = 24387, normalized size = 25.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (3*a^2*c^2*f^3 - 6*a^2*c*d*f^2*cosh(1) + 3*a^2*d^2*f*cosh(1)^2 + 3*a^2*d^2*
f*sinh(1)^2 - 3*(a^2*d^2*f^3*x^2 - a^2*c^2*f^3 + 2*(a^2*d^2*f^2*x + a^2*c*d
*f^2)*cosh(1) + 2*(a^2*d^2*f^2*x + a^2*c*d*f^2)*sinh(1))*cosh(d*x + c)^4 -
3*(a^2*d^2*f^3*x^2 - a^2*c^2*f^3 + 2*(a^2*d^2*f^2*x + a^2*c*d*f^2)*cosh(1)
+ 2*(a^2*d^2*f^2*x + a^2*c*d*f^2)*sinh(1))*sinh(d*x + c)^4 + 2*(a*b*d^3*f^3
*x^3 + 3*a*b*d^3*f^2*x^2*cosh(1) + 3*a*b*d^3*f*x*cosh(1)^2 + a*b*d^3*cosh(1
)^3 + a*b*d^3*sinh(1)^3 + 3*(a*b*d^3*f*x + a*b*d^3*cosh(1))*sinh(1)^2 + 3*(
a*b*d^3*f^2*x^2 + 2*a*b*d^3*f*x*cosh(1) + a*b*d^3*cosh(1)^2)*sinh(1))*cosh(
d*x + c)^3 + 2*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*f^2*x^2*cosh(1) + 3*a*b*d^3*f*x
*cosh(1)^2 + a*b*d^3*cosh(1)^3 + a*b*d^3*sinh(1)^3 + 3*(a*b*d^3*f*x + a*b*d
^3*cosh(1))*sinh(1)^2 - 6*(a^2*d^2*f^3*x^2 - a^2*c^2*f^3 + 2*(a^2*d^2*f^2*x
 + a^2*c*d*f^2)*cosh(1) + 2*(a^2*d^2*f^2*x + a^2*c*d*f^2)*sinh(1))*cosh(d*x
 + c) + 3*(a*b*d^3*f^2*x^2 + 2*a*b*d^3*f*x*cosh(1) + a*b*d^3*cosh(1)^2)*sin
h(1))*sinh(d*x + c)^3 - (2*a^2*d^3*f^3*x^3 - 3*a^2*d^2*f^3*x^2 + 2*a^2*d^3*
cosh(1)^3 + 2*a^2*d^3*sinh(1)^3 + 6*a^2*c^2*f^3 + 3*(2*a^2*d^3*f*x + a^2*d^
2*f)*cosh(1)^2 + 3*(2*a^2*d^3*f*x + 2*a^2*d^3*cosh(1) + a^2*d^2*f)*sinh(1)^
2 + 6*(a^2*d^3*f^2*x^2 - a^2*d^2*f^2*x - 2*a^2*c*d*f^2)*cosh(1) + 6*(a^2*d^
3*f^2*x^2 - a^2*d^2*f^2*x + a^2*d^3*cosh(1)^2 - 2*a^2*c*d*f^2 + (2*a^2*d^3*
f*x + a^2*d^2*f)*cosh(1))*sinh(1))*cosh(d*x + c)^2 - (2*a^2*d^3*f^3*x^3 - 3
*a^2*d^2*f^3*x^2 + 2*a^2*d^3*cosh(1)^3 + 2*a^2*d^3*sinh(1)^3 + 6*a^2*c^2*f^
3 + 3*(2*a^2*d^3*f*x + a^2*d^2*f)*cosh(1)^2 + 18*(a^2*d^2*f^3*x^2 - a^2*c^2
*f^3 + 2*(a^2*d^2*f^2*x + a^2*c*d*f^2)*cosh(1) + 2*(a^2*d^2*f^2*x + a^2*c*d
*f^2)*sinh(1))*cosh(d*x + c)^2 + 3*(2*a^2*d^3*f*x + 2*a^2*d^3*cosh(1) + a^2
*d^2*f)*sinh(1)^2 + 6*(a^2*d^3*f^2*x^2 - a^2*d^2*f^2*x - 2*a^2*c*d*f^2)*cos
h(1) - 6*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*f^2*x^2*cosh(1) + 3*a*b*d^3*f*x*cosh(
1)^2 + a*b*d^3*cosh(1)^3 + a*b*d^3*sinh(1)^3 + 3*(a*b*d^3*f*x + a*b*d^3*cos
h(1))*sinh(1)^2 + 3*(a*b*d^3*f^2*x^2 + 2*a*b*d^3*f*x*cosh(1) + a*b*d^3*cosh
(1)^2)*sinh(1))*cosh(d*x + c) + 6*(a^2*d^3*f^2*x^2 - a^2*d^2*f^2*x + a^2*d^
3*cosh(1)^2 - 2*a^2*c*d*f^2 + (2*a^2*d^3*f*x + a^2*d^2*f)*cosh(1))*sinh(1))
```

```
*sinh(d*x + c)^2 - 2*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*f^2*x^2*cosh(1) + 3*a*b*d
^3*f*x*cosh(1)^2 + a*b*d^3*cosh(1)^3 + a*b*d^3*sinh(1)^3 + 3*(a*b*d^3*f*x +
 a*b*d^3*cosh(1))*sinh(1)^2 + 3*(a*b*d^3*f^2*x^2 + 2*a*b*d^3*f*x*cosh(1) +
a*b*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c) - 3*((a^2 + b^2)*d^2*f^3*x^2 + 2*
(a^2 + b^2)*d^2*f^2*x*cosh(1) + (a^2 + b^2)*d^2*f*cosh(1)^2 + (a^2 + b^2)*d
^2*f*sinh(1)^2 + ((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*f^2*x*cosh(1)
 + (a^2 + b^2)*d^2*f*cosh(1)^2 + (a^2 + b^2)*d^2*f*sinh(1)^2 + 2*((a^2 + b^
2)*d^2*f^2*x + (a^2 + b^2)*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^4 + 4*((a^
2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*f^2*x*cosh(1) + (a^2 + b^2)*d^2*f*
cosh(1)^2 + (a^2 + b^2)*d^2*f*sinh(1)^2 + 2*((a^2 + b^2)*d^2*f^2*x + (a^2 +
 b^2)*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 + b^2)*
d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*f^2*x*cosh(1) + (a^2 + b^2)*d^2*f*cosh(1)^2
 + (a^2 + b^2)*d^2*f*sinh(1)^2 + 2*((a^2 + b^2)*d^2*f^2*x + (a^2 + b^2)*d^2
*f*cosh(1))*sinh(1))*sinh(d*x + c)^4 - 2*((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2
+ b^2)*d^2*f^2*x*cosh(1) + (a^2 + b^2)*d^2*f*cosh(1)^2 + (a^2 + b^2)*d^2*f*
sinh(1)^2 + 2*((a^2 + b^2)*d^2*f^2*x + (a^2 + b^2)*d^2*f*cosh(1))*sinh(1))*
cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*f^2*x*cosh
(1) + (a^2 + b^2)*d^2*f*cosh(1)^2 + (a^2 + b^2)*d^2*f*sinh(1)^2 - 3*((a^2 +
 b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*f^2*x*cosh(1) + (a^2 + b^2)*d^2*f*cos
h(1)^2 + (a^2 + b^2)*d^2*f*sinh(1)^2 + 2*((a^2 + b^2)*d^2*f^2*x + (a^2 + b^
2)*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*((a^2 + b^2)*d^2*f^2*x + (a^
2 + b^2)*d^2*f*cosh(1))*sinh(1))*sinh(d*x + c)^2 + 2*((a^2 + b^2)*d^2*f^2*x
 + (a^2 + b^2)*d^2*f*cosh(1))*sinh(1) + 4*(((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^
2 + b^2)*d^2*f^2*x*cosh(1) + (a^2 + b^2)*d^2*f*cosh(1)^2 + (a^2 + b^2)*d^2*
f*sinh(1)^2 + 2*((a^2 + b^2)*d^2*f^2*x + (a^2 + b^2)*d^2*f*cosh(1))*sinh(1)
)*cosh(d*x + c)^3 - ((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*f^2*x*cosh
(1) + (a^2 + b^2)*d^2*f*cosh(1)^2 + (a^2 + b^2)*d^2*f*sinh(1)^2 + 2*((a^2 +
 b^2)*d^2*f^2*x + (a^2 + b^2)*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c))*sinh(d
*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*((a^2 + b^2)*d^2*f^3*x^2
 + 2*(a^2 + b^2)*d^2*f^2*x*cosh(1) + (a^2 + b^2)*d^2*f*cosh(1)^2 + (a^2 + b
^2)*d^2*f*sinh(1)^2 + ((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*f^2*x*co
sh(1) + (a^2 + b^2)*d^2*f*cosh(1)^2 + (a^2 + b^2)*d^2*f*sinh(1)^2 + 2*((a^2
 + b^2)*d^2*f^2*x + (a^2 + b^2)*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^4 + 4
*((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*f^2*x*cosh(1) + (a^2 + b^2)*d
^2*f*cosh(1)^2 + (a^2 + b^2)*d^2*f*sinh(1)^2 + 2*((a^2 + b^2)*d^2*f^2*x + (
a^2 + b^2)*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 +
b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*f^2*x*cosh...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*coth(d*x+c)**3/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)**3*coth(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{coth}(c+d\,x)^3\,{(e+f\,x)}^3}{a+b\,\mathrm{sinh}\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)),x)
[Out] int((coth(c + d*x)^3*(e + f*x)^3)/(a + b*sinh(c + d*x)), x)
```

3.487 $\int \frac{(e+fx)^2\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=689

$$\frac{efx}{ad}+\frac{f^2x^2}{2ad}-\frac{(e+fx)^3}{3af}-\frac{b^2(e+fx)^3}{3a^3f}+\frac{(a^2+b^2)(e+fx)^3}{3a^3f}+\frac{4bf(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2}-\frac{f(e+fx)\coth(c}{ad^2}$$

```
[Out] e*f*x/a/d+1/2*f^2*x^2/a/d-1/3*(f*x+e)^3/a/f-1/3*b^2*(f*x+e)^3/a^3/f+1/3*(a^
2+b^2)*(f*x+e)^3/a^3/f+4*b*f*(f*x+e)*arctanh(exp(d*x+c))/a^2/d^2-f*(f*x+e)*
coth(d*x+c)/a/d^2-1/2*(f*x+e)^2*coth(d*x+c)^2/a/d+b*(f*x+e)^2*csch(d*x+c)/a
^2/d+(f*x+e)^2*ln(1-exp(2*d*x+2*c))/a/d+b^2*(f*x+e)^2*ln(1-exp(2*d*x+2*c))/
a^3/d+f^2*ln(sinh(d*x+c))/a/d^3-(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a
^2+b^2)^(1/2)))/a^3/d-(a^2+b^2)*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1
/2)))/a^3/d+2*b*f^2*polylog(2,-exp(d*x+c))/a^2/d^3-2*b*f^2*polylog(2,exp(d*
x+c))/a^2/d^3+f*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d^2+b^2*f*(f*x+e)*polyl
og(2,exp(2*d*x+2*c))/a^3/d^2-2*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/
(a-(a^2+b^2)^(1/2)))/a^3/d^2-2*(a^2+b^2)*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/
(a+(a^2+b^2)^(1/2)))/a^3/d^2-1/2*f^2*polylog(3,exp(2*d*x+2*c))/a/d^3-1/2*b^
2*f^2*polylog(3,exp(2*d*x+2*c))/a^3/d^3+2*(a^2+b^2)*f^2*polylog(3,-b*exp(d*
x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d^3+2*(a^2+b^2)*f^2*polylog(3,-b*exp(d*x+c)/(
a+(a^2+b^2)^(1/2)))/a^3/d^3
```

Rubi [A]

time = 1.19, antiderivative size = 689, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 20, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5688, 3801, 3556, 3797, 2221, 2611, 2320, 6724, 5704, 5558, 3377, 2717, 5560, 4267, 2317, 2438, 5554, 3391, 5684, 5680}

$$-\frac{b^2f^2\text{Li}_3\left(e^{2(c+dx)}\right)}{2a^3d^3}+\frac{b^2f(e+fx)\text{Li}_2\left(e^{2(c+dx)}\right)}{a^3d^2}+\frac{b^2(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{a^3d}-\frac{b^2(e+fx)^3}{3a^3f}+\frac{2bf^2\text{Li}_2\left(-e^{c+dx}\right)}{a^2d^3}-\frac{2bf^2\text{Li}_2\left(e^{c+dx}\right)}{a^2d^3}+\frac{4bf(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2}+\frac{b(e+fx)^2\text{csch}(c+dx)}{a^2d}+\frac{2f^2(a^2+b^2)\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^3}+\frac{2f^2(a^2+b^2)\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^3}-\frac{2f(a^2+b^2)(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}-\frac{2f(a^2+b^2)(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2}-\frac{(a^2+b^2)(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^3d}-\frac{(a^2+b^2)(e+fx)^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{a^3d}+\frac{(a^2+b^2)(e+fx)^3}{3a^3f}-\frac{f^2\text{Li}_3\left(e^{2(c+dx)}\right)}{2ad^3}+\frac{f^2\log(\sinh(c+dx))}{ad^3}+\frac{f(e+fx)\text{Li}_2\left(e^{2(c+dx)}\right)}{ad^2}-\frac{f(e+fx)\coth(c+dx)}{ad^2}+\frac{(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{ad}-\frac{(e+fx)^2\coth^2(c+dx)}{2ad}+\frac{efx}{ad}+\frac{f^2x^2}{2ad}-\frac{(e+fx)^3}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e*f*x)/(a*d) + (f^2*x^2)/(2*a*d) - (e + f*x)^3/(3*a*f) - (b^2*(e + f*x)^3)
/(3*a^3*f) + ((a^2 + b^2)*(e + f*x)^3)/(3*a^3*f) + (4*b*f*(e + f*x)*ArcTanh
[E^(c + d*x)])/(a^2*d^2) - (f*(e + f*x)*Coth[c + d*x])/(a*d^2) - ((e + f*x)
^2*Coth[c + d*x]^2)/(2*a*d) + (b*(e + f*x)^2*Csch[c + d*x])/(a^2*d) - ((a^2
 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*d)
 - ((a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])
/(a^3*d) + ((e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a*d) + (b^2*(e + f*x)^2*
Log[1 - E^(2*(c + d*x))])/(a^3*d) + (f^2*Log[Sinh[c + d*x]])/(a*d^3) + (2*b
*f^2*PolyLog[2, -E^(c + d*x)])/(a^2*d^3) - (2*b*f^2*PolyLog[2, E^(c + d*x)]
)/(a^2*d^3) - (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a -
```

```
Sqrt[a^2 + b^2]))])/(a^3*d^2) - (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) + (f*(e + f*x)*PolyLog[2, E
^(2*(c + d*x))])/(a*d^2) + (b^2*f*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a
^3*d^2) + (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/(a^3*d^3) + (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/(a^3*d^3) - (f^2*PolyLog[3, E^(2*(c + d*x))])/(2*a*d^3) -
 (b^2*f^2*PolyLog[3, E^(2*(c + d*x))])/(2*a^3*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
```

```
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5688

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x]^
(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5704

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
```

```
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\coth^3(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^2\coth^2(c+dx)}{2ad} + \frac{\int (e+fx)^2\coth(c+dx)\,dx}{a} - \frac{b\int (e+fx)^2 \mathrm{c}}{}$$

$$= -\frac{(e+fx)^3}{3af} - \frac{f(e+fx)\coth(c+dx)}{ad^2} - \frac{(e+fx)^2\coth^2(c+dx)}{2ad} - \frac{2\int \frac{e^2}{}}{}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{(e+fx)^3}{3af} - \frac{f(e+fx)\coth(c+dx)}{ad^2} - \frac{(e+fx)^2\coth^2(c+}{2ad}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{(e+fx)^3}{3af} - \frac{b^2(e+fx)^3}{3a^3f} + \frac{(a^2+b^2)(e+fx)^3}{3a^3f} + \frac{4bf(e+}{}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{(e+fx)^3}{3af} - \frac{b^2(e+fx)^3}{3a^3f} + \frac{(a^2+b^2)(e+fx)^3}{3a^3f} + \frac{4bf(e+}{}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{(e+fx)^3}{3af} - \frac{b^2(e+fx)^3}{3a^3f} + \frac{(a^2+b^2)(e+fx)^3}{3a^3f} + \frac{4bf(e+}{}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{(e+fx)^3}{3af} - \frac{b^2(e+fx)^3}{3a^3f} + \frac{(a^2+b^2)(e+fx)^3}{3a^3f} + \frac{4bf(e+}{}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{(e+fx)^3}{3af} - \frac{b^2(e+fx)^3}{3a^3f} + \frac{(a^2+b^2)(e+fx)^3}{3a^3f} + \frac{4bf(e+}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1748 vs. $2(689) = 1378$.

time = 30.37, size = 1748, normalized size = 2.54

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*(e + f*x)^2*Csch[c])/(a^2*d) + ((-e^2 - 2*e*f*x - f^2*x^2)*Csch[c/2 + (d
*x)/2]^2)/(8*a*d) - (12*a^2*d^3*e^2*E^(2*c)*x + 12*b^2*d^3*e^2*E^(2*c)*x +
12*a^2*d*E^(2*c)*f^2*x + 12*a^2*d^3*e*E^(2*c)*f*x^2 + 12*b^2*d^3*e*E^(2*c)*
f*x^2 + 4*a^2*d^3*E^(2*c)*f^2*x^3 + 4*b^2*d^3*E^(2*c)*f^2*x^3 + 24*a*b*d*e*
f*ArcTanh[E^(c + d*x)] - 24*a*b*d*e*E^(2*c)*f*ArcTanh[E^(c + d*x)] - 12*a*b
*d*f^2*x*Log[1 - E^(c + d*x)] + 12*a*b*d*E^(2*c)*f^2*x*Log[1 - E^(c + d*x)]
 + 12*a*b*d*f^2*x*Log[1 + E^(c + d*x)] - 12*a*b*d*E^(2*c)*f^2*x*Log[1 + E^(
c + d*x)] + 6*a^2*d^2*e^2*Log[1 - E^(2*(c + d*x))] + 6*b^2*d^2*e^2*Log[1 -
E^(2*(c + d*x))] - 6*a^2*d^2*e^2*E^(2*c)*Log[1 - E^(2*(c + d*x))] - 6*b^2*d
^2*e^2*E^(2*c)*Log[1 - E^(2*(c + d*x))] + 6*a^2*f^2*Log[1 - E^(2*(c + d*x))
] - 6*a^2*E^(2*c)*f^2*Log[1 - E^(2*(c + d*x))] + 12*a^2*d^2*e*f*x*Log[1 - E
^(2*(c + d*x))] + 12*b^2*d^2*e*f*x*Log[1 - E^(2*(c + d*x))] - 12*a^2*d^2*e*
E^(2*c)*f*x*Log[1 - E^(2*(c + d*x))] - 12*b^2*d^2*e*E^(2*c)*f*x*Log[1 - E^(
2*(c + d*x))] + 6*a^2*d^2*f^2*x^2*Log[1 - E^(2*(c + d*x))] + 6*b^2*d^2*f^2*
x^2*Log[1 - E^(2*(c + d*x))] - 6*a^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(2*(c +
d*x))] - 6*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(2*(c + d*x))] - 12*a*b*(-1 +
E^(2*c))*f^2*PolyLog[2, -E^(c + d*x)] + 12*a*b*(-1 + E^(2*c))*f^2*PolyLog[2
, E^(c + d*x)] + 6*a^2*d*e*f*PolyLog[2, E^(2*(c + d*x))] + 6*b^2*d*e*f*Poly
Log[2, E^(2*(c + d*x))] - 6*a^2*d*e*E^(2*c)*f*PolyLog[2, E^(2*(c + d*x))] -
 6*b^2*d*e*E^(2*c)*f*PolyLog[2, E^(2*(c + d*x))] + 6*a^2*d*f^2*x*PolyLog[2,
 E^(2*(c + d*x))] + 6*b^2*d*f^2*x*PolyLog[2, E^(2*(c + d*x))] - 6*a^2*d*E^(
2*c)*f^2*x*PolyLog[2, E^(2*(c + d*x))] - 6*b^2*d*E^(2*c)*f^2*x*PolyLog[2, E
^(2*(c + d*x))] - 3*a^2*f^2*PolyLog[3, E^(2*(c + d*x))] - 3*b^2*f^2*PolyLog
[3, E^(2*(c + d*x))] + 3*a^2*E^(2*c)*f^2*PolyLog[3, E^(2*(c + d*x))] + 3*b^
2*E^(2*c)*f^2*PolyLog[3, E^(2*(c + d*x))])/(6*a^3*d^3*(-1 + E^(2*c))) + ((a
^2 + b^2)*((2*E^(2*c)*x*(3*e^2 + 3*e*f*x + f^2*x^2))/(-1 + E^(2*c)) - (3*(d
^2*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*d^2*e*f*x*Log[1
+ (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + d^2*f^2*x^2*Log[
1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*d^2*e*f*x*Lo
g[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + d^2*f^2*x^2*
Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*d*f*(e +
 f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))]
+ 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*
E^(2*c)]))] - 2*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2
)*E^(2*c)]))] - 2*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b
^2)*E^(2*c)]))]))/d^3))/(3*a^3) + ((e^2 + 2*e*f*x + f^2*x^2)*Sech[c/2 + (d*
x)/2]^2)/(8*a*d) + (Sech[c/2]*Sech[c/2 + (d*x)/2]*(-(b*d*e^2*Sinh[(d*x)/2])
 - a*e*f*Sinh[(d*x)/2] - 2*b*d*e*f*x*Sinh[(d*x)/2] - a*f^2*x*Sinh[(d*x)/2]
```

```
- b*d*f^2*x^2*Sinh[(d*x)/2]))/(2*a^2*d^2) + (Csch[c/2]*Csch[c/2 + (d*x)/2]*
(-(b*d*e^2*Sinh[(d*x)/2]) + a*e*f*Sinh[(d*x)/2] - 2*b*d*e*f*x*Sinh[(d*x)/2]
 + a*f^2*x*Sinh[(d*x)/2] - b*d*f^2*x^2*Sinh[(d*x)/2]))/(2*a^2*d^2)
```

Maple [F]
time = 3.04, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \left(\coth^3 (dx+c)\right)}{a+b\sinh (dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
[Out] int((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
[Out] -(2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2*e^(-
2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + (a^2 + b^2)*log(-2*a*e^(-d*
x - c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) - (a^2 + b^2)*log(e^(-d*x - c) + 1
)/(a^3*d) - (a^2 + b^2)*log(e^(-d*x - c) - 1)/(a^3*d))*e^2 + 2*(a*f^2*x + a
*f*e + (b*d*f^2*x^2*e^(3*c) + 2*b*d*f*x*e^(3*c + 1))*e^(3*d*x) - (a*d*f^2*x
^2*e^(2*c) + a*f*e^(2*c + 1) + (a*f^2*e^(2*c) + 2*a*d*f*e^(2*c + 1))*x)*e^(
2*d*x) - (b*d*f^2*x^2*e^c + 2*b*d*f*x*e^(c + 1))*e^(d*x))/(a^2*d^2*e^(4*d*x
 + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) - (2*b*d*f*e + a*f^2)*x/(a^2
*d^2) + (2*b*d*f*e - a*f^2)*x/(a^2*d^2) + (2*b*d*f*e + a*f^2)*log(e^(d*x +
c) + 1)/(a^2*d^3) - (2*b*d*f*e - a*f^2)*log(e^(d*x + c) - 1)/(a^2*d^3) + (d
^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(
d*x + c)))*(a^2*f^2 + b^2*f^2)/(a^3*d^3) + (d^2*x^2*log(-e^(d*x + c) + 1) +
 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*(a^2*f^2 + b^2*f^2)/
(a^3*d^3) + 2*(a*b*f^2 + (a^2*d*f + b^2*d*f)*e)*(d*x*log(e^(d*x + c) + 1) +
 dilog(-e^(d*x + c)))/(a^3*d^3) - 2*(a*b*f^2 - (a^2*d*f + b^2*d*f)*e)*(d*x*
log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^3*d^3) - 1/3*((a^2*f^2 + b^2
*f^2)*d^3*x^3 + 3*(a*b*f^2 + (a^2*d*f + b^2*d*f)*e)*d^2*x^2)/(a^3*d^3) - 1/
3*((a^2*f^2 + b^2*f^2)*d^3*x^3 - 3*(a*b*f^2 - (a^2*d*f + b^2*d*f)*e)*d^2*x^
2)/(a^3*d^3) + integrate(-2*((a^2*b*f^2 + b^3*f^2)*x^2 + 2*(a^2*b*f + b^3*f
)*x*e - ((a^3*f^2*e^c + a*b^2*f^2*e^c)*x^2 + 2*(a^3*f*e^c + a*b^2*f*e^c)*x*
e)*e^(d*x))/(a^3*b*e^(2*d*x + 2*c) + 2*a^4*e^(d*x + c) - a^3*b), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11172 vs. $2(664) = 1328$.
time = 0.46, size = 11172, normalized size = 16.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*a^2*c*f^2 - 2*a^2*d*f*cosh(1) + 2*(a^2*d*f^2*x + a^2*c*f^2)*cosh(d*x +
c)^4 - 2*a^2*d*f*sinh(1) + 2*(a^2*d*f^2*x + a^2*c*f^2)*sinh(d*x + c)^4 - 2*
(a*b*d^2*f^2*x^2 + 2*a*b*d^2*f*x*cosh(1) + a*b*d^2*cosh(1)^2 + a*b*d^2*sinh
(1)^2 + 2*(a*b*d^2*f*x + a*b*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^3 - 2*(a*b
*d^2*f^2*x^2 + 2*a*b*d^2*f*x*cosh(1) + a*b*d^2*cosh(1)^2 + a*b*d^2*sinh(1)^
2 - 4*(a^2*d*f^2*x + a^2*c*f^2)*cosh(d*x + c) + 2*(a*b*d^2*f*x + a*b*d^2*co
sh(1))*sinh(1))*sinh(d*x + c)^3 + 2*(a^2*d^2*f^2*x^2 - a^2*d*f^2*x + a^2*d^
2*cosh(1)^2 + a^2*d^2*sinh(1)^2 - 2*a^2*c*f^2 + (2*a^2*d^2*f*x + a^2*d*f)*c
osh(1) + (2*a^2*d^2*f*x + 2*a^2*d^2*cosh(1) + a^2*d*f)*sinh(1))*cosh(d*x +
c)^2 + 2*(a^2*d^2*f^2*x^2 - a^2*d*f^2*x + a^2*d^2*cosh(1)^2 + a^2*d^2*sinh(
1)^2 - 2*a^2*c*f^2 + 6*(a^2*d*f^2*x + a^2*c*f^2)*cosh(d*x + c)^2 + (2*a^2*d
^2*f*x + a^2*d*f)*cosh(1) - 3*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*f*x*cosh(1) + a*
b*d^2*cosh(1)^2 + a*b*d^2*sinh(1)^2 + 2*(a*b*d^2*f*x + a*b*d^2*cosh(1))*sin
h(1))*cosh(d*x + c) + (2*a^2*d^2*f*x + 2*a^2*d^2*cosh(1) + a^2*d*f)*sinh(1)
)*sinh(d*x + c)^2 + 2*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*f*x*cosh(1) + a*b*d^2*co
sh(1)^2 + a*b*d^2*sinh(1)^2 + 2*(a*b*d^2*f*x + a*b*d^2*cosh(1))*sinh(1))*co
sh(d*x + c) + 2*((a^2 + b^2)*d*f^2*x + ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d
*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*cosh(d*x + c)^4 + 4*((a^2 + b^2)*d*f^
2*x + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*cosh(d*x + c)*sinh
(d*x + c)^3 + ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*
d*f*sinh(1))*sinh(d*x + c)^4 + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*si
nh(1) - 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*
sinh(1))*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(1)
 + (a^2 + b^2)*d*f*sinh(1) - 3*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(
1) + (a^2 + b^2)*d*f*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^2 +
 b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*cosh(d*x
 + c)^3 - ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*
sinh(1))*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x
+ c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1
) + 2*((a^2 + b^2)*d*f^2*x + ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(1)
 + (a^2 + b^2)*d*f*sinh(1))*cosh(d*x + c)^4 + 4*((a^2 + b^2)*d*f^2*x + (a^2
 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^
3 + ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1
))*sinh(d*x + c)^4 + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1) - 2*
((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*c
osh(d*x + c)^2 - 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(1) + (a^2 +
```

```
b^2)*d*f*sinh(1) - 3*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(1) + (a^2
+ b^2)*d*f*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^2 + b^2)*d*f^
2*x + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*cosh(d*x + c)^3 -
((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*c
osh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*((a^
2 + b^2)*d*f^2*x + ((a^2 + b^2)*d*f^2*x - a*b*f^2 + (a^2 + b^2)*d*f*cosh(1)
 + (a^2 + b^2)*d*f*sinh(1))*cosh(d*x + c)^4 + 4*((a^2 + b^2)*d*f^2*x - a*b*
f^2 + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*cosh(d*x + c)*sinh
(d*x + c)^3 + ((a^2 + b^2)*d*f^2*x - a*b*f^2 + (a^2 + b^2)*d*f*cosh(1) + (a
^2 + b^2)*d*f*sinh(1))*sinh(d*x + c)^4 - a*b*f^2 + (a^2 + b^2)*d*f*cosh(1)
+ (a^2 + b^2)*d*f*sinh(1) - 2*((a^2 + b^2)*d*f^2*x - a*b*f^2 + (a^2 + b^2)*
d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*f
^2*x - a*b*f^2 + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1) - 3*((a^
2 + b^2)*d*f^2*x - a*b*f^2 + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh
(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^2 + b^2)*d*f^2*x - a*b*f^2 +
 (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*cosh(d*x + c)^3 - ((a^2
 + b^2)*d*f^2*x - a*b*f^2 + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(
1))*cosh(d*x + c))*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)) - 2*
((a^2 + b^2)*d*f^2*x + ((a^2 + b^2)*d*f^2*x + a*b*f^2 + (a^2 + b^2)*d*f*cos
h(1) + (a^2 + b^2)*d*f*sinh(1))*cosh(d*x + c)^4 + 4*((a^2 + b^2)*d*f^2*x +
a*b*f^2 + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*cosh(d*x + c)*
sinh(d*x + c)^3 + ((a^2 + b^2)*d*f^2*x + a*b*f^2 + (a^2 + b^2)*d*f*cosh(1)
+ (a^2 + b^2)*d*f*sinh(1))*sinh(d*x + c)^4 + a*b*f^2 + (a^2 + b^2)*d*f*cosh
(1) + (a^2 + b^2)*d*f*sinh(1) - 2*((a^2 + b^2)*d*f^2*x + a*b*f^2 + (a^2 + b
^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*cosh(d*x + c)^2 - 2*((a^2 + b^2)
*d*f^2*x + a*b*f^2 + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1) - 3*
((a^2 + b^2)*d*f^2*x + a*b*f^2 + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*
sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^2 + b^2)*d*f^2*x + a*b*f
^2 + (a^2 + b^2)*d*f*cosh(1) + (a^2 + b^2)*d*f*sinh(1))*cosh(d*x + c)^3 - (
(a^2 + b^2)*d*f^2*x + a*b*f^2 + (a^2 + b^2)*d*f...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e + fx)^2 \coth^3 (c + dx)}{a + b \sinh (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*coth(d*x+c)**3/(a+b*sinh(d*x+c)),x)
[Out] Integral((e + f*x)**2*coth(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(c+d\,x)^3\,(e+f\,x)^2}{a+b\sinh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((coth(c + d*x)^3*(e + f*x)^2)/(a + b*sinh(c + d*x)), x)
```

3.488 $\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=435

$$\frac{fx}{2ad}-\frac{(e+fx)^2}{2af}-\frac{b^2(e+fx)^2}{2a^3f}+\frac{(a^2+b^2)(e+fx)^2}{2a^3f}+\frac{bf\tanh^{-1}(\cosh(c+dx))}{a^2d^2}-\frac{f\coth(c+dx)}{2ad^2}-$$

```
[Out] 1/2*f*x/a/d-1/2*(f*x+e)^2/a/f-1/2*b^2*(f*x+e)^2/a^3/f+1/2*(a^2+b^2)*(f*x+e)
^2/a^3/f+b*f*arctanh(cosh(d*x+c))/a^2/d^2-1/2*f*coth(d*x+c)/a/d^2-1/2*(f*x+
e)*coth(d*x+c)^2/a/d+b*(f*x+e)*csch(d*x+c)/a^2/d+(f*x+e)*ln(1-exp(2*d*x+2*c
))/a/d+b^2*(f*x+e)*ln(1-exp(2*d*x+2*c))/a^3/d-(a^2+b^2)*(f*x+e)*ln(1+b*exp(
d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/d-(a^2+b^2)*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a
^2+b^2)^(1/2)))/a^3/d+1/2*f*polylog(2,exp(2*d*x+2*c))/a/d^2+1/2*b^2*f*polyl
og(2,exp(2*d*x+2*c))/a^3/d^2-(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^
2)^(1/2)))/a^3/d^2-(a^2+b^2)*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))
/a^3/d^2
```

Rubi [A]
time = 0.66, antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 18, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {5688, 3801, 3554, 8, 3797, 2221, 2317, 2438, 5704, 5558, 3377, 2718, 5560, 3855, 5554, 2715, 5684, 5680}

$$\frac{b^2f\text{Li}_2\left(e^{2(c+dx)}\right)}{2a^3d^2}+\frac{b^2(e+fx)\log\left(1-e^{2(c+dx)}\right)}{a^3d}-\frac{b^2(e+fx)^2}{2a^3f}+\frac{bf\tanh^{-1}(\cosh(c+dx))}{a^2d^2}+\frac{b(e+fx)\text{csch}(c+dx)}{a^2d}-\frac{f(a^2+b^2)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}-\frac{f(a^2+b^2)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a^3d^2}-\frac{(a^2+b^2)(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^3d}-\frac{(a^2+b^2)(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{a^3d}+\frac{(a^2+b^2)(e+fx)^2}{2a^3f}+\frac{f\text{Li}_2\left(e^{2(c+dx)}\right)}{2ad^2}-\frac{f\coth(c+dx)}{2ad^2}+\frac{(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad}-\frac{(e+fx)\coth^2(c+dx)}{2ad}+\frac{fx}{2ad}-\frac{(e+fx)^2}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (f*x)/(2*a*d) - (e + f*x)^2/(2*a*f) - (b^2*(e + f*x)^2)/(2*a^3*f) + ((a^2 +
 b^2)*(e + f*x)^2)/(2*a^3*f) + (b*f*ArcTanh[Cosh[c + d*x]])/(a^2*d^2) - (f*
Coth[c + d*x])/(2*a*d^2) - ((e + f*x)*Coth[c + d*x]^2)/(2*a*d) + (b*(e + f*
x)*Csch[c + d*x])/(a^2*d) - ((a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])])/(a^3*d) - ((a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*d) + ((e + f*x)*Log[1 - E^(2*(c + d*x))]
)/(a*d) + (b^2*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a^3*d) - ((a^2 + b^2)*f
*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2) - ((a^2 +
b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) + (f
*PolyLog[2, E^(2*(c + d*x))])/(2*a*d^2) + (b^2*f*PolyLog[2, E^(2*(c + d*x))
])/(2*a^3*d^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
```

```
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5558

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5684

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[-a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[(e + f*x)^m*(Cosh[c + d
*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5688

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Cosh[c + d*x]*(Coth[c + d*x]^
(n - 1)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 5704

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Cosh[c + d*x]^(p + 1)*(Coth[c + d*x]^(n - 1)/(a + b*Sinh[
c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\coth^3(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)\coth^2(c+dx)}{2ad} + \frac{\int (e+fx)\coth(c+dx)\,dx}{a} - \frac{b\int (e+fx)\cosh(}{}$$

$$= -\frac{(e+fx)^2}{2af} - \frac{f\coth(c+dx)}{2ad^2} - \frac{(e+fx)\coth^2(c+dx)}{2ad} - \frac{2\int \frac{e^{2(c+dx)}(e+fx)}{1-e^{2(c+dx)}}}{a}$$

$$= \frac{fx}{2ad} - \frac{(e+fx)^2}{2af} - \frac{f\coth(c+dx)}{2ad^2} - \frac{(e+fx)\coth^2(c+dx)}{2ad} + \frac{b(e+fx)c}{a}$$

$$= \frac{fx}{2ad} - \frac{(e+fx)^2}{2af} - \frac{b^2(e+fx)^2}{2a^3f} + \frac{(a^2+b^2)(e+fx)^2}{2a^3f} + \frac{bf\tanh^{-1}(\cosh(c}{a^2d^2}$$

$$= \frac{fx}{2ad} - \frac{(e+fx)^2}{2af} - \frac{b^2(e+fx)^2}{2a^3f} + \frac{(a^2+b^2)(e+fx)^2}{2a^3f} + \frac{bf\tanh^{-1}(\cosh(c}{a^2d^2}$$

$$= \frac{fx}{2ad} - \frac{(e+fx)^2}{2af} - \frac{b^2(e+fx)^2}{2a^3f} + \frac{(a^2+b^2)(e+fx)^2}{2a^3f} + \frac{bf\tanh^{-1}(\cosh(c}{a^2d^2}$$

$$= \frac{fx}{2ad} - \frac{(e+fx)^2}{2af} - \frac{b^2(e+fx)^2}{2a^3f} + \frac{(a^2+b^2)(e+fx)^2}{2a^3f} + \frac{bf\tanh^{-1}(\cosh(c}{a^2d^2}$$

<u>Mathematica</u> [**A**]
time = 3.40, size = 455, normalized size = 1.05

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a*(-(a*f) + 2*b*d*(e + f*x))*Coth[(c + d*x)/2] - a^2*d*(e + f*x)*Csch[(c
 + d*x)/2]^2 + 8*a^2*d*e*Log[Sinh[c + d*x]] + 8*b^2*d*e*Log[Sinh[c + d*x]]
- 8*a^2*c*f*Log[Sinh[c + d*x]] - 8*b^2*c*f*Log[Sinh[c + d*x]] - 8*a*b*f*Log
[Tanh[(c + d*x)/2]] + 4*a^2*f*((c + d*x)*(c + d*x + 2*Log[1 - E^(-2*(c + d*
x))]) - PolyLog[2, E^(-2*(c + d*x))]) + 4*b^2*f*((c + d*x)*(c + d*x + 2*Log
[1 - E^(-2*(c + d*x))]) - PolyLog[2, E^(-2*(c + d*x))]) - 8*(a^2 + b^2)*(-1
/2*(f*(c + d*x)^2) + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d*e*Log
```

```
[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLog[2, (b*E^(c
 + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2]))]]) + a^2*d*(e + f*x)*Sech[(c + d*x)/2]^2 - 2*a*(a*f + 2*b*d*(e
 + f*x))*Tanh[(c + d*x)/2])/(8*a^3*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1097 vs. $2(407) = 814$.
time = 8.26, size = 1098, normalized size = 2.52

method	result
risch	$\frac{b^{2} f \operatorname{dilog}\left(e^{d x+c}+1\right)}{d^{2} a^{3}}-\frac{f b^{2} \ln \left(\frac{b e^{d x+c}+\sqrt{a^{2}+b^{2}}+a}{a+\sqrt{a^{2}+b^{2}}}\right) c}{d^{2} a^{3}}-\frac{f \operatorname{dilog}\left(\frac{b e^{d x+c}+\sqrt{a^{2}+b^{2}}+a}{a+\sqrt{a^{2}+b^{2}}}\right)}{d^{2} a}-\frac{f b^{2} \ln \left(\frac{-b e^{d x+c}+\sqrt{a^{2}+\cdots}}{-a+\sqrt{a^{2}+\cdots}}\cdots\right.}{d a^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2/a^3*f*b^2*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c
-1/d/a^3*f*b^2*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x
-1/d/a^3*f*b^2*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1
/d/a^3*b^2*e*ln(exp(d*x+c)+1)-1/d/a^3*b^2*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x
+c)-b)+1/d/a^3*b^2*e*ln(exp(d*x+c)-1)+1/d^2/a^2*b*f*ln(exp(d*x+c)+1)-1/d^2/
a^2*b*f*ln(exp(d*x+c)-1)+1/d^2*f/a*dilog(exp(d*x+c)+1)-1/d^2*f*dilog(exp(d*
x+c))/a+1/d/a^3*b^2*f*ln(exp(d*x+c)+1)*x-1/d^2/a^3*b^2*f*ln((-b*exp(d*x+c)+
(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d^2/a^3*b^2*f*c*ln(b*exp(2*d*x
+2*c)+2*a*exp(d*x+c)-b)-1/d^2/a^3*b^2*f*c*ln(exp(d*x+c)-1)-(-2*b*d*f*x*exp(
3*d*x+3*c)+2*a*d*f*x*exp(2*d*x+2*c)-2*b*d*e*exp(3*d*x+3*c)+2*a*d*e*exp(2*d*
x+2*c)+2*b*d*f*x*exp(d*x+c)+a*f*exp(2*d*x+2*c)+2*b*d*e*exp(d*x+c)-f*a)/d^2/
a^2/(exp(2*d*x+2*c)-1)^2-1/d^2/a*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/
(-a+(a^2+b^2)^(1/2)))-1/d^2/a*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(
a^2+b^2)^(1/2)))-1/d/a*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/d*e/a*ln(e
xp(d*x+c)-1)+1/d*e/a*ln(exp(d*x+c)+1)+1/d*f/a*ln(exp(d*x+c)+1)*x-1/d^2*f*c/
a*ln(exp(d*x+c)-1)-1/d^2/a^3*b^2*f*dilog(exp(d*x+c))+1/d^2/a^3*b^2*f*dilog(
exp(d*x+c)+1)-1/d^2/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2
)))*c*f-1/d^2/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*
c*f+1/d^2/a*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d^2/a^3*f*b^2*dilog
((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2/a^3*f*b^2*di
log((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d/a*ln((b*exp(d
*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*f*x-1/d/a*ln((-b*exp(d*x+c)+(
a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*f*x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")

```
[Out] -(a^2*d*integrate(x/(a^3*d*e^(d*x + c) + a^3*d), x) + b^2*d*integrate(x/(a^
3*d*e^(d*x + c) + a^3*d), x) - a^2*d*integrate(x/(a^3*d*e^(d*x + c) - a^3*d
), x) - b^2*d*integrate(x/(a^3*d*e^(d*x + c) - a^3*d), x) + a*b*((d*x + c)/
(a^3*d^2) - log(e^(d*x + c) + 1)/(a^3*d^2)) - a*b*((d*x + c)/(a^3*d^2) - lo
g(e^(d*x + c) - 1)/(a^3*d^2)) - (2*b*d*x*e^(3*d*x + 3*c) - 2*b*d*x*e^(d*x +
 c) - (2*a*d*x*e^(2*c) + a*e^(2*c))*e^(2*d*x) + a)/(a^2*d^2*e^(4*d*x + 4*c)
 - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) - integrate(2*((a^3*e^c + a*b^2*e^c
)*x*e^(d*x) - (a^2*b + b^3)*x)/(a^3*b*e^(2*d*x + 2*c) + 2*a^4*e^(d*x + c) -
 a^3*b), x))*f - (2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*
c))/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + (a^2 + b^2)
*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) - (a^2 + b^2)*log(
e^(-d*x - c) + 1)/(a^3*d) - (a^2 + b^2)*log(e^(-d*x - c) - 1)/(a^3*d))*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4064 vs. $2(412) = 824$.
time = 0.39, size = 4064, normalized size = 9.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")

```
[Out] (2*(a*b*d*f*x + a*b*d*cosh(1) + a*b*d*sinh(1))*cosh(d*x + c)^3 + 2*(a*b*d*f
*x + a*b*d*cosh(1) + a*b*d*sinh(1))*sinh(d*x + c)^3 + a^2*f - (2*a^2*d*f*x
+ 2*a^2*d*cosh(1) + 2*a^2*d*sinh(1) + a^2*f)*cosh(d*x + c)^2 - (2*a^2*d*f*x
 + 2*a^2*d*cosh(1) + 2*a^2*d*sinh(1) + a^2*f - 6*(a*b*d*f*x + a*b*d*cosh(1)
 + a*b*d*sinh(1))*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(a*b*d*f*x + a*b*d*cos
h(1) + a*b*d*sinh(1))*cosh(d*x + c) - ((a^2 + b^2)*f*cosh(d*x + c)^4 + 4*(a
^2 + b^2)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + b^2)*f*sinh(d*x + c)^4 -
 2*(a^2 + b^2)*f*cosh(d*x + c)^2 + 2*(3*(a^2 + b^2)*f*cosh(d*x + c)^2 - (a^
2 + b^2)*f)*sinh(d*x + c)^2 + (a^2 + b^2)*f + 4*((a^2 + b^2)*f*cosh(d*x + c
)^3 - (a^2 + b^2)*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) +
a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
 - b)/b + 1) - ((a^2 + b^2)*f*cosh(d*x + c)^4 + 4*(a^2 + b^2)*f*cosh(d*x +
c)*sinh(d*x + c)^3 + (a^2 + b^2)*f*sinh(d*x + c)^4 - 2*(a^2 + b^2)*f*cosh(d
*x + c)^2 + 2*(3*(a^2 + b^2)*f*cosh(d*x + c)^2 - (a^2 + b^2)*f)*sinh(d*x +
c)^2 + (a^2 + b^2)*f + 4*((a^2 + b^2)*f*cosh(d*x + c)^3 - (a^2 + b^2)*f*cos
h(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*co
sh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + ((a^2 +
b^2)*f*cosh(d*x + c)^4 + 4*(a^2 + b^2)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a
^2 + b^2)*f*sinh(d*x + c)^4 - 2*(a^2 + b^2)*f*cosh(d*x + c)^2 + 2*(3*(a^2 +
 b^2)*f*cosh(d*x + c)^2 - (a^2 + b^2)*f)*sinh(d*x + c)^2 + (a^2 + b^2)*f +
```

```
4*((a^2 + b^2)*f*cosh(d*x + c)^3 - (a^2 + b^2)*f*cosh(d*x + c))*sinh(d*x +
c))*dilog(cosh(d*x + c) + sinh(d*x + c)) + ((a^2 + b^2)*f*cosh(d*x + c)^4 +
 4*(a^2 + b^2)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + b^2)*f*sinh(d*x + c
)^4 - 2*(a^2 + b^2)*f*cosh(d*x + c)^2 + 2*(3*(a^2 + b^2)*f*cosh(d*x + c)^2
- (a^2 + b^2)*f)*sinh(d*x + c)^2 + (a^2 + b^2)*f + 4*((a^2 + b^2)*f*cosh(d*
x + c)^3 - (a^2 + b^2)*f*cosh(d*x + c))*sinh(d*x + c))*dilog(-cosh(d*x + c)
 - sinh(d*x + c)) + (((a^2 + b^2)*c*f - (a^2 + b^2)*d*cosh(1) - (a^2 + b^2)
*d*sinh(1))*cosh(d*x + c)^4 + 4*((a^2 + b^2)*c*f - (a^2 + b^2)*d*cosh(1) -
(a^2 + b^2)*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 + b^2)*c*f - (
a^2 + b^2)*d*cosh(1) - (a^2 + b^2)*d*sinh(1))*sinh(d*x + c)^4 + (a^2 + b^2)
*c*f - (a^2 + b^2)*d*cosh(1) - 2*((a^2 + b^2)*c*f - (a^2 + b^2)*d*cosh(1) -
 (a^2 + b^2)*d*sinh(1))*cosh(d*x + c)^2 - (a^2 + b^2)*d*sinh(1) - 2*((a^2 +
 b^2)*c*f - (a^2 + b^2)*d*cosh(1) - 3*((a^2 + b^2)*c*f - (a^2 + b^2)*d*cosh
(1) - (a^2 + b^2)*d*sinh(1))*cosh(d*x + c)^2 - (a^2 + b^2)*d*sinh(1))*sinh(
d*x + c)^2 + 4*(((a^2 + b^2)*c*f - (a^2 + b^2)*d*cosh(1) - (a^2 + b^2)*d*si
nh(1))*cosh(d*x + c)^3 - ((a^2 + b^2)*c*f - (a^2 + b^2)*d*cosh(1) - (a^2 +
b^2)*d*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*s
inh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (((a^2 + b^2)*c*f - (a^2
+ b^2)*d*cosh(1) - (a^2 + b^2)*d*sinh(1))*cosh(d*x + c)^4 + 4*((a^2 + b^2)*
c*f - (a^2 + b^2)*d*cosh(1) - (a^2 + b^2)*d*sinh(1))*cosh(d*x + c)*sinh(d*x
 + c)^3 + ((a^2 + b^2)*c*f - (a^2 + b^2)*d*cosh(1) - (a^2 + b^2)*d*sinh(1))
*sinh(d*x + c)^4 + (a^2 + b^2)*c*f - (a^2 + b^2)*d*cosh(1) - 2*((a^2 + b^2)
*c*f - (a^2 + b^2)*d*cosh(1) - (a^2 + b^2)*d*sinh(1))*cosh(d*x + c)^2 - (a^
2 + b^2)*d*sinh(1) - 2*((a^2 + b^2)*c*f - (a^2 + b^2)*d*cosh(1) - 3*((a^2 +
 b^2)*c*f - (a^2 + b^2)*d*cosh(1) - (a^2 + b^2)*d*sinh(1))*cosh(d*x + c)^2
- (a^2 + b^2)*d*sinh(1))*sinh(d*x + c)^2 + 4*(((a^2 + b^2)*c*f - (a^2 + b^2
)*d*cosh(1) - (a^2 + b^2)*d*sinh(1))*cosh(d*x + c)^3 - ((a^2 + b^2)*c*f - (
a^2 + b^2)*d*cosh(1) - (a^2 + b^2)*d*sinh(1))*cosh(d*x + c))*sinh(d*x + c))
*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*
a) - (((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)^4 + 4*((a^2 + b^2
)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 + b^2)*d*f
*x + (a^2 + b^2)*c*f)*sinh(d*x + c)^4 + (a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f
 - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)^2 - 2*((a^2 + b^2)
*d*f*x + (a^2 + b^2)*c*f - 3*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 + 4*(((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*
x + c)^3 - ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c))*sinh(d*x +
c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*
f)*cosh(d*x + c)^4 + 4*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)*
sinh(d*x + c)^3 + ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*sinh(d*x + c)^4 + (
a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f
)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f - 3*((a^2 + b^2)
*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^2 + b^2
)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)^3 - ((a^2 + b^2)*d*f*x + (a^2 + b^
2)*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x +
```

```
c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + ((
(a^2 + b^2)*d*f*x + a*b*f + (a^2 + b^2)*d*cosh(...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*coth(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(c+d\,x)^3\,(e+f\,x)}{a+b\sinh(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)),x)
```

```
[Out] int((coth(c + d*x)^3*(e + f*x))/(a + b*sinh(c + d*x)), x)
```

3.489 $\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{b\operatorname{csch}(c+dx)}{a^2d}-\frac{\operatorname{csch}^2(c+dx)}{2ad}+\frac{(a^2+b^2)\log(\sinh(c+dx))}{a^3d}-\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{a^3d}$$

```
[Out] b*csch(d*x+c)/a^2/d-1/2*csch(d*x+c)^2/a/d+(a^2+b^2)*ln(sinh(d*x+c))/a^3/d-(
a^2+b^2)*ln(a+b*sinh(d*x+c))/a^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2800, 908}

$$\frac{b\operatorname{csch}(c+dx)}{a^2d}+\frac{(a^2+b^2)\log(\sinh(c+dx))}{a^3d}-\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{a^3d}-\frac{\operatorname{csch}^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*Csch[c + d*x])/(a^2*d) - Csch[c + d*x]^2/(2*a*d) + ((a^2 + b^2)*Log[Sinh
[c + d*x]])/(a^3*d) - ((a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/(a^3*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{-b^2-x^2}{x^3(a+x)}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{b^2}{ax^3}+\frac{b^2}{a^2x^2}+\frac{-a^2-b^2}{a^3x}+\frac{a^2+b^2}{a^3(a+x)}\right)dx, x, b\sinh(c+dx)\right)}{d}$$

$$= \frac{b\text{csch}(c+dx)}{a^2d} - \frac{\text{csch}^2(c+dx)}{2ad} + \frac{(a^2+b^2)\log(\sinh(c+dx))}{a^3d} - \frac{(a^2+b^2)\log(a \ldots}{a \ldots}$$

Mathematica [A]

time = 0.12, size = 64, normalized size = 0.80

$$\frac{2ab\text{csch}(c+dx) - a^2\text{csch}^2(c+dx) + 2(a^2+b^2)\left(\log(\sinh(c+dx)) - \log(a+b\sinh(c+dx))\right)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a*b*Csch[c + d*x] - a^2*Csch[c + d*x]^2 + 2*(a^2 + b^2)*(Log[Sinh[c + d*
x]] - Log[a + b*Sinh[c + d*x]]))/(2*a^3*d)
```

Maple [A]

time = 2.15, size = 143, normalized size = 1.79

method	result
derivativedivides	$\frac{-\frac{\frac{a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(4a^2+4b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{b}{2a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{\left(-4a^2-4b^2\right)\ln \ldots}{\ldots}}{d}$
default	$\frac{-\frac{\frac{a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(4a^2+4b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{b}{2a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{\left(-4a^2-4b^2\right)\ln \ldots}{\ldots}}{d}$
risch	$-\frac{2\,e^{dx+c}\left(-b\,e^{2dx+2c}+a\,e^{dx+c}+b\right)}{a^2d\left(e^{2dx+2c}-1\right)^2}+\frac{\ln\left(e^{2dx+2c}-1\right)}{ad}+\frac{b^2\ln\left(e^{2dx+2c}-1\right)}{a^3d}-\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}-1\right)}{ad}-\frac{b^2\ln\left(\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4/a^2*(1/2*a*tanh(1/2*d*x+1/2*c)^2+2*b*tanh(1/2*d*x+1/2*c))-1/8/a/t
anh(1/2*d*x+1/2*c)^2+1/4/a^3*(4*a^2+4*b^2)*ln(tanh(1/2*d*x+1/2*c))+1/2*b/a^
2/tanh(1/2*d*x+1/2*c)+1/4/a^3*(-4*a^2-4*b^2)*ln(a*tanh(1/2*d*x+1/2*c)^2-2*b
*tanh(1/2*d*x+1/2*c)-a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(78) = 156$.
time = 0.28, size = 173, normalized size = 2.16

$$-\frac{2\left(be^{(-dx-c)} - ae^{(-2\,dx-2\,c)} - be^{(-3\,dx-3\,c)}\right)}{\left(2\,a^{2}e^{(-2\,dx-2\,c)} - a^{2}e^{(-4\,dx-4\,c)} - a^{2}\right)d} - \frac{(a^{2}+b^{2})\log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{a^{3}d} + \frac{(a^{2}+b^{2})\log\left(e^{(-dx-c)}+1\right)}{a^{3}d} + \frac{(a^{2}+b^{2})\log\left(e^{(-dx-c)}-1\right)}{a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2*e^(-2
*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - (a^2 + b^2)*log(-2*a*e^(-d*x
 - c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) + (a^2 + b^2)*log(e^(-d*x - c) + 1)
/(a^3*d) + (a^2 + b^2)*log(e^(-d*x - c) - 1)/(a^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(78) = 156$.
time = 0.35, size = 617, normalized size = 7.71

$$\frac{2\,ab\cosh(dx+c)^{3} + 2\,ab\sinh(dx+c)^{3} - 2\,a^{2}\cosh(dx+c)^{2} - 2\,ab\cosh(dx+c) + 2\left(3\,ab\cosh(dx+c) - a^{2}\right)\sinh(dx+c)^{2} - \left((a^{2}+b^{2})\cosh(dx+c)^{4} + 4\,(a^{2}+b^{2})\cosh(dx+c)\sinh(dx+c)^{3} + (a^{2}+b^{2})\sinh(dx+c)^{4} - 2\,(a^{2}+b^{2})\cosh(dx+c)^{2} + 2\left(3\,(a^{2}+b^{2})\cosh(dx+c)^{2} - a^{2} - b^{2}\right)\sinh(dx+c)^{2} + a^{2} + b^{2} + 4\left((a^{2}+b^{2})\cosh(dx+c)^{3} - (a^{2}+b^{2})\cosh(dx+c)\right)\sinh(dx+c)\right)\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right) + \left((a^{2}+b^{2})\cosh(dx+c)^{4} + 4\,(a^{2}+b^{2})\cosh(dx+c)\sinh(dx+c)^{3} + (a^{2}+b^{2})\sinh(dx+c)^{4} - 2\,(a^{2}+b^{2})\cosh(dx+c)^{2} + 2\left(3\,(a^{2}+b^{2})\cosh(dx+c)^{2} - a^{2} - b^{2}\right)\sinh(dx+c)^{2} + a^{2} + b^{2} + 4\left((a^{2}+b^{2})\cosh(dx+c)^{3} - (a^{2}+b^{2})\cosh(dx+c)\right)\sinh(dx+c)\right)\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right) + 2\left(3\,ab\cosh(dx+c)^{2} - 2\,a^{2}\cosh(dx+c) - ab\right)\sinh(dx+c)}{a^{3}d\cosh(dx+c)^{4} + 4\,a^{3}d\cosh(dx+c)\sinh(dx+c)^{3} + a^{3}d\sinh(dx+c)^{4} - 2\,a^{3}d\cosh(dx+c)^{2} + a^{3}d + 2\left(3\,a^{3}d\cosh(dx+c)^{2} - a^{3}d\right)\sinh(dx+c)^{2} + 4\left(a^{3}d\cosh(dx+c)^{3} - a^{3}d\cosh(dx+c)\right)\sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*a*b*cosh(d*x + c)^3 + 2*a*b*sinh(d*x + c)^3 - 2*a^2*cosh(d*x + c)^2 - 2*
a*b*cosh(d*x + c) + 2*(3*a*b*cosh(d*x + c) - a^2)*sinh(d*x + c)^2 - ((a^2 +
 b^2)*cosh(d*x + c)^4 + 4*(a^2 + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2
+ b^2)*sinh(d*x + c)^4 - 2*(a^2 + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + b^2)*c
osh(d*x + c)^2 - a^2 - b^2)*sinh(d*x + c)^2 + a^2 + b^2 + 4*((a^2 + b^2)*co
sh(d*x + c)^3 - (a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x
 + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + ((a^2 + b^2)*cosh(d*x + c)^4
+ 4*(a^2 + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + b^2)*sinh(d*x + c)^4
 - 2*(a^2 + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + b^2)*cosh(d*x + c)^2 - a^2 -
 b^2)*sinh(d*x + c)^2 + a^2 + b^2 + 4*((a^2 + b^2)*cosh(d*x + c)^3 - (a^2 +
 b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - si
nh(d*x + c))) + 2*(3*a*b*cosh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - a*b)*sinh(
d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)*sinh(d*x + c)^3 +
a^3*d*sinh(d*x + c)^4 - 2*a^3*d*cosh(d*x + c)^2 + a^3*d + 2*(3*a^3*d*cosh(d
*x + c)^2 - a^3*d)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^3 - a^3*d*cosh(
d*x + c))*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{3}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(78) = 156.
time = 0.48, size = 184, normalized size = 2.30

$$\frac{\frac{2\,(a^2+b^2)\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a^3}-\frac{2\,(a^2b+b^3)\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^3b}-\frac{3\,a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+3\,b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-4\,ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)+4\,a^2}{a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^2 + b^2)*log(abs(e^(d*x + c) - e^(-d*x - c)))/a^3 - 2*(a^2*b + b^
3)*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^3*b) - (3*a^2*(e^(d*x
+ c) - e^(-d*x - c))^2 + 3*b^2*(e^(d*x + c) - e^(-d*x - c))^2 - 4*a*b*(e^(d
*x + c) - e^(-d*x - c)) + 4*a^2)/(a^3*(e^(d*x + c) - e^(-d*x - c))^2))/d
```

Mupad [B]
time = 1.01, size = 1329, normalized size = 16.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3/(a + b*sinh(c + d*x)),x)
```

```
[Out] ((2*atan((a^2*(-a^6*d^2)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b^2*(-a^6*
d^2)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(2*a^3*d*(a^2 + b^2)^2) + ((a^7*d
 + a^5*b^2*d)*(-a^6*d^2)^(1/2))/(2*a^6*d^2*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2
)) - (a^6*b^2*exp(2*c)*exp(2*d*x)*(-a^6*d^2)^(1/2)*((4*(a^2 + 2*b^2)*(a^4 +
 b^4 + 2*a^2*b^2))/(a^9*b^2*d*(a^2 + b^2)^2) + (2*(2*a^4*b^3*d + 2*a^6*b*d)
*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^11*b^3*d^2*((a^2 + b^2)^2)^(1/2)*(a^2 +
b^2)) + (4*(a^2*(-a^6*d^2)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b^2*(-a^
6*d^2)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/
(a^9*b^2*d*(a^2 + b^2)^2*(-a^6*d^2)^(1/2)) + (4*(a^7*d + a^5*b^2*d)*(a^4 +
b^4 + 2*a^2*b^2)^(1/2))/(a^12*b^2*d^2*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2))))/
(8*(a^4 + b^4 + 2*a^2*b^2)^(1/2)) + (a^6*b^2*exp(3*c)*exp(3*d*x)*((2*(a^7*d
 + a^5*b^2*d)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^11*b^3*d^2*((a^2 + b^2)^2)^
(1/2)*(a^2 + b^2)) - (2*(a^2 + 2*b^2)*(a^2*(-a^6*d^2)^(1/2)*(a^4 + b^4 + 2*
a^2*b^2)^(1/2) + 2*b^2*(-a^6*d^2)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))*(a^4
 + b^4 + 2*a^2*b^2)^(1/2))/(a^10*b^3*d*(a^2 + b^2)^2*(-a^6*d^2)^(1/2)))*(-a
^6*d^2)^(1/2))/(8*(a^4 + b^4 + 2*a^2*b^2)^(1/2)) - (a^6*b^2*exp(d*x)*exp(c)
*(-a^6*d^2)^(1/2)*((8*(a^4 + b^4 + 2*a^2*b^2))/(a^8*b*d*(a^2 + b^2)^2) - (4
*(2*a^4*b^3*d + 2*a^6*b*d)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^12*b^2*d^2*((a
```

```
^2 + b^2)^2)^(1/2)*(a^2 + b^2)) + (2*(a^7*d + a^5*b^2*d)*(a^4 + b^4 + 2*a^2
*b^2)^(1/2))/(a^11*b^3*d^2*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)) - (2*(a^2 + 2
*b^2)*(a^2*(-a^6*d^2)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b^2*(-a^6*d^2
)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^10
*b^3*d*(a^2 + b^2)^2*(-a^6*d^2)^(1/2)))))/(8*(a^4 + b^4 + 2*a^2*b^2)^(1/2)))
 - 2*atan((4*a^6*b*d*(a^2 + b^2)^2*(-a^6*d^2)^(1/2) + 4*a^4*b^3*d*(a^2 + b^
2)^2*(-a^6*d^2)^(1/2))*(1/(8*a^5*b*d^2*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)^3)
 - exp(d*x)*exp(c)*(1/(16*a^4*b^2*d^2*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)^3)
- (a^2 + 2*b^2)^2/(16*a^8*b^2*d^2*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)^3)) + (
a^2 + 2*b^2)/(8*a^7*b*d^2*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)^3))))*(a^4 + b^
4 + 2*a^2*b^2)^(1/2))/(-a^6*d^2)^(1/2) - (2/(a*d) - (2*b*exp(c + d*x))/(a^2
*d))/(exp(2*c + 2*d*x) - 1) - 2/(a*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x)
 + 1))
```

3.490 $\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Coth[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] Defer[Int][Coth[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Coth[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3\left(dx+c\right)}{\left(fx+e\right)\left(a+b\sinh\left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(a*f - 2*(b*d*f*x*e^(3*c) + b*d*e^(3*c + 1))*e^(3*d*x) + (2*a*d*f*x*e^(2*c
) - a*f*e^(2*c) + 2*a*d*e^(2*c + 1))*e^(2*d*x) + 2*(b*d*f*x*e^c + b*d*e^(c
+ 1))*e^(d*x))/(a^2*d^2*f^2*x^2 + 2*a^2*d^2*f*x*e + a^2*d^2*e^2 + (a^2*d^2*
f^2*x^2*e^(4*c) + 2*a^2*d^2*f*x*e^(4*c + 1) + a^2*d^2*e^(4*c + 2))*e^(4*d*x
) - 2*(a^2*d^2*f^2*x^2*e^(2*c) + 2*a^2*d^2*f*x*e^(2*c + 1) + a^2*d^2*e^(2*c
 + 2))*e^(2*d*x)) + integrate(-(a*b*d*f*e + a^2*f^2 + (a^2*d^2*f^2 + b^2*d^
2*f^2)*x^2 + (a*b*d*f^2 + 2*(a^2*d^2*f + b^2*d^2*f)*e)*x + (a^2*d^2 + b^2*d
^2)*e^2)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*f^2*x^2*e + 3*a^3*d^2*f*x*e^2 + a^3*d
^2*e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*f^2*x^2*e^(c + 1) + 3*a^3*d^2*f*x
*e^(c + 2) + a^3*d^2*e^(c + 3))*e^(d*x)), x) - integrate(-(a*b*d*f*e - a^2*
f^2 - (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + (a*b*d*f^2 - 2*(a^2*d^2*f + b^2*d^2
*f)*e)*x - (a^2*d^2 + b^2*d^2)*e^2)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*f^2*x^2*e
+ 3*a^3*d^2*f*x*e^2 + a^3*d^2*e^3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*f^2*x^
2*e^(c + 1) + 3*a^3*d^2*f*x*e^(c + 2) + a^3*d^2*e^(c + 3))*e^(d*x)), x) + i
ntegrate(2*(a^2*b + b^3 - (a^3*e^c + a*b^2*e^c)*e^(d*x))/(a^3*b*f*x + a^3*b
*e - (a^3*b*f*x*e^(2*c) + a^3*b*e^(2*c + 1))*e^(2*d*x) - 2*(a^4*f*x*e^c + a
^4*e^(c + 1))*e^(d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(coth(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{3}\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)**3/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\coth(c+d\,x)^3}{(e+f\,x)\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(coth(c + d*x)^3/((e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.491 $\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx) \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1795

result too large to display

```
[Out] 3*f^2*(f*x+e)*ln(1-exp(2*d*x+2*c))/a/d^3+3*I*b^3*f*(f*x+e)^2*polylog(2,-I*e
xp(d*x+c))/a^2/(a^2+b^2)/d^2+6*I*b^3*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/a^
2/(a^2+b^2)/d^3+3/2*b^4*f*(f*x+e)^2*polylog(2,-exp(2*d*x+2*c))/a^3/(a^2+b^2
)/d^2-3/2*b^4*f^2*(f*x+e)*polylog(3,-exp(2*d*x+2*c))/a^3/(a^2+b^2)/d^3-3/4*
f^3*polylog(4,exp(2*d*x+2*c))/a/d^4-1/2*(f*x+e)^3*coth(d*x+c)^2/a/d-3/2*f*(
f*x+e)^2*polylog(2,exp(2*d*x+2*c))/a/d^2+3/2*f^2*(f*x+e)*polylog(3,exp(2*d*
x+2*c))/a/d^3+6*b*f*(f*x+e)^2*arctanh(exp(d*x+c))/a^2/d^2+6*b*f^2*(f*x+e)*p
olylog(2,-exp(d*x+c))/a^2/d^3-6*b*f^2*(f*x+e)*polylog(2,exp(d*x+c))/a^2/d^3
+3/2*b^2*f*(f*x+e)^2*polylog(2,exp(2*d*x+2*c))/a^3/d^2-3/2*b^2*f^2*(f*x+e)*
polylog(3,exp(2*d*x+2*c))/a^3/d^3-3/2*f*(f*x+e)^2/a/d^2-3/2*b^2*f*(f*x+e)^2
*polylog(2,-exp(2*d*x+2*c))/a^3/d^2+3/2*b^2*f^2*(f*x+e)*polylog(3,-exp(2*d*
x+2*c))/a^3/d^3+3/4*b^4*f^3*polylog(4,-exp(2*d*x+2*c))/a^3/(a^2+b^2)/d^4-6*
I*b*f^3*polylog(4,-I*exp(d*x+c))/a^2/d^4+2*(f*x+e)^3*arctanh(exp(2*d*x+2*c)
)/a/d+3/4*f^3*polylog(4,-exp(2*d*x+2*c))/a/d^4+3/2*f^3*polylog(2,exp(2*d*x+
2*c))/a/d^4+1/2*(f*x+e)^3/a/d-b^4*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^
(1/2)))/a^3/(a^2+b^2)/d-b^4*(f*x+e)^3*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2))
)/a^3/(a^2+b^2)/d-6*b^4*f^3*polylog(4,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^
3/(a^2+b^2)/d^4-6*b^4*f^3*polylog(4,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/
(a^2+b^2)/d^4+b^4*(f*x+e)^3*ln(1+exp(2*d*x+2*c))/a^3/(a^2+b^2)/d+6*I*b*f^3*
polylog(4,I*exp(d*x+c))/a^2/d^4-3*I*b*f*(f*x+e)^2*polylog(2,-I*exp(d*x+c))/
a^2/d^2-6*I*b*f^2*(f*x+e)*polylog(3,I*exp(d*x+c))/a^2/d^3-6*I*b^3*f^3*polyl
og(4,I*exp(d*x+c))/a^2/(a^2+b^2)/d^4-3/2*f*(f*x+e)^2*coth(d*x+c)/a/d^2-6*b*
f^3*polylog(3,-exp(d*x+c))/a^2/d^4+6*b*f^3*polylog(3,exp(d*x+c))/a^2/d^4+3/
4*b^2*f^3*polylog(4,exp(2*d*x+2*c))/a^3/d^4+2*b*(f*x+e)^3*arctan(exp(d*x+c)
)/a^2/d-2*b^2*(f*x+e)^3*arctanh(exp(2*d*x+2*c))/a^3/d+b*(f*x+e)^3*csch(d*x+
c)/a^2/d-3/4*b^2*f^3*polylog(4,-exp(2*d*x+2*c))/a^3/d^4-2*b^3*(f*x+e)^3*arc
tan(exp(d*x+c))/a^2/(a^2+b^2)/d+3/2*f*(f*x+e)^2*polylog(2,-exp(2*d*x+2*c))/
a/d^2-3/2*f^2*(f*x+e)*polylog(3,-exp(2*d*x+2*c))/a/d^3+3*I*b*f*(f*x+e)^2*po
lylog(2,I*exp(d*x+c))/a^2/d^2+6*I*b*f^2*(f*x+e)*polylog(3,-I*exp(d*x+c))/a^
2/d^3+6*I*b^3*f^3*polylog(4,-I*exp(d*x+c))/a^2/(a^2+b^2)/d^4-3*I*b^3*f*(f*x
+e)^2*polylog(2,I*exp(d*x+c))/a^2/(a^2+b^2)/d^2-6*I*b^3*f^2*(f*x+e)*polylog
(3,-I*exp(d*x+c))/a^2/(a^2+b^2)/d^3-3*b^4*f*(f*x+e)^2*polylog(2,-b*exp(d*x+
c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)/d^2-3*b^4*f*(f*x+e)^2*polylog(2,-b*ex
p(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)/d^2+6*b^4*f^2*(f*x+e)*polylog(3
,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)/d^3+6*b^4*f^2*(f*x+e)*pol
ylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)/d^3
```

Rubi [A]

time = 2.42, antiderivative size = 1795, normalized size of antiderivative = 1.00, number of steps used = 87, number of rules used = 28, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules used = {5708, 2700, 14, 5570, 6873, 12, 6874, 3801, 3797, 2221, 2317, 2438, 32, 2631, 4267, 2611, 6744, 2320, 6724, 2701, 327, 213, 5313, 4265, 5569, 5692, 5680, 3799}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-3*f*(e + f*x)^2)/(2*a*d^2) + (e + f*x)^3/(2*a*d) + (2*b*(e + f*x)^3*ArcTa
n[E^(c + d*x)])/(a^2*d) - (2*b^3*(e + f*x)^3*ArcTan[E^(c + d*x)])/(a^2*(a^2
 + b^2)*d) + (6*b*f*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^2*d^2) + (2*(e + f
*x)^3*ArcTanh[E^(2*c + 2*d*x)])/(a*d) - (2*b^2*(e + f*x)^3*ArcTanh[E^(2*c +
 2*d*x)])/(a^3*d) - (3*f*(e + f*x)^2*Coth[c + d*x])/(2*a*d^2) - ((e + f*x)^
3*Coth[c + d*x]^2)/(2*a*d) + (b*(e + f*x)^3*Csch[c + d*x])/(a^2*d) - (b^4*(
e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)
*d) - (b^4*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3
*(a^2 + b^2)*d) + (3*f^2*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a*d^3) + (b^4
*(e + f*x)^3*Log[1 + E^(2*(c + d*x))])/(a^3*(a^2 + b^2)*d) + (6*b*f^2*(e +
f*x)*PolyLog[2, -E^(c + d*x)])/(a^2*d^3) - ((3*I)*b*f*(e + f*x)^2*PolyLog[2
, (-I)*E^(c + d*x)])/(a^2*d^2) + ((3*I)*b^3*f*(e + f*x)^2*PolyLog[2, (-I)*E
^(c + d*x)])/(a^2*(a^2 + b^2)*d^2) + ((3*I)*b*f*(e + f*x)^2*PolyLog[2, I*E^
(c + d*x)])/(a^2*d^2) - ((3*I)*b^3*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])
/(a^2*(a^2 + b^2)*d^2) - (6*b*f^2*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^2*d
^3) - (3*b^4*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/(a^3*(a^2 + b^2)*d^2) - (3*b^4*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^2) + (3*b^4*f*(e + f*x)^2*
PolyLog[2, -E^(2*(c + d*x))])/(2*a^3*(a^2 + b^2)*d^2) + (3*f^3*PolyLog[2, E
^(2*(c + d*x))])/(2*a*d^4) + (3*f*(e + f*x)^2*PolyLog[2, -E^(2*c + 2*d*x)])
/(2*a*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a^3*d^2)
 - (3*f*(e + f*x)^2*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d^2) + (3*b^2*f*(e +
f*x)^2*PolyLog[2, E^(2*c + 2*d*x)])/(2*a^3*d^2) - (6*b*f^3*PolyLog[3, -E^(c
 + d*x)])/(a^2*d^4) + ((6*I)*b*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/
(a^2*d^3) - ((6*I)*b^3*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(a^2*(a^
2 + b^2)*d^3) - ((6*I)*b*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(a^2*d^3)
 + ((6*I)*b^3*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^3
) + (6*b*f^3*PolyLog[3, E^(c + d*x)])/(a^2*d^4) + (6*b^4*f^2*(e + f*x)*Poly
Log[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^3) + (
6*b^4*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
a^3*(a^2 + b^2)*d^3) - (3*b^4*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(
2*a^3*(a^2 + b^2)*d^3) - (3*f^2*(e + f*x)*PolyLog[3, -E^(2*c + 2*d*x)])/(2*
a*d^3) + (3*b^2*f^2*(e + f*x)*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a^3*d^3) + (
3*f^2*(e + f*x)*PolyLog[3, E^(2*c + 2*d*x)])/(2*a*d^3) - (3*b^2*f^2*(e + f*
x)*PolyLog[3, E^(2*c + 2*d*x)])/(2*a^3*d^3) - ((6*I)*b*f^3*PolyLog[4, (-I)*
E^(c + d*x)])/(a^2*d^4) + ((6*I)*b^3*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(a^2
```

```
*(a^2 + b^2)*d^4) + ((6*I)*b*f^3*PolyLog[4, I*E^(c + d*x)])/(a^2*d^4) - ((6
*I)*b^3*f^3*PolyLog[4, I*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^4) - (6*b^4*f^3*P
olyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^4)
- (6*b^4*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^
2 + b^2)*d^4) + (3*b^4*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*a^3*(a^2 + b^2)
*d^4) + (3*f^3*PolyLog[4, -E^(2*c + 2*d*x)])/(4*a*d^4) - (3*b^2*f^3*PolyLog
[4, -E^(2*c + 2*d*x)])/(4*a^3*d^4) - (3*f^3*PolyLog[4, E^(2*c + 2*d*x)])/(4
*a*d^4) + (3*b^2*f^3*PolyLog[4, E^(2*c + 2*d*x)])/(4*a^3*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2631

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a +
b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5313

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcTan[u])/(d*(m + 1))), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 + u^2)), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
```

```
!FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m + 1, x]]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5708

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\text{csch}^3(c+dx)\text{sech}(c+dx)\,dx}{a} -$$

$$= -\frac{(e+fx)^3\coth^2(c+dx)}{2ad} - \frac{(e+fx)^3\log(\tanh(c+dx))}{ad} -$$

$$= \frac{b(e+fx)^3\tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{(e+fx)^3\coth^2(c+dx)}{2ad} +$$

$$= \frac{b(e+fx)^3\tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{(e+fx)^3\coth^2(c+dx)}{2ad} +$$

$$= \frac{b^4(e+fx)^4}{4a^3\left(a^2+b^2\right)f} + \frac{b(e+fx)^3\tan^{-1}(\sinh(c+dx))}{a^2d} -$$

$$= \frac{b^4(e+fx)^4}{4a^3\left(a^2+b^2\right)f} + \frac{b(e+fx)^3\tan^{-1}(\sinh(c+dx))}{a^2d} -$$

$$= -\frac{2b^3(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d} +$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{2b^3(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d} +$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{2b^3(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d} +$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} + \frac{2b(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a^2d} -$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} + \frac{2b(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a^2d} -$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} + \frac{2b(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a^2d} -$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} + \frac{2b(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a^2d} -$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} + \frac{2b(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a^2d} -$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 10559 vs. 2(1795) = 3590.
time = 62.92, size = 10559, normalized size = 5.88

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 2.48, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c)^3 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -(b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + a^3*b^2)*d) +
 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a
^2 + b^2)*d) + 2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))
/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + (a^2 - b^2)*lo
g(e^(-d*x - c) + 1)/(a^3*d) + (a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d))*e^
3 + (3*a*f^3*x^2 + 6*a*f^2*x*e + 3*a*f*e^2 + 2*(b*d*f^3*x^3*e^(3*c) + 3*b*d
*f^2*x^2*e^(3*c + 1) + 3*b*d*f*x*e^(3*c + 2))*e^(3*d*x) - (2*a*d*f^3*x^3*e^
(2*c) + 3*(a*f^3*e^(2*c) + 2*a*d*f^2*e^(2*c + 1))*x^2 + 3*a*f*e^(2*c + 2) +
 6*(a*d*f*e^(2*c + 2) + a*f^2*e^(2*c + 1))*x)*e^(2*d*x) - 2*(b*d*f^3*x^3*e^
c + 3*b*d*f^2*x^2*e^(c + 1) + 3*b*d*f*x*e^(c + 2))*e^(d*x))/(a^2*d^2*e^(4*d
*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) - 3*(b*d*f*e^2 + a*f^2*e)*
x/(a^2*d^2) + 3*(b*d*f*e^2 - a*f^2*e)*x/(a^2*d^2) + 3*(b*d*f*e^2 + a*f^2*e)
*log(e^(d*x + c) + 1)/(a^2*d^3) - 3*(b*d*f*e^2 - a*f^2*e)*log(e^(d*x + c) -
```

```
1)/(a^2*d^3) - (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c
)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*(a^2*f^3
- b^2*f^3)/(a^3*d^4) - (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(
d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*(a^2
*f^3 - b^2*f^3)/(a^3*d^4) + 3*(a*b*f^3 - (a^2*d*f^2 - b^2*d*f^2)*e)*(d^2*x^
2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x +
 c)))/(a^3*d^4) - 3*(a*b*f^3 + (a^2*d*f^2 - b^2*d*f^2)*e)*(d^2*x^2*log(-e^(
d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))/(a^3*
d^4) + 3*(2*a*b*d*f^2*e + a^2*f^3 - (a^2*d^2*f - b^2*d^2*f)*e^2)*(d*x*log(e
^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^3*d^4) - 3*(2*a*b*d*f^2*e - a^2*f
^3 + (a^2*d^2*f - b^2*d^2*f)*e^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x
 + c)))/(a^3*d^4) + 1/4*((a^2*f^3 - b^2*f^3)*d^4*x^4 + 4*(a*b*f^3 + (a^2*d*
f^2 - b^2*d*f^2)*e)*d^3*x^3 + 6*(2*a*b*d*f^2*e - a^2*f^3 + (a^2*d^2*f - b^2
*d^2*f)*e^2)*d^2*x^2)/(a^3*d^4) + 1/4*((a^2*f^3 - b^2*f^3)*d^4*x^4 - 4*(a*b
*f^3 - (a^2*d*f^2 - b^2*d*f^2)*e)*d^3*x^3 - 6*(2*a*b*d*f^2*e + a^2*f^3 - (a
^2*d^2*f - b^2*d^2*f)*e^2)*d^2*x^2)/(a^3*d^4) + integrate(2*(b^5*f^3*x^3 +
3*b^5*f^2*x^2*e + 3*b^5*f*x*e^2 - (a*b^4*f^3*x^3*e^c + 3*a*b^4*f^2*x^2*e^(c
 + 1) + 3*a*b^4*f*x*e^(c + 2))*e^(d*x))/(a^5*b + a^3*b^3 - (a^5*b*e^(2*c) +
 a^3*b^3*e^(2*c))*e^(2*d*x) - 2*(a^6*e^c + a^4*b^2*e^c)*e^(d*x)), x) + inte
grate(-2*(a*f^3*x^3 + 3*a*f^2*x^2*e + 3*a*f*x*e^2 - (b*f^3*x^3*e^c + 3*b*f^
2*x^2*e^(c + 1) + 3*b*f*x*e^(c + 2))*e^(d*x))/(a^2 + b^2 + (a^2*e^(2*c) + b
^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 43440 vs. $2(1674) = 3348$.
time = 1.11, size = 43440, normalized size = 24.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] (3*(a^4 + a^2*b^2)*c^2*f^3 - 6*(a^4 + a^2*b^2)*c*d*f^2*cosh(1) + 3*(a^4 + a
^2*b^2)*d^2*f*cosh(1)^2 + 3*(a^4 + a^2*b^2)*d^2*f*sinh(1)^2 - 3*((a^4 + a^2
*b^2)*d^2*f^3*x^2 - (a^4 + a^2*b^2)*c^2*f^3 + 2*((a^4 + a^2*b^2)*d^2*f^2*x
+ (a^4 + a^2*b^2)*c*d*f^2)*cosh(1) + 2*((a^4 + a^2*b^2)*d^2*f^2*x + (a^4 +
a^2*b^2)*c*d*f^2)*sinh(1))*cosh(d*x + c)^4 - 3*((a^4 + a^2*b^2)*d^2*f^3*x^2
 - (a^4 + a^2*b^2)*c^2*f^3 + 2*((a^4 + a^2*b^2)*d^2*f^2*x + (a^4 + a^2*b^2)
*c*d*f^2)*cosh(1) + 2*((a^4 + a^2*b^2)*d^2*f^2*x + (a^4 + a^2*b^2)*c*d*f^2)
*sinh(1))*sinh(d*x + c)^4 + 2*((a^3*b + a*b^3)*d^3*f^3*x^3 + 3*(a^3*b + a*b
^3)*d^3*f^2*x^2*cosh(1) + 3*(a^3*b + a*b^3)*d^3*f*x*cosh(1)^2 + (a^3*b + a*
b^3)*d^3*cosh(1)^3 + (a^3*b + a*b^3)*d^3*sinh(1)^3 + 3*((a^3*b + a*b^3)*d^3
*f*x + (a^3*b + a*b^3)*d^3*cosh(1))*sinh(1)^2 + 3*((a^3*b + a*b^3)*d^3*f^2*
x^2 + 2*(a^3*b + a*b^3)*d^3*f*x*cosh(1) + (a^3*b + a*b^3)*d^3*cosh(1)^2)*si
```

nh(1))*cosh(d*x + c)^3 + 2*((a^3*b + a*b^3)*d^3*f^3*x^3 + 3*(a^3*b + a*b^3)
*d^3*f^2*x^2*cosh(1) + 3*(a^3*b + a*b^3)*d^3*f*x*cosh(1)^2 + (a^3*b + a*b^3
)*d^3*cosh(1)^3 + (a^3*b + a*b^3)*d^3*sinh(1)^3 + 3*((a^3*b + a*b^3)*d^3*f*
x + (a^3*b + a*b^3)*d^3*cosh(1))*sinh(1)^2 - 6*((a^4 + a^2*b^2)*d^2*f^3*x^2
 - (a^4 + a^2*b^2)*c^2*f^3 + 2*((a^4 + a^2*b^2)*d^2*f^2*x + (a^4 + a^2*b^2)
*c*d*f^2)*cosh(1) + 2*((a^4 + a^2*b^2)*d^2*f^2*x + (a^4 + a^2*b^2)*c*d*f^2)
*sinh(1))*cosh(d*x + c) + 3*((a^3*b + a*b^3)*d^3*f^2*x^2 + 2*(a^3*b + a*b^3
)*d^3*f*x*cosh(1) + (a^3*b + a*b^3)*d^3*cosh(1)^2)*sinh(1))*sinh(d*x + c)^3
 - (2*(a^4 + a^2*b^2)*d^3*f^3*x^3 - 3*(a^4 + a^2*b^2)*d^2*f^3*x^2 + 2*(a^4
+ a^2*b^2)*d^3*cosh(1)^3 + 2*(a^4 + a^2*b^2)*d^3*sinh(1)^3 + 6*(a^4 + a^2*b
^2)*c^2*f^3 + 3*(2*(a^4 + a^2*b^2)*d^3*f*x + (a^4 + a^2*b^2)*d^2*f)*cosh(1)
^2 + 3*(2*(a^4 + a^2*b^2)*d^3*f*x + 2*(a^4 + a^2*b^2)*d^3*cosh(1) + (a^4 +
a^2*b^2)*d^2*f)*sinh(1)^2 + 6*((a^4 + a^2*b^2)*d^3*f^2*x^2 - (a^4 + a^2*b^2
)*d^2*f^2*x - 2*(a^4 + a^2*b^2)*c*d*f^2)*cosh(1) + 6*((a^4 + a^2*b^2)*d^3*f
^2*x^2 - (a^4 + a^2*b^2)*d^2*f^2*x + (a^4 + a^2*b^2)*d^3*cosh(1)^2 - 2*(a^4
 + a^2*b^2)*c*d*f^2 + (2*(a^4 + a^2*b^2)*d^3*f*x + (a^4 + a^2*b^2)*d^2*f)*c
osh(1))*sinh(1))*cosh(d*x + c)^2 - (2*(a^4 + a^2*b^2)*d^3*f^3*x^3 - 3*(a^4
+ a^2*b^2)*d^2*f^3*x^2 + 2*(a^4 + a^2*b^2)*d^3*cosh(1)^3 + 2*(a^4 + a^2*b^2
)*d^3*sinh(1)^3 + 6*(a^4 + a^2*b^2)*c^2*f^3 + 3*(2*(a^4 + a^2*b^2)*d^3*f*x
+ (a^4 + a^2*b^2)*d^2*f)*cosh(1)^2 + 18*((a^4 + a^2*b^2)*d^2*f^3*x^2 - (a^4
 + a^2*b^2)*c^2*f^3 + 2*((a^4 + a^2*b^2)*d^2*f^2*x + (a^4 + a^2*b^2)*c*d*f^
2)*cosh(1) + 2*((a^4 + a^2*b^2)*d^2*f^2*x + (a^4 + a^2*b^2)*c*d*f^2)*sinh(1
))*cosh(d*x + c)^2 + 3*(2*(a^4 + a^2*b^2)*d^3*f*x + 2*(a^4 + a^2*b^2)*d^3*c
osh(1) + (a^4 + a^2*b^2)*d^2*f)*sinh(1)^2 + 6*((a^4 + a^2*b^2)*d^3*f^2*x^2
- (a^4 + a^2*b^2)*d^2*f^2*x - 2*(a^4 + a^2*b^2)*c*d*f^2)*cosh(1) - 6*((a^3*
b + a*b^3)*d^3*f^3*x^3 + 3*(a^3*b + a*b^3)*d^3*f^2*x^2*cosh(1) + 3*(a^3*b +
 a*b^3)*d^3*f*x*cosh(1)^2 + (a^3*b + a*b^3)*d^3*cosh(1)^3 + (a^3*b + a*b^3)
*d^3*sinh(1)^3 + 3*((a^3*b + a*b^3)*d^3*f*x + (a^3*b + a*b^3)*d^3*cosh(1))*
sinh(1)^2 + 3*((a^3*b + a*b^3)*d^3*f^2*x^2 + 2*(a^3*b + a*b^3)*d^3*f*x*cosh
(1) + (a^3*b + a*b^3)*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c) + 6*((a^4 + a^2
*b^2)*d^3*f^2*x^2 - (a^4 + a^2*b^2)*d^2*f^2*x + (a^4 + a^2*b^2)*d^3*cosh(1)
^2 - 2*(a^4 + a^2*b^2)*c*d*f^2 + (2*(a^4 + a^2*b^2)*d^3*f*x + (a^4 + a^2*b^
2)*d^2*f)*cosh(1))*sinh(1))*sinh(d*x + c)^2 - 2*((a^3*b + a*b^3)*d^3*f^3*x^
3 + 3*(a^3*b + a*b^3)*d^3*f^2*x^2*cosh(1) + 3*(a^3*b + a*b^3)*d^3*f*x*cosh(
1)^2 + (a^3*b + a*b^3)*d^3*cosh(1)^3 + (a^3*b + a*b^3)*d^3*sinh(1)^3 + 3*((
a^3*b + a*b^3)*d^3*f*x + (a^3*b + a*b^3)*d^3*cosh(1))*sinh(1)^2 + 3*((a^3*b
 + a*b^3)*d^3*f^2*x^2 + 2*(a^3*b + a*b^3)*d^3*f*x*cosh(1) + (a^3*b + a*b^3)
*d^3*cosh(1)^2)*sinh(1))*cosh(d*x + c) - 3*(b^4*d^2*f^3*x^2 + 2*b^4*d^2*f^2
*x*cosh(1) + b^4*d^2*f*cosh(1)^2 + b^4*d^2*f*sinh(1)^2 + (b^4*d^2*f^3*x^2 +
 2*b^4*d^2*f^2*x*cosh(1) + b^4*d^2*f*cosh(1)^2 + b^4*d^2*f*sinh(1)^2 + 2*(b
^4*d^2*f^2*x + b^4*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)^4 + 4*(b^4*d^2*f^3
*x^2 + 2*b^4*d^2*f^2*x*cosh(1) + b^4*d^2*f*cosh(1)^2 + b^4*d^2*f*sinh(1)^2
+ 2*(b^4*d^2*f^2*x + b^4*d^2*f*cosh(1))*sinh(1))*cosh(d*x + c)*sinh(d*x + c
)^3 + (b^4*d^2*f^3*x^2 + 2*b^4*d^2*f^2*x*cosh(1) + b^4*d^2*f*cosh(1)^2 + b^
4*d^2*f*sinh(1)^2 + 2*(b^4*d^2*f^2*x + b^4*d^2*f*cosh(1))*sinh(1))*sinh(d*x

```
 + c)^4 - 2*(b^4*d^2*f^3*x^2 + 2*b^4*d^2*f^2*x*cosh(1) + b^4*d^2*f*cosh(1)^
2 + b^4*d^2*f*sinh(1)^2 + 2*(b^4*d^2*f^2*x + b^4*d^2*f*cosh(1))*sinh(1))*co
sh(d*x + c)^2 - 2*(b^4*d^2*f^3*x^2 + 2*b^4*d^2*f^2*x*cosh(1) + b^4*d^2*f*co
sh(1)^2 + b^4*d^2*f*sinh(1)^2 - 3*(b^4*d^2*f^3*x^2 + 2*b^4*d^2*f^2*x*cosh(1
) + b^4*d^2*f*cosh(1)^2 + b^4*d^2*f*sinh(1)^2 + 2*(b^4*d^2*f^2*x + b^4*d^2*
f*cosh(1))*sinh(1))*cosh(d*x + c)^2 + 2*(b^4*d^2*f^2*x + b^4*d^2*f*cosh(1))
*sinh(1))*sinh(d*x + c)^2 + 2*(b^4*d^2*f^2*x + b^4*d^2*f*cosh(1))*sinh(1) +
 4*((b^4*d^2*f^3*x^2 + 2*b^4*d^2*f^2*x*cosh(1) + b^4*d^2*f*cosh(1)^2 + b^4*
d^2*f*sinh(1)^2 + 2*(b^4*d^2*f^2*x + b^4*d^2*f*cosh(1))*sinh(1))*cosh(d*x +
 c)^3 - (b^4*d^2*f^3*x^2 + 2*b^4*d^2*f^2*x*cosh...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)**3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^3}{\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)^3\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3/(cosh(c + d*x)*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
[Out] int((e + f*x)^3/(cosh(c + d*x)*sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

3.492 $\int \frac{(e+fx)^2\mathbf{csch}^3(c+dx)\mathbf{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1219

$$\frac{efx}{ad}+\frac{f^2x^2}{2ad}+\frac{2b(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{a^2d}-\frac{2b^3(e+fx)^2\text{ArcTan}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d}+\frac{4bf(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2}+\frac{2(e+ \cdots}{}$$

```
[Out] 2*I*b^3*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/a^2/(a^2+b^2)/d^2+1/2*f^2*polylo
g(3,exp(2*d*x+2*c))/a/d^3+1/2*f^2*x^2/a/d-1/2*(f*x+e)^2*coth(d*x+c)^2/a/d+f
^2*ln(sinh(d*x+c))/a/d^3+4*b*f*(f*x+e)*arctanh(exp(d*x+c))/a^2/d^2-2*b^3*(f
*x+e)^2*arctan(exp(d*x+c))/a^2/(a^2+b^2)/d+2*(f*x+e)^2*arctanh(exp(2*d*x+2*
c))/a/d-1/2*f^2*polylog(3,-exp(2*d*x+2*c))/a/d^3+b^4*(f*x+e)^2*ln(1+exp(2*d
*x+2*c))/a^3/(a^2+b^2)/d-b^4*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)
))/a^3/(a^2+b^2)/d-b^4*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3
/(a^2+b^2)/d+2*b^4*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^
2+b^2)/d^3+2*b^4*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+
b^2)/d^3-b^2*f*(f*x+e)*polylog(2,-exp(2*d*x+2*c))/a^3/d^2+b^2*f*(f*x+e)*pol
ylog(2,exp(2*d*x+2*c))/a^3/d^2-1/2*b^4*f^2*polylog(3,-exp(2*d*x+2*c))/a^3/(
a^2+b^2)/d^3-2*I*b*f^2*polylog(3,I*exp(d*x+c))/a^2/d^3+b^4*f*(f*x+e)*polylo
g(2,-exp(2*d*x+2*c))/a^3/(a^2+b^2)/d^2+2*I*b*f^2*polylog(3,-I*exp(d*x+c))/a
^2/d^3-2*I*b*f*(f*x+e)*polylog(2,-I*exp(d*x+c))/a^2/d^2-2*I*b^3*f^2*polylog
(3,-I*exp(d*x+c))/a^2/(a^2+b^2)/d^3+e*f*x/a/d+2*b*f^2*polylog(2,-exp(d*x+c)
)/a^2/d^3-2*b*f^2*polylog(2,exp(d*x+c))/a^2/d^3-1/2*b^2*f^2*polylog(3,exp(2
*d*x+2*c))/a^3/d^3+2*b*(f*x+e)^2*arctan(exp(d*x+c))/a^2/d-2*b^2*(f*x+e)^2*a
rctanh(exp(2*d*x+2*c))/a^3/d-f*(f*x+e)*coth(d*x+c)/a/d^2+b*(f*x+e)^2*csch(d
*x+c)/a^2/d+1/2*b^2*f^2*polylog(3,-exp(2*d*x+2*c))/a^3/d^3+f*(f*x+e)*polylo
g(2,-exp(2*d*x+2*c))/a/d^2-f*(f*x+e)*polylog(2,exp(2*d*x+2*c))/a/d^2+2*I*b*
f*(f*x+e)*polylog(2,I*exp(d*x+c))/a^2/d^2+2*I*b^3*f^2*polylog(3,I*exp(d*x+c
))/a^2/(a^2+b^2)/d^3-2*I*b^3*f*(f*x+e)*polylog(2,I*exp(d*x+c))/a^2/(a^2+b^2
)/d^2-2*b^4*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2
+b^2)/d^2-2*b^4*f*(f*x+e)*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/
(a^2+b^2)/d^2
```

Rubi [A]
time = 1.73, antiderivative size = 1219, normalized size of antiderivative = 1.00, number of steps used = 71, number of rules used = 26, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {5708, 2700, 14, 5570, 6873, 12, 6874, 3801, 3556, 2631, 4267, 2611, 2320, 6724, 2701, 327, 213, 5313, 4265, 2317, 2438, 5569, 5692, 5680, 2221, 3799}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e*f*x)/(a*d) + (f^2*x^2)/(2*a*d) + (2*b*(e + f*x)^2*ArcTan[E^(c + d*x)])/(
a^2*d) - (2*b^3*(e + f*x)^2*ArcTan[E^(c + d*x)])/(a^2*(a^2 + b^2)*d) + (4*b
*f*(e + f*x)*ArcTanh[E^(c + d*x)])/(a^2*d^2) + (2*(e + f*x)^2*ArcTanh[E^(2*
c + 2*d*x)])/(a*d) - (2*b^2*(e + f*x)^2*ArcTanh[E^(2*c + 2*d*x)])/(a^3*d) -
 (f*(e + f*x)*Coth[c + d*x])/(a*d^2) - ((e + f*x)^2*Coth[c + d*x]^2)/(2*a*d
) + (b*(e + f*x)^2*Csch[c + d*x])/(a^2*d) - (b^4*(e + f*x)^2*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)*d) - (b^4*(e + f*x)^2*Lo
g[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)*d) + (b^4*(e
 + f*x)^2*Log[1 + E^(2*(c + d*x))])/(a^3*(a^2 + b^2)*d) + (f^2*Log[Sinh[c +
 d*x]])/(a*d^3) + (2*b*f^2*PolyLog[2, -E^(c + d*x)])/(a^2*d^3) - ((2*I)*b*f
*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*d^2) + ((2*I)*b^3*f*(e + f*x)
*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^2) + ((2*I)*b*f*(e + f*x)
*PolyLog[2, I*E^(c + d*x)])/(a^2*d^2) - ((2*I)*b^3*f*(e + f*x)*PolyLog[2, I
*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^2) - (2*b*f^2*PolyLog[2, E^(c + d*x)])/(a
^2*d^3) - (2*b^4*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/(a^3*(a^2 + b^2)*d^2) - (2*b^4*f*(e + f*x)*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^2) + (b^4*f*(e + f*x)*Poly
Log[2, -E^(2*(c + d*x))])/(a^3*(a^2 + b^2)*d^2) + (f*(e + f*x)*PolyLog[2, -
E^(2*c + 2*d*x)])/(a*d^2) - (b^2*f*(e + f*x)*PolyLog[2, -E^(2*c + 2*d*x)])/
(a^3*d^2) - (f*(e + f*x)*PolyLog[2, E^(2*c + 2*d*x)])/(a*d^2) + (b^2*f*(e +
 f*x)*PolyLog[2, E^(2*c + 2*d*x)])/(a^3*d^2) + ((2*I)*b*f^2*PolyLog[3, (-I)
*E^(c + d*x)])/(a^2*d^3) - ((2*I)*b^3*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(a^
2*(a^2 + b^2)*d^3) - ((2*I)*b*f^2*PolyLog[3, I*E^(c + d*x)])/(a^2*d^3) + ((
2*I)*b^3*f^2*PolyLog[3, I*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^3) + (2*b^4*f^2*
PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^3)
 + (2*b^4*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a
^2 + b^2)*d^3) - (b^4*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*a^3*(a^2 + b^2)*
d^3) - (f^2*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a*d^3) + (b^2*f^2*PolyLog[3, -
E^(2*c + 2*d*x)])/(2*a^3*d^3) + (f^2*PolyLog[3, E^(2*c + 2*d*x)])/(2*a*d^3)
 - (b^2*f^2*PolyLog[3, E^(2*c + 2*d*x)])/(2*a^3*d^3)
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 14</u>

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

<u>Rule 213</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2631

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a +
b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5313

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcTan[u])/(d*(m + 1))), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 + u^2)), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5708

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{csch}^3(c+dx)\text{sech}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+}}{a}$$

$$= -\frac{(e+fx)^2\coth^2(c+dx)}{2ad} - \frac{(e+fx)^2\log(\tanh(c+dx))}{ad} - \frac{b\int}{}$$

$$= \frac{b(e+fx)^2\tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{(e+fx)^2\coth^2(c+dx)}{2ad} + \frac{b(}{}$$

$$= \frac{b(e+fx)^2\tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{(e+fx)^2\coth^2(c+dx)}{2ad} + \frac{b(}{}$$

$$= \frac{b^4(e+fx)^3}{3a^3(a^2+b^2)f} + \frac{b(e+fx)^2\tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{2b^2(e+fx}{}$$

$$= \frac{b^4(e+fx)^3}{3a^3(a^2+b^2)f} + \frac{b(e+fx)^2\tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{2b^2(e+fx}{}$$

$$= -\frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a^2(a^2+b^2)d} + \frac{b(e+fx)^2\tan^{-1}(\sinh(c+dx))}{a^2d}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a^2(a^2+b^2)d} + \frac{4bf(e+fx)\tanh^{-}}{a^2d^2}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a^2(a^2+b^2)d} + \frac{4bf(e+fx)\tanh^{-}}{a^2d^2}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} + \frac{2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^3(e+fx)^2\tan^{-1}}{a^2(a^2+b^2)}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} + \frac{2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^3(e+fx)^2\tan^{-1}}{a^2(a^2+b^2)}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} + \frac{2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^3(e+fx)^2\tan^{-1}}{a^2(a^2+b^2)}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} + \frac{2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^3(e+fx)^2\tan^{-1}}{a^2(a^2+b^2)}$$

Mathematica [A]
time = 30.44, size = 2337, normalized size = 1.92

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] ((-e^2 - 2*e*f*x - f^2*x^2)*Csch[c/2 + (d*x)/2]^2)/(8*a*d) + (-12*a*d^3*e^2
*E^(2*c)*x + 12*a*d^3*e^2*(1 + E^(2*c))*x + 12*a*d^3*e*f*x^2 + 4*a*d^3*f^2*
x^3 + 12*b*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] - 6*a*d^2*e^2*(1 + E^(
2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*b*d*e*(1 + E^(2*c))*f*(d*
x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - PolyLog[2, (-I)*E^(c
+ d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*d*e*(1 + E^(2*c))*f*(2*d*x*(d*x
- Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))]) + (6*I)*b*(1 +
E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Log[1 + I*E^(c + d*x
)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2, I*E^(c + d*x)] +
 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d*x)]) - a*(1 + E^(
2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))]) - 6*d*x*PolyLog[2
, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))]))/(6*(a^2 + b^2)*d^3*(
1 + E^(2*c))) - (-12*a^2*d^3*e^2*E^(2*c)*x + 12*b^2*d^3*e^2*E^(2*c)*x + 12*
a^2*d*E^(2*c)*f^2*x - 12*a^2*d^3*e*E^(2*c)*f*x^2 + 12*b^2*d^3*e*E^(2*c)*f*x
^2 - 4*a^2*d^3*E^(2*c)*f^2*x^3 + 4*b^2*d^3*E^(2*c)*f^2*x^3 + 24*a*b*d*e*f*A
rcTanh[E^(c + d*x)] - 24*a*b*d*e*E^(2*c)*f*ArcTanh[E^(c + d*x)] - 12*a*b*d*
f^2*x*Log[1 - E^(c + d*x)] + 12*a*b*d*E^(2*c)*f^2*x*Log[1 - E^(c + d*x)] +
12*a*b*d*f^2*x*Log[1 + E^(c + d*x)] - 12*a*b*d*E^(2*c)*f^2*x*Log[1 + E^(c +
 d*x)] - 6*a^2*d^2*e^2*Log[1 - E^(2*(c + d*x))] + 6*b^2*d^2*e^2*Log[1 - E^(
2*(c + d*x))] + 6*a^2*d^2*e^2*E^(2*c)*Log[1 - E^(2*(c + d*x))] - 6*b^2*d^2*
e^2*E^(2*c)*Log[1 - E^(2*(c + d*x))] + 6*a^2*f^2*Log[1 - E^(2*(c + d*x))] -
 6*a^2*E^(2*c)*f^2*Log[1 - E^(2*(c + d*x))] - 12*a^2*d^2*e*f*x*Log[1 - E^(2
*(c + d*x))] + 12*b^2*d^2*e*f*x*Log[1 - E^(2*(c + d*x))] + 12*a^2*d^2*e*E^(
2*c)*f*x*Log[1 - E^(2*(c + d*x))] - 12*b^2*d^2*e*E^(2*c)*f*x*Log[1 - E^(2*(
c + d*x))] - 6*a^2*d^2*f^2*x^2*Log[1 - E^(2*(c + d*x))] + 6*b^2*d^2*f^2*x^2
*Log[1 - E^(2*(c + d*x))] + 6*a^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(2*(c + d*x
))] - 6*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(2*(c + d*x))] - 12*a*b*(-1 + E^(
2*c))*f^2*PolyLog[2, -E^(c + d*x)] + 12*a*b*(-1 + E^(2*c))*f^2*PolyLog[2, E
^(c + d*x)] - 6*a^2*d*e*f*PolyLog[2, E^(2*(c + d*x))] + 6*b^2*d*e*f*PolyLog
[2, E^(2*(c + d*x))] + 6*a^2*d*e*E^(2*c)*f*PolyLog[2, E^(2*(c + d*x))] - 6*
b^2*d*e*E^(2*c)*f*PolyLog[2, E^(2*(c + d*x))] - 6*a^2*d*f^2*x*PolyLog[2, E^
(2*(c + d*x))] + 6*b^2*d*f^2*x*PolyLog[2, E^(2*(c + d*x))] + 6*a^2*d*E^(2*c
)*f^2*x*PolyLog[2, E^(2*(c + d*x))] - 6*b^2*d*E^(2*c)*f^2*x*PolyLog[2, E^(2
*(c + d*x))] + 3*a^2*f^2*PolyLog[3, E^(2*(c + d*x))] - 3*b^2*f^2*PolyLog[3,
 E^(2*(c + d*x))] - 3*a^2*E^(2*c)*f^2*PolyLog[3, E^(2*(c + d*x))] + 3*b^2*E
^(2*c)*f^2*PolyLog[3, E^(2*(c + d*x))])/(6*a^3*d^3*(-1 + E^(2*c))) + (b^4*(
```

```
(2*E^(2*c)*x*(3*e^2 + 3*e*f*x + f^2*x^2))/(-1 + E^(2*c)) - (3*(d^2*e^2*Log[
2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 2*d^2*e*f*x*Log[1 + (b*E^(2*c
 + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + d^2*f^2*x^2*Log[1 + (b*E^(2
*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*d^2*e*f*x*Log[1 + (b*E^
(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + d^2*f^2*x^2*Log[1 + (b*
E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 2*d*f*(e + f*x)*PolyL
og[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 2*d*f*(e
+ f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]
 - 2*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])
)] - 2*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)
]))]))/d^3))/(3*a^3*(a^2 + b^2)) + ((-3*a^3*d*e^2*x - 3*a^3*d*e*f*x^2 - a^3
*d*f^2*x^3 + 3*a^2*b*e^2*Cosh[c] + 3*b^3*e^2*Cosh[c] + 6*a^2*b*e*f*x*Cosh[c
] + 6*b^3*e*f*x*Cosh[c] + 3*a^2*b*f^2*x^2*Cosh[c] + 3*b^3*f^2*x^2*Cosh[c])*
Csch[c/2]*Sech[c/2]*Sech[c])/(6*a^2*(a^2 + b^2)*d) + ((e^2 + 2*e*f*x + f^2*
x^2)*Sech[c/2 + (d*x)/2]^2)/(8*a*d) + (Sech[c/2]*Sech[c/2 + (d*x)/2]*(-(b*d
*e^2*Sinh[(d*x)/2]) - a*e*f*Sinh[(d*x)/2] - 2*b*d*e*f*x*Sinh[(d*x)/2] - a*f
^2*x*Sinh[(d*x)/2] - b*d*f^2*x^2*Sinh[(d*x)/2]))/(2*a^2*d^2) + (Csch[c/2]*C
sch[c/2 + (d*x)/2]*(-(b*d*e^2*Sinh[(d*x)/2]) + a*e*f*Sinh[(d*x)/2] - 2*b*d*
e*f*x*Sinh[(d*x)/2] + a*f^2*x*Sinh[(d*x)/2] - b*d*f^2*x^2*Sinh[(d*x)/2]))/(
2*a^2*d^2)
```

Maple [F]
time = 2.99, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^3 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -(b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + a^3*b^2)*d) +
 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a
^2 + b^2)*d) + 2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))
/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + (a^2 - b^2)*lo
g(e^(-d*x - c) + 1)/(a^3*d) + (a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d))*e^
```

```
2 + 2*(a*f^2*x + a*f*e + (b*d*f^2*x^2*e^(3*c) + 2*b*d*f*x*e^(3*c + 1))*e^(3
*d*x) - (a*d*f^2*x^2*e^(2*c) + a*f*e^(2*c + 1) + (a*f^2*e^(2*c) + 2*a*d*f*e
^(2*c + 1))*x)*e^(2*d*x) - (b*d*f^2*x^2*e^c + 2*b*d*f*x*e^(c + 1))*e^(d*x))
/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) - (2*b*d*f
*e + a*f^2)*x/(a^2*d^2) + (2*b*d*f*e - a*f^2)*x/(a^2*d^2) + (2*b*d*f*e + a*
f^2)*log(e^(d*x + c) + 1)/(a^2*d^3) - (2*b*d*f*e - a*f^2)*log(e^(d*x + c) -
 1)/(a^2*d^3) - (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) -
 2*polylog(3, -e^(d*x + c)))*(a^2*f^2 - b^2*f^2)/(a^3*d^3) - (d^2*x^2*log(-
e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*(a
^2*f^2 - b^2*f^2)/(a^3*d^3) + 2*(a*b*f^2 - (a^2*d*f - b^2*d*f)*e)*(d*x*log(
e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^3*d^3) - 2*(a*b*f^2 + (a^2*d*f -
 b^2*d*f)*e)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^3*d^3) + 1
/3*((a^2*f^2 - b^2*f^2)*d^3*x^3 + 3*(a*b*f^2 + (a^2*d*f - b^2*d*f)*e)*d^2*x
^2)/(a^3*d^3) + 1/3*((a^2*f^2 - b^2*f^2)*d^3*x^3 - 3*(a*b*f^2 - (a^2*d*f -
b^2*d*f)*e)*d^2*x^2)/(a^3*d^3) + integrate(2*(b^5*f^2*x^2 + 2*b^5*f*x*e - (
a*b^4*f^2*x^2*e^c + 2*a*b^4*f*x*e^(c + 1))*e^(d*x))/(a^5*b + a^3*b^3 - (a^5
*b*e^(2*c) + a^3*b^3*e^(2*c))*e^(2*d*x) - 2*(a^6*e^c + a^4*b^2*e^c)*e^(d*x)
), x) + integrate(-2*(a*f^2*x^2 + 2*a*f*x*e - (b*f^2*x^2*e^c + 2*b*f*x*e^(c
 + 1))*e^(d*x))/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 19402 vs. $2(1142) = 2284$.
time = 0.65, size = 19402, normalized size = 15.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -(2*((a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*c*f^2)*cosh(d*x + c)^4 + 2*(
(a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*c*f^2)*sinh(d*x + c)^4 + 2*(a^4 +
 a^2*b^2)*c*f^2 - 2*(a^4 + a^2*b^2)*d*f*cosh(1) - 2*((a^3*b + a*b^3)*d^2*f^
2*x^2 + 2*(a^3*b + a*b^3)*d^2*f*x*cosh(1) + (a^3*b + a*b^3)*d^2*cosh(1)^2 +
 (a^3*b + a*b^3)*d^2*sinh(1)^2 + 2*((a^3*b + a*b^3)*d^2*f*x + (a^3*b + a*b^
3)*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^3 - 2*(a^4 + a^2*b^2)*d*f*sinh(1) -
2*((a^3*b + a*b^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*f*x*cosh(1) + (a^3*b
 + a*b^3)*d^2*cosh(1)^2 + (a^3*b + a*b^3)*d^2*sinh(1)^2 - 4*((a^4 + a^2*b^2
)*d*f^2*x + (a^4 + a^2*b^2)*c*f^2)*cosh(d*x + c) + 2*((a^3*b + a*b^3)*d^2*f
*x + (a^3*b + a*b^3)*d^2*cosh(1))*sinh(1))*sinh(d*x + c)^3 + 2*((a^4 + a^2*
b^2)*d^2*f^2*x^2 - (a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*d^2*cosh(1)^2
+ (a^4 + a^2*b^2)*d^2*sinh(1)^2 - 2*(a^4 + a^2*b^2)*c*f^2 + (2*(a^4 + a^2*b
^2)*d^2*f*x + (a^4 + a^2*b^2)*d*f)*cosh(1) + (2*(a^4 + a^2*b^2)*d^2*f*x + 2
*(a^4 + a^2*b^2)*d^2*cosh(1) + (a^4 + a^2*b^2)*d*f)*sinh(1))*cosh(d*x + c)^
2 + 2*((a^4 + a^2*b^2)*d^2*f^2*x^2 - (a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b
```

```
^2)*d^2*cosh(1)^2 + (a^4 + a^2*b^2)*d^2*sinh(1)^2 - 2*(a^4 + a^2*b^2)*c*f^2
 + 6*((a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*c*f^2)*cosh(d*x + c)^2 + (2
*(a^4 + a^2*b^2)*d^2*f*x + (a^4 + a^2*b^2)*d*f)*cosh(1) - 3*((a^3*b + a*b^3
)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*f*x*cosh(1) + (a^3*b + a*b^3)*d^2*cos
h(1)^2 + (a^3*b + a*b^3)*d^2*sinh(1)^2 + 2*((a^3*b + a*b^3)*d^2*f*x + (a^3*
b + a*b^3)*d^2*cosh(1))*sinh(1))*cosh(d*x + c) + (2*(a^4 + a^2*b^2)*d^2*f*x
 + 2*(a^4 + a^2*b^2)*d^2*cosh(1) + (a^4 + a^2*b^2)*d*f)*sinh(1))*sinh(d*x +
 c)^2 + 2*((a^3*b + a*b^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*f*x*cosh(1)
+ (a^3*b + a*b^3)*d^2*cosh(1)^2 + (a^3*b + a*b^3)*d^2*sinh(1)^2 + 2*((a^3*b
 + a*b^3)*d^2*f*x + (a^3*b + a*b^3)*d^2*cosh(1))*sinh(1))*cosh(d*x + c) + 2
*(b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1) + (b^4*d*f^2*x + b^4*d*f*
cosh(1) + b^4*d*f*sinh(1))*cosh(d*x + c)^4 + 4*(b^4*d*f^2*x + b^4*d*f*cosh(
1) + b^4*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*d*f^2*x + b^4*d*
f*cosh(1) + b^4*d*f*sinh(1))*sinh(d*x + c)^4 - 2*(b^4*d*f^2*x + b^4*d*f*cos
h(1) + b^4*d*f*sinh(1))*cosh(d*x + c)^2 - 2*(b^4*d*f^2*x + b^4*d*f*cosh(1)
+ b^4*d*f*sinh(1) - 3*(b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1))*cos
h(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f
*sinh(1))*cosh(d*x + c)^3 - (b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1
))*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) +
 (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*
(b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1) + (b^4*d*f^2*x + b^4*d*f*c
osh(1) + b^4*d*f*sinh(1))*cosh(d*x + c)^4 + 4*(b^4*d*f^2*x + b^4*d*f*cosh(1
) + b^4*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*d*f^2*x + b^4*d*f
*cosh(1) + b^4*d*f*sinh(1))*sinh(d*x + c)^4 - 2*(b^4*d*f^2*x + b^4*d*f*cosh
(1) + b^4*d*f*sinh(1))*cosh(d*x + c)^2 - 2*(b^4*d*f^2*x + b^4*d*f*cosh(1) +
 b^4*d*f*sinh(1) - 3*(b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1))*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f*
sinh(1))*cosh(d*x + c)^3 - (b^4*d*f^2*x + b^4*d*f*cosh(1) + b^4*d*f*sinh(1)
)*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) -
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(
(a^4 - b^4)*d*f^2*x + ((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*f*cosh(1) + (a^4
 - b^4)*d*f*sinh(1) + (a^3*b + a*b^3)*f^2)*cosh(d*x + c)^4 + 4*((a^4 - b^4)
*d*f^2*x + (a^4 - b^4)*d*f*cosh(1) + (a^4 - b^4)*d*f*sinh(1) + (a^3*b + a*b
^3)*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 - b^4)*d*f^2*x + (a^4 - b^4)
*d*f*cosh(1) + (a^4 - b^4)*d*f*sinh(1) + (a^3*b + a*b^3)*f^2)*sinh(d*x + c)
^4 + (a^4 - b^4)*d*f*cosh(1) + (a^4 - b^4)*d*f*sinh(1) + (a^3*b + a*b^3)*f^
2 - 2*((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*f*cosh(1) + (a^4 - b^4)*d*f*sinh
(1) + (a^3*b + a*b^3)*f^2)*cosh(d*x + c)^2 - 2*((a^4 - b^4)*d*f^2*x + (a^4
- b^4)*d*f*cosh(1) + (a^4 - b^4)*d*f*sinh(1) + (a^3*b + a*b^3)*f^2 - 3*((a^
4 - b^4)*d*f^2*x + (a^4 - b^4)*d*f*cosh(1) + (a^4 - b^4)*d*f*sinh(1) + (a^3
*b + a*b^3)*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^4 - b^4)*d*f^2*x
 + (a^4 - b^4)*d*f*cosh(1) + (a^4 - b^4)*d*f*sinh(1) + (a^3*b + a*b^3)*f^2)
*cosh(d*x + c)^3 - ((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*f*cosh(1) + (a^4 -
b^4)*d*f*sinh(1) + (a^3*b + a*b^3)*f^2)*cosh(d*x + c))*sinh(d*x + c))*dilog
(cosh(d*x + c) + sinh(d*x + c)) - 2*(a^4*d*f^2*x + I*a^3*b*d*f^2*x + a^4*d*
```

```
f*cosh(1) + I*a^3*b*d*f*cosh(1) + a^4*d*f*sinh(1) + I*a^3*b*d*f*sinh(1) + (
a^4*d*f^2*x + I*a^3*b*d*f^2*x + a^4*d*f*cosh(1) + I*a^3*b*d*f*cosh(1) + a^4
*d*f*sinh(1) + I*a^3*b*d*f*sinh(1))*cosh(d*x + c)^4 + 4*(a^4*d*f^2*x + I*a^
3*b*d*f^2*x + a^4*d*f*cosh(1) + I*a^3*b*d*f*cosh(1) + a^4*d*f*sinh(1) + I*a
^3*b*d*f*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*d*f^2*x + I*a^3*b*d*
f^2*x + a^4*d*f*cosh(1) + I*a^3*b*d*f*cosh(1) + a^4*d*f*sinh(1) + I*a^3*b*d
*f*sinh(1))*sinh(d*x + c)^4 - 2*(a^4*d*f^2*x + ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)**3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^2}{\cosh\left(c+d\,x\right)\,\sinh\left(c+d\,x\right)^3\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(cosh(c + d*x)*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((e + f*x)^2/(cosh(c + d*x)*sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

3.493 $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=762

$$\frac{fx}{2ad}+\frac{2bfx\text{ArcTan}\left(e^{c+dx}\right)}{a^2d}-\frac{2b^3(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d}-\frac{bfx\text{ArcTan}(\sinh(c+dx))}{a^2d}+\frac{b(e+fx)\text{ArcTan}(\sinh(}{a^2d}$$

[Out] -1/2*f*polylog(2,exp(2*d*x+2*c))/a/d^2+1/2*f*x/a/d-1/2*(f*x+e)*coth(d*x+c)^2/a/d+1/2*f*polylog(2,-exp(2*d*x+2*c))/a/d^2-1/2*f*coth(d*x+c)/a/d^2-(f*x+e)*ln(tanh(d*x+c))/a/d+f*x*ln(tanh(d*x+c))/a/d+b^4*(f*x+e)*ln(1+exp(2*d*x+2*c))/a^3/(a^2+b^2)/d-b^4*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)/d-b^4*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)/d-b^4*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)/d^2-b^4*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)/d^2+2*b*f*x*arctan(exp(d*x+c))/a^2/d-2*b^3*(f*x+e)*arctan(exp(d*x+c))/a^2/(a^2+b^2)/d-b*f*x*arctan(sinh(d*x+c))/a^2/d+I*b*f*polylog(2,I*exp(d*x+c))/a^2/d^2+1/2*b^4*f*polylog(2,-exp(2*d*x+2*c))/a^3/(a^2+b^2)/d^2-I*b*f*polylog(2,-I*exp(d*x+c))/a^2/d^2+I*b^3*f*polylog(2,-I*exp(d*x+c))/a^2/(a^2+b^2)/d^2-I*b^3*f*polylog(2,I*exp(d*x+c))/a^2/(a^2+b^2)/d^2+1/2*b^2*f*polylog(2,exp(2*d*x+2*c))/a^3/d^2+b*(f*x+e)*arctan(sinh(d*x+c))/a^2/d-2*b^2*(f*x+e)*arctanh(exp(2*d*x+2*c))/a^3/d+b*f*arctanh(cosh(d*x+c))/a^2/d^2+b*(f*x+e)*csch(d*x+c)/a^2/d-1/2*b^2*f*polylog(2,-exp(2*d*x+2*c))/a^3/d^2+2*f*x*arctanh(exp(2*d*x+2*c))/a/d

Rubi [A]
time = 0.82, antiderivative size = 762, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 23, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.719$, Rules used = {5708, 2700, 14, 5570, 3554, 8, 2628, 12, 4267, 2317, 2438, 2701, 327, 213, 5311, 4265, 3855, 5569, 5692, 5680, 2221, 6874, 3799}

Antiderivative was successfully verified.

[In] Int[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]

[Out] (f*x)/(2*a*d) + (2*b*f*x*ArcTan[E^(c + d*x)])/(a^2*d) - (2*b^3*(e + f*x)*ArcTan[E^(c + d*x)])/(a^2*(a^2 + b^2)*d) - (b*f*x*ArcTan[Sinh[c + d*x]])/(a^2*d) + (b*(e + f*x)*ArcTan[Sinh[c + d*x]])/(a^2*d) + (2*f*x*ArcTanh[E^(2*c + 2*d*x)])/(a*d) - (2*b^2*(e + f*x)*ArcTanh[E^(2*c + 2*d*x)])/(a^3*d) + (b*f*ArcTanh[Cosh[c + d*x]])/(a^2*d^2) - (f*Coth[c + d*x])/(2*a*d^2) - ((e + f*x)*Coth[c + d*x]^2)/(2*a*d) + (b*(e + f*x)*Csch[c + d*x])/(a^2*d) - (b^4*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)*d) - (b^4*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*(a^2

```
 + b^2)*d) + (b^4*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a^3*(a^2 + b^2)*d) +
 (f*x*Log[Tanh[c + d*x]])/(a*d) - ((e + f*x)*Log[Tanh[c + d*x]])/(a*d) - (I
*b*f*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*d^2) + (I*b^3*f*PolyLog[2, (-I)*E^(
c + d*x)])/(a^2*(a^2 + b^2)*d^2) + (I*b*f*PolyLog[2, I*E^(c + d*x)])/(a^2*d
^2) - (I*b^3*f*PolyLog[2, I*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^2) - (b^4*f*Po
lyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^2) -
 (b^4*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b
^2)*d^2) + (b^4*f*PolyLog[2, -E^(2*(c + d*x))])/(2*a^3*(a^2 + b^2)*d^2) + (
f*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a*d^2) - (b^2*f*PolyLog[2, -E^(2*c + 2*d
*x)])/(2*a^3*d^2) - (f*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d^2) + (b^2*f*Poly
Log[2, E^(2*c + 2*d*x)])/(2*a^3*d^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5311

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5708

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{csch}^3(c+dx)\text{sech}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{s}}{a+b\sinh(c+d}}{a}$$

$$= -\frac{(e+fx)\coth^2(c+dx)}{2ad} - \frac{(e+fx)\log(\tanh(c+dx))}{ad} - \frac{b\int(e}{}$$

$$= \frac{b(e+fx)\tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{(e+fx)\coth^2(c+dx)}{2ad} + \frac{b(e}{}$$

$$= \frac{b(e+fx)\tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{f\coth(c+dx)}{2ad^2} - \frac{(e+fx)\text{co}}{2a}$$

$$= \frac{fx}{2ad} + \frac{b^4(e+fx)^2}{2a^3(a^2+b^2)f} - \frac{bfx\tan^{-1}(\sinh(c+dx))}{a^2d} + \frac{b(e+fx)\text{t}}{}$$

$$= \frac{fx}{2ad} + \frac{b^4(e+fx)^2}{2a^3(a^2+b^2)f} - \frac{bfx\tan^{-1}(\sinh(c+dx))}{a^2d} + \frac{b(e+fx)\text{t}}{}$$

$$= \frac{fx}{2ad} + \frac{2bfx\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2(a^2+b^2)d} - \frac{bfx\text{t}}{}$$

$$= \frac{fx}{2ad} + \frac{2bfx\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2(a^2+b^2)d} - \frac{bfx\text{t}}{}$$

$$= \frac{fx}{2ad} + \frac{2bfx\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2(a^2+b^2)d} - \frac{bfx\text{t}}{}$$

$$= \frac{fx}{2ad} + \frac{2bfx\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2(a^2+b^2)d} - \frac{bfx\text{t}}{}$$

Mathematica [A]
time = 7.18, size = 913, normalized size = 1.20

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] ((2*b*d*e*Cosh[(c + d*x)/2] - a*f*Cosh[(c + d*x)/2] - 2*b*c*f*Cosh[(c + d*x
)/2] + 2*b*f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(4*a^2*d^2) +
((-(d*e) + c*f - f*(c + d*x))*Csch[(c + d*x)/2]^2)/(8*a*d^2) - (e*Log[Sinh[
c + d*x]])/(a*d) + (b^2*e*Log[Sinh[c + d*x]])/(a^3*d) + (c*f*Log[Sinh[c + d
*x]])/(a*d^2) - (b^2*c*f*Log[Sinh[c + d*x]])/(a^3*d^2) - (b*f*Log[Tanh[(c +
 d*x)/2]])/(a^2*d^2) + (I*f*(I*(c + d*x)*Log[1 - E^(-2*(c + d*x))] - (I/2)*
(-(c + d*x)^2 + PolyLog[2, E^(-2*(c + d*x))])))/(a*d^2) - (I*b^2*f*(I*(c +
d*x)*Log[1 - E^(-2*(c + d*x))] - (I/2)*(-(c + d*x)^2 + PolyLog[2, E^(-2*(c
+ d*x))])))/(a^3*d^2) - (b^4*(-1/2*(f*(c + d*x)^2) + f*(c + d*x)*Log[1 + (b
*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2])] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[
c + d*x]] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLo
g[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^3*(a^2 + b^2)*d^2) + (-
(a*d*e*(c + d*x)) + a*c*f*(c + d*x) - (a*f*(c + d*x)^2)/2 + 2*b*d*e*ArcTan[
Cosh[c + d*x] + Sinh[c + d*x]] - 2*b*c*f*ArcTan[Cosh[c + d*x] + Sinh[c + d*
x]] + 2*b*f*(c + d*x)*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]] + a*d*e*Log[1 +
 Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]] - a*c*f*Log[1 + Cosh[2*(c + d*x)] +
 Sinh[2*(c + d*x)]] + a*f*(c + d*x)*Log[1 + Cosh[2*(c + d*x)] + Sinh[2*(c +
 d*x)]] - I*b*f*PolyLog[2, (-I)*(Cosh[c + d*x] + Sinh[c + d*x])] + I*b*f*Po
lyLog[2, I*(Cosh[c + d*x] + Sinh[c + d*x])] + (a*f*PolyLog[2, -Cosh[2*(c +
d*x)] - Sinh[2*(c + d*x)]])/2)/((a^2 + b^2)*d^2) + ((d*e - c*f + f*(c + d*x
))*Sech[(c + d*x)/2]^2)/(8*a*d^2) + (Sech[(c + d*x)/2]*(-2*b*d*e*Sinh[(c +
d*x)/2] - a*f*Sinh[(c + d*x)/2] + 2*b*c*f*Sinh[(c + d*x)/2] - 2*b*f*(c + d*
x)*Sinh[(c + d*x)/2]))/(4*a^2*d^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1477 vs. 2(714) = 1428.
time = 8.66, size = 1478, normalized size = 1.94

method	result	size
risch	Expression too large to display	1478

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVER
BOSE)
```

```
[Out] 4/d*e/(4*a^2+4*b^2)*a*ln(1+exp(2*d*x+2*c))+1/d/a^3*b^2*e*ln(exp(d*x+c)+1)+1
/d/a^3*b^2*e*ln(exp(d*x+c)-1)+1/d^2/a^2*b*f*ln(exp(d*x+c)+1)-1/d^2/a^2*b*f*
ln(exp(d*x+c)-1)+4/d*a*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x+4/d*a*f/(4*a^2+
4*b^2)*ln(1+I*exp(d*x+c))*x+4/d^2*a*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*c-1/
d/a^3*b^4*e/(a^2+b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+4/d^2*a*f/(4*a^
2+4*b^2)*ln(1+I*exp(d*x+c))*c+1/d^2/a^2*f*b^4/(a^2+b^2)^(3/2)*arctanh(1/2*(
2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d^2/a^2*b^2*f/(a^2+b^2)^(1/2)*arctan
h(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d^2/a^3*f*b^4/(a^2+b^2)*dilog
((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2/a^3*f*b^4/(a^2
```

```
+b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-4/d^2*a
*f*c/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))-8/d^2*f*c/(4*a^2+4*b^2)*b*arctan(ex
p(d*x+c))-4*I/d^2*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))*b+4*I/d^2*f/(4*a^2+
4*b^2)*dilog(1-I*exp(d*x+c))*b+1/d^2*b^2*f/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b
*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+8/d*e/(4*a^2+4*b^2)*b*arctan(exp(d*x+c))-
1/d^2/a^3*f*b^4/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2
)^(1/2)))*c-1/d^2/a^3*f*b^4/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(
a+(a^2+b^2)^(1/2)))*c-1/d/a^3*f*b^4/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(
1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d/a^3*f*b^4/(a^2+b^2)*ln((b*exp(d*x+c)+(a
^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2/a^3*f*b^4*c/(a^2+b^2)*ln(b*ex
p(2*d*x+2*c)+2*a*exp(d*x+c)-b)+4*I/d*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*b*x
+4*I/d^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*b*c-4*I/d*f/(4*a^2+4*b^2)*ln(1+
I*exp(d*x+c))*b*x-4*I/d^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*b*c-1/d^2*f/a*
dilog(exp(d*x+c)+1)+1/d^2*f*dilog(exp(d*x+c))/a+1/d/a^3*b^2*f*ln(exp(d*x+c)
+1)*x-1/d^2/a^3*b^2*f*c*ln(exp(d*x+c)-1)-(-2*b*d*f*x*exp(3*d*x+3*c)+2*a*d*f
*x*exp(2*d*x+2*c)-2*b*d*e*exp(3*d*x+3*c)+2*a*d*e*exp(2*d*x+2*c)+2*b*d*f*x*e
xp(d*x+c)+a*f*exp(2*d*x+2*c)+2*b*d*e*exp(d*x+c)-f*a)/d^2/a^2/(exp(2*d*x+2*c
)-1)^2-1/d*e/a*ln(exp(d*x+c)-1)-1/d*e/a*ln(exp(d*x+c)+1)-1/d*f/a*ln(exp(d*x
+c)+1)*x+1/d^2*f*c/a*ln(exp(d*x+c)-1)-1/d^2/a^3*b^2*f*dilog(exp(d*x+c))+1/d
^2/a^3*b^2*f*dilog(exp(d*x+c)+1)+4/d^2*a*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+
c))+4/d^2*a*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] (16*a^2*d*integrate(1/16*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 16*b^2*d*integ
rate(1/16*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 16*a^2*d*integrate(1/16*x/(a^
3*d*e^(d*x + c) - a^3*d), x) + 16*b^2*d*integrate(1/16*x/(a^3*d*e^(d*x + c)
 - a^3*d), x) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) + 1)/(a^3*d^2))
+ a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) - 1)/(a^3*d^2)) + (2*b*d*x*e^(
3*d*x + 3*c) - 2*b*d*x*e^(d*x + c) - (2*a*d*x*e^(2*c) + a*e^(2*c))*e^(2*d*x
) + a)/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) + 16
*integrate(-1/8*(a*b^4*x*e^(d*x + c) - b^5*x)/(a^5*b + a^3*b^3 - (a^5*b*e^(
2*c) + a^3*b^3*e^(2*c))*e^(2*d*x) - 2*(a^6*e^c + a^4*b^2*e^c)*e^(d*x)), x)
+ 16*integrate(1/8*(b*x*e^(d*x + c) - a*x)/(a^2 + b^2 + (a^2*e^(2*c) + b^2*
e^(2*c))*e^(2*d*x)), x))*f - (b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c
) - b)/((a^5 + a^3*b^2)*d) + 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*l
og(e^(-2*d*x - 2*c) + 1)/((a^2 + b^2)*d) + 2*(b*e^(-d*x - c) - a*e^(-2*d*x
- 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c
```

```
) - a^2)*d) + (a^2 - b^2)*log(e^(-d*x - c) + 1)/(a^3*d) + (a^2 - b^2)*log(e
^(-d*x - c) - 1)/(a^3*d))*e
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6634 vs. $2(704) = 1408$.
time = 0.48, size = 6634, normalized size = 8.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] (2*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*cosh(1) + (a^3*b + a*b^3)*d*s
inh(1))*cosh(d*x + c)^3 + 2*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*cosh
(1) + (a^3*b + a*b^3)*d*sinh(1))*sinh(d*x + c)^3 - (2*(a^4 + a^2*b^2)*d*f*x
 + 2*(a^4 + a^2*b^2)*d*cosh(1) + 2*(a^4 + a^2*b^2)*d*sinh(1) + (a^4 + a^2*b
^2)*f)*cosh(d*x + c)^2 - (2*(a^4 + a^2*b^2)*d*f*x + 2*(a^4 + a^2*b^2)*d*cos
h(1) + 2*(a^4 + a^2*b^2)*d*sinh(1) + (a^4 + a^2*b^2)*f - 6*((a^3*b + a*b^3)
*d*f*x + (a^3*b + a*b^3)*d*cosh(1) + (a^3*b + a*b^3)*d*sinh(1))*cosh(d*x +
c))*sinh(d*x + c)^2 + (a^4 + a^2*b^2)*f - 2*((a^3*b + a*b^3)*d*f*x + (a^3*b
 + a*b^3)*d*cosh(1) + (a^3*b + a*b^3)*d*sinh(1))*cosh(d*x + c) - (b^4*f*cos
h(d*x + c)^4 + 4*b^4*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^4*f*sinh(d*x + c)^
4 - 2*b^4*f*cosh(d*x + c)^2 + b^4*f + 2*(3*b^4*f*cosh(d*x + c)^2 - b^4*f)*s
inh(d*x + c)^2 + 4*(b^4*f*cosh(d*x + c)^3 - b^4*f*cosh(d*x + c))*sinh(d*x +
 c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^4*f*cosh(d*x + c)^4 + 4*b^4
*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^4*f*sinh(d*x + c)^4 - 2*b^4*f*cosh(d*x
 + c)^2 + b^4*f + 2*(3*b^4*f*cosh(d*x + c)^2 - b^4*f)*sinh(d*x + c)^2 + 4*(
b^4*f*cosh(d*x + c)^3 - b^4*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d
*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2) - b)/b + 1) - ((a^4 - b^4)*f*cosh(d*x + c)^4 + 4*(a^4 - b^4)*f*c
osh(d*x + c)*sinh(d*x + c)^3 + (a^4 - b^4)*f*sinh(d*x + c)^4 - 2*(a^4 - b^4
)*f*cosh(d*x + c)^2 + 2*(3*(a^4 - b^4)*f*cosh(d*x + c)^2 - (a^4 - b^4)*f)*s
inh(d*x + c)^2 + (a^4 - b^4)*f + 4*((a^4 - b^4)*f*cosh(d*x + c)^3 - (a^4 -
b^4)*f*cosh(d*x + c))*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)) +
 (a^4*f + I*a^3*b*f + (a^4*f + I*a^3*b*f)*cosh(d*x + c)^4 + 4*(a^4*f + I*a^
3*b*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*f + I*a^3*b*f)*sinh(d*x + c)^4
- 2*(a^4*f + I*a^3*b*f)*cosh(d*x + c)^2 - 2*(a^4*f + I*a^3*b*f - 3*(a^4*f +
 I*a^3*b*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^4*f + I*a^3*b*f)*cosh(
d*x + c)^3 - (a^4*f + I*a^3*b*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(I*cosh
(d*x + c) + I*sinh(d*x + c)) + (a^4*f - I*a^3*b*f + (a^4*f - I*a^3*b*f)*cos
h(d*x + c)^4 + 4*(a^4*f - I*a^3*b*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*f
 - I*a^3*b*f)*sinh(d*x + c)^4 - 2*(a^4*f - I*a^3*b*f)*cosh(d*x + c)^2 - 2*(
a^4*f - I*a^3*b*f - 3*(a^4*f - I*a^3*b*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2
```

```
+ 4*((a^4*f - I*a^3*b*f)*cosh(d*x + c)^3 - (a^4*f - I*a^3*b*f)*cosh(d*x + c
))*sinh(d*x + c))*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) - ((a^4 - b^4)*
f*cosh(d*x + c)^4 + 4*(a^4 - b^4)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 -
b^4)*f*sinh(d*x + c)^4 - 2*(a^4 - b^4)*f*cosh(d*x + c)^2 + 2*(3*(a^4 - b^4)
*f*cosh(d*x + c)^2 - (a^4 - b^4)*f)*sinh(d*x + c)^2 + (a^4 - b^4)*f + 4*((a
^4 - b^4)*f*cosh(d*x + c)^3 - (a^4 - b^4)*f*cosh(d*x + c))*sinh(d*x + c))*d
ilog(-cosh(d*x + c) - sinh(d*x + c)) + (b^4*c*f - b^4*d*cosh(1) - b^4*d*sin
h(1) + (b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*cosh(d*x + c)^4 + 4*(b^4*c
*f - b^4*d*cosh(1) - b^4*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*c*
f - b^4*d*cosh(1) - b^4*d*sinh(1))*sinh(d*x + c)^4 - 2*(b^4*c*f - b^4*d*cos
h(1) - b^4*d*sinh(1))*cosh(d*x + c)^2 - 2*(b^4*c*f - b^4*d*cosh(1) - b^4*d*
sinh(1) - 3*(b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*cosh(d*x + c)^2)*sinh
(d*x + c)^2 + 4*((b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*cosh(d*x + c)^3
- (b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*l
og(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
 + (b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1) + (b^4*c*f - b^4*d*cosh(1) - b^
4*d*sinh(1))*cosh(d*x + c)^4 + 4*(b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*
cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*s
inh(d*x + c)^4 - 2*(b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1))*cosh(d*x + c)^
2 - 2*(b^4*c*f - b^4*d*cosh(1) - b^4*d*sinh(1) - 3*(b^4*c*f - b^4*d*cosh(1)
 - b^4*d*sinh(1))*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*c*f - b^4*d*co
sh(1) - b^4*d*sinh(1))*cosh(d*x + c)^3 - (b^4*c*f - b^4*d*cosh(1) - b^4*d*s
inh(1))*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x
+ c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^4*d*f*x + b^4*c*f + (b^4*d*f*x
 + b^4*c*f)*cosh(d*x + c)^4 + 4*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)*sinh(d*
x + c)^3 + (b^4*d*f*x + b^4*c*f)*sinh(d*x + c)^4 - 2*(b^4*d*f*x + b^4*c*f)*
cosh(d*x + c)^2 - 2*(b^4*d*f*x + b^4*c*f - 3*(b^4*d*f*x + b^4*c*f)*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 + 4*((b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^3 - (b^4*
d*f*x + b^4*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*si
nh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b
)/b) - (b^4*d*f*x + b^4*c*f + (b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^4 + 4*(b^
4*d*f*x + b^4*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*d*f*x + b^4*c*f)*si
nh(d*x + c)^4 - 2*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^2 - 2*(b^4*d*f*x + b^
4*c*f - 3*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*
d*f*x + b^4*c*f)*cosh(d*x + c)^3 - (b^4*d*f*x +...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6439 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\cosh\left(c+d\,x\right)\,{\sinh\left(c+d\,x\right)}^3\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(cosh(c + d*x)*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((e + f*x)/(cosh(c + d*x)*sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

3.494 $\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{b\operatorname{ArcTan}(\sinh(c+dx))}{(a^2+b^2)\,d}+\frac{b\operatorname{csch}(c+dx)}{a^2d}-\frac{\operatorname{csch}^2(c+dx)}{2ad}+\frac{a\log(\cosh(c+dx))}{(a^2+b^2)\,d}-\frac{(a^2-b^2)\log(\sinh(c+dx))}{a^3d}-$$

```
[Out] b*arctan(sinh(d*x+c))/(a^2+b^2)/d+b*csch(d*x+c)/a^2/d-1/2*csch(d*x+c)^2/a/d
+a*ln(cosh(d*x+c))/(a^2+b^2)/d-(a^2-b^2)*ln(sinh(d*x+c))/a^3/d-b^4*ln(a+b*s
inh(d*x+c))/a^3/(a^2+b^2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2916, 12, 908, 649, 209, 266}

$$\frac{b\operatorname{ArcTan}(\sinh(c+dx))}{d\,(a^2+b^2)}+\frac{a\log(\cosh(c+dx))}{d\,(a^2+b^2)}+\frac{b\operatorname{csch}(c+dx)}{a^2d}-\frac{(a^2-b^2)\log(\sinh(c+dx))}{a^3d}-\frac{b^4\log(a+b\sinh(c+dx))}{a^3d\,(a^2+b^2)}-\frac{\operatorname{csch}^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d) + (b*Csch[c + d*x])/(a^2*d) - Csc
h[c + d*x]^2/(2*a*d) + (a*Log[Cosh[c + d*x]])/((a^2 + b^2)*d) - ((a^2 - b^2
)*Log[Sinh[c + d*x]])/(a^3*d) - (b^4*Log[a + b*Sinh[c + d*x]])/(a^3*(a^2 +
b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b\text{Subst}\left(\int \frac{b^3}{x^3(a+x)(-b^2-x^2)}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= -\frac{b^4\text{Subst}\left(\int \frac{1}{x^3(a+x)(-b^2-x^2)}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= -\frac{b^4\text{Subst}\left(\int \left(-\frac{1}{ab^2x^3} + \frac{1}{a^2b^2x^2} + \frac{a^2-b^2}{a^3b^4x} + \frac{1}{a^3(a^2+b^2)(a+x)} + \frac{-b^2-ax}{b^4(a^2+b^2)(b^2+x^2)}\right) \, d}{d}$$

$$= \frac{b\operatorname{csch}(c+dx)}{a^2d} - \frac{\operatorname{csch}^2(c+dx)}{2ad} - \frac{(a^2-b^2)\log(\sinh(c+dx))}{a^3d} - \frac{b^4\log(a -}{a^3}$$

$$= \frac{b\operatorname{csch}(c+dx)}{a^2d} - \frac{\operatorname{csch}^2(c+dx)}{2ad} - \frac{(a^2-b^2)\log(\sinh(c+dx))}{a^3d} - \frac{b^4\log(a -}{a^3}$$

$$= \frac{b\tan^{-1}(\sinh(c+dx))}{(a^2+b^2)\,d} + \frac{b\operatorname{csch}(c+dx)}{a^2d} - \frac{\operatorname{csch}^2(c+dx)}{2ad} + \frac{a\log(\cosh(c +}{(a^2+b^2)\,d}$$

Mathematica [A]

time = 0.31, size = 164, normalized size = 1.26

$$\frac{\frac{2b\operatorname{csch}(c+dx)}{a^2} - \frac{\operatorname{csch}^2(c+dx)}{a} - \frac{2(a-b)(a+b)\log(\sinh(c+dx))}{a^3} + \frac{\left(a-\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\sinh(c+dx)\right)}{a^2+b^2} - \frac{2b^4\log(a+b\sinh(c+dx))}{a^3(a^2+b^2)} + \frac{\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\sinh(c+dx)\right)}{a^2+b^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((2*b*Csch[c + d*x])/a^2 - Csch[c + d*x]^2/a - (2*(a - b)*(a + b)*Log[Sinh[
c + d*x]])/a^3 + ((a - Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[c + d*x]])/(a^2
+ b^2) - (2*b^4*Log[a + b*Sinh[c + d*x]])/(a^3*(a^2 + b^2)) + ((a + Sqrt[-b
^2])*Log[Sqrt[-b^2] + b*Sinh[c + d*x]])/(a^2 + b^2))/(2*d)
```

Maple [A]
time = 1.77, size = 186, normalized size = 1.43

method	result
derivativedivides	$\frac{-\frac{\frac{a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{b^4\ln\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a^3\left(a^2+b^2\right)}+\frac{4a\ln\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+8b\arctan\left(\text{ta}\right.}{4a^2+4b^2}}{d}$
default	$\frac{-\frac{\frac{a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{b^4\ln\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a^3\left(a^2+b^2\right)}+\frac{4a\ln\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+8b\arctan\left(\text{ta}\right.}{4a^2+4b^2}}{d}$
risch	$-\frac{2a\,d^2x}{a^2d^2+b^2d^2}-\frac{2adc}{a^2d^2+b^2d^2}+\frac{2b^4x}{a^3(a^2+b^2)}+\frac{2b^4c}{a^3d(a^2+b^2)}+\frac{2x}{a}+\frac{2c}{ad}-\frac{2b^2x}{a^3}-\frac{2b^2c}{a^3d}-\frac{2\,\mathrm{e}^{dx+c}\left(-b\,\mathrm{e}^{2dx+2c}\right.}{a^2d\left(\mathrm{e}^{2dx+2c}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4/a^2*(1/2*a*tanh(1/2*d*x+1/2*c)^2+2*b*tanh(1/2*d*x+1/2*c))-b^4/a^3
/(a^2+b^2)*ln(a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)-a)+1/4/(a^2+b
^2)*(4*a*ln(tanh(1/2*d*x+1/2*c)^2+1)+8*b*arctan(tanh(1/2*d*x+1/2*c)))-1/8/a
/tanh(1/2*d*x+1/2*c)^2+1/4/a^3*(-4*a^2+4*b^2)*ln(tanh(1/2*d*x+1/2*c))+1/2*b
/a^2/tanh(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.48, size = 236, normalized size = 1.82

$$-\frac{b^4\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^5+a^3b^2)d}-\frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d}+\frac{a\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d}-\frac{2\left(be^{(-dx-c)}-ae^{(-2\,dx-2\,c)}-be^{(-3\,dx-3\,c)}\right)}{(2\,a^2e^{(-2\,dx-2\,c)}-a^2e^{(-4\,dx-4\,c)}-a^2)d}-\frac{(a^2-b^2)\log\left(e^{(-dx-c)}+1\right)}{a^3d}-\frac{(a^2-b^2)\log\left(e^{(-dx-c)}-1\right)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + a^3*b^2)*d) -
2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(e^(-2*d*x - 2*c) + 1)/((a^
2 + b^2)*d) - 2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/
((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - (a^2 - b^2)*log
(e^(-d*x - c) + 1)/(a^3*d) - (a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. $2(128) = 256$.

time = 0.45, size = 1035, normalized size = 7.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] (2*(a^3*b + a*b^3)*cosh(d*x + c)^3 + 2*(a^3*b + a*b^3)*sinh(d*x + c)^3 - 2*
(a^4 + a^2*b^2)*cosh(d*x + c)^2 - 2*(a^4 + a^2*b^2 - 3*(a^3*b + a*b^3)*cosh
(d*x + c))*sinh(d*x + c)^2 + 2*(a^3*b*cosh(d*x + c)^4 + 4*a^3*b*cosh(d*x +
c)*sinh(d*x + c)^3 + a^3*b*sinh(d*x + c)^4 - 2*a^3*b*cosh(d*x + c)^2 + a^3*
b + 2*(3*a^3*b*cosh(d*x + c)^2 - a^3*b)*sinh(d*x + c)^2 + 4*(a^3*b*cosh(d*x
 + c)^3 - a^3*b*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d
*x + c)) - 2*(a^3*b + a*b^3)*cosh(d*x + c) - (b^4*cosh(d*x + c)^4 + 4*b^4*c
osh(d*x + c)*sinh(d*x + c)^3 + b^4*sinh(d*x + c)^4 - 2*b^4*cosh(d*x + c)^2
+ b^4 + 2*(3*b^4*cosh(d*x + c)^2 - b^4)*sinh(d*x + c)^2 + 4*(b^4*cosh(d*x +
 c)^3 - b^4*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh
(d*x + c) - sinh(d*x + c))) + (a^4*cosh(d*x + c)^4 + 4*a^4*cosh(d*x + c)*si
nh(d*x + c)^3 + a^4*sinh(d*x + c)^4 - 2*a^4*cosh(d*x + c)^2 + a^4 + 2*(3*a^
4*cosh(d*x + c)^2 - a^4)*sinh(d*x + c)^2 + 4*(a^4*cosh(d*x + c)^3 - a^4*cos
h(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x +
c))) - ((a^4 - b^4)*cosh(d*x + c)^4 + 4*(a^4 - b^4)*cosh(d*x + c)*sinh(d*x
+ c)^3 + (a^4 - b^4)*sinh(d*x + c)^4 + a^4 - b^4 - 2*(a^4 - b^4)*cosh(d*x +
 c)^2 - 2*(a^4 - b^4 - 3*(a^4 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(
(a^4 - b^4)*cosh(d*x + c)^3 - (a^4 - b^4)*cosh(d*x + c))*sinh(d*x + c))*log
(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - 2*(a^3*b + a*b^3 - 3*(a
^3*b + a*b^3)*cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*cosh(d*x + c))*sinh(d*x +
 c))/((a^5 + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a^5 + a^3*b^2)*d*cosh(d*x + c)
*sinh(d*x + c)^3 + (a^5 + a^3*b^2)*d*sinh(d*x + c)^4 - 2*(a^5 + a^3*b^2)*d*
cosh(d*x + c)^2 + 2*(3*(a^5 + a^3*b^2)*d*cosh(d*x + c)^2 - (a^5 + a^3*b^2)*
d)*sinh(d*x + c)^2 + (a^5 + a^3*b^2)*d + 4*((a^5 + a^3*b^2)*d*cosh(d*x + c)
^3 - (a^5 + a^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3436 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(128) = 256.

time = 0.50, size = 263, normalized size = 2.02

$$-\frac{\frac{2b^5\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2a\right|\right)}{a^5b+a^3b^3}-\frac{\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)b}{a^2+b^2}-\frac{a\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^2+b^2}+\frac{2(a^2-b^2)\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a^3}-\frac{3a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-3b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)-4a^2}{a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^5*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^5*b + a^3*b^3
) - (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*b/(a^2 + b^2) -
 a*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2 + b^2) + 2*(a^2 - b^2)*log(
abs(e^(d*x + c) - e^(-d*x - c)))/a^3 - (3*a^2*(e^(d*x + c) - e^(-d*x - c))^
2 - 3*b^2*(e^(d*x + c) - e^(-d*x - c))^2 + 4*a*b*(e^(d*x + c) - e^(-d*x - c
)) - 4*a^2)/(a^3*(e^(d*x + c) - e^(-d*x - c))^2))/d
```

Mupad [B]
time = 3.61, size = 196, normalized size = 1.51

$$\frac{\ln\left(e^{c+d\,x}+1{}\mathrm{i}\right)}{a\,d-b\,d\,1{}\mathrm{i}}-\frac{\frac{2}{a\,d}-\frac{2\,b\,e^{c+d\,x}}{a^2\,d}}{e^{2\,c+2\,d\,x}-1}-\frac{2}{a\,d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}-\frac{b^4\,\ln\left(2\,a\,e^{c+d\,x}-b+b\,e^{2\,c+2\,d\,x}\right)}{d\,a^5+d\,a^3\,b^2}-\frac{\ln\left(e^{2\,c+2\,d\,x}-1\right)\,\left(a^2-b^2\right)}{a^3\,d}+\frac{\ln\left(1+e^{c+d\,x}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{-b\,d+a\,d\,1{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

```
[Out] log(exp(c + d*x) + 1i)/(a*d - b*d*1i) - (2/(a*d) - (2*b*exp(c + d*x))/(a^2*
d))/(exp(2*c + 2*d*x) - 1) + (log(exp(c + d*x)*1i + 1)*1i)/(a*d*1i - b*d) -
 2/(a*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (b^4*log(2*a*exp(c +
 d*x) - b + b*exp(2*c + 2*d*x)))/(a^5*d + a^3*b^2*d) - (log(exp(2*c + 2*d*x
) - 1)*(a^2 - b^2))/(a^3*d)
```

3.495 $\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=37

$$\operatorname{Int}\left(\frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csch[c + d*x]^3*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Csch[c + d*x]^3*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 118.32, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csch[c + d*x]^3*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] Integrate[(Csch[c + d*x]^3*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(dx+c)^3 \operatorname{sech}(dx+c)}{(fx+e)\,(a+b\sinh\,(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -(a*f - 2*(b*d*f*x*e^(3*c) + b*d*e^(3*c + 1))*e^(3*d*x) + (2*a*d*f*x*e^(2*c
) - a*f*e^(2*c) + 2*a*d*e^(2*c + 1))*e^(2*d*x) + 2*(b*d*f*x*e^c + b*d*e^(c
+ 1))*e^(d*x))/(a^2*d^2*f^2*x^2 + 2*a^2*d^2*f*x*e + a^2*d^2*e^2 + (a^2*d^2*
f^2*x^2*e^(4*c) + 2*a^2*d^2*f*x*e^(4*c + 1) + a^2*d^2*e^(4*c + 2))*e^(4*d*x
) - 2*(a^2*d^2*f^2*x^2*e^(2*c) + 2*a^2*d^2*f*x*e^(2*c + 1) + a^2*d^2*e^(2*c
 + 2))*e^(2*d*x)) - 16*integrate(1/16*(a*b*d*f*e + a^2*f^2 - (a^2*d^2*f^2 -
 b^2*d^2*f^2)*x^2 + (a*b*d*f^2 - 2*(a^2*d^2*f - b^2*d^2*f)*e)*x - (a^2*d^2
- b^2*d^2)*e^2)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*f^2*x^2*e + 3*a^3*d^2*f*x*e^2
+ a^3*d^2*e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*f^2*x^2*e^(c + 1) + 3*a^3*
d^2*f*x*e^(c + 2) + a^3*d^2*e^(c + 3))*e^(d*x)), x) + 16*integrate(1/16*(a*
b*d*f*e - a^2*f^2 + (a^2*d^2*f^2 - b^2*d^2*f^2)*x^2 + (a*b*d*f^2 + 2*(a^2*d
^2*f - b^2*d^2*f)*e)*x + (a^2*d^2 - b^2*d^2)*e^2)/(a^3*d^2*f^3*x^3 + 3*a^3*
d^2*f^2*x^2*e + 3*a^3*d^2*f*x*e^2 + a^3*d^2*e^3 + (a^3*d^2*f^3*x^3*e^c + 3*
a^3*d^2*f^2*x^2*e^(c + 1) + 3*a^3*d^2*f*x*e^(c + 2) + a^3*d^2*e^(c + 3))*e^
(d*x)), x) + 16*integrate(-1/8*(a*b^4*e^(d*x + c) - b^5)/((a^5*b*f + a^3*b^
3*f)*x + (a^5*b + a^3*b^3)*e - ((a^5*b*f*e^(2*c) + a^3*b^3*f*e^(2*c))*x + (
a^5*b*e^(2*c) + a^3*b^3*e^(2*c))*e)*e^(2*d*x) - 2*((a^6*f*e^c + a^4*b^2*f*e
^c)*x + (a^6*e^c + a^4*b^2*e^c)*e)*e^(d*x)), x) + 16*integrate(1/8*(b*e^(d*
x + c) - a)/((a^2*f + b^2*f)*x + (a^2 + b^2)*e + ((a^2*f*e^(2*c) + b^2*f*e^
(2*c))*x + (a^2*e^(2*c) + b^2*e^(2*c))*e)*e^(2*d*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(csch(d*x + c)^3*sech(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3437 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cosh\left(c+d\,x\right)\,{\sinh\left(c+d\,x\right)}^3\,\left(e+f\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(1/(cosh(c + d*x)*sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.496 $\int \frac{(e+fx)^2\mathbf{csch}^3(c+dx)\mathbf{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1245

$$\frac{2b(e+fx)^2}{a^2d}-\frac{b^3(e+fx)^2}{a^2\left(a^2+b^2\right)d}+\frac{4f^2x\text{ArcTan}\left(e^{c+dx}\right)}{ad^2}-\frac{4b^2f(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{a^3d^2}+\frac{4b^4f(e+fx)\text{ArcTan}\left(e^{c+}\right.}{a^3\left(a^2+b^2\right)d^2}$$

```
[Out] 3*(f*x+e)^2*arctanh(exp(d*x+c))/a/d-3*f^2*polylog(3,-exp(d*x+c))/a/d^3+3*f^
2*polylog(3,exp(d*x+c))/a/d^3-f^2*arctanh(cosh(d*x+c))/a/d^3-b^3*(f*x+e)^2/
a^2/(a^2+b^2)/d+4*f^2*x*arctan(exp(d*x+c))/a/d^2+2*e*f*arctan(sinh(d*x+c))/
a/d^2+2*b*(f*x+e)^2*coth(2*d*x+2*c)/a^2/d-e*f*csch(d*x+c)/a/d^2-f^2*x*csch(
d*x+c)/a/d^2-1/2*b*f^2*polylog(2,exp(4*d*x+4*c))/a^2/d^3+b^2*(f*x+e)^2*sech
(d*x+c)/a^3/d-1/2*(f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/a/d-2*I*f^2*polylog(2
,-I*exp(d*x+c))/a/d^3-2*b^2*(f*x+e)^2*arctanh(exp(d*x+c))/a^3/d+2*b*(f*x+e)
^2/a^2/d+3*f*(f*x+e)*polylog(2,-exp(d*x+c))/a/d^2-3*f*(f*x+e)*polylog(2,exp
(d*x+c))/a/d^2-3/2*(f*x+e)^2*sech(d*x+c)/a/d-2*b*f*(f*x+e)*ln(1-exp(4*d*x+4
*c))/a^2/d^2-b^5*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+
b^2)^(3/2)/d+b^5*(f*x+e)^2*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+
b^2)^(3/2)/d+2*b^5*f^2*polylog(3,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^
2+b^2)^(3/2)/d^3-2*b^5*f^2*polylog(3,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3
/(a^2+b^2)^(3/2)/d^3-4*b^2*f*(f*x+e)*arctan(exp(d*x+c))/a^3/d^2+2*I*f^2*pol
ylog(2,I*exp(d*x+c))/a/d^3+b^3*f^2*polylog(2,-exp(2*d*x+2*c))/a^2/(a^2+b^2)
/d^3-b^4*(f*x+e)^2*sech(d*x+c)/a^3/(a^2+b^2)/d-b^3*(f*x+e)^2*tanh(d*x+c)/a^
2/(a^2+b^2)/d-2*I*b^2*f^2*polylog(2,I*exp(d*x+c))/a^3/d^3-2*b^2*f*(f*x+e)*p
olylog(2,-exp(d*x+c))/a^3/d^2+2*b^2*f*(f*x+e)*polylog(2,exp(d*x+c))/a^3/d^2
+4*b^4*f*(f*x+e)*arctan(exp(d*x+c))/a^3/(a^2+b^2)/d^2+2*I*b^2*f^2*polylog(2
,-I*exp(d*x+c))/a^3/d^3-2*I*b^4*f^2*polylog(2,-I*exp(d*x+c))/a^3/(a^2+b^2)/
d^3+2*b^2*f^2*polylog(3,-exp(d*x+c))/a^3/d^3-2*b^2*f^2*polylog(3,exp(d*x+c)
)/a^3/d^3+2*I*b^4*f^2*polylog(2,I*exp(d*x+c))/a^3/(a^2+b^2)/d^3+2*b^3*f*(f*
x+e)*ln(1+exp(2*d*x+2*c))/a^2/(a^2+b^2)/d^2-2*b^5*f*(f*x+e)*polylog(2,-b*ex
p(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(3/2)/d^2+2*b^5*f*(f*x+e)*polyl
og(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(3/2)/d^2
```

Rubi [A]
time = 2.59, antiderivative size = 1245, normalized size of antiderivative = 1.00, number of steps used = 88, number of rules used = 33, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {5708, 2702, 294, 327, 213, 5570, 6820, 12, 6874, 6408, 4267, 2611, 2320, 6724, 4218, 464, 209, 4265, 2317, 2438, 2701, 5311, 3855, 5569, 4269, 3797, 2221, 6873, 5692, 3403, 2296, 3799, 5559}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*b*(e + f*x)^2)/(a^2*d) - (b^3*(e + f*x)^2)/(a^2*(a^2 + b^2)*d) + (4*f^2*
x*ArcTan[E^(c + d*x)])/(a*d^2) - (4*b^2*f*(e + f*x)*ArcTan[E^(c + d*x)])/(a
^3*d^2) + (4*b^4*f*(e + f*x)*ArcTan[E^(c + d*x)])/(a^3*(a^2 + b^2)*d^2) + (
2*e*f*ArcTan[Sinh[c + d*x]])/(a*d^2) + (3*(e + f*x)^2*ArcTanh[E^(c + d*x)])
/(a*d) - (2*b^2*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^3*d) - (f^2*ArcTanh[Co
sh[c + d*x]])/(a*d^3) + (2*b*(e + f*x)^2*Coth[2*c + 2*d*x])/(a^2*d) - (e*f*
Csch[c + d*x])/(a*d^2) - (f^2*x*Csch[c + d*x])/(a*d^2) - (b^5*(e + f*x)^2*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)^(3/2)*d) +
(b^5*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*(a^2
+ b^2)^(3/2)*d) + (2*b^3*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a^2*(a^2 +
b^2)*d^2) - (2*b*f*(e + f*x)*Log[1 - E^(4*(c + d*x))])/(a^2*d^2) + (3*f*(e
+ f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2, -E
^(c + d*x)])/(a^3*d^2) - ((2*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^3) +
 ((2*I)*b^2*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a^3*d^3) - ((2*I)*b^4*f^2*Po
lyLog[2, (-I)*E^(c + d*x)])/(a^3*(a^2 + b^2)*d^3) + ((2*I)*f^2*PolyLog[2, I
*E^(c + d*x)])/(a*d^3) - ((2*I)*b^2*f^2*PolyLog[2, I*E^(c + d*x)])/(a^3*d^3
) + ((2*I)*b^4*f^2*PolyLog[2, I*E^(c + d*x)])/(a^3*(a^2 + b^2)*d^3) - (3*f*
(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^2) + (2*b^2*f*(e + f*x)*PolyLog[2,
E^(c + d*x)])/(a^3*d^2) - (2*b^5*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)^(3/2)*d^2) + (2*b^5*f*(e + f*x)*Po
lyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)^(3/2)*
d^2) + (b^3*f^2*PolyLog[2, -E^(2*(c + d*x))])/(a^2*(a^2 + b^2)*d^3) - (b*f^
2*PolyLog[2, E^(4*(c + d*x))])/(2*a^2*d^3) - (3*f^2*PolyLog[3, -E^(c + d*x)
])/(a*d^3) + (2*b^2*f^2*PolyLog[3, -E^(c + d*x)])/(a^3*d^3) + (3*f^2*PolyLo
g[3, E^(c + d*x)])/(a*d^3) - (2*b^2*f^2*PolyLog[3, E^(c + d*x)])/(a^3*d^3)
+ (2*b^5*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*(a^
2 + b^2)^(3/2)*d^3) - (2*b^5*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2]))])/(a^3*(a^2 + b^2)^(3/2)*d^3) - (3*(e + f*x)^2*Sech[c + d*x])/(2*
a*d) + (b^2*(e + f*x)^2*Sech[c + d*x])/(a^3*d) - (b^4*(e + f*x)^2*Sech[c +
d*x])/(a^3*(a^2 + b^2)*d) - ((e + f*x)^2*Csch[c + d*x]^2*Sech[c + d*x])/(2*
a*d) - (b^3*(e + f*x)^2*Tanh[c + d*x])/(a^2*(a^2 + b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4218

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5311

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5708

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{csch}^3(c+dx)\text{sech}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\text{csch}^2(c+\ldots}{a+b\sinh\ldots}}{a}$$

$$= \frac{3(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{3(e+fx)^2\text{sech}(c+dx)}{2ad} -$$

$$= \frac{3(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{3(e+fx)^2\text{sech}(c+dx)}{2ad} -$$

$$= \frac{3(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{b^2(e+fx)^2\tanh^{-1}(\cosh(c\ldots}{a^3d}$$

$$= \frac{2b(e+fx)^2}{a^2d} + \frac{3(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{b^2(e+fx)^2\ldots}{\ldots}$$

$$= \frac{2b(e+fx)^2}{a^2d} + \frac{3(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{b^2(e+fx)^2\ldots}{\ldots}$$

$$= \frac{2b(e+fx)^2}{a^2d} - \frac{b^2(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{a^3d} + \frac{2b(e+fx)\ldots}{\ldots}$$

$$= \frac{2b(e+fx)^2}{a^2d} - \frac{b^3(e+fx)^2}{a^2(a^2+b^2)d} + \frac{4b^4f(e+fx)\tan^{-1}(e^{c+dx})}{a^3(a^2+b^2)d^2} - \ldots$$

$$= \frac{2b(e+fx)^2}{a^2d} - \frac{b^3(e+fx)^2}{a^2(a^2+b^2)d} - \frac{4b^2f(e+fx)\tan^{-1}(e^{c+dx})}{a^3d^2} + \ldots$$

$$= \frac{2b(e+fx)^2}{a^2d} - \frac{b^3(e+fx)^2}{a^2(a^2+b^2)d} - \frac{4b^2f(e+fx)\tan^{-1}(e^{c+dx})}{a^3d^2} + \ldots$$

$$= \frac{2b(e+fx)^2}{a^2d} - \frac{b^3(e+fx)^2}{a^2(a^2+b^2)d} + \frac{6f^2x\tan^{-1}(e^{c+dx})}{ad^2} - \frac{4b^2f(e+\ldots}{\ldots}$$

$$= \frac{2b(e+fx)^2}{a^2d} - \frac{b^3(e+fx)^2}{a^2(a^2+b^2)d} + \frac{6f^2x\tan^{-1}(e^{c+dx})}{ad^2} - \frac{4b^2f(e+\ldots}{\ldots}$$

$$= \frac{2b(e+fx)^2}{a^2d} - \frac{b^3(e+fx)^2}{a^2(a^2+b^2)d} + \frac{6f^2x\tan^{-1}(e^{c+dx})}{ad^2} - \frac{4b^2f(e+\ldots}{\ldots}$$

$$= \frac{2b(e+fx)^2}{a^2d} - \frac{b^3(e+fx)^2}{a^2(a^2+b^2)d} + \frac{6f^2x\tan^{-1}(e^{c+dx})}{ad^2} - \frac{4b^2f(e+\ldots}{\ldots}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3762 vs. 2(1245) = 2490.
time = 21.60, size = 3762, normalized size = 3.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x
]),x]
```

```
[Out] (-2*b^5*e*f*(((-I)*Pi*ArcTanh[(-b + a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/
Sqrt[a^2 + b^2] - (2*((-I)*c + Pi/2 - I*d*x)*ArcTanh[((a - I*b)*Cot[((-I)*c
 + Pi/2 - I*d*x)/2])/Sqrt[-a^2 - b^2]] - 2*((-I)*c + ArcCos[((-I)*a)/b])*Ar
cTanh[((-a - I*b)*Tan[((-I)*c + Pi/2 - I*d*x)/2])/Sqrt[-a^2 - b^2]] + (ArcC
os[((-I)*a)/b] - (2*I)*(ArcTanh[((a - I*b)*Cot[((-I)*c + Pi/2 - I*d*x)/2])/
Sqrt[-a^2 - b^2]] - ArcTanh[((-a - I*b)*Tan[((-I)*c + Pi/2 - I*d*x)/2])/Sqr
t[-a^2 - b^2]]))*Log[Sqrt[-a^2 - b^2]/(Sqrt[2]*Sqrt[(-I)*b]*E^((I/2)*((-I)*
c + Pi/2 - I*d*x))*Sqrt[a + b*Sinh[c + d*x]])] + (ArcCos[((-I)*a)/b] + (2*I
)*(ArcTanh[((a - I*b)*Cot[((-I)*c + Pi/2 - I*d*x)/2])/Sqrt[-a^2 - b^2]] - A
rcTanh[((-a - I*b)*Tan[((-I)*c + Pi/2 - I*d*x)/2])/Sqrt[-a^2 - b^2]]))*Log[
(Sqrt[-a^2 - b^2]*E^((I/2)*((-I)*c + Pi/2 - I*d*x)))/(Sqrt[2]*Sqrt[(-I)*b]*
Sqrt[a + b*Sinh[c + d*x]])] - (ArcCos[((-I)*a)/b] + (2*I)*ArcTanh[((-a - I*
b)*Tan[((-I)*c + Pi/2 - I*d*x)/2])/Sqrt[-a^2 - b^2]])*Log[1 - (I*(a - I*Sqr
t[-a^2 - b^2])*(a - I*b - Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x)/2]))
/(b*(a - I*b + Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x)/2]))] + (-ArcCo
s[((-I)*a)/b] + (2*I)*ArcTanh[((-a - I*b)*Tan[((-I)*c + Pi/2 - I*d*x)/2])/S
qrt[-a^2 - b^2]])*Log[1 - (I*(a + I*Sqrt[-a^2 - b^2])*(a - I*b - Sqrt[-a^2
 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x)/2]))/(b*(a - I*b + Sqrt[-a^2 - b^2]*Tan[
((-I)*c + Pi/2 - I*d*x)/2]))] + I*(PolyLog[2, (I*(a - I*Sqrt[-a^2 - b^2])*(
a - I*b - Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x)/2]))/(b*(a - I*b + S
qrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x)/2]))] - PolyLog[2, (I*(a + I*Sq
rt[-a^2 - b^2])*(a - I*b - Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x)/2])
)/(b*(a - I*b + Sqrt[-a^2 - b^2]*Tan[((-I)*c + Pi/2 - I*d*x)/2]))]))/Sqrt[-
a^2 - b^2]))/(a^3*(a^2 + b^2)*d^2) + (8*a*b*d^2*e*E^(2*c)*f*x + 4*a*b*d^2*E
^(2*c)*f^2*x^2 - 6*a^2*d^2*e^2*ArcTanh[E^(c + d*x)] + 4*b^2*d^2*e^2*ArcTanh
[E^(c + d*x)] + 6*a^2*d^2*e^2*E^(2*c)*ArcTanh[E^(c + d*x)] - 4*b^2*d^2*e^2*
E^(2*c)*ArcTanh[E^(c + d*x)] + 4*a^2*f^2*ArcTanh[E^(c + d*x)] - 4*a^2*E^(2*
c)*f^2*ArcTanh[E^(c + d*x)] + 6*a^2*d^2*e*f*x*Log[1 - E^(c + d*x)] - 4*b^2*
d^2*e*f*x*Log[1 - E^(c + d*x)] - 6*a^2*d^2*e*E^(2*c)*f*x*Log[1 - E^(c + d*x
)] + 4*b^2*d^2*e*E^(2*c)*f*x*Log[1 - E^(c + d*x)] + 3*a^2*d^2*f^2*x^2*Log[1
 - E^(c + d*x)] - 2*b^2*d^2*f^2*x^2*Log[1 - E^(c + d*x)] - 3*a^2*d^2*E^(2*c
)*f^2*x^2*Log[1 - E^(c + d*x)] + 2*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d
*x)] - 6*a^2*d^2*e*f*x*Log[1 + E^(c + d*x)] + 4*b^2*d^2*e*f*x*Log[1 + E^(c
+ d*x)] + 6*a^2*d^2*e*E^(2*c)*f*x*Log[1 + E^(c + d*x)] - 4*b^2*d^2*e*E^(2*c
)*f*x*Log[1 + E^(c + d*x)] - 3*a^2*d^2*f^2*x^2*Log[1 + E^(c + d*x)] + 2*b^2
```

```
*d^2*f^2*x^2*Log[1 + E^(c + d*x)] + 3*a^2*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(c
+ d*x)] - 2*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] + 4*a*b*d*e*f*Log[
1 - E^(2*(c + d*x))] - 4*a*b*d*e*E^(2*c)*f*Log[1 - E^(2*(c + d*x))] + 4*a*b
*d*f^2*x*Log[1 - E^(2*(c + d*x))] - 4*a*b*d*E^(2*c)*f^2*x*Log[1 - E^(2*(c +
 d*x))] + 2*(3*a^2 - 2*b^2)*d*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -E^(c +
 d*x)] - 2*(3*a^2 - 2*b^2)*d*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, E^(c + d
*x)] + 2*a*b*f^2*PolyLog[2, E^(2*(c + d*x))] - 2*a*b*E^(2*c)*f^2*PolyLog[2,
 E^(2*(c + d*x))] + 6*a^2*f^2*PolyLog[3, -E^(c + d*x)] - 4*b^2*f^2*PolyLog[
3, -E^(c + d*x)] - 6*a^2*E^(2*c)*f^2*PolyLog[3, -E^(c + d*x)] + 4*b^2*E^(2*
c)*f^2*PolyLog[3, -E^(c + d*x)] - 6*a^2*f^2*PolyLog[3, E^(c + d*x)] + 4*b^2
*f^2*PolyLog[3, E^(c + d*x)] + 6*a^2*E^(2*c)*f^2*PolyLog[3, E^(c + d*x)] -
4*b^2*E^(2*c)*f^2*PolyLog[3, E^(c + d*x)])/(2*a^3*d^3*(-1 + E^(2*c))) - (b^
5*f^2*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^2*x^2*Log
[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, (b*E^(c + d*
x))/(-a + Sqrt[a^2 + b^2])] - 2*d*x*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[
a^2 + b^2]))] - 2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*Po
lyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^3*(a^2 + b^2)^(3/2)
*d^3) - (2*b*e*f*Sech[c]*(Cosh[c]*Log[Cosh[c]*Cosh[d*x] + Sinh[c]*Sinh[d*x]
] - d*x*Sinh[c]))/((a^2 + b^2)*d^2*(Cosh[c]^2 - Sinh[c]^2)) + (4*a*e*f*ArcT
an[(Sinh[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]])/((a^2 +
b^2)*d^2*Sqrt[Cosh[c]^2 - Sinh[c]^2]) - (b*f^2*Csch[c]*((d^2*x^2)/E^ArcTanh
[Coth[c]] - (I*Coth[c]*(-(d*x*(-Pi + (2*I)*ArcTanh[Coth[c]])) - Pi*Log[1 +
E^(2*d*x)] - 2*(I*d*x + I*ArcTanh[Coth[c]])*Log[1 - E^((2*I)*(I*d*x + I*Arc
Tanh[Coth[c]]))] + Pi*Log[Cosh[d*x]] + (2*I)*ArcTanh[Coth[c]]*Log[I*Sinh[d*
x + ArcTanh[Coth[c]]]] + I*PolyLog[2, E^((2*I)*(I*d*x + I*ArcTanh[Coth[c]])
)]))/Sqrt[1 - Coth[c]^2])*Sech[c])/((a^2 + b^2)*d^3*Sqrt[Csch[c]^2*(-Cosh[c
]^2 + Sinh[c]^2)]) - (2*b^5*e^2*ArcTan[(b*Cosh[c] + (-a + b*Sinh[c])*Tanh[(
d*x)/2])/Sqrt[-a^2 - b^2*Cosh[c]^2 + b^2*Sinh[c]^2]])/(a^3*(a^2 + b^2)*d*Sq
rt[-a^2 - b^2*Cosh[c]^2 + b^2*Sinh[c]^2]) + (2*a*f^2*(((-I)*Csch[c]*(I*(d*x
 + ArcTanh[Coth[c]])*(Log[1 - E^(-(d*x) - ArcTanh[Coth[c]])] - Log[1 + E^(-
(d*x) - ArcTanh[Coth[c]])]) + I*(PolyLog[2, -E^(-(d*x) - ArcTanh[Coth[c]])]
 - PolyLog[2, E^(-(d*x) - ArcTanh[Coth[c]])])))...
```

Maple [F]
time = 2.92, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^3 \operatorname{sech}(dx+c)^2}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="maxima")
```

```
[Out] 2*b*f*(2*(d*x + c)/((a^2 + b^2)*d^2) - log(e^(2*d*x + 2*c) + 1)/((a^2 + b^2
)*d^2))*e + 4*a*f^2*integrate(x*e^(d*x + c)/(a^2*d*e^(2*d*x + 2*c) + b^2*d*
e^(2*d*x + 2*c) + a^2*d + b^2*d), x) + 4*b*f^2*integrate(x/(a^2*d*e^(2*d*x
+ 2*c) + b^2*d*e^(2*d*x + 2*c) + a^2*d + b^2*d), x) - 1/2*(2*b^5*log((b*e^(
-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((
a^5 + a^3*b^2)*sqrt(a^2 + b^2)*d) + 2*(4*a^2*b*e^(-2*d*x - 2*c) + 2*b^3*e^(
-4*d*x - 4*c) - 4*a^2*b - 2*b^3 + (3*a^3 + a*b^2)*e^(-d*x - c) - 2*(a^3 - a
*b^2)*e^(-3*d*x - 3*c) + (3*a^3 + a*b^2)*e^(-5*d*x - 5*c))/((a^4 + a^2*b^2
- (a^4 + a^2*b^2)*e^(-2*d*x - 2*c) - (a^4 + a^2*b^2)*e^(-4*d*x - 4*c) + (a^
4 + a^2*b^2)*e^(-6*d*x - 6*c))*d) - (3*a^2 - 2*b^2)*log(e^(-d*x - c) + 1)/(
a^3*d) + (3*a^2 - 2*b^2)*log(e^(-d*x - c) - 1)/(a^3*d))*e^2 + 4*a*f*arctan(
e^(d*x + c))*e/((a^2 + b^2)*d^2) - (2*(2*a^2*b*d*f^2 + b^3*d*f^2)*x^2 + 4*(
2*a^2*b*d*f + b^3*d*f)*x*e + ((3*a^3*d*f^2*e^(5*c) + a*b^2*d*f^2*e^(5*c))*x
^2 + 2*(a^3*f^2*e^(5*c) + a*b^2*f^2*e^(5*c) + (3*a^3*d*f*e^(5*c) + a*b^2*d*
f*e^(5*c))*e)*x + 2*(a^3*f*e^(5*c) + a*b^2*f*e^(5*c))*e)*e^(5*d*x) - 2*(b^3
*d*f^2*x^2*e^(4*c) + 2*b^3*d*f*x*e^(4*c + 1))*e^(4*d*x) - 2*((a^3*d*f^2*e^(
3*c) - a*b^2*d*f^2*e^(3*c))*x^2 + 2*(a^3*d*f*e^(3*c) - a*b^2*d*f*e^(3*c))*x
*e)*e^(3*d*x) - 4*(a^2*b*d*f^2*x^2*e^(2*c) + 2*a^2*b*d*f*x*e^(2*c + 1))*e^(
2*d*x) + ((3*a^3*d*f^2*e^c + a*b^2*d*f^2*e^c)*x^2 - 2*(a^3*f^2*e^c + a*b^2*
f^2*e^c - (3*a^3*d*f*e^c + a*b^2*d*f*e^c)*e)*x - 2*(a^3*f*e^c + a*b^2*f*e^c
)*e)*e^(d*x))/(a^4*d^2 + a^2*b^2*d^2 + (a^4*d^2*e^(6*c) + a^2*b^2*d^2*e^(6*
c))*e^(6*d*x) - (a^4*d^2*e^(4*c) + a^2*b^2*d^2*e^(4*c))*e^(4*d*x) - (a^4*d^
2*e^(2*c) + a^2*b^2*d^2*e^(2*c))*e^(2*d*x)) + (2*b*d*f*e + a*f^2)*x/(a^2*d^
2) + (2*b*d*f*e - a*f^2)*x/(a^2*d^2) - (2*b*d*f*e + a*f^2)*log(e^(d*x + c)
+ 1)/(a^2*d^3) - (2*b*d*f*e - a*f^2)*log(e^(d*x + c) - 1)/(a^2*d^3) + 1/2*(
d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^
(d*x + c)))*(3*a^2*f^2 - 2*b^2*f^2)/(a^3*d^3) - 1/2*(d^2*x^2*log(-e^(d*x +
c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*(3*a^2*f^2
- 2*b^2*f^2)/(a^3*d^3) - (2*a*b*f^2 - (3*a^2*d*f - 2*b^2*d*f)*e)*(d*x*log(e
^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^3*d^3) - (2*a*b*f^2 + (3*a^2*d*f
- 2*b^2*d*f)*e)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^3*d^3)
+ 1/6*((3*a^2*f^2 - 2*b^2*f^2)*d^3*x^3 + 3*(2*a*b*f^2 + (3*a^2*d*f - 2*b^2*
d*f)*e)*d^2*x^2)/(a^3*d^3) - 1/6*((3*a^2*f^2 - 2*b^2*f^2)*d^3*x^3 - 3*(2*a*
b*f^2 - (3*a^2*d*f - 2*b^2*d*f)*e)*d^2*x^2)/(a^3*d^3) - integrate(-2*(b^5*f
^2*x^2*e^c + 2*b^5*f*x*e^(c + 1))*e^(d*x)/(a^5*b + a^3*b^3 - (a^5*b*e^(2*c)
 + a^3*b^3*e^(2*c))*e^(2*d*x) - 2*(a^6*e^c + a^4*b^2*e^c)*e^(d*x)), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 41729 vs. $2(1176) = 2352$.
time = 0.98, size = 41729, normalized size = 33.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="fricas")
```

```
[Out] 1/2*(4*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d^2*f^2*x^2 - (2*a^5*b + 3*a^3*b^3 +
a*b^5)*c^2*f^2 + 2*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d^2*f*x + (2*a^5*b + 3*a^
3*b^3 + a*b^5)*c*d*f)*cosh(1) + 2*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d^2*f*x +
(2*a^5*b + 3*a^3*b^3 + a*b^5)*c*d*f)*sinh(1))*cosh(d*x + c)^6 + 4*((2*a^5*b
 + 3*a^3*b^3 + a*b^5)*d^2*f^2*x^2 - (2*a^5*b + 3*a^3*b^3 + a*b^5)*c^2*f^2 +
 2*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d^2*f*x + (2*a^5*b + 3*a^3*b^3 + a*b^5)*c
*d*f)*cosh(1) + 2*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d^2*f*x + (2*a^5*b + 3*a^3
*b^3 + a*b^5)*c*d*f)*sinh(1))*sinh(d*x + c)^6 - 2*((3*a^6 + 4*a^4*b^2 + a^2
*b^4)*d^2*f^2*x^2 + 2*(a^6 + 2*a^4*b^2 + a^2*b^4)*d*f^2*x + (3*a^6 + 4*a^4*
b^2 + a^2*b^4)*d^2*cosh(1)^2 + (3*a^6 + 4*a^4*b^2 + a^2*b^4)*d^2*sinh(1)^2
+ 2*((3*a^6 + 4*a^4*b^2 + a^2*b^4)*d^2*f*x + (a^6 + 2*a^4*b^2 + a^2*b^4)*d*
f)*cosh(1) + 2*((3*a^6 + 4*a^4*b^2 + a^2*b^4)*d^2*f*x + (3*a^6 + 4*a^4*b^2
+ a^2*b^4)*d^2*cosh(1) + (a^6 + 2*a^4*b^2 + a^2*b^4)*d*f)*sinh(1))*cosh(d*x
 + c)^5 - 2*((3*a^6 + 4*a^4*b^2 + a^2*b^4)*d^2*f^2*x^2 + 2*(a^6 + 2*a^4*b^2
 + a^2*b^4)*d*f^2*x + (3*a^6 + 4*a^4*b^2 + a^2*b^4)*d^2*cosh(1)^2 + (3*a^6
+ 4*a^4*b^2 + a^2*b^4)*d^2*sinh(1)^2 + 2*((3*a^6 + 4*a^4*b^2 + a^2*b^4)*d^2
*f*x + (a^6 + 2*a^4*b^2 + a^2*b^4)*d*f)*cosh(1) - 12*((2*a^5*b + 3*a^3*b^3
+ a*b^5)*d^2*f^2*x^2 - (2*a^5*b + 3*a^3*b^3 + a*b^5)*c^2*f^2 + 2*((2*a^5*b
+ 3*a^3*b^3 + a*b^5)*d^2*f*x + (2*a^5*b + 3*a^3*b^3 + a*b^5)*c*d*f)*cosh(1)
 + 2*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d^2*f*x + (2*a^5*b + 3*a^3*b^3 + a*b^5)
*c*d*f)*sinh(1))*cosh(d*x + c) + 2*((3*a^6 + 4*a^4*b^2 + a^2*b^4)*d^2*f*x +
 (3*a^6 + 4*a^4*b^2 + a^2*b^4)*d^2*cosh(1) + (a^6 + 2*a^4*b^2 + a^2*b^4)*d*
f)*sinh(1))*sinh(d*x + c)^5 - 4*(2*a^5*b + 3*a^3*b^3 + a*b^5)*c^2*f^2 + 8*(
2*a^5*b + 3*a^3*b^3 + a*b^5)*c*d*f*cosh(1) - 4*(2*a^5*b + 3*a^3*b^3 + a*b^5
)*d^2*cosh(1)^2 - 4*(2*(a^5*b + a^3*b^3)*d^2*f^2*x^2 - (2*a^5*b + 3*a^3*b^3
 + a*b^5)*c^2*f^2 - (a^3*b^3 + a*b^5)*d^2*cosh(1)^2 - (a^3*b^3 + a*b^5)*d^2
*sinh(1)^2 + 2*(2*(a^5*b + a^3*b^3)*d^2*f*x + (2*a^5*b + 3*a^3*b^3 + a*b^5)
*c*d*f)*cosh(1) + 2*(2*(a^5*b + a^3*b^3)*d^2*f*x + (2*a^5*b + 3*a^3*b^3 + a
*b^5)*c*d*f - (a^3*b^3 + a*b^5)*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^4 - 4*(
2*a^5*b + 3*a^3*b^3 + a*b^5)*d^2*sinh(1)^2 - 2*(4*(a^5*b + a^3*b^3)*d^2*f^2
*x^2 - 2*(2*a^5*b + 3*a^3*b^3 + a*b^5)*c^2*f^2 - 2*(a^3*b^3 + a*b^5)*d^2*co
sh(1)^2 - 2*(a^3*b^3 + a*b^5)*d^2*sinh(1)^2 - 30*((2*a^5*b + 3*a^3*b^3 + a*
b^5)*d^2*f^2*x^2 - (2*a^5*b + 3*a^3*b^3 + a*b^5)*c^2*f^2 + 2*((2*a^5*b + 3*
a^3*b^3 + a*b^5)*d^2*f*x + (2*a^5*b + 3*a^3*b^3 + a*b^5)*c*d*f)*cosh(1) + 2
*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d^2*f*x + (2*a^5*b + 3*a^3*b^3 + a*b^5)*c*d
```

```
*f)*sinh(1))*cosh(d*x + c)^2 + 4*(2*(a^5*b + a^3*b^3)*d^2*f*x + (2*a^5*b +
3*a^3*b^3 + a*b^5)*c*d*f)*cosh(1) + 5*((3*a^6 + 4*a^4*b^2 + a^2*b^4)*d^2*f^
2*x^2 + 2*(a^6 + 2*a^4*b^2 + a^2*b^4)*d*f^2*x + (3*a^6 + 4*a^4*b^2 + a^2*b^
4)*d^2*cosh(1)^2 + (3*a^6 + 4*a^4*b^2 + a^2*b^4)*d^2*sinh(1)^2 + 2*((3*a^6
+ 4*a^4*b^2 + a^2*b^4)*d^2*f*x + (a^6 + 2*a^4*b^2 + a^2*b^4)*d*f)*cosh(1) +
 2*((3*a^6 + 4*a^4*b^2 + a^2*b^4)*d^2*f*x + (3*a^6 + 4*a^4*b^2 + a^2*b^4)*d
^2*cosh(1) + (a^6 + 2*a^4*b^2 + a^2*b^4)*d*f)*sinh(1))*cosh(d*x + c) + 4*(2
*(a^5*b + a^3*b^3)*d^2*f*x + (2*a^5*b + 3*a^3*b^3 + a*b^5)*c*d*f - (a^3*b^3
 + a*b^5)*d^2*cosh(1))*sinh(1))*sinh(d*x + c)^4 + 4*((a^6 - a^2*b^4)*d^2*f^
2*x^2 + 2*(a^6 - a^2*b^4)*d^2*f*x*cosh(1) + (a^6 - a^2*b^4)*d^2*cosh(1)^2 +
 (a^6 - a^2*b^4)*d^2*sinh(1)^2 + 2*((a^6 - a^2*b^4)*d^2*f*x + (a^6 - a^2*b^
4)*d^2*cosh(1))*sinh(1))*cosh(d*x + c)^3 + 4*((a^6 - a^2*b^4)*d^2*f^2*x^2 +
 2*(a^6 - a^2*b^4)*d^2*f*x*cosh(1) + (a^6 - a^2*b^4)*d^2*cosh(1)^2 + (a^6 -
 a^2*b^4)*d^2*sinh(1)^2 + 20*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d^2*f^2*x^2 - (
2*a^5*b + 3*a^3*b^3 + a*b^5)*c^2*f^2 + 2*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d^2
*f*x + (2*a^5*b + 3*a^3*b^3 + a*b^5)*c*d*f)*cosh(1) + 2*((2*a^5*b + 3*a^3*b
^3 + a*b^5)*d^2*f*x + (2*a^5*b + 3*a^3*b^3 + a*b^5)*c*d*f)*sinh(1))*cosh(d*
x + c)^3 - 5*((3*a^6 + 4*a^4*b^2 + a^2*b^4)*d^2*f^2*x^2 + 2*(a^6 + 2*a^4*b^
2 + a^2*b^4)*d*f^2*x + (3*a^6 + 4*a^4*b^2 + a^2*b^4)*d^2*cosh(1)^2 + (3*a^6
 + 4*a^4*b^2 + a^2*b^4)*d^2*sinh(1)^2 + 2*((3*a^6 + 4*a^4*b^2 + a^2*b^4)*d^
2*f*x + (a^6 + 2*a^4*b^2 + a^2*b^4)*d*f)*cosh(1) + 2*((3*a^6 + 4*a^4*b^2 +
a^2*b^4)*d^2*f*x + (3*a^6 + 4*a^4*b^2 + a^2*b^4)*d^2*cosh(1) + (a^6 + 2*a^4
*b^2 + a^2*b^4)*d*f)*sinh(1))*cosh(d*x + c)^2 - 4*(2*(a^5*b + a^3*b^3)*d^2*
f^2*x^2 - (2*a^5*b + 3*a^3*b^3 + a*b^5)*c^2*f^2 - (a^3*b^3 + a*b^5)*d^2*cos
h(1)^2 - (a^3*b^3 + a*b^5)*d^2*sinh(1)^2 + 2*(2*(a^5*b + a^3*b^3)*d^2*f*x +
 (2*a^5*b + 3*a^3*b^3 + a*b^5)*c*d*f)*cosh(1) + 2*(2*(a^5*b + a^3*b^3)*d^2*
f*x + (2*a^5*b + 3*a^3*b^3 + a*b^5)*c*d*f - (a^3*b^3 + a*b^5)*d^2*cosh(1))*
sinh(1))*cosh(d*x + c) + 2*((a^6 - a^2*b^4)*d^2*f*x + (a^6 - a^2*b^4)*d^2*c
osh(1))*sinh(1))*sinh(d*x + c)^3 - 4*((a^3*b^3 + a*b^5)*d^2*f^2*x^2 - (2*a^
5*b + 3*a^3*b^3 + a*b^5)*c^2*f^2 - 2*(a^5*b + a^3*b^3)*d^2*cosh(1)^2 - 2*(a
^5*b + a^3*b^3)*d^2*sinh(1)^2 + 2*((a^3*b^3 + a*b^5)*d^2*f*x + (2*a^5*b + 3
*a^3*b^3 + a*b^5)*c*d*f)*cosh(1) + 2*((a^3*b^3 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)**3*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 127.92Not invertible
 Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^2}{\cosh\left(c+d\,x\right)^2\,\sinh\left(c+d\,x\right)^3\,\left(a+b\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2/(cosh(c + d*x)^2*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((e + f*x)^2/(cosh(c + d*x)^2*sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

3.497 $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=699

$$\frac{f\operatorname{ArcTan}(\sinh(c+dx))}{ad^2}-\frac{b^2f\operatorname{ArcTan}(\sinh(c+dx))}{a^3d^2}+\frac{b^4f\operatorname{ArcTan}(\sinh(c+dx))}{a^3(a^2+b^2)d^2}+\frac{3fx\tanh^{-1}(e^{c+dx})}{ad}-\frac{2b^2f}{}$$

[Out] 3/2*f*polylog(2,-exp(d*x+c))/a/d^2-3/2*f*polylog(2,exp(d*x+c))/a/d^2-1/2*f*csch(d*x+c)/a/d^2-3/2*f*x*arctanh(cosh(d*x+c))/a/d+2*b*(f*x+e)*coth(2*d*x+2*c)/a^2/d-1/2*(f*x+e)*csch(d*x+c)^2*sech(d*x+c)/a/d+3/2*(f*x+e)*arctanh(cosh(d*x+c))/a/d-3/2*(f*x+e)*sech(d*x+c)/a/d+f*arctan(sinh(d*x+c))/a/d^2-b^2*f*arctan(sinh(d*x+c))/a^3/d^2-b^2*(f*x+e)*arctanh(cosh(d*x+c))/a^3/d-b*f*ln(sinh(2*d*x+2*c))/a^2/d^2+b^2*(f*x+e)*sech(d*x+c)/a^3/d+b^4*f*arctan(sinh(d*x+c))/a^3/(a^2+b^2)/d^2+b^2*f*x*arctanh(cosh(d*x+c))/a^3/d+b^3*f*ln(cosh(d*x+c))/a^2/(a^2+b^2)/d^2-b^5*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(3/2)/d+b^5*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(3/2)/d-2*b^2*f*x*arctanh(exp(d*x+c))/a^3/d-b^2*f*polylog(2,-exp(d*x+c))/a^3/d^2+b^2*f*polylog(2,exp(d*x+c))/a^3/d^2-b^5*f*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(3/2)/d^2+b^5*f*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^(3/2)/d^2-b^4*(f*x+e)*sech(d*x+c)/a^3/(a^2+b^2)/d-b^3*(f*x+e)*tanh(d*x+c)/a^2/(a^2+b^2)/d+3*f*x*arctanh(exp(d*x+c))/a/d

Rubi [A]
time = 1.02, antiderivative size = 699, normalized size of antiderivative = 1.00, number of steps used = 44, number of rules used = 22, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {5708, 2702, 294, 327, 213, 5570, 6406, 12, 4267, 2317, 2438, 3855, 2701, 5569, 4269, 3556, 5692, 3403, 2296, 2221, 6874, 5559}

Antiderivative was successfully verified.

[In] Int[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]

[Out] (f*ArcTan[Sinh[c + d*x]])/(a*d^2) - (b^2*f*ArcTan[Sinh[c + d*x]])/(a^3*d^2) + (b^4*f*ArcTan[Sinh[c + d*x]])/(a^3*(a^2 + b^2)*d^2) + (3*f*x*ArcTanh[E^(c + d*x)])/(a*d) - (2*b^2*f*x*ArcTanh[E^(c + d*x)])/(a^3*d) - (3*f*x*ArcTanh[Cosh[c + d*x]])/(2*a*d) + (b^2*f*x*ArcTanh[Cosh[c + d*x]])/(a^3*d) + (3*(e + f*x)*ArcTanh[Cosh[c + d*x]])/(2*a*d) - (b^2*(e + f*x)*ArcTanh[Cosh[c + d*x]])/(a^3*d) + (2*b*(e + f*x)*Coth[2*c + 2*d*x])/(a^2*d) - (f*Csch[c + d*x])/(2*a*d^2) - (b^5*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)^(3/2)*d) + (b^5*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)^(3/2)*d) + (b^3*f*Log[Cosh[c + d*x]])/

```
(a^2*(a^2 + b^2)*d^2) - (b*f*Log[Sinh[2*c + 2*d*x]])/(a^2*d^2) + (3*f*PolyL
og[2, -E^(c + d*x)])/(2*a*d^2) - (b^2*f*PolyLog[2, -E^(c + d*x)])/(a^3*d^2)
 - (3*f*PolyLog[2, E^(c + d*x)])/(2*a*d^2) + (b^2*f*PolyLog[2, E^(c + d*x)]
)/(a^3*d^2) - (b^5*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/
(a^3*(a^2 + b^2)^(3/2)*d^2) + (b^5*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2]))])/(a^3*(a^2 + b^2)^(3/2)*d^2) - (3*(e + f*x)*Sech[c + d*x])/(
2*a*d) + (b^2*(e + f*x)*Sech[c + d*x])/(a^3*d) - (b^4*(e + f*x)*Sech[c + d*
x])/(a^3*(a^2 + b^2)*d) - ((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x])/(2*a*d)
 - (b^3*(e + f*x)*Tanh[c + d*x])/(a^2*(a^2 + b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5708

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
```

```
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6406

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(1 - u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{s}}{a+b\sinh(c+}}{a}$$

$$= \frac{3(e+fx)\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{3(e+fx)\text{sech}(c+dx)}{2ad} - \frac{(e}{}$$

$$= \frac{3(e+fx)\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{3(e+fx)\text{sech}(c+dx)}{2ad} - \frac{(e}{}$$

$$= \frac{3f\tan^{-1}(\sinh(c+dx))}{2ad^2} - \frac{3fx\tanh^{-1}(\cosh(c+dx))}{2ad} + \frac{3(e+f}{}$$

$$= \frac{3f\tan^{-1}(\sinh(c+dx))}{2ad^2} - \frac{3fx\tanh^{-1}(\cosh(c+dx))}{2ad} + \frac{3(e+f}{}$$

$$= \frac{f\tan^{-1}(\sinh(c+dx))}{ad^2} - \frac{b^2f\tan^{-1}(\sinh(c+dx))}{a^3d^2} + \frac{3fx\tanh^{-1}}{ad}$$

$$= \frac{f\tan^{-1}(\sinh(c+dx))}{ad^2} - \frac{b^2f\tan^{-1}(\sinh(c+dx))}{a^3d^2} + \frac{3fx\tanh^{-1}}{ad}$$

$$= \frac{f\tan^{-1}(\sinh(c+dx))}{ad^2} - \frac{b^2f\tan^{-1}(\sinh(c+dx))}{a^3d^2} + \frac{b^4f\tan^{-1}(\text{s}}{a^3(a^2}$$

$$= \frac{f\tan^{-1}(\sinh(c+dx))}{ad^2} - \frac{b^2f\tan^{-1}(\sinh(c+dx))}{a^3d^2} + \frac{b^4f\tan^{-1}(\text{s}}{a^3(a^2}$$

$$= \frac{f\tan^{-1}(\sinh(c+dx))}{ad^2} - \frac{b^2f\tan^{-1}(\sinh(c+dx))}{a^3d^2} + \frac{b^4f\tan^{-1}(\text{s}}{a^3(a^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.43, size = 863, normalized size = 1.23

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (2*a*f*ArcTan[Tanh[(c + d*x)/2]])/(d*(a^2*d + b^2*d)) + ((2*b*d*e*Cosh[(c +
 d*x)/2] - a*f*Cosh[(c + d*x)/2] - 2*b*c*f*Cosh[(c + d*x)/2] + 2*b*f*(c + d
*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(4*a^2*d^2) + ((-(d*e) + c*f - f*
(c + d*x))*Csch[(c + d*x)/2]^2)/(8*a*d^2) - (b*f*Log[Cosh[c + d*x]])/((a^2
+ b^2)*d^2) - (b*f*Log[Sinh[c + d*x]])/(a^2*d^2) - (3*e*Log[Tanh[(c + d*x)/
2]])/(2*a*d) + (b^2*e*Log[Tanh[(c + d*x)/2]])/(a^3*d) + (3*c*f*Log[Tanh[(c
+ d*x)/2]])/(2*a*d^2) - (b^2*c*f*Log[Tanh[(c + d*x)/2]])/(a^3*d^2) + (((3*I
)/2)*f*(I*(c + d*x)*(Log[1 - E^(-c - d*x)] - Log[1 + E^(-c - d*x)]) + I*(Po
lyLog[2, -E^(-c - d*x)] - PolyLog[2, E^(-c - d*x)])))/(a*d^2) - (I*b^2*f*(I
*(c + d*x)*(Log[1 - E^(-c - d*x)] - Log[1 + E^(-c - d*x)]) + I*(PolyLog[2,
-E^(-c - d*x)] - PolyLog[2, E^(-c - d*x)])))/(a^3*d^2) + (b^5*(2*d*e*ArcTan
h[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*c*f*ArcTanh[(a + b*E^(c + d*x))/
Sqrt[a^2 + b^2]] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
)] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - f*PolyLog
[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))]))/(a^3*(a^2 + b^2)^(3/2)*d^2) + ((-(d*e) + c*f -
f*(c + d*x))*Sech[(c + d*x)/2]^2)/(8*a*d^2) + (Sech[(c + d*x)/2]*(2*b*d*e*S
inh[(c + d*x)/2] + a*f*Sinh[(c + d*x)/2] - 2*b*c*f*Sinh[(c + d*x)/2] + 2*b*
f*(c + d*x)*Sinh[(c + d*x)/2]))/(4*a^2*d^2) + (Sech[c + d*x]*(-(a*d*e) + a*
c*f - a*f*(c + d*x) + b*d*e*Sinh[c + d*x] - b*c*f*Sinh[c + d*x] + b*f*(c +
d*x)*Sinh[c + d*x]))/((a^2 + b^2)*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2766 vs. $2(659) = 1318$.
time = 11.23, size = 2767, normalized size = 3.96

method	result	size
risch	Expression too large to display	2767

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/2/d/(a^2+b^2)*b^2*f/a*ln(exp(d*x+c)+1)*x+1/2/d^2/(a^2+b^2)*b^2*f*c/a*ln(e
xp(d*x+c)-1)-3/2/d*a*b*e/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(
a^2+b^2)^(1/2))-1/2/d/a*b^3*e/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2
*a)/(a^2+b^2)^(1/2))-1/d^2*a*b*f/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c
)+2*a)/(a^2+b^2)^(1/2))+1/2/d^2/a*b^3*f*c/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*
exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)^(3/2)/d^2/a^3*b^5*f*c*arctanh(
1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)^(5/2)/d^2/a*b^5*f*ln(
(-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/(a^2+b^2)^(5/2)
/d^2/a*b^5*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/(
a^2+b^2)/d/a^3*b^4*e*ln(exp(d*x+c)-1)-1/(a^2+b^2)/d/a^3*b^4*e*ln(exp(d*x+c)
+1)-1/(a^2+b^2)/d^2/a^3*b^4*f*dilog(exp(d*x+c))-1/(a^2+b^2)/d^2/a^3*b^4*f*d
ilog(exp(d*x+c)+1)+2/(a^2+b^2)/d^2/a^2*b^3*f*ln(exp(d*x+c))-1/(a^2+b^2)/d^2
```

/a^2*b^3*f*ln(exp(d*x+c)-1)-1/(a^2+b^2)/d^2/a^2*b^3*f*ln(exp(d*x+c)+1)-4/(a
^2+b^2)/d^2*b^3*f/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))-1/d^2/a*b^3*f/(a^2+b^2
)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/(a^2+b^2)^(5/2)
/d^2*a*f*b^3*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-3/2/(a^2+b
^2)^(5/2)/d^2/a*f*b^5*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-(
3*a^3*d*f*x*exp(5*d*x+5*c)+a*b^2*d*f*x*exp(5*d*x+5*c)+3*a^3*d*e*exp(5*d*x+5
*c)+a*b^2*d*e*exp(5*d*x+5*c)-2*b^3*d*f*x*exp(4*d*x+4*c)-2*a^3*d*f*x*exp(3*d
*x+3*c)+a^3*f*exp(5*d*x+5*c)+2*a*b^2*d*f*x*exp(3*d*x+3*c)+a*b^2*f*exp(5*d*x
+5*c)-2*b^3*d*e*exp(4*d*x+4*c)-2*a^3*d*e*exp(3*d*x+3*c)-4*a^2*b*d*f*x*exp(2
*d*x+2*c)+2*a*b^2*d*e*exp(3*d*x+3*c)+3*a^3*d*f*x*exp(d*x+c)-4*a^2*b*d*e*exp
(2*d*x+2*c)+a*b^2*d*f*x*exp(d*x+c)+3*a^3*d*e*exp(d*x+c)+4*a^2*b*d*f*x+a*b^2
*d*e*exp(d*x+c)+2*b^3*d*f*x-a^3*f*exp(d*x+c)+4*a^2*b*d*e-a*b^2*f*exp(d*x+c)
+2*b^3*d*e)/d^2/a^2/(exp(2*d*x+2*c)-1)^2/(a^2+b^2)/(1+exp(2*d*x+2*c))+8/d^2
/(a^2+b^2)*a^3*f/(4*a^2+4*b^2)*arctan(exp(d*x+c))+1/2/d^2/(a^2+b^2)*b^2*f/a
*dilog(exp(d*x+c)+1)+1/2/d^2/(a^2+b^2)*b^2*f*dilog(exp(d*x+c))/a-1/(a^2+b^2
)^(5/2)/d^2/a^3*b^7*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1
/2)))*c+1/(a^2+b^2)^(5/2)/d^2/a^3*b^7*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)
/(a+(a^2+b^2)^(1/2)))*c-1/(a^2+b^2)^(5/2)/d^2/a^3*b^7*f*c*arctanh(1/2*(2*b*
exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)^(5/2)/d/a^3*b^7*f*ln((-b*exp(d
*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/(a^2+b^2)^(5/2)/d/a^3*b^
7*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/(a^2+b^2)^
(5/2)/d/a*b^5*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*
x+1/(a^2+b^2)^(5/2)/d/a*b^5*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b
^2)^(1/2)))*x-3/2/(a^2+b^2)^(5/2)/d^2*a^3*f*c*b*arctanh(1/2*(2*b*exp(d*x+c)
+2*a)/(a^2+b^2)^(1/2))+3/2/d^2/(a^2+b^2)*a*f*c*ln(exp(d*x+c)-1)+1/2/d/(a^2+
b^2)*b^2*e/a*ln(exp(d*x+c)+1)-1/2/d/(a^2+b^2)*b^2*e/a*ln(exp(d*x+c)-1)+3/2/
d/(a^2+b^2)*ln(exp(d*x+c)+1)*a*f*x+3/2/d^2*a*b*f*c/(a^2+b^2)^(3/2)*arctanh(
1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)^(5/2)/d*a*b^3*e*arcta
nh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+3/2/(a^2+b^2)^(5/2)/d*a^3*e*b*
arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+3/2/(a^2+b^2)^(5/2)/d/a*b
^5*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)^(5/2)/d^
2*a^3*f*b*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)^(5/
2)/d^2*a*f*b^3*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2+b^2
)^(5/2)/d^2/a*f*b^5*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-4/(a^
2+b^2)/d^2*a^2*f/(4*a^2+4*b^2)*b*ln(1+exp(2*d*x+2*c))-1/(a^2+b^2)/d^2/a^3*b
^4*f*c*ln(exp(d*x+c)-1)+1/(a^2+b^2)^(3/2)/d/a^3*b^5*e*arctanh(1/2*(2*b*exp(
d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)^(5/2)/d/a^3*b^7*e*arctanh(1/2*(2*b
*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)/d/a^3*b^4*f*ln(exp(d*x+c)+1)*
x+8/(a^2+b^2)/d^2*a*b^2*f/(4*a^2+4*b^2)*arctan(exp(d*x+c))+1/(a^2+b^2)^(5/2
)/d^2/a^3*b^7*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))
-1/(a^2+b^2)^(5/2)/d^2/a^3*b^7*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-
a+(a^2+b^2)^(1/2)))+1/(a^2+b^2)^(5/2)/d^2/a*b^5*f*dilog((b*exp(d*x+c)+(a^2+
b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/(a^2+b^2)^(5/2)/d^2/a*b^5*f*dilog((-b*
exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+3/2/d/(a^2+b^2)*a*e*ln(
exp(d*x+c)+1)-3/2/d/(a^2+b^2)*a*e*ln(exp(d*x+c)-1)+3/2/d^2/(a^2+b^2)*a*f*di

```
log(exp(d*x+c)+1)+3/2/d^2/(a^2+b^2)*a*f*dilog(exp(d*x+c))+4/(a^2+b^2)/d^2*b
*f*ln(exp(d*x+c))-1/(a^2+b^2)/d^2*b*f*ln(exp(d*x+c)-1)-1/(a^2+b^2)/d^2*b*f*
ln(exp(d*x+c)+1)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -(32*b^5*integrate(-1/16*x*e^(d*x + c)/(a^5*b + a^3*b^3 - (a^5*b*e^(2*c) +
a^3*b^3*e^(2*c))*e^(2*d*x) - 2*(a^6*e^c + a^4*b^2*e^c)*e^(d*x)), x) + 96*a^
2*d*integrate(1/64*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 64*b^2*d*integrate(1
/64*x/(a^3*d*e^(d*x + c) + a^3*d), x) + 96*a^2*d*integrate(1/64*x/(a^3*d*e^
(d*x + c) - a^3*d), x) - 64*b^2*d*integrate(1/64*x/(a^3*d*e^(d*x + c) - a^3
*d), x) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) + 1)/(a^3*d^2)) - a*b*
((d*x + c)/(a^3*d^2) - log(e^(d*x + c) - 1)/(a^3*d^2)) - (2*b^3*d*x*e^(4*d*
x + 4*c) + 4*a^2*b*d*x*e^(2*d*x + 2*c) + 2*(a^3*d*e^(3*c) - a*b^2*d*e^(3*c)
)*x*e^(3*d*x) - 2*(2*a^2*b*d + b^3*d)*x - (a^3*e^(5*c) + a*b^2*e^(5*c) + (3
*a^3*d*e^(5*c) + a*b^2*d*e^(5*c))*x)*e^(5*d*x) + (a^3*e^c + a*b^2*e^c - (3*
a^3*d*e^c + a*b^2*d*e^c)*x)*e^(d*x))/(a^4*d^2 + a^2*b^2*d^2 + (a^4*d^2*e^(6
*c) + a^2*b^2*d^2*e^(6*c))*e^(6*d*x) - (a^4*d^2*e^(4*c) + a^2*b^2*d^2*e^(4*
c))*e^(4*d*x) - (a^4*d^2*e^(2*c) + a^2*b^2*d^2*e^(2*c))*e^(2*d*x)) - 2*b*x/
((a^2 + b^2)*d) - 2*a*arctan(e^(d*x + c))/((a^2 + b^2)*d^2) + b*log(e^(2*d*
x + 2*c) + 1)/((a^2 + b^2)*d^2))*f - 1/2*(2*b^5*log((b*e^(-d*x - c) - a - s
qrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 + b^2)))/((a^5 + a^3*b^2)*sq
rt(a^2 + b^2)*d) + 2*(4*a^2*b*e^(-2*d*x - 2*c) + 2*b^3*e^(-4*d*x - 4*c) - 4
*a^2*b - 2*b^3 + (3*a^3 + a*b^2)*e^(-d*x - c) - 2*(a^3 - a*b^2)*e^(-3*d*x -
 3*c) + (3*a^3 + a*b^2)*e^(-5*d*x - 5*c))/((a^4 + a^2*b^2 - (a^4 + a^2*b^2)
*e^(-2*d*x - 2*c) - (a^4 + a^2*b^2)*e^(-4*d*x - 4*c) + (a^4 + a^2*b^2)*e^(-
6*d*x - 6*c))*d) - (3*a^2 - 2*b^2)*log(e^(-d*x - c) + 1)/(a^3*d) + (3*a^2 -
 2*b^2)*log(e^(-d*x - c) - 1)/(a^3*d))*e
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 12895 vs. $2(663) = 1326$.
time = 0.49, size = 12895, normalized size = 18.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/2*(4*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d*f*x + (a^5*b + 2*a^3*b^3 + a*b^5)*c
*f)*cosh(d*x + c)^6 + 4*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d*f*x + (a^5*b + 2*a
^3*b^3 + a*b^5)*c*f)*sinh(d*x + c)^6 - 2*((3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*f
*x + (3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*cosh(1) + (3*a^6 + 4*a^4*b^2 + a^2*b^4
)*d*sinh(1) + (a^6 + 2*a^4*b^2 + a^2*b^4)*f)*cosh(d*x + c)^5 - 2*((3*a^6 +
4*a^4*b^2 + a^2*b^4)*d*f*x + (3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*cosh(1) + (3*a
^6 + 4*a^4*b^2 + a^2*b^4)*d*sinh(1) + (a^6 + 2*a^4*b^2 + a^2*b^4)*f - 12*((
2*a^5*b + 3*a^3*b^3 + a*b^5)*d*f*x + (a^5*b + 2*a^3*b^3 + a*b^5)*c*f)*cosh(
d*x + c))*sinh(d*x + c)^5 - 4*(2*(a^5*b + a^3*b^3)*d*f*x + (a^5*b + 2*a^3*b
^3 + a*b^5)*c*f - (a^3*b^3 + a*b^5)*d*cosh(1) - (a^3*b^3 + a*b^5)*d*sinh(1)
)*cosh(d*x + c)^4 - 2*(4*(a^5*b + a^3*b^3)*d*f*x + 2*(a^5*b + 2*a^3*b^3 + a
*b^5)*c*f - 2*(a^3*b^3 + a*b^5)*d*cosh(1) - 30*((2*a^5*b + 3*a^3*b^3 + a*b^
5)*d*f*x + (a^5*b + 2*a^3*b^3 + a*b^5)*c*f)*cosh(d*x + c)^2 - 2*(a^3*b^3 +
a*b^5)*d*sinh(1) + 5*((3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*f*x + (3*a^6 + 4*a^4*
b^2 + a^2*b^4)*d*cosh(1) + (3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*sinh(1) + (a^6 +
 2*a^4*b^2 + a^2*b^4)*f)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*((a^6 - a^2*b^4
)*d*f*x + (a^6 - a^2*b^4)*d*cosh(1) + (a^6 - a^2*b^4)*d*sinh(1))*cosh(d*x +
 c)^3 + 4*((a^6 - a^2*b^4)*d*f*x + 20*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d*f*x
+ (a^5*b + 2*a^3*b^3 + a*b^5)*c*f)*cosh(d*x + c)^3 + (a^6 - a^2*b^4)*d*cosh
(1) - 5*((3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*f*x + (3*a^6 + 4*a^4*b^2 + a^2*b^4
)*d*cosh(1) + (3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*sinh(1) + (a^6 + 2*a^4*b^2 +
a^2*b^4)*f)*cosh(d*x + c)^2 + (a^6 - a^2*b^4)*d*sinh(1) - 4*(2*(a^5*b + a^3
*b^3)*d*f*x + (a^5*b + 2*a^3*b^3 + a*b^5)*c*f - (a^3*b^3 + a*b^5)*d*cosh(1)
 - (a^3*b^3 + a*b^5)*d*sinh(1))*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^5*b +
 2*a^3*b^3 + a*b^5)*c*f - 4*(2*a^5*b + 3*a^3*b^3 + a*b^5)*d*cosh(1) - 4*((a
^3*b^3 + a*b^5)*d*f*x + (a^5*b + 2*a^3*b^3 + a*b^5)*c*f - 2*(a^5*b + a^3*b^
3)*d*cosh(1) - 2*(a^5*b + a^3*b^3)*d*sinh(1))*cosh(d*x + c)^2 - 4*(2*a^5*b
+ 3*a^3*b^3 + a*b^5)*d*sinh(1) + 4*(15*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d*f*x
 + (a^5*b + 2*a^3*b^3 + a*b^5)*c*f)*cosh(d*x + c)^4 - (a^3*b^3 + a*b^5)*d*f
*x - 5*((3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*f*x + (3*a^6 + 4*a^4*b^2 + a^2*b^4)
*d*cosh(1) + (3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*sinh(1) + (a^6 + 2*a^4*b^2 + a
^2*b^4)*f)*cosh(d*x + c)^3 - (a^5*b + 2*a^3*b^3 + a*b^5)*c*f + 2*(a^5*b + a
^3*b^3)*d*cosh(1) - 6*(2*(a^5*b + a^3*b^3)*d*f*x + (a^5*b + 2*a^3*b^3 + a*b
^5)*c*f - (a^3*b^3 + a*b^5)*d*cosh(1) - (a^3*b^3 + a*b^5)*d*sinh(1))*cosh(d
*x + c)^2 + 2*(a^5*b + a^3*b^3)*d*sinh(1) + 3*((a^6 - a^2*b^4)*d*f*x + (a^6
 - a^2*b^4)*d*cosh(1) + (a^6 - a^2*b^4)*d*sinh(1))*cosh(d*x + c))*sinh(d*x
+ c)^2 - 2*(b^6*f*cosh(d*x + c)^6 + 6*b^6*f*cosh(d*x + c)*sinh(d*x + c)^5 +
 b^6*f*sinh(d*x + c)^6 - b^6*f*cosh(d*x + c)^4 - b^6*f*cosh(d*x + c)^2 + b^
6*f + (15*b^6*f*cosh(d*x + c)^2 - b^6*f)*sinh(d*x + c)^4 + 4*(5*b^6*f*cosh(
d*x + c)^3 - b^6*f*cosh(d*x + c))*sinh(d*x + c)^3 + (15*b^6*f*cosh(d*x + c)
^4 - 6*b^6*f*cosh(d*x + c)^2 - b^6*f)*sinh(d*x + c)^2 + 2*(3*b^6*f*cosh(d*x
 + c)^5 - 2*b^6*f*cosh(d*x + c)^3 - b^6*f*cosh(d*x + c))*sinh(d*x + c))*sqr
t((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
 c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b^6*f*cosh(d*
x + c)^6 + 6*b^6*f*cosh(d*x + c)*sinh(d*x + c)^5 + b^6*f*sinh(d*x + c)^6 -
```

```
b^6*f*cosh(d*x + c)^4 - b^6*f*cosh(d*x + c)^2 + b^6*f + (15*b^6*f*cosh(d*x
+ c)^2 - b^6*f)*sinh(d*x + c)^4 + 4*(5*b^6*f*cosh(d*x + c)^3 - b^6*f*cosh(d
*x + c))*sinh(d*x + c)^3 + (15*b^6*f*cosh(d*x + c)^4 - 6*b^6*f*cosh(d*x + c
)^2 - b^6*f)*sinh(d*x + c)^2 + 2*(3*b^6*f*cosh(d*x + c)^5 - 2*b^6*f*cosh(d*
x + c)^3 - b^6*f*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog(
(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sq
rt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b^6*c*f - b^6*d*cosh(1) - b^6*d*sinh(1
) + (b^6*c*f - b^6*d*cosh(1) - b^6*d*sinh(1))*cosh(d*x + c)^6 + 6*(b^6*c*f
- b^6*d*cosh(1) - b^6*d*sinh(1))*cosh(d*x + c)*sinh(d*x + c)^5 + (b^6*c*f -
 b^6*d*cosh(1) - b^6*d*sinh(1))*sinh(d*x + c)^6 - (b^6*c*f - b^6*d*cosh(1)
- b^6*d*sinh(1))*cosh(d*x + c)^4 - (b^6*c*f - b^6*d*cosh(1) - b^6*d*sinh(1)
 - 15*(b^6*c*f - b^6*d*cosh(1) - b^6*d*sinh(1))*cosh(d*x + c)^2)*sinh(d*x +
 c)^4 + 4*(5*(b^6*c*f - b^6*d*cosh(1) - b^6*d*sinh(1))*cosh(d*x + c)^3 - (b
^6*c*f - b^6*d*cosh(1) - b^6*d*sinh(1))*cosh(d*x + c))*sinh(d*x + c)^3 - (b
^6*c*f - b^6*d*cosh(1) - b^6*d*sinh(1))*cosh(d*x + c)^2 - (b^6*c*f - b^6*d*
cosh(1) - b^6*d*sinh(1) - 15*(b^6*c*f - b^6*d*cosh(1) - b^6*d*sinh(1))*cosh
(d*x + c)^4 + 6*(b^6*c*f - b^6*d*cosh(1) - b^6*d*sinh(1))*cosh(d*x + c)^2)*
sinh(d*x + c)^2 + 2*(3*(b^6*c*f - b^6*d*cosh(1) - b^6*d*sinh(1))*cosh(d*x +
 c)^5 - 2*(b^6*c*f - b^6*d*cosh(1) - b^6*d*sinh(1))*cosh(d*x + c)^3 - (b^6*
c*f - b^6*d*cosh(1) - b^6*d*sinh(1))*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b
^2)/b^2) + 2*a) + 2*(b^6*c*f - b^6*d*cosh(1) - ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**3*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{{\mathrm{cosh}\left(c+d\,x\right)}^2\,{\mathrm{sinh}\left(c+d\,x\right)}^3\,\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(cosh(c + d*x)^2*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
[Out] int((e + f*x)/(cosh(c + d*x)^2*sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

3.498 $\int \frac{\mathrm{csch}^3(c+dx)\mathrm{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=206

$$\frac{3\tanh^{-1}(\cosh(c+dx))}{2ad}-\frac{b^2\tanh^{-1}(\cosh(c+dx))}{a^3d}+\frac{2b^5\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)^{3/2}d}+\frac{b\coth(c+dx)}{a^2d}-\frac{3\mathrm{sech}(}{2a}$$

```
[Out] 3/2*arctanh(cosh(d*x+c))/a/d-b^2*arctanh(cosh(d*x+c))/a^3/d+2*b^5*arctanh((
b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/a^3/(a^2+b^2)^(3/2)/d+b*coth(d*x+
c)/a^2/d-3/2*sech(d*x+c)/a/d+b^2*sech(d*x+c)/a^3/d-1/2*csch(d*x+c)^2*sech(d
*x+c)/a/d-b^3*sech(d*x+c)*(b+a*sinh(d*x+c))/a^3/(a^2+b^2)/d+b*tanh(d*x+c)/a
^2/d
```

Rubi [A]
time = 0.29, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {2977, 2702, 327, 213, 2700, 14, 294, 2775, 12, 2739, 632, 210}

$$\frac{b^2\mathrm{sech}(c+dx)}{a^3d}-\frac{b^2\tanh^{-1}(\cosh(c+dx))}{a^3d}+\frac{b\tanh(c+dx)}{a^2d}+\frac{b\coth(c+dx)}{a^2d}+\frac{2b^5\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3d\left(a^2+b^2\right)^{3/2}}-\frac{b^3\mathrm{sech}(c+dx)(a\sinh(c+dx)+b)}{a^3d\left(a^2+b^2\right)}-\frac{3\mathrm{sech}(c+dx)}{2ad}+\frac{3\tanh^{-1}(\cosh(c+dx))}{2ad}-\frac{\mathrm{csch}^2(c+dx)\mathrm{sech}(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Cosh[c + d*x]])/(2*a*d) - (b^2*ArcTanh[Cosh[c + d*x]])/(a^3*d) +
 (2*b^5*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^3*(a^2 + b^2
)^(3/2)*d) + (b*Coth[c + d*x])/(a^2*d) - (3*Sech[c + d*x])/(2*a*d) + (b^2*S
ech[c + d*x])/(a^3*d) - (Csch[c + d*x]^2*Sech[c + d*x])/(2*a*d) - (b^3*Sech
[c + d*x]*(b + a*Sinh[c + d*x]))/(a^3*(a^2 + b^2)*d) + (b*Tanh[c + d*x])/(a
^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\left(i\int \left(\frac{ib^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a^3} - \frac{ib\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a^2}\right.\right.$$

$$= \frac{\int \operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)\,dx}{a} - \frac{b\int \operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)\,dx}{a^2}$$

$$= -\frac{b^3\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a^3(a^2+b^2)\,d} - \frac{b^3\int \frac{b^2}{a+b\sinh(c+dx)}\,dx}{a^3(a^2+b^2)} - \frac{\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{b^2\operatorname{sech}(c+dx)}{a^3d} - \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{2ad} - \frac{b^3\operatorname{sech}(c+dx)(b+a\sin}{a^3(a^2+b^2)}$$

$$= -\frac{b^2\tanh^{-1}(\cosh(c+dx))}{a^3d} + \frac{b\coth(c+dx)}{a^2d} - \frac{3\operatorname{sech}(c+dx)}{2ad} + \frac{b^2\operatorname{sech}(}{a}$$

$$= \frac{3\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{b^2\tanh^{-1}(\cosh(c+dx))}{a^3d} + \frac{b\coth(c+dx)}{a^2d} -$$

$$= \frac{3\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{b^2\tanh^{-1}(\cosh(c+dx))}{a^3d} + \frac{2b^5\tanh^{-1}\left(\frac{b-a}{\sqrt{}}\right.}{a^3(a^2+b}$$

Mathematica [A]

time = 1.76, size = 185, normalized size = 0.90

$$\frac{\frac{16b^5\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{a^3(-a^2-b^2)^{3/2}}+\frac{4b\coth\left(\frac{1}{2}(c+dx)\right)}{a^2}-\frac{\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{a}-\frac{4(3a^2-2b^2)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a^3}-\frac{\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{a}+\frac{8\operatorname{sech}(c+dx)(-a+b\sinh(c+dx))}{a^2+b^2}+\frac{4b\tanh\left(\frac{1}{2}(c+dx)\right)}{a^2}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((16*b^5*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(a^3*(-a^2 - b
^2)^(3/2)) + (4*b*Coth[(c + d*x)/2])/a^2 - Csch[(c + d*x)/2]^2/a - (4*(3*a^
2 - 2*b^2)*Log[Tanh[(c + d*x)/2]])/a^3 - Sech[(c + d*x)/2]^2/a + (8*Sech[c
+ d*x]*(-a + b*Sinh[c + d*x]))/(a^2 + b^2) + (4*b*Tanh[(c + d*x)/2])/a^2)/(
8*d)
```

Maple [A]

time = 2.79, size = 183, normalized size = 0.89

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{2b^5\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(-6a^2+4b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+}{d}$
default	$\frac{\frac{\frac{a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{2b^5\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(-6a^2+4b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+}{d}$
risch	$-\frac{3a^3e^{5dx+5c}+ab^2e^{5dx+5c}-2b^3e^{4dx+4c}-2a^3e^{3dx+3c}+2ab^2e^{3dx+3c}-4a^2be^{2dx+2c}+3a^3e^{dx+c}+ab^2e^{dx+c}+4a^2b+2b^3}{da^2\left(e^{2dx+2c}-1\right)^2\left(a^2+b^2\right)\left(1+e^{2dx+2c}\right)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a^2*(1/2*a*tanh(1/2*d*x+1/2*c)^2+2*b*tanh(1/2*d*x+1/2*c))-2/a^3*b^
5/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)
)-1/8/a/tanh(1/2*d*x+1/2*c)^2+1/4/a^3*(-6*a^2+4*b^2)*ln(tanh(1/2*d*x+1/2*c)
)+1/2*b/a^2/tanh(1/2*d*x+1/2*c)+2/(a^2+b^2)*(b*tanh(1/2*d*x+1/2*c)-a)/(tanh
(1/2*d*x+1/2*c)^2+1))
```

Maxima [A]

time = 0.49, size = 334, normalized size = 1.62

$$-\frac{b^5\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{(a^5+a^3b^2)\sqrt{a^2+b^2}\,d}-\frac{4\,a^2be^{(-2\,dx-2\,c)}+2\,b^3e^{(-4\,dx-4\,c)}-4\,a^2b-2\,b^3+(3\,a^3+ab^2)e^{(-dx-c)}-2\,(a^3-ab^2)e^{(-3\,dx-3\,c)}+(3\,a^3+ab^2)e^{(-5\,dx-5\,c)}}{(a^4+a^2b^2-(a^4+a^2b^2)e^{(-2\,dx-2\,c)}-(a^4+a^2b^2)e^{(-4\,dx-4\,c)}+(a^4+a^2b^2)e^{(-6\,dx-6\,c)})d}+\frac{(3\,a^2-2\,b^2)\log\left(e^{(-dx-c)}+1\right)}{2\,a^3d}-\frac{(3\,a^2-2\,b^2)\log\left(e^{(-dx-c)}-1\right)}{2\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxim
a")
```

```
[Out] -b^5*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(
a^2 + b^2)))/((a^5 + a^3*b^2)*sqrt(a^2 + b^2)*d) - (4*a^2*b*e^(-2*d*x - 2*c
) + 2*b^3*e^(-4*d*x - 4*c) - 4*a^2*b - 2*b^3 + (3*a^3 + a*b^2)*e^(-d*x - c)
 - 2*(a^3 - a*b^2)*e^(-3*d*x - 3*c) + (3*a^3 + a*b^2)*e^(-5*d*x - 5*c))/((a
^4 + a^2*b^2 - (a^4 + a^2*b^2)*e^(-2*d*x - 2*c) - (a^4 + a^2*b^2)*e^(-4*d*x
 - 4*c) + (a^4 + a^2*b^2)*e^(-6*d*x - 6*c))*d) + 1/2*(3*a^2 - 2*b^2)*log(e^
(-d*x - c) + 1)/(a^3*d) - 1/2*(3*a^2 - 2*b^2)*log(e^(-d*x - c) - 1)/(a^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2653 vs. $2(197) = 394$.
time = 0.57, size = 2653, normalized size = 12.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="frica
s")
```

```
[Out] -1/2*(8*a^5*b + 12*a^3*b^3 + 4*a*b^5 + 2*(3*a^6 + 4*a^4*b^2 + a^2*b^4)*cosh
(d*x + c)^5 + 2*(3*a^6 + 4*a^4*b^2 + a^2*b^4)*sinh(d*x + c)^5 - 4*(a^3*b^3
+ a*b^5)*cosh(d*x + c)^4 - 2*(2*a^3*b^3 + 2*a*b^5 - 5*(3*a^6 + 4*a^4*b^2 +
a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(a^6 - a^2*b^4)*cosh(d*x + c)^3
 - 4*(a^6 - a^2*b^4 - 5*(3*a^6 + 4*a^4*b^2 + a^2*b^4)*cosh(d*x + c)^2 + 4*(
a^3*b^3 + a*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*(a^5*b + a^3*b^3)*cosh(
d*x + c)^2 - 4*(2*a^5*b + 2*a^3*b^3 - 5*(3*a^6 + 4*a^4*b^2 + a^2*b^4)*cosh(
d*x + c)^3 + 6*(a^3*b^3 + a*b^5)*cosh(d*x + c)^2 + 3*(a^6 - a^2*b^4)*cosh(d
*x + c))*sinh(d*x + c)^2 - 2*(b^5*cosh(d*x + c)^6 + 6*b^5*cosh(d*x + c)*sin
h(d*x + c)^5 + b^5*sinh(d*x + c)^6 - b^5*cosh(d*x + c)^4 - b^5*cosh(d*x + c
)^2 + b^5 + (15*b^5*cosh(d*x + c)^2 - b^5)*sinh(d*x + c)^4 + 4*(5*b^5*cosh(
d*x + c)^3 - b^5*cosh(d*x + c))*sinh(d*x + c)^3 + (15*b^5*cosh(d*x + c)^4 -
 6*b^5*cosh(d*x + c)^2 - b^5)*sinh(d*x + c)^2 + 2*(3*b^5*cosh(d*x + c)^5 -
2*b^5*cosh(d*x + c)^3 - b^5*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)*l
og((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2
 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*c
osh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*(3*a
^6 + 4*a^4*b^2 + a^2*b^4)*cosh(d*x + c) - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2
*b^6)*cosh(d*x + c)^6 + 6*(3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(d*x +
c)*sinh(d*x + c)^5 + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*sinh(d*x + c)^6
+ 3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^
6)*cosh(d*x + c)^4 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6 - 15*(3*a^6 + 4*a
^4*b^2 - a^2*b^4 - 2*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(3*a^6 +
4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(d*x + c)^3 - (3*a^6 + 4*a^4*b^2 - a^2*b^4
```

```
 - 2*b^6)*cosh(d*x + c))*sinh(d*x + c)^3 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2
*b^6)*cosh(d*x + c)^2 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6 - 15*(3*a^6 +
4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(d*x + c)^4 + 6*(3*a^6 + 4*a^4*b^2 - a^2*b
^4 - 2*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(3*a^6 + 4*a^4*b^2 - a^
2*b^4 - 2*b^6)*cosh(d*x + c)^5 - 2*(3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*co
sh(d*x + c)^3 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(d*x + c))*sinh(d
*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((3*a^6 + 4*a^4*b^2 - a^2
*b^4 - 2*b^6)*cosh(d*x + c)^6 + 6*(3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*cos
h(d*x + c)*sinh(d*x + c)^5 + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*sinh(d*x
 + c)^6 + 3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6 - (3*a^6 + 4*a^4*b^2 - a^2*b^
4 - 2*b^6)*cosh(d*x + c)^4 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6 - 15*(3*a
^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(
3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(d*x + c)^3 - (3*a^6 + 4*a^4*b^2 -
 a^2*b^4 - 2*b^6)*cosh(d*x + c))*sinh(d*x + c)^3 - (3*a^6 + 4*a^4*b^2 - a^2
*b^4 - 2*b^6)*cosh(d*x + c)^2 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6 - 15*(
3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(d*x + c)^4 + 6*(3*a^6 + 4*a^4*b^2
 - a^2*b^4 - 2*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(3*a^6 + 4*a^4*
b^2 - a^2*b^4 - 2*b^6)*cosh(d*x + c)^5 - 2*(3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2
*b^6)*cosh(d*x + c)^3 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(d*x + c)
)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(3*a^6 + 4*a^4*
b^2 + a^2*b^4 + 5*(3*a^6 + 4*a^4*b^2 + a^2*b^4)*cosh(d*x + c)^4 - 8*(a^3*b^
3 + a*b^5)*cosh(d*x + c)^3 - 6*(a^6 - a^2*b^4)*cosh(d*x + c)^2 - 8*(a^5*b +
 a^3*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh
(d*x + c)^6 + 6*(a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5
 + (a^7 + 2*a^5*b^2 + a^3*b^4)*d*sinh(d*x + c)^6 - (a^7 + 2*a^5*b^2 + a^3*b
^4)*d*cosh(d*x + c)^4 + (15*(a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^2 -
 (a^7 + 2*a^5*b^2 + a^3*b^4)*d)*sinh(d*x + c)^4 - (a^7 + 2*a^5*b^2 + a^3*b^
4)*d*cosh(d*x + c)^2 + 4*(5*(a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^3 -
 (a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^7 +
2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^4 - 6*(a^7 + 2*a^5*b^2 + a^3*b^4)*d*co
sh(d*x + c)^2 - (a^7 + 2*a^5*b^2 + a^3*b^4)*d)*sinh(d*x + c)^2 + (a^7 + 2*a
^5*b^2 + a^3*b^4)*d + 2*(3*(a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^5 -
2*(a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^3 - (a^7 + 2*a^5*b^2 + a^3*b^
4)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6439 deep
```

Giac [A]

time = 0.43, size = 224, normalized size = 1.09

$$-\frac{\frac{2\,b^5\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^5+a^3b^2)\sqrt{a^2+b^2}}+\frac{4\,(ae^{(dx+c)}+b)}{(a^2+b^2)(e^{(2\,dx+2\,c)}+1)}-\frac{(3\,a^2-2\,b^2)\log(e^{(dx+c)}+1)}{a^3}+\frac{(3\,a^2-2\,b^2)\log(|e^{(dx+c)}-1|)}{a^3}+\frac{2\,(ae^{(3\,dx+3\,c)}-2\,be^{(2\,dx+2\,c)}+ae^{(dx+c)}+2\,b)}{a^2\left(e^{(2\,dx+2\,c)}-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac"
)
```

```
[Out] -1/2*(2*b^5*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d
*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/((a^5 + a^3*b^2)*sqrt(a^2 + b^2)) + 4*(
a*e^(d*x + c) + b)/((a^2 + b^2)*(e^(2*d*x + 2*c) + 1)) - (3*a^2 - 2*b^2)*lo
g(e^(d*x + c) + 1)/a^3 + (3*a^2 - 2*b^2)*log(abs(e^(d*x + c) - 1))/a^3 + 2*
(a*e^(3*d*x + 3*c) - 2*b*e^(2*d*x + 2*c) + a*e^(d*x + c) + 2*b)/(a^2*(e^(2*
d*x + 2*c) - 1)^2))/d
```

Mupad [B]
time = 3.47, size = 531, normalized size = 2.58

$$\frac{b^5\,\ln\left(2\,a^4\,b-4\,a^5\,e^{c+d\,x}+b^5+3\,a^2\,b^3+4\,a^2\,e^{c+d\,x}\,\sqrt{{\left(a^2+b^2\right)}^3}+b^2\,e^{c+d\,x}\,\sqrt{{\left(a^2+b^2\right)}^3}-7\,a^3\,b^2\,e^{c+d\,x}-2\,a\,b\,\sqrt{{\left(a^2+b^2\right)}^3}-3\,a\,b^4\,e^{c+d\,x}\right)\,\sqrt{{\left(a^2+b^2\right)}^3}}{d\,a^9+3\,d\,a^7\,b^2+3\,d\,a^5\,b^4+d\,a^3\,b^6}-\frac{\frac{e^{c+d\,x}}{a\,d}-\frac{2\,\left(a^2\,b+b^3\right)}{a^2\,d\,\left(a^2+b^2\right)}}{e^{2\,c+2\,d\,x}-1}-\frac{\frac{2\,b}{d\,\left(a^2+b^2\right)}+\frac{2\,a\,e^{c+d\,x}}{d\,\left(a^2+b^2\right)}}{e^{2\,c+2\,d\,x}+1}-\frac{\ln\left(e^{c+d\,x}-1\right)\,\left(3\,a^2-2\,b^2\right)}{2\,a^3\,d}+\frac{\ln\left(e^{c+d\,x}+1\right)\,\left(3\,a^2-2\,b^2\right)}{2\,a^3\,d}-\frac{b^5\,\ln\left(4\,a^5\,e^{c+d\,x}-2\,a^4\,b-b^5-3\,a^2\,b^3+4\,a^2\,e^{c+d\,x}\,\sqrt{{\left(a^2+b^2\right)}^3}+b^2\,e^{c+d\,x}\,\sqrt{{\left(a^2+b^2\right)}^3}+7\,a^3\,b^2\,e^{c+d\,x}-2\,a\,b\,\sqrt{{\left(a^2+b^2\right)}^3}+3\,a\,b^4\,e^{c+d\,x}\right)\,\sqrt{{\left(a^2+b^2\right)}^3}}{d\,a^9+3\,d\,a^7\,b^2+3\,d\,a^5\,b^4+d\,a^3\,b^6}-\frac{2\,e^{c+d\,x}}{a\,d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

```
[Out] (b^5*log(2*a^4*b - 4*a^5*exp(c + d*x) + b^5 + 3*a^2*b^3 + 4*a^2*exp(c + d*x
)*((a^2 + b^2)^3)^(1/2) + b^2*exp(c + d*x)*((a^2 + b^2)^3)^(1/2) - 7*a^3*b^
2*exp(c + d*x) - 2*a*b*((a^2 + b^2)^3)^(1/2) - 3*a*b^4*exp(c + d*x))*((a^2
+ b^2)^3)^(1/2))/(a^9*d + a^3*b^6*d + 3*a^5*b^4*d + 3*a^7*b^2*d) - (exp(c +
 d*x)/(a*d) - (2*(a^2*b + b^3))/(a^2*d*(a^2 + b^2)))/(exp(2*c + 2*d*x) - 1)
 - ((2*b)/(d*(a^2 + b^2)) + (2*a*exp(c + d*x))/(d*(a^2 + b^2)))/(exp(2*c +
2*d*x) + 1) - (log(exp(c + d*x) - 1)*(3*a^2 - 2*b^2))/(2*a^3*d) + (log(exp(
c + d*x) + 1)*(3*a^2 - 2*b^2))/(2*a^3*d) - (b^5*log(4*a^5*exp(c + d*x) - 2*
a^4*b - b^5 - 3*a^2*b^3 + 4*a^2*exp(c + d*x)*((a^2 + b^2)^3)^(1/2) + b^2*ex
p(c + d*x)*((a^2 + b^2)^3)^(1/2) + 7*a^3*b^2*exp(c + d*x) - 2*a*b*((a^2 + b
^2)^3)^(1/2) + 3*a*b^4*exp(c + d*x))*((a^2 + b^2)^3)^(1/2))/(a^9*d + a^3*b^
6*d + 3*a^5*b^4*d + 3*a^7*b^2*d) - (2*exp(c + d*x))/(a*d*(exp(4*c + 4*d*x)
- 2*exp(2*c + 2*d*x) + 1))
```

3.499 $\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=39

$$\operatorname{Int}\left(\frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csch[c + d*x]^3*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Csch[c + d*x]^3*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A]
time = 138.80, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Csch[c + d*x]^3*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]
)),x]
```

```
[Out] Integrate[(Csch[c + d*x]^3*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]
)), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(dx+c)^{3} \operatorname{sech}(dx+c)^{2}}{(fx+e)\,(a+b\sinh{(dx+c)})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -32*b^5*integrate(-1/16*e^(d*x + c)/((a^5*b*f + a^3*b^3*f)*x + (a^5*b + a^3
*b^3)*e - ((a^5*b*f*e^(2*c) + a^3*b^3*f*e^(2*c))*x + (a^5*b*e^(2*c) + a^3*b
^3*e^(2*c))*e)*e^(2*d*x) - 2*((a^6*f*e^c + a^4*b^2*f*e^c)*x + (a^6*e^c + a^
4*b^2*e^c)*e)*e^(d*x)), x) - (2*(2*a^2*b*d*f + b^3*d*f)*x + 2*(2*a^2*b*d +
b^3*d)*e - (a^3*f*e^(5*c) + a*b^2*f*e^(5*c) - (3*a^3*d*f*e^(5*c) + a*b^2*d*
f*e^(5*c))*x - (3*a^3*d*e^(5*c) + a*b^2*d*e^(5*c))*e)*e^(5*d*x) - 2*(b^3*d*
f*x*e^(4*c) + b^3*d*e^(4*c + 1))*e^(4*d*x) - 2*((a^3*d*f*e^(3*c) - a*b^2*d*
f*e^(3*c))*x + (a^3*d*e^(3*c) - a*b^2*d*e^(3*c))*e)*e^(3*d*x) - 4*(a^2*b*d*
f*x*e^(2*c) + a^2*b*d*e^(2*c + 1))*e^(2*d*x) + (a^3*f*e^c + a*b^2*f*e^c + (
3*a^3*d*f*e^c + a*b^2*d*f*e^c)*x + (3*a^3*d*e^c + a*b^2*d*e^c)*e)*e^(d*x))/
((a^4*d^2*f^2 + a^2*b^2*d^2*f^2)*x^2 + 2*(a^4*d^2*f + a^2*b^2*d^2*f)*x*e +
(a^4*d^2 + a^2*b^2*d^2)*e^2 + ((a^4*d^2*f^2*e^(6*c) + a^2*b^2*d^2*f^2*e^(6*
c))*x^2 + 2*(a^4*d^2*f*e^(6*c) + a^2*b^2*d^2*f*e^(6*c))*x*e + (a^4*d^2*e^(6
*c) + a^2*b^2*d^2*e^(6*c))*e^2)*e^(6*d*x) - ((a^4*d^2*f^2*e^(4*c) + a^2*b^2
*d^2*f^2*e^(4*c))*x^2 + 2*(a^4*d^2*f*e^(4*c) + a^2*b^2*d^2*f*e^(4*c))*x*e +
 (a^4*d^2*e^(4*c) + a^2*b^2*d^2*e^(4*c))*e^2)*e^(4*d*x) - ((a^4*d^2*f^2*e^(
2*c) + a^2*b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^4*d^2*f*e^(2*c) + a^2*b^2*d^2*f*
e^(2*c))*x*e + (a^4*d^2*e^(2*c) + a^2*b^2*d^2*e^(2*c))*e^2)*e^(2*d*x)) - 32
*integrate(1/64*(2*a*b*d*f*e + 2*a^2*f^2 - (3*a^2*d^2*f^2 - 2*b^2*d^2*f^2)*
x^2 + 2*(a*b*d*f^2 - (3*a^2*d^2*f - 2*b^2*d^2*f)*e)*x - (3*a^2*d^2 - 2*b^2*
d^2)*e^2)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*f^2*x^2*e + 3*a^3*d^2*f*x*e^2 + a^3*
d^2*e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*f^2*x^2*e^(c + 1) + 3*a^3*d^2*f*
x*e^(c + 2) + a^3*d^2*e^(c + 3))*e^(d*x)), x) - 32*integrate(1/64*(2*a*b*d*
f*e - 2*a^2*f^2 + (3*a^2*d^2*f^2 - 2*b^2*d^2*f^2)*x^2 + 2*(a*b*d*f^2 + (3*a
^2*d^2*f - 2*b^2*d^2*f)*e)*x + (3*a^2*d^2 - 2*b^2*d^2)*e^2)/(a^3*d^2*f^3*x^
```

```
3 + 3*a^3*d^2*f^2*x^2*e + 3*a^3*d^2*f*x*e^2 + a^3*d^2*e^3 + (a^3*d^2*f^3*x^
3*e^c + 3*a^3*d^2*f^2*x^2*e^(c + 1) + 3*a^3*d^2*f*x*e^(c + 2) + a^3*d^2*e^(
c + 3))*e^(d*x)), x) - 32*integrate(1/16*(a*f*e^(d*x + c) + b*f)/((a^2*d*f^
2 + b^2*d*f^2)*x^2 + 2*(a^2*d*f + b^2*d*f)*x*e + (a^2*d + b^2*d)*e^2 + ((a^
2*d*f^2*e^(2*c) + b^2*d*f^2*e^(2*c))*x^2 + 2*(a^2*d*f*e^(2*c) + b^2*d*f*e^(
2*c))*x*e + (a^2*d*e^(2*c) + b^2*d*e^(2*c))*e^2)*e^(2*d*x)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] integral(csch(d*x + c)^3*sech(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(
d*x + c)), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*sech(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6440 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cosh\left(c+d\,x\right)^2\,\sinh\left(c+d\,x\right)^3\,\left(e+f\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))),x)
```

```
[Out] int(1/(cosh(c + d*x)^2*sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

3.500 $\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1122

$$\frac{b^2fx}{2a^3d}+\frac{3bfx\text{ArcTan}\left(e^{c+dx}\right)}{a^2d}-\frac{2b^5(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)^2d}-\frac{b^3(e+fx)\text{ArcTan}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d}-\frac{3bfx\text{ArcTan}(\sinh(c}{2a^2d}$$

```
[Out] -f*polylog(2,exp(2*d*x+2*c))/a/d^2+4*(f*x+e)*arctanh(exp(2*d*x+2*c))/a/d+f*
polylog(2,-exp(2*d*x+2*c))/a/d^2+b^2*(f*x+e)*ln(tanh(d*x+c))/a^3/d+1/2*b^2*
f*x/a^3/d-2*(f*x+e)*coth(2*d*x+2*c)*csch(2*d*x+2*c)/a/d+1/2*b*f*sech(d*x+c)
/a^2/d^2-1/2*b^2*f*tanh(d*x+c)/a^3/d^2-1/2*b^2*(f*x+e)*tanh(d*x+c)^2/a^3/d-
f*csch(2*d*x+2*c)/a/d^2+3*b*f*x*arctan(exp(d*x+c))/a^2/d-b^3*(f*x+e)*arctan
(exp(d*x+c))/a^2/(a^2+b^2)/d-3/2*b*f*x*arctan(sinh(d*x+c))/a^2/d+b^6*(f*x+e
)*ln(1+exp(2*d*x+2*c))/a^3/(a^2+b^2)^2/d-b^6*(f*x+e)*ln(1+b*exp(d*x+c)/(a-(
a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^2/d-b^6*(f*x+e)*ln(1+b*exp(d*x+c)/(a+(a^2+b^
2)^(1/2)))/a^3/(a^2+b^2)^2/d-b^2*f*x*ln(tanh(d*x+c))/a^3/d-b^6*f*polylog(2,
-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^2/d^2-b^6*f*polylog(2,-b*e
xp(d*x+c)/(a+(a^2+b^2)^(1/2)))/a^3/(a^2+b^2)^2/d^2+I*b^5*f*polylog(2,-I*exp
(d*x+c))/a^2/(a^2+b^2)^2/d^2+3/2*I*b*f*polylog(2,I*exp(d*x+c))/a^2/d^2-1/2*
b^3*(f*x+e)*sech(d*x+c)*tanh(d*x+c)/a^2/(a^2+b^2)/d-I*b^5*f*polylog(2,I*exp
(d*x+c))/a^2/(a^2+b^2)^2/d^2+1/2*b^2*f*polylog(2,exp(2*d*x+2*c))/a^3/d^2-1/
2*I*b^3*f*polylog(2,I*exp(d*x+c))/a^2/(a^2+b^2)/d^2+3/2*b*(f*x+e)*arctan(si
nh(d*x+c))/a^2/d+b*f*arctanh(cosh(d*x+c))/a^2/d^2+3/2*b*(f*x+e)*csch(d*x+c)
/a^2/d-1/2*b^2*f*polylog(2,-exp(2*d*x+2*c))/a^3/d^2-2*b^5*(f*x+e)*arctan(ex
p(d*x+c))/a^2/(a^2+b^2)^2/d-2*b^2*f*x*arctanh(exp(2*d*x+2*c))/a^3/d+1/2*b^6
*f*polylog(2,-exp(2*d*x+2*c))/a^3/(a^2+b^2)^2/d^2-1/2*b^3*f*sech(d*x+c)/a^2
/(a^2+b^2)/d^2-1/2*b^4*(f*x+e)*sech(d*x+c)^2/a^3/(a^2+b^2)/d-1/2*b*(f*x+e)*
csch(d*x+c)*sech(d*x+c)^2/a^2/d+1/2*b^4*f*tanh(d*x+c)/a^3/(a^2+b^2)/d^2-3/2
*I*b*f*polylog(2,-I*exp(d*x+c))/a^2/d^2+1/2*I*b^3*f*polylog(2,-I*exp(d*x+c)
)/a^2/(a^2+b^2)/d^2
```

Rubi [A]

time = 1.37, antiderivative size = 1122, normalized size of antiderivative = 1.00, number of steps used = 65, number of rules used = 28, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules used = {5708, 5569, 4270, 4267, 2317, 2438, 2701, 294, 327, 213, 5570, 5311, 12, 4265, 3855, 2702, 2700, 14, 2628, 3554, 8, 5692, 5680, 2221, 6874, 3799, 5559, 3852}

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b^2*f*x)/(2*a^3*d) + (3*b*f*x*ArcTan[E^(c + d*x)])/(a^2*d) - (2*b^5*(e + f
*x)*ArcTan[E^(c + d*x)])/(a^2*(a^2 + b^2)^2*d) - (b^3*(e + f*x)*ArcTan[E^(c
```

```
 + d*x)])/(a^2*(a^2 + b^2)*d) - (3*b*f*x*ArcTan[Sinh[c + d*x]])/(2*a^2*d) +
 (3*b*(e + f*x)*ArcTan[Sinh[c + d*x]])/(2*a^2*d) - (2*b^2*f*x*ArcTanh[E^(2*
c + 2*d*x)])/(a^3*d) + (4*(e + f*x)*ArcTanh[E^(2*c + 2*d*x)])/(a*d) + (b*f*
ArcTanh[Cosh[c + d*x]])/(a^2*d^2) + (3*b*(e + f*x)*Csch[c + d*x])/(2*a^2*d)
 - (f*Csch[2*c + 2*d*x])/(a*d^2) - (2*(e + f*x)*Coth[2*c + 2*d*x]*Csch[2*c
+ 2*d*x])/(a*d) - (b^6*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])])/(a^3*(a^2 + b^2)^2*d) - (b^6*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2])])/(a^3*(a^2 + b^2)^2*d) + (b^6*(e + f*x)*Log[1 + E^(2*(c +
d*x))])/(a^3*(a^2 + b^2)^2*d) - (b^2*f*x*Log[Tanh[c + d*x]])/(a^3*d) + (b^2
*(e + f*x)*Log[Tanh[c + d*x]])/(a^3*d) - (((3*I)/2)*b*f*PolyLog[2, (-I)*E^(
c + d*x)])/(a^2*d^2) + (I*b^5*f*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*(a^2 + b
^2)^2*d^2) + ((I/2)*b^3*f*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^
2) + (((3*I)/2)*b*f*PolyLog[2, I*E^(c + d*x)])/(a^2*d^2) - (I*b^5*f*PolyLog
[2, I*E^(c + d*x)])/(a^2*(a^2 + b^2)^2*d^2) - ((I/2)*b^3*f*PolyLog[2, I*E^(
c + d*x)])/(a^2*(a^2 + b^2)*d^2) - (b^6*f*PolyLog[2, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)^2*d^2) - (b^6*f*PolyLog[2, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)^2*d^2) + (b^6*f*PolyLog[2
, -E^(2*(c + d*x))])/(2*a^3*(a^2 + b^2)^2*d^2) + (f*PolyLog[2, -E^(2*c + 2*
d*x)])/(a*d^2) - (b^2*f*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a^3*d^2) - (f*Poly
Log[2, E^(2*c + 2*d*x)])/(a*d^2) + (b^2*f*PolyLog[2, E^(2*c + 2*d*x)])/(2*a
^3*d^2) + (b*f*Sech[c + d*x])/(2*a^2*d^2) - (b^3*f*Sech[c + d*x])/(2*a^2*(a
^2 + b^2)*d^2) - (b^4*(e + f*x)*Sech[c + d*x]^2)/(2*a^3*(a^2 + b^2)*d) - (b
*(e + f*x)*Csch[c + d*x]*Sech[c + d*x]^2)/(2*a^2*d) - (b^2*f*Tanh[c + d*x])
/(2*a^3*d^2) + (b^4*f*Tanh[c + d*x])/(2*a^3*(a^2 + b^2)*d^2) - (b^3*(e + f*
x)*Sech[c + d*x]*Tanh[c + d*x])/(2*a^2*(a^2 + b^2)*d) - (b^2*(e + f*x)*Tanh
[c + d*x]^2)/(2*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
```

```
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
```

```
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5311

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 5559

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Sech[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5570

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5692

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[(e + f
*x)^m*(Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x])), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5708

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[(e + f*x)^m*Sech[c + d*x]^p*(Csch[c + d*x]^(n - 1)/(a + b*Sinh[c + d*
x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\text{csch}^2(c+d}{a+b\sinh(}}{a}$$

$$= \frac{8\int (e+fx)\text{csch}^3(2c+2dx)\,dx}{a} - \frac{b\int (e+fx)\text{csch}^2(c+dx)\text{sec}}{a^2}$$

$$= \frac{3b(e+fx)\tan^{-1}(\sinh(c+dx))}{2a^2d} + \frac{3b(e+fx)\text{csch}(c+dx)}{2a^2d} -$$

$$= \frac{3b(e+fx)\tan^{-1}(\sinh(c+dx))}{2a^2d} + \frac{4(e+fx)\tanh^{-1}\left(e^{2c+2dx}\right)}{ad}$$

$$= -\frac{3bfx\tan^{-1}(\sinh(c+dx))}{2a^2d} + \frac{3b(e+fx)\tan^{-1}(\sinh(c+dx))}{2a^2d}$$

$$= \frac{b^6(e+fx)^2}{2a^3\left(a^2+b^2\right)^2 f} - \frac{3bfx\tan^{-1}(\sinh(c+dx))}{2a^2d} + \frac{3b(e+fx)\tan}{2}$$

$$= \frac{b^2fx}{2a^3d} + \frac{b^6(e+fx)^2}{2a^3\left(a^2+b^2\right)^2 f} + \frac{3bfx\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{3bfx\tan^{-1}(\text{s}}{2a}$$

$$= \frac{b^2fx}{2a^3d} + \frac{3bfx\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^5(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)^2 d} - \frac{b^3(}{}$$

$$= \frac{b^2fx}{2a^3d} + \frac{3bfx\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^5(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)^2 d} - \frac{b^3(}{}$$

$$= \frac{b^2fx}{2a^3d} + \frac{3bfx\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^5(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)^2 d} - \frac{b^3(}{}$$

$$= \frac{b^2fx}{2a^3d} + \frac{3bfx\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^5(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)^2 d} - \frac{b^3(}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2870 vs. 2(1122) = 2244.
time = 8.53, size = 2870, normalized size = 2.56

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] 8*(((I/16)*(2*a^6 + 3*a^4*b^2 + b^6)*(d*e - c*f)*(c + d*x))/(a^3*(a^2 + b^2
)^2*d^2) + ((I/32)*(2*a^6 + 3*a^4*b^2 + b^6)*f*(c + d*x)^2)/(a^3*(a^2 + b^2
)^2*d^2) + (a^3*e*ArcTanh[1 - (2*I)*Tanh[(c + d*x)/2]])/(2*(a^2 + b^2)^2*d)
 + (3*a*b^2*e*ArcTanh[1 - (2*I)*Tanh[(c + d*x)/2]])/(4*(a^2 + b^2)^2*d) - (
b^6*e*ArcTanh[1 - (2*I)*Tanh[(c + d*x)/2]])/(4*a^3*(a^2 + b^2)^2*d) - (a^3*
c*f*ArcTanh[1 - (2*I)*Tanh[(c + d*x)/2]])/(2*(a^2 + b^2)^2*d^2) - (3*a*b^2*
c*f*ArcTanh[1 - (2*I)*Tanh[(c + d*x)/2]])/(4*(a^2 + b^2)^2*d^2) + (b^6*c*f*
ArcTanh[1 - (2*I)*Tanh[(c + d*x)/2]])/(4*a^3*(a^2 + b^2)^2*d^2) - (e*Log[Co
sh[(c + d*x)/2]])/(4*a*d) + (b^2*e*Log[Cosh[(c + d*x)/2]])/(8*a^3*d) + (c*f
*Log[Cosh[(c + d*x)/2]])/(4*a*d^2) - (b^2*c*f*Log[Cosh[(c + d*x)/2]])/(8*a^
3*d^2) + (a^3*e*((-1/2*I)*(c + d*x) + Log[Cosh[(c + d*x)/2] + I*Sinh[(c + d
*x)/2]]))/(4*(a^2 + b^2)^2*d) + (3*a*b^2*e*((-1/2*I)*(c + d*x) + Log[Cosh[(
c + d*x)/2] + I*Sinh[(c + d*x)/2]]))/(8*(a^2 + b^2)^2*d) - (a^3*c*f*((-1/2*
I)*(c + d*x) + Log[Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]]))/(4*(a^2 + b^2
)^2*d^2) - (3*a*b^2*c*f*((-1/2*I)*(c + d*x) + Log[Cosh[(c + d*x)/2] + I*Sin
h[(c + d*x)/2]]))/(8*(a^2 + b^2)^2*d^2) + (b^6*e*((-I)*(c + d*x) + 2*ArcTan
h[1 - (2*I)*Tanh[(c + d*x)/2]] + Log[-1 + Cosh[c + d*x] + I*Sinh[c + d*x]])
)/(16*a^3*(a^2 + b^2)^2*d) - (b^6*c*f*((-I)*(c + d*x) + 2*ArcTanh[1 - (2*I)
*Tanh[(c + d*x)/2]] + Log[-1 + Cosh[c + d*x] + I*Sinh[c + d*x]]))/(16*a^3*(
a^2 + b^2)^2*d^2) - (b*f*Log[Tanh[(c + d*x)/2]])/(8*a^2*d^2) - ((I/2)*f*((-
1/8*I)*(c + d*x)^2 - (I/2)*(c + d*x)*Log[1 + E^(-c - d*x)] + (I/2)*PolyLog[
2, -E^(-c - d*x)]))/(a*d^2) + ((I/4)*b^2*f*((-1/8*I)*(c + d*x)^2 - (I/2)*(c
 + d*x)*Log[1 + E^(-c - d*x)] + (I/2)*PolyLog[2, -E^(-c - d*x)]))/(a^3*d^2)
 + (b^6*f*((-1/2*I)*(c + d*x)^2 + (I/4)*(3*Pi*(c + d*x) + (1 - I)*(c + d*x)
^2 + 2*(Pi - (2*I)*(c + d*x))*Log[1 + I*E^(-c - d*x)] - 4*Pi*Log[1 + E^(c +
 d*x)] - 2*Pi*Log[-Cos[(Pi + (2*I)*(c + d*x))/4]] + 4*Pi*Log[Cosh[(c + d*x)
/2]] + (4*I)*PolyLog[2, (-I)*E^(-c - d*x)])))/(8*a^3*(a^2 + b^2)^2*d^2) - (
(I/4)*a^3*f*((c + d*x)^2/4 + (-3*Pi*(c + d*x) - (1 - I)*(c + d*x)^2 - 2*(Pi
 - (2*I)*(c + d*x))*Log[1 + I*E^(-c - d*x)] + 4*Pi*Log[1 + E^(c + d*x)] + 2
*Pi*Log[-Cos[(Pi + (2*I)*(c + d*x))/4]] - 4*Pi*Log[Cosh[(c + d*x)/2]] - (4*
I)*PolyLog[2, (-I)*E^(-c - d*x)])/4 - (I/2)*(-1/2*(c + d*x)^2 + 2*(c + d*x)
*Log[1 - E^(c + d*x)] + 2*PolyLog[2, E^(c + d*x)])))/((a^2 + b^2)^2*d^2) -
(((3*I)/8)*a*b^2*f*((c + d*x)^2/4 + (-3*Pi*(c + d*x) - (1 - I)*(c + d*x)^2
- 2*(Pi - (2*I)*(c + d*x))*Log[1 + I*E^(-c - d*x)] + 4*Pi*Log[1 + E^(c + d*
x)] + 2*Pi*Log[-Cos[(Pi + (2*I)*(c + d*x))/4]] - 4*Pi*Log[Cosh[(c + d*x)/2]
] - (4*I)*PolyLog[2, (-I)*E^(-c - d*x)])/4 - (I/2)*(-1/2*(c + d*x)^2 + 2*(c
 + d*x)*Log[1 - E^(c + d*x)] + 2*PolyLog[2, E^(c + d*x)])))/((a^2 + b^2)^2*
d^2) + ((I/8)*b^6*f*((c + d*x)^2/4 + (-3*Pi*(c + d*x) - (1 - I)*(c + d*x)^2
 - 2*(Pi - (2*I)*(c + d*x))*Log[1 + I*E^(-c - d*x)] + 4*Pi*Log[1 + E^(c + d
*x)] + 2*Pi*Log[-Cos[(Pi + (2*I)*(c + d*x))/4]] - 4*Pi*Log[Cosh[(c + d*x)/2
```

```
]] - (4*I)*PolyLog[2, (-I)*E^(-c - d*x)])/4 - (I/2)*(-1/2*(c + d*x)^2 + 2*(
c + d*x)*Log[1 - E^(c + d*x)] + 2*PolyLog[2, E^(c + d*x)])))/(a^3*(a^2 + b^
2)^2*d^2) - (b^6*(-1/2*(f*(c + d*x)^2) + f*(c + d*x)*Log[1 + (b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2])] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] +
f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(8*a^3*(a^2 + b^2)^2*d^2) - ((I/2)*a^3
*f*((E^((I/4)*Pi)*(c + d*x)^2)/4 - ((Pi*(c + d*x))/4 - Pi*Log[1 + E^(c + d*
x)] - 2*(Pi/4 + (I/2)*(c + d*x))*Log[1 - E^((2*I)*(Pi/4 + (I/2)*(c + d*x)))
] + Pi*Log[Cosh[(c + d*x)/2]] + (Pi*Log[Sin[Pi/4 + (I/2)*(c + d*x)]])/2 + I
*PolyLog[2, E^((2*I)*(Pi/4 + (I/2)*(c + d*x)))])/Sqrt[2]))/(Sqrt[2]*(a^2 +
b^2)^2*d^2) - (((3*I)/4)*a*b^2*f*((E^((I/4)*Pi)*(c + d*x)^2)/4 - ((Pi*(c +
d*x))/4 - Pi*Log[1 + E^(c + d*x)] - 2*(Pi/4 + (I/2)*(c + d*x))*Log[1 - E^((
2*I)*(Pi/4 + (I/2)*(c + d*x)))] + Pi*Log[Cosh[(c + d*x)/2]] + (Pi*Log[Sin[P
i/4 + (I/2)*(c + d*x)]])/2 + I*PolyLog[2, E^((2*I)*(Pi/4 + (I/2)*(c + d*x))
)])/Sqrt[2]))/(Sqrt[2]*(a^2 + b^2)^2*d^2) + (b*(3*a^2 + 5*b^2)*(2*(d*e - c*
f + f*(c + d*x))*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]] - I*f*PolyLog[2, (-I
)*(Cosh[c + d*x] + Sinh[c + d*x])] + I*f*PolyLog[2, I*(Cosh[c + d*x] + Sinh
[c + d*x])]))/(16*(a^2 + b^2)^2*d^2) + (Csch[c + d*x]^2*Sech[c + d*x]^2*(-4
*a*b^2*d*e + 4*a*b^2*c*f - 4*a*b^2*f*(c + d*x) - 2*a^2*b*f*Cosh[c + d*x] -
8*a^3*d*e*Cosh[2*(c + d*x)] - 4*a*b^2*d*e*Cosh[2*(c + d*x)] + 8*a^3*c*f*Cos
h[2*(c + d*x)] + 4*a*b^2*c*f*Cosh[2*(c + d*x)] - 8*a^3*f*(c + d*x)*Cosh[2*(
c + d*x)] - 4*a*b^2*f*(c + d*x)*Cosh[2*(c + d*x)] + 2*a^2*b*f*Cosh[3*(c + d
*x)] - 2*a^2*b*d*e*Sinh[c + d*x] + 4*b^3*d*e*Sinh[c + d*x] + 2*a^2*b*c*f*Si
nh[c + d*x] - 4*b^3*c*f*Sinh[c + d*x] - 2*a^2*b*f*(c + d*x)*Sinh[c + d*x] +
 4*b^3*f*(c + d*x)*Sinh[c + d*x] - 4*a^3*f*Sinh[2*(c + d*x)] - 2*a*b^2*f*Si
nh[2*(c + d*x)] + 6*a^2*b*d*e*Sinh[3*(c + d*x)]...
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3562 vs. 2(1045) = 2090.
time = 11.30, size = 3563, normalized size = 3.18

method	result	size
risch	Expression too large to display	3563

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNV
ERBOSE)
```

```
[Out] -1/d/(a^2+b^2)*b^2*f/a*ln(exp(d*x+c)+1)*x+10*I/(a^2+b^2)/d^2*b^3*f/(4*a^2+4
*b^2)*ln(1-I*exp(d*x+c))*c-10*I/(a^2+b^2)/d^2*b^3*f/(4*a^2+4*b^2)*ln(1+I*ex
p(d*x+c))*c+10*I/(a^2+b^2)/d*b^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x-6*I/(
a^2+b^2)/d^2*a^2*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))*b-12/d^2/(a^2+b^2)*a
^2*f*c/(4*a^2+4*b^2)*b*arctan(exp(d*x+c))+12/d/(a^2+b^2)*b^2*f/(4*a^2+4*b^2
)*ln(1-I*exp(d*x+c))*a*x+12/d^2/(a^2+b^2)*b^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*
```

x+c))*a*c+12/d/(a^2+b^2)*b^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*a*x+12/d^2/(a^2+b^2)*b^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*a*c+1/2/d^2/(a^2+b^2)^(5/2)*a^2*b^2*f*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-12/d^2/(a^2+b^2)*b^2*f*c/(4*a^2+4*b^2)*a*ln(1+exp(2*d*x+2*c))+1/d^2/(a^2+b^2)*b^2*f*c/a*ln(exp(d*x+c)-1)+6*I/(a^2+b^2)/d*a^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*b*x-6*I/(a^2+b^2)/d^2*a^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*b*c+6*I/(a^2+b^2)/d^2*a^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*b*c-6*I/(a^2+b^2)/d*a^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*b*x-(-2*a^3*f*exp(2*d*x+2*c)+2*a^3*f*exp(6*d*x+6*c)+2*b^3*d*f*x*exp(d*x+c)+3*a^2*b*d*e*exp(d*x+c)+2*b^3*d*e*exp(d*x+c)-a^2*b*f*exp(d*x+c)+2*b^3*d*f*x*exp(3*d*x+3*c)+4*a^3*d*f*x*exp(2*d*x+2*c)-a^2*b*d*e*exp(3*d*x+3*c)+2*a*b^2*d*e*exp(2*d*x+2*c)-2*b^3*d*f*x*exp(7*d*x+7*c)+4*a^3*d*f*x*exp(6*d*x+6*c)-3*a^2*b*d*e*exp(7*d*x+7*c)+2*a*b^2*d*e*exp(6*d*x+6*c)-2*b^3*d*f*x*exp(5*d*x+5*c)+a^2*b*d*e*exp(5*d*x+5*c)+4*a*b^2*d*e*exp(4*d*x+4*c)+3*a^2*b*d*f*x*exp(d*x+c)-a*b^2*f+2*a*b^2*d*f*x*exp(2*d*x+2*c)-a^2*b*d*f*x*exp(3*d*x+3*c)-3*a^2*b*d*f*x*exp(7*d*x+7*c)+2*a*b^2*d*f*x*exp(6*d*x+6*c)+a^2*b*d*f*x*exp(5*d*x+5*c)+4*a*b^2*d*f*x*exp(4*d*x+4*c)+2*b^3*d*e*exp(3*d*x+3*c)+4*a^3*d*e*exp(2*d*x+2*c)+a^2*b*f*exp(3*d*x+3*c)-a*b^2*f*exp(2*d*x+2*c)-2*b^3*d*e*exp(7*d*x+7*c)+4*a^3*d*e*exp(6*d*x+6*c)-a^2*b*f*exp(7*d*x+7*c)+a*b^2*f*exp(6*d*x+6*c)-2*b^3*d*e*exp(5*d*x+5*c)+a^2*b*f*exp(5*d*x+5*c)+a*b^2*f*exp(4*d*x+4*c))/d^2/(a^2+b^2)/(1+exp(2*d*x+2*c))^2/a^2/(exp(2*d*x+2*c)-1)^2-20/d^2/(a^2+b^2)*b^3*f*c/(4*a^2+4*b^2)*arctan(exp(d*x+c))-1/2/d^2/(a^2+b^2)^(3/2)*b^2*f*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+12/d/(a^2+b^2)*a^2*e/(4*a^2+4*b^2)*b*arctan(exp(d*x+c))-1/2/d/(a^2+b^2)^(5/2)*e*a^2*b^2*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+12/d/(a^2+b^2)*b^2*e/(4*a^2+4*b^2)*a*ln(1+exp(2*d*x+2*c))-1/d^2/a^2*f*b^4/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d^2*b^2*f/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)/d/a^3*b^4*e*ln(exp(d*x+c)-1)+1/(a^2+b^2)/d/a^3*b^4*e*ln(exp(d*x+c)+1)-1/(a^2+b^2)/d^2/a^3*b^4*f*dilog(exp(d*x+c))+1/(a^2+b^2)/d^2/a^3*b^4*f*dilog(exp(d*x+c)+1)-1/(a^2+b^2)/d^2/a^2*b^3*f*ln(exp(d*x+c)-1)+1/(a^2+b^2)/d^2/a^2*b^3*f*ln(exp(d*x+c)+1)+8/d^2/(a^2+b^2)*a^3*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))+1/2/d/(a^2+b^2)^(3/2)*b^2*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+8/d/(a^2+b^2)*a^3*e/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))-1/2/d/(a^2+b^2)^(5/2)*b^4*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+20/d/(a^2+b^2)*b^3*e/(4*a^2+4*b^2)*arctan(exp(d*x+c))+8/d^2/(a^2+b^2)*a^3*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))-1/d^2/(a^2+b^2)*b^2*f/a*dilog(exp(d*x+c)+1)+1/d^2/(a^2+b^2)*b^2*f*dilog(exp(d*x+c))/a+6*I/(a^2+b^2)/d^2*a^2*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))*b-10*I/(a^2+b^2)/d*b^3*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x+8/d/(a^2+b^2)*a^3*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x+8/d^2/(a^2+b^2)*a^3*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*c+8/d/(a^2+b^2)*a^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x+8/d^2/(a^2+b^2)*a^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*c+12/d^2/(a^2+b^2)*b^2*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))*a+12/d^2/(a^2+b^2)*b^2*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))*a-8/d^2/(a^2+b^2)*a^3*f*c/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))+1/2/d^2/(a^2+b^2)^(5/2)*b^4*f*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)^(5/2)/d^2/a^2*b^6*f*ar

```
ctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)^(5/2)/d^2*a^2*b
^2*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)^2/d/a^3*
b^6*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+10*I/(a^
2+b^2)/d^2*b^3*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))-10*I/(a^2+b^2)/d^2*b^3
*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))+2/d^2/(a^2+b^2)*a*f*c*ln(exp(d*x+c)-
1)-1/d/(a^2+b^2)*b^2*e/a*ln(exp(d*x+c)+1)-1/d/(a^2+b^2)*b^2*e/a*ln(exp(d*x+
c)-1)-2/d/(a^2+b^2)*ln(exp(d*x+c)+1)*a*f*x-1/(a^2+b^2)^2/d/a^3*b^6*f*ln((b*
exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/(a^2+b^2)^2/d^2/a^3*
b^6*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/(a^2+b^2)^2/d^2/a^3*b^6*f*l
n((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/(a^2+b^2)^2/d
^2/a^3*b^6*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/(
a^2+b^2)^2/d^2/a^3*b^6*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b
^2)^(1/2)))-1/(a^2+b^2)^2/d^2/a^3*b^6*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)
+a)/(a+(a^2+b^2)^(1/2)))-1/(a^2+b^2)^2/d/a^3*b^6*e*ln(b*exp(2*d*x+2*c)+2*a*
exp(d*x+c)-b)+2/(a^2+b^2)^(5/2)/d^2*b^4*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/
(a^2+b^2)^(1/2))-1/(a^2+b^2)/d^2/a^3*b^4*f*c*ln...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] (128*a^2*d*integrate(1/64*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 64*b^2*d*inte
grate(1/64*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 128*a^2*d*integrate(1/64*x/(
a^3*d*e^(d*x + c) - a^3*d), x) + 64*b^2*d*integrate(1/64*x/(a^3*d*e^(d*x +
c) - a^3*d), x) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) + 1)/(a^3*d^2)
) + a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) - 1)/(a^3*d^2)) + (a*b^2 + (
a^2*b*e^(7*c) + (3*a^2*b*d*e^(7*c) + 2*b^3*d*e^(7*c))*x)*e^(7*d*x) - (2*a^3
*e^(6*c) + a*b^2*e^(6*c) + 2*(2*a^3*d*e^(6*c) + a*b^2*d*e^(6*c))*x)*e^(6*d*
x) - (a^2*b*e^(5*c) + (a^2*b*d*e^(5*c) - 2*b^3*d*e^(5*c))*x)*e^(5*d*x) - (4
*a*b^2*d*x*e^(4*c) + a*b^2*e^(4*c))*e^(4*d*x) - (a^2*b*e^(3*c) - (a^2*b*d*e
^(3*c) - 2*b^3*d*e^(3*c))*x)*e^(3*d*x) + (2*a^3*e^(2*c) + a*b^2*e^(2*c) - 2
*(2*a^3*d*e^(2*c) + a*b^2*d*e^(2*c))*x)*e^(2*d*x) + (a^2*b*e^c - (3*a^2*b*d
*e^c + 2*b^3*d*e^c)*x)*e^(d*x))/(a^4*d^2 + a^2*b^2*d^2 + (a^4*d^2*e^(8*c) +
 a^2*b^2*d^2*e^(8*c))*e^(8*d*x) - 2*(a^4*d^2*e^(4*c) + a^2*b^2*d^2*e^(4*c))
*e^(4*d*x)) + 64*integrate(-1/32*(a*b^6*x*e^(d*x + c) - b^7*x)/(a^7*b + 2*a
^5*b^3 + a^3*b^5 - (a^7*b*e^(2*c) + 2*a^5*b^3*e^(2*c) + a^3*b^5*e^(2*c))*e^
(2*d*x) - 2*(a^8*e^c + 2*a^6*b^2*e^c + a^4*b^4*e^c)*e^(d*x)), x) + 64*integ
rate(1/64*((3*a^2*b*e^c + 5*b^3*e^c)*x*e^(d*x) - 2*(2*a^3 + 3*a*b^2)*x)/(a^
4 + 2*a^2*b^2 + b^4 + (a^4*e^(2*c) + 2*a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*
d*x)), x))*f - (b^6*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^7 +
```

```
 2*a^5*b^2 + a^3*b^4)*d) + (3*a^2*b + 5*b^3)*arctan(e^(-d*x - c))/((a^4 + 2
*a^2*b^2 + b^4)*d) - (2*a^3 + 3*a*b^2)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*
a^2*b^2 + b^4)*d) + (4*a*b^2*e^(-4*d*x - 4*c) - (3*a^2*b + 2*b^3)*e^(-d*x -
 c) + 2*(2*a^3 + a*b^2)*e^(-2*d*x - 2*c) + (a^2*b - 2*b^3)*e^(-3*d*x - 3*c)
 - (a^2*b - 2*b^3)*e^(-5*d*x - 5*c) + 2*(2*a^3 + a*b^2)*e^(-6*d*x - 6*c) +
(3*a^2*b + 2*b^3)*e^(-7*d*x - 7*c))/((a^4 + a^2*b^2 - 2*(a^4 + a^2*b^2)*e^(
-4*d*x - 4*c) + (a^4 + a^2*b^2)*e^(-8*d*x - 8*c))*d) + (2*a^2 - b^2)*log(e^
(-d*x - c) + 1)/(a^3*d) + (2*a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d))*e
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 19958 vs. 2(1034) = 2068.
time = 0.64, size = 19958, normalized size = 17.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/2*(2*((3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*d*f*x + (3*a^5*b + 5*a^3*b^3 + 2*a*
b^5)*d*cosh(1) + (3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*d*sinh(1) + (a^5*b + a^3*b
^3)*f)*cosh(d*x + c)^7 + 2*((3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*d*f*x + (3*a^5*
b + 5*a^3*b^3 + 2*a*b^5)*d*cosh(1) + (3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*d*sinh
(1) + (a^5*b + a^3*b^3)*f)*sinh(d*x + c)^7 - 2*(2*(2*a^6 + 3*a^4*b^2 + a^2*
b^4)*d*f*x + 2*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*d*cosh(1) + 2*(2*a^6 + 3*a^4*b
^2 + a^2*b^4)*d*sinh(1) + (2*a^6 + 3*a^4*b^2 + a^2*b^4)*f)*cosh(d*x + c)^6
- 2*(2*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*d*f*x + 2*(2*a^6 + 3*a^4*b^2 + a^2*b^4
)*d*cosh(1) + 2*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*d*sinh(1) + (2*a^6 + 3*a^4*b^
2 + a^2*b^4)*f - 7*((3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*d*f*x + (3*a^5*b + 5*a^
3*b^3 + 2*a*b^5)*d*cosh(1) + (3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*d*sinh(1) + (a
^5*b + a^3*b^3)*f)*cosh(d*x + c))*sinh(d*x + c)^6 - 2*((a^5*b - a^3*b^3 - 2
*a*b^5)*d*f*x + (a^5*b - a^3*b^3 - 2*a*b^5)*d*cosh(1) + (a^5*b - a^3*b^3 -
2*a*b^5)*d*sinh(1) + (a^5*b + a^3*b^3)*f)*cosh(d*x + c)^5 - 2*((a^5*b - a^3
*b^3 - 2*a*b^5)*d*f*x + (a^5*b - a^3*b^3 - 2*a*b^5)*d*cosh(1) - 21*((3*a^5*
b + 5*a^3*b^3 + 2*a*b^5)*d*f*x + (3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*d*cosh(1)
+ (3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*d*sinh(1) + (a^5*b + a^3*b^3)*f)*cosh(d*x
 + c)^2 + (a^5*b - a^3*b^3 - 2*a*b^5)*d*sinh(1) + (a^5*b + a^3*b^3)*f + 6*(
2*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*d*f*x + 2*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*d*c
osh(1) + 2*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*d*sinh(1) + (2*a^6 + 3*a^4*b^2 + a
^2*b^4)*f)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(4*(a^4*b^2 + a^2*b^4)*d*f*x
+ 4*(a^4*b^2 + a^2*b^4)*d*cosh(1) + 4*(a^4*b^2 + a^2*b^4)*d*sinh(1) + (a^4*
b^2 + a^2*b^4)*f)*cosh(d*x + c)^4 - 2*(4*(a^4*b^2 + a^2*b^4)*d*f*x - 35*((3
*a^5*b + 5*a^3*b^3 + 2*a*b^5)*d*f*x + (3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*d*cos
h(1) + (3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*d*sinh(1) + (a^5*b + a^3*b^3)*f)*cos
h(d*x + c)^3 + 4*(a^4*b^2 + a^2*b^4)*d*cosh(1) + 15*(2*(2*a^6 + 3*a^4*b^2 +
```

```
 a^2*b^4)*d*f*x + 2*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*d*cosh(1) + 2*(2*a^6 + 3*
a^4*b^2 + a^2*b^4)*d*sinh(1) + (2*a^6 + 3*a^4*b^2 + a^2*b^4)*f)*cosh(d*x +
c)^2 + 4*(a^4*b^2 + a^2*b^4)*d*sinh(1) + (a^4*b^2 + a^2*b^4)*f + 5*((a^5*b
- a^3*b^3 - 2*a*b^5)*d*f*x + (a^5*b - a^3*b^3 - 2*a*b^5)*d*cosh(1) + (a^5*b
 - a^3*b^3 - 2*a*b^5)*d*sinh(1) + (a^5*b + a^3*b^3)*f)*cosh(d*x + c))*sinh(
d*x + c)^4 + 2*((a^5*b - a^3*b^3 - 2*a*b^5)*d*f*x + (a^5*b - a^3*b^3 - 2*a*
b^5)*d*cosh(1) + (a^5*b - a^3*b^3 - 2*a*b^5)*d*sinh(1) - (a^5*b + a^3*b^3)*
f)*cosh(d*x + c)^3 + 2*(35*((3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*d*f*x + (3*a^5*
b + 5*a^3*b^3 + 2*a*b^5)*d*cosh(1) + (3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*d*sinh
(1) + (a^5*b + a^3*b^3)*f)*cosh(d*x + c)^4 + (a^5*b - a^3*b^3 - 2*a*b^5)*d*
f*x - 20*(2*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*d*f*x + 2*(2*a^6 + 3*a^4*b^2 + a^
2*b^4)*d*cosh(1) + 2*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*d*sinh(1) + (2*a^6 + 3*a
^4*b^2 + a^2*b^4)*f)*cosh(d*x + c)^3 + (a^5*b - a^3*b^3 - 2*a*b^5)*d*cosh(1
) - 10*((a^5*b - a^3*b^3 - 2*a*b^5)*d*f*x + (a^5*b - a^3*b^3 - 2*a*b^5)*d*c
osh(1) + (a^5*b - a^3*b^3 - 2*a*b^5)*d*sinh(1) + (a^5*b + a^3*b^3)*f)*cosh(
d*x + c)^2 + (a^5*b - a^3*b^3 - 2*a*b^5)*d*sinh(1) - (a^5*b + a^3*b^3)*f -
4*(4*(a^4*b^2 + a^2*b^4)*d*f*x + 4*(a^4*b^2 + a^2*b^4)*d*cosh(1) + 4*(a^4*b
^2 + a^2*b^4)*d*sinh(1) + (a^4*b^2 + a^2*b^4)*f)*cosh(d*x + c))*sinh(d*x +
c)^3 - 2*(2*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*d*f*x + 2*(2*a^6 + 3*a^4*b^2 + a^
2*b^4)*d*cosh(1) + 2*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*d*sinh(1) - (2*a^6 + 3*a
^4*b^2 + a^2*b^4)*f)*cosh(d*x + c)^2 + 2*(21*((3*a^5*b + 5*a^3*b^3 + 2*a*b^
5)*d*f*x + (3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*d*cosh(1) + (3*a^5*b + 5*a^3*b^3
 + 2*a*b^5)*d*sinh(1) + (a^5*b + a^3*b^3)*f)*cosh(d*x + c)^5 - 15*(2*(2*a^6
 + 3*a^4*b^2 + a^2*b^4)*d*f*x + 2*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*d*cosh(1) +
 2*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*d*sinh(1) + (2*a^6 + 3*a^4*b^2 + a^2*b^4)*
f)*cosh(d*x + c)^4 - 2*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*d*f*x - 10*((a^5*b - a
^3*b^3 - 2*a*b^5)*d*f*x + (a^5*b - a^3*b^3 - 2*a*b^5)*d*cosh(1) + (a^5*b -
a^3*b^3 - 2*a*b^5)*d*sinh(1) + (a^5*b + a^3*b^3)*f)*cosh(d*x + c)^3 - 2*(2*
a^6 + 3*a^4*b^2 + a^2*b^4)*d*cosh(1) - 6*(4*(a^4*b^2 + a^2*b^4)*d*f*x + 4*(
a^4*b^2 + a^2*b^4)*d*cosh(1) + 4*(a^4*b^2 + a^2*b^4)*d*sinh(1) + (a^4*b^2 +
 a^2*b^4)*f)*cosh(d*x + c)^2 - 2*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*d*sinh(1) +
(2*a^6 + 3*a^4*b^2 + a^2*b^4)*f + 3*((a^5*b - a^3*b^3 - 2*a*b^5)*d*f*x + (a
^5*b - a^3*b^3 - 2*a*b^5)*d*cosh(1) + (a^5*b - a^3*b^3 - 2*a*b^5)*d*sinh(1)
 - (a^5*b + a^3*b^3)*f)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(a^4*b^2 + a^2*b
^4)*f - 2*((3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*d*f*x + (3*a^5*b + 5*a^3*b^3 + 2
*a*b^5)*d*cosh(1) + (3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*d*sinh(1) - (a^5*b + a^
3*b^3)*f)*cosh(d*x + c) - 2*(b^6*f*cosh(d*x + c)^8 + 56*b^6*f*cosh(d*x + c)
^3*sinh(d*x + c)^5 + 28*b^6*f*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*b^6*f*cos
h(d*x + c)*sinh(d*x + c)^7 + b^6*f*sinh(d*x + c)^8 - 2*b^6*f*cosh(d*x + c)^
4 + b^6*f + 2*(35*b^6*f*cosh(d*x + c)^4 - b^6*f)*sinh(d*x + c)^4 + 8*(7*b^6
*f*cosh(d*x + c)^5 - b^6*f*cosh(d*x + c))*sinh(...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**3*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\cosh\left(c+d\,x\right)^3\,\sinh\left(c+d\,x\right)^3\,\left(a+b\,\sinh\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(cosh(c + d*x)^3*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

```
[Out] int((e + f*x)/(cosh(c + d*x)^3*sinh(c + d*x)^3*(a + b*sinh(c + d*x))), x)
```

3.501 $\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=211

$$\frac{b\text{ArcTan}(\sinh(c+dx))}{2(a^2+b^2)d}+\frac{b(a^2+2b^2)\text{ArcTan}(\sinh(c+dx))}{(a^2+b^2)^2d}+\frac{b\operatorname{csch}(c+dx)}{a^2d}-\frac{\operatorname{csch}^2(c+dx)}{2ad}+\frac{a(2a^2+3b^2)\,\mathrm{lo}}{(a^2-}$$

```
[Out] 1/2*b*arctan(sinh(d*x+c))/(a^2+b^2)/d+b*(a^2+2*b^2)*arctan(sinh(d*x+c))/(a^
2+b^2)^2/d+b*csch(d*x+c)/a^2/d-1/2*csch(d*x+c)^2/a/d+a*(2*a^2+3*b^2)*ln(cos
h(d*x+c))/(a^2+b^2)^2/d-(2*a^2-b^2)*ln(sinh(d*x+c))/a^3/d-b^6*ln(a+b*sinh(d
*x+c))/a^3/(a^2+b^2)^2/d-1/2*sech(d*x+c)^2*(a-b*sinh(d*x+c))/(a^2+b^2)/d
```

Rubi [A]
time = 0.26, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2916, 12, 908, 653, 209, 649, 266}

$$\frac{b(a^2+2b^2)\text{ArcTan}(\sinh(c+dx))}{d(a^2+b^2)^2}+\frac{b\text{ArcTan}(\sinh(c+dx))}{2d(a^2+b^2)}+\frac{a(2a^2+3b^2)\log(\cosh(c+dx))}{d(a^2+b^2)^2}-\frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))}{2d(a^2+b^2)}+\frac{b\operatorname{csch}(c+dx)}{a^2d}-\frac{(2a^2-b^2)\log(\sinh(c+dx))}{a^3d}-\frac{b^6\log(a+b\sinh(c+dx))}{a^3d(a^2+b^2)^2}-\frac{\operatorname{csch}^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[c + d*x]^3*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/(2*(a^2 + b^2)*d) + (b*(a^2 + 2*b^2)*ArcTan[Sinh[
c + d*x]])/((a^2 + b^2)^2*d) + (b*Csch[c + d*x])/(a^2*d) - Csch[c + d*x]^2/
(2*a*d) + (a*(2*a^2 + 3*b^2)*Log[Cosh[c + d*x]])/((a^2 + b^2)^2*d) - ((2*a^
2 - b^2)*Log[Sinh[c + d*x]])/(a^3*d) - (b^6*Log[a + b*Sinh[c + d*x]])/(a^3*
(a^2 + b^2)^2*d) - (Sech[c + d*x]^2*(a - b*Sinh[c + d*x]))/(2*(a^2 + b^2)*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b^3\text{Subst}\left(\int \frac{b^3}{x^3(a+x)(-b^2-x^2)^2}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= \frac{b^6\text{Subst}\left(\int \frac{1}{x^3(a+x)(-b^2-x^2)^2}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= \frac{b^6\text{Subst}\left(\int \left(\frac{1}{ab^4x^3} - \frac{1}{a^2b^4x^2} + \frac{-2a^2+b^2}{a^3b^6x} - \frac{1}{a^3(a^2+b^2)^2(a+x)} + \frac{b^2+ax}{b^4(a^2+b^2)(b^2+x^2)^2}\right.\right.}{d}$$

$$= \frac{b\operatorname{csch}(c+dx)}{a^2d} - \frac{\operatorname{csch}^2(c+dx)}{2ad} - \frac{(2a^2-b^2)\log(\sinh(c+dx))}{a^3d} - \frac{b^6\log}{}$$

$$= \frac{b\operatorname{csch}(c+dx)}{a^2d} - \frac{\operatorname{csch}^2(c+dx)}{2ad} - \frac{(2a^2-b^2)\log(\sinh(c+dx))}{a^3d} - \frac{b^6\log}{}$$

$$= \frac{b\tan^{-1}(\sinh(c+dx))}{2(a^2+b^2)d} + \frac{b(a^2+2b^2)\tan^{-1}(\sinh(c+dx))}{(a^2+b^2)^2d} + \frac{b\operatorname{csch}(c+d}{a^2d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.62, size = 237, normalized size = 1.12

$$\frac{\frac{b\text{ArcTan}(\sinh(c+dx))}{a^2+b^2} + \frac{2b\operatorname{csch}(c+dx)}{a^2} - \frac{\operatorname{csch}^2(c+dx)}{a} + \frac{(a-ib)(2a^2+iab+2b^2)\log(i-\sinh(c+dx))}{(a^2+b^2)^2} - \frac{2(2a^2-b^2)\log(\sinh(c+dx))}{a^3} + \frac{(a+ib)(2a^2-iab+2b^2)\log(i+\sinh(c+dx))}{(a^2+b^2)^2} - \frac{2b^6\log(a+b\sinh(c+dx))}{a^3(a^2+b^2)^2} - \frac{a\operatorname{sech}^2(c+dx)}{a^2+b^2} + \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{a^2+b^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[c + d*x]^3*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((b*ArcTan[Sinh[c + d*x]])/(a^2 + b^2) + (2*b*Csch[c + d*x])/a^2 - Csch[c +
 d*x]^2/a + ((a - I*b)*(2*a^2 + I*a*b + 2*b^2)*Log[I - Sinh[c + d*x]])/(a^2
 + b^2)^2 - (2*(2*a^2 - b^2)*Log[Sinh[c + d*x]])/a^3 + ((a + I*b)*(2*a^2 -
I*a*b + 2*b^2)*Log[I + Sinh[c + d*x]])/(a^2 + b^2)^2 - (2*b^6*Log[a + b*Sin
h[c + d*x]])/(a^3*(a^2 + b^2)^2) - (a*Sech[c + d*x]^2)/(a^2 + b^2) + (b*Sec
h[c + d*x]*Tanh[c + d*x]])/(a^2 + b^2))/(2*d)
```

Maple [A]
time = 2.02, size = 292, normalized size = 1.38

method	result
derivativedivides	$\frac{-\frac{\frac{a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2} - \frac{b^6\ln\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^2+b^2\right)^2a^3} + \frac{2\left(\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\right.\right.}{\left(\right.}}{}$

default	$-\frac{\frac{a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{b^6\ln\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{\left(a^2+b^2\right)^2a^3}+\frac{2\left(\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^3\right.\right.}{\left(\mathrm{ta}\right.}$
risch	$-\frac{4a^3d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{4a^3dc}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{6a\,b^2d^2x}{a^4d^2+2a^2b^2d^2+b^4d^2}-\frac{6a\,b^2dc}{a^4d^2+2a^2b^2d^2+b^4d^2}+\frac{4x}{a}+\frac{4c}{ad}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4/a^2*(1/2*a*tanh(1/2*d*x+1/2*c)^2+2*b*tanh(1/2*d*x+1/2*c))-b^6/(a^
2+b^2)^2/a^3*ln(a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)-a)+2/(a^2+b
^2)^2*(((-1/2*a^2*b-1/2*b^3)*tanh(1/2*d*x+1/2*c)^3+(a^3+a*b^2)*tanh(1/2*d*x
+1/2*c)^2+(1/2*a^2*b+1/2*b^3)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2+1
)^2+1/4*(4*a^3+6*a*b^2)*ln(tanh(1/2*d*x+1/2*c)^2+1)+1/2*(3*a^2*b+5*b^3)*arc
tan(tanh(1/2*d*x+1/2*c)))-1/8/a/tanh(1/2*d*x+1/2*c)^2+1/4/a^3*(-8*a^2+4*b^2
)*ln(tanh(1/2*d*x+1/2*c))+1/2*b/a^2/tanh(1/2*d*x+1/2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(206) = 412$.
time = 0.50, size = 418, normalized size = 1.98

$$-\frac{b^6\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^7+2\,a^5b^2+a^3b^4)d}-\frac{(3\,a^2b+5\,b^3)\arctan\left(e^{(-dx-c)}\right)}{(a^4+2\,a^2b^2+b^4)d}+\frac{(2\,a^3+3\,ab^2)\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^4+2\,a^2b^2+b^4)d}-\frac{4\,ab^2e^{(-4\,dx-4\,c)}-(3\,a^2b+2\,b^3)e^{(-dx-c)}+2\,(2\,a^3+ab^2)e^{(-2\,dx-2\,c)}+(a^2b-2\,b^3)e^{(-3\,dx-3\,c)}-(a^2b-2\,b^3)e^{(-5\,dx-5\,c)}+2\,(2\,a^3+ab^2)e^{(-6\,dx-6\,c)}+(3\,a^2b+2\,b^3)e^{(-7\,dx-7\,c)}}{(a^4+a^2b^2-2\,(a^4+a^2b^2)e^{(-4\,dx-4\,c)}+(a^4+a^2b^2)e^{(-8\,dx-8\,c)})d}-\frac{(2\,a^2-b^2)\log\left(e^{(-dx-c)}+1\right)}{a^3d}-\frac{(2\,a^2-b^2)\log\left(e^{(-dx-c)}-1\right)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxim
a")
```

```
[Out] -b^6*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^7 + 2*a^5*b^2 + a^
3*b^4)*d) - (3*a^2*b + 5*b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)
*d) + (2*a^3 + 3*a*b^2)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*
d) - (4*a*b^2*e^(-4*d*x - 4*c) - (3*a^2*b + 2*b^3)*e^(-d*x - c) + 2*(2*a^3
+ a*b^2)*e^(-2*d*x - 2*c) + (a^2*b - 2*b^3)*e^(-3*d*x - 3*c) - (a^2*b - 2*b
^3)*e^(-5*d*x - 5*c) + 2*(2*a^3 + a*b^2)*e^(-6*d*x - 6*c) + (3*a^2*b + 2*b^
3)*e^(-7*d*x - 7*c))/((a^4 + a^2*b^2 - 2*(a^4 + a^2*b^2)*e^(-4*d*x - 4*c) +
 (a^4 + a^2*b^2)*e^(-8*d*x - 8*c))*d) - (2*a^2 - b^2)*log(e^(-d*x - c) + 1)
/(a^3*d) - (2*a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3148 vs. $2(206) = 412$.
time = 0.72, size = 3148, normalized size = 14.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="frica
s")

[Out] ((3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*cosh(d*x + c)^7 + (3*a^5*b + 5*a^3*b^3 + 2
*a*b^5)*sinh(d*x + c)^7 - 2*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*cosh(d*x + c)^6 -
 (4*a^6 + 6*a^4*b^2 + 2*a^2*b^4 - 7*(3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*cosh(d*
x + c))*sinh(d*x + c)^6 - (a^5*b - a^3*b^3 - 2*a*b^5)*cosh(d*x + c)^5 - (a^
5*b - a^3*b^3 - 2*a*b^5 - 21*(3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*cosh(d*x + c)^
2 + 12*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 - 4*(a^
4*b^2 + a^2*b^4)*cosh(d*x + c)^4 - (4*a^4*b^2 + 4*a^2*b^4 - 35*(3*a^5*b + 5
*a^3*b^3 + 2*a*b^5)*cosh(d*x + c)^3 + 30*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*cosh
(d*x + c)^2 + 5*(a^5*b - a^3*b^3 - 2*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^4
+ (a^5*b - a^3*b^3 - 2*a*b^5)*cosh(d*x + c)^3 + (a^5*b - a^3*b^3 - 2*a*b^5
+ 35*(3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*cosh(d*x + c)^4 - 40*(2*a^6 + 3*a^4*b^
2 + a^2*b^4)*cosh(d*x + c)^3 - 10*(a^5*b - a^3*b^3 - 2*a*b^5)*cosh(d*x + c)
^2 - 16*(a^4*b^2 + a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(2*a^6 + 3*a
^4*b^2 + a^2*b^4)*cosh(d*x + c)^2 - (4*a^6 + 6*a^4*b^2 + 2*a^2*b^4 - 21*(3*
a^5*b + 5*a^3*b^3 + 2*a*b^5)*cosh(d*x + c)^5 + 30*(2*a^6 + 3*a^4*b^2 + a^2*
b^4)*cosh(d*x + c)^4 + 10*(a^5*b - a^3*b^3 - 2*a*b^5)*cosh(d*x + c)^3 + 24*
(a^4*b^2 + a^2*b^4)*cosh(d*x + c)^2 - 3*(a^5*b - a^3*b^3 - 2*a*b^5)*cosh(d*
x + c))*sinh(d*x + c)^2 + ((3*a^5*b + 5*a^3*b^3)*cosh(d*x + c)^8 + 56*(3*a^
5*b + 5*a^3*b^3)*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*(3*a^5*b + 5*a^3*b^3)
*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*(3*a^5*b + 5*a^3*b^3)*cosh(d*x + c)*si
nh(d*x + c)^7 + (3*a^5*b + 5*a^3*b^3)*sinh(d*x + c)^8 + 3*a^5*b + 5*a^3*b^3
 - 2*(3*a^5*b + 5*a^3*b^3)*cosh(d*x + c)^4 - 2*(3*a^5*b + 5*a^3*b^3 - 35*(3
*a^5*b + 5*a^3*b^3)*cosh(d*x + c)^4)*sinh(d*x + c)^4 + 8*(7*(3*a^5*b + 5*a^
3*b^3)*cosh(d*x + c)^5 - (3*a^5*b + 5*a^3*b^3)*cosh(d*x + c))*sinh(d*x + c)
^3 + 4*(7*(3*a^5*b + 5*a^3*b^3)*cosh(d*x + c)^6 - 3*(3*a^5*b + 5*a^3*b^3)*c
osh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^5*b + 5*a^3*b^3)*cosh(d*x + c)^7
- (3*a^5*b + 5*a^3*b^3)*cosh(d*x + c)^3)*sinh(d*x + c))*arctan(cosh(d*x + c
) + sinh(d*x + c)) - (3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*cosh(d*x + c) - (b^6*c
osh(d*x + c)^8 + 56*b^6*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*b^6*cosh(d*x +
 c)^2*sinh(d*x + c)^6 + 8*b^6*cosh(d*x + c)*sinh(d*x + c)^7 + b^6*sinh(d*x
+ c)^8 - 2*b^6*cosh(d*x + c)^4 + b^6 + 2*(35*b^6*cosh(d*x + c)^4 - b^6)*sin
h(d*x + c)^4 + 8*(7*b^6*cosh(d*x + c)^5 - b^6*cosh(d*x + c))*sinh(d*x + c)^
3 + 4*(7*b^6*cosh(d*x + c)^6 - 3*b^6*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(
b^6*cosh(d*x + c)^7 - b^6*cosh(d*x + c)^3)*sinh(d*x + c))*log(2*(b*sinh(d*x
 + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + ((2*a^6 + 3*a^4*b^2)*cosh(d*x
 + c)^8 + 56*(2*a^6 + 3*a^4*b^2)*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*(2*a^
6 + 3*a^4*b^2)*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*(2*a^6 + 3*a^4*b^2)*cosh
(d*x + c)*sinh(d*x + c)^7 + (2*a^6 + 3*a^4*b^2)*sinh(d*x + c)^8 + 2*a^6 + 3
*a^4*b^2 - 2*(2*a^6 + 3*a^4*b^2)*cosh(d*x + c)^4 - 2*(2*a^6 + 3*a^4*b^2 - 3
5*(2*a^6 + 3*a^4*b^2)*cosh(d*x + c)^4)*sinh(d*x + c)^4 + 8*(7*(2*a^6 + 3*a^
4*b^2)*cosh(d*x + c)^5 - (2*a^6 + 3*a^4*b^2)*cosh(d*x + c))*sinh(d*x + c)^3
 + 4*(7*(2*a^6 + 3*a^4*b^2)*cosh(d*x + c)^6 - 3*(2*a^6 + 3*a^4*b^2)*cosh(d*
```

```
x + c)^2)*sinh(d*x + c)^2 + 8*((2*a^6 + 3*a^4*b^2)*cosh(d*x + c)^7 - (2*a^6
 + 3*a^4*b^2)*cosh(d*x + c)^3)*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x
 + c) - sinh(d*x + c))) - ((2*a^6 + 3*a^4*b^2 - b^6)*cosh(d*x + c)^8 + 56*(
2*a^6 + 3*a^4*b^2 - b^6)*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*(2*a^6 + 3*a^
4*b^2 - b^6)*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*(2*a^6 + 3*a^4*b^2 - b^6)*
cosh(d*x + c)*sinh(d*x + c)^7 + (2*a^6 + 3*a^4*b^2 - b^6)*sinh(d*x + c)^8 +
 2*a^6 + 3*a^4*b^2 - b^6 - 2*(2*a^6 + 3*a^4*b^2 - b^6)*cosh(d*x + c)^4 - 2*
(2*a^6 + 3*a^4*b^2 - b^6 - 35*(2*a^6 + 3*a^4*b^2 - b^6)*cosh(d*x + c)^4)*si
nh(d*x + c)^4 + 8*(7*(2*a^6 + 3*a^4*b^2 - b^6)*cosh(d*x + c)^5 - (2*a^6 + 3
*a^4*b^2 - b^6)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(2*a^6 + 3*a^4*b^2 -
b^6)*cosh(d*x + c)^6 - 3*(2*a^6 + 3*a^4*b^2 - b^6)*cosh(d*x + c)^2)*sinh(d*
x + c)^2 + 8*((2*a^6 + 3*a^4*b^2 - b^6)*cosh(d*x + c)^7 - (2*a^6 + 3*a^4*b^
2 - b^6)*cosh(d*x + c)^3)*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c)
 - sinh(d*x + c))) + (7*(3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*cosh(d*x + c)^6 - 3
*a^5*b - 5*a^3*b^3 - 2*a*b^5 - 12*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*cosh(d*x +
c)^5 - 5*(a^5*b - a^3*b^3 - 2*a*b^5)*cosh(d*x + c)^4 - 16*(a^4*b^2 + a^2*b^
4)*cosh(d*x + c)^3 + 3*(a^5*b - a^3*b^3 - 2*a*b^5)*cosh(d*x + c)^2 - 4*(2*a
^6 + 3*a^4*b^2 + a^2*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^7 + 2*a^5*b^2 +
 a^3*b^4)*d*cosh(d*x + c)^8 + 56*(a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c
)^3*sinh(d*x + c)^5 + 28*(a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^2*sinh
(d*x + c)^6 + 8*(a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7
 + (a^7 + 2*a^5*b^2 + a^3*b^4)*d*sinh(d*x + c)^8 - 2*(a^7 + 2*a^5*b^2 + a^3
*b^4)*d*cosh(d*x + c)^4 + 2*(35*(a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)
^4 - (a^7 + 2*a^5*b^2 + a^3*b^4)*d)*sinh(d*x + c)^4 + 8*(7*(a^7 + 2*a^5*b^2
 + a^3*b^4)*d*cosh(d*x + c)^5 - (a^7 + 2*a^5*b^...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 464 vs. 2(206) = 412.
time = 0.45, size = 464, normalized size = 2.20

$$-\frac{\frac{4\,b^7\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^7b+2\,a^5b^3+a^3b^5}-\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(3\,a^2b+5\,b^3\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,\left(2\,a^3+3\,ab^2\right)\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,\left(2\,a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+3\,ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-2\,a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)-2\,b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)+12\,a^3+16\,ab^2\right)}{\left(a^4+2\,a^2b^2+b^4\right)\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}+\frac{4\,\left(2\,a^2-b^2\right)\log\left(\left|e^{(dx+c)}-e^{(-dx-c)}\right|\right)}{a^3}-\frac{2\,\left(6\,a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-3\,b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\,ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)-4\,a^2\right)}{a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac"
)
```

```
[Out] -1/4*(4*b^7*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^7*b + 2*a^5*b
^3 + a^3*b^5) - (pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(3*
a^2*b + 5*b^3)/(a^4 + 2*a^2*b^2 + b^4) - 2*(2*a^3 + 3*a*b^2)*log((e^(d*x +
c) - e^(-d*x - c))^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 2*(2*a^3*(e^(d*x + c) -
 e^(-d*x - c))^2 + 3*a*b^2*(e^(d*x + c) - e^(-d*x - c))^2 - 2*a^2*b*(e^(d*x
 + c) - e^(-d*x - c)) - 2*b^3*(e^(d*x + c) - e^(-d*x - c)) + 12*a^3 + 16*a*
b^2)/((a^4 + 2*a^2*b^2 + b^4)*((e^(d*x + c) - e^(-d*x - c))^2 + 4)) + 4*(2*
a^2 - b^2)*log(abs(e^(d*x + c) - e^(-d*x - c)))/a^3 - 2*(6*a^2*(e^(d*x + c)
 - e^(-d*x - c))^2 - 3*b^2*(e^(d*x + c) - e^(-d*x - c))^2 + 4*a*b*(e^(d*x +
 c) - e^(-d*x - c)) - 4*a^2)/(a^3*(e^(d*x + c) - e^(-d*x - c))^2))/d
```

Mupad [B]
time = 6.91, size = 554, normalized size = 2.63

$$-\frac{\frac{4b^5}{a\,d\,(a^2b^3+b^5)}-\frac{4b^4e^{3c+3dx}}{d\,(a^2b^3+b^5)}+\frac{4b^4e^{c+dx}}{d\,(a^2b^3+b^5)}+\frac{4b^3e^{2c+2dx}\,(2a^2+b^2)}{a\,d\,(a^2b^3+b^5)}}{e^{8c+8dx}-2e^{4c+4dx}+1}-\frac{\frac{4\,(a^2b^5+b^7)}{a\,d\,(a^2b^3+b^5)\,(a^2+b^2)}+\frac{2e^{2c+2dx}\,(2a^4b^3+3a^2b^5+b^7)}{a\,d\,(a^2b^3+b^5)\,(a^2+b^2)}-\frac{e^{3c+3dx}\,(3a^4b^4+5a^2b^6+2b^8)}{a^2d\,(a^2b^3+b^5)\,(a^2+b^2)}-\frac{b^4e^{c+dx}\,(-a^4+a^2b^2+2b^4)}{a^2d\,(a^2b^3+b^5)\,(a^2+b^2)}}{e^{4c+4dx}-1}+\frac{\ln\left(1+e^{c+dx}\,1i\right)\,(4a+b\,5i)}{2\,(d\,a^2+2i\,d\,a\,b-d\,b^2)}-\frac{b^6\ln\left(2a\,e^{c+dx}-b+b\,e^{2c+2dx}\right)}{d\,a^7+2d\,a^5b^2+d\,a^3b^4}+\frac{\ln\left(e^{c+dx}+1i\right)\,(5b+a\,4i)}{2\,(1i\,d\,a^2+2d\,a\,b-1i\,d\,b^2)}-\frac{\ln\left(e^{2c+2dx}-1\right)\,(2a^2-b^2)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*sinh(c + d*x)^3*(a + b*sinh(c + d*x))),x)
```

```
[Out] (log(exp(c + d*x)*1i + 1)*(4*a + b*5i))/(2*(a^2*d - b^2*d + a*b*d*2i)) - ((
4*(b^7 + a^2*b^5))/(a*d*(b^5 + a^2*b^3)*(a^2 + b^2)) + (2*exp(2*c + 2*d*x)*
(b^7 + 3*a^2*b^5 + 2*a^4*b^3))/(a*d*(b^5 + a^2*b^3)*(a^2 + b^2)) - (exp(3*c
 + 3*d*x)*(2*b^8 + 5*a^2*b^6 + 3*a^4*b^4))/(a^2*d*(b^5 + a^2*b^3)*(a^2 + b^
2)) - (b^4*exp(c + d*x)*(2*b^4 - a^4 + a^2*b^2))/(a^2*d*(b^5 + a^2*b^3)*(a^
2 + b^2)))/(exp(4*c + 4*d*x) - 1) - ((4*b^5)/(a*d*(b^5 + a^2*b^3)) - (4*b^4
*exp(3*c + 3*d*x))/(d*(b^5 + a^2*b^3)) + (4*b^4*exp(c + d*x))/(d*(b^5 + a^2
*b^3)) + (4*b^3*exp(2*c + 2*d*x)*(2*a^2 + b^2))/(a*d*(b^5 + a^2*b^3)))/(exp
(8*c + 8*d*x) - 2*exp(4*c + 4*d*x) + 1) - (b^6*log(2*a*exp(c + d*x) - b + b
*exp(2*c + 2*d*x)))/(a^7*d + a^3*b^4*d + 2*a^5*b^2*d) + (log(exp(c + d*x) +
 1i)*(a*4i + 5*b))/(2*(a^2*d*1i - b^2*d*1i + 2*a*b*d)) - (log(exp(2*c + 2*d
*x) - 1)*(2*a^2 - b^2))/(a^3*d)
```

3.502 $\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=39

$$\operatorname{Int}\left(\frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csch[c + d*x]^3*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Csch[c + d*x]^3*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F]
time = 180.02, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] Integrate[(Csch[c + d*x]^3*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x]
)),x]
```

```
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(dx+c)^3\operatorname{sech}(dx+c)^3}{(fx+e)\,(a+b\sinh\,(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -(a*b^2*f + (a^2*b*f*e^(7*c) - (3*a^2*b*d*f*e^(7*c) + 2*b^3*d*f*e^(7*c))*x
- (3*a^2*b*d*e^(7*c) + 2*b^3*d*e^(7*c))*e)*e^(7*d*x) - (2*a^3*f*e^(6*c) + a
*b^2*f*e^(6*c) - 2*(2*a^3*d*f*e^(6*c) + a*b^2*d*f*e^(6*c))*x - 2*(2*a^3*d*e
^(6*c) + a*b^2*d*e^(6*c))*e)*e^(6*d*x) - (a^2*b*f*e^(5*c) - (a^2*b*d*f*e^(5
*c) - 2*b^3*d*f*e^(5*c))*x - (a^2*b*d*e^(5*c) - 2*b^3*d*e^(5*c))*e)*e^(5*d*
x) + (4*a*b^2*d*f*x*e^(4*c) - a*b^2*f*e^(4*c) + 4*a*b^2*d*e^(4*c + 1))*e^(4
*d*x) - (a^2*b*f*e^(3*c) + (a^2*b*d*f*e^(3*c) - 2*b^3*d*f*e^(3*c))*x + (a^2
*b*d*e^(3*c) - 2*b^3*d*e^(3*c))*e)*e^(3*d*x) + (2*a^3*f*e^(2*c) + a*b^2*f*e
^(2*c) + 2*(2*a^3*d*f*e^(2*c) + a*b^2*d*f*e^(2*c))*x + 2*(2*a^3*d*e^(2*c) +
 a*b^2*d*e^(2*c))*e)*e^(2*d*x) + (a^2*b*f*e^c + (3*a^2*b*d*f*e^c + 2*b^3*d*
f*e^c)*x + (3*a^2*b*d*e^c + 2*b^3*d*e^c)*e)*e^(d*x))/((a^4*d^2*f^2 + a^2*b^
2*d^2*f^2)*x^2 + 2*(a^4*d^2*f + a^2*b^2*d^2*f)*x*e + (a^4*d^2 + a^2*b^2*d^2
)*e^2 + ((a^4*d^2*f^2*e^(8*c) + a^2*b^2*d^2*f^2*e^(8*c))*x^2 + 2*(a^4*d^2*f
*e^(8*c) + a^2*b^2*d^2*f*e^(8*c))*x*e + (a^4*d^2*e^(8*c) + a^2*b^2*d^2*e^(8
*c))*e^2)*e^(8*d*x) - 2*((a^4*d^2*f^2*e^(4*c) + a^2*b^2*d^2*f^2*e^(4*c))*x^
2 + 2*(a^4*d^2*f*e^(4*c) + a^2*b^2*d^2*f*e^(4*c))*x*e + (a^4*d^2*e^(4*c) +
a^2*b^2*d^2*e^(4*c))*e^2)*e^(4*d*x)) - 64*integrate(1/64*(a*b*d*f*e + a^2*f
^2 - (2*a^2*d^2*f^2 - b^2*d^2*f^2)*x^2 + (a*b*d*f^2 - 2*(2*a^2*d^2*f - b^2*
d^2*f)*e)*x - (2*a^2*d^2 - b^2*d^2)*e^2)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*f^2*x
^2*e + 3*a^3*d^2*f*x*e^2 + a^3*d^2*e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*f
^2*x^2*e^(c + 1) + 3*a^3*d^2*f*x*e^(c + 2) + a^3*d^2*e^(c + 3))*e^(d*x)), x
) + 64*integrate(1/64*(a*b*d*f*e - a^2*f^2 + (2*a^2*d^2*f^2 - b^2*d^2*f^2)*
x^2 + (a*b*d*f^2 + 2*(2*a^2*d^2*f - b^2*d^2*f)*e)*x + (2*a^2*d^2 - b^2*d^2)
*e^2)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*f^2*x^2*e + 3*a^3*d^2*f*x*e^2 + a^3*d^2*
e^3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*f^2*x^2*e^(c + 1) + 3*a^3*d^2*f*x*e^
(c + 2) + a^3*d^2*e^(c + 3))*e^(d*x)), x) + 64*integrate(1/64*(2*a^3*f^2 +
2*a*b^2*f^2 - 2*(2*a^3*d^2*f^2 + 3*a*b^2*d^2*f^2)*x^2 - 4*(2*a^3*d^2*f + 3*
a*b^2*d^2*f)*x*e - 2*(2*a^3*d^2 + 3*a*b^2*d^2)*e^2 - (2*a^2*b*f^2*e^c + 2*b
^3*f^2*e^c - (3*a^2*b*d^2*f^2*e^c + 5*b^3*d^2*f^2*e^c)*x^2 - 2*(3*a^2*b*d^2
*f*e^c + 5*b^3*d^2*f*e^c)*x*e - (3*a^2*b*d^2*e^c + 5*b^3*d^2*e^c)*e^2)*e^(d
*x))/((a^4*d^2*f^3 + 2*a^2*b^2*d^2*f^3 + b^4*d^2*f^3)*x^3 + 3*(a^4*d^2*f^2
```

```
+ 2*a^2*b^2*d^2*f^2 + b^4*d^2*f^2)*x^2*e + 3*(a^4*d^2*f + 2*a^2*b^2*d^2*f +
 b^4*d^2*f)*x*e^2 + (a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2)*e^3 + ((a^4*d^2*f^3
*e^(2*c) + 2*a^2*b^2*d^2*f^3*e^(2*c) + b^4*d^2*f^3*e^(2*c))*x^3 + 3*(a^4*d^
2*f^2*e^(2*c) + 2*a^2*b^2*d^2*f^2*e^(2*c) + b^4*d^2*f^2*e^(2*c))*x^2*e + 3*
(a^4*d^2*f*e^(2*c) + 2*a^2*b^2*d^2*f*e^(2*c) + b^4*d^2*f*e^(2*c))*x*e^2 + (
a^4*d^2*e^(2*c) + 2*a^2*b^2*d^2*e^(2*c) + b^4*d^2*e^(2*c))*e^3)*e^(2*d*x)),
 x) + 64*integrate(-1/32*(a*b^6*e^(d*x + c) - b^7)/((a^7*b*f + 2*a^5*b^3*f
+ a^3*b^5*f)*x + (a^7*b + 2*a^5*b^3 + a^3*b^5)*e - ((a^7*b*f*e^(2*c) + 2*a^
5*b^3*f*e^(2*c) + a^3*b^5*f*e^(2*c))*x + (a^7*b*e^(2*c) + 2*a^5*b^3*e^(2*c)
 + a^3*b^5*e^(2*c))*e)*e^(2*d*x) - 2*((a^8*f*e^c + 2*a^6*b^2*f*e^c + a^4*b^
4*f*e^c)*x + (a^8*e^c + 2*a^6*b^2*e^c + a^4*b^4*e^c)*e)*e^(d*x)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*sech(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\cosh\left(c+d\,x\right)}^3\,{\sinh\left(c+d\,x\right)}^3\,\left(e+f\,x\right)\,\left(a+b\,\sinh\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))),x)
[Out] int(1/(cosh(c + d*x)^3*sinh(c + d*x)^3*(e + f*x)*(a + b*sinh(c + d*x))), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```